

# Computer algebra independent integration tests

## 2-Exponentials/2.3-Exponential-functions

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3.277	$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx$	968
3.278	$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx$	971
3.279	$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx$	974
3.280	$\int F^{a+b(c+dx)^3} (c+dx)^m dx$	977
3.281	$\int F^{a+b(c+dx)^3} (c+dx)^{17} dx$	980
3.282	$\int F^{a+b(c+dx)^3} (c+dx)^{14} dx$	984
3.283	$\int F^{a+b(c+dx)^3} (c+dx)^{11} dx$	987
3.284	$\int F^{a+b(c+dx)^3} (c+dx)^8 dx$	991
3.285	$\int F^{a+b(c+dx)^3} (c+dx)^5 dx$	994
3.286	$\int F^{a+b(c+dx)^3} (c+dx)^2 dx$	997
3.287	$\int \frac{F^{a+b(c+dx)^3}}{c+dx} dx$	999
3.288	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx$	1001



3.289	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} dx$	1004
3.290	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}} dx$	1007
3.291	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}} dx$	1010
3.292	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} dx$	1012
3.293	$\int F^{a+b(c+dx)^3} (c+dx)^3 dx$	1014
3.294	$\int F^{a+b(c+dx)^3} (c+dx) dx$	1017
3.295	$\int F^{a+b(c+dx)^3} dx$	1020
3.296	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx$	1023
3.297	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3} dx$	1026
3.298	$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx$	1029
3.299	$\int f^{a+b\sqrt{c+dx}} dx$	1032
3.300	$\int f^{a+b\sqrt[3]{c+dx}} dx$	1036
3.301	$\int F^{a+\frac{b}{c+dx}} (c+dx)^m dx$	1040
3.302	$\int F^{a+\frac{b}{c+dx}} (c+dx)^4 dx$	1042
3.303	$\int F^{a+\frac{b}{c+dx}} (c+dx)^3 dx$	1045
3.304	$\int F^{a+\frac{b}{c+dx}} (c+dx)^2 dx$	1048
3.305	$\int F^{a+\frac{b}{c+dx}} (c+dx) dx$	1051
3.306	$\int F^{a+\frac{b}{c+dx}} dx$	1054
3.307	$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx$	1057
3.308	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx$	1059
3.309	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx$	1062
3.310	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4} dx$	1065
3.311	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx$	1068
3.312	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6} dx$	1071
3.313	$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} dx$	1074
3.314	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^m dx$	1077
3.315	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^9 dx$	1080
3.316	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^7 dx$	1083
3.317	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^5 dx$	1086
3.318	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^3 dx$	1089
3.319	$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx) dx$	1092
3.320	$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx$	1095
3.321	$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx$	1097
3.322	$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5} dx$	1100
3.323	$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx$	1103

3.324	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9} dx \dots \dots \dots$	1107
3.325	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^{11}} dx \dots \dots \dots$	1111
3.326	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^{13}} dx \dots \dots \dots$	1114
3.327	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} (c+dx)^{10} dx \dots \dots \dots$	1118
3.328	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} (c+dx)^8 dx \dots \dots \dots$	1121
3.329	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} (c+dx)^6 dx \dots \dots \dots$	1124
3.330	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} (c+dx)^4 dx \dots \dots \dots$	1128
3.331	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} (c+dx)^2 dx \dots \dots \dots$	1131
3.332	$\int F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2} dx \dots \dots \dots$	1134
3.333	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^2} dx \dots \dots \dots$	1137
3.334	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^4} dx \dots \dots \dots$	1140
3.335	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^6} dx \dots \dots \dots$	1143
3.336	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^8} dx \dots \dots \dots$	1146
3.337	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^{10}} dx \dots \dots \dots$	1149
3.338	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^{12}} dx \dots \dots \dots$	1153
3.339	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}}{(c+dx)^{14}} dx \dots \dots \dots$	1156
3.340	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^m dx \dots \dots \dots$	1159
3.341	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^{14} dx \dots \dots \dots$	1162
3.342	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^{11} dx \dots \dots \dots$	1165
3.343	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^8 dx \dots \dots \dots$	1168
3.344	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^5 dx \dots \dots \dots$	1171
3.345	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^2 dx \dots \dots \dots$	1174
3.346	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{c+dx}}{(c+dx)^3} dx \dots \dots \dots$	1177
3.347	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^4}}{(c+dx)^4} dx \dots \dots \dots$	1179
3.348	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^7}}{(c+dx)^7} dx \dots \dots \dots$	1182
3.349	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^{10}}}{(c+dx)^{10}} dx \dots \dots \dots$	1185
3.350	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^{13}}}{(c+dx)^{13}} dx \dots \dots \dots$	1188
3.351	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^{16}}}{(c+dx)^{16}} dx \dots \dots \dots$	1192
3.352	$\int \frac{F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^{19}}}{(c+dx)^{19}} dx \dots \dots \dots$	1196
3.353	$\int F^{\frac{a+\frac{b}{(c+dx)^3}}{(c+dx)^3} (c+dx)^3 dx \dots \dots \dots$	1200

3.354	$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx) dx$	1203
3.355	$\int F^{a+\frac{b}{(c+dx)^3}} dx$	1206
3.356	$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2} dx$	1209
3.357	$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx$	1212
3.358	$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5} dx$	1215
3.359	$\int F^{a+b(c+dx)^n}(c+dx)^m dx$	1218
3.360	$\int F^{a+b(c+dx)^n}(c+dx)^3 dx$	1220
3.361	$\int F^{a+b(c+dx)^n}(c+dx)^2 dx$	1222
3.362	$\int F^{a+b(c+dx)^n}(c+dx) dx$	1224
3.363	$\int F^{a+b(c+dx)^n} dx$	1226
3.364	$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$	1228
3.365	$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx$	1230
3.366	$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} dx$	1233
3.367	$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx$	1236
3.368	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n} dx$	1239
3.369	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n} dx$	1242
3.370	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n} dx$	1245
3.371	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n} dx$	1248
3.372	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n} dx$	1251
3.373	$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n} dx$	1254
3.374	$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$	1256
3.375	$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n} dx$	1258
3.376	$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n} dx$	1261
3.377	$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n} dx$	1264
3.378	$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n} dx$	1267
3.379	$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n} dx$	1269
3.380	$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}} dx$	1271
3.381	$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}} dx$	1274
3.382	$\int F^{a+b(c+dx)^2}(e+fx)^5 dx$	1277
3.383	$\int F^{a+b(c+dx)^2}(e+fx)^4 dx$	1282
3.384	$\int F^{a+b(c+dx)^2}(e+fx)^3 dx$	1287
3.385	$\int F^{a+b(c+dx)^2}(e+fx)^2 dx$	1291
3.386	$\int F^{a+b(c+dx)^2}(e+fx) dx$	1295
3.387	$\int F^{a+b(c+dx)^2} dx$	1298
3.388	$\int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$	1301
3.389	$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx$	1303
3.390	$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$	1306
3.391	$\int e^{e(c+dx)^3}(a+bx)^3 dx$	1309
3.392	$\int e^{e(c+dx)^3}(a+bx)^2 dx$	1312
3.393	$\int e^{e(c+dx)^3}(a+bx) dx$	1315
3.394	$\int e^{e(c+dx)^3} dx$	1318
3.395	$\int \frac{e^{e(c+dx)^3}}{a+bx} dx$	1320
3.396	$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} dx$	1322

3.397	$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx$	1325
3.398	$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2} dx$	1328
3.399	$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} dx$	1332
3.400	$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4} dx$	1337
3.401	$\int e^{\frac{e}{c+dx}} (a+bx)^4 dx$	1342
3.402	$\int e^{\frac{e}{c+dx}} (a+bx)^3 dx$	1346
3.403	$\int e^{\frac{e}{c+dx}} (a+bx)^2 dx$	1350
3.404	$\int e^{\frac{e}{c+dx}} (a+bx) dx$	1353
3.405	$\int e^{\frac{e}{c+dx}} dx$	1356
3.406	$\int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx$	1359
3.407	$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} dx$	1362
3.408	$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3} dx$	1366
3.409	$\int e^{\frac{e}{(c+dx)^2}} (a+bx)^3 dx$	1370
3.410	$\int e^{\frac{e}{(c+dx)^2}} (a+bx)^2 dx$	1374
3.411	$\int e^{\frac{e}{(c+dx)^2}} (a+bx) dx$	1378
3.412	$\int e^{\frac{e}{(c+dx)^2}} dx$	1381
3.413	$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx$	1384
3.414	$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx$	1386
3.415	$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$	1388
3.416	$\int e^{\frac{e}{(c+dx)^3}} (a+bx)^3 dx$	1390
3.417	$\int e^{\frac{e}{(c+dx)^3}} (a+bx)^2 dx$	1394
3.418	$\int e^{\frac{e}{(c+dx)^3}} (a+bx) dx$	1397
3.419	$\int e^{\frac{e}{(c+dx)^3}} dx$	1400
3.420	$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx$	1403
3.421	$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$	1405
3.422	$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx$	1407
3.423	$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2} dx$	1410
3.424	$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3} dx$	1414
3.425	$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4} dx$	1419
3.426	$\int f^{a+bx+cx^2} x^3 dx$	1426
3.427	$\int f^{a+bx+cx^2} x^2 dx$	1430
3.428	$\int f^{a+bx+cx^2} x dx$	1434
3.429	$\int f^{a+bx+cx^2} dx$	1437
3.430	$\int \frac{f^{a+bx+cx^2}}{x} dx$	1440
3.431	$\int \frac{f^{a+bx+cx^2}}{x^2} dx$	1442

3.432	$\int e^{a+bx-cx^2} x^3 dx$	1444
3.433	$\int e^{a+bx-cx^2} x^2 dx$	1447
3.434	$\int e^{a+bx-cx^2} x dx$	1450
3.435	$\int e^{a+bx-cx^2} dx$	1453
3.436	$\int \frac{e^{a+bx-cx^2}}{x} dx$	1456
3.437	$\int \frac{e^{a+bx-cx^2}}{x^2} dx$	1458
3.438	$\int e^{(a+bx)(c+dx)} x^3 dx$	1460
3.439	$\int e^{(a+bx)(c+dx)} x^2 dx$	1464
3.440	$\int e^{(a+bx)(c+dx)} x dx$	1468
3.441	$\int e^{(a+bx)(c+dx)} dx$	1471
3.442	$\int \frac{e^{(a+bx)(c+dx)}}{x} dx$	1474
3.443	$\int \frac{e^{(a+bx)(c+dx)}}{x^2} dx$	1476
3.444	$\int f^{a+bx+cx^2} (d+ex)^3 dx$	1479
3.445	$\int f^{a+bx+cx^2} (d+ex)^2 dx$	1483
3.446	$\int f^{a+bx+cx^2} (d+ex) dx$	1487
3.447	$\int \frac{f^{a+bx+cx^2}}{d+ex} dx$	1490
3.448	$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$	1492
3.449	$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx$	1495
3.450	$\int f^{a+bx+cx^2} (b+2cx)^3 dx$	1498
3.451	$\int f^{a+bx+cx^2} (b+2cx)^2 dx$	1501
3.452	$\int f^{a+bx+cx^2} (b+2cx) dx$	1504
3.453	$\int \frac{f^{a+bx+cx^2}}{b+2cx} dx$	1506
3.454	$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} dx$	1508
3.455	$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3} dx$	1511
3.456	$\int f^{bx+cx^2} (b+2cx)^3 dx$	1514
3.457	$\int f^{bx+cx^2} (b+2cx)^2 dx$	1517
3.458	$\int f^{bx+cx^2} (b+2cx) dx$	1520
3.459	$\int \frac{f^{bx+cx^2}}{b+2cx} dx$	1522
3.460	$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx$	1524
3.461	$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx$	1527
3.462	$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$	1530
3.463	$\int \frac{e^{a+bx}}{x(c+dx^2)} dx$	1533
3.464	$\int \frac{e^{a+bx}}{c+dx^2} dx$	1536
3.465	$\int \frac{e^{a+bx} x}{c+dx^2} dx$	1539
3.466	$\int \frac{e^{a+bx} x^2}{c+dx^2} dx$	1542
3.467	$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$	1545
3.468	$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$	1549
3.469	$\int \frac{e^{d+ex}}{a+bx+cx^2} dx$	1552
3.470	$\int \frac{e^{d+ex} x}{a+bx+cx^2} dx$	1555
3.471	$\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx$	1558
3.472	$\int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx$	1562
3.473	$\int \frac{4^x}{a+2^x b} dx$	1567

3.474	$\int \frac{2^{2x}}{a+2^x b} dx$	1570
3.475	$\int \frac{4^x}{a-2^{2x} b} dx$	1573
3.476	$\int \frac{2^{2x}}{a-2^{2x} b} dx$	1576
3.477	$\int \frac{4^x}{a+2^{-x} b} dx$	1579
3.478	$\int \frac{2^{2x}}{a+2^{-x} b} dx$	1582
3.479	$\int \frac{4^x}{a-2^{-x} b} dx$	1585
3.480	$\int \frac{2^{2x}}{a-2^{-x} b} dx$	1588
3.481	$\int \frac{2^x}{a+4^x b} dx$	1591
3.482	$\int \frac{2^x}{a+2^{2x} b} dx$	1594
3.483	$\int \frac{2^x}{a-4^x b} dx$	1597
3.484	$\int \frac{2^x}{a-2^{2x} b} dx$	1600
3.485	$\int \frac{2^x}{a+4^{-x} b} dx$	1603
3.486	$\int \frac{2^x}{a+2^{-2x} b} dx$	1606
3.487	$\int \frac{2^x}{a-4^{-x} b} dx$	1609
3.488	$\int \frac{2^x}{a-2^{-2x} b} dx$	1612
3.489	$\int \frac{2^x}{\sqrt{a+4^x b}} dx$	1615
3.490	$\int \frac{2^x}{\sqrt{a+2^{2x} b}} dx$	1618
3.491	$\int \frac{2^x}{\sqrt{a-4^x b}} dx$	1621
3.492	$\int \frac{2^x}{\sqrt{a-2^{2x} b}} dx$	1624
3.493	$\int \frac{2^x}{\sqrt{a+4^{-x} b}} dx$	1627
3.494	$\int \frac{2^x}{\sqrt{a+2^{-2x} b}} dx$	1630
3.495	$\int \frac{2^x}{\sqrt{a-4^{-x} b}} dx$	1633
3.496	$\int \frac{2^x}{\sqrt{a-2^{-2x} b}} dx$	1636
3.497	$\int \frac{4^x}{\sqrt{a+2^x b}} dx$	1639
3.498	$\int \frac{2^{2x}}{\sqrt{a+2^x b}} dx$	1642
3.499	$\int \frac{4^x}{\sqrt{a-2^x b}} dx$	1645
3.500	$\int \frac{2^{2x}}{\sqrt{a-2^x b}} dx$	1648
3.501	$\int \frac{4^x}{\sqrt{a+2^{-x} b}} dx$	1651
3.502	$\int \frac{2^{2x}}{\sqrt{a+2^{-x} b}} dx$	1654
3.503	$\int \frac{4^x}{\sqrt{a-2^{-x} b}} dx$	1657
3.504	$\int \frac{2^{2x}}{\sqrt{a-2^{-x} b}} dx$	1660
3.505	$\int \frac{1}{1+2e^x+e^{2x}} dx$	1663
3.506	$\int \frac{1}{2+3e^x+e^{2x}} dx$	1666
3.507	$\int \frac{1}{-1+e^x+e^{2x}} dx$	1669
3.508	$\int \frac{1}{3+3e^x+e^{2x}} dx$	1672
3.509	$\int \frac{1}{a+be^x+ce^{2x}} dx$	1675
3.510	$\int \frac{x}{1+2e^x+e^{2x}} dx$	1679
3.511	$\int \frac{x}{2+3e^x+e^{2x}} dx$	1683
3.512	$\int \frac{x}{-1+e^x+e^{2x}} dx$	1686
3.513	$\int \frac{x}{3+3e^x+e^{2x}} dx$	1689
3.514	$\int \frac{x}{a+be^x+ce^{2x}} dx$	1692
3.515	$\int \frac{x^2}{1+2e^x+e^{2x}} dx$	1696

3.516	$\int \frac{x^2}{2+3e^x+e^{2x}} dx$	1700
3.517	$\int \frac{x^2}{-1+e^x+e^{2x}} dx$	1703
3.518	$\int \frac{x^2}{3+3e^x+e^{2x}} dx$	1707
3.519	$\int \frac{x^2}{a+be^x+ce^{2x}} dx$	1711
3.520	$\int \frac{1}{1+2fc+dx+fc^2+2dx} dx$	1715
3.521	$\int \frac{1}{a+bf^c+dx+cf^2c+2dx} dx$	1718
3.522	$\int \frac{1}{a+bf^g+hx+cf^2(g+hx)} dx$	1722
3.523	$\int \frac{x}{1+2fc+dx+fc^2+2dx} dx$	1726
3.524	$\int \frac{x}{a+bf^c+dx+cf^2c+2dx} dx$	1730
3.525	$\int \frac{x^2}{1+2fc+dx+fc^2+2dx} dx$	1734
3.526	$\int \frac{x^2}{a+bf^c+dx+cf^2c+2dx} dx$	1739
3.527	$\int \frac{d+ef^g+hx}{a+bf^g+hx+cf^2g+2hx} dx$	1743
3.528	$\int \frac{d+ef^g+hx}{a+bf^g+hx+cf^2(g+hx)} dx$	1747
3.529	$\int \frac{1}{2+e^{-x}+e^x} dx$	1751
3.530	$\int \frac{x}{2+e^{-x}+e^x} dx$	1753
3.531	$\int \frac{x^2}{2+e^{-x}+e^x} dx$	1756
3.532	$\int \frac{1}{2+f^{-c}-dx+fc+dx} dx$	1759
3.533	$\int \frac{x}{2+f^{-c}-dx+fc+dx} dx$	1762
3.534	$\int \frac{x^2}{2+f^{-c}-dx+fc+dx} dx$	1766
3.535	$\int \frac{1}{2+3^{-x}+3^x} dx$	1770
3.536	$\int \frac{1}{1-e^{-x}+2e^x} dx$	1773
3.537	$\int \frac{1}{a+be^{-x}+ce^x} dx$	1776
3.538	$\int \frac{x}{a+be^{-x}+ce^x} dx$	1779
3.539	$\int \frac{x^2}{a+be^{-x}+ce^x} dx$	1782
3.540	$\int \frac{1}{a+bf^{-c}-dx+cf^c+dx} dx$	1786
3.541	$\int \frac{x}{a+bf^{-c}-dx+cf^c+dx} dx$	1789
3.542	$\int \frac{x^2}{a+bf^{-c}-dx+cf^c+dx} dx$	1793
3.543	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{f+gx}}\right)^n}{df+(ef+dg)x+egx^2} dx$	1797
3.544	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{f+gx}}\right)^3}{df+(ef+dg)x+egx^2} dx$	1800
3.545	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{f+gx}}\right)^2}{df+(ef+dg)x+egx^2} dx$	1803
3.546	$\int \frac{a+bF\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}{df+(ef+dg)x+egx^2} dx$	1806
3.547	$\int \frac{1}{df+(ef+dg)x+egx^2} dx$	1809
3.548	$\int \frac{1}{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{f+gx}}\right)(df+(ef+dg)x+egx^2)} dx$	1812

3.549	$\int \frac{1}{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{f+gx}}\right)^2 (df+(ef+dg)x+egx^2)} dx$	1815
3.550	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}}\right)^n}{d^2-e^2x^2} dx$	1818
3.551	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}}\right)^3}{d^2-e^2x^2} dx$	1821
3.552	$\int \frac{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}}\right)^2}{d^2-e^2x^2} dx$	1824
3.553	$\int \frac{a+bF\sqrt{df-efx}}{d^2-e^2x^2} dx$	1827
3.554	$\int \frac{1}{d^2-e^2x^2} dx$	1830
3.555	$\int \frac{1}{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}}\right) (d^2-e^2x^2)} dx$	1832
3.556	$\int \frac{1}{\left(\frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}}\right)^2 (d^2-e^2x^2)} dx$	1835
3.557	$\int \frac{\left(\frac{\sqrt{1-ax}}{F\sqrt{1+ax}}\right)^n}{1-a^2x^2} dx$	1838
3.558	$\int \frac{F\sqrt{1+ax}}{1-a^2x^2} dx$	1841
3.559	$\int \frac{F\sqrt{1+ax}}{1-a^2x^2} dx$	1844
3.560	$\int \frac{F\sqrt{1+ax}}{1-a^2x^2} dx$	1847
3.561	$\int \frac{F\sqrt{1+ax}}{1-a^2x^2} dx$	1850
3.562	$\int \frac{F\sqrt{1+ax}}{1-a^2x^2} dx$	1853
3.563	$\int a^x b^x x^2 dx$	1856
3.564	$\int a^x b^x x dx$	1861
3.565	$\int a^x b^x dx$	1864
3.566	$\int \frac{a^x b^x}{x} dx$	1867
3.567	$\int \frac{a^x b^x}{x^2} dx$	1869
3.568	$\int \frac{a^x b^x}{x^3} dx$	1872
3.569	$\int a^x b^x c^x dx$	1875
3.570	$\int a^x b^{-x} dx$	1878
3.571	$\int a^x b^{-x} x^2 dx$	1881
3.572	$\int \frac{(d+e^{h+ix})(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}} dx$	1885
3.573	$\int \frac{(d+e^{h+ix})(f+gx)^2}{a+be^{h+ix}+ce^{2h+2ix}} dx$	1891
3.574	$\int \frac{(d+e^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}} dx$	1896
3.575	$\int \frac{d+e^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}} dx$	1901
3.576	$\int \frac{d+e^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)} dx$	1905
3.577	$\int \frac{d+e^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2} dx$	1908



3.578	$\int \frac{(be - aee^{c+dx})x}{be - 2aee^{c+dx} - bee^{2(c+dx)}} dx$	1911
3.579	$\int F^{a+b \log(c+dx^n)} x^2 dx$	1915
3.580	$\int F^{a+b \log(c+dx^n)} x dx$	1918
3.581	$\int F^{a+b \log(c+dx^n)} dx$	1921
3.582	$\int \frac{F^{a+b \log(c+dx^n)}}{x} dx$	1924
3.583	$\int \frac{F^{a+b \log(c+dx^n)}}{x^2} dx$	1927
3.584	$\int \frac{F^{a+b \log(c+dx^n)}}{x^3} dx$	1930
3.585	$\int F^{a+b \log(c+dx^n)} (dx)^m dx$	1933
3.586	$\int e^{\log^2((d+ex)^n)} (d+ex)^m dx$	1936
3.587	$\int F^{f(a+b \log^2(c(d+ex)^n))} (dg+egx)^m dx$	1939
3.588	$\int F^{f(a+b \log^2(c(d+ex)^n))} (dg+egx)^2 dx$	1942
3.589	$\int F^{f(a+b \log^2(c(d+ex)^n))} (dg+egx) dx$	1945
3.590	$\int F^{f(a+b \log^2(c(d+ex)^n))} dx$	1948
3.591	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{dg+egx} dx$	1951
3.592	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^2} dx$	1954
3.593	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^3} dx$	1957
3.594	$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx$	1960
3.595	$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^3 dx$	1962
3.596	$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^2 dx$	1965
3.597	$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx) dx$	1968
3.598	$\int F^{f(a+b \log^2(c(d+ex)^n))} dx$	1971
3.599	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx$	1974
3.600	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx$	1976
3.601	$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx$	1978
3.602	$\int F^{f(a+b \log(c(d+ex)^n))^2} (dg+egx)^m dx$	1980
3.603	$\int F^{f(a+b \log(c(d+ex)^n))^2} (dg+egx)^2 dx$	1984
3.604	$\int F^{f(a+b \log(c(d+ex)^n))^2} (dg+egx) dx$	1988
3.605	$\int F^{f(a+b \log(c(d+ex)^n))^2} dx$	1992
3.606	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{dg+egx} dx$	1995
3.607	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(dg+egx)^2} dx$	1998
3.608	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(dg+egx)^3} dx$	2002
3.609	$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx$	2006
3.610	$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^3 dx$	2008
3.611	$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^2 dx$	2012
3.612	$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx) dx$	2016
3.613	$\int F^{f(a+b \log(c(d+ex)^n))^2} dx$	2019
3.614	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx$	2022
3.615	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx$	2024

3.616	$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx$	2026
3.617	$\int F^{a+bx+cx^3} (b+3cx^2) dx$	2028
3.618	$\int \frac{1}{F^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^2} dx$	2030
3.619	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^m dx$	2033
3.620	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^3 dx$	2035
3.621	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^2 dx$	2039
3.622	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2) dx$	2042
3.623	$\int e^{a+bx+cx^2} (b+2cx) dx$	2045
3.624	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{a+bx+cx^2} dx$	2047
3.625	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^2} dx$	2049
3.626	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^3} dx$	2052
3.627	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^{7/2} dx$	2055
3.628	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^{5/2} dx$	2058
3.629	$\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^{3/2} dx$	2061
3.630	$\int e^{a+bx+cx^2} (b+2cx) \sqrt{a+bx+cx^2} dx$	2064
3.631	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{\sqrt{a+bx+cx^2}} dx$	2067
3.632	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^{3/2}} dx$	2070
3.633	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^{5/2}} dx$	2073
3.634	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^{7/2}} dx$	2076
3.635	$\int \frac{e^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^{9/2}} dx$	2079
3.636	$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$	2082
3.637	$\int \frac{e^x}{4+e^{2x}} dx$	2085
3.638	$\int \frac{e^x}{1-e^{2x}} dx$	2088
3.639	$\int \frac{e^x}{3-4e^{2x}} dx$	2090
3.640	$\int e^x \sqrt{3-4e^{2x}} dx$	2093
3.641	$\int e^{x^2} x^3 dx$	2096
3.642	$\int e^x \sqrt{1-e^{2x}} dx$	2099
3.643	$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} dx$	2102
3.644	$\int \frac{e^x}{-4+e^{2x}} dx$	2105
3.645	$\int e^{2-x^2} x dx$	2108
3.646	$\int (e^x - x^e) dx$	2110
3.647	$\int \frac{-1+e^{2x}}{3+e^{2x}} dx$	2112
3.648	$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$	2115
3.649	$\int \frac{e^{2x}}{1+e^{4x}} dx$	2117
3.650	$\int \frac{1}{-3e^x+e^{2x}} dx$	2120
3.651	$\int \frac{e^x(-2+e^x)}{1+e^x} dx$	2123
3.652	$\int \frac{e^x}{-1+e^{2x}} dx$	2125
3.653	$\int \frac{e^x}{1+e^{2x}} dx$	2127
3.654	$\int \frac{e^{-x}+e^x}{-e^{-x}+e^x} dx$	2129

3.655	$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x} dx$	2132
3.656	$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}} dx$	2135
3.657	$\int \frac{e^x}{\sqrt{1+e^{2x}}} dx$	2138
3.658	$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}} dx$	2140
3.659	$\int \frac{x}{\sqrt{-1+e^{2x^2}}} dx$	2142
3.660	$\int e^x \sqrt{9+e^{2x}} dx$	2145
3.661	$\int e^x \sqrt{1+e^{2x}} dx$	2148
3.662	$\int \frac{e^{x^2} x}{1+e^{2x^2}} dx$	2151
3.663	$\int e^{x^{3/2}} x^2 dx$	2154
3.664	$\int \frac{e^x}{\sqrt{-3+e^{2x}}} dx$	2157
3.665	$\int \frac{e^x}{16-e^{2x}} dx$	2160
3.666	$\int \frac{e^{5x}}{1+e^{10x}} dx$	2163
3.667	$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}} dx$	2166
3.668	$\int e^{4x^3} x^2 \cos(7x^3) dx$	2169
3.669	$\int e^{1+x^2} x dx$	2172
3.670	$\int e^{1+x^3} x^2 dx$	2174
3.671	$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$	2176
3.672	$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} dx$	2178
3.673	$\int e^{3x} (-8+2x^3+x^5) dx$	2180
3.674	$\int (e^x+x)^2 dx$	2183
3.675	$\int e^{-4x} (e^x+e^{2x}+e^{3x}) dx$	2186
3.676	$\int \frac{e^x}{1+2e^x+e^{2x}} dx$	2189
3.677	$\int e^{-x} \cos(3x) dx$	2192
3.678	$\int \frac{e^{2x}}{2+3e^x+e^{2x}} dx$	2194
3.679	$\int \frac{e^{2x}}{1+e^x} dx$	2197
3.680	$\int e^{3x} \cos(5x) dx$	2200
3.681	$\int e^x \operatorname{sech}(e^x) dx$	2202
3.682	$\int \frac{e^{-x}}{1+2e^x} dx$	2204
3.683	$\int e^x \cos(4+3x) dx$	2207
3.684	$\int e^x \sec^3(1-e^x) dx$	2209
3.685	$\int (e^{-x}+e^x)x dx$	2212
3.686	$\int \frac{e^x}{2+3e^x+e^{2x}} dx$	2215
3.687	$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}} dx$	2218
3.688	$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}} dx$	2221
3.689	$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}} dx$	2224
3.690	$\int e^x (-5x+x^2) dx$	2227
3.691	$\int e^{3x} (-x+x^2) dx$	2230
3.692	$\int e^{x^x} x^{2x} (1+\log(x)) dx$	2233
3.693	$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x} dx$	2235
3.694	$\int x^{-2-\frac{1}{x}} (1-\log(x)) dx$	2238
3.695	$\int (a+be^x)^2 dx$	2240
3.696	$\int (a+be^x)^3 dx$	2243
3.697	$\int (a+be^x)^4 dx$	2246
3.698	$\int \frac{1}{\sqrt{a+be^{c+dx}}} dx$	2249

3.699	$\int \frac{1}{\sqrt{-a+be^{c+dx}}} dx$	2252
3.700	$\int \sqrt{a+be^{c+dx}} dx$	2255
3.701	$\int \sqrt{-a+be^{c+dx}} dx$	2258
3.702	$\int e^{6x} \sin(3x) dx$	2261
3.703	$\int \frac{e^{3x}}{1+e^{2x}} dx$	2263
3.704	$\int \frac{e^{3x}}{-1+e^{2x}} dx$	2266
3.705	$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}} dx$	2269
3.706	$\int \frac{e^x}{-1-8e^x+e^{2x}} dx$	2272
3.707	$\int e^{7x} x^3 dx$	2275
3.708	$\int e^{8-2x} x^3 dx$	2278
3.709	$\int e^x \sqrt{9-e^{2x}} dx$	2281
3.710	$\int e^{6x} \sqrt{9-e^{2x}} dx$	2284
3.711	$\int \frac{e^{6x}}{(9-e^x)^{5/2}} dx$	2287
3.712	$\int (2-7e^{x^4})^5 x^3 dx$	2290
3.713	$\int e^{x^2} \sqrt{1-e^{2x^2}} x dx$	2293
3.714	$\int e^{x^3} (1-e^{4x^3})^2 x^2 dx$	2296
3.715	$\int e^{e^x+x} dx$	2299
3.716	$\int e^{e^{e^x}+e^x+x} dx$	2301
3.717	$\int (e^{-x}+e^x)^2 dx$	2303
3.718	$\int \frac{1}{e^{-x}+e^x} dx$	2306
3.719	$\int \frac{1}{(e^{-x}+e^x)^2} dx$	2308
3.720	$\int \frac{1}{-e^{-x}+e^x} dx$	2311
3.721	$\int \frac{1}{(-e^{-x}+e^x)^2} dx$	2313
3.722	$\int e^x (-e^{-x}+e^x)^2 dx$	2316
3.723	$\int e^x (-e^{-x}+e^x)^3 dx$	2319
3.724	$\int \frac{1+4^x}{1+2^x} dx$	2322
3.725	$\int \frac{1+4^x}{1+2^{-x}} dx$	2325
3.726	$\int \left( \frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x} \right) dx$	2328
3.727	$\int e^{-x^2} (x^4+x^6+x^8) dx$	2331
3.728	$\int \frac{1}{-e^x+e^{3x}} dx$	2334
3.729	$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx$	2337
3.730	$\int \frac{e^{x^2} x^3}{(1+x^2)^2} dx$	2340
3.731	$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx$	2343
3.732	$\int \frac{1+e^x}{\sqrt{e^x+x}} dx$	2346
3.733	$\int \frac{1+e^x}{e^x+x} dx$	2348
3.734	$\int \frac{e^{x^2}}{x^2} dx$	2350
3.735	$\int \frac{e^{x^2}(1+4x^4)}{x^2} dx$	2353
3.736	$\int \sqrt{f^x(a+bx)^2} dx$	2356
3.737	$\int 3^{1+x^2} x dx$	2360
3.738	$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx$	2362
3.739	$\int \frac{2^{\frac{1}{x}}}{x^2} dx$	2364
3.740	$\int (2^{-x}+2^x) dx$	2366
3.741	$\int e^{-4x} (2-3x+x^2) dx$	2368

3.742	$\int (k^{x/2} + x^{\sqrt{k}}) dx$	2371
3.743	$\int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx$	2374
3.744	$\int \left( \frac{1}{\sqrt{e^x+x}} + \frac{e^x}{\sqrt{e^x+x}} \right) dx$	2376
3.745	$\int \left( \frac{(1+e^x)x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx$	2378
3.746	$\int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx$	2381
3.747	$\int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx$	2384
3.748	$\int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx$	2386
3.749	$\int \frac{e^x x}{\sqrt{e^x+x}} dx$	2388
3.750	$\int \left( \frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}} + \frac{4}{5}x\sqrt{5e^x+x^3} \right) dx$	2390
3.751	$\int \frac{e^x x^2}{\sqrt{5e^x+x^3}} dx$	2393
3.752	$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}} dx$	2395
3.753	$\int \left( -\frac{1}{\sqrt[3]{e^x+x}} + \frac{x}{\sqrt[3]{e^x+x}} - (e^x+x)^{2/3} \right) dx$	2397
3.754	$\int \frac{x}{\sqrt[3]{e^x+x}} dx$	2400
3.755	$\int \frac{5x+e^x(3+2x)}{\sqrt[3]{e^x+x}} dx$	2402
3.756	$\int \left( \frac{2x}{\sqrt[3]{e^x+x}} + \frac{2e^x x}{\sqrt[3]{e^x+x}} + 3(e^x+x)^{2/3} \right) dx$	2405
3.757	$\int e^x(-e^{-x}+e^x)(e^{-x}+e^x)^2 dx$	2408
3.758	$\int \frac{x}{e^x+x} dx$	2411
3.759	$\int \frac{x^2}{\sqrt{e^x+x}} dx$	2413
3.760	$\int \frac{e^x}{e^x+x} dx$	2415
3.761	$\int \frac{e^x}{e^x+x^2} dx$	2417
3.762	$\int \frac{F0(x)}{x+F0(x)} dx$	2419
3.763	$\int \frac{F0(x)}{x^2+F0(x)} dx$	2421
3.764	$\int \frac{F0(x)}{(x+F0(x))^2} dx$	2423
3.765	$\int \frac{F0(x)}{(x^2+F0(x))^2} dx$	2425
3.766	$\int (aF^{c+dx})^m (bF^{e+fx})^n dx$	2427
3.767	$\int e^{a+c+bx^n+dx^n} dx$	2430
3.768	$\int f^{a+bx^n} g^{c+dx^n} dx$	2433
3.769	$\int e^{x^n} x^m dx$	2436
3.770	$\int f^{x^n} x^m dx$	2439
3.771	$\int e^{(a+bx)^n} (a+bx)^m dx$	2442
3.772	$\int f^{(a+bx)^n} (a+bx)^m dx$	2444
3.773	$\int e^{(a+bx)^3} x dx$	2446
3.774	$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}} dx$	2449



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 774 ]. This is test number [ 55 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric<sub>2</sub>F<sub>1</sub> functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.97 ( 766 )	% 1.03 ( 8 )
Mathematica	% 97.03 ( 751 )	% 2.97 ( 23 )
Maple	% 80.23 ( 621 )	% 19.77 ( 153 )
Maxima	% 59.69 ( 462 )	% 40.31 ( 312 )
Fricas	% 86.05 ( 666 )	% 13.95 ( 108 )
Sympy	% 39.79 ( 308 )	% 60.21 ( 466 )
Giac	% 44.7 ( 346 )	% 55.3 ( 428 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

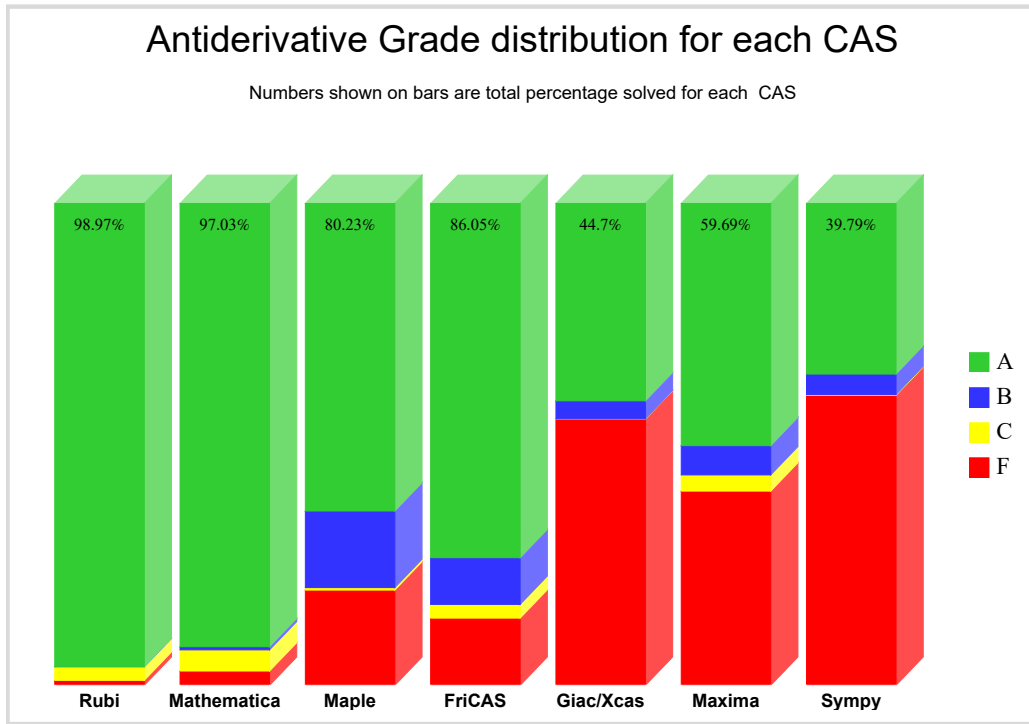
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

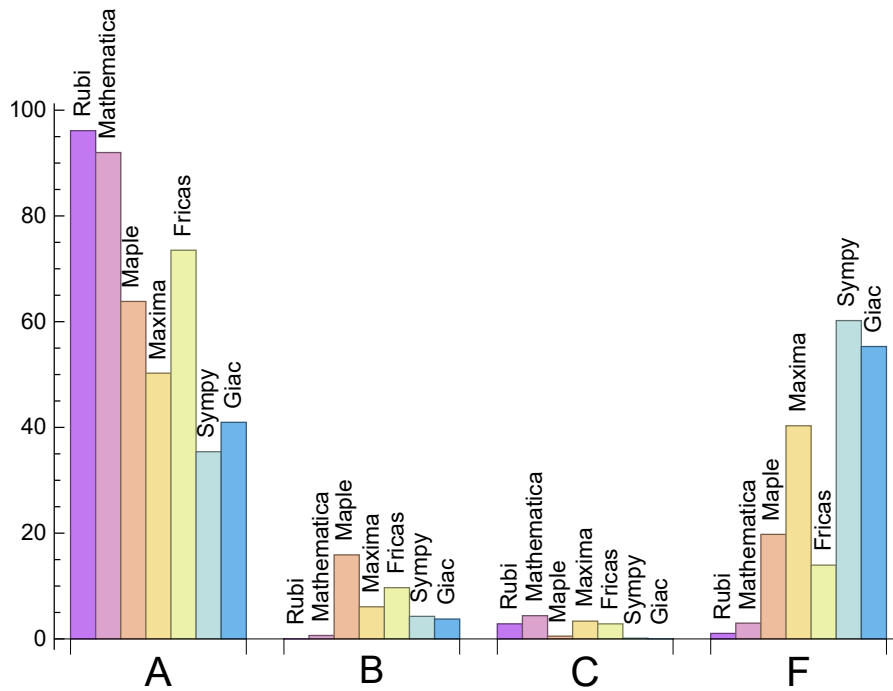
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.12	0.	2.84	1.03
Mathematica	91.99	0.65	4.39	2.97
Maple	63.82	15.89	0.52	19.77
Maxima	50.26	6.07	3.36	40.31
Fricas	73.51	9.69	2.84	13.95
Sympy	35.4	4.26	0.13	60.21
Giac	40.96	3.75	0.	55.3



The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	68.28	0.89	41.5	1.
Mathematica	0.14	60.94	0.82	36.	0.93
Maple	0.05	137.74	1.83	49.	1.
Maxima	1.01	186.15	2.71	31.	1.12
Fricas	1.31	256.	3.27	131.	2.52
Sympy	2.68	71.21	1.33	24.	0.9
Giac	1.01	115.75	2.22	28.	1.13

## 1.4 list of integrals that has no closed form antiderivative

{199, 200, 201, 205, 206, 207, 213, 214, 229, 230, 231, 237, 238, 239, 240, 241, 243, 244, 245, 246, 251, 252, 253, 388, 389, 390, 395, 396, 413, 414, 415, 420, 421, 430, 431, 436, 437, 442, 443, 447, 448, 449, 543, 548, 549, 550, 555, 556, 576, 577, 594, 599, 600, 601, 609, 614, 615, 616, 747, 748, 749, 751, 754, 758, 759, 760, 761, 762, 763, 764, 765}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {66}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

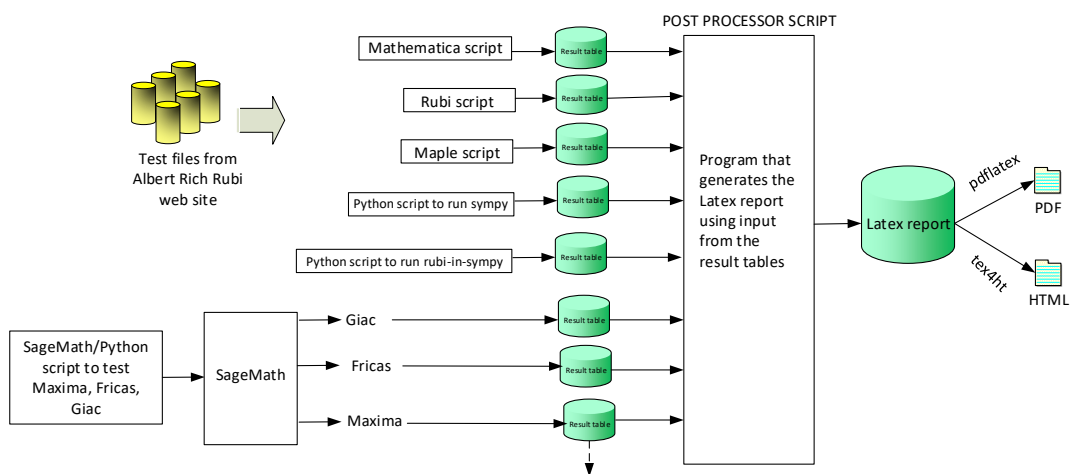
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774 }

B grade: { }

C grade: { 70, 71, 96, 97, 126, 127, 139, 140, 165, 166, 255, 256, 281, 282, 312, 313, 325, 326, 351,

352, 368, 369 }

F grade: { 595, 596, 597, 610, 611, 612, 692, 694 }

## 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 543, 547, 548, 549, 550, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 569, 570, 571, 574, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774 }

B grade: { 572, 573, 578, 631, 636 }

C grade: { 70, 71, 96, 97, 126, 127, 139, 140, 165, 166, 183, 184, 255, 256, 281, 282, 312, 313, 325, 326, 351, 352, 368, 369, 370, 371, 372, 462, 463, 464, 465, 466, 663, 728 }

F grade: { 16, 17, 399, 400, 408, 423, 424, 425, 524, 526, 541, 542, 544, 545, 546, 551, 552, 553, 567, 568, 586, 587, 602 }

## 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 44, 47, 48, 51, 52, 54, 55, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 161, 162, 163, 164, 165, 166, 179, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 237, 238, 239, 240, 241, 243, 244, 245, 246, 251, 252, 253, 258, 259, 260, 261, 262, 263, 264, 273, 274, 275, 276, 277, 278, 279, 285, 286, 305, 306, 307, 308, 309, 310, 311, 319, 320, 321, 331, 332, 333, 334, 335, 336, 337, 338, 339, 347, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 380, 381, 386, 387, 388, 389, 390,



395, 396, 397, 398, 399, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 420, 421, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 469, 473, 474, 475, 476, 477, 478, 479, 480, 487, 488, 493, 494, 495, 496, 497, 498, 499, 500, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 520, 523, 525, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 543, 547, 548, 549, 550, 555, 556, 563, 564, 565, 569, 570, 571, 576, 577, 594, 599, 600, 601, 609, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766 }

B grade: { 12, 33, 35, 37, 43, 69, 80, 81, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 128, 129, 130, 154, 155, 156, 157, 158, 159, 160, 167, 168, 169, 170, 171, 172, 173, 242, 255, 256, 257, 265, 266, 267, 268, 269, 270, 271, 272, 281, 282, 283, 284, 302, 303, 304, 312, 313, 315, 316, 317, 318, 322, 323, 324, 325, 326, 327, 328, 329, 330, 348, 349, 350, 351, 352, 378, 379, 382, 383, 384, 385, 400, 401, 402, 422, 423, 424, 425, 444, 465, 466, 467, 468, 470, 471, 472, 481, 482, 483, 484, 485, 486, 521, 522, 524, 527, 528, 540, 541, 554, 574, 575, 578, 636, 644, 665, 692, 704 }

C grade: { 566, 567, 568, 767 }

F grade: { 17, 45, 46, 49, 50, 53, 56, 57, 64, 65, 66, 67, 68, 174, 175, 176, 177, 178, 180, 181, 182, 202, 203, 204, 208, 209, 210, 211, 212, 232, 233, 234, 235, 236, 247, 248, 249, 250, 254, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 314, 340, 341, 342, 343, 344, 345, 346, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 391, 392, 393, 394, 416, 417, 418, 419, 489, 490, 491, 492, 501, 502, 503, 504, 517, 518, 519, 526, 539, 542, 544, 545, 546, 551, 552, 553, 557, 558, 559, 560, 561, 562, 572, 573, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 595, 596, 597, 598, 602, 603, 604, 605, 606, 607, 608, 610, 611, 612, 613, 619, 667, 726, 768, 769, 770, 771, 772, 773, 774 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 18, 19, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 40, 41, 42, 58, 59, 60, 61, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 128, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 205, 206, 207, 213, 214, 215, 229, 230, 231, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 260, 273, 286, 299, 300, 308, 321, 322, 347, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 395, 396, 413, 414, 415, 420, 421, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 447, 448, 449, 452, 458, 473, 474, 475, 476, 477, 478, 479, 480, 493, 494, 495, 496, 497, 498, 499, 500, 505, 506, 507, 508, 510, 511, 515, 516, 520, 523, 525, 529, 530, 531, 532, 533, 534, 535, 536, 543, 547, 548, 549, 550, 555, 556, 563, 564, 566, 567, 568, 571, 576, 577, 594, 599, 600, 601, 609, 614, 615, 616, 617, 618, 623, 637, 639, 640, 641, 642, 643, 645, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 719, 721, 722, 723, 724, 725, 727, 728, 730, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 774 }

B grade: { 20, 21, 24, 25, 35, 39, 197, 267, 268, 269, 270, 271, 272, 281, 282, 283, 284, 285, 323, 324, 325, 326, 348, 349, 350, 351, 352, 382, 383, 384, 385, 386, 444, 445, 446, 451, 457, 554, 636, 638, 644, 652, 665, 704, 718, 720, 731 }

C grade: { 123, 124, 125, 126, 127, 136, 137, 138, 139, 140, 162, 163, 164, 165, 166, 255, 256, 257, 258, 259, 450, 456, 620, 621, 622, 735 }

F grade: { 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 66, 67, 68, 80, 81, 106, 107, 116, 117, 129, 130, 155, 156, 202, 203, 204, 208, 209, 210, 211, }

212, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 233, 234, 235, 236, 247, 248, 249, 250, 254, 261, 262, 263, 264, 265, 266, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 374, 375, 376, 377, 378, 379, 380, 381, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 416, 417, 418, 419, 422, 423, 424, 425, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 501, 502, 503, 504, 509, 512, 513, 514, 517, 518, 519, 521, 522, 524, 526, 527, 528, 537, 538, 539, 540, 541, 542, 544, 545, 546, 551, 552, 553, 557, 558, 559, 560, 561, 562, 565, 569, 570, 572, 573, 574, 575, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 595, 596, 597, 598, 602, 603, 604, 605, 606, 607, 608, 610, 611, 612, 613, 619, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 646, 698, 699, 700, 701, 726, 729, 771, 772, 773 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 43, 44, 47, 51, 54, 55, 58, 59, 60, 62, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 179, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 257, 258, 259, 260, 261, 267, 268, 270, 271, 272, 273, 274, 275, 280, 284, 285, 286, 294, 295, 299, 300, 304, 305, 306, 307, 308, 309, 310, 311, 317, 318, 319, 320, 322, 329, 330, 331, 332, 333, 334, 338, 339, 356, 357, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 520, 521, 522, 523, 524, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 547, 548, 554, 555, 557, 558, 559, 560, 563, 564, 565, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 637, 640, 641, 642, 643, 645, 646, 647, 649, 650, 651, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 683, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 757, 758, 760, 761, 762, 763, 764, 765, 766, 773 }

B grade: { 20, 24, 39, 48, 52, 63, 236, 255, 256, 262, 263, 264, 269, 276, 277, 278, 279, 281, 282, 283, 287, 288, 289, 290, 293, 296, 297, 298, 312, 313, 321, 323, 324, 325, 326, 327, 328, 335, 336, 337, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 358, 399, 400, 408, 418, 419, 424, 425, 636, 638, 639, 644, 648, 652, 657, 665, 681, 684, 704, 706, 720, 728 }

C grade: { 41, 42, 45, 46, 49, 50, 53, 56, 57, 61, 64, 515, 516, 517, 518, 519, 525, 526, 539, 542, 572, 573 }

F grade: { 66, 67, 68, 80, 81, 106, 107, 115, 116, 117, 128, 129, 130, 154, 155, 156, 174, 175, 176, 177, 178, 180, 181, 182, 193, 194, 216, 217, 247, 248, 249, 250, 265, 266, 291, 292, 301, 302, 303, 314, 315, 316, 340, 341, 342, 359, 360, 361, 362, 363, 365, 366, 367, 378, 379, 401, 402, 543, 544, 545, 546, 549, 550, 551, 552, 553, 556, 561, 562, 579, 580, 581, 582, 583, 584, 585, 627, 628, 629, 630, 631, 632, 633, 634, 635, 732, 736, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 759, 767, 768, 769, 770, 771, 772, 774 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 14, 18, 19, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 34, 36, 43, 47, 51, 54, 58, 59, 62, 70, 71, 72, 73, 74, 75, 96, 97, 98, 99, 100, 101, 114, 122, 123, 124, 125, 126, 127, 135, 136, 137, 138, 139, 140, 161, 162, 163, 164, 165, 166, 185, 194, 199, 200, 201, 205, 206, 207, 213, 215, 229, 237, 241, 243, 246, 251, 252, 253, 255, 256, 257, 258, 259, 260, 281, 282, 283, 284, 285, 286, 299, 308, 309, 310, 311, 321, 322, 347, 388, 389, 395, 430, 431, 435, 436, 437, 442, 443, 447, 448, 449, 450, 452, 456, 458, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 498, 499, 500, 505, 506, 507, 508, 509, 520, 521, 522, 527, 528, 529, 530, 532, 533, 535, 536, 537, 540, 555, 563, 564, 565, 569, 571, 575, 590, 598, 617, 618, 620, 621, 622, 623, 624, 630, 632, 636, 639, 640, 641, 642, 644, 645, 646, 647, 648, 650, 651, 654, 655, 656, 657, 658, 663, 664, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 683, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 702, 706, 707, 708, 709, 710, 711, 712, 714, 715, 716, 717, 719, 721, 722, 723, 724, 725, 727, 729, 730, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 747, 748, 749, 751, 752, 754, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766 }

B grade: { 20, 24, 35, 37, 38, 39, 312, 313, 323, 324, 325, 326, 348, 349, 350, 351, 352, 547, 554, 631, 637, 638, 649, 652, 653, 662, 665, 666, 703, 704, 718, 720, 728 }

C grade: { 769 }

F grade: { 6, 7, 12, 13, 15, 16, 17, 26, 40, 41, 42, 44, 45, 46, 48, 49, 50, 52, 53, 55, 56, 57, 60, 61, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 128, 129, 130, 131, 132, 133, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 202, 203, 204, 208, 209, 210, 211, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 242, 244, 245, 247, 248, 249, 250, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 314, 315, 316, 317, 318, 319, 320, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 432, 433, 434, 438, 439, 440, 441, 444, 445, 446, 451, 453, 454, 455, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 493, 494, 495, 496, 501, 502, 503, 504, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 523, 524, 525, 526, 531, 534, 538, 539, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 556, 557, 558, 559, 560, 561, 562, 566, 567, 568, 570, 572, 573, 574, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 619, 625, 626, 627, 628, 629, 633, 634, 635, 643, 659, 660, 661, 681, 684, 689, 698, 699, 700, 701, 705, 713, 726, 731, 744, 745, 746, 750, 753, 755, 756, 767, 768, 770, 771, 772, 773, 774 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 43, 47, 51, 54, 58, 62, 65, 70, 71, 72, 73, 75, 82, 83, 84, 85, 86, 87, 88, 97, 98, 99, 101, 114, 122, 135, 161, 185, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 213, 214, 215, 229, 230, 231, 237, 238, 239, 240, 241, 243, 244, 245, 246, 251, 252, 253, 255, 256, 257, 258, 259, 260, 267, 268, 269, 270, 271, 272, 273, 286, 308, 321, 347, 373, 382, 383, 384, 385, 386, 387, 388, 389, 390, 395, 396, 413, 414, 415, 420, 421, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 474, 476, 478, 480, 482, 484, 486, 488, 490, 492, 494, 496, 498, 500, 502, 504, 505, 506, 507, 508, 509, 529, 530, 535, 536, 537, 543, 548, 549, 550, 555, 556, 575, 576, 577, 586, 590, 594, 598, 599, 600, 601, 605, 609, 613, 614, 615, 616, 617, 618, 621, 622, 623, 630, 637, 640, 641, 642, 643, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 707, 708, 709, 710, 711, 712, 713, 714, 715, 717, 718, 719, 721, 722, 723, 727, 728, 730, 731, 732, 733, 735, 737, 738, 739, 740, 741, 742, 743, 747, 748, 749, 751, 752, 754,

757, 758, 759, 760, 761, 762, 763, 764, 765, 766 }

B grade: { 39, 74, 100, 283, 284, 285, 299, 300, 323, 554, 563, 564, 565, 569, 570, 571, 620, 636, 638, 639, 644, 652, 657, 665, 704, 706, 720, 729, 736 }

C grade: { }

F grade: { 17, 33, 40, 41, 42, 44, 45, 46, 48, 49, 50, 52, 53, 55, 56, 57, 59, 60, 61, 63, 64, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 202, 203, 204, 208, 209, 210, 211, 212, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 233, 234, 235, 236, 242, 247, 248, 249, 250, 254, 261, 262, 263, 264, 265, 266, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 378, 379, 380, 381, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 416, 417, 418, 419, 422, 423, 424, 425, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 477, 479, 481, 483, 485, 487, 489, 491, 493, 495, 497, 499, 501, 503, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 538, 539, 540, 541, 542, 544, 545, 546, 547, 551, 552, 553, 557, 558, 559, 560, 561, 562, 566, 567, 568, 572, 573, 574, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 591, 592, 593, 595, 596, 597, 602, 603, 604, 606, 607, 608, 610, 611, 612, 619, 624, 625, 626, 627, 628, 629, 631, 632, 633, 634, 635, 692, 716, 724, 725, 726, 734, 744, 745, 746, 750, 753, 755, 756, 767, 768, 769, 770, 771, 772, 773, 774 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	12	27	8	12
normalized size	1	1.	1.	0.83	1.	2.25	0.67	1.
time (sec)	N/A	0.017	0.006	0.001	1.137	1.448	0.123	1.284

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	24	8	16
normalized size	1	1.	1.	1.	1.25	2.	0.67	1.33
time (sec)	N/A	0.02	0.005	0.002	1.051	1.439	0.124	1.205

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	50	19	31
normalized size	1	1.	1.	0.96	1.25	2.08	0.79	1.29
time (sec)	N/A	0.069	0.012	0.004	1.064	1.466	0.177	1.203

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	24	41	14	26
normalized size	1	1.	1.	1.	1.26	2.16	0.74	1.37
time (sec)	N/A	0.036	0.006	0.003	1.101	1.472	0.219	1.281

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	0	50	56	26
normalized size	1	1.	0.95	1.	0.	2.5	2.8	1.3
time (sec)	N/A	0.02	0.019	0.002	0.	1.541	1.263	1.305

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	0	81	0	41
normalized size	1	1.	0.97	0.97	0.	2.53	0.	1.28
time (sec)	N/A	0.07	0.034	0.002	0.	1.547	0.	1.214

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	27	0	77	0	35
normalized size	1	1.	0.96	1.	0.	2.85	0.	1.3
time (sec)	N/A	0.035	0.024	0.003	0.	1.555	0.	1.285

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	36	12	23
normalized size	1	1.	1.	1.06	1.38	2.25	0.75	1.44
time (sec)	N/A	0.021	0.006	0.001	1.151	1.5	0.128	1.284

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	33	38	55	24	41
normalized size	1	1.	1.	1.18	1.36	1.96	0.86	1.46
time (sec)	N/A	0.071	0.009	0.009	1.063	1.479	0.516	1.274

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	50	17	32
normalized size	1	1.	1.	1.04	1.35	2.17	0.74	1.39
time (sec)	N/A	0.036	0.005	0.	1.034	1.505	0.182	1.226

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	0	62	82	32
normalized size	1	1.	1.	1.04	0.	2.58	3.42	1.33
time (sec)	N/A	0.023	0.024	0.003	0.	1.551	1.425	1.267

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	81	0	100	0	82
normalized size	1	1.	0.97	2.25	0.	2.78	0.	2.28
time (sec)	N/A	0.073	0.043	0.025	0.	1.597	0.	1.269

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	32	0	89	0	42
normalized size	1	1.	0.97	1.03	0.	2.87	0.	1.35
time (sec)	N/A	0.035	0.03	0.001	0.	1.539	0.	1.228

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	25	0	66	80	32
normalized size	1	1.	0.96	1.	0.	2.64	3.2	1.28
time (sec)	N/A	0.037	0.065	0.001	0.	1.508	3.214	1.212

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	52	0	66	0	32
normalized size	1	1.	0.97	1.41	0.	1.78	0.	0.86
time (sec)	N/A	0.059	0.042	0.015	0.	1.579	0.	1.264

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	42	0	122	0	58
normalized size	1	1.	0.	1.02	0.	2.98	0.	1.41
time (sec)	N/A	0.065	0.29	0.008	0.	1.561	0.	1.352

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	186	0	0
normalized size	1	1.	0.	0.	0.	2.32	0.	0.
time (sec)	N/A	0.135	0.303	0.802	0.	1.589	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	43	20	28
normalized size	1	1.	1.	0.95	1.23	1.95	0.91	1.27
time (sec)	N/A	0.032	0.015	0.005	1.195	1.521	0.134	1.344

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	26	38	70	24	35
normalized size	1	1.	0.89	0.96	1.41	2.59	0.89	1.3
time (sec)	N/A	0.034	0.024	0.007	1.064	1.456	0.164	1.303

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	82	78	37	27
normalized size	1	1.	1.	1.38	3.9	3.71	1.76	1.29
time (sec)	N/A	0.023	0.009	0.006	1.116	1.416	0.124	1.246

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	29	115	105	51	27
normalized size	1	1.	0.71	0.85	3.38	3.09	1.5	0.79
time (sec)	N/A	0.036	0.021	0.006	1.089	1.529	0.147	1.202

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	26	34	59	29	35
normalized size	1	1.	0.97	0.84	1.1	1.9	0.94	1.13
time (sec)	N/A	0.034	0.016	0.003	1.136	1.491	0.141	1.299

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	32	46	92	32	43
normalized size	1	1.	0.84	0.86	1.24	2.49	0.86	1.16
time (sec)	N/A	0.038	0.026	0.006	1.05	1.488	0.134	1.271

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	33	90	89	41	32
normalized size	1	1.	1.	1.43	3.91	3.87	1.78	1.39
time (sec)	N/A	0.027	0.009	0.006	1.029	1.457	0.159	1.25

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	33	123	117	54	32
normalized size	1	1.	0.74	0.87	3.24	3.08	1.42	0.84
time (sec)	N/A	0.037	0.018	0.004	1.12	1.483	0.178	1.342

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	27	43	66	0	39
normalized size	1	1.	0.74	0.64	1.02	1.57	0.	0.93
time (sec)	N/A	0.048	0.019	0.016	1.154	1.45	0.	1.264

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	20	43	15	20
normalized size	1	1.	1.	1.5	1.25	2.69	0.94	1.25
time (sec)	N/A	0.019	0.008	0.009	1.124	1.479	0.126	1.251

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	39	41	73	39	41
normalized size	1	1.	0.97	1.22	1.28	2.28	1.22	1.28
time (sec)	N/A	0.034	0.018	0.006	1.151	1.485	0.184	1.271

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	57	63	111	73	63
normalized size	1	1.	0.92	1.1	1.21	2.13	1.4	1.21
time (sec)	N/A	0.042	0.025	0.006	1.159	1.492	0.191	1.3

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	47	43	95	49	51
normalized size	1	1.	0.85	1.18	1.08	2.38	1.22	1.27
time (sec)	N/A	0.044	0.038	0.01	1.043	1.534	0.362	1.263



Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	67	77	198	78	80
normalized size	1	1.	0.8	1.1	1.26	3.25	1.28	1.31
time (sec)	N/A	0.057	0.108	0.01	1.101	1.521	0.202	1.286

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	86	115	328	114	103
normalized size	1	1.	0.83	1.04	1.39	3.95	1.37	1.24
time (sec)	N/A	0.068	0.133	0.01	1.151	1.598	0.236	1.277

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	91	100	397	51	0
normalized size	1	1.	1.	1.82	2.	7.94	1.02	0.
time (sec)	N/A	0.078	0.025	0.046	1.591	1.797	1.603	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	47	43	72	42	50
normalized size	1	1.	1.	1.38	1.26	2.12	1.24	1.47
time (sec)	N/A	0.081	0.017	0.01	1.156	1.785	0.915	1.47

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	171	171	374	253	104
normalized size	1	1.	0.76	1.94	1.94	4.25	2.88	1.18
time (sec)	N/A	0.068	0.064	0.062	1.737	1.887	1.656	1.235

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	76	112	123	218	89
normalized size	1	1.	0.79	1.25	1.84	2.02	3.57	1.46
time (sec)	N/A	0.06	0.042	0.016	1.131	1.624	1.676	1.21

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	86	212	211	501	366	165
normalized size	1	1.	0.68	1.67	1.66	3.94	2.88	1.3
time (sec)	N/A	0.08	0.062	0.066	1.733	1.627	2.286	1.214

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	18	15	4
normalized size	1	1.	1.	1.	1.	4.5	3.75	1.
time (sec)	N/A	0.017	0.003	0.003	1.8	1.501	0.105	1.227

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	20	50	15	22
normalized size	1	1.	1.	1.	5.	12.5	3.75	5.5
time (sec)	N/A	0.019	0.003	0.002	1.208	1.5	0.107	1.191

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	45	34	42	104	0	0
normalized size	1	1.	1.67	1.26	1.56	3.85	0.	0.
time (sec)	N/A	0.058	0.032	0.007	1.211	1.524	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	60	51	65	154	0	0
normalized size	1	1.	1.5	1.27	1.62	3.85	0.	0.
time (sec)	N/A	0.1	0.033	0.006	1.138	1.456	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	89	74	96	236	0	0
normalized size	1	1.	1.29	1.07	1.39	3.42	0.	0.
time (sec)	N/A	0.121	0.038	0.005	1.203	1.499	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	53	0	190	24	28
normalized size	1	1.	1.	1.77	0.	6.33	0.8	0.93
time (sec)	N/A	0.029	0.008	0.028	0.	1.566	0.465	1.249

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	134	0	247	0	0
normalized size	1	1.	0.98	1.22	0.	2.25	0.	0.
time (sec)	N/A	0.106	0.063	0.041	0.	1.528	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	168	0	0	402	0	0
normalized size	1	1.	0.91	0.	0.	2.18	0.	0.
time (sec)	N/A	0.179	0.05	0.04	0.	1.557	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	224	0	0	552	0	0
normalized size	1	1.	0.84	0.	0.	2.06	0.	0.
time (sec)	N/A	0.23	0.051	0.036	0.	1.587	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	82	0	373	53	66
normalized size	1	1.	0.9	1.39	0.	6.32	0.9	1.12
time (sec)	N/A	0.039	0.05	0.031	0.	1.53	0.378	1.173

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	271	195	0	670	0	0
normalized size	1	1.	1.58	1.13	0.	3.9	0.	0.
time (sec)	N/A	0.158	0.117	0.053	0.	1.537	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	477	0	0	873	0	0
normalized size	1	1.	1.43	0.	0.	2.62	0.	0.
time (sec)	N/A	0.365	0.122	0.133	0.	1.592	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	434	0	0	1245	0	0
normalized size	1	1.	0.87	0.	0.	2.49	0.	0.
time (sec)	N/A	0.514	0.26	0.161	0.	1.642	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	94	0	593	85	82
normalized size	1	1.	0.81	1.12	0.	7.06	1.01	0.98
time (sec)	N/A	0.051	0.05	0.043	0.	1.569	0.469	1.225

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	184	223	0	1068	0	0
normalized size	1	1.	0.83	1.	0.	4.79	0.	0.
time (sec)	N/A	0.217	0.306	0.069	0.	1.604	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	353	0	0	1747	0	0
normalized size	1	1.	0.84	0.	0.	4.16	0.	0.
time (sec)	N/A	0.582	0.49	0.143	0.	1.685	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	22	0	190	26	28
normalized size	1	1.	1.	0.73	0.	6.33	0.87	0.93
time (sec)	N/A	0.021	0.008	0.004	0.	1.622	0.264	1.212

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	134	0	247	0	0
normalized size	1	1.	0.98	1.22	0.	2.25	0.	0.
time (sec)	N/A	0.095	0.062	0.041	0.	1.729	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	168	0	0	402	0	0
normalized size	1	1.	0.91	0.	0.	2.18	0.	0.
time (sec)	N/A	0.167	0.05	0.029	0.	1.928	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	224	0	0	552	0	0
normalized size	1	1.	0.84	0.	0.	2.06	0.	0.
time (sec)	N/A	0.22	0.051	0.033	0.	1.783	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	31	54	22	27
normalized size	1	1.	1.05	0.95	1.41	2.45	1.	1.23
time (sec)	N/A	0.02	0.02	0.001	1.099	1.547	0.247	1.239

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	73	144	54	0
normalized size	1	1.	0.76	0.89	1.16	2.29	0.86	0.
time (sec)	N/A	0.078	0.057	0.016	1.064	1.504	0.297	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	90	91	117	370	0	0
normalized size	1	1.	0.92	0.93	1.19	3.78	0.	0.
time (sec)	N/A	0.169	0.062	0.029	1.109	1.536	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	124	119	149	582	0	0
normalized size	1	1.	0.97	0.93	1.16	4.55	0.	0.
time (sec)	N/A	0.225	0.081	0.035	1.181	1.51	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	70	78	0	586	87	89
normalized size	1	1.	0.8	0.9	0.	6.74	1.	1.02
time (sec)	N/A	0.046	0.053	0.008	0.	1.556	0.299	1.244

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	209	209	0	774	0	0
normalized size	1	1.	1.07	1.07	0.	3.95	0.	0.
time (sec)	N/A	0.503	0.209	0.062	0.	1.601	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	254	0	0	1474	0	0
normalized size	1	1.	0.8	0.	0.	4.66	0.	0.
time (sec)	N/A	1.163	0.467	0.132	0.	1.67	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	93	0	122	385	0	177
normalized size	1	1.	0.98	0.	1.28	4.05	0.	1.86
time (sec)	N/A	0.181	0.065	0.027	1.08	1.554	0.	1.194

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	92	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.045	0.092	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.16	0.025	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.144	0.037	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	140	51	126	0	0
normalized size	1	1.	1.	3.04	1.11	2.74	0.	0.
time (sec)	N/A	0.024	0.012	0.035	1.226	1.567	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	24	24	76	124	194	95	107
normalized size	1	0.31	0.31	0.97	1.59	2.49	1.22	1.37
time (sec)	N/A	0.025	0.003	0.01	1.166	1.547	0.161	1.322

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	24	24	64	104	161	82	90
normalized size	1	0.37	0.37	0.98	1.6	2.48	1.26	1.38
time (sec)	N/A	0.024	0.003	0.008	1.127	1.528	0.253	1.259

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	53	52	84	128	68	74
normalized size	1	1.	0.62	0.6	0.98	1.49	0.79	0.86
time (sec)	N/A	0.093	0.011	0.006	1.137	1.502	0.142	1.25

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	41	40	63	100	54	58
normalized size	1	1.	0.66	0.65	1.02	1.61	0.87	0.94
time (sec)	N/A	0.063	0.008	0.007	1.167	1.535	0.125	1.275

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	43	72	41	932
normalized size	1	1.	0.66	0.64	0.98	1.64	0.93	21.18
time (sec)	N/A	0.038	0.007	0.002	1.131	1.525	0.115	1.291

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	41	24	24
normalized size	1	1.	1.	0.95	1.2	2.05	1.2	1.2
time (sec)	N/A	0.013	0.002	0.002	1.047	1.523	0.121	1.171

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	18	35	0	0
normalized size	1	1.	1.	1.07	1.2	2.33	0.	0.
time (sec)	N/A	0.021	0.002	0.011	1.287	1.731	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	35	24	82	0	0
normalized size	1	1.	0.91	1.	0.69	2.34	0.	0.
time (sec)	N/A	0.045	0.01	0.02	1.172	1.836	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	48	57	30	113	0	0
normalized size	1	1.	0.83	0.98	0.52	1.95	0.	0.
time (sec)	N/A	0.066	0.018	0.026	1.222	1.754	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	79	30	140	0	0
normalized size	1	1.	0.73	0.98	0.37	1.73	0.	0.
time (sec)	N/A	0.091	0.024	0.033	1.249	1.851	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	101	0	0	0	0
normalized size	1	1.	1.	4.21	0.	0.	0.	0.
time (sec)	N/A	0.022	0.002	0.044	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	123	0	0	0	0
normalized size	1	1.	1.	5.12	0.	0.	0.	0.
time (sec)	N/A	0.023	0.002	0.066	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	164	171	320	0	157
normalized size	1	1.	1.	4.82	5.03	9.41	0.	4.62
time (sec)	N/A	0.023	0.006	0.123	1.048	1.804	0.	1.2

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	142	151	275	0	140
normalized size	1	1.	1.	4.18	4.44	8.09	0.	4.12
time (sec)	N/A	0.023	0.006	0.054	1.077	1.737	0.	1.246

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	120	131	243	0	124
normalized size	1	1.	0.74	0.94	1.02	1.9	0.	0.97
time (sec)	N/A	0.142	0.042	0.039	1.079	1.755	0.	1.221

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	98	111	209	0	108
normalized size	1	1.	0.79	0.93	1.06	1.99	0.	1.03
time (sec)	N/A	0.088	0.035	0.029	1.034	1.753	0.	1.247

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	76	90	177	0	92
normalized size	1	1.	0.87	0.93	1.1	2.16	0.	1.12
time (sec)	N/A	0.059	0.027	0.023	1.141	1.713	0.	1.331



Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	72	139	0	77
normalized size	1	1.	1.	0.92	1.22	2.36	0.	1.31
time (sec)	N/A	0.034	0.023	0.022	1.056	1.83	0.	1.257

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	26	34	93	0	35
normalized size	1	1.	1.	0.7	0.92	2.51	0.	0.95
time (sec)	N/A	0.007	0.005	0.016	1.06	1.784	0.	1.223

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	38	103	0	0
normalized size	1	1.	1.	0.9	0.78	2.1	0.	0.
time (sec)	N/A	0.031	0.014	0.022	1.234	1.787	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	67	38	157	0	0
normalized size	1	1.	0.85	0.92	0.52	2.15	0.	0.
time (sec)	N/A	0.055	0.041	0.027	1.207	1.783	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	77	89	38	192	0	0
normalized size	1	1.	0.8	0.93	0.4	2.	0.	0.
time (sec)	N/A	0.081	0.034	0.034	1.231	1.732	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	89	111	38	223	0	0
normalized size	1	1.	0.75	0.93	0.32	1.87	0.	0.
time (sec)	N/A	0.108	0.041	0.044	1.176	1.741	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	133	38	258	0	0
normalized size	1	1.	1.	3.91	1.12	7.59	0.	0.
time (sec)	N/A	0.021	0.006	0.058	1.194	1.748	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	155	38	297	0	0
normalized size	1	1.	1.	4.56	1.12	8.74	0.	0.
time (sec)	N/A	0.021	0.005	0.091	1.302	1.827	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	140	51	126	0	0
normalized size	1	1.	1.	3.04	1.11	2.74	0.	0.
time (sec)	N/A	0.021	0.011	0.033	1.225	1.772	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	24	24	76	124	196	95	0
normalized size	1	0.31	0.31	0.97	1.59	2.51	1.22	0.
time (sec)	N/A	0.022	0.003	0.013	1.232	1.732	0.161	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	24	24	64	104	162	82	142
normalized size	1	0.37	0.37	0.98	1.6	2.49	1.26	2.18
time (sec)	N/A	0.023	0.003	0.01	1.033	1.726	0.148	1.236

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	53	52	84	128	68	112
normalized size	1	1.	0.63	0.62	1.	1.52	0.81	1.33
time (sec)	N/A	0.095	0.011	0.008	1.313	1.795	0.136	1.228

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	41	40	63	100	54	82
normalized size	1	1.	0.61	0.6	0.94	1.49	0.81	1.22
time (sec)	N/A	0.07	0.009	0.008	1.09	1.825	0.128	1.239

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	43	72	41	932
normalized size	1	1.	0.66	0.64	0.98	1.64	0.93	21.18
time (sec)	N/A	0.044	0.007	0.005	1.144	1.83	0.122	1.355

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	41	24	24
normalized size	1	1.	1.	0.95	1.2	2.05	1.2	1.2
time (sec)	N/A	0.022	0.002	0.003	1.072	1.89	0.11	1.246

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	18	35	0	0
normalized size	1	1.	1.	2.73	1.2	2.33	0.	0.
time (sec)	N/A	0.021	0.002	0.017	1.217	1.71	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	97	24	82	0	0
normalized size	1	1.	0.91	2.77	0.69	2.34	0.	0.
time (sec)	N/A	0.042	0.009	0.027	1.233	1.795	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	48	141	30	113	0	0
normalized size	1	1.	0.83	2.43	0.52	1.95	0.	0.
time (sec)	N/A	0.066	0.019	0.035	1.172	1.767	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	177	30	140	0	0
normalized size	1	1.	0.73	2.19	0.37	1.73	0.	0.
time (sec)	N/A	0.09	0.023	0.041	1.18	1.735	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	213	0	0	0	0
normalized size	1	1.	1.	8.88	0.	0.	0.	0.
time (sec)	N/A	0.021	0.002	0.054	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	249	0	0	0	0
normalized size	1	1.	1.	10.38	0.	0.	0.	0.
time (sec)	N/A	0.021	0.002	0.06	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	106	38	139	0	0
normalized size	1	1.	1.	3.12	1.12	4.09	0.	0.
time (sec)	N/A	0.023	0.006	0.022	1.204	1.656	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	109	38	134	0	0
normalized size	1	1.	1.	3.21	1.12	3.94	0.	0.
time (sec)	N/A	0.022	0.006	0.025	1.272	1.83	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	75	38	86	0	0
normalized size	1	1.	1.	2.21	1.12	2.53	0.	0.
time (sec)	N/A	0.013	0.005	0.016	1.298	1.701	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	78	35	86	0	0
normalized size	1	1.	1.	2.44	1.09	2.69	0.	0.
time (sec)	N/A	0.004	0.004	0.015	1.29	1.789	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	100	38	96	0	0
normalized size	1	1.	1.	2.94	1.12	2.82	0.	0.
time (sec)	N/A	0.022	0.004	0.022	1.232	1.811	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	102	38	107	0	0
normalized size	1	1.	1.	3.	1.12	3.15	0.	0.
time (sec)	N/A	0.021	0.004	0.023	1.216	1.71	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	22	7	11
normalized size	1	1.	1.	0.82	1.	2.	0.64	1.
time (sec)	N/A	0.014	0.002	0.003	1.184	1.797	0.085	1.241

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	136	47	0	0	0
normalized size	1	1.	1.	3.89	1.34	0.	0.	0.
time (sec)	N/A	0.018	0.007	0.036	1.271	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	121	0	0	0	0
normalized size	1	1.	1.	5.5	0.	0.	0.	0.
time (sec)	N/A	0.019	0.002	0.073	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	0	0	0	0
normalized size	1	1.	1.	4.71	0.	0.	0.	0.
time (sec)	N/A	0.02	0.002	0.064	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	53	77	30	135	0	0
normalized size	1	1.	0.67	0.97	0.38	1.71	0.	0.
time (sec)	N/A	0.059	0.024	0.066	1.169	1.776	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	40	55	28	107	0	0
normalized size	1	1.	0.71	0.98	0.5	1.91	0.	0.
time (sec)	N/A	0.035	0.017	0.062	1.216	1.754	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	24	68	0	0
normalized size	1	1.	1.	1.11	0.86	2.43	0.	0.
time (sec)	N/A	0.022	0.007	0.059	1.364	1.766	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	18	28	0	0
normalized size	1	1.	1.	1.15	1.38	2.15	0.	0.
time (sec)	N/A	0.018	0.002	0.056	1.192	1.735	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	39	20	24
normalized size	1	1.	1.	1.06	1.33	2.17	1.11	1.33
time (sec)	N/A	0.018	0.003	0.	1.129	1.694	0.113	1.299

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	27	49	28	68	22	0
normalized size	1	1.	0.69	1.26	0.72	1.74	0.56	0.
time (sec)	N/A	0.036	0.006	0.008	1.193	1.89	0.117	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	73	30	101	39	0
normalized size	1	1.	0.67	1.2	0.49	1.66	0.64	0.
time (sec)	N/A	0.06	0.008	0.012	1.272	1.72	0.134	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	53	96	28	130	53	0
normalized size	1	1.	0.65	1.17	0.34	1.59	0.65	0.
time (sec)	N/A	0.082	0.01	0.013	1.229	1.696	0.145	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	22	22	119	30	162	66	0
normalized size	1	0.34	0.34	1.83	0.46	2.49	1.02	0.
time (sec)	N/A	0.018	0.003	0.013	1.288	1.828	0.156	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	21	21	142	28	194	80	0
normalized size	1	0.27	0.27	1.84	0.36	2.52	1.04	0.
time (sec)	N/A	0.019	0.003	0.016	1.161	1.931	0.162	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	169	51	0	0	0
normalized size	1	1.	1.	3.67	1.11	0.	0.	0.
time (sec)	N/A	0.024	0.01	0.038	1.392	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	123	0	0	0	0
normalized size	1	1.	1.	5.12	0.	0.	0.	0.
time (sec)	N/A	0.025	0.002	0.063	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	101	0	0	0	0
normalized size	1	1.	1.	4.21	0.	0.	0.	0.
time (sec)	N/A	0.025	0.002	0.042	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	57	79	30	149	0	0
normalized size	1	1.	0.7	0.98	0.37	1.84	0.	0.
time (sec)	N/A	0.089	0.02	0.031	1.27	1.917	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	44	57	30	117	0	0
normalized size	1	1.	0.76	0.98	0.52	2.02	0.	0.
time (sec)	N/A	0.063	0.014	0.026	1.204	1.772	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	35	24	89	0	0
normalized size	1	1.	0.91	1.	0.69	2.54	0.	0.
time (sec)	N/A	0.036	0.005	0.024	1.297	1.693	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	18	36	0	0
normalized size	1	1.	1.	1.07	1.2	2.4	0.	0.
time (sec)	N/A	0.023	0.002	0.019	1.267	1.76	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	50	29	24
normalized size	1	1.	1.	0.95	1.2	2.5	1.45	1.2
time (sec)	N/A	0.021	0.004	0.002	1.009	1.765	0.118	1.731

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	32	52	30	82	29	0
normalized size	1	1.	0.73	1.18	0.68	1.86	0.66	0.
time (sec)	N/A	0.045	0.007	0.011	1.175	1.664	0.122	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	45	74	30	115	44	0
normalized size	1	1.	0.73	1.19	0.48	1.85	0.71	0.
time (sec)	N/A	0.069	0.009	0.016	1.166	1.655	0.136	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	58	98	30	142	58	0
normalized size	1	1.	0.67	1.14	0.35	1.65	0.67	0.
time (sec)	N/A	0.096	0.01	0.017	1.262	1.729	0.143	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	24	24	121	30	176	71	0
normalized size	1	0.35	0.35	1.75	0.43	2.55	1.03	0.
time (sec)	N/A	0.022	0.003	0.02	1.181	1.652	0.159	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	24	24	144	30	209	85	0
normalized size	1	0.29	0.29	1.76	0.37	2.55	1.04	0.
time (sec)	N/A	0.024	0.003	0.023	1.184	1.752	0.169	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	155	38	298	0	0
normalized size	1	1.	1.	4.56	1.12	8.76	0.	0.
time (sec)	N/A	0.024	0.004	0.101	1.206	1.854	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	133	38	259	0	0
normalized size	1	1.	1.	3.91	1.12	7.62	0.	0.
time (sec)	N/A	0.024	0.004	0.054	1.204	1.75	0.	0.



Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	86	111	38	224	0	0
normalized size	1	1.	0.72	0.93	0.32	1.88	0.	0.
time (sec)	N/A	0.125	0.036	0.04	1.226	1.786	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	74	89	38	192	0	0
normalized size	1	1.	0.77	0.93	0.4	2.	0.	0.
time (sec)	N/A	0.088	0.032	0.033	1.282	1.821	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	67	38	153	0	0
normalized size	1	1.	0.82	0.92	0.52	2.1	0.	0.
time (sec)	N/A	0.061	0.024	0.028	1.166	1.807	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	35	104	0	0
normalized size	1	1.	1.	0.9	0.71	2.12	0.	0.
time (sec)	N/A	0.034	0.011	0.024	1.232	1.785	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	28	46	92	0	0
normalized size	1	1.	1.	0.72	1.18	2.36	0.	0.
time (sec)	N/A	0.027	0.006	0.023	1.146	2.025	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	58	38	151	0	0
normalized size	1	1.	1.	0.92	0.6	2.4	0.	0.
time (sec)	N/A	0.053	0.019	0.03	1.225	2.056	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	80	38	192	0	0
normalized size	1	1.	0.86	0.93	0.44	2.23	0.	0.
time (sec)	N/A	0.079	0.049	0.035	1.164	2.049	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	86	102	38	227	0	0
normalized size	1	1.	0.79	0.94	0.35	2.08	0.	0.
time (sec)	N/A	0.112	0.056	0.044	1.244	2.117	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	100	124	38	258	0	0
normalized size	1	1.	0.76	0.94	0.29	1.95	0.	0.
time (sec)	N/A	0.16	0.086	0.058	1.224	1.913	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	146	38	293	0	0
normalized size	1	1.	1.	4.29	1.12	8.62	0.	0.
time (sec)	N/A	0.026	0.005	0.08	1.254	1.788	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	168	38	338	0	0
normalized size	1	1.	1.	4.94	1.12	9.94	0.	0.
time (sec)	N/A	0.024	0.006	0.125	1.195	1.82	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	169	51	0	0	0
normalized size	1	1.	1.	3.67	1.11	0.	0.	0.
time (sec)	N/A	0.025	0.01	0.037	1.343	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	249	0	0	0	0
normalized size	1	1.	1.	10.38	0.	0.	0.	0.
time (sec)	N/A	0.028	0.002	0.056	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	213	0	0	0	0
normalized size	1	1.	1.	8.88	0.	0.	0.	0.
time (sec)	N/A	0.033	0.002	0.052	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	57	177	30	149	0	0
normalized size	1	1.	0.7	2.19	0.37	1.84	0.	0.
time (sec)	N/A	0.115	0.02	0.043	1.299	1.807	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	44	141	30	117	0	0
normalized size	1	1.	0.76	2.43	0.52	2.02	0.	0.
time (sec)	N/A	0.076	0.015	0.04	1.189	1.775	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	97	24	88	0	0
normalized size	1	1.	0.91	2.77	0.69	2.51	0.	0.
time (sec)	N/A	0.048	0.005	0.034	1.173	1.771	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	18	36	0	0
normalized size	1	1.	1.	2.73	1.2	2.4	0.	0.
time (sec)	N/A	0.022	0.002	0.029	1.275	1.708	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	50	29	24
normalized size	1	1.	1.	0.95	1.2	2.5	1.45	1.2
time (sec)	N/A	0.022	0.004	0.001	1.115	1.778	0.109	1.334

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	32	52	30	82	29	0
normalized size	1	1.	0.73	1.18	0.68	1.86	0.66	0.
time (sec)	N/A	0.046	0.007	0.013	1.301	1.762	0.116	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	75	30	115	44	0
normalized size	1	1.	0.67	1.12	0.45	1.72	0.66	0.
time (sec)	N/A	0.07	0.009	0.018	1.122	1.766	0.129	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	97	30	142	58	0
normalized size	1	1.	0.7	1.17	0.36	1.71	0.7	0.
time (sec)	N/A	0.094	0.011	0.022	1.28	1.73	0.144	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	24	24	121	30	178	71	0
normalized size	1	0.35	0.35	1.75	0.43	2.58	1.03	0.
time (sec)	N/A	0.023	0.003	0.028	1.308	1.655	0.152	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	24	24	84	30	211	85	0
normalized size	1	0.29	0.29	1.02	0.37	2.57	1.04	0.
time (sec)	N/A	0.023	0.003	0.029	1.187	2.015	0.161	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	120	38	157	0	0
normalized size	1	1.	1.	3.53	1.12	4.62	0.	0.
time (sec)	N/A	0.025	0.004	0.036	1.248	2.106	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	115	38	149	0	0
normalized size	1	1.	1.	3.38	1.12	4.38	0.	0.
time (sec)	N/A	0.025	0.004	0.034	1.187	2.047	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	38	112	0	0
normalized size	1	1.	1.	3.09	1.12	3.29	0.	0.
time (sec)	N/A	0.014	0.004	0.029	1.17	2.14	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	98	35	100	0	0
normalized size	1	1.	1.	3.06	1.09	3.12	0.	0.
time (sec)	N/A	0.005	0.003	0.029	1.248	1.809	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	82	38	88	0	0
normalized size	1	1.	1.	2.41	1.12	2.59	0.	0.
time (sec)	N/A	0.024	0.005	0.03	1.202	1.594	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	78	38	88	0	0
normalized size	1	1.	1.	2.29	1.12	2.59	0.	0.
time (sec)	N/A	0.022	0.005	0.03	1.226	1.595	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	112	38	144	0	0
normalized size	1	1.	1.	3.29	1.12	4.24	0.	0.
time (sec)	N/A	0.023	0.005	0.037	1.229	1.634	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	63	0	0	0
normalized size	1	1.	1.	0.	1.37	0.	0.	0.
time (sec)	N/A	0.026	0.012	0.056	1.228	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	55	0	0	0
normalized size	1	1.	1.	0.	1.41	0.	0.	0.
time (sec)	N/A	0.025	0.006	0.027	1.229	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	55	0	0	0
normalized size	1	1.	1.	0.	1.41	0.	0.	0.
time (sec)	N/A	0.025	0.006	0.044	1.246	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	55	0	0	0
normalized size	1	1.	1.	0.	1.41	0.	0.	0.
time (sec)	N/A	0.015	0.006	0.036	1.265	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	47	0	0	0
normalized size	1	1.	1.	0.	1.34	0.	0.	0.
time (sec)	N/A	0.004	0.005	0.025	1.24	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	19	20	32	0	0
normalized size	1	1.	1.	1.27	1.33	2.13	0.	0.
time (sec)	N/A	0.023	0.002	0.157	1.141	1.514	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	50	0	0	0
normalized size	1	1.	1.	0.	1.35	0.	0.	0.
time (sec)	N/A	0.024	0.004	0.038	1.202	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	53	0	0	0
normalized size	1	1.	1.	0.	1.36	0.	0.	0.
time (sec)	N/A	0.025	0.004	0.02	1.215	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	53	0	0	0
normalized size	1	1.	1.	0.	1.36	0.	0.	0.
time (sec)	N/A	0.024	0.004	0.023	1.198	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	24	44	69	122	0	0
normalized size	1	1.	0.34	0.62	0.97	1.72	0.	0.
time (sec)	N/A	0.076	0.004	0.02	1.128	1.534	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	25	56	46	88	0	0
normalized size	1	1.	0.56	1.24	1.02	1.96	0.	0.
time (sec)	N/A	0.048	0.004	0.031	1.147	1.556	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	27	57	39	27
normalized size	1	1.	1.	1.25	1.35	2.85	1.95	1.35
time (sec)	N/A	0.023	0.004	0.022	1.163	1.568	161.134	1.254

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	19	20	32	0	0
normalized size	1	1.	1.	1.27	1.33	2.13	0.	0.
time (sec)	N/A	0.023	0.002	0.	1.269	1.554	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	20	43	27	101	0	0
normalized size	1	1.	0.53	1.13	0.71	2.66	0.	0.
time (sec)	N/A	0.049	0.004	0.09	1.223	1.579	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	25	70	34	149	0	0
normalized size	1	1.	0.35	0.99	0.48	2.1	0.	0.
time (sec)	N/A	0.075	0.004	0.098	1.233	1.499	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	39	96	45	228	0	0
normalized size	1	1.	0.38	0.92	0.43	2.19	0.	0.
time (sec)	N/A	0.11	0.011	0.084	1.204	1.645	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	39	67	45	182	0	0
normalized size	1	1.	0.53	0.91	0.61	2.46	0.	0.
time (sec)	N/A	0.065	0.009	0.043	1.284	1.603	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	51	109	0	0
normalized size	1	1.	1.	0.74	1.19	2.53	0.	0.
time (sec)	N/A	0.036	0.009	0.05	1.142	1.525	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	39	59	47	208	0	0
normalized size	1	1.	0.59	0.89	0.71	3.15	0.	0.
time (sec)	N/A	0.065	0.007	0.061	1.208	1.61	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	39	88	47	0	0	0
normalized size	1	1.	0.41	0.92	0.49	0.	0.	0.
time (sec)	N/A	0.096	0.008	0.054	1.337	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	0	10	14
normalized size	1	1.	0.69	0.62	0.75	0.	0.62	0.88
time (sec)	N/A	0.008	0.005	0.003	1.045	0.	0.09	1.217

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	96	249	370	270	0	184
normalized size	1	1.	0.47	1.23	1.82	1.33	0.	0.91
time (sec)	N/A	0.229	0.106	0.073	1.328	1.6	0.	1.246

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	83	168	302	239	0	144
normalized size	1	1.	0.59	1.2	2.16	1.71	0.	1.03
time (sec)	N/A	0.129	0.066	0.033	1.401	1.524	0.	1.242

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	80	182	173	0	104
normalized size	1	1.	0.93	1.18	2.68	2.54	0.	1.53
time (sec)	N/A	0.058	0.028	0.027	1.344	1.514	0.	1.227

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	54	117	0	45
normalized size	1	1.	1.	1.	1.32	2.85	0.	1.1
time (sec)	N/A	0.008	0.004	0.026	1.141	1.554	0.	1.206



Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.114	0.016	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.307	0.023	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.409	0.029	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	373	0	0
normalized size	1	1.	0.92	0.	0.	3.11	0.	0.
time (sec)	N/A	0.089	0.2	0.025	0.	1.555	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	288	0	0
normalized size	1	1.	0.93	0.	0.	3.13	0.	0.
time (sec)	N/A	0.049	0.047	0.018	0.	1.567	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	151	0	0
normalized size	1	1.	1.	0.	0.	3.43	0.	0.
time (sec)	N/A	0.006	0.009	0.016	0.	1.523	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.296	0.016	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	132	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	1.088	0.023	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	261	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	1.389	0.03	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	164	0	0	348	0	0
normalized size	1	1.	0.9	0.	0.	1.9	0.	0.
time (sec)	N/A	0.183	0.18	0.025	0.	1.511	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	138	0	0	312	0	0
normalized size	1	1.	1.	0.	0.	2.26	0.	0.
time (sec)	N/A	0.149	0.135	0.019	0.	1.515	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	0	0	277	0	0
normalized size	1	1.	0.9	0.	0.	2.8	0.	0.
time (sec)	N/A	0.121	0.081	0.017	0.	1.551	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	203	0	0
normalized size	1	1.	0.92	0.	0.	2.54	0.	0.
time (sec)	N/A	0.068	0.031	0.015	0.	1.467	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	101	0	0
normalized size	1	1.	1.	0.	0.	2.66	0.	0.
time (sec)	N/A	0.009	0.005	0.011	0.	1.478	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.24	0.013	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.181	0.027	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	26	29	26	58	34	28
normalized size	1	1.	0.65	0.72	0.65	1.45	0.85	0.7
time (sec)	N/A	0.012	0.012	0.003	1.051	1.452	0.182	1.265

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	241	517	0	0	0	0
normalized size	1	1.	0.83	1.78	0.	0.	0.	0.
time (sec)	N/A	0.288	0.19	0.112	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	179	359	0	0	0	0
normalized size	1	1.	0.67	1.33	0.	0.	0.	0.
time (sec)	N/A	0.254	0.154	0.081	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	128	227	0	263	0	0
normalized size	1	1.	0.56	0.99	0.	1.15	0.	0.
time (sec)	N/A	0.223	0.113	0.072	0.	1.546	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	82	126	0	163	0	0
normalized size	1	1.	0.68	1.05	0.	1.36	0.	0.
time (sec)	N/A	0.116	0.066	0.068	0.	1.552	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	0	89	0	0
normalized size	1	1.	1.	1.27	0.	2.17	0.	0.
time (sec)	N/A	0.029	0.015	0.06	0.	1.517	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	0	89	0	0
normalized size	1	1.	1.	1.15	0.	2.17	0.	0.
time (sec)	N/A	0.131	0.031	0.112	0.	1.567	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	80	0	131	0	0
normalized size	1	1.	1.	1.18	0.	1.93	0.	0.
time (sec)	N/A	0.397	0.102	0.093	0.	1.54	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	115	226	0	238	0	0
normalized size	1	1.	0.69	1.36	0.	1.43	0.	0.
time (sec)	N/A	0.718	0.185	0.097	0.	1.547	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	195	343	0	481	0	0
normalized size	1	1.	0.47	0.83	0.	1.16	0.	0.
time (sec)	N/A	0.435	0.203	0.076	0.	1.856	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	148	228	0	366	0	0
normalized size	1	1.	0.51	0.78	0.	1.26	0.	0.
time (sec)	N/A	0.304	0.137	0.046	0.	1.935	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	131	175	0	312	0	0
normalized size	1	1.	0.64	0.85	0.	1.51	0.	0.
time (sec)	N/A	0.215	0.103	0.04	0.	1.888	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	93	0	248	0	0
normalized size	1	1.	0.8	0.84	0.	2.23	0.	0.
time (sec)	N/A	0.119	0.053	0.032	0.	1.827	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	65	0	158	0	0
normalized size	1	1.	1.	1.05	0.	2.55	0.	0.
time (sec)	N/A	0.039	0.02	0.025	0.	1.565	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.07	0.034	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.214	0.048	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.576	0.064	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	219	0	0	585	0	0
normalized size	1	1.	0.92	0.	0.	2.45	0.	0.
time (sec)	N/A	0.201	0.193	0.046	0.	1.609	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	167	0	0	516	0	0
normalized size	1	1.	0.91	0.	0.	2.8	0.	0.
time (sec)	N/A	0.149	0.363	0.042	0.	1.638	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	127	0	0	450	0	0
normalized size	1	1.	0.89	0.	0.	3.17	0.	0.
time (sec)	N/A	0.118	0.064	0.037	0.	1.599	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	354	0	0
normalized size	1	1.	0.93	0.	0.	3.85	0.	0.
time (sec)	N/A	0.052	0.067	0.033	0.	1.579	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	208	0	0
normalized size	1	1.	1.	0.	0.	4.73	0.	0.
time (sec)	N/A	0.005	0.008	0.025	0.	1.549	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.061	0.03	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.29	0.042	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.036	0.059	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.241	0.036	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.14	0.033	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	117	49	105	0	0
normalized size	1	1.	0.88	2.85	1.2	2.56	0.	0.
time (sec)	N/A	0.022	0.008	0.036	1.26	1.532	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.042	0.039	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.082	0.043	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.077	0.051	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.052	0.027	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	183	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.15	0.018	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	136	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.069	0.043	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.033	0.031	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.006	0.023	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.035	0.015	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.032	0.04	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.035	0.013	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	163	0	0
normalized size	1	1.	1.	0.	0.	2.67	0.	0.
time (sec)	N/A	0.065	0.036	0.057	0.	1.587	0.	0.



Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	C	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	31	31	579	7426	999	796	196
normalized size	1	0.3	0.3	5.51	70.72	9.51	7.58	1.87
time (sec)	N/A	0.073	0.008	0.021	3.58	1.548	0.399	1.331

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	C	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	31	31	396	5261	687	558	167
normalized size	1	0.35	0.35	4.5	59.78	7.81	6.34	1.9
time (sec)	N/A	0.069	0.008	0.015	3.095	1.564	0.319	1.318

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	72	249	3461	446	366	139
normalized size	1	1.	0.57	1.98	27.47	3.54	2.9	1.1
time (sec)	N/A	0.256	0.041	0.009	2.707	1.58	0.262	1.673

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	138	2030	265	214	111
normalized size	1	1.	0.62	1.52	22.31	2.91	2.35	1.22
time (sec)	N/A	0.176	0.033	0.005	2.152	1.543	0.213	1.27

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	40	63	965	142	100	82
normalized size	1	1.	0.65	1.02	15.56	2.29	1.61	1.32
time (sec)	N/A	0.105	0.022	0.007	1.688	1.497	0.172	1.271

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	36	34	76	36	34
normalized size	1	1.	1.	1.33	1.26	2.81	1.33	1.26
time (sec)	N/A	0.036	0.007	0.003	1.016	1.536	0.137	1.223

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	73	0	0
normalized size	1	1.	1.	1.05	0.	3.32	0.	0.
time (sec)	N/A	0.068	0.006	0.021	0.	1.509	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	53	0	220	0	0
normalized size	1	1.	0.89	1.	0.	4.15	0.	0.
time (sec)	N/A	0.132	0.039	0.034	0.	1.533	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	64	86	0	389	0	0
normalized size	1	1.	0.74	0.99	0.	4.47	0.	0.
time (sec)	N/A	0.198	0.077	0.048	0.	1.55	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	119	0	616	0	0
normalized size	1	1.	0.65	0.98	0.	5.09	0.	0.
time (sec)	N/A	0.258	0.094	0.064	0.	1.525	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	152	0	0	0	0
normalized size	1	1.	1.	4.9	0.	0.	0.	0.
time (sec)	N/A	0.063	0.007	0.091	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	185	0	0	0	0
normalized size	1	1.	1.	5.97	0.	0.	0.	0.
time (sec)	N/A	0.064	0.007	0.137	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1896	8659	1355	0	263
normalized size	1	1.	1.	38.69	176.71	27.65	0.	5.37
time (sec)	N/A	0.066	0.028	0.504	3.669	1.639	0.	1.299

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1359	6310	992	0	235
normalized size	1	1.	1.	27.73	128.78	20.24	0.	4.8
time (sec)	N/A	0.067	0.025	0.222	3.17	1.6	0.	1.223

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	153	914	4327	713	0	207
normalized size	1	1.	0.85	5.11	24.17	3.98	0.	1.16
time (sec)	N/A	0.331	0.371	0.123	2.636	1.596	0.	1.303

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	126	561	2712	493	0	178
normalized size	1	1.	0.87	3.87	18.7	3.4	0.	1.23
time (sec)	N/A	0.23	0.144	0.081	2.161	1.524	0.	1.271

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	90	300	1463	331	0	150
normalized size	1	1.	0.81	2.7	13.18	2.98	0.	1.35
time (sec)	N/A	0.154	0.094	0.056	1.786	1.537	0.	1.34

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	131	583	220	0	123
normalized size	1	1.	1.	1.7	7.57	2.86	0.	1.6
time (sec)	N/A	0.082	0.043	0.043	1.402	1.578	0.	1.211

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	58	78	123	0	49
normalized size	1	1.	1.	1.32	1.77	2.8	0.	1.11
time (sec)	N/A	0.011	0.006	0.028	1.028	1.546	0.	1.227

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	62	0	192	0	0
normalized size	1	1.	0.94	0.93	0.	2.87	0.	0.
time (sec)	N/A	0.079	0.045	0.049	0.	1.53	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	81	96	0	373	0	0
normalized size	1	1.	0.79	0.94	0.	3.66	0.	0.
time (sec)	N/A	0.145	0.097	0.049	0.	1.564	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	97	129	0	621	0	0
normalized size	1	1.	0.71	0.95	0.	4.57	0.	0.
time (sec)	N/A	0.219	0.12	0.064	0.	1.525	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	112	162	0	913	0	0
normalized size	1	1.	0.66	0.95	0.	5.37	0.	0.
time (sec)	N/A	0.286	0.152	0.09	0.	1.609	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	195	0	1273	0	0
normalized size	1	1.	1.	3.98	0.	25.98	0.	0.
time (sec)	N/A	0.061	0.03	0.122	0.	1.77	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	228	0	1729	0	0
normalized size	1	1.	1.	4.65	0.	35.29	0.	0.
time (sec)	N/A	0.062	0.03	0.182	0.	2.059	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	188	0	0
normalized size	1	1.	1.	0.	0.	3.08	0.	0.
time (sec)	N/A	0.061	0.04	0.057	0.	1.906	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	31	31	857	1712	1513	1171	0
normalized size	1	0.3	0.3	8.16	16.3	14.41	11.15	0.
time (sec)	N/A	0.067	0.01	0.023	1.671	1.891	0.549	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	31	31	584	1180	1019	823	0
normalized size	1	0.35	0.35	6.64	13.41	11.58	9.35	0.
time (sec)	N/A	0.067	0.009	0.019	1.714	1.848	0.44	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	75	365	749	640	537	1782
normalized size	1	1.	0.6	2.94	6.04	5.16	4.33	14.37
time (sec)	N/A	0.284	0.061	0.013	1.603	1.735	0.349	1.324

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	56	200	416	371	306	952
normalized size	1	1.	0.58	2.08	4.33	3.86	3.19	9.92
time (sec)	N/A	0.209	0.042	0.008	1.589	1.535	0.275	1.281

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	40	89	180	190	144	1203
normalized size	1	1.	0.65	1.44	2.9	3.06	2.32	19.4
time (sec)	N/A	0.138	0.025	0.007	1.566	1.563	0.21	1.421

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	48	34	100	46	34
normalized size	1	1.	1.	1.78	1.26	3.7	1.7	1.26
time (sec)	N/A	0.067	0.008	0.005	1.025	1.549	0.159	1.228

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	97	0	0
normalized size	1	1.	1.	0.	0.	4.41	0.	0.
time (sec)	N/A	0.066	0.006	0.036	0.	1.631	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	315	0	0
normalized size	1	1.	0.89	0.	0.	5.94	0.	0.
time (sec)	N/A	0.125	0.032	0.054	0.	1.498	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	64	0	0	567	0	0
normalized size	1	1.	0.74	0.	0.	6.52	0.	0.
time (sec)	N/A	0.193	0.076	0.084	0.	1.589	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	80	0	0	907	0	0
normalized size	1	1.	0.66	0.	0.	7.5	0.	0.
time (sec)	N/A	0.26	0.1	0.101	0.	1.555	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.008	0.154	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.008	0.187	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	282	0	0
normalized size	1	1.	1.	0.	0.	5.76	0.	0.
time (sec)	N/A	0.064	0.025	0.047	0.	1.626	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	157	0	0
normalized size	1	1.	1.	0.	0.	3.2	0.	0.
time (sec)	N/A	0.041	0.023	0.026	0.	1.581	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	157	0	0
normalized size	1	1.	1.	0.	0.	3.34	0.	0.
time (sec)	N/A	0.007	0.016	0.017	0.	1.544	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	250	0	0
normalized size	1	1.	1.	0.	0.	5.1	0.	0.
time (sec)	N/A	0.064	0.016	0.04	0.	1.549	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	301	0	0
normalized size	1	1.	1.	0.	0.	6.14	0.	0.
time (sec)	N/A	0.063	0.017	0.044	0.	1.541	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	501	0	0
normalized size	1	1.	1.	0.	0.	10.22	0.	0.
time (sec)	N/A	0.063	0.023	0.054	0.	1.618	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	42	0	58	117	76	1436
normalized size	1	1.	0.66	0.	0.91	1.83	1.19	22.44
time (sec)	N/A	0.032	0.034	0.005	1.013	1.522	0.757	2.132

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	0	84	167	0	1804
normalized size	1	1.	0.6	0.	0.84	1.67	0.	18.04
time (sec)	N/A	0.064	0.041	0.005	1.014	1.573	0.	1.528

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.018	0.069	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	534	0	0	0	0
normalized size	1	1.	1.	18.41	0.	0.	0.	0.
time (sec)	N/A	0.045	0.006	0.122	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	368	0	0	0	0
normalized size	1	1.	1.	13.14	0.	0.	0.	0.
time (sec)	N/A	0.044	0.006	0.086	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	76	234	0	270	0	0
normalized size	1	1.	0.64	1.97	0.	2.27	0.	0.
time (sec)	N/A	0.133	0.068	0.085	0.	1.567	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	133	0	180	0	0
normalized size	1	1.	0.68	1.56	0.	2.12	0.	0.
time (sec)	N/A	0.08	0.046	0.083	0.	1.597	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	61	0	116	0	0
normalized size	1	1.	0.91	1.33	0.	2.52	0.	0.
time (sec)	N/A	0.052	0.025	0.076	0.	1.539	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	0	42	0	0
normalized size	1	1.	1.	1.1	0.	2.1	0.	0.
time (sec)	N/A	0.044	0.005	0.079	0.	1.602	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	34	63	34	34
normalized size	1	1.	1.	1.04	1.36	2.52	1.36	1.36
time (sec)	N/A	0.042	0.007	0.003	1.	1.542	0.372	1.34

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	106	0	117	44	0
normalized size	1	1.	0.72	1.86	0.	2.05	0.77	0.
time (sec)	N/A	0.085	0.017	0.021	0.	1.592	0.198	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	60	169	0	212	102	0
normalized size	1	1.	0.67	1.88	0.	2.36	1.13	0.
time (sec)	N/A	0.133	0.025	0.028	0.	1.625	0.236	0.



Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	76	243	0	328	177	0
normalized size	1	1.	0.62	1.99	0.	2.69	1.45	0.
time (sec)	N/A	0.185	0.034	0.038	0.	1.574	0.275	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	29	29	329	0	479	272	0
normalized size	1	0.32	0.32	3.58	0.	5.21	2.96	0.
time (sec)	N/A	0.049	0.006	0.048	0.	1.662	0.318	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	28	28	427	0	663	388	0
normalized size	1	0.26	0.26	3.95	0.	6.14	3.59	0.
time (sec)	N/A	0.053	0.006	0.059	0.	1.654	0.361	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.034	0.08	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	961	0	0	0	0
normalized size	1	1.	1.	31.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.008	0.096	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	646	0	0	0	0
normalized size	1	1.	1.	20.84	0.	0.	0.	0.
time (sec)	N/A	0.051	0.008	0.064	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	395	0	486	0	0
normalized size	1	1.	0.79	3.26	0.	4.02	0.	0.
time (sec)	N/A	0.186	0.166	0.05	0.	1.666	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	208	0	315	0	0
normalized size	1	1.	0.82	2.39	0.	3.62	0.	0.
time (sec)	N/A	0.124	0.042	0.04	0.	1.621	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	86	0	211	0	0
normalized size	1	1.	0.89	1.62	0.	3.98	0.	0.
time (sec)	N/A	0.071	0.023	0.03	0.	1.6	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	69	0	0
normalized size	1	1.	1.	1.05	0.	3.14	0.	0.
time (sec)	N/A	0.045	0.006	0.026	0.	1.61	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	115	54	34
normalized size	1	1.	1.	0.96	1.26	4.26	2.	1.26
time (sec)	N/A	0.043	0.009	0.005	0.991	1.577	0.44	1.183

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	47	185	136	217	82	0
normalized size	1	1.	0.76	2.98	2.19	3.5	1.32	0.
time (sec)	N/A	0.087	0.025	0.037	1.074	1.544	0.27	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	64	301	281	386	189	2236
normalized size	1	1.	0.7	3.31	3.09	4.24	2.08	24.57
time (sec)	N/A	0.137	0.033	0.061	1.028	1.811	0.351	1.342

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	81	444	471	610	333	0
normalized size	1	1.	0.64	3.52	3.74	4.84	2.64	0.
time (sec)	N/A	0.191	0.042	0.093	1.05	2.007	0.431	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	31	31	609	710	907	518	0
normalized size	1	0.32	0.32	6.34	7.4	9.45	5.4	0.
time (sec)	N/A	0.045	0.007	0.145	1.084	1.975	0.537	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	31	31	797	999	1283	745	0
normalized size	1	0.27	0.27	7.05	8.84	11.35	6.59	0.
time (sec)	N/A	0.046	0.007	0.189	1.074	2.178	0.763	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1173	0	1285	0	0
normalized size	1	1.	1.	23.94	0.	26.22	0.	0.
time (sec)	N/A	0.048	0.029	0.14	0.	1.683	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	826	0	934	0	0
normalized size	1	1.	1.	16.86	0.	19.06	0.	0.
time (sec)	N/A	0.049	0.026	0.079	0.	1.641	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	113	543	0	663	0	0
normalized size	1	1.	0.66	3.19	0.	3.9	0.	0.
time (sec)	N/A	0.234	0.128	0.059	0.	1.61	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	97	324	0	458	0	0
normalized size	1	1.	0.71	2.38	0.	3.37	0.	0.
time (sec)	N/A	0.168	0.096	0.046	0.	1.688	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	79	169	0	306	0	0
normalized size	1	1.	0.77	1.66	0.	3.	0.	0.
time (sec)	N/A	0.116	0.076	0.036	0.	1.554	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	74	0	209	0	0
normalized size	1	1.	0.94	1.1	0.	3.12	0.	0.
time (sec)	N/A	0.064	0.034	0.029	0.	1.542	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	116	0	0
normalized size	1	1.	1.	0.76	0.	2.52	0.	0.
time (sec)	N/A	0.054	0.009	0.031	0.	1.54	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	76	0	275	0	0
normalized size	1	1.	1.	0.94	0.	3.4	0.	0.
time (sec)	N/A	0.102	0.046	0.054	0.	1.544	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	109	0	443	0	0
normalized size	1	1.	0.83	0.95	0.	3.85	0.	0.
time (sec)	N/A	0.151	0.093	0.078	0.	1.651	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	111	142	0	670	0	0
normalized size	1	1.	0.74	0.95	0.	4.5	0.	0.
time (sec)	N/A	0.208	0.128	0.113	0.	1.685	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	175	0	950	0	0
normalized size	1	1.	0.69	0.96	0.	5.19	0.	0.
time (sec)	N/A	0.265	0.154	0.155	0.	1.792	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	208	0	1300	0	0
normalized size	1	1.	1.	4.24	0.	26.53	0.	0.
time (sec)	N/A	0.045	0.041	0.254	0.	2.011	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	241	0	1750	0	0
normalized size	1	1.	1.	4.92	0.	35.71	0.	0.
time (sec)	N/A	0.045	0.026	0.341	0.	2.082	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.038	0.094	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.01	0.146	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.009	0.114	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	0	0	707	0	0
normalized size	1	1.	0.79	0.	0.	5.84	0.	0.
time (sec)	N/A	0.191	0.207	0.095	0.	1.688	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	0	0	454	0	0
normalized size	1	1.	0.82	0.	0.	5.22	0.	0.
time (sec)	N/A	0.139	0.06	0.083	0.	1.692	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	300	0	0
normalized size	1	1.	0.89	0.	0.	5.66	0.	0.
time (sec)	N/A	0.091	0.036	0.07	0.	1.672	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	90	0	0
normalized size	1	1.	1.	0.	0.	4.09	0.	0.
time (sec)	N/A	0.044	0.007	0.048	0.	1.623	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	161	66	34
normalized size	1	1.	1.	0.96	1.26	5.96	2.44	1.26
time (sec)	N/A	0.043	0.012	0.003	0.974	1.578	0.521	1.388

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	47	261	194	312	114	0
normalized size	1	1.	0.76	4.21	3.13	5.03	1.84	0.
time (sec)	N/A	0.087	0.029	0.061	1.034	1.593	0.339	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	64	434	405	563	270	0
normalized size	1	1.	0.67	4.52	4.22	5.86	2.81	0.
time (sec)	N/A	0.135	0.044	0.112	1.051	1.625	0.47	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	641	684	900	484	0
normalized size	1	1.	0.59	5.21	5.56	7.32	3.93	0.
time (sec)	N/A	0.186	0.033	0.185	1.085	1.816	0.629	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	31	31	889	1040	1381	760	0
normalized size	1	0.32	0.32	9.26	10.83	14.39	7.92	0.
time (sec)	N/A	0.044	0.009	0.29	1.11	2.02	1.066	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	31	31	733	1465	1979	1096	0
normalized size	1	0.27	0.27	6.49	12.96	17.51	9.7	0.
time (sec)	N/A	0.045	0.008	0.041	1.175	2.42	3.413	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	402	0	0
normalized size	1	1.	1.	0.	0.	8.2	0.	0.
time (sec)	N/A	0.048	0.028	0.072	0.	1.589	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	313	0	0
normalized size	1	1.	1.	0.	0.	6.39	0.	0.
time (sec)	N/A	0.027	0.021	0.058	0.	1.633	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	284	0	0
normalized size	1	1.	1.	0.	0.	6.04	0.	0.
time (sec)	N/A	0.007	0.012	0.029	0.	1.625	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	147	0	0
normalized size	1	1.	1.	0.	0.	3.	0.	0.
time (sec)	N/A	0.046	0.028	0.053	0.	1.624	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	144	0	0
normalized size	1	1.	1.	0.	0.	2.94	0.	0.
time (sec)	N/A	0.044	0.028	0.059	0.	1.591	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	350	0	0
normalized size	1	1.	1.	0.	0.	7.14	0.	0.
time (sec)	N/A	0.044	0.036	0.077	0.	1.683	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.021	0.067	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.015	0.046	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.013	0.067	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.012	0.052	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	0.009	0.024	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	0	49	0	0
normalized size	1	1.	1.	1.18	0.	2.23	0.	0.
time (sec)	N/A	0.035	0.004	0.31	0.	1.571	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.012	0.066	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.01	0.032	0.	0.	0.	0.



Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.01	0.031	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	32	32	113	174	298	0	0
normalized size	1	0.28	0.28	0.99	1.53	2.61	0.	0.
time (sec)	N/A	0.038	0.007	0.027	1.024	1.62	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	31	31	95	146	250	0	0
normalized size	1	0.33	0.33	1.01	1.55	2.66	0.	0.
time (sec)	N/A	0.04	0.007	0.022	1.032	1.574	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	32	77	117	201	0	0
normalized size	1	1.	0.23	0.56	0.85	1.47	0.	0.
time (sec)	N/A	0.168	0.006	0.019	1.043	1.602	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	31	59	89	157	0	0
normalized size	1	1.	0.31	0.59	0.89	1.57	0.	0.
time (sec)	N/A	0.12	0.007	0.02	1.045	1.567	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	32	74	61	112	0	0
normalized size	1	1.	0.51	1.17	0.97	1.78	0.	0.
time (sec)	N/A	0.074	0.006	0.056	1.015	1.578	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	32	36	70	0	36
normalized size	1	1.	1.	1.19	1.33	2.59	0.	1.33
time (sec)	N/A	0.037	0.007	0.034	0.968	1.531	0.	1.408

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	0	49	0	0
normalized size	1	1.	1.	1.18	0.	2.23	0.	0.
time (sec)	N/A	0.036	0.002	0.	0.	1.58	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	27	61	0	147	0	0
normalized size	1	1.	0.48	1.09	0.	2.62	0.	0.
time (sec)	N/A	0.073	0.006	0.12	0.	1.506	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	32	99	0	205	0	0
normalized size	1	1.	0.32	0.99	0.	2.05	0.	0.
time (sec)	N/A	0.114	0.007	0.129	0.	1.583	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	31	137	0	247	0	0
normalized size	1	1.	0.22	0.99	0.	1.78	0.	0.
time (sec)	N/A	0.164	0.007	0.089	0.	1.55	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	175	0	0	0	0
normalized size	1	1.	1.	5.47	0.	0.	0.	0.
time (sec)	N/A	0.043	0.007	0.089	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	213	0	0	0	0
normalized size	1	1.	1.	6.87	0.	0.	0.	0.
time (sec)	N/A	0.039	0.007	0.095	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	0	128	0	0
normalized size	1	1.	1.	0.77	0.	2.72	0.	0.
time (sec)	N/A	0.065	0.012	0.177	0.	1.652	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	34	0	124	0	0
normalized size	1	1.	1.	0.72	0.	2.64	0.	0.
time (sec)	N/A	0.048	0.007	0.079	0.	1.603	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	412	1657	2030	1118	0	1272
normalized size	1	1.	0.8	3.2	3.92	2.16	0.	2.46
time (sec)	N/A	0.943	0.606	0.089	1.906	1.667	0.	1.297

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	220	1063	1463	775	0	869
normalized size	1	1.	0.57	2.73	3.76	1.99	0.	2.23
time (sec)	N/A	0.654	0.419	0.075	1.717	1.666	0.	1.247

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	148	617	965	467	0	575
normalized size	1	1.	0.57	2.39	3.74	1.81	0.	2.23
time (sec)	N/A	0.436	0.227	0.063	1.538	1.558	0.	1.306

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	105	324	583	324	0	348
normalized size	1	1.	0.62	1.91	3.43	1.91	0.	2.05
time (sec)	N/A	0.314	0.142	0.045	1.377	1.579	0.	1.269

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	74	132	269	201	0	171
normalized size	1	1.	0.91	1.63	3.32	2.48	0.	2.11
time (sec)	N/A	0.147	0.067	0.035	1.217	1.542	0.	1.274

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	58	78	123	0	49
normalized size	1	1.	1.	1.32	1.77	2.8	0.	1.11
time (sec)	N/A	0.012	0.006	0.002	1.016	1.583	0.	1.252

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.288	0.04	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.748	0.044	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	1.121	0.052	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	167	0	0	514	0	0
normalized size	1	1.	0.94	0.	0.	2.9	0.	0.
time (sec)	N/A	0.156	0.213	0.046	0.	1.538	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	117	0	0	385	0	0
normalized size	1	1.	0.93	0.	0.	3.06	0.	0.
time (sec)	N/A	0.107	0.085	0.033	0.	1.532	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	250	0	0
normalized size	1	1.	0.93	0.	0.	2.72	0.	0.
time (sec)	N/A	0.06	0.06	0.023	0.	1.496	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	120	0	0
normalized size	1	1.	1.	0.	0.	3.	0.	0.
time (sec)	N/A	0.005	0.006	0.012	0.	1.539	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.385	0.044	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	2.055	0.041	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	106	0	185	0	0
normalized size	1	1.	0.93	1.49	0.	2.61	0.	0.
time (sec)	N/A	0.405	0.124	0.162	0.	1.578	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	191	0	375	0	0
normalized size	1	1.	1.	1.65	0.	3.23	0.	0.
time (sec)	N/A	1.011	0.34	0.128	0.	1.644	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	0	506	0	1116	0	0
normalized size	1	1.	0.	1.9	0.	4.18	0.	0.
time (sec)	N/A	1.914	0.653	0.15	0.	1.636	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	460	460	0	922	0	2736	0	0
normalized size	1	1.	0.	2.	0.	5.95	0.	0.
time (sec)	N/A	3.751	0.627	0.2	0.	1.814	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	468	1146	0	0	0	0
normalized size	1	1.	1.35	3.31	0.	0.	0.	0.
time (sec)	N/A	0.362	0.487	0.013	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	292	682	0	0	0	0
normalized size	1	1.	0.91	2.13	0.	0.	0.	0.
time (sec)	N/A	0.317	0.316	0.01	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	170	356	0	408	0	0
normalized size	1	1.	0.67	1.4	0.	1.6	0.	0.
time (sec)	N/A	0.256	0.2	0.007	0.	1.541	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	150	0	178	0	0
normalized size	1	1.	0.73	1.2	0.	1.42	0.	0.
time (sec)	N/A	0.128	0.094	0.006	0.	1.551	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	42	0	70	0	0
normalized size	1	1.	1.	1.14	0.	1.89	0.	0.
time (sec)	N/A	0.03	0.013	0.002	0.	1.526	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	79	0	138	0	0
normalized size	1	1.	0.9	1.27	0.	2.23	0.	0.
time (sec)	N/A	0.203	0.063	0.011	0.	1.573	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	97	0	312	0	0
normalized size	1	1.	0.98	0.91	0.	2.92	0.	0.
time (sec)	N/A	0.545	0.163	0.01	0.	1.59	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	0	240	0	1007	0	0
normalized size	1	1.	0.	1.	0.	4.2	0.	0.
time (sec)	N/A	1.037	0.401	0.01	0.	1.645	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	243	560	0	635	0	0
normalized size	1	1.	0.75	1.74	0.	1.97	0.	0.
time (sec)	N/A	0.339	0.308	0.018	0.	1.646	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	176	313	0	420	0	0
normalized size	1	1.	0.82	1.46	0.	1.95	0.	0.
time (sec)	N/A	0.233	0.188	0.009	0.	1.575	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	85	140	0	265	0	0
normalized size	1	1.	0.77	1.26	0.	2.39	0.	0.
time (sec)	N/A	0.129	0.116	0.007	0.	1.476	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	48	0	139	0	0
normalized size	1	1.	1.	0.96	0.	2.78	0.	0.
time (sec)	N/A	0.04	0.016	0.005	0.	1.545	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.037	0.091	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.216	0.103	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.083	0.14	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	195	0	0	736	0	0
normalized size	1	1.	0.95	0.	0.	3.57	0.	0.
time (sec)	N/A	0.189	0.146	0.098	0.	1.648	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	136	0	0	551	0	0
normalized size	1	1.	0.9	0.	0.	3.65	0.	0.
time (sec)	N/A	0.133	0.076	0.076	0.	1.573	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	85	0	0	370	0	0
normalized size	1	1.	0.92	0.	0.	4.02	0.	0.
time (sec)	N/A	0.057	0.085	0.059	0.	1.531	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	189	0	0
normalized size	1	1.	1.	0.	0.	4.72	0.	0.
time (sec)	N/A	0.006	0.006	0.017	0.	1.574	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.031	0.076	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.298	0.083	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	103	432	0	270	0	0
normalized size	1	1.	0.99	4.15	0.	2.6	0.	0.
time (sec)	N/A	1.046	0.317	0.242	0.	1.609	0.	0.



Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	0	580	0	456	0	0
normalized size	1	1.	0.	3.65	0.	2.87	0.	0.
time (sec)	N/A	2.559	0.985	0.17	0.	1.539	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	366	366	0	2014	0	1507	0	0
normalized size	1	1.	0.	5.5	0.	4.12	0.	0.
time (sec)	N/A	4.757	0.503	0.213	0.	1.732	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	634	634	0	4671	0	4427	0	0
normalized size	1	1.	0.	7.37	0.	6.98	0.	0.
time (sec)	N/A	9.405	0.951	0.27	0.	1.946	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	122	218	271	278	0	185
normalized size	1	1.	0.56	1.	1.25	1.28	0.	0.85
time (sec)	N/A	0.231	0.177	0.086	1.162	1.571	0.	1.251

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	104	163	224	238	0	146
normalized size	1	1.	0.63	0.99	1.37	1.45	0.	0.89
time (sec)	N/A	0.093	0.116	0.031	1.176	1.557	0.	1.223

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	79	144	184	0	108
normalized size	1	1.	1.	0.98	1.78	2.27	0.	1.33
time (sec)	N/A	0.038	0.064	0.029	1.124	1.538	0.	1.199

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	50	68	142	0	68
normalized size	1	1.	1.	0.89	1.21	2.54	0.	1.21
time (sec)	N/A	0.015	0.013	0.025	0.984	1.554	0.	1.273

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.111	0.014	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.257	0.021	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	91	194	244	217	0	140
normalized size	1	1.	0.5	1.07	1.35	1.2	0.	0.77
time (sec)	N/A	0.179	0.218	0.011	1.152	1.543	0.	1.266

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	79	111	204	181	0	108
normalized size	1	1.	0.59	0.83	1.52	1.35	0.	0.81
time (sec)	N/A	0.082	0.114	0.004	1.161	1.535	0.	1.267

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	68	53	132	149	0	78
normalized size	1	1.	1.03	0.8	2.	2.26	0.	1.18
time (sec)	N/A	0.032	0.046	0.003	1.128	1.528	0.	1.383

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	34	43	101	41	51
normalized size	1	1.	1.05	0.77	0.98	2.3	0.93	1.16
time (sec)	N/A	0.011	0.012	0.003	0.979	1.512	0.79	1.342

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.121	0.012	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.221	0.016	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	191	368	360	460	0	338
normalized size	1	1.	0.64	1.24	1.21	1.55	0.	1.14
time (sec)	N/A	0.636	0.354	0.011	1.198	1.511	0.	1.164

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	144	212	298	328	0	205
normalized size	1	1.	0.67	0.98	1.38	1.52	0.	0.95
time (sec)	N/A	0.271	0.178	0.006	1.179	1.582	0.	1.24

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	116	102	193	251	0	140
normalized size	1	1.	1.08	0.95	1.8	2.35	0.	1.31
time (sec)	N/A	0.104	0.088	0.004	1.169	1.528	0.	1.224

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	78	171	0	92
normalized size	1	1.	1.	0.88	1.15	2.51	0.	1.35
time (sec)	N/A	0.025	0.016	0.003	1.009	1.531	0.	1.214

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.545	0.015	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	127	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	0.379	0.017	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	169	550	728	458	0	541
normalized size	1	1.	0.64	2.07	2.74	1.72	0.	2.03
time (sec)	N/A	0.324	0.275	0.052	1.359	1.56	0.	1.364

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	123	307	448	311	0	340
normalized size	1	1.	0.65	1.62	2.37	1.65	0.	1.8
time (sec)	N/A	0.108	0.175	0.043	1.247	1.59	0.	1.286

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	96	131	216	203	0	184
normalized size	1	1.	1.07	1.46	2.4	2.26	0.	2.04
time (sec)	N/A	0.041	0.103	0.033	1.142	1.568	0.	1.254

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.209	0.037	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.583	0.04	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	205	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.833	0.049	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	45	728	99	85	59
normalized size	1	1.	0.69	1.	16.18	2.2	1.89	1.31
time (sec)	N/A	0.056	0.125	0.004	1.321	1.519	0.153	1.296

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	99	448	190	0	119
normalized size	1	1.	1.1	1.27	5.74	2.44	0.	1.53
time (sec)	N/A	0.056	0.105	0.039	1.255	1.539	0.	1.338

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	23	38	24	23
normalized size	1	1.	1.	1.06	1.35	2.24	1.41	1.35
time (sec)	N/A	0.017	0.038	0.001	0.97	1.557	0.116	1.233

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	0	105	0	0
normalized size	1	1.	1.	1.03	0.	2.69	0.	0.
time (sec)	N/A	0.037	0.055	0.023	0.	1.507	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	96	101	0	205	0	0
normalized size	1	1.	1.14	1.2	0.	2.44	0.	0.
time (sec)	N/A	0.055	0.098	0.062	0.	1.543	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	79	88	0	234	0	0
normalized size	1	1.	1.14	1.28	0.	3.39	0.	0.
time (sec)	N/A	0.071	0.088	0.031	0.	1.567	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	29	44	724	93	83	54
normalized size	1	1.	0.67	1.02	16.84	2.16	1.93	1.26
time (sec)	N/A	0.043	0.095	0.004	1.341	1.527	0.15	1.319

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	90	444	171	0	104
normalized size	1	1.	1.12	1.2	5.92	2.28	0.	1.39
time (sec)	N/A	0.058	0.062	0.036	1.242	1.54	0.	1.236

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	32	22	22
normalized size	1	1.	1.	1.06	1.38	2.	1.38	1.38
time (sec)	N/A	0.014	0.027	0.002	0.994	1.52	0.113	1.261

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	33	0	92	0	0
normalized size	1	1.	1.	0.89	0.	2.49	0.	0.
time (sec)	N/A	0.029	0.033	0.017	0.	1.562	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	94	87	0	186	0	0
normalized size	1	1.	1.16	1.07	0.	2.3	0.	0.
time (sec)	N/A	0.049	0.059	0.034	0.	1.63	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	77	74	0	215	0	0
normalized size	1	1.	1.17	1.12	0.	3.26	0.	0.
time (sec)	N/A	0.063	0.049	0.027	0.	1.53	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	133	142	0	266	0	0
normalized size	1	1.	0.92	0.98	0.	1.83	0.	0.
time (sec)	N/A	0.352	0.244	0.023	0.	1.529	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	93	112	0	167	0	0
normalized size	1	1.	0.84	1.01	0.	1.5	0.	0.
time (sec)	N/A	0.247	0.121	0.018	0.	1.559	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	94	102	0	192	0	0
normalized size	1	1.	0.8	0.86	0.	1.63	0.	0.
time (sec)	N/A	0.119	0.054	0.01	0.	1.513	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	83	323	0	144	0	0
normalized size	1	1.	0.83	3.23	0.	1.44	0.	0.
time (sec)	N/A	0.129	0.054	0.012	0.	1.574	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	120	660	0	212	0	0
normalized size	1	1.	0.91	5.	0.	1.61	0.	0.
time (sec)	N/A	0.235	0.122	0.017	0.	1.57	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	232	561	0	682	0	0
normalized size	1	1.	1.09	2.65	0.	3.22	0.	0.
time (sec)	N/A	0.62	1.131	0.023	0.	1.582	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	163	369	0	533	0	0
normalized size	1	1.	0.96	2.18	0.	3.15	0.	0.
time (sec)	N/A	0.411	0.555	0.018	0.	1.617	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	127	169	0	420	0	0
normalized size	1	1.	0.92	1.22	0.	3.04	0.	0.
time (sec)	N/A	0.194	0.153	0.011	0.	1.572	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	153	685	0	491	0	0
normalized size	1	1.	0.97	4.34	0.	3.11	0.	0.
time (sec)	N/A	0.214	0.179	0.011	0.	1.533	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	217	1730	0	583	0	0
normalized size	1	1.	1.17	9.3	0.	3.13	0.	0.
time (sec)	N/A	0.408	0.519	0.017	0.	1.643	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	268	3532	0	709	0	0
normalized size	1	1.	1.16	15.22	0.	3.06	0.	0.
time (sec)	N/A	0.51	0.639	0.022	0.	1.651	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	35	41	55	41	0
normalized size	1	1.	0.9	1.17	1.37	1.83	1.37	0.
time (sec)	N/A	0.037	0.019	0.013	1.44	1.565	0.311	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	35	41	55	31	42
normalized size	1	1.	0.9	1.17	1.37	1.83	1.03	1.4
time (sec)	N/A	0.034	0.012	0.009	0.965	1.545	0.152	1.211

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	37	45	57	44	0
normalized size	1	1.	0.81	1.16	1.41	1.78	1.38	0.
time (sec)	N/A	0.038	0.022	0.017	1.466	1.574	0.321	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	37	45	57	34	46
normalized size	1	1.	0.81	1.16	1.41	1.78	1.06	1.44
time (sec)	N/A	0.035	0.013	0.009	0.967	1.597	0.155	1.174

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	36	54	80	90	76	0
normalized size	1	1.	0.62	0.93	1.38	1.55	1.31	0.
time (sec)	N/A	0.058	0.033	0.015	1.451	1.587	0.362	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	36	54	80	90	68	65
normalized size	1	1.	0.62	0.93	1.38	1.55	1.17	1.12
time (sec)	N/A	0.051	0.018	0.01	0.992	1.577	0.192	1.217



Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	38	55	78	90	76	0
normalized size	1	1.	0.66	0.95	1.34	1.55	1.31	0.
time (sec)	N/A	0.054	0.029	0.012	1.453	1.543	0.36	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	38	55	78	90	68	68
normalized size	1	1.	0.66	0.95	1.34	1.55	1.17	1.17
time (sec)	N/A	0.052	0.017	0.011	0.982	1.526	0.194	1.241

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	53	0	190	29	0
normalized size	1	1.	1.	1.77	0.	6.33	0.97	0.
time (sec)	N/A	0.03	0.007	0.029	0.	1.565	0.326	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	53	0	190	24	28
normalized size	1	1.	1.	1.77	0.	6.33	0.8	0.93
time (sec)	N/A	0.029	0.004	0.026	0.	1.513	0.17	1.146

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	49	0	189	29	0
normalized size	1	1.	1.	1.63	0.	6.3	0.97	0.
time (sec)	N/A	0.028	0.007	0.026	0.	1.554	0.34	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	49	0	189	24	32
normalized size	1	1.	1.	1.63	0.	6.3	0.8	1.07
time (sec)	N/A	0.03	0.005	0.024	0.	1.559	0.182	1.267

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	74	0	212	39	0
normalized size	1	1.	0.93	1.72	0.	4.93	0.91	0.
time (sec)	N/A	0.043	0.02	0.029	0.	1.553	0.202	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	74	0	212	39	51
normalized size	1	1.	0.93	1.72	0.	4.93	0.91	1.19
time (sec)	N/A	0.04	0.008	0.029	0.	1.551	0.202	1.229

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	211	39	0
normalized size	1	1.	0.93	1.63	0.	4.91	0.91	0.
time (sec)	N/A	0.043	0.018	0.032	0.	1.627	0.205	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	211	39	53
normalized size	1	1.	0.93	1.63	0.	4.91	0.91	1.23
time (sec)	N/A	0.043	0.009	0.029	0.	1.654	0.206	1.37

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	0	0	198	85	0
normalized size	1	1.	1.06	0.	0.	6.39	2.74	0.
time (sec)	N/A	0.039	0.013	0.028	0.	1.626	0.756	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	0	0	198	85	42
normalized size	1	1.	1.06	0.	0.	6.39	2.74	1.35
time (sec)	N/A	0.038	0.004	0.02	0.	1.622	0.824	1.246

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	224	82	0
normalized size	1	1.	1.06	0.	0.	7.	2.56	0.
time (sec)	N/A	0.04	0.013	0.027	0.	1.582	0.771	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	224	82	49
normalized size	1	1.	1.06	0.	0.	7.	2.56	1.53
time (sec)	N/A	0.04	0.004	0.02	0.	1.655	0.855	1.372

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	40	26	62	0	0
normalized size	1	1.	1.46	1.67	1.08	2.58	0.	0.
time (sec)	N/A	0.048	0.031	0.029	1.651	1.53	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	40	32	62	0	39
normalized size	1	1.	1.46	1.67	1.33	2.58	0.	1.62
time (sec)	N/A	0.047	0.006	0.016	0.967	1.575	0.	1.297

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	38	44	28	62	0	0
normalized size	1	1.	1.52	1.76	1.12	2.48	0.	0.
time (sec)	N/A	0.05	0.031	0.024	1.65	1.614	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	38	44	34	62	0	45
normalized size	1	1.	1.52	1.76	1.36	2.48	0.	1.8
time (sec)	N/A	0.05	0.007	0.016	0.98	1.499	0.	1.314

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	29	92	65	56	0
normalized size	1	1.	0.66	0.66	2.09	1.48	1.27	0.
time (sec)	N/A	0.04	0.022	0.015	1.462	1.551	0.969	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	29	51	65	58	42
normalized size	1	1.	0.66	0.66	1.16	1.48	1.32	0.95
time (sec)	N/A	0.041	0.012	0.01	0.974	1.564	0.956	1.289

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	29	96	68	58	0
normalized size	1	1.	0.65	0.63	2.09	1.48	1.26	0.
time (sec)	N/A	0.043	0.022	0.015	1.483	1.519	0.986	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	29	54	68	60	45
normalized size	1	1.	0.65	0.63	1.17	1.48	1.3	0.98
time (sec)	N/A	0.044	0.012	0.01	0.978	1.499	0.953	1.156

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	111	0	0	373	0	0
normalized size	1	1.	1.19	0.	0.	4.01	0.	0.
time (sec)	N/A	0.074	0.086	0.024	0.	1.54	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	111	0	0	373	0	127
normalized size	1	1.	1.19	0.	0.	4.01	0.	1.37
time (sec)	N/A	0.073	0.035	0.01	0.	1.588	0.	1.343

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	115	0	0	374	0	0
normalized size	1	1.	1.2	0.	0.	3.9	0.	0.
time (sec)	N/A	0.076	0.081	0.022	0.	1.531	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	115	0	0	374	0	136
normalized size	1	1.	1.2	0.	0.	3.9	0.	1.42
time (sec)	N/A	0.077	0.013	0.013	0.	1.596	0.	1.377

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	20	70	14	20
normalized size	1	1.	1.	1.06	1.18	4.12	0.82	1.18
time (sec)	N/A	0.015	0.014	0.008	0.978	1.56	0.08	1.33

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	55	17	24
normalized size	1	1.	1.	0.88	1.	2.29	0.71	1.
time (sec)	N/A	0.02	0.01	0.007	0.963	1.495	0.106	1.272

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	44	35	58	163	22	62
normalized size	1	1.	0.79	0.62	1.04	2.91	0.39	1.11
time (sec)	N/A	0.032	0.025	0.006	1.472	1.528	0.122	1.296

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	46	117	24	46
normalized size	1	1.	1.	0.84	1.05	2.66	0.55	1.05
time (sec)	N/A	0.036	0.018	0.006	1.463	1.525	0.121	1.269

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	66	0	518	63	85
normalized size	1	1.	0.99	0.99	0.	7.73	0.94	1.27
time (sec)	N/A	0.065	0.108	0.008	0.	1.536	0.285	1.212

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	38	50	140	0	0
normalized size	1	1.	0.86	0.86	1.14	3.18	0.	0.
time (sec)	N/A	0.128	0.055	0.012	0.986	1.517	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	41	51	117	0	0
normalized size	1	1.	0.91	0.76	0.94	2.17	0.	0.
time (sec)	N/A	0.124	0.003	0.006	0.981	1.562	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	120	183	0	302	0	0
normalized size	1	1.	0.67	1.02	0.	1.68	0.	0.
time (sec)	N/A	0.188	0.088	0.01	0.	1.576	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	144	235	0	317	0	0
normalized size	1	1.	0.71	1.15	0.	1.55	0.	0.
time (sec)	N/A	0.195	0.09	0.01	0.	1.609	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	205	378	0	649	0	0
normalized size	1	1.	0.74	1.37	0.	2.35	0.	0.
time (sec)	N/A	0.429	0.21	0.013	0.	1.529	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	65	84	212	0	0
normalized size	1	1.	0.79	0.9	1.17	2.94	0.	0.
time (sec)	N/A	0.23	0.092	0.033	0.986	1.5	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	62	80	185	0	0
normalized size	1	1.	1.	0.81	1.04	2.4	0.	0.
time (sec)	N/A	0.222	0.01	0.006	0.989	1.486	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	172	0	0	468	0	0
normalized size	1	1.	0.66	0.	0.	1.81	0.	0.
time (sec)	N/A	0.296	0.151	0.028	0.	1.499	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	216	0	0	520	0	0
normalized size	1	1.	0.74	0.	0.	1.77	0.	0.
time (sec)	N/A	0.309	0.165	0.026	0.	1.654	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	407	0	0	977	0	0
normalized size	1	1.	1.04	0.	0.	2.5	0.	0.
time (sec)	N/A	0.665	0.184	0.029	0.	1.554	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	68	74	159	34	0
normalized size	1	1.	0.92	1.7	1.85	3.98	0.85	0.
time (sec)	N/A	0.027	0.034	0.013	0.99	1.603	0.116	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	93	547	0	711	104	0
normalized size	1	1.	0.99	5.82	0.	7.56	1.11	0.
time (sec)	N/A	0.108	0.138	0.085	0.	1.684	0.421	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	93	546	0	711	104	0
normalized size	1	1.	0.99	5.81	0.	7.56	1.11	0.
time (sec)	N/A	0.093	0.144	0.076	0.	1.801	0.424	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	143	154	356	0	0
normalized size	1	1.	0.92	1.49	1.6	3.71	0.	0.
time (sec)	N/A	0.268	0.176	0.059	1.005	1.541	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	0	855	0	1176	0	0
normalized size	1	1.	0.	2.53	0.	3.48	0.	0.
time (sec)	N/A	0.686	5.06	0.078	0.	1.512	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	123	232	211	522	0	0
normalized size	1	1.	0.85	1.6	1.46	3.6	0.	0.
time (sec)	N/A	0.421	0.231	0.061	1.012	1.368	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	484	484	0	0	0	1623	0	0
normalized size	1	1.	0.	0.	0.	3.35	0.	0.
time (sec)	N/A	0.875	2.776	0.126	0.	1.493	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	993	0	755	139	0
normalized size	1	1.	0.99	9.64	0.	7.33	1.35	0.
time (sec)	N/A	0.157	0.158	0.134	0.	1.338	0.879	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	993	0	755	139	0
normalized size	1	1.	0.99	9.64	0.	7.33	1.35	0.
time (sec)	N/A	0.141	0.033	0.002	0.	1.429	0.894	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	19	7	11
normalized size	1	1.	1.	1.	1.22	2.11	0.78	1.22
time (sec)	N/A	0.011	0.008	0.004	0.961	1.246	0.071	1.229

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	59	14	38
normalized size	1	1.	1.	0.95	1.2	2.95	0.7	1.9
time (sec)	N/A	0.134	0.025	0.009	0.988	1.383	0.089	1.422

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	32	41	103	0	0
normalized size	1	1.	0.97	0.94	1.21	3.03	0.	0.
time (sec)	N/A	0.25	0.047	0.02	0.981	1.318	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	30	51	19	0
normalized size	1	1.	1.	1.25	1.5	2.55	0.95	0.
time (sec)	N/A	0.02	0.017	0.011	0.954	1.252	0.102	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	64	77	147	42	0
normalized size	1	1.	0.88	1.28	1.54	2.94	0.84	0.
time (sec)	N/A	0.288	0.065	0.014	0.99	1.291	0.137	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	63	134	109	274	0	0
normalized size	1	1.	0.84	1.79	1.45	3.65	0.	0.
time (sec)	N/A	0.491	0.112	0.06	1.033	1.349	0.	0.



Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	19	35	12	18
normalized size	1	1.	1.	1.08	1.46	2.69	0.92	1.38
time (sec)	N/A	0.012	0.01	0.003	0.987	1.228	0.088	1.199

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	26	53	17	24
normalized size	1	1.	0.7	0.78	1.13	2.3	0.74	1.04
time (sec)	N/A	0.019	0.011	0.006	0.988	1.177	0.112	1.226

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	0	298	36	47
normalized size	1	1.	1.	1.	0.	8.28	1.	1.31
time (sec)	N/A	0.057	0.024	0.006	0.	1.261	0.23	1.144

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	123	181	0	504	0	0
normalized size	1	1.	0.77	1.14	0.	3.17	0.	0.
time (sec)	N/A	0.304	0.067	0.008	0.	1.362	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	185	0	0	759	0	0
normalized size	1	1.	0.76	0.	0.	3.11	0.	0.
time (sec)	N/A	0.499	0.037	0.017	0.	1.385	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	135	0	424	66	0
normalized size	1	1.	1.	2.87	0.	9.02	1.4	0.
time (sec)	N/A	0.064	0.054	0.029	0.	1.376	0.328	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	0	433	0	845	0	0
normalized size	1	1.	0.	2.13	0.	4.16	0.	0.
time (sec)	N/A	0.407	0.45	0.067	0.	1.396	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	1164	0	0
normalized size	1	1.	0.	0.	0.	3.75	0.	0.
time (sec)	N/A	0.66	0.192	0.115	0.	1.413	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	52	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.663	0.092	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	1.51	0.076	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	1.388	0.071	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.508	0.086	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	37	49	58	128	0
normalized size	1	1.	0.72	1.03	1.36	1.61	3.56	0.
time (sec)	N/A	0.015	0.01	0.007	0.975	1.615	0.307	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	52	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.378	0.076	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	52	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	1.272	0.072	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.719	0.04	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.327	1.319	0.023	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.303	1.268	0.02	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.454	0.024	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	42	55	20	51
normalized size	1	1.	1.	2.29	3.	3.93	1.43	3.64
time (sec)	N/A	0.008	0.003	0.006	0.975	1.201	0.133	1.307

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.47	0.021	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	1.407	0.021	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	0	0	62	0	0
normalized size	1	1.	1.	0.	0.	0.81	0.	0.
time (sec)	N/A	0.241	0.317	0.064	0.	2.878	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	62	0	0
normalized size	1	1.	1.	0.	0.	2.14	0.	0.
time (sec)	N/A	0.106	0.253	0.02	0.	2.497	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	62	0	0
normalized size	1	1.	1.	0.	0.	2.14	0.	0.
time (sec)	N/A	0.108	0.246	0.018	0.	2.43	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	59	0	0
normalized size	1	1.	1.	0.	0.	2.11	0.	0.
time (sec)	N/A	0.096	0.23	0.019	0.	2.452	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.243	0.019	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.245	0.02	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	35	69	90	197	279	3617
normalized size	1	1.	0.71	1.41	1.84	4.02	5.69	73.82
time (sec)	N/A	0.064	0.029	0.007	0.977	1.284	2.36	1.431

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	26	25	50	101	97	1377
normalized size	1	1.	0.84	0.81	1.61	3.26	3.13	44.42
time (sec)	N/A	0.029	0.012	0.005	0.992	1.357	1.178	1.266

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	36	24	327
normalized size	1	1.	1.	1.07	0.	2.57	1.71	23.36
time (sec)	N/A	0.012	0.006	0.003	0.	1.248	0.565	1.255

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	10	56	11	34	0	0
normalized size	1	1.	1.25	7.	1.38	4.25	0.	0.
time (sec)	N/A	0.042	0.011	0.034	1.093	1.226	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	A	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	0	160	22	84	0	0
normalized size	1	1.	0.	6.15	0.85	3.23	0.	0.
time (sec)	N/A	0.062	0.049	0.036	1.111	1.272	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	A	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	0	225	26	167	0	0
normalized size	1	1.	0.	4.41	0.51	3.27	0.	0.
time (sec)	N/A	0.085	0.057	0.044	1.097	1.332	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	20	0	54	41	429
normalized size	1	1.	1.11	1.05	0.	2.84	2.16	22.58
time (sec)	N/A	0.041	0.016	0.005	0.	1.288	2.651	1.48

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	39	0	312
normalized size	1	1.	1.	1.06	0.	2.17	0.	17.33
time (sec)	N/A	0.021	0.01	0.003	0.	1.284	0.	1.496

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	73	97	200	333	2510
normalized size	1	1.	0.7	1.2	1.59	3.28	5.46	41.15
time (sec)	N/A	0.07	0.028	0.007	1.033	1.312	1.66	1.316

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	770	770	2441	0	0	4178	0	0
normalized size	1	1.	3.17	0.	0.	5.43	0.	0.
time (sec)	N/A	1.374	4.479	0.331	0.	1.766	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	599	599	1412	0	0	2732	0	0
normalized size	1	1.	2.36	0.	0.	4.56	0.	0.
time (sec)	N/A	1.002	2.523	0.208	0.	1.591	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	428	677	1261	0	1513	0	0
normalized size	1	1.	1.58	2.95	0.	3.54	0.	0.
time (sec)	N/A	0.583	1.762	0.032	0.	1.348	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	183	0	686	116	161
normalized size	1	1.	0.99	1.93	0.	7.22	1.22	1.69
time (sec)	N/A	0.151	0.167	0.003	0.	1.441	0.711	1.292

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.018	0.412	0.156	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.87	5.761	0.197	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	398	285	0	593	0	0
normalized size	1	1.	2.65	1.9	0.	3.95	0.	0.
time (sec)	N/A	0.672	0.413	0.037	0.	1.314	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	85	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.141	0.043	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	85	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.108	0.042	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	83	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.087	0.069	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.102	0.086	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	81	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.118	0.044	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	85	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.118	0.044	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	94	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.159	0.072	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	151	0	76
normalized size	1	1.	0.	0.	0.	1.99	0.	1.
time (sec)	N/A	0.157	0.085	0.079	0.	1.022	0.	1.328

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	136	0	0	0	382	0	0
normalized size	1	0.99	0.	0.	0.	2.79	0.	0.
time (sec)	N/A	0.39	0.157	80.773	0.	1.027	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	316	0	0
normalized size	1	1.	1.	0.	0.	2.57	0.	0.
time (sec)	N/A	0.249	0.236	0.302	0.	1.066	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	293	0	0
normalized size	1	1.	1.	0.	0.	2.55	0.	0.
time (sec)	N/A	0.207	0.491	0.307	0.	0.995	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	309	532	136
normalized size	1	1.	1.	0.	0.	2.62	4.51	1.15
time (sec)	N/A	0.127	0.095	0.125	0.	1.021	107.124	1.291



Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	146	0	0
normalized size	1	1.	1.	0.	0.	2.18	0.	0.
time (sec)	N/A	0.144	0.037	180.	0.	0.981	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	0	0	315	0	0
normalized size	1	1.	1.	0.	0.	2.6	0.	0.
time (sec)	N/A	0.234	0.202	0.533	0.	1.034	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	117	0	0	296	0	0
normalized size	1	1.	0.99	0.	0.	2.51	0.	0.
time (sec)	N/A	0.231	0.207	0.614	0.	1.06	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.775	0.861	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	502	0	396	0	0	1319	0	0
normalized size	1	0.	0.79	0.	0.	2.63	0.	0.
time (sec)	N/A	0.376	1.551	0.552	0.	1.093	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	372	0	303	0	0	961	0	0
normalized size	1	0.	0.81	0.	0.	2.58	0.	0.
time (sec)	N/A	0.306	0.703	0.458	0.	1.038	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	242	0	204	0	0	610	0	0
normalized size	1	0.	0.84	0.	0.	2.52	0.	0.
time (sec)	N/A	0.23	0.374	0.372	0.	1.06	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	309	532	136
normalized size	1	1.	1.	0.	0.	2.62	4.51	1.15
time (sec)	N/A	0.094	0.023	0.013	0.	0.986	105.909	1.361

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.517	0.386	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	2.484	0.468	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	3.302	0.657	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	152	0	0	0	443	0	0
normalized size	1	0.99	0.	0.	0.	2.9	0.	0.
time (sec)	N/A	0.757	0.238	180.	0.	1.029	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	0	0	355	0	0
normalized size	1	1.	0.97	0.	0.	2.67	0.	0.
time (sec)	N/A	0.411	0.343	0.441	0.	1.003	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	120	0	0	333	0	0
normalized size	1	1.	0.98	0.	0.	2.73	0.	0.
time (sec)	N/A	0.369	0.704	0.353	0.	1.076	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	123	0	0	350	0	157
normalized size	1	1.	0.98	0.	0.	2.78	0.	1.25
time (sec)	N/A	0.232	0.092	0.194	0.	1.009	0.	1.346

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	0	0	159	0	0
normalized size	1	1.	0.84	0.	0.	2.27	0.	0.
time (sec)	N/A	0.268	0.046	180.	0.	1.002	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	354	0	0
normalized size	1	1.	0.98	0.	0.	2.77	0.	0.
time (sec)	N/A	0.412	0.257	0.521	0.	1.014	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	121	0	0	335	0	0
normalized size	1	1.	0.96	0.	0.	2.66	0.	0.
time (sec)	N/A	0.405	0.245	0.665	0.	1.091	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	1.402	0.888	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	535	0	434	0	0	1469	0	0
normalized size	1	0.	0.81	0.	0.	2.75	0.	0.
time (sec)	N/A	0.493	2.38	0.517	0.	1.099	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	397	0	331	0	0	1076	0	0
normalized size	1	0.	0.83	0.	0.	2.71	0.	0.
time (sec)	N/A	0.367	1.047	0.483	0.	1.121	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	257	0	221	0	0	689	0	0
normalized size	1	0.	0.86	0.	0.	2.68	0.	0.
time (sec)	N/A	0.306	0.506	0.373	0.	1.356	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	123	0	0	350	0	157
normalized size	1	1.	0.98	0.	0.	2.78	0.	1.25
time (sec)	N/A	0.143	0.06	0.037	0.	1.318	0.	1.305

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.959	0.553	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	4.509	0.579	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	6.77	0.786	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	23	38	24	23
normalized size	1	1.	1.	1.06	1.35	2.24	1.41	1.35
time (sec)	N/A	0.052	0.052	0.043	0.96	0.924	0.124	1.339

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	27	45	32	27
normalized size	1	1.	0.95	1.05	1.35	2.25	1.6	1.35
time (sec)	N/A	0.166	0.384	0.042	0.978	0.779	0.679	1.301

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	0	0	57	0	0
normalized size	1	1.	0.9	0.	0.	1.16	0.	0.
time (sec)	N/A	0.2	0.052	1.413	0.	0.732	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	49	145	3214	261	160	360
normalized size	1	1.	0.54	1.61	35.71	2.9	1.78	4.
time (sec)	N/A	0.182	0.037	0.039	2.137	0.832	0.236	1.264

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	64	1651	136	68	161
normalized size	1	1.	0.56	1.	25.8	2.12	1.06	2.52
time (sec)	N/A	0.161	0.031	0.039	1.683	0.809	0.181	1.312

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	23	24	676	58	22	59
normalized size	1	1.	0.61	0.63	17.79	1.53	0.58	1.55
time (sec)	N/A	0.096	0.029	0.037	1.37	0.825	0.135	1.193

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	28	10	15
normalized size	1	1.	1.	1.	1.25	2.33	0.83	1.25
time (sec)	N/A	0.019	0.037	0.036	0.974	0.706	0.101	1.257

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	19	0	28	10	0
normalized size	1	1.	0.91	1.73	0.	2.55	0.91	0.
time (sec)	N/A	0.177	0.024	0.036	0.	0.806	34.115	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	45	0	109	0	0
normalized size	1	1.	0.92	1.18	0.	2.87	0.	0.
time (sec)	N/A	0.197	0.05	0.037	0.	0.809	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	70	0	252	0	0
normalized size	1	1.	0.69	0.97	0.	3.5	0.	0.
time (sec)	N/A	0.241	0.069	0.041	0.	0.906	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	47	119	0	0	0	0
normalized size	1	1.	0.33	0.84	0.	0.	0.	0.
time (sec)	N/A	0.633	0.196	0.042	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	46	94	0	0	0	0
normalized size	1	1.	0.41	0.84	0.	0.	0.	0.
time (sec)	N/A	0.457	0.124	0.041	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	47	69	0	0	0	0
normalized size	1	1.	0.57	0.84	0.	0.	0.	0.
time (sec)	N/A	0.363	0.11	0.041	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	44	0	0	78	63
normalized size	1	1.	0.88	0.85	0.	0.	1.5	1.21
time (sec)	N/A	0.235	0.048	0.04	0.	0.	12.855	1.255

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	46	18	0	0	49	0
normalized size	1	1.	2.19	0.86	0.	0.	2.33	0.
time (sec)	N/A	0.264	0.059	0.043	0.	0.	5.721	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	62	45	0	0	80	0
normalized size	1	1.	1.22	0.88	0.	0.	1.57	0.
time (sec)	N/A	0.315	0.095	0.039	0.	0.	9.462	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	70	0	0	0	0
normalized size	1	1.	0.91	0.82	0.	0.	0.	0.
time (sec)	N/A	0.348	0.127	0.041	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	91	95	0	0	0	0
normalized size	1	1.	0.79	0.83	0.	0.	0.	0.
time (sec)	N/A	0.355	0.151	0.051	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	103	120	0	0	0	0
normalized size	1	1.	0.71	0.83	0.	0.	0.	0.
time (sec)	N/A	0.379	0.194	0.041	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	42	37	19	55	7	19
normalized size	1	1.	5.25	4.62	2.38	6.88	0.88	2.38
time (sec)	N/A	0.028	0.017	0.073	1.458	0.756	0.895	1.327

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	8	9	28	15	9
normalized size	1	1.	1.	0.67	0.75	2.33	1.25	0.75
time (sec)	N/A	0.019	0.003	0.02	1.46	0.811	0.107	1.237

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	20	50	15	22
normalized size	1	1.	1.	1.	5.	12.5	3.75	5.5
time (sec)	N/A	0.02	0.003	0.018	0.962	0.736	0.1	1.275

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	35	90	15	41
normalized size	1	1.	1.	0.7	1.75	4.5	0.75	2.05
time (sec)	N/A	0.022	0.006	0.019	1.46	0.709	0.116	1.314

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	26	34	103	42	34
normalized size	1	1.	1.	0.72	0.94	2.86	1.17	0.94
time (sec)	N/A	0.029	0.015	0.059	1.461	0.825	1.483	1.214

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	15	31	10	15
normalized size	1	1.	0.64	0.55	0.68	1.41	0.45	0.68
time (sec)	N/A	0.021	0.002	0.019	0.973	0.721	0.084	1.142

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	21	27	95	24	27
normalized size	1	1.	0.9	0.72	0.93	3.28	0.83	0.93
time (sec)	N/A	0.027	0.012	0.054	1.46	0.892	1.277	1.216

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	61	0	28
normalized size	1	1.	1.	0.79	1.14	4.36	0.	2.
time (sec)	N/A	0.036	0.01	0.062	1.443	0.882	0.	1.284

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	20	51	15	22
normalized size	1	1.	1.	1.33	1.67	4.25	1.25	1.83
time (sec)	N/A	0.02	0.003	0.027	0.962	0.741	0.102	1.249

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	14	26	8	14
normalized size	1	1.	1.	0.85	1.08	2.	0.62	1.08
time (sec)	N/A	0.009	0.002	0.018	0.963	0.763	0.084	1.268

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	0	43	10	20
normalized size	1	1.	1.	1.	0.	2.69	0.62	1.25
time (sec)	N/A	0.005	0.005	0.036	0.	0.862	0.08	1.165



Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	18	42	14	18
normalized size	1	1.	1.	1.	1.	2.33	0.78	1.
time (sec)	N/A	0.026	0.009	0.025	0.958	0.947	0.087	1.201

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	59	3	4
normalized size	1	1.	1.	1.	1.	14.75	0.75	1.
time (sec)	N/A	0.023	0.004	0.058	1.45	0.77	0.672	1.365

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	28	17	9
normalized size	1	1.	1.	0.8	0.9	2.8	1.7	0.9
time (sec)	N/A	0.02	0.003	0.02	1.497	0.751	0.109	1.177

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	23	59	17	24
normalized size	1	1.	0.85	0.74	0.85	2.19	0.63	0.89
time (sec)	N/A	0.019	0.018	0.027	0.975	0.774	0.1	1.248

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	30	10	14
normalized size	1	1.	1.	0.92	1.17	2.5	0.83	1.17
time (sec)	N/A	0.035	0.01	0.02	1.006	0.771	0.093	1.286

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	20	51	15	22
normalized size	1	1.	1.	1.	3.33	8.5	2.5	3.67
time (sec)	N/A	0.019	0.002	0.018	0.978	0.909	0.102	1.233

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	18	15	4
normalized size	1	1.	1.	1.	1.	4.5	3.75	1.
time (sec)	N/A	0.019	0.003	0.02	1.882	0.87	0.105	1.253

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	14	17	14	31	8	16
normalized size	1	1.17	1.17	1.42	1.17	2.58	0.67	1.33
time (sec)	N/A	0.04	0.008	0.027	0.964	0.962	0.092	1.264

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	12	12	14	11	31	8	15
normalized size	1	1.2	1.2	1.4	1.1	3.1	0.8	1.5
time (sec)	N/A	0.037	0.008	0.026	0.979	0.8	0.09	1.331

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	30	19	36	10	19
normalized size	1	1.	1.	1.67	1.06	2.	0.56	1.06
time (sec)	N/A	0.044	0.008	0.03	0.982	0.852	0.095	1.207

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	42	3	22
normalized size	1	1.	1.	1.	1.	10.5	0.75	5.5
time (sec)	N/A	0.021	0.004	0.065	1.457	0.771	0.62	1.226

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	26	8	11
normalized size	1	1.	1.	0.82	1.	2.36	0.73	1.
time (sec)	N/A	0.024	0.003	0.024	0.968	0.679	0.193	1.259

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	18	45	0	18
normalized size	1	1.	1.	0.78	1.	2.5	0.	1.
time (sec)	N/A	0.061	0.016	0.06	1.459	0.789	0.	1.236

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	21	27	84	0	39
normalized size	1	1.	0.97	0.68	0.87	2.71	0.	1.26
time (sec)	N/A	0.025	0.01	0.059	1.46	0.778	0.	1.247

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	19	24	84	0	39
normalized size	1	1.	0.89	0.7	0.89	3.11	0.	1.44
time (sec)	N/A	0.023	0.009	0.058	1.488	0.658	0.	1.226

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	28	17	9
normalized size	1	1.	1.	0.8	0.9	2.8	1.7	0.9
time (sec)	N/A	0.126	0.016	0.021	1.495	0.815	0.129	1.259

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	13	17	15	42	24	15
normalized size	1	1.	0.46	0.61	0.54	1.5	0.86	0.54
time (sec)	N/A	0.042	0.001	0.016	0.969	0.695	4.295	1.244

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	22	42	10	22
normalized size	1	1.	1.	0.81	1.38	2.62	0.62	1.38
time (sec)	N/A	0.024	0.004	0.06	0.966	0.738	0.802	1.224

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	20	50	15	22
normalized size	1	1.	1.	1.33	1.67	4.17	1.25	1.83
time (sec)	N/A	0.022	0.003	0.023	0.967	0.788	0.106	1.217

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	28	17	9
normalized size	1	1.	1.	0.8	0.9	2.8	1.7	0.9
time (sec)	N/A	0.021	0.003	0.02	1.456	0.857	0.113	1.739

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	0	12	54	8	24
normalized size	1	1.	1.	0.	0.86	3.86	0.57	1.71
time (sec)	N/A	0.023	0.004	0.122	1.457	0.597	0.946	1.232

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	53	39	77	32	34
normalized size	1	1.	0.8	1.51	1.11	2.2	0.91	0.97
time (sec)	N/A	0.179	0.053	0.052	0.969	0.868	2.243	1.239

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	23	7	11
normalized size	1	1.	1.	0.82	1.	2.09	0.64	1.
time (sec)	N/A	0.009	0.002	0.019	0.968	0.866	0.085	1.268

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	23	7	11
normalized size	1	1.	1.	0.82	1.	2.09	0.64	1.
time (sec)	N/A	0.016	0.002	0.019	0.956	0.862	0.084	1.254

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	7	8
normalized size	1	1.	1.	0.78	0.89	2.	0.78	0.89
time (sec)	N/A	0.01	0.002	0.023	0.962	0.82	0.192	1.185

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	20	7	8
normalized size	1	1.	1.	0.78	0.89	2.22	0.78	0.89
time (sec)	N/A	0.011	0.002	0.02	0.959	0.804	0.372	1.27

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	32	80	92	31	42
normalized size	1	1.	0.5	0.47	1.18	1.35	0.46	0.62
time (sec)	N/A	0.107	0.021	0.024	0.984	0.853	0.101	1.254

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	26	53	20	26
normalized size	1	1.	0.93	0.79	0.93	1.89	0.71	0.93
time (sec)	N/A	0.024	0.015	0.019	0.981	0.863	0.096	1.235

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	20	26	53	22	24
normalized size	1	1.	0.88	0.77	1.	2.04	0.85	0.92
time (sec)	N/A	0.022	0.011	0.021	0.976	0.76	0.105	1.213

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	19	7	11
normalized size	1	1.	1.	1.	1.22	2.11	0.78	1.22
time (sec)	N/A	0.024	0.007	0.023	0.971	0.806	0.079	1.243

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	23	62	20	23
normalized size	1	1.	0.74	0.81	0.85	2.3	0.74	0.85
time (sec)	N/A	0.01	0.027	0.03	0.963	0.824	0.484	1.24

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	42	14	20
normalized size	1	1.	1.	0.94	1.18	2.47	0.82	1.18
time (sec)	N/A	0.032	0.013	0.026	0.98	0.809	0.121	1.265

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.022	0.008	0.02	0.978	0.872	0.086	1.225

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	63	26	26
normalized size	1	1.	0.81	0.81	0.96	2.33	0.96	0.96
time (sec)	N/A	0.011	0.027	0.026	0.985	0.921	0.308	1.239

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	5	5	82	0	8
normalized size	1	1.	1.	1.	1.	16.4	0.	1.6
time (sec)	N/A	0.014	0.006	0.02	0.97	0.908	0.	1.207

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	22	62	17	26
normalized size	1	1.	1.	1.05	1.05	2.95	0.81	1.24
time (sec)	N/A	0.025	0.015	0.025	0.976	0.958	0.097	1.197

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	63	24	26
normalized size	1	1.	0.81	0.81	0.96	2.33	0.89	0.96
time (sec)	N/A	0.01	0.042	0.023	0.972	0.679	0.311	1.225

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	28	53	157	0	55
normalized size	1	1.	1.	0.82	1.56	4.62	0.	1.62
time (sec)	N/A	0.03	0.016	0.305	0.965	0.736	0.	1.206

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	20	23	22	46	14	22
normalized size	1	1.	0.77	0.88	0.85	1.77	0.54	0.85
time (sec)	N/A	0.017	0.021	0.021	0.966	0.848	0.096	1.218

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	10	14	18	41	12	18
normalized size	1	1.	0.67	0.93	1.2	2.73	0.8	1.2
time (sec)	N/A	0.029	0.007	0.026	0.973	0.896	0.111	1.211

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	23	46	22	23
normalized size	1	1.	0.74	0.67	0.85	1.7	0.81	0.85
time (sec)	N/A	0.026	0.01	0.02	0.992	0.781	1.896	1.195

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	23	46	22	23
normalized size	1	1.	0.74	0.67	0.85	1.7	0.81	0.85
time (sec)	N/A	0.026	0.01	0.021	0.96	0.772	2.628	1.235

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	50	65	173	0	66
normalized size	1	1.	0.87	0.81	1.05	2.79	0.	1.06
time (sec)	N/A	0.052	0.041	0.068	1.46	0.82	0.	1.208

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	26	28	10	15
normalized size	1	1.	0.63	0.63	1.37	1.47	0.53	0.79
time (sec)	N/A	0.043	0.027	0.022	0.981	0.721	0.087	1.263

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	17	38	45	15	22
normalized size	1	1.	0.59	0.53	1.19	1.41	0.47	0.69
time (sec)	N/A	0.051	0.028	0.022	0.976	0.808	0.096	1.216

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	11	0	11	22	14	26	8	0
normalized size	1	0.	1.	2.	1.27	2.36	0.73	0.
time (sec)	N/A	0.145	0.043	0.04	1.142	0.826	0.516	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.026	0.001	0.023	0.99	0.823	0.116	1.213

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	9	0	9	18	12	30	12	22
normalized size	1	0.	1.	2.	1.33	3.33	1.33	2.44
time (sec)	N/A	0.073	0.02	0.032	1.168	0.815	0.338	1.298

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	28	50	22	28
normalized size	1	1.	1.	0.96	1.12	2.	0.88	1.12
time (sec)	N/A	0.014	0.009	0.037	0.968	0.832	0.118	1.3

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	45	80	37	45
normalized size	1	1.	1.	0.9	1.12	2.	0.92	1.12
time (sec)	N/A	0.02	0.012	0.039	0.955	0.769	0.141	1.237

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	48	61	107	51	61
normalized size	1	1.	1.	0.91	1.15	2.02	0.96	1.15
time (sec)	N/A	0.026	0.017	0.036	0.978	0.865	0.162	1.559

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	0	208	0	39
normalized size	1	1.	1.	0.81	0.	6.5	0.	1.22
time (sec)	N/A	0.027	0.013	0.189	0.	0.909	0.	1.235

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	28	0	207	0	36
normalized size	1	1.	1.	0.82	0.	6.09	0.	1.06
time (sec)	N/A	0.029	0.013	0.191	0.	0.816	0.	1.318

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	0	278	0	59
normalized size	1	1.	0.96	0.79	0.	5.25	0.	1.11
time (sec)	N/A	0.035	0.017	0.182	0.	0.791	0.	1.304

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	48	0	277	0	61
normalized size	1	1.	0.96	0.84	0.	4.86	0.	1.07
time (sec)	N/A	0.036	0.017	0.155	0.	0.786	0.	1.257

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	23	65	24	23
normalized size	1	1.	0.74	0.81	0.85	2.41	0.89	0.85
time (sec)	N/A	0.011	0.023	0.022	0.957	0.917	0.319	1.261



Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	27	19	11
normalized size	1	1.	1.	0.9	1.1	2.7	1.9	1.1
time (sec)	N/A	0.023	0.006	0.021	1.449	0.857	0.121	1.196

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	18	23	58	19	24
normalized size	1	1.	1.	1.8	2.3	5.8	1.9	2.4
time (sec)	N/A	0.024	0.006	0.022	0.959	0.871	0.116	1.222

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	28	0	28
normalized size	1	1.	1.	0.83	1.06	1.56	0.	1.56
time (sec)	N/A	0.026	0.012	0.06	1.	0.92	0.	1.312

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	18	35	127	17	45
normalized size	1	1.	0.95	0.9	1.75	6.35	0.85	2.25
time (sec)	N/A	0.043	0.012	0.021	1.459	0.917	0.133	1.273

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	22	28	63	20	28
normalized size	1	1.	0.55	0.5	0.64	1.43	0.45	0.64
time (sec)	N/A	0.035	0.007	0.021	0.964	0.808	0.089	1.408

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	26	24	41	61	24	31
normalized size	1	1.	0.5	0.46	0.79	1.17	0.46	0.6
time (sec)	N/A	0.039	0.01	0.02	0.962	0.83	0.094	1.275

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	23	30	97	29	30
normalized size	1	1.	0.97	0.7	0.91	2.94	0.88	0.91
time (sec)	N/A	0.026	0.012	0.057	1.439	0.714	1.353	1.287

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	33	46	50	95	41	72
normalized size	1	1.	0.66	0.92	1.	1.9	0.82	1.44
time (sec)	N/A	0.036	0.02	0.021	0.975	0.873	3.564	1.313

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	48	62	82	163	61	101
normalized size	1	1.	0.59	0.77	1.01	2.01	0.75	1.25
time (sec)	N/A	0.045	0.027	0.057	0.983	0.863	60.025	1.285

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	59	131	49	59
normalized size	1	1.	1.	0.85	1.07	2.38	0.89	1.07
time (sec)	N/A	0.086	0.029	0.021	0.974	0.926	0.142	1.266

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	27	35	113	0	35
normalized size	1	1.	0.91	0.77	1.	3.23	0.	1.
time (sec)	N/A	0.16	0.028	0.058	1.468	0.932	0.	1.306

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	24	31	63	24	31
normalized size	1	1.	0.91	0.75	0.97	1.97	0.75	0.97
time (sec)	N/A	0.209	0.024	0.021	0.959	0.87	0.144	1.254

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	4	12	3	4
normalized size	1	1.	1.	0.8	0.8	2.4	0.6	0.8
time (sec)	N/A	0.005	0.005	0.017	0.964	0.798	0.638	1.295

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	5	5	18	5	0
normalized size	1	1.	1.	0.71	0.71	2.57	0.71	0.
time (sec)	N/A	0.015	0.011	0.021	0.964	0.84	0.975	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	17	22	57	17	32
normalized size	1	1.	0.91	0.77	1.	2.59	0.77	1.45
time (sec)	N/A	0.018	0.012	0.02	0.968	0.737	0.1	1.204

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	9	18	15	4
normalized size	1	1.	1.	1.	2.25	4.5	3.75	1.
time (sec)	N/A	0.011	0.003	0.02	1.451	0.761	0.103	1.222

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	14	27	10	14
normalized size	1	1.	1.	0.85	1.08	2.08	0.77	1.08
time (sec)	N/A	0.012	0.009	0.022	0.966	0.798	0.075	1.316

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	26	51	15	22
normalized size	1	1.	1.	1.	4.33	8.5	2.5	3.67
time (sec)	N/A	0.01	0.002	0.02	0.968	0.795	0.101	1.27

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	11	14	27	10	14
normalized size	1	1.	0.73	0.73	0.93	1.8	0.67	0.93
time (sec)	N/A	0.014	0.011	0.018	0.961	0.889	0.075	1.299

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	28	51	15	23
normalized size	1	1.	1.	0.82	1.27	2.32	0.68	1.05
time (sec)	N/A	0.028	0.007	0.023	0.967	0.843	0.111	1.303

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	25	32	74	26	41
normalized size	1	1.	0.94	0.81	1.03	2.39	0.84	1.32
time (sec)	N/A	0.037	0.013	0.026	0.968	0.856	0.134	1.205

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	27	30	57	29	0
normalized size	1	1.	0.95	1.23	1.36	2.59	1.32	0.
time (sec)	N/A	0.027	0.012	0.027	1.453	0.651	0.29	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	40	54	63	42	0
normalized size	1	1.	0.68	1.18	1.59	1.85	1.24	0.
time (sec)	N/A	0.031	0.021	0.03	1.455	0.843	0.316	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	70	0	0
normalized size	1	1.	1.	0.	0.	3.04	0.	0.
time (sec)	N/A	0.048	0.075	0.137	0.	0.734	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	41	51	100	101	54	47
normalized size	1	1.	0.62	0.77	1.52	1.53	0.82	0.71
time (sec)	N/A	0.184	0.023	0.025	0.976	0.755	158.37	1.224

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	19	20	26	74	20	27
normalized size	1	1.	1.58	1.67	2.17	6.17	1.67	2.25
time (sec)	N/A	0.013	0.006	0.026	0.968	0.822	0.112	1.223

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	12	0	28	8	43
normalized size	1	1.	0.75	0.75	0.	1.75	0.5	2.69
time (sec)	N/A	0.074	0.033	0.019	0.	0.788	0.094	1.248

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	18	31	10	18
normalized size	1	1.	1.	0.88	1.12	1.94	0.62	1.12
time (sec)	N/A	0.058	0.035	0.02	0.975	0.883	0.092	1.305

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	23	100	103	0	45
normalized size	1	1.	1.	0.7	3.03	3.12	0.	1.36
time (sec)	N/A	0.032	0.016	0.065	1.017	0.887	0.	1.247

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	0	8	11
normalized size	1	1.	1.	0.82	1.	0.	0.73	1.
time (sec)	N/A	0.025	0.008	0.066	0.966	0.	0.148	1.287

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	19	5	7
normalized size	1	1.	1.	1.	1.17	3.17	0.83	1.17
time (sec)	N/A	0.02	0.03	0.017	0.98	0.899	0.095	1.225

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	26	46	14	0
normalized size	1	1.	1.	0.89	1.37	2.42	0.74	0.
time (sec)	N/A	0.015	0.004	0.022	1.057	0.801	0.745	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	49	31	12	27
normalized size	1	1.	0.84	0.84	2.58	1.63	0.63	1.42
time (sec)	N/A	0.1	0.014	0.019	1.059	0.854	0.088	1.245

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	41	60	85	0	94	1910
normalized size	1	1.	0.73	1.07	1.52	0.	1.68	34.11
time (sec)	N/A	0.041	0.039	0.043	1.01	0.	0.148	1.306

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	14	18	32	10	18
normalized size	1	1.	0.8	0.93	1.2	2.13	0.67	1.2
time (sec)	N/A	0.009	0.002	0.024	0.965	0.761	0.094	1.191

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	16	27	10	15
normalized size	1	1.	1.	0.86	1.14	1.93	0.71	1.07
time (sec)	N/A	0.011	0.003	0.018	0.963	0.648	0.139	1.23

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	23	8	15
normalized size	1	1.	1.	1.09	1.36	2.09	0.73	1.36
time (sec)	N/A	0.011	0.002	0.022	0.974	0.722	0.098	1.296

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	38	17	27
normalized size	1	1.	1.	1.05	1.35	1.9	0.85	1.35
time (sec)	N/A	0.006	0.003	0.018	0.98	0.795	0.113	1.209

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	19	46	49	15	22
normalized size	1	1.	0.59	0.59	1.44	1.53	0.47	0.69
time (sec)	N/A	0.044	0.019	0.019	1.008	0.701	0.092	1.264

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	111	36	36
normalized size	1	1.	1.	0.85	1.09	3.36	1.09	1.09
time (sec)	N/A	0.009	0.016	0.037	0.977	0.792	0.102	1.274

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	12	15	30	10	15
normalized size	1	1.	1.	0.57	0.71	1.43	0.48	0.71
time (sec)	N/A	0.011	0.003	0.019	0.949	0.883	0.139	1.215

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	0	0	0
normalized size	1	1.	1.	0.82	1.	0.	0.	0.
time (sec)	N/A	0.041	0.005	0.042	1.094	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	22	0	0	0
normalized size	1	1.	1.	0.83	1.83	0.	0.	0.
time (sec)	N/A	0.259	0.092	0.048	1.138	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	12	0	0	0
normalized size	1	1.	1.	0.83	1.	0.	0.	0.
time (sec)	N/A	0.13	0.045	0.039	1.083	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.122	0.033	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.07	0.032	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.097	0.029	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	31	0	0	0
normalized size	1	1.	1.	0.8	1.55	0.	0.	0.
time (sec)	N/A	0.597	0.161	0.085	1.135	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.188	0.041	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	11	0	12	11
normalized size	1	1.	1.	0.69	0.85	0.	0.92	0.85
time (sec)	N/A	0.024	0.009	0.028	0.947	0.	0.221	1.238

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	11	0	0	0
normalized size	1	1.	1.	0.69	0.85	0.	0.	0.
time (sec)	N/A	0.066	0.005	0.042	1.1	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.048	0.017	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	22	0	0	0
normalized size	1	1.	1.	0.83	1.83	0.	0.	0.
time (sec)	N/A	0.337	0.092	0.029	1.084	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	22	0	0	0
normalized size	1	1.	1.	0.83	1.83	0.	0.	0.
time (sec)	N/A	0.144	0.047	0.043	1.105	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	32	74	22	38
normalized size	1	1.	1.	0.74	1.03	2.39	0.71	1.23
time (sec)	N/A	0.049	0.008	0.02	0.977	0.674	0.117	1.257

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.354	0.024	0.	0.	0.	0.



Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.048	0.028	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.018	0.029	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.033	0.028	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.038	0.02	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.044	0.021	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.017	0.02	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.019	0.022	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	88	124	143	63
normalized size	1	1.	1.	1.03	2.44	3.44	3.97	1.75
time (sec)	N/A	0.099	0.046	0.038	1.155	0.914	65.84	1.681

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	241	49	0	0	0
normalized size	1	1.	1.	6.51	1.32	0.	0.	0.
time (sec)	N/A	0.035	0.016	0.24	1.09	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	68	0	0	0
normalized size	1	1.	1.	0.	1.36	0.	0.	0.
time (sec)	N/A	0.044	0.022	0.352	1.245	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	51	0	105	0
normalized size	1	1.	1.	0.	1.38	0.	2.84	0.
time (sec)	N/A	0.017	0.008	0.165	1.049	0.	1.875	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	57	0	0	0
normalized size	1	1.	1.	0.	1.39	0.	0.	0.
time (sec)	N/A	0.016	0.008	0.181	1.119	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.014	0.294	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.014	0.34	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	203	0	0
normalized size	1	1.	0.92	0.	0.	2.54	0.	0.
time (sec)	N/A	0.044	0.038	0.051	0.	0.754	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	28	0	0	0
normalized size	1	1.	1.	0.	1.65	0.	0.	0.
time (sec)	N/A	0.635	0.22	0.026	1.092	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [64] had the largest ratio of [ 0.7368 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.	13	0.154
2	A	2	2	1.	13	0.154
3	A	3	3	1.	19	0.158
4	A	2	2	1.	21	0.095
5	A	2	2	1.	13	0.154
6	A	3	3	1.	19	0.158
7	A	2	2	1.	21	0.095
8	A	2	2	1.	13	0.154
9	A	3	3	1.	19	0.158
10	A	2	2	1.	21	0.095
11	A	2	2	1.	13	0.154
12	A	3	3	1.	19	0.158
13	A	2	2	1.	21	0.095
14	A	2	2	1.	17	0.118
15	A	3	3	1.	17	0.176
16	A	2	2	1.	29	0.069
17	A	3	3	1.	44	0.068
18	A	3	2	1.	15	0.133
19	A	3	2	1.	15	0.133
20	A	2	2	1.	15	0.133

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	2	1.	15	0.133
22	A	3	2	1.	17	0.118
23	A	3	2	1.	17	0.118
24	A	2	2	1.	17	0.118
25	A	3	2	1.	17	0.118
26	A	3	2	1.	19	0.105
27	A	3	2	1.	16	0.125
28	A	3	2	1.	18	0.111
29	A	3	2	1.	18	0.111
30	A	3	2	1.	18	0.111
31	A	3	2	1.	18	0.111
32	A	3	2	1.	18	0.111
33	A	2	2	1.	22	0.091
34	A	3	3	1.	23	0.13
35	A	3	3	1.	23	0.13
36	A	3	2	1.	23	0.087
37	A	4	3	1.	23	0.13
38	A	2	2	1.	13	0.154
39	A	2	2	1.	15	0.133
40	A	3	5	1.	16	0.312
41	A	8	7	1.	18	0.389
42	A	10	8	1.	18	0.444
43	A	2	2	1.	15	0.133
44	A	6	7	1.	16	0.438
45	A	9	8	1.	18	0.444
46	A	11	9	1.	18	0.5
47	A	3	3	1.	15	0.2
48	A	8	7	1.	16	0.438
49	A	16	12	1.	18	0.667
50	A	21	11	1.	18	0.611
51	A	4	3	1.	15	0.2
52	A	11	7	1.	16	0.438
53	A	24	12	1.	18	0.667
54	A	2	2	1.	15	0.133
55	A	6	6	1.	17	0.353
56	A	9	7	1.	19	0.368
57	A	11	8	1.	19	0.421
58	A	2	2	1.	15	0.133
59	A	6	6	1.	17	0.353
60	A	6	6	1.	19	0.316
61	A	7	7	1.	19	0.368
62	A	4	4	1.	15	0.267
63	A	22	9	1.	17	0.529
64	A	43	14	1.	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	3	3	1.	25	0.12
66	A	2	2	1.	25	0.08
67	A	2	2	1.	34	0.059
68	A	2	2	1.	34	0.059
69	A	1	1	1.	13	0.077
70	C	1	1	0.31	13	0.077
71	C	1	1	0.37	13	0.077
72	A	4	2	1.	13	0.154
73	A	3	2	1.	13	0.154
74	A	2	2	1.	13	0.154
75	A	1	1	1.	11	0.091
76	A	1	1	1.	13	0.077
77	A	2	2	1.	13	0.154
78	A	3	2	1.	13	0.154
79	A	4	2	1.	13	0.154
80	A	1	1	1.	13	0.077
81	A	1	1	1.	13	0.077
82	A	1	1	1.	13	0.077
83	A	1	1	1.	13	0.077
84	A	5	2	1.	13	0.154
85	A	4	2	1.	13	0.154
86	A	3	2	1.	13	0.154
87	A	2	2	1.	13	0.154
88	A	1	1	1.	9	0.111
89	A	2	2	1.	13	0.154
90	A	3	2	1.	13	0.154
91	A	4	2	1.	13	0.154
92	A	5	2	1.	13	0.154
93	A	1	1	1.	13	0.077
94	A	1	1	1.	13	0.077
95	A	1	1	1.	13	0.077
96	C	1	1	0.31	13	0.077
97	C	1	1	0.37	13	0.077
98	A	4	2	1.	13	0.154
99	A	3	2	1.	13	0.154
100	A	2	2	1.	13	0.154
101	A	1	1	1.	13	0.077
102	A	1	1	1.	13	0.077
103	A	2	2	1.	13	0.154
104	A	3	2	1.	13	0.154
105	A	4	2	1.	13	0.154
106	A	1	1	1.	13	0.077
107	A	1	1	1.	13	0.077
108	A	1	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	1	1	1.	13	0.077
110	A	1	1	1.	11	0.091
111	A	1	1	1.	9	0.111
112	A	1	1	1.	13	0.077
113	A	1	1	1.	13	0.077
114	A	1	1	1.	11	0.091
115	A	1	1	1.	13	0.077
116	A	1	1	1.	13	0.077
117	A	1	1	1.	13	0.077
118	A	4	3	1.	13	0.231
119	A	3	3	1.	11	0.273
120	A	2	2	1.	9	0.222
121	A	1	1	1.	13	0.077
122	A	1	1	1.	13	0.077
123	A	2	2	1.	13	0.154
124	A	3	2	1.	13	0.154
125	A	4	2	1.	13	0.154
126	C	1	1	0.34	13	0.077
127	C	1	1	0.27	13	0.077
128	A	1	1	1.	13	0.077
129	A	1	1	1.	13	0.077
130	A	1	1	1.	13	0.077
131	A	4	2	1.	13	0.154
132	A	3	2	1.	13	0.154
133	A	2	2	1.	11	0.182
134	A	1	1	1.	13	0.077
135	A	1	1	1.	13	0.077
136	A	2	2	1.	13	0.154
137	A	3	2	1.	13	0.154
138	A	4	2	1.	13	0.154
139	C	1	1	0.35	13	0.077
140	C	1	1	0.29	13	0.077
141	A	1	1	1.	13	0.077
142	A	1	1	1.	13	0.077
143	A	6	4	1.	13	0.308
144	A	5	4	1.	13	0.308
145	A	4	4	1.	13	0.308
146	A	3	3	1.	9	0.333
147	A	2	2	1.	13	0.154
148	A	3	3	1.	13	0.231
149	A	4	3	1.	13	0.231
150	A	5	3	1.	13	0.231
151	A	6	3	1.	13	0.231
152	A	1	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	1	1	1.	13	0.077
154	A	1	1	1.	13	0.077
155	A	1	1	1.	13	0.077
156	A	1	1	1.	13	0.077
157	A	4	2	1.	13	0.154
158	A	3	2	1.	13	0.154
159	A	2	2	1.	13	0.154
160	A	1	1	1.	13	0.077
161	A	1	1	1.	13	0.077
162	A	2	2	1.	13	0.154
163	A	3	2	1.	13	0.154
164	A	4	2	1.	13	0.154
165	C	1	1	0.35	13	0.077
166	C	1	1	0.29	13	0.077
167	A	1	1	1.	13	0.077
168	A	1	1	1.	13	0.077
169	A	1	1	1.	11	0.091
170	A	1	1	1.	9	0.111
171	A	1	1	1.	13	0.077
172	A	1	1	1.	13	0.077
173	A	1	1	1.	13	0.077
174	A	1	1	1.	13	0.077
175	A	1	1	1.	13	0.077
176	A	1	1	1.	13	0.077
177	A	1	1	1.	11	0.091
178	A	1	1	1.	9	0.111
179	A	1	1	1.	13	0.077
180	A	1	1	1.	13	0.077
181	A	1	1	1.	13	0.077
182	A	1	1	1.	13	0.077
183	A	3	2	1.	17	0.118
184	A	2	2	1.	17	0.118
185	A	1	1	1.	15	0.067
186	A	1	1	1.	13	0.077
187	A	2	2	1.	17	0.118
188	A	3	2	1.	17	0.118
189	A	4	3	1.	19	0.158
190	A	3	3	1.	19	0.158
191	A	2	2	1.	19	0.105
192	A	3	3	1.	19	0.158
193	A	4	3	1.	19	0.158
194	A	2	2	1.	7	0.286
195	A	8	4	1.	15	0.267
196	A	6	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	4	3	1.	13	0.231
198	A	1	1	1.	11	0.091
199	A	0	0	0.	0	0.
200	A	0	0	0.	0	0.
201	A	0	0	0.	0	0.
202	A	5	4	1.	15	0.267
203	A	4	3	1.	13	0.231
204	A	1	1	1.	11	0.091
205	A	0	0	0.	0	0.
206	A	0	0	0.	0	0.
207	A	0	0	0.	0	0.
208	A	8	5	1.	33	0.152
209	A	7	5	1.	33	0.152
210	A	6	5	1.	33	0.152
211	A	5	4	1.	31	0.129
212	A	2	2	1.	29	0.069
213	A	0	0	0.	0	0.
214	A	0	0	0.	0	0.
215	A	3	3	1.	11	0.273
216	A	13	5	1.	15	0.333
217	A	12	5	1.	15	0.333
218	A	11	4	1.	15	0.267
219	A	7	4	1.	13	0.308
220	A	2	2	1.	11	0.182
221	A	4	4	1.	15	0.267
222	A	9	7	1.	15	0.467
223	A	18	7	1.	15	0.467
224	A	19	6	1.	15	0.4
225	A	14	6	1.	15	0.4
226	A	11	6	1.	15	0.4
227	A	7	6	1.	13	0.462
228	A	3	3	1.	11	0.273
229	A	0	0	0.	0	0.
230	A	0	0	0.	0	0.
231	A	0	0	0.	0	0.
232	A	8	5	1.	15	0.333
233	A	7	5	1.	15	0.333
234	A	6	5	1.	15	0.333
235	A	4	3	1.	13	0.231
236	A	1	1	1.	11	0.091
237	A	0	0	0.	0	0.
238	A	0	0	0.	0	0.
239	A	0	0	0.	0	0.
240	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
241	A	0	0	0.	0	0.
242	A	1	1	1.	13	0.077
243	A	0	0	0.	0	0.
244	A	0	0	0.	0	0.
245	A	0	0	0.	0	0.
246	A	0	0	0.	0	0.
247	A	6	3	1.	15	0.2
248	A	5	3	1.	15	0.2
249	A	4	3	1.	13	0.231
250	A	1	1	1.	11	0.091
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	1	1	1.	21	0.048
255	C	1	1	0.3	21	0.048
256	C	1	1	0.35	21	0.048
257	A	4	2	1.	21	0.095
258	A	3	2	1.	21	0.095
259	A	2	2	1.	21	0.095
260	A	1	1	1.	19	0.053
261	A	1	1	1.	21	0.048
262	A	2	2	1.	21	0.095
263	A	3	2	1.	21	0.095
264	A	4	2	1.	21	0.095
265	A	1	1	1.	21	0.048
266	A	1	1	1.	21	0.048
267	A	1	1	1.	21	0.048
268	A	1	1	1.	21	0.048
269	A	5	2	1.	21	0.095
270	A	4	2	1.	21	0.095
271	A	3	2	1.	21	0.095
272	A	2	2	1.	21	0.095
273	A	1	1	1.	13	0.077
274	A	2	2	1.	21	0.095
275	A	3	2	1.	21	0.095
276	A	4	2	1.	21	0.095
277	A	5	2	1.	21	0.095
278	A	1	1	1.	21	0.048
279	A	1	1	1.	21	0.048
280	A	1	1	1.	21	0.048
281	C	1	1	0.3	21	0.048
282	C	1	1	0.35	21	0.048
283	A	4	2	1.	21	0.095
284	A	3	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
285	A	2	2	1.	21	0.095
286	A	1	1	1.	21	0.048
287	A	1	1	1.	21	0.048
288	A	2	2	1.	21	0.095
289	A	3	2	1.	21	0.095
290	A	4	2	1.	21	0.095
291	A	1	1	1.	21	0.048
292	A	1	1	1.	21	0.048
293	A	1	1	1.	21	0.048
294	A	1	1	1.	19	0.053
295	A	1	1	1.	13	0.077
296	A	1	1	1.	21	0.048
297	A	1	1	1.	21	0.048
298	A	1	1	1.	21	0.048
299	A	3	3	1.	15	0.2
300	A	4	3	1.	15	0.2
301	A	1	1	1.	21	0.048
302	A	1	1	1.	21	0.048
303	A	1	1	1.	21	0.048
304	A	4	3	1.	21	0.143
305	A	3	3	1.	19	0.158
306	A	2	2	1.	13	0.154
307	A	1	1	1.	21	0.048
308	A	1	1	1.	21	0.048
309	A	2	2	1.	21	0.095
310	A	3	2	1.	21	0.095
311	A	4	2	1.	21	0.095
312	C	1	1	0.32	21	0.048
313	C	1	1	0.26	21	0.048
314	A	1	1	1.	21	0.048
315	A	1	1	1.	21	0.048
316	A	1	1	1.	21	0.048
317	A	4	2	1.	21	0.095
318	A	3	2	1.	21	0.095
319	A	2	2	1.	19	0.105
320	A	1	1	1.	21	0.048
321	A	1	1	1.	21	0.048
322	A	2	2	1.	21	0.095
323	A	3	2	1.	21	0.095
324	A	4	2	1.	21	0.095
325	C	1	1	0.32	21	0.048
326	C	1	1	0.27	21	0.048
327	A	1	1	1.	21	0.048
328	A	1	1	1.	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	6	4	1.	21	0.19
330	A	5	4	1.	21	0.19
331	A	4	4	1.	21	0.19
332	A	3	3	1.	13	0.231
333	A	2	2	1.	21	0.095
334	A	3	3	1.	21	0.143
335	A	4	3	1.	21	0.143
336	A	5	3	1.	21	0.143
337	A	6	3	1.	21	0.143
338	A	1	1	1.	21	0.048
339	A	1	1	1.	21	0.048
340	A	1	1	1.	21	0.048
341	A	1	1	1.	21	0.048
342	A	1	1	1.	21	0.048
343	A	4	2	1.	21	0.095
344	A	3	2	1.	21	0.095
345	A	2	2	1.	21	0.095
346	A	1	1	1.	21	0.048
347	A	1	1	1.	21	0.048
348	A	2	2	1.	21	0.095
349	A	3	2	1.	21	0.095
350	A	4	2	1.	21	0.095
351	C	1	1	0.32	21	0.048
352	C	1	1	0.27	21	0.048
353	A	1	1	1.	21	0.048
354	A	1	1	1.	19	0.053
355	A	1	1	1.	13	0.077
356	A	1	1	1.	21	0.048
357	A	1	1	1.	21	0.048
358	A	1	1	1.	21	0.048
359	A	1	1	1.	21	0.048
360	A	1	1	1.	21	0.048
361	A	1	1	1.	21	0.048
362	A	1	1	1.	19	0.053
363	A	1	1	1.	13	0.077
364	A	1	1	1.	21	0.048
365	A	1	1	1.	21	0.048
366	A	1	1	1.	21	0.048
367	A	1	1	1.	21	0.048
368	C	1	1	0.28	25	0.04
369	C	1	1	0.33	25	0.04
370	A	4	2	1.	25	0.08
371	A	3	2	1.	25	0.08
372	A	2	2	1.	25	0.08

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	1	1	1.	23	0.043
374	A	1	1	1.	21	0.048
375	A	2	2	1.	25	0.08
376	A	3	2	1.	25	0.08
377	A	4	2	1.	25	0.08
378	A	1	1	1.	25	0.04
379	A	1	1	1.	25	0.04
380	A	2	2	1.	25	0.08
381	A	2	2	1.	26	0.077
382	A	14	4	1.	21	0.19
383	A	11	4	1.	21	0.19
384	A	8	4	1.	21	0.19
385	A	6	4	1.	21	0.19
386	A	4	3	1.	19	0.158
387	A	1	1	1.	13	0.077
388	A	0	0	0.	0	0.
389	A	0	0	0.	0	0.
390	A	0	0	0.	0	0.
391	A	6	4	1.	19	0.21
392	A	5	4	1.	19	0.21
393	A	4	3	1.	17	0.176
394	A	1	1	1.	11	0.091
395	A	0	0	0.	0	0.
396	A	0	0	0.	0	0.
397	A	4	4	1.	21	0.19
398	A	9	7	1.	21	0.333
399	A	18	7	1.	21	0.333
400	A	36	7	1.	21	0.333
401	A	13	5	1.	19	0.263
402	A	12	5	1.	19	0.263
403	A	11	4	1.	19	0.21
404	A	7	4	1.	17	0.235
405	A	2	2	1.	11	0.182
406	A	4	4	1.	19	0.21
407	A	9	7	1.	19	0.368
408	A	18	7	1.	19	0.368
409	A	14	6	1.	19	0.316
410	A	11	6	1.	19	0.316
411	A	7	6	1.	17	0.353
412	A	3	3	1.	11	0.273
413	A	0	0	0.	0	0.
414	A	0	0	0.	0	0.
415	A	0	0	0.	0	0.
416	A	7	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
417	A	6	5	1.	19	0.263
418	A	4	3	1.	17	0.176
419	A	1	1	1.	11	0.091
420	A	0	0	0.	0	0.
421	A	0	0	0.	0	0.
422	A	5	5	1.	26	0.192
423	A	12	8	1.	26	0.308
424	A	24	8	1.	26	0.308
425	A	48	8	1.	26	0.308
426	A	10	4	1.	16	0.25
427	A	6	4	1.	16	0.25
428	A	3	3	1.	14	0.214
429	A	2	2	1.	12	0.167
430	A	0	0	0.	0	0.
431	A	0	0	0.	0	0.
432	A	10	4	1.	17	0.235
433	A	6	4	1.	17	0.235
434	A	3	3	1.	15	0.2
435	A	2	2	1.	13	0.154
436	A	0	0	0.	0	0.
437	A	0	0	0.	0	0.
438	A	11	5	1.	17	0.294
439	A	7	5	1.	17	0.294
440	A	4	4	1.	15	0.267
441	A	3	3	1.	13	0.231
442	A	0	0	0.	0	0.
443	A	0	0	0.	0	0.
444	A	10	4	1.	20	0.2
445	A	6	4	1.	20	0.2
446	A	3	3	1.	18	0.167
447	A	0	0	0.	0	0.
448	A	0	0	0.	0	0.
449	A	0	0	0.	0	0.
450	A	2	2	1.	21	0.095
451	A	3	3	1.	21	0.143
452	A	1	1	1.	19	0.053
453	A	1	1	1.	21	0.048
454	A	3	3	1.	21	0.143
455	A	2	2	1.	21	0.095
456	A	2	2	1.	20	0.1
457	A	3	3	1.	20	0.15
458	A	1	1	1.	18	0.056
459	A	1	1	1.	20	0.05
460	A	3	3	1.	20	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	2	2	1.	20	0.1
462	A	8	4	1.	20	0.2
463	A	7	2	1.	20	0.1
464	A	4	2	1.	17	0.118
465	A	4	2	1.	18	0.111
466	A	7	4	1.	20	0.2
467	A	9	3	1.	23	0.13
468	A	7	2	1.	23	0.087
469	A	4	2	1.	20	0.1
470	A	4	2	1.	21	0.095
471	A	7	3	1.	23	0.13
472	A	9	4	1.	23	0.174
473	A	3	2	1.	13	0.154
474	A	3	2	1.	15	0.133
475	A	3	2	1.	14	0.143
476	A	3	2	1.	16	0.125
477	A	3	2	1.	15	0.133
478	A	3	2	1.	17	0.118
479	A	3	2	1.	16	0.125
480	A	3	2	1.	18	0.111
481	A	2	2	1.	13	0.154
482	A	2	2	1.	15	0.133
483	A	2	2	1.	14	0.143
484	A	2	2	1.	16	0.125
485	A	4	4	1.	15	0.267
486	A	4	4	1.	15	0.267
487	A	4	4	1.	16	0.25
488	A	4	4	1.	16	0.25
489	A	3	3	1.	15	0.2
490	A	3	3	1.	17	0.176
491	A	3	3	1.	16	0.188
492	A	3	3	1.	18	0.167
493	A	2	2	1.	17	0.118
494	A	2	2	1.	17	0.118
495	A	2	2	1.	18	0.111
496	A	2	2	1.	18	0.111
497	A	3	2	1.	15	0.133
498	A	3	2	1.	17	0.118
499	A	3	2	1.	16	0.125
500	A	3	2	1.	18	0.111
501	A	5	4	1.	17	0.235
502	A	5	4	1.	19	0.21
503	A	5	4	1.	18	0.222
504	A	5	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	3	2	1.	14	0.143
506	A	6	5	1.	14	0.357
507	A	6	5	1.	12	0.417
508	A	7	7	1.	14	0.5
509	A	7	7	1.	16	0.438
510	A	11	11	1.	16	0.688
511	A	9	5	1.	16	0.312
512	A	9	5	1.	14	0.357
513	A	9	5	1.	16	0.312
514	A	9	5	1.	18	0.278
515	A	12	10	1.	18	0.556
516	A	11	6	1.	18	0.333
517	A	11	6	1.	16	0.375
518	A	11	6	1.	18	0.333
519	A	11	6	1.	20	0.3
520	A	3	2	1.	23	0.087
521	A	7	7	1.	25	0.28
522	A	7	7	1.	24	0.292
523	A	11	11	1.	25	0.44
524	A	9	5	1.	27	0.185
525	A	12	10	1.	27	0.37
526	A	11	6	1.	29	0.207
527	A	7	6	1.	37	0.162
528	A	7	6	1.	36	0.167
529	A	2	2	1.	12	0.167
530	A	7	7	1.	14	0.5
531	A	7	7	1.	16	0.438
532	A	2	2	1.	21	0.095
533	A	7	7	1.	23	0.304
534	A	7	7	1.	25	0.28
535	A	2	2	1.	12	0.167
536	A	4	3	1.	16	0.188
537	A	4	4	1.	16	0.25
538	A	8	5	1.	18	0.278
539	A	10	6	1.	20	0.3
540	A	4	4	1.	25	0.16
541	A	8	5	1.	27	0.185
542	A	10	6	1.	29	0.207
543	A	0	0	0.	0	0.
544	A	6	3	1.	50	0.06
545	A	5	3	1.	50	0.06
546	A	4	3	1.	48	0.062
547	A	3	2	1.	21	0.095
548	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
549	A	0	0	0.	0	0.
550	A	0	0	0.	0	0.
551	A	6	3	1.	47	0.064
552	A	5	3	1.	47	0.064
553	A	4	3	1.	45	0.067
554	A	1	1	1.	14	0.071
555	A	0	0	0.	0	0.
556	A	0	0	0.	0	0.
557	A	3	3	1.	37	0.081
558	A	2	2	1.	36	0.056
559	A	2	2	1.	36	0.056
560	A	2	2	1.	35	0.057
561	A	2	2	1.	36	0.056
562	A	2	2	1.	36	0.056
563	A	4	3	1.	10	0.3
564	A	3	3	1.	8	0.375
565	A	2	2	1.	7	0.286
566	A	2	2	1.	10	0.2
567	A	3	3	1.	10	0.3
568	A	4	3	1.	10	0.3
569	A	3	2	1.	10	0.2
570	A	2	2	1.	9	0.222
571	A	4	3	1.	12	0.25
572	A	13	7	1.	44	0.159
573	A	11	6	1.	44	0.136
574	A	9	5	1.	42	0.119
575	A	7	6	1.	37	0.162
576	A	0	0	0.	0	0.
577	A	0	0	0.	0	0.
578	A	9	5	1.	47	0.106
579	A	4	4	1.	18	0.222
580	A	4	4	1.	16	0.25
581	A	4	4	1.	14	0.286
582	A	4	4	1.	18	0.222
583	A	4	4	1.	18	0.222
584	A	4	4	1.	18	0.222
585	A	4	4	1.	20	0.2
586	A	4	3	1.	20	0.15
587	A	4	3	0.99	31	0.097
588	A	5	4	1.	31	0.129
589	A	5	4	1.	29	0.138
590	A	4	3	1.	20	0.15
591	A	4	3	1.	31	0.097
592	A	5	4	1.	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
593	A	5	4	1.	31	0.129
594	A	0	0	0.	0	0.
595	F	0	0	N/A	0	N/A
596	F	0	0	N/A	0	N/A
597	F	0	0	N/A	0	N/A
598	A	4	3	1.	20	0.15
599	A	0	0	0.	0	0.
600	A	0	0	0.	0	0.
601	A	0	0	0.	0	0.
602	A	8	7	0.99	31	0.226
603	A	8	7	1.	31	0.226
604	A	8	7	1.	29	0.241
605	A	7	6	1.	20	0.3
606	A	8	7	1.	31	0.226
607	A	8	7	1.	31	0.226
608	A	8	7	1.	31	0.226
609	A	0	0	0.	0	0.
610	F	0	0	N/A	0	N/A
611	F	0	0	N/A	0	N/A
612	F	0	0	N/A	0	N/A
613	A	7	6	1.	20	0.3
614	A	0	0	0.	0	0.
615	A	0	0	0.	0	0.
616	A	0	0	0.	0	0.
617	A	1	1	1.	21	0.048
618	A	1	1	1.	33	0.03
619	A	2	2	1.	31	0.065
620	A	5	3	1.	31	0.097
621	A	4	3	1.	31	0.097
622	A	3	3	1.	29	0.103
623	A	1	1	1.	19	0.053
624	A	2	2	1.	31	0.065
625	A	3	3	1.	31	0.097
626	A	4	3	1.	31	0.097
627	A	7	4	1.	33	0.121
628	A	6	4	1.	33	0.121
629	A	5	4	1.	33	0.121
630	A	4	4	1.	33	0.121
631	A	3	3	1.	33	0.091
632	A	4	4	1.	33	0.121
633	A	5	4	1.	33	0.121
634	A	6	4	1.	33	0.121
635	A	7	4	1.	33	0.121
636	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
637	A	2	2	1.	13	0.154
638	A	2	2	1.	15	0.133
639	A	2	2	1.	15	0.133
640	A	3	3	1.	17	0.176
641	A	2	2	1.	9	0.222
642	A	3	3	1.	17	0.176
643	A	3	3	1.	18	0.167
644	A	2	2	1.	13	0.154
645	A	1	1	1.	11	0.091
646	A	2	1	1.	9	0.111
647	A	3	2	1.	17	0.118
648	A	2	2	1.	17	0.118
649	A	2	2	1.	15	0.133
650	A	3	2	1.	13	0.154
651	A	3	2	1.	16	0.125
652	A	2	2	1.	13	0.154
653	A	2	2	1.	13	0.154
654	A	4	3	1.17	23	0.13
655	A	4	3	1.2	23	0.13
656	A	4	3	1.	27	0.111
657	A	2	2	1.	15	0.133
658	A	1	1	1.	17	0.059
659	A	4	4	1.	15	0.267
660	A	3	3	1.	15	0.2
661	A	3	3	1.	15	0.2
662	A	3	3	1.	18	0.167
663	A	3	3	1.	11	0.273
664	A	3	3	1.	15	0.2
665	A	2	2	1.	15	0.133
666	A	2	2	1.	15	0.133
667	A	2	2	1.	17	0.118
668	A	2	2	1.	17	0.118
669	A	1	1	1.	9	0.111
670	A	1	1	1.	11	0.091
671	A	1	1	1.	13	0.077
672	A	1	1	1.	13	0.077
673	A	13	3	1.	16	0.188
674	A	5	3	1.	7	0.429
675	A	3	2	1.	20	0.1
676	A	2	2	1.	18	0.111
677	A	1	1	1.	10	0.1
678	A	4	3	1.	20	0.15
679	A	3	2	1.	13	0.154
680	A	1	1	1.	10	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
681	A	2	2	1.	8	0.25
682	A	3	2	1.	15	0.133
683	A	1	1	1.	10	0.1
684	A	3	3	1.	14	0.214
685	A	6	3	1.	11	0.273
686	A	4	3	1.	18	0.167
687	A	3	2	1.	15	0.133
688	A	3	2	1.	15	0.133
689	A	4	4	1.	32	0.125
690	A	8	4	1.	11	0.364
691	A	8	4	1.	13	0.308
692	F	0	0	N/A	0	N/A
693	A	2	2	1.	23	0.087
694	F	0	0	N/A	0	N/A
695	A	3	2	1.	9	0.222
696	A	3	2	1.	9	0.222
697	A	3	2	1.	9	0.222
698	A	3	3	1.	15	0.2
699	A	3	3	1.	17	0.176
700	A	4	4	1.	15	0.267
701	A	4	4	1.	17	0.235
702	A	1	1	1.	10	0.1
703	A	3	3	1.	15	0.2
704	A	3	3	1.	15	0.2
705	A	2	2	1.	17	0.118
706	A	3	3	1.	18	0.167
707	A	4	2	1.	9	0.222
708	A	4	2	1.	11	0.182
709	A	3	3	1.	17	0.176
710	A	3	2	1.	19	0.105
711	A	3	2	1.	17	0.118
712	A	4	3	1.	15	0.2
713	A	4	4	1.	22	0.182
714	A	4	3	1.	22	0.136
715	A	2	2	1.	7	0.286
716	A	3	2	1.	12	0.167
717	A	4	3	1.	11	0.273
718	A	2	2	1.	11	0.182
719	A	2	2	1.	11	0.182
720	A	2	2	1.	13	0.154
721	A	2	2	1.	13	0.154
722	A	3	2	1.	17	0.118
723	A	4	3	1.	17	0.176
724	A	3	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	A	3	2	1.	15	0.133
726	A	3	2	1.	25	0.08
727	A	15	4	1.	18	0.222
728	A	3	3	1.	13	0.231
729	A	6	4	1.	15	0.267
730	A	1	1	1.	16	0.062
731	A	3	3	1.	19	0.158
732	A	1	1	1.	15	0.067
733	A	1	1	1.	13	0.077
734	A	2	2	1.	9	0.222
735	A	6	4	1.	16	0.25
736	A	3	2	1.	15	0.133
737	A	1	1	1.	9	0.111
738	A	1	1	1.	13	0.077
739	A	1	1	1.	9	0.111
740	A	3	1	1.	9	0.111
741	A	8	3	1.	14	0.214
742	A	2	1	1.	15	0.067
743	A	1	1	1.	13	0.077
744	A	2	1	1.	23	0.043
745	A	6	3	1.	28	0.107
746	A	4	2	1.	37	0.054
747	A	0	0	0.	0	0.
748	A	0	0	0.	0	0.
749	A	0	0	0.	0	0.
750	A	4	2	1.	50	0.04
751	A	0	0	0.	0	0.
752	A	1	1	1.	16	0.062
753	A	2	1	1.	34	0.029
754	A	0	0	0.	0	0.
755	A	8	4	1.	23	0.174
756	A	4	2	1.	39	0.051
757	A	3	2	1.	26	0.077
758	A	0	0	0.	0	0.
759	A	0	0	0.	0	0.
760	A	0	0	0.	0	0.
761	A	0	0	0.	0	0.
762	A	0	0	0.	0	0.
763	A	0	0	0.	0	0.
764	A	0	0	0.	0	0.
765	A	0	0	0.	0	0.
766	A	4	3	1.	23	0.13
767	A	2	2	1.	15	0.133
768	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
769	A	1	1	1.	9	0.111
770	A	1	1	1.	9	0.111
771	A	1	1	1.	17	0.059
772	A	1	1	1.	17	0.059
773	A	4	3	1.	11	0.273
774	A	8	4	1.	43	0.093



# Chapter 3

## Listing of integrals

### 3.1

$$\int \frac{e^x}{4+6e^x} dx$$

Optimal. Leaf size=12

$$\frac{1}{6} \log(3e^x + 2)$$

[Out] Log[2 + 3\*E^x]/6

---

**Rubi [A]** time = 0.0168879, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2246, 31}

$$\frac{1}{6} \log(3e^x + 2)$$

Antiderivative was successfully verified.

[In] Int[E^x/(4 + 6\*E^x), x]

[Out] Log[2 + 3\*E^x]/6

### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned} \int \frac{e^x}{4+6e^x} dx &= \text{Subst} \left( \int \frac{1}{4+6x} dx, x, e^x \right) \\ &= \frac{1}{6} \log(2 + 3e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0056498, size = 12, normalized size = 1.

$$\frac{1}{6} \log(3e^x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(4 + 6\*E^x), x]

[Out] Log[2 + 3\*E^x]/6

---

**Maple [A]** time = 0.001, size = 10, normalized size = 0.8

$$\frac{\ln(2 + 3e^x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(4+6\*exp(x)), x)

[Out] 1/6\*ln(2+3\*exp(x))

---

**Maxima [A]** time = 1.13738, size = 12, normalized size = 1.

$$\frac{1}{6} \log(3e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(4+6\*exp(x)), x, algorithm="maxima")

[Out] 1/6\*log(3\*e^x + 2)

---

**Fricas [A]** time = 1.44771, size = 27, normalized size = 2.25

$$\frac{1}{6} \log(3e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(4+6\*exp(x)), x, algorithm="fricas")

[Out] 1/6\*log(3\*e^x + 2)

---

**Sympy [A]** time = 0.12272, size = 8, normalized size = 0.67

$$\frac{\log\left(e^x + \frac{2}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(exp(x)/(4+6*exp(x)),x)
```

```
[Out] log(exp(x) + 2/3)/6
```

---

**Giac [A]** time = 1.28391, size = 12, normalized size = 1.

$$\frac{1}{6} \log(3e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(4+6*exp(x)),x, algorithm="giac")
```

```
[Out] 1/6*log(3*e^x + 2)
```

### 3.2 $\int \frac{e^x}{a+be^x} dx$

**Optimal.** Leaf size=12

$$\frac{\log(a + be^x)}{b}$$

[Out] Log[a + b\*E^x]/b

**Rubi [A]** time = 0.0197353, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2246, 31}

$$\frac{\log(a + be^x)}{b}$$

Antiderivative was successfully verified.

[In] Int[E^x/(a + b\*E^x), x]

[Out] Log[a + b\*E^x]/b

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{a + be^x} dx &= \text{Subst} \left( \int \frac{1}{a + bx} dx, x, e^x \right) \\ &= \frac{\log(a + be^x)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0049704, size = 12, normalized size = 1.

$$\frac{\log(a + be^x)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(a + b\*E^x), x]

[Out] Log[a + b\*E^x]/b

**Maple [A]** time = 0.002, size = 12, normalized size = 1.

$$\frac{\ln(a + be^x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(a+b\*exp(x)),x)

[Out] ln(a+b\*exp(x))/b

---

**Maxima [A]** time = 1.05088, size = 15, normalized size = 1.25

$$\frac{\log(be^x + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a+b\*exp(x)),x, algorithm="maxima")

[Out] log(b\*e^x + a)/b

---

**Fricas [A]** time = 1.43886, size = 24, normalized size = 2.

$$\frac{\log(be^x + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a+b\*exp(x)),x, algorithm="fricas")

[Out] log(b\*e^x + a)/b

---

**Sympy [A]** time = 0.124032, size = 8, normalized size = 0.67

$$\frac{\log\left(\frac{a}{b} + e^x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a+b\*exp(x)),x)

[Out] log(a/b + exp(x))/b

---

**Giac [A]** time = 1.20525, size = 16, normalized size = 1.33

$$\frac{\log(|be^x + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*e^x + a))/b
```

### 3.3 $\int \frac{e^{dx}}{a+be^{c+dx}} dx$

**Optimal.** Leaf size=24

$$\frac{e^{-c} \log(a + be^{c+dx})}{bd}$$

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d\*E^c)

**Rubi [A]** time = 0.0692306, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2247, 2246, 31}

$$\frac{e^{-c} \log(a + be^{c+dx})}{bd}$$

Antiderivative was successfully verified.

[In] Int[E^(d\*x)/(a + b\*E^(c + d\*x)), x]

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d\*E^c)

#### Rule 2247

Int[((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.))^(p\_.)\*((G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))))^(m\_.), x\_Symbol] :> Dist[(G^(h\*(f + g\*x)))^m/(F^(e\*(c + d\*x)))^n, Int[(F^(e\*(c + d\*x)))^n\*(a + b\*(F^(e\*(c + d\*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d\*e\*n\*Log[F], g\*h\*m\*Log[G]]

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{dx}}{a+be^{c+dx}} dx &= e^{-c} \int \frac{e^{c+dx}}{a+be^{c+dx}} dx \\ &= \frac{e^{-c} \text{Subst}\left(\int \frac{1}{a+bx} dx, x, e^{c+dx}\right)}{d} \\ &= \frac{e^{-c} \log(a + be^{c+dx})}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0117087, size = 24, normalized size = 1.

$$\frac{e^{-c} \log(a + be^{c+dx})}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[E^(d\*x)/(a + b\*E^(c + d\*x)), x]

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d\*E^c)

**Maple [A]** time = 0.004, size = 23, normalized size = 1.

$$\frac{\ln(a + be^{dx}e^c)}{bde^c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(d\*x)/(a+b\*exp(d\*x+c)), x)

[Out] 1/d\*ln(a+b\*exp(d\*x)\*exp(c))/b/exp(c)

**Maxima [A]** time = 1.06388, size = 30, normalized size = 1.25

$$\frac{e^{(-c)} \log(be^{(dx+c)} + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)/(a+b\*exp(d\*x+c)), x, algorithm="maxima")

[Out] e^(-c)\*log(b\*e^(d\*x + c) + a)/(b\*d)

**Fricas [A]** time = 1.46597, size = 50, normalized size = 2.08

$$\frac{e^{(-c)} \log(be^{(dx+c)} + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)/(a+b\*exp(d\*x+c)), x, algorithm="fricas")

[Out] e^(-c)\*log(b\*e^(d\*x + c) + a)/(b\*d)

**Sympy [A]** time = 0.177016, size = 19, normalized size = 0.79

$$\frac{e^{-c} \log\left(\frac{ae^{-c}}{b} + e^{dx}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)/(a+b\*exp(d\*x+c)),x)

[Out] exp(-c)\*log(a\*exp(-c)/b + exp(d\*x))/(b\*d)

**Giac [A]** time = 1.2031, size = 31, normalized size = 1.29

$$\frac{e^{(-c)} \log(|be^{(dx+c)} + a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)/(a+b\*exp(d\*x+c)),x, algorithm="giac")

[Out] e^(-c)\*log(abs(b\*e^(d\*x + c) + a))/(b\*d)

### 3.4 $\int \frac{e^{c+dx}}{a+be^{c+dx}} dx$

**Optimal.** Leaf size=19

$$\frac{\log(a + be^{c+dx})}{bd}$$

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d)

**Rubi [A]** time = 0.0359485, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 31}

$$\frac{\log(a + be^{c+dx})}{bd}$$

Antiderivative was successfully verified.

[In] Int[E^(c + d\*x)/(a + b\*E^(c + d\*x)),x]

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d)

#### Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^{c+dx}}{a + be^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx} dx, x, e^{c+dx}\right)}{d} \\ &= \frac{\log(a + be^{c+dx})}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0063149, size = 19, normalized size = 1.

$$\frac{\log(a + be^{c+dx})}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c + d\*x)/(a + b\*E^(c + d\*x)),x]

[Out] Log[a + b\*E^(c + d\*x)]/(b\*d)



---

**Maple [A]** time = 0.003, size = 19, normalized size = 1.

$$\frac{\ln(a + be^{dx+c})}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(d\*x+c)/(a+b\*exp(d\*x+c)), x)

[Out] ln(a+b\*exp(d\*x+c))/b/d

---

**Maxima [A]** time = 1.10066, size = 24, normalized size = 1.26

$$\frac{\log(be^{dx+c} + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)/(a+b\*exp(d\*x+c)), x, algorithm="maxima")

[Out] log(b\*e^(d\*x + c) + a)/(b\*d)

---

**Fricas [A]** time = 1.47187, size = 41, normalized size = 2.16

$$\frac{\log(be^{dx+c} + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)/(a+b\*exp(d\*x+c)), x, algorithm="fricas")

[Out] log(b\*e^(d\*x + c) + a)/(b\*d)

---

**Sympy [A]** time = 0.218728, size = 14, normalized size = 0.74

$$\frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)/(a+b\*exp(d\*x+c)), x)

[Out] log(a/b + exp(c + d\*x))/(b\*d)

---

**Giac [A]** time = 1.28083, size = 26, normalized size = 1.37

$$\frac{\log(|be^{dx+c} + a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)/(a+b*exp(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*e^(d*x + c) + a))/(b*d)
```

### 3.5 $\int e^x (a + be^x)^n dx$

**Optimal.** Leaf size=20

$$\frac{(a + be^x)^{n+1}}{b(n+1)}$$

[Out] (a + b\*E^x)^(1 + n)/(b\*(1 + n))

**Rubi [A]** time = 0.019931, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2246, 32}

$$\frac{(a + be^x)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[E^x\*(a + b\*E^x)^n,x]

[Out] (a + b\*E^x)^(1 + n)/(b\*(1 + n))

**Rule 2246**

Int[((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

**Rule 32**

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int e^x (a + be^x)^n dx &= \text{Subst} \left( \int (a + bx)^n dx, x, e^x \right) \\ &= \frac{(a + be^x)^{1+n}}{b(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.018944, size = 19, normalized size = 0.95

$$\frac{(a + be^x)^{n+1}}{bn + b}$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*(a + b\*E^x)^n,x]

[Out] (a + b\*E^x)^(1 + n)/(b + b\*n)

**Maple [A]** time = 0.002, size = 20, normalized size = 1.

$$\frac{(a + be^x)^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(a+b\*exp(x))^n,x)

[Out] (a+b\*exp(x))^(1+n)/b/(1+n)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(a+b\*exp(x))^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.54118, size = 50, normalized size = 2.5

$$\frac{(be^x + a)(be^x + a)^n}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(a+b\*exp(x))^n,x, algorithm="fricas")

[Out] (b\*e^x + a)\*(b\*e^x + a)^n/(b\*n + b)

**Sympy [A]** time = 1.26341, size = 56, normalized size = 2.8

$$\begin{cases} \frac{e^x}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n e^x & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + e^x\right)}{b} & \text{for } n = -1 \\ \frac{a(a+be^x)^n}{bn+b} + \frac{b(a+be^x)^n e^x}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(a+b\*exp(x))\*\*n,x)

[Out] Piecewise((exp(x)/a, Eq(b, 0) & Eq(n, -1)), (a\*\*n\*exp(x), Eq(b, 0)), (log(a/b + exp(x))/b, Eq(n, -1)), (a\*(a + b\*exp(x))\*\*n/(b\*n + b) + b\*(a + b\*exp(x))\*\*n\*exp(x)/(b\*n + b), True))

**Giac [A]** time = 1.30517, size = 26, normalized size = 1.3

$$\frac{(be^x + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(a+b*exp(x))^n,x, algorithm="giac")
```

```
[Out] (b*e^x + a)^(n + 1)/(b*(n + 1))
```

### 3.6 $\int e^{dx} (a + be^{c+dx})^n dx$

**Optimal.** Leaf size=32

$$\frac{e^{-c} (a + be^{c+dx})^{n+1}}{bd(n+1)}$$

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(b\*d\*E^c\*(1 + n))

**Rubi [A]** time = 0.0696912, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2247, 2246, 32}

$$\frac{e^{-c} (a + be^{c+dx})^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(d\*x)\*(a + b\*E^(c + d\*x))^n,x]

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(b\*d\*E^c\*(1 + n))

#### Rule 2247

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] := Dist[(G^(h*(f + g*x)))^m/(F^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log[F], g*h*m*Log[G]]
```

#### Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned} \int e^{dx} (a + be^{c+dx})^n dx &= e^{-c} \int e^{c+dx} (a + be^{c+dx})^n dx \\ &= \frac{e^{-c} \text{Subst}\left(\int (a + bx)^n dx, x, e^{c+dx}\right)}{d} \\ &= \frac{e^{-c} (a + be^{c+dx})^{1+n}}{bd(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.034068, size = 31, normalized size = 0.97

$$\frac{e^{-c} (a + be^{c+dx})^{n+1}}{bdn + bd}$$

Antiderivative was successfully verified.

[In] Integrate[E^(d\*x)\*(a + b\*E^(c + d\*x))^n,x]

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(E^c\*(b\*d + b\*d\*n))

**Maple [A]** time = 0.002, size = 31, normalized size = 1.

$$\frac{(a + be^{dx}e^c)^{1+n}}{bde^c(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(d\*x)\*(a+b\*exp(d\*x+c))^n,x)

[Out] 1/d\*(a+b\*exp(d\*x)\*exp(c))^(1+n)/b/exp(c)/(1+n)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)\*(a+b\*exp(d\*x+c))^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.54729, size = 81, normalized size = 2.53

$$\frac{(be^{(dx)} + ae^{(-c)})(be^{(dx+c)} + a)^n}{bdn + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x)\*(a+b\*exp(d\*x+c))^n,x, algorithm="fricas")

[Out] (b\*e^(d\*x) + a\*e^(-c))\*(b\*e^(d\*x + c) + a)^n/(b\*d\*n + b\*d)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x)*(a+b*exp(d*x+c))**n,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.21372, size = 41, normalized size = 1.28

$$\frac{(be^{dx+c} + a)^{n+1} e^{-c}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x)*(a+b*exp(d*x+c))^n,x, algorithm="giac")
```

```
[Out] (b*e^(d*x + c) + a)^(n + 1)*e^(-c)/(b*d*(n + 1))
```



### 3.7 $\int e^{c+dx} (a + be^{c+dx})^n dx$

**Optimal.** Leaf size=27

$$\frac{(a + be^{c+dx})^{n+1}}{bd(n+1)}$$

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(b\*d\*(1 + n))

**Rubi [A]** time = 0.0350901, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 32}

$$\frac{(a + be^{c+dx})^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(c + d\*x)\*(a + b\*E^(c + d\*x))^n,x]

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(b\*d\*(1 + n))

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int e^{c+dx} (a + be^{c+dx})^n dx &= \frac{\text{Subst}\left(\int (a + bx)^n dx, x, e^{c+dx}\right)}{d} \\ &= \frac{(a + be^{c+dx})^{1+n}}{bd(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0242155, size = 26, normalized size = 0.96

$$\frac{(a + be^{c+dx})^{n+1}}{bdn + bd}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c + d\*x)\*(a + b\*E^(c + d\*x))^n,x]

[Out] (a + b\*E^(c + d\*x))^(1 + n)/(b\*d + b\*d\*n)

---

**Maple [A]** time = 0.003, size = 27, normalized size = 1.

$$\frac{(a + be^{dx+c})^{1+n}}{bd(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(d\*x+c)\*(a+b\*exp(d\*x+c))^n,x)

[Out] (a+b\*exp(d\*x+c))^(1+n)/b/d/(1+n)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)\*(a+b\*exp(d\*x+c))^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.55541, size = 77, normalized size = 2.85

$$\frac{(be^{(dx+c)} + a)(be^{(dx+c)} + a)^n}{bdn + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)\*(a+b\*exp(d\*x+c))^n,x, algorithm="fricas")

[Out] (b\*e^(d\*x + c) + a)\*(b\*e^(d\*x + c) + a)^n/(b\*d\*n + b\*d)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(d\*x+c)\*(a+b\*exp(d\*x+c))\*\*n,x)

[Out] Timed out

---

**Giac [A]** time = 1.28504, size = 35, normalized size = 1.3

$$\frac{(be^{(dx+c)} + a)^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*(a+b*exp(d*x+c))^n,x, algorithm="giac")
```

```
[Out] (b*e^(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

$$3.8 \quad \int \frac{F^x}{a+bF^x} dx$$

**Optimal.** Leaf size=16

$$\frac{\log(a+bF^x)}{b \log(F)}$$

[Out] Log[a + b\*F^x]/(b\*Log[F])

**Rubi [A]** time = 0.0205769, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2246, 31}

$$\frac{\log(a+bF^x)}{b \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^x/(a + b\*F^x), x]

[Out] Log[a + b\*F^x]/(b\*Log[F])

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{F^x}{a+bF^x} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx} dx, x, F^x\right)}{\log(F)} \\ &= \frac{\log(a+bF^x)}{b \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0059151, size = 16, normalized size = 1.

$$\frac{\log(a+bF^x)}{b \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^x/(a + b\*F^x), x]

[Out] Log[a + b\*F^x]/(b\*Log[F])

---

**Maple [A]** time = 0.001, size = 17, normalized size = 1.1

$$\frac{\ln(a + bF^x)}{b \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^x/(a+b\*F^x),x)

[Out] ln(a+b\*F^x)/b/ln(F)

---

**Maxima [A]** time = 1.15051, size = 22, normalized size = 1.38

$$\frac{\log(F^x b + a)}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^x/(a+b\*F^x),x, algorithm="maxima")

[Out] log(F^x\*b + a)/(b\*log(F))

---

**Fricas [A]** time = 1.4996, size = 36, normalized size = 2.25

$$\frac{\log(F^x b + a)}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^x/(a+b\*F^x),x, algorithm="fricas")

[Out] log(F^x\*b + a)/(b\*log(F))

---

**Sympy [A]** time = 0.127843, size = 12, normalized size = 0.75

$$\frac{\log\left(F^x + \frac{a}{b}\right)}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*x/(a+b\*F\*\*x),x)

[Out] log(F\*\*x + a/b)/(b\*log(F))

---

**Giac [A]** time = 1.28401, size = 23, normalized size = 1.44

$$\frac{\log(|F^x b + a|)}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^x/(a+b*F^x),x, algorithm="giac")
```

```
[Out] log(abs(F^x*b + a))/(b*log(F))
```

### 3.9 $\int \frac{F^{dx}}{a+bF^{c+dx}} dx$

**Optimal.** Leaf size=28

$$\frac{F^{-c} \log(a + bF^{c+dx})}{bd \log(F)}$$

[Out] Log[a + bF^(c + d\*x)]/(b\*d\*F^c\*Log[F])

**Rubi [A]** time = 0.0707328, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2247, 2246, 31}

$$\frac{F^{-c} \log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(d\*x)/(a + b\*F^(c + d\*x)), x]

[Out] Log[a + b\*F^(c + d\*x)]/(b\*d\*F^c\*Log[F])

#### Rule 2247

Int[((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_)\*((G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))))^(m\_), x\_Symbol] :> Dist[(G^(h\*(f + g\*x)))^m/(F^(e\*(c + d\*x)))^n, Int[(F^(e\*(c + d\*x)))^n\*(a + b\*(F^(e\*(c + d\*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d\*e\*n\*Log[F], g\*h\*m\*Log[G]]

#### Rule 2246

Int[((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{dx}}{a+bF^{c+dx}} dx &= F^{-c} \int \frac{F^{c+dx}}{a+bF^{c+dx}} dx \\ &= \frac{F^{-c} \text{Subst}\left(\int \frac{1}{a+bx} dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= \frac{F^{-c} \log(a + bF^{c+dx})}{bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0089136, size = 28, normalized size = 1.

$$\frac{F^{-c} \log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(d\*x)/(a + b\*F^(c + d\*x)), x]

[Out] Log[a + b\*F^(c + d\*x)]/(b\*d\*F^c\*Log[F])

**Maple [A]** time = 0.009, size = 33, normalized size = 1.2

$$\frac{\ln(a + be^{c \ln(F)} e^{d \ln(F)x})}{F^c b \ln(F) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x)/(a+b\*F^(d\*x+c)), x)

[Out] 1/(F^c)/b/ln(F)/d\*ln(a+b\*exp(c\*ln(F))\*exp(d\*ln(F)\*x))

**Maxima [A]** time = 1.06323, size = 38, normalized size = 1.36

$$\frac{\log(F^{dx+cb} + a)}{F^c b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)/(a+b\*F^(d\*x+c)), x, algorithm="maxima")

[Out] log(F^(d\*x + c)\*b + a)/(F^c\*b\*d\*log(F))

**Fricas [A]** time = 1.47925, size = 55, normalized size = 1.96

$$\frac{\log(F^{dx+cb} + a)}{F^c b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)/(a+b\*F^(d\*x+c)), x, algorithm="fricas")

[Out] log(F^(d\*x + c)\*b + a)/(F^c\*b\*d\*log(F))

**Sympy [A]** time = 0.516429, size = 24, normalized size = 0.86

$$\frac{e^{-c \log(F)} \log\left(F^{c+dx} + \frac{a}{b}\right)}{bd \log(F)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x)/(a+b\*F\*\*(d\*x+c)),x)

[Out] exp(-c\*log(F))\*log(F\*\*(c + d\*x) + a/b)/(b\*d\*log(F))

**Giac [A]** time = 1.27375, size = 41, normalized size = 1.46

$$\frac{\log(|F^{dx}F^c b + a|)}{F^c b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)/(a+b\*F^(d\*x+c)),x, algorithm="giac")

[Out] log(abs(F^(d\*x)\*F^c\*b + a))/(F^c\*b\*d\*log(F))

### 3.10 $\int \frac{F^{c+dx}}{a+bF^{c+dx}} dx$

**Optimal.** Leaf size=23

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

[Out] Log[a + bF^(c + d\*x)]/(b\*d\*Log[F])

**Rubi [A]** time = 0.0358378, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 31}

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c + d\*x)/(a + bF^(c + d\*x)),x]

[Out] Log[a + bF^(c + d\*x)]/(b\*d\*Log[F])

#### Rule 2246

Int[((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx}}{a+bF^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx} dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= \frac{\log(a + bF^{c+dx})}{bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0052869, size = 23, normalized size = 1.

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c + d\*x)/(a + bF^(c + d\*x)),x]

[Out]  $\text{Log}[a + bF^{(c + d*x)}]/(b*d*\text{Log}[F])$

---

**Maple [A]** time = 0., size = 24, normalized size = 1.

$$\frac{\ln(a + bF^{dx+c})}{bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(d*x+c)}/(a+bF^{(d*x+c)}), x)$

[Out]  $\ln(a+bF^{(d*x+c)})/b/d/\ln(F)$

---

**Maxima [A]** time = 1.03403, size = 31, normalized size = 1.35

$$\frac{\log(F^{dx+c}b + a)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(d*x+c)}/(a+bF^{(d*x+c)}), x, \text{algorithm}="maxima")$

[Out]  $\log(F^{(d*x + c)*b + a})/(b*d*\log(F))$

---

**Fricas [A]** time = 1.50509, size = 50, normalized size = 2.17

$$\frac{\log(F^{dx+c}b + a)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(d*x+c)}/(a+bF^{(d*x+c)}), x, \text{algorithm}="fricas")$

[Out]  $\log(F^{(d*x + c)*b + a})/(b*d*\log(F))$

---

**Sympy [A]** time = 0.181558, size = 17, normalized size = 0.74

$$\frac{\log(F^{c+dx} + \frac{a}{b})}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{**}(d*x+c)/(a+bF^{**}(d*x+c)), x)$

[Out]  $\log(F^{**}(c + d*x) + a/b)/(b*d*\log(F))$

---

**Giac [A]** time = 1.22573, size = 32, normalized size = 1.39

$$\frac{\log(|F^{dx+c}b + a|)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(F^(d*x + c)*b + a))/(b*d*log(F))
```

### 3.11 $\int F^x (a + bF^x)^n dx$

**Optimal.** Leaf size=24

$$\frac{(a + bF^x)^{n+1}}{b(n+1)\log(F)}$$

[Out]  $(a + bF^x)^{(1 + n)}/(b*(1 + n)*\text{Log}[F])$

**Rubi [A]** time = 0.0229501, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2246, 32}

$$\frac{(a + bF^x)^{n+1}}{b(n+1)\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^x\*(a + bF^x)^n,x]

[Out]  $(a + bF^x)^{(1 + n)}/(b*(1 + n)*\text{Log}[F])$

#### Rule 2246

Int[((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int F^x (a + bF^x)^n dx &= \frac{\text{Subst}\left(\int (a + bx)^n dx, x, F^x\right)}{\log(F)} \\ &= \frac{(a + bF^x)^{1+n}}{b(1+n)\log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0240604, size = 24, normalized size = 1.

$$\frac{(a + bF^x)^{n+1}}{bn \log(F) + b \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^x\*(a + bF^x)^n,x]

[Out]  $(a + bF^x)^{(1 + n)}/(b*\text{Log}[F] + b*n*\text{Log}[F])$

---

**Maple [A]** time = 0.003, size = 25, normalized size = 1.

$$\frac{(a + bF^x)^{1+n}}{b(1+n)\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^x\*(a+b\*F^x)^n,x)

[Out] (a+b\*F^x)^(1+n)/b/(1+n)/ln(F)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^x\*(a+b\*F^x)^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.55113, size = 62, normalized size = 2.58

$$\frac{(F^x b + a)(F^x b + a)^n}{(bn + b)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^x\*(a+b\*F^x)^n,x, algorithm="fricas")

[Out] (F^x\*b + a)\*(F^x\*b + a)^n/((b\*n + b)\*log(F))

---

**Sympy [A]** time = 1.425, size = 82, normalized size = 3.42

$$\begin{cases} \frac{x}{a} & \text{for } F = 1 \wedge b = 0 \wedge n = -1 \\ x(a+b)^n & \text{for } F = 1 \\ \frac{F^x a^n}{\log(F)} & \text{for } b = 0 \\ \frac{\log\left(F^x + \frac{a}{b}\right)}{\log(F)} & \text{for } n = -1 \\ \frac{b \log(F)}{bn \log(F) + b \log(F)} + \frac{a(F^x b + a)^n}{bn \log(F) + b \log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*x\*(a+b\*F\*\*x)\*\*n,x)

[Out] Piecewise((x/a, Eq(F, 1) & Eq(b, 0) & Eq(n, -1)), (x\*(a + b)\*\*n, Eq(F, 1)), (F\*\*x\*a\*\*n/log(F), Eq(b, 0)), (log(F\*\*x + a/b)/(b\*log(F)), Eq(n, -1)), (F\*\*x\*b\*(F\*\*x\*b + a)\*\*n/(b\*n\*log(F) + b\*log(F)) + a\*(F\*\*x\*b + a)\*\*n/(b\*n\*log(F)

```
) + b*log(F)), True))
```

---

**Giac [A]** time = 1.26729, size = 32, normalized size = 1.33

$$\frac{(F^x b + a)^{n+1}}{b(n+1)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^x*(a+b*F^x)^n,x, algorithm="giac")
```

```
[Out] (F^x*b + a)^(n + 1)/(b*(n + 1)*log(F))
```

### 3.12 $\int F^{dx} (a + bF^{c+dx})^n dx$

**Optimal.** Leaf size=36

$$\frac{F^{-c} (a + bF^{c+dx})^{n+1}}{bd(n+1)\log(F)}$$

[Out]  $(a + bF^{(c + d*x)})^{(1 + n)}/(b*d*F^c*(1 + n)*\text{Log}[F])$

**Rubi [A]** time = 0.0734713, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2247, 2246, 32}

$$\frac{F^{-c} (a + bF^{c+dx})^{n+1}}{bd(n+1)\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(d*x)}*(a + bF^{(c + d*x)})^n, x]$

[Out]  $(a + bF^{(c + d*x)})^{(1 + n)}/(b*d*F^c*(1 + n)*\text{Log}[F])$

#### Rule 2247

$\text{Int}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)})^{(p_.)}*((G_.)^{((h_.)*((f_.) + (g_.)*(x_)))})^{(m_.)}, x\_Symbol] := \text{Dist}[(G^{(h*(f + g*x))})^m/(F^{(e*(c + d*x))})^n, \text{Int}[(F^{(e*(c + d*x))})^n*(a + b*(F^{(e*(c + d*x))})^n)^p, x], x] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, m, n, p\}, x] \&\& \text{EqQ}[d*e*n*\text{Log}[F], g*h*m*\text{Log}[G]]$

#### Rule 2246

$\text{Int}[(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)}*(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)})^{(p_.)}, x\_Symbol] := \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[(a + b*x)^p, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n, p\}, x]$

#### Rule 32

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int F^{dx} (a + bF^{c+dx})^n dx &= F^{-c} \int F^{c+dx} (a + bF^{c+dx})^n dx \\ &= \frac{F^{-c} \text{Subst}\left(\int (a + bx)^n dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= \frac{F^{-c} (a + bF^{c+dx})^{1+n}}{bd(1+n)\log(F)} \end{aligned}$$



**Mathematica [A]** time = 0.0428595, size = 35, normalized size = 0.97

$$\frac{F^{-c} (a + bF^{c+dx})^{n+1}}{bdn \log(F) + bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(d\*x)\*(a + b\*F^(c + d\*x))^n,x]

[Out] (a + b\*F^(c + d\*x))^(1 + n)/(F^c\*(b\*d\*Log[F] + b\*d\*n\*Log[F]))

**Maple [B]** time = 0.025, size = 81, normalized size = 2.3

$$\frac{e^{d \ln(F)x} e^{n \ln(a + b e^{c \ln(F)} e^{d \ln(F)x})}}{d \ln(F) (1 + n)} + \frac{a e^{n \ln(a + b e^{c \ln(F)} e^{d \ln(F)x})}}{b F^c d \ln(F) (1 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x)\*(a+b\*F^(d\*x+c))^n,x)

[Out] 1/ln(F)/d/(1+n)\*exp(d\*ln(F)\*x)\*exp(n\*ln(a+b\*exp(c\*ln(F))\*exp(d\*ln(F)\*x)))+1/(F^c)/ln(F)/b/d/(1+n)\*a\*exp(n\*ln(a+b\*exp(c\*ln(F))\*exp(d\*ln(F)\*x)))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)\*(a+b\*F^(d\*x+c))^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.59747, size = 100, normalized size = 2.78

$$\frac{(F^{dx+cb} + a)^n \left( \frac{F^{dx+cb}}{F^c} + \frac{a}{F^c} \right)}{(bdn + bd) \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)\*(a+b\*F^(d\*x+c))^n,x, algorithm="fricas")

[Out] (F^(d\*x + c)\*b + a)^n\*(F^(d\*x + c)\*b/F^c + a/F^c)/((b\*d\*n + b\*d)\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x)\*(a+b\*F\*\*(d\*x+c))\*\*n,x)

[Out] Timed out

**Giac [A]** time = 1.26914, size = 82, normalized size = 2.28

$$\frac{(F^{dx}F^c b + a)^n F^{dx}F^c b + (F^{dx}F^c b + a)^n a}{(bn + b)F^c d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x)\*(a+b\*F^(d\*x+c))^n,x, algorithm="giac")

[Out] ((F^(d\*x)\*F^c\*b + a)^n\*F^(d\*x)\*F^c\*b + (F^(d\*x)\*F^c\*b + a)^n\*a)/((b\*n + b)\*F^c\*d\*log(F))

$$3.13 \quad \int F^{c+dx} (a + bF^{c+dx})^n dx$$

**Optimal.** Leaf size=31

$$\frac{(a + bF^{c+dx})^{n+1}}{bd(n+1)\log(F)}$$

[Out] (a + bF^(c + d\*x))^(1 + n)/(b\*d\*(1 + n)\*Log[F])

**Rubi [A]** time = 0.0351259, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 32}

$$\frac{(a + bF^{c+dx})^{n+1}}{bd(n+1)\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c + d\*x)\*(a + bF^(c + d\*x))^n,x]

[Out] (a + bF^(c + d\*x))^(1 + n)/(b\*d\*(1 + n)\*Log[F])

#### Rule 2246

Int[((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_))^(p\_), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int F^{c+dx} (a + bF^{c+dx})^n dx &= \frac{\text{Subst}\left(\int (a + bx)^n dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= \frac{(a + bF^{c+dx})^{1+n}}{bd(1+n)\log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0303065, size = 30, normalized size = 0.97

$$\frac{(a + bF^{c+dx})^{n+1}}{bdn \log(F) + bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c + d\*x)\*(a + bF^(c + d\*x))^n,x]

[Out]  $(a + bF^{(c + dx)})^{(1 + n)}/(b*d*\text{Log}[F] + b*d*n*\text{Log}[F])$

**Maple [A]** time = 0.001, size = 32, normalized size = 1.

$$\frac{(a + bF^{dx+c})^{1+n}}{bd(1+n)\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(d*x+c)*(a+b*F^(d*x+c))^n,x)`

[Out]  $(a+b*F^{(d*x+c)})^{(1+n)}/b/d/(1+n)/\ln(F)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(d*x+c)*(a+b*F^(d*x+c))^n,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.53902, size = 89, normalized size = 2.87

$$\frac{(F^{dx+cb} + a)(F^{dx+cb} + a)^n}{(bdn + bd)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(d*x+c)*(a+b*F^(d*x+c))^n,x, algorithm="fricas")`

[Out]  $(F^{(d*x + c)*b + a}*(F^{(d*x + c)*b + a})^n)/((b*d*n + b*d)*\log(F))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*(a+b*F**(d*x+c))**n,x)`

[Out] Timed out

**Giac [A]** time = 1.22773, size = 42, normalized size = 1.35

$$\frac{(F^{dx+cb} + a)^{n+1}}{bd(n+1)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(d*x+c)*(a+b*F^(d*x+c))^n,x, algorithm="giac")
```

```
[Out] (F^(d*x + c)*b + a)^(n + 1)/(b*d*(n + 1)*log(F))
```

### 3.14 $\int (e^x)^n (a + b(e^x)^n)^p dx$

**Optimal.** Leaf size=25

$$\frac{(a + b(e^x)^n)^{p+1}}{bn(p+1)}$$

[Out] (a + b\*(E^x)^n)^(1 + p)/(b\*n\*(1 + p))

**Rubi [A]** time = 0.0373275, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2246, 32}

$$\frac{(a + b(e^x)^n)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^x)^n\*(a + b\*(E^x)^n)^p,x]

[Out] (a + b\*(E^x)^n)^(1 + p)/(b\*n\*(1 + p))

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (e^x)^n (a + b(e^x)^n)^p dx &= \frac{\text{Subst}\left(\int (a + bx)^p dx, x, (e^x)^n\right)}{n} \\ &= \frac{(a + b(e^x)^n)^{1+p}}{bn(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0650999, size = 24, normalized size = 0.96

$$\frac{(a + b(e^x)^n)^{p+1}}{bnp + bn}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x)^n\*(a + b\*(E^x)^n)^p,x]

[Out]  $(a + b \cdot (E^x)^n)^{(1 + p)} / (b \cdot n + b \cdot n \cdot p)$

**Maple [A]** time = 0.001, size = 25, normalized size = 1.

$$\frac{(a + b(e^x)^n)^{1+p}}{bn(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)^n*(a+b*exp(x)^n)^p,x)`

[Out]  $(a+b \cdot \exp(x)^n)^{(1+p)} / b/n/(1+p)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.50802, size = 66, normalized size = 2.64

$$\frac{(be^{(nx)} + a)(be^{(nx)} + a)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="fricas")`

[Out]  $(b \cdot e^{(n \cdot x)} + a) \cdot (b \cdot e^{(n \cdot x)} + a)^p / (b \cdot n \cdot p + b \cdot n)$

**Sympy [A]** time = 3.21396, size = 80, normalized size = 3.2

$$\left\{ \begin{array}{ll} \frac{x}{a} & \text{for } b = 0 \wedge n = 0 \wedge p = -1 \\ \frac{a^p (e^x)^n}{n} & \text{for } b = 0 \\ x (a + b)^p & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b} + (e^x)^n\right)}{bn} & \text{for } p = -1 \\ \frac{a(a+b(e^x)^n)^p}{bnp+bn} + \frac{b(a+b(e^x)^n)^p (e^x)^n}{bnp+bn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)**n*(a+b*exp(x)**n)**p,x)`

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*exp(x)**n/n, Eq(b, 0)), (x*(a + b)**p, Eq(n, 0)), (log(a/b + exp(x)**n)/(b*n), Eq(p, -1)), (a*(a + b*exp(x)**n)**p/(b*n*p + b*n) + b*(a + b*exp(x)**n)**p*exp(x)**n/(b*n*p + b*n), True))
```

---

**Giac [A]** time = 1.21202, size = 32, normalized size = 1.28

$$\frac{(be^{nx} + a)^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="giac")
```

```
[Out] (b*e^(n*x) + a)^(p + 1)/(b*n*(p + 1))
```



### 3.15 $\int e^{nx} (a + b(e^x)^n)^p dx$

**Optimal.** Leaf size=37

$$\frac{e^{nx} (e^x)^{-n} (a + b(e^x)^n)^{p+1}}{bn(p+1)}$$

[Out]  $(E^{(n*x)}*(a + b*(E^x)^n)^{(1 + p)})/(b*(E^x)^{n*n*(1 + p)})$

**Rubi [A]** time = 0.0589087, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2247, 2246, 32}

$$\frac{e^{nx} (e^x)^{-n} (a + b(e^x)^n)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(n\*x)\*(a + b\*(E^x)^n)^p,x]

[Out]  $(E^{(n*x)}*(a + b*(E^x)^n)^{(1 + p)})/(b*(E^x)^{n*n*(1 + p)})$

#### Rule 2247

Int[((a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)^(p\_)\*((G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)))^(m\_), x\_Symbol] :> Dist[(G^(h\*(f + g\*x)))^m/(F^(e\*(c + d\*x)))^n, Int[(F^(e\*(c + d\*x)))^n\*(a + b\*(F^(e\*(c + d\*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d\*e\*n\*Log[F], g\*h\*m\*Log[G]]

#### Rule 2246

Int[((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)\*((a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int e^{nx} (a + b(e^x)^n)^p dx &= (e^{nx} (e^x)^{-n}) \int (e^x)^n (a + b(e^x)^n)^p dx \\ &= \frac{(e^{nx} (e^x)^{-n}) \text{Subst}(\int (a + bx)^p dx, x, (e^x)^n)}{n} \\ &= \frac{e^{nx} (e^x)^{-n} (a + b(e^x)^n)^{1+p}}{bn(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0424461, size = 36, normalized size = 0.97

$$\frac{e^{nx} (e^x)^{-n} (a + b (e^x)^n)^{p+1}}{bnp + bn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n\*x)\*(a + b\*(E^x)^n)^p,x]

[Out] (E^(n\*x)\*(a + b\*(E^x)^n)^(1 + p))/((E^x)^n\*(b\*n + b\*n\*p))

**Maple [A]** time = 0.015, size = 52, normalized size = 1.4

$$\frac{e^{nx} e^{p \ln(a + b e^{nx})}}{n(1+p)} + \frac{a e^{p \ln(a + b e^{nx})}}{bn(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n\*x)\*(a+b\*exp(x)^n)^p,x)

[Out] 1/n/(1+p)\*exp(n\*x)\*exp(p\*ln(a+b\*exp(n\*x)))+a/b/n/(1+p)\*exp(p\*ln(a+b\*exp(n\*x)))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n\*x)\*(a+b\*exp(x)^n)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.57852, size = 66, normalized size = 1.78

$$\frac{(be^{(nx)} + a)(be^{(nx)} + a)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n\*x)\*(a+b\*exp(x)^n)^p,x, algorithm="fricas")

[Out] (b\*e^(n\*x) + a)\*(b\*e^(n\*x) + a)^p/(b\*n\*p + b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b (e^x)^n)^p e^{nx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n\*x)\*(a+b\*exp(x)\*\*n)\*\*p,x)

[Out] Integral((a + b\*exp(x)\*\*n)\*\*p\*exp(n\*x), x)

**Giac [A]** time = 1.26427, size = 32, normalized size = 0.86

$$\frac{(be^{nx} + a)^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n\*x)\*(a+b\*exp(x)^n)^p,x, algorithm="giac")

[Out] (b\*e^(n\*x) + a)^(p + 1)/(b\*n\*(p + 1))

$$3.16 \quad \int \left( F^{e(c+dx)} \right)^n \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p dx$$

**Optimal.** Leaf size=41

$$\frac{\left( a + b \left( F^{e(c+dx)} \right)^n \right)^{p+1}}{bden(p+1) \log(F)}$$

[Out]  $(a + b*(F^{e*(c + d*x)})^n)^{(1 + p)}/(b*d*e*n*(1 + p)*\text{Log}[F])$

**Rubi [A]** time = 0.0646444, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2246, 32}

$$\frac{\left( a + b \left( F^{e(c+dx)} \right)^n \right)^{p+1}}{bden(p+1) \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{e*(c + d*x)})^n*(a + b*(F^{e*(c + d*x)})^n)^p, x]$

[Out]  $(a + b*(F^{e*(c + d*x)})^n)^{(1 + p)}/(b*d*e*n*(1 + p)*\text{Log}[F])$

#### Rule 2246

$\text{Int}[(F^{e*(c + d*x)})^n*(a + b*(F^{e*(c + d*x)})^n)^p, x] \text{ :> Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[(a + b*x)^p, x], x, (F^{e*(c + d*x)})^n], x] \text{ ;/; FreeQ}\{F, a, b, c, d, e, n, p\}, x]$

#### Rule 32

$\text{Int}[(a + b*x)^m, x] \text{ :> Simp}[(a + b*x)^{m+1}/(b*(m+1)), x] \text{ ;/; FreeQ}\{a, b, m\}, x] \text{ \&\& NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \left( F^{e(c+dx)} \right)^n \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p dx &= \frac{\text{Subst} \left( \int (a + bx)^p dx, x, \left( F^{e(c+dx)} \right)^n \right)}{den \log(F)} \\ &= \frac{\left( a + b \left( F^{e(c+dx)} \right)^n \right)^{1+p}}{bden(1+p) \log(F)} \end{aligned}$$

**Mathematica [F]** time = 0.289949, size = 0, normalized size = 0.

$$\int \left( F^{e(c+dx)} \right)^n \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[(F^{e*(c + d*x)})^n*(a + b*(F^{e*(c + d*x)})^n)^p, x]$

[Out] Integrate[(F^(e\*(c + d\*x)))^n\*(a + b\*(F^(e\*(c + d\*x)))^n)^p, x]

**Maple [A]** time = 0.008, size = 42, normalized size = 1.

$$\frac{\left(a + b \left(F^{e(dx+c)}\right)^n\right)^{1+p}}{bden(1+p)\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(e\*(d\*x+c)))^n\*(a+b\*(F^(e\*(d\*x+c)))^n)^p,x)

[Out] (a+b\*(F^(e\*(d\*x+c)))^n)^(1+p)/b/d/e/n/(1+p)/ln(F)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^(e\*(d\*x+c)))^n\*(a+b\*(F^(e\*(d\*x+c)))^n)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.56138, size = 122, normalized size = 2.98

$$\frac{\left(F^{denx+cen}b + a\right)\left(F^{denx+cen}b + a\right)^P}{\left(bdenp + bden\right)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^(e\*(d\*x+c)))^n\*(a+b\*(F^(e\*(d\*x+c)))^n)^p,x, algorithm="fricas")

[Out] (F^(d\*e\*n\*x + c\*e\*n)\*b + a)\*(F^(d\*e\*n\*x + c\*e\*n)\*b + a)^p/((b\*d\*e\*n\*p + b\*d\*e\*n)\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F\*\*(e\*(d\*x+c)))\*\*n\*(a+b\*(F\*\*(e\*(d\*x+c)))\*\*n)\*\*p,x)

[Out] Timed out

---

**Giac [A]** time = 1.35188, size = 58, normalized size = 1.41

$$\frac{(F^{dnxe+cn}b+a)^{p+1}e^{-1}}{bdn(p+1)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^(e\*(d\*x+c)))^n\*(a+b\*(F^(e\*(d\*x+c))))^p,x, algorithm="giac")

[Out] (F^(d\*n\*x\*e + c\*n\*e)\*b + a)^(p + 1)\*e^(-1)/(b\*d\*n\*(p + 1)\*log(F))

$$3.17 \quad \int \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}} dx$$

**Optimal.** Leaf size=80

$$\frac{\left( F^{e(c+dx)} \right)^{-n} \left( a + b \left( F^{e(c+dx)} \right)^n \right)^{p+1} \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}}}{bden(p+1) \log(F)}$$

[Out] ((a + b\*(F^(e\*(c + d\*x))))^n)^(1 + p)\*(G^(h\*(f + g\*x)))^((d\*e\*n\*Log[F])/(g\*h\*Log[G]))/(b\*d\*e\*(F^(e\*(c + d\*x))))^n\*n\*(1 + p)\*Log[F]

**Rubi [A]** time = 0.134639, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$ , Rules used = {2247, 2246, 32}

$$\frac{\left( F^{e(c+dx)} \right)^{-n} \left( a + b \left( F^{e(c+dx)} \right)^n \right)^{p+1} \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}}}{bden(p+1) \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(e\*(c + d\*x))))^n]^p\*(G^(h\*(f + g\*x)))^((d\*e\*n\*Log[F])/(g\*h\*Log[G])), x]

[Out] ((a + b\*(F^(e\*(c + d\*x))))^n)^(1 + p)\*(G^(h\*(f + g\*x)))^((d\*e\*n\*Log[F])/(g\*h\*Log[G]))/(b\*d\*e\*(F^(e\*(c + d\*x))))^n\*n\*(1 + p)\*Log[F]

#### Rule 2247

Int[((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.))^(p\_.)\*((G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))))^(m\_.), x\_Symbol] :> Dist[(G^(h\*(f + g\*x)))^m/(F^(e\*(c + d\*x)))^n, Int[(F^(e\*(c + d\*x)))^n\*(a + b\*(F^(e\*(c + d\*x))))^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d\*e\*n\*Log[F], g\*h\*m\*Log[G]]

#### Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}} dx &= \left( \left( F^{e(c+dx)} \right)^{-n} \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}} \right) \int \left( F^{e(c+dx)} \right)^n \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p dx \\ &= \frac{\left( \left( F^{e(c+dx)} \right)^{-n} \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}} \right) \text{Subst} \left( \int (a + bx)^p dx, x, \left( F^{e(c+dx)} \right)^n \right)}{den \log(F)} \\ &= \frac{\left( F^{e(c+dx)} \right)^{-n} \left( a + b \left( F^{e(c+dx)} \right)^n \right)^{1+p} \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}}}{b den (1 + p) \log(F)} \end{aligned}$$

**Mathematica [F]** time = 0.302933, size = 0, normalized size = 0.

$$\int \left( a + b \left( F^{e(c+dx)} \right)^n \right)^p \left( G^{h(f+gx)} \right)^{\frac{den \log(F)}{gh \log(G)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*(F^(e\*(c + d\*x)))^n)^p\*(G^(h\*(f + g\*x)))^((d\*e\*n\*Log[F])/(g\*h\*Log[G])), x]

[Out] Integrate[(a + b\*(F^(e\*(c + d\*x)))^n)^p\*(G^(h\*(f + g\*x)))^((d\*e\*n\*Log[F])/(g\*h\*Log[G])), x]

**Maple [F]** time = 0.802, size = 0, normalized size = 0.

$$\int \left( a + b \left( F^{e(dx+c)} \right)^n \right)^p \left( G^{h(gx+f)} \right)^{\frac{nde \ln(F)}{gh \ln(G)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*(F^(e\*(d\*x+c)))^n)^p\*(G^(h\*(g\*x+f)))^(d\*e\*n\*ln(F)/g/h/ln(G)), x)

[Out] int((a+b\*(F^(e\*(d\*x+c)))^n)^p\*(G^(h\*(g\*x+f)))^(d\*e\*n\*ln(F)/g/h/ln(G)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( \left( F^{(dx+c)e} \right)^n b + a \right)^p \left( G^{(gx+f)h} \right)^{\frac{den \log(F)}{gh \log(G)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(e\*(d\*x+c)))^n)^p\*(G^(h\*(g\*x+f)))^(d\*e\*n\*log(F)/g/h/log(G)), x, algorithm="maxima")

[Out] integrate(((F^((d\*x + c)\*e))^n\*b + a)^p\*(G^((g\*x + f)\*h))^(d\*e\*n\*log(F)/(g\*h\*log(G))), x)



**Fricas [A]** time = 1.58859, size = 186, normalized size = 2.32

$$\frac{\left( F^{\frac{denx+cen}{s}} b + F^{\frac{def-ceg}{s}} a \right) \left( F^{denx+cen} b + a \right)^p}{(bdenp + bden) \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(e\*(d\*x+c))))^n)^p\*(G^(h\*(g\*x+f)))^(d\*e\*n\*log(F)/g/h/log(G)),x, algorithm="fricas")

[Out] (F^(d\*e\*n\*x + c\*e\*n)\*F^((d\*e\*f - c\*e\*g)\*n/g)\*b + F^((d\*e\*f - c\*e\*g)\*n/g)\*a) \* (F^(d\*e\*n\*x + c\*e\*n)\*b + a)^p / ((b\*d\*e\*n\*p + b\*d\*e\*n)\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(e\*(d\*x+c))))\*\*n)\*\*p\*(G\*\*(h\*(g\*x+f)))\*\*((d\*e\*n\*ln(F)/g/h/ln(G))),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( \left( F^{(dx+c)e} \right)^n b + a \right)^p \left( G^{(gx+f)h} \right)^{\frac{den \log(F)}{gh \log(G)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(e\*(d\*x+c))))^n)^p\*(G^(h\*(g\*x+f)))^(d\*e\*n\*log(F)/g/h/log(G)),x, algorithm="giac")

[Out] integrate(((F^((d\*x + c)\*e))^n\*b + a)^p\*(G^((g\*x + f)\*h))^(d\*e\*n\*log(F)/(g\*h\*log(G))), x)

$$3.18 \quad \int \frac{e^{2x}}{a+be^x} dx$$

**Optimal.** Leaf size=22

$$\frac{e^x}{b} - \frac{a \log(a + be^x)}{b^2}$$

[Out]  $E^x/b - (a*\text{Log}[a + b*E^x])/b^2$

**Rubi [A]** time = 0.0320372, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{e^x}{b} - \frac{a \log(a + be^x)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(a + b\*E^x), x]

[Out]  $E^x/b - (a*\text{Log}[a + b*E^x])/b^2$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] :> With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{a+be^x} dx &= \text{Subst} \left( \int \frac{x}{a+bx} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{1}{b} - \frac{a}{b(a+bx)} \right) dx, x, e^x \right) \\ &= \frac{e^x}{b} - \frac{a \log(a + be^x)}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0152858, size = 22, normalized size = 1.

$$\frac{e^x}{b} - \frac{a \log(a + be^x)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(a + b\*E^x),x]

[Out] E^x/b - (a\*Log[a + b\*E^x])/b^2

**Maple [A]** time = 0.005, size = 21, normalized size = 1.

$$\frac{e^x}{b} - \frac{a \ln(a + be^x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(a+b\*exp(x)),x)

[Out] exp(x)/b-a\*ln(a+b\*exp(x))/b^2

**Maxima [A]** time = 1.19547, size = 27, normalized size = 1.23

$$\frac{e^x}{b} - \frac{a \log(be^x + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x)),x, algorithm="maxima")

[Out] e^x/b - a\*log(b\*e^x + a)/b^2

**Fricas [A]** time = 1.52141, size = 43, normalized size = 1.95

$$\frac{be^x - a \log(be^x + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x)),x, algorithm="fricas")

[Out] (b\*e^x - a\*log(b\*e^x + a))/b^2

**Sympy [A]** time = 0.134158, size = 20, normalized size = 0.91

$$-\frac{a \log\left(\frac{a}{b} + e^x\right)}{b^2} + \begin{cases} \frac{e^x}{b} & \text{for } b \neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x)),x)

[Out] -a\*log(a/b + exp(x))/b\*\*2 + Piecewise((exp(x)/b, Ne(b, 0)), (x/b, True))

**Giac [A]** time = 1.3437, size = 28, normalized size = 1.27

$$\frac{e^x}{b} - \frac{a \log(|be^x + a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] e^x/b - a*log(abs(b*e^x + a))/b^2
```

$$3.19 \quad \int \frac{e^{2x}}{(a+be^x)^2} dx$$

**Optimal.** Leaf size=27

$$\frac{a}{b^2(a+be^x)} + \frac{\log(a+be^x)}{b^2}$$

[Out] a/(b^2\*(a + b\*E^x)) + Log[a + b\*E^x]/b^2

**Rubi [A]** time = 0.0337294, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{a}{b^2(a+be^x)} + \frac{\log(a+be^x)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(a + b\*E^x)^2,x]

[Out] a/(b^2\*(a + b\*E^x)) + Log[a + b\*E^x]/b^2

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{(a+be^x)^2} dx &= \text{Subst} \left( \int \frac{x}{(a+bx)^2} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)} \right) dx, x, e^x \right) \\ &= \frac{a}{b^2(a+be^x)} + \frac{\log(a+be^x)}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0240013, size = 24, normalized size = 0.89

$$\frac{\frac{a}{a+be^x} + \log(a+be^x)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(a + b\*E^x)^2,x]

[Out] (a/(a + b\*E^x) + Log[a + b\*E^x])/b^2

**Maple [A]** time = 0.007, size = 26, normalized size = 1.

$$\frac{a}{b^2(a + be^x)} + \frac{\ln(a + be^x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(a+b\*exp(x))^2,x)

[Out] a/b^2/(a+b\*exp(x))+ln(a+b\*exp(x))/b^2

**Maxima [A]** time = 1.06359, size = 38, normalized size = 1.41

$$\frac{a}{b^3e^x + ab^2} + \frac{\log(be^x + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^2,x, algorithm="maxima")

[Out] a/(b^3\*e^x + a\*b^2) + log(b\*e^x + a)/b^2

**Fricas [A]** time = 1.45637, size = 70, normalized size = 2.59

$$\frac{(be^x + a)\log(be^x + a) + a}{b^3e^x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^2,x, algorithm="fricas")

[Out] ((b\*e^x + a)\*log(b\*e^x + a) + a)/(b^3\*e^x + a\*b^2)

**Sympy [A]** time = 0.164065, size = 24, normalized size = 0.89

$$\frac{a}{ab^2 + b^3e^x} + \frac{\log\left(\frac{a}{b} + e^x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))\*\*2,x)

[Out] a/(a\*b\*\*2 + b\*\*3\*exp(x)) + log(a/b + exp(x))/b\*\*2

**Giac [A]** time = 1.30259, size = 35, normalized size = 1.3

$$\frac{\log(|be^x + a|)}{b^2} + \frac{a}{(be^x + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(a+b*exp(x))^2,x, algorithm="giac")
```

```
[Out] log(abs(b*e^x + a))/b^2 + a/((b*e^x + a)*b^2)
```

$$3.20 \quad \int \frac{e^{2x}}{(a+be^x)^3} dx$$

**Optimal.** Leaf size=21

$$\frac{e^{2x}}{2a(a+be^x)^2}$$

[Out] E^(2\*x)/(2\*a\*(a + b\*E^x)^2)

**Rubi [A]** time = 0.0226021, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 37}

$$\frac{e^{2x}}{2a(a+be^x)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(a + b\*E^x)^3, x]

[Out] E^(2\*x)/(2\*a\*(a + b\*E^x)^2)

#### Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(g*h*Log[G])/(d*e*Log[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 37

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{e^{2x}}{(a+be^x)^3} dx = \text{Subst} \left( \int \frac{x}{(a+bx)^3} dx, x, e^x \right) = \frac{e^{2x}}{2a(a+be^x)^2}$$

**Mathematica [A]** time = 0.0092525, size = 21, normalized size = 1.

$$\frac{e^{2x}}{2a(a+be^x)^2}$$

Antiderivative was successfully verified.



[In] Integrate[E^(2\*x)/(a + b\*E^x)^3,x]

[Out] E^(2\*x)/(2\*a\*(a + b\*E^x)^2)

**Maple [A]** time = 0.006, size = 29, normalized size = 1.4

$$\frac{a}{2b^2(a+be^x)^2} - \frac{1}{b^2(a+be^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(a+b\*exp(x))^3,x)

[Out] 1/2\*a/b^2/(a+b\*exp(x))^2-1/b^2/(a+b\*exp(x))

**Maxima [B]** time = 1.11568, size = 82, normalized size = 3.9

$$-\frac{be^x}{b^4e^{(2x)} + 2ab^3e^x + a^2b^2} - \frac{a}{2(b^4e^{(2x)} + 2ab^3e^x + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^3,x, algorithm="maxima")

[Out] -b\*e^x/(b^4\*e^(2\*x) + 2\*a\*b^3\*e^x + a^2\*b^2) - 1/2\*a/(b^4\*e^(2\*x) + 2\*a\*b^3\*e^x + a^2\*b^2)

**Fricas [B]** time = 1.41622, size = 78, normalized size = 3.71

$$\frac{2be^x + a}{2(b^4e^{(2x)} + 2ab^3e^x + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^3,x, algorithm="fricas")

[Out] -1/2\*(2\*b\*e^x + a)/(b^4\*e^(2\*x) + 2\*a\*b^3\*e^x + a^2\*b^2)

**Sympy [B]** time = 0.123841, size = 37, normalized size = 1.76

$$\frac{-a - 2be^x}{2a^2b^2 + 4ab^3e^x + 2b^4e^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))\*\*3,x)

[Out] (-a - 2\*b\*exp(x))/(2\*a\*\*2\*b\*\*2 + 4\*a\*b\*\*3\*exp(x) + 2\*b\*\*4\*exp(2\*x))

---

**Giac [A]** time = 1.24601, size = 27, normalized size = 1.29

$$\frac{2be^x + a}{2(be^x + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^3,x, algorithm="giac")

[Out] -1/2\*(2\*b\*e^x + a)/((b\*e^x + a)^2\*b^2)

$$3.21 \quad \int \frac{e^{2x}}{(a+be^x)^4} dx$$

**Optimal.** Leaf size=34

$$\frac{a}{3b^2(a+be^x)^3} - \frac{1}{2b^2(a+be^x)^2}$$

[Out] a/(3\*b^2\*(a + b\*E^x)^3) - 1/(2\*b^2\*(a + b\*E^x)^2)

**Rubi [A]** time = 0.0359505, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{a}{3b^2(a+be^x)^3} - \frac{1}{2b^2(a+be^x)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(a + b\*E^x)^4,x]

[Out] a/(3\*b^2\*(a + b\*E^x)^3) - 1/(2\*b^2\*(a + b\*E^x)^2)

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{(a+be^x)^4} dx &= \text{Subst} \left( \int \frac{x}{(a+bx)^4} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -\frac{a}{b(a+bx)^4} + \frac{1}{b(a+bx)^3} \right) dx, x, e^x \right) \\ &= \frac{a}{3b^2(a+be^x)^3} - \frac{1}{2b^2(a+be^x)^2} \end{aligned}$$

**Mathematica [A]** time = 0.0211904, size = 24, normalized size = 0.71

$$-\frac{a+3be^x}{6b^2(a+be^x)^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(a + b\*E^x)^4,x]

[Out] -(a + 3\*b\*E^x)/(6\*b^2\*(a + b\*E^x)^3)

**Maple [A]** time = 0.006, size = 29, normalized size = 0.9

$$\frac{a}{3b^2(a+be^x)^3} - \frac{1}{2b^2(a+be^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(a+b\*exp(x))^4,x)

[Out] 1/3\*a/b^2/(a+b\*exp(x))^3-1/2/b^2/(a+b\*exp(x))^2

**Maxima [B]** time = 1.08893, size = 115, normalized size = 3.38

$$-\frac{be^x}{2(b^5e^{3x} + 3ab^4e^{2x} + 3a^2b^3e^x + a^3b^2)} - \frac{a}{6(b^5e^{3x} + 3ab^4e^{2x} + 3a^2b^3e^x + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^4,x, algorithm="maxima")

[Out] -1/2\*b\*e^x/(b^5\*e^(3\*x) + 3\*a\*b^4\*e^(2\*x) + 3\*a^2\*b^3\*e^x + a^3\*b^2) - 1/6\*a/(b^5\*e^(3\*x) + 3\*a\*b^4\*e^(2\*x) + 3\*a^2\*b^3\*e^x + a^3\*b^2)

**Fricas [A]** time = 1.52872, size = 105, normalized size = 3.09

$$-\frac{3be^x + a}{6(b^5e^{3x} + 3ab^4e^{2x} + 3a^2b^3e^x + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))^4,x, algorithm="fricas")

[Out] -1/6\*(3\*b\*e^x + a)/(b^5\*e^(3\*x) + 3\*a\*b^4\*e^(2\*x) + 3\*a^2\*b^3\*e^x + a^3\*b^2)

**Sympy [A]** time = 0.146506, size = 51, normalized size = 1.5

$$\frac{-a - 3be^x}{6a^3b^2 + 18a^2b^3e^x + 18ab^4e^{2x} + 6b^5e^{3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(a+b\*exp(x))\*\*4,x)

```
[Out] (-a - 3*b*exp(x))/(6*a**3*b**2 + 18*a**2*b**3*exp(x) + 18*a*b**4*exp(2*x) +
6*b**5*exp(3*x))
```

---

**Giac [A]** time = 1.20175, size = 27, normalized size = 0.79

$$\frac{3be^x + a}{6(be^x + a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(a+b*exp(x))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*e^x + a)/((b*e^x + a)^3*b^2)
```

### 3.22 $\int \frac{e^{4x}}{a+be^{2x}} dx$

**Optimal.** Leaf size=31

$$\frac{e^{2x}}{2b} - \frac{a \log(a + be^{2x})}{2b^2}$$

[Out]  $E^{(2*x)/(2*b)} - (a*\text{Log}[a + b*E^{(2*x)}])/(2*b^2)$

**Rubi [A]** time = 0.0338813, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 43}

$$\frac{e^{2x}}{2b} - \frac{a \log(a + be^{2x})}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(4*x)/(a + b*E^{(2*x)})}, x]$

[Out]  $E^{(2*x)/(2*b)} - (a*\text{Log}[a + b*E^{(2*x)}])/(2*b^2)$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))^{(p_.)*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}}$ , x\_Symbol]  $\rightarrow$  With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^{(f\*h - (c\*g\*h)/d})/(d\*e\*Log[F]), Subst[Int[x^{(Numerator[m] - 1)\*(a + b\*x^{Denominator[m]})^p}, x], x, F^{((e\*(c + d\*x))/Denominator[m])}], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})$ , x\_Symbol]  $\rightarrow$  Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{a+be^{2x}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{a+bx} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{1}{b} - \frac{a}{b(a+bx)} \right) dx, x, e^{2x} \right) \\ &= \frac{e^{2x}}{2b} - \frac{a \log(a + be^{2x})}{2b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0155778, size = 30, normalized size = 0.97

$$\frac{1}{2} \left( \frac{e^{2x}}{b} - \frac{a \log(a + be^{2x})}{b^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/(a + b\*E^(2\*x)),x]

[Out] (E^(2\*x)/b - (a\*Log[a + b\*E^(2\*x)])/b^2)/2

**Maple [A]** time = 0.003, size = 26, normalized size = 0.8

$$\frac{(e^x)^2}{2b} - \frac{a \ln(a + b(e^x)^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(a+b\*exp(2\*x)),x)

[Out] 1/2/b\*exp(x)^2-1/2\*a/b^2\*ln(a+b\*exp(x)^2)

**Maxima [A]** time = 1.13591, size = 34, normalized size = 1.1

$$\frac{e^{(2x)}}{2b} - \frac{a \log(b e^{(2x)} + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x)),x, algorithm="maxima")

[Out] 1/2\*e^(2\*x)/b - 1/2\*a\*log(b\*e^(2\*x) + a)/b^2

**Fricas [A]** time = 1.49142, size = 59, normalized size = 1.9

$$\frac{b e^{(2x)} - a \log(b e^{(2x)} + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*(b\*e^(2\*x) - a\*log(b\*e^(2\*x) + a))/b^2

**Sympy [A]** time = 0.140633, size = 29, normalized size = 0.94

$$-\frac{a \log\left(\frac{a}{b} + e^{2x}\right)}{2b^2} + \begin{cases} \frac{e^{2x}}{2b} & \text{for } 2b \neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x)),x)

[Out] -a\*log(a/b + exp(2\*x))/(2\*b\*\*2) + Piecewise((exp(2\*x)/(2\*b), Ne(2\*b, 0)), (x/b, True))

---

**Giac [A]** time = 1.29887, size = 35, normalized size = 1.13

$$\frac{e^{(2x)}}{2b} - \frac{a \log(|be^{(2x)} + a|)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x)),x, algorithm="giac")

[Out] 1/2\*e^(2\*x)/b - 1/2\*a\*log(abs(b\*e^(2\*x) + a))/b^2



$$3.23 \quad \int \frac{e^{4x}}{(a+be^{2x})^2} dx$$

**Optimal.** Leaf size=37

$$\frac{a}{2b^2(a+be^{2x})} + \frac{\log(a+be^{2x})}{2b^2}$$

[Out] a/(2\*b^2\*(a + b\*E^(2\*x))) + Log[a + b\*E^(2\*x)]/(2\*b^2)

**Rubi [A]** time = 0.037703, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 43}

$$\frac{a}{2b^2(a+be^{2x})} + \frac{\log(a+be^{2x})}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x)/(a + b\*E^(2\*x))^2, x]

[Out] a/(2\*b^2\*(a + b\*E^(2\*x))) + Log[a + b\*E^(2\*x)]/(2\*b^2)

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{(a+be^{2x})^2} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{(a+bx)^2} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)} \right) dx, x, e^{2x} \right) \\ &= \frac{a}{2b^2(a+be^{2x})} + \frac{\log(a+be^{2x})}{2b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0255883, size = 31, normalized size = 0.84

$$\frac{\frac{a}{a+be^{2x}} + \log(a+be^{2x})}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/(a + b\*E^(2\*x))^2,x]

[Out] (a/(a + b\*E^(2\*x)) + Log[a + b\*E^(2\*x)])/(2\*b^2)

**Maple [A]** time = 0.006, size = 32, normalized size = 0.9

$$\frac{\ln(a + b(e^x)^2)}{2b^2} + \frac{a}{2b^2(a + b(e^x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(a+b\*exp(2\*x))^2,x)

[Out] 1/2/b^2\*ln(a+b\*exp(x)^2)+1/2\*a/b^2/(a+b\*exp(x)^2)

**Maxima [A]** time = 1.04988, size = 46, normalized size = 1.24

$$\frac{a}{2(b^3e^{2x} + ab^2)} + \frac{\log(b e^{2x} + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^2,x, algorithm="maxima")

[Out] 1/2\*a/(b^3\*e^(2\*x) + a\*b^2) + 1/2\*log(b\*e^(2\*x) + a)/b^2

**Fricas [A]** time = 1.48779, size = 92, normalized size = 2.49

$$\frac{(be^{2x} + a)\log(b e^{2x} + a) + a}{2(b^3e^{2x} + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^2,x, algorithm="fricas")

[Out] 1/2\*((b\*e^(2\*x) + a)\*log(b\*e^(2\*x) + a) + a)/(b^3\*e^(2\*x) + a\*b^2)

**Sympy [A]** time = 0.133682, size = 32, normalized size = 0.86

$$\frac{a}{2ab^2 + 2b^3e^{2x}} + \frac{\log\left(\frac{a}{b} + e^{2x}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))\*\*2,x)

[Out]  $a/(2*a*b**2 + 2*b**3*exp(2*x)) + \log(a/b + exp(2*x))/(2*b**2)$

---

**Giac [A]** time = 1.27111, size = 43, normalized size = 1.16

$$\frac{\log(|be^{(2x)} + a|)}{2b^2} + \frac{a}{2(be^{(2x)} + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(4*x)/(a+b*exp(2*x))^2,x, algorithm="giac")`

[Out]  $1/2*\log(\text{abs}(b*e^{(2*x)} + a))/b^2 + 1/2*a/((b*e^{(2*x)} + a)*b^2)$

$$3.24 \quad \int \frac{e^{4x}}{(a+be^{2x})^3} dx$$

**Optimal.** Leaf size=23

$$\frac{e^{4x}}{4a(a+be^{2x})^2}$$

[Out]  $E^{(4*x)/(4*a*(a + b*E^{(2*x)})^2)$

**Rubi [A]** time = 0.0267732, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 37}

$$\frac{e^{4x}}{4a(a+be^{2x})^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(4*x)/(a + b*E^{(2*x)})^3}, x]$

[Out]  $E^{(4*x)/(4*a*(a + b*E^{(2*x)})^2)$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(p_.)*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))})}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] || \text{GeQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 37

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)*((c_.) + (d_.)*(x_.)^{(n_.)})}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m + 1)*(c + d*x)^{(n + 1)}}/((b*c - a*d)*(m + 1)), x] /; \text{FreeQ}[\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{(a+be^{2x})^3} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{(a+bx)^3} dx, x, e^{2x} \right) \\ &= \frac{e^{4x}}{4a(a+be^{2x})^2} \end{aligned}$$

**Mathematica [A]** time = 0.009315, size = 23, normalized size = 1.

$$\frac{e^{4x}}{4a(a+be^{2x})^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/(a + b\*E^(2\*x))^3,x]

[Out] E^(4\*x)/(4\*a\*(a + b\*E^(2\*x))^2)

**Maple [A]** time = 0.006, size = 33, normalized size = 1.4

$$\frac{a}{4b^2(a + b(e^x)^2)^2} - \frac{1}{2b^2(a + b(e^x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(a+b\*exp(2\*x))^3,x)

[Out] 1/4\*a/b^2/(a+b\*exp(x)^2)^2-1/2/b^2/(a+b\*exp(x)^2)

**Maxima [B]** time = 1.02851, size = 90, normalized size = 3.91

$$-\frac{be^{2x}}{2(b^4e^{4x} + 2ab^3e^{2x} + a^2b^2)} - \frac{a}{4(b^4e^{4x} + 2ab^3e^{2x} + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^3,x, algorithm="maxima")

[Out] -1/2\*b\*e^(2\*x)/(b^4\*e^(4\*x) + 2\*a\*b^3\*e^(2\*x) + a^2\*b^2) - 1/4\*a/(b^4\*e^(4\*x) + 2\*a\*b^3\*e^(2\*x) + a^2\*b^2)

**Fricas [B]** time = 1.45707, size = 89, normalized size = 3.87

$$\frac{2be^{2x} + a}{4(b^4e^{4x} + 2ab^3e^{2x} + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^3,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*e^(2\*x) + a)/(b^4\*e^(4\*x) + 2\*a\*b^3\*e^(2\*x) + a^2\*b^2)

**Sympy [B]** time = 0.15944, size = 41, normalized size = 1.78

$$\frac{-a - 2be^{2x}}{4a^2b^2 + 8ab^3e^{2x} + 4b^4e^{4x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))\*\*3,x)

[Out]  $(-a - 2*b*\exp(2*x))/(4*a**2*b**2 + 8*a*b**3*\exp(2*x) + 4*b**4*\exp(4*x))$

---

**Giac [A]** time = 1.25, size = 32, normalized size = 1.39

$$-\frac{2be^{(2x)} + a}{4\left(be^{(2x)} + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(4*x)/(a+b*exp(2*x))^3,x, algorithm="giac")`

[Out]  $-1/4*(2*b*e^{(2*x)} + a)/((b*e^{(2*x)} + a)^2*b^2)$

$$3.25 \quad \int \frac{e^{4x}}{(a+be^{2x})^4} dx$$

**Optimal.** Leaf size=38

$$\frac{a}{6b^2(a+be^{2x})^3} - \frac{1}{4b^2(a+be^{2x})^2}$$

[Out] a/(6\*b^2\*(a + b\*E^(2\*x))^3) - 1/(4\*b^2\*(a + b\*E^(2\*x))^2)

**Rubi [A]** time = 0.0366779, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 43}

$$\frac{a}{6b^2(a+be^{2x})^3} - \frac{1}{4b^2(a+be^{2x})^2}$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x)/(a + b\*E^(2\*x))^4,x]

[Out] a/(6\*b^2\*(a + b\*E^(2\*x))^3) - 1/(4\*b^2\*(a + b\*E^(2\*x))^2)

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{(a+be^{2x})^4} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{(a+bx)^4} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{a}{b(a+bx)^4} + \frac{1}{b(a+bx)^3} \right) dx, x, e^{2x} \right) \\ &= \frac{a}{6b^2(a+be^{2x})^3} - \frac{1}{4b^2(a+be^{2x})^2} \end{aligned}$$

**Mathematica [A]** time = 0.0181263, size = 28, normalized size = 0.74

$$-\frac{a+3be^{2x}}{12b^2(a+be^{2x})^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/(a + b\*E^(2\*x))^4,x]

[Out] -(a + 3\*b\*E^(2\*x))/(12\*b^2\*(a + b\*E^(2\*x))^3)

**Maple [A]** time = 0.004, size = 33, normalized size = 0.9

$$-\frac{1}{4b^2(a + b(e^x)^2)^2} + \frac{a}{6b^2(a + b(e^x)^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(a+b\*exp(2\*x))^4,x)

[Out] -1/4/b^2/(a+b\*exp(x)^2)^2+1/6\*a/b^2/(a+b\*exp(x)^2)^3

**Maxima [B]** time = 1.1199, size = 123, normalized size = 3.24

$$-\frac{be^{2x}}{4(b^5e^{6x} + 3ab^4e^{4x} + 3a^2b^3e^{2x} + a^3b^2)} - \frac{a}{12(b^5e^{6x} + 3ab^4e^{4x} + 3a^2b^3e^{2x} + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^4,x, algorithm="maxima")

[Out] -1/4\*b\*e^(2\*x)/(b^5\*e^(6\*x) + 3\*a\*b^4\*e^(4\*x) + 3\*a^2\*b^3\*e^(2\*x) + a^3\*b^2) - 1/12\*a/(b^5\*e^(6\*x) + 3\*a\*b^4\*e^(4\*x) + 3\*a^2\*b^3\*e^(2\*x) + a^3\*b^2)

**Fricas [A]** time = 1.48252, size = 117, normalized size = 3.08

$$-\frac{3be^{2x} + a}{12(b^5e^{6x} + 3ab^4e^{4x} + 3a^2b^3e^{2x} + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^4,x, algorithm="fricas")

[Out] -1/12\*(3\*b\*e^(2\*x) + a)/(b^5\*e^(6\*x) + 3\*a\*b^4\*e^(4\*x) + 3\*a^2\*b^3\*e^(2\*x) + a^3\*b^2)

**Sympy [A]** time = 0.178177, size = 54, normalized size = 1.42

$$\frac{-a - 3be^{2x}}{12a^3b^2 + 36a^2b^3e^{2x} + 36ab^4e^{4x} + 12b^5e^{6x}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))\*\*4,x)

[Out] (-a - 3\*b\*exp(2\*x))/(12\*a\*\*3\*b\*\*2 + 36\*a\*\*2\*b\*\*3\*exp(2\*x) + 36\*a\*b\*\*4\*exp(4\*x) + 12\*b\*\*5\*exp(6\*x))

**Giac [A]** time = 1.34235, size = 32, normalized size = 0.84

$$-\frac{3be^{2x} + a}{12(be^{2x} + a)^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^4,x, algorithm="giac")

[Out] -1/12\*(3\*b\*e^(2\*x) + a)/((b\*e^(2\*x) + a)^3\*b^2)

$$3.26 \quad \int \frac{e^{4x}}{(a+be^{2x})^{2/3}} dx$$

**Optimal.** Leaf size=42

$$\frac{3(a+be^{2x})^{4/3}}{8b^2} - \frac{3a\sqrt[3]{a+be^{2x}}}{2b^2}$$

[Out]  $(-3*a*(a + b*E^{(2*x)})^{(1/3)})/(2*b^2) + (3*(a + b*E^{(2*x)})^{(4/3)})/(8*b^2)$

**Rubi [A]** time = 0.0478876, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2248, 43}

$$\frac{3(a+be^{2x})^{4/3}}{8b^2} - \frac{3a\sqrt[3]{a+be^{2x}}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x)/(a + b\*E^(2\*x))^(2/3), x]

[Out]  $(-3*a*(a + b*E^{(2*x)})^{(1/3)})/(2*b^2) + (3*(a + b*E^{(2*x)})^{(4/3)})/(8*b^2)$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{(a+be^{2x})^{2/3}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{(a+bx)^{2/3}} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{a}{b(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{b} \right) dx, x, e^{2x} \right) \\ &= -\frac{3a\sqrt[3]{a+be^{2x}}}{2b^2} + \frac{3(a+be^{2x})^{4/3}}{8b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0193891, size = 31, normalized size = 0.74

$$\frac{3(be^{2x} - 3a)\sqrt[3]{a+be^{2x}}}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/(a + b\*E^(2\*x))^(2/3), x]

[Out] (3\*(-3\*a + b\*E^(2\*x))\*(a + b\*E^(2\*x))^(1/3))/(8\*b^2)

**Maple [A]** time = 0.016, size = 27, normalized size = 0.6

$$-\frac{-3be^{2x} + 9a}{8b^2} \sqrt[3]{a + be^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(a+b\*exp(2\*x))^(2/3), x)

[Out] -3/8\*(a+b\*exp(2\*x))^(1/3)\*(-b\*exp(2\*x)+3\*a)/b^2

**Maxima [A]** time = 1.15377, size = 43, normalized size = 1.02

$$\frac{3\left(be^{(2x)} + a\right)^{\frac{4}{3}}}{8b^2} - \frac{3\left(be^{(2x)} + a\right)^{\frac{1}{3}}a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^(2/3), x, algorithm="maxima")

[Out] 3/8\*(b\*e^(2\*x) + a)^(4/3)/b^2 - 3/2\*(b\*e^(2\*x) + a)^(1/3)\*a/b^2

**Fricas [A]** time = 1.44982, size = 66, normalized size = 1.57

$$\frac{3\left(be^{(2x)} + a\right)^{\frac{1}{3}}\left(be^{(2x)} - 3a\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^(2/3), x, algorithm="fricas")

[Out] 3/8\*(b\*e^(2\*x) + a)^(1/3)\*(b\*e^(2\*x) - 3\*a)/b^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{4x}}{\left(a + be^{2x}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))\*\*(2/3), x)

[Out] Integral(exp(4\*x)/(a + b\*exp(2\*x))\*\*(2/3), x)

---

**Giac [A]** time = 1.26407, size = 39, normalized size = 0.93

$$\frac{3 \left( (be^{2x} + a)^{\frac{4}{3}} - 4 (be^{2x} + a)^{\frac{1}{3}} a \right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(a+b\*exp(2\*x))^(2/3),x, algorithm="giac")

[Out] 3/8\*((b\*e^(2\*x) + a)^(4/3) - 4\*(b\*e^(2\*x) + a)^(1/3)\*a)/b^2

### 3.27 $\int e^{-nx} (a + be^{nx}) dx$

**Optimal.** Leaf size=16

$$bx - \frac{ae^{-nx}}{n}$$

[Out]  $-(a/(E^{(n*x)*n})) + b*x$

**Rubi [A]** time = 0.0185339, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2248, 43}

$$bx - \frac{ae^{-nx}}{n}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(n\*x))/E^(n\*x), x]

[Out]  $-(a/(E^{(n*x)*n})) + b*x$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_ .) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int e^{-nx} (a + be^{nx}) dx &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{ae^{-nx}}{n} + bx \end{aligned}$$

**Mathematica [A]** time = 0.0082154, size = 16, normalized size = 1.

$$bx - \frac{ae^{-nx}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(n\*x))/E^(n\*x), x]

[Out] -(a/(E^(n\*x)\*n)) + b\*x

**Maple [A]** time = 0.009, size = 24, normalized size = 1.5

$$\frac{b \ln(e^{nx})}{n} - \frac{a}{ne^{nx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(n\*x))/exp(n\*x), x)

[Out] 1/n\*b\*ln(exp(n\*x))-a/exp(n\*x)/n

**Maxima [A]** time = 1.12373, size = 20, normalized size = 1.25

$$bx - \frac{ae^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))/exp(n\*x), x, algorithm="maxima")

[Out] b\*x - a\*e^(-n\*x)/n

**Fricas [A]** time = 1.47915, size = 43, normalized size = 2.69

$$\frac{(bnxe^{(nx)} - a)e^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))/exp(n\*x), x, algorithm="fricas")

[Out] (b\*n\*x\*e^(n\*x) - a)\*e^(-n\*x)/n

**Sympy [A]** time = 0.125903, size = 15, normalized size = 0.94

$$bx + \begin{cases} -\frac{ae^{-nx}}{n} & \text{for } n \neq 0 \\ ax & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))/exp(n\*x), x)

[Out] b\*x + Piecewise((-a\*exp(-n\*x)/n, Ne(n, 0)), (a\*x, True))

---

**Giac [A]** time = 1.25101, size = 20, normalized size = 1.25

$$bx - \frac{ae^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))/exp(n\*x),x, algorithm="giac")

[Out] b\*x - a\*e^(-n\*x)/n

### 3.28 $\int e^{-nx} (a + be^{nx})^2 dx$

**Optimal.** Leaf size=32

$$-\frac{a^2 e^{-nx}}{n} + 2abx + \frac{b^2 e^{nx}}{n}$$

[Out]  $-(a^2/(E^{(n*x)*n})) + (b^2*E^{(n*x)})/n + 2*a*b*x$

**Rubi [A]** time = 0.0342845, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 43}

$$-\frac{a^2 e^{-nx}}{n} + 2abx + \frac{b^2 e^{nx}}{n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*E^{(n*x)})^2/E^{(n*x)}, x]$

[Out]  $-(a^2/(E^{(n*x)*n})) + (b^2*E^{(n*x)})/n + 2*a*b*x$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int e^{-nx} (a + be^{nx})^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^2} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(b^2 + \frac{a^2}{x^2} + \frac{2ab}{x}\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{a^2 e^{-nx}}{n} + \frac{b^2 e^{nx}}{n} + 2abx \end{aligned}$$

**Mathematica [A]** time = 0.0178364, size = 31, normalized size = 0.97

$$\frac{a^2 (-e^{-nx}) + 2abnx + b^2 e^{nx}}{n}$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*E^(n\*x))^2/E^(n\*x), x]

[Out]  $(-a^2/E^{n*x}) + b^2*E^{n*x} + 2*a*b*n*x)/n$

**Maple [A]** time = 0.006, size = 39, normalized size = 1.2

$$\frac{b^2 e^{nx}}{n} + 2 \frac{ab \ln(e^{nx})}{n} - \frac{a^2}{n e^{nx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(n\*x))^2/exp(n\*x), x)

[Out]  $b^2*\exp(n*x)/n+2/n*a*b*\ln(\exp(n*x))-a^2/\exp(n*x)/n$

**Maxima [A]** time = 1.15055, size = 41, normalized size = 1.28

$$2 abx + \frac{b^2 e^{(nx)}}{n} - \frac{a^2 e^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))^2/exp(n\*x), x, algorithm="maxima")

[Out]  $2*a*b*x + b^2*e^{(n*x)}/n - a^2*e^{(-n*x)}/n$

**Fricas [A]** time = 1.48498, size = 73, normalized size = 2.28

$$\frac{(2 abnxe^{(nx)} + b^2e^{(2nx)} - a^2)e^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))^2/exp(n\*x), x, algorithm="fricas")

[Out]  $(2*a*b*n*x*e^{(n*x)} + b^2*e^{(2*n*x)} - a^2)*e^{(-n*x)}/n$

**Sympy [A]** time = 0.183723, size = 39, normalized size = 1.22

$$2abx + \begin{cases} \frac{-a^2 ne^{-nx} + b^2 ne^{nx}}{n^2} & \text{for } n^2 \neq 0 \\ x(a^2 + b^2) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))\*\*2/exp(n\*x), x)

[Out]  $2*a*b*x + \text{Piecewise}((( -a**2*n*\exp(-n*x) + b**2*n*\exp(n*x))/n**2, \text{Ne}(n**2, 0)), (x*(a**2 + b**2), \text{True}))$

---

**Giac [A]** time = 1.27072, size = 41, normalized size = 1.28

$$2abx + \frac{b^2e^{nx}}{n} - \frac{a^2e^{-nx}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))^2/exp(n\*x),x, algorithm="giac")

[Out] 2\*a\*b\*x + b^2\*e^(n\*x)/n - a^2\*e^(-n\*x)/n

### 3.29 $\int e^{-nx} (a + be^{nx})^3 dx$

**Optimal.** Leaf size=52

$$3a^2bx - \frac{a^3e^{-nx}}{n} + \frac{3ab^2e^{nx}}{n} + \frac{b^3e^{2nx}}{2n}$$

[Out]  $-(a^3/(E^{(n*x)*n})) + (3*a*b^2*E^{(n*x)})/n + (b^3*E^{(2*n*x)})/(2*n) + 3*a^2*b*x$

**Rubi [A]** time = 0.0416476, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 43}

$$3a^2bx - \frac{a^3e^{-nx}}{n} + \frac{3ab^2e^{nx}}{n} + \frac{b^3e^{2nx}}{2n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + bE^{(n*x)})^3/E^{(n*x)}, x]$

[Out]  $-(a^3/(E^{(n*x)*n})) + (3*a*b^2*E^{(n*x)})/n + (b^3*E^{(2*n*x)})/(2*n) + 3*a^2*b*x$

#### Rule 2248

$\text{Int}[(a + b*(F)^{(e*(c + d*x))})^{(p)}*(G)^{(h*(f + g*x))}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{(e*(c + d*x))/\text{Denominator}[m]}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])]$

#### Rubi steps

$$\begin{aligned} \int e^{-nx} (a + be^{nx})^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^2} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2b}{x} + b^3x\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{a^3e^{-nx}}{n} + \frac{3ab^2e^{nx}}{n} + \frac{b^3e^{2nx}}{2n} + 3a^2bx \end{aligned}$$

**Mathematica [A]** time = 0.0249187, size = 48, normalized size = 0.92

$$\frac{3a^2bnx + a^3(-e^{-nx}) + 3ab^2e^{nx} + \frac{1}{2}b^3e^{2nx}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(n\*x))^3/E^(n\*x), x]

[Out]  $(-a^3/E^{(n*x)}) + 3*a*b^2*E^{(n*x)} + (b^3*E^{(2*n*x)})/2 + 3*a^2*b*n*x)/n$

**Maple [A]** time = 0.006, size = 57, normalized size = 1.1

$$\frac{b^3 (e^{nx})^2}{2n} + 3 \frac{ab^2 e^{nx}}{n} + 3 \frac{a^2 b \ln(e^{nx})}{n} - \frac{a^3}{ne^{nx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(n\*x))^3/exp(n\*x), x)

[Out]  $1/2/n*b^3*exp(n*x)^2+3*a*b^2*exp(n*x)/n+3/n*a^2*b*\ln(exp(n*x))-a^3/exp(n*x)/n$

**Maxima [A]** time = 1.15863, size = 63, normalized size = 1.21

$$3a^2bx + \frac{b^3 e^{(2nx)}}{2n} + \frac{3ab^2 e^{(nx)}}{n} - \frac{a^3 e^{(-nx)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))^3/exp(n\*x), x, algorithm="maxima")

[Out]  $3*a^2*b*x + 1/2*b^3*e^{(2*n*x)}/n + 3*a*b^2*e^{(n*x)}/n - a^3*e^{(-n*x)}/n$

**Fricas [A]** time = 1.49152, size = 111, normalized size = 2.13

$$\frac{(6a^2bnxe^{(nx)} + b^3e^{(3nx)} + 6ab^2e^{(2nx)} - 2a^3)e^{(-nx)}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))^3/exp(n\*x), x, algorithm="fricas")

[Out]  $1/2*(6*a^2*b*n*x*e^{(n*x)} + b^3*e^{(3*n*x)} + 6*a*b^2*e^{(2*n*x)} - 2*a^3)*e^{(-n*x)}/n$

**Sympy [A]** time = 0.190626, size = 73, normalized size = 1.4

$$3a^2bx + \begin{cases} \frac{-2a^3n^2e^{-nx}+6ab^2n^2e^{nx}+b^3n^2e^{2nx}}{2n^3} & \text{for } 2n^3 \neq 0 \\ x(a^3 + 3ab^2 + b^3) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(n\*x))\*\*3/exp(n\*x), x)

```
[Out] 3*a**2*b*x + Piecewise((( -2*a**3*n**2*exp(-n*x) + 6*a*b**2*n**2*exp(n*x) +
b**3*n**2*exp(2*n*x))/(2*n**3), Ne(2*n**3, 0)), (x*(a**3 + 3*a*b**2 + b**3)
, True))
```

**Giac [A]** time = 1.30001, size = 63, normalized size = 1.21

$$3a^2bx + \frac{b^3e^{2nx}}{2n} + \frac{3ab^2e^{nx}}{n} - \frac{a^3e^{-nx}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^3/exp(n*x),x, algorithm="giac")
```

```
[Out] 3*a^2*b*x + 1/2*b^3*e^(2*n*x)/n + 3*a*b^2*e^(n*x)/n - a^3*e^(-n*x)/n
```

### 3.30 $\int \frac{e^{-nx}}{a+be^{nx}} dx$

**Optimal.** Leaf size=40

$$\frac{b \log(a + be^{nx})}{a^2 n} - \frac{bx}{a^2} - \frac{e^{-nx}}{an}$$

[Out]  $-(1/(aE^{(n*x)*n})) - (b*x)/a^2 + (b*Log[a + bE^{(n*x)}])/(a^2*n)$

**Rubi [A]** time = 0.0438791, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 44}

$$\frac{b \log(a + be^{nx})}{a^2 n} - \frac{bx}{a^2} - \frac{e^{-nx}}{an}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(E^{(n*x)}*(a + bE^{(n*x)})), x]$

[Out]  $-(1/(aE^{(n*x)*n})) - (b*x)/a^2 + (b*Log[a + bE^{(n*x)}])/(a^2*n)$

#### Rule 2248

$\text{Int}[(a + b \cdot (F)^{(e \cdot (c + d \cdot x))})^{(p)} \cdot (G)^{(h \cdot (f + g \cdot x))}], x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g \cdot h \cdot \text{Log}[G]) / (d \cdot e \cdot \text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m] \cdot G^{(f \cdot h - (c \cdot g \cdot h) / d)}) / (d \cdot e \cdot \text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1) \cdot (a + b \cdot x^{\text{Denominator}[m]})^p}, x], x, F^{(e \cdot (c + d \cdot x)) / \text{Denominator}[m]}], x] /; \text{LeQ}[m, -1] \mid \mid \text{GeQ}[m, 1]] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 44

$\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \& \& \text{NeQ}[b \cdot c - a \cdot d, 0] \& \& \text{ILtQ}[m, 0] \& \& \text{IntegerQ}[n] \& \& !(\text{IGtQ}[n, 0] \& \& \text{LtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-nx}}{a + be^{nx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{e^{-nx}}{an} - \frac{bx}{a^2} + \frac{b \log(a + be^{nx})}{a^2 n} \end{aligned}$$

**Mathematica [A]** time = 0.0382528, size = 34, normalized size = 0.85

$$-\frac{-b \log(a + be^{nx}) + ae^{-nx} + bnx}{a^2 n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(n\*x)\*(a + b\*E^(n\*x))),x]

[Out]  $-\left(\frac{a}{E^{n x}} + b n x - b \operatorname{Log}[a + b E^{n x}]\right) / \left(a^2 n\right)$

**Maple [A]** time = 0.01, size = 47, normalized size = 1.2

$$-\frac{1}{a e^{n x n}} - \frac{b \ln \left(e^{n x}\right)}{a^2 n} + \frac{b \ln \left(a + b e^{n x}\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(n\*x)/(a+b\*exp(n\*x)),x)

[Out]  $-1/a/\exp(n x) / n - 1/n * b / a^2 * \ln(\exp(n x)) + b * \ln(a + b * \exp(n x)) / a^2 / n$

**Maxima [A]** time = 1.04324, size = 43, normalized size = 1.08

$$-\frac{e^{(-n x)}}{a n} + \frac{b \log \left(a e^{(-n x)} + b\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x)),x, algorithm="maxima")

[Out]  $-e^{(-n x)} / \left(a * n\right) + b * \log \left(a * e^{(-n x)} + b\right) / \left(a^2 * n\right)$

**Fricas [A]** time = 1.53373, size = 95, normalized size = 2.38

$$\frac{\left(b n x e^{(n x)} - b e^{(n x)} \log \left(b e^{(n x)} + a\right) + a\right) e^{(-n x)}}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x)),x, algorithm="fricas")

[Out]  $-\left(b * n * x * e^{(n x)} - b * e^{(n x)} * \log \left(b * e^{(n x)} + a\right) + a\right) * e^{(-n x)} / \left(a^2 * n\right)$

**Sympy [A]** time = 0.361883, size = 49, normalized size = 1.22

$$\begin{cases} -\frac{e^{-n x}}{a n} & \text{for } a n \neq 0 \\ x \left(\frac{a n}{a^2} + \frac{a-b}{a^2}\right) & \text{otherwise} \end{cases} - \frac{b x}{a^2} + \frac{b \log \left(\frac{a}{b} + e^{n x}\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x)),x)

[Out]  $\operatorname{Piecewise}\left(\left(-\exp(-n x) / \left(a * n\right), \operatorname{Ne}\left(a * n, 0\right)\right), \left(x * \left(b / a^{** 2} + \left(a - b\right) / a^{** 2}\right), \operatorname{True}\right)\right) - b * x / a^{** 2} + b * \log \left(a / b + \exp(n x)\right) / \left(a^{** 2} * n\right)$

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**Giac [A]** time = 1.26291, size = 51, normalized size = 1.27

$$-\frac{\frac{bnx}{a^2} + \frac{e^{-nx}}{a} - \frac{b \log(|be^{nx}+a|)}{a^2}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x)),x, algorithm="giac")

[Out] -(b\*n\*x/a^2 + e^(-n\*x)/a - b\*log(abs(b\*e^(n\*x) + a))/a^2)/n



$$3.31 \quad \int \frac{e^{-nx}}{(a+be^{nx})^2} dx$$

**Optimal.** Leaf size=61

$$-\frac{b}{a^2n(a+be^{nx})} + \frac{2b \log(a+be^{nx})}{a^3n} - \frac{2bx}{a^3} - \frac{e^{-nx}}{a^2n}$$

[Out]  $-(1/(a^2E^{(n*x)*n})) - b/(a^2*(a + bE^{(n*x)})*n) - (2*b*x)/a^3 + (2*b*Log[a + bE^{(n*x)}])/(a^3*n)$

**Rubi [A]** time = 0.0572723, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 44}

$$-\frac{b}{a^2n(a+be^{nx})} + \frac{2b \log(a+be^{nx})}{a^3n} - \frac{2bx}{a^3} - \frac{e^{-nx}}{a^2n}$$

Antiderivative was successfully verified.

[In] Int[1/(E^{(n\*x)\*(a + bE^{(n\*x)})^2}), x]

[Out]  $-(1/(a^2E^{(n*x)*n})) - b/(a^2*(a + bE^{(n*x)})*n) - (2*b*x)/a^3 + (2*b*Log[a + bE^{(n*x)}])/(a^3*n)$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b\*c - a\*d, 0] & & ILtQ[m, 0] & & IntegerQ[n] & & !(IGtQ[n, 0] & & LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-nx}}{(a+be^{nx})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{e^{-nx}}{a^2n} - \frac{b}{a^2(a+be^{nx})n} - \frac{2bx}{a^3} + \frac{2b \log(a+be^{nx})}{a^3n} \end{aligned}$$

**Mathematica [A]** time = 0.107794, size = 49, normalized size = 0.8

$$\frac{a\left(\frac{b}{a+be^{nx}} + e^{-nx}\right) - 2b \log(a+be^{nx}) + 2bnx}{a^3n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(n\*x)\*(a + b\*E^(n\*x))^2),x]

[Out] -((a\*(E^(-(n\*x)) + b/(a + b\*E^(n\*x)))) + 2\*b\*n\*x - 2\*b\*Log[a + b\*E^(n\*x)])/(a^3\*n))

**Maple [A]** time = 0.01, size = 67, normalized size = 1.1

$$-\frac{1}{a^2 e^{nx} n} - 2 \frac{b \ln(e^{nx})}{a^3 n} - \frac{b}{a^2 (a + b e^{nx}) n} + 2 \frac{b \ln(a + b e^{nx})}{a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(n\*x)/(a+b\*exp(n\*x))^2,x)

[Out] -1/a^2/exp(n\*x)/n-2/n/a^3\*b\*ln(exp(n\*x))-b/a^2/(a+b\*exp(n\*x))/n+2\*b\*ln(a+b\*exp(n\*x))/a^3/n

**Maxima [A]** time = 1.10071, size = 77, normalized size = 1.26

$$\frac{b^2}{(a^4 e^{-nx} + a^3 b)n} - \frac{e^{-nx}}{a^2 n} + \frac{2 b \log(a e^{-nx} + b)}{a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^2,x, algorithm="maxima")

[Out] b^2/((a^4\*e^(-n\*x) + a^3\*b)\*n) - e^(-n\*x)/(a^2\*n) + 2\*b\*log(a\*e^(-n\*x) + b)/(a^3\*n)

**Fricas [A]** time = 1.52061, size = 198, normalized size = 3.25

$$-\frac{2 b^2 n x e^{(2 n x)} + a^2 + 2 (a b n x + a b) e^{(n x)} - 2 (b^2 e^{(2 n x)} + a b e^{(n x)}) \log(b e^{(n x)} + a)}{a^3 b n e^{(2 n x)} + a^4 n e^{(n x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^2,x, algorithm="fricas")

[Out] -(2\*b^2\*n\*x\*e^(2\*n\*x) + a^2 + 2\*(a\*b\*n\*x + a\*b)\*e^(n\*x) - 2\*(b^2\*e^(2\*n\*x) + a\*b\*e^(n\*x))\*log(b\*e^(n\*x) + a))/(a^3\*b\*n\*e^(2\*n\*x) + a^4\*n\*e^(n\*x))

**Sympy [A]** time = 0.201925, size = 78, normalized size = 1.28

$$-\frac{b}{a^3 n + a^2 b n e^{nx}} + \begin{cases} -\frac{e^{-nx}}{a^2 n} \\ x \left( \frac{2b}{a^3} + \frac{a-2b}{a^3} \right) \end{cases} \begin{matrix} \text{for } a^2 n \neq 0 \\ \text{otherwise} \end{matrix} - \frac{2bx}{a^3} + \frac{2b \log\left(\frac{a}{b} + e^{nx}\right)}{a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))\*\*2,x)

[Out]  $-b/(a^{3n} + a^{2n}b^n \exp(n*x)) + \text{Piecewise}((- \exp(-n*x)/(a^{2n}), \text{Ne}(a^{2n}, 0)), (x*(2b/a^3 + (a - 2b)/a^3), \text{True})) - 2*b*x/a^3 + 2*b*\log(a/b + \exp(n*x))/(a^{3n})$

**Giac [A]** time = 1.28641, size = 80, normalized size = 1.31

$$\frac{\frac{2bnx}{a^3} - \frac{2b \log(|be^{nx}+a|)}{a^3} + \frac{2be^{nx}+a}{(be^{2nx}+ae^{nx})a^2}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^2,x, algorithm="giac")

[Out]  $-(2*b*n*x/a^3 - 2*b*\log(\text{abs}(b*e^{(n*x)} + a))/a^3 + (2*b*e^{(n*x)} + a)/((b*e^{(2*n*x)} + a*e^{(n*x)})*a^2))/n$

$$3.32 \quad \int \frac{e^{-nx}}{(a+be^{nx})^3} dx$$

**Optimal.** Leaf size=83

$$-\frac{2b}{a^3n(a+be^{nx})} - \frac{b}{2a^2n(a+be^{nx})^2} + \frac{3b \log(a+be^{nx})}{a^4n} - \frac{3bx}{a^4} - \frac{e^{-nx}}{a^3n}$$

[Out]  $-(1/(a^3E^{(n*x)*n})) - b/(2*a^2*(a + b*E^{(n*x)})^2*n) - (2*b)/(a^3*(a + b*E^{(n*x)})*n) - (3*b*x)/a^4 + (3*b*Log[a + b*E^{(n*x)}])/(a^4*n)$

**Rubi [A]** time = 0.0683861, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 44}

$$-\frac{2b}{a^3n(a+be^{nx})} - \frac{b}{2a^2n(a+be^{nx})^2} + \frac{3b \log(a+be^{nx})}{a^4n} - \frac{3bx}{a^4} - \frac{e^{-nx}}{a^3n}$$

Antiderivative was successfully verified.

[In] Int[1/(E^{(n\*x)\*(a + b\*E^{(n\*x)})^3}), x]

[Out]  $-(1/(a^3E^{(n*x)*n})) - b/(2*a^2*(a + b*E^{(n*x)})^2*n) - (2*b)/(a^3*(a + b*E^{(n*x)})*n) - (3*b*x)/a^4 + (3*b*Log[a + b*E^{(n*x)}])/(a^4*n)$

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b\*c - a\*d, 0] & & ILtQ[m, 0] & & IntegerQ[n] & & !(IGtQ[n, 0] & & LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-nx}}{(a+be^{nx})^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3} dx, x, e^{nx}\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3b}{a^4x} + \frac{b^2}{a^2(a+bx)^3} + \frac{2b^2}{a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)}\right) dx, x, e^{nx}\right)}{n} \\ &= -\frac{e^{-nx}}{a^3n} - \frac{b}{2a^2(a+be^{nx})^2n} - \frac{2b}{a^3(a+be^{nx})n} - \frac{3bx}{a^4} + \frac{3b \log(a+be^{nx})}{a^4n} \end{aligned}$$

**Mathematica [A]** time = 0.133166, size = 69, normalized size = 0.83

$$\frac{\frac{a^2b}{(a+be^{nx})^2} + \frac{4ab}{a+be^{nx}} - 6b \log(a+be^{nx}) + 2ae^{-nx} + 6bnx}{2a^4n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(n\*x)\*(a + b\*E^(n\*x))^3), x]

[Out]  $-\frac{(2a)/E^{nx} + (a^2b)/(a + bE^{nx})^2 + (4ab)/(a + bE^{nx}) + 6bnx - 6b \operatorname{Log}[a + bE^{nx}]}{(2a^4n)}$

**Maple [A]** time = 0.01, size = 86, normalized size = 1.

$$-\frac{1}{a^3 e^{nx} n} - 3 \frac{b \ln(e^{nx})}{na^4} - \frac{b}{2a^2(a + be^{nx})^2 n} + 3 \frac{b \ln(a + be^{nx})}{na^4} - 2 \frac{b}{a^3(a + be^{nx})n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(n\*x)/(a+b\*exp(n\*x))^3, x)

[Out]  $-1/a^3/\exp(n*x)/n - 3/n/a^4*b*\ln(\exp(n*x)) - 1/2*b/a^2/(a+b*\exp(n*x))^2/n + 3*b*1n(a+b*\exp(n*x))/a^4/n - 2*b/a^3/(a+b*\exp(n*x))/n$

**Maxima [A]** time = 1.15054, size = 115, normalized size = 1.39

$$\frac{6ab^2e^{-nx} + 5b^3}{2(a^5be^{-nx} + a^6e^{-2nx} + a^4b^2)n} - \frac{e^{-nx}}{a^3n} + \frac{3b \log(ae^{-nx} + b)}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^3, x, algorithm="maxima")

[Out]  $1/2*(6*a*b^2*e^{-n*x} + 5*b^3)/((2*a^5*b*e^{-n*x} + a^6*e^{-2*n*x} + a^4*b^2)*n) - e^{-n*x}/(a^3*n) + 3*b*\log(a*e^{-n*x} + b)/(a^4*n)$

**Fricas [A]** time = 1.59791, size = 328, normalized size = 3.95

$$\frac{6b^3nxe^{3nx} + 2a^3 + 6(2ab^2nx + ab^2)e^{2nx} + 3(2a^2bnx + 3a^2b)e^{nx} - 6(b^3e^{3nx} + 2ab^2e^{2nx} + a^2be^{nx}) \log(b e^{nx} + a)}{2(a^4b^2ne^{3nx} + 2a^5bne^{2nx} + a^6ne^{nx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^3, x, algorithm="fricas")

[Out]  $-1/2*(6*b^3*n*x*e^{3*n*x} + 2*a^3 + 6*(2*a*b^2*n*x + a*b^2)*e^{2*n*x} + 3*(2*a^2*b*n*x + 3*a^2*b)*e^{n*x} - 6*(b^3*e^{3*n*x} + 2*a*b^2*e^{2*n*x} + a^2*b*e^{n*x})*\log(b*e^{n*x} + a))/(a^4*b^2*n*e^{3*n*x} + 2*a^5*b*n*e^{2*n*x} + a^6*n*e^{n*x})$

**Sympy [A]** time = 0.235815, size = 114, normalized size = 1.37

$$\frac{-5ab - 4b^2e^{nx}}{2a^5n + 4a^4bne^{nx} + 2a^3b^2ne^{2nx}} + \begin{cases} -\frac{e^{-nx}}{a^3n} & \text{for } a^3n \neq 0 \\ x\left(\frac{3b}{a^4} + \frac{a-3b}{a^4}\right) & \text{otherwise} \end{cases} - \frac{3bx}{a^4} + \frac{3b \log\left(\frac{a}{b} + e^{nx}\right)}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))\*\*3,x)

[Out]  $(-5*a*b - 4*b**2*exp(n*x))/(2*a**5*n + 4*a**4*b*n*exp(n*x) + 2*a**3*b**2*n*exp(2*n*x)) + \text{Piecewise}((-exp(-n*x)/(a**3*n), \text{Ne}(a**3*n, 0)), (x*(3*b/a**4 + (a - 3*b)/a**4), \text{True})) - 3*b*x/a**4 + 3*b*log(a/b + exp(n*x))/(a**4*n)$

**Giac [A]** time = 1.27695, size = 103, normalized size = 1.24

$$\frac{\frac{6bnx}{a^4} - \frac{6b \log(|be^{nx}+a|)}{a^4} + \frac{(6ab^2e^{2nx}+9a^2be^{nx}+2a^3)e^{-nx}}{(be^{nx}+a)^2 a^4}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(n\*x)/(a+b\*exp(n\*x))^3,x, algorithm="giac")

[Out]  $-1/2*(6*b*n*x/a^4 - 6*b*log(abs(b*e^{n*x} + a))/a^4 + (6*a*b^2*e^{2*n*x} + 9*a^2*b*e^{n*x} + 2*a^3)*e^{-n*x}/((b*e^{n*x} + a)^2*a^4))/n$

$$3.33 \quad \int \frac{f^{a+bx}}{c+df^{e+2bx}} dx$$

**Optimal.** Leaf size=50

$$\frac{f^{a-\frac{e}{2}} \tan^{-1} \left( \frac{\sqrt{d} f^{\frac{bx+\frac{e}{2}}{2}}}{\sqrt{c}} \right)}{b\sqrt{c}\sqrt{d} \log(f)}$$

[Out] (f^(a - e/2)\*ArcTan[(Sqrt[d]\*f^(e/2 + b\*x))/Sqrt[c]])/(b\*Sqrt[c]\*Sqrt[d]\*Log[f])

**Rubi [A]** time = 0.077644, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2249, 205}

$$\frac{f^{a-\frac{e}{2}} \tan^{-1} \left( \frac{\sqrt{d} f^{\frac{bx+\frac{e}{2}}{2}}}{\sqrt{c}} \right)}{b\sqrt{c}\sqrt{d} \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x)/(c + d\*f^(e + 2\*b\*x)), x]

[Out] (f^(a - e/2)\*ArcTan[(Sqrt[d]\*f^(e/2 + b\*x))/Sqrt[c]])/(b\*Sqrt[c]\*Sqrt[d]\*Log[f])

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx}}{c+df^{e+2bx}} dx &= \frac{\text{Subst} \left( \int \frac{1}{c+df^{-2a+ex^2}} dx, x, f^{a+bx} \right)}{b \log(f)} \\ &= \frac{f^{a-\frac{e}{2}} \tan^{-1} \left( \frac{\sqrt{d} f^{\frac{e}{2}+bx}}{\sqrt{c}} \right)}{b\sqrt{c}\sqrt{d} \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0250429, size = 50, normalized size = 1.

$$\frac{f^{a-\frac{e}{2}} \tan^{-1} \left( \frac{\sqrt{d} f^{\frac{e}{2}+bx}}{\sqrt{c}} \right)}{b\sqrt{c}\sqrt{d} \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x)/(c + d\*f^(e + 2\*b\*x)),x]

[Out] (f^(a - e/2)\*ArcTan[(Sqrt[d]\*f^(e/2 + b\*x))/Sqrt[c]])/(b\*Sqrt[c]\*Sqrt[d]\*Log[f])

**Maple [B]** time = 0.046, size = 91, normalized size = 1.8

$$-\frac{f^a}{2b \ln(f)} \ln\left(f^{bx+a} - f^a c \frac{1}{\sqrt{-f^e cd}}\right) \frac{1}{\sqrt{-f^e cd}} + \frac{f^a}{2b \ln(f)} \ln\left(f^{bx+a} + f^a c \frac{1}{\sqrt{-f^e cd}}\right) \frac{1}{\sqrt{-f^e cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x+a)/(c+d\*f^(2\*b\*x+e)),x)

[Out] -1/2/(-f^e\*c\*d)^(1/2)\*f^a/b/ln(f)\*ln(f^(b\*x+a))-1/(-f^e\*c\*d)^(1/2)\*f^a\*c+1/2/(-f^e\*c\*d)^(1/2)\*f^a/b/ln(f)\*ln(f^(b\*x+a))+1/(-f^e\*c\*d)^(1/2)\*f^a\*c

**Maxima [A]** time = 1.59135, size = 100, normalized size = 2.

$$\frac{f^a \log\left(\frac{df^{bx+a+e}-\sqrt{-cdf^e}f^a}{df^{bx+a+e}+\sqrt{-cdf^e}f^a}\right)}{2\sqrt{-cdf^e}b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="maxima")

[Out] 1/2\*f^a\*log((d\*f^(b\*x + a + e) - sqrt(-c\*d\*f^e)\*f^a)/(d\*f^(b\*x + a + e) + sqrt(-c\*d\*f^e)\*f^a))/(sqrt(-c\*d\*f^e)\*b\*log(f))

**Fricas [A]** time = 1.79711, size = 397, normalized size = 7.94

$$\left[ -\frac{\sqrt{-cdf^{-2a+e}} \log\left(\frac{df^{2bx+2a}f^{-2a+e}-2\sqrt{-cdf^{-2a+e}}f^{bx+a}-c}{df^{2bx+2a}f^{-2a+e}+c}\right)}{2bcd f^{-2a+e} \log(f)}, -\frac{\sqrt{cdf^{-2a+e}} \arctan\left(\frac{\sqrt{cdf^{-2a+e}}}{df^{bx+a}f^{-2a+e}}\right)}{bcd f^{-2a+e} \log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="fricas")

[Out] [-1/2\*sqrt(-c\*d\*f^(-2\*a + e))\*log((d\*f^(2\*b\*x + 2\*a)\*f^(-2\*a + e) - 2\*sqrt(-c\*d\*f^(-2\*a + e))\*f^(b\*x + a) - c)/(d\*f^(2\*b\*x + 2\*a)\*f^(-2\*a + e) + c))/(b\*c\*d\*f^(-2\*a + e)\*log(f)), -sqrt(c\*d\*f^(-2\*a + e))\*arctan(sqrt(c\*d\*f^(-2\*a + e))/(d\*f^(b\*x + a)\*f^(-2\*a + e)))/(b\*c\*d\*f^(-2\*a + e)\*log(f)]]



**Sympy [A]** time = 1.60283, size = 51, normalized size = 1.02

$$\text{RootSum}\left(4z^2b^2cde^{e\log(f)}\log(f)^2 + e^{2a\log(f)}, (i \mapsto i\log(2ibc\log(f) + f^{a+bx}))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x+a)/(c+d\*f\*\*(2\*b\*x+e)), x)

[Out] RootSum(4\*\_z\*\*2\*b\*\*2\*c\*d\*exp(e\*log(f))\*log(f)\*\*2 + exp(2\*a\*log(f)), Lambda(\_i, \_i\*log(2\*\_i\*b\*c\*log(f) + f\*\*(a + b\*x))))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx+a}}{df^{2bx+e} + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x+a)/(c+d\*f^(2\*b\*x+e)), x, algorithm="giac")

[Out] integrate(f^(b\*x + a)/(d\*f^(2\*b\*x + e) + c), x)

$$3.34 \quad \int \frac{f^{a+2bx}}{c+df^{e+2bx}} dx$$

**Optimal.** Leaf size=34

$$\frac{f^{a-e} \log(df^{2bx+e} + c)}{2bd \log(f)}$$

[Out] (f^(a - e)\*Log[c + d\*f^(e + 2\*b\*x)])/(2\*b\*d\*Log[f])

**Rubi [A]** time = 0.0811105, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2247, 2246, 31}

$$\frac{f^{a-e} \log(df^{2bx+e} + c)}{2bd \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + 2\*b\*x)/(c + d\*f^(e + 2\*b\*x)),x]

[Out] (f^(a - e)\*Log[c + d\*f^(e + 2\*b\*x)])/(2\*b\*d\*Log[f])

#### Rule 2247

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] := Dist[(G^(h*(f + g*x)))^m/(F^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log[F], g*h*m*Log[G]]
```

#### Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+2bx}}{c+df^{e+2bx}} dx &= f^{a-e} \int \frac{f^{e+2bx}}{c+df^{e+2bx}} dx \\ &= \frac{f^{a-e} \text{Subst}\left(\int \frac{1}{c+dx} dx, x, f^{e+2bx}\right)}{2b \log(f)} \\ &= \frac{f^{a-e} \log(c + df^{e+2bx})}{2bd \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.017154, size = 34, normalized size = 1.

$$\frac{f^{a-e} \log(df^{2bx+e} + c)}{2bd \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + 2\*b\*x)/(c + d\*f^(e + 2\*b\*x)), x]

[Out] (f^(a - e)\*Log[c + d\*f^(e + 2\*b\*x)])/(2\*b\*d\*Log[f])

**Maple [A]** time = 0.01, size = 47, normalized size = 1.4

$$\frac{f^a \ln\left(c + d e^{-\ln(f)a + \ln(f)e} e^{(2bx+a)\ln(f)}\right)}{2 f^e d \ln(f) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(2\*b\*x+a)/(c+d\*f^(2\*b\*x+e)), x)

[Out] 1/2/(f^e)/d/ln(f)/b\*f^a\*ln(c+d\*exp(-ln(f)\*a+ln(f)\*e)\*exp((2\*b\*x+a)\*ln(f)))

**Maxima [A]** time = 1.1564, size = 43, normalized size = 1.26

$$\frac{f^{a-e} \log(df^{2bx+e} + c)}{2bd \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(2\*b\*x+a)/(c+d\*f^(2\*b\*x+e)), x, algorithm="maxima")

[Out] 1/2\*f^(a - e)\*log(d\*f^(2\*b\*x + e) + c)/(b\*d\*log(f))

**Fricas [A]** time = 1.78473, size = 72, normalized size = 2.12

$$\frac{f^{a-e} \log(df^{2bx+e} + c)}{2bd \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(2\*b\*x+a)/(c+d\*f^(2\*b\*x+e)), x, algorithm="fricas")

[Out] 1/2\*f^(a - e)\*log(d\*f^(2\*b\*x + e) + c)/(b\*d\*log(f))

**Sympy [A]** time = 0.914736, size = 42, normalized size = 1.24

$$\frac{e^{(a-e)\log(f)} \log\left(\frac{c e^{a\log(f)} e^{-e\log(f)}}{d} + f^{a+2bx}\right)}{2bd \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(2\*b\*x+a)/(c+d\*f\*\*(2\*b\*x+e)),x)

[Out] exp((a - e)\*log(f))\*log(c\*exp(a\*log(f))\*exp(-e\*log(f))/d + f\*\*(a + 2\*b\*x))/(2\*b\*d\*log(f))

**Giac [A]** time = 1.47007, size = 50, normalized size = 1.47

$$\frac{f^a \log(|df^{2bx} f^e + c|)}{2bd f^e \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(2\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="giac")

[Out] 1/2\*f^a\*log(abs(d\*f^(2\*b\*x)\*f^e + c))/(b\*d\*f^e\*log(f))

$$3.35 \quad \int \frac{f^{a+3bx}}{c+df^{e+2bx}} dx$$

**Optimal.** Leaf size=88

$$\frac{f^{\frac{1}{2}(2a-3e)+\frac{1}{2}(2bx+e)}}{bd \log(f)} - \frac{\sqrt{c} f^{a-\frac{3e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{bd^{3/2} \log(f)}$$

[Out]  $f^{((2*a - 3*e)/2 + (e + 2*b*x)/2)/(b*d*\text{Log}[f])} - (\text{Sqrt}[c]*f^{(a - (3*e)/2)*\text{ArcTan}[(\text{Sqrt}[d]*f^{(e + 2*b*x)/2})/\text{Sqrt}[c]])/(b*d^{(3/2)*\text{Log}[f])}$

**Rubi [A]** time = 0.067568, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2248, 321, 205}

$$\frac{f^{\frac{1}{2}(2a-3e)+\frac{1}{2}(2bx+e)}}{bd \log(f)} - \frac{\sqrt{c} f^{a-\frac{3e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{bd^{3/2} \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + 3\*b\*x)/(c + d\*f^(e + 2\*b\*x)), x]

[Out]  $f^{((2*a - 3*e)/2 + (e + 2*b*x)/2)/(b*d*\text{Log}[f])} - (\text{Sqrt}[c]*f^{(a - (3*e)/2)*\text{ArcTan}[(\text{Sqrt}[d]*f^{(e + 2*b*x)/2})/\text{Sqrt}[c]])/(b*d^{(3/2)*\text{Log}[f])}$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+3bx}}{c + df^{e+2bx}} dx &= \frac{f^{a-\frac{3e}{2}} \operatorname{Subst}\left(\int \frac{x^2}{c+dx^2} dx, x, f^{\frac{1}{2}(e+2bx)}\right)}{b \log(f)} \\ &= \frac{f^{\frac{1}{2}(2a-3e)+\frac{1}{2}(e+2bx)}}{bd \log(f)} - \frac{\left(cf^{a-\frac{3e}{2}}\right) \operatorname{Subst}\left(\int \frac{1}{c+dx^2} dx, x, f^{\frac{1}{2}(e+2bx)}\right)}{bd \log(f)} \\ &= \frac{f^{\frac{1}{2}(2a-3e)+\frac{1}{2}(e+2bx)}}{bd \log(f)} - \frac{\sqrt{c} f^{a-\frac{3e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(e+2bx)}}{\sqrt{c}}\right)}{bd^{3/2} \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0636177, size = 67, normalized size = 0.76

$$\frac{f^a \left( \frac{f^{bx-e}}{d} - \frac{\sqrt{c} f^{-3e/2} \tan^{-1}\left(\frac{\sqrt{d} f^{bx+\frac{e}{2}}}{\sqrt{c}}\right)}{d^{3/2}} \right)}{b \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + 3\*b\*x)/(c + d\*f^(e + 2\*b\*x)),x]

[Out] (f^a\*(f^(-e + b\*x))/d - (Sqrt[c]\*ArcTan[(Sqrt[d]\*f^(e/2 + b\*x))/Sqrt[c]])/(d^(3/2)\*f^((3\*e)/2)))/(b\*Log[f])

**Maple [B]** time = 0.062, size = 171, normalized size = 1.9

$$\frac{1}{d \ln(f) b} f^{bx+\frac{a}{3}} \left(f^{\frac{e}{2}}\right)^{-2} \left(f^{-\frac{a}{3}}\right)^{-2} + \frac{1}{2bd^2 \ln(f)} \sqrt{-cd} \ln\left(f^{bx+\frac{a}{3}} - \frac{1}{d} \sqrt{-cd} \left(f^{\frac{e}{2}}\right)^{-1} \left(f^{-\frac{a}{3}}\right)^{-1}\right) \left(f^{-\frac{a}{3}}\right)^{-3} \left(f^{\frac{e}{2}}\right)^{-3} - \frac{1}{2bd^2 \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(3\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x)

[Out] 1/(f^(1/2\*e))^2/(f^(-1/3\*a))^2/d/ln(f)/b\*f^(b\*x+1/3\*a)+1/2/d^2\*(-c\*d)^(1/2)/b/(f^(-1/3\*a))^3/(f^(1/2\*e))^3/ln(f)\*ln(f^(b\*x+1/3\*a)-1/d\*(-c\*d)^(1/2))/(f^(-1/3\*a))/(f^(1/2\*e))-1/2/d^2\*(-c\*d)^(1/2)/b/(f^(-1/3\*a))^3/(f^(1/2\*e))^3/ln(f)\*ln(f^(b\*x+1/3\*a)+1/d\*(-c\*d)^(1/2))/(f^(-1/3\*a))/(f^(1/2\*e))

**Maxima [B]** time = 1.73656, size = 171, normalized size = 1.94

$$-\frac{cf^{a-e} \log\left(\frac{d(f^{3bx+a})^{\frac{1}{3}} f^{e-\sqrt{-cd}} f^{\frac{1}{3}a}}{d(f^{3bx+a})^{\frac{1}{3}} f^{e+\sqrt{-cd}} f^{\frac{1}{3}a}}\right)}{2\sqrt{-cd} f^e b d \log(f)} + \frac{(f^{3bx+a})^{\frac{1}{3}} f^{\frac{2}{3}a-e}}{bd \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(3\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="maxima")

[Out]  $-1/2*c*f^{(a - e)}*\log((d*(f^{(3*b*x + a)})^{(1/3)}*f^e - \sqrt{-c*d*f^e}*f^{(1/3*a)})/(d*(f^{(3*b*x + a)})^{(1/3)}*f^e + \sqrt{-c*d*f^e}*f^{(1/3*a)}))/(\sqrt{-c*d*f^e}*b*d*\log(f)) + (f^{(3*b*x + a)})^{(1/3)}*f^{(2/3*a - e)}/(b*d*\log(f))$

**Fricas [A]** time = 1.88714, size = 374, normalized size = 4.25

$$\left[ \frac{f^{a-\frac{3}{2}e} \sqrt{\frac{-c}{d}} \log\left(\frac{2df^{bx+\frac{1}{2}e} \sqrt{\frac{-c}{d}} - df^{2bx+e+c}}{df^{2bx+e+c}}\right) + 2f^{bx+\frac{1}{2}e} f^{a-\frac{3}{2}e}}{2bd \log(f)}, \frac{f^{a-\frac{3}{2}e} \sqrt{\frac{-c}{d}} \arctan\left(\frac{df^{bx+\frac{1}{2}e} \sqrt{\frac{-c}{d}}}{c}\right) - f^{bx+\frac{1}{2}e} f^{a-\frac{3}{2}e}}{bd \log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")`

[Out]  $[1/2*(f^{(a - 3/2*e)}*\sqrt{-c/d}*\log(-(2*d*f^{(b*x + 1/2*e)}*\sqrt{-c/d} - d*f^{(2*b*x + e) + c})/(d*f^{(2*b*x + e) + c})) + 2*f^{(b*x + 1/2*e)}*f^{(a - 3/2*e)})/(b*d*\log(f)), -(f^{(a - 3/2*e)}*\sqrt{c/d}*\arctan(d*f^{(b*x + 1/2*e)}*\sqrt{c/d}/c) - f^{(b*x + 1/2*e)}*f^{(a - 3/2*e)})/(b*d*\log(f))]$

**Sympy [B]** time = 1.6555, size = 253, normalized size = 2.88

$$\left\{ \begin{array}{ll} \frac{e^{\frac{2a \log(f)}{3}} e^{-e \log(f)} e^{\frac{(a+3bx) \log(f)}{3}}}{bd \log(f)} & \text{for } bde^{e \log(f)} \log(f) \neq 0 \\ \frac{x \left( c^2 e^{\frac{10a \log(f)}{3}} + 2cde^{\frac{8a \log(f)}{3}} e^{e \log(f)} + d^2 e^{2a \log(f)} e^{2e \log(f)} \right)}{c^2 de^{\frac{8a \log(f)}{3}} e^{e \log(f)} + 2cd^2 e^{2a \log(f)} e^{2e \log(f)} + d^3 e^{\frac{4a \log(f)}{3}} e^{3e \log(f)}} & \text{otherwise} \end{array} \right. + \text{RootSum}\left(4z^2 b^2 d^3 e^{3e \log(f)} \log\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(3*b*x+a)/(c+d*f**(2*b*x+e)),x)`

[Out] `Piecewise((exp(2*a*log(f)/3)*exp(-e*log(f))*exp((a + 3*b*x)*log(f)/3)/(b*d*log(f)), Ne(b*d*exp(e*log(f))*log(f), 0)), (x*(c**2*exp(10*a*log(f)/3) + 2*c*d*exp(8*a*log(f)/3)*exp(e*log(f)) + d**2*exp(2*a*log(f))*exp(2*e*log(f)))/(c**2*d*exp(8*a*log(f)/3)*exp(e*log(f)) + 2*c*d**2*exp(2*a*log(f))*exp(2*e*log(f)) + d**3*exp(4*a*log(f)/3)*exp(3*e*log(f))), True)) + RootSum(4*_z**2*b**2*d**3*exp(3*e*log(f))*log(f)**2 + c*exp(2*a*log(f)), Lambda(_i, _i*log(-2*_i*b*d*exp(-2*a*log(f)/3)*exp(e*log(f))*log(f) + exp((a + 3*b*x)*log(f)/3))))`

**Giac [A]** time = 1.23517, size = 104, normalized size = 1.18

$$-f^a \left( \frac{c \arctan\left(\frac{df^{bx} f^e}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e} b d f^e \log(f)} - \frac{f^{bx}}{b d f^e \log(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

```
[Out] -f^a*(c*arctan(d*f^(b*x)*f^e/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*b*d*f^e*log(f))  
- f^(b*x)/(b*d*f^e*log(f)))
```



$$3.36 \quad \int \frac{f^{a+4bx}}{c+df^{e+2bx}} dx$$

**Optimal.** Leaf size=61

$$\frac{f^{a+2bx-e}}{2bd \log(f)} - \frac{cf^{a-2e} \log(df^{2bx+e} + c)}{2bd^2 \log(f)}$$

[Out]  $f^{(a - e + 2*b*x)/(2*b*d*Log[f])} - (c*f^{(a - 2*e)*Log[c + d*f^{(e + 2*b*x)]})/(2*b*d^2*Log[f])$

**Rubi [A]** time = 0.060494, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2248, 43}

$$\frac{f^{a+2bx-e}}{2bd \log(f)} - \frac{cf^{a-2e} \log(df^{2bx+e} + c)}{2bd^2 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + 4\*b\*x)/(c + d\*f^(e + 2\*b\*x)), x]

[Out]  $f^{(a - e + 2*b*x)/(2*b*d*Log[f])} - (c*f^{(a - 2*e)*Log[c + d*f^{(e + 2*b*x)]})/(2*b*d^2*Log[f])$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+4bx}}{c+df^{e+2bx}} dx &= \frac{f^{a-2e} \text{Subst}\left(\int \frac{x}{c+dx} dx, x, f^{e+2bx}\right)}{2b \log(f)} \\ &= \frac{f^{a-2e} \text{Subst}\left(\int \left(\frac{1}{d} - \frac{c}{d(c+dx)}\right) dx, x, f^{e+2bx}\right)}{2b \log(f)} \\ &= \frac{f^{a-e+2bx}}{2bd \log(f)} - \frac{cf^{a-2e} \log(c + df^{e+2bx})}{2bd^2 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.04202, size = 48, normalized size = 0.79

$$\frac{f^{a-2e} (df^{2bx+e} - c \log(df^{2bx+e} + c))}{2bd^2 \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + 4\*b\*x)/(c + d\*f^(e + 2\*b\*x)),x]

[Out] (f^(a - 2\*e)\*(d\*f^(e + 2\*b\*x) - c\*Log[c + d\*f^(e + 2\*b\*x)]))/(2\*b\*d^2\*Log[f])

**Maple [A]** time = 0.016, size = 76, normalized size = 1.3

$$\frac{e^{(2bx+e)\ln(f)}}{2(f^e)^2 \ln(f) bd} \left(f^{\frac{a}{2}}\right)^2 - \frac{c \ln\left(c + de^{(2bx+e)\ln(f)}\right)}{2d^2b \ln(f) (f^e)^2} \left(f^{\frac{a}{2}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(4\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x)

[Out] 1/2/(f^e)^2/ln(f)/b/d\*(f^(1/2\*a))^2\*exp((2\*b\*x+e)\*ln(f))-1/2/ln(f)/b/d^2\*c/(f^e)^2\*(f^(1/2\*a))^2\*ln(c+d\*exp((2\*b\*x+e)\*ln(f)))

**Maxima [A]** time = 1.13063, size = 112, normalized size = 1.84

$$-\frac{cf^{a-2e} \log\left(d\sqrt{f^{4bx+a}}f^{-\frac{1}{2}a+e} + c\right)}{2bd^2 \log(f)} + \frac{\left(d\sqrt{f^{4bx+a}}f^{-\frac{1}{2}a+e} + c\right)f^{a-2e}}{2bd^2 \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(4\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="maxima")

[Out] -1/2\*c\*f^(a - 2\*e)\*log(d\*sqrt(f^(4\*b\*x + a))\*f^(-1/2\*a + e) + c)/(b\*d^2\*log(f)) + 1/2\*(d\*sqrt(f^(4\*b\*x + a))\*f^(-1/2\*a + e) + c)\*f^(a - 2\*e)/(b\*d^2\*log(f))

**Fricas [A]** time = 1.62445, size = 123, normalized size = 2.02

$$\frac{df^{2bx+e}f^{a-2e} - cf^{a-2e} \log(df^{2bx+e} + c)}{2bd^2 \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(4\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="fricas")

[Out] 1/2\*(d\*f^(2\*b\*x + e)\*f^(a - 2\*e) - c\*f^(a - 2\*e)\*log(d\*f^(2\*b\*x + e) + c))/(b\*d^2\*log(f))

**Sympy [A]** time = 1.67611, size = 218, normalized size = 3.57

$$\begin{cases} \frac{e^{\frac{a \log(f)}{2}} e^{-e \log(f)} \sqrt{e^{(a+4bx) \log(f)}}}{2bd \log(f)} & \text{for } 2bde^{e \log(f)} \log(f) \neq 0 \\ x \left( \frac{c^2 e^{\frac{3a \log(f)}{2}} + 2cde^{a \log(f)} e^{e \log(f)} + d^2 e^{\frac{a \log(f)}{2}} e^{2e \log(f)}}{c^2 d e^{a \log(f)} e^{e \log(f)} + 2cd^2 e^{\frac{a \log(f)}{2}} e^{2e \log(f)} + d^3 e^{3e \log(f)}} \right) & \text{otherwise} \end{cases} - \frac{ce^{(a-2e) \log(f)} \log\left(\frac{ce^{\frac{a \log(f)}{2}} e^{-e \log(f)}}{d} + \sqrt{\dots}\right)}{2bd^2 \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(4\*b\*x+a)/(c+d\*f\*\*(2\*b\*x+e)),x)

[Out] Piecewise((exp(a\*log(f)/2)\*exp(-e\*log(f))\*sqrt(exp((a + 4\*b\*x)\*log(f)))/(2\*b\*d\*log(f)), Ne(2\*b\*d\*exp(e\*log(f))\*log(f), 0)), (x\*(c\*\*2\*exp(3\*a\*log(f)/2) + 2\*c\*d\*exp(a\*log(f))\*exp(e\*log(f)) + d\*\*2\*exp(a\*log(f)/2)\*exp(2\*e\*log(f)))/(c\*\*2\*d\*exp(a\*log(f))\*exp(e\*log(f)) + 2\*c\*d\*\*2\*exp(a\*log(f)/2)\*exp(2\*e\*log(f)) + d\*\*3\*exp(3\*e\*log(f))), True)) - c\*exp((a - 2\*e)\*log(f))\*log(c\*exp(a\*log(f)/2)\*exp(-e\*log(f))/d + sqrt(exp((a + 4\*b\*x)\*log(f))))/(2\*b\*d\*\*2\*log(f))

**Giac [A]** time = 1.20974, size = 89, normalized size = 1.46

$$\frac{1}{2} f^a \left( \frac{f^{2bx}}{bd f^e \log(f)} - \frac{c \log(|df^{2bx} f^e + c|)}{bd^2 f^{2e} \log(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(4\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="giac")

[Out] 1/2\*f^a\*(f^(2\*b\*x)/(b\*d\*f^e\*log(f)) - c\*log(abs(d\*f^(2\*b\*x)\*f^e + c))/(b\*d^2\*f^(2\*e)\*log(f)))

$$3.37 \quad \int \frac{f^{a+5bx}}{c+df^{e+2bx}} dx$$

**Optimal.** Leaf size=127

$$\frac{c^{3/2} f^{a-\frac{5e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{bd^{5/2} \log(f)} - \frac{c f^{\frac{1}{2}(2a-5e)+\frac{1}{2}(2bx+e)}}{bd^2 \log(f)} + \frac{f^{\frac{1}{2}(2a-5e)+\frac{3}{2}(2bx+e)}}{3bd \log(f)}$$

[Out]  $-\left(\frac{c f^{\left(\frac{2a-5e}{2} + \frac{e+2bx}{2}\right)}}{b d^2 \log[f]}\right) + f^{\left(\frac{2a-5e}{2} + \frac{3(e+2bx)}{2}\right)} / (3 b d \log[f]) + \frac{c^{\frac{3}{2}} f^{a-\frac{5e}{2}} \operatorname{ArcTan}\left[\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right]}{b d^{\frac{5}{2}} \log[f]}$

**Rubi [A]** time = 0.079692, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2248, 302, 205}

$$\frac{c^{3/2} f^{a-\frac{5e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{bd^{5/2} \log(f)} - \frac{c f^{\frac{1}{2}(2a-5e)+\frac{1}{2}(2bx+e)}}{bd^2 \log(f)} + \frac{f^{\frac{1}{2}(2a-5e)+\frac{3}{2}(2bx+e)}}{3bd \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + 5\*b\*x)/(c + d\*f^(e + 2\*b\*x)),x]

[Out]  $-\left(\frac{c f^{\left(\frac{2a-5e}{2} + \frac{e+2bx}{2}\right)}}{b d^2 \log[f]}\right) + f^{\left(\frac{2a-5e}{2} + \frac{3(e+2bx)}{2}\right)} / (3 b d \log[f]) + \frac{c^{\frac{3}{2}} f^{a-\frac{5e}{2}} \operatorname{ArcTan}\left[\frac{\sqrt{d} f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right]}{b d^{\frac{5}{2}} \log[f]}$

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+5bx}}{c + df^{e+2bx}} dx &= \frac{f^{a-\frac{5e}{2}} \operatorname{Subst}\left(\int \frac{x^4}{c+dx^2} dx, x, f^{\frac{1}{2}(e+2bx)}\right)}{b \log(f)} \\
&= \frac{f^{a-\frac{5e}{2}} \operatorname{Subst}\left(\int \left(-\frac{c}{d^2} + \frac{x^2}{d} + \frac{c^2}{d^2(c+dx^2)}\right) dx, x, f^{\frac{1}{2}(e+2bx)}\right)}{b \log(f)} \\
&= -\frac{cf^{\frac{1}{2}(2a-5e)+\frac{1}{2}(e+2bx)}}{bd^2 \log(f)} + \frac{f^{\frac{1}{2}(2a-5e)+\frac{3}{2}(e+2bx)}}{3bd \log(f)} + \frac{\left(c^2 f^{a-\frac{5e}{2}}\right) \operatorname{Subst}\left(\int \frac{1}{c+dx^2} dx, x, f^{\frac{1}{2}(e+2bx)}\right)}{bd^2 \log(f)} \\
&= -\frac{cf^{\frac{1}{2}(2a-5e)+\frac{1}{2}(e+2bx)}}{bd^2 \log(f)} + \frac{f^{\frac{1}{2}(2a-5e)+\frac{3}{2}(e+2bx)}}{3bd \log(f)} + \frac{c^{3/2} f^{a-\frac{5e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{\frac{1}{2}(e+2bx)}}{\sqrt{c}}\right)}{bd^{5/2} \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0622489, size = 86, normalized size = 0.68

$$\frac{3c^{3/2} f^{a-\frac{5e}{2}} \tan^{-1}\left(\frac{\sqrt{d} f^{bx+\frac{e}{2}}}{\sqrt{c}}\right) + \sqrt{d} f^{a+bx-2e} (df^{2bx+e} - 3c)}{3bd^{5/2} \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + 5\*b\*x)/(c + d\*f^(e + 2\*b\*x)), x]

[Out] (Sqrt[d]\*f^(a - 2\*e + b\*x)\*(-3\*c + d\*f^(e + 2\*b\*x)) + 3\*c^(3/2)\*f^(a - (5\*e)/2)\*ArcTan[(Sqrt[d]\*f^(e/2 + b\*x))/Sqrt[c]])/(3\*b\*d^(5/2)\*Log[f])

**Maple [B]** time = 0.066, size = 212, normalized size = 1.7

$$\frac{1}{3 \ln(f) bd} \left(f^{bx+\frac{a}{5}}\right)^3 \left(f^{\frac{e}{2}}\right)^{-2} \left(f^{-\frac{a}{5}}\right)^{-2} - \frac{c}{d^2 \ln(f) b} f^{bx+\frac{a}{5}} \left(f^{\frac{e}{2}}\right)^{-4} \left(f^{-\frac{a}{5}}\right)^{-4} + \frac{c}{2bd^3 \ln(f)} \sqrt{-cd} \ln\left(f^{bx+\frac{a}{5}} + \frac{1}{d} \sqrt{-cd} \left(f^{\frac{e}{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(5\*b\*x+a)/(c+d\*f^(2\*b\*x+e)), x)

[Out] 1/3/(f^(1/2\*e))^2/(f^(-1/5\*a))^2/d/ln(f)/b\*(f^(b\*x+1/5\*a))^3-c/(f^(1/2\*e))^4/(f^(-1/5\*a))^4/d^2/ln(f)/b\*f^(b\*x+1/5\*a)+1/2/d^3\*(-c\*d)^(1/2)\*c/b/(f^(-1/5\*a))^5/(f^(1/2\*e))^5/ln(f)\*ln(f^(b\*x+1/5\*a)+1/d\*(-c\*d)^(1/2))/(f^(-1/5\*a))/(f^(1/2\*e))-1/2/d^3\*(-c\*d)^(1/2)\*c/b/(f^(-1/5\*a))^5/(f^(1/2\*e))^5/ln(f)\*ln(f^(b\*x+1/5\*a)-1/d\*(-c\*d)^(1/2))/(f^(-1/5\*a))/(f^(1/2\*e))

**Maxima [A]** time = 1.73309, size = 211, normalized size = 1.66

$$\frac{c^2 f^{a-2e} \log\left(\frac{d(f^{5bx+a})^{\frac{1}{5}} f^{e-\sqrt{-cd}} f^{\frac{1}{5}a}}{d(f^{5bx+a})^{\frac{1}{5}} f^{e+\sqrt{-cd}} f^{\frac{1}{5}a}}\right)}{2\sqrt{-cd} f^e b d^2 \log(f)} + \frac{d(f^{5bx+a})^{\frac{3}{5}} f^{\frac{2}{5}a+e} - 3c(f^{5bx+a})^{\frac{1}{5}} f^{\frac{4}{5}a}}{3bd^2 f^{2e} \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*f^(a - 2*e)*log((d*(f^(5*b*x + a))^(1/5)*f^e - sqrt(-c*d*f^e)*f^(1/5*a))/(d*(f^(5*b*x + a))^(1/5)*f^e + sqrt(-c*d*f^e)*f^(1/5*a)))/(sqrt(-c*d*f^e)*b*d^2*log(f)) + 1/3*(d*(f^(5*b*x + a))^(3/5)*f^(2/5*a + e) - 3*c*(f^(5*b*x + a))^(1/5)*f^(4/5*a))/(b*d^2*f^(2*e)*log(f))
```

**Fricas [A]** time = 1.62651, size = 501, normalized size = 3.94

$$\left[ \frac{3cf^{a-\frac{5}{2}e}\sqrt{-\frac{c}{d}}\log\left(\frac{2df^{bx+\frac{1}{2}e}\sqrt{-\frac{c}{d}}+df^{2bx+e}-c}{df^{2bx+e}+c}\right)+2df^{3bx+\frac{3}{2}e}f^{a-\frac{5}{2}e}-6cf^{bx+\frac{1}{2}e}f^{a-\frac{5}{2}e}}{6bd^2\log(f)}, \frac{3cf^{a-\frac{5}{2}e}\sqrt{\frac{c}{d}}\arctan\left(\frac{df^{bx+\frac{1}{2}e}\sqrt{\frac{c}{d}}}{c}\right)+df^{3bx+\frac{3}{2}e}}{3bd^2\log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*f^(a - 5/2*e)*sqrt(-c/d)*log((2*d*f^(b*x + 1/2*e)*sqrt(-c/d) + d*f^(2*b*x + e) - c)/(d*f^(2*b*x + e) + c)) + 2*d*f^(3*b*x + 3/2*e)*f^(a - 5/2*e) - 6*c*f^(b*x + 1/2*e)*f^(a - 5/2*e))/(b*d^2*log(f)), 1/3*(3*c*f^(a - 5/2*e)*sqrt(c/d)*arctan(d*f^(b*x + 1/2*e)*sqrt(c/d)/c) + d*f^(3*b*x + 3/2*e)*f^(a - 5/2*e) - 3*c*f^(b*x + 1/2*e)*f^(a - 5/2*e))/(b*d^2*log(f))]
```

**Sympy [B]** time = 2.28615, size = 366, normalized size = 2.88

$$\left\{ \begin{array}{ll} \frac{\left(-3bcde\frac{4a\log(f)}{5}e^{e\log(f)}e^{\frac{(a+5bx)\log(f)}{5}}\log(f)+bd^2e^{\frac{2a\log(f)}{5}}e^{2e\log(f)}e^{\frac{3(a+5bx)\log(f)}{5}}\log(f)\right)e^{-3e\log(f)}}{3b^2d^3\log(f)^2} & \text{for } 3b^2d^3e^{3e\log(f)}\log(f)^2 \neq 0 \\ x\left(\frac{c^3e^{\frac{16a\log(f)}{5}}+c^2de^{\frac{14a\log(f)}{5}}e^{e\log(f)}-cd^2e^{\frac{12a\log(f)}{5}}e^{2e\log(f)}-d^3e^{2a\log(f)}e^{3e\log(f)}}{c^2d^2e^{\frac{12a\log(f)}{5}}e^{2e\log(f)}+2cd^3e^{2a\log(f)}e^{3e\log(f)}+d^4e^{\frac{8a\log(f)}{5}}e^{4e\log(f)}}\right) & \text{otherwise} \end{array} \right. + R$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(5*b*x+a)/(c+d*f**(2*b*x+e)),x)
```

```
[Out] Piecewise(((( -3*b*c*d*exp(4*a*log(f)/5)*exp(e*log(f))*exp((a + 5*b*x)*log(f)/5)*log(f) + b*d**2*exp(2*a*log(f)/5)*exp(2*e*log(f))*exp(3*(a + 5*b*x)*log(f)/5)*log(f))*exp(-3*e*log(f))/(3*b**2*d**3*log(f)**2), Ne(3*b**2*d**3*exp(3*e*log(f))*log(f)**2, 0)), (-x*(c**3*exp(16*a*log(f)/5) + c**2*d*exp(14*a*log(f)/5)*exp(e*log(f)) - c*d**2*exp(12*a*log(f)/5)*exp(2*e*log(f)) - d**3*exp(2*a*log(f))*exp(3*e*log(f)))/(c**2*d**2*exp(12*a*log(f)/5)*exp(2*e*log(f)) + 2*c*d**3*exp(2*a*log(f))*exp(3*e*log(f)) + d**4*exp(8*a*log(f)/5)*exp(4*e*log(f))), True)) + RootSum(4*_z**2*b**2*d**5*exp(5*e*log(f))*log(f)**2 + c**3*exp(2*a*log(f)), Lambda(_i, _i*log(2*_i*b*d**2*exp(-4*a*log(f)/5)*exp(2*e*log(f))*log(f)/c + exp((a + 5*b*x)*log(f)/5))))
```

**Giac [A]** time = 1.21433, size = 165, normalized size = 1.3

$$\frac{1}{3} f^a \left( \frac{3 c^2 \arctan\left(\frac{d f^{bx} f^e}{\sqrt{cd f^e}}\right)}{\sqrt{cd f^e} b d^2 f^{2e} \log(f)} + \frac{b^2 d^2 f^{3bx} f^{2e} \log(f)^2 - 3 b^2 c d f^{bx} f^e \log(f)^2}{b^3 d^3 f^{3e} \log(f)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(5\*b\*x+a)/(c+d\*f^(2\*b\*x+e)),x, algorithm="giac")

[Out] 1/3\*f^a\*(3\*c^2\*arctan(d\*f^(b\*x)\*f^e/sqrt(c\*d\*f^e))/(sqrt(c\*d\*f^e)\*b\*d^2\*f^(2\*e)\*log(f)) + (b^2\*d^2\*f^(3\*b\*x)\*f^(2\*e)\*log(f)^2 - 3\*b^2\*c\*d\*f^(b\*x)\*f^e\*log(f)^2)/(b^3\*d^3\*f^(3\*e)\*log(f)^3)

$$3.38 \quad \int \frac{e^x}{1+e^{2x}} dx$$

**Optimal.** Leaf size=4

$$\tan^{-1}(e^x)$$

[Out] ArcTan[E^x]

**Rubi [A]** time = 0.0173531, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 203}

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + E^(2\*x)), x]

[Out] ArcTan[E^x]

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\int \frac{e^x}{1+e^{2x}} dx = \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, e^x \right) = \tan^{-1}(e^x)$$

**Mathematica [A]** time = 0.0027281, size = 4, normalized size = 1.

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + E^(2\*x)), x]

[Out] ArcTan[E^x]



**Maple [A]** time = 0.003, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+exp(2*x)),x)`

[Out] `arctan(exp(x))`

---

**Maxima [A]** time = 1.80007, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")`

[Out] `arctan(e^x)`

---

**Fricas [A]** time = 1.50125, size = 18, normalized size = 4.5

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")`

[Out] `arctan(e^x)`

---

**Sympy [B]** time = 0.104927, size = 15, normalized size = 3.75

$$\text{RootSum}(4z^2 + 1, (i \mapsto i \log(2i + e^x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x)`

[Out] `RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))`

---

**Giac [A]** time = 1.22691, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")`

[Out] `arctan(e^x)`

$$3.39 \quad \int \frac{e^x}{1-e^{2x}} dx$$

**Optimal.** Leaf size=4

$$\tanh^{-1}(e^x)$$

[Out] ArcTanh[E^x]

**Rubi [A]** time = 0.0194058, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 206}

$$\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - E^(2\*x)),x]

[Out] ArcTanh[E^x]

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\int \frac{e^x}{1-e^{2x}} dx = \text{Subst} \left( \int \frac{1}{1-x^2} dx, x, e^x \right) = \tanh^{-1}(e^x)$$

**Mathematica [A]** time = 0.0033732, size = 4, normalized size = 1.

$$\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 - E^(2\*x)),x]

[Out] ArcTanh[E^x]

**Maple [A]** time = 0.002, size = 4, normalized size = 1.

$$\operatorname{Artanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1-exp(2*x)),x)`

[Out] `arctanh(exp(x))`

---

**Maxima [B]** time = 1.20756, size = 20, normalized size = 5.

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="maxima")`

[Out] `1/2*log(e^x + 1) - 1/2*log(e^x - 1)`

---

**Fricas [B]** time = 1.50029, size = 50, normalized size = 12.5

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="fricas")`

[Out] `1/2*log(e^x + 1) - 1/2*log(e^x - 1)`

---

**Sympy [B]** time = 0.107427, size = 15, normalized size = 3.75

$$-\frac{\log(e^x - 1)}{2} + \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x)`

[Out] `-log(exp(x) - 1)/2 + log(exp(x) + 1)/2`

---

**Giac [B]** time = 1.1911, size = 22, normalized size = 5.5

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="giac")`

[Out] `1/2*log(e^x + 1) - 1/2*log(abs(e^x - 1))`

### 3.40 $\int \frac{e^x x}{1-e^{2x}} dx$

**Optimal.** Leaf size=27

$$\frac{1}{2}\text{PolyLog}(2, -e^x) - \frac{1}{2}\text{PolyLog}(2, e^x) + x \tanh^{-1}(e^x)$$

[Out] x\*ArcTanh[E^x] + PolyLog[2, -E^x]/2 - PolyLog[2, E^x]/2

**Rubi [A]** time = 0.0581555, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2249, 206, 2245, 2282, 5912}

$$\frac{1}{2}\text{PolyLog}(2, -e^x) - \frac{1}{2}\text{PolyLog}(2, e^x) + x \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(E^x\*x)/(1 - E^(2\*x)),x]

[Out] x\*ArcTanh[E^x] + PolyLog[2, -E^x]/2 - PolyLog[2, E^x]/2

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 2245

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*((a\_) + (b\_)\*(F\_)^(v\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)\*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2\*e\*(c + d\*x)] && GtQ[m, 0] && ILtQ[p, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)^(v\_)] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

#### Rule 5912

Int[((a\_) + ArcTanh[(c\_)\*(x\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /

; FreeQ[{a, b, c}, x]

### Rubi steps

$$\begin{aligned}\int \frac{e^x x}{1 - e^{2x}} dx &= x \tanh^{-1}(e^x) - \int \tanh^{-1}(e^x) dx \\ &= x \tanh^{-1}(e^x) - \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x} dx, x, e^x\right) \\ &= x \tanh^{-1}(e^x) + \frac{\text{Li}_2(-e^x)}{2} - \frac{\text{Li}_2(e^x)}{2}\end{aligned}$$

**Mathematica [A]** time = 0.0323428, size = 45, normalized size = 1.67

$$\frac{1}{2}\text{PolyLog}(2, -e^x) - \frac{1}{2}\text{PolyLog}(2, e^x) - \frac{1}{2}x \log(1 - e^x) + \frac{1}{2}x \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x\*x)/(1 - E^(2\*x)), x]

[Out] -(x\*Log[1 - E^x])/2 + (x\*Log[1 + E^x])/2 + PolyLog[2, -E^x]/2 - PolyLog[2, E^x]/2

**Maple [A]** time = 0.007, size = 34, normalized size = 1.3

$$\frac{x \ln(1 + e^x)}{2} + \frac{\text{polylog}(2, -e^x)}{2} - \frac{x \ln(1 - e^x)}{2} - \frac{\text{polylog}(2, e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*x/(1-exp(2\*x)), x)

[Out] 1/2\*x\*ln(1+exp(x))+1/2\*polylog(2,-exp(x))-1/2\*x\*ln(1-exp(x))-1/2\*polylog(2,exp(x))

**Maxima [A]** time = 1.21072, size = 42, normalized size = 1.56

$$\frac{1}{2}x \log(e^x + 1) - \frac{1}{2}x \log(-e^x + 1) + \frac{1}{2}\text{Li}_2(-e^x) - \frac{1}{2}\text{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x/(1-exp(2\*x)), x, algorithm="maxima")

[Out] 1/2\*x\*log(e^x + 1) - 1/2\*x\*log(-e^x + 1) + 1/2\*dilog(-e^x) - 1/2\*dilog(e^x)

**Fricas [A]** time = 1.52436, size = 104, normalized size = 3.85

$$\frac{1}{2}x \log(e^x + 1) - \frac{1}{2}x \log(-e^x + 1) + \frac{1}{2}\text{Li}_2(-e^x) - \frac{1}{2}\text{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x/(1-exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*x\*log(e^x + 1) - 1/2\*x\*log(-e^x + 1) + 1/2\*dilog(-e^x) - 1/2\*dilog(e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{xe^x}{e^{2x}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x/(1-exp(2\*x)),x)

[Out] -Integral(x\*exp(x)/(exp(2\*x) - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{xe^x}{e^{(2x)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x/(1-exp(2\*x)),x, algorithm="giac")

[Out] integrate(-x\*e^x/(e^(2\*x) - 1), x)

### 3.41 $\int \frac{e^x x^2}{1-e^{2x}} dx$

**Optimal.** Leaf size=40

$$x \operatorname{PolyLog}(2, -e^x) - x \operatorname{PolyLog}(2, e^x) - \operatorname{PolyLog}(3, -e^x) + \operatorname{PolyLog}(3, e^x) + x^2 \tanh^{-1}(e^x)$$

[Out]  $x^2 \operatorname{ArcTanh}[E^x] + x \operatorname{PolyLog}[2, -E^x] - x \operatorname{PolyLog}[2, E^x] - \operatorname{PolyLog}[3, -E^x] + \operatorname{PolyLog}[3, E^x]$

**Rubi [A]** time = 0.100404, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {2249, 206, 2245, 6213, 2531, 2282, 6589}

$$x \operatorname{PolyLog}(2, -e^x) - x \operatorname{PolyLog}(2, e^x) - \operatorname{PolyLog}(3, -e^x) + \operatorname{PolyLog}(3, e^x) + x^2 \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^x * x^2) / (1 - E^{(2*x)}), x]$

[Out]  $x^2 \operatorname{ArcTanh}[E^x] + x \operatorname{PolyLog}[2, -E^x] - x \operatorname{PolyLog}[2, E^x] - \operatorname{PolyLog}[3, -E^x] + \operatorname{PolyLog}[3, E^x]$

#### Rule 2249

$\operatorname{Int}[(a + (b \cdot (F_1)^{(e \cdot (c + d \cdot x))})^p) \cdot (G_1)^{(h \cdot (f + g \cdot x))}], x_{\text{Symbol}}] := \operatorname{With}[\{m = \operatorname{FullSimplify}[(d \cdot e \cdot \operatorname{Log}[F]) / (g \cdot h \cdot \operatorname{Log}[G])], \operatorname{Dist}[\operatorname{Denominator}[m] / (g \cdot h \cdot \operatorname{Log}[G]), \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Denominator}[m] - 1) \cdot (a + b \cdot F_1^{(c \cdot e - (d \cdot e \cdot f) / g)} \cdot x^{\operatorname{Numerator}[m]})^p}, x], x, G_1^{(h \cdot (f + g \cdot x)) / \operatorname{Denominator}[m]}], x] /; \operatorname{LtQ}[m, -1] \parallel \operatorname{GtQ}[m, 1] /; \operatorname{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 206

$\operatorname{Int}[(a + (b \cdot (x_1)^2)^{-1}), x_{\text{Symbol}}] := \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] \cdot x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

#### Rule 2245

$\operatorname{Int}[(F_1)^{(e \cdot (c + d \cdot x))} \cdot (a + (b \cdot (F_1)^v)^p) \cdot (x_1)^m], x_{\text{Symbol}}] := \operatorname{With}[\{u = \operatorname{IntHide}[F_1^{(e \cdot (c + d \cdot x))} \cdot (a + b \cdot F_1^v)^p], \operatorname{Dist}[x^m, u, x] - \operatorname{Dist}[m, \operatorname{Int}[x^{(m-1)} \cdot u, x], x] /; \operatorname{FreeQ}[\{F, a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[v, 2 \cdot e \cdot (c + d \cdot x)] \&\& \operatorname{GtQ}[m, 0] \&\& \operatorname{ILtQ}[p, 0]$

#### Rule 6213

$\operatorname{Int}[\operatorname{ArcTanh}[(a + (b \cdot (f_1)^{(c + d \cdot x)})] \cdot (x_1)^m], x_{\text{Symbol}}] := \operatorname{Dist}[1/2, \operatorname{Int}[x^m \cdot \operatorname{Log}[1 + a + b \cdot f_1^{(c + d \cdot x)}], x], x] - \operatorname{Dist}[1/2, \operatorname{Int}[x^m \cdot \operatorname{Log}[1 - a - b \cdot f_1^{(c + d \cdot x)}], x], x] /; \operatorname{FreeQ}[\{a, b, c, d, f\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e \cdot (F_1)^{(c \cdot (a + b \cdot x))})^n] \cdot (f + g \cdot x)^m], x_{\text{Symbol}}] := -\operatorname{Simp}[(f + g \cdot x)^m \cdot \operatorname{PolyLog}[2, -(e \cdot (F_1^{(c \cdot (a + b \cdot x))})^n)] / (b \cdot c \cdot n \cdot \operatorname{Log}[F]), x] + \operatorname{Dist}[(g \cdot m) / (b \cdot c \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(f + g \cdot x)^{(m-1)}], x]$

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{e^x x^2}{1 - e^{2x}} dx &= x^2 \tanh^{-1}(e^x) - 2 \int x \tanh^{-1}(e^x) dx \\ &= x^2 \tanh^{-1}(e^x) + \int x \log(1 - e^x) dx - \int x \log(1 + e^x) dx \\ &= x^2 \tanh^{-1}(e^x) + x \operatorname{Li}_2(-e^x) - x \operatorname{Li}_2(e^x) - \int \operatorname{Li}_2(-e^x) dx + \int \operatorname{Li}_2(e^x) dx \\ &= x^2 \tanh^{-1}(e^x) + x \operatorname{Li}_2(-e^x) - x \operatorname{Li}_2(e^x) - \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, x, e^x\right) + \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(x)}{x} dx, x, e^x\right) \\ &= x^2 \tanh^{-1}(e^x) + x \operatorname{Li}_2(-e^x) - x \operatorname{Li}_2(e^x) - \operatorname{Li}_3(-e^x) + \operatorname{Li}_3(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0327251, size = 60, normalized size = 1.5

$$x \operatorname{PolyLog}(2, -e^x) - x \operatorname{PolyLog}(2, e^x) - \operatorname{PolyLog}(3, -e^x) + \operatorname{PolyLog}(3, e^x) - \frac{1}{2} x^2 \log(1 - e^x) + \frac{1}{2} x^2 \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*x^2)/(1 - E^(2*x)), x]
```

```
[Out] -(x^2*Log[1 - E^x])/2 + (x^2*Log[1 + E^x])/2 + x*PolyLog[2, -E^x] - x*PolyLog[2, E^x] - PolyLog[3, -E^x] + PolyLog[3, E^x]
```

**Maple [A]** time = 0.006, size = 51, normalized size = 1.3

$$\frac{x^2 \ln(1 + e^x)}{2} + x \operatorname{polylog}(2, -e^x) - \operatorname{polylog}(3, -e^x) - \frac{x^2 \ln(1 - e^x)}{2} - x \operatorname{polylog}(2, e^x) + \operatorname{polylog}(3, e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2/(1-exp(2*x)), x)
```

```
[Out] 1/2*x^2*ln(1+exp(x))+x*polylog(2,-exp(x))-polylog(3,-exp(x))-1/2*x^2*ln(1-exp(x))-x*polylog(2,exp(x))+polylog(3,exp(x))
```



**Maxima [A]** time = 1.1383, size = 65, normalized size = 1.62

$$\frac{1}{2}x^2 \log(e^x + 1) - \frac{1}{2}x^2 \log(-e^x + 1) + x\text{Li}_2(-e^x) - x\text{Li}_2(e^x) - \text{Li}_3(-e^x) + \text{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x^2/(1-exp(2\*x)),x, algorithm="maxima")

[Out] 1/2\*x^2\*log(e^x + 1) - 1/2\*x^2\*log(-e^x + 1) + x\*dilog(-e^x) - x\*dilog(e^x) - polylog(3, -e^x) + polylog(3, e^x)

**Fricas [C]** time = 1.45584, size = 154, normalized size = 3.85

$$\frac{1}{2}x^2 \log(e^x + 1) - \frac{1}{2}x^2 \log(-e^x + 1) + x\text{Li}_2(-e^x) - x\text{Li}_2(e^x) - \text{polylog}(3, -e^x) + \text{polylog}(3, e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x^2/(1-exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*x^2\*log(e^x + 1) - 1/2\*x^2\*log(-e^x + 1) + x\*dilog(-e^x) - x\*dilog(e^x) - polylog(3, -e^x) + polylog(3, e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{x^2 e^x}{e^{2x} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x\*\*2/(1-exp(2\*x)),x)

[Out] -Integral(x\*\*2\*exp(x)/(exp(2\*x) - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 e^x}{e^{(2.x)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*x^2/(1-exp(2\*x)),x, algorithm="giac")

[Out] integrate(-x^2\*e^x/(e^(2\*x) - 1), x)

### 3.42 $\int \frac{e^x x^3}{1-e^{2x}} dx$

**Optimal.** Leaf size=69

$$\frac{3}{2}x^2 \text{PolyLog}(2, -e^x) - \frac{3}{2}x^2 \text{PolyLog}(2, e^x) - 3x \text{PolyLog}(3, -e^x) + 3x \text{PolyLog}(3, e^x) + 3 \text{PolyLog}(4, -e^x) - 3 \text{PolyLog}(4, e^x)$$

[Out]  $x^3 \text{ArcTanh}[E^x] + (3x^2 \text{PolyLog}[2, -E^x])/2 - (3x^2 \text{PolyLog}[2, E^x])/2 - 3x \text{PolyLog}[3, -E^x] + 3x \text{PolyLog}[3, E^x] + 3 \text{PolyLog}[4, -E^x] - 3 \text{PolyLog}[4, E^x]$

**Rubi [A]** time = 0.121056, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {2249, 206, 2245, 6213, 2531, 6609, 2282, 6589}

$$\frac{3}{2}x^2 \text{PolyLog}(2, -e^x) - \frac{3}{2}x^2 \text{PolyLog}(2, e^x) - 3x \text{PolyLog}(3, -e^x) + 3x \text{PolyLog}(3, e^x) + 3 \text{PolyLog}(4, -e^x) - 3 \text{PolyLog}(4, e^x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^x x^3)/(1 - E^{(2x)}), x]$

[Out]  $x^3 \text{ArcTanh}[E^x] + (3x^2 \text{PolyLog}[2, -E^x])/2 - (3x^2 \text{PolyLog}[2, E^x])/2 - 3x \text{PolyLog}[3, -E^x] + 3x \text{PolyLog}[3, E^x] + 3 \text{PolyLog}[4, -E^x] - 3 \text{PolyLog}[4, E^x]$

#### Rule 2249

$\text{Int}[(a + (b \cdot (F_1)^{(c + d \cdot x)}))^{(p)} \cdot (G_1)^{(h \cdot (f + g \cdot x))}], x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(d \cdot e \cdot \text{Log}[F]) / (g \cdot h \cdot \text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m] / (g \cdot h \cdot \text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)} \cdot (a + b \cdot F_1^{(c \cdot e - (d \cdot e \cdot f) / g)} \cdot x^{\text{Numerator}[m]})^p, x], x, G_1^{(h \cdot (f + g \cdot x)) / \text{Denominator}[m]}], x] /; \text{LtQ}[m, -1] \parallel \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 206

$\text{Int}[(a + (b \cdot x^2))^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rule 2245

$\text{Int}[(F_1)^{(c + d \cdot x)} \cdot (a + (b \cdot (F_1)^{(v)})^{(p)})^{(m)}], x\_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[F_1^{(c + d \cdot x)} \cdot (a + b \cdot F_1^v)^p, x]\}, \text{Dist}[x^m, u, x] - \text{Dist}[m, \text{Int}[x^{(m-1)} \cdot u, x], x] /; \text{FreeQ}[\{F, a, b, c, d, e\}, x] \&\& \text{EqQ}[v, 2 \cdot e \cdot (c + d \cdot x)] \&\& \text{GtQ}[m, 0] \&\& \text{ILtQ}[p, 0]$

#### Rule 6213

$\text{Int}[\text{ArcTanh}[(a + (b \cdot (f_1)^{(c + d \cdot x)})^{(m)}], x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[x^m \cdot \text{Log}[1 + a + b \cdot f_1^{(c + d \cdot x)}], x], x] - \text{Dist}[1/2, \text{Int}[x^m \cdot \text{Log}[1 - a - b \cdot f_1^{(c + d \cdot x)}], x], x] /; \text{FreeQ}[\{a, b, c, d, f\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^x x^3}{1 - e^{2x}} dx &= x^3 \tanh^{-1}(e^x) - 3 \int x^2 \tanh^{-1}(e^x) dx \\
&= x^3 \tanh^{-1}(e^x) + \frac{3}{2} \int x^2 \log(1 - e^x) dx - \frac{3}{2} \int x^2 \log(1 + e^x) dx \\
&= x^3 \tanh^{-1}(e^x) + \frac{3}{2} x^2 \text{Li}_2(-e^x) - \frac{3}{2} x^2 \text{Li}_2(e^x) - 3 \int x \text{Li}_2(-e^x) dx + 3 \int x \text{Li}_2(e^x) dx \\
&= x^3 \tanh^{-1}(e^x) + \frac{3}{2} x^2 \text{Li}_2(-e^x) - \frac{3}{2} x^2 \text{Li}_2(e^x) - 3x \text{Li}_3(-e^x) + 3x \text{Li}_3(e^x) + 3 \int \text{Li}_3(-e^x) dx - 3 \int \text{Li}_3(e^x) dx \\
&= x^3 \tanh^{-1}(e^x) + \frac{3}{2} x^2 \text{Li}_2(-e^x) - \frac{3}{2} x^2 \text{Li}_2(e^x) - 3x \text{Li}_3(-e^x) + 3x \text{Li}_3(e^x) + 3 \text{Subst} \left( \int \frac{\text{Li}_3(-x)}{x} dx, x, e^x \right) \\
&= x^3 \tanh^{-1}(e^x) + \frac{3}{2} x^2 \text{Li}_2(-e^x) - \frac{3}{2} x^2 \text{Li}_2(e^x) - 3x \text{Li}_3(-e^x) + 3x \text{Li}_3(e^x) + 3 \text{Li}_4(-e^x) - 3 \text{Li}_4(e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.0384728, size = 89, normalized size = 1.29

$$\frac{3}{2} x^2 \text{PolyLog}(2, -e^x) - \frac{3}{2} x^2 \text{PolyLog}(2, e^x) - 3x \text{PolyLog}(3, -e^x) + 3x \text{PolyLog}(3, e^x) + 3 \text{PolyLog}(4, -e^x) - 3 \text{PolyLog}(4, e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*x^3)/(1 - E^(2*x)), x]
```

```
[Out] -(x^3*Log[1 - E^x])/2 + (x^3*Log[1 + E^x])/2 + (3*x^2*PolyLog[2, -E^x])/2 -
(3*x^2*PolyLog[2, E^x])/2 - 3*x*PolyLog[3, -E^x] + 3*x*PolyLog[3, E^x] + 3
*PolyLog[4, -E^x] - 3*PolyLog[4, E^x]
```

---

**Maple [A]** time = 0.005, size = 74, normalized size = 1.1

$$\frac{x^3 \ln(1 + e^x)}{2} + \frac{3x^2 \operatorname{polylog}(2, -e^x)}{2} - 3x \operatorname{polylog}(3, -e^x) + 3 \operatorname{polylog}(4, -e^x) - \frac{x^3 \ln(1 - e^x)}{2} - \frac{3x^2 \operatorname{polylog}(2, e^x)}{2} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*x^3/(1-exp(2*x)),x)`

[Out] `1/2*x^3*ln(1+exp(x))+3/2*x^2*polylog(2,-exp(x))-3*x*polylog(3,-exp(x))+3*polylog(4,-exp(x))-1/2*x^3*ln(1-exp(x))-3/2*x^2*polylog(2,exp(x))+3*x*polylog(3,exp(x))-3*polylog(4,exp(x))`

---

**Maxima [A]** time = 1.20274, size = 96, normalized size = 1.39

$$\frac{1}{2} x^3 \log(e^x + 1) - \frac{1}{2} x^3 \log(-e^x + 1) + \frac{3}{2} x^2 \operatorname{Li}_2(-e^x) - \frac{3}{2} x^2 \operatorname{Li}_2(e^x) - 3x \operatorname{Li}_3(-e^x) + 3x \operatorname{Li}_3(e^x) + 3 \operatorname{Li}_4(-e^x) - 3 \operatorname{Li}_4(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="maxima")`

[Out] `1/2*x^3*log(e^x + 1) - 1/2*x^3*log(-e^x + 1) + 3/2*x^2*dilog(-e^x) - 3/2*x^2*dilog(e^x) - 3*x*polylog(3, -e^x) + 3*x*polylog(3, e^x) + 3*polylog(4, -e^x) - 3*polylog(4, e^x)`

---

**Fricas [C]** time = 1.49903, size = 236, normalized size = 3.42

$$\frac{1}{2} x^3 \log(e^x + 1) - \frac{1}{2} x^3 \log(-e^x + 1) + \frac{3}{2} x^2 \operatorname{Li}_2(-e^x) - \frac{3}{2} x^2 \operatorname{Li}_2(e^x) - 3x \operatorname{polylog}(3, -e^x) + 3x \operatorname{polylog}(3, e^x) + 3 \operatorname{polylog}(4, -e^x) - 3 \operatorname{polylog}(4, e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="fricas")`

[Out] `1/2*x^3*log(e^x + 1) - 1/2*x^3*log(-e^x + 1) + 3/2*x^2*dilog(-e^x) - 3/2*x^2*dilog(e^x) - 3*x*polylog(3, -e^x) + 3*x*polylog(3, e^x) + 3*polylog(4, -e^x) - 3*polylog(4, e^x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3 e^x}{e^{2x} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x**3/(1-exp(2*x)),x)`

[Out] `-Integral(x**3*exp(x)/(exp(2*x) - 1), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 e^x}{e^{(2x)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*e^x/(e^(2*x) - 1), x)
```

### 3.43 $\int \frac{f^x}{a+bf^{2x}} dx$

**Optimal.** Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

[Out] ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

**Rubi [A]** time = 0.029278, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^x/(a + b\*f^(2\*x)),x]

[Out] ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{f^x}{a+bf^{2x}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, f^x\right)}{\log(f)} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0083495, size = 30, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^x/(a + b\*f^(2\*x)),x]

[Out] ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

**Maple [B]** time = 0.028, size = 53, normalized size = 1.8

$$-\frac{1}{2 \ln(f)} \ln\left(f^x - a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{2 \ln(f)} \ln\left(f^x + a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x/(a+b\*f^(2\*x)),x)

[Out]  $-1/2/(-a*b)^{(1/2)}/\ln(f)*\ln(f^x-1/(-a*b)^{(1/2)}*a)+1/2/(-a*b)^{(1/2)}/\ln(f)*\ln(f^x+1/(-a*b)^{(1/2)}*a)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.56626, size = 190, normalized size = 6.33

$$\left[ -\frac{\sqrt{-ab} \log\left(\frac{bf^{2x}-2\sqrt{-ab}f^x-a}{bf^{2x}+a}\right)}{2ab \log(f)}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}}{bf^x}\right)}{ab \log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x)),x, algorithm="fricas")

[Out]  $[-1/2*\sqrt{-a*b}*\log((b*f^{2*x} - 2*\sqrt{-a*b}*f^x - a)/(b*f^{2*x} + a))/(a*b*\log(f)), -\sqrt{a*b}*\arctan(\sqrt{a*b}/(b*f^x))/(a*b*\log(f))]$

**Sympy [A]** time = 0.465495, size = 24, normalized size = 0.8

$$\frac{\text{RootSum}\left(4z^2ab + 1, \left(i \mapsto i \log(2ia + f^x)\right)\right)}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x/(a+b\*f\*\*(2\*x)),x)

[Out]  $\text{RootSum}(4*_z^{**2}*a*b + 1, \text{Lambda}(_i, _i*\log(2*_i*a + f**x)))/\log(f)$

---

**Giac [A]** time = 1.24852, size = 28, normalized size = 0.93

$$\frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{\sqrt{ab}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^x/(a+b*f^(2*x)),x, algorithm="giac")`

[Out]  $\arctan(b*f^x/\text{sqrt}(a*b))/(\text{sqrt}(a*b)*\log(f))$



$$3.44 \quad \int \frac{f^x x}{a + b f^{2x}} dx$$

**Optimal.** Leaf size=110

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

[Out] (x\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - ((I/2)\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((I/2)\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2)

**Rubi [A]** time = 0.10602, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {2249, 205, 2245, 12, 2282, 4848, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Int[(f^x\*x)/(a + b\*f^(2\*x)),x]

[Out] (x\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - ((I/2)\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((I/2)\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2)

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*(f\_ + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2245

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*((a\_) + (b\_)\*(F\_)^(v\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)\*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2\*e\*(c + d\*x)] && GtQ[m, 0] && ILtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 4848

```
Int[((a_) + ArcTan[(c_)*(x)]*(b_))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{f^x x}{a + b f^{2x}} dx &= \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \int \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} dx \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{\int \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{\sqrt{a} \sqrt{b} \log^2(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i \text{Subst}\left(\int \frac{\log\left(1 - \frac{i \sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{2 \sqrt{a} \sqrt{b} \log^2(f)} + \frac{i \text{Subst}\left(\int \frac{\log\left(1 + \frac{i \sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{2 \sqrt{a} \sqrt{b} \log^2(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2 \sqrt{a} \sqrt{b} \log^2(f)} + \frac{i \text{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2 \sqrt{a} \sqrt{b} \log^2(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0627689, size = 108, normalized size = 0.98

$$\frac{i \left( -\text{PolyLog}\left(2, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right) + \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right) + x \log(f) \left( \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right) - \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right) \right) \right)}{2 \sqrt{a} \sqrt{b} \log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f^x*x)/(a + b*f^(2*x)), x]
```

```
[Out] ((I/2)*(x*Log[f]*(Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]]) - PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Log[f]^2)
```

**Maple [A]** time = 0.041, size = 134, normalized size = 1.2

$$\frac{x}{2 \ln(f)} \ln\left(\left(-bf^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} - \frac{x}{2 \ln(f)} \ln\left(\left(bf^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{2 (\ln(f))^2} \operatorname{dilog}\left(\left(-bf^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} - \frac{1}{2 (\ln(f))^2} \operatorname{dilog}\left(\left(bf^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x/(a+b\*f^(2\*x)),x)

[Out] 1/2/ln(f)\*x/(-a\*b)^(1/2)\*ln((-b\*f^x+(-a\*b)^(1/2))/(-a\*b)^(1/2))-1/2/ln(f)\*x/(-a\*b)^(1/2)\*ln((b\*f^x+(-a\*b)^(1/2))/(-a\*b)^(1/2))+1/2/ln(f)^2/(-a\*b)^(1/2)\*dilog((-b\*f^x+(-a\*b)^(1/2))/(-a\*b)^(1/2))-1/2/ln(f)^2/(-a\*b)^(1/2)\*dilog((b\*f^x+(-a\*b)^(1/2))/(-a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.52764, size = 247, normalized size = 2.25

$$\frac{x \sqrt{-\frac{b}{a}} \log\left(f^x \sqrt{-\frac{b}{a}} + 1\right) \log(f) - x \sqrt{-\frac{b}{a}} \log\left(-f^x \sqrt{-\frac{b}{a}} + 1\right) \log(f) - \sqrt{-\frac{b}{a}} \operatorname{Li}_2\left(f^x \sqrt{-\frac{b}{a}}\right) + \sqrt{-\frac{b}{a}} \operatorname{Li}_2\left(-f^x \sqrt{-\frac{b}{a}}\right)}{2b \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x)),x, algorithm="fricas")

[Out] -1/2\*(x\*sqrt(-b/a)\*log(f^x\*sqrt(-b/a) + 1)\*log(f) - x\*sqrt(-b/a)\*log(-f^x\*sqrt(-b/a) + 1)\*log(f) - sqrt(-b/a)\*dilog(f^x\*sqrt(-b/a)) + sqrt(-b/a)\*dilog(-f^x\*sqrt(-b/a)))/(b\*log(f)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x}{a + b f^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x/(a+b\*f\*\*(2\*x)),x)

[Out] Integral(f\*\*x\*x/(a + b\*f\*\*(2\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x}{b f^{2x} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x)),x, algorithm="giac")

[Out] integrate(f^x\*x/(b\*f^(2\*x) + a), x)

$$3.45 \quad \int \frac{f^x x^2}{a + b f^{2x}} dx$$

**Optimal.** Leaf size=184

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log(f)}$$

[Out] (x^2\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (I\*x\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*x\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - (I\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3)

**Rubi [A]** time = 0.178665, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {2249, 205, 2245, 12, 5143, 2531, 2282, 6589}

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b} \log(f)}$$

Antiderivative was successfully verified.

[In] Int[(f^x\*x^2)/(a + b\*f^(2\*x)), x]

[Out] (x^2\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (I\*x\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*x\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - (I\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3)

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2245

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))\*((a\_.) + (b\_.)\*(F\_)^(v\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)\*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2\*e\*(c + d\*x)] && GtQ[m, 0] && ILtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 5143

Int[ArcTan[(a\_.) + (b\_.)\*(f\_)^((c\_.) + (d\_.)\*(x\_))]\*(x\_)^(m\_.), x\_Symbol] :> Dist[I/2, Int[x^m\*Log[1 - I\*a - I\*b\*f^(c + d\*x)], x], x] - Dist[I/2, Int[x^m\*Log[1 + I\*a + I\*b\*f^(c + d\*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{f^x x^2}{a + b f^{2x}} dx &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - 2 \int \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} dx \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{2 \int x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i \int x \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log(f)} + \frac{i \int x \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i x \operatorname{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i x \operatorname{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i \int \operatorname{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^2(f)} - \frac{i \int \operatorname{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^2(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i x \operatorname{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i x \operatorname{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{i \sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{i \sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{\sqrt{a} \sqrt{b} \log^3(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{i x \operatorname{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i x \operatorname{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^2(f)} + \frac{i \operatorname{Li}_3\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{i \operatorname{Li}_3\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0495243, size = 168, normalized size = 0.91

$$\frac{i \left( 2 \operatorname{PolyLog} \left( 3, -\frac{i\sqrt{b}f^x}{\sqrt{a}} \right) - 2 \operatorname{PolyLog} \left( 3, \frac{i\sqrt{b}f^x}{\sqrt{a}} \right) - 2x \log(f) \operatorname{PolyLog} \left( 2, -\frac{i\sqrt{b}f^x}{\sqrt{a}} \right) + 2x \log(f) \operatorname{PolyLog} \left( 2, \frac{i\sqrt{b}f^x}{\sqrt{a}} \right) + x^2 \right)}{2\sqrt{a}\sqrt{b}\log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x^2)/(a + b\*f^(2\*x)), x]

[Out] ((I/2)\*(x^2\*Log[f]^2\*Log[1 - (I\*Sqrt[b]\*f^x)/Sqrt[a]] - x^2\*Log[f]^2\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]] - 2\*x\*Log[f]\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]] + 2\*x\*Log[f]\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]] + 2\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]] - 2\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]]))/(Sqrt[a]\*Sqrt[b]\*Log[f]^3)

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{a + b f^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x^2/(a+b\*f^(2\*x)), x)

[Out] int(f^x\*x^2/(a+b\*f^(2\*x)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.55749, size = 402, normalized size = 2.18

$$\frac{x^2 \sqrt{-\frac{b}{a}} \log \left( f^x \sqrt{-\frac{b}{a}} + 1 \right) \log(f)^2 - x^2 \sqrt{-\frac{b}{a}} \log \left( -f^x \sqrt{-\frac{b}{a}} + 1 \right) \log(f)^2 - 2x \sqrt{-\frac{b}{a}} \operatorname{Li}_2 \left( f^x \sqrt{-\frac{b}{a}} \right) \log(f) + 2x \sqrt{-\frac{b}{a}} \operatorname{Li}_2 \left( -f^x \sqrt{-\frac{b}{a}} \right) \log(f)}{2b \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x)), x, algorithm="fricas")

[Out] -1/2\*(x^2\*sqrt(-b/a)\*log(f^x\*sqrt(-b/a) + 1)\*log(f)^2 - x^2\*sqrt(-b/a)\*log(-f^x\*sqrt(-b/a) + 1)\*log(f)^2 - 2\*x\*sqrt(-b/a)\*dilog(f^x\*sqrt(-b/a))\*log(f) + 2\*x\*sqrt(-b/a)\*dilog(-f^x\*sqrt(-b/a))\*log(f) + 2\*sqrt(-b/a)\*polylog(3, f^x\*sqrt(-b/a)) - 2\*sqrt(-b/a)\*polylog(3, -f^x\*sqrt(-b/a)))/(b\*log(f)^3)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{a + b f^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x\*\*2/(a+b\*f\*\*(2\*x)),x)

[Out] Integral(f\*\*x\*x\*\*2/(a + b\*f\*\*(2\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{b f^{2x} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x)),x, algorithm="giac")

[Out] integrate(f^x\*x^2/(b\*f^(2\*x) + a), x)



$$3.46 \quad \int \frac{f^x x^3}{a + b f^{2x}} dx$$

**Optimal.** Leaf size=268

$$\frac{3ix^2 \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix \text{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix \text{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i \text{PolyLog}\left(4, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i \text{PolyLog}\left(4, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}$$

[Out] (x^3\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (((3\*I)/2)\*x^2\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (((3\*I)/2)\*x^2\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((3\*I)\*x\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*x\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*PolyLog[4, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4) + ((3\*I)\*PolyLog[4, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

**Rubi [A]** time = 0.22996, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {2249, 205, 2245, 12, 5143, 2531, 6609, 2282, 6589}

$$\frac{3ix^2 \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix \text{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix \text{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i \text{PolyLog}\left(4, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i \text{PolyLog}\left(4, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}$$

Antiderivative was successfully verified.

[In] Int[(f^x\*x^3)/(a + b\*f^(2\*x)), x]

[Out] (x^3\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (((3\*I)/2)\*x^2\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (((3\*I)/2)\*x^2\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((3\*I)\*x\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*x\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*PolyLog[4, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4) + ((3\*I)\*PolyLog[4, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2245

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*((a\_) + (b\_)\*(F\_)^(v\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)]^p, x}], Di

```
st[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] :=> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{f^x x^3}{a + b f^{2x}} dx &= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - 3 \int \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} dx \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{3 \int x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{(3i) \int x^2 \log\left(1 - \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{2\sqrt{a} \sqrt{b} \log(f)} + \frac{(3i) \int x^2 \log\left(1 + \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{2\sqrt{a} \sqrt{b} \log(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{(3i) \int x \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^2(f)} - \frac{(3i) \int x \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^2(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{(3i) \int \text{Li}_3\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^3(f)} + \frac{(3i) \int \text{Li}_3\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^3(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{(3i) \int \text{Li}_3\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^3(f)} + \frac{(3i) \int \text{Li}_3\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) dx}{\sqrt{a} \sqrt{b} \log^3(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} \log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^3(f)} - \frac{3i \text{Li}_4\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^4(f)} + \frac{3i \text{Li}_4\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log^4(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0514449, size = 224, normalized size = 0.84

$$\frac{i\left(-3x^2 \log^2(f) \text{PolyLog}\left(2, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) + 3x^2 \log^2(f) \text{PolyLog}\left(2, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) - 6 \text{PolyLog}\left(4, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) + 6 \text{PolyLog}\left(4, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right)\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x^3)/(a + b\*f^(2\*x)), x]

[Out] ((I/2)\*(x^3\*Log[f]^3\*Log[1 - (I\*Sqrt[b]\*f^x)/Sqrt[a]] - x^3\*Log[f]^3\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]] - 3\*x^2\*Log[f]^2\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]] + 3\*x^2\*Log[f]^2\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]] + 6\*x\*Log[f]\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]] - 6\*x\*Log[f]\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]] - 6\*PolyLog[4, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]] + 6\*PolyLog[4, (I\*Sqrt[b]\*f^x)/Sqrt[a]]))/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{a + b f^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x^3/(a+b\*f^(2\*x)), x)

[Out] int(f^x\*x^3/(a+b\*f^(2\*x)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.58735, size = 552, normalized size = 2.06

$$x^3 \sqrt{\frac{-b}{a}} \log\left(f^x \sqrt{\frac{-b}{a}} + 1\right) \log(f)^3 - x^3 \sqrt{\frac{-b}{a}} \log\left(-f^x \sqrt{\frac{-b}{a}} + 1\right) \log(f)^3 - 3x^2 \sqrt{\frac{-b}{a}} \operatorname{Li}_2\left(f^x \sqrt{\frac{-b}{a}}\right) \log(f)^2 + 3x^2 \sqrt{\frac{-b}{a}} \operatorname{Li}_2\left(-f^x \sqrt{\frac{-b}{a}}\right) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x)),x, algorithm="fricas")

[Out] 
$$\frac{-1/2*(x^3*\sqrt{-b/a}*\log(f^x*\sqrt{-b/a} + 1)*\log(f)^3 - x^3*\sqrt{-b/a}*\log(-f^x*\sqrt{-b/a} + 1)*\log(f)^3 - 3*x^2*\sqrt{-b/a}*\operatorname{dilog}(f^x*\sqrt{-b/a})*\log(f)^2 + 3*x^2*\sqrt{-b/a}*\operatorname{dilog}(-f^x*\sqrt{-b/a})*\log(f)^2 + 6*x*\sqrt{-b/a}*\log(f)*\operatorname{polylog}(3, f^x*\sqrt{-b/a}) - 6*x*\sqrt{-b/a}*\log(f)*\operatorname{polylog}(3, -f^x*\sqrt{-b/a}) - 6*\sqrt{-b/a}*\operatorname{polylog}(4, f^x*\sqrt{-b/a}) + 6*\sqrt{-b/a}*\operatorname{polylog}(4, -f^x*\sqrt{-b/a}))}{(b*\log(f)^4)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{a + b f^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x\*\*3/(a+b\*f\*\*(2\*x)),x)

[Out] Integral(f\*\*x\*x\*\*3/(a + b\*f\*\*(2\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{b f^{2x} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x)),x, algorithm="giac")

[Out] integrate(f^x\*x^3/(b\*f^(2\*x) + a), x)

$$3.47 \quad \int \frac{f^x}{(a+bf^{2x})^2} dx$$

**Optimal.** Leaf size=59

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} + \frac{f^x}{2a\log(f)(a+bf^{2x})}$$

[Out]  $f^x/(2*a*(a + b*f^(2*x))*Log[f]) + ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*Log[f])$

**Rubi [A]** time = 0.0394861, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} + \frac{f^x}{2a\log(f)(a+bf^{2x})}$$

Antiderivative was successfully verified.

[In] Int[f^x/(a + b\*f^(2\*x))^2,x]

[Out]  $f^x/(2*a*(a + b*f^(2*x))*Log[f]) + ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*Log[f])$

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^x}{(a + bf^{2x})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, f^x\right)}{\log(f)} \\ &= \frac{f^x}{2a(a + bf^{2x})\log(f)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, f^x\right)}{2a\log(f)} \\ &= \frac{f^x}{2a(a + bf^{2x})\log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0501102, size = 53, normalized size = 0.9

$$\frac{\frac{f^x}{a^2+abf^{2x}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}}{2\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^x/(a + b\*f^(2\*x))^2,x]

[Out] (f^x/(a^2 + a\*b\*f^(2\*x)) + ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]]/(a^(3/2)\*Sqrt[b]))/(2\*Log[f])

**Maple [A]** time = 0.031, size = 82, normalized size = 1.4

$$\frac{f^x}{2 \ln(f) a (a + b (f^x)^2)} - \frac{1}{4 \ln(f) a} \ln\left(f^x - a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{4 \ln(f) a} \ln\left(f^x + a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x/(a+b\*f^(2\*x))^2,x)

[Out] 1/2/ln(f)/a\*f^x/(a+b\*(f^x)^2)-1/4/(-a\*b)^(1/2)/a/ln(f)\*ln(f^x-1/(-a\*b)^(1/2)\*a)+1/4/(-a\*b)^(1/2)/a/ln(f)\*ln(f^x+1/(-a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.53019, size = 373, normalized size = 6.32

$$\left[ \frac{2abf^x - (\sqrt{-abb}f^{2x} + \sqrt{-aba}) \log\left(\frac{bf^{2x} - 2\sqrt{-ab}f^x - a}{bf^{2x} + a}\right)}{4(a^2b^2f^{2x}\log(f) + a^3b\log(f))}, \frac{abf^x - (\sqrt{abb}f^{2x} + \sqrt{aba}) \arctan\left(\frac{\sqrt{ab}}{bf^x}\right)}{2(a^2b^2f^{2x}\log(f) + a^3b\log(f))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^2,x, algorithm="fricas")

[Out] [1/4\*(2\*a\*b\*f^x - (sqrt(-a\*b))\*b\*f^(2\*x) + sqrt(-a\*b)\*a)\*log((b\*f^(2\*x) - 2\*sqrt(-a\*b)\*f^x - a)/(b\*f^(2\*x) + a))/(a^2\*b^2\*f^(2\*x)\*log(f) + a^3\*b\*log(f)), 1/2\*(a\*b\*f^x - (sqrt(a\*b))\*b\*f^(2\*x) + sqrt(a\*b)\*a)\*arctan(sqrt(a\*b)/(b\*f^x))/(a^2\*b^2\*f^(2\*x)\*log(f) + a^3\*b\*log(f))]

**Sympy [A]** time = 0.378443, size = 53, normalized size = 0.9

$$\frac{f^x}{2a^2 \log(f) + 2abf^{2x} \log(f)} + \frac{\text{RootSum}\left(16z^2a^3b + 1, (i \mapsto i \log(4ia^2 + f^x))\right)}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x/(a+b\*f\*\*(2\*x))\*\*2,x)

[Out] f\*\*x/(2\*a\*\*2\*log(f) + 2\*a\*b\*f\*\*(2\*x)\*log(f)) + RootSum(16\*\_z\*\*2\*a\*\*3\*b + 1, Lambda(\_i, \_i\*log(4\*\_i\*a\*\*2 + f\*\*x)))/log(f)

**Giac [A]** time = 1.17298, size = 66, normalized size = 1.12

$$\frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{2\sqrt{aba}\log(f)} + \frac{f^x}{2(bf^{2x} + a)a\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^2,x, algorithm="giac")

[Out] 1/2\*arctan(b\*f^x/sqrt(a\*b))/(sqrt(a\*b)\*a\*log(f)) + 1/2\*f^x/((b\*f^(2\*x) + a)\*a\*log(f))

### 3.48 $\int \frac{f^x x}{(a + b f^{2x})^2} dx$

**Optimal.** Leaf size=172

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} - \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} + \frac{xf^x}{2a \log(f)(a + bf^{2x})}$$

[Out]  $-\operatorname{ArcTan}\left[\frac{\sqrt{b}f^x}{\sqrt{a}}\right]/(2a^{3/2}\sqrt{b}\log^2(f)) + (f^x x)/(2a(a + bf^{2x})\log(f)) + (x \operatorname{ArcTan}\left[\frac{\sqrt{b}f^x}{\sqrt{a}}\right])/ (2a^{3/2}\sqrt{b}\log(f)) - ((I/4) \operatorname{PolyLog}[2, ((-I)\sqrt{b}f^x)/\sqrt{a}])/(a^{3/2}\sqrt{b}\log^2(f)) + ((I/4) \operatorname{PolyLog}[2, (I\sqrt{b}f^x)/\sqrt{a}])/(a^{3/2}\sqrt{b}\log^2(f))$

**Rubi [A]** time = 0.158377, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {2249, 199, 205, 2245, 2282, 4848, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} - \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} + \frac{xf^x}{2a \log(f)(a + bf^{2x})}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(f^x x)/(a + b f^{2x})^2, x]$

[Out]  $-\operatorname{ArcTan}\left[\frac{\sqrt{b}f^x}{\sqrt{a}}\right]/(2a^{3/2}\sqrt{b}\log^2(f)) + (f^x x)/(2a(a + bf^{2x})\log(f)) + (x \operatorname{ArcTan}\left[\frac{\sqrt{b}f^x}{\sqrt{a}}\right])/ (2a^{3/2}\sqrt{b}\log(f)) - ((I/4) \operatorname{PolyLog}[2, ((-I)\sqrt{b}f^x)/\sqrt{a}])/(a^{3/2}\sqrt{b}\log^2(f)) + ((I/4) \operatorname{PolyLog}[2, (I\sqrt{b}f^x)/\sqrt{a}])/(a^{3/2}\sqrt{b}\log^2(f))$

#### Rule 2249

$\operatorname{Int}[(a + b \cdot (F)^{(e \cdot (c + d \cdot x))})^{(p)} \cdot (G)^{(h \cdot (f + g \cdot x))}], x_{\text{Symbol}}] \rightarrow \operatorname{With}[m = \operatorname{FullSimplify}[(d \cdot e \cdot \log[F]) / (g \cdot h \cdot \log[G])], \operatorname{Dist}[\operatorname{Denominator}[m] / (g \cdot h \cdot \log[G]), \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Denominator}[m] - 1)} \cdot (a + b \cdot F^{(c \cdot e - (d \cdot e \cdot f) / g)} \cdot x^{\operatorname{Numerator}[m]})^p, x], x, G^{(h \cdot (f + g \cdot x))} / \operatorname{Denominator}[m]], x] /; \operatorname{LtQ}[m, -1] \parallel \operatorname{GtQ}[m, 1]] /; \operatorname{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 199

$\operatorname{Int}[(a + b \cdot (x)^{(n)})^{(p)}], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(x \cdot (a + b \cdot x^n)^{(p+1)}) / (a \cdot n \cdot (p+1)), x] + \operatorname{Dist}[(n \cdot (p+1) + 1) / (a \cdot n \cdot (p+1)), \operatorname{Int}[(a + b \cdot x^n)^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& (\operatorname{IntegerQ}[2 \cdot p] \parallel (n == 2 \&\& \operatorname{IntegerQ}[4 \cdot p]) \parallel (n == 2 \&\& \operatorname{IntegerQ}[3 \cdot p]) \parallel \operatorname{Denominator}[p + 1/n] < \operatorname{Denominator}[p])$

#### Rule 205

$\operatorname{Int}[(a + b \cdot (x)^2)^{-1}], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] \cdot \operatorname{ArcTan}[x / \operatorname{Rt}[a/b, 2]]) / a, x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b]$

#### Rule 2245



```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_), x_Symbol] := With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Di
st[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e
}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{f^x x}{(a + b f^{2x})^2} dx &= \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \int \left( \frac{f^x}{2a(a + b f^{2x}) \log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \right) dx \\ &= \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{\int \frac{f^x}{a + b f^{2x}} dx}{2a \log(f)} - \frac{\int \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{2a^{3/2} \sqrt{b} \log(f)} \\ &= \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{\text{Subst}\left(\int \frac{1}{a + b x^2} dx, x, f^x\right)}{2a \log^2(f)} - \frac{\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{2a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{i \text{Subst}\left(\int \frac{\log\left(1 - \frac{i \sqrt{b} x}{\sqrt{a}}\right)}{x} dx, x, f^x\right)}{4a^{3/2} \sqrt{b} \log^2(f)} + \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{i \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} + \frac{i \text{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \end{aligned}$$

**Mathematica [A]** time = 0.116771, size = 271, normalized size = 1.58

$$\frac{-\frac{i \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} - \frac{i x \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} + \frac{i x^2}{2 \sqrt{a}}}{2 \sqrt{b}} + \frac{\frac{i \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{i x \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} - \frac{i x^2}{2 \sqrt{a}}}{2 \sqrt{b}} + \frac{x f^x}{2a \log(f) (a + b f^{2x})} - \frac{\left(\frac{b f^{2x}}{a} + 1\right) \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2 \sqrt{a} \sqrt{b} \log^2(f) (a + b f^{2x})}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f^x*x)/(a + b*f^(2*x))^2, x]
```

```
[Out] -((1 + (b*f^(2*x))/a)*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*(a
+ b*f^(2*x))*Log[f]^2) + (f^x*x)/(2*a*(a + b*f^(2*x))*Log[f]) + (((I/2)*x^
2)/Sqrt[a] - (I*x*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]) - (I*P
olyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/(2*Sqrt[b]) + ((
(-I/2)*x^2)/Sqrt[a] + (I*x*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f
])) + (I*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/(2*Sqrt[b
])/ (2*a)
```

**Maple [A]** time = 0.053, size = 195, normalized size = 1.1

$$\frac{f^x x}{2 \ln(f) a (a + b (f^x)^2)} + \frac{x}{4 \ln(f) a} \ln\left(\left(-b f^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} - \frac{x}{4 \ln(f) a} \ln\left(\left(b f^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^x*x/(a+b*f^(2*x))^2,x)
```

```
[Out] 1/2/ln(f)/a*f^x*x/(a+b*(f^x)^2)+1/4/ln(f)/a*x/(-a*b)^(1/2)*ln((-b*f^x+(-a*b
)^(1/2))/(-a*b)^(1/2))-1/4/ln(f)/a*x/(-a*b)^(1/2)*ln((b*f^x+(-a*b)^(1/2))/(-
-a*b)^(1/2))+1/4/ln(f)^2/a/(-a*b)^(1/2)*dilog((-b*f^x+(-a*b)^(1/2))/(-a*b)^(
1/2))-1/4/ln(f)^2/a/(-a*b)^(1/2)*dilog((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-
1/2/ln(f)^2/a/(a*b)^(1/2)*arctan(b*f^x/(a*b)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^x*x/(a+b*f^(2*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 1.53727, size = 670, normalized size = 3.9

$$2 b f^x x \log(f) + \left(b f^{2x} \sqrt{-\frac{b}{a}} + a \sqrt{-\frac{b}{a}}\right) \text{Li}_2\left(f^x \sqrt{-\frac{b}{a}}\right) - \left(b f^{2x} \sqrt{-\frac{b}{a}} + a \sqrt{-\frac{b}{a}}\right) \text{Li}_2\left(-f^x \sqrt{-\frac{b}{a}}\right) - \left(b f^{2x} \sqrt{-\frac{b}{a}} + a \sqrt{-\frac{b}{a}}\right) \log\left(2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^x*x/(a+b*f^(2*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*f^x*x*log(f) + (b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*dilog(f^x*sq
rt(-b/a)) - (b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) - (
b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*log(2*b*f^x + 2*a*sqrt(-b/a)) + (b*f^(
2*x)*sqrt(-b/a) + a*sqrt(-b/a))*log(2*b*f^x - 2*a*sqrt(-b/a)) - (b*f^(2*x)*
x*sqrt(-b/a)*log(f) + a*x*sqrt(-b/a)*log(f))*log(f^x*sqrt(-b/a) + 1) + (b*f
^(2*x)*x*sqrt(-b/a)*log(f) + a*x*sqrt(-b/a)*log(f))*log(-f^x*sqrt(-b/a) + 1
))/(a*b^2*f^(2*x)*log(f)^2 + a^2*b*log(f)^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^x x}{2a^2 \log(f) + 2abf^{2x} \log(f)} + \frac{\int -\frac{f^x}{a+bf^{2x}} dx + \int \frac{f^x \log(f)}{a+bf^{2x}} dx}{2a \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x/(a+b\*f\*\*(2\*x))\*\*2,x)

[Out] f\*\*x\*x/(2\*a\*\*2\*log(f) + 2\*a\*b\*f\*\*(2\*x)\*log(f)) + (Integral(-f\*\*x/(a + b\*f\*\*(2\*x)), x) + Integral(f\*\*x\*x\*log(f)/(a + b\*f\*\*(2\*x)), x))/(2\*a\*log(f))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x}{(bf^{2x} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x))^2,x, algorithm="giac")

[Out] integrate(f^x\*x/(b\*f^(2\*x) + a)^2, x)

$$3.49 \quad \int \frac{f^x x^2}{(a + b f^{2x})^2} dx$$

**Optimal.** Leaf size=333

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} - \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}$$

[Out]  $-\left(\frac{x \operatorname{ArcTan}\left[\frac{\sqrt{b} f^x}{\sqrt{a}}\right]}{a^{3/2} \sqrt{b} \log^2(f)}\right) + \left(\frac{f^x x^2}{2 a (a + b f^{2x}) \log(f)} + \frac{x^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} f^x}{\sqrt{a}}\right]}{2 a^{3/2} \sqrt{b} \log(f)} + \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[2, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) - \left(\frac{1}{2}\right) x \operatorname{PolyLog}\left[2, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^2(f)\right) - \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[2, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) + \left(\frac{1}{2}\right) x \operatorname{PolyLog}\left[2, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^2(f)\right) + \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[3, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) - \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[3, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right)\right)$

**Rubi [A]** time = 0.364925, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {2249, 199, 205, 2245, 14, 12, 2282, 4848, 2391, 5143, 2531, 6589}

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} - \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{f^x x^2}{(a + b f^{2x})^2}, x\right]$

[Out]  $-\left(\frac{x \operatorname{ArcTan}\left[\frac{\sqrt{b} f^x}{\sqrt{a}}\right]}{a^{3/2} \sqrt{b} \log^2(f)}\right) + \left(\frac{f^x x^2}{2 a (a + b f^{2x}) \log(f)} + \frac{x^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} f^x}{\sqrt{a}}\right]}{2 a^{3/2} \sqrt{b} \log(f)} + \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[2, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) - \left(\frac{1}{2}\right) x \operatorname{PolyLog}\left[2, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^2(f)\right) - \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[2, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) + \left(\frac{1}{2}\right) x \operatorname{PolyLog}\left[2, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^2(f)\right) + \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[3, \left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right) - \left(\frac{1}{2}\right) \operatorname{PolyLog}\left[3, \left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)\right] / \left(a^{3/2} \sqrt{b} \log^3(f)\right)\right)$

### Rule 2249

$\operatorname{Int}\left[\left((a_{\cdot}) + (b_{\cdot}) \cdot (F_{\cdot})^{\left((e_{\cdot}) \cdot ((c_{\cdot}) + (d_{\cdot}) \cdot (x_{\cdot}))\right)}\right)^{(p_{\cdot})} \cdot (G_{\cdot})^{\left((h_{\cdot}) \cdot ((f_{\cdot}) + (g_{\cdot}) \cdot (x_{\cdot}))\right)}\right], x_{\text{Symbol}}] \rightarrow \operatorname{With}\left[\{m = \operatorname{FullSimplify}\left[\frac{d \cdot e \cdot \log(F)}{g \cdot h \cdot \log(G)}\right]\}, \operatorname{Dist}\left[\frac{\operatorname{Denominator}[m]}{g \cdot h \cdot \log(G)}, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(\operatorname{Denominator}[m] - 1\right)} \cdot (a + b \cdot F^{\left(c \cdot e - \frac{d \cdot e \cdot f}{g}\right)} \cdot x^{\operatorname{Numerator}[m]}\right)^p, x\right], x, G^{\left(\frac{h \cdot (f + g \cdot x)}{\operatorname{Denominator}[m]}\right)}\right], x\right] /; \operatorname{LtQ}[m, -1] \parallel \operatorname{GtQ}[m, 1] /; \operatorname{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

### Rule 199

$\operatorname{Int}\left[\left((a_{\cdot}) + (b_{\cdot}) \cdot (x_{\cdot})^{(n_{\cdot})}\right)^{(p_{\cdot})}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}\left[\frac{x \cdot (a + b \cdot x^n)^{(p+1)}}{a \cdot n \cdot (p+1)}, x\right] + \operatorname{Dist}\left[\frac{n \cdot (p+1) + 1}{a \cdot n \cdot (p+1)}, \operatorname{Int}\left[(a + b \cdot x^n)^{(p+1)}, x\right], x\right] /; \operatorname{FreeQ}\{a, b\}, x \parallel \operatorname{IGtQ}[n, 0] \parallel \operatorname{LtQ}[p, -1] \parallel (\operatorname{IntegerQ}[2 \cdot p] \parallel (n == 2 \parallel \operatorname{IntegerQ}[4 \cdot p]) \parallel (n == 2 \parallel \operatorname{IntegerQ}[3 \cdot p]) \parallel \operatorname{Denomin}$

ator[p + 1/n] < Denominator[p])

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2245

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*((a\_) + (b\_)\*(F\_)^(v\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)\*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2\*e\*(c + d\*x)] && GtQ[m, 0] && ILtQ[p, 0]

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 4848

Int[((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x] /; FreeQ[{a, b, c}, x]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 5143

Int[ArcTan[(a\_) + (b\_)\*(f\_)^((c\_) + (d\_)\*(x\_))]\*(x\_)^(m\_), x\_Symbol] := Dist[I/2, Int[x^m\*Log[1 - I\*a - I\*b\*f^(c + d\*x)], x], x] - Dist[I/2, Int[x^m\*Log[1 + I\*a + I\*b\*f^(c + d\*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_)]\*((f\_) + (g\_)\*(x\_)^(m\_)), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

**Rule 6589**

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

**Rubi steps**

$$\begin{aligned}
\int \frac{f^x x^2}{(a + b f^{2x})^2} dx &= \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - 2 \int x \left( \frac{f^x}{2a(a + b f^{2x}) \log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \right) dx \\
&= \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - 2 \int \left( \frac{f^x x}{2a(a + b f^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \right) dx \\
&= \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{\int \frac{f^x x}{a + b f^{2x}} dx}{a \log(f)} - \frac{\int x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{a^{3/2} \sqrt{b} \log(f)} \\
&= -\frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} dx}{a \log(f)} - \frac{i \int x \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \\
&= -\frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{i x \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{i x \text{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \\
&= -\frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{i x \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{i x \text{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \\
&= -\frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{i x \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{i x \text{Li}_2\left(\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \\
&= -\frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{i \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^3(f)} - \frac{i x \text{Li}_2\left(-\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.121893, size = 477, normalized size = 1.43

$$\frac{-\frac{i \text{PolyLog}\left(2, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} - \frac{i x \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} + \frac{i x^2}{2 \sqrt{a}}}{2 \sqrt{b}} + \frac{\frac{i \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{i x \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} - \frac{i x^2}{2 \sqrt{a}}}{2 \sqrt{b}} + \frac{-\frac{2 i x \text{PolyLog}\left(2, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{2 i \text{PolyLog}\left(3, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^3(f)} - \frac{i x^2 \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} + \frac{2 i x \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{2 i \text{PolyLog}\left(3, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^3(f)} - \frac{i x^2 \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)}}{2 \sqrt{b}} + \frac{-\frac{2 i x \text{PolyLog}\left(2, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{2 i \text{PolyLog}\left(3, -\frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^3(f)} - \frac{i x^2 \log\left(1 + \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)} + \frac{2 i x \text{PolyLog}\left(2, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^2(f)} + \frac{2 i \text{PolyLog}\left(3, \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log^3(f)} - \frac{i x^2 \log\left(1 - \frac{i \sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \log(f)}}{2 \sqrt{b}}}{a \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x^2)/(a + b\*f^(2\*x))^2,x]

[Out] (f^x\*x^2)/(2\*a\*(a + b\*f^(2\*x))\*Log[f]) - (((I/2)\*x^2)/Sqrt[a] - (I\*x\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[a])/Sqrt[a]\*Log[f] - (I\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Log[f]^2))/(2\*Sqrt[b]) + (((-I/2)\*x^2)/Sqrt[a] + (I\*x\*Log[1 - (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[a])/Sqrt[a]\*Log[f] + (I\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/(Sqrt[a]\*Log[f]^2))/(2\*Sqrt[b])/(a\*Log[f]) + (((I/3)\*x^3)/Sqrt[a] - (I\*x^2\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[a])/Sqrt[a]\*Log[f]

$$- ((2*I)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2) + ((2*I)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^3)/(2*Sqrt[b]) + (((-I/3)*x^3)/Sqrt[a] + (I*x^2*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]) + ((2*I)*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2) - ((2*I)*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^3)/(2*Sqrt[b]))/(2*a)$$

**Maple [F]** time = 0.133, size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{(a + b f^{2x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x^2/(a+b\*f^(2\*x))^2,x)

[Out] int(f^x\*x^2/(a+b\*f^(2\*x))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.5916, size = 873, normalized size = 2.62

$$2bf^x x^2 \log(f)^2 + 2 \left( (bx \log(f) - b) f^{2x} \sqrt{-\frac{b}{a}} + (ax \log(f) - a) \sqrt{-\frac{b}{a}} \right) \text{Li}_2 \left( f^x \sqrt{-\frac{b}{a}} \right) - 2 \left( (bx \log(f) - b) f^{2x} \sqrt{-\frac{b}{a}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x))^2,x, algorithm="fricas")

[Out] 1/4\*(2\*b\*f^x\*x^2\*log(f)^2 + 2\*((b\*x\*log(f) - b)\*f^(2\*x)\*sqrt(-b/a) + (a\*x\*log(f) - a)\*sqrt(-b/a))\*dilog(f^x\*sqrt(-b/a)) - 2\*((b\*x\*log(f) - b)\*f^(2\*x)\*sqrt(-b/a) + (a\*x\*log(f) - a)\*sqrt(-b/a))\*dilog(-f^x\*sqrt(-b/a)) - ((b\*x^2\*log(f)^2 - 2\*b\*x\*log(f))\*f^(2\*x)\*sqrt(-b/a) + (a\*x^2\*log(f)^2 - 2\*a\*x\*log(f))\*sqrt(-b/a))\*log(f^x\*sqrt(-b/a) + 1) + ((b\*x^2\*log(f)^2 - 2\*b\*x\*log(f))\*f^(2\*x)\*sqrt(-b/a) + (a\*x^2\*log(f)^2 - 2\*a\*x\*log(f))\*sqrt(-b/a))\*log(-f^x\*sqrt(-b/a) + 1) - 2\*(b\*f^(2\*x)\*sqrt(-b/a) + a\*sqrt(-b/a))\*polylog(3, f^x\*sqrt(-b/a)) + 2\*(b\*f^(2\*x)\*sqrt(-b/a) + a\*sqrt(-b/a))\*polylog(3, -f^x\*sqrt(-b/a)))/(a\*b^2\*f^(2\*x)\*log(f)^3 + a^2\*b\*log(f)^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^x x^2}{2a^2 \log(f) + 2abf^{2x} \log(f)} + \frac{\int -\frac{2f^x x}{a+bf^{2x}} dx + \int \frac{f^x x^2 \log(f)}{a+bf^{2x}} dx}{2a \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x\*\*2/(a+b\*f\*\*(2\*x))\*\*2,x)

[Out] f\*\*x\*x\*\*2/(2\*a\*\*2\*log(f) + 2\*a\*b\*f\*\*(2\*x)\*log(f)) + (Integral(-2\*f\*\*x\*x/(a + b\*f\*\*(2\*x)), x) + Integral(f\*\*x\*x\*\*2\*log(f)/(a + b\*f\*\*(2\*x)), x))/(2\*a\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{(bf^{2x} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x))^2,x, algorithm="giac")

[Out] integrate(f^x\*x^2/(b\*f^(2\*x) + a)^2, x)



$$3.50 \quad \int \frac{f^x x^3}{(a + b f^{2x})^2} dx$$

**Optimal.** Leaf size=501

$$-\frac{3ix^2 \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{3ix \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} - \frac{3ix \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{3ix \text{Poly}}{2a^3}$$

[Out]  $(-3x^2 \text{ArcTan}[\text{Sqrt}[b]f^x/\text{Sqrt}[a]])/(2a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) + (f^x x^3)/(2a(a + bf^{(2x)})\text{Log}[f]) + (x^3 \text{ArcTan}[\text{Sqrt}[b]f^x/\text{Sqrt}[a]])/(2a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]) + (((3I)/2)x \text{PolyLog}[2, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) - (((3I)/4)x^2 \text{PolyLog}[2, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) - (((3I)/2)x \text{PolyLog}[2, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) + (((3I)/4)x^2 \text{PolyLog}[2, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) - (((3I)/2)\text{PolyLog}[3, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) + (((3I)/2)x \text{PolyLog}[3, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) + (((3I)/2)\text{PolyLog}[3, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) - (((3I)/2)x \text{PolyLog}[3, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) - (((3I)/2)\text{PolyLog}[4, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) + (((3I)/2)\text{PolyLog}[4, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4)$

**Rubi [A]** time = 0.513821, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {2249, 199, 205, 2245, 14, 12, 5143, 2531, 2282, 6589, 6609}

$$-\frac{3ix^2 \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{3ix \text{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} - \frac{3ix \text{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{3ix \text{Poly}}{2a^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f^x x^3)/(a + b f^{(2x)})^2, x]$

[Out]  $(-3x^2 \text{ArcTan}[\text{Sqrt}[b]f^x/\text{Sqrt}[a]])/(2a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) + (f^x x^3)/(2a(a + bf^{(2x)})\text{Log}[f]) + (x^3 \text{ArcTan}[\text{Sqrt}[b]f^x/\text{Sqrt}[a]])/(2a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]) + (((3I)/2)x \text{PolyLog}[2, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) - (((3I)/4)x^2 \text{PolyLog}[2, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) - (((3I)/2)x \text{PolyLog}[2, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) + (((3I)/4)x^2 \text{PolyLog}[2, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^2) - (((3I)/2)\text{PolyLog}[3, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) + (((3I)/2)x \text{PolyLog}[3, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) + (((3I)/2)\text{PolyLog}[3, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) - (((3I)/2)x \text{PolyLog}[3, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^3) - (((3I)/2)\text{PolyLog}[4, ((-I)\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4) + (((3I)/2)\text{PolyLog}[4, (I\text{Sqrt}[b]f^x/\text{Sqrt}[a])])/(a^{(3/2)}\text{Sqrt}[b]\text{Log}[f]^4)$

**Rule 2249**

$\text{Int}[(a_ + (b_ \cdot)(F_ )^((e_ )((c_ ) + (d_ ) \cdot(x_ ))))^((p_ ) \cdot(G_ )^((h_ )((f_ ) + (g_ ) \cdot(x_ ))))], x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(d \cdot e \cdot \text{Log}[F])/ (g \cdot h \cdot \text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/ (g \cdot h \cdot \text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)} \cdot (a + b \cdot F^{(c \cdot e - (d \cdot e \cdot f)/g)} \cdot x^{\text{Numerator}[m]})^p, x], x, G^{((h \cdot (f + g \cdot x))/ \text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] || \text{GtQ}[m, 1]] /; \text{FreeQ}\{F, G, a, b, c, d, e,$

f, g, h, p}, x]

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 2245

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) \* ((a\_.) + (b\_.)\*(F\_)^(v\_))^(p\_) \* (x\_)^(m\_), x\_Symbol] := With[{u = IntHide[F^(e\*(c + d\*x))\*(a + b\*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)\*u, x], x]] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2\*e\*(c + d\*x)] && GtQ[m, 0] && ILtQ[p, 0]

### Rule 14

Int[(u\_) \* ((c\_.)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_.)\*(v\_)] /; FreeQ[b, x]

### Rule 5143

Int[ArcTan[(a\_.) + (b\_.)\*(f\_)^((c\_.) + (d\_.)\*(x\_))]\*(x\_)^(m\_), x\_Symbol] := Dist[I/2, Int[x^m\*Log[1 - I\*a - I\*b\*f^(c + d\*x)], x], x] - Dist[I/2, Int[x^m\*Log[1 + I\*a + I\*b\*f^(c + d\*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)] \* ((f\_.) + (g\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_.)^((c_.)*((a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/ (b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{f^x x^3}{(a + b f^{2x})^2} dx &= \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - 3 \int x^2 \left( \frac{f^x}{2a(a + b f^{2x}) \log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \right) dx \\ &= \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - 3 \int \left( \frac{f^x x^2}{2a(a + b f^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} \right) dx \\ &= \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{3 \int \frac{f^x x^2}{a + b f^{2x}} dx}{2a \log(f)} - \frac{3 \int x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right) dx}{2a^{3/2} \sqrt{b} \log(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{3 \int \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} \log(f)} dx}{a \log(f)} - \frac{(3i) \int x^2 \log\left(\frac{a + b f^{2x}}{a}\right) dx}{4a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{3ix \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^3(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{3ix \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^3(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \\ &= -\frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^2(f)} + \frac{f^x x^3}{2a(a + b f^{2x}) \log(f)} + \frac{x^3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log(f)} + \frac{3ix \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{b} \log^3(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{b} \log^2(f)} \end{aligned}$$

**Mathematica [A]** time = 0.259529, size = 434, normalized size = 0.87

$$\frac{6i \text{PolyLog}\left(3, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{6i \text{PolyLog}\left(3, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{6i \text{PolyLog}\left(4, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{6i \text{PolyLog}\left(4, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{3ix \log(f)(x \log(f) - 2) \text{PolyLog}\left(2, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{3ix \log(f)(x \log(f) - 2) \text{PolyLog}\left(2, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x^3)/(a + b\*f^(2\*x))^2,x]

[Out] ((2\*Sqrt[a]\*f^x\*x^3\*Log[f]^3)/(a + b\*f^(2\*x)) - ((3\*I)\*x^2\*Log[f]^2\*Log[1 - (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + (I\*x^3\*Log[f]^3\*Log[1 - (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + ((3\*I)\*x^2\*Log[f]^2\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] - (I\*x^3\*Log[f]^3\*Log[1 + (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] - ((3\*I)\*x\*Log[f]\*(-2 + x\*Log[f])\*PolyLog[2, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + ((3\*I)\*x\*Log[f]\*(-2 + x\*Log[f])\*PolyLog[2, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] - ((6\*I)\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + ((6\*I)\*x\*Log[f]\*PolyLog[3, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + ((6\*I)\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] - ((6\*I)\*x\*Log[f]\*PolyLog[3, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] - ((6\*I)\*PolyLog[4, ((-I)\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b] + ((6\*I)\*PolyLog[4, (I\*Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b])/(4\*a^(3/2)\*Log[f]^4)

**Maple [F]** time = 0.161, size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{(a + b f^{2x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x^3/(a+b\*f^(2\*x))^2,x)

[Out] int(f^x\*x^3/(a+b\*f^(2\*x))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.64195, size = 1245, normalized size = 2.49

$$2bf^x x^3 \log(f)^3 + 3 \left( (bx^2 \log(f)^2 - 2bx \log(f)) f^{2x} \sqrt{-\frac{b}{a}} + (ax^2 \log(f)^2 - 2ax \log(f)) \sqrt{-\frac{b}{a}} \right) \text{Li}_2 \left( f^x \sqrt{-\frac{b}{a}} \right) - 3 \left( (bx^2 \log(f)^2 - 2bx \log(f)) f^{2x} \sqrt{-\frac{b}{a}} + (ax^2 \log(f)^2 - 2ax \log(f)) \sqrt{-\frac{b}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x))^2,x, algorithm="fricas")

[Out] 1/4\*(2\*b\*f^x\*x^3\*log(f)^3 + 3\*((b\*x^2\*log(f)^2 - 2\*b\*x\*log(f))\*f^(2\*x)\*sqrt(-b/a) + (a\*x^2\*log(f)^2 - 2\*a\*x\*log(f))\*sqrt(-b/a))\*dilog(f^x\*sqrt(-b/a)) - 3\*((b\*x^2\*log(f)^2 - 2\*b\*x\*log(f))\*f^(2\*x)\*sqrt(-b/a) + (a\*x^2\*log(f)^2 - 2\*a\*x\*log(f))\*sqrt(-b/a))\*dilog(-f^x\*sqrt(-b/a)) - ((b\*x^3\*log(f)^3 - 3\*b\*

$$x^2 \log(f)^2 f^{2x} \sqrt{-b/a} + (a x^3 \log(f)^3 - 3 a x^2 \log(f)^2) \sqrt{-b/a} \log(f^x \sqrt{-b/a} + 1) + ((b x^3 \log(f)^3 - 3 b x^2 \log(f)^2) f^{2x} \sqrt{-b/a} + (a x^3 \log(f)^3 - 3 a x^2 \log(f)^2) \sqrt{-b/a}) \log(-f^x \sqrt{-b/a} + 1) + 6 (b f^{2x} \sqrt{-b/a} + a \sqrt{-b/a}) \operatorname{polylog}(4, f^x \sqrt{-b/a}) - 6 (b f^{2x} \sqrt{-b/a} + a \sqrt{-b/a}) \operatorname{polylog}(4, -f^x \sqrt{-b/a}) - 6 ((b x \log(f) - b) f^{2x} \sqrt{-b/a} + (a x \log(f) - a) \sqrt{-b/a}) \operatorname{polylog}(3, f^x \sqrt{-b/a}) + 6 ((b x \log(f) - b) f^{2x} \sqrt{-b/a} + (a x \log(f) - a) \sqrt{-b/a}) \operatorname{polylog}(3, -f^x \sqrt{-b/a}) / (a b^2 f^{2x} \log(f)^4 + a^2 b \log(f)^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^x x^3}{2a^2 \log(f) + 2abf^{2x} \log(f)} + \frac{\int -\frac{3f^x x^2}{a+bf^{2x}} dx + \int \frac{f^x x^3 \log(f)}{a+bf^{2x}} dx}{2a \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x\*\*3/(a+b\*f\*\*(2\*x))\*\*2,x)

[Out] f\*\*x\*x\*\*3/(2\*a\*\*2\*log(f) + 2\*a\*b\*f\*\*(2\*x)\*log(f)) + (Integral(-3\*f\*\*x\*x\*\*2/(a + b\*f\*\*(2\*x)), x) + Integral(f\*\*x\*x\*\*3\*log(f)/(a + b\*f\*\*(2\*x)), x))/(2\*a\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{(bf^{2x} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^3/(a+b\*f^(2\*x))^2,x, algorithm="giac")

[Out] integrate(f^x\*x^3/(b\*f^(2\*x) + a)^2, x)

### 3.51 $\int \frac{f^x}{(a+bf^{2x})^3} dx$

**Optimal.** Leaf size=84

$$\frac{3f^x}{8a^2 \log(f)(a+bf^{2x})} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log(f)} + \frac{f^x}{4a \log(f)(a+bf^{2x})^2}$$

[Out]  $f^x/(4*a*(a + b*f^(2*x))^2*\text{Log}[f]) + (3*f^x)/(8*a^2*(a + b*f^(2*x))*\text{Log}[f])$   
 $+ (3*\text{ArcTan}[(\text{Sqrt}[b]*f^x)/\text{Sqrt}[a]])/(8*a^(5/2)*\text{Sqrt}[b]*\text{Log}[f])$

**Rubi [A]** time = 0.0510675, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 199, 205}

$$\frac{3f^x}{8a^2 \log(f)(a+bf^{2x})} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log(f)} + \frac{f^x}{4a \log(f)(a+bf^{2x})^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^x/(a + b*f^(2*x))^3, x]$

[Out]  $f^x/(4*a*(a + b*f^(2*x))^2*\text{Log}[f]) + (3*f^x)/(8*a^2*(a + b*f^(2*x))*\text{Log}[f])$   
 $+ (3*\text{ArcTan}[(\text{Sqrt}[b]*f^x)/\text{Sqrt}[a]])/(8*a^(5/2)*\text{Sqrt}[b]*\text{Log}[f])$

#### Rule 2249

$\text{Int}[(a_ + (b_)*(F_)^((e_)*((c_ + (d_)*(x_))))^(p_))*(G_)^((h_)*((f_ + (g_)*(x_))))], x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(d*e*\text{Log}[F])/(g*h*\text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*\text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)*(a + b*f^{(c*e - (d*e*f)/g)*x^{\text{Numerator}[m]})}]]^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] || \text{GtQ}[m, 1] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 199

$\text{Int}[(a_ + (b_)*(x_)^{(n_}))^(p_)], x\_Symbol] := -\text{Simp}[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + \text{Dist}[(n*(p + 1) + 1)/(a*n*(p + 1)), \text{Int}[(a + b*x^n)^(p + 1), x], x] /; \text{FreeQ}\{a, b\}, x \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[2*p] || (n == 2 \&\& \text{IntegerQ}[4*p]) || (n == 2 \&\& \text{IntegerQ}[3*p]) || \text{Denominator}[p + 1/n] < \text{Denominator}[p])$

#### Rule 205

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}], x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \&\& \text{PosQ}[a/b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{f^x}{(a + bf^{2x})^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3} dx, x, f^x\right)}{\log(f)} \\
&= \frac{f^x}{4a(a + bf^{2x})^2 \log(f)} + \frac{3 \text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, f^x\right)}{4a \log(f)} \\
&= \frac{f^x}{4a(a + bf^{2x})^2 \log(f)} + \frac{3f^x}{8a^2(a + bf^{2x}) \log(f)} + \frac{3 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, f^x\right)}{8a^2 \log(f)} \\
&= \frac{f^x}{4a(a + bf^{2x})^2 \log(f)} + \frac{3f^x}{8a^2(a + bf^{2x}) \log(f)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0501984, size = 68, normalized size = 0.81

$$\frac{\frac{\sqrt{a}f^x(5a+3bf^{2x})}{(a+bf^{2x})^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2} \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^x/(a + b\*f^(2\*x))^3,x]

[Out] ((Sqrt[a]\*f^x\*(5\*a + 3\*b\*f^(2\*x)))/(a + b\*f^(2\*x))^2 + (3\*ArcTan[(Sqrt[b]\*f^x)/Sqrt[a]])/Sqrt[b])/(8\*a^(5/2)\*Log[f])

**Maple [A]** time = 0.043, size = 94, normalized size = 1.1

$$\frac{f^x \left(3b(f^x)^2 + 5a\right)}{8 \ln(f) a^2 \left(a + b(f^x)^2\right)^2} - \frac{3}{16 \ln(f) a^2} \ln\left(f^x - a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{3}{16 \ln(f) a^2} \ln\left(f^x + a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x/(a+b\*f^(2\*x))^3,x)

[Out] 1/8\*f^x\*(3\*b\*(f^x)^2+5\*a)/ln(f)/a^2/(a+b\*(f^x)^2)^2-3/16/(-a\*b)^(1/2)/a^2/ln(f)\*ln(f^x-1/(-a\*b)^(1/2)\*a)+3/16/(-a\*b)^(1/2)/a^2/ln(f)\*ln(f^x+1/(-a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.56869, size = 593, normalized size = 7.06

$$\left[ \frac{6ab^2f^{3x} + 10a^2bf^x - 3(\sqrt{-abb^2}f^{4x} + 2\sqrt{-abab}f^{2x} + \sqrt{-aba^2}) \log\left(\frac{bf^{2x} - 2\sqrt{-ab}f^x - a}{bf^{2x} + a}\right)}{16(a^3b^3f^{4x} \log(f) + 2a^4b^2f^{2x} \log(f) + a^5b \log(f))}, \frac{3ab^2f^{3x} + 5a^2bf^x - 3(\sqrt{abb^2}f^{4x} + 2\sqrt{abab}f^{2x} + \sqrt{aba^2}) \log\left(\frac{bf^{2x} + 2\sqrt{ab}f^x + a}{bf^{2x} + a}\right)}{8(a^3b^3f^{4x} \log(f) + 2a^4b^2f^{2x} \log(f) + a^5b \log(f))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^3,x, algorithm="fricas")

[Out] [1/16\*(6\*a\*b^2\*f^(3\*x) + 10\*a^2\*b\*f^x - 3\*(sqrt(-a\*b)\*b^2\*f^(4\*x) + 2\*sqrt(-a\*b)\*a\*b\*f^(2\*x) + sqrt(-a\*b)\*a^2)\*log((b\*f^(2\*x) - 2\*sqrt(-a\*b)\*f^x - a)/(b\*f^(2\*x) + a)))/(a^3\*b^3\*f^(4\*x)\*log(f) + 2\*a^4\*b^2\*f^(2\*x)\*log(f) + a^5\*b\*log(f)), 1/8\*(3\*a\*b^2\*f^(3\*x) + 5\*a^2\*b\*f^x - 3\*(sqrt(a\*b)\*b^2\*f^(4\*x) + 2\*sqrt(a\*b)\*a\*b\*f^(2\*x) + sqrt(a\*b)\*a^2)\*arctan(sqrt(a\*b)/(b\*f^x)))/(a^3\*b^3\*f^(4\*x)\*log(f) + 2\*a^4\*b^2\*f^(2\*x)\*log(f) + a^5\*b\*log(f))]

**Sympy [A]** time = 0.46923, size = 85, normalized size = 1.01

$$\frac{5af^x + 3bf^{3x}}{8a^4 \log(f) + 16a^3bf^{2x} \log(f) + 8a^2b^2f^{4x} \log(f)} + \frac{\text{RootSum}\left(256z^2a^5b + 9, \left(i \mapsto i \log\left(\frac{16ia^3}{3} + f^x\right)\right)\right)}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x/(a+b\*f\*\*(2\*x))\*\*3,x)

[Out] (5\*a\*f\*\*x + 3\*b\*f\*\*(3\*x))/(8\*a\*\*4\*log(f) + 16\*a\*\*3\*b\*f\*\*(2\*x)\*log(f) + 8\*a\*\*2\*b\*\*2\*f\*\*(4\*x)\*log(f)) + RootSum(256\*\_z\*\*2\*a\*\*5\*b + 9, Lambda(\_i, \_i\*log(16\*\_i\*a\*\*3/3 + f\*\*x)))/log(f)

**Giac [A]** time = 1.22519, size = 82, normalized size = 0.98

$$\frac{3 \arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{8\sqrt{aba^2} \log(f)} + \frac{3bf^{3x} + 5af^x}{8(bf^{2x} + a)^2 a^2 \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x/(a+b\*f^(2\*x))^3,x, algorithm="giac")

[Out] 3/8\*arctan(b\*f^x/sqrt(a\*b))/(sqrt(a\*b)\*a^2\*log(f)) + 1/8\*(3\*b\*f^(3\*x) + 5\*a\*f^x)/((b\*f^(2\*x) + a)^2\*a^2\*log(f))



$$3.52 \quad \int \frac{f^x x}{(a + b f^{2x})^3} dx$$

**Optimal.** Leaf size=223

$$\frac{3i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)} + \frac{3i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)} - \frac{f^x}{8a^2 \log^2(f)(a + b f^{2x})} + \frac{3x f^x}{8a^2 \log(f)(a + b f^{2x})} - \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^2(f)}$$

```
[Out] -f^x/(8*a^2*(a + b*f^(2*x))*Log[f]^2) - ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2*a^(5/2)*Sqrt[b]*Log[f]^2) + (f^x*x)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f^x*x)/(8*a^2*(a + b*f^(2*x))*Log[f]) + (3*x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*Log[f]) - (((3*I)/16)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/16)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2)
```

**Rubi [A]** time = 0.217052, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {2249, 199, 205, 2245, 2282, 4848, 2391}

$$\frac{3i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)} + \frac{3i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)} - \frac{f^x}{8a^2 \log^2(f)(a + b f^{2x})} + \frac{3x f^x}{8a^2 \log(f)(a + b f^{2x})} - \frac{\tan^{-1}\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Int[(f^x*x)/(a + b*f^(2*x))^3,x]
```

```
[Out] -f^x/(8*a^2*(a + b*f^(2*x))*Log[f]^2) - ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2*a^(5/2)*Sqrt[b]*Log[f]^2) + (f^x*x)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f^x*x)/(8*a^2*(a + b*f^(2*x))*Log[f]) + (3*x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*Log[f]) - (((3*I)/16)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/16)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2)
```

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2245

```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] := With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Di
st[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e
}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{f^x x}{(a + b f^{2x})^3} dx &= \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - \int \left( \frac{f^x}{4a(a + b f^{2x})^2 \log(f)} + \frac{f^x}{8a^2(a + b f^{2x}) \log(f)} \right) dx \\ &= \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - \frac{3 \int \frac{f^x}{a + b f^{2x}} dx}{8a^2 \log(f)} - \frac{\int \frac{f^x}{(a + b f^{2x})^2} dx}{4a \log(f)} \\ &= \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{a + b x^2} dx, x, f^x\right)}{8a^2 \log^2(f)} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{(a + b x^2)^2} dx, x, f^x\right)}{4a \log(f)} \\ &= -\frac{f^x}{8a^2(a + b f^{2x}) \log^2(f)} - \frac{3 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} \\ &= -\frac{f^x}{8a^2(a + b f^{2x}) \log^2(f)} - \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{2a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.305697, size = 184, normalized size = 0.83

$$\frac{6i \left( -\operatorname{PolyLog}\left(2, -\frac{i\sqrt{b} f^x}{\sqrt{a}}\right) + \operatorname{PolyLog}\left(2, \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) + x \log(f) \left( \log\left(1 - \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) - \log\left(1 + \frac{i\sqrt{b} f^x}{\sqrt{a}}\right) \right) \right)}{\sqrt{a}\sqrt{b}} + \frac{8ax f^x \log(f)}{(a + b f^{2x})^2} + \frac{4f^x(3x \log(f) - 1)}{a + b f^{2x}} - \frac{16 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$


---


$$32a^2 \log^2(f)$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x)/(a + b\*f^(2\*x))^3,x]

[Out] 
$$\frac{(-16 \operatorname{ArcTan}[\frac{\sqrt{b} f^x}{\sqrt{a}}]) / (\sqrt{a} \sqrt{b}) + (8 a f^x \operatorname{Log}[f]) / (a + b f^{2x})^2 + (4 f^x (-1 + 3 x \operatorname{Log}[f])) / (a + b f^{2x}) + ((6 I) (x \operatorname{Log}[f] (\operatorname{Log}[1 - (I \sqrt{b} f^x) / \sqrt{a}] - \operatorname{Log}[1 + (I \sqrt{b} f^x) / \sqrt{a}]) - \operatorname{PolyLog}[2, (-I \sqrt{b} f^x) / \sqrt{a}] + \operatorname{PolyLog}[2, (I \sqrt{b} f^x) / \sqrt{a}]) / (\sqrt{a} \sqrt{b}))}{(32 a^2 \operatorname{Log}[f]^2)}$$

**Maple [A]** time = 0.069, size = 223, normalized size = 1.

$$\frac{f^x \left( 3 \ln(f) b x (f^x)^2 + 5 \ln(f) a x - b (f^x)^2 - a \right)}{8 (\ln(f))^2 a^2 (a + b (f^x)^2)^2} - \frac{1}{2 (\ln(f))^2 a^2} \arctan\left(b f^x \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} + \frac{3x}{16 \ln(f) a^2} \ln\left(\left(-b f^x + \dots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^x\*x/(a+b\*f^(2\*x))^3,x)

[Out] 
$$\frac{1}{8} f^x (3 \ln(f) b x (f^x)^2 + 5 \ln(f) a x - b (f^x)^2 - a) / \ln(f)^2 a^2 (a + b (f^x)^2)^2 - \frac{1}{2 (\ln(f))^2 a^2} \arctan\left(b f^x / (a b)^{1/2}\right) + \frac{3}{16} \ln(f) / a^2 x / (-a b)^{1/2} \ln\left(\frac{-b f^x + (-a b)^{1/2}}{(-a b)^{1/2}}\right) - \frac{3}{16} \ln(f) / a^2 x / (-a b)^{1/2} \ln\left(\frac{b f^x + (-a b)^{1/2}}{(-a b)^{1/2}}\right) + \frac{3}{16} \ln(f)^2 / a^2 (-a b)^{1/2} \operatorname{dilog}\left(\frac{-b f^x + (-a b)^{1/2}}{(-a b)^{1/2}}\right) - \frac{3}{16} \ln(f)^2 / a^2 (-a b)^{1/2} \operatorname{dilog}\left(\frac{b f^x + (-a b)^{1/2}}{(-a b)^{1/2}}\right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.60431, size = 1068, normalized size = 4.79

$$2 \left( 3 b^2 x \log(f) - b^2 \right) f^{3x} + 2 \left( 5 a b x \log(f) - a b \right) f^x + 3 \left( b^2 f^{4x} \sqrt{-\frac{b}{a}} + 2 a b f^{2x} \sqrt{-\frac{b}{a}} + a^2 \sqrt{-\frac{b}{a}} \right) \operatorname{Li}_2\left(f^x \sqrt{-\frac{b}{a}}\right) - 3 \left( b^2 f^{4x} \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x))^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{16} (2 (3 b^2 x \log(f) - b^2) f^{3x} + 2 (5 a b x \log(f) - a b) f^x + 3 (b^2 f^{4x} \sqrt{-\frac{b}{a}} + 2 a b f^{2x} \sqrt{-\frac{b}{a}} + a^2 \sqrt{-\frac{b}{a}}) \operatorname{Li}_2\left(f^x \sqrt{-\frac{b}{a}}\right) - 3 (b^2 f^{4x} \sqrt{-\frac{b}{a}} + 2 a b f^{2x} \sqrt{-\frac{b}{a}} + a^2 \sqrt{-\frac{b}{a}}) \operatorname{dilog}\left(\frac{f^x \sqrt{-\frac{b}{a}}}{\sqrt{-\frac{b}{a}}}\right) - 3 (b^2 f^{4x} \sqrt{-\frac{b}{a}} + 2 a b f^{2x} \sqrt{-\frac{b}{a}} + a^2 \sqrt{-\frac{b}{a}}) \operatorname{dilog}\left(\frac{-f^x \sqrt{-\frac{b}{a}}}{\sqrt{-\frac{b}{a}}}\right) - 4 (b^2 f^{4x} \sqrt{-\frac{b}{a}} + 2 a b f^{2x} \sqrt{-\frac{b}{a}} + a^2 \sqrt{-\frac{b}{a}}) \log\left(2 b f^x + 2 a \sqrt{-\frac{b}{a}}\right) + 4 (b^2 f^{4x} \dots))$$

$$(4x)\sqrt{-b/a} + 2abf^{2x}\sqrt{-b/a} + a^2\sqrt{-b/a})\log(2bf^x - 2a\sqrt{-b/a}) - 3(b^2f^{4x}x\sqrt{-b/a}\log(f) + 2abf^{2x}x\sqrt{-b/a}\log(f) + a^2x\sqrt{-b/a}\log(f))\log(f^x\sqrt{-b/a} + 1) + 3(b^2f^{4x}x\sqrt{-b/a}\log(f) + 2abf^{2x}x\sqrt{-b/a}\log(f) + a^2x\sqrt{-b/a}\log(f))\log(-f^x\sqrt{-b/a} + 1))/(a^2b^3f^{4x}\log(f)^2 + 2a^3b^2f^{2x}\log(f)^2 + a^4b\log(f)^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^{3x}(3bx\log(f) - b) + f^x(5ax\log(f) - a)}{8a^4\log(f)^2 + 16a^3bf^{2x}\log(f)^2 + 8a^2b^2f^{4x}\log(f)^2} + \frac{\int -\frac{4f^x}{a+bf^{2x}} dx + \int \frac{3f^{3x}\log(f)}{a+bf^{2x}} dx}{8a^2\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x/(a+b\*f\*\*(2\*x))\*\*3,x)

[Out] (f\*\*(3\*x)\*(3\*b\*x\*log(f) - b) + f\*\*x\*(5\*a\*x\*log(f) - a))/(8\*a\*\*4\*log(f)\*\*2 + 16\*a\*\*3\*b\*f\*\*(2\*x)\*log(f)\*\*2 + 8\*a\*\*2\*b\*\*2\*f\*\*(4\*x)\*log(f)\*\*2) + (Integral(-4\*f\*\*x/(a + b\*f\*\*(2\*x)), x) + Integral(3\*f\*\*x\*x\*log(f)/(a + b\*f\*\*(2\*x)), x))/(8\*a\*\*2\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x}{(bf^{2x} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x/(a+b\*f^(2\*x))^3,x, algorithm="giac")

[Out] integrate(f^x\*x/(b\*f^(2\*x) + a)^3, x)

$$3.53 \quad \int \frac{f^x x^2}{(a + b f^{2x})^3} dx$$

**Optimal.** Leaf size=420

$$-\frac{3ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b} \log^3(f)} + \frac{3ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^2(f)} - \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b} \log^3(f)} + \frac{3i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^3(f)}$$

```
[Out] ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(4*a^(5/2)*Sqrt[b]*Log[f]^3) - (f^x*x)/(4*a^2
*(a + b*f^(2*x))*Log[f]^2) - (x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqr
t[b]*Log[f]^2) + (f^x*x^2)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f^x*x^2)/(8*
a^2*(a + b*f^(2*x))*Log[f]) + (3*x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(8*a^(5
/2)*Sqrt[b]*Log[f]) + ((I/2)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/
2)*Sqrt[b]*Log[f]^3) - (((3*I)/8)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])
/(a^(5/2)*Sqrt[b]*Log[f]^2) - ((I/2)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(
a^(5/2)*Sqrt[b]*Log[f]^3) + (((3*I)/8)*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]
])/ (a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/8)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sq
rt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3) - (((3*I)/8)*PolyLog[3, (I*Sqrt[b]*f^x)/
Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3)
```

**Rubi [A]** time = 0.582367, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {2249, 199, 205, 2245, 14, 2282, 4848, 2391, 12, 5143, 2531, 6589}

$$-\frac{3ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^2(f)} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b} \log^3(f)} + \frac{3ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^2(f)} - \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b} \log^3(f)} + \frac{3i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b} \log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Int[(f^x*x^2)/(a + b*f^(2*x))^3, x]
```

```
[Out] ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(4*a^(5/2)*Sqrt[b]*Log[f]^3) - (f^x*x)/(4*a^2
*(a + b*f^(2*x))*Log[f]^2) - (x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqr
t[b]*Log[f]^2) + (f^x*x^2)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f^x*x^2)/(8*
a^2*(a + b*f^(2*x))*Log[f]) + (3*x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(8*a^(5
/2)*Sqrt[b]*Log[f]) + ((I/2)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(5/
2)*Sqrt[b]*Log[f]^3) - (((3*I)/8)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])
/(a^(5/2)*Sqrt[b]*Log[f]^2) - ((I/2)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(
a^(5/2)*Sqrt[b]*Log[f]^3) + (((3*I)/8)*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]
])/ (a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/8)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sq
rt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3) - (((3*I)/8)*PolyLog[3, (I*Sqrt[b]*f^x)/
Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3)
```

**Rule 2249**

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*(f_
.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
f, g, h, p}, x]
```

**Rule 199**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1) / (a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2245

```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_))) * ((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(m_), x_Symbol] := With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Rule 14

```
Int[(u_) * ((c_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 + I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_) /; FreeQ[b, x]]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] := Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)] * ((f_.) + (g_.)*(x_)^(m_.)), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{f^x x^2}{(a + b f^{2x})^3} dx &= \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - 2 \int x \left( \frac{f^x}{4a(a + b f^{2x})^2 \log(f)} \right. \\
 &= \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - 2 \int \left( \frac{f^x x}{4a(a + b f^{2x})^2 \log(f)} \right. \\
 &= \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} - \frac{3 \int \frac{f^x x}{a + b f^{2x}} dx}{4a^2 \log(f)} - \frac{\int \frac{f^x x}{(a + b f^{2x})^2} dx}{2a \log(f)} \\
 &= -\frac{f^x x}{4a^2(a + b f^{2x}) \log^2(f)} - \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} \\
 &= -\frac{f^x x}{4a^2(a + b f^{2x}) \log^2(f)} - \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} \\
 &= -\frac{f^x x}{4a^2(a + b f^{2x}) \log^2(f)} - \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} + \frac{3x^2 \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{8a^{5/2} \sqrt{b} \log(f)} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{5/2} \sqrt{b} \log^3(f)} - \frac{f^x x}{4a^2(a + b f^{2x}) \log^2(f)} - \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{4a^{5/2} \sqrt{b} \log^3(f)} - \frac{f^x x}{4a^2(a + b f^{2x}) \log^2(f)} - \frac{x \tan^{-1}\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{a^{5/2} \sqrt{b} \log^2(f)} + \frac{f^x x^2}{4a(a + b f^{2x})^2 \log(f)} + \frac{3f^x x^2}{8a^2(a + b f^{2x}) \log(f)}
 \end{aligned}$$

**Mathematica [A]** time = 0.490146, size = 353, normalized size = 0.84

$$\frac{3i \left( 2 \operatorname{PolyLog} \left( 3, -\frac{i\sqrt{b} f^x}{\sqrt{a}} \right) - 2 \operatorname{PolyLog} \left( 3, \frac{i\sqrt{b} f^x}{\sqrt{a}} \right) - 2x \log(f) \operatorname{PolyLog} \left( 2, -\frac{i\sqrt{b} f^x}{\sqrt{a}} \right) + 2x \log(f) \operatorname{PolyLog} \left( 2, \frac{i\sqrt{b} f^x}{\sqrt{a}} \right) + x^2 \log^2(f) \log \left( 1 - \frac{i\sqrt{b} f^x}{\sqrt{a}} \right) - x^2 \log^2(f) \log \left( 1 + \frac{i\sqrt{b} f^x}{\sqrt{a}} \right) \right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(f^x\*x^2)/(a + b\*f^(2\*x))^3,x]

```
[Out] ((4*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (4*a*f^x*x^2*Log[f]^2)/(a + b*f^(2*x))^2 + (2*f^x*x*Log[f]*(-2 + 3*x*Log[f]))/(a + b*f^(2*x)) - ((8*I)*(x*Log[f]*(Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]]) - PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]) + ((3*I)*(x^2*Log[f]^2*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - x^2*Log[f]^2*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]] - 2*x*Log[f]*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + 2*x*Log[f]*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]] + 2*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] - 2*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]))/(16*a^2*Log[f]^3)
```

**Maple [F]** time = 0.143, size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{(a + b f^{2x})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^x*x^2/(a+b*f^(2*x))^3,x)
```

```
[Out] int(f^x*x^2/(a+b*f^(2*x))^3,x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^x*x^2/(a+b*f^(2*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 1.68545, size = 1747, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^x*x^2/(a+b*f^(2*x))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(3*b^2*x^2*log(f)^2 - 2*b^2*x*log(f))*f^(3*x) + 2*(5*a*b*x^2*log(f)^2 - 2*a*b*x*log(f))*f^x + 2*((3*b^2*x*log(f) - 4*b^2)*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x*log(f) - 4*a*b)*f^(2*x)*sqrt(-b/a) + (3*a^2*x*log(f) - 4*a^2)*sqrt(-b/a))*dilog(f^x*sqrt(-b/a)) - 2*((3*b^2*x*log(f) - 4*b^2)*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x*log(f) - 4*a*b)*f^(2*x)*sqrt(-b/a) + (3*a^2*x*log(f) - 4*a^2)*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) + 2*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*log(2*b*f^x + 2*a*sqrt(-b/a)) - 2*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*log(2*b*f^x - 2*a*sqrt(-b/a)) - ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (3*a^2*x^2*log(f)^2 - 8*a^2*x*log(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a) + 1) + ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (3*a^2*x^2*log(f)^2 - 8*a^2*x*log(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a) + 1) + ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (3*a^2*x^2*log(f)^2 - 8*a^2*x*log(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a) - 1) + ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^(4*x)*sqrt(-b/a) + 2*(3*a*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (3*a^2*x^2*log(f)^2 - 8*a^2*x*log(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a) - 1)
```



$$\begin{aligned} &)^2 - 8abx \log(f) f^{2x} \sqrt{-b/a} + (3a^2 x^2 \log(f)^2 - 8a^2 x \log(f)) \sqrt{-b/a} \log(-f^x \sqrt{-b/a} + 1) \\ &- 6(b^2 f^{4x} \sqrt{-b/a} + 2ab f^{2x} \sqrt{-b/a} + a^2 \sqrt{-b/a}) \operatorname{polylog}(3, f^x \sqrt{-b/a}) + 6(b^2 f^{4x} \sqrt{-b/a} \\ &+ 2ab f^{2x} \sqrt{-b/a} + a^2 \sqrt{-b/a}) \operatorname{polylog}(3, -f^x \sqrt{-b/a}) / (a^2 b^3 f^{4x} \log(f)^3 + 2a^3 b^2 f^{2x} \log(f)^3 \\ &+ a^4 b \log(f)^3) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^{3x} (3bx^2 \log(f) - 2bx) + f^x (5ax^2 \log(f) - 2ax)}{8a^4 \log(f)^2 + 16a^3 b f^{2x} \log(f)^2 + 8a^2 b^2 f^{4x} \log(f)^2} + \frac{\int \frac{2f^x}{a+bf^{2x}} dx + \int -\frac{8f^x \log(f)}{a+bf^{2x}} dx + \int \frac{3f^{x^2} \log(f)^2}{a+bf^{2x}} dx}{8a^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*x\*x\*\*2/(a+b\*f\*\*(2\*x))\*\*3,x)

[Out] (f\*\*(3\*x)\*(3\*b\*x\*\*2\*log(f) - 2\*b\*x) + f\*\*x\*(5\*a\*x\*\*2\*log(f) - 2\*a\*x))/(8\*a\*\*4\*log(f)\*\*2 + 16\*a\*\*3\*b\*f\*\*(2\*x)\*log(f)\*\*2 + 8\*a\*\*2\*b\*\*2\*f\*\*(4\*x)\*log(f)\*\*2) + (Integral(2\*f\*\*x/(a + b\*f\*\*(2\*x)), x) + Integral(-8\*f\*\*x\*x\*log(f)/(a + b\*f\*\*(2\*x)), x) + Integral(3\*f\*\*x\*x\*\*2\*log(f)\*\*2/(a + b\*f\*\*(2\*x)), x))/(8\*a\*\*2\*log(f)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{(bf^{2x} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^x\*x^2/(a+b\*f^(2\*x))^3,x, algorithm="giac")

[Out] integrate(f^x\*x^2/(b\*f^(2\*x) + a)^3, x)

$$3.54 \quad \int \frac{1}{bf^{-x}+af^x} dx$$

**Optimal.** Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

[Out] ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

**Rubi [A]** time = 0.0211419, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2282, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Int[(b/f^x + a\*f^x)^(-1), x]

[Out] ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{bf^{-x}+af^x} dx &= \frac{\text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, f^x\right)}{\log(f)} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0077143, size = 30, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[(b/f^x + a\*f^x)^(-1),x]

[Out] ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]]/(Sqrt[a]\*Sqrt[b]\*Log[f])

**Maple [A]** time = 0.004, size = 22, normalized size = 0.7

$$\frac{1}{\ln(f)} \arctan\left(af^x \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/(f^x)+a\*f^x),x)

[Out] 1/ln(f)/(a\*b)^(1/2)\*arctan(a\*f^x/(a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.62191, size = 190, normalized size = 6.33

$$\left[ \frac{\sqrt{-ab} \log\left(\frac{af^{2x} - 2\sqrt{-ab}f^x - b}{af^{2x} + b}\right)}{2ab \log(f)}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}}{af^x}\right)}{ab \log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x),x, algorithm="fricas")

[Out] [-1/2\*sqrt(-a\*b)\*log((a\*f^(2\*x) - 2\*sqrt(-a\*b)\*f^x - b)/(a\*f^(2\*x) + b))/(a\*b\*log(f)), -sqrt(a\*b)\*arctan(sqrt(a\*b)/(a\*f^x))/(a\*b\*log(f))]

**Sympy [A]** time = 0.263538, size = 26, normalized size = 0.87

$$\frac{\text{RootSum}\left(4z^2ab + 1, \left(i \mapsto i \log(-2ia + f^{-x})\right)\right)}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f\*\*x)+a\*f\*\*x),x)

[Out]  $\text{RootSum}(4*_z^{**2}*a*b + 1, \text{Lambda}(_i, _i*\log(-2*_i*a + f**(-x))))/\log(f)$

---

**Giac [A]** time = 1.21153, size = 28, normalized size = 0.93

$$\frac{\arctan\left(\frac{af^x}{\sqrt{ab}}\right)}{\sqrt{ab}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b/(f^x)+a*f^x),x, algorithm="giac")`

[Out]  $\arctan(a*f^x/\text{sqrt}(a*b))/(\text{sqrt}(a*b)*\log(f))$

$$3.55 \quad \int \frac{x}{bf^{-x}+af^x} dx$$

**Optimal.** Leaf size=110

$$-\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

[Out] (x\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - ((I/2)\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((I/2)\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2)

**Rubi [A]** time = 0.0952606, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2282, 205, 2266, 12, 4848, 2391}

$$-\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Int[x/(b/f^x + a\*f^x), x]

[Out] (x\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - ((I/2)\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((I/2)\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2)

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2266

Int[(x\_)^(m\_)/((b\_)\*(F\_)^(v\_) + (a\_)\*(F\_)^(c\_) + (d\_)\*(x\_)), x\_Symbol] := With[{u = IntHide[1/(a\*F^(c) + b\*F^v), x]}, Simp[x^m\*u, x] - Dist[m, Int[x^(m-1)\*u, x], x] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c + d\*x)] && GtQ[m, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 4848

Int[((a\_) + ArcTan[(c\_)\*(x\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 +

$I*c*x]/x, x], x]) /; \text{FreeQ}[\{a, b, c\}, x]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_)*(d\_)+(e\_)*(x\_)^{(n\_)}]]/(x\_), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

### Rubi steps

$$\begin{aligned} \int \frac{x}{bf^{-x} + af^x} dx &= \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \int \frac{\tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} dx \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{\int \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{b}}\right)}{x} dx, x, f^x\right)}{\sqrt{a}\sqrt{b} \log^2(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{i \text{Subst}\left(\int \frac{\log\left(1 - \frac{i\sqrt{ax}}{\sqrt{b}}\right)}{x} dx, x, f^x\right)}{2\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \text{Subst}\left(\int \frac{\log\left(1 + \frac{i\sqrt{ax}}{\sqrt{b}}\right)}{x} dx, x, f^x\right)}{2\sqrt{a}\sqrt{b} \log^2(f)} \\ &= \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{i \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b} \log^2(f)} \end{aligned}$$

**Mathematica [A]** time = 0.062469, size = 108, normalized size = 0.98

$$\frac{i\left(-\text{PolyLog}\left(2, -\frac{i\sqrt{af^x}}{\sqrt{b}}\right) + \text{PolyLog}\left(2, \frac{i\sqrt{af^x}}{\sqrt{b}}\right) + x \log(f) \left(\log\left(1 - \frac{i\sqrt{af^x}}{\sqrt{b}}\right) - \log\left(1 + \frac{i\sqrt{af^x}}{\sqrt{b}}\right)\right)\right)}{2\sqrt{a}\sqrt{b} \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b/f^x + a\*f^x), x]

[Out]  $\left(\frac{I}{2}\right) * (x * \text{Log}[f] * (\text{Log}[1 - (I * \text{Sqrt}[a] * f^x) / \text{Sqrt}[b]] - \text{Log}[1 + (I * \text{Sqrt}[a] * f^x) / \text{Sqrt}[b]])) - \text{PolyLog}[2, ((-I) * \text{Sqrt}[a] * f^x) / \text{Sqrt}[b]] + \text{PolyLog}[2, (I * \text{Sqrt}[a] * f^x) / \text{Sqrt}[b]]) / (\text{Sqrt}[a] * \text{Sqrt}[b] * \text{Log}[f]^2)$

**Maple [A]** time = 0.041, size = 134, normalized size = 1.2

$$\frac{x}{2 \ln(f)} \ln\left(\left(-af^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} - \frac{x}{2 \ln(f)} \ln\left(\left(af^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{2 (\ln(f))^2} \text{dilog}\left(\left(-af^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} - \frac{1}{2 (\ln(f))^2} \text{dilog}\left(\left(af^x + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b/(f^x)+a\*f^x), x)

[Out]  $\frac{1}{2} / \ln(f) * x / (-a*b)^{(1/2)} * \ln((-a*f^x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)}) - 1/2 / \ln(f) * x / (-a*b)^{(1/2)} * \ln((a*f^x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)}) + 1/2 / \ln(f)^2 / (-a*b)^{(1/2)} * \text{dilog}((-a*f^x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)}) - 1/2 / \ln(f)^2 / (-a*b)^{(1/2)} * \text{dilog}((a*f^x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)})$

$$(a*f^x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.72942, size = 247, normalized size = 2.25

$$\frac{x\sqrt{-\frac{a}{b}}\log\left(f^x\sqrt{-\frac{a}{b}}+1\right)\log(f)-x\sqrt{-\frac{a}{b}}\log\left(-f^x\sqrt{-\frac{a}{b}}+1\right)\log(f)-\sqrt{-\frac{a}{b}}\text{Li}_2\left(f^x\sqrt{-\frac{a}{b}}\right)+\sqrt{-\frac{a}{b}}\text{Li}_2\left(-f^x\sqrt{-\frac{a}{b}}\right)}{2a\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x),x, algorithm="fricas")

[Out]  $-1/2*(x*\sqrt{-a/b}*\log(f^x*\sqrt{-a/b} + 1)*\log(f) - x*\sqrt{-a/b}*\log(-f^x*\sqrt{-a/b} + 1)*\log(f) - \sqrt{-a/b}*\text{dilog}(f^x*\sqrt{-a/b}) + \sqrt{-a/b}*\text{dilog}(-f^x*\sqrt{-a/b}))/ (a*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x}{af^{2x} + b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f\*\*x)+a\*f\*\*x),x)

[Out] Integral(f\*\*x\*x/(a\*f\*\*(2\*x) + b), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{af^x + \frac{b}{f^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x),x, algorithm="giac")

[Out] integrate(x/(a\*f^x + b/f^x), x)

### 3.56 $\int \frac{x^2}{bf^{-x}+af^x} dx$

**Optimal.** Leaf size=184

$$-\frac{ix\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{ix\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i\text{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{i\text{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

[Out] (x^2\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (I\*x\*PolyLog[2, ((-1)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*x\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*PolyLog[3, ((-1)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - (I\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3)

**Rubi [A]** time = 0.167025, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {2282, 205, 2266, 12, 5143, 2531, 6589}

$$-\frac{ix\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{ix\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i\text{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{i\text{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(b/f^x + a\*f^x), x]

[Out] (x^2\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (I\*x\*PolyLog[2, ((-1)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*x\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (I\*PolyLog[3, ((-1)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - (I\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 2266

```
Int[(x_)^(m_.)/((b_.)*(F_)^(v_) + (a_.)*(F_)^((c_.) + (d_.)*(x_))), x_Symbol] := With[{u = IntHide[1/(a*F^(c + d*x) + b*F^v), x]}, Simp[x^m*u, x] - Dist[m, Int[x^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c + d*x)] && GtQ[m, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```



Rule 5143

Int[ArcTan[(a\_.) + (b\_.)\*(f\_)^((c\_.) + (d\_.)\*(x\_))]\*(x\_)^(m\_.), x\_Symbol] :> Dist[I/2, Int[x^m\*Log[1 - I\*a - I\*b\*f^(c + d\*x)], x], x] - Dist[I/2, Int[x^m\*Log[1 + I\*a + I\*b\*f^(c + d\*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IntegerQ[m] && m > 0

Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{bf^{-x} + af^x} dx &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - 2 \int \frac{x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} dx \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{2 \int x \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{i \int x \log\left(1 - \frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log(f)} + \frac{i \int x \log\left(1 + \frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{ix \operatorname{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{ix \operatorname{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \int \operatorname{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log^2(f)} - \frac{i \int \operatorname{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b} \log^2(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{ix \operatorname{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{ix \operatorname{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{i\sqrt{ax}}{\sqrt{b}}\right)}{x} dx, x, f^x\right)}{\sqrt{a}\sqrt{b} \log^3(f)} - \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{i\sqrt{ax}}{\sqrt{b}}\right)}{x} dx, x, f^x\right)}{\sqrt{a}\sqrt{b} \log^3(f)} \\
 &= \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log(f)} - \frac{ix \operatorname{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{ix \operatorname{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^2(f)} + \frac{i \operatorname{Li}_3\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)} - \frac{i \operatorname{Li}_3\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b} \log^3(f)}
 \end{aligned}$$

**Mathematica [A]** time = 0.049838, size = 168, normalized size = 0.91

$$\frac{i \left( 2 \operatorname{PolyLog}\left(3, -\frac{i\sqrt{af^x}}{\sqrt{b}}\right) - 2 \operatorname{PolyLog}\left(3, \frac{i\sqrt{af^x}}{\sqrt{b}}\right) - 2x \log(f) \operatorname{PolyLog}\left(2, -\frac{i\sqrt{af^x}}{\sqrt{b}}\right) + 2x \log(f) \operatorname{PolyLog}\left(2, \frac{i\sqrt{af^x}}{\sqrt{b}}\right) + x^2 \right)}{2\sqrt{a}\sqrt{b} \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(b/f^x + a\*f^x), x]

[Out] ((I/2)\*(x^2\*Log[f]^2\*Log[1 - (I\*Sqrt[a]\*f^x)/Sqrt[b]] - x^2\*Log[f]^2\*Log[1 + (I\*Sqrt[a]\*f^x)/Sqrt[b]] - 2\*x\*Log[f]\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] + 2\*x\*Log[f]\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]] + 2\*PolyLog[3, ((-I)\*S

$\text{qrt}[a]*f^x)/\text{Sqrt}[b]] - 2*\text{PolyLog}[3, (I*\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Log}[f]^3)$

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int x^2 \left( \frac{b}{f^x} + af^x \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b/(f^x)+a*f^x),x)`

[Out] `int(x^2/(b/(f^x)+a*f^x),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b/(f^x)+a*f^x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.92755, size = 402, normalized size = 2.18

$$\frac{x^2 \sqrt{-\frac{a}{b}} \log\left(f^x \sqrt{-\frac{a}{b}} + 1\right) \log(f)^2 - x^2 \sqrt{-\frac{a}{b}} \log\left(-f^x \sqrt{-\frac{a}{b}} + 1\right) \log(f)^2 - 2x \sqrt{-\frac{a}{b}} \text{Li}_2\left(f^x \sqrt{-\frac{a}{b}}\right) \log(f) + 2x \sqrt{-\frac{a}{b}} \text{Li}_2\left(-f^x \sqrt{-\frac{a}{b}}\right) \log(f)}{2a \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b/(f^x)+a*f^x),x, algorithm="fricas")`

[Out] `-1/2*(x^2*sqrt(-a/b)*log(f^x*sqrt(-a/b) + 1)*log(f)^2 - x^2*sqrt(-a/b)*log(-f^x*sqrt(-a/b) + 1)*log(f)^2 - 2*x*sqrt(-a/b)*dilog(f^x*sqrt(-a/b))*log(f) + 2*x*sqrt(-a/b)*dilog(-f^x*sqrt(-a/b))*log(f) + 2*sqrt(-a/b)*polylog(3, f^x*sqrt(-a/b)) - 2*sqrt(-a/b)*polylog(3, -f^x*sqrt(-a/b)))/(a*log(f)^3)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^2}{af^{2x} + b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b/(f**x)+a*f**x),x)`

[Out] Integral( $f^{**x}x^{**2}/(a*f^{**}(2*x) + b)$ , x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{af^x + \frac{b}{f^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^2/(b/(f^x)+a*f^x)$ ,x, algorithm="giac")

[Out] integrate( $x^2/(a*f^x + b/f^x)$ , x)

$$3.57 \quad \int \frac{x^3}{bf^{-x}+af^x} dx$$

**Optimal.** Leaf size=268

$$-\frac{3ix^2\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix\text{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix\text{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i\text{PolyLog}\left(4, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i\text{PolyLog}\left(4, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}$$

[Out] (x^3\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (((3\*I)/2)\*x^2\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (((3\*I)/2)\*x^2\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((3\*I)\*x\*PolyLog[3, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*x\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*PolyLog[4, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4) + ((3\*I)\*PolyLog[4, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

**Rubi [A]** time = 0.219538, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {2282, 205, 2266, 12, 5143, 2531, 6609, 6589}

$$-\frac{3ix^2\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix\text{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix\text{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i\text{PolyLog}\left(4, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i\text{PolyLog}\left(4, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b/f^x + a\*f^x), x]

[Out] (x^3\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]) - (((3\*I)/2)\*x^2\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + (((3\*I)/2)\*x^2\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^2) + ((3\*I)\*x\*PolyLog[3, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*x\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^3) - ((3\*I)\*PolyLog[4, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4) + ((3\*I)\*PolyLog[4, (I\*Sqrt[a]\*f^x)/Sqrt[b]])/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

#### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :=> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 2266

Int[(x\_)^(m\_.)/((b\_.)\*(F\_)^(v\_) + (a\_.)\*(F\_)^((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :=> With[{u = IntHide[1/(a\*f^(c + d\*x) + b\*f^v), x]}, Simp[x^m\*u, x] - Dist[m, Int[x^(m-1)\*u, x], x] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c +

$d*x)] \&\& \text{GtQ}[m, 0]$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \text{:>} \text{Dist}[a, \text{Int}[u, x], x] \text{/; FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_) \text{/; FreeQ}[b, x]]$

### Rule 5143

$\text{Int}[\text{ArcTan}[(a_*) + (b_*)*(f_)^((c_*) + (d_*)*(x_)))]*(x_)^(m_), x\_Symbol] \text{:>} \text{Dist}[I/2, \text{Int}[x^m*\text{Log}[1 - I*a - I*b*f^(c + d*x)], x], x] - \text{Dist}[I/2, \text{Int}[x^m*\text{Log}[1 + I*a + I*b*f^(c + d*x)], x], x] \text{/; FreeQ}[\{a, b, c, d, f\}, x] \&\& \text{IntegerQ}[m] \&\& m > 0$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_*)*(F_)^((c_*)*(a_*) + (b_*)*(x_)))]*(f_*) + (g_*)*(x_)^(m_), x\_Symbol] \text{:>} -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^(m-1)*\text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)], x], x] \text{/; FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 6609

$\text{Int}[(e_*) + (f_*)*(x_)^(m_*)*\text{PolyLog}[n_, (d_*)*(F_)^((c_*)*(a_*) + (b_*)*(x_)))]*(x_)^(p_), x\_Symbol] \text{:>} \text{Simp}[(e + f*x)^m*\text{PolyLog}[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m)/(b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^(m-1)*\text{PolyLog}[n + 1, d*(F^(c*(a + b*x)))^p], x], x] \text{/; FreeQ}[\{F, a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_*)*(a_*) + (b_*)*(x_)^(p_)]/((d_*) + (e_*)*(x_)), x\_Symbol] \text{:>} \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] \text{/; FreeQ}[\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{bf^{-x} + af^x} dx &= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - 3 \int \frac{x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} dx \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3 \int x^2 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{(3i) \int x^2 \log\left(1 - \frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{2\sqrt{a}\sqrt{b}\log(f)} + \frac{(3i) \int x^2 \log\left(1 + \frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{2\sqrt{a}\sqrt{b}\log(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{(3i) \int x \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^2(f)} - \frac{(3i) \int x \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^2(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{(3i) \int \text{Li}_4\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{(3i) \int \text{Li}_4\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^4(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{(3i) \int \text{Li}_4\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{(3i) \int \text{Li}_4\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right) dx}{\sqrt{a}\sqrt{b}\log^4(f)} \\
&= \frac{x^3 \tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2 \text{Li}_2\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2 \text{Li}_2\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix \text{Li}_3\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix \text{Li}_3\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i \text{Li}_4\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i \text{Li}_4\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.050695, size = 224, normalized size = 0.84

$$\frac{i\left(-3x^2 \log^2(f) \text{PolyLog}\left(2, -\frac{i\sqrt{af^x}}{\sqrt{b}}\right) + 3x^2 \log^2(f) \text{PolyLog}\left(2, \frac{i\sqrt{af^x}}{\sqrt{b}}\right) - 6 \text{PolyLog}\left(4, -\frac{i\sqrt{af^x}}{\sqrt{b}}\right) + 6 \text{PolyLog}\left(4, \frac{i\sqrt{af^x}}{\sqrt{b}}\right) + 6i \text{Li}_4\left(-\frac{i\sqrt{af^x}}{\sqrt{b}}\right) - 6i \text{Li}_4\left(\frac{i\sqrt{af^x}}{\sqrt{b}}\right)\right)}{2\sqrt{a}\sqrt{b}\log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b/f^x + a\*f^x), x]

[Out] ((I/2)\*(x^3\*Log[f]^3\*Log[1 - (I\*Sqrt[a]\*f^x)/Sqrt[b]] - x^3\*Log[f]^3\*Log[1 + (I\*Sqrt[a]\*f^x)/Sqrt[b]] - 3\*x^2\*Log[f]^2\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] + 3\*x^2\*Log[f]^2\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]] + 6\*x\*Log[f]\*PolyLog[3, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] - 6\*x\*Log[f]\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]] - 6\*PolyLog[4, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] + 6\*PolyLog[4, (I\*Sqrt[a]\*f^x)/Sqrt[b]]))/(Sqrt[a]\*Sqrt[b]\*Log[f]^4)

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int x^3 \left(\frac{b}{f^x} + af^x\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b/(f^x)+a\*f^x), x)

[Out] int(x^3/(b/(f^x)+a\*f^x), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.78342, size = 552, normalized size = 2.06

$$x^3 \sqrt{-\frac{a}{b}} \log\left(f^x \sqrt{-\frac{a}{b}} + 1\right) \log(f)^3 - x^3 \sqrt{-\frac{a}{b}} \log\left(-f^x \sqrt{-\frac{a}{b}} + 1\right) \log(f)^3 - 3x^2 \sqrt{-\frac{a}{b}} \operatorname{Li}_2\left(f^x \sqrt{-\frac{a}{b}}\right) \log(f)^2 + 3x^2 \sqrt{-\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x),x, algorithm="fricas")

[Out] 
$$-1/2*(x^3*\sqrt{-a/b}*\log(f^x*\sqrt{-a/b} + 1)*\log(f)^3 - x^3*\sqrt{-a/b}*\log(-f^x*\sqrt{-a/b} + 1)*\log(f)^3 - 3*x^2*\sqrt{-a/b}*\operatorname{dilog}(f^x*\sqrt{-a/b})*\log(f)^2 + 3*x^2*\sqrt{-a/b}*\operatorname{dilog}(-f^x*\sqrt{-a/b})*\log(f)^2 + 6*x*\sqrt{-a/b}*\log(f)*\operatorname{polylog}(3, f^x*\sqrt{-a/b}) - 6*x*\sqrt{-a/b}*\log(f)*\operatorname{polylog}(3, -f^x*\sqrt{-a/b}) - 6*\sqrt{-a/b}*\operatorname{polylog}(4, f^x*\sqrt{-a/b}) + 6*\sqrt{-a/b}*\operatorname{polylog}(4, -f^x*\sqrt{-a/b}))/ (a*\log(f)^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^x x^3}{af^{2x} + b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b/(f\*\*x)+a\*f\*\*x),x)

[Out] Integral(f\*\*x\*x\*\*3/(a\*f\*\*(2\*x) + b), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{af^x + \frac{b}{f^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x),x, algorithm="giac")

[Out] integrate(x^3/(a\*f^x + b/f^x), x)

$$3.58 \quad \int \frac{1}{(bf^{-x} + af^x)^2} dx$$

**Optimal.** Leaf size=22

$$-\frac{1}{2a \log(f) (af^{2x} + b)}$$

[Out] -1/(2\*a\*(b + a\*f^(2\*x))\*Log[f])

**Rubi [A]** time = 0.0199484, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2282, 261}

$$-\frac{1}{2a \log(f) (af^{2x} + b)}$$

Antiderivative was successfully verified.

[In] Int[(b/f^x + a\*f^x)^(-2), x]

[Out] -1/(2\*a\*(b + a\*f^(2\*x))\*Log[f])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

#### Rubi steps

$$\int \frac{1}{(bf^{-x} + af^x)^2} dx = \frac{\text{Subst}\left(\int \frac{x}{(b+ax^2)^2} dx, x, f^x\right)}{\log(f)} = -\frac{1}{2a(b + af^{2x}) \log(f)}$$

**Mathematica [A]** time = 0.020285, size = 23, normalized size = 1.05

$$-\frac{1}{2a^2 f^{2x} \log(f) + 2ab \log(f)}$$

Antiderivative was successfully verified.



[In] Integrate[(b/f^x + a\*f^x)^(-2),x]

[Out]  $-(2*a*b*\text{Log}[f] + 2*a^2*f^{(2*x)}*\text{Log}[f])^{-1}$

**Maple [A]** time = 0.001, size = 21, normalized size = 1.

$$-\frac{1}{2 \ln(f) a \left( a (f^x)^2 + b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/(f^x)+a\*f^x)^2,x)

[Out]  $-1/2/\ln(f)/a/(a*(f^x)^2+b)$

**Maxima [A]** time = 1.09949, size = 31, normalized size = 1.41

$$\frac{1}{2 \left( ab + \frac{b^2}{f^{2x}} \right) \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^2,x, algorithm="maxima")

[Out]  $1/2/((a*b + b^2/f^{(2*x)})*\log(f))$

**Fricas [A]** time = 1.54727, size = 54, normalized size = 2.45

$$-\frac{1}{2 \left( a^2 f^{2x} \log(f) + ab \log(f) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^2,x, algorithm="fricas")

[Out]  $-1/2/(a^2*f^{(2*x)}*\log(f) + a*b*\log(f))$

**Sympy [A]** time = 0.246905, size = 22, normalized size = 1.

$$\frac{1}{2ab \log(f) + 2b^2 f^{-2x} \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f\*\*x)+a\*f\*\*x)\*\*2,x)

[Out]  $1/(2*a*b*\log(f) + 2*b**2*f**(-2*x)*\log(f))$

---

**Giac [A]** time = 1.23908, size = 27, normalized size = 1.23

$$-\frac{1}{2(a f^{2x} + b)a \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^2,x, algorithm="giac")

[Out] -1/2/((a\*f^(2\*x) + b)\*a\*log(f))

$$3.59 \quad \int \frac{x}{(bf^{-x} + af^x)^2} dx$$

**Optimal.** Leaf size=63

$$-\frac{\log(af^{2x} + b)}{4ab \log^2(f)} - \frac{x}{2a \log(f)(af^{2x} + b)} + \frac{x}{2ab \log(f)}$$

[Out] x/(2\*a\*b\*Log[f]) - x/(2\*a\*(b + a\*f^(2\*x))\*Log[f]) - Log[b + a\*f^(2\*x)]/(4\*a\*b\*Log[f]^2)

**Rubi [A]** time = 0.0783578, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{\log(af^{2x} + b)}{4ab \log^2(f)} - \frac{x}{2a \log(f)(af^{2x} + b)} + \frac{x}{2ab \log(f)}$$

Antiderivative was successfully verified.

[In] Int[x/(b/f^x + a\*f^x)^2,x]

[Out] x/(2\*a\*b\*Log[f]) - x/(2\*a\*(b + a\*f^(2\*x))\*Log[f]) - Log[b + a\*f^(2\*x)]/(4\*a\*b\*Log[f]^2)

#### Rule 2283

Int[(u\_.)\*((a\_.)\*(F\_)^(v\_) + (b\_.)\*(F\_)^(w\_))^(n\_), x\_Symbol] := Int[u\*F^(n\*v)\*(a + b\*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ[n, 0] && LinearQ[{v, w}, x]

#### Rule 2191

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1)/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_.)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x}{(bf^{-x} + af^x)^2} dx &= \int \frac{f^{2x}x}{(b + af^{2x})^2} dx \\ &= -\frac{x}{2a(b + af^{2x})\log(f)} + \frac{\int \frac{1}{b+af^{2x}} dx}{2a\log(f)} \\ &= -\frac{x}{2a(b + af^{2x})\log(f)} + \frac{\text{Subst}\left(\int \frac{1}{x(b+ax)} dx, x, f^{2x}\right)}{4a\log^2(f)} \\ &= -\frac{x}{2a(b + af^{2x})\log(f)} - \frac{\text{Subst}\left(\int \frac{1}{b+ax} dx, x, f^{2x}\right)}{4b\log^2(f)} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, f^{2x}\right)}{4ab\log^2(f)} \\ &= \frac{x}{2ab\log(f)} - \frac{x}{2a(b + af^{2x})\log(f)} - \frac{\log(b + af^{2x})}{4ab\log^2(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0572333, size = 48, normalized size = 0.76

$$\frac{\frac{2xf^{2x}\log(f)}{af^{2x}+b} - \frac{\log(af^{2x}+b)}{a}}{4b\log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b/f^x + a\*f^x)^2,x]

[Out] ((2\*f^(2\*x)\*x\*Log[f])/(b + a\*f^(2\*x)) - Log[b + a\*f^(2\*x)]/a)/(4\*b\*Log[f]^2)

**Maple [A]** time = 0.016, size = 56, normalized size = 0.9

$$\frac{x\left(e^{x\ln(f)}\right)^2}{2b\ln(f)\left(\left(e^{x\ln(f)}\right)^2 a + b\right)} - \frac{\ln\left(\left(e^{x\ln(f)}\right)^2 a + b\right)}{4\left(\ln(f)\right)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b/(f^x)+a\*f^x)^2,x)

[Out] 1/2/b/ln(f)\*x\*exp(x\*ln(f))^2/(exp(x\*ln(f))^2\*a+b)-1/4/ln(f)^2/a/b\*ln(exp(x\*ln(f))^2\*a+b)

**Maxima [A]** time = 1.06419, size = 73, normalized size = 1.16

$$\frac{f^{2x}x}{2(abf^{2x}\log(f) + b^2\log(f))} - \frac{\log\left(\frac{af^{2x}+b}{a}\right)}{4ab\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x)^2,x, algorithm="maxima")

[Out] 1/2\*f^(2\*x)\*x/(a\*b\*f^(2\*x)\*log(f) + b^2\*log(f)) - 1/4\*log((a\*f^(2\*x) + b)/a)/(a\*b\*log(f)^2)

**Fricas [A]** time = 1.50443, size = 144, normalized size = 2.29

$$\frac{2af^{2x}x\log(f) - (af^{2x} + b)\log(af^{2x} + b)}{4(a^2bf^{2x}\log(f)^2 + ab^2\log(f)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x)^2,x, algorithm="fricas")

[Out] 1/4\*(2\*a\*f^(2\*x)\*x\*log(f) - (a\*f^(2\*x) + b)\*log(a\*f^(2\*x) + b))/(a^2\*b\*f^(2\*x)\*log(f)^2 + a\*b^2\*log(f)^2)

**Sympy [A]** time = 0.296525, size = 54, normalized size = 0.86

$$\frac{x}{2ab\log(f) + 2b^2f^{-2x}\log(f)} - \frac{x}{2ab\log(f)} - \frac{\log\left(\frac{a}{b} + f^{-2x}\right)}{4ab\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f\*\*x)+a\*f\*\*x)\*\*2,x)

[Out] x/(2\*a\*b\*log(f) + 2\*b\*\*2\*f\*\*(-2\*x)\*log(f)) - x/(2\*a\*b\*log(f)) - log(a/b + f\*\*(-2\*x))/(4\*a\*b\*log(f)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(af^x + \frac{b}{f^x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/(f^x)+a\*f^x)^2,x, algorithm="giac")

[Out] integrate(x/(a\*f^x + b/f^x)^2, x)

$$3.60 \quad \int \frac{x^2}{(bf^{-x} + af^x)^2} dx$$

**Optimal.** Leaf size=98

$$-\frac{\text{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{4ab \log^3(f)} - \frac{x^2}{2a \log(f)(af^{2x} + b)} - \frac{x \log\left(\frac{af^{2x}}{b} + 1\right)}{2ab \log^2(f)} + \frac{x^2}{2ab \log(f)}$$

[Out]  $x^2/(2*a*b*\text{Log}[f]) - x^2/(2*a*(b + a*f^{(2*x)})*\text{Log}[f]) - (x*\text{Log}[1 + (a*f^{(2*x)})/b])/(2*a*b*\text{Log}[f]^2) - \text{PolyLog}[2, -((a*f^{(2*x)})/b)]/(4*a*b*\text{Log}[f]^3)$

**Rubi [A]** time = 0.169159, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2283, 2191, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{4ab \log^3(f)} - \frac{x^2}{2a \log(f)(af^{2x} + b)} - \frac{x \log\left(\frac{af^{2x}}{b} + 1\right)}{2ab \log^2(f)} + \frac{x^2}{2ab \log(f)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(b/f^x + a\*f^x)^2, x]

[Out]  $x^2/(2*a*b*\text{Log}[f]) - x^2/(2*a*(b + a*f^{(2*x)})*\text{Log}[f]) - (x*\text{Log}[1 + (a*f^{(2*x)})/b])/(2*a*b*\text{Log}[f]^2) - \text{PolyLog}[2, -((a*f^{(2*x)})/b)]/(4*a*b*\text{Log}[f]^3)$

#### Rule 2283

Int[(u\_.)\*((a\_.)\*(F\_)^(v\_) + (b\_.)\*(F\_)^(w\_))^(n\_), x\_Symbol] :> Int[u\*F^(n\*v)\*(a + b\*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ[n, 0] && LinearQ[{v, w}, x]

#### Rule 2191

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(bf^{-x} + af^x)^2} dx &= \int \frac{f^{2x}x^2}{(b + af^{2x})^2} dx \\
&= -\frac{x^2}{2a(b + af^{2x})\log(f)} + \frac{\int \frac{x}{b+af^{2x}} dx}{a\log(f)} \\
&= \frac{x^2}{2ab\log(f)} - \frac{x^2}{2a(b + af^{2x})\log(f)} - \frac{\int \frac{f^{2x}x}{b+af^{2x}} dx}{b\log(f)} \\
&= \frac{x^2}{2ab\log(f)} - \frac{x^2}{2a(b + af^{2x})\log(f)} - \frac{x\log\left(1 + \frac{af^{2x}}{b}\right)}{2ab\log^2(f)} + \frac{\int \log\left(1 + \frac{af^{2x}}{b}\right) dx}{2ab\log^2(f)} \\
&= \frac{x^2}{2ab\log(f)} - \frac{x^2}{2a(b + af^{2x})\log(f)} - \frac{x\log\left(1 + \frac{af^{2x}}{b}\right)}{2ab\log^2(f)} + \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{ax}{b}\right)}{x} dx, x, f^{2x}\right)}{4ab\log^3(f)} \\
&= \frac{x^2}{2ab\log(f)} - \frac{x^2}{2a(b + af^{2x})\log(f)} - \frac{x\log\left(1 + \frac{af^{2x}}{b}\right)}{2ab\log^2(f)} - \frac{\text{Li}_2\left(-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0624651, size = 90, normalized size = 0.92

$$\frac{2x \log(f) \left( ax f^{2x} \log(f) - (af^{2x} + b) \log\left(\frac{af^{2x}}{b} + 1\right) \right) - (af^{2x} + b) \text{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{4ab \log^3(f) (af^{2x} + b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b/f^x + a*f^x)^2,x]
```

```
[Out] (2*x*Log[f]*(a*f^(2*x))*x*Log[f] - (b + a*f^(2*x))*Log[1 + (a*f^(2*x))/b]) -
(b + a*f^(2*x))*PolyLog[2, -((a*f^(2*x))/b)]/(4*a*b*(b + a*f^(2*x))*Log[f
]^3)
```

**Maple [A]** time = 0.029, size = 91, normalized size = 0.9

$$-\frac{x^2}{2 \ln(f) a (a (f^x)^2 + b)} + \frac{x^2}{2 \ln(f) ab} - \frac{x}{2 (\ln(f))^2 ab} \ln\left(1 + \frac{af^{2x}}{b}\right) - \frac{1}{4 ab (\ln(f))^3} \text{polylog}\left(2, -\frac{af^{2x}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b/(f^x)+a*f^x)^2,x)`

[Out] 
$$-1/2/\ln(f)*x^2/a/(a*(f^x)^2+b)+1/2*x^2/a/b/\ln(f)-1/2*x*\ln(1+a*f^(2*x)/b)/a/b/\ln(f)^2-1/4*polylog(2,-a*f^(2*x)/b)/a/b/\ln(f)^3$$

**Maxima [A]** time = 1.10902, size = 117, normalized size = 1.19

$$\frac{x^2}{2(a^2 f^{2x} \log(f) + ab \log(f))} + \frac{\log(f^x)^2}{2ab \log(f)^3} - \frac{2 \log(f^x) \log\left(\frac{af^{2x}}{b} + 1\right) + \text{Li}_2\left(-\frac{af^{2x}}{b}\right)}{4ab \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="maxima")`

[Out] 
$$-1/2*x^2/(a^2*f^(2*x)*\log(f) + a*b*\log(f)) + 1/2*\log(f^x)^2/(a*b*\log(f)^3) - 1/4*(2*\log(f^x)*\log(a*f^(2*x)/b + 1) + \text{dilog}(-a*f^(2*x)/b))/(a*b*\log(f)^3)$$

**Fricas [A]** time = 1.53634, size = 370, normalized size = 3.78

$$\frac{af^{2x}x^2 \log(f)^2 - (af^{2x} + b)\text{Li}_2\left(f^x \sqrt{-\frac{a}{b}}\right) - (af^{2x} + b)\text{Li}_2\left(-f^x \sqrt{-\frac{a}{b}}\right) - (af^{2x}x \log(f) + bx \log(f)) \log\left(f^x \sqrt{-\frac{a}{b}} + 1\right)}{2\left(a^2bf^{2x} \log(f)^3 + ab^2 \log(f)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="fricas")`

[Out] 
$$1/2*(a*f^(2*x)*x^2*\log(f)^2 - (a*f^(2*x) + b)*\text{dilog}(f^x*\sqrt{-a/b}) - (a*f^(2*x) + b)*\text{dilog}(-f^x*\sqrt{-a/b}) - (a*f^(2*x)*x*\log(f) + b*x*\log(f))*\log(f^x*\sqrt{-a/b} + 1) - (a*f^(2*x)*x*\log(f) + b*x*\log(f))*\log(-f^x*\sqrt{-a/b} + 1))/(a^2*b*f^(2*x)*\log(f)^3 + a*b^2*\log(f)^3)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2ab \log(f) + 2b^2 f^{-2x} \log(f)} - \frac{\int \frac{f^{2x}}{af^{2x}+b} dx}{b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b/(f**x)+a*f**x)**2,x)`

[Out] 
$$x**2/(2*a*b*\log(f) + 2*b**2*f**(-2*x)*\log(f)) - \text{Integral}(f**(2*x)*x/(a*f**(2*x) + b), x)/(b*\log(f))$$



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(af^x + \frac{b}{f^x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*f^x + b/f^x)^2, x)
```

### 3.61 $\int \frac{x^3}{(bf^{-x}+af^x)^2} dx$

**Optimal.** Leaf size=128

$$-\frac{3x \operatorname{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{4ab \log^3(f)} + \frac{3 \operatorname{PolyLog}\left(3, -\frac{af^{2x}}{b}\right)}{8ab \log^4(f)} - \frac{3x^2 \log\left(\frac{af^{2x}}{b} + 1\right)}{4ab \log^2(f)} - \frac{x^3}{2a \log(f)(af^{2x} + b)} + \frac{x^3}{2ab \log(f)}$$

[Out]  $x^3/(2*a*b*\operatorname{Log}[f]) - x^3/(2*a*(b + a*f^{(2*x)})*\operatorname{Log}[f]) - (3*x^2*\operatorname{Log}[1 + (a*f^{(2*x)})/b])/(4*a*b*\operatorname{Log}[f]^2) - (3*x*\operatorname{PolyLog}[2, -((a*f^{(2*x)})/b)])/(4*a*b*\operatorname{Log}[f]^3) + (3*\operatorname{PolyLog}[3, -((a*f^{(2*x)})/b)])/(8*a*b*\operatorname{Log}[f]^4)$

**Rubi [A]** time = 0.225486, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {2283, 2191, 2184, 2190, 2531, 2282, 6589}

$$-\frac{3x \operatorname{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{4ab \log^3(f)} + \frac{3 \operatorname{PolyLog}\left(3, -\frac{af^{2x}}{b}\right)}{8ab \log^4(f)} - \frac{3x^2 \log\left(\frac{af^{2x}}{b} + 1\right)}{4ab \log^2(f)} - \frac{x^3}{2a \log(f)(af^{2x} + b)} + \frac{x^3}{2ab \log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(b/f^x + a*f^x)^2, x]$

[Out]  $x^3/(2*a*b*\operatorname{Log}[f]) - x^3/(2*a*(b + a*f^{(2*x)})*\operatorname{Log}[f]) - (3*x^2*\operatorname{Log}[1 + (a*f^{(2*x)})/b])/(4*a*b*\operatorname{Log}[f]^2) - (3*x*\operatorname{PolyLog}[2, -((a*f^{(2*x)})/b)])/(4*a*b*\operatorname{Log}[f]^3) + (3*\operatorname{PolyLog}[3, -((a*f^{(2*x)})/b)])/(8*a*b*\operatorname{Log}[f]^4)$

#### Rule 2283

$\operatorname{Int}[(u_.)*((a_.)*(F_)^{(v_)} + (b_.)*(F_)^{(w_)}))^{(n_)}, x\_Symbol] \rightarrow \operatorname{Int}[u*F^{(n*v)}*(a + b*F^{\operatorname{ExpandToSum}[w - v, x]})^n, x] /; \operatorname{FreeQ}\{F, a, b, n\}, x] \&\& \operatorname{ILtQ}[n, 0] \&\& \operatorname{LinearQ}\{v, w\}, x]$

#### Rule 2191

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}}*((a_.) + (b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}})^{(p_.)}}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*(a + b*(F^{(g*(e + f*x))})^n)^{(p + 1)}/(b*f*g*n*(p + 1)*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*(p + 1)*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m - 1)}*(a + b*(F^{(g*(e + f*x))})^n)^{(p + 1)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, m, n, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

#### Rule 2184

$\operatorname{Int}[(c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}})^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \operatorname{Dist}[b/a, \operatorname{Int}[(c + d*x)^m*(F^{(g*(e + f*x))})^n]/(a + b*(F^{(g*(e + f*x))})^n), x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2190

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}}*((c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}})^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Di}$

st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{(bf^{-x} + af^x)^2} dx &= \int \frac{f^{2x}x^3}{(b + af^{2x})^2} dx \\ &= -\frac{x^3}{2a(b + af^{2x})\log(f)} + \frac{3 \int \frac{x^2}{b + af^{2x}} dx}{2a\log(f)} \\ &= \frac{x^3}{2ab\log(f)} - \frac{x^3}{2a(b + af^{2x})\log(f)} - \frac{3 \int \frac{f^{2x}x^2}{b + af^{2x}} dx}{2b\log(f)} \\ &= \frac{x^3}{2ab\log(f)} - \frac{x^3}{2a(b + af^{2x})\log(f)} - \frac{3x^2 \log\left(1 + \frac{af^{2x}}{b}\right)}{4ab\log^2(f)} + \frac{3 \int x \log\left(1 + \frac{af^{2x}}{b}\right) dx}{2ab\log^2(f)} \\ &= \frac{x^3}{2ab\log(f)} - \frac{x^3}{2a(b + af^{2x})\log(f)} - \frac{3x^2 \log\left(1 + \frac{af^{2x}}{b}\right)}{4ab\log^2(f)} - \frac{3x\text{Li}_2\left(-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)} + \frac{3 \int \text{Li}_2\left(-\frac{af^{2x}}{b}\right) dx}{4ab\log^3(f)} \\ &= \frac{x^3}{2ab\log(f)} - \frac{x^3}{2a(b + af^{2x})\log(f)} - \frac{3x^2 \log\left(1 + \frac{af^{2x}}{b}\right)}{4ab\log^2(f)} - \frac{3x\text{Li}_2\left(-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)} + \frac{3 \text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{ax}{b}\right)}{x} dx\right)}{8ab\log^4(f)} \\ &= \frac{x^3}{2ab\log(f)} - \frac{x^3}{2a(b + af^{2x})\log(f)} - \frac{3x^2 \log\left(1 + \frac{af^{2x}}{b}\right)}{4ab\log^2(f)} - \frac{3x\text{Li}_2\left(-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)} + \frac{3\text{Li}_3\left(-\frac{af^{2x}}{b}\right)}{8ab\log^4(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0809213, size = 124, normalized size = 0.97

$$\frac{3 \left( -\frac{x \text{PolyLog}\left(2, -\frac{af^{2x}}{b}\right)}{2b\log^2(f)} + \frac{\text{PolyLog}\left(3, -\frac{af^{2x}}{b}\right)}{4b\log^3(f)} - \frac{x^2 \log\left(\frac{af^{2x}}{b} + 1\right)}{2b\log(f)} + \frac{x^3}{3b} \right)}{2a\log(f)} - \frac{x^3}{2a\log(f)(af^{2x} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b/f^x + a\*f^x)^2,x]

[Out]  $-x^3/(2*a*(b + a*f^{(2*x)})*\text{Log}[f]) + (3*(x^3/(3*b) - (x^2*\text{Log}[1 + (a*f^{(2*x)})/b]))/(2*b*\text{Log}[f]) - (x*\text{PolyLog}[2, -(a*f^{(2*x)})/b]))/(2*b*\text{Log}[f]^2) + \text{PolyLog}[3, -(a*f^{(2*x)})/b]/(4*b*\text{Log}[f]^3))/(2*a*\text{Log}[f])$

**Maple [A]** time = 0.035, size = 119, normalized size = 0.9

$$-\frac{x^3}{2 \ln(f) a (a (f^x)^2 + b)} + \frac{x^3}{2 \ln(f) a b} - \frac{3 x^2}{4 (\ln(f))^2 a b} \ln\left(1 + \frac{a f^{2x}}{b}\right) - \frac{3 x}{4 a b (\ln(f))^3} \text{polylog}\left(2, -\frac{a f^{2x}}{b}\right) + \frac{3}{8 a b (\ln(f))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b/(f^x)+a\*f^x)^2,x)

[Out]  $-1/2/\ln(f)*x^3/a/(a*(f^x)^2+b)+1/2*x^3/a/b/\ln(f)-3/4*x^2*\ln(1+a*f^{(2*x)}/b)/a/b/\ln(f)^2-3/4*x*\text{polylog}(2,-a*f^{(2*x)}/b)/a/b/\ln(f)^3+3/8*\text{polylog}(3,-a*f^{(2*x)}/b)/a/b/\ln(f)^4$

**Maxima [A]** time = 1.18148, size = 149, normalized size = 1.16

$$-\frac{x^3}{2(a^2 f^{2x} \log(f) + ab \log(f))} + \frac{\log(f^x)^3}{2ab \log(f)^4} - \frac{3\left(2 \log(f^x)^2 \log\left(\frac{a f^{2x}}{b} + 1\right) + 2 \text{Li}_2\left(-\frac{a f^{2x}}{b}\right) \log(f^x) - \text{Li}_3\left(-\frac{a f^{2x}}{b}\right)\right)}{8ab \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x)^2,x, algorithm="maxima")

[Out]  $-1/2*x^3/(a^2*f^{(2*x)}*\log(f) + a*b*\log(f)) + 1/2*\log(f^x)^3/(a*b*\log(f)^4) - 3/8*(2*\log(f^x)^2*\log(a*f^{(2*x)}/b + 1) + 2*dilog(-a*f^{(2*x)}/b)*\log(f^x) - \text{polylog}(3, -a*f^{(2*x)}/b))/(a*b*\log(f)^4)$

**Fricas [C]** time = 1.50973, size = 582, normalized size = 4.55

$$2af^{2x}x^3 \log(f)^3 - 6(a f^{2x} x \log(f) + b x \log(f)) \text{Li}_2\left(f^x \sqrt{-\frac{a}{b}}\right) - 6(a f^{2x} x \log(f) + b x \log(f)) \text{Li}_2\left(-f^x \sqrt{-\frac{a}{b}}\right) - 3(a f^{2x} x^2 \log(f)^2 + b x^2 \log(f)^2) \log(f^x \sqrt{-\frac{a}{b}}) - 3(a f^{2x} x^2 \log(f)^2 + b x^2 \log(f)^2) \log(-f^x \sqrt{-\frac{a}{b}}) + 6(a f^{2x} x \log(f) + b x \log(f)) \text{polylog}(3, f^x \sqrt{-\frac{a}{b}}) + 6(a f^{2x} x \log(f) + b x \log(f)) \text{polylog}(3, -f^x \sqrt{-\frac{a}{b}})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x)^2,x, algorithm="fricas")

[Out]  $1/4*(2*a*f^{(2*x)}*x^3*\log(f)^3 - 6*(a*f^{(2*x)}*x*\log(f) + b*x*\log(f))*dilog(f^x*\sqrt{-a/b}) - 6*(a*f^{(2*x)}*x*\log(f) + b*x*\log(f))*dilog(-f^x*\sqrt{-a/b}) - 3*(a*f^{(2*x)}*x^2*\log(f)^2 + b*x^2*\log(f)^2)*\log(f^x*\sqrt{-a/b} + 1) - 3*(a*f^{(2*x)}*x^2*\log(f)^2 + b*x^2*\log(f)^2)*\log(-f^x*\sqrt{-a/b} + 1) + 6*(a*f^{(2*x)}*x*\log(f) + b*x*\log(f))*\text{polylog}(3, f^x*\sqrt{-a/b}) + 6*(a*f^{(2*x)}*x*\log(f) + b*x*\log(f))*\text{polylog}(3, -f^x*\sqrt{-a/b})$

$\sqrt{-a/b})/(a^2*b*f^{(2*x)}*\log(f)^4 + a*b^2*\log(f)^4)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{2ab \log(f) + 2b^2 f^{-2x} \log(f)} - \frac{3 \int \frac{f^{2x} x^2}{af^{2x} + b} dx}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b/(f\*\*x)+a\*f\*\*x)\*\*2,x)

[Out] x\*\*3/(2\*a\*b\*log(f) + 2\*b\*\*2\*f\*\*(-2\*x)\*log(f)) - 3\*Integral(f\*\*(2\*x)\*x\*\*2/(a\*f\*\*(2\*x) + b), x)/(2\*b\*log(f))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(af^x + \frac{b}{f^x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b/(f^x)+a\*f^x)^2,x, algorithm="giac")

[Out] integrate(x^3/(a\*f^x + b/f^x)^2, x)

$$3.62 \quad \int \frac{1}{(bf^{-x} + af^x)^3} dx$$

**Optimal.** Leaf size=87

$$\frac{\tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} + \frac{f^x}{8ab\log(f)(af^{2x} + b)} - \frac{f^x}{4a\log(f)(af^{2x} + b)^2}$$

[Out]  $-f^x/(4*a*(b + a*f^{(2*x)})^2*\text{Log}[f]) + f^x/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]) + \text{ArcTan}[(\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]]/(8*a^{(3/2)}*b^{(3/2)}*\text{Log}[f])$

**Rubi [A]** time = 0.0463815, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2282, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{af^x}}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} + \frac{f^x}{8ab\log(f)(af^{2x} + b)} - \frac{f^x}{4a\log(f)(af^{2x} + b)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(b/f^x + a*f^x)^{-3}, x]$

[Out]  $-f^x/(4*a*(b + a*f^{(2*x)})^2*\text{Log}[f]) + f^x/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]) + \text{ArcTan}[(\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]]/(8*a^{(3/2)}*b^{(3/2)}*\text{Log}[f])$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 288

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bf^{-x} + af^x)^3} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(b+ax^2)^3} dx, x, f^x\right)}{\log(f)} \\
&= -\frac{f^x}{4a(b + af^{2x})^2 \log(f)} + \frac{\text{Subst}\left(\int \frac{1}{(b+ax^2)^2} dx, x, f^x\right)}{4a \log(f)} \\
&= -\frac{f^x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x}{8ab(b + af^{2x}) \log(f)} + \frac{\text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, f^x\right)}{8ab \log(f)} \\
&= -\frac{f^x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x}{8ab(b + af^{2x}) \log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0529376, size = 70, normalized size = 0.8

$$\frac{\frac{\sqrt{a}\sqrt{b}f^x(af^{2x}-b)}{(af^{2x}+b)^2} + \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[(b/f^x + a\*f^x)^(-3), x]

[Out] ((Sqrt[a]\*Sqrt[b]\*f^x\*(-b + a\*f^(2\*x)))/(b + a\*f^(2\*x))^2 + ArcTan[(Sqrt[a]\*f^x/Sqrt[b])]/(8\*a^(3/2)\*b^(3/2)\*Log[f])

**Maple [A]** time = 0.008, size = 78, normalized size = 0.9

$$\frac{(f^x)^3}{8 \ln(f) (a(f^x)^2 + b)^2 b} - \frac{f^x}{8 \ln(f) (a(f^x)^2 + b)^2 a} + \frac{1}{8 b \ln(f) a} \arctan\left(a f^x \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/(f^x)+a\*f^x)^3, x)

[Out] 1/8/ln(f)/(a\*(f^x)^2+b)^2/b\*(f^x)^3-1/8/ln(f)/(a\*(f^x)^2+b)^2/a\*f^x+1/8/ln(f)/b/a/(a\*b)^(1/2)\*arctan(a\*f^x/(a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.55599, size = 586, normalized size = 6.74

$$\left[ \frac{2a^2bf^{3x} - 2ab^2f^x - (\sqrt{-aba^2}f^{4x} + 2\sqrt{-abab}f^{2x} + \sqrt{-abb^2}) \log\left(\frac{af^{2x} - 2\sqrt{-ab}f^x - b}{af^{2x} + b}\right)}{16(a^4b^2f^{4x} \log(f) + 2a^3b^3f^{2x} \log(f) + a^2b^4 \log(f))}, \frac{a^2bf^{3x} - ab^2f^x - (\sqrt{aba^2}f^{4x} + 2\sqrt{abab}f^{2x} + \sqrt{abb^2}) \log\left(\frac{af^{2x} - 2\sqrt{-ab}f^x - b}{af^{2x} + b}\right)}{8(a^4b^2f^{4x} \log(f) + 2a^3b^3f^{2x} \log(f) + a^2b^4 \log(f))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^3,x, algorithm="fricas")

[Out] [1/16\*(2\*a^2\*b\*f^(3\*x) - 2\*a\*b^2\*f^x - (sqrt(-a\*b)\*a^2\*f^(4\*x) + 2\*sqrt(-a\*b)\*a\*b\*f^(2\*x) + sqrt(-a\*b)\*b^2)\*log((a\*f^(2\*x) - 2\*sqrt(-a\*b)\*f^x - b)/(a\*f^(2\*x) + b)))/(a^4\*b^2\*f^(4\*x)\*log(f) + 2\*a^3\*b^3\*f^(2\*x)\*log(f) + a^2\*b^4\*log(f)), 1/8\*(a^2\*b\*f^(3\*x) - a\*b^2\*f^x - (sqrt(a\*b)\*a^2\*f^(4\*x) + 2\*sqrt(a\*b)\*a\*b\*f^(2\*x) + sqrt(a\*b)\*b^2)\*arctan(sqrt(a\*b)/(a\*f^x)))/(a^4\*b^2\*f^(4\*x)\*log(f) + 2\*a^3\*b^3\*f^(2\*x)\*log(f) + a^2\*b^4\*log(f))]

**Sympy [A]** time = 0.299246, size = 87, normalized size = 1.

$$\frac{af^{-x} - bf^{-3x}}{8a^3b \log(f) + 16a^2b^2f^{-2x} \log(f) + 8ab^3f^{-4x} \log(f)} + \frac{\text{RootSum}\left(256z^2a^3b^3 + 1, (i \mapsto i \log(-16ia^2b + f^{-x}))\right)}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f\*\*x)+a\*f\*\*x)\*\*3,x)

[Out] (a\*f\*\*(-x) - b\*f\*\*(-3\*x))/(8\*a\*\*3\*b\*log(f) + 16\*a\*\*2\*b\*\*2\*f\*\*(-2\*x)\*log(f) + 8\*a\*b\*\*3\*f\*\*(-4\*x)\*log(f)) + RootSum(256\*\_z\*\*2\*a\*\*3\*b\*\*3 + 1, Lambda(\_i, \_i\*log(-16\*\_i\*a\*\*2\*b + f\*\*(-x))))/log(f)

**Giac [A]** time = 1.24434, size = 89, normalized size = 1.02

$$\frac{\arctan\left(\frac{af^x}{\sqrt{ab}}\right)}{8\sqrt{abab} \log(f)} + \frac{af^{3x} - bf^x}{8(af^{2x} + b)^2 ab \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/(f^x)+a\*f^x)^3,x, algorithm="giac")

[Out] 1/8\*arctan(a\*f^x/sqrt(a\*b))/(sqrt(a\*b)\*a\*b\*log(f)) + 1/8\*(a\*f^(3\*x) - b\*f^x)/((a\*f^(2\*x) + b)^2\*a\*b\*log(f))



$$3.63 \quad \int \frac{x}{(bf^{-x} + af^x)^3} dx$$

**Optimal.** Leaf size=196

$$-\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)} + \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} + \frac{f^x}{8ab\log^2(f)(af^{2x} + b)} + \frac{xf^x}{8ab\log(f)(af^{2x} + b)}$$

[Out]  $f^x/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]^2) - (f^x*x)/(4*a*(b + a*f^{(2*x)})^2*\text{Log}[f]) + (f^x*x)/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]) + (x*\text{ArcTan}[(\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(8*a^{(3/2)}*b^{(3/2)}*\text{Log}[f]) - ((I/16)*\text{PolyLog}[2, ((-I)*\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(a^{(3/2)}*b^{(3/2)}*\text{Log}[f]^2) + ((I/16)*\text{PolyLog}[2, (I*\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(a^{(3/2)}*b^{(3/2)}*\text{Log}[f]^2)$

**Rubi [A]** time = 0.502889, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$ , Rules used = {2283, 2254, 2249, 199, 205, 2245, 2282, 4848, 2391}

$$-\frac{i\text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)} + \frac{i\text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} + \frac{f^x}{8ab\log^2(f)(af^{2x} + b)} + \frac{xf^x}{8ab\log(f)(af^{2x} + b)}$$

Antiderivative was successfully verified.

[In] Int[x/(b/f^x + a\*f^x)^3,x]

[Out]  $f^x/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]^2) - (f^x*x)/(4*a*(b + a*f^{(2*x)})^2*\text{Log}[f]) + (f^x*x)/(8*a*b*(b + a*f^{(2*x)})*\text{Log}[f]) + (x*\text{ArcTan}[(\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(8*a^{(3/2)}*b^{(3/2)}*\text{Log}[f]) - ((I/16)*\text{PolyLog}[2, ((-I)*\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(a^{(3/2)}*b^{(3/2)}*\text{Log}[f]^2) + ((I/16)*\text{PolyLog}[2, (I*\text{Sqrt}[a]*f^x)/\text{Sqrt}[b]])/(a^{(3/2)}*b^{(3/2)}*\text{Log}[f]^2)$

#### Rule 2283

Int[(u\_.)\*((a\_.)\*(F\_)^(v\_) + (b\_.)\*(F\_)^(w\_))^(n\_), x\_Symbol] := Int[u\*F^(n\*v)\*(a + b\*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ[n, 0] && LinearQ[{v, w}, x]

#### Rule 2254

Int[((a\_.) + (b\_.)\*(F\_)^(u\_))^(p\_.)\*((c\_.) + (d\_.)\*(F\_)^(v\_))^(q\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := With[{w = ExpandIntegrand[(e + f\*x)^m, (a + b\*F^u)^p\*(c + d\*F^v)^q, x]}, Int[w, x] /; SumQ[w] /; FreeQ[{F, a, b, c, d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simplify[u/v]]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^(h\_.)\*((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2245

```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(m_), x_Symbol] := With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Dist[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := Simp[a*Log[x], x] + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 + I*c*x]/x, x], x) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(bf^{-x} + af^x)^3} dx &= \int \frac{f^{3x}x}{(b + af^{2x})^3} dx \\
&= \int \left( -\frac{bf^x x}{a(b + af^{2x})^3} + \frac{f^x x}{a(b + af^{2x})^2} \right) dx \\
&= \frac{\int \frac{f^x x}{(b + af^{2x})^2} dx}{a} - \frac{b \int \frac{f^x x}{(b + af^{2x})^3} dx}{a} \\
&= -\frac{f^x x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x}{8ab(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{\int \left( \frac{f^x}{2b(b + af^{2x}) \log(f)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{ab}} \right) dx}{a} \\
&= -\frac{f^x x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x}{8ab(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} + \frac{\int \frac{f^x}{(b + af^{2x})^2} dx}{4a \log(f)} + \frac{3 \int \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right) dx}{8a^{3/2}b^{3/2} \log(f)} \\
&= -\frac{f^x x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x}{8ab(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} + \frac{\text{Subst}\left(\int \frac{1}{(b + ax^2)^2} dx, x, f^x\right)}{4a \log^2(f)} \\
&= \frac{f^x}{8ab(b + af^{2x}) \log^2(f)} - \frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log^2(f)} - \frac{f^x x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x}{8ab(b + af^{2x}) \log(f)} + \frac{\int \frac{1}{(b + ax^2)^2} dx, x, f^x}{4a \log^2(f)} \\
&= \frac{f^x}{8ab(b + af^{2x}) \log^2(f)} - \frac{f^x x}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x}{8ab(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{\int \frac{1}{(b + ax^2)^2} dx, x, f^x}{4a \log^2(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.208664, size = 209, normalized size = 1.07

$$\frac{-\frac{i \text{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}} + \frac{i \text{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}} + \frac{2\sqrt{a}f^x}{abf^{2x} + b^2} + \frac{2\sqrt{ax}f^x \log(f)}{abf^{2x} + b^2} + \frac{ix \log(f) \log\left(1 - \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}} - \frac{ix \log(f) \log\left(1 + \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}} - \frac{4\sqrt{ax}f^x \log(f)}{(af^{2x} + b)^2}}{16a^{3/2} \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b/f^x + a\*f^x)^3, x]

[Out]  $\left(\frac{2\sqrt{a}f^x}{b^2 + a b f^{2x}} - \frac{4\sqrt{a}f^x x \log(f)}{(b + a f^{2x})^2} + \frac{2\sqrt{a}f^x x \log(f)}{(b^2 + a b f^{2x})} + \frac{(x \log(f) \log(1 - \frac{i\sqrt{a}f^x}{\sqrt{b}}) - (x \log(f) \log(1 + \frac{i\sqrt{a}f^x}{\sqrt{b}}))}{b^{3/2}} - \frac{(x \log(f) \log(1 - \frac{i\sqrt{a}f^x}{\sqrt{b}}) - (x \log(f) \log(1 + \frac{i\sqrt{a}f^x}{\sqrt{b}})))}{b^{3/2}} - \frac{(x \log(f) \log(1 - \frac{i\sqrt{a}f^x}{\sqrt{b}}) - (x \log(f) \log(1 + \frac{i\sqrt{a}f^x}{\sqrt{b}})))}{b^{3/2}} + \frac{(x \log(f) \log(1 - \frac{i\sqrt{a}f^x}{\sqrt{b}}) - (x \log(f) \log(1 + \frac{i\sqrt{a}f^x}{\sqrt{b}})))}{b^{3/2}}\right) / (16 a^{3/2} \log(f)^2)$

**Maple [A]** time = 0.062, size = 209, normalized size = 1.1

$$\frac{f^x \left( (f^x)^2 \ln(f) a x - \ln(f) b x + a (f^x)^2 + b \right)}{8 (\ln(f))^2 a b (a (f^x)^2 + b)^2} + \frac{x}{16 b \ln(f) a} \ln \left( \left( -a f^x + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}} \right) \frac{1}{\sqrt{-ab}} - \frac{x}{16 b \ln(f) a} \ln \left( \left( a f^x + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}} \right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b/(f^x)+a*f^x)^3,x)`

[Out]  $\frac{1}{8}f^x((f^x)^2 \ln(f) a x - \ln(f) b x + a(f^x)^2 + b) / \ln(f)^2 / b / a / (a(f^x)^2 + b)^2 + 1/16 / \ln(f) / a / b x / (-a b)^{1/2} \ln((-a f^x + (-a b)^{1/2}) / (-a b)^{1/2}) - 1/16 / \ln(f) / a / b x / (-a b)^{1/2} \ln((a f^x + (-a b)^{1/2}) / (-a b)^{1/2}) + 1/16 / \ln(f)^2 / a / b / (-a b)^{1/2} \operatorname{dilog}((-a f^x + (-a b)^{1/2}) / (-a b)^{1/2}) - 1/16 / \ln(f)^2 / a / b / (-a b)^{1/2} \operatorname{dilog}((a f^x + (-a b)^{1/2}) / (-a b)^{1/2})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.60115, size = 774, normalized size = 3.95

$$2(a^2 x \log(f) + a^2) f^{3x} - 2(abx \log(f) - ab) f^x + \left(a^2 f^{4x} \sqrt{-\frac{a}{b}} + 2abf^{2x} \sqrt{-\frac{a}{b}} + b^2 \sqrt{-\frac{a}{b}}\right) \operatorname{Li}_2\left(f^x \sqrt{-\frac{a}{b}}\right) - \left(a^2 f^{4x} \sqrt{-\frac{a}{b}} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{16} * (2 * (a^2 * x * \log(f) + a^2) * f^{(3 * x)} - 2 * (a * b * x * \log(f) - a * b) * f^x + (a^2 * f^{(4 * x)} * \sqrt{-a/b} + 2 * a * b * f^{(2 * x)} * \sqrt{-a/b} + b^2 * \sqrt{-a/b}) * \operatorname{dilog}(f^x * \sqrt{-a/b}) - (a^2 * f^{(4 * x)} * \sqrt{-a/b} + 2 * a * b * f^{(2 * x)} * \sqrt{-a/b} + b^2 * \sqrt{-a/b}) * \operatorname{dilog}(-f^x * \sqrt{-a/b}) - (a^2 * f^{(4 * x)} * x * \sqrt{-a/b} * \log(f) + 2 * a * b * f^{(2 * x)} * x * \sqrt{-a/b} * \log(f) + b^2 * x * \sqrt{-a/b} * \log(f)) * \log(f^x * \sqrt{-a/b} + 1) + (a^2 * f^{(4 * x)} * x * \sqrt{-a/b} * \log(f) + 2 * a * b * f^{(2 * x)} * x * \sqrt{-a/b} * \log(f) + b^2 * x * \sqrt{-a/b} * \log(f)) * \log(-f^x * \sqrt{-a/b} + 1)) / (a^4 * b * f^{(4 * x)} * \log(f)^2 + 2 * a^3 * b^2 * f^{(2 * x)} * \log(f)^2 + a^2 * b^3 * \log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^{-x} (ax \log(f) + a) + f^{-3x} (-bx \log(f) + b)}{8a^3 b \log(f)^2 + 16a^2 b^2 f^{-2x} \log(f)^2 + 8ab^3 f^{-4x} \log(f)^2} + \frac{\int \frac{f^x}{af^{2x+b}} dx}{8ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b/(f**x)+a*f**x)**3,x)`

[Out]  $(f^{(-x)} * (a * x * \log(f) + a) + f^{(-3 * x)} * (-b * x * \log(f) + b)) / (8 * a^3 * b * \log(f)^2 + 16 * a^2 * b^2 * f^{(-2 * x)} * \log(f)^2 + 8 * a * b^3 * f^{(-4 * x)} * \log(f)^2) + \operatorname{Inte}$

```
gral(f**x*x/(a*f**(2*x) + b), x)/(8*a*b)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a f^x + \frac{b}{f^x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/(a*f^x + b/f^x)^3, x)
```

$$3.64 \quad \int \frac{x^2}{(bf^{-x} + af^x)^3} dx$$

**Optimal.** Leaf size=316

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} - \frac{1}{4}$$

[Out]  $-\operatorname{ArcTan}\left[\frac{\sqrt{a}f^x}{\sqrt{b}}\right]/(4a^{3/2}b^{3/2}\log[f]^3) + (f^x x)/(4a^*b*(b + a*f^{(2*x)})*\log[f]^2) - (f^x*x^2)/(4*a*(b + a*f^{(2*x)})^2*\log[f]) + (f^x*x^2)/(8*a*b*(b + a*f^{(2*x)})*\log[f]) + (x^2*\operatorname{ArcTan}\left[\frac{\sqrt{a}f^x}{\sqrt{b}}\right])/(8*a^{(3/2)}*b^{(3/2)}*\log[f]) - ((1/8)*x*\operatorname{PolyLog}[2, ((-1)*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^2) + ((1/8)*x*\operatorname{PolyLog}[2, (1*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^2) + ((1/8)*\operatorname{PolyLog}[3, ((-1)*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^3) - ((1/8)*\operatorname{PolyLog}[3, (1*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^3)$

**Rubi [A]** time = 1.16332, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 14, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$ , Rules used = {2283, 2254, 2249, 199, 205, 2245, 14, 2282, 4848, 2391, 12, 5143, 2531, 6589}

$$-\frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} - \frac{1}{4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(b/f^x + a*f^x)^3, x]$

[Out]  $-\operatorname{ArcTan}\left[\frac{\sqrt{a}f^x}{\sqrt{b}}\right]/(4a^{3/2}b^{3/2}\log[f]^3) + (f^x x)/(4a^*b*(b + a*f^{(2*x)})*\log[f]^2) - (f^x*x^2)/(4*a*(b + a*f^{(2*x)})^2*\log[f]) + (f^x*x^2)/(8*a*b*(b + a*f^{(2*x)})*\log[f]) + (x^2*\operatorname{ArcTan}\left[\frac{\sqrt{a}f^x}{\sqrt{b}}\right])/(8*a^{(3/2)}*b^{(3/2)}*\log[f]) - ((1/8)*x*\operatorname{PolyLog}[2, ((-1)*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^2) + ((1/8)*x*\operatorname{PolyLog}[2, (1*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^2) + ((1/8)*\operatorname{PolyLog}[3, ((-1)*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^3) - ((1/8)*\operatorname{PolyLog}[3, (1*\sqrt{a}f^x)/\sqrt{b}])/(a^{(3/2)}*b^{(3/2)}*\log[f]^3)$

#### Rule 2283

$\operatorname{Int}[(u_*)*((a_*)*(F_)^{(v_*)} + (b_*)*(F_)^{(w_*)})^{(n_*)}, x\_Symbol] := \operatorname{Int}[u*F^{(n*v)}*(a + b*F^{\operatorname{ExpandToSum}[w - v, x]})^n, x] /; \operatorname{FreeQ}\{F, a, b, n, x\} \ \&\& \ \operatorname{ILtQ}[n, 0] \ \&\& \ \operatorname{LinearQ}\{v, w, x\}$

#### Rule 2254

$\operatorname{Int}[(a_*) + (b_*)*(F_)^{(u_*)})^{(p_*)}*((c_*) + (d_*)*(F_)^{(v_*)})^{(q_*)}*((e_*) + (f_*)*(x_*)^{(m_*)}), x\_Symbol] := \operatorname{With}\{w = \operatorname{ExpandIntegrand}[e + f*x]^m, (a + b*F^u)^p*(c + d*F^v)^q, x\}, \operatorname{Int}[w, x] /; \operatorname{SumQ}[w] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, m\}, x\} \ \&\& \ \operatorname{IntegersQ}[p, q] \ \&\& \ \operatorname{LinearQ}\{u, v, x\} \ \&\& \ \operatorname{RationalQ}[\operatorname{Simplify}[u/v]]$

#### Rule 2249

```
Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_))))^(p_)*(G_)^((h_)*((f_
.) + (g_)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
f, g, h, p}, x]
```

### Rule 199

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

### Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 2245

```
Int[(F_)^((e_)*((c_) + (d_)*(x_)))*((a_) + (b_)*(F_)^(v_))^(p_)*(x_)^(
m_), x_Symbol] := With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Di
st[x^m, u, x] - Dist[m, Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e
}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

### Rule 14

```
Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 4848

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^2}{(bf^{-x} + af^x)^3} dx &= \int \frac{f^{3x}x^2}{(b + af^{2x})^3} dx \\
&= \int \left( -\frac{bf^x x^2}{a(b + af^{2x})^3} + \frac{f^x x^2}{a(b + af^{2x})^2} \right) dx \\
&= \frac{\int \frac{f^x x^2}{(b + af^{2x})^2} dx}{a} - \frac{b \int \frac{f^x x^2}{(b + af^{2x})^3} dx}{a} \\
&= -\frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{2 \int x \left( \frac{f^x}{2b(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{2b(b + af^{2x}) \log(f)} \right) dx}{a} \\
&= -\frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{2 \int \left( \frac{f^x x}{2b(b + af^{2x}) \log(f)} + \frac{x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{2b(b + af^{2x}) \log(f)} \right) dx}{a} \\
&= -\frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} + \frac{\int \frac{f^x x}{(b + af^{2x})^2} dx}{2a \log(f)} + \frac{3 \int x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right) dx}{4a^{3/2}b^{3/2} \log(f)} \\
&= \frac{f^x x}{4ab(b + af^{2x}) \log^2(f)} - \frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{2 \int \frac{f^x x}{(b + af^{2x})^2} dx}{2a \log(f)} - \frac{3 \int x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right) dx}{4a^{3/2}b^{3/2} \log(f)} \\
&= \frac{f^x x}{4ab(b + af^{2x}) \log^2(f)} - \frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{2 \int \frac{f^x x}{(b + af^{2x})^2} dx}{2a \log(f)} - \frac{3 \int x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right) dx}{4a^{3/2}b^{3/2} \log(f)} \\
&= \frac{f^x x}{4ab(b + af^{2x}) \log^2(f)} - \frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} + \frac{x^2 \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2} \log(f)} - \frac{2 \int \frac{f^x x}{(b + af^{2x})^2} dx}{2a \log(f)} - \frac{3 \int x \tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right) dx}{4a^{3/2}b^{3/2} \log(f)} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2} \log^3(f)} + \frac{f^x x}{4ab(b + af^{2x}) \log^2(f)} - \frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2} \log^3(f)} + \frac{f^x x}{4ab(b + af^{2x}) \log^2(f)} - \frac{f^x x^2}{4a(b + af^{2x})^2 \log(f)} + \frac{f^x x^2}{8ab(b + af^{2x}) \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.467264, size = 254, normalized size = 0.8

$$\frac{3i \left( 2 \operatorname{PolyLog} \left( 3, -\frac{i\sqrt{a}f^x}{\sqrt{b}} \right) - 2 \operatorname{PolyLog} \left( 3, \frac{i\sqrt{a}f^x}{\sqrt{b}} \right) - 2x \log(f) \operatorname{PolyLog} \left( 2, -\frac{i\sqrt{a}f^x}{\sqrt{b}} \right) + 2x \log(f) \operatorname{PolyLog} \left( 2, \frac{i\sqrt{a}f^x}{\sqrt{b}} \right) + x^2 \log^2(f) \log \left( 1 - \frac{i\sqrt{a}f^x}{\sqrt{b}} \right) - x^2 \log^2(f) \log \left( 1 + \frac{i\sqrt{a}f^x}{\sqrt{b}} \right) \right)}{b^{3/2}}$$


---


$$48a^{3/2} \log^3(f)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(b/f^x + a\*f^x)^3,x]

[Out] ((-12\*ArcTan[(Sqrt[a]\*f^x)/Sqrt[b]])/b^(3/2) - (12\*Sqrt[a]\*f^x\*x^2\*Log[f]^2)/(b + a\*f^(2\*x))^2 + (6\*Sqrt[a]\*f^x\*x\*Log[f]\*(2 + x\*Log[f]))/(b\*(b + a\*f^(2\*x))))/b^(3/2)

2\*x))) + ((3\*I)\*(x^2\*Log[f]^2\*Log[1 - (I\*Sqrt[a]\*f^x)/Sqrt[b]] - x^2\*Log[f]^2\*Log[1 + (I\*Sqrt[a]\*f^x)/Sqrt[b]] - 2\*x\*Log[f]\*PolyLog[2, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] + 2\*x\*Log[f]\*PolyLog[2, (I\*Sqrt[a]\*f^x)/Sqrt[b]] + 2\*PolyLog[3, ((-I)\*Sqrt[a]\*f^x)/Sqrt[b]] - 2\*PolyLog[3, (I\*Sqrt[a]\*f^x)/Sqrt[b]]))/b^(3/2))/(48\*a^(3/2)\*Log[f]^3)

**Maple [F]** time = 0.132, size = 0, normalized size = 0.

$$\int x^2 \left( \frac{b}{f^x} + af^x \right)^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b/(f^x)+a\*f^x)^3,x)

[Out] int(x^2/(b/(f^x)+a\*f^x)^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b/(f^x)+a\*f^x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.67007, size = 1474, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b/(f^x)+a\*f^x)^3,x, algorithm="fricas")

[Out] 1/16\*(2\*(a^2\*x^2\*log(f)^2 + 2\*a^2\*x\*log(f))\*f^(3\*x) - 2\*(a\*b\*x^2\*log(f)^2 - 2\*a\*b\*x\*log(f))\*f^x + 2\*(a^2\*f^(4\*x)\*x\*sqrt(-a/b)\*log(f) + 2\*a\*b\*f^(2\*x)\*x\*sqrt(-a/b)\*log(f) + b^2\*x\*sqrt(-a/b)\*log(f))\*dilog(f^x\*sqrt(-a/b)) - 2\*(a^2\*f^(4\*x)\*x\*sqrt(-a/b)\*log(f) + 2\*a\*b\*f^(2\*x)\*x\*sqrt(-a/b)\*log(f) + b^2\*x\*sqrt(-a/b)\*log(f))\*dilog(-f^x\*sqrt(-a/b)) - 2\*(a^2\*f^(4\*x)\*sqrt(-a/b) + 2\*a\*b\*f^(2\*x)\*sqrt(-a/b) + b^2\*sqrt(-a/b))\*log(2\*a\*f^x + 2\*b\*sqrt(-a/b)) + 2\*(a^2\*f^(4\*x)\*sqrt(-a/b) + 2\*a\*b\*f^(2\*x)\*sqrt(-a/b) + b^2\*sqrt(-a/b))\*log(2\*a\*f^x - 2\*b\*sqrt(-a/b)) - (a^2\*f^(4\*x)\*x^2\*sqrt(-a/b)\*log(f)^2 + 2\*a\*b\*f^(2\*x)\*x^2\*sqrt(-a/b)\*log(f)^2 + b^2\*x^2\*sqrt(-a/b)\*log(f)^2)\*log(f^x\*sqrt(-a/b) + 1) + (a^2\*f^(4\*x)\*x^2\*sqrt(-a/b)\*log(f)^2 + 2\*a\*b\*f^(2\*x)\*x^2\*sqrt(-a/b)\*log(f)^2 + b^2\*x^2\*sqrt(-a/b)\*log(f)^2)\*log(-f^x\*sqrt(-a/b) + 1) - 2\*(a^2\*f^(4\*x)\*sqrt(-a/b) + 2\*a\*b\*f^(2\*x)\*sqrt(-a/b) + b^2\*sqrt(-a/b))\*polylog(3, f^x\*sqrt(-a/b)) + 2\*(a^2\*f^(4\*x)\*sqrt(-a/b) + 2\*a\*b\*f^(2\*x)\*sqrt(-a/b) + b^2\*sqrt(-a/b))\*polylog(3, -f^x\*sqrt(-a/b)))/(a^4\*b\*f^(4\*x)\*log(f)^3 + 2\*a^3\*b^2\*f^(2\*x)\*log(f)^3 + a^2\*b^3\*log(f)^3)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{f^{-x} (ax^2 \log(f) + 2ax) + f^{-3x} (-bx^2 \log(f) + 2bx)}{8a^3b \log(f)^2 + 16a^2b^2 f^{-2x} \log(f)^2 + 8ab^3 f^{-4x} \log(f)^2} + \frac{\int -\frac{2f^x}{af^{2x}+b} dx + \int \frac{f^x x^2 \log(f)^2}{af^{2x}+b} dx}{8ab \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(b/(f\*\*x)+a\*f\*\*x)\*\*3,x)

[Out] (f\*\*(-x)\*(a\*x\*\*2\*log(f) + 2\*a\*x) + f\*\*(-3\*x)\*(-b\*x\*\*2\*log(f) + 2\*b\*x))/(8\*a\*\*3\*b\*log(f)\*\*2 + 16\*a\*\*2\*b\*\*2\*f\*\*(-2\*x)\*log(f)\*\*2 + 8\*a\*b\*\*3\*f\*\*(-4\*x)\*log(f)\*\*2) + (Integral(-2\*f\*\*x/(a\*f\*\*(2\*x) + b), x) + Integral(f\*\*x\*x\*\*2\*log(f)\*\*2/(a\*f\*\*(2\*x) + b), x))/(8\*a\*b\*log(f)\*\*2)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(af^x + \frac{b}{f^x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b/(f^x)+a\*f^x)^3,x, algorithm="giac")

[Out] integrate(x^2/(a\*f^x + b/f^x)^3, x)

### 3.65 $\int f^{a+bx+cx^2} g^{d+ex+fx^2} dx$

**Optimal.** Leaf size=95

$$\frac{\sqrt{\pi} f^a g^d \exp\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right) \operatorname{Erfi}\left(\frac{b \log(f) + 2x(c \log(f) + f \log(g)) + e \log(g)}{2\sqrt{c \log(f) + f \log(g)}}\right)}{2\sqrt{c \log(f) + f \log(g)}}$$

[Out] (f^a\*g^d\*Sqrt[Pi]\*Erfi[(b\*Log[f] + e\*Log[g] + 2\*x\*(c\*Log[f] + f\*Log[g]))/(2\*Sqrt[c\*Log[f] + f\*Log[g]])])/(2\*E^((b\*Log[f] + e\*Log[g])^2/(4\*(c\*Log[f] + f\*Log[g]))) \* Sqrt[c\*Log[f] + f\*Log[g]])

**Rubi [A]** time = 0.180561, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2287, 2234, 2204}

$$\frac{\sqrt{\pi} f^a g^d \exp\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right) \operatorname{Erfi}\left(\frac{b \log(f) + 2x(c \log(f) + f \log(g)) + e \log(g)}{2\sqrt{c \log(f) + f \log(g)}}\right)}{2\sqrt{c \log(f) + f \log(g)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*g^(d + e\*x + f\*x^2),x]

[Out] (f^a\*g^d\*Sqrt[Pi]\*Erfi[(b\*Log[f] + e\*Log[g] + 2\*x\*(c\*Log[f] + f\*Log[g]))/(2\*Sqrt[c\*Log[f] + f\*Log[g]])])/(2\*E^((b\*Log[f] + e\*Log[g])^2/(4\*(c\*Log[f] + f\*Log[g]))) \* Sqrt[c\*Log[f] + f\*Log[g]])

#### Rule 2287

Int[(u\_.)\*(F\_)^(v\_.)\*(G\_)^(w\_.), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2234

Int[(F\_)^(a\_. + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2} g^{d+ex+fx^2} dx &= \int \exp\left(a \log(f) + d \log(g) + x(b \log(f) + e \log(g)) + x^2(c \log(f) + f \log(g))\right) dx \\ &= \left(\exp\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right) f^a g^d\right) \int \exp\left(\frac{(b \log(f) + e \log(g) + 2x(c \log(f) + f \log(g)))^2}{4(c \log(f) + f \log(g))}\right) dx \\ &= \frac{\exp\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right) f^a g^d \sqrt{\pi} \operatorname{erfi}\left(\frac{b \log(f) + e \log(g) + 2x(c \log(f) + f \log(g))}{2\sqrt{c \log(f) + f \log(g)}}\right)}{2\sqrt{c \log(f) + f \log(g)}} \end{aligned}$$

**Mathematica [A]** time = 0.0646792, size = 93, normalized size = 0.98

$$\frac{\sqrt{\pi} f^a g^d \exp\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right) \operatorname{Erfi}\left(\frac{\log(f)(b + 2cx) + \log(g)(e + 2fx)}{2\sqrt{c \log(f) + f \log(g)}}\right)}{2\sqrt{c \log(f) + f \log(g)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*g^(d + e\*x + f\*x^2), x]

[Out] (f^a\*g^d\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Log[f] + (e + 2\*f\*x)\*Log[g])/(2\*Sqrt[c\*Log[f] + f\*Log[g]])])/(2\*E^((b\*Log[f] + e\*Log[g])^2/(4\*(c\*Log[f] + f\*Log[g]))) \* Sqrt[c\*Log[f] + f\*Log[g]])

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} g^{fx^2+ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*g^(f\*x^2+e\*x+d), x)

[Out] int(f^(c\*x^2+b\*x+a)\*g^(f\*x^2+e\*x+d), x)

**Maxima [A]** time = 1.07988, size = 122, normalized size = 1.28

$$\frac{\sqrt{\pi} f^a g^d \operatorname{erf}\left(\sqrt{-c \log(f) - f \log(g)} x - \frac{b \log(f) + e \log(g)}{2\sqrt{-c \log(f) - f \log(g)}}\right) e^{\left(-\frac{(b \log(f) + e \log(g))^2}{4(c \log(f) + f \log(g))}\right)}}{2\sqrt{-c \log(f) - f \log(g)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*g^(f\*x^2+e\*x+d), x, algorithm="maxima")

[Out] 1/2\*sqrt(pi)\*f^a\*g^d\*erf(sqrt(-c\*log(f) - f\*log(g))\*x - 1/2\*(b\*log(f) + e\*log(g))/sqrt(-c\*log(f) - f\*log(g)))\*e^(-1/4\*(b\*log(f) + e\*log(g))^2/(c\*log(f) + f\*log(g)))/sqrt(-c\*log(f) - f\*log(g))

**Fricas [A]** time = 1.55353, size = 385, normalized size = 4.05

$$\frac{\sqrt{\pi} \sqrt{-c \log(f) - f \log(g)} \operatorname{erf}\left(\frac{((2cx+b) \log(f) + (2fx+e) \log(g)) \sqrt{-c \log(f) - f \log(g)}}{2(c \log(f) + f \log(g))}\right) e^{\left(-\frac{(b^2-4ac) \log(f)^2 - 2(2cd-be+2af) \log(f) \log(g) + (e^2-4ad) \log(g)^2}{4(c \log(f) + f \log(g))}\right)}}{2(c \log(f) + f \log(g))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*g^(f\*x^2+e\*x+d), x, algorithm="fricas")

```
[Out] -1/2*sqrt(pi)*sqrt(-c*log(f) - f*log(g))*erf(1/2*((2*c*x + b)*log(f) + (2*f*x + e)*log(g))*sqrt(-c*log(f) - f*log(g))/(c*log(f) + f*log(g)))*e^(-1/4*(b^2 - 4*a*c)*log(f)^2 - 2*(2*c*d - b*e + 2*a*f)*log(f)*log(g) + (e^2 - 4*d*f)*log(g)^2)/(c*log(f) + f*log(g))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} g^{d+ex+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*g**(f*x**2+e*x+d), x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*g**(d + e*x + f*x**2), x)
```

**Giac [A]** time = 1.19354, size = 177, normalized size = 1.86

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log(f) - f \log(g)} \left(2x + \frac{b \log(f) + e \log(g)}{c \log(f) + f \log(g)}\right)\right) e^{\left(-\frac{b^2 \log(f)^2 - 4ac \log(f)^2 - 4cd \log(f) \log(g) - 4af \log(f) \log(g) + 2be \log(f) \log(g) - 4df \log(g)^2}{4(c \log(f) + f \log(g))}\right)}}{2 \sqrt{-c \log(f) - f \log(g)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d), x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f*log(g))*(2*x + (b*log(f) + e*log(g))/(c*log(f) + f*log(g))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f)*log(g) - 4*a*f*log(f)*log(g) + 2*b*e*log(f)*log(g) - 4*d*f*log(g)^2 + e^2*log(g)^2)/(c*log(f) + f*log(g)))/sqrt(-c*log(f) - f*log(g))
```

### 3.66 $\int F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^n dx$

**Optimal.** Leaf size=106

$$\frac{F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^n \left(\frac{bG^{h(f+gx)}}{a} + 1\right)^{-n} {}_2F_1\left(-n, \frac{de \log(F)}{gh \log(G)}; \frac{de \log(F)}{gh \log(G)} + 1; -\frac{bG^{h(f+gx)}}{a}\right)}{de \log(F)}$$

[Out] (F^(e\*(c + d\*x))\*(a + b\*G^(h\*(f + g\*x))))^n\*Hypergeometric2F1[-n, (d\*e\*Log[F])/(g\*h\*Log[G]), 1 + (d\*e\*Log[F])/(g\*h\*Log[G]), -(b\*G^(h\*(f + g\*x)))/a)]/(d\*e\*(1 + (b\*G^(h\*(f + g\*x)))/a)^n\*Log[F])

**Rubi [A]** time = 0.0930117, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2252, 2251}

$$\frac{F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^n \left(\frac{bG^{h(f+gx)}}{a} + 1\right)^{-n} {}_2F_1\left(-n, \frac{de \log(F)}{gh \log(G)}; \frac{de \log(F)}{gh \log(G)} + 1; -\frac{bG^{h(f+gx)}}{a}\right)}{de \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(e\*(c + d\*x))\*(a + b\*G^(h\*(f + g\*x))))^n,x]

[Out] (F^(e\*(c + d\*x))\*(a + b\*G^(h\*(f + g\*x))))^n\*Hypergeometric2F1[-n, (d\*e\*Log[F])/(g\*h\*Log[G]), 1 + (d\*e\*Log[F])/(g\*h\*Log[G]), -(b\*G^(h\*(f + g\*x)))/a)]/(d\*e\*(1 + (b\*G^(h\*(f + g\*x)))/a)^n\*Log[F])

#### Rule 2252

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Dist[(a + b\*F^(e\*(c + d\*x)))^p/(1 + (b/a)\*F^(e\*(c + d\*x)))^p, Int[G^(h\*(f + g\*x))\*(1 + (b\*F^(e\*(c + d\*x)))/a)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 2251

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Simp[(a^p\*G^(h\*(f + g\*x))\*Hypergeometric2F1[-p, (g\*h\*Log[G])/(d\*e\*Log[F]), (g\*h\*Log[G])/(d\*e\*Log[F]) + 1, Simplify[-((b\*F^(e\*(c + d\*x)))/a])])]/(g\*h\*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^n dx &= \left( \left(a + bG^{h(f+gx)}\right)^n \left(1 + \frac{bG^{h(f+gx)}}{a}\right)^{-n} \right) \int F^{e(c+dx)} \left(1 + \frac{bG^{h(f+gx)}}{a}\right)^n dx \\ &= \frac{F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^n \left(1 + \frac{bG^{h(f+gx)}}{a}\right)^{-n} {}_2F_1\left(-n, \frac{de \log(F)}{gh \log(G)}; 1 + \frac{de \log(F)}{gh \log(G)}; -\frac{bG^{h(f+gx)}}{a}\right)}{de \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0453719, size = 92, normalized size = 0.87

$$\frac{F^{e(c+dx)} \left(a + bG^{h(f+gx)}\right)^{n+1} {}_2F_1\left(1, n + \frac{de \log(F)}{gh \log(G)} + 1; \frac{de \log(F)}{gh \log(G)} + 1; -\frac{bG^{h(f+gx)}}{a}\right)}{ade \log(F)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[F^(e\*(c + d\*x))\*(a + b\*G^(h\*(f + g\*x)))^n,x]

[Out] (F^(e\*(c + d\*x))\*(a + b\*G^(h\*(f + g\*x)))^(1 + n)\*Hypergeometric2F1[1, 1 + n + (d\*e\*Log[F])/(g\*h\*Log[G]), 1 + (d\*e\*Log[F])/(g\*h\*Log[G]), -(b\*G^(h\*(f + g\*x)))/a])/((a\*d\*e\*Log[F])

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int F^{e(dx+c)} \left( a + bG^{h(gx+f)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e\*(d\*x+c))\*(a+b\*G^(h\*(g\*x+f)))^n,x)

[Out] int(F^(e\*(d\*x+c))\*(a+b\*G^(h\*(g\*x+f)))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( G^{(gx+f)h} b + a \right)^n F^{(dx+c)e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*(a+b\*G^(h\*(g\*x+f)))^n,x, algorithm="maxima")

[Out] integrate((G^((g\*x + f)\*h)\*b + a)^n \* F^((d\*x + c)\*e), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(G^{ghx+fh}b + a\right)^n F^{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*(a+b\*G^(h\*(g\*x+f)))^n,x, algorithm="fricas")

[Out] integral((G^(g\*h\*x + f\*h)\*b + a)^n \* F^(d\*e\*x + c\*e), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e\*(d\*x+c))\*(a+b\*G\*\*(h\*(g\*x+f)))\*\*n,x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( G^{(g*x+f)*h} b + a \right)^n F^{(d*x+c)*e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*(a+b\*G^(h\*(g\*x+f)))^n,x, algorithm="giac")

[Out] integrate((G^((g\*x + f)\*h)\*b + a)^n\*F^((d\*x + c)\*e), x)

$$3.67 \quad \int \frac{F^{e(c+dx)} H^{t(r+sx)}}{a+bF^{e(c+dx)}} dx$$

**Optimal.** Leaf size=75

$$\frac{H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}; -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)}$$

[Out]  $(H^{(t*(r + s*x))*Hypergeometric2F1[1, -((s*t*Log[H])/(d*e*Log[F]))], 1 - (s*t*Log[H])/(d*e*Log[F]), -(a/(b*F^{(e*(c + d*x))})])]/(b*s*t*Log[H])$

**Rubi [A]** time = 0.134529, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2256, 2251}

$$\frac{H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}; -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)}$$

Antiderivative was successfully verified.

[In] Int[(F^(e\*(c + d\*x))\*H^(t\*(r + s\*x)))/(a + b\*F^(e\*(c + d\*x))),x]

[Out]  $(H^{(t*(r + s*x))*Hypergeometric2F1[1, -((s*t*Log[H])/(d*e*Log[F]))], 1 - (s*t*Log[H])/(d*e*Log[F]), -(a/(b*F^{(e*(c + d*x))})])]/(b*s*t*Log[H])$

#### Rule 2256

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_)))\*(H\_)^((t\_.)\*((r\_.) + (s\_.)\*(x\_))), x\_Symbol] :> Dist[G^((f - (c\*g)/d)\*h), Int[H^(t\*(r + s\*x))\* (b + a/F^(e\*(c + d\*x)))^p, x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h, r, s, t}, x] && EqQ[d\*e\*p\*Log[F] + g\*h\*Log[G], 0] && IntegerQ[p]

#### Rule 2251

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[(a^p\*G^(h\*(f + g\*x))\*Hypergeometric2F1[-p, (g\*h\*Log[G])/(d\*e\*Log[F]), (g\*h\*Log[G])/(d\*e\*Log[F]) + 1, Simplify[-((b\*F^(e\*(c + d\*x)))/a])]/(g\*h\*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{F^{e(c+dx)} H^{t(r+sx)}}{a+bF^{e(c+dx)}} dx &= \int \frac{H^{t(r+sx)}}{b+aF^{-e(c+dx)}} dx \\ &= \frac{H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}; -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)} \end{aligned}$$

**Mathematica [A]** time = 0.16019, size = 75, normalized size = 1.

$$\frac{H^{t(r+sx)} \left( {}_2F_1\left(1, \frac{st \log(H)}{de \log(F)}; \frac{st \log(H)}{de \log(F)} + 1; -\frac{bF^{e(c+dx)}}{a}\right) - 1 \right)}{bst \log(H)}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(e\*(c + d\*x))\*H^(t\*(r + s\*x)))/(a + b\*F^(e\*(c + d\*x))),x]

[Out] -((H^(t\*(r + s\*x))\*(-1 + Hypergeometric2F1[1, (s\*t\*Log[H])/(d\*e\*Log[F]), 1 + (s\*t\*Log[H])/(d\*e\*Log[F]), -(b\*F^(e\*(c + d\*x)))/a])))/(b\*s\*t\*Log[H])

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int \frac{F^{e(dx+c)} H^{t(sx+r)}}{a + bF^{e(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e\*(d\*x+c))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x)

[Out] int(F^(e\*(d\*x+c))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-H^{rt} a^2 de \int \frac{H^{stx}}{a^2 bde \log(F) - a^2 bst \log(H) + (F^{2ce} b^3 de \log(F) - F^{2ce} b^3 st \log(H)) F^{2dex} + 2(F^{ce} ab^2 de \log(F) - F^{ce} ab^2 st \log(H)) F^{dex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="maxima")

[Out] -H^(r\*t)\*a^2\*d\*e\*integrate(H^(s\*t\*x)/(a^2\*b\*d\*e\*log(F) - a^2\*b\*s\*t\*log(H) + (F^(2\*c\*e)\*b^3\*d\*e\*log(F) - F^(2\*c\*e)\*b^3\*s\*t\*log(H))\*F^(2\*d\*e\*x) + 2\*(F^(c\*e)\*a\*b^2\*d\*e\*log(F) - F^(c\*e)\*a\*b^2\*s\*t\*log(H))\*F^(d\*e\*x)), x)\*log(F) + (H^(r\*t)\*a\*d\*e\*log(F) + (F^(c\*e)\*H^(r\*t)\*b\*d\*e\*log(F) - F^(c\*e)\*H^(r\*t)\*b\*s\*t\*log(H))\*F^(d\*e\*x))\*H^(s\*t\*x)/(a\*b\*d\*e\*s\*t\*log(F)\*log(H) - a\*b\*s^2\*t^2\*log(H)^2 + (F^(c\*e)\*b^2\*d\*e\*s\*t\*log(F)\*log(H) - F^(c\*e)\*b^2\*s^2\*t^2\*log(H)^2)\*F^(d\*e\*x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{dex+ce} H^{stx+rt}}{F^{dex+ce} b + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="fricas")

[Out] integral(F^(d\*e\*x + c\*e)\*H^(s\*t\*x + r\*t)/(F^(d\*e\*x + c\*e)\*b + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e(c+dx)} H^{t(r+sx)}}{F^{ce} F^{dex} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e\*(d\*x+c))\*H\*\*(t\*(s\*x+r))/(a+b\*F\*\*(e\*(d\*x+c))),x)

[Out] Integral(F\*\*(e\*(c + d\*x))\*H\*\*(t\*(r + s\*x))/(F\*\*(c\*e)\*F\*\*(d\*e\*x)\*b + a), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)e} H^{(sx+r)t}}{F^{(dx+c)e} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+c))\*H^(t\*(s\*x+r))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="giac")

[Out] integrate(F^((d\*x + c)\*e)\*H^((s\*x + r)\*t)/(F^((d\*x + c)\*e)\*b + a), x)

$$3.68 \quad \int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a+bF^{e(c+dx)}} dx$$

**Optimal.** Leaf size=85

$$\frac{F^{-e(c-f)} H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}, -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)}$$

[Out] (H^(t\*(r + s\*x))\*Hypergeometric2F1[1, -((s\*t\*Log[H])/(d\*e\*Log[F])), 1 - (s\*t\*Log[H])/(d\*e\*Log[F]), -(a/(b\*F^(e\*(c + d\*x))))])/(b\*F^(e\*(c - f))\*s\*t\*Log[H])

**Rubi [A]** time = 0.132062, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2256, 2251}

$$\frac{F^{-e(c-f)} H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}, -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)}$$

Antiderivative was successfully verified.

[In] Int[(F^(e\*(f + d\*x))\*H^(t\*(r + s\*x)))/(a + b\*F^(e\*(c + d\*x))),x]

[Out] (H^(t\*(r + s\*x))\*Hypergeometric2F1[1, -((s\*t\*Log[H])/(d\*e\*Log[F])), 1 - (s\*t\*Log[H])/(d\*e\*Log[F]), -(a/(b\*F^(e\*(c + d\*x))))])/(b\*F^(e\*(c - f))\*s\*t\*Log[H])

#### Rule 2256

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_)))\*(H\_)^((t\_)\*((r\_) + (s\_)\*(x\_))), x\_Symbol] :> Dist[G^((f - (c\*g)/d)\*h), Int[H^(t\*(r + s\*x))\*(b + a/F^(e\*(c + d\*x)))^p, x], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h, r, s, t}, x] && EqQ[d\*e\*p\*Log[F] + g\*h\*Log[G], 0] && IntegerQ[p]

#### Rule 2251

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] :> Simp[(a^p\*G^(h\*(f + g\*x))\*Hypergeometric2F1[-p, (g\*h\*Log[G])/(d\*e\*Log[F]), (g\*h\*Log[G])/(d\*e\*Log[F]) + 1, Simplify[-((b\*F^(e\*(c + d\*x)))/a])])/(g\*h\*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a+bF^{e(c+dx)}} dx &= F^{-e(c-f)} \int \frac{H^{t(r+sx)}}{b+aF^{-e(c+dx)}} dx \\ &= \frac{F^{-e(c-f)} H^{t(r+sx)} {}_2F_1\left(1, -\frac{st \log(H)}{de \log(F)}; 1 - \frac{st \log(H)}{de \log(F)}, -\frac{aF^{-e(c+dx)}}{b}\right)}{bst \log(H)} \end{aligned}$$

**Mathematica [A]** time = 0.143593, size = 84, normalized size = 0.99

$$\frac{F^{e(f-c)} H^{t(r+sx)} \left( {}_2F_1\left(1, \frac{st \log(H)}{de \log(F)}, \frac{st \log(H)}{de \log(F)} + 1; -\frac{bF^{e(c+dx)}}{a}\right) - 1 \right)}{bst \log(H)}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(e\*(f + d\*x))\*H^(t\*(r + s\*x)))/(a + b\*F^(e\*(c + d\*x))),x]

[Out] -((F^(e\*(-c + f))\*H^(t\*(r + s\*x))\*(-1 + Hypergeometric2F1[1, (s\*t\*Log[H])/(d\*e\*Log[F]), 1 + (s\*t\*Log[H])/(d\*e\*Log[F]), -(b\*F^(e\*(c + d\*x)))/a]))/(b\*s\*t\*Log[H]))

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{F^{e(dx+f)} H^{t(sx+r)}}{a + bF^{e(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e\*(d\*x+f))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x

[Out] int(F^(e\*(d\*x+f))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-F^{ef} H^{rt} a^2 de \int \frac{H^{stx}}{F^{ce} a^2 bde \log(F) - F^{ce} a^2 bst \log(H) + (F^{3ce} b^3 de \log(F) - F^{3ce} b^3 st \log(H)) F^{2dex} + 2(F^{2ce} ab^2 de \log(F) - F^{2ce} ab^2 st \log(H)) F^{dex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+f))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="maxima")

[Out] -F^(e\*f)\*H^(r\*t)\*a^2\*d\*e\*integrate(H^(s\*t\*x)/(F^(c\*e)\*a^2\*b\*d\*e\*log(F) - F^(c\*e)\*a^2\*b\*s\*t\*log(H) + (F^(3\*c\*e)\*b^3\*d\*e\*log(F) - F^(3\*c\*e)\*b^3\*s\*t\*log(H))\*F^(2\*d\*e\*x) + 2\*(F^(2\*c\*e)\*a\*b^2\*d\*e\*log(F) - F^(2\*c\*e)\*a\*b^2\*s\*t\*log(H))\*F^(d\*e\*x)), x)\*log(F) + (F^(e\*f)\*H^(r\*t)\*a\*d\*e\*log(F) + (F^(c\*e + e\*f)\*H^(r\*t)\*b\*d\*e\*log(F) - F^(c\*e + e\*f)\*H^(r\*t)\*b\*s\*t\*log(H))\*F^(d\*e\*x))\*H^(s\*t\*x)/(F^(c\*e)\*a\*b\*d\*e\*s\*t\*log(F)\*log(H) - F^(c\*e)\*a\*b\*s^2\*t^2\*log(H)^2 + (F^(2\*c\*e)\*b^2\*d\*e\*s\*t\*log(F)\*log(H) - F^(2\*c\*e)\*b^2\*s^2\*t^2\*log(H)^2)\*F^(d\*e\*x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{dex+ef} H^{stx+rt}}{F^{dex+ceb} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+f))\*H^(t\*(s\*x+r)))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="fricas")

[Out] integral(F^(d\*e\*x + e\*f)\*H^(s\*t\*x + r\*t)/(F^(d\*e\*x + c\*e)\*b + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e(dx+f)} H^{t(r+sx)}}{F^{ce} F^{dex} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e\*(d\*x+f))\*H\*\*(t\*(s\*x+r))/(a+b\*F\*\*(e\*(d\*x+c))),x)

[Out] Integral(F\*\*(e\*(d\*x + f))\*H\*\*(t\*(r + s\*x))/(F\*\*(c\*e)\*F\*\*(d\*e\*x)\*b + a), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+f)e} H^{(sx+r)t}}{F^{(dx+c)e} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e\*(d\*x+f))\*H^(t\*(s\*x+r))/(a+b\*F^(e\*(d\*x+c))),x, algorithm="giac")

[Out] integrate(F^((d\*x + f)\*e)\*H^((s\*x + r)\*t)/(F^((d\*x + c)\*e)\*b + a), x)

### 3.69 $\int f^{a+bx^2} x^m dx$

**Optimal.** Leaf size=46

$$-\frac{1}{2} f^a x^{m+1} (-bx^2 \log(f))^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -bx^2 \log(f)\right)$$

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/2, -(b x^2 \text{Log}[f])]) * (- (b x^2 \text{Log}[f]))^{((-1-m)/2)}/2$

**Rubi [A]** time = 0.0239831, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{2} f^a x^{m+1} (-bx^2 \log(f))^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -bx^2 \log(f)\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^m, x]

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/2, -(b x^2 \text{Log}[f])]) * (- (b x^2 \text{Log}[f]))^{((-1-m)/2)}/2$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m+1)\*Gamma[(m+1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m+1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x^m dx = -\frac{1}{2} f^a x^{1+m} \Gamma\left(\frac{1+m}{2}, -bx^2 \log(f)\right) (-bx^2 \log(f))^{\frac{1}{2}(-1-m)}$$

**Mathematica [A]** time = 0.0121115, size = 46, normalized size = 1.

$$-\frac{1}{2} f^a x^{m+1} (-bx^2 \log(f))^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -bx^2 \log(f)\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^m, x]

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/2, -(b x^2 \text{Log}[f])]) * (- (b x^2 \text{Log}[f]))^{((-1-m)/2)}/2$

**Maple [B]** time = 0.035, size = 140, normalized size = 3.

$$\frac{f^a}{2} (-b)^{-\frac{m}{2}-\frac{1}{2}} (\ln(f))^{-\frac{m}{2}-\frac{1}{2}} \left( 2 \frac{x^{1+m} (-b)^{m/2+1/2} (\ln(f))^{m/2+1/2} (m/2+1/2) (-bx^2 \ln(f))^{-m/2-1/2} \Gamma(m/2+1/2)}{1+m} + 2 \frac{x^{1+m} (-b)^{m/2+1/2} (\ln(f))^{m/2+1/2} (m/2+1/2) (-bx^2 \ln(f))^{-m/2-1/2} \Gamma(m/2+1/2)}{1+m} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)*x^m,x)`

[Out]  $\frac{1}{2}f^a(-b)^{-1/2m-1/2}\ln(f)^{-1/2m-1/2}\frac{2}{(1+m)}x^{(1+m)}(-b)^{(1/2m+1/2)}\ln(f)^{(1/2m+1/2)}(1/2m+1/2)(-b*x^2*\ln(f))^{-1/2m-1/2}\text{GAMMA}(1/2m+1/2)+2/(1+m)*x^{(1+m)}(-b)^{(1/2m+1/2)}\ln(f)^{(1/2m+1/2)}(-1/2m-1/2)(-b*x^2*\ln(f))^{-1/2m-1/2}\text{GAMMA}(1/2m+1/2,-b*x^2*\ln(f))$

**Maxima [A]** time = 1.22648, size = 51, normalized size = 1.11

$$-\frac{1}{2}(-bx^2 \log(f))^{-\frac{1}{2}m-\frac{1}{2}} f^a x^{m+1} \Gamma\left(\frac{1}{2}m + \frac{1}{2}, -bx^2 \log(f)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^m,x, algorithm="maxima")`

[Out]  $-1/2*(-b*x^2*\log(f))^{-1/2m-1/2}*f^a*x^{(m+1)}*\text{gamma}(1/2m+1/2,-b*x^2*\log(f))$

**Fricas [A]** time = 1.56654, size = 126, normalized size = 2.74

$$\frac{e^{\left(-\frac{1}{2}(m-1)\log(-b\log(f))+a\log(f)\right)}\Gamma\left(\frac{1}{2}m + \frac{1}{2}, -bx^2 \log(f)\right)}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^m,x, algorithm="fricas")`

[Out]  $1/2*e^{(-1/2*(m-1)*\log(-b*\log(f))+a*\log(f))*\text{gamma}(1/2m+1/2,-b*x^2*\log(f))/(b*\log(f))}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)*x**m,x)`

[Out] `Integral(f**(a + b*x**2)*x**m, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^2+a} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)*x^m,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)*x^m, x)
```

### 3.70 $\int f^{a+bx^2} x^{11} dx$

**Optimal.** Leaf size=78

$$\frac{f^{a+bx^2} (-b^5 x^{10} \log^5(f) + 5b^4 x^8 \log^4(f) - 20b^3 x^6 \log^3(f) + 60b^2 x^4 \log^2(f) - 120bx^2 \log(f) + 120)}{2b^6 \log^6(f)}$$

[Out]  $-(f^{(a + b*x^2)}*(120 - 120*b*x^2*Log[f] + 60*b^2*x^4*Log[f]^2 - 20*b^3*x^6*Log[f]^3 + 5*b^4*x^8*Log[f]^4 - b^5*x^{10}*Log[f]^5))/(2*b^6*Log[f]^6)$

**Rubi [C]** time = 0.0254629, antiderivative size = 24, normalized size of antiderivative = 0.31, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}(6, -bx^2 \log(f))}{2b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^11,x]

[Out]  $-(f^a*\text{Gamma}[6, -(b*x^2*\text{Log}[f])])/(2*b^6*\text{Log}[f]^6)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x^{11} dx = -\frac{f^a \Gamma(6, -bx^2 \log(f))}{2b^6 \log^6(f)}$$

**Mathematica [C]** time = 0.0030158, size = 24, normalized size = 0.31

$$\frac{f^a \text{Gamma}(6, -bx^2 \log(f))}{2b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^11,x]

[Out]  $-(f^a*\text{Gamma}[6, -(b*x^2*\text{Log}[f])])/(2*b^6*\text{Log}[f]^6)$

**Maple [A]** time = 0.01, size = 76, normalized size = 1.

$$\frac{(b^5 x^{10} (\ln(f))^5 - 5b^4 x^8 (\ln(f))^4 + 20b^3 x^6 (\ln(f))^3 - 60b^2 x^4 (\ln(f))^2 + 120bx^2 \ln(f) - 120) f^{bx^2+a}}{2 (\ln(f))^6 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)*x^11,x)`

[Out]  $\frac{1}{2}(b^5x^{10}\ln(f)^5 - 5b^4x^8\ln(f)^4 + 20b^3x^6\ln(f)^3 - 60b^2x^4\ln(f)^2 + 120bx^2\ln(f) - 120)f^{bx^2+a}/\ln(f)^6/b^6$

**Maxima [A]** time = 1.16641, size = 124, normalized size = 1.59

$$\frac{(b^5 f^a x^{10} \log(f)^5 - 5 b^4 f^a x^8 \log(f)^4 + 20 b^3 f^a x^6 \log(f)^3 - 60 b^2 f^a x^4 \log(f)^2 + 120 b f^a x^2 \log(f) - 120 f^a) f^{bx^2}}{2 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^11,x, algorithm="maxima")`

[Out]  $\frac{1}{2}(b^5 f^a x^{10} \log(f)^5 - 5 b^4 f^a x^8 \log(f)^4 + 20 b^3 f^a x^6 \log(f)^3 - 60 b^2 f^a x^4 \log(f)^2 + 120 b f^a x^2 \log(f) - 120 f^a) f^{bx^2+a}/(b^6 \log(f)^6)$

**Fricas [A]** time = 1.54697, size = 194, normalized size = 2.49

$$\frac{(b^5 x^{10} \log(f)^5 - 5 b^4 x^8 \log(f)^4 + 20 b^3 x^6 \log(f)^3 - 60 b^2 x^4 \log(f)^2 + 120 b x^2 \log(f) - 120) f^{bx^2+a}}{2 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^11,x, algorithm="fricas")`

[Out]  $\frac{1}{2}(b^5 x^{10} \log(f)^5 - 5 b^4 x^8 \log(f)^4 + 20 b^3 x^6 \log(f)^3 - 60 b^2 x^4 \log(f)^2 + 120 b x^2 \log(f) - 120) f^{bx^2+a}/(b^6 \log(f)^6)$

**Sympy [A]** time = 0.16055, size = 95, normalized size = 1.22

$$\begin{cases} \frac{f^{a+bx^2}(b^5 x^{10} \log(f)^5 - 5 b^4 x^8 \log(f)^4 + 20 b^3 x^6 \log(f)^3 - 60 b^2 x^4 \log(f)^2 + 120 b x^2 \log(f) - 120)}{2 b^6 \log(f)^6} & \text{for } 2 b^6 \log(f)^6 \neq 0 \\ \frac{x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)*x**11,x)`

[Out] `Piecewise((f**(a + b*x**2)*(b**5*x**10*log(f)**5 - 5*b**4*x**8*log(f)**4 + 20*b**3*x**6*log(f)**3 - 60*b**2*x**4*log(f)**2 + 120*b*x**2*log(f) - 120)/(2*b**6*log(f)**6), Ne(2*b**6*log(f)**6, 0)), (x**12/12, True))`

**Giac [A]** time = 1.32157, size = 107, normalized size = 1.37

$$\frac{\left(b^5 x^{10} \log(f)^5 - 5 b^4 x^8 \log(f)^4 + 20 b^3 x^6 \log(f)^3 - 60 b^2 x^4 \log(f)^2 + 120 b x^2 \log(f) - 120\right) e^{(b x^2 \log(f) + a \log(f))}}{2 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^11,x, algorithm="giac")

[Out] 1/2\*(b^5\*x^10\*log(f)^5 - 5\*b^4\*x^8\*log(f)^4 + 20\*b^3\*x^6\*log(f)^3 - 60\*b^2\*x^4\*log(f)^2 + 120\*b\*x^2\*log(f) - 120)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^6\*log(f)^6)

### 3.71 $\int f^{a+bx^2} x^9 dx$

**Optimal.** Leaf size=65

$$\frac{f^{a+bx^2} (b^4 x^8 \log^4(f) - 4b^3 x^6 \log^3(f) + 12b^2 x^4 \log^2(f) - 24bx^2 \log(f) + 24)}{2b^5 \log^5(f)}$$

[Out] (f^(a + b\*x^2)\*(24 - 24\*b\*x^2\*Log[f] + 12\*b^2\*x^4\*Log[f]^2 - 4\*b^3\*x^6\*Log[f]^3 + b^4\*x^8\*Log[f]^4))/(2\*b^5\*Log[f]^5)

**Rubi [C]** time = 0.0240763, antiderivative size = 24, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}(5, -bx^2 \log(f))}{2b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^9, x]

[Out] (f^a\*Gamma[5, -(b\*x^2\*Log[f])])/(2\*b^5\*Log[f]^5)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x^9 dx = \frac{f^a \Gamma(5, -bx^2 \log(f))}{2b^5 \log^5(f)}$$

**Mathematica [C]** time = 0.0028788, size = 24, normalized size = 0.37

$$\frac{f^a \text{Gamma}(5, -bx^2 \log(f))}{2b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^9, x]

[Out] (f^a\*Gamma[5, -(b\*x^2\*Log[f])])/(2\*b^5\*Log[f]^5)

**Maple [A]** time = 0.008, size = 64, normalized size = 1.

$$\frac{f^{bx^2+a} (24 - 24bx^2 \ln(f) + 12b^2x^4 (\ln(f))^2 - 4b^3x^6 (\ln(f))^3 + b^4x^8 (\ln(f))^4)}{2b^5 (\ln(f))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)*x^9,x)`

[Out]  $\frac{1}{2}f^{(b*x^2+a)}*(24-24*b*x^2*\ln(f)+12*b^2*x^4*\ln(f)^2-4*b^3*x^6*\ln(f)^3+b^4*x^8*\ln(f)^4)/b^5/\ln(f)^5$

**Maxima [A]** time = 1.12678, size = 104, normalized size = 1.6

$$\frac{(b^4 f^a x^8 \log(f)^4 - 4 b^3 f^a x^6 \log(f)^3 + 12 b^2 f^a x^4 \log(f)^2 - 24 b f^a x^2 \log(f) + 24 f^a) f^{bx^2}}{2 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^9,x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(b^4*x^8*\log(f)^4 - 4*b^3*f^a*x^6*\log(f)^3 + 12*b^2*f^a*x^4*\log(f)^2 - 24*b*f^a*x^2*\log(f) + 24*f^a)*f^{(b*x^2)}/(b^5*\log(f)^5)$

**Fricas [A]** time = 1.528, size = 161, normalized size = 2.48

$$\frac{(b^4 x^8 \log(f)^4 - 4 b^3 x^6 \log(f)^3 + 12 b^2 x^4 \log(f)^2 - 24 b x^2 \log(f) + 24) f^{bx^2+a}}{2 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^9,x, algorithm="fricas")`

[Out]  $\frac{1}{2}*(b^4*x^8*\log(f)^4 - 4*b^3*x^6*\log(f)^3 + 12*b^2*x^4*\log(f)^2 - 24*b*x^2*\log(f) + 24)*f^{(b*x^2 + a)}/(b^5*\log(f)^5)$

**Sympy [A]** time = 0.253196, size = 82, normalized size = 1.26

$$\begin{cases} \frac{f^{a+bx^2}(b^4 x^8 \log(f)^4 - 4 b^3 x^6 \log(f)^3 + 12 b^2 x^4 \log(f)^2 - 24 b x^2 \log(f) + 24)}{2 b^5 \log(f)^5} & \text{for } 2 b^5 \log(f)^5 \neq 0 \\ \frac{x^{10}}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)*x**9,x)`

[Out] `Piecewise((f**(a + b*x**2)*(b**4*x**8*log(f)**4 - 4*b**3*x**6*log(f)**3 + 12*b**2*x**4*log(f)**2 - 24*b*x**2*log(f) + 24)/(2*b**5*log(f)**5), Ne(2*b**5*log(f)**5, 0)), (x**10/10, True))`

**Giac [A]** time = 1.25853, size = 90, normalized size = 1.38

$$\frac{\left(b^4 x^8 \log(f)^4 - 4 b^3 x^6 \log(f)^3 + 12 b^2 x^4 \log(f)^2 - 24 b x^2 \log(f) + 24\right) e^{(b x^2 \log(f) + a \log(f))}}{2 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^9,x, algorithm="giac")

[Out] 1/2\*(b^4\*x^8\*log(f)^4 - 4\*b^3\*x^6\*log(f)^3 + 12\*b^2\*x^4\*log(f)^2 - 24\*b\*x^2\*log(f) + 24)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^5\*log(f)^5)



## 3.72 $\int f^{a+bx^2} x^7 dx$

**Optimal.** Leaf size=86

$$-\frac{3x^4 f^{a+bx^2}}{2b^2 \log^2(f)} + \frac{3x^2 f^{a+bx^2}}{b^3 \log^3(f)} - \frac{3f^{a+bx^2}}{b^4 \log^4(f)} + \frac{x^6 f^{a+bx^2}}{2b \log(f)}$$

[Out]  $(-3*f^{(a + b*x^2)})/(b^4*\text{Log}[f]^4) + (3*f^{(a + b*x^2)}*x^2)/(b^3*\text{Log}[f]^3) - (3*f^{(a + b*x^2)}*x^4)/(2*b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^6)/(2*b*\text{Log}[f])$

**Rubi [A]** time = 0.0929452, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$-\frac{3x^4 f^{a+bx^2}}{2b^2 \log^2(f)} + \frac{3x^2 f^{a+bx^2}}{b^3 \log^3(f)} - \frac{3f^{a+bx^2}}{b^4 \log^4(f)} + \frac{x^6 f^{a+bx^2}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^7,x]

[Out]  $(-3*f^{(a + b*x^2)})/(b^4*\text{Log}[f]^4) + (3*f^{(a + b*x^2)}*x^2)/(b^3*\text{Log}[f]^3) - (3*f^{(a + b*x^2)}*x^4)/(2*b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^6)/(2*b*\text{Log}[f])$

### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^7 dx &= \frac{f^{a+bx^2} x^6}{2b \log(f)} - \frac{3 \int f^{a+bx^2} x^5 dx}{b \log(f)} \\ &= -\frac{3f^{a+bx^2} x^4}{2b^2 \log^2(f)} + \frac{f^{a+bx^2} x^6}{2b \log(f)} + \frac{6 \int f^{a+bx^2} x^3 dx}{b^2 \log^2(f)} \\ &= \frac{3f^{a+bx^2} x^2}{b^3 \log^3(f)} - \frac{3f^{a+bx^2} x^4}{2b^2 \log^2(f)} + \frac{f^{a+bx^2} x^6}{2b \log(f)} - \frac{6 \int f^{a+bx^2} x dx}{b^3 \log^3(f)} \\ &= -\frac{3f^{a+bx^2}}{b^4 \log^4(f)} + \frac{3f^{a+bx^2} x^2}{b^3 \log^3(f)} - \frac{3f^{a+bx^2} x^4}{2b^2 \log^2(f)} + \frac{f^{a+bx^2} x^6}{2b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0105276, size = 53, normalized size = 0.62

$$\frac{f^{a+bx^2} (b^3 x^6 \log^3(f) - 3b^2 x^4 \log^2(f) + 6bx^2 \log(f) - 6)}{2b^4 \log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^7,x]

[Out] (f^(a + b\*x^2)\*(-6 + 6\*b\*x^2\*Log[f] - 3\*b^2\*x^4\*Log[f]^2 + b^3\*x^6\*Log[f]^3))/ (2\*b^4\*Log[f]^4)

**Maple [A]** time = 0.006, size = 52, normalized size = 0.6

$$\frac{(b^3 x^6 (\ln(f))^3 - 3b^2 x^4 (\ln(f))^2 + 6bx^2 \ln(f) - 6) f^{bx^2+a}}{2b^4 (\ln(f))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^7,x)

[Out] 1/2\*(b^3\*x^6\*ln(f)^3-3\*b^2\*x^4\*ln(f)^2+6\*b\*x^2\*ln(f)-6)\*f^(b\*x^2+a)/ln(f)^4/b^4

**Maxima [A]** time = 1.13667, size = 84, normalized size = 0.98

$$\frac{(b^3 f^a x^6 \log(f)^3 - 3b^2 f^a x^4 \log(f)^2 + 6bf^a x^2 \log(f) - 6f^a) f^{bx^2}}{2b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^7,x, algorithm="maxima")

[Out] 1/2\*(b^3\*f^a\*x^6\*log(f)^3 - 3\*b^2\*f^a\*x^4\*log(f)^2 + 6\*b\*f^a\*x^2\*log(f) - 6\*f^a)\*f^(b\*x^2)/(b^4\*log(f)^4)

**Fricas [A]** time = 1.50238, size = 128, normalized size = 1.49

$$\frac{(b^3 x^6 \log(f)^3 - 3b^2 x^4 \log(f)^2 + 6bx^2 \log(f) - 6) f^{bx^2+a}}{2b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^7,x, algorithm="fricas")

[Out] 1/2\*(b^3\*x^6\*log(f)^3 - 3\*b^2\*x^4\*log(f)^2 + 6\*b\*x^2\*log(f) - 6)\*f^(b\*x^2 + a)/(b^4\*log(f)^4)

---

**Sympy [A]** time = 0.141825, size = 68, normalized size = 0.79

$$\begin{cases} \frac{f^{a+bx^2}(b^3x^6\log(f)^3 - 3b^2x^4\log(f)^2 + 6bx^2\log(f) - 6)}{2b^4\log(f)^4} & \text{for } 2b^4\log(f)^4 \neq 0 \\ \frac{x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*7,x)

[Out] Piecewise((f\*\*(a + b\*x\*\*2)\*(b\*\*3\*x\*\*6\*log(f)\*\*3 - 3\*b\*\*2\*x\*\*4\*log(f)\*\*2 + 6\*b\*x\*\*2\*log(f) - 6)/(2\*b\*\*4\*log(f)\*\*4), Ne(2\*b\*\*4\*log(f)\*\*4, 0)), (x\*\*8/8, True))

---

**Giac [A]** time = 1.25016, size = 74, normalized size = 0.86

$$\frac{(b^3x^6\log(f)^3 - 3b^2x^4\log(f)^2 + 6bx^2\log(f) - 6)e^{(bx^2\log(f)+a\log(f))}}{2b^4\log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^7,x, algorithm="giac")

[Out] 1/2\*(b^3\*x^6\*log(f)^3 - 3\*b^2\*x^4\*log(f)^2 + 6\*b\*x^2\*log(f) - 6)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^4\*log(f)^4)

### 3.73 $\int f^{a+bx^2} x^5 dx$

**Optimal.** Leaf size=62

$$-\frac{x^2 f^{a+bx^2}}{b^2 \log^2(f)} + \frac{f^{a+bx^2}}{b^3 \log^3(f)} + \frac{x^4 f^{a+bx^2}}{2b \log(f)}$$

[Out]  $f^{(a + b*x^2)}/(b^3*\text{Log}[f]^3) - (f^{(a + b*x^2)}*x^2)/(b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^4)/(2*b*\text{Log}[f])$

**Rubi [A]** time = 0.0628613, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$-\frac{x^2 f^{a+bx^2}}{b^2 \log^2(f)} + \frac{f^{a+bx^2}}{b^3 \log^3(f)} + \frac{x^4 f^{a+bx^2}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^5,x]

[Out]  $f^{(a + b*x^2)}/(b^3*\text{Log}[f]^3) - (f^{(a + b*x^2)}*x^2)/(b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^4)/(2*b*\text{Log}[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^5 dx &= \frac{f^{a+bx^2} x^4}{2b \log(f)} - \frac{2 \int f^{a+bx^2} x^3 dx}{b \log(f)} \\ &= -\frac{f^{a+bx^2} x^2}{b^2 \log^2(f)} + \frac{f^{a+bx^2} x^4}{2b \log(f)} + \frac{2 \int f^{a+bx^2} x dx}{b^2 \log^2(f)} \\ &= \frac{f^{a+bx^2}}{b^3 \log^3(f)} - \frac{f^{a+bx^2} x^2}{b^2 \log^2(f)} + \frac{f^{a+bx^2} x^4}{2b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0084153, size = 41, normalized size = 0.66

$$\frac{f^{a+bx^2} (b^2 x^4 \log^2(f) - 2bx^2 \log(f) + 2)}{2b^3 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^5,x]

[Out] (f^(a + b\*x^2)\*(2 - 2\*b\*x^2\*Log[f] + b^2\*x^4\*Log[f]^2))/(2\*b^3\*Log[f]^3)

**Maple [A]** time = 0.007, size = 40, normalized size = 0.7

$$\frac{\left(b^2 x^4 (\ln(f))^2 - 2 b x^2 \ln(f) + 2\right) f^{b x^2 + a}}{2 (\ln(f))^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^5,x)

[Out] 1/2\*(b^2\*x^4\*ln(f)^2-2\*b\*x^2\*ln(f)+2)\*f^(b\*x^2+a)/ln(f)^3/b^3

**Maxima [A]** time = 1.16652, size = 63, normalized size = 1.02

$$\frac{\left(b^2 f^a x^4 \log(f)^2 - 2 b f^a x^2 \log(f) + 2 f^a\right) f^{b x^2}}{2 b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^5,x, algorithm="maxima")

[Out] 1/2\*(b^2\*f^a\*x^4\*log(f)^2 - 2\*b\*f^a\*x^2\*log(f) + 2\*f^a)\*f^(b\*x^2)/(b^3\*log(f)^3)

**Fricas [A]** time = 1.53527, size = 100, normalized size = 1.61

$$\frac{\left(b^2 x^4 \log(f)^2 - 2 b x^2 \log(f) + 2\right) f^{b x^2 + a}}{2 b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^5,x, algorithm="fricas")

[Out] 1/2\*(b^2\*x^4\*log(f)^2 - 2\*b\*x^2\*log(f) + 2)\*f^(b\*x^2 + a)/(b^3\*log(f)^3)

**Sympy [A]** time = 0.124658, size = 54, normalized size = 0.87

$$\begin{cases} \frac{f^{a+bx^2} \left(b^2 x^4 \log(f)^2 - 2 b x^2 \log(f) + 2\right)}{2 b^3 \log(f)^3} & \text{for } 2 b^3 \log(f)^3 \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*5,x)

[Out] Piecewise((f\*\*(a + b\*x\*\*2)\*(b\*\*2\*x\*\*4\*log(f)\*\*2 - 2\*b\*x\*\*2\*log(f) + 2)/(2\*b\*\*3\*log(f)\*\*3), Ne(2\*b\*\*3\*log(f)\*\*3, 0)), (x\*\*6/6, True))

---

**Giac [A]** time = 1.27472, size = 58, normalized size = 0.94

$$\frac{\left(b^2 x^4 \log(f)^2 - 2 b x^2 \log(f) + 2\right) e^{(b x^2 \log(f) + a \log(f))}}{2 b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^5,x, algorithm="giac")

[Out] 1/2\*(b^2\*x^4\*log(f)^2 - 2\*b\*x^2\*log(f) + 2)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^3\*log(f)^3)

### 3.74 $\int f^{a+bx^2} x^3 dx$

**Optimal.** Leaf size=44

$$\frac{x^2 f^{a+bx^2}}{2b \log(f)} - \frac{f^{a+bx^2}}{2b^2 \log^2(f)}$$

[Out]  $-f^{(a + b*x^2)}/(2*b^2*Log[f]^2) + (f^{(a + b*x^2)}*x^2)/(2*b*Log[f])$

**Rubi [A]** time = 0.0380601, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{x^2 f^{a+bx^2}}{2b \log(f)} - \frac{f^{a+bx^2}}{2b^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^3,x]

[Out]  $-f^{(a + b*x^2)}/(2*b^2*Log[f]^2) + (f^{(a + b*x^2)}*x^2)/(2*b*Log[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^3 dx &= \frac{f^{a+bx^2} x^2}{2b \log(f)} - \frac{\int f^{a+bx^2} x dx}{b \log(f)} \\ &= -\frac{f^{a+bx^2}}{2b^2 \log^2(f)} + \frac{f^{a+bx^2} x^2}{2b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0065671, size = 29, normalized size = 0.66

$$\frac{f^{a+bx^2} (bx^2 \log(f) - 1)}{2b^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^3,x]

[Out] (f^(a + b\*x^2)\*(-1 + b\*x^2\*Log[f]))/(2\*b^2\*Log[f]^2)

**Maple [A]** time = 0.002, size = 28, normalized size = 0.6

$$\frac{(bx^2 \ln(f) - 1) f^{bx^2+a}}{2 (\ln(f))^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^3,x)

[Out] 1/2\*(b\*x^2\*ln(f)-1)\*f^(b\*x^2+a)/b^2/ln(f)^2

**Maxima [A]** time = 1.13082, size = 43, normalized size = 0.98

$$\frac{(bf^ax^2 \log(f) - f^a)f^{bx^2}}{2b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^3,x, algorithm="maxima")

[Out] 1/2\*(b\*f^a\*x^2\*log(f) - f^a)\*f^(b\*x^2)/(b^2\*log(f)^2)

**Fricas [A]** time = 1.52543, size = 72, normalized size = 1.64

$$\frac{(bx^2 \log(f) - 1)f^{bx^2+a}}{2b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^3,x, algorithm="fricas")

[Out] 1/2\*(b\*x^2\*log(f) - 1)\*f^(b\*x^2 + a)/(b^2\*log(f)^2)

**Sympy [A]** time = 0.114831, size = 41, normalized size = 0.93

$$\begin{cases} \frac{f^{a+bx^2}(bx^2 \log(f)-1)}{2b^2 \log(f)^2} & \text{for } 2b^2 \log(f)^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*3,x)



```
[Out] Piecewise((f**(a + b*x**2)*(b*x**2*log(f) - 1)/(2*b**2*log(f)**2), Ne(2*b**2*log(f)**2, 0)), (x**4/4, True))
```

**Giac [B]** time = 1.29085, size = 932, normalized size = 21.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)*x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*((pi*b*x^2*sgn(f) - pi*b*x^2)*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) + (pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)*(b*x^2*log(abs(f)) - 1)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2))*cos(-1/2*pi*b*x^2*sgn(f) + 1/2*pi*b*x^2 - 1/2*pi*a*sgn(f) + 1/2*pi*a) + ((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)*(pi*b*x^2*sgn(f) - pi*b*x^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) - 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))*(b*x^2*log(abs(f)) - 1)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2))*sin(-1/2*pi*b*x^2*sgn(f) + 1/2*pi*b*x^2 - 1/2*pi*a*sgn(f) + 1/2*pi*a))*e^(b*x^2*log(abs(f)) + a*log(abs(f))) - 1/4*((2*b*i*x^2*log(abs(f)) - pi*b*x^2*sgn(f) + pi*b*x^2 - 2*i)*e^(1/2*(pi*b*x^2*(sgn(f) - 1) + pi*a*(sgn(f) - 1))*i)/(2*pi*b^2*i*log(abs(f))*sgn(f) - 2*pi*b^2*i*log(abs(f)) + pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2) + (2*b*i*x^2*log(abs(f)) + pi*b*x^2*sgn(f) - pi*b*x^2 - 2*i)*e^(-1/2*(pi*b*x^2*(sgn(f) - 1) + pi*a*(sgn(f) - 1))*i)/(2*pi*b^2*i*log(abs(f))*sgn(f) - 2*pi*b^2*i*log(abs(f)) - pi^2*b^2*sgn(f) + pi^2*b^2 - 2*b^2*log(abs(f))^2))*e^(b*x^2*log(abs(f)) + a*log(abs(f)))/i
```

### 3.75 $\int f^{a+bx^2} x dx$

**Optimal.** Leaf size=20

$$\frac{f^{a+bx^2}}{2b \log(f)}$$

[Out]  $f^{(a + b*x^2)/(2*b*Log[f])}$

**Rubi [A]** time = 0.0129456, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2209}

$$\frac{f^{a+bx^2}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x,x]

[Out]  $f^{(a + b*x^2)/(2*b*Log[f])}$

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x dx = \frac{f^{a+bx^2}}{2b \log(f)}$$

**Mathematica [A]** time = 0.0023447, size = 20, normalized size = 1.

$$\frac{f^{a+bx^2}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x,x]

[Out]  $f^{(a + b*x^2)/(2*b*Log[f])}$

**Maple [A]** time = 0.002, size = 19, normalized size = 1.

$$\frac{f^{bx^2+a}}{2b \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)*x,x)`

[Out]  $1/2*f^(b*x^2+a)/b/\ln(f)$

**Maxima [A]** time = 1.04665, size = 24, normalized size = 1.2

$$\frac{f^{bx^2+a}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x,x, algorithm="maxima")`

[Out]  $1/2*f^(b*x^2 + a)/(b*\log(f))$

**Fricas [A]** time = 1.52332, size = 41, normalized size = 2.05

$$\frac{f^{bx^2+a}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x,x, algorithm="fricas")`

[Out]  $1/2*f^(b*x^2 + a)/(b*\log(f))$

**Sympy [A]** time = 0.121413, size = 24, normalized size = 1.2

$$\begin{cases} \frac{f^{a+bx^2}}{2b \log(f)} & \text{for } 2b \log(f) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)*x,x)`

[Out] `Piecewise((f**(a + b*x**2)/(2*b*log(f)), Ne(2*b*log(f), 0)), (x**2/2, True))`

**Giac [A]** time = 1.17054, size = 24, normalized size = 1.2

$$\frac{f^{bx^2+a}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)*x,x, algorithm="giac")
```

```
[Out] 1/2*f^(b*x^2 + a)/(b*log(f))
```

$$3.76 \quad \int \frac{f^{a+bx^2}}{x} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{2}f^a \text{Ei}(bx^2 \log(f))$$

[Out] (f^a\*ExpIntegralEi[b\*x^2\*Log[f]])/2

**Rubi [A]** time = 0.0212704, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$\frac{1}{2}f^a \text{Ei}(bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x,x]

[Out] (f^a\*ExpIntegralEi[b\*x^2\*Log[f]])/2

**Rule 2210**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^2}}{x} dx = \frac{1}{2}f^a \text{Ei}(bx^2 \log(f))$$

**Mathematica [A]** time = 0.0019738, size = 15, normalized size = 1.

$$\frac{1}{2}f^a \text{Ei}(bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x,x]

[Out] (f^a\*ExpIntegralEi[b\*x^2\*Log[f]])/2

**Maple [A]** time = 0.011, size = 16, normalized size = 1.1

$$-\frac{f^a \text{Ei}(1, -bx^2 \ln(f))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)/x,x)`

[Out] `-1/2*f^a*Ei(1,-b*x^2*ln(f))`

**Maxima [A]** time = 1.28702, size = 18, normalized size = 1.2

$$\frac{1}{2} f^a \operatorname{Ei}(bx^2 \log(f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x,x, algorithm="maxima")`

[Out] `1/2*f^a*Ei(b*x^2*log(f))`

**Fricas [A]** time = 1.73133, size = 35, normalized size = 2.33

$$\frac{1}{2} f^a \operatorname{Ei}(bx^2 \log(f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x,x, algorithm="fricas")`

[Out] `1/2*f^a*Ei(b*x^2*log(f))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)/x,x)`

[Out] `Integral(f**(a + b*x**2)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x,x, algorithm="giac")`

[Out] `integrate(f^(b*x^2 + a)/x, x)`

$$3.77 \quad \int \frac{f^{a+bx^2}}{x^3} dx$$

**Optimal.** Leaf size=35

$$\frac{1}{2}bf^a \log(f)\text{Ei}(bx^2 \log(f)) - \frac{f^{a+bx^2}}{2x^2}$$

[Out]  $-f^{(a + b*x^2)}/(2*x^2) + (b*f^a*\text{ExpIntegralEi}[b*x^2*\text{Log}[f]]*\text{Log}[f])/2$

**Rubi [A]** time = 0.0448627, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{2}bf^a \log(f)\text{Ei}(bx^2 \log(f)) - \frac{f^{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^3,x]

[Out]  $-f^{(a + b*x^2)}/(2*x^2) + (b*f^a*\text{ExpIntegralEi}[b*x^2*\text{Log}[f]]*\text{Log}[f])/2$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^3} dx &= -\frac{f^{a+bx^2}}{2x^2} + (b \log(f)) \int \frac{f^{a+bx^2}}{x} dx \\ &= -\frac{f^{a+bx^2}}{2x^2} + \frac{1}{2}bf^a \text{Ei}(bx^2 \log(f)) \log(f) \end{aligned}$$

**Mathematica [A]** time = 0.0098074, size = 32, normalized size = 0.91

$$\frac{1}{2}f^a \left( b \log(f)\text{Ei}(bx^2 \log(f)) - \frac{f^{bx^2}}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^3,x]

[Out]  $(f^a * (-f^{(b*x^2)/x^2}) + b * \text{ExpIntegralEi}[b*x^2*\text{Log}[f]] * \text{Log}[f]))/2$

**Maple [A]** time = 0.02, size = 35, normalized size = 1.

$$-\frac{f^a f^{bx^2}}{2x^2} - \frac{f^a \ln(f) b \text{Ei}(1, -bx^2 \ln(f))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)/x^3,x)`

[Out]  $-1/2*f^a/x^2*f^{(b*x^2)} - 1/2*f^a*\ln(f)*b*\text{Ei}(1, -b*x^2*\ln(f))$

**Maxima [A]** time = 1.17222, size = 24, normalized size = 0.69

$$\frac{1}{2} b f^a \Gamma(-1, -bx^2 \log(f)) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^3,x, algorithm="maxima")`

[Out]  $1/2*b*f^a*\text{gamma}(-1, -b*x^2*\log(f))*\log(f)$

**Fricas [A]** time = 1.83609, size = 82, normalized size = 2.34

$$\frac{b f^a x^2 \text{Ei}(bx^2 \log(f)) \log(f) - f^{bx^2+a}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^3,x, algorithm="fricas")`

[Out]  $1/2*(b*f^a*x^2*\text{Ei}(b*x^2*\log(f))*\log(f) - f^{(b*x^2 + a)})/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)/x**3,x)`

[Out] `Integral(f**(a + b*x**2)/x**3, x)`



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^3, x)
```

$$3.78 \quad \int \frac{f^{a+bx^2}}{x^5} dx$$

**Optimal.** Leaf size=58

$$\frac{1}{4}b^2 f^a \log^2(f) \text{Ei}(bx^2 \log(f)) - \frac{f^{a+bx^2}}{4x^4} - \frac{b \log(f) f^{a+bx^2}}{4x^2}$$

[Out]  $-f^{(a + b*x^2)}/(4*x^4) - (b*f^{(a + b*x^2)}*Log[f])/(4*x^2) + (b^2*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^2)/4$

**Rubi [A]** time = 0.0659541, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{4}b^2 f^a \log^2(f) \text{Ei}(bx^2 \log(f)) - \frac{f^{a+bx^2}}{4x^4} - \frac{b \log(f) f^{a+bx^2}}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^5,x]

[Out]  $-f^{(a + b*x^2)}/(4*x^4) - (b*f^{(a + b*x^2)}*Log[f])/(4*x^2) + (b^2*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^2)/4$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^5} dx &= -\frac{f^{a+bx^2}}{4x^4} + \frac{1}{2}(b \log(f)) \int \frac{f^{a+bx^2}}{x^3} dx \\ &= -\frac{f^{a+bx^2}}{4x^4} - \frac{b f^{a+bx^2} \log(f)}{4x^2} + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+bx^2}}{x} dx \\ &= -\frac{f^{a+bx^2}}{4x^4} - \frac{b f^{a+bx^2} \log(f)}{4x^2} + \frac{1}{4}b^2 f^a \text{Ei}(bx^2 \log(f)) \log^2(f) \end{aligned}$$

**Mathematica [A]** time = 0.0184995, size = 48, normalized size = 0.83

$$\frac{f^a (b^2 x^4 \log^2(f) \text{Ei}(bx^2 \log(f)) - f^{bx^2} (bx^2 \log(f) + 1))}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^5,x]

[Out] (f^a\*(b^2\*x^4\*ExpIntegralEi[b\*x^2\*Log[f]]\*Log[f]^2 - f^(b\*x^2)\*(1 + b\*x^2\*Log[f])))/(4\*x^4)

**Maple [A]** time = 0.026, size = 57, normalized size = 1.

$$-\frac{f^a f^{bx^2}}{4x^4} - \frac{f^a \ln(f) b f^{bx^2}}{4x^2} - \frac{f^a (\ln(f))^2 b^2 \text{Ei}(1, -bx^2 \ln(f))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^5,x)

[Out] -1/4\*f^a/x^4\*f^(b\*x^2)-1/4\*f^a\*ln(f)\*b/x^2\*f^(b\*x^2)-1/4\*f^a\*ln(f)^2\*b^2\*Ei(1,-b\*x^2\*ln(f))

**Maxima [A]** time = 1.22192, size = 30, normalized size = 0.52

$$-\frac{1}{2} b^2 f^a \Gamma(-2, -bx^2 \log(f)) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^5,x, algorithm="maxima")

[Out] -1/2\*b^2\*f^a\*gamma(-2, -b\*x^2\*log(f))\*log(f)^2

**Fricas [A]** time = 1.75448, size = 113, normalized size = 1.95

$$\frac{b^2 f^a x^4 \text{Ei}(bx^2 \log(f)) \log(f)^2 - (bx^2 \log(f) + 1) f^{bx^2+a}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^5,x, algorithm="fricas")

[Out] 1/4\*(b^2\*f^a\*x^4\*Ei(b\*x^2\*log(f))\*log(f)^2 - (b\*x^2\*log(f) + 1)\*f^(b\*x^2 + a))/x^4

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**2+a)/x**5,x)
```

```
[Out] Integral(f**(a + b*x**2)/x**5, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^5, x)
```

$$3.79 \quad \int \frac{f^{a+bx^2}}{x^7} dx$$

**Optimal.** Leaf size=81

$$\frac{1}{12} b^3 f^a \log^3(f) \text{Ei}(bx^2 \log(f)) - \frac{b^2 \log^2(f) f^{a+bx^2}}{12x^2} - \frac{f^{a+bx^2}}{6x^6} - \frac{b \log(f) f^{a+bx^2}}{12x^4}$$

[Out]  $-f^{(a + b*x^2)}/(6*x^6) - (b*f^{(a + b*x^2)}*Log[f])/(12*x^4) - (b^2*f^{(a + b*x^2)}*Log[f]^2)/(12*x^2) + (b^3*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^3)/12$

**Rubi [A]** time = 0.09055, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{12} b^3 f^a \log^3(f) \text{Ei}(bx^2 \log(f)) - \frac{b^2 \log^2(f) f^{a+bx^2}}{12x^2} - \frac{f^{a+bx^2}}{6x^6} - \frac{b \log(f) f^{a+bx^2}}{12x^4}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^7, x]

[Out]  $-f^{(a + b*x^2)}/(6*x^6) - (b*f^{(a + b*x^2)}*Log[f])/(12*x^4) - (b^2*f^{(a + b*x^2)}*Log[f]^2)/(12*x^2) + (b^3*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^3)/12$

#### Rule 2214

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^7} dx &= -\frac{f^{a+bx^2}}{6x^6} + \frac{1}{3}(b \log(f)) \int \frac{f^{a+bx^2}}{x^5} dx \\ &= -\frac{f^{a+bx^2}}{6x^6} - \frac{b f^{a+bx^2} \log(f)}{12x^4} + \frac{1}{6}(b^2 \log^2(f)) \int \frac{f^{a+bx^2}}{x^3} dx \\ &= -\frac{f^{a+bx^2}}{6x^6} - \frac{b f^{a+bx^2} \log(f)}{12x^4} - \frac{b^2 f^{a+bx^2} \log^2(f)}{12x^2} + \frac{1}{6}(b^3 \log^3(f)) \int \frac{f^{a+bx^2}}{x} dx \\ &= -\frac{f^{a+bx^2}}{6x^6} - \frac{b f^{a+bx^2} \log(f)}{12x^4} - \frac{b^2 f^{a+bx^2} \log^2(f)}{12x^2} + \frac{1}{12} b^3 f^a \text{Ei}(bx^2 \log(f)) \log^3(f) \end{aligned}$$

**Mathematica [A]** time = 0.0237001, size = 59, normalized size = 0.73

$$\frac{f^a (b^3 x^6 \log^3(f) \text{Ei}(bx^2 \log(f)) - f^{bx^2} (b^2 x^4 \log^2(f) + bx^2 \log(f) + 2))}{12x^6}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^7,x]

[Out] (f^a\*(b^3\*x^6\*ExpIntegralEi[b\*x^2\*Log[f]]\*Log[f]^3 - f^(b\*x^2)\*(2 + b\*x^2\*Log[f] + b^2\*x^4\*Log[f]^2)))/(12\*x^6)

**Maple [A]** time = 0.033, size = 79, normalized size = 1.

$$-\frac{f^a f^{bx^2}}{6x^6} - \frac{f^a \ln(f) b f^{bx^2}}{12x^4} - \frac{f^a (\ln(f))^2 b^2 f^{bx^2}}{12x^2} - \frac{f^a (\ln(f))^3 b^3 \text{Ei}(1, -bx^2 \ln(f))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^7,x)

[Out] -1/6\*f^a/x^6\*f^(b\*x^2)-1/12\*f^a\*ln(f)\*b/x^4\*f^(b\*x^2)-1/12\*f^a\*ln(f)^2\*b^2/x^2\*f^(b\*x^2)-1/12\*f^a\*ln(f)^3\*b^3\*Ei(1,-b\*x^2\*ln(f))

**Maxima [A]** time = 1.24856, size = 30, normalized size = 0.37

$$\frac{1}{2} b^3 f^a \Gamma(-3, -bx^2 \log(f)) \log(f)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^7,x, algorithm="maxima")

[Out] 1/2\*b^3\*f^a\*gamma(-3, -b\*x^2\*log(f))\*log(f)^3

**Fricas [A]** time = 1.85051, size = 140, normalized size = 1.73

$$\frac{b^3 f^a x^6 \text{Ei}(bx^2 \log(f)) \log(f)^3 - (b^2 x^4 \log(f)^2 + bx^2 \log(f) + 2) f^{bx^2+a}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^7,x, algorithm="fricas")

[Out] 1/12\*(b^3\*f^a\*x^6\*Ei(b\*x^2\*log(f))\*log(f)^3 - (b^2\*x^4\*log(f)^2 + b\*x^2\*log(f) + 2)\*f^(b\*x^2 + a))/x^6

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**2+a)/x**7,x)
```

```
[Out] Integral(f**(a + b*x**2)/x**7, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^7, x)
```

$$3.80 \quad \int \frac{f^{a+bx^2}}{x^9} dx$$

**Optimal.** Leaf size=24

$$-\frac{1}{2}b^4 f^a \log^4(f) \Gamma(-4, -bx^2 \log(f))$$

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^2 \text{Log}[f])]) \text{Log}[f]^4 / 2$

**Rubi [A]** time = 0.0221914, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{2}b^4 f^a \log^4(f) \Gamma(-4, -bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^9, x]

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^2 \text{Log}[f])]) \text{Log}[f]^4 / 2$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^2}}{x^9} dx = -\frac{1}{2}b^4 f^a \Gamma(-4, -bx^2 \log(f)) \log^4(f)$$

**Mathematica [A]** time = 0.002278, size = 24, normalized size = 1.

$$-\frac{1}{2}b^4 f^a \log^4(f) \Gamma(-4, -bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^9, x]

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^2 \text{Log}[f])]) \text{Log}[f]^4 / 2$

**Maple [B]** time = 0.044, size = 101, normalized size = 4.2

$$\frac{f^a f^{bx^2}}{8x^8} - \frac{f^a \ln(f) b f^{bx^2}}{24x^6} - \frac{f^a (\ln(f))^2 b^2 f^{bx^2}}{48x^4} - \frac{f^a (\ln(f))^3 b^3 f^{bx^2}}{48x^2} - \frac{f^a (\ln(f))^4 b^4 \text{Ei}(1, -bx^2 \ln(f))}{48}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(f^(b*x^2+a)/x^9,x)
```

```
[Out] -1/8*f^a/x^8*f^(b*x^2)-1/24*f^a*ln(f)*b/x^6*f^(b*x^2)-1/48*f^a*ln(f)^2*b^2/
x^4*f^(b*x^2)-1/48*f^a*ln(f)^3*b^3/x^2*f^(b*x^2)-1/48*f^a*ln(f)^4*b^4*Ei(1,
-b*x^2*ln(f))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: TypeError
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^9,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**2+a)/x**9,x)
```

```
[Out] Integral(f**(a + b*x**2)/x**9, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^9,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^9, x)
```

$$3.81 \quad \int \frac{f^{a+bx^2}}{x^{11}} dx$$

**Optimal.** Leaf size=24

$$\frac{1}{2} b^5 f^a \log^5(f) \Gamma(-5, -bx^2 \log(f))$$

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^2\*Log[f])])\*Log[f]^5/2

**Rubi [A]** time = 0.0228089, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{2} b^5 f^a \log^5(f) \Gamma(-5, -bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^11,x]

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^2\*Log[f])])\*Log[f]^5/2

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^2}}{x^{11}} dx = \frac{1}{2} b^5 f^a \Gamma(-5, -bx^2 \log(f)) \log^5(f)$$

**Mathematica [A]** time = 0.0024617, size = 24, normalized size = 1.

$$\frac{1}{2} b^5 f^a \log^5(f) \Gamma(-5, -bx^2 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^11,x]

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^2\*Log[f])])\*Log[f]^5/2

**Maple [B]** time = 0.066, size = 123, normalized size = 5.1

$$\frac{f^a f^{bx^2}}{10 x^{10}} - \frac{f^a \ln(f) b f^{bx^2}}{40 x^8} - \frac{f^a (\ln(f))^2 b^2 f^{bx^2}}{120 x^6} - \frac{f^a (\ln(f))^3 b^3 f^{bx^2}}{240 x^4} - \frac{f^a (\ln(f))^4 b^4 f^{bx^2}}{240 x^2} - \frac{f^a (\ln(f))^5 b^5 \text{Ei}(1, -bx^2 \ln(f))}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)/x^11,x)`

[Out] 
$$-1/10*f^a/x^{10}*f^{(b*x^2)}-1/40*f^a*\ln(f)*b/x^8*f^{(b*x^2)}-1/120*f^a*\ln(f)^2*b^2/x^6*f^{(b*x^2)}-1/240*f^a*\ln(f)^3*b^3/x^4*f^{(b*x^2)}-1/240*f^a*\ln(f)^4*b^4/x^2*f^{(b*x^2)}-1/240*f^a*\ln(f)^5*b^5*Ei(1,-b*x^2*\ln(f))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^11,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^11,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)/x**11,x)`

[Out] `Integral(f**(a + b*x**2)/x**11, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^11,x, algorithm="giac")`

[Out] `integrate(f^(b*x^2 + a)/x^11, x)`

### 3.82 $\int f^{a+bx^2} x^{12} dx$

**Optimal.** Leaf size=34

$$\frac{x^{13} f^a \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{13/2}}$$

[Out]  $-(f^a x^{13} \Gamma[13/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(13/2)})$

**Rubi [A]** time = 0.0225019, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{x^{13} f^a \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^12,x]

[Out]  $-(f^a x^{13} \Gamma[13/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(13/2)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x^{12} dx = -\frac{f^a x^{13} \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{13/2}}$$

**Mathematica [A]** time = 0.0056599, size = 34, normalized size = 1.

$$\frac{x^{13} f^a \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^12,x]

[Out]  $-(f^a x^{13} \Gamma[13/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(13/2)})$

**Maple [A]** time = 0.123, size = 164, normalized size = 4.8

$$\frac{f^a f^{bx^2} x^{11}}{2b \ln(f)} - \frac{11 f^a x^9 f^{bx^2}}{4 (\ln(f))^2 b^2} + \frac{99 f^a x^7 f^{bx^2}}{8 (\ln(f))^3 b^3} - \frac{693 f^a x^5 f^{bx^2}}{16 b^4 (\ln(f))^4} + \frac{3465 f^a x^3 f^{bx^2}}{32 b^5 (\ln(f))^5} - \frac{10395 f^a x f^{bx^2}}{64 (\ln(f))^6 b^6} + \frac{10395 f^a \sqrt{\pi}}{128 (\ln(f))^6 b^6} E$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^12,x)

[Out] 1/2\*f^a\*f^(b\*x^2)\*x^11/ln(f)/b-11/4\*f^a/ln(f)^2/b^2\*x^9\*f^(b\*x^2)+99/8\*f^a/ln(f)^3/b^3\*x^7\*f^(b\*x^2)-693/16\*f^a/ln(f)^4/b^4\*x^5\*f^(b\*x^2)+3465/32\*f^a/ln(f)^5/b^5\*x^3\*f^(b\*x^2)-10395/64\*f^a/ln(f)^6/b^6\*x\*f^(b\*x^2)+10395/128\*f^a/ln(f)^6/b^6\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.04814, size = 171, normalized size = 5.03

$$\frac{(32 b^5 f^a x^{11} \log(f)^5 - 176 b^4 f^a x^9 \log(f)^4 + 792 b^3 f^a x^7 \log(f)^3 - 2772 b^2 f^a x^5 \log(f)^2 + 6930 b f^a x^3 \log(f) - 10395 f^a) x^{12}}{64 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^12,x, algorithm="maxima")

[Out] 1/64\*(32\*b^5\*f^a\*x^11\*log(f)^5 - 176\*b^4\*f^a\*x^9\*log(f)^4 + 792\*b^3\*f^a\*x^7\*log(f)^3 - 2772\*b^2\*f^a\*x^5\*log(f)^2 + 6930\*b\*f^a\*x^3\*log(f) - 10395\*f^a\*x^2)/(b^6\*log(f)^6) + 10395/128\*sqrt(pi)\*f^a\*erf(sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b^6\*log(f)^6)

**Fricas [A]** time = 1.80379, size = 320, normalized size = 9.41

$$\frac{10395 \sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right) - 2 \left(32 b^6 x^{11} \log(f)^6 - 176 b^5 x^9 \log(f)^5 + 792 b^4 x^7 \log(f)^4 - 2772 b^3 x^5 \log(f)^3 + 6930 b^2 x^3 \log(f)^2 - 10395 b x \log(f)\right) f^{b x^2 + a}}{128 b^7 \log(f)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^12,x, algorithm="fricas")

[Out] -1/128\*(10395\*sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))\*x) - 2\*(32\*b^6\*x^11\*log(f)^6 - 176\*b^5\*x^9\*log(f)^5 + 792\*b^4\*x^7\*log(f)^4 - 2772\*b^3\*x^5\*log(f)^3 + 6930\*b^2\*x^3\*log(f)^2 - 10395\*b\*x\*log(f))\*f^(b\*x^2 + a))/(b^7\*log(f)^7)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*12,x)

[Out] Timed out

**Giac [A]** time = 1.20004, size = 157, normalized size = 4.62

$$\frac{10395 \sqrt{\pi} f^a \operatorname{erf}\left(-\sqrt{-b \log(f)} x\right)}{128 \sqrt{-b \log(f)} b^6 \log(f)^6} + \frac{\left(32 b^5 x^{11} \log(f)^5 - 176 b^4 x^9 \log(f)^4 + 792 b^3 x^7 \log(f)^3 - 2772 b^2 x^5 \log(f)^2 + 6930 b x^3 \log(f) - 10395 x\right) e^{(b x^2 \log(f) + a \log(f))}}{64 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^12,x, algorithm="giac")

[Out] -10395/128\*sqrt(pi)\*f^a\*erf(-sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b^6\*log(f)^6) + 1/64\*(32\*b^5\*x^11\*log(f)^5 - 176\*b^4\*x^9\*log(f)^4 + 792\*b^3\*x^7\*log(f)^3 - 2772\*b^2\*x^5\*log(f)^2 + 6930\*b\*x^3\*log(f) - 10395\*x)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^6\*log(f)^6)

### 3.83 $\int f^{a+bx^2} x^{10} dx$

**Optimal.** Leaf size=34

$$\frac{x^{11} f^a \Gamma\left(\frac{11}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{11/2}}$$

[Out]  $-(f^a x^{11} \Gamma[11/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(11/2)})$

**Rubi [A]** time = 0.0225988, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{x^{11} f^a \Gamma\left(\frac{11}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^10,x]

[Out]  $-(f^a x^{11} \Gamma[11/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(11/2)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^2} x^{10} dx = -\frac{f^a x^{11} \Gamma\left(\frac{11}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{11/2}}$$

**Mathematica [A]** time = 0.0057456, size = 34, normalized size = 1.

$$\frac{x^{11} f^a \Gamma\left(\frac{11}{2}, -bx^2 \log(f)\right)}{2(-bx^2 \log(f))^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^10,x]

[Out]  $-(f^a x^{11} \Gamma[11/2, -(b x^2 \text{Log}[f])]) / (2 * (-(b x^2 \text{Log}[f]))^{(11/2)})$

**Maple [A]** time = 0.054, size = 142, normalized size = 4.2

$$\frac{f^a x^9 f^{bx^2}}{2b \ln(f)} - \frac{9 f^a x^7 f^{bx^2}}{4 (\ln(f))^2 b^2} + \frac{63 f^a x^5 f^{bx^2}}{8 (\ln(f))^3 b^3} - \frac{315 f^a x^3 f^{bx^2}}{16 b^4 (\ln(f))^4} + \frac{945 f^a x f^{bx^2}}{32 b^5 (\ln(f))^5} - \frac{945 f^a \sqrt{\pi}}{64 b^5 (\ln(f))^5} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^10,x)

[Out] 1/2\*f^a/ln(f)/b\*x^9\*f^(b\*x^2)-9/4\*f^a/ln(f)^2/b^2\*x^7\*f^(b\*x^2)+63/8\*f^a/ln(f)^3/b^3\*x^5\*f^(b\*x^2)-315/16\*f^a/ln(f)^4/b^4\*x^3\*f^(b\*x^2)+945/32\*f^a/ln(f)^5/b^5\*x\*f^(b\*x^2)-945/64\*f^a/ln(f)^5/b^5\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.0775, size = 151, normalized size = 4.44

$$\frac{(16 b^4 f^a x^9 \log(f)^4 - 72 b^3 f^a x^7 \log(f)^3 + 252 b^2 f^a x^5 \log(f)^2 - 630 b f^a x^3 \log(f) + 945 f^a x) f^{bx^2}}{32 b^5 \log(f)^5} - \frac{945 \sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right)}{64 \sqrt{-b \log(f)} b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^10,x, algorithm="maxima")

[Out] 1/32\*(16\*b^4\*f^a\*x^9\*log(f)^4 - 72\*b^3\*f^a\*x^7\*log(f)^3 + 252\*b^2\*f^a\*x^5\*log(f)^2 - 630\*b\*f^a\*x^3\*log(f) + 945\*f^a\*x)\*f^(b\*x^2)/(b^5\*log(f)^5) - 945/64\*sqrt(pi)\*f^a\*erf(sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b^5\*log(f)^5)

**Fricas [A]** time = 1.73718, size = 275, normalized size = 8.09

$$\frac{945 \sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right) + 2 \left(16 b^5 x^9 \log(f)^5 - 72 b^4 x^7 \log(f)^4 + 252 b^3 x^5 \log(f)^3 - 630 b^2 x^3 \log(f)^2 + 945 b x \log(f)\right) f^{bx^2 + a}}{64 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^10,x, algorithm="fricas")

[Out] 1/64\*(945\*sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))\*x) + 2\*(16\*b^5\*x^9\*log(f)^5 - 72\*b^4\*x^7\*log(f)^4 + 252\*b^3\*x^5\*log(f)^3 - 630\*b^2\*x^3\*log(f)^2 + 945\*b\*x\*log(f))\*f^(b\*x^2 + a))/(b^6\*log(f)^6)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} x^{10} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*10,x)



[Out] Integral(f\*\*(a + b\*x\*\*2)\*x\*\*10, x)

---

**Giac [A]** time = 1.24632, size = 140, normalized size = 4.12

$$\frac{945 \sqrt{\pi} f^a \operatorname{erf}\left(-\sqrt{-b \log(f)} x\right)}{64 \sqrt{-b \log(f)} b^5 \log(f)^5} + \frac{\left(16 b^4 x^9 \log(f)^4 - 72 b^3 x^7 \log(f)^3 + 252 b^2 x^5 \log(f)^2 - 630 b x^3 \log(f) + 945 x\right) e^{(b x^2 \log(f) + a \log(f))}}{32 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^10,x, algorithm="giac")

[Out] 945/64\*sqrt(pi)\*f^a\*erf(-sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b^5\*log(f)^5) + 1/32\*(16\*b^4\*x^9\*log(f)^4 - 72\*b^3\*x^7\*log(f)^3 + 252\*b^2\*x^5\*log(f)^2 - 630\*b\*x^3\*log(f) + 945\*x)\*e^(b\*x^2\*log(f) + a\*log(f))/(b^5\*log(f)^5)

### 3.84 $\int f^{a+bx^2} x^8 dx$

**Optimal.** Leaf size=128

$$\frac{105\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)}{32b^{9/2}\log^2(f)} - \frac{7x^5f^{a+bx^2}}{4b^2\log^2(f)} + \frac{35x^3f^{a+bx^2}}{8b^3\log^3(f)} - \frac{105xf^{a+bx^2}}{16b^4\log^4(f)} + \frac{x^7f^{a+bx^2}}{2b\log(f)}$$

[Out] (105\*f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]])/(32\*b^(9/2)\*Log[f]^(9/2)) - (105\*f^(a + b\*x^2)\*x)/(16\*b^4\*Log[f]^4) + (35\*f^(a + b\*x^2)\*x^3)/(8\*b^3\*Log[f]^3) - (7\*f^(a + b\*x^2)\*x^5)/(4\*b^2\*Log[f]^2) + (f^(a + b\*x^2)\*x^7)/(2\*b\*Log[f])

**Rubi [A]** time = 0.142399, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2204}

$$\frac{105\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)}{32b^{9/2}\log^2(f)} - \frac{7x^5f^{a+bx^2}}{4b^2\log^2(f)} + \frac{35x^3f^{a+bx^2}}{8b^3\log^3(f)} - \frac{105xf^{a+bx^2}}{16b^4\log^4(f)} + \frac{x^7f^{a+bx^2}}{2b\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^8,x]

[Out] (105\*f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]])/(32\*b^(9/2)\*Log[f]^(9/2)) - (105\*f^(a + b\*x^2)\*x)/(16\*b^4\*Log[f]^4) + (35\*f^(a + b\*x^2)\*x^3)/(8\*b^3\*Log[f]^3) - (7\*f^(a + b\*x^2)\*x^5)/(4\*b^2\*Log[f]^2) + (f^(a + b\*x^2)\*x^7)/(2\*b\*Log[f])

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int f^{a+bx^2} x^8 dx &= \frac{f^{a+bx^2} x^7}{2b \log(f)} - \frac{7 \int f^{a+bx^2} x^6 dx}{2b \log(f)} \\
&= -\frac{7 f^{a+bx^2} x^5}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^7}{2b \log(f)} + \frac{35 \int f^{a+bx^2} x^4 dx}{4b^2 \log^2(f)} \\
&= \frac{35 f^{a+bx^2} x^3}{8b^3 \log^3(f)} - \frac{7 f^{a+bx^2} x^5}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^7}{2b \log(f)} - \frac{105 \int f^{a+bx^2} x^2 dx}{8b^3 \log^3(f)} \\
&= -\frac{105 f^{a+bx^2} x}{16b^4 \log^4(f)} + \frac{35 f^{a+bx^2} x^3}{8b^3 \log^3(f)} - \frac{7 f^{a+bx^2} x^5}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^7}{2b \log(f)} + \frac{105 \int f^{a+bx^2} dx}{16b^4 \log^4(f)} \\
&= \frac{105 f^a \sqrt{\pi} \operatorname{erfi}(\sqrt{bx} \sqrt{\log(f)})}{32b^{9/2} \log^{\frac{9}{2}}(f)} - \frac{105 f^{a+bx^2} x}{16b^4 \log^4(f)} + \frac{35 f^{a+bx^2} x^3}{8b^3 \log^3(f)} - \frac{7 f^{a+bx^2} x^5}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^7}{2b \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0421417, size = 95, normalized size = 0.74

$$\frac{f^a \left( 2\sqrt{bx} \sqrt{\log(f)} f^{bx^2} \left( 8b^3 x^6 \log^3(f) - 28b^2 x^4 \log^2(f) + 70bx^2 \log(f) - 105 \right) + 105\sqrt{\pi} \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)}) \right)}{32b^{9/2} \log^{\frac{9}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^8,x]

[Out] (f^a\*(105\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]] + 2\*Sqrt[b]\*f^(b\*x^2)\*x\*Sqrt[Log[f]]\*(-105 + 70\*b\*x^2\*Log[f] - 28\*b^2\*x^4\*Log[f]^2 + 8\*b^3\*x^6\*Log[f]^3)))/(32\*b^(9/2)\*Log[f]^(9/2))

**Maple [A]** time = 0.039, size = 120, normalized size = 0.9

$$\frac{f^a x^7 f^{bx^2}}{2b \ln(f)} - \frac{7 f^a x^5 f^{bx^2}}{4 (\ln(f))^2 b^2} + \frac{35 f^a x^3 f^{bx^2}}{8 (\ln(f))^3 b^3} - \frac{105 f^a x f^{bx^2}}{16 b^4 (\ln(f))^4} + \frac{105 f^a \sqrt{\pi}}{32 b^4 (\ln(f))^4} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^8,x)

[Out] 1/2\*f^a/ln(f)/b\*x^7\*f^(b\*x^2)-7/4\*f^a/ln(f)^2/b^2\*x^5\*f^(b\*x^2)+35/8\*f^a/ln(f)^3/b^3\*x^3\*f^(b\*x^2)-105/16\*f^a/ln(f)^4/b^4\*x\*f^(b\*x^2)+105/32\*f^a/ln(f)^4/b^4\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.07931, size = 131, normalized size = 1.02

$$\frac{\left( 8 b^3 f^a x^7 \log(f)^3 - 28 b^2 f^a x^5 \log(f)^2 + 70 b f^a x^3 \log(f) - 105 f^a x \right) f^{bx^2}}{16 b^4 \log(f)^4} + \frac{105 \sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right)}{32 \sqrt{-b \log(f)} b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^8,x, algorithm="maxima")

[Out]  $\frac{1}{16}(8b^3f^ax^7\log(f)^3 - 28b^2f^ax^5\log(f)^2 + 70bf^ax^3\log(f) - 105f^ax)f^{bx^2}/(b^4\log(f)^4) + 105/32\sqrt{\pi}f^a\operatorname{erf}(\sqrt{-b\log(f)}x)/(\sqrt{-b\log(f)}b^4\log(f)^4)$

**Fricas [A]** time = 1.75507, size = 243, normalized size = 1.9

$$\frac{105\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right) - 2\left(8b^4x^7\log(f)^4 - 28b^3x^5\log(f)^3 + 70b^2x^3\log(f)^2 - 105bx\log(f)\right)f^b}{32b^5\log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^8,x, algorithm="fricas")`

[Out]  $-1/32(105\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}(\sqrt{-b\log(f)}x) - 2(8b^4x^7\log(f)^4 - 28b^3x^5\log(f)^3 + 70b^2x^3\log(f)^2 - 105bx\log(f))f^{bx^2+a})/(b^5\log(f)^5)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2}x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)*x**8,x)`

[Out] `Integral(f**(a + b*x**2)*x**8, x)`

**Giac [A]** time = 1.2209, size = 124, normalized size = 0.97

$$-\frac{105\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-b\log(f)}x\right)}{32\sqrt{-b\log(f)}b^4\log(f)^4} + \frac{\left(8b^3x^7\log(f)^3 - 28b^2x^5\log(f)^2 + 70bx^3\log(f) - 105x\right)e^{(bx^2\log(f)+a\log(f))}}{16b^4\log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)*x^8,x, algorithm="giac")`

[Out]  $-105/32\sqrt{\pi}f^a\operatorname{erf}(-\sqrt{-b\log(f)}x)/(\sqrt{-b\log(f)}b^4\log(f)^4) + 1/16(8b^3x^7\log(f)^3 - 28b^2x^5\log(f)^2 + 70bx^3\log(f) - 105x)e^{(bx^2\log(f) + a\log(f))}/(b^4\log(f)^4)$

### 3.85 $\int f^{a+bx^2} x^6 dx$

**Optimal.** Leaf size=105

$$-\frac{15\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)}{16b^{7/2}\log^2(f)} - \frac{5x^3f^{a+bx^2}}{4b^2\log^2(f)} + \frac{15xf^{a+bx^2}}{8b^3\log^3(f)} + \frac{x^5f^{a+bx^2}}{2b\log(f)}$$

[Out]  $(-15*f^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*x*\sqrt{\log[f]}])/(16*b^{(7/2)}*\log[f]^{(7/2)}) + (15*f^{(a + b*x^2)}*x)/(8*b^3*\log[f]^3) - (5*f^{(a + b*x^2)}*x^3)/(4*b^2*\log[f]^2) + (f^{(a + b*x^2)}*x^5)/(2*b*\log[f])$

**Rubi [A]** time = 0.0875476, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2204}

$$-\frac{15\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)}{16b^{7/2}\log^2(f)} - \frac{5x^3f^{a+bx^2}}{4b^2\log^2(f)} + \frac{15xf^{a+bx^2}}{8b^3\log^3(f)} + \frac{x^5f^{a+bx^2}}{2b\log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x^2)}*x^6, x]$

[Out]  $(-15*f^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*x*\sqrt{\log[f]}])/(16*b^{(7/2)}*\log[f]^{(7/2)}) + (15*f^{(a + b*x^2)}*x)/(8*b^3*\log[f]^3) - (5*f^{(a + b*x^2)}*x^3)/(4*b^2*\log[f]^2) + (f^{(a + b*x^2)}*x^5)/(2*b*\log[f])$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m - n + 1)}*F^{(a + b*(c + d*x)^n)}/(b*d*n*\log[F]), x] - \operatorname{Dist}[(m - n + 1)/(b*n*\log[F]), \operatorname{Int}[(c + d*x)^{(m - n)}*F^{(a + b*(c + d*x)^n)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{IntegerQ}[(2*(m + 1))/n] \&\& \operatorname{LtQ}[0, (m + 1)/n, 5] \&\& \operatorname{IntegerQ}[n] \&\& (\operatorname{LtQ}[0, n, m + 1] \mid \mid \operatorname{LtQ}[m, n, 0])$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\sqrt{\pi}*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\log[F], 2]])/(2*d*\operatorname{Rt}[b*\log[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^6 dx &= \frac{f^{a+bx^2} x^5}{2b\log(f)} - \frac{5 \int f^{a+bx^2} x^4 dx}{2b\log(f)} \\ &= -\frac{5f^{a+bx^2} x^3}{4b^2\log^2(f)} + \frac{f^{a+bx^2} x^5}{2b\log(f)} + \frac{15 \int f^{a+bx^2} x^2 dx}{4b^2\log^2(f)} \\ &= \frac{15f^{a+bx^2} x}{8b^3\log^3(f)} - \frac{5f^{a+bx^2} x^3}{4b^2\log^2(f)} + \frac{f^{a+bx^2} x^5}{2b\log(f)} - \frac{15 \int f^{a+bx^2} dx}{8b^3\log^3(f)} \\ &= -\frac{15f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)}{16b^{7/2}\log^2(f)} + \frac{15f^{a+bx^2} x}{8b^3\log^3(f)} - \frac{5f^{a+bx^2} x^3}{4b^2\log^2(f)} + \frac{f^{a+bx^2} x^5}{2b\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0347835, size = 83, normalized size = 0.79

$$\frac{f^a \left( 2\sqrt{bx}\sqrt{\log(f)} f^{bx^2} \left( 4b^2x^4 \log^2(f) - 10bx^2 \log(f) + 15 \right) - 15\sqrt{\pi} \operatorname{Erfi} \left( \sqrt{bx}\sqrt{\log(f)} \right) \right)}{16b^{7/2} \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^6,x]

[Out] (f^a\*(-15\*sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]] + 2\*sqrt[b]\*f^(b\*x^2)\*x\*sqrt[Log[f]]\*(15 - 10\*b\*x^2\*Log[f] + 4\*b^2\*x^4\*Log[f]^2)))/(16\*b^(7/2)\*Log[f]^(7/2))

**Maple [A]** time = 0.029, size = 98, normalized size = 0.9

$$\frac{f^a x^5 f^{bx^2}}{2b \ln(f)} - \frac{5 f^a x^3 f^{bx^2}}{4 (\ln(f))^2 b^2} + \frac{15 f^a x f^{bx^2}}{8 (\ln(f))^3 b^3} - \frac{15 f^a \sqrt{\pi}}{16 (\ln(f))^3 b^3} \operatorname{Erf} \left( \sqrt{-b \ln(f)} x \right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^6,x)

[Out] 1/2\*f^a/ln(f)/b\*x^5\*f^(b\*x^2)-5/4\*f^a/ln(f)^2/b^2\*x^3\*f^(b\*x^2)+15/8\*f^a/ln(f)^3/b^3\*x\*f^(b\*x^2)-15/16\*f^a/ln(f)^3/b^3\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.03444, size = 111, normalized size = 1.06

$$\frac{\left( 4b^2 f^a x^5 \log(f)^2 - 10b f^a x^3 \log(f) + 15 f^a x \right) f^{bx^2}}{8b^3 \log(f)^3} - \frac{15 \sqrt{\pi} f^a \operatorname{erf} \left( \sqrt{-b \log(f)} x \right)}{16 \sqrt{-b \log(f)} b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^6,x, algorithm="maxima")

[Out] 1/8\*(4\*b^2\*f^a\*x^5\*log(f)^2 - 10\*b\*f^a\*x^3\*log(f) + 15\*f^a\*x)\*f^(b\*x^2)/(b^3\*log(f)^3) - 15/16\*sqrt(pi)\*f^a\*erf(sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b^3\*log(f)^3)

**Fricas [A]** time = 1.75294, size = 209, normalized size = 1.99

$$\frac{15 \sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf} \left( \sqrt{-b \log(f)} x \right) + 2 \left( 4b^3 x^5 \log(f)^3 - 10b^2 x^3 \log(f)^2 + 15bx \log(f) \right) f^{bx^2+a}}{16b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^6,x, algorithm="fricas")

```
[Out] 1/16*(15*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) + 2*(4*b^3*x^5
*log(f)^3 - 10*b^2*x^3*log(f)^2 + 15*b*x*log(f))*f^(b*x^2 + a))/(b^4*log(f)
^4)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**2+a)*x**6,x)
```

```
[Out] Integral(f**(a + b*x**2)*x**6, x)
```

**Giac [A]** time = 1.24671, size = 108, normalized size = 1.03

$$\frac{15 \sqrt{\pi} f^a \operatorname{erf}\left(-\sqrt{-b \log(f)} x\right)}{16 \sqrt{-b \log(f)} b^3 \log(f)^3} + \frac{\left(4 b^2 x^5 \log(f)^2 - 10 b x^3 \log(f) + 15 x\right) e^{(b x^2 \log(f) + a \log(f))}}{8 b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)*x^6,x, algorithm="giac")
```

```
[Out] 15/16*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^3*log(f)^3) +
1/8*(4*b^2*x^5*log(f)^2 - 10*b*x^3*log(f) + 15*x)*e^(b*x^2*log(f) + a*log(
f))/(b^3*log(f)^3)
```

### 3.86 $\int f^{a+bx^2} x^4 dx$

**Optimal.** Leaf size=82

$$\frac{3\sqrt{\pi}f^a \operatorname{Erfi}(\sqrt{bx}\sqrt{\log(f)})}{8b^{5/2} \log^{\frac{5}{2}}(f)} - \frac{3xf^{a+bx^2}}{4b^2 \log^2(f)} + \frac{x^3 f^{a+bx^2}}{2b \log(f)}$$

[Out]  $(3*f^a*\sqrt{\text{Pi}}*\operatorname{Erfi}[\sqrt{b}*x*\sqrt{\text{Log}[f]}])/(8*b^{(5/2)}*\text{Log}[f]^{(5/2)}) - (3*f^{(a + b*x^2)}*x)/(4*b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^3)/(2*b*\text{Log}[f])$

**Rubi [A]** time = 0.0593823, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2204}

$$\frac{3\sqrt{\pi}f^a \operatorname{Erfi}(\sqrt{bx}\sqrt{\log(f)})}{8b^{5/2} \log^{\frac{5}{2}}(f)} - \frac{3xf^{a+bx^2}}{4b^2 \log^2(f)} + \frac{x^3 f^{a+bx^2}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^4, x]

[Out]  $(3*f^a*\sqrt{\text{Pi}}*\operatorname{Erfi}[\sqrt{b}*x*\sqrt{\text{Log}[f]}])/(8*b^{(5/2)}*\text{Log}[f]^{(5/2)}) - (3*f^{(a + b*x^2)}*x)/(4*b^2*\text{Log}[f]^2) + (f^{(a + b*x^2)}*x^3)/(2*b*\text{Log}[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^4 dx &= \frac{f^{a+bx^2} x^3}{2b \log(f)} - \frac{3 \int f^{a+bx^2} x^2 dx}{2b \log(f)} \\ &= -\frac{3f^{a+bx^2} x}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^3}{2b \log(f)} + \frac{3 \int f^{a+bx^2} dx}{4b^2 \log^2(f)} \\ &= \frac{3f^a \sqrt{\pi} \operatorname{erfi}(\sqrt{bx}\sqrt{\log(f)})}{8b^{5/2} \log^{\frac{5}{2}}(f)} - \frac{3f^{a+bx^2} x}{4b^2 \log^2(f)} + \frac{f^{a+bx^2} x^3}{2b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0269446, size = 71, normalized size = 0.87

$$\frac{f^a \left( 3\sqrt{\pi} \operatorname{Erfi}(\sqrt{bx}\sqrt{\log(f)}) + 2\sqrt{bx}\sqrt{\log(f)} f^{bx^2} (2bx^2 \log(f) - 3) \right)}{8b^{5/2} \log^{\frac{5}{2}}(f)}$$



Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^4,x]

[Out]  $(f^a(3\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}x\sqrt{\log[f]}] + 2\sqrt{b}f^{(b*x^2)}x\sqrt{\log[f]}(-3 + 2*b*x^2*\log[f])))/(8*b^{(5/2)}*\log[f]^{(5/2)})$

**Maple [A]** time = 0.023, size = 76, normalized size = 0.9

$$\frac{f^a x^3 f^{bx^2}}{2b \ln(f)} - \frac{3 f^a x f^{bx^2}}{4 (\ln(f))^2 b^2} + \frac{3 f^a \sqrt{\pi}}{8 (\ln(f))^2 b^2} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^4,x)

[Out]  $1/2*f^a/\ln(f)/b*x^3*f^{(b*x^2)}-3/4*f^a/\ln(f)^2/b^2*x*f^{(b*x^2)}+3/8*f^a/\ln(f)^2/b^2*\pi^{(1/2)/(-b*\ln(f))^{(1/2)}*erf((-b*\ln(f))^{(1/2)}*x)$

**Maxima [A]** time = 1.14074, size = 90, normalized size = 1.1

$$\frac{(2bf^ax^3\log(f) - 3f^ax)f^{bx^2}}{4b^2\log(f)^2} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)}{8\sqrt{-b\log(f)}b^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^4,x, algorithm="maxima")

[Out]  $1/4*(2*b*f^a*x^3*\log(f) - 3*f^a*x)*f^{(b*x^2)}/(b^2*\log(f)^2) + 3/8*\sqrt{\pi}*f^a*\operatorname{erf}(\sqrt{-b*\log(f)}*x)/(\sqrt{-b*\log(f)}*b^2*\log(f)^2)$

**Fricas [A]** time = 1.71272, size = 177, normalized size = 2.16

$$\frac{3\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right) - 2\left(2b^2x^3\log(f)^2 - 3bx\log(f)\right)f^{bx^2+a}}{8b^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^4,x, algorithm="fricas")

[Out]  $-1/8*(3*\sqrt{\pi}*\sqrt{-b*\log(f)}*f^a*\operatorname{erf}(\sqrt{-b*\log(f)}*x) - 2*(2*b^2*x^3*\log(f)^2 - 3*b*x*\log(f))*f^{(b*x^2 + a)})/(b^3*\log(f)^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*4,x)

[Out] Integral(f\*\*(a + b\*x\*\*2)\*x\*\*4, x)

**Giac [A]** time = 1.33054, size = 92, normalized size = 1.12

$$-\frac{3\sqrt{\pi}f^a \operatorname{erf}\left(-\sqrt{-b \log(f)}x\right)}{8\sqrt{-b \log(f)}b^2 \log(f)^2} + \frac{(2bx^3 \log(f) - 3x)e^{(bx^2 \log(f) + a \log(f))}}{4b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^4,x, algorithm="giac")

[Out]  $-\frac{3}{8}\sqrt{\pi}f^a \operatorname{erf}(-\sqrt{-b \log(f)}x)/(\sqrt{-b \log(f)}b^2 \log(f)^2) + \frac{1}{4}(2bx^3 \log(f) - 3x)e^{(bx^2 \log(f) + a \log(f))}/(b^2 \log(f)^2)$

### 3.87 $\int f^{a+bx^2} x^2 dx$

**Optimal.** Leaf size=59

$$\frac{x f^{a+bx^2}}{2b \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{4b^{3/2} \log^{\frac{3}{2}}(f)}$$

[Out]  $-(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x \sqrt{\log[f]}]) / (4 b^{(3/2)} \log[f]^{(3/2)}) + (f^{(a + b x^2)} x) / (2 b \log[f])$

**Rubi [A]** time = 0.0344495, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2204}

$$\frac{x f^{a+bx^2}}{2b \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{4b^{3/2} \log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)\*x^2,x]

[Out]  $-(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x \sqrt{\log[f]}]) / (4 b^{(3/2)} \log[f]^{(3/2)}) + (f^{(a + b x^2)} x) / (2 b \log[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^2} x^2 dx &= \frac{f^{a+bx^2} x}{2b \log(f)} - \frac{\int f^{a+bx^2} dx}{2b \log(f)} \\ &= -\frac{f^a \sqrt{\pi} \operatorname{erfi}(\sqrt{bx} \sqrt{\log(f)})}{4b^{3/2} \log^{\frac{3}{2}}(f)} + \frac{f^{a+bx^2} x}{2b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0225355, size = 59, normalized size = 1.

$$\frac{x f^{a+bx^2}}{2b \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{4b^{3/2} \log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)\*x^2,x]

[Out]  $-(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x \sqrt{\log[f]}]) / (4 b^{3/2} \log[f]^{3/2}) + (f^{a+b x^2} x) / (2 b \log[f])$

**Maple [A]** time = 0.022, size = 54, normalized size = 0.9

$$\frac{f^a x f^{bx^2}}{2 b \ln(f)} - \frac{f^a \sqrt{\pi}}{4 b \ln(f)} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)\*x^2,x)

[Out]  $1/2 * f^a / \ln(f) / b * x * f^{(b*x^2)} - 1/4 * f^a / \ln(f) / b * \pi^{1/2} / (-b * \ln(f))^{1/2} * \operatorname{erf}((-b * \ln(f))^{1/2} * x)$

**Maxima [A]** time = 1.0562, size = 72, normalized size = 1.22

$$\frac{f^{bx^2} f^a x}{2 b \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right)}{4 \sqrt{-b \log(f)} b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^2,x, algorithm="maxima")

[Out]  $1/2 * f^{(b*x^2)} * f^a * x / (b * \log(f)) - 1/4 * \sqrt{\pi} * f^a * \operatorname{erf}(\sqrt{-b * \log(f)} * x) / (\sqrt{-b * \log(f)} * b * \log(f))$

**Fricas [A]** time = 1.83004, size = 139, normalized size = 2.36

$$\frac{2 b f^{bx^2+a} x \log(f) + \sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x\right)}{4 b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^2,x, algorithm="fricas")

[Out]  $1/4 * (2 * b * f^{(b*x^2 + a)} * x * \log(f) + \sqrt{\pi} * \sqrt{-b * \log(f)} * f^a * \operatorname{erf}(\sqrt{-b * \log(f)} * x)) / (b^2 * \log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)\*x\*\*2,x)

[Out] Integral(f\*\*(a + b\*x\*\*2)\*x\*\*2, x)

---

**Giac [A]** time = 1.25691, size = 77, normalized size = 1.31

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(-\sqrt{-b \log(f)} x\right)}{4 \sqrt{-b \log(f)} b \log(f)} + \frac{x e^{(b x^2 \log(f) + a \log(f))}}{2 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)\*x^2,x, algorithm="giac")

[Out] 1/4\*sqrt(pi)\*f^a\*erf(-sqrt(-b\*log(f))\*x)/(sqrt(-b\*log(f))\*b\*log(f)) + 1/2\*x\*e^(b\*x^2\*log(f) + a\*log(f))/(b\*log(f))

### 3.88 $\int f^{a+bx^2} dx$

**Optimal.** Leaf size=37

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{2\sqrt{b} \sqrt{\log(f)}}$$

[Out] (f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]])/(2\*Sqrt[b]\*Sqrt[Log[f]])

**Rubi [A]** time = 0.006949, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2204}

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{2\sqrt{b} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2), x]

[Out] (f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]])/(2\*Sqrt[b]\*Sqrt[Log[f]])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)) ^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\int f^{a+bx^2} dx = \frac{f^a \sqrt{\pi} \operatorname{erfi}(\sqrt{bx} \sqrt{\log(f)})}{2\sqrt{b} \sqrt{\log(f)}}$$

**Mathematica [A]** time = 0.0045511, size = 37, normalized size = 1.

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)})}{2\sqrt{b} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2), x]

[Out] (f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]])/(2\*Sqrt[b]\*Sqrt[Log[f]])

**Maple [A]** time = 0.016, size = 26, normalized size = 0.7

$$\frac{f^a \sqrt{\pi}}{2} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a),x)`

[Out]  $1/2*f^a*\pi^{(1/2)/(-b*\ln(f))^{(1/2)}*erf((-b*\ln(f))^{(1/2)*x)}$

**Maxima [A]** time = 1.06032, size = 34, normalized size = 0.92

$$\frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-b \log(f)}x\right)}{2\sqrt{-b \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a),x, algorithm="maxima")`

[Out]  $1/2*\sqrt{\pi}*f^a*\operatorname{erf}(\sqrt{-b*\log(f)}*x)/\sqrt{-b*\log(f)}$

**Fricas [A]** time = 1.78363, size = 93, normalized size = 2.51

$$-\frac{\sqrt{\pi}\sqrt{-b \log(f)}f^a \operatorname{erf}\left(\sqrt{-b \log(f)}x\right)}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a),x, algorithm="fricas")`

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*\log(f)}*f^a*\operatorname{erf}(\sqrt{-b*\log(f)}*x)/(b*\log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a),x)`

[Out] `Integral(f**(a + b*x**2), x)`

**Giac [A]** time = 1.22289, size = 35, normalized size = 0.95

$$-\frac{\sqrt{\pi}f^a \operatorname{erf}\left(-\sqrt{-b \log(f)}x\right)}{2\sqrt{-b \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/sqrt(-b*log(f))
```



$$3.89 \quad \int \frac{f^{a+bx^2}}{x^2} dx$$

**Optimal.** Leaf size=49

$$\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{x}$$

[Out]  $-(f^{(a + b*x^2)}/x) + \operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x*\operatorname{Sqrt}[\operatorname{Log}[f]]]*\operatorname{Sqrt}[\operatorname{Log}[f]]$

**Rubi [A]** time = 0.0314357, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2204}

$$\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x^2)}/x^2, x]$

[Out]  $-(f^{(a + b*x^2)}/x) + \operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x*\operatorname{Sqrt}[\operatorname{Log}[f]]]*\operatorname{Sqrt}[\operatorname{Log}[f]]$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}*F^{(a + b*(c + d*x)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c + d*x)^{m+n}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^2} dx &= -\frac{f^{a+bx^2}}{x} + (2b \log(f)) \int f^{a+bx^2} dx \\ &= -\frac{f^{a+bx^2}}{x} + \sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right)\sqrt{\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0142022, size = 49, normalized size = 1.

$$\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^2,x]

[Out]  $-(f^{a + b*x^2}/x) + \text{Sqrt}[b]*f^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*x*\text{Sqrt}[\text{Log}[f]]]*\text{Sqrt}[\text{Log}[f]]$

**Maple [A]** time = 0.022, size = 44, normalized size = 0.9

$$-\frac{f^a f^{bx^2}}{x} + f^a \ln(f) b \sqrt{\pi} \text{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^2,x)

[Out]  $-f^a/x*f^{(b*x^2)}+f^a*\ln(f)*b*\text{Pi}^{(1/2)/(-b*\ln(f))^{(1/2)}}*\text{erf}((-b*\ln(f))^{(1/2)}*x)$

**Maxima [A]** time = 1.23406, size = 38, normalized size = 0.78

$$-\frac{\sqrt{-bx^2 \log(f)} f^a \Gamma\left(-\frac{1}{2}, -bx^2 \log(f)\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^2,x, algorithm="maxima")

[Out]  $-1/2*\text{sqrt}(-b*x^2*\log(f))*f^a*\text{gamma}(-1/2, -b*x^2*\log(f))/x$

**Fricas [A]** time = 1.78717, size = 103, normalized size = 2.1

$$\frac{\sqrt{\pi} \sqrt{-b \log(f)} f^a x \text{erf}\left(\sqrt{-b \log(f)} x\right) + f^{bx^2+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^2,x, algorithm="fricas")

[Out]  $-(\text{sqrt}(\text{pi})*\text{sqrt}(-b*\log(f))*f^a*x*\text{erf}(\text{sqrt}(-b*\log(f))*x) + f^{(b*x^2 + a)})/x$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)/x\*\*2,x)

[Out] Integral(f\*\*(a + b\*x\*\*2)/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^2,x, algorithm="giac")

[Out] integrate(f^(b\*x^2 + a)/x^2, x)

### 3.90 $\int \frac{f^{a+bx^2}}{x^4} dx$

**Optimal.** Leaf size=73

$$\frac{2}{3}\sqrt{\pi}b^{3/2}f^a \log^{\frac{3}{2}}(f)\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{3x^3} - \frac{2b \log(f)f^{a+bx^2}}{3x}$$

[Out]  $-f^{(a + b*x^2)}/(3*x^3) - (2*b*f^{(a + b*x^2)}*Log[f])/(3*x) + (2*b^{(3/2)}*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x*\operatorname{Sqrt}[\operatorname{Log}[f]]]*\operatorname{Log}[f]^{(3/2)})/3$

**Rubi [A]** time = 0.0548927, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2204}

$$\frac{2}{3}\sqrt{\pi}b^{3/2}f^a \log^{\frac{3}{2}}(f)\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{3x^3} - \frac{2b \log(f)f^{a+bx^2}}{3x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x^2)}/x^4, x]$

[Out]  $-f^{(a + b*x^2)}/(3*x^3) - (2*b*f^{(a + b*x^2)}*Log[f])/(3*x) + (2*b^{(3/2)}*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x*\operatorname{Sqrt}[\operatorname{Log}[f]]]*\operatorname{Log}[f]^{(3/2)})/3$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \operatorname{IntegerQ}[(2*(m + 1))/n] \ \&\& \operatorname{LtQ}[-4, (m + 1)/n, 5] \ \&\& \operatorname{IntegerQ}[n] \ \&\& ((\operatorname{GtQ}[n, 0] \ \&\& \operatorname{LtQ}[m, -1]) \ \|\ (\operatorname{GtQ}[-n, 0] \ \&\& \operatorname{LeQ}[-n, m + 1]))$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^4} dx &= -\frac{f^{a+bx^2}}{3x^3} + \frac{1}{3}(2b \log(f)) \int \frac{f^{a+bx^2}}{x^2} dx \\ &= -\frac{f^{a+bx^2}}{3x^3} - \frac{2bf^{a+bx^2} \log(f)}{3x} + \frac{1}{3}(4b^2 \log^2(f)) \int f^{a+bx^2} dx \\ &= -\frac{f^{a+bx^2}}{3x^3} - \frac{2bf^{a+bx^2} \log(f)}{3x} + \frac{2}{3}b^{3/2}f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) \log^{\frac{3}{2}}(f) \end{aligned}$$

**Mathematica [A]** time = 0.0408832, size = 62, normalized size = 0.85

$$\frac{1}{3}f^a \left( 2\sqrt{\pi}b^{3/2} \log^{\frac{3}{2}}(f)\operatorname{Erfi}\left(\sqrt{bx}\sqrt{\log(f)}\right) - \frac{f^{bx^2}(2bx^2 \log(f) + 1)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^4,x]

[Out]  $(f^a*(2*b^{(3/2)}*Sqrt[\pi]*Erfi[Sqrt[b]*x*Sqrt[\log[f]]])*Log[f]^{(3/2)} - (f^{(b*x^2)}*(1 + 2*b*x^2*Log[f]))/x^3)/3$

**Maple [A]** time = 0.027, size = 67, normalized size = 0.9

$$-\frac{f^a f^{bx^2}}{3x^3} - \frac{2 f^a \ln(f) b f^{bx^2}}{3x} + \frac{2 f^a (\ln(f))^2 b^2 \sqrt{\pi}}{3} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^4,x)

[Out]  $-1/3*f^a/x^3*f^{(b*x^2)}-2/3*f^a*\ln(f)*b/x*f^{(b*x^2)}+2/3*f^a*\ln(f)^2*b^2*\pi^{(1/2)}/(-b*\ln(f))^{(1/2)}*\operatorname{erf}((-b*\ln(f))^{(1/2)}*x)$

**Maxima [A]** time = 1.20686, size = 38, normalized size = 0.52

$$-\frac{(-bx^2 \log(f))^{\frac{3}{2}} f^a \Gamma\left(-\frac{3}{2}, -bx^2 \log(f)\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^4,x, algorithm="maxima")

[Out]  $-1/2*(-b*x^2*\log(f))^{(3/2)}*f^a*\gamma(-3/2, -b*x^2*\log(f))/x^3$

**Fricas [A]** time = 1.78261, size = 157, normalized size = 2.15

$$\frac{2\sqrt{\pi}\sqrt{-b\log(f)}bf^ax^3\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)+(2bx^2\log(f)+1)f^{bx^2+a}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^4,x, algorithm="fricas")

[Out]  $-1/3*(2*\sqrt{\pi}*\sqrt{-b*\log(f)}*b*f^a*x^3*\operatorname{erf}(\sqrt{-b*\log(f)}*x)*\log(f) + (2*b*x^2*\log(f) + 1)*f^{(b*x^2 + a)})/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**2+a)/x**4,x)
```

```
[Out] Integral(f**(a + b*x**2)/x**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^4, x)
```

### 3.91 $\int \frac{f^{a+bx^2}}{x^6} dx$

**Optimal.** Leaf size=96

$$\frac{4}{15} \sqrt{\pi} b^{5/2} f^a \log^5(f) \operatorname{Erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) - \frac{4b^2 \log^2(f) f^{a+bx^2}}{15x} - \frac{f^{a+bx^2}}{5x^5} - \frac{2b \log(f) f^{a+bx^2}}{15x^3}$$

[Out]  $-f^{(a + b*x^2)}/(5*x^5) - (2*b*f^{(a + b*x^2)}*Log[f])/(15*x^3) - (4*b^2*f^{(a + b*x^2)}*Log[f]^2)/(15*x) + (4*b^{(5/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x*sqrt[Log[f]]]*Log[f]^{(5/2)})/15$

**Rubi [A]** time = 0.0805698, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2204}

$$\frac{4}{15} \sqrt{\pi} b^{5/2} f^a \log^5(f) \operatorname{Erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) - \frac{4b^2 \log^2(f) f^{a+bx^2}}{15x} - \frac{f^{a+bx^2}}{5x^5} - \frac{2b \log(f) f^{a+bx^2}}{15x^3}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^6,x]

[Out]  $-f^{(a + b*x^2)}/(5*x^5) - (2*b*f^{(a + b*x^2)}*Log[f])/(15*x^3) - (4*b^2*f^{(a + b*x^2)}*Log[f]^2)/(15*x) + (4*b^{(5/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x*sqrt[Log[f]]]*Log[f]^{(5/2)})/15$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^6} dx &= -\frac{f^{a+bx^2}}{5x^5} + \frac{1}{5}(2b \log(f)) \int \frac{f^{a+bx^2}}{x^4} dx \\ &= -\frac{f^{a+bx^2}}{5x^5} - \frac{2bf^{a+bx^2} \log(f)}{15x^3} + \frac{1}{15}(4b^2 \log^2(f)) \int \frac{f^{a+bx^2}}{x^2} dx \\ &= -\frac{f^{a+bx^2}}{5x^5} - \frac{2bf^{a+bx^2} \log(f)}{15x^3} - \frac{4b^2 f^{a+bx^2} \log^2(f)}{15x} + \frac{1}{15}(8b^3 \log^3(f)) \int f^{a+bx^2} dx \\ &= -\frac{f^{a+bx^2}}{5x^5} - \frac{2bf^{a+bx^2} \log(f)}{15x^3} - \frac{4b^2 f^{a+bx^2} \log^2(f)}{15x} + \frac{4}{15} b^{5/2} f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) \log^5(f) \end{aligned}$$

**Mathematica [A]** time = 0.0343455, size = 77, normalized size = 0.8

$$\frac{f^a \left( 4\sqrt{\pi} b^{5/2} x^5 \log^2(f) \operatorname{Erfi}(\sqrt{bx} \sqrt{\log(f)}) - f^{bx^2} (4b^2 x^4 \log^2(f) + 2bx^2 \log(f) + 3) \right)}{15x^5}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^6, x]

[Out] (f^a\*(4\*b^(5/2)\*Sqrt[Pi]\*x^5\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]]\*Log[f]^(5/2) - f^(b\*x^2)\*(3 + 2\*b\*x^2\*Log[f] + 4\*b^2\*x^4\*Log[f]^2))/(15\*x^5)

**Maple [A]** time = 0.034, size = 89, normalized size = 0.9

$$-\frac{f^a f^{bx^2}}{5x^5} - \frac{2f^a \ln(f) b f^{bx^2}}{15x^3} - \frac{4f^a (\ln(f))^2 b^2 f^{bx^2}}{15x} + \frac{4f^a (\ln(f))^3 b^3 \sqrt{\pi}}{15} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^6, x)

[Out] -1/5\*f^a/x^5\*f^(b\*x^2)-2/15\*f^a\*ln(f)\*b/x^3\*f^(b\*x^2)-4/15\*f^a\*ln(f)^2\*b^2/x\*f^(b\*x^2)+4/15\*f^a\*ln(f)^3\*b^3\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.23129, size = 38, normalized size = 0.4

$$-\frac{(-bx^2 \log(f))^{\frac{5}{2}} f^a \Gamma\left(-\frac{5}{2}, -bx^2 \log(f)\right)}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^6, x, algorithm="maxima")

[Out] -1/2\*(-b\*x^2\*log(f))^(5/2)\*f^a\*gamma(-5/2, -b\*x^2\*log(f))/x^5

**Fricas [A]** time = 1.73238, size = 192, normalized size = 2.

$$\frac{4\sqrt{\pi}\sqrt{-b\log(f)}b^2f^ax^5\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^2+\left(4b^2x^4\log(f)^2+2bx^2\log(f)+3\right)f^{bx^2+a}}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^6, x, algorithm="fricas")

[Out] -1/15\*(4\*sqrt(pi)\*sqrt(-b\*log(f))\*b^2\*f^a\*x^5\*erf(sqrt(-b\*log(f))\*x)\*log(f)^2 + (4\*b^2\*x^4\*log(f)^2 + 2\*b\*x^2\*log(f) + 3)\*f^(b\*x^2 + a))/x^5



---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)/x\*\*6,x)

[Out] Integral(f\*\*(a + b\*x\*\*2)/x\*\*6, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^6,x, algorithm="giac")

[Out] integrate(f^(b\*x^2 + a)/x^6, x)

### 3.92 $\int \frac{f^{a+bx^2}}{x^8} dx$

**Optimal.** Leaf size=119

$$\frac{8}{105} \sqrt{\pi} b^{7/2} f^a \log^7(f) \operatorname{Erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) - \frac{8b^3 \log^3(f) f^{a+bx^2}}{105x} - \frac{4b^2 \log^2(f) f^{a+bx^2}}{105x^3} - \frac{f^{a+bx^2}}{7x^7} - \frac{2b \log(f) f^{a+bx^2}}{35x^5}$$

[Out]  $-f^{(a + b*x^2)}/(7*x^7) - (2*b*f^{(a + b*x^2)}*Log[f])/(35*x^5) - (4*b^2*f^{(a + b*x^2)}*Log[f]^2)/(105*x^3) - (8*b^3*f^{(a + b*x^2)}*Log[f]^3)/(105*x) + (8*b^{(7/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x*sqrt[Log[f]]]*Log[f]^{(7/2)})/105$

**Rubi [A]** time = 0.107872, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2204}

$$\frac{8}{105} \sqrt{\pi} b^{7/2} f^a \log^7(f) \operatorname{Erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) - \frac{8b^3 \log^3(f) f^{a+bx^2}}{105x} - \frac{4b^2 \log^2(f) f^{a+bx^2}}{105x^3} - \frac{f^{a+bx^2}}{7x^7} - \frac{2b \log(f) f^{a+bx^2}}{35x^5}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^8, x]

[Out]  $-f^{(a + b*x^2)}/(7*x^7) - (2*b*f^{(a + b*x^2)}*Log[f])/(35*x^5) - (4*b^2*f^{(a + b*x^2)}*Log[f]^2)/(105*x^3) - (8*b^3*f^{(a + b*x^2)}*Log[f]^3)/(105*x) + (8*b^{(7/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x*sqrt[Log[f]]]*Log[f]^{(7/2)})/105$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^2}}{x^8} dx &= -\frac{f^{a+bx^2}}{7x^7} + \frac{1}{7}(2b \log(f)) \int \frac{f^{a+bx^2}}{x^6} dx \\ &= -\frac{f^{a+bx^2}}{7x^7} - \frac{2bf^{a+bx^2} \log(f)}{35x^5} + \frac{1}{35} (4b^2 \log^2(f)) \int \frac{f^{a+bx^2}}{x^4} dx \\ &= -\frac{f^{a+bx^2}}{7x^7} - \frac{2bf^{a+bx^2} \log(f)}{35x^5} - \frac{4b^2 f^{a+bx^2} \log^2(f)}{105x^3} + \frac{1}{105} (8b^3 \log^3(f)) \int \frac{f^{a+bx^2}}{x^2} dx \\ &= -\frac{f^{a+bx^2}}{7x^7} - \frac{2bf^{a+bx^2} \log(f)}{35x^5} - \frac{4b^2 f^{a+bx^2} \log^2(f)}{105x^3} - \frac{8b^3 f^{a+bx^2} \log^3(f)}{105x} + \frac{1}{105} (16b^4 \log^4(f)) \int f^{a+bx^2} dx \\ &= -\frac{f^{a+bx^2}}{7x^7} - \frac{2bf^{a+bx^2} \log(f)}{35x^5} - \frac{4b^2 f^{a+bx^2} \log^2(f)}{105x^3} - \frac{8b^3 f^{a+bx^2} \log^3(f)}{105x} + \frac{8}{105} b^{7/2} f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{bx} \sqrt{\log(f)}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0414604, size = 89, normalized size = 0.75

$$\frac{f^a \left( 8\sqrt{\pi} b^{7/2} x^7 \log^2(f) \operatorname{Erfi} \left( \sqrt{bx} \sqrt{\log(f)} \right) - f^{bx^2} \left( 8b^3 x^6 \log^3(f) + 4b^2 x^4 \log^2(f) + 6bx^2 \log(f) + 15 \right) \right)}{105x^7}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^8, x]

[Out] (f^a\*(8\*b^(7/2)\*Sqrt[Pi]\*x^7\*Erfi[Sqrt[b]\*x\*Sqrt[Log[f]]]\*Log[f]^(7/2) - f^(b\*x^2)\*(15 + 6\*b\*x^2\*Log[f] + 4\*b^2\*x^4\*Log[f]^2 + 8\*b^3\*x^6\*Log[f]^3))/(105\*x^7)

**Maple [A]** time = 0.044, size = 111, normalized size = 0.9

$$\frac{f^a f^{bx^2}}{7x^7} - \frac{2f^a \ln(f) b f^{bx^2}}{35x^5} - \frac{4f^a (\ln(f))^2 b^2 f^{bx^2}}{105x^3} - \frac{8f^a (\ln(f))^3 b^3 f^{bx^2}}{105x} + \frac{8f^a (\ln(f))^4 b^4 \sqrt{\pi}}{105} \operatorname{Erf} \left( \sqrt{-b \ln(f)} x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^8, x)

[Out] -1/7\*f^a/x^7\*f^(b\*x^2)-2/35\*f^a\*ln(f)\*b/x^5\*f^(b\*x^2)-4/105\*f^a\*ln(f)^2\*b^2/x^3\*f^(b\*x^2)-8/105\*f^a\*ln(f)^3\*b^3/x\*f^(b\*x^2)+8/105\*f^a\*ln(f)^4\*b^4\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x)

**Maxima [A]** time = 1.17596, size = 38, normalized size = 0.32

$$-\frac{(-bx^2 \log(f))^{\frac{7}{2}} f^a \Gamma\left(-\frac{7}{2}, -bx^2 \log(f)\right)}{2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^8, x, algorithm="maxima")

[Out] -1/2\*(-b\*x^2\*log(f))^(7/2)\*f^a\*gamma(-7/2, -b\*x^2\*log(f))/x^7

**Fricas [A]** time = 1.74106, size = 223, normalized size = 1.87

$$\frac{8\sqrt{\pi}\sqrt{-b\log(f)}b^3f^ax^7\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^3+\left(8b^3x^6\log(f)^3+4b^2x^4\log(f)^2+6bx^2\log(f)+15\right)f^{bx^2+a}}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^8, x, algorithm="fricas")

[Out] -1/105\*(8\*sqrt(pi)\*sqrt(-b\*log(f))\*b^3\*f^a\*x^7\*erf(sqrt(-b\*log(f))\*x)\*log(f)^3 + (8\*b^3\*x^6\*log(f)^3 + 4\*b^2\*x^4\*log(f)^2 + 6\*b\*x^2\*log(f) + 15)\*f^(b\*

$x^2 + a)/x^7$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)/x\*\*8,x)

[Out] Integral(f\*\*(a + b\*x\*\*2)/x\*\*8, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^8,x, algorithm="giac")

[Out] integrate(f^(b\*x^2 + a)/x^8, x)

$$3.93 \quad \int \frac{f^{a+bx^2}}{x^{10}} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a (-bx^2 \log(f))^{9/2} \text{Gamma}\left(-\frac{9}{2}, -bx^2 \log(f)\right)}{2x^9}$$

[Out]  $-(f^a \text{Gamma}[-9/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(9/2)} / (2*x^9)$

**Rubi [A]** time = 0.0206185, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a (-bx^2 \log(f))^{9/2} \text{Gamma}\left(-\frac{9}{2}, -bx^2 \log(f)\right)}{2x^9}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^10, x]

[Out]  $-(f^a \text{Gamma}[-9/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(9/2)} / (2*x^9)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^2}}{x^{10}} dx = -\frac{f^a \Gamma\left(-\frac{9}{2}, -bx^2 \log(f)\right) (-bx^2 \log(f))^{9/2}}{2x^9}$$

**Mathematica [A]** time = 0.0056244, size = 34, normalized size = 1.

$$\frac{f^a (-bx^2 \log(f))^{9/2} \text{Gamma}\left(-\frac{9}{2}, -bx^2 \log(f)\right)}{2x^9}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^10, x]

[Out]  $-(f^a \text{Gamma}[-9/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(9/2)} / (2*x^9)$

**Maple [A]** time = 0.058, size = 133, normalized size = 3.9

$$\frac{f^a f^{bx^2}}{9x^9} - \frac{2f^a \ln(f) b f^{bx^2}}{63x^7} - \frac{4f^a (\ln(f))^2 b^2 f^{bx^2}}{315x^5} - \frac{8f^a (\ln(f))^3 b^3 f^{bx^2}}{945x^3} - \frac{16f^a (\ln(f))^4 b^4 f^{bx^2}}{945x} + \frac{16f^a (\ln(f))^5 b^5 f^{bx^2}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^2+a)/x^10,x)`

[Out] 
$$-1/9*f^a/x^9*f^(b*x^2)-2/63*f^a*\ln(f)*b/x^7*f^(b*x^2)-4/315*f^a*\ln(f)^2*b^2/x^5*f^(b*x^2)-8/945*f^a*\ln(f)^3*b^3/x^3*f^(b*x^2)-16/945*f^a*\ln(f)^4*b^4/x*f^(b*x^2)+16/945*f^a*\ln(f)^5*b^5*\pi^{1/2}/(-b*\ln(f))^{1/2}*erf((-b*\ln(f))^{1/2}*x)$$

**Maxima [A]** time = 1.19433, size = 38, normalized size = 1.12

$$\frac{(-bx^2 \log(f))^{\frac{9}{2}} f^a \Gamma\left(-\frac{9}{2}, -bx^2 \log(f)\right)}{2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^10,x, algorithm="maxima")`

[Out] 
$$-1/2*(-b*x^2*\log(f))^{9/2}*f^a*\gamma(-9/2, -b*x^2*\log(f))/x^9$$

**Fricas [A]** time = 1.74763, size = 258, normalized size = 7.59

$$\frac{16\sqrt{\pi}\sqrt{-b\log(f)}b^4f^ax^9\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^4+\left(16b^4x^8\log(f)^4+8b^3x^6\log(f)^3+12b^2x^4\log(f)^2+30bx^2\log(f)+105\right)f^{a+bx^2}}{945x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^2+a)/x^10,x, algorithm="fricas")`

[Out] 
$$-1/945*(16*\sqrt{\pi}*\sqrt{-b*\log(f)}*b^4*f^a*x^9*\operatorname{erf}(\sqrt{-b*\log(f)}*x)*\log(f)^4+(16*b^4*x^8*\log(f)^4+8*b^3*x^6*\log(f)^3+12*b^2*x^4*\log(f)^2+30*b*x^2*\log(f)+105)*f^{(b*x^2+a)})/x^9$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^2}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**2+a)/x**10,x)`

[Out] `Integral(f**(a + b*x**2)/x**10, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^10,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^10, x)
```

$$3.94 \quad \int \frac{f^{a+bx^2}}{x^{12}} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a (-bx^2 \log(f))^{11/2} \text{Gamma}\left(-\frac{11}{2}, -bx^2 \log(f)\right)}{2x^{11}}$$

[Out]  $-(f^a \text{Gamma}[-11/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(11/2)} / (2*x^{11})$

**Rubi [A]** time = 0.0208784, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a (-bx^2 \log(f))^{11/2} \text{Gamma}\left(-\frac{11}{2}, -bx^2 \log(f)\right)}{2x^{11}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^2)/x^12,x]

[Out]  $-(f^a \text{Gamma}[-11/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(11/2)} / (2*x^{11})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^2}}{x^{12}} dx = -\frac{f^a \Gamma\left(-\frac{11}{2}, -bx^2 \log(f)\right) (-bx^2 \log(f))^{11/2}}{2x^{11}}$$

**Mathematica [A]** time = 0.0054607, size = 34, normalized size = 1.

$$\frac{f^a (-bx^2 \log(f))^{11/2} \text{Gamma}\left(-\frac{11}{2}, -bx^2 \log(f)\right)}{2x^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^2)/x^12,x]

[Out]  $-(f^a \text{Gamma}[-11/2, -(b*x^2*\text{Log}[f])]) * (-(b*x^2*\text{Log}[f]))^{(11/2)} / (2*x^{11})$

**Maple [A]** time = 0.091, size = 155, normalized size = 4.6

$$\frac{f^a f^{bx^2}}{11 x^{11}} - \frac{2 f^a \ln(f) b f^{bx^2}}{99 x^9} - \frac{4 f^a (\ln(f))^2 b^2 f^{bx^2}}{693 x^7} - \frac{8 f^a (\ln(f))^3 b^3 f^{bx^2}}{3465 x^5} - \frac{16 f^a (\ln(f))^4 b^4 f^{bx^2}}{10395 x^3} - \frac{32 f^a (\ln(f))^5 b^5 f^{bx^2}}{10395 x}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^2+a)/x^12,x)

[Out] 
$$\frac{-1/11*f^a/x^{11}*f^{(b*x^2)}-2/99*f^a*\ln(f)*b/x^9*f^{(b*x^2)}-4/693*f^a*\ln(f)^2*b^2/x^7*f^{(b*x^2)}-8/3465*f^a*\ln(f)^3*b^3/x^5*f^{(b*x^2)}-16/10395*f^a*\ln(f)^4*b^4/x^3*f^{(b*x^2)}-32/10395*f^a*\ln(f)^5*b^5/x*f^{(b*x^2)}+32/10395*f^a*\ln(f)^6*b^6*\pi^{(1/2)}/(-b*\ln(f))^{(1/2)}*erf((-b*\ln(f))^{(1/2)}*x)}{2*x^{11}}$$

**Maxima [A]** time = 1.30194, size = 38, normalized size = 1.12

$$\frac{(-bx^2 \log(f))^{\frac{11}{2}} f^a \Gamma\left(-\frac{11}{2}, -bx^2 \log(f)\right)}{2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^12,x, algorithm="maxima")

[Out] 
$$-1/2*(-b*x^2*\log(f))^{(11/2)}*f^a*\gamma(-11/2, -b*x^2*\log(f))/x^{11}$$

**Fricas [A]** time = 1.82728, size = 297, normalized size = 8.74

$$\frac{32\sqrt{\pi}\sqrt{-b\log(f)}b^5f^ax^{11}\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^5+\left(32b^5x^{10}\log(f)^5+16b^4x^8\log(f)^4+24b^3x^6\log(f)^3+60b^2x^4\log(f)^2+210bx^2\log(f)+945\right)f^{(b*x^2+a)}}{10395x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^2+a)/x^12,x, algorithm="fricas")

[Out] 
$$-1/10395*(32*\sqrt{\pi}*\sqrt{-b*\log(f)}*b^5*f^a*x^{11}*\operatorname{erf}(\sqrt{-b*\log(f)}*x)*\log(f)^5+(32*b^5*x^{10}*\log(f)^5+16*b^4*x^8*\log(f)^4+24*b^3*x^6*\log(f)^3+60*b^2*x^4*\log(f)^2+210*b*x^2*\log(f)+945)*f^{(b*x^2+a)})/x^{11}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*2+a)/x\*\*12,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^2+a}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^2+a)/x^12,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^2 + a)/x^12, x)
```

### 3.95 $\int f^{a+bx^3} x^m dx$

**Optimal.** Leaf size=46

$$-\frac{1}{3} f^a x^{m+1} (-bx^3 \log(f))^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -bx^3 \log(f)\right)$$

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/3, -(b x^3 \text{Log}[f])]) * (-(b x^3 \text{Log}[f]))^{((-1-m)/3)}/3$

**Rubi [A]** time = 0.0214375, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{3} f^a x^{m+1} (-bx^3 \log(f))^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -bx^3 \log(f)\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^m,x]

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/3, -(b x^3 \text{Log}[f])]) * (-(b x^3 \text{Log}[f]))^{((-1-m)/3)}/3$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x^m dx = -\frac{1}{3} f^a x^{1+m} \Gamma\left(\frac{1+m}{3}, -bx^3 \log(f)\right) (-bx^3 \log(f))^{\frac{1}{3}(-1-m)}$$

**Mathematica [A]** time = 0.0111653, size = 46, normalized size = 1.

$$-\frac{1}{3} f^a x^{m+1} (-bx^3 \log(f))^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -bx^3 \log(f)\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^m,x]

[Out]  $-(f^a x^{(1+m)} \text{Gamma}[(1+m)/3, -(b x^3 \text{Log}[f])]) * (-(b x^3 \text{Log}[f]))^{((-1-m)/3)}/3$

**Maple [B]** time = 0.033, size = 140, normalized size = 3.

$$\frac{f^a}{3} (-b)^{-\frac{m}{3}-\frac{1}{3}} (\ln(f))^{-\frac{m}{3}-\frac{1}{3}} \left( 3 \frac{x^{1+m} (-b)^{1/3+m/3} (\ln(f))^{1/3+m/3} (1/3+m/3) (-bx^3 \ln(f))^{-m/3-1/3} \Gamma(1/3+m/3)}{1+m} + 3 \frac{x^{1+m}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)*x^m,x)`

[Out]  $\frac{1}{3}f^a(-b)^{-1/3m-1/3}\ln(f)^{-1/3m-1/3}\left(\frac{3}{1+m}x^{1+m}(-b)^{1/3+1/3m}\ln(f)^{1/3+1/3m}\left(\frac{1}{3+1/3m}\right)(-bx^3\ln(f))^{-1/3m-1/3}\Gamma\left(\frac{1}{3+1/3m}\right)+\frac{3}{1+m}x^{1+m}(-b)^{1/3+1/3m}\ln(f)^{1/3+1/3m}\left(-\frac{1}{3m-1/3}\right)(-bx^3\ln(f))^{-1/3m-1/3}\Gamma\left(\frac{1}{3+1/3m},-bx^3\ln(f)\right)\right)$

**Maxima [A]** time = 1.2255, size = 51, normalized size = 1.11

$$-\frac{1}{3}(-bx^3 \log(f))^{-\frac{1}{3}m-\frac{1}{3}} f^a x^{m+1} \Gamma\left(\frac{1}{3}m + \frac{1}{3}, -bx^3 \log(f)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^m,x, algorithm="maxima")`

[Out]  $-\frac{1}{3}(-bx^3 \log(f))^{-1/3m-1/3} f^a x^{m+1} \text{gamma}(1/3m+1/3, -bx^3 \log(f))$

**Fricas [A]** time = 1.77222, size = 126, normalized size = 2.74

$$\frac{e^{\left(-\frac{1}{3}(m-2)\log(-b\log(f))+a\log(f)\right)} \Gamma\left(\frac{1}{3}m + \frac{1}{3}, -bx^3 \log(f)\right)}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^m,x, algorithm="fricas")`

[Out]  $\frac{1}{3}e^{(-1/3(m-2)\log(-b\log(f))+a\log(f))} \text{gamma}(1/3m+1/3, -bx^3 \log(f)) / (b \log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^3} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)*x**m,x)`

[Out] `Integral(f**(a + b*x**3)*x**m, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^3+a} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^m,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)*x^m, x)
```

### 3.96 $\int f^{a+bx^3} x^{17} dx$

**Optimal.** Leaf size=78

$$\frac{f^{a+bx^3} \left( -b^5 x^{15} \log^5(f) + 5b^4 x^{12} \log^4(f) - 20b^3 x^9 \log^3(f) + 60b^2 x^6 \log^2(f) - 120bx^3 \log(f) + 120 \right)}{3b^6 \log^6(f)}$$

[Out]  $-(f^{(a + b*x^3)}*(120 - 120*b*x^3*\text{Log}[f] + 60*b^2*x^6*\text{Log}[f]^2 - 20*b^3*x^9*\text{Log}[f]^3 + 5*b^4*x^{12}*\text{Log}[f]^4 - b^5*x^{15}*\text{Log}[f]^5))/(3*b^6*\text{Log}[f]^6)$

**Rubi [C]** time = 0.0224891, antiderivative size = 24, normalized size of antiderivative = 0.31, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}(6, -bx^3 \log(f))}{3b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^17, x]

[Out]  $-(f^a*\text{Gamma}[6, -(b*x^3*\text{Log}[f])])/(3*b^6*\text{Log}[f]^6)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x^{17} dx = -\frac{f^a \Gamma(6, -bx^3 \log(f))}{3b^6 \log^6(f)}$$

**Mathematica [C]** time = 0.0028409, size = 24, normalized size = 0.31

$$\frac{f^a \text{Gamma}(6, -bx^3 \log(f))}{3b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^17, x]

[Out]  $-(f^a*\text{Gamma}[6, -(b*x^3*\text{Log}[f])])/(3*b^6*\text{Log}[f]^6)$

**Maple [A]** time = 0.013, size = 76, normalized size = 1.

$$\frac{(b^5 x^{15} (\ln(f))^5 - 5b^4 x^{12} (\ln(f))^4 + 20b^3 x^9 (\ln(f))^3 - 60b^2 x^6 (\ln(f))^2 + 120bx^3 \ln(f) - 120) f^{bx^3+a}}{3b^6 (\ln(f))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^17,x)

[Out]  $\frac{1}{3}(b^5x^{15}\ln(f)^5 - 5b^4x^{12}\ln(f)^4 + 20b^3x^9\ln(f)^3 - 60b^2x^6\ln(f)^2 + 120bx^3\ln(f) - 120)f^{bx^3+a}/b^6/\ln(f)^6$

**Maxima [A]** time = 1.2323, size = 124, normalized size = 1.59

$$\frac{(b^5 f^a x^{15} \log(f)^5 - 5 b^4 f^a x^{12} \log(f)^4 + 20 b^3 f^a x^9 \log(f)^3 - 60 b^2 f^a x^6 \log(f)^2 + 120 b f^a x^3 \log(f) - 120 f^a) f^{bx^3}}{3 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^17,x, algorithm="maxima")

[Out]  $\frac{1}{3}(b^5 f^a x^{15} \log(f)^5 - 5 b^4 f^a x^{12} \log(f)^4 + 20 b^3 f^a x^9 \log(f)^3 - 60 b^2 f^a x^6 \log(f)^2 + 120 b f^a x^3 \log(f) - 120 f^a) f^{bx^3+a} / (b^6 \log(f)^6)$

**Fricas [A]** time = 1.73179, size = 196, normalized size = 2.51

$$\frac{(b^5 x^{15} \log(f)^5 - 5 b^4 x^{12} \log(f)^4 + 20 b^3 x^9 \log(f)^3 - 60 b^2 x^6 \log(f)^2 + 120 b x^3 \log(f) - 120) f^{bx^3+a}}{3 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^17,x, algorithm="fricas")

[Out]  $\frac{1}{3}(b^5x^{15}\log(f)^5 - 5b^4x^{12}\log(f)^4 + 20b^3x^9\log(f)^3 - 60b^2x^6\log(f)^2 + 120bx^3\log(f) - 120)f^{bx^3+a}/(b^6\log(f)^6)$

**Sympy [A]** time = 0.161398, size = 95, normalized size = 1.22

$$\begin{cases} \frac{f^{a+bx^3}(b^5x^{15}\log(f)^5 - 5b^4x^{12}\log(f)^4 + 20b^3x^9\log(f)^3 - 60b^2x^6\log(f)^2 + 120bx^3\log(f) - 120)}{3b^6\log(f)^6} & \text{for } 3b^6\log(f)^6 \neq 0 \\ \frac{x^{18}}{18} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*17,x)

[Out] Piecewise((f\*\*(a + b\*x\*\*3)\*(b\*\*5\*x\*\*15\*log(f)\*\*5 - 5\*b\*\*4\*x\*\*12\*log(f)\*\*4 + 20\*b\*\*3\*x\*\*9\*log(f)\*\*3 - 60\*b\*\*2\*x\*\*6\*log(f)\*\*2 + 120\*b\*x\*\*3\*log(f) - 120)/(3\*b\*\*6\*log(f)\*\*6), Ne(3\*b\*\*6\*log(f)\*\*6, 0)), (x\*\*18/18, True))

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^17,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.97 $\int f^{a+bx^3} x^{14} dx$

**Optimal.** Leaf size=65

$$\frac{f^{a+bx^3} (b^4 x^{12} \log^4(f) - 4b^3 x^9 \log^3(f) + 12b^2 x^6 \log^2(f) - 24bx^3 \log(f) + 24)}{3b^5 \log^5(f)}$$

[Out] (f^(a + b\*x^3)\*(24 - 24\*b\*x^3\*Log[f] + 12\*b^2\*x^6\*Log[f]^2 - 4\*b^3\*x^9\*Log[f]^3 + b^4\*x^12\*Log[f]^4))/(3\*b^5\*Log[f]^5)

**Rubi [C]** time = 0.0231564, antiderivative size = 24, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}(5, -bx^3 \log(f))}{3b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^14, x]

[Out] (f^a\*Gamma[5, -(b\*x^3\*Log[f])])/(3\*b^5\*Log[f]^5)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x^{14} dx = \frac{f^a \Gamma(5, -bx^3 \log(f))}{3b^5 \log^5(f)}$$

**Mathematica [C]** time = 0.0029752, size = 24, normalized size = 0.37

$$\frac{f^a \text{Gamma}(5, -bx^3 \log(f))}{3b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^14, x]

[Out] (f^a\*Gamma[5, -(b\*x^3\*Log[f])])/(3\*b^5\*Log[f]^5)

**Maple [A]** time = 0.01, size = 64, normalized size = 1.

$$\frac{f^{bx^3+a} (24 - 24bx^3 \ln(f) + 12b^2x^6 (\ln(f))^2 - 4b^3x^9 (\ln(f))^3 + b^4x^{12} (\ln(f))^4)}{3b^5 (\ln(f))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)*x^14,x)`

[Out]  $\frac{1}{3}f^{(b*x^3+a)}*(24-24*b*x^3*\ln(f)+12*b^2*x^6*\ln(f)^2-4*b^3*x^9*\ln(f)^3+b^4*x^{12}*\ln(f)^4)/b^5/\ln(f)^5$

**Maxima [A]** time = 1.03267, size = 104, normalized size = 1.6

$$\frac{(b^4 f^a x^{12} \log(f)^4 - 4 b^3 f^a x^9 \log(f)^3 + 12 b^2 f^a x^6 \log(f)^2 - 24 b f^a x^3 \log(f) + 24 f^a) f^{bx^3}}{3 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^14,x, algorithm="maxima")`

[Out]  $\frac{1}{3}*(b^4*f^a*x^{12}*\log(f)^4 - 4*b^3*f^a*x^9*\log(f)^3 + 12*b^2*f^a*x^6*\log(f)^2 - 24*b*f^a*x^3*\log(f) + 24*f^a)*f^{(b*x^3)}/(b^5*\log(f)^5)$

**Fricas [A]** time = 1.72558, size = 162, normalized size = 2.49

$$\frac{(b^4 x^{12} \log(f)^4 - 4 b^3 x^9 \log(f)^3 + 12 b^2 x^6 \log(f)^2 - 24 b x^3 \log(f) + 24) f^{bx^3+a}}{3 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^14,x, algorithm="fricas")`

[Out]  $\frac{1}{3}*(b^4*x^{12}*\log(f)^4 - 4*b^3*x^9*\log(f)^3 + 12*b^2*x^6*\log(f)^2 - 24*b*x^3*\log(f) + 24)*f^{(b*x^3 + a)}/(b^5*\log(f)^5)$

**Sympy [A]** time = 0.148227, size = 82, normalized size = 1.26

$$\begin{cases} \frac{f^{a+bx^3}(b^4 x^{12} \log(f)^4 - 4 b^3 x^9 \log(f)^3 + 12 b^2 x^6 \log(f)^2 - 24 b x^3 \log(f) + 24)}{3 b^5 \log(f)^5} & \text{for } 3 b^5 \log(f)^5 \neq 0 \\ \frac{x^{15}}{15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)*x**14,x)`

[Out] `Piecewise((f**(a + b*x**3)*(b**4*x**12*log(f)**4 - 4*b**3*x**9*log(f)**3 + 12*b**2*x**6*log(f)**2 - 24*b*x**3*log(f) + 24)/(3*b**5*log(f)**5), Ne(3*b**5*log(f)**5, 0)), (x**15/15, True))`

**Giac [A]** time = 1.23566, size = 142, normalized size = 2.18

$$\frac{b^4 f^{bx^3} f^a x^{12} \log(f)^4 - 4b^3 f^{bx^3} f^a x^9 \log(f)^3 + 12b^2 f^{bx^3} f^a x^6 \log(f)^2 - 24b f^{bx^3} f^a x^3 \log(f) + 24 f^{bx^3} f^a}{3b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^14,x, algorithm="giac")

[Out] 1/3\*(b^4\*f^(b\*x^3)\*f^a\*x^12\*log(f)^4 - 4\*b^3\*f^(b\*x^3)\*f^a\*x^9\*log(f)^3 + 12\*b^2\*f^(b\*x^3)\*f^a\*x^6\*log(f)^2 - 24\*b\*f^(b\*x^3)\*f^a\*x^3\*log(f) + 24\*f^(b\*x^3)\*f^a)/(b^5\*log(f)^5)

### 3.98 $\int f^{a+bx^3} x^{11} dx$

**Optimal.** Leaf size=84

$$-\frac{x^6 f^{a+bx^3}}{b^2 \log^2(f)} + \frac{2x^3 f^{a+bx^3}}{b^3 \log^3(f)} - \frac{2f^{a+bx^3}}{b^4 \log^4(f)} + \frac{x^9 f^{a+bx^3}}{3b \log(f)}$$

[Out]  $(-2*f^{(a + b*x^3)})/(b^4*Log[f]^4) + (2*f^{(a + b*x^3)}*x^3)/(b^3*Log[f]^3) - (f^{(a + b*x^3)}*x^6)/(b^2*Log[f]^2) + (f^{(a + b*x^3)}*x^9)/(3*b*Log[f])$

**Rubi [A]** time = 0.0950641, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$-\frac{x^6 f^{a+bx^3}}{b^2 \log^2(f)} + \frac{2x^3 f^{a+bx^3}}{b^3 \log^3(f)} - \frac{2f^{a+bx^3}}{b^4 \log^4(f)} + \frac{x^9 f^{a+bx^3}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^11,x]

[Out]  $(-2*f^{(a + b*x^3)})/(b^4*Log[f]^4) + (2*f^{(a + b*x^3)}*x^3)/(b^3*Log[f]^3) - (f^{(a + b*x^3)}*x^6)/(b^2*Log[f]^2) + (f^{(a + b*x^3)}*x^9)/(3*b*Log[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^3} x^{11} dx &= \frac{f^{a+bx^3} x^9}{3b \log(f)} - \frac{3 \int f^{a+bx^3} x^8 dx}{b \log(f)} \\ &= -\frac{f^{a+bx^3} x^6}{b^2 \log^2(f)} + \frac{f^{a+bx^3} x^9}{3b \log(f)} + \frac{6 \int f^{a+bx^3} x^5 dx}{b^2 \log^2(f)} \\ &= \frac{2f^{a+bx^3} x^3}{b^3 \log^3(f)} - \frac{f^{a+bx^3} x^6}{b^2 \log^2(f)} + \frac{f^{a+bx^3} x^9}{3b \log(f)} - \frac{6 \int f^{a+bx^3} x^2 dx}{b^3 \log^3(f)} \\ &= -\frac{2f^{a+bx^3}}{b^4 \log^4(f)} + \frac{2f^{a+bx^3} x^3}{b^3 \log^3(f)} - \frac{f^{a+bx^3} x^6}{b^2 \log^2(f)} + \frac{f^{a+bx^3} x^9}{3b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.01129, size = 53, normalized size = 0.63

$$\frac{f^{a+bx^3} (b^3 x^9 \log^3(f) - 3b^2 x^6 \log^2(f) + 6bx^3 \log(f) - 6)}{3b^4 \log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^11,x]

[Out] (f^(a + b\*x^3)\*(-6 + 6\*b\*x^3\*Log[f] - 3\*b^2\*x^6\*Log[f]^2 + b^3\*x^9\*Log[f]^3))/ (3\*b^4\*Log[f]^4)

**Maple [A]** time = 0.008, size = 52, normalized size = 0.6

$$\frac{(b^3 x^9 (\ln(f))^3 - 3b^2 x^6 (\ln(f))^2 + 6bx^3 \ln(f) - 6) f^{bx^3+a}}{3b^4 (\ln(f))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^11,x)

[Out] 1/3\*(b^3\*x^9\*ln(f)^3-3\*b^2\*x^6\*ln(f)^2+6\*b\*x^3\*ln(f)-6)\*f^(b\*x^3+a)/ln(f)^4/b^4

**Maxima [A]** time = 1.31256, size = 84, normalized size = 1.

$$\frac{(b^3 f^a x^9 \log(f)^3 - 3b^2 f^a x^6 \log(f)^2 + 6bf^a x^3 \log(f) - 6f^a) f^{bx^3}}{3b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^11,x, algorithm="maxima")

[Out] 1/3\*(b^3\*f^a\*x^9\*log(f)^3 - 3\*b^2\*f^a\*x^6\*log(f)^2 + 6\*b\*f^a\*x^3\*log(f) - 6\*f^a)\*f^(b\*x^3)/(b^4\*log(f)^4)

**Fricas [A]** time = 1.79495, size = 128, normalized size = 1.52

$$\frac{(b^3 x^9 \log(f)^3 - 3b^2 x^6 \log(f)^2 + 6bx^3 \log(f) - 6) f^{bx^3+a}}{3b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^11,x, algorithm="fricas")

[Out] 1/3\*(b^3\*x^9\*log(f)^3 - 3\*b^2\*x^6\*log(f)^2 + 6\*b\*x^3\*log(f) - 6)\*f^(b\*x^3 + a)/(b^4\*log(f)^4)

---

**Sympy [A]** time = 0.135558, size = 68, normalized size = 0.81

$$\begin{cases} \frac{f^{a+bx^3}(b^3x^9 \log(f)^3 - 3b^2x^6 \log(f)^2 + 6bx^3 \log(f) - 6)}{3b^4 \log(f)^4} & \text{for } 3b^4 \log(f)^4 \neq 0 \\ \frac{x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*11,x)

[Out] Piecewise((f\*\*(a + b\*x\*\*3)\*(b\*\*3\*x\*\*9\*log(f)\*\*3 - 3\*b\*\*2\*x\*\*6\*log(f)\*\*2 + 6\*b\*x\*\*3\*log(f) - 6)/(3\*b\*\*4\*log(f)\*\*4), Ne(3\*b\*\*4\*log(f)\*\*4, 0)), (x\*\*12/12, True))

---

**Giac [A]** time = 1.22751, size = 112, normalized size = 1.33

$$\frac{b^3 f^{bx^3} f^a x^9 \log(f)^3 - 3 b^2 f^{bx^3} f^a x^6 \log(f)^2 + 6 b f^{bx^3} f^a x^3 \log(f) - 6 f^{bx^3} f^a}{3 b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^11,x, algorithm="giac")

[Out] 1/3\*(b^3\*f^(b\*x^3)\*f^a\*x^9\*log(f)^3 - 3\*b^2\*f^(b\*x^3)\*f^a\*x^6\*log(f)^2 + 6\*b\*f^(b\*x^3)\*f^a\*x^3\*log(f) - 6\*f^(b\*x^3)\*f^a)/(b^4\*log(f)^4)

### 3.99 $\int f^{a+bx^3} x^8 dx$

**Optimal.** Leaf size=67

$$-\frac{2x^3 f^{a+bx^3}}{3b^2 \log^2(f)} + \frac{2f^{a+bx^3}}{3b^3 \log^3(f)} + \frac{x^6 f^{a+bx^3}}{3b \log(f)}$$

[Out]  $(2*f^{(a + b*x^3)})/(3*b^3*Log[f]^3) - (2*f^{(a + b*x^3)}*x^3)/(3*b^2*Log[f]^2) + (f^{(a + b*x^3)}*x^6)/(3*b*Log[f])$

**Rubi [A]** time = 0.0702023, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$-\frac{2x^3 f^{a+bx^3}}{3b^2 \log^2(f)} + \frac{2f^{a+bx^3}}{3b^3 \log^3(f)} + \frac{x^6 f^{a+bx^3}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^8, x]

[Out]  $(2*f^{(a + b*x^3)})/(3*b^3*Log[f]^3) - (2*f^{(a + b*x^3)}*x^3)/(3*b^2*Log[f]^2) + (f^{(a + b*x^3)}*x^6)/(3*b*Log[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^3} x^8 dx &= \frac{f^{a+bx^3} x^6}{3b \log(f)} - \frac{2 \int f^{a+bx^3} x^5 dx}{b \log(f)} \\ &= -\frac{2f^{a+bx^3} x^3}{3b^2 \log^2(f)} + \frac{f^{a+bx^3} x^6}{3b \log(f)} + \frac{2 \int f^{a+bx^3} x^2 dx}{b^2 \log^2(f)} \\ &= \frac{2f^{a+bx^3}}{3b^3 \log^3(f)} - \frac{2f^{a+bx^3} x^3}{3b^2 \log^2(f)} + \frac{f^{a+bx^3} x^6}{3b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.008727, size = 41, normalized size = 0.61

$$\frac{f^{a+bx^3} (b^2 x^6 \log^2(f) - 2bx^3 \log(f) + 2)}{3b^3 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^8,x]

[Out] (f^(a + b\*x^3)\*(2 - 2\*b\*x^3\*Log[f] + b^2\*x^6\*Log[f]^2))/(3\*b^3\*Log[f]^3)

**Maple [A]** time = 0.008, size = 40, normalized size = 0.6

$$\frac{\left(b^2x^6(\ln(f))^2 - 2bx^3\ln(f) + 2\right)f^{bx^3+a}}{3(\ln(f))^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^8,x)

[Out] 1/3\*(b^2\*x^6\*ln(f)^2-2\*b\*x^3\*ln(f)+2)\*f^(b\*x^3+a)/ln(f)^3/b^3

**Maxima [A]** time = 1.09035, size = 63, normalized size = 0.94

$$\frac{\left(b^2f^ax^6\log(f)^2 - 2bf^ax^3\log(f) + 2f^a\right)f^{bx^3}}{3b^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^8,x, algorithm="maxima")

[Out] 1/3\*(b^2\*f^a\*x^6\*log(f)^2 - 2\*b\*f^a\*x^3\*log(f) + 2\*f^a)\*f^(b\*x^3)/(b^3\*log(f)^3)

**Fricas [A]** time = 1.82486, size = 100, normalized size = 1.49

$$\frac{\left(b^2x^6\log(f)^2 - 2bx^3\log(f) + 2\right)f^{bx^3+a}}{3b^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^8,x, algorithm="fricas")

[Out] 1/3\*(b^2\*x^6\*log(f)^2 - 2\*b\*x^3\*log(f) + 2)\*f^(b\*x^3 + a)/(b^3\*log(f)^3)

**Sympy [A]** time = 0.128307, size = 54, normalized size = 0.81

$$\begin{cases} \frac{f^{a+bx^3}\left(b^2x^6\log(f)^2-2bx^3\log(f)+2\right)}{3b^3\log(f)^3} & \text{for } 3b^3\log(f)^3 \neq 0 \\ \frac{x^9}{9} & \text{otherwise} \end{cases}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*8,x)

[Out] Piecewise((f\*\*(a + b\*x\*\*3)\*(b\*\*2\*x\*\*6\*log(f)\*\*2 - 2\*b\*x\*\*3\*log(f) + 2)/(3\*b\*\*3\*log(f)\*\*3), Ne(3\*b\*\*3\*log(f)\*\*3, 0)), (x\*\*9/9, True))

**Giac [A]** time = 1.23852, size = 82, normalized size = 1.22

$$\frac{b^2 f^{bx^3} f^a x^6 \log(f)^2 - 2 b f^{bx^3} f^a x^3 \log(f) + 2 f^{bx^3} f^a}{3 b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^8,x, algorithm="giac")

[Out] 1/3\*(b^2\*f^(b\*x^3)\*f^a\*x^6\*log(f)^2 - 2\*b\*f^(b\*x^3)\*f^a\*x^3\*log(f) + 2\*f^(b\*x^3)\*f^a)/(b^3\*log(f)^3)

### 3.100 $\int f^{a+bx^3} x^5 dx$

**Optimal.** Leaf size=44

$$\frac{x^3 f^{a+bx^3}}{3b \log(f)} - \frac{f^{a+bx^3}}{3b^2 \log^2(f)}$$

[Out]  $-f^{(a + b*x^3)}/(3*b^2*\text{Log}[f]^2) + (f^{(a + b*x^3)}*x^3)/(3*b*\text{Log}[f])$

**Rubi [A]** time = 0.0439159, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{x^3 f^{a+bx^3}}{3b \log(f)} - \frac{f^{a+bx^3}}{3b^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*x^3)}*x^5, x]$

[Out]  $-f^{(a + b*x^3)}/(3*b^2*\text{Log}[f]^2) + (f^{(a + b*x^3)}*x^3)/(3*b*\text{Log}[f])$

#### Rule 2212

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] :> \text{Simp}[(c + d*x)^{(m - n + 1)}*F^{(a + b*(c + d*x)^n)}/(b*d*n*\text{Log}[F]), x] - \text{Dist}[(m - n + 1)/(b*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((e_.) + (f_.)*(x_.))^{(m_.)}], x\_Symbol] :> \text{Simp}[(e + f*x)^n * F^{(a + b*(c + d*x)^n)}/(b*f*n*(c + d*x)^n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^3} x^5 dx &= \frac{f^{a+bx^3} x^3}{3b \log(f)} - \frac{\int f^{a+bx^3} x^2 dx}{b \log(f)} \\ &= -\frac{f^{a+bx^3}}{3b^2 \log^2(f)} + \frac{f^{a+bx^3} x^3}{3b \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0066651, size = 29, normalized size = 0.66

$$\frac{f^{a+bx^3} (bx^3 \log(f) - 1)}{3b^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^5,x]

[Out] (f^(a + b\*x^3)\*(-1 + b\*x^3\*Log[f]))/(3\*b^2\*Log[f]^2)

**Maple [A]** time = 0.005, size = 28, normalized size = 0.6

$$\frac{(bx^3 \ln(f) - 1) f^{bx^3+a}}{3 (\ln(f))^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^5,x)

[Out] 1/3\*(b\*x^3\*ln(f)-1)\*f^(b\*x^3+a)/ln(f)^2/b^2

**Maxima [A]** time = 1.14378, size = 43, normalized size = 0.98

$$\frac{(bf^ax^3 \log(f) - f^a) f^{bx^3}}{3 b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^5,x, algorithm="maxima")

[Out] 1/3\*(b\*f^a\*x^3\*log(f) - f^a)\*f^(b\*x^3)/(b^2\*log(f)^2)

**Fricas [A]** time = 1.83048, size = 72, normalized size = 1.64

$$\frac{(bx^3 \log(f) - 1) f^{bx^3+a}}{3 b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^5,x, algorithm="fricas")

[Out] 1/3\*(b\*x^3\*log(f) - 1)\*f^(b\*x^3 + a)/(b^2\*log(f)^2)

**Sympy [A]** time = 0.122187, size = 41, normalized size = 0.93

$$\begin{cases} \frac{f^{a+bx^3}(bx^3 \log(f)-1)}{3b^2 \log(f)^2} & \text{for } 3b^2 \log(f)^2 \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*5,x)

```
[Out] Piecewise((f**(a + b*x**3)*(b*x**3*log(f) - 1)/(3*b**2*log(f)**2), Ne(3*b**2*log(f)**2, 0)), (x**6/6, True))
```

**Giac [B]** time = 1.35529, size = 932, normalized size = 21.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^5,x, algorithm="giac")
```

```
[Out] 1/3*(2*((b*x^3*log(abs(f)) - 1)*(pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))
^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f)) ^2)^2 + 4*(pi*b^2*log(abs(f))
*sgn(f) - pi*b^2*log(abs(f)))^2) + (pi*b*x^3*sgn(f) - pi*b*x^3)*(pi*b^2*log(abs(f))
*sgn(f) - pi*b^2*log(abs(f)))/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f)) ^2)
^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2))*cos(-1/2*pi*b*x^3*sgn(f)
+ 1/2*pi*b*x^3 - 1/2*pi*a*sgn(f) + 1/2*pi*a) + ((pi*b*x^3*sgn(f) - pi*b*x^3)*(pi^2*b^2*sgn(f)
- pi^2*b^2 + 2*b^2*log(abs(f)) ^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))
^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) - 4*(b*x^3*log(abs(f))
- 1)*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))/((pi^2*b^2*sgn(f) - pi^2*b^2
+ 2*b^2*log(abs(f)) ^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2))
*sin(-1/2*pi*b*x^3*sgn(f) + 1/2*pi*b*x^3 - 1/2*pi*a*sgn(f) + 1/2*pi*a))*e^(b*x^3*log(abs(f))
+ a*log(abs(f))) - 1/6*((2*b*i*x^3*log(abs(f)) - pi*b*x^3*sgn(f) + pi*b*x^3 - 2*i)*e^(1/2*(pi*b*x^3*(sgn(f)
- 1) + pi*a*(sgn(f) - 1))*i)/(2*pi*b^2*i*log(abs(f))*sgn(f) - 2*pi*b^2*i*log(abs(f))
+ pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f)) ^2) + (2*b*i*x^3*log(abs(f)) + pi*b*x^3*sgn(f)
- pi*b*x^3 - 2*i)*e^(-1/2*(pi*b*x^3*(sgn(f) - 1) + pi*a*(sgn(f) - 1))*i)/(2*pi*b^2*i*log(abs(f))*sgn(f)
- 2*pi*b^2*i*log(abs(f)) - pi^2*b^2*sgn(f) + pi^2*b^2 - 2*b^2*log(abs(f)) ^2))*e^(b*x^3*log(abs(f))
+ a*log(abs(f)))/i
```

### 3.101 $\int f^{a+bx^3} x^2 dx$

**Optimal.** Leaf size=20

$$\frac{f^{a+bx^3}}{3b \log(f)}$$

[Out]  $f^{(a + b*x^3)/(3*b*Log[f])}$

**Rubi [A]** time = 0.0220422, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$\frac{f^{a+bx^3}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*x^3)}*x^2, x]$

[Out]  $f^{(a + b*x^3)/(3*b*Log[f])}$

#### Rule 2209

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[\frac{(e + f*x)^n * F^{(a + b*(c + d*x)^n)}}{(b*f*n*(c + d*x)^n * \text{Log}[F])}, x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\int f^{a+bx^3} x^2 dx = \frac{f^{a+bx^3}}{3b \log(f)}$$

**Mathematica [A]** time = 0.0024945, size = 20, normalized size = 1.

$$\frac{f^{a+bx^3}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[f^{(a + b*x^3)}*x^2, x]$

[Out]  $f^{(a + b*x^3)/(3*b*Log[f])}$

**Maple [A]** time = 0.003, size = 19, normalized size = 1.

$$\frac{f^{bx^3+a}}{3b \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)*x^2,x)`

[Out]  $1/3*f^{(b*x^3+a)}/b/\ln(f)$

**Maxima [A]** time = 1.07176, size = 24, normalized size = 1.2

$$\frac{f^{bx^3+a}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^2,x, algorithm="maxima")`

[Out]  $1/3*f^{(b*x^3 + a)}/(b*\log(f))$

**Fricas [A]** time = 1.89043, size = 41, normalized size = 2.05

$$\frac{f^{bx^3+a}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)*x^2,x, algorithm="fricas")`

[Out]  $1/3*f^{(b*x^3 + a)}/(b*\log(f))$

**Sympy [A]** time = 0.109959, size = 24, normalized size = 1.2

$$\begin{cases} \frac{f^{a+bx^3}}{3b \log(f)} & \text{for } 3b \log(f) \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)*x**2,x)`

[Out] `Piecewise((f**(a + b*x**3)/(3*b*log(f)), Ne(3*b*log(f), 0)), (x**3/3, True))`

**Giac [A]** time = 1.24623, size = 24, normalized size = 1.2

$$\frac{f^{bx^3+a}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*f^(b*x^3 + a)/(b*log(f))
```

$$3.102 \quad \int \frac{f^{a+bx^3}}{x} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{3}f^a \operatorname{Ei}(bx^3 \log(f))$$

[Out] (f^a\*ExpIntegralEi[b\*x^3\*Log[f]])/3

**Rubi [A]** time = 0.0206387, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$\frac{1}{3}f^a \operatorname{Ei}(bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x, x]

[Out] (f^a\*ExpIntegralEi[b\*x^3\*Log[f]])/3

**Rule 2210**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^3}}{x} dx = \frac{1}{3}f^a \operatorname{Ei}(bx^3 \log(f))$$

**Mathematica [A]** time = 0.001954, size = 15, normalized size = 1.

$$\frac{1}{3}f^a \operatorname{Ei}(bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x, x]

[Out] (f^a\*ExpIntegralEi[b\*x^3\*Log[f]])/3

**Maple [B]** time = 0.017, size = 41, normalized size = 2.7

$$\frac{f^a \left( 3 \ln(x) + \ln(-b) + \ln(\ln(f)) - \ln(-bx^3 \ln(f)) - \operatorname{Ei}(1, -bx^3 \ln(f)) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(f^(b*x^3+a)/x,x)`

[Out] `1/3*f^a*(3*ln(x)+ln(-b)+ln(ln(f))-ln(-b*x^3*ln(f))-Ei(1,-b*x^3*ln(f)))`

**Maxima [A]** time = 1.21716, size = 18, normalized size = 1.2

$$\frac{1}{3} f^a \operatorname{Ei}(bx^3 \log(f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x,x, algorithm="maxima")`

[Out] `1/3*f^a*Ei(b*x^3*log(f))`

**Fricas [A]** time = 1.70961, size = 35, normalized size = 2.33

$$\frac{1}{3} f^a \operatorname{Ei}(bx^3 \log(f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x,x, algorithm="fricas")`

[Out] `1/3*f^a*Ei(b*x^3*log(f))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)/x,x)`

[Out] `Integral(f**(a + b*x**3)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x,x, algorithm="giac")`

[Out] `integrate(f^(b*x^3 + a)/x, x)`

### 3.103 $\int \frac{f^{a+bx^3}}{x^4} dx$

**Optimal.** Leaf size=35

$$\frac{1}{3}bf^a \log(f)\text{Ei}(bx^3 \log(f)) - \frac{f^{a+bx^3}}{3x^3}$$

[Out]  $-f^{(a + b*x^3)}/(3*x^3) + (b*f^a*\text{ExpIntegralEi}[b*x^3*\text{Log}[f]]*\text{Log}[f])/3$

**Rubi [A]** time = 0.0420116, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{3}bf^a \log(f)\text{Ei}(bx^3 \log(f)) - \frac{f^{a+bx^3}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^4,x]

[Out]  $-f^{(a + b*x^3)}/(3*x^3) + (b*f^a*\text{ExpIntegralEi}[b*x^3*\text{Log}[f]]*\text{Log}[f])/3$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^3}}{x^4} dx &= -\frac{f^{a+bx^3}}{3x^3} + (b \log(f)) \int \frac{f^{a+bx^3}}{x} dx \\ &= -\frac{f^{a+bx^3}}{3x^3} + \frac{1}{3}bf^a \text{Ei}(bx^3 \log(f)) \log(f) \end{aligned}$$

**Mathematica [A]** time = 0.0092715, size = 32, normalized size = 0.91

$$\frac{1}{3}f^a \left( b \log(f) \text{Ei}(bx^3 \log(f)) - \frac{f^{bx^3}}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^4,x]

[Out]  $(f^a * (-f^{(b*x^3)/x^3} + b * \text{ExpIntegralEi}[b*x^3 * \text{Log}[f]] * \text{Log}[f])) / 3$

**Maple [B]** time = 0.027, size = 97, normalized size = 2.8

$$-\frac{f^a b \ln(f)}{3} \left( \frac{1}{bx^3 \ln(f)} + 1 - 3 \ln(x) - \ln(-b) - \ln(\ln(f)) - \frac{2 + 2bx^3 \ln(f)}{2bx^3 \ln(f)} + \frac{e^{bx^3 \ln(f)}}{bx^3 \ln(f)} + \ln(-bx^3 \ln(f)) + \text{Ei}\left(\frac{bx^3 \ln(f)}{\ln(f)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(f^{(b*x^3+a)}/x^4, x)$

[Out]  $-1/3*f^a*b*\ln(f)*(1/x^3/b/\ln(f)+1-3*\ln(x)-\ln(-b)-\ln(\ln(f))-1/2/b/x^3/\ln(f)*(2+2*b*x^3*\ln(f))+1/b/x^3/\ln(f)*\exp(b*x^3*\ln(f))+\ln(-b*x^3*\ln(f))+\text{Ei}(1,-b*x^3*\ln(f)))$

**Maxima [A]** time = 1.23272, size = 24, normalized size = 0.69

$$\frac{1}{3} b f^a \Gamma(-1, -bx^3 \log(f)) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x^3+a)}/x^4, x, \text{algorithm}="maxima")$

[Out]  $1/3*b*f^a*\text{gamma}(-1, -b*x^3*\log(f))*\log(f)$

**Fricas [A]** time = 1.79514, size = 82, normalized size = 2.34

$$\frac{b f^a x^3 \text{Ei}(bx^3 \log(f)) \log(f) - f^{bx^3+a}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x^3+a)}/x^4, x, \text{algorithm}="fricas")$

[Out]  $1/3*(b*f^a*x^3*\text{Ei}(b*x^3*\log(f))*\log(f) - f^{(b*x^3 + a)})/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x**3+a)}/x**4, x)$

[Out]  $\text{Integral}(f^{(a + b*x**3)}/x**4, x)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^4, x)
```

### 3.104 $\int \frac{f^{a+bx^3}}{x^7} dx$

**Optimal.** Leaf size=58

$$\frac{1}{6}b^2 f^a \log^2(f) \text{Ei}(bx^3 \log(f)) - \frac{f^{a+bx^3}}{6x^6} - \frac{b \log(f) f^{a+bx^3}}{6x^3}$$

[Out]  $-f^{(a + b*x^3)}/(6*x^6) - (b*f^{(a + b*x^3)}*Log[f])/(6*x^3) + (b^2*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^2)/6$

**Rubi [A]** time = 0.0658597, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{6}b^2 f^a \log^2(f) \text{Ei}(bx^3 \log(f)) - \frac{f^{a+bx^3}}{6x^6} - \frac{b \log(f) f^{a+bx^3}}{6x^3}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^7, x]

[Out]  $-f^{(a + b*x^3)}/(6*x^6) - (b*f^{(a + b*x^3)}*Log[f])/(6*x^3) + (b^2*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^2)/6$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^3}}{x^7} dx &= -\frac{f^{a+bx^3}}{6x^6} + \frac{1}{2}(b \log(f)) \int \frac{f^{a+bx^3}}{x^4} dx \\ &= -\frac{f^{a+bx^3}}{6x^6} - \frac{b f^{a+bx^3} \log(f)}{6x^3} + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+bx^3}}{x} dx \\ &= -\frac{f^{a+bx^3}}{6x^6} - \frac{b f^{a+bx^3} \log(f)}{6x^3} + \frac{1}{6}b^2 f^a \text{Ei}(bx^3 \log(f)) \log^2(f) \end{aligned}$$

**Mathematica [A]** time = 0.0185699, size = 48, normalized size = 0.83

$$\frac{f^a (b^2 x^6 \log^2(f) \text{Ei}(bx^3 \log(f)) - f^{bx^3} (bx^3 \log(f) + 1))}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^7,x]

[Out] (f^a\*(b^2\*x^6\*ExpIntegralEi[b\*x^3\*Log[f]]\*Log[f]^2 - f^(b\*x^3)\*(1 + b\*x^3\*Log[f])))/(6\*x^6)

**Maple [B]** time = 0.035, size = 141, normalized size = 2.4

$$\frac{f^a b^2 (\ln(f))^2}{3} \left( -\frac{1}{2 b^2 x^6 (\ln(f))^2} - \frac{1}{b x^3 \ln(f)} - \frac{3}{4} + \frac{3 \ln(x)}{2} + \frac{\ln(-b)}{2} + \frac{\ln(\ln(f))}{2} + \frac{9 b^2 x^6 (\ln(f))^2 + 12 b x^3 \ln(f)}{12 b^2 x^6 (\ln(f))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)/x^7,x)

[Out] 1/3\*f^a\*b^2\*ln(f)^2\*(-1/2/x^6/b^2/ln(f)^2-1/x^3/b/ln(f)-3/4+3/2\*ln(x)+1/2\*ln(-b)+1/2\*ln(ln(f))+1/12/b^2/x^6/ln(f)^2\*(9\*b^2\*x^6\*ln(f)^2+12\*b\*x^3\*ln(f)+6)-1/6/b^2/x^6/ln(f)^2\*(3+3\*b\*x^3\*ln(f))\*exp(b\*x^3\*ln(f))-1/2\*ln(-b\*x^3\*ln(f))-1/2\*Ei(1,-b\*x^3\*ln(f)))

**Maxima [A]** time = 1.17171, size = 30, normalized size = 0.52

$$-\frac{1}{3} b^2 f^a \Gamma(-2, -b x^3 \log(f)) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)/x^7,x, algorithm="maxima")

[Out] -1/3\*b^2\*f^a\*gamma(-2, -b\*x^3\*log(f))\*log(f)^2

**Fricas [A]** time = 1.76726, size = 113, normalized size = 1.95

$$\frac{b^2 f^a x^6 \text{Ei}(b x^3 \log(f)) \log(f)^2 - (b x^3 \log(f) + 1) f^{b x^3 + a}}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)/x^7,x, algorithm="fricas")

[Out] 1/6\*(b^2\*f^a\*x^6\*Ei(b\*x^3\*log(f))\*log(f)^2 - (b\*x^3\*log(f) + 1)\*f^(b\*x^3 + a))/x^6

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**3+a)/x**7,x)
```

```
[Out] Integral(f**(a + b*x**3)/x**7, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^7,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^7, x)
```

### 3.105 $\int \frac{f^{a+bx^3}}{x^{10}} dx$

**Optimal.** Leaf size=81

$$\frac{1}{18} b^3 f^a \log^3(f) \text{Ei}(bx^3 \log(f)) - \frac{b^2 \log^2(f) f^{a+bx^3}}{18x^3} - \frac{f^{a+bx^3}}{9x^9} - \frac{b \log(f) f^{a+bx^3}}{18x^6}$$

[Out]  $-f^{(a + b*x^3)}/(9*x^9) - (b*f^{(a + b*x^3)}*Log[f])/(18*x^6) - (b^2*f^{(a + b*x^3)}*Log[f]^2)/(18*x^3) + (b^3*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^3)/18$

**Rubi [A]** time = 0.0897108, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{18} b^3 f^a \log^3(f) \text{Ei}(bx^3 \log(f)) - \frac{b^2 \log^2(f) f^{a+bx^3}}{18x^3} - \frac{f^{a+bx^3}}{9x^9} - \frac{b \log(f) f^{a+bx^3}}{18x^6}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^10, x]

[Out]  $-f^{(a + b*x^3)}/(9*x^9) - (b*f^{(a + b*x^3)}*Log[f])/(18*x^6) - (b^2*f^{(a + b*x^3)}*Log[f]^2)/(18*x^3) + (b^3*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^3)/18$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx^3}}{x^{10}} dx &= -\frac{f^{a+bx^3}}{9x^9} + \frac{1}{3}(b \log(f)) \int \frac{f^{a+bx^3}}{x^7} dx \\ &= -\frac{f^{a+bx^3}}{9x^9} - \frac{b f^{a+bx^3} \log(f)}{18x^6} + \frac{1}{6}(b^2 \log^2(f)) \int \frac{f^{a+bx^3}}{x^4} dx \\ &= -\frac{f^{a+bx^3}}{9x^9} - \frac{b f^{a+bx^3} \log(f)}{18x^6} - \frac{b^2 f^{a+bx^3} \log^2(f)}{18x^3} + \frac{1}{6}(b^3 \log^3(f)) \int \frac{f^{a+bx^3}}{x} dx \\ &= -\frac{f^{a+bx^3}}{9x^9} - \frac{b f^{a+bx^3} \log(f)}{18x^6} - \frac{b^2 f^{a+bx^3} \log^2(f)}{18x^3} + \frac{1}{18} b^3 f^a \text{Ei}(bx^3 \log(f)) \log^3(f) \end{aligned}$$

**Mathematica [A]** time = 0.023417, size = 59, normalized size = 0.73

$$\frac{f^a (b^3 x^9 \log^3(f) \text{Ei}(bx^3 \log(f)) - f^{bx^3} (b^2 x^6 \log^2(f) + bx^3 \log(f) + 2))}{18x^9}$$



Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^10,x]

[Out] (f^a\*(b^3\*x^9\*ExpIntegralEi[b\*x^3\*Log[f]]\*Log[f]^3 - f^(b\*x^3)\*(2 + b\*x^3\*Log[f] + b^2\*x^6\*Log[f]^2)))/(18\*x^9)

**Maple [B]** time = 0.041, size = 177, normalized size = 2.2

$$\frac{f^a b^3 (\ln(f))^3}{3} \left( \frac{1}{3 b^3 x^9 (\ln(f))^3} + \frac{1}{2 b^2 x^6 (\ln(f))^2} + \frac{1}{2 b x^3 \ln(f)} + \frac{11}{36} - \frac{\ln(x)}{2} - \frac{\ln(-b)}{6} - \frac{\ln(\ln(f))}{6} - \frac{22 b^3 x^9}{18 x^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)/x^10,x)

[Out] -1/3\*f^a\*b^3\*ln(f)^3\*(1/3/x^9/b^3/ln(f)^3+1/2/x^6/b^2/ln(f)^2+1/2/x^3/b/ln(f)+11/36-1/2\*ln(x)-1/6\*ln(-b)-1/6\*ln(ln(f))-1/72/b^3/x^9/ln(f)^3\*(22\*b^3\*x^9\*ln(f)^3+36\*b^2\*x^6\*ln(f)^2+36\*b\*x^3\*ln(f)+24)+1/24/b^3/x^9/ln(f)^3\*(4\*b^2\*x^6\*ln(f)^2+4\*b\*x^3\*ln(f)+8)\*exp(b\*x^3\*ln(f))+1/6\*ln(-b\*x^3\*ln(f))+1/6\*Ei(1,-b\*x^3\*ln(f)))

**Maxima [A]** time = 1.18032, size = 30, normalized size = 0.37

$$\frac{1}{3} b^3 f^a \Gamma(-3, -b x^3 \log(f)) \log(f)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)/x^10,x, algorithm="maxima")

[Out] 1/3\*b^3\*f^a\*gamma(-3, -b\*x^3\*log(f))\*log(f)^3

**Fricas [A]** time = 1.73534, size = 140, normalized size = 1.73

$$\frac{b^3 f^a x^9 \text{Ei}(b x^3 \log(f)) \log(f)^3 - (b^2 x^6 \log(f)^2 + b x^3 \log(f) + 2) f^{b x^3 + a}}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)/x^10,x, algorithm="fricas")

[Out] 1/18\*(b^3\*f^a\*x^9\*Ei(b\*x^3\*log(f))\*log(f)^3 - (b^2\*x^6\*log(f)^2 + b\*x^3\*log(f) + 2)\*f^(b\*x^3 + a))/x^9

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x**3+a)/x**10,x)
```

```
[Out] Integral(f**(a + b*x**3)/x**10, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^10,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^10, x)
```

$$3.106 \quad \int \frac{f^{a+bx^3}}{x^{13}} dx$$

**Optimal.** Leaf size=24

$$-\frac{1}{3}b^4 f^a \log^4(f) \Gamma(-4, -bx^3 \log(f))$$

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^3 \log[f])]) \log[f]^4 / 3$

**Rubi [A]** time = 0.0209979, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{3}b^4 f^a \log^4(f) \Gamma(-4, -bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^13, x]

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^3 \log[f])]) \log[f]^4 / 3$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^3}}{x^{13}} dx = -\frac{1}{3}b^4 f^a \Gamma(-4, -bx^3 \log(f)) \log^4(f)$$

**Mathematica [A]** time = 0.0023959, size = 24, normalized size = 1.

$$-\frac{1}{3}b^4 f^a \log^4(f) \Gamma(-4, -bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^13, x]

[Out]  $-(b^4 f^a \Gamma[-4, -(b x^3 \log[f])]) \log[f]^4 / 3$

**Maple [B]** time = 0.054, size = 213, normalized size = 8.9

$$\frac{f^a b^4 (\ln(f))^4}{3} \left( -\frac{1}{4 b^4 x^{12} (\ln(f))^4} - \frac{1}{3 b^3 x^9 (\ln(f))^3} - \frac{1}{4 b^2 x^6 (\ln(f))^2} - \frac{1}{6 b x^3 \ln(f)} - \frac{25}{288} + \frac{\ln(x)}{8} + \frac{\ln(-b)}{24} + \frac{\ln(\dots)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(f^{(b*x^3+a)}/x^{13},x)$

[Out]  $\frac{1}{3}f^a b^4 \ln(f)^4 \left( -\frac{1}{4}x^{12}/b^4/\ln(f)^4 - \frac{1}{3}x^9/b^3/\ln(f)^3 - \frac{1}{4}x^6/b^2/\ln(f)^2 - \frac{1}{6}x^3/b/\ln(f) - \frac{25}{288} + \frac{1}{8}\ln(x) + \frac{1}{24}\ln(-b) + \frac{1}{24}\ln(\ln(f)) + \frac{1}{1440}b^4/x^{12}/\ln(f)^4 \right) + \frac{125b^4x^{12}\ln(f)^4 + 240b^3x^9\ln(f)^3 + 360b^2x^6\ln(f)^2 + 480bx^3\ln(f) + 360}{120b^4x^{12}/\ln(f)^4} + \frac{5b^3x^9\ln(f)^3 + 5b^2x^6\ln(f)^2 + 10bx^3\ln(f) + 30}{120b^4x^{12}/\ln(f)^4} \exp(bx^3\ln(f)) - \frac{1}{24}\ln(-bx^3\ln(f)) - \frac{1}{24}\text{Ei}(1, -bx^3\ln(f))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x^3+a)}/x^{13},x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x^3+a)}/x^{13},x, \text{algorithm}=\text{"fricas"})$

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x**3+a)}/x**13,x)$

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(b*x^3+a)}/x^{13},x, \text{algorithm}=\text{"giac"})$

[Out]  $\text{integrate}(f^{(b*x^3 + a)}/x^{13}, x)$

$$3.107 \quad \int \frac{f^{a+bx^3}}{x^{16}} dx$$

**Optimal.** Leaf size=24

$$\frac{1}{3}b^5 f^a \log^5(f) \Gamma(-5, -bx^3 \log(f))$$

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^3\*Log[f])])\*Log[f]^5)/3

**Rubi [A]** time = 0.0209877, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{3}b^5 f^a \log^5(f) \Gamma(-5, -bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^16, x]

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^3\*Log[f])])\*Log[f]^5)/3

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^3}}{x^{16}} dx = \frac{1}{3}b^5 f^a \Gamma(-5, -bx^3 \log(f)) \log^5(f)$$

**Mathematica [A]** time = 0.0023457, size = 24, normalized size = 1.

$$\frac{1}{3}b^5 f^a \log^5(f) \Gamma(-5, -bx^3 \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^16, x]

[Out] (b^5\*f^a\*Gamma[-5, -(b\*x^3\*Log[f])])\*Log[f]^5)/3

**Maple [B]** time = 0.06, size = 249, normalized size = 10.4

$$\frac{f^a b^5 (\ln(f))^5}{3} \left( \frac{1}{5 b^5 x^{15} (\ln(f))^5} + \frac{1}{4 b^4 x^{12} (\ln(f))^4} + \frac{1}{6 b^3 x^9 (\ln(f))^3} + \frac{1}{12 b^2 x^6 (\ln(f))^2} + \frac{1}{24 b x^3 \ln(f)} + \frac{137}{7200} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)/x^16,x)`

[Out] 
$$-1/3*f^a*b^5*\ln(f)^5*(1/5/x^{15}/b^5/\ln(f)^5+1/4/x^{12}/b^4/\ln(f)^4+1/6/x^9/b^3/\ln(f)^3+1/12/x^6/b^2/\ln(f)^2+1/24/x^3/b/\ln(f))+137/7200-1/40*\ln(x)-1/120*\ln(-b)-1/120*\ln(\ln(f))-1/7200/b^5/x^{15}/\ln(f)^5*(137*b^5*x^{15}*\ln(f)^5+300*b^4*x^{12}*\ln(f)^4+600*b^3*x^9*\ln(f)^3+1200*b^2*x^6*\ln(f)^2+1800*b*x^3*\ln(f)+1440)+1/720/b^5/x^{15}/\ln(f)^5*(6*b^4*x^{12}*\ln(f)^4+6*b^3*x^9*\ln(f)^3+12*b^2*x^6*\ln(f)^2+36*b*x^3*\ln(f)+144)*\exp(b*x^3*\ln(f))+1/120*\ln(-b*x^3*\ln(f))+1/120*Ei(1,-b*x^3*\ln(f))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^16,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^16,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)/x**16,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^16,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^16, x)
```

### 3.108 $\int f^{a+bx^3} x^4 dx$

**Optimal.** Leaf size=34

$$-\frac{x^5 f^a \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{5/3}}$$

[Out]  $-(f^a x^5 \Gamma[5/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(5/3)})$

**Rubi [A]** time = 0.0229117, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{x^5 f^a \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^4,x]

[Out]  $-(f^a x^5 \Gamma[5/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(5/3)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x^4 dx = -\frac{f^a x^5 \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{5/3}}$$

**Mathematica [A]** time = 0.0056367, size = 34, normalized size = 1.

$$-\frac{x^5 f^a \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^4,x]

[Out]  $-(f^a x^5 \Gamma[5/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(5/3)})$



**Maple [B]** time = 0.022, size = 106, normalized size = 3.1

$$\frac{f^a}{3} \left( -\frac{2x^2\Gamma(2/3)}{3b} (-b)^{\frac{5}{3}} (\ln(f))^{\frac{2}{3}} (-bx^3 \ln(f))^{-\frac{2}{3}} + \frac{x^2 e^{bx^3 \ln(f)}}{b} (-b)^{\frac{5}{3}} (\ln(f))^{\frac{2}{3}} + \frac{2x^2}{3b} (-b)^{\frac{5}{3}} (\ln(f))^{\frac{2}{3}} \Gamma\left(\frac{2}{3}, -bx^3 \ln(f)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^4,x)

[Out] 1/3\*f^a/(-b)^(5/3)/ln(f)^(5/3)\*(-2/3\*x^2\*(-b)^(5/3)\*ln(f)^(2/3)/b\*GAMMA(2/3)/(-b\*x^3\*ln(f))^(2/3)+x^2\*(-b)^(5/3)\*ln(f)^(2/3)/b\*exp(b\*x^3\*ln(f))+2/3\*x^2\*(-b)^(5/3)\*ln(f)^(2/3)/b/(-b\*x^3\*ln(f))^(2/3)\*GAMMA(2/3,-b\*x^3\*ln(f))

**Maxima [A]** time = 1.20381, size = 38, normalized size = 1.12

$$\frac{f^a x^5 \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3 (-bx^3 \log(f))^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^4,x, algorithm="maxima")

[Out] -1/3\*f^a\*x^5\*gamma(5/3, -b\*x^3\*log(f))/(-b\*x^3\*log(f))^(5/3)

**Fricas [A]** time = 1.65594, size = 139, normalized size = 4.09

$$\frac{3bf^{bx^3+a}x^2 \log(f) - 2(-b \log(f))^{\frac{1}{3}} f^a \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right)}{9b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^4,x, algorithm="fricas")

[Out] 1/9\*(3\*b\*f^(b\*x^3 + a)\*x^2\*log(f) - 2\*(-b\*log(f))^(1/3)\*f^a\*gamma(2/3, -b\*x^3\*log(f)))/(b^2\*log(f)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^3} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*4,x)

[Out] Integral(f\*\*(a + b\*x\*\*3)\*x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^3+a} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^4,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)*x^4, x)
```

### 3.109 $\int f^{a+bx^3} x^3 dx$

**Optimal.** Leaf size=34

$$-\frac{x^4 f^a \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{4/3}}$$

[Out]  $-(f^a x^4 \Gamma[4/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(4/3)})$

**Rubi [A]** time = 0.0224622, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{x^4 f^a \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x^3,x]

[Out]  $-(f^a x^4 \Gamma[4/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(4/3)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x^3 dx = -\frac{f^a x^4 \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{4/3}}$$

**Mathematica [A]** time = 0.0056656, size = 34, normalized size = 1.

$$-\frac{x^4 f^a \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x^3,x]

[Out]  $-(f^a x^4 \Gamma[4/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(4/3)})$

**Maple [B]** time = 0.025, size = 109, normalized size = 3.2

$$-\frac{f^a}{3b} \left( -\frac{2x\pi\sqrt{3}}{9b\Gamma(2/3)} (-b)^{\frac{4}{3}} \sqrt[3]{\ln(f)} \frac{1}{\sqrt[3]{-bx^3 \ln(f)}} + \frac{xe^{bx^3 \ln(f)}}{b} (-b)^{\frac{4}{3}} \sqrt[3]{\ln(f)} + \frac{x}{3b} (-b)^{\frac{4}{3}} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -bx^3 \ln(f)\right) \right) \frac{1}{\sqrt[3]{-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x^3,x)

[Out]  $-1/3*f^a/b/\ln(f)^{(4/3)}/(-b)^{(1/3)}*(-2/9*x*(-b)^{(4/3)}*\ln(f)^{(1/3)}/b*\text{Pi}*3^{(1/2)}/\text{GAMMA}(2/3)/(-b*x^3*\ln(f))^{(1/3)}+x*(-b)^{(4/3)}*\ln(f)^{(1/3)}/b*\exp(b*x^3*\ln(f))+1/3*x*(-b)^{(4/3)}*\ln(f)^{(1/3)}/b/(-b*x^3*\ln(f))^{(1/3)}*\text{GAMMA}(1/3,-b*x^3*\ln(f))$

**Maxima [A]** time = 1.27188, size = 38, normalized size = 1.12

$$\frac{f^a x^4 \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3 (-bx^3 \log(f))^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^3,x, algorithm="maxima")

[Out]  $-1/3*f^a*x^4*\text{gamma}(4/3, -b*x^3*\log(f))/(-b*x^3*\log(f))^{(4/3)}$

**Fricas [A]** time = 1.82996, size = 134, normalized size = 3.94

$$\frac{3bf^{bx^3+a}x \log(f) - (-b \log(f))^{\frac{2}{3}} f^a \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{9b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x^3,x, algorithm="fricas")

[Out]  $1/9*(3*b*f^(b*x^3 + a)*x*\log(f) - (-b*\log(f))^{(2/3)}*f^a*\text{gamma}(1/3, -b*x^3*\log(f)))/(b^2*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^3} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x\*\*3,x)

[Out] Integral(f\*\*(a + b\*x\*\*3)\*x\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^3+a} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)*x^3, x)
```

### 3.110 $\int f^{a+bx^3} x dx$

**Optimal.** Leaf size=34

$$-\frac{x^2 f^a \text{Gamma}\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{2/3}}$$

[Out]  $-(f^a x^2 \text{Gamma}[2/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(2/3)})$

**Rubi [A]** time = 0.013284, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2218}

$$-\frac{x^2 f^a \text{Gamma}\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)\*x, x]

[Out]  $-(f^a x^2 \text{Gamma}[2/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(2/3)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^3} x dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{2/3}}$$

**Mathematica [A]** time = 0.0049437, size = 34, normalized size = 1.

$$-\frac{x^2 f^a \text{Gamma}\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3(-bx^3 \log(f))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)\*x, x]

[Out]  $-(f^a x^2 \text{Gamma}[2/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(2/3)})$

**Maple [B]** time = 0.016, size = 75, normalized size = 2.2

$$\frac{f^a}{3} \left( x^2 \Gamma\left(\frac{2}{3}\right) (-b)^{\frac{2}{3}} (\ln(f))^{\frac{2}{3}} (-bx^3 \ln(f))^{-\frac{2}{3}} - x^2 (-b)^{\frac{2}{3}} (\ln(f))^{\frac{2}{3}} \Gamma\left(\frac{2}{3}, -bx^3 \ln(f)\right) (-bx^3 \ln(f))^{-\frac{2}{3}} \right) (-b)^{-\frac{2}{3}} (\ln(f))^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b\*x^3+a)\*x, x)

[Out] 1/3\*f^a/(-b)^(2/3)/ln(f)^(2/3)\*(x^2\*(-b)^(2/3)\*ln(f)^(2/3)\*GAMMA(2/3)/(-b\*x^3\*ln(f))^(2/3)-x^2\*(-b)^(2/3)\*ln(f)^(2/3)/(-b\*x^3\*ln(f))^(2/3)\*GAMMA(2/3,-b\*x^3\*ln(f)))

**Maxima [A]** time = 1.29837, size = 38, normalized size = 1.12

$$\frac{f^a x^2 \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3 (-bx^3 \log(f))^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x, x, algorithm="maxima")

[Out] -1/3\*f^a\*x^2\*gamma(2/3, -b\*x^3\*log(f))/(-b\*x^3\*log(f))^(2/3)

**Fricas [A]** time = 1.70051, size = 86, normalized size = 2.53

$$\frac{(-b \log(f))^{\frac{1}{3}} f^a \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b\*x^3+a)\*x, x, algorithm="fricas")

[Out] 1/3\*(-b\*log(f))^(1/3)\*f^a\*gamma(2/3, -b\*x^3\*log(f))/(b\*log(f))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^3} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(b\*x\*\*3+a)\*x, x)

[Out] Integral(f\*\*(a + b\*x\*\*3)\*x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^3+a} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)*x,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)*x, x)
```



### 3.111 $\int f^{a+bx^3} dx$

**Optimal.** Leaf size=32

$$\frac{xf^a \text{Gamma}\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

[Out]  $-(f^a x \text{Gamma}[1/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(1/3)})$

**Rubi [A]** time = 0.0038035, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2208}

$$\frac{xf^a \text{Gamma}\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3), x]

[Out]  $-(f^a x \text{Gamma}[1/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(1/3)})$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := -Simp[(F^a \* (c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])]) / (d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int f^{a+bx^3} dx = -\frac{f^a x \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

**Mathematica [A]** time = 0.0043517, size = 32, normalized size = 1.

$$\frac{xf^a \text{Gamma}\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3), x]

[Out]  $-(f^a x \text{Gamma}[1/3, -(b x^3 \text{Log}[f])]) / (3 * (-(b x^3 \text{Log}[f]))^{(1/3)})$

**Maple [B]** time = 0.015, size = 78, normalized size = 2.4

$$\frac{f^a}{3} \left( \frac{2x\pi\sqrt{3}}{3\Gamma(2/3)} \sqrt[3]{-b} \sqrt[3]{\ln(f)} \frac{1}{\sqrt[3]{-bx^3 \ln(f)}} - x \sqrt[3]{-b} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -bx^3 \ln(f)\right) \frac{1}{\sqrt[3]{-bx^3 \ln(f)}} \right) \frac{1}{\sqrt[3]{-b}} \frac{1}{\sqrt[3]{\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a),x)`

[Out]  $\frac{1}{3}f^a/(-b)^{1/3}/\ln(f)^{1/3}*(2/3*x*(-b)^{1/3}*\ln(f)^{1/3}*Pi*3^{1/2})/GAMMA(2/3)/(-b*x^3*\ln(f))^{1/3}-x*(-b)^{1/3}*\ln(f)^{1/3}/(-b*x^3*\ln(f))^{1/3}*GAMMA(1/3,-b*x^3*\ln(f))$

**Maxima [A]** time = 1.28956, size = 35, normalized size = 1.09

$$-\frac{f^a x \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3 \left(-bx^3 \log(f)\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a),x, algorithm="maxima")`

[Out]  $-1/3*f^a*x*\gamma(1/3, -b*x^3*\log(f))/(-b*x^3*\log(f))^{1/3}$

**Fricas [A]** time = 1.78871, size = 86, normalized size = 2.69

$$\frac{(-b \log(f))^{\frac{2}{3}} f^a \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a),x, algorithm="fricas")`

[Out]  $1/3*(-b*\log(f))^{2/3}*f^a*\gamma(1/3, -b*x^3*\log(f))/(b*\log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a),x)`

[Out] `Integral(f**(a + b*x**3), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^3+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a), x)
```

$$3.112 \quad \int \frac{f^{a+bx^3}}{x^2} dx$$

**Optimal.** Leaf size=34

$$-\frac{f^a \sqrt[3]{-bx^3 \log(f)} \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

[Out]  $-(f^a \Gamma[-1/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(1/3)} / (3*x)$

**Rubi [A]** time = 0.0216656, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{f^a \sqrt[3]{-bx^3 \log(f)} \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^2,x]

[Out]  $-(f^a \Gamma[-1/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(1/3)} / (3*x)$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^3}}{x^2} dx = -\frac{f^a \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right) \sqrt[3]{-bx^3 \log(f)}}{3x}$$

**Mathematica [A]** time = 0.0037348, size = 34, normalized size = 1.

$$-\frac{f^a \sqrt[3]{-bx^3 \log(f)} \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^2,x]

[Out]  $-(f^a \Gamma[-1/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(1/3)} / (3*x)$

**Maple [B]** time = 0.022, size = 100, normalized size = 2.9

$$\frac{f^a}{3} \sqrt[3]{-b} \sqrt[3]{\ln(f)} \left( 3 \frac{x^2 (\ln(f))^{2/3} b \Gamma(2/3)}{\sqrt[3]{-b} (-bx^3 \ln(f))^{2/3}} - 3 \frac{e^{bx^3 \ln(f)}}{x \sqrt[3]{-b} \sqrt[3]{\ln(f)}} - 3 \frac{x^2 (\ln(f))^{2/3} b \Gamma(2/3, -bx^3 \ln(f))}{\sqrt[3]{-b} (-bx^3 \ln(f))^{2/3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)/x^2,x)`

[Out]  $\frac{1}{3}f^a(-b)^{1/3}\ln(f)^{1/3}\left(3x^2/(-b)^{1/3}\ln(f)^{2/3}b\Gamma(2/3)/(-bx^3\ln(f))^{2/3}-3/x/(-b)^{1/3}/\ln(f)^{1/3}\exp(bx^3\ln(f))-3x^2/(-b)^{1/3}\ln(f)^{2/3}b/(-bx^3\ln(f))^{2/3}\Gamma(2/3,-bx^3\ln(f))\right)$

**Maxima [A]** time = 1.23225, size = 38, normalized size = 1.12

$$\frac{(-bx^3 \log(f))^{\frac{1}{3}} f^a \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^2,x, algorithm="maxima")`

[Out]  $-1/3*(-bx^3*\log(f))^{1/3}*f^a*\gamma(-1/3, -bx^3*\log(f))/x$

**Fricas [A]** time = 1.81136, size = 96, normalized size = 2.82

$$\frac{(-b \log(f))^{\frac{1}{3}} f^a x \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right) - f^{bx^3+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^2,x, algorithm="fricas")`

[Out]  $((-b*\log(f))^{1/3}*f^a*x*\gamma(2/3, -bx^3*\log(f)) - f^{(bx^3 + a)})/x$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)/x**2,x)`

[Out] `Integral(f**(a + b*x**3)/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^2, x)
```

$$3.113 \quad \int \frac{f^{a+bx^3}}{x^3} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a (-bx^3 \log(f))^{2/3} \text{Gamma}\left(-\frac{2}{3}, -bx^3 \log(f)\right)}{3x^2}$$

[Out]  $-(f^a \text{Gamma}[-2/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(2/3)} / (3*x^2)$

**Rubi [A]** time = 0.0208568, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a (-bx^3 \log(f))^{2/3} \text{Gamma}\left(-\frac{2}{3}, -bx^3 \log(f)\right)}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^3)/x^3,x]

[Out]  $-(f^a \text{Gamma}[-2/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(2/3)} / (3*x^2)$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+bx^3}}{x^3} dx = -\frac{f^a \Gamma\left(-\frac{2}{3}, -bx^3 \log(f)\right) (-bx^3 \log(f))^{2/3}}{3x^2}$$

**Mathematica [A]** time = 0.003847, size = 34, normalized size = 1.

$$\frac{f^a (-bx^3 \log(f))^{2/3} \text{Gamma}\left(-\frac{2}{3}, -bx^3 \log(f)\right)}{3x^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^3)/x^3,x]

[Out]  $-(f^a \text{Gamma}[-2/3, -(b*x^3 \text{Log}[f])]) * (-(b*x^3 \text{Log}[f]))^{(2/3)} / (3*x^2)$

**Maple [B]** time = 0.023, size = 102, normalized size = 3.

$$-\frac{f^a b}{3} (\ln(f))^{\frac{2}{3}} \left( \frac{bx\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)} \sqrt[3]{\ln(f)} (-b)^{-\frac{2}{3}} \frac{1}{\sqrt[3]{-bx^3 \ln(f)}} - \frac{3e^{bx^3 \ln(f)}}{2x^2} (-b)^{-\frac{2}{3}} (\ln(f))^{-\frac{2}{3}} - \frac{3bx}{2} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -bx^3 \ln(f)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(b*x^3+a)/x^3,x)`

[Out] 
$$-1/3*f^a*b*\ln(f)^{(2/3)/(-b)^{(1/3)}*(x/(-b)^{(2/3)}*\ln(f)^{(1/3)*b*\text{Pi}*3^{(1/2)}/\text{GAMMA}(2/3)/(-b*x^3*\ln(f))^{(1/3)}-3/2/x^2/(-b)^{(2/3)}/\ln(f)^{(2/3)*\exp(b*x^3*\ln(f))-3/2*x/(-b)^{(2/3)}*\ln(f)^{(1/3)*b/(-b*x^3*\ln(f))^{(1/3)*\text{GAMMA}(1/3,-b*x^3*\ln(f))}}$$

**Maxima [A]** time = 1.21619, size = 38, normalized size = 1.12

$$\frac{(-bx^3 \log(f))^{\frac{2}{3}} f^a \Gamma\left(-\frac{2}{3}, -bx^3 \log(f)\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^3,x, algorithm="maxima")`

[Out] 
$$-1/3*(-b*x^3*\log(f))^{(2/3)}*f^a*\text{gamma}(-2/3, -b*x^3*\log(f))/x^2$$

**Fricas [A]** time = 1.71017, size = 107, normalized size = 3.15

$$\frac{(-b \log(f))^{\frac{2}{3}} f^a x^2 \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right) - f^{bx^3+a}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(b*x^3+a)/x^3,x, algorithm="fricas")`

[Out] 
$$1/2*((-b*\log(f))^{(2/3)}*f^a*x^2*\text{gamma}(1/3, -b*x^3*\log(f)) - f^{(b*x^3 + a)})/x^2$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^3}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x**3+a)/x**3,x)`

[Out] `Integral(f**(a + b*x**3)/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^3+a}}{x^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x^3+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^3 + a)/x^3, x)
```

### 3.114 $\int e^{4x^3} x^2 dx$

**Optimal.** Leaf size=11

$$\frac{e^{4x^3}}{12}$$

[Out]  $E^{(4*x^3)}/12$

**Rubi [A]** time = 0.0143383, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2209}

$$\frac{e^{4x^3}}{12}$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x^3)\*x^2,x]

[Out] E^(4\*x^3)/12

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x]
/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\int e^{4x^3} x^2 dx = \frac{e^{4x^3}}{12}$$

**Mathematica [A]** time = 0.001619, size = 11, normalized size = 1.

$$\frac{e^{4x^3}}{12}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x^3)\*x^2,x]

[Out] E^(4\*x^3)/12

**Maple [A]** time = 0.003, size = 9, normalized size = 0.8

$$\frac{e^{4x^3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x^3)\*x^2,x)

[Out] 1/12\*exp(4\*x^3)

**Maxima [A]** time = 1.18386, size = 11, normalized size = 1.

$$\frac{1}{12} e^{(4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x^3)\*x^2,x, algorithm="maxima")

[Out] 1/12\*e^(4\*x^3)

**Fricas [A]** time = 1.79657, size = 22, normalized size = 2.

$$\frac{1}{12} e^{(4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x^3)\*x^2,x, algorithm="fricas")

[Out] 1/12\*e^(4\*x^3)

**Sympy [A]** time = 0.084682, size = 7, normalized size = 0.64

$$\frac{e^{4x^3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x\*\*3)\*x\*\*2,x)

[Out] exp(4\*x\*\*3)/12

**Giac [A]** time = 1.24131, size = 11, normalized size = 1.

$$\frac{1}{12} e^{(4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x^3)\*x^2,x, algorithm="giac")

[Out] 1/12\*e^(4\*x^3)

### 3.115 $\int f^{a+\frac{b}{x}} x^m dx$

**Optimal.** Leaf size=35

$$f^a x^{m+1} \left( -\frac{b \log(f)}{x} \right)^{m+1} \text{Gamma} \left( -m-1, -\frac{b \log(f)}{x} \right)$$

[Out]  $f^a x^{(1+m)} \text{Gamma}[-1-m, -((b \text{Log}[f])/x)] * (-((b \text{Log}[f])/x))^{(1+m)}$

**Rubi [A]** time = 0.0179775, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$f^a x^{m+1} \left( -\frac{b \log(f)}{x} \right)^{m+1} \text{Gamma} \left( -m-1, -\frac{b \log(f)}{x} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)\*x^m, x]

[Out]  $f^a x^{(1+m)} \text{Gamma}[-1-m, -((b \text{Log}[f])/x)] * (-((b \text{Log}[f])/x))^{(1+m)}$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x}} x^m dx = f^a x^{1+m} \Gamma \left( -1-m, -\frac{b \log(f)}{x} \right) \left( -\frac{b \log(f)}{x} \right)^{1+m}$$

**Mathematica [A]** time = 0.0071684, size = 35, normalized size = 1.

$$f^a x^{m+1} \left( -\frac{b \log(f)}{x} \right)^{m+1} \text{Gamma} \left( -m-1, -\frac{b \log(f)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)\*x^m, x]

[Out]  $f^a x^{(1+m)} \text{Gamma}[-1-m, -((b \text{Log}[f])/x)] * (-((b \text{Log}[f])/x))^{(1+m)}$

**Maple [B]** time = 0.036, size = 136, normalized size = 3.9

$$f^a (-b)^m (\ln(f))^{1+m} b \left( -\frac{x^m (-b)^{-m} (\ln(f))^{-m} \Gamma(-m) \left( -\frac{b \ln(f)}{x} \right)^m}{1+m} + \frac{x^{1+m} (-b)^{-m} (\ln(f))^{-m-1} e^{\frac{b \ln(f)}{x}}}{(1+m)b} + \frac{x^m (-b)^{-m} (\ln(f))}{1+m} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x)*x^m,x)`

[Out]  $f^a(-b)^m \ln(f)^{(1+m)} b(-1/(1+m)) x^m (-b)^{-m} \ln(f)^{-m} \text{GAMMA}(-m) (-b \ln(f)/x)^{m+1/(1+m)} x^{(1+m)} (-b)^{-m} \ln(f)^{-m-1} / b \exp(b \ln(f)/x) + 1/(1+m) x^m (-b)^{-m} \ln(f)^{-m} (-b \ln(f)/x)^m \text{GAMMA}(-m, -b \ln(f)/x)$

**Maxima [A]** time = 1.27095, size = 47, normalized size = 1.34

$$f^a x^{m+1} \left( -\frac{b \log(f)}{x} \right)^{m+1} \Gamma \left( -m-1, -\frac{b \log(f)}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^m,x, algorithm="maxima")`

[Out]  $f^a x^{(m+1)} (-b \log(f)/x)^{(m+1)} \text{gamma}(-m-1, -b \log(f)/x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( f^{\frac{ax+b}{x}} x^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^m,x, algorithm="fricas")`

[Out] `integral(f^((a*x + b)/x)*x^m, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x)*x**m,x)`

[Out] `Integral(f**(a + b/x)*x**m, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^m,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x)*x^m, x)`

### 3.116 $\int f^{a+\frac{b}{x}} x^4 dx$

**Optimal.** Leaf size=22

$$-b^5 f^a \log^5(f) \text{Gamma}\left(-5, -\frac{b \log(f)}{x}\right)$$

[Out]  $-(b^5 f^a \text{Gamma}[-5, -(b \text{Log}[f])/x]) \text{Log}[f]^5$

**Rubi [A]** time = 0.0189867, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-b^5 f^a \log^5(f) \text{Gamma}\left(-5, -\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b/x)} x^4, x]$

[Out]  $-(b^5 f^a \text{Gamma}[-5, -(b \text{Log}[f])/x]) \text{Log}[f]^5$

#### Rule 2218

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((e_.) + (f_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow -\text{Simp}[(F^a*(e + f*x)^{(m + 1)}*\text{Gamma}[(m + 1)/n, -(b*(c + d*x)^n*\text{Log}[F]])]/(f*n*(-(b*(c + d*x)^n*\text{Log}[F]))^{((m + 1)/n)}), x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\int f^{a+\frac{b}{x}} x^4 dx = -b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x}\right) \log^5(f)$$

**Mathematica [A]** time = 0.0023878, size = 22, normalized size = 1.

$$-b^5 f^a \log^5(f) \text{Gamma}\left(-5, -\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[f^{(a + b/x)} x^4, x]$

[Out]  $-(b^5 f^a \text{Gamma}[-5, -(b \text{Log}[f])/x]) \text{Log}[f]^5$

**Maple [B]** time = 0.073, size = 121, normalized size = 5.5

$$\frac{f^a x^5}{5} f^{\frac{b}{x}} + \frac{b \ln(f) f^a x^4}{20} f^{\frac{b}{x}} + \frac{(\ln(f))^2 b^2 f^a x^3}{60} f^{\frac{b}{x}} + \frac{(\ln(f))^3 b^3 f^a x^2}{120} f^{\frac{b}{x}} + \frac{b^4 (\ln(f))^4 f^a x}{120} f^{\frac{b}{x}} + \frac{b^5 (\ln(f))^5 f^a}{120} \text{Ei}\left(1, -\frac{b \log(f)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x)*x^4,x)`

[Out]  $\frac{1}{5}f^a f^{b/x} x^5 + \frac{1}{20}b \ln(f) f^a f^{b/x} x^4 + \frac{1}{60}b^2 \ln(f)^2 f^a f^{b/x} x^3 + \frac{1}{120}b^3 \ln(f)^3 f^a f^{b/x} x^2 + \frac{1}{120}b^4 \ln(f)^4 f^a f^{b/x} x + \frac{1}{120}b^5 \ln(f)^5 f^a \operatorname{Ei}(1, -b \ln(f)/x)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^4,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^4,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x)*x**4,x)`

[Out] `Integral(f**(a + b/x)*x**4, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^4,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x)*x^4, x)`

### 3.117 $\int f^{a+\frac{b}{x}} x^3 dx$

**Optimal.** Leaf size=21

$$b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x}\right)$$

[Out]  $b^4 f^a \Gamma[-4, -(b \log[f])/x] \log[f]^4$

**Rubi [A]** time = 0.0198408, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)\*x^3,x]

[Out]  $b^4 f^a \Gamma[-4, -(b \log[f])/x] \log[f]^4$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :- Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x}} x^3 dx = b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x}\right) \log^4(f)$$

**Mathematica [A]** time = 0.0022257, size = 21, normalized size = 1.

$$b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)\*x^3,x]

[Out]  $b^4 f^a \Gamma[-4, -(b \log[f])/x] \log[f]^4$

**Maple [B]** time = 0.064, size = 99, normalized size = 4.7

$$\frac{f^a x^4}{4} f^{\frac{b}{x}} + \frac{b \ln(f) f^a x^3}{12} f^{\frac{b}{x}} + \frac{(\ln(f))^2 b^2 f^a x^2}{24} f^{\frac{b}{x}} + \frac{(\ln(f))^3 b^3 f^a x}{24} f^{\frac{b}{x}} + \frac{b^4 (\ln(f))^4 f^a}{24} \text{Ei}\left(1, -\frac{b \ln(f)}{x}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x)*x^3,x)`

[Out]  $\frac{1}{4}f^a f^{b/x} x^4 + \frac{1}{12}b \ln(f) f^a f^{b/x} x^3 + \frac{1}{24}b^2 \ln(f)^2 f^a f^{b/x} x^2 + \frac{1}{24}b^3 \ln(f)^3 f^a f^{b/x} x + \frac{1}{24}b^4 \ln(f)^4 f^a \operatorname{Ei}(1, -b \ln(f)/x)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^3,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^3,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x)*x**3,x)`

[Out] `Integral(f**(a + b/x)*x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)*x^3,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x)*x^3, x)`

### 3.118 $\int f^{a+\frac{b}{x}} x^2 dx$

**Optimal.** Leaf size=79

$$-\frac{1}{6}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) + \frac{1}{6}b^2 x \log^2(f) f^{a+\frac{b}{x}} + \frac{1}{3}x^3 f^{a+\frac{b}{x}} + \frac{1}{6}bx^2 \log(f) f^{a+\frac{b}{x}}$$

[Out] (f^(a + b/x)\*x^3)/3 + (b\*f^(a + b/x)\*x^2\*Log[f])/6 + (b^2\*f^(a + b/x)\*x\*Log[f]^2)/6 - (b^3\*f^a\*ExpIntegralEi[(b\*Log[f])/x]\*Log[f]^3)/6

**Rubi [A]** time = 0.0592223, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2214, 2206, 2210}

$$-\frac{1}{6}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) + \frac{1}{6}b^2 x \log^2(f) f^{a+\frac{b}{x}} + \frac{1}{3}x^3 f^{a+\frac{b}{x}} + \frac{1}{6}bx^2 \log(f) f^{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)\*x^2,x]

[Out] (f^(a + b/x)\*x^3)/3 + (b\*f^(a + b/x)\*x^2\*Log[f])/6 + (b^2\*f^(a + b/x)\*x\*Log[f]^2)/6 - (b^3\*f^a\*ExpIntegralEi[(b\*Log[f])/x]\*Log[f]^3)/6

#### Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

#### Rule 2206

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] := Simp[((c + d*x)*F^(a + b*(c + d*x)^n))/d, x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && I LtQ[n, 0]
```

#### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned}
\int f^{a+\frac{b}{x}} x^2 dx &= \frac{1}{3} f^{a+\frac{b}{x}} x^3 + \frac{1}{3} (b \log(f)) \int f^{a+\frac{b}{x}} x dx \\
&= \frac{1}{3} f^{a+\frac{b}{x}} x^3 + \frac{1}{6} b f^{a+\frac{b}{x}} x^2 \log(f) + \frac{1}{6} (b^2 \log^2(f)) \int f^{a+\frac{b}{x}} dx \\
&= \frac{1}{3} f^{a+\frac{b}{x}} x^3 + \frac{1}{6} b f^{a+\frac{b}{x}} x^2 \log(f) + \frac{1}{6} b^2 f^{a+\frac{b}{x}} x \log^2(f) + \frac{1}{6} (b^3 \log^3(f)) \int \frac{f^{a+\frac{b}{x}}}{x} dx \\
&= \frac{1}{3} f^{a+\frac{b}{x}} x^3 + \frac{1}{6} b f^{a+\frac{b}{x}} x^2 \log(f) + \frac{1}{6} b^2 f^{a+\frac{b}{x}} x \log^2(f) - \frac{1}{6} b^3 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \log^3(f)
\end{aligned}$$

**Mathematica [A]** time = 0.0238828, size = 53, normalized size = 0.67

$$\frac{1}{6} f^a \left( x f^{b/x} (b^2 \log^2(f) + b x \log(f) + 2x^2) - b^3 \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)\*x^2,x]

[Out] (f^a\*(-(b^3\*ExpIntegralEi[(b\*Log[f])/x]\*Log[f]^3) + f^(b/x)\*x\*(2\*x^2 + b\*x\*Log[f] + b^2\*Log[f]^2)))/6

**Maple [A]** time = 0.066, size = 77, normalized size = 1.

$$\frac{f^a x^3}{3} f^{\frac{b}{x}} + \frac{b \ln(f) f^a x^2}{6} f^{\frac{b}{x}} + \frac{(\ln(f))^2 b^2 f^a x}{6} f^{\frac{b}{x}} + \frac{(\ln(f))^3 b^3 f^a}{6} \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)\*x^2,x)

[Out] 1/3\*f^a\*f^(b/x)\*x^3+1/6\*b\*ln(f)\*f^a\*f^(b/x)\*x^2+1/6\*b^2\*ln(f)^2\*f^a\*f^(b/x)\*x+1/6\*b^3\*ln(f)^3\*f^a\*Ei(1,-b\*ln(f)/x)

**Maxima [A]** time = 1.16895, size = 30, normalized size = 0.38

$$-b^3 f^a \Gamma\left(-3, -\frac{b \log(f)}{x}\right) \log(f)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x^2,x, algorithm="maxima")

[Out] -b^3\*f^a\*gamma(-3, -b\*log(f)/x)\*log(f)^3

**Fricas [A]** time = 1.77556, size = 135, normalized size = 1.71

$$-\frac{1}{6} b^3 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \log(f)^3 + \frac{1}{6} (b^2 x \log(f)^2 + b x^2 \log(f) + 2x^3) f^{\frac{ax+b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x^2,x, algorithm="fricas")

[Out]  $-1/6*b^3*f^a*Ei(b*\log(f)/x)*\log(f)^3 + 1/6*(b^2*x*\log(f)^2 + b*x^2*\log(f) + 2*x^3)*f^{(a*x + b)/x}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}}x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)\*x\*\*2,x)

[Out] Integral(f\*\*(a + b/x)\*x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}}x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x^2,x, algorithm="giac")

[Out] integrate(f^(a + b/x)\*x^2, x)

### 3.119 $\int f^{a+\frac{b}{x}} x dx$

**Optimal.** Leaf size=56

$$-\frac{1}{2}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) + \frac{1}{2}x^2 f^{a+\frac{b}{x}} + \frac{1}{2}bx \log(f) f^{a+\frac{b}{x}}$$

[Out]  $(f^{(a + b/x)*x^2})/2 + (b*f^{(a + b/x)*x}*Log[f])/2 - (b^2*f^a*ExpIntegralEi[(b*Log[f])/x]*Log[f]^2)/2$

**Rubi [A]** time = 0.0349944, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2214, 2206, 2210}

$$-\frac{1}{2}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) + \frac{1}{2}x^2 f^{a+\frac{b}{x}} + \frac{1}{2}bx \log(f) f^{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)\*x,x]

[Out]  $(f^{(a + b/x)*x^2})/2 + (b*f^{(a + b/x)*x}*Log[f])/2 - (b^2*f^a*ExpIntegralEi[(b*Log[f])/x]*Log[f]^2)/2$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n] && LtQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x}} x dx &= \frac{1}{2}f^{a+\frac{b}{x}}x^2 + \frac{1}{2}(b \log(f)) \int f^{a+\frac{b}{x}} dx \\ &= \frac{1}{2}f^{a+\frac{b}{x}}x^2 + \frac{1}{2}bf^{a+\frac{b}{x}}x \log(f) + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+\frac{b}{x}}}{x} dx \\ &= \frac{1}{2}f^{a+\frac{b}{x}}x^2 + \frac{1}{2}bf^{a+\frac{b}{x}}x \log(f) - \frac{1}{2}b^2 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \log^2(f) \end{aligned}$$

**Mathematica [A]** time = 0.0174743, size = 40, normalized size = 0.71

$$\frac{1}{2}f^a \left( x f^{b/x} (b \log(f) + x) - b^2 \log^2(f) \operatorname{Ei} \left( \frac{b \log(f)}{x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)\*x,x]

[Out] (f^a\*(-(b^2\*ExpIntegralEi[(b\*Log[f])/x]\*Log[f]^2) + f^(b/x)\*x\*(x + b\*Log[f])))/2

**Maple [A]** time = 0.062, size = 55, normalized size = 1.

$$\frac{f^a x^2}{2} f^{\frac{b}{x}} + \frac{b \ln(f) f^a x}{2} f^{\frac{b}{x}} + \frac{(\ln(f))^2 b^2 f^a}{2} \operatorname{Ei} \left( 1, -\frac{b \ln(f)}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)\*x,x)

[Out] 1/2\*f^a\*f^(b/x)\*x^2+1/2\*b\*ln(f)\*f^a\*f^(b/x)\*x+1/2\*b^2\*ln(f)^2\*f^a\*Ei(1,-b\*ln(f)/x)

**Maxima [A]** time = 1.21554, size = 28, normalized size = 0.5

$$b^2 f^a \Gamma \left( -2, -\frac{b \log(f)}{x} \right) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x,x, algorithm="maxima")

[Out] b^2\*f^a\*gamma(-2, -b\*log(f)/x)\*log(f)^2

**Fricas [A]** time = 1.75354, size = 107, normalized size = 1.91

$$-\frac{1}{2} b^2 f^a \operatorname{Ei} \left( \frac{b \log(f)}{x} \right) \log(f)^2 + \frac{1}{2} (bx \log(f) + x^2) f^{\frac{ax+b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x,x, algorithm="fricas")

[Out] -1/2\*b^2\*f^a\*Ei(b\*log(f)/x)\*log(f)^2 + 1/2\*(b\*x\*log(f) + x^2)\*f^((a\*x + b)/x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)\*x,x)

[Out] Integral(f\*\*(a + b/x)\*x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)\*x,x, algorithm="giac")

[Out] integrate(f^(a + b/x)\*x, x)

### 3.120 $\int f^{a+\frac{b}{x}} dx$

**Optimal.** Leaf size=28

$$xf^{a+\frac{b}{x}} - bf^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

[Out]  $f^{(a + b/x)*x} - b*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x]*\operatorname{Log}[f]$

**Rubi [A]** time = 0.022346, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2206, 2210}

$$xf^{a+\frac{b}{x}} - bf^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x)}, x]$

[Out]  $f^{(a + b/x)*x} - b*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x]*\operatorname{Log}[f]$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.))}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[2/n] \ \&\& \ \operatorname{LtQ}[n, 0]$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.))}/((e_.) + (f_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]])/(f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x}} dx &= f^{a+\frac{b}{x}} x + (b \log(f)) \int \frac{f^{a+\frac{b}{x}}}{x} dx \\ &= f^{a+\frac{b}{x}} x - bf^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \log(f) \end{aligned}$$

**Mathematica [A]** time = 0.0070814, size = 28, normalized size = 1.

$$xf^{a+\frac{b}{x}} - bf^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[f^{(a + b/x)}, x]$

[Out]  $f^{(a + b/x)*x} - b*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x]*\operatorname{Log}[f]$



---

**Maple [A]** time = 0.059, size = 31, normalized size = 1.1

$$b \ln(f) f^a \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x}\right) + f^a f^{\frac{b}{x}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x), x)

[Out] b\*ln(f)\*f^a\*Ei(1, -b\*ln(f)/x)+f^a\*f^(b/x)\*x

---

**Maxima [A]** time = 1.36428, size = 24, normalized size = 0.86

$$-b f^a \Gamma\left(-1, -\frac{b \log(f)}{x}\right) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x), x, algorithm="maxima")

[Out] -b\*f^a\*gamma(-1, -b\*log(f)/x)\*log(f)

---

**Fricas [A]** time = 1.76561, size = 68, normalized size = 2.43

$$-b f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right) \log(f) + f^{\frac{ax+b}{x}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x), x, algorithm="fricas")

[Out] -b\*f^a\*Ei(b\*log(f)/x)\*log(f) + f^((a\*x + b)/x)\*x

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x), x)

[Out] Integral(f\*\*(a + b/x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x), x)
```

$$3.121 \quad \int \frac{f^{a+\frac{b}{x}}}{x} dx$$

**Optimal.** Leaf size=13

$$-f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x])$

**Rubi [A]** time = 0.0183578, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$-f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x])$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}/((e_.) + (f_.)*(x_)), x_ \text{ Symbol}] :> \operatorname{Simp}[F^a \operatorname{ExpIntegralEi}[b*(c + d*x)^n \operatorname{Log}[F]]/(f*n), x] /;$  Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x}}}{x} dx = -f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

**Mathematica [A]** time = 0.0019576, size = 13, normalized size = 1.

$$-f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[f^{(a + b/x)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x])$

**Maple [A]** time = 0.056, size = 15, normalized size = 1.2

$$f^a \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x)/x,x)`

[Out] `f^a*Ei(1,-b*ln(f)/x)`

**Maxima [A]** time = 1.19228, size = 18, normalized size = 1.38

$$-f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)/x,x, algorithm="maxima")`

[Out] `-f^a*Ei(b*log(f)/x)`

**Fricas [A]** time = 1.73477, size = 28, normalized size = 2.15

$$-f^a \operatorname{Ei}\left(\frac{b \log(f)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)/x,x, algorithm="fricas")`

[Out] `-f^a*Ei(b*log(f)/x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x)/x,x)`

[Out] `Integral(f**(a + b/x)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)/x,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x)/x, x)`

$$3.122 \quad \int \frac{f^{a+\frac{b}{x}}}{x^2} dx$$

**Optimal.** Leaf size=18

$$-\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

[Out]  $-(f^{(a + b/x)/(b*\text{Log}[f])})$

**Rubi [A]** time = 0.018051, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$-\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b/x)/x^2}, x]$

[Out]  $-(f^{(a + b/x)/(b*\text{Log}[f])})$

**Rule 2209**

$\text{Int}[(F\_)^{((a\_.) + (b\_.)*((c\_.) + (d\_.)*(x\_))^n)}*((e\_.) + (f\_.)*(x\_))^m], x\_Symbol] :> \text{Simp}[(e + f*x)^n * F^{(a + b*(c + d*x)^n)} / (b*f*n*(c + d*x)^n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x}}}{x^2} dx = -\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

**Mathematica [A]** time = 0.0033678, size = 18, normalized size = 1.

$$-\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[f^{(a + b/x)/x^2}, x]$

[Out]  $-(f^{(a + b/x)/(b*\text{Log}[f])})$

**Maple [A]** time = 0., size = 19, normalized size = 1.1

$$-\frac{1}{b \ln(f)} f^{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x)/x^2,x)`

[Out] `-f^(a+b/x)/b/ln(f)`

**Maxima [A]** time = 1.12925, size = 24, normalized size = 1.33

$$-\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)/x^2,x, algorithm="maxima")`

[Out] `-f^(a + b/x)/(b*log(f))`

**Fricas [A]** time = 1.69438, size = 39, normalized size = 2.17

$$-\frac{f^{\frac{ax+b}{x}}}{b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x)/x^2,x, algorithm="fricas")`

[Out] `-f^((a*x + b)/x)/(b*log(f))`

**Sympy [A]** time = 0.113452, size = 20, normalized size = 1.11

$$\begin{cases} -\frac{f^{a+\frac{b}{x}}}{b \log(f)} & \text{for } b \log(f) \neq 0 \\ -\frac{1}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x)/x**2,x)`

[Out] `Piecewise((-f**(a + b/x)/(b*log(f)), Ne(b*log(f), 0)), (-1/x, True))`

**Giac [A]** time = 1.29895, size = 24, normalized size = 1.33

$$-\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] -f^(a + b/x)/(b*log(f))
```

$$3.123 \quad \int \frac{f^{a+\frac{b}{x}}}{x^3} dx$$

**Optimal.** Leaf size=39

$$\frac{f^{a+\frac{b}{x}}}{b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx \log(f)}$$

[Out]  $f^{(a + b/x)/(b^2 * \text{Log}[f]^2)} - f^{(a + b/x)/(b * x * \text{Log}[f])}$

**Rubi [A]** time = 0.0364967, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{f^{a+\frac{b}{x}}}{b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)/x^3, x]

[Out]  $f^{(a + b/x)/(b^2 * \text{Log}[f]^2)} - f^{(a + b/x)/(b * x * \text{Log}[f])}$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x}}}{x^3} dx &= -\frac{f^{a+\frac{b}{x}}}{bx \log(f)} - \int \frac{f^{a+\frac{b}{x}}}{x^2} dx \\ &= \frac{f^{a+\frac{b}{x}}}{b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0060578, size = 27, normalized size = 0.69

$$\frac{f^{a+\frac{b}{x}}(x - b \log(f))}{b^2 x \log^2(f)}$$



Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)/x^3,x]

[Out] (f^(a + b/x)\*(x - b\*Log[f]))/(b^2\*x\*Log[f]^2)

**Maple [A]** time = 0.008, size = 49, normalized size = 1.3

$$\frac{1}{x^2} \left( \frac{x^2}{(\ln(f))^2 b^2} e^{\left(\frac{a+b}{x}\right) \ln(f)} - \frac{x}{b \ln(f)} e^{\left(\frac{a+b}{x}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)/x^3,x)

[Out] (1/b^2/ln(f)^2\*x^2\*exp((a+b/x)\*ln(f))-1/b/ln(f)\*x\*exp((a+b/x)\*ln(f)))/x^2

**Maxima [C]** time = 1.19299, size = 28, normalized size = 0.72

$$\frac{f^a \Gamma\left(2, -\frac{b \log(f)}{x}\right)}{b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^3,x, algorithm="maxima")

[Out] f^a\*gamma(2, -b\*log(f)/x)/(b^2\*log(f)^2)

**Fricas [A]** time = 1.89046, size = 68, normalized size = 1.74

$$-\frac{(b \log(f) - x) f^{\frac{ax+b}{x}}}{b^2 x \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^3,x, algorithm="fricas")

[Out] -(b\*log(f) - x)\*f^((a\*x + b)/x)/(b^2\*x\*log(f)^2)

**Sympy [A]** time = 0.11709, size = 22, normalized size = 0.56

$$\frac{f^{a+\frac{b}{x}} (-b \log(f) + x)}{b^2 x \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x)/x**3,x)
```

```
[Out] f**(a + b/x)*(-b*log(f) + x)/(b**2*x*log(f)**2)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x)/x^3, x)
```

$$3.124 \quad \int \frac{f^{a+\frac{b}{x}}}{x^4} dx$$

**Optimal.** Leaf size=61

$$\frac{2f^{a+\frac{b}{x}}}{b^2x \log^2(f)} - \frac{2f^{a+\frac{b}{x}}}{b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)}$$

[Out]  $(-2*f^{(a + b/x)})/(b^3*\text{Log}[f]^3) + (2*f^{(a + b/x)})/(b^2*x*\text{Log}[f]^2) - f^{(a + b/x)}/(b*x^2*\text{Log}[f])$

**Rubi [A]** time = 0.0604829, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{2f^{a+\frac{b}{x}}}{b^2x \log^2(f)} - \frac{2f^{a+\frac{b}{x}}}{b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)/x^4,x]

[Out]  $(-2*f^{(a + b/x)})/(b^3*\text{Log}[f]^3) + (2*f^{(a + b/x)})/(b^2*x*\text{Log}[f]^2) - f^{(a + b/x)}/(b*x^2*\text{Log}[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x}}}{x^4} dx &= -\frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)} - \frac{2 \int \frac{f^{a+\frac{b}{x}}}{x^3} dx}{b \log(f)} \\ &= \frac{2f^{a+\frac{b}{x}}}{b^2x \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)} + \frac{2 \int \frac{f^{a+\frac{b}{x}}}{x^2} dx}{b^2 \log^2(f)} \\ &= -\frac{2f^{a+\frac{b}{x}}}{b^3 \log^3(f)} + \frac{2f^{a+\frac{b}{x}}}{b^2x \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0081107, size = 41, normalized size = 0.67

$$\frac{f^{a+\frac{b}{x}} \left( b^2 \log^2(f) - 2bx \log(f) + 2x^2 \right)}{b^3 x^2 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)/x^4,x]

[Out] -((f^(a + b/x)\*(2\*x^2 - 2\*b\*x\*Log[f] + b^2\*Log[f]^2))/(b^3\*x^2\*Log[f]^3))

**Maple [A]** time = 0.012, size = 73, normalized size = 1.2

$$\frac{1}{x^3} \left( -2 \frac{x^3}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x}\right) \ln(f)} + 2 \frac{x^2}{(\ln(f))^2 b^2} e^{\left(\frac{a+b}{x}\right) \ln(f)} - \frac{x}{b \ln(f)} e^{\left(\frac{a+b}{x}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)/x^4,x)

[Out] (-2/b^3/ln(f)^3\*x^3\*exp((a+b/x)\*ln(f))+2/b^2/ln(f)^2\*x^2\*exp((a+b/x)\*ln(f))-1/b/ln(f)\*x\*exp((a+b/x)\*ln(f)))/x^3

**Maxima [C]** time = 1.27157, size = 30, normalized size = 0.49

$$\frac{f^a \Gamma\left(3, -\frac{b \log(f)}{x}\right)}{b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^4,x, algorithm="maxima")

[Out] -f^a\*gamma(3, -b\*log(f)/x)/(b^3\*log(f)^3)

**Fricas [A]** time = 1.72028, size = 101, normalized size = 1.66

$$\frac{\left( b^2 \log(f)^2 - 2bx \log(f) + 2x^2 \right) f^{\frac{ax+b}{x}}}{b^3 x^2 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^4,x, algorithm="fricas")

[Out] -(b^2\*log(f)^2 - 2\*b\*x\*log(f) + 2\*x^2)\*f^((a\*x + b)/x)/(b^3\*x^2\*log(f)^3)

**Sympy [A]** time = 0.133591, size = 39, normalized size = 0.64

$$\frac{f^{a+\frac{b}{x}} \left( -b^2 \log(f)^2 + 2bx \log(f) - 2x^2 \right)}{b^3 x^2 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)/x\*\*4,x)

[Out] f\*\*(a + b/x)\*(-b\*\*2\*log(f)\*\*2 + 2\*b\*x\*log(f) - 2\*x\*\*2)/(b\*\*3\*x\*\*2\*log(f)\*\*3)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^4,x, algorithm="giac")

[Out] integrate(f^(a + b/x)/x^4, x)

$$3.125 \quad \int \frac{f^{a+\frac{b}{x}}}{x^5} dx$$

**Optimal.** Leaf size=82

$$\frac{3f^{a+\frac{b}{x}}}{b^2x^2\log^2(f)} - \frac{6f^{a+\frac{b}{x}}}{b^3x\log^3(f)} + \frac{6f^{a+\frac{b}{x}}}{b^4\log^4(f)} - \frac{f^{a+\frac{b}{x}}}{bx^3\log(f)}$$

[Out]  $(6*f^{(a + b/x)})/(b^4*\text{Log}[f]^4) - (6*f^{(a + b/x)})/(b^3*x*\text{Log}[f]^3) + (3*f^{(a + b/x)})/(b^2*x^2*\text{Log}[f]^2) - f^{(a + b/x)}/(b*x^3*\text{Log}[f])$

**Rubi [A]** time = 0.0815529, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{3f^{a+\frac{b}{x}}}{b^2x^2\log^2(f)} - \frac{6f^{a+\frac{b}{x}}}{b^3x\log^3(f)} + \frac{6f^{a+\frac{b}{x}}}{b^4\log^4(f)} - \frac{f^{a+\frac{b}{x}}}{bx^3\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)/x^5,x]

[Out]  $(6*f^{(a + b/x)})/(b^4*\text{Log}[f]^4) - (6*f^{(a + b/x)})/(b^3*x*\text{Log}[f]^3) + (3*f^{(a + b/x)})/(b^2*x^2*\text{Log}[f]^2) - f^{(a + b/x)}/(b*x^3*\text{Log}[f])$

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x}}}{x^5} dx &= -\frac{f^{a+\frac{b}{x}}}{bx^3 \log(f)} - \frac{3 \int \frac{f^{a+\frac{b}{x}}}{x^4} dx}{b \log(f)} \\
&= \frac{3f^{a+\frac{b}{x}}}{b^2x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^3 \log(f)} + \frac{6 \int \frac{f^{a+\frac{b}{x}}}{x^3} dx}{b^2 \log^2(f)} \\
&= -\frac{6f^{a+\frac{b}{x}}}{b^3x \log^3(f)} + \frac{3f^{a+\frac{b}{x}}}{b^2x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^3 \log(f)} - \frac{6 \int \frac{f^{a+\frac{b}{x}}}{x^2} dx}{b^3 \log^3(f)} \\
&= \frac{6f^{a+\frac{b}{x}}}{b^4 \log^4(f)} - \frac{6f^{a+\frac{b}{x}}}{b^3x \log^3(f)} + \frac{3f^{a+\frac{b}{x}}}{b^2x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^3 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.010215, size = 53, normalized size = 0.65

$$\frac{f^{a+\frac{b}{x}} (3b^2x \log^2(f) - b^3 \log^3(f) - 6bx^2 \log(f) + 6x^3)}{b^4x^3 \log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)/x^5,x]

[Out] (f^(a + b/x)\*(6\*x^3 - 6\*b\*x^2\*Log[f] + 3\*b^2\*x\*Log[f]^2 - b^3\*Log[f]^3))/(b^4\*x^3\*Log[f]^4)

**Maple [A]** time = 0.013, size = 96, normalized size = 1.2

$$\frac{1}{x^4} \left( 6 \frac{x^4}{b^4 (\ln(f))^4} e^{\left(\frac{a+b}{x}\right) \ln(f)} - 6 \frac{x^3}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x}\right) \ln(f)} + 3 \frac{x^2}{(\ln(f))^2 b^2} e^{\left(\frac{a+b}{x}\right) \ln(f)} - \frac{x}{b \ln(f)} e^{\left(\frac{a+b}{x}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)/x^5,x)

[Out] (6/b^4/ln(f)^4\*x^4\*exp((a+b/x)\*ln(f))-6/b^3/ln(f)^3\*x^3\*exp((a+b/x)\*ln(f))+3/b^2/ln(f)^2\*x^2\*exp((a+b/x)\*ln(f))-1/b/ln(f)\*x\*exp((a+b/x)\*ln(f)))/x^4

**Maxima [C]** time = 1.22899, size = 28, normalized size = 0.34

$$\frac{f^a \Gamma\left(4, -\frac{b \log(f)}{x}\right)}{b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^5,x, algorithm="maxima")

[Out] f^a\*gamma(4, -b\*log(f)/x)/(b^4\*log(f)^4)

---

**Fricas [A]** time = 1.69579, size = 130, normalized size = 1.59

$$\frac{\left(b^3 \log(f)^3 - 3b^2x \log(f)^2 + 6bx^2 \log(f) - 6x^3\right) f^{\frac{ax+b}{x}}}{b^4x^3 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^5,x, algorithm="fricas")

[Out]  $-(b^3 \log(f)^3 - 3b^2x \log(f)^2 + 6bx^2 \log(f) - 6x^3) f^{(ax+b)/x} / (b^4x^3 \log(f)^4)$

---

**Sympy [A]** time = 0.145166, size = 53, normalized size = 0.65

$$\frac{f^{a+\frac{b}{x}} \left(-b^3 \log(f)^3 + 3b^2x \log(f)^2 - 6bx^2 \log(f) + 6x^3\right)}{b^4x^3 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)/x\*\*5,x)

[Out]  $f^{(a+b/x)} (-b^3 \log(f)^3 + 3b^2x \log(f)^2 - 6bx^2 \log(f) + 6x^3) / (b^4x^3 \log(f)^4)$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^5,x, algorithm="giac")

[Out] integrate(f^(a + b/x)/x^5, x)



$$3.126 \quad \int \frac{f^{a+\frac{b}{x}}}{x^6} dx$$

**Optimal.** Leaf size=65

$$\frac{f^{a+\frac{b}{x}} (12b^2x^2 \log^2(f) - 4b^3x \log^3(f) + b^4 \log^4(f) - 24bx^3 \log(f) + 24x^4)}{b^5x^4 \log^5(f)}$$

[Out] -((f^(a + b/x)\*(24\*x^4 - 24\*b\*x^3\*Log[f] + 12\*b^2\*x^2\*Log[f]^2 - 4\*b^3\*x\*Log[f]^3 + b^4\*Log[f]^4))/(b^5\*x^4\*Log[f]^5))

**Rubi [C]** time = 0.0183533, antiderivative size = 22, normalized size of antiderivative = 0.34, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x}\right)}{b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)/x^6, x]

[Out] -((f^a\*Gamma[5, -((b\*Log[f])/x)])/(b^5\*Log[f]^5))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x}}}{x^6} dx = -\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x}\right)}{b^5 \log^5(f)}$$

**Mathematica [C]** time = 0.0027207, size = 22, normalized size = 0.34

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x}\right)}{b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)/x^6, x]

[Out] -((f^a\*Gamma[5, -((b\*Log[f])/x)])/(b^5\*Log[f]^5))

**Maple [A]** time = 0.013, size = 119, normalized size = 1.8

$$\frac{1}{x^5} \left( -24 \frac{x^5}{b^5 (\ln(f))^5} e^{\left(\frac{a+b}{x}\right) \ln(f)} + 24 \frac{x^4}{b^4 (\ln(f))^4} e^{\left(\frac{a+b}{x}\right) \ln(f)} - 12 \frac{x^3}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x}\right) \ln(f)} + 4 \frac{x^2}{(\ln(f))^2 b^2} e^{\left(\frac{a+b}{x}\right) \ln(f)} - \frac{x}{\ln(f) b} e^{\left(\frac{a+b}{x}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)/x^6,x)

[Out] (-24/b^5/ln(f)^5\*x^5\*exp((a+b/x)\*ln(f))+24/b^4/ln(f)^4\*x^4\*exp((a+b/x)\*ln(f))-12/b^3/ln(f)^3\*x^3\*exp((a+b/x)\*ln(f))+4/b^2/ln(f)^2\*x^2\*exp((a+b/x)\*ln(f))-1/b/ln(f)\*x\*exp((a+b/x)\*ln(f)))/x^5

**Maxima [C]** time = 1.28846, size = 30, normalized size = 0.46

$$-\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x}\right)}{b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^6,x, algorithm="maxima")

[Out] -f^a\*gamma(5, -b\*log(f)/x)/(b^5\*log(f)^5)

**Fricas [A]** time = 1.8285, size = 162, normalized size = 2.49

$$\frac{\left(b^4 \log(f)^4 - 4b^3 x \log(f)^3 + 12b^2 x^2 \log(f)^2 - 24bx^3 \log(f) + 24x^4\right) f^{\frac{ax+b}{x}}}{b^5 x^4 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^6,x, algorithm="fricas")

[Out] -(b^4\*log(f)^4 - 4\*b^3\*x\*log(f)^3 + 12\*b^2\*x^2\*log(f)^2 - 24\*b\*x^3\*log(f) + 24\*x^4)\*f^((a\*x + b)/x)/(b^5\*x^4\*log(f)^5)

**Sympy [A]** time = 0.155604, size = 66, normalized size = 1.02

$$\frac{f^{a+\frac{b}{x}} \left(-b^4 \log(f)^4 + 4b^3 x \log(f)^3 - 12b^2 x^2 \log(f)^2 + 24bx^3 \log(f) - 24x^4\right)}{b^5 x^4 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)/x\*\*6,x)

```
[Out] f**(a + b/x)*(-b**4*log(f)**4 + 4*b**3*x*log(f)**3 - 12*b**2*x**2*log(f)**2
+ 24*b*x**3*log(f) - 24*x**4)/(b**5*x**4*log(f)**5)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x)/x^6, x)
```

$$3.127 \quad \int \frac{f^{a+\frac{b}{x}}}{x^7} dx$$

**Optimal.** Leaf size=77

$$\frac{f^{a+\frac{b}{x}} (60b^2x^3 \log^2(f) - 20b^3x^2 \log^3(f) + 5b^4x \log^4(f) - b^5 \log^5(f) - 120bx^4 \log(f) + 120x^5)}{b^6x^5 \log^6(f)}$$

[Out] (f^(a + b/x)\*(120\*x^5 - 120\*b\*x^4\*Log[f] + 60\*b^2\*x^3\*Log[f]^2 - 20\*b^3\*x^2\*Log[f]^3 + 5\*b^4\*x\*Log[f]^4 - b^5\*Log[f]^5))/(b^6\*x^5\*Log[f]^6)

**Rubi [C]** time = 0.0190488, antiderivative size = 21, normalized size of antiderivative = 0.27, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x}\right)}{b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x)/x^7, x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x))]/(b^6\*Log[f]^6)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x}}}{x^7} dx = \frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x}\right)}{b^6 \log^6(f)}$$

**Mathematica [C]** time = 0.002575, size = 21, normalized size = 0.27

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x}\right)}{b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x)/x^7, x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x))]/(b^6\*Log[f]^6)

**Maple [A]** time = 0.016, size = 142, normalized size = 1.8

$$\frac{1}{x^6} \left( 120 \frac{x^6}{b^6 (\ln(f))^6} e^{\left(\frac{a+b}{x}\right) \ln(f)} - 120 \frac{x^5}{b^5 (\ln(f))^5} e^{\left(\frac{a+b}{x}\right) \ln(f)} + 60 \frac{x^4}{b^4 (\ln(f))^4} e^{\left(\frac{a+b}{x}\right) \ln(f)} - 20 \frac{x^3}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x)/x^7,x)

[Out] (120/b^6/ln(f)^6\*x^6\*exp((a+b/x)\*ln(f))-120/b^5/ln(f)^5\*x^5\*exp((a+b/x)\*ln(f))+60/b^4/ln(f)^4\*x^4\*exp((a+b/x)\*ln(f))-20/b^3/ln(f)^3\*x^3\*exp((a+b/x)\*ln(f))+5/b^2/ln(f)^2\*x^2\*exp((a+b/x)\*ln(f))-1/b/ln(f)\*x\*exp((a+b/x)\*ln(f)))/x^6

**Maxima [C]** time = 1.16134, size = 28, normalized size = 0.36

$$\frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x}\right)}{b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^7,x, algorithm="maxima")

[Out] f^a\*gamma(6, -b\*log(f)/x)/(b^6\*log(f)^6)

**Fricas [A]** time = 1.93082, size = 194, normalized size = 2.52

$$\frac{\left(b^5 \log(f)^5 - 5b^4x \log(f)^4 + 20b^3x^2 \log(f)^3 - 60b^2x^3 \log(f)^2 + 120bx^4 \log(f) - 120x^5\right) f^{\frac{ax+b}{x}}}{b^6x^5 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x)/x^7,x, algorithm="fricas")

[Out] -(b^5\*log(f)^5 - 5\*b^4\*x\*log(f)^4 + 20\*b^3\*x^2\*log(f)^3 - 60\*b^2\*x^3\*log(f)^2 + 120\*b\*x^4\*log(f) - 120\*x^5)\*f^((a\*x + b)/x)/(b^6\*x^5\*log(f)^6)

**Sympy [A]** time = 0.162109, size = 80, normalized size = 1.04

$$\frac{f^{a+\frac{b}{x}} \left(-b^5 \log(f)^5 + 5b^4x \log(f)^4 - 20b^3x^2 \log(f)^3 + 60b^2x^3 \log(f)^2 - 120bx^4 \log(f) + 120x^5\right)}{b^6x^5 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x)/x\*\*7,x)

```
[Out] f**(a + b/x)*(-b**5*log(f)**5 + 5*b**4*x*log(f)**4 - 20*b**3*x**2*log(f)**3
+ 60*b**2*x**3*log(f)**2 - 120*b*x**4*log(f) + 120*x**5)/(b**6*x**5*log(f)
**6)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x)/x^7, x)
```

$$3.128 \quad \int f^{a+\frac{b}{x^2}} x^m dx$$

**Optimal.** Leaf size=46

$$\frac{1}{2} f^a x^{m+1} \left( -\frac{b \log(f)}{x^2} \right)^{\frac{m+1}{2}} \text{Gamma} \left( \frac{1}{2}(-m-1), -\frac{b \log(f)}{x^2} \right)$$

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^((1 + m)/2))/2

**Rubi [A]** time = 0.0236666, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{2} f^a x^{m+1} \left( -\frac{b \log(f)}{x^2} \right)^{\frac{m+1}{2}} \text{Gamma} \left( \frac{1}{2}(-m-1), -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^m,x]

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^((1 + m)/2))/2

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^2}} x^m dx = \frac{1}{2} f^a x^{1+m} \Gamma \left( \frac{1}{2}(-1-m), -\frac{b \log(f)}{x^2} \right) \left( -\frac{b \log(f)}{x^2} \right)^{\frac{1+m}{2}}$$

**Mathematica [A]** time = 0.0102872, size = 46, normalized size = 1.

$$\frac{1}{2} f^a x^{m+1} \left( -\frac{b \log(f)}{x^2} \right)^{\frac{m+1}{2}} \text{Gamma} \left( \frac{1}{2}(-m-1), -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^m,x]

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^((1 + m)/2))/2

**Maple [B]** time = 0.038, size = 169, normalized size = 3.7

$$-\frac{f^a}{2} (-b)^{\frac{m}{2}+\frac{1}{2}} (\ln(f))^{\frac{m}{2}+\frac{1}{2}} \left( 2 \frac{x^{-1+m} (-b)^{-m/2-1/2} (\ln(f))^{1/2-m/2} b \Gamma(1/2 - m/2) \left( -\frac{b \ln(f)}{x^2} \right)^{-1/2+m/2}}{1+m} - 2 \frac{x^{1+m} (-b)^{-m/2-1/2}}{1+m} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^m,x)

[Out]  $-1/2*f^a*(-b)^{(1/2*m+1/2)}*\ln(f)^{(1/2*m+1/2)}*(2/(1+m)*x^{(-1+m)}*(-b)^{(-1/2*m-1/2)}*\ln(f)^{(1/2-1/2*m)}*b*(-b*\ln(f)/x^2)^{(-1/2+1/2*m)}*GAMMA(1/2-1/2*m)-2/(1+m)*x^{(1+m)}*(-b)^{(-1/2*m-1/2)}*\ln(f)^{(-1/2*m-1/2)}*\exp(b*\ln(f)/x^2)-2/(1+m)*x^{(-1+m)}*(-b)^{(-1/2*m-1/2)}*\ln(f)^{(1/2-1/2*m)}*b*(-b*\ln(f)/x^2)^{(-1/2+1/2*m)}*GAMMA(1/2-1/2*m,-b*\ln(f)/x^2)$

**Maxima [A]** time = 1.3925, size = 51, normalized size = 1.11

$$\frac{1}{2} f^a x^{m+1} \left( -\frac{b \log(f)}{x^2} \right)^{\frac{1}{2} m + \frac{1}{2}} \Gamma \left( -\frac{1}{2} m - \frac{1}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^m,x, algorithm="maxima")

[Out]  $1/2*f^a*x^{(m+1)}*(-b*\log(f)/x^2)^{(1/2*m+1/2)}*\gamma(-1/2*m-1/2,-b*\log(f)/x^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( f^{\frac{ax^2+b}{x^2}} x^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^m,x, algorithm="fricas")

[Out] integral(f^((a\*x^2 + b)/x^2)\*x^m, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)\*x\*\*m,x)

[Out] Integral(f\*\*(a + b/x\*\*2)\*x\*\*m, x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^m,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)\*x^m, x)

$$3.129 \quad \int f^{a+\frac{b}{x^2}} x^9 dx$$

**Optimal.** Leaf size=24

$$-\frac{1}{2}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right)$$

[Out]  $-(b^5 f^a \Gamma[-5, -(b \text{Log}[f])/x^2]) \text{Log}[f]^5/2$

**Rubi [A]** time = 0.0247295, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{2}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^9, x]

[Out]  $-(b^5 f^a \Gamma[-5, -(b \text{Log}[f])/x^2]) \text{Log}[f]^5/2$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int f^{a+\frac{b}{x^2}} x^9 dx = -\frac{1}{2}b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right) \log^5(f)$$

**Mathematica [A]** time = 0.0023841, size = 24, normalized size = 1.

$$-\frac{1}{2}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^9, x]

[Out]  $-(b^5 f^a \Gamma[-5, -(b \text{Log}[f])/x^2]) \text{Log}[f]^5/2$

**Maple [B]** time = 0.063, size = 123, normalized size = 5.1

$$\frac{f^a x^{10}}{10} f^{\frac{b}{x^2}} + \frac{f^a \ln(f) b x^8}{40} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^2 b^2 x^6}{120} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^3 b^3 x^4}{240} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^4 b^4 x^2}{240} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^5 b^5}{240} \text{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)*x^9,x)`

[Out]  $\frac{1}{10}f^a x^{10} f^{b/x^2} + \frac{1}{40}f^a \ln(f) b x^8 f^{b/x^2} + \frac{1}{120}f^a \ln(f)^2 b^2 x^6 f^{b/x^2} + \frac{1}{240}f^a \ln(f)^3 b^3 x^4 f^{b/x^2} + \frac{1}{240}f^a \ln(f)^4 b^4 x^2 f^{b/x^2} + \frac{1}{240}f^a \ln(f)^5 b^5 \text{Ei}(1, -b \ln(f)/x^2)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^9,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^9,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)*x**9,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^9 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^9,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^2)*x^9, x)`

$$3.130 \quad \int f^{a+\frac{b}{x^2}} x^7 dx$$

**Optimal.** Leaf size=24

$$\frac{1}{2} b^4 f^a \log^4(f) \text{Gamma}\left(-4, -\frac{b \log(f)}{x^2}\right)$$

[Out] (b^4\*f^a\*Gamma[-4, -((b\*Log[f])/x^2)]\*Log[f]^4)/2

**Rubi [A]** time = 0.0246971, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{2} b^4 f^a \log^4(f) \text{Gamma}\left(-4, -\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^7, x]

[Out] (b^4\*f^a\*Gamma[-4, -((b\*Log[f])/x^2)]\*Log[f]^4)/2

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :- Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F]))]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^2}} x^7 dx = \frac{1}{2} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^2}\right) \log^4(f)$$

**Mathematica [A]** time = 0.0024848, size = 24, normalized size = 1.

$$\frac{1}{2} b^4 f^a \log^4(f) \text{Gamma}\left(-4, -\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^7, x]

[Out] (b^4\*f^a\*Gamma[-4, -((b\*Log[f])/x^2)]\*Log[f]^4)/2

**Maple [B]** time = 0.042, size = 101, normalized size = 4.2

$$\frac{f^a x^8}{8} f^{\frac{b}{x^2}} + \frac{f^a \ln(f) b x^6}{24} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^2 b^2 x^4}{48} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^3 b^3 x^2}{48} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^4 b^4}{48} \text{Ei}\left(1, -\frac{b \ln(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)*x^7,x)`

[Out]  $\frac{1}{8}f^a x^8 f^{b/x^2} + \frac{1}{24}f^a \ln(f) b x^6 f^{b/x^2} + \frac{1}{48}f^a \ln(f)^2 b^2 x^4 f^{b/x^2} + \frac{1}{48}f^a \ln(f)^3 b^3 x^2 f^{b/x^2} + \frac{1}{48}f^a \ln(f)^4 b^4 \text{Ei}(1, -b \ln(f)/x^2)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^7,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^7,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)*x**7,x)`

[Out] `Integral(f**(a + b/x**2)*x**7, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^7,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^2)*x^7, x)`

### 3.131 $\int f^{a+\frac{b}{x^2}} x^5 dx$

**Optimal.** Leaf size=81

$$-\frac{1}{12}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) + \frac{1}{12}b^2 x^2 \log^2(f) f^{a+\frac{b}{x^2}} + \frac{1}{6}x^6 f^{a+\frac{b}{x^2}} + \frac{1}{12}bx^4 \log(f) f^{a+\frac{b}{x^2}}$$

[Out]  $(f^{(a + b/x^2)}*x^6)/6 + (b*f^{(a + b/x^2)}*x^4*\operatorname{Log}[f])/12 + (b^2*f^{(a + b/x^2)}*x^2*\operatorname{Log}[f]^2)/12 - (b^3*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f]^3)/12$

**Rubi [A]** time = 0.0892356, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$-\frac{1}{12}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) + \frac{1}{12}b^2 x^2 \log^2(f) f^{a+\frac{b}{x^2}} + \frac{1}{6}x^6 f^{a+\frac{b}{x^2}} + \frac{1}{12}bx^4 \log(f) f^{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}*x^5, x]$

[Out]  $(f^{(a + b/x^2)}*x^6)/6 + (b*f^{(a + b/x^2)}*x^4*\operatorname{Log}[f])/12 + (b^2*f^{(a + b/x^2)}*x^2*\operatorname{Log}[f]^2)/12 - (b^3*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f]^3)/12$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{IntegerQ}[(2*(m + 1))/n] \ \&\& \ \operatorname{LtQ}[-4, (m + 1)/n, 5] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ ((\operatorname{GtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m, -1]) \ || \ (\operatorname{GtQ}[-n, 0] \ \&\& \ \operatorname{LeQ}[-n, m + 1]))$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n, x\} \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^2}} x^5 dx &= \frac{1}{6}f^{a+\frac{b}{x^2}} x^6 + \frac{1}{3}(b \log(f)) \int f^{a+\frac{b}{x^2}} x^3 dx \\ &= \frac{1}{6}f^{a+\frac{b}{x^2}} x^6 + \frac{1}{12}b f^{a+\frac{b}{x^2}} x^4 \log(f) + \frac{1}{6}(b^2 \log^2(f)) \int f^{a+\frac{b}{x^2}} x dx \\ &= \frac{1}{6}f^{a+\frac{b}{x^2}} x^6 + \frac{1}{12}b f^{a+\frac{b}{x^2}} x^4 \log(f) + \frac{1}{12}b^2 f^{a+\frac{b}{x^2}} x^2 \log^2(f) + \frac{1}{6}(b^3 \log^3(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx \\ &= \frac{1}{6}f^{a+\frac{b}{x^2}} x^6 + \frac{1}{12}b f^{a+\frac{b}{x^2}} x^4 \log(f) + \frac{1}{12}b^2 f^{a+\frac{b}{x^2}} x^2 \log^2(f) - \frac{1}{12}b^3 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \log^3(f) \end{aligned}$$

**Mathematica [A]** time = 0.0204158, size = 57, normalized size = 0.7

$$\frac{1}{12}f^a \left( x^2 f^{\frac{b}{x^2}} (b^2 \log^2(f) + bx^2 \log(f) + 2x^4) - b^3 \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^5,x]

[Out] (f^a\*(-(b^3\*ExpIntegralEi[(b\*Log[f])/x^2]\*Log[f]^3) + f^(b/x^2)\*x^2\*(2\*x^4 + b\*x^2\*Log[f] + b^2\*Log[f]^2)))/12

**Maple [A]** time = 0.031, size = 79, normalized size = 1.

$$\frac{f^a x^6}{6} f^{\frac{b}{x^2}} + \frac{f^a \ln(f) b x^4}{12} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^2 b^2 x^2}{12} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^3 b^3}{12} \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^5,x)

[Out] 1/6\*f^a\*x^6\*f^(b/x^2)+1/12\*f^a\*ln(f)\*b\*x^4\*f^(b/x^2)+1/12\*f^a\*ln(f)^2\*b^2\*x^2\*f^(b/x^2)+1/12\*f^a\*ln(f)^3\*b^3\*Ei(1,-b\*ln(f)/x^2)

**Maxima [A]** time = 1.27026, size = 30, normalized size = 0.37

$$-\frac{1}{2} b^3 f^a \Gamma\left(-3, -\frac{b \log(f)}{x^2}\right) \log(f)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^5,x, algorithm="maxima")

[Out] -1/2\*b^3\*f^a\*gamma(-3, -b\*log(f)/x^2)\*log(f)^3

**Fricas [A]** time = 1.91733, size = 149, normalized size = 1.84

$$-\frac{1}{12} b^3 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \log(f)^3 + \frac{1}{12} \left(2x^6 + bx^4 \log(f) + b^2 x^2 \log(f)^2\right) f^{\frac{ax^2+b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^5,x, algorithm="fricas")

[Out] -1/12\*b^3\*f^a\*Ei(b\*log(f)/x^2)\*log(f)^3 + 1/12\*(2\*x^6 + b\*x^4\*log(f) + b^2\*x^2\*log(f)^2)\*f^((a\*x^2 + b)/x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x**2)*x**5,x)
```

```
[Out] Integral(f**(a + b/x**2)*x**5, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)*x^5,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)*x^5, x)
```



$$3.132 \quad \int f^{a+\frac{b}{x^2}} x^3 dx$$

**Optimal.** Leaf size=58

$$-\frac{1}{4}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) + \frac{1}{4}x^4 f^{a+\frac{b}{x^2}} + \frac{1}{4}bx^2 \log(f) f^{a+\frac{b}{x^2}}$$

[Out]  $(f^{(a + b/x^2)}*x^4)/4 + (b*f^{(a + b/x^2)}*x^2*\operatorname{Log}[f])/4 - (b^2*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f]^2)/4$

**Rubi [A]** time = 0.062517, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$-\frac{1}{4}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) + \frac{1}{4}x^4 f^{a+\frac{b}{x^2}} + \frac{1}{4}bx^2 \log(f) f^{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}*x^3, x]$

[Out]  $(f^{(a + b/x^2)}*x^4)/4 + (b*f^{(a + b/x^2)}*x^2*\operatorname{Log}[f])/4 - (b^2*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f]^2)/4$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.))], x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^2}} x^3 dx &= \frac{1}{4}f^{a+\frac{b}{x^2}} x^4 + \frac{1}{2}(b \log(f)) \int f^{a+\frac{b}{x^2}} x dx \\ &= \frac{1}{4}f^{a+\frac{b}{x^2}} x^4 + \frac{1}{4}bf^{a+\frac{b}{x^2}} x^2 \log(f) + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx \\ &= \frac{1}{4}f^{a+\frac{b}{x^2}} x^4 + \frac{1}{4}bf^{a+\frac{b}{x^2}} x^2 \log(f) - \frac{1}{4}b^2 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \log^2(f) \end{aligned}$$

**Mathematica [A]** time = 0.0141431, size = 44, normalized size = 0.76

$$\frac{1}{4}f^a \left( x^2 f^{\frac{b}{x^2}} (b \log(f) + x^2) - b^2 \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^3,x]

[Out] (f^a\*(-(b^2\*ExpIntegralEi[(b\*Log[f])/x^2]\*Log[f]^2) + f^(b/x^2)\*x^2\*(x^2 + b\*Log[f]))) / 4

**Maple [A]** time = 0.026, size = 57, normalized size = 1.

$$\frac{f^a x^4}{4} f^{\frac{b}{x^2}} + \frac{f^a \ln(f) b x^2}{4} f^{\frac{b}{x^2}} + \frac{f^a (\ln(f))^2 b^2}{4} \text{Ei}\left(1, -\frac{b \ln(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^3,x)

[Out] 1/4\*f^a\*x^4\*f^(b/x^2)+1/4\*f^a\*ln(f)\*b\*x^2\*f^(b/x^2)+1/4\*f^a\*ln(f)^2\*b^2\*Ei(1,-b\*ln(f)/x^2)

**Maxima [A]** time = 1.20352, size = 30, normalized size = 0.52

$$\frac{1}{2} b^2 f^a \Gamma\left(-2, -\frac{b \log(f)}{x^2}\right) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^3,x, algorithm="maxima")

[Out] 1/2\*b^2\*f^a\*gamma(-2, -b\*log(f)/x^2)\*log(f)^2

**Fricas [A]** time = 1.77214, size = 117, normalized size = 2.02

$$-\frac{1}{4} b^2 f^a \text{Ei}\left(\frac{b \log(f)}{x^2}\right) \log(f)^2 + \frac{1}{4} (x^4 + b x^2 \log(f)) f^{\frac{ax^2+b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^3,x, algorithm="fricas")

[Out] -1/4\*b^2\*f^a\*Ei(b\*log(f)/x^2)\*log(f)^2 + 1/4\*(x^4 + b\*x^2\*log(f))\*f^((a\*x^2 + b)/x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x**2)*x**3,x)
```

```
[Out] Integral(f**(a + b/x**2)*x**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)*x^3, x)
```

### 3.133 $\int f^{a+\frac{b}{x^2}} x dx$

**Optimal.** Leaf size=35

$$\frac{1}{2}x^2 f^{a+\frac{b}{x^2}} - \frac{1}{2}b f^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

[Out]  $(f^{(a + b/x^2)}*x^2)/2 - (b*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f])/2$

**Rubi [A]** time = 0.0359106, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2214, 2210}

$$\frac{1}{2}x^2 f^{a+\frac{b}{x^2}} - \frac{1}{2}b f^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}*x, x]$

[Out]  $(f^{(a + b/x^2)}*x^2)/2 - (b*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^2]*\operatorname{Log}[f])/2$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.))], x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]])/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^2}} x dx &= \frac{1}{2}f^{a+\frac{b}{x^2}}x^2 + (b \log(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx \\ &= \frac{1}{2}f^{a+\frac{b}{x^2}}x^2 - \frac{1}{2}b f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \log(f) \end{aligned}$$

**Mathematica [A]** time = 0.0050389, size = 32, normalized size = 0.91

$$\frac{1}{2}f^a \left( x^2 f^{\frac{b}{x^2}} - b \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right) \right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[f^{(a + b/x^2)}*x, x]$

[Out]  $(f^a * (f^{(b/x^2)} * x^2 - b * \text{ExpIntegralEi}[(b * \text{Log}[f])/x^2] * \text{Log}[f]))/2$

**Maple [A]** time = 0.024, size = 35, normalized size = 1.

$$\frac{f^a x^2}{2} f^{\frac{b}{x^2}} + \frac{f^a \ln(f) b}{2} \text{Ei}\left(1, -\frac{b \ln(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)*x,x)`

[Out]  $1/2 * f^a * x^2 * f^{(b/x^2)} + 1/2 * f^a * \ln(f) * b * \text{Ei}(1, -b * \ln(f)/x^2)$

**Maxima [A]** time = 1.29685, size = 24, normalized size = 0.69

$$-\frac{1}{2} b f^a \Gamma\left(-1, -\frac{b \log(f)}{x^2}\right) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x,x, algorithm="maxima")`

[Out]  $-1/2 * b * f^a * \text{gamma}(-1, -b * \log(f)/x^2) * \log(f)$

**Fricas [A]** time = 1.69334, size = 89, normalized size = 2.54

$$-\frac{1}{2} b f^a \text{Ei}\left(\frac{b \log(f)}{x^2}\right) \log(f) + \frac{1}{2} f^{\frac{ax^2+b}{x^2}} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x,x, algorithm="fricas")`

[Out]  $-1/2 * b * f^a * \text{Ei}(b * \log(f)/x^2) * \log(f) + 1/2 * f^{((a * x^2 + b)/x^2)} * x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a + \frac{b}{x^2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)*x,x)`

[Out] `Integral(f**(a + b/x**2)*x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)*x,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)*x, x)
```

$$3.134 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^2])/2$

**Rubi [A]** time = 0.0234397, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$-\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^2])/2$

Rule 2210

$\operatorname{Int}[(F\_)^{(a\_)} + (b\_)*((c\_)+ (d\_)*(x\_))^n]/((e\_)+ (f\_)*(x\_)), x\_ \text{Symbol}] :> \operatorname{Simp}[(F^a \operatorname{ExpIntegralEi}[b*(c + d*x)^n \operatorname{Log}[F]])/(f*n), x] /;$  Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} dx = -\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

**Mathematica [A]** time = 0.0020256, size = 15, normalized size = 1.

$$-\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[f^{(a + b/x^2)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^2])/2$

**Maple [A]** time = 0.019, size = 16, normalized size = 1.1

$$\frac{f^a}{2} \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)/x,x)`

[Out]  $\frac{1}{2}f^a \operatorname{Ei}\left(1, -b \ln(f)/x^2\right)$

**Maxima [A]** time = 1.26697, size = 18, normalized size = 1.2

$$-\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x,x, algorithm="maxima")`

[Out]  $-1/2*f^a*\operatorname{Ei}(b*\log(f)/x^2)$

**Fricas [A]** time = 1.76026, size = 36, normalized size = 2.4

$$-\frac{1}{2}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x,x, algorithm="fricas")`

[Out]  $-1/2*f^a*\operatorname{Ei}(b*\log(f)/x^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)/x,x)`

[Out] `Integral(f**(a + b/x**2)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^2)/x, x)`



$$3.135 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx$$

**Optimal.** Leaf size=20

$$-\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

[Out]  $-f^{(a + b/x^2)}/(2*b*Log[f])$

**Rubi [A]** time = 0.0214927, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$-\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^3,x]

[Out]  $-f^{(a + b/x^2)}/(2*b*Log[f])$

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx = -\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

**Mathematica [A]** time = 0.0037513, size = 20, normalized size = 1.

$$-\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^3,x]

[Out]  $-f^{(a + b/x^2)}/(2*b*Log[f])$

**Maple [A]** time = 0.002, size = 19, normalized size = 1.

$$-\frac{1}{2b \ln(f)} f^{a+\frac{b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)/x^3,x)`

[Out]  $-1/2*f^{(a+b/x^2)}/b/\ln(f)$

**Maxima [A]** time = 1.00888, size = 24, normalized size = 1.2

$$-\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^3,x, algorithm="maxima")`

[Out]  $-1/2*f^{(a + b/x^2)}/(b*\log(f))$

**Fricas [A]** time = 1.76488, size = 50, normalized size = 2.5

$$-\frac{f^{\frac{ax^2+b}{x^2}}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^3,x, algorithm="fricas")`

[Out]  $-1/2*f^{((a*x^2 + b)/x^2)}/(b*\log(f))$

**Sympy [A]** time = 0.11768, size = 29, normalized size = 1.45

$$\begin{cases} -\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)} & \text{for } 2b \log(f) \neq 0 \\ -\frac{1}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)/x**3,x)`

[Out] `Piecewise((-f**(a + b/x**2)/(2*b*log(f)), Ne(2*b*log(f), 0)), (-1/(2*x**2), True))`

**Giac [A]** time = 1.73057, size = 24, normalized size = 1.2

$$-\frac{f^{a+\frac{b}{x^2}}}{2b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*f^(a + b/x^2)/(b*log(f))
```

$$3.136 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx$$

**Optimal.** Leaf size=44

$$\frac{f^{a+\frac{b}{x^2}}}{2b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2 \log(f)}$$

[Out]  $f^{(a + b/x^2)/(2*b^2*Log[f]^2)} - f^{(a + b/x^2)/(2*b*x^2*Log[f])}$

**Rubi [A]** time = 0.0451656, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{f^{a+\frac{b}{x^2}}}{2b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^5,x]

[Out]  $f^{(a + b/x^2)/(2*b^2*Log[f]^2)} - f^{(a + b/x^2)/(2*b*x^2*Log[f])}$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^2 \log(f)} - \frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx}{b \log(f)} \\ &= \frac{f^{a+\frac{b}{x^2}}}{2b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0069059, size = 32, normalized size = 0.73

$$\frac{f^{a+\frac{b}{x^2}} (x^2 - b \log(f))}{2b^2 x^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^5,x]

[Out] (f^(a + b/x^2)\*(x^2 - b\*Log[f]))/(2\*b^2\*x^2\*Log[f]^2)

**Maple [A]** time = 0.011, size = 52, normalized size = 1.2

$$\frac{1}{x^4} \left( \frac{x^4}{2 (\ln(f))^2 b^2} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} - \frac{x^2}{2 b \ln(f)} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^5,x)

[Out] (1/2/b^2/ln(f)^2\*x^4\*exp((a+b/x^2)\*ln(f))-1/2/b/ln(f)\*x^2\*exp((a+b/x^2)\*ln(f)))/x^4

**Maxima [C]** time = 1.17494, size = 30, normalized size = 0.68

$$\frac{f^a \Gamma\left(2, -\frac{b \log(f)}{x^2}\right)}{2 b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^5,x, algorithm="maxima")

[Out] 1/2\*f^a\*gamma(2, -b\*log(f)/x^2)/(b^2\*log(f)^2)

**Fricas [A]** time = 1.66412, size = 82, normalized size = 1.86

$$\frac{(x^2 - b \log(f)) f^{\frac{ax^2+b}{x^2}}}{2 b^2 x^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^5,x, algorithm="fricas")

[Out] 1/2\*(x^2 - b\*log(f))\*f^((a\*x^2 + b)/x^2)/(b^2\*x^2\*log(f)^2)

**Sympy [A]** time = 0.122252, size = 29, normalized size = 0.66

$$\frac{f^{a + \frac{b}{x^2}} (-b \log(f) + x^2)}{2 b^2 x^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*5,x)

[Out] f\*\*(a + b/x\*\*2)\*(-b\*log(f) + x\*\*2)/(2\*b\*\*2\*x\*\*2\*log(f)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^5,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^5, x)

$$3.137 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^7} dx$$

**Optimal.** Leaf size=62

$$\frac{f^{a+\frac{b}{x^2}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^4 \log(f)}$$

[Out]  $-(f^{(a + b/x^2)} / (b^3 * \text{Log}[f]^3)) + f^{(a + b/x^2)} / (b^2 * x^2 * \text{Log}[f]^2) - f^{(a + b/x^2)} / (2 * b * x^4 * \text{Log}[f])$

**Rubi [A]** time = 0.0693236, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{f^{a+\frac{b}{x^2}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^4 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^7, x]

[Out]  $-(f^{(a + b/x^2)} / (b^3 * \text{Log}[f]^3)) + f^{(a + b/x^2)} / (b^2 * x^2 * \text{Log}[f]^2) - f^{(a + b/x^2)} / (2 * b * x^4 * \text{Log}[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x^2}}}{x^7} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^4 \log(f)} - \frac{2 \int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx}{b \log(f)} \\ &= \frac{f^{a+\frac{b}{x^2}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^4 \log(f)} + \frac{2 \int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx}{b^2 \log^2(f)} \\ &= -\frac{f^{a+\frac{b}{x^2}}}{b^3 \log^3(f)} + \frac{f^{a+\frac{b}{x^2}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^4 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0089231, size = 45, normalized size = 0.73

$$\frac{f^{a+\frac{b}{x^2}} (b^2 \log^2(f) - 2bx^2 \log(f) + 2x^4)}{2b^3x^4 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^7,x]

[Out] -(f^(a + b/x^2)\*(2\*x^4 - 2\*b\*x^2\*Log[f] + b^2\*Log[f]^2))/(2\*b^3\*x^4\*Log[f]^3)

**Maple [A]** time = 0.016, size = 74, normalized size = 1.2

$$\frac{1}{x^6} \left( \frac{x^4}{(\ln(f))^2 b^2} e^{(a+\frac{b}{x^2})\ln(f)} - \frac{x^6}{(\ln(f))^3 b^3} e^{(a+\frac{b}{x^2})\ln(f)} - \frac{x^2}{2b \ln(f)} e^{(a+\frac{b}{x^2})\ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^7,x)

[Out] (1/b^2/ln(f)^2\*x^4\*exp((a+b/x^2)\*ln(f))-1/b^3/ln(f)^3\*x^6\*exp((a+b/x^2)\*ln(f))-1/2/b/ln(f)\*x^2\*exp((a+b/x^2)\*ln(f)))/x^6

**Maxima [C]** time = 1.16573, size = 30, normalized size = 0.48

$$\frac{f^a \Gamma\left(3, -\frac{b \log(f)}{x^2}\right)}{2b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^7,x, algorithm="maxima")

[Out] -1/2\*f^a\*gamma(3, -b\*log(f)/x^2)/(b^3\*log(f)^3)

**Fricas [A]** time = 1.65512, size = 115, normalized size = 1.85

$$\frac{(2x^4 - 2bx^2 \log(f) + b^2 \log(f)^2) f^{\frac{ax^2+b}{x^2}}}{2b^3x^4 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^7,x, algorithm="fricas")

[Out] -1/2\*(2\*x^4 - 2\*b\*x^2\*log(f) + b^2\*log(f)^2)\*f^((a\*x^2 + b)/x^2)/(b^3\*x^4\*log(f)^3)



---

**Sympy [A]** time = 0.13622, size = 44, normalized size = 0.71

$$\frac{f^{a+\frac{b}{x^2}} \left( -b^2 \log(f)^2 + 2bx^2 \log(f) - 2x^4 \right)}{2b^3x^4 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*7,x)

[Out] f\*\*(a + b/x\*\*2)\*(-b\*\*2\*log(f)\*\*2 + 2\*b\*x\*\*2\*log(f) - 2\*x\*\*4)/(2\*b\*\*3\*x\*\*4\*log(f)\*\*3)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^7,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^7, x)

$$3.138 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^9} dx$$

**Optimal.** Leaf size=86

$$\frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4\log^2(f)} - \frac{3f^{a+\frac{b}{x^2}}}{b^3x^2\log^3(f)} + \frac{3f^{a+\frac{b}{x^2}}}{b^4\log^4(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^6\log(f)}$$

[Out]  $(3f^{(a + b/x^2)})/(b^4*\text{Log}[f]^4) - (3f^{(a + b/x^2)})/(b^3*x^2*\text{Log}[f]^3) + (3f^{(a + b/x^2)})/(2*b^2*x^4*\text{Log}[f]^2) - f^{(a + b/x^2)}/(2*b*x^6*\text{Log}[f])$

**Rubi [A]** time = 0.0957998, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4\log^2(f)} - \frac{3f^{a+\frac{b}{x^2}}}{b^3x^2\log^3(f)} + \frac{3f^{a+\frac{b}{x^2}}}{b^4\log^4(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^6\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^9,x]

[Out]  $(3f^{(a + b/x^2)})/(b^4*\text{Log}[f]^4) - (3f^{(a + b/x^2)})/(b^3*x^2*\text{Log}[f]^3) + (3f^{(a + b/x^2)})/(2*b^2*x^4*\text{Log}[f]^2) - f^{(a + b/x^2)}/(2*b*x^6*\text{Log}[f])$

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^2}}}{x^9} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^6 \log(f)} - \frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^7} dx}{b \log(f)} \\
&= \frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^6 \log(f)} + \frac{6 \int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx}{b^2 \log^2(f)} \\
&= -\frac{3f^{a+\frac{b}{x^2}}}{b^3x^2 \log^3(f)} + \frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^6 \log(f)} - \frac{6 \int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx}{b^3 \log^3(f)} \\
&= \frac{3f^{a+\frac{b}{x^2}}}{b^4 \log^4(f)} - \frac{3f^{a+\frac{b}{x^2}}}{b^3x^2 \log^3(f)} + \frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^6 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0103556, size = 58, normalized size = 0.67

$$\frac{f^{a+\frac{b}{x^2}} (3b^2x^2 \log^2(f) - b^3 \log^3(f) - 6bx^4 \log(f) + 6x^6)}{2b^4x^6 \log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^9,x]

[Out] (f^(a + b/x^2)\*(6\*x^6 - 6\*b\*x^4\*Log[f] + 3\*b^2\*x^2\*Log[f]^2 - b^3\*Log[f]^3))/(2\*b^4\*x^6\*Log[f]^4)

**Maple [A]** time = 0.017, size = 98, normalized size = 1.1

$$\frac{1}{x^8} \left( 3 \frac{x^8}{b^4 (\ln(f))^4} e^{(a+\frac{b}{x^2}) \ln(f)} - 3 \frac{x^6}{(\ln(f))^3 b^3} e^{(a+\frac{b}{x^2}) \ln(f)} + \frac{3x^4}{2 (\ln(f))^2 b^2} e^{(a+\frac{b}{x^2}) \ln(f)} - \frac{x^2}{2b \ln(f)} e^{(a+\frac{b}{x^2}) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^9,x)

[Out] (3/b^4/ln(f)^4\*x^8\*exp((a+b/x^2)\*ln(f))-3/b^3/ln(f)^3\*x^6\*exp((a+b/x^2)\*ln(f))+3/2/b^2/ln(f)^2\*x^4\*exp((a+b/x^2)\*ln(f))-1/2/b/ln(f)\*x^2\*exp((a+b/x^2)\*ln(f)))/x^8

**Maxima [C]** time = 1.2621, size = 30, normalized size = 0.35

$$\frac{f^a \Gamma\left(4, -\frac{b \log(f)}{x^2}\right)}{2b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^9,x, algorithm="maxima")

[Out]  $\frac{1}{2}f^a \text{gamma}(4, -b \log(f)/x^2) / (b^4 \log(f)^4)$

**Fricas [A]** time = 1.72871, size = 142, normalized size = 1.65

$$\frac{\left(6x^6 - 6bx^4 \log(f) + 3b^2x^2 \log(f)^2 - b^3 \log(f)^3\right) f^{\frac{ax^2+b}{x^2}}}{2b^4x^6 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^9,x, algorithm="fricas")`

[Out]  $\frac{1}{2}(6x^6 - 6bx^4 \log(f) + 3b^2x^2 \log(f)^2 - b^3 \log(f)^3) f^{(ax^2 + b)/x^2} / (b^4x^6 \log(f)^4)$

**Sympy [A]** time = 0.142659, size = 58, normalized size = 0.67

$$\frac{f^{a+\frac{b}{x^2}} \left(-b^3 \log(f)^3 + 3b^2x^2 \log(f)^2 - 6bx^4 \log(f) + 6x^6\right)}{2b^4x^6 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)/x**9,x)`

[Out]  $f^{(a + b/x^2)} (-b^3 \log(f)^3 + 3b^2x^2 \log(f)^2 - 6bx^4 \log(f) + 6x^6) / (2b^4x^6 \log(f)^4)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^9,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^2)/x^9, x)`

$$3.139 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx$$

**Optimal.** Leaf size=69

$$\frac{f^{a+\frac{b}{x^2}} (12b^2x^4 \log^2(f) - 4b^3x^2 \log^3(f) + b^4 \log^4(f) - 24bx^6 \log(f) + 24x^8)}{2b^5x^8 \log^5(f)}$$

[Out]  $-(f^{(a + b/x^2)}*(24*x^8 - 24*b*x^6*Log[f] + 12*b^2*x^4*Log[f]^2 - 4*b^3*x^2*Log[f]^3 + b^4*Log[f]^4))/(2*b^5*x^8*Log[f]^5)$

**Rubi [C]** time = 0.022182, antiderivative size = 24, normalized size of antiderivative = 0.35, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x^2}\right)}{2b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^11,x]

[Out]  $-(f^a*\text{Gamma}[5, -((b*\text{Log}[f])/x^2)))/(2*b^5*\text{Log}[f]^5)$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx = -\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x^2}\right)}{2b^5 \log^5(f)}$$

**Mathematica [C]** time = 0.0028538, size = 24, normalized size = 0.35

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x^2}\right)}{2b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^11,x]

[Out]  $-(f^a*\text{Gamma}[5, -((b*\text{Log}[f])/x^2)))/(2*b^5*\text{Log}[f]^5)$

**Maple [A]** time = 0.02, size = 121, normalized size = 1.8

$$\frac{1}{x^{10}} \left( -12 \frac{x^{10}}{b^5 (\ln(f))^5} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} + 12 \frac{x^8}{b^4 (\ln(f))^4} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} - 6 \frac{x^6}{(\ln(f))^3 b^3} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} + 2 \frac{x^4}{(\ln(f))^2 b^2} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^11,x)

[Out] (-12/b^5/ln(f)^5\*x^10\*exp((a+b/x^2)\*ln(f))+12/b^4/ln(f)^4\*x^8\*exp((a+b/x^2)\*ln(f))-6/b^3/ln(f)^3\*x^6\*exp((a+b/x^2)\*ln(f))+2/b^2/ln(f)^2\*x^4\*exp((a+b/x^2)\*ln(f))-1/2/b/ln(f)\*x^2\*exp((a+b/x^2)\*ln(f)))/x^10

**Maxima [C]** time = 1.18113, size = 30, normalized size = 0.43

$$\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x^2}\right)}{2 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^11,x, algorithm="maxima")

[Out] -1/2\*f^a\*gamma(5, -b\*log(f)/x^2)/(b^5\*log(f)^5)

**Fricas [A]** time = 1.65185, size = 176, normalized size = 2.55

$$\frac{\left(24x^8 - 24bx^6 \log(f) + 12b^2x^4 \log(f)^2 - 4b^3x^2 \log(f)^3 + b^4 \log(f)^4\right) f^{\frac{ax^2+b}{x^2}}}{2b^5x^8 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^11,x, algorithm="fricas")

[Out] -1/2\*(24\*x^8 - 24\*b\*x^6\*log(f) + 12\*b^2\*x^4\*log(f)^2 - 4\*b^3\*x^2\*log(f)^3 + b^4\*log(f)^4)\*f^((a\*x^2 + b)/x^2)/(b^5\*x^8\*log(f)^5)

**Sympy [A]** time = 0.158997, size = 71, normalized size = 1.03

$$\frac{f^{a+\frac{b}{x^2}} \left( -b^4 \log(f)^4 + 4b^3x^2 \log(f)^3 - 12b^2x^4 \log(f)^2 + 24bx^6 \log(f) - 24x^8 \right)}{2b^5x^8 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*11,x)

```
[Out] f**(a + b/x**2)*(-b**4*log(f)**4 + 4*b**3*x**2*log(f)**3 - 12*b**2*x**4*log
(f)**2 + 24*b*x**6*log(f) - 24*x**8)/(2*b**5*x**8*log(f)**5)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)/x^11,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)/x^11, x)
```

$$3.140 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx$$

**Optimal.** Leaf size=82

$$\frac{f^{a+\frac{b}{x^2}} (60b^2x^6 \log^2(f) - 20b^3x^4 \log^3(f) + 5b^4x^2 \log^4(f) - b^5 \log^5(f) - 120bx^8 \log(f) + 120x^{10})}{2b^6x^{10} \log^6(f)}$$

[Out] (f^(a + b/x^2)\*(120\*x^10 - 120\*b\*x^8\*Log[f] + 60\*b^2\*x^6\*Log[f]^2 - 20\*b^3\*x^4\*Log[f]^3 + 5\*b^4\*x^2\*Log[f]^4 - b^5\*Log[f]^5))/(2\*b^6\*x^10\*Log[f]^6)

**Rubi [C]** time = 0.0240109, antiderivative size = 24, normalized size of antiderivative = 0.29, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x^2}\right)}{2b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^13,x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x^2)])/(2\*b^6\*Log[f]^6)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx = \frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x^2}\right)}{2b^6 \log^6(f)}$$

**Mathematica [C]** time = 0.0027983, size = 24, normalized size = 0.29

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x^2}\right)}{2b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^13,x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x^2)])/(2\*b^6\*Log[f]^6)



**Maple [A]** time = 0.023, size = 144, normalized size = 1.8

$$\frac{1}{x^{12}} \left( 60 \frac{x^{12}}{b^6 (\ln(f))^6} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} - 60 \frac{x^{10}}{b^5 (\ln(f))^5} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} + 30 \frac{x^8}{b^4 (\ln(f))^4} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} - 10 \frac{x^6}{(\ln(f))^3 b^3} e^{\left(a + \frac{b}{x^2}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^13,x)

[Out] (60/b^6/ln(f)^6\*x^12\*exp((a+b/x^2)\*ln(f))-60/b^5/ln(f)^5\*x^10\*exp((a+b/x^2)\*ln(f))+30/b^4/ln(f)^4\*x^8\*exp((a+b/x^2)\*ln(f))-10/b^3/ln(f)^3\*x^6\*exp((a+b/x^2)\*ln(f))+5/2/b^2/ln(f)^2\*x^4\*exp((a+b/x^2)\*ln(f))-1/2/b/ln(f)\*x^2\*exp((a+b/x^2)\*ln(f)))/x^12

**Maxima [C]** time = 1.18374, size = 30, normalized size = 0.37

$$\frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x^2}\right)}{2 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^13,x, algorithm="maxima")

[Out] 1/2\*f^a\*gamma(6, -b\*log(f)/x^2)/(b^6\*log(f)^6)

**Fricas [A]** time = 1.75167, size = 209, normalized size = 2.55

$$\frac{\left(120 x^{10} - 120 b x^8 \log(f) + 60 b^2 x^6 \log(f)^2 - 20 b^3 x^4 \log(f)^3 + 5 b^4 x^2 \log(f)^4 - b^5 \log(f)^5\right) f^{\frac{ax^2+b}{x^2}}}{2 b^6 x^{10} \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^13,x, algorithm="fricas")

[Out] 1/2\*(120\*x^10 - 120\*b\*x^8\*log(f) + 60\*b^2\*x^6\*log(f)^2 - 20\*b^3\*x^4\*log(f)^3 + 5\*b^4\*x^2\*log(f)^4 - b^5\*log(f)^5)\*f^((a\*x^2 + b)/x^2)/(b^6\*x^10\*log(f)^6)

**Sympy [A]** time = 0.168652, size = 85, normalized size = 1.04

$$\frac{f^{a+\frac{b}{x^2}} \left(-b^5 \log(f)^5 + 5b^4 x^2 \log(f)^4 - 20b^3 x^4 \log(f)^3 + 60b^2 x^6 \log(f)^2 - 120b x^8 \log(f) + 120x^{10}\right)}{2b^6 x^{10} \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*13,x)

```
[Out] f**(a + b/x**2)*(-b**5*log(f)**5 + 5*b**4*x**2*log(f)**4 - 20*b**3*x**4*log
(f)**3 + 60*b**2*x**6*log(f)**2 - 120*b*x**8*log(f) + 120*x**10)/(2*b**6*x*
*10*log(f)**6)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)/x^13, x)
```

$$3.141 \quad \int f^{a+\frac{b}{x^2}} x^{10} dx$$

**Optimal.** Leaf size=34

$$\frac{1}{2} x^{11} f^a \left( -\frac{b \log(f)}{x^2} \right)^{11/2} \text{Gamma} \left( -\frac{11}{2}, -\frac{b \log(f)}{x^2} \right)$$

[Out] (f^a\*x^11\*Gamma[-11/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^(11/2))/2

**Rubi [A]** time = 0.0241223, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{2} x^{11} f^a \left( -\frac{b \log(f)}{x^2} \right)^{11/2} \text{Gamma} \left( -\frac{11}{2}, -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^10,x]

[Out] (f^a\*x^11\*Gamma[-11/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^(11/2))/2

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^2}} x^{10} dx = \frac{1}{2} f^a x^{11} \Gamma \left( -\frac{11}{2}, -\frac{b \log(f)}{x^2} \right) \left( -\frac{b \log(f)}{x^2} \right)^{11/2}$$

**Mathematica [A]** time = 0.0043307, size = 34, normalized size = 1.

$$\frac{1}{2} x^{11} f^a \left( -\frac{b \log(f)}{x^2} \right)^{11/2} \text{Gamma} \left( -\frac{11}{2}, -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^10,x]

[Out] (f^a\*x^11\*Gamma[-11/2, -((b\*Log[f])/x^2)]\*(-((b\*Log[f])/x^2))^(11/2))/2

**Maple [A]** time = 0.101, size = 155, normalized size = 4.6

$$\frac{f^a x^{11}}{11} f^{\frac{b}{x^2}} + \frac{2 f^a \ln(f) b x^9}{99} f^{\frac{b}{x^2}} + \frac{4 f^a (\ln(f))^2 b^2 x^7}{693} f^{\frac{b}{x^2}} + \frac{8 f^a (\ln(f))^3 b^3 x^5}{3465} f^{\frac{b}{x^2}} + \frac{16 f^a (\ln(f))^4 b^4 x^3}{10395} f^{\frac{b}{x^2}} + \frac{32 f^a (\ln(f))^5 b^5 x}{10395} f^{\frac{b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)*x^10,x)`

[Out]  $\frac{1}{11}f^a x^{11} f^{(b/x^2)} + \frac{2}{99}f^a \ln(f) b x^9 f^{(b/x^2)} + \frac{4}{693}f^a \ln(f)^2 b^2 x^7 f^{(b/x^2)} + \frac{8}{3465}f^a \ln(f)^3 b^3 x^5 f^{(b/x^2)} + \frac{16}{10395}f^a \ln(f)^4 b^4 x^3 f^{(b/x^2)} + \frac{32}{10395}f^a \ln(f)^5 b^5 x f^{(b/x^2)} - \frac{32}{10395}f^a \ln(f)^6 b^6 \pi^{(1/2)} / (-b \ln(f))^{(1/2)} \operatorname{erf}((-b \ln(f))^{(1/2)}) / x$

**Maxima [A]** time = 1.20637, size = 38, normalized size = 1.12

$$\frac{1}{2} f^a x^{11} \left( -\frac{b \log(f)}{x^2} \right)^{\frac{11}{2}} \Gamma \left( -\frac{11}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^10,x, algorithm="maxima")`

[Out]  $\frac{1}{2} f^a x^{11} (-b \log(f) / x^2)^{(11/2)} \operatorname{gamma}(-11/2, -b \log(f) / x^2)$

**Fricas [A]** time = 1.85431, size = 298, normalized size = 8.76

$$\frac{32}{10395} \sqrt{\pi} \sqrt{-b \log(f)} b^5 f^a \operatorname{erf} \left( \frac{\sqrt{-b \log(f)}}{x} \right) \log(f)^5 + \frac{1}{10395} (945 x^{11} + 210 b x^9 \log(f) + 60 b^2 x^7 \log(f)^2 + 24 b^3 x^5 \log(f)^3 + 16 b^4 x^3 \log(f)^4 + 32 b^5 x \log(f)^5) f^{(a x^2 + b) / x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^10,x, algorithm="fricas")`

[Out]  $\frac{32}{10395} \sqrt{\pi} \sqrt{-b \log(f)} b^5 f^a \operatorname{erf}(\sqrt{-b \log(f)} / x) \log(f)^5 + \frac{1}{10395} (945 x^{11} + 210 b x^9 \log(f) + 60 b^2 x^7 \log(f)^2 + 24 b^3 x^5 \log(f)^3 + 16 b^4 x^3 \log(f)^4 + 32 b^5 x \log(f)^5) f^{(a x^2 + b) / x^2}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)*x**10,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a + \frac{b}{x^2}} x^{10} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)*x^10,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)*x^10, x)
```

$$3.142 \quad \int f^{a+\frac{b}{x^2}} x^8 dx$$

**Optimal.** Leaf size=34

$$\frac{1}{2} x^9 f^a \left( -\frac{b \log(f)}{x^2} \right)^{9/2} \text{Gamma} \left( -\frac{9}{2}, -\frac{b \log(f)}{x^2} \right)$$

[Out] (f^a\*x^9\*Gamma[-9/2, -(b\*Log[f])/x^2])\*(-(b\*Log[f])/x^2)^(9/2))/2

**Rubi [A]** time = 0.0236792, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{2} x^9 f^a \left( -\frac{b \log(f)}{x^2} \right)^{9/2} \text{Gamma} \left( -\frac{9}{2}, -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^8,x]

[Out] (f^a\*x^9\*Gamma[-9/2, -(b\*Log[f])/x^2])\*(-(b\*Log[f])/x^2)^(9/2))/2

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^2}} x^8 dx = \frac{1}{2} f^a x^9 \Gamma \left( -\frac{9}{2}, -\frac{b \log(f)}{x^2} \right) \left( -\frac{b \log(f)}{x^2} \right)^{9/2}$$

**Mathematica [A]** time = 0.004138, size = 34, normalized size = 1.

$$\frac{1}{2} x^9 f^a \left( -\frac{b \log(f)}{x^2} \right)^{9/2} \text{Gamma} \left( -\frac{9}{2}, -\frac{b \log(f)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^8,x]

[Out] (f^a\*x^9\*Gamma[-9/2, -(b\*Log[f])/x^2])\*(-(b\*Log[f])/x^2)^(9/2))/2

**Maple [A]** time = 0.054, size = 133, normalized size = 3.9

$$\frac{f^a x^9}{9} f^{\frac{b}{x^2}} + \frac{2 f^a \ln(f) b x^7}{63} f^{\frac{b}{x^2}} + \frac{4 f^a (\ln(f))^2 b^2 x^5}{315} f^{\frac{b}{x^2}} + \frac{8 f^a (\ln(f))^3 b^3 x^3}{945} f^{\frac{b}{x^2}} + \frac{16 f^a (\ln(f))^4 b^4 x}{945} f^{\frac{b}{x^2}} - \frac{16 f^a (\ln(f))}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^2)*x^8,x)`

[Out]  $\frac{1}{9}f^a x^9 f^{(b/x^2)} + \frac{2}{63}f^a \ln(f) b x^7 f^{(b/x^2)} + \frac{4}{315}f^a \ln(f)^2 b^2 x^5 f^{(b/x^2)} + \frac{8}{945}f^a \ln(f)^3 b^3 x^3 f^{(b/x^2)} + \frac{16}{945}f^a \ln(f)^4 b^4 x f^{(b/x^2)} - \frac{16}{945}f^a \ln(f)^5 b^5 \pi^{(1/2)} / (-b \ln(f))^{(1/2)} \operatorname{erf}((-b \ln(f))^{(1/2)}) / x$

**Maxima [A]** time = 1.20365, size = 38, normalized size = 1.12

$$\frac{1}{2} f^a x^9 \left( -\frac{b \log(f)}{x^2} \right)^{\frac{9}{2}} \Gamma \left( -\frac{9}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^8,x, algorithm="maxima")`

[Out]  $\frac{1}{2}f^a x^9 (-b \log(f)/x^2)^{(9/2)} \operatorname{gamma}(-9/2, -b \log(f)/x^2)$

**Fricas [A]** time = 1.75018, size = 259, normalized size = 7.62

$$\frac{16}{945} \sqrt{\pi} \sqrt{-b \log(f)} b^4 f^a \operatorname{erf} \left( \frac{\sqrt{-b \log(f)}}{x} \right) \log(f)^4 + \frac{1}{945} (105 x^9 + 30 b x^7 \log(f) + 12 b^2 x^5 \log(f)^2 + 8 b^3 x^3 \log(f)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)*x^8,x, algorithm="fricas")`

[Out]  $\frac{16}{945} \sqrt{\pi} \sqrt{-b \log(f)} b^4 f^a \operatorname{erf}(\sqrt{-b \log(f)}/x) \log(f)^4 + \frac{1}{945} (105 x^9 + 30 b x^7 \log(f) + 12 b^2 x^5 \log(f)^2 + 8 b^3 x^3 \log(f)^3 + 16 b^4 x \log(f)^4) f^{(a x^2 + b)/x^2}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)*x**8,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)*x^8,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)*x^8, x)
```



### 3.143 $\int f^{a+\frac{b}{x^2}} x^6 dx$

**Optimal.** Leaf size=119

$$-\frac{8}{105}\sqrt{\pi}b^{7/2}f^a \log^{\frac{7}{2}}(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) + \frac{8}{105}b^3x \log^3(f)f^{a+\frac{b}{x^2}} + \frac{4}{105}b^2x^3 \log^2(f)f^{a+\frac{b}{x^2}} + \frac{1}{7}x^7f^{a+\frac{b}{x^2}} + \frac{2}{35}bx^5 \log(f)$$

[Out]  $(f^{(a + b/x^2)}x^7)/7 + (2*b*f^{(a + b/x^2)}x^5*\operatorname{Log}[f])/35 + (4*b^2*f^{(a + b/x^2)}x^3*\operatorname{Log}[f]^2)/105 + (8*b^3*f^{(a + b/x^2)}x*\operatorname{Log}[f]^3)/105 - (8*b^{(7/2)}*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x]*\operatorname{Log}[f]^{(7/2)})/105$

**Rubi [A]** time = 0.125317, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{8}{105}\sqrt{\pi}b^{7/2}f^a \log^{\frac{7}{2}}(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) + \frac{8}{105}b^3x \log^3(f)f^{a+\frac{b}{x^2}} + \frac{4}{105}b^2x^3 \log^2(f)f^{a+\frac{b}{x^2}} + \frac{1}{7}x^7f^{a+\frac{b}{x^2}} + \frac{2}{35}bx^5 \log(f)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^6,x]

[Out]  $(f^{(a + b/x^2)}x^7)/7 + (2*b*f^{(a + b/x^2)}x^5*\operatorname{Log}[f])/35 + (4*b^2*f^{(a + b/x^2)}x^3*\operatorname{Log}[f]^2)/105 + (8*b^3*f^{(a + b/x^2)}x*\operatorname{Log}[f]^3)/105 - (8*b^{(7/2)}*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x]*\operatorname{Log}[f]^{(7/2)})/105$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int f^{a+\frac{b}{x^2}} x^6 dx &= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{1}{7} (2b \log(f)) \int f^{a+\frac{b}{x^2}} x^4 dx \\
&= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{1}{35} (4b^2 \log^2(f)) \int f^{a+\frac{b}{x^2}} x^2 dx \\
&= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{4}{105} b^2 f^{a+\frac{b}{x^2}} x^3 \log^2(f) + \frac{1}{105} (8b^3 \log^3(f)) \int f^{a+\frac{b}{x^2}} dx \\
&= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{4}{105} b^2 f^{a+\frac{b}{x^2}} x^3 \log^2(f) + \frac{8}{105} b^3 f^{a+\frac{b}{x^2}} x \log^3(f) + \frac{1}{105} (16b^4 \log^4(f)) \int f^{a+\frac{b}{x^2}} dx \\
&= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{4}{105} b^2 f^{a+\frac{b}{x^2}} x^3 \log^2(f) + \frac{8}{105} b^3 f^{a+\frac{b}{x^2}} x \log^3(f) - \frac{1}{105} (16b^4 \log^4(f)) \int f^{a+\frac{b}{x^2}} dx \\
&= \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{4}{105} b^2 f^{a+\frac{b}{x^2}} x^3 \log^2(f) + \frac{8}{105} b^3 f^{a+\frac{b}{x^2}} x \log^3(f) - \frac{8}{105} b^{7/2} f^a \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0364954, size = 86, normalized size = 0.72

$$\frac{1}{105} f^a \left( x f^{\frac{b}{x^2}} (4b^2 x^2 \log^2(f) + 8b^3 \log^3(f) + 6bx^4 \log(f) + 15x^6) - 8\sqrt{\pi} b^{7/2} \log^{\frac{7}{2}}(f) \operatorname{Erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^6,x]

[Out] (f^a\*(-8\*b^(7/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(7/2) + f^(b/x^2)\*x\*(15\*x^6 + 6\*b\*x^4\*Log[f] + 4\*b^2\*x^2\*Log[f]^2 + 8\*b^3\*Log[f]^3))/105

**Maple [A]** time = 0.04, size = 111, normalized size = 0.9

$$\frac{f^a x^7}{7} f^{\frac{b}{x^2}} + \frac{2 f^a \ln(f) b x^5}{35} f^{\frac{b}{x^2}} + \frac{4 f^a (\ln(f))^2 b^2 x^3}{105} f^{\frac{b}{x^2}} + \frac{8 f^a (\ln(f))^3 b^3 x}{105} f^{\frac{b}{x^2}} - \frac{8 f^a (\ln(f))^4 b^4 \sqrt{\pi}}{105} \operatorname{Erf} \left( \frac{1}{x} \sqrt{-b \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^6,x)

[Out] 1/7\*f^a\*x^7\*f^(b/x^2)+2/35\*f^a\*ln(f)\*b\*x^5\*f^(b/x^2)+4/105\*f^a\*ln(f)^2\*b^2\*x^3\*f^(b/x^2)+8/105\*f^a\*ln(f)^3\*b^3\*x\*f^(b/x^2)-8/105\*f^a\*ln(f)^4\*b^4\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.22611, size = 38, normalized size = 0.32

$$\frac{1}{2} f^a x^7 \left( -\frac{b \log(f)}{x^2} \right)^{\frac{7}{2}} \Gamma \left( \frac{7}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^6,x, algorithm="maxima")

[Out]  $\frac{1}{2}f^a x^7 (-b \log(f)/x^2)^{7/2} \Gamma(-7/2, -b \log(f)/x^2)$

**Fricas [A]** time = 1.78589, size = 224, normalized size = 1.88

$$\frac{8}{105} \sqrt{\pi} \sqrt{-b \log(f)} b^3 f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) \log(f)^3 + \frac{1}{105} \left(15x^7 + 6bx^5 \log(f) + 4b^2x^3 \log(f)^2 + 8b^3x \log(f)^3\right) f^{\left(\frac{a+b}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^6,x, algorithm="fricas")

[Out]  $\frac{8}{105} \sqrt{\pi} \sqrt{-b \log(f)} b^3 f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) \log(f)^3 + \frac{1}{105} (15x^7 + 6bx^5 \log(f) + 4b^2x^3 \log(f)^2 + 8b^3x \log(f)^3) f^{\left(\frac{a+b}{x^2}\right)}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)\*x\*\*6,x)

[Out] Integral(f\*\*(a + b/x\*\*2)\*x\*\*6, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^6,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)\*x^6, x)

### 3.144 $\int f^{a+\frac{b}{x^2}} x^4 dx$

**Optimal.** Leaf size=96

$$-\frac{4}{15}\sqrt{\pi}b^{5/2}f^a\log^2(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)+\frac{4}{15}b^2x\log^2(f)f^{a+\frac{b}{x^2}}+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}+\frac{2}{15}bx^3\log(f)f^{a+\frac{b}{x^2}}$$

[Out] (f^(a + b/x^2)\*x^5)/5 + (2\*b\*f^(a + b/x^2)\*x^3\*Log[f])/15 + (4\*b^2\*f^(a + b/x^2)\*x\*Log[f]^2)/15 - (4\*b^(5/2)\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(5/2))/15

**Rubi [A]** time = 0.0881798, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{4}{15}\sqrt{\pi}b^{5/2}f^a\log^2(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)+\frac{4}{15}b^2x\log^2(f)f^{a+\frac{b}{x^2}}+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}+\frac{2}{15}bx^3\log(f)f^{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^4, x]

[Out] (f^(a + b/x^2)\*x^5)/5 + (2\*b\*f^(a + b/x^2)\*x^3\*Log[f])/15 + (4\*b^2\*f^(a + b/x^2)\*x\*Log[f]^2)/15 - (4\*b^(5/2)\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(5/2))/15

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int f^{a+\frac{b}{x^2}} x^4 dx &= \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{1}{5} (2b \log(f)) \int f^{a+\frac{b}{x^2}} x^2 dx \\
&= \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{2}{15} b f^{a+\frac{b}{x^2}} x^3 \log(f) + \frac{1}{15} (4b^2 \log^2(f)) \int f^{a+\frac{b}{x^2}} dx \\
&= \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{2}{15} b f^{a+\frac{b}{x^2}} x^3 \log(f) + \frac{4}{15} b^2 f^{a+\frac{b}{x^2}} x \log^2(f) + \frac{1}{15} (8b^3 \log^3(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx \\
&= \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{2}{15} b f^{a+\frac{b}{x^2}} x^3 \log(f) + \frac{4}{15} b^2 f^{a+\frac{b}{x^2}} x \log^2(f) - \frac{1}{15} (8b^3 \log^3(f)) \text{Subst} \left( \int f^{a+bx^2} dx, x, \frac{1}{x} \right) \\
&= \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{2}{15} b f^{a+\frac{b}{x^2}} x^3 \log(f) + \frac{4}{15} b^2 f^{a+\frac{b}{x^2}} x \log^2(f) - \frac{4}{15} b^{5/2} f^a \sqrt{\pi} \text{erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right) \log^{\frac{5}{2}}(f)
\end{aligned}$$

**Mathematica [A]** time = 0.032388, size = 74, normalized size = 0.77

$$\frac{1}{15} f^a \left( x f^{\frac{b}{x^2}} (4b^2 \log^2(f) + 2bx^2 \log(f) + 3x^4) - 4\sqrt{\pi} b^{5/2} \log^{\frac{5}{2}}(f) \text{Erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^4,x]

[Out] (f^a\*(-4\*b^(5/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(5/2) + f^(b/x^2)\*x\*(3\*x^4 + 2\*b\*x^2\*Log[f] + 4\*b^2\*Log[f]^2))/15

**Maple [A]** time = 0.033, size = 89, normalized size = 0.9

$$\frac{f^a x^5}{5} f^{\frac{b}{x^2}} + \frac{2 f^a \ln(f) b x^3}{15} f^{\frac{b}{x^2}} + \frac{4 f^a (\ln(f))^2 b^2 x}{15} f^{\frac{b}{x^2}} - \frac{4 f^a (\ln(f))^3 b^3 \sqrt{\pi}}{15} \text{Erf} \left( \frac{1}{x} \sqrt{-b \ln(f)} \right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^4,x)

[Out] 1/5\*f^a\*x^5\*f^(b/x^2)+2/15\*f^a\*ln(f)\*b\*x^3\*f^(b/x^2)+4/15\*f^a\*ln(f)^2\*b^2\*x\*f^(b/x^2)-4/15\*f^a\*ln(f)^3\*b^3\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.28243, size = 38, normalized size = 0.4

$$\frac{1}{2} f^a x^5 \left( -\frac{b \log(f)}{x^2} \right)^{\frac{5}{2}} \Gamma \left( -\frac{5}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^4,x, algorithm="maxima")

[Out] 1/2\*f^a\*x^5\*(-b\*log(f)/x^2)^(5/2)\*gamma(-5/2, -b\*log(f)/x^2)

---

**Fricas [A]** time = 1.82148, size = 192, normalized size = 2.

$$\frac{4}{15} \sqrt{\pi} \sqrt{-b \log(f)} b^2 f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) \log(f)^2 + \frac{1}{15} \left(3x^5 + 2bx^3 \log(f) + 4b^2x \log(f)^2\right) f^{\frac{ax^2+b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^4,x, algorithm="fricas")

[Out] 4/15\*sqrt(pi)\*sqrt(-b\*log(f))\*b^2\*f^a\*erf(sqrt(-b\*log(f))/x)\*log(f)^2 + 1/15\*(3\*x^5 + 2\*b\*x^3\*log(f) + 4\*b^2\*x\*log(f)^2)\*f^((a\*x^2 + b)/x^2)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)\*x\*\*4,x)

[Out] Integral(f\*\*(a + b/x\*\*2)\*x\*\*4, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^4,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)\*x^4, x)

### 3.145 $\int f^{a+\frac{b}{x^2}} x^2 dx$

**Optimal.** Leaf size=73

$$-\frac{2}{3}\sqrt{\pi}b^{3/2}f^a\log^{\frac{3}{2}}(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}+\frac{2}{3}bx\log(f)f^{a+\frac{b}{x^2}}$$

[Out] (f^(a + b/x^2)\*x^3)/3 + (2\*b\*f^(a + b/x^2)\*x\*Log[f])/3 - (2\*b^(3/2)\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(3/2))/3

**Rubi [A]** time = 0.0608214, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{2}{3}\sqrt{\pi}b^{3/2}f^a\log^{\frac{3}{2}}(f)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}+\frac{2}{3}bx\log(f)f^{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)\*x^2,x]

[Out] (f^(a + b/x^2)\*x^3)/3 + (2\*b\*f^(a + b/x^2)\*x\*Log[f])/3 - (2\*b^(3/2)\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(3/2))/3

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int f^{a+\frac{b}{x^2}} x^2 dx &= \frac{1}{3} f^{a+\frac{b}{x^2}} x^3 + \frac{1}{3} (2b \log(f)) \int f^{a+\frac{b}{x^2}} dx \\
&= \frac{1}{3} f^{a+\frac{b}{x^2}} x^3 + \frac{2}{3} b f^{a+\frac{b}{x^2}} x \log(f) + \frac{1}{3} (4b^2 \log^2(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx \\
&= \frac{1}{3} f^{a+\frac{b}{x^2}} x^3 + \frac{2}{3} b f^{a+\frac{b}{x^2}} x \log(f) - \frac{1}{3} (4b^2 \log^2(f)) \text{Subst} \left( \int f^{a+bx^2} dx, x, \frac{1}{x} \right) \\
&= \frac{1}{3} f^{a+\frac{b}{x^2}} x^3 + \frac{2}{3} b f^{a+\frac{b}{x^2}} x \log(f) - \frac{2}{3} b^{3/2} f^a \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right) \log^{\frac{3}{2}}(f)
\end{aligned}$$

**Mathematica [A]** time = 0.0243976, size = 60, normalized size = 0.82

$$\frac{1}{3} f^a \left( x f^{\frac{b}{x^2}} (2b \log(f) + x^2) - 2\sqrt{\pi} b^{3/2} \log^{\frac{3}{2}}(f) \operatorname{Erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)\*x^2,x]

[Out] (f^a\*(-2\*b^(3/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Log[f]^(3/2) + f^(b/x^2)\*x\*(x^2 + 2\*b\*Log[f]))/3

**Maple [A]** time = 0.028, size = 67, normalized size = 0.9

$$\frac{f^a x^3}{3} f^{\frac{b}{x^2}} + \frac{2 f^a \ln(f) b x}{3} f^{\frac{b}{x^2}} - \frac{2 f^a (\ln(f))^2 b^2 \sqrt{\pi}}{3} \operatorname{Erf} \left( \frac{1}{x} \sqrt{-b \ln(f)} \right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)\*x^2,x)

[Out] 1/3\*f^a\*x^3\*f^(b/x^2)+2/3\*f^a\*ln(f)\*b\*x\*f^(b/x^2)-2/3\*f^a\*ln(f)^2\*b^2\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.16611, size = 38, normalized size = 0.52

$$\frac{1}{2} f^a x^3 \left( -\frac{b \log(f)}{x^2} \right)^{\frac{3}{2}} \Gamma \left( -\frac{3}{2}, -\frac{b \log(f)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^2,x, algorithm="maxima")

[Out] 1/2\*f^a\*x^3\*(-b\*log(f)/x^2)^(3/2)\*gamma(-3/2, -b\*log(f)/x^2)



**Fricas [A]** time = 1.80747, size = 153, normalized size = 2.1

$$\frac{2}{3} \sqrt{\pi} \sqrt{-b \log(f)} b f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) \log(f) + \frac{1}{3} (x^3 + 2 b x \log(f)) f^{\frac{ax^2+b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^2,x, algorithm="fricas")

[Out] 2/3\*sqrt(pi)\*sqrt(-b\*log(f))\*b\*f^a\*erf(sqrt(-b\*log(f))/x)\*log(f) + 1/3\*(x^3 + 2\*b\*x\*log(f))\*f^((a\*x^2 + b)/x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)\*x\*\*2,x)

[Out] Integral(f\*\*(a + b/x\*\*2)\*x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)\*x^2,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)\*x^2, x)

### 3.146 $\int f^{a+\frac{b}{x^2}} dx$

**Optimal.** Leaf size=49

$$xf^{a+\frac{b}{x^2}} - \sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)$$

[Out]  $f^{(a + b/x^2)*x} - \operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x]*\operatorname{Sqrt}[\operatorname{Log}[f]]$

**Rubi [A]** time = 0.0342323, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2206, 2211, 2204}

$$xf^{a+\frac{b}{x^2}} - \sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}, x]$

[Out]  $f^{(a + b/x^2)*x} - \operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x]*\operatorname{Sqrt}[\operatorname{Log}[f]]$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)}), x\_Symbol] := \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n*F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[2/n] \ \&\& \ \operatorname{LtQ}[n, 0]$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}), x\_Symbol] := \operatorname{Dist}[1/(d*(m + 1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n\}, x \ \&\& \ \operatorname{EqQ}[n, 2*(m + 1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(2)}), x\_Symbol] := \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^2}} dx &= f^{a+\frac{b}{x^2}}x + (2b \log(f)) \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx \\ &= f^{a+\frac{b}{x^2}}x - (2b \log(f)) \operatorname{Subst}\left(\int f^{a+bx^2} dx, x, \frac{1}{x}\right) \\ &= f^{a+\frac{b}{x^2}}x - \sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)\sqrt{\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0109012, size = 49, normalized size = 1.

$$x f^{a+\frac{b}{x^2}} - \sqrt{\pi} \sqrt{b} f^a \sqrt{\log(f)} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2), x]

[Out] f^(a + b/x^2)\*x - Sqrt[b]\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x]\*Sqrt[Log[f]]

**Maple [A]** time = 0.024, size = 44, normalized size = 0.9

$$f^a x f^{\frac{b}{x^2}} - f^a \ln(f) b \sqrt{\pi} \operatorname{Erf}\left(\frac{1}{x} \sqrt{-b \ln(f)}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2), x)

[Out] f^a\*x\*f^(b/x^2)-f^a\*ln(f)\*b\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.23155, size = 35, normalized size = 0.71

$$\frac{1}{2} f^a x \sqrt{-\frac{b \log(f)}{x^2}} \Gamma\left(-\frac{1}{2}, -\frac{b \log(f)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2), x, algorithm="maxima")

[Out] 1/2\*f^a\*x\*sqrt(-b\*log(f)/x^2)\*gamma(-1/2, -b\*log(f)/x^2)

**Fricas [A]** time = 1.78514, size = 104, normalized size = 2.12

$$\sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + f^{\frac{ax^2+b}{x^2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2), x, algorithm="fricas")

[Out] sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))/x) + f^((a\*x^2 + b)/x^2)\*x

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2),x)

[Out] Integral(f\*\*(a + b/x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2),x, algorithm="giac")

[Out] integrate(f^(a + b/x^2), x)

$$3.147 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx$$

**Optimal.** Leaf size=39

$$-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

[Out]  $-(f^a \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[(\operatorname{Sqrt}[b] \operatorname{Sqrt}[\operatorname{Log}[f]])/x]) / (2 \operatorname{Sqrt}[b] \operatorname{Sqrt}[\operatorname{Log}[f]])$

**Rubi [A]** time = 0.0274652, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2211, 2204}

$$-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^2)}/x^2, x]$

[Out]  $-(f^a \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[(\operatorname{Sqrt}[b] \operatorname{Sqrt}[\operatorname{Log}[f]])/x]) / (2 \operatorname{Sqrt}[b] \operatorname{Sqrt}[\operatorname{Log}[f]])$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m + 1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n\}, x \ \&\& \ \operatorname{EqQ}[n, 2*(m + 1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[(c + d*x) \operatorname{Rt}[b \operatorname{Log}[F], 2]]) / (2*d \operatorname{Rt}[b \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx &= -\operatorname{Subst}\left(\int f^{a+bx^2} dx, x, \frac{1}{x}\right) \\ &= -\frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.0060323, size = 39, normalized size = 1.

$$-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^2,x]

[Out]  $-(f^a \sqrt{\pi}) \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right) / (2 \sqrt{b} \sqrt{\log(f)})$

**Maple [A]** time = 0.023, size = 28, normalized size = 0.7

$$-\frac{f^a \sqrt{\pi}}{2} \operatorname{Erf}\left(\frac{1}{x} \sqrt{-b \ln(f)}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^2,x)

[Out]  $-1/2 * f^a * \pi^{(1/2)} / (-b * \ln(f))^{(1/2)} * \operatorname{erf}((-b * \ln(f))^{(1/2)} / x)$

**Maxima [A]** time = 1.14587, size = 46, normalized size = 1.18

$$-\frac{\sqrt{\pi} f^a \left( \operatorname{erf}\left(\sqrt{-\frac{b \log(f)}{x^2}}\right) - 1 \right)}{2 x \sqrt{-\frac{b \log(f)}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^2,x, algorithm="maxima")

[Out]  $-1/2 * \operatorname{sqrt}(\pi) * f^a * (\operatorname{erf}(\operatorname{sqrt}(-b * \log(f) / x^2)) - 1) / (x * \operatorname{sqrt}(-b * \log(f) / x^2))$

**Fricas [A]** time = 2.02545, size = 92, normalized size = 2.36

$$\frac{\sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right)}{2 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^2,x, algorithm="fricas")

[Out]  $1/2 * \operatorname{sqrt}(\pi) * \operatorname{sqrt}(-b * \log(f)) * f^a * \operatorname{erf}(\operatorname{sqrt}(-b * \log(f)) / x) / (b * \log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a + \frac{b}{x^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x**2)/x**2,x)
```

```
[Out] Integral(f**(a + b/x**2)/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)/x^2, x)
```

$$3.148 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx$$

**Optimal.** Leaf size=63

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}$$

[Out] (f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(4\*b^(3/2)\*Log[f]^(3/2)) - f^(a + b/x^2)/(2\*b\*x\*Log[f])

**Rubi [A]** time = 0.0526033, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2212, 2211, 2204}

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^4, x]

[Out] (f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(4\*b^(3/2)\*Log[f]^(3/2)) - f^(a + b/x^2)/(2\*b\*x\*Log[f])

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2211

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)} - \frac{\int \frac{f^{\frac{a+b}{x^2}}}{x^2} dx}{2b \log(f)} \\
&= -\frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)} + \frac{\text{Subst}\left(\int f^{a+bx^2} dx, x, \frac{1}{x}\right)}{2b \log(f)} \\
&= \frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0191198, size = 63, normalized size = 1.

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^4,x]

[Out] (f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(4\*b^(3/2)\*Log[f]^(3/2)) - f^(a + b/x^2)/(2\*b\*x\*Log[f])

**Maple [A]** time = 0.03, size = 58, normalized size = 0.9

$$-\frac{f^a}{2 \ln(f) b x} f^{\frac{b}{x^2}} + \frac{f^a \sqrt{\pi}}{4 b \ln(f)} \operatorname{Erf}\left(\frac{1}{x} \sqrt{-b \ln(f)}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^4,x)

[Out] -1/2\*f^a\*f^(b/x^2)/x/b/ln(f)+1/4\*f^a/ln(f)/b\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.22468, size = 38, normalized size = 0.6

$$\frac{f^a \Gamma\left(\frac{3}{2}, -\frac{b \log(f)}{x^2}\right)}{2 x^3 \left(-\frac{b \log(f)}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^4,x, algorithm="maxima")

[Out] 1/2\*f^a\*gamma(3/2, -b\*log(f)/x^2)/(x^3\*(-b\*log(f)/x^2)^(3/2))

---

**Fricas [A]** time = 2.05611, size = 151, normalized size = 2.4

$$\frac{\sqrt{\pi}\sqrt{-b\log(f)}f^ax\operatorname{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)+2bf^{\frac{ax^2+b}{x^2}}\log(f)}{4b^2x\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^4,x, algorithm="fricas")

[Out] -1/4\*(sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*x\*erf(sqrt(-b\*log(f))/x) + 2\*b\*f^((a\*x^2 + b)/x^2)\*log(f))/(b^2\*x\*log(f)^2)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^4,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^4, x)

$$3.149 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx$$

**Optimal.** Leaf size=86

$$-\frac{3\sqrt{\pi}f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{8b^{5/2}\log^5(f)} + \frac{3f^{a+\frac{b}{x^2}}}{4b^2x\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

[Out]  $(-3*f^a*\sqrt{\text{Pi}}*\operatorname{Erfi}[(\sqrt{b}*\sqrt{\text{Log}[f]})/x])/(8*b^{(5/2)}*\text{Log}[f]^{(5/2)}) + (3*f^{(a + b/x^2)})/(4*b^2*x*\text{Log}[f]^2) - f^{(a + b/x^2)}/(2*b*x^3*\text{Log}[f])$

**Rubi [A]** time = 0.0790159, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2212, 2211, 2204}

$$-\frac{3\sqrt{\pi}f^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{8b^{5/2}\log^5(f)} + \frac{3f^{a+\frac{b}{x^2}}}{4b^2x\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^6, x]

[Out]  $(-3*f^a*\sqrt{\text{Pi}}*\operatorname{Erfi}[(\sqrt{b}*\sqrt{\text{Log}[f]})/x])/(8*b^{(5/2)}*\text{Log}[f]^{(5/2)}) + (3*f^{(a + b/x^2)})/(4*b^2*x*\text{Log}[f]^2) - f^{(a + b/x^2)}/(2*b*x^3*\text{Log}[f])$

#### Rule 2212

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^(n\_))\*((c\_) + (d\_)\*(x\_)^(m\_)), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2211

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^(n\_))\*((c\_) + (d\_)\*(x\_)^(m\_)), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^3 \log(f)} - \frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx}{2b \log(f)} \\
&= \frac{3f^{a+\frac{b}{x^2}}}{4b^2x \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3 \log(f)} + \frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx}{4b^2 \log^2(f)} \\
&= \frac{3f^{a+\frac{b}{x^2}}}{4b^2x \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3 \log(f)} - \frac{3 \text{Subst}\left(\int f^{a+bx^2} dx, x, \frac{1}{x}\right)}{4b^2 \log^2(f)} \\
&= -\frac{3f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)}{8b^{5/2} \log^{5/2}(f)} + \frac{3f^{a+\frac{b}{x^2}}}{4b^2x \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.049416, size = 74, normalized size = 0.86

$$\frac{f^{a+\frac{b}{x^2}} (3x^2 - 2b \log(f))}{4b^2x^3 \log^2(f)} - \frac{3\sqrt{\pi} f^a \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)}{8b^{5/2} \log^{5/2}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^6, x]

[Out]  $(-3f^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(f)})/x]) / (8b^{5/2} \log^{5/2}(f)) + (f^{a+b/x^2} (3x^2 - 2b \log(f))) / (4b^2x^3 \log^2(f))$

**Maple [A]** time = 0.035, size = 80, normalized size = 0.9

$$-\frac{f^a}{2bx^3 \ln(f)} f^{\frac{b}{x^2}} + \frac{3f^a}{4(\ln(f))^2 b^2x} f^{\frac{b}{x^2}} - \frac{3f^a \sqrt{\pi}}{8(\ln(f))^2 b^2} \operatorname{Erf}\left(\frac{1}{x} \sqrt{-b \ln(f)}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^6, x)

[Out]  $-1/2*f^a*f^{(b/x^2)}/x^3/b/\ln(f)+3/4*f^a/\ln(f)^2/b^2*f^{(b/x^2)}/x-3/8*f^a/\ln(f)^2/b^2*\pi^{(1/2)/(-b*\ln(f))^{(1/2)*\operatorname{erf}((-b*\ln(f))^{(1/2)}/x)}$

**Maxima [A]** time = 1.16363, size = 38, normalized size = 0.44

$$\frac{f^a \Gamma\left(\frac{5}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^5 \left(-\frac{b \log(f)}{x^2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^6, x, algorithm="maxima")

[Out]  $\frac{1}{2}f^a \gamma\left(\frac{5}{2}, -b \log(f)/x^2\right) / \left(x^5 (-b \log(f)/x^2)^{5/2}\right)$

**Fricas [A]** time = 2.04876, size = 192, normalized size = 2.23

$$\frac{3 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^3 \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 \left(3 b x^2 \log(f) - 2 b^2 \log(f)^2\right) f^{\frac{ax^2+b}{x^2}}}{8 b^3 x^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^6,x, algorithm="fricas")`

[Out]  $\frac{1}{8} (3 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^3 \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 (3 b x^2 \log(f) - 2 b^2 \log(f)^2) f^{\frac{ax^2+b}{x^2}}) / (b^3 x^3 \log(f)^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**2)/x**6,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^2)/x^6,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^2)/x^6, x)`

$$3.150 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx$$

**Optimal.** Leaf size=109

$$\frac{15\sqrt{\pi}f^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{16b^{7/2}\log^2(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3\log^2(f)} - \frac{15f^{a+\frac{b}{x^2}}}{8b^3x\log^3(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}$$

[Out] (15\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(16\*b^(7/2)\*Log[f]^(7/2)) - (15\*f^(a + b/x^2))/(8\*b^3\*x\*Log[f]^3) + (5\*f^(a + b/x^2))/(4\*b^2\*x^3\*Log[f]^2) - f^(a + b/x^2)/(2\*b\*x^5\*Log[f])

**Rubi [A]** time = 0.112402, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2212, 2211, 2204}

$$\frac{15\sqrt{\pi}f^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{16b^{7/2}\log^2(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3\log^2(f)} - \frac{15f^{a+\frac{b}{x^2}}}{8b^3x\log^3(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^8, x]

[Out] (15\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(16\*b^(7/2)\*Log[f]^(7/2)) - (15\*f^(a + b/x^2))/(8\*b^3\*x\*Log[f]^3) + (5\*f^(a + b/x^2))/(4\*b^2\*x^3\*Log[f]^2) - f^(a + b/x^2)/(2\*b\*x^5\*Log[f])

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^5 \log(f)} - \frac{5 \int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx}{2b \log(f)} \\
&= \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5 \log(f)} + \frac{15 \int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx}{4b^2 \log^2(f)} \\
&= -\frac{15f^{a+\frac{b}{x^2}}}{8b^3x \log^3(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5 \log(f)} - \frac{15 \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx}{8b^3 \log^3(f)} \\
&= -\frac{15f^{a+\frac{b}{x^2}}}{8b^3x \log^3(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5 \log(f)} + \frac{15 \text{Subst} \left( \int f^{a+bx^2} dx, x, \frac{1}{x} \right)}{8b^3 \log^3(f)} \\
&= \frac{15f^a \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{\log(f)}}{x} \right)}{16b^{7/2} \log^{\frac{7}{2}}(f)} - \frac{15f^{a+\frac{b}{x^2}}}{8b^3x \log^3(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0557144, size = 86, normalized size = 0.79

$$\frac{15\sqrt{\pi}f^a \operatorname{Erfi} \left( \frac{\sqrt{b}\sqrt{\log(f)}}{x} \right)}{16b^{7/2} \log^{\frac{7}{2}}(f)} - \frac{f^{a+\frac{b}{x^2}} (4b^2 \log^2(f) - 10bx^2 \log(f) + 15x^4)}{8b^3x^5 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^8, x]

[Out] (15\*f^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x])/(16\*b^(7/2)\*Log[f]^(7/2)) - (f^(a + b/x^2)\*(15\*x^4 - 10\*b\*x^2\*Log[f] + 4\*b^2\*Log[f]^2))/(8\*b^3\*x^5\*Log[f]^3)

**Maple [A]** time = 0.044, size = 102, normalized size = 0.9

$$-\frac{f^a}{2x^5b \ln(f)} f^{\frac{b}{x^2}} + \frac{5f^a}{4b^2x^3 (\ln(f))^2} f^{\frac{b}{x^2}} - \frac{15f^a}{8b^3x (\ln(f))^3} f^{\frac{b}{x^2}} + \frac{15f^a \sqrt{\pi}}{16 (\ln(f))^3 b^3} \operatorname{Erf} \left( \frac{1}{x} \sqrt{-b \ln(f)} \right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^8, x)

[Out] -1/2\*f^a\*f^(b/x^2)/x^5/b/ln(f)+5/4\*f^a/ln(f)^2/b^2\*f^(b/x^2)/x^3-15/8\*f^a/ln(f)^3/b^3\*f^(b/x^2)/x+15/16\*f^a/ln(f)^3/b^3\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.24372, size = 38, normalized size = 0.35

$$\frac{f^a \Gamma \left( \frac{7}{2}, -\frac{b \log(f)}{x^2} \right)}{2x^7 \left( -\frac{b \log(f)}{x^2} \right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^8,x, algorithm="maxima")

[Out]  $\frac{1}{2}f^a\gamma(7/2, -b\log(f)/x^2)/(x^7*(-b\log(f)/x^2)^{(7/2)})$

**Fricas [A]** time = 2.11687, size = 227, normalized size = 2.08

$$\frac{15\sqrt{\pi}\sqrt{-b\log(f)}f^ax^5\operatorname{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)+2\left(15bx^4\log(f)-10b^2x^2\log(f)^2+4b^3\log(f)^3\right)f^{\frac{ax^2+b}{x^2}}}{16b^4x^5\log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^8,x, algorithm="fricas")

[Out]  $-1/16*(15*\sqrt{\pi}*\sqrt{-b*\log(f)}*f^a*x^5*\operatorname{erf}(\sqrt{-b*\log(f)}/x) + 2*(15*b*x^4*\log(f) - 10*b^2*x^2*\log(f)^2 + 4*b^3*\log(f)^3)*f^{(a*x^2 + b)/x^2})/(b^4*x^5*\log(f)^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*8,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^8,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^8, x)



$$3.151 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^{10}} dx$$

**Optimal.** Leaf size=132

$$-\frac{105\sqrt{\pi}f^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{32b^{9/2}\log^2(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5\log^2(f)} - \frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3\log^3(f)} + \frac{105f^{a+\frac{b}{x^2}}}{16b^4x\log^4(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7\log(f)}$$

[Out]  $(-105*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x])/(32*b^{(9/2)}*\operatorname{Log}[f]^{(9/2)}) + (105*f^{(a+b/x^2)})/(16*b^4*x*\operatorname{Log}[f]^4) - (35*f^{(a+b/x^2)})/(8*b^3*x^3*\operatorname{Log}[f]^3) + (7*f^{(a+b/x^2)})/(4*b^2*x^5*\operatorname{Log}[f]^2) - f^{(a+b/x^2)}/(2*b*x^7*\operatorname{Log}[f])$

**Rubi [A]** time = 0.160033, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2212, 2211, 2204}

$$-\frac{105\sqrt{\pi}f^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{32b^{9/2}\log^2(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5\log^2(f)} - \frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3\log^3(f)} + \frac{105f^{a+\frac{b}{x^2}}}{16b^4x\log^4(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7\log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a+b/x^2)}/x^{10}, x]$

[Out]  $(-105*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[f]])/x])/(32*b^{(9/2)}*\operatorname{Log}[f]^{(9/2)}) + (105*f^{(a+b/x^2)})/(16*b^4*x*\operatorname{Log}[f]^4) - (35*f^{(a+b/x^2)})/(8*b^3*x^3*\operatorname{Log}[f]^3) + (7*f^{(a+b/x^2)})/(4*b^2*x^5*\operatorname{Log}[f]^2) - f^{(a+b/x^2)}/(2*b*x^7*\operatorname{Log}[f])$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m-n+1)}*F^{(a+b*(c+d*x)^n)}/(b*d*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(m-n+1)/(b*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-n)}*F^{(a+b*(c+d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{IntegerQ}[(2*(m+1))/n] \&\& \operatorname{LtQ}[0, (m+1)/n, 5] \&\& \operatorname{IntegerQ}[n] \&\& (\operatorname{LtQ}[0, n, m+1] \mid\mid \operatorname{LtQ}[m, n, 0])$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{(a+b*x^2)}, x], x, (c+d*x)^{(m+1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n\}, x \&\& \operatorname{EqQ}[n, 2*(m+1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}} dx &= -\frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)} - \frac{7 \int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx}{2b \log(f)} \\
&= \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)} + \frac{35 \int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx}{4b^2 \log^2(f)} \\
&= -\frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3 \log^3(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)} - \frac{105 \int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx}{8b^3 \log^3(f)} \\
&= \frac{105f^{a+\frac{b}{x^2}}}{16b^4x \log^4(f)} - \frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3 \log^3(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)} + \frac{105 \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx}{16b^4 \log^4(f)} \\
&= \frac{105f^{a+\frac{b}{x^2}}}{16b^4x \log^4(f)} - \frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3 \log^3(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)} - \frac{105 \operatorname{Subst}\left(\int f^{a+bx^2} dx, x, \frac{1}{x}\right)}{16b^4 \log^4(f)} \\
&= -\frac{105f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{32b^{9/2} \log^{\frac{9}{2}}(f)} + \frac{105f^{a+\frac{b}{x^2}}}{16b^4x \log^4(f)} - \frac{35f^{a+\frac{b}{x^2}}}{8b^3x^3 \log^3(f)} + \frac{7f^{a+\frac{b}{x^2}}}{4b^2x^5 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^7 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0864489, size = 100, normalized size = 0.76

$$\frac{f^a \left( \frac{2\sqrt{b}\sqrt{\log(f)}f^{\frac{b}{x^2}}(28b^2x^2 \log^2(f) - 8b^3 \log^3(f) - 70bx^4 \log(f) + 105x^6)}{x^7} - 105\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \right)}{32b^{9/2} \log^{\frac{9}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^10,x]

[Out] (f^a\*(-105\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[f]])/x] + (2\*Sqrt[b]\*f^(b/x^2)\*Sqrt[Log[f]]\*(105\*x^6 - 70\*b\*x^4\*Log[f] + 28\*b^2\*x^2\*Log[f]^2 - 8\*b^3\*Log[f]^3))/x^7))/(32\*b^(9/2)\*Log[f]^(9/2))

**Maple [A]** time = 0.058, size = 124, normalized size = 0.9

$$-\frac{f^a}{2x^7 b \ln(f)} f^{\frac{b}{x^2}} + \frac{7f^a}{4(\ln(f))^2 b^2 x^5} f^{\frac{b}{x^2}} - \frac{35f^a}{8b^3 x^3 (\ln(f))^3} f^{\frac{b}{x^2}} + \frac{105f^a}{16b^4 x (\ln(f))^4} f^{\frac{b}{x^2}} - \frac{105f^a \sqrt{\pi}}{32b^4 (\ln(f))^4} \operatorname{Erf}\left(\frac{1}{x} \sqrt{-b \ln(f)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^10,x)

[Out] -1/2\*f^a\*f^(b/x^2)/x^7/b/ln(f)+7/4\*f^a/ln(f)^2/b^2\*f^(b/x^2)/x^5-35/8\*f^a/ln(f)^3/b^3\*f^(b/x^2)/x^3+105/16\*f^a/ln(f)^4/b^4\*f^(b/x^2)/x-105/32\*f^a/ln(f)^4/b^4\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)/x)

**Maxima [A]** time = 1.2237, size = 38, normalized size = 0.29

$$\frac{f^a \Gamma\left(\frac{9}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^9 \left(-\frac{b \log(f)}{x^2}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^10,x, algorithm="maxima")

[Out] 1/2\*f^a\*gamma(9/2, -b\*log(f)/x^2)/(x^9\*(-b\*log(f)/x^2)^(9/2))

**Fricas [A]** time = 1.91263, size = 258, normalized size = 1.95

$$\frac{105 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^7 \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 \left(105 b x^6 \log(f) - 70 b^2 x^4 \log(f)^2 + 28 b^3 x^2 \log(f)^3 - 8 b^4 \log(f)^4\right) f^{\frac{ax^2}{x}}}{32 b^5 x^7 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^10,x, algorithm="fricas")

[Out] 1/32\*(105\*sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*x^7\*erf(sqrt(-b\*log(f))/x) + 2\*(105\*b\*x^6\*log(f) - 70\*b^2\*x^4\*log(f)^2 + 28\*b^3\*x^2\*log(f)^3 - 8\*b^4\*log(f)^4)\*f^((a\*x^2 + b)/x^2))/(b^5\*x^7\*log(f)^5)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^10,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^10, x)

$$3.152 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^{12}} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{11} \left(-\frac{b \log(f)}{x^2}\right)^{11/2}}$$

[Out] (f^a\*Gamma[11/2, -((b\*Log[f])/x^2)))/(2\*x^11\*(-((b\*Log[f])/x^2))^(11/2))

**Rubi [A]** time = 0.0256291, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{11} \left(-\frac{b \log(f)}{x^2}\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^12, x]

[Out] (f^a\*Gamma[11/2, -((b\*Log[f])/x^2)))/(2\*x^11\*(-((b\*Log[f])/x^2))^(11/2))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F]))]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}} dx = \frac{f^a \Gamma\left(\frac{11}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{11} \left(-\frac{b \log(f)}{x^2}\right)^{11/2}}$$

**Mathematica [A]** time = 0.0052913, size = 34, normalized size = 1.

$$\frac{f^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{11} \left(-\frac{b \log(f)}{x^2}\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^12, x]

[Out] (f^a\*Gamma[11/2, -((b\*Log[f])/x^2)))/(2\*x^11\*(-((b\*Log[f])/x^2))^(11/2))

---

**Maple [A]** time = 0.08, size = 146, normalized size = 4.3

$$-\frac{f^a}{2bx^9 \ln(f)} f^{\frac{b}{x^2}} + \frac{9f^a}{4(\ln(f))^2 b^2 x^7} f^{\frac{b}{x^2}} - \frac{63f^a}{8(\ln(f))^3 b^3 x^5} f^{\frac{b}{x^2}} + \frac{315f^a}{16b^4 x^3 (\ln(f))^4} f^{\frac{b}{x^2}} - \frac{945f^a}{32b^5 (\ln(f))^5 x} f^{\frac{b}{x^2}} + \frac{945}{64b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^12,x)

[Out]  $-1/2*f^a*f^{(b/x^2)}/x^9/b/\ln(f)+9/4*f^a/\ln(f)^2/b^2*f^{(b/x^2)}/x^7-63/8*f^a/\ln(f)^3/b^3*f^{(b/x^2)}/x^5+315/16*f^a/\ln(f)^4/b^4*f^{(b/x^2)}/x^3-945/32*f^a/\ln(f)^5/b^5*f^{(b/x^2)}/x+945/64*f^a/\ln(f)^5/b^5*\text{Pi}^{(1/2)}/(-b*\ln(f))^{(1/2)}*\text{erf}((-b*\ln(f))^{(1/2)})/x$

---

**Maxima [A]** time = 1.25357, size = 38, normalized size = 1.12

$$\frac{f^a \Gamma\left(\frac{11}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{11} \left(-\frac{b \log(f)}{x^2}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^12,x, algorithm="maxima")

[Out]  $1/2*f^a*\text{gamma}(11/2, -b*\log(f)/x^2)/(x^{11}*(-b*\log(f)/x^2)^{(11/2)})$

---

**Fricas [A]** time = 1.78804, size = 293, normalized size = 8.62

$$\frac{945 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^9 \text{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 \left(945 b x^8 \log(f) - 630 b^2 x^6 \log(f)^2 + 252 b^3 x^4 \log(f)^3 - 72 b^4 x^2 \log(f)^4 + 16 b^5 \log(f)^5\right) f^{(a*x^2 + b)/x^2}}{64 b^6 x^9 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^12,x, algorithm="fricas")

[Out]  $-1/64*(945*\text{sqrt}(\text{pi})*\text{sqrt}(-b*\log(f))*f^a*x^9*\text{erf}(\text{sqrt}(-b*\log(f))/x) + 2*(945*b*x^8*\log(f) - 630*b^2*x^6*\log(f)^2 + 252*b^3*x^4*\log(f)^3 - 72*b^4*x^2*\log(f)^4 + 16*b^5*\log(f)^5)*f^{(a*x^2 + b)/x^2})/(b^6*x^9*\log(f)^6)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*2)/x\*\*12,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^12,x, algorithm="giac")

[Out] integrate(f^(a + b/x^2)/x^12, x)

$$3.153 \quad \int \frac{f^{a+\frac{b}{x^2}}}{x^{14}} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{13} \left(-\frac{b \log(f)}{x^2}\right)^{13/2}}$$

[Out] (f^a\*Gamma[13/2, -((b\*Log[f])/x^2)])/(2\*x^13\*(-((b\*Log[f])/x^2))^(13/2))

**Rubi [A]** time = 0.0240466, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{13} \left(-\frac{b \log(f)}{x^2}\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^2)/x^14,x]

[Out] (f^a\*Gamma[13/2, -((b\*Log[f])/x^2)])/(2\*x^13\*(-((b\*Log[f])/x^2))^(13/2))

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}} dx = \frac{f^a \Gamma\left(\frac{13}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{13} \left(-\frac{b \log(f)}{x^2}\right)^{13/2}}$$

**Mathematica [A]** time = 0.0057157, size = 34, normalized size = 1.

$$\frac{f^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{13} \left(-\frac{b \log(f)}{x^2}\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^2)/x^14,x]

[Out] (f^a\*Gamma[13/2, -((b\*Log[f])/x^2)])/(2\*x^13\*(-((b\*Log[f])/x^2))^(13/2))

---

**Maple [A]** time = 0.125, size = 168, normalized size = 4.9

$$-\frac{f^a}{2x^{11}b\ln(f)}f^{\frac{b}{x^2}} + \frac{11f^a}{4b^2x^9(\ln(f))^2}f^{\frac{b}{x^2}} - \frac{99f^a}{8(\ln(f))^3b^3x^7}f^{\frac{b}{x^2}} + \frac{693f^a}{16b^4(\ln(f))^4x^5}f^{\frac{b}{x^2}} - \frac{3465f^a}{32b^5(\ln(f))^5x^3}f^{\frac{b}{x^2}} + \frac{10395f^a}{64(\ln(f))^6b^6x}f^{\frac{b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^2)/x^14,x)

[Out]  $-\frac{1}{2}f^a f^{\frac{b}{x^2}}/x^{11}/b/\ln(f) + \frac{11}{4}f^a f^{\frac{b}{x^2}}/\ln(f)^2/b^2/x^9 - \frac{99}{8}f^a f^{\frac{b}{x^2}}/\ln(f)^3/b^3/x^7 + \frac{693}{16}f^a f^{\frac{b}{x^2}}/\ln(f)^4/b^4/x^5 - \frac{3465}{32}f^a f^{\frac{b}{x^2}}/\ln(f)^5/b^5/x^3 + \frac{10395}{64}f^a f^{\frac{b}{x^2}}/\ln(f)^6/b^6/x - \frac{10395}{128}f^a f^{\frac{b}{x^2}}/\ln(f)^6/b^6 \cdot \frac{\pi^{1/2}}{(-b \ln(f))^{1/2}} \operatorname{erf}\left(\frac{-b \ln(f)}{x}\right)$

---

**Maxima [A]** time = 1.19514, size = 38, normalized size = 1.12

$$\frac{f^a \Gamma\left(\frac{13}{2}, -\frac{b \log(f)}{x^2}\right)}{2x^{13} \left(-\frac{b \log(f)}{x^2}\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^14,x, algorithm="maxima")

[Out]  $\frac{1}{2}f^a \cdot \frac{\Gamma(13/2, -b \log(f)/x^2)}{(x^{13} (-b \log(f)/x^2)^{13/2}}$

---

**Fricas [A]** time = 1.82045, size = 338, normalized size = 9.94

$$\frac{10395 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^{11} \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 \left(10395 b x^{10} \log(f) - 6930 b^2 x^8 \log(f)^2 + 2772 b^3 x^6 \log(f)^3 - 792 b^4 x^4 \log(f)^4 + 176 b^5 x^2 \log(f)^5 - 32 b^6 \log(f)^6\right) f^{\frac{b}{x^2}}}{128 b^7 x^{11} \log(f)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^2)/x^14,x, algorithm="fricas")

[Out]  $\frac{1}{128} \left(10395 \sqrt{\pi} \sqrt{-b \log(f)} f^a x^{11} \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) + 2 \left(10395 b x^{10} \log(f) - 6930 b^2 x^8 \log(f)^2 + 2772 b^3 x^6 \log(f)^3 - 792 b^4 x^4 \log(f)^4 + 176 b^5 x^2 \log(f)^5 - 32 b^6 \log(f)^6\right) f^{\frac{b}{x^2}}\right) / (b^7 x^{11} \log(f)^7)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(f**(a+b/x**2)/x**14,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^2)/x^14,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^2)/x^14, x)
```

### 3.154 $\int f^{a+\frac{b}{x^3}} x^m dx$

**Optimal.** Leaf size=46

$$\frac{1}{3} f^a x^{m+1} \left( -\frac{b \log(f)}{x^3} \right)^{\frac{m+1}{3}} \text{Gamma} \left( \frac{1}{3}(-m-1), -\frac{b \log(f)}{x^3} \right)$$

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^((1 + m)/3))/3

**Rubi [A]** time = 0.0250903, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{3} f^a x^{m+1} \left( -\frac{b \log(f)}{x^3} \right)^{\frac{m+1}{3}} \text{Gamma} \left( \frac{1}{3}(-m-1), -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^m,x]

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^((1 + m)/3))/3

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^3}} x^m dx = \frac{1}{3} f^a x^{1+m} \Gamma \left( \frac{1}{3}(-1-m), -\frac{b \log(f)}{x^3} \right) \left( -\frac{b \log(f)}{x^3} \right)^{\frac{1+m}{3}}$$

**Mathematica [A]** time = 0.0102192, size = 46, normalized size = 1.

$$\frac{1}{3} f^a x^{m+1} \left( -\frac{b \log(f)}{x^3} \right)^{\frac{m+1}{3}} \text{Gamma} \left( \frac{1}{3}(-m-1), -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^m,x]

[Out] (f^a\*x^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^((1 + m)/3))/3

**Maple [B]** time = 0.037, size = 169, normalized size = 3.7

$$-\frac{f^a}{3}(-b)^{\frac{1}{3}+\frac{m}{3}}(\ln(f))^{\frac{1}{3}+\frac{m}{3}}\left(3\frac{x^{-2+m}(-b)^{-m/3-1/3}(\ln(f))^{2/3-m/3}b\Gamma(2/3-m/3)\left(-\frac{b\ln(f)}{x^3}\right)^{-2/3+m/3}}{1+m}-3\frac{x^{1+m}(-b)^{-m/3-1/3}}{1+m}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)\*x^m,x)

[Out]  $-1/3*f^a*(-b)^{(1/3+1/3*m)}*\ln(f)^{(1/3+1/3*m)}*(3/(1+m)*x^{(-2+m)}*(-b)^{(-1/3*m-1/3)}*\ln(f)^{(2/3-1/3*m)}*b*(-b*\ln(f)/x^3)^{(-2/3+1/3*m)}*GAMMA(2/3-1/3*m)-3/(1+m)*x^{(1+m)}*(-b)^{(-1/3*m-1/3)}*\ln(f)^{(-1/3*m-1/3)}*\exp(b*\ln(f)/x^3)-3/(1+m)*x^{(-2+m)}*(-b)^{(-1/3*m-1/3)}*\ln(f)^{(2/3-1/3*m)}*b*(-b*\ln(f)/x^3)^{(-2/3+1/3*m)}*GAMMA(2/3-1/3*m,-b*\ln(f)/x^3)$

**Maxima [A]** time = 1.3426, size = 51, normalized size = 1.11

$$\frac{1}{3}f^a x^{m+1} \left(-\frac{b \log(f)}{x^3}\right)^{\frac{1}{3}m+\frac{1}{3}} \Gamma\left(-\frac{1}{3}m-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^m,x, algorithm="maxima")

[Out]  $1/3*f^a*x^{(m+1)}*(-b*\log(f)/x^3)^{(1/3*m+1/3)}*\gamma(-1/3*m-1/3,-b*\log(f)/x^3)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{\frac{ax^3+b}{x^3}}x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^m,x, algorithm="fricas")

[Out] integral(f^((a\*x^3 + b)/x^3)\*x^m, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}}x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)\*x\*\*m,x)

[Out] Integral(f\*\*(a + b/x\*\*3)\*x\*\*m, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^m,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)\*x^m, x)

$$3.155 \quad \int f^{a+\frac{b}{x^3}} x^{14} dx$$

**Optimal.** Leaf size=24

$$-\frac{1}{3}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right)$$

[Out]  $-(b^5 f^a \Gamma[-5, -((b \text{Log}[f])/x^3)] \text{Log}[f]^5)/3$

**Rubi [A]** time = 0.0279187, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$-\frac{1}{3}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^14,x]

[Out]  $-(b^5 f^a \Gamma[-5, -((b \text{Log}[f])/x^3)] \text{Log}[f]^5)/3$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int f^{a+\frac{b}{x^3}} x^{14} dx = -\frac{1}{3}b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right) \log^5(f)$$

**Mathematica [A]** time = 0.0023997, size = 24, normalized size = 1.

$$-\frac{1}{3}b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^14,x]

[Out]  $-(b^5 f^a \Gamma[-5, -((b \text{Log}[f])/x^3)] \text{Log}[f]^5)/3$

**Maple [B]** time = 0.056, size = 249, normalized size = 10.4

$$\frac{f^a b^5 (\ln(f))^5}{3} \left( \frac{x^{15}}{5 b^5 (\ln(f))^5} + \frac{x^{12}}{4 b^4 (\ln(f))^4} + \frac{x^9}{6 (\ln(f))^3 b^3} + \frac{x^6}{12 (\ln(f))^2 b^2} + \frac{x^3}{24 b \ln(f)} + \frac{137}{7200} + \frac{\ln(x)}{40} - \frac{\ln}{1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)*x^14,x)`

[Out]  $\frac{1}{3}f^a b^5 \ln(f)^5 \left( \frac{1}{5}x^{15}/b^5/\ln(f)^5 + \frac{1}{4}x^{12}/b^4/\ln(f)^4 + \frac{1}{6}x^9/b^3/\ln(f)^3 + \frac{1}{12}x^6/b^2/\ln(f)^2 + \frac{1}{24}x^3/b/\ln(f) + \frac{137}{7200} + \frac{1}{40}\ln(x) - \frac{1}{120}\ln(-b) - \frac{1}{120}\ln(\ln(f)) - \frac{1}{7200}/b^5/\ln(f)^5 \right) x^{15} + 300b^4 \ln(f)^4/x^{12} + 600b^3 \ln(f)^3/x^9 + 1200b^2 \ln(f)^2/x^6 + 1800b \ln(f)/x^3 + 1440 + \frac{1}{720}/b^5/\ln(f)^5 x^{15} (6b^4 \ln(f)^4/x^{12} + 6b^3 \ln(f)^3/x^9 + 12b^2 \ln(f)^2/x^6 + 36b \ln(f)/x^3 + 144) \exp(b \ln(f)/x^3) + \frac{1}{120}\ln(-b \ln(f)/x^3) + \frac{1}{120}\text{Ei}(1, -b \ln(f)/x^3)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^14,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^14,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)*x**14,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)*x^14,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x^14, x)
```

$$3.156 \quad \int f^{a+\frac{b}{x^3}} x^{11} dx$$

**Optimal.** Leaf size=24

$$\frac{1}{3} b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right)$$

[Out] (b^4\*f^a\*Gamma[-4, -(b\*Log[f])/x^3])\*Log[f]^4/3

**Rubi [A]** time = 0.0330287, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{3} b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^11,x]

[Out] (b^4\*f^a\*Gamma[-4, -(b\*Log[f])/x^3])\*Log[f]^4/3

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :- Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^3}} x^{11} dx = \frac{1}{3} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right) \log^4(f)$$

**Mathematica [A]** time = 0.0023848, size = 24, normalized size = 1.

$$\frac{1}{3} b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^11,x]

[Out] (b^4\*f^a\*Gamma[-4, -(b\*Log[f])/x^3])\*Log[f]^4/3

**Maple [B]** time = 0.052, size = 213, normalized size = 8.9

$$-\frac{f^a b^4 (\ln(f))^4}{3} \left( -\frac{x^{12}}{4 b^4 (\ln(f))^4} - \frac{x^9}{3 (\ln(f))^3 b^3} - \frac{x^6}{4 (\ln(f))^2 b^2} - \frac{x^3}{6 b \ln(f)} - \frac{25}{288} - \frac{\ln(x)}{8} + \frac{\ln(-b)}{24} + \frac{\ln(\ln(f))}{24} + \dots \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)*x^11,x)`

[Out] 
$$-1/3*f^a*b^4*\ln(f)^4*(-1/4*x^{12}/b^4/\ln(f)^4-1/3*x^9/b^3/\ln(f)^3-1/4*x^6/b^2/\ln(f)^2-1/6*x^3/b/\ln(f)-25/288-1/8*\ln(x)+1/24*\ln(-b)+1/24*\ln(\ln(f)))+1/1440/b^4/\ln(f)^4*x^{12}*(125*b^4*\ln(f)^4/x^{12}+240*b^3*\ln(f)^3/x^9+360*b^2*\ln(f)^2/x^6+480*b*\ln(f)/x^3+360)-1/120/b^4/\ln(f)^4*x^{12}*(5*b^3*\ln(f)^3/x^9+5*b^2*\ln(f)^2/x^6+10*b*\ln(f)/x^3+30)*\exp(b*\ln(f)/x^3)-1/24*\ln(-b*\ln(f)/x^3)-1/24*Ei(1,-b*\ln(f)/x^3)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^11,x, algorithm="maxima")`

[Out] Exception raised: TypeError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^11,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)*x**11,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^11,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^3)*x^11, x)`

### 3.157 $\int f^{a+\frac{b}{x^3}} x^8 dx$

**Optimal.** Leaf size=81

$$-\frac{1}{18}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) + \frac{1}{18}b^2 x^3 \log^2(f) f^{a+\frac{b}{x^3}} + \frac{1}{9}x^9 f^{a+\frac{b}{x^3}} + \frac{1}{18}bx^6 \log(f) f^{a+\frac{b}{x^3}}$$

[Out]  $(f^{(a + b/x^3)}*x^9)/9 + (b*f^{(a + b/x^3)}*x^6*\operatorname{Log}[f])/18 + (b^2*f^{(a + b/x^3)}*x^3*\operatorname{Log}[f]^2)/18 - (b^3*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^3]*\operatorname{Log}[f]^3)/18$

**Rubi [A]** time = 0.114718, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$-\frac{1}{18}b^3 f^a \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) + \frac{1}{18}b^2 x^3 \log^2(f) f^{a+\frac{b}{x^3}} + \frac{1}{9}x^9 f^{a+\frac{b}{x^3}} + \frac{1}{18}bx^6 \log(f) f^{a+\frac{b}{x^3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^3)}*x^8, x]$

[Out]  $(f^{(a + b/x^3)}*x^9)/9 + (b*f^{(a + b/x^3)}*x^6*\operatorname{Log}[f])/18 + (b^2*f^{(a + b/x^3)}*x^3*\operatorname{Log}[f]^2)/18 - (b^3*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^3]*\operatorname{Log}[f]^3)/18$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{IntegerQ}[(2*(m + 1))/n] \ \&\& \ \operatorname{LtQ}[-4, (m + 1)/n, 5] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ ((\operatorname{GtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m, -1]) \ || \ (\operatorname{GtQ}[-n, 0] \ \&\& \ \operatorname{LeQ}[-n, m + 1]))$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.))], x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]])/(f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n, x\} \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^3}} x^8 dx &= \frac{1}{9}f^{a+\frac{b}{x^3}} x^9 + \frac{1}{3}(b \log(f)) \int f^{a+\frac{b}{x^3}} x^5 dx \\ &= \frac{1}{9}f^{a+\frac{b}{x^3}} x^9 + \frac{1}{18}bf^{a+\frac{b}{x^3}} x^6 \log(f) + \frac{1}{6}(b^2 \log^2(f)) \int f^{a+\frac{b}{x^3}} x^2 dx \\ &= \frac{1}{9}f^{a+\frac{b}{x^3}} x^9 + \frac{1}{18}bf^{a+\frac{b}{x^3}} x^6 \log(f) + \frac{1}{18}b^2 f^{a+\frac{b}{x^3}} x^3 \log^2(f) + \frac{1}{6}(b^3 \log^3(f)) \int \frac{f^{a+\frac{b}{x^3}}}{x} dx \\ &= \frac{1}{9}f^{a+\frac{b}{x^3}} x^9 + \frac{1}{18}bf^{a+\frac{b}{x^3}} x^6 \log(f) + \frac{1}{18}b^2 f^{a+\frac{b}{x^3}} x^3 \log^2(f) - \frac{1}{18}b^3 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \log^3(f) \end{aligned}$$

**Mathematica [A]** time = 0.019582, size = 57, normalized size = 0.7

$$\frac{1}{18}f^a \left( x^3 f^{\frac{b}{x^3}} (b^2 \log^2(f) + bx^3 \log(f) + 2x^6) - b^3 \log^3(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^8,x]

[Out] (f^a\*(-(b^3\*ExpIntegralEi[(b\*Log[f])/x^3]\*Log[f]^3) + f^(b/x^3)\*x^3\*(2\*x^6 + b\*x^3\*Log[f] + b^2\*Log[f]^2)))/18

**Maple [B]** time = 0.043, size = 177, normalized size = 2.2

$$\frac{f^a b^3 (\ln(f))^3}{3} \left( \frac{x^9}{3 (\ln(f))^3 b^3} + \frac{x^6}{2 (\ln(f))^2 b^2} + \frac{x^3}{2 b \ln(f)} + \frac{11}{36} + \frac{\ln(x)}{2} - \frac{\ln(-b)}{6} - \frac{\ln(\ln(f))}{6} - \frac{x^9}{72 (\ln(f))^3 b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)\*x^8,x)

[Out] 1/3\*f^a\*b^3\*ln(f)^3\*(1/3\*x^9/b^3/ln(f)^3+1/2\*x^6/b^2/ln(f)^2+1/2\*x^3/b/ln(f)+11/36+1/2\*ln(x)-1/6\*ln(-b)-1/6\*ln(ln(f))-1/72/b^3/ln(f)^3\*x^9\*(22\*b^3\*ln(f)^3/x^9+36\*b^2\*ln(f)^2/x^6+36\*b\*ln(f)/x^3+24)+1/24/b^3/ln(f)^3\*x^9\*(4\*b^2\*ln(f)^2/x^6+4\*b\*ln(f)/x^3+8)\*exp(b\*ln(f)/x^3)+1/6\*ln(-b\*ln(f)/x^3)+1/6\*Ei(1,-b\*ln(f)/x^3))

**Maxima [A]** time = 1.29929, size = 30, normalized size = 0.37

$$-\frac{1}{3} b^3 f^a \Gamma\left(-3, -\frac{b \log(f)}{x^3}\right) \log(f)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^8,x, algorithm="maxima")

[Out] -1/3\*b^3\*f^a\*gamma(-3, -b\*log(f)/x^3)\*log(f)^3

**Fricas [A]** time = 1.80731, size = 149, normalized size = 1.84

$$-\frac{1}{18} b^3 f^a \text{Ei}\left(\frac{b \log(f)}{x^3}\right) \log(f)^3 + \frac{1}{18} (2x^9 + bx^6 \log(f) + b^2 x^3 \log(f)^2) f^{\frac{ax^3+b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^8,x, algorithm="fricas")

[Out] -1/18\*b^3\*f^a\*Ei(b\*log(f)/x^3)\*log(f)^3 + 1/18\*(2\*x^9 + b\*x^6\*log(f) + b^2\*x^3\*log(f)^2)\*f^((a\*x^3 + b)/x^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x**3)*x**8,x)
```

```
[Out] Integral(f**(a + b/x**3)*x**8, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)*x^8,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x^8, x)
```

$$3.158 \quad \int f^{a+\frac{b}{x^3}} x^5 dx$$

**Optimal.** Leaf size=58

$$-\frac{1}{6}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) + \frac{1}{6}x^6 f^{a+\frac{b}{x^3}} + \frac{1}{6}bx^3 \log(f) f^{a+\frac{b}{x^3}}$$

[Out]  $(f^{(a + b/x^3)}x^6)/6 + (b*f^{(a + b/x^3)}*x^3*\operatorname{Log}[f])/6 - (b^2*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^3]*\operatorname{Log}[f]^2)/6$

**Rubi [A]** time = 0.0760661, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$-\frac{1}{6}b^2 f^a \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) + \frac{1}{6}x^6 f^{a+\frac{b}{x^3}} + \frac{1}{6}bx^3 \log(f) f^{a+\frac{b}{x^3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^3)}x^5, x]$

[Out]  $(f^{(a + b/x^3)}x^6)/6 + (b*f^{(a + b/x^3)}*x^3*\operatorname{Log}[f])/6 - (b^2*f^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[f])/x^3]*\operatorname{Log}[f]^2)/6$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}F^{(a + b*(c + d*x)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c + d*x)^{m+n}F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^3}} x^5 dx &= \frac{1}{6}f^{a+\frac{b}{x^3}} x^6 + \frac{1}{2}(b \log(f)) \int f^{a+\frac{b}{x^3}} x^2 dx \\ &= \frac{1}{6}f^{a+\frac{b}{x^3}} x^6 + \frac{1}{6}bf^{a+\frac{b}{x^3}} x^3 \log(f) + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+\frac{b}{x^3}}}{x} dx \\ &= \frac{1}{6}f^{a+\frac{b}{x^3}} x^6 + \frac{1}{6}bf^{a+\frac{b}{x^3}} x^3 \log(f) - \frac{1}{6}b^2 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \log^2(f) \end{aligned}$$

**Mathematica [A]** time = 0.0147116, size = 44, normalized size = 0.76

$$\frac{1}{6}f^a \left( x^3 f^{\frac{b}{x^3}} (b \log(f) + x^3) - b^2 \log^2(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^5,x]

[Out] (f^a\*(-(b^2\*ExpIntegralEi[(b\*Log[f])/x^3]\*Log[f]^2) + f^(b/x^3)\*x^3\*(x^3 + b\*Log[f]))) / 6

**Maple [B]** time = 0.04, size = 141, normalized size = 2.4

$$-\frac{f^a b^2 (\ln(f))^2}{3} \left( \frac{x^6}{2 (\ln(f))^2 b^2} - \frac{x^3}{b \ln(f)} - \frac{3}{4} - \frac{3 \ln(x)}{2} + \frac{\ln(-b)}{2} + \frac{\ln(\ln(f))}{2} + \frac{x^6}{12 (\ln(f))^2 b^2} \left( 9 \frac{(\ln(f))^2 b^2}{x^6} + 12 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)\*x^5,x)

[Out] -1/3\*f^a\*b^2\*ln(f)^2\*(-1/2\*x^6/b^2/ln(f)^2-x^3/b/ln(f)-3/4-3/2\*ln(x)+1/2\*ln(-b)+1/2\*ln(ln(f))+1/12/b^2/ln(f)^2\*x^6\*(9\*b^2\*ln(f)^2/x^6+12\*b\*ln(f)/x^3+6)-1/6/b^2/ln(f)^2\*x^6\*(3+3\*b\*ln(f)/x^3)\*exp(b\*ln(f)/x^3)-1/2\*ln(-b\*ln(f)/x^3)-1/2\*Ei(1,-b\*ln(f)/x^3))

**Maxima [A]** time = 1.18913, size = 30, normalized size = 0.52

$$\frac{1}{3} b^2 f^a \Gamma\left(-2, -\frac{b \log(f)}{x^3}\right) \log(f)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^5,x, algorithm="maxima")

[Out] 1/3\*b^2\*f^a\*gamma(-2, -b\*log(f)/x^3)\*log(f)^2

**Fricas [A]** time = 1.77484, size = 117, normalized size = 2.02

$$-\frac{1}{6} b^2 f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \log(f)^2 + \frac{1}{6} (x^6 + b x^3 \log(f)) f^{\frac{a x^3 + b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^5,x, algorithm="fricas")

[Out] -1/6\*b^2\*f^a\*Ei(b\*log(f)/x^3)\*log(f)^2 + 1/6\*(x^6 + b\*x^3\*log(f))\*f^((a\*x^3 + b)/x^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(a+b/x**3)*x**5,x)
```

```
[Out] Integral(f**(a + b/x**3)*x**5, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)*x^5,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x^5, x)
```

### 3.159 $\int f^{a+\frac{b}{x^3}} x^2 dx$

**Optimal.** Leaf size=35

$$\frac{1}{3}x^3 f^{a+\frac{b}{x^3}} - \frac{1}{3}b f^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

[Out]  $(f^{(a + b/x^3)}x^3)/3 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^3]*Log[f])/3$

**Rubi [A]** time = 0.0481365, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2214, 2210}

$$\frac{1}{3}x^3 f^{a+\frac{b}{x^3}} - \frac{1}{3}b f^a \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^2,x]

[Out]  $(f^{(a + b/x^3)}x^3)/3 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^3]*Log[f])/3$

#### Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

#### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned} \int f^{a+\frac{b}{x^3}} x^2 dx &= \frac{1}{3} f^{a+\frac{b}{x^3}} x^3 + (b \log(f)) \int \frac{f^{a+\frac{b}{x^3}}}{x} dx \\ &= \frac{1}{3} f^{a+\frac{b}{x^3}} x^3 - \frac{1}{3} b f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \log(f) \end{aligned}$$

**Mathematica [A]** time = 0.0052936, size = 32, normalized size = 0.91

$$\frac{1}{3} f^a \left( x^3 f^{\frac{b}{x^3}} - b \log(f) \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^2,x]



[Out]  $(f^a \cdot (f^{(b/x^3)} \cdot x^3 - b \cdot \text{ExpIntegralEi}[(b \cdot \text{Log}[f])/x^3] \cdot \text{Log}[f]))/3$

**Maple [B]** time = 0.034, size = 97, normalized size = 2.8

$$\frac{f^a b \ln(f)}{3} \left( \frac{x^3}{b \ln(f)} + 1 + 3 \ln(x) - \ln(-b) - \ln(\ln(f)) - \frac{x^3}{2 b \ln(f)} \left( 2 + 2 \frac{b \ln(f)}{x^3} \right) + \frac{x^3}{b \ln(f)} e^{\frac{b \ln(f)}{x^3}} + \ln \left( -\frac{b \ln(f)}{x^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)*x^2,x)`

[Out]  $1/3 \cdot f^a \cdot b \cdot \ln(f) \cdot (x^3/b/\ln(f) + 1 + 3 \cdot \ln(x) - \ln(-b) - \ln(\ln(f)) - 1/2/b/\ln(f) \cdot x^3 \cdot (2 + 2 \cdot b \cdot \ln(f)/x^3) + 1/b/\ln(f) \cdot x^3 \cdot \exp(b \cdot \ln(f)/x^3) + \ln(-b \cdot \ln(f)/x^3) + \text{Ei}(1, -b \cdot \ln(f)/x^3))$

**Maxima [A]** time = 1.17347, size = 24, normalized size = 0.69

$$-\frac{1}{3} b f^a \Gamma \left( -1, -\frac{b \log(f)}{x^3} \right) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^2,x, algorithm="maxima")`

[Out]  $-1/3 \cdot b \cdot f^a \cdot \text{gamma}(-1, -b \cdot \log(f)/x^3) \cdot \log(f)$

**Fricas [A]** time = 1.77106, size = 88, normalized size = 2.51

$$\frac{1}{3} f^{\frac{ax^3+b}{x^3}} x^3 - \frac{1}{3} b f^a \text{Ei} \left( \frac{b \log(f)}{x^3} \right) \log(f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^2,x, algorithm="fricas")`

[Out]  $1/3 \cdot f^{((a \cdot x^3 + b)/x^3)} \cdot x^3 - 1/3 \cdot b \cdot f^a \cdot \text{Ei}(b \cdot \log(f)/x^3) \cdot \log(f)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)*x**2,x)`

[Out] `Integral(f**(a + b/x**3)*x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^2,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)\*x^2, x)

$$3.160 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{3}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^3])/3$

**Rubi [A]** time = 0.0223782, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$-\frac{1}{3}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b/x^3)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^3])/3$

**Rule 2210**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^n))}/((e_.) + (f_.)*(x_)), x_{\text{Symbol}}] :> \operatorname{Simp}[(F^a \operatorname{ExpIntegralEi}[b*(c + d*x)^n \operatorname{Log}[F]])/(f*n), x] /;$  Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^3}}}{x} dx = -\frac{1}{3}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

**Mathematica [A]** time = 0.0020762, size = 15, normalized size = 1.

$$-\frac{1}{3}f^a \operatorname{Ei}\left(\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[f^{(a + b/x^3)}/x, x]$

[Out]  $-(f^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[f])/x^3])/3$

**Maple [B]** time = 0.029, size = 41, normalized size = 2.7

$$-\frac{f^a}{3} \left( -3 \ln(x) + \ln(-b) + \ln(\ln(f)) - \ln\left(-\frac{b \ln(f)}{x^3}\right) - \operatorname{Ei}\left(1, -\frac{b \ln(f)}{x^3}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)/x,x)`

[Out]  $-1/3*f^a*(-3*\ln(x)+\ln(-b)+\ln(\ln(f))-\ln(-b*\ln(f)/x^3)-\text{Ei}(1,-b*\ln(f)/x^3))$

**Maxima [A]** time = 1.27502, size = 18, normalized size = 1.2

$$-\frac{1}{3}f^a\text{Ei}\left(\frac{b\log(f)}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x,x, algorithm="maxima")`

[Out]  $-1/3*f^a*\text{Ei}(b*\log(f)/x^3)$

**Fricas [A]** time = 1.70802, size = 36, normalized size = 2.4

$$-\frac{1}{3}f^a\text{Ei}\left(\frac{b\log(f)}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x,x, algorithm="fricas")`

[Out]  $-1/3*f^a*\text{Ei}(b*\log(f)/x^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)/x,x)`

[Out] `Integral(f**(a + b/x**3)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^3)/x, x)`

$$3.161 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx$$

**Optimal.** Leaf size=20

$$-\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

[Out]  $-f^{(a + b/x^3)}/(3*b*Log[f])$

**Rubi [A]** time = 0.0215353, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$-\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b/x^3)}/x^4, x]$

[Out]  $-f^{(a + b/x^3)}/(3*b*Log[f])$

**Rule 2209**

$\text{Int}[(F\_)^{((a\_.) + (b\_.)*((c\_.) + (d\_.)*(x\_))^n))*((e\_.) + (f\_.)*(x\_))^m], x\_Symbol] :> \text{Simp}[(e + f*x)^n * F^{(a + b*(c + d*x)^n)} / (b*f*n*(c + d*x)^n * \text{Log}[F]), x] /;$   $\text{FreeQ}\{F, a, b, c, d, e, f, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx = -\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

**Mathematica [A]** time = 0.0037118, size = 20, normalized size = 1.

$$-\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[f^{(a + b/x^3)}/x^4, x]$

[Out]  $-f^{(a + b/x^3)}/(3*b*Log[f])$

**Maple [A]** time = 0.001, size = 19, normalized size = 1.

$$-\frac{1}{3b \ln(f)} f^{a+\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)/x^4,x)`

[Out]  $-1/3*f^{(a+b/x^3)}/b/\ln(f)$

**Maxima [A]** time = 1.11471, size = 24, normalized size = 1.2

$$-\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x^4,x, algorithm="maxima")`

[Out]  $-1/3*f^{(a + b/x^3)}/(b*\log(f))$

**Fricas [A]** time = 1.77764, size = 50, normalized size = 2.5

$$-\frac{f^{\frac{ax^3+b}{x^3}}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x^4,x, algorithm="fricas")`

[Out]  $-1/3*f^{((a*x^3 + b)/x^3)}/(b*\log(f))$

**Sympy [A]** time = 0.109273, size = 29, normalized size = 1.45

$$\begin{cases} -\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)} & \text{for } 3b \log(f) \neq 0 \\ -\frac{1}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)/x**4,x)`

[Out] `Piecewise((-f**(a + b/x**3)/(3*b*log(f)), Ne(3*b*log(f), 0)), (-1/(3*x**3), True))`

**Giac [A]** time = 1.33449, size = 24, normalized size = 1.2

$$-\frac{f^{a+\frac{b}{x^3}}}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*f^(a + b/x^3)/(b*log(f))
```

$$3.162 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx$$

**Optimal.** Leaf size=44

$$\frac{f^{a+\frac{b}{x^3}}}{3b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^3 \log(f)}$$

[Out]  $f^{(a + b/x^3)/(3*b^2*\text{Log}[f]^2)} - f^{(a + b/x^3)/(3*b*x^3*\text{Log}[f])}$

**Rubi [A]** time = 0.04618, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{f^{a+\frac{b}{x^3}}}{3b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^3 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^7,x]

[Out]  $f^{(a + b/x^3)/(3*b^2*\text{Log}[f]^2)} - f^{(a + b/x^3)/(3*b*x^3*\text{Log}[f])}$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx &= -\frac{f^{a+\frac{b}{x^3}}}{3bx^3 \log(f)} - \frac{\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx}{b \log(f)} \\ &= \frac{f^{a+\frac{b}{x^3}}}{3b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^3 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0070246, size = 32, normalized size = 0.73

$$\frac{f^{a+\frac{b}{x^3}} (x^3 - b \log(f))}{3b^2 x^3 \log^2(f)}$$



Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^7,x]

[Out] (f^(a + b/x^3)\*(x^3 - b\*Log[f]))/(3\*b^2\*x^3\*Log[f]^2)

**Maple [A]** time = 0.013, size = 52, normalized size = 1.2

$$\frac{1}{x^6} \left( \frac{x^6}{3 (\ln(f))^2 b^2} e^{\left(a + \frac{b}{x^3}\right) \ln(f)} - \frac{x^3}{3 b \ln(f)} e^{\left(a + \frac{b}{x^3}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^7,x)

[Out] (1/3/b^2/ln(f)^2\*x^6\*exp((a+b/x^3)\*ln(f))-1/3/b/ln(f)\*x^3\*exp((a+b/x^3)\*ln(f)))/x^6

**Maxima [C]** time = 1.30106, size = 30, normalized size = 0.68

$$\frac{f^a \Gamma\left(2, -\frac{b \log(f)}{x^3}\right)}{3 b^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^7,x, algorithm="maxima")

[Out] 1/3\*f^a\*gamma(2, -b\*log(f)/x^3)/(b^2\*log(f)^2)

**Fricas [A]** time = 1.76167, size = 82, normalized size = 1.86

$$\frac{(x^3 - b \log(f)) f^{\frac{ax^3+b}{x^3}}}{3 b^2 x^3 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^7,x, algorithm="fricas")

[Out] 1/3\*(x^3 - b\*log(f))\*f^((a\*x^3 + b)/x^3)/(b^2\*x^3\*log(f)^2)

**Sympy [A]** time = 0.116286, size = 29, normalized size = 0.66

$$\frac{f^{a + \frac{b}{x^3}} (-b \log(f) + x^3)}{3 b^2 x^3 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*7,x)

[Out] f\*\*(a + b/x\*\*3)\*(-b\*log(f) + x\*\*3)/(3\*b\*\*2\*x\*\*3\*log(f)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^7,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)/x^7, x)

$$3.163 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^{10}} dx$$

**Optimal.** Leaf size=67

$$\frac{2f^{a+\frac{b}{x^3}}}{3b^2x^3 \log^2(f)} - \frac{2f^{a+\frac{b}{x^3}}}{3b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^6 \log(f)}$$

[Out]  $(-2*f^{(a + b/x^3)})/(3*b^3*Log[f]^3) + (2*f^{(a + b/x^3)})/(3*b^2*x^3*Log[f]^2) - f^{(a + b/x^3)}/(3*b*x^6*Log[f])$

**Rubi [A]** time = 0.0703232, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{2f^{a+\frac{b}{x^3}}}{3b^2x^3 \log^2(f)} - \frac{2f^{a+\frac{b}{x^3}}}{3b^3 \log^3(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^6 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^10,x]

[Out]  $(-2*f^{(a + b/x^3)})/(3*b^3*Log[f]^3) + (2*f^{(a + b/x^3)})/(3*b^2*x^3*Log[f]^2) - f^{(a + b/x^3)}/(3*b*x^6*Log[f])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+\frac{b}{x^3}}}{x^{10}} dx &= -\frac{f^{a+\frac{b}{x^3}}}{3bx^6 \log(f)} - \frac{2 \int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx}{b \log(f)} \\ &= \frac{2f^{a+\frac{b}{x^3}}}{3b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^6 \log(f)} + \frac{2 \int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx}{b^2 \log^2(f)} \\ &= -\frac{2f^{a+\frac{b}{x^3}}}{3b^3 \log^3(f)} + \frac{2f^{a+\frac{b}{x^3}}}{3b^2x^3 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^6 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0087973, size = 45, normalized size = 0.67

$$\frac{f^{a+\frac{b}{x^3}} (b^2 \log^2(f) - 2bx^3 \log(f) + 2x^6)}{3b^3x^6 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^10,x]

[Out] -(f^(a + b/x^3)\*(2\*x^6 - 2\*b\*x^3\*Log[f] + b^2\*Log[f]^2))/(3\*b^3\*x^6\*Log[f]^3)

**Maple [A]** time = 0.018, size = 75, normalized size = 1.1

$$\frac{1}{x^9} \left( -\frac{2x^9}{3(\ln(f))^3 b^3} e^{\left(\frac{b}{x^3}\right) \ln(f)} + \frac{2x^6}{3(\ln(f))^2 b^2} e^{\left(\frac{b}{x^3}\right) \ln(f)} - \frac{x^3}{3b \ln(f)} e^{\left(\frac{b}{x^3}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^10,x)

[Out] (-2/3/b^3/ln(f)^3\*x^9\*exp((a+b/x^3)\*ln(f))+2/3/b^2/ln(f)^2\*x^6\*exp((a+b/x^3)\*ln(f))-1/3/b/ln(f)\*x^3\*exp((a+b/x^3)\*ln(f)))/x^9

**Maxima [C]** time = 1.12247, size = 30, normalized size = 0.45

$$\frac{f^a \Gamma\left(3, -\frac{b \log(f)}{x^3}\right)}{3b^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^10,x, algorithm="maxima")

[Out] -1/3\*f^a\*gamma(3, -b\*log(f)/x^3)/(b^3\*log(f)^3)

**Fricas [A]** time = 1.76628, size = 115, normalized size = 1.72

$$\frac{\left(2x^6 - 2bx^3 \log(f) + b^2 \log(f)^2\right) f^{\frac{ax^3+b}{x^3}}}{3b^3x^6 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^10,x, algorithm="fricas")

[Out] -1/3\*(2\*x^6 - 2\*b\*x^3\*log(f) + b^2\*log(f)^2)\*f^((a\*x^3 + b)/x^3)/(b^3\*x^6\*log(f)^3)

---

**Sympy [A]** time = 0.129388, size = 44, normalized size = 0.66

$$\frac{f^{a+\frac{b}{x^3}} \left( -b^2 \log(f)^2 + 2bx^3 \log(f) - 2x^6 \right)}{3b^3 x^6 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*10,x)

[Out] f\*\*(a + b/x\*\*3)\*(-b\*\*2\*log(f)\*\*2 + 2\*b\*x\*\*3\*log(f) - 2\*x\*\*6)/(3\*b\*\*3\*x\*\*6\*log(f)\*\*3)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^10,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)/x^10, x)

$$3.164 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^{13}} dx$$

**Optimal.** Leaf size=83

$$\frac{f^{a+\frac{b}{x^3}}}{b^2 x^6 \log^2(f)} - \frac{2f^{a+\frac{b}{x^3}}}{b^3 x^3 \log^3(f)} + \frac{2f^{a+\frac{b}{x^3}}}{b^4 \log^4(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)}$$

[Out]  $(2f^{(a + b/x^3)})/(b^4 \text{Log}[f]^4) - (2f^{(a + b/x^3)})/(b^3 x^3 \text{Log}[f]^3) + f^{(a + b/x^3)}/(b^2 x^6 \text{Log}[f]^2) - f^{(a + b/x^3)}/(3b x^9 \text{Log}[f])$

**Rubi [A]** time = 0.0942408, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2212, 2209}

$$\frac{f^{a+\frac{b}{x^3}}}{b^2 x^6 \log^2(f)} - \frac{2f^{a+\frac{b}{x^3}}}{b^3 x^3 \log^3(f)} + \frac{2f^{a+\frac{b}{x^3}}}{b^4 \log^4(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^13,x]

[Out]  $(2f^{(a + b/x^3)})/(b^4 \text{Log}[f]^4) - (2f^{(a + b/x^3)})/(b^3 x^3 \text{Log}[f]^3) + f^{(a + b/x^3)}/(b^2 x^6 \text{Log}[f]^2) - f^{(a + b/x^3)}/(3b x^9 \text{Log}[f])$

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}} dx &= -\frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)} - \frac{3 \int \frac{f^{a+\frac{b}{x^3}}}{x^{10}} dx}{b \log(f)} \\
&= \frac{f^{a+\frac{b}{x^3}}}{b^2 x^6 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)} + \frac{6 \int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx}{b^2 \log^2(f)} \\
&= -\frac{2f^{a+\frac{b}{x^3}}}{b^3 x^3 \log^3(f)} + \frac{f^{a+\frac{b}{x^3}}}{b^2 x^6 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)} - \frac{6 \int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx}{b^3 \log^3(f)} \\
&= \frac{2f^{a+\frac{b}{x^3}}}{b^4 \log^4(f)} - \frac{2f^{a+\frac{b}{x^3}}}{b^3 x^3 \log^3(f)} + \frac{f^{a+\frac{b}{x^3}}}{b^2 x^6 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^9 \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0105382, size = 58, normalized size = 0.7

$$\frac{f^{a+\frac{b}{x^3}} (3b^2 x^3 \log^2(f) - b^3 \log^3(f) - 6bx^6 \log(f) + 6x^9)}{3b^4 x^9 \log^4(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^13,x]

[Out] (f^(a + b/x^3)\*(6\*x^9 - 6\*b\*x^6\*Log[f] + 3\*b^2\*x^3\*Log[f]^2 - b^3\*Log[f]^3))/(3\*b^4\*x^9\*Log[f]^4)

**Maple [A]** time = 0.022, size = 97, normalized size = 1.2

$$\frac{1}{x^{12}} \left( \frac{x^6}{(\ln(f))^2 b^2} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} + 2 \frac{x^{12}}{b^4 (\ln(f))^4} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} - 2 \frac{x^9}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} - \frac{x^3}{3 b \ln(f)} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^13,x)

[Out] (1/b^2/ln(f)^2\*x^6\*exp((a+b/x^3)\*ln(f))+2/b^4/ln(f)^4\*x^12\*exp((a+b/x^3)\*ln(f))-2/b^3/ln(f)^3\*x^9\*exp((a+b/x^3)\*ln(f))-1/3/b/ln(f)\*x^3\*exp((a+b/x^3)\*ln(f)))/x^12

**Maxima [C]** time = 1.2797, size = 30, normalized size = 0.36

$$\frac{f^a \Gamma\left(4, -\frac{b \log(f)}{x^3}\right)}{3 b^4 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^13,x, algorithm="maxima")

[Out]  $\frac{1}{3}f^a \gamma(4, -b \log(f)/x^3)/(b^4 \log(f)^4)$

**Fricas [A]** time = 1.73014, size = 142, normalized size = 1.71

$$\frac{(6x^9 - 6bx^6 \log(f) + 3b^2x^3 \log(f)^2 - b^3 \log(f)^3) f^{\frac{ax^3+b}{x^3}}}{3b^4x^9 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x^13,x, algorithm="fricas")`

[Out]  $\frac{1}{3}(6x^9 - 6bx^6 \log(f) + 3b^2x^3 \log(f)^2 - b^3 \log(f)^3) f^{(ax^3 + b)/x^3} / (b^4x^9 \log(f)^4)$

**Sympy [A]** time = 0.14431, size = 58, normalized size = 0.7

$$\frac{f^{a+\frac{b}{x^3}} \left( -b^3 \log(f)^3 + 3b^2x^3 \log(f)^2 - 6bx^6 \log(f) + 6x^9 \right)}{3b^4x^9 \log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)/x**13,x)`

[Out]  $f^{(a + b/x^3)} (-b^3 \log(f)^3 + 3b^2x^3 \log(f)^2 - 6bx^6 \log(f) + 6x^9) / (3b^4x^9 \log(f)^4)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)/x^13,x, algorithm="giac")`

[Out] `integrate(f^(a + b/x^3)/x^13, x)`



$$3.165 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^{16}} dx$$

**Optimal.** Leaf size=69

$$\frac{f^{a+\frac{b}{x^3}} (12b^2x^6 \log^2(f) - 4b^3x^3 \log^3(f) + b^4 \log^4(f) - 24bx^9 \log(f) + 24x^{12})}{3b^5x^{12} \log^5(f)}$$

[Out]  $-(f^{(a + b/x^3)}(24*x^{12} - 24*b*x^9*Log[f] + 12*b^2*x^6*Log[f]^2 - 4*b^3*x^3*Log[f]^3 + b^4*Log[f]^4))/(3*b^5*x^{12}*Log[f]^5)$

**Rubi [C]** time = 0.0226648, antiderivative size = 24, normalized size of antiderivative = 0.35, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x^3}\right)}{3b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^16,x]

[Out]  $-(f^a*\text{Gamma}[5, -((b*\text{Log}[f])/x^3)])/(3*b^5*\text{Log}[f]^5)$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}} dx = -\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x^3}\right)}{3b^5 \log^5(f)}$$

**Mathematica [C]** time = 0.0027375, size = 24, normalized size = 0.35

$$\frac{f^a \text{Gamma}\left(5, -\frac{b \log(f)}{x^3}\right)}{3b^5 \log^5(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^16,x]

[Out]  $-(f^a*\text{Gamma}[5, -((b*\text{Log}[f])/x^3)])/(3*b^5*\text{Log}[f]^5)$

**Maple [A]** time = 0.028, size = 121, normalized size = 1.8

$$\frac{1}{x^{15}} \left( -8 \frac{x^{15}}{b^5 (\ln(f))^5} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} + 8 \frac{x^{12}}{b^4 (\ln(f))^4} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} - 4 \frac{x^9}{(\ln(f))^3 b^3} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} + \frac{4 x^6}{3 (\ln(f))^2 b^2} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} - \frac{4 x^3}{3 \ln(f) b} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} - \frac{4}{3 b} e^{\left(\frac{a+b}{x^3}\right) \ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^16,x)

[Out] (-8/b^5/ln(f)^5\*x^15\*exp((a+b/x^3)\*ln(f))+8/b^4/ln(f)^4\*x^12\*exp((a+b/x^3)\*ln(f))-4/b^3/ln(f)^3\*x^9\*exp((a+b/x^3)\*ln(f))+4/3/b^2/ln(f)^2\*x^6\*exp((a+b/x^3)\*ln(f))-1/3/b/ln(f)\*x^3\*exp((a+b/x^3)\*ln(f)))/x^15

**Maxima [C]** time = 1.30783, size = 30, normalized size = 0.43

$$\frac{f^a \Gamma\left(5, -\frac{b \log(f)}{x^3}\right)}{3 b^5 \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^16,x, algorithm="maxima")

[Out] -1/3\*f^a\*gamma(5, -b\*log(f)/x^3)/(b^5\*log(f)^5)

**Fricas [A]** time = 1.65528, size = 178, normalized size = 2.58

$$\frac{\left(24 x^{12} - 24 b x^9 \log(f) + 12 b^2 x^6 \log(f)^2 - 4 b^3 x^3 \log(f)^3 + b^4 \log(f)^4\right) f^{\frac{a x^3 + b}{x^3}}}{3 b^5 x^{12} \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^16,x, algorithm="fricas")

[Out] -1/3\*(24\*x^12 - 24\*b\*x^9\*log(f) + 12\*b^2\*x^6\*log(f)^2 - 4\*b^3\*x^3\*log(f)^3 + b^4\*log(f)^4)\*f^((a\*x^3 + b)/x^3)/(b^5\*x^12\*log(f)^5)

**Sympy [A]** time = 0.152325, size = 71, normalized size = 1.03

$$\frac{f^{a+\frac{b}{x^3}} \left( -b^4 \log(f)^4 + 4b^3 x^3 \log(f)^3 - 12b^2 x^6 \log(f)^2 + 24bx^9 \log(f) - 24x^{12} \right)}{3b^5 x^{12} \log(f)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*16,x)

```
[Out] f**(a + b/x**3)*(-b**4*log(f)**4 + 4*b**3*x**3*log(f)**3 - 12*b**2*x**6*log
(f)**2 + 24*b*x**9*log(f) - 24*x**12)/(3*b**5*x**12*log(f)**5)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)/x^16,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)/x^16, x)
```

$$3.166 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^{19}} dx$$

**Optimal.** Leaf size=82

$$\frac{f^{a+\frac{b}{x^3}} (60b^2x^9 \log^2(f) - 20b^3x^6 \log^3(f) + 5b^4x^3 \log^4(f) - b^5 \log^5(f) - 120bx^{12} \log(f) + 120x^{15})}{3b^6x^{15} \log^6(f)}$$

[Out] (f^(a + b/x^3)\*(120\*x^15 - 120\*b\*x^12\*Log[f] + 60\*b^2\*x^9\*Log[f]^2 - 20\*b^3\*x^6\*Log[f]^3 + 5\*b^4\*x^3\*Log[f]^4 - b^5\*Log[f]^5))/(3\*b^6\*x^15\*Log[f]^6)

**Rubi [C]** time = 0.0228049, antiderivative size = 24, normalized size of antiderivative = 0.29, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x^3}\right)}{3b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^19,x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x^3))]/(3\*b^6\*Log[f]^6)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F]))]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F])))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}} dx = \frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x^3}\right)}{3b^6 \log^6(f)}$$

**Mathematica [C]** time = 0.0029883, size = 24, normalized size = 0.29

$$\frac{f^a \text{Gamma}\left(6, -\frac{b \log(f)}{x^3}\right)}{3b^6 \log^6(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^19,x]

[Out] (f^a\*Gamma[6, -((b\*Log[f])/x^3))]/(3\*b^6\*Log[f]^6)

**Maple [A]** time = 0.029, size = 84, normalized size = 1.

$$\frac{-120x^{15} + 120bx^{12}\ln(f) - 60b^2x^9(\ln(f))^2 + 20b^3x^6(\ln(f))^3 - 5b^4x^3(\ln(f))^4 + b^5(\ln(f))^5}{3(\ln(f))^6 b^6 x^{15}} f^{\frac{ax^3+b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^19,x)

[Out] -1/3\*(-120\*x^15+120\*b\*x^12\*ln(f)-60\*b^2\*x^9\*ln(f)^2+20\*b^3\*x^6\*ln(f)^3-5\*b^4\*x^3\*ln(f)^4+b^5\*ln(f)^5)/ln(f)^6/b^6/x^15\*f^((a\*x^3+b)/x^3)

**Maxima [C]** time = 1.18733, size = 30, normalized size = 0.37

$$\frac{f^a \Gamma\left(6, -\frac{b \log(f)}{x^3}\right)}{3 b^6 \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^19,x, algorithm="maxima")

[Out] 1/3\*f^a\*gamma(6, -b\*log(f)/x^3)/(b^6\*log(f)^6)

**Fricas [A]** time = 2.01496, size = 211, normalized size = 2.57

$$\frac{(120x^{15} - 120bx^{12}\log(f) + 60b^2x^9\log(f)^2 - 20b^3x^6\log(f)^3 + 5b^4x^3\log(f)^4 - b^5\log(f)^5)f^{\frac{ax^3+b}{x^3}}}{3b^6x^{15}\log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^19,x, algorithm="fricas")

[Out] 1/3\*(120\*x^15 - 120\*b\*x^12\*log(f) + 60\*b^2\*x^9\*log(f)^2 - 20\*b^3\*x^6\*log(f)^3 + 5\*b^4\*x^3\*log(f)^4 - b^5\*log(f)^5)\*f^((a\*x^3 + b)/x^3)/(b^6\*x^15\*log(f)^6)

**Sympy [A]** time = 0.161392, size = 85, normalized size = 1.04

$$\frac{f^{a+\frac{b}{x^3}} \left(-b^5 \log(f)^5 + 5b^4x^3 \log(f)^4 - 20b^3x^6 \log(f)^3 + 60b^2x^9 \log(f)^2 - 120bx^{12} \log(f) + 120x^{15}\right)}{3b^6x^{15} \log(f)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*19,x)

[Out] f\*\*(a + b/x\*\*3)\*(-b\*\*5\*log(f)\*\*5 + 5\*b\*\*4\*x\*\*3\*log(f)\*\*4 - 20\*b\*\*3\*x\*\*6\*log(f)\*\*3 + 60\*b\*\*2\*x\*\*9\*log(f)\*\*2 - 120\*b\*x\*\*12\*log(f) + 120\*x\*\*15)/(3\*b\*\*6\*x

```
**15*log(f)**6)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)/x^19,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)/x^19, x)
```

$$3.167 \quad \int f^{a+\frac{b}{x^3}} x^4 dx$$

**Optimal.** Leaf size=34

$$\frac{1}{3} x^5 f^a \left( -\frac{b \log(f)}{x^3} \right)^{5/3} \text{Gamma} \left( -\frac{5}{3}, -\frac{b \log(f)}{x^3} \right)$$

[Out] (f^a\*x^5\*Gamma[-5/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(5/3)/3

**Rubi [A]** time = 0.0252712, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{3} x^5 f^a \left( -\frac{b \log(f)}{x^3} \right)^{5/3} \text{Gamma} \left( -\frac{5}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^4,x]

[Out] (f^a\*x^5\*Gamma[-5/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(5/3)/3

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^3}} x^4 dx = \frac{1}{3} f^a x^5 \Gamma \left( -\frac{5}{3}, -\frac{b \log(f)}{x^3} \right) \left( -\frac{b \log(f)}{x^3} \right)^{5/3}$$

**Mathematica [A]** time = 0.0041903, size = 34, normalized size = 1.

$$\frac{1}{3} x^5 f^a \left( -\frac{b \log(f)}{x^3} \right)^{5/3} \text{Gamma} \left( -\frac{5}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^4,x]

[Out] (f^a\*x^5\*Gamma[-5/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(5/3)/3

**Maple [B]** time = 0.036, size = 120, normalized size = 3.5

$$-\frac{f^a}{3} (-b)^{\frac{5}{3}} (\ln(f))^{\frac{5}{3}} \left( \frac{3b^2\pi\sqrt{3}}{5x\Gamma(2/3)} \sqrt[3]{\ln(f)} (-b)^{-\frac{5}{3}} \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} - \frac{3x^5}{5} \left( \frac{3b\ln(f)}{2x^3} + 1 \right) e^{\frac{b\ln(f)}{x^3}} (-b)^{-\frac{5}{3}} (\ln(f))^{-\frac{5}{3}} - \frac{9b^2}{10x} \sqrt[3]{\ln(f)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)*x^4,x)`

[Out] 
$$-1/3*f^a*(-b)^{(5/3)}*\ln(f)^{(5/3)}*(3/5/x/(-b)^{(5/3)}*\ln(f)^{(1/3)}*b^2*\pi*3^{(1/2)})/\text{GAMMA}(2/3)/(-b*\ln(f)/x^3)^{(1/3)}-3/5*x^5/(-b)^{(5/3)}/\ln(f)^{(5/3)}*(3/2*b*\ln(f)/x^3+1)*\exp(b*\ln(f)/x^3)-9/10/x/(-b)^{(5/3)}*\ln(f)^{(1/3)}*b^2/(-b*\ln(f)/x^3)^{(1/3)}*\text{GAMMA}(1/3,-b*\ln(f)/x^3)$$

**Maxima [A]** time = 1.24825, size = 38, normalized size = 1.12

$$\frac{1}{3} f^a x^5 \left( -\frac{b \log(f)}{x^3} \right)^{\frac{5}{3}} \Gamma \left( -\frac{5}{3}, -\frac{b \log(f)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^4,x, algorithm="maxima")`

[Out] 
$$1/3*f^a*x^5*(-b*\log(f)/x^3)^{(5/3)}*\text{gamma}(-5/3, -b*\log(f)/x^3)$$

**Fricas [A]** time = 2.10641, size = 157, normalized size = 4.62

$$-\frac{3}{10} (-b \log(f))^{\frac{2}{3}} b f^a \Gamma \left( \frac{1}{3}, -\frac{b \log(f)}{x^3} \right) \log(f) + \frac{1}{10} (2x^5 + 3bx^2 \log(f)) f^{\frac{ax^3+b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x^4,x, algorithm="fricas")`

[Out] 
$$-3/10*(-b*\log(f))^{(2/3)}*b*f^a*\text{gamma}(1/3, -b*\log(f)/x^3)*\log(f) + 1/10*(2*x^5 + 3*b*x^2*\log(f))*f^{(a*x^3 + b)/x^3}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)*x**4,x)`

[Out] `Integral(f**(a + b/x**3)*x**4, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(f^(a+b/x^3)*x^4,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x^4, x)
```

$$3.168 \quad \int f^{a+\frac{b}{x^3}} x^3 dx$$

**Optimal.** Leaf size=34

$$\frac{1}{3} x^4 f^a \left( -\frac{b \log(f)}{x^3} \right)^{4/3} \text{Gamma} \left( -\frac{4}{3}, -\frac{b \log(f)}{x^3} \right)$$

[Out] (f^a\*x^4\*Gamma[-4/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(4/3)/3

**Rubi [A]** time = 0.0247654, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{1}{3} x^4 f^a \left( -\frac{b \log(f)}{x^3} \right)^{4/3} \text{Gamma} \left( -\frac{4}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x^3, x]

[Out] (f^a\*x^4\*Gamma[-4/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(4/3)/3

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F]))^(m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^3}} x^3 dx = \frac{1}{3} f^a x^4 \Gamma \left( -\frac{4}{3}, -\frac{b \log(f)}{x^3} \right) \left( -\frac{b \log(f)}{x^3} \right)^{4/3}$$

**Mathematica [A]** time = 0.0035358, size = 34, normalized size = 1.

$$\frac{1}{3} x^4 f^a \left( -\frac{b \log(f)}{x^3} \right)^{4/3} \text{Gamma} \left( -\frac{4}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x^3, x]

[Out] (f^a\*x^4\*Gamma[-4/3, -(b\*Log[f])/x^3])\*(-(b\*Log[f])/x^3)^(4/3)/3

**Maple [B]** time = 0.034, size = 115, normalized size = 3.4

$$\frac{f^a b}{3} (\ln(f))^{\frac{4}{3}} \sqrt[3]{-b} \left( \frac{9 b^2 \Gamma(2/3)}{4 x^2} (\ln(f))^{\frac{2}{3}} (-b)^{-\frac{4}{3}} \left( -\frac{b \ln(f)}{x^3} \right)^{-\frac{2}{3}} - \frac{3 x^4}{4} \left( 3 \frac{b \ln(f)}{x^3} + 1 \right) e^{\frac{b \ln(f)}{x^3}} (-b)^{-\frac{4}{3}} (\ln(f))^{-\frac{4}{3}} - \frac{9 b^2}{4 x^2} (\ln(f))^{\frac{4}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)\*x^3,x)

[Out]  $\frac{1}{3}f^a b \ln(f)^{4/3} (-b)^{1/3} (9/4/x^2/(-b)^{4/3} \ln(f)^{2/3} b^2 \text{GAMMA}(2/3)/(-b \ln(f)/x^3)^{2/3} - 3/4 x^4/(-b)^{4/3} \ln(f)^{4/3} (3 b \ln(f)/x^3 + 1) \exp(b \ln(f)/x^3) - 9/4/x^2/(-b)^{4/3} \ln(f)^{2/3} b^2/(-b \ln(f)/x^3)^{2/3} \text{GAMMA}(2/3, -b \ln(f)/x^3))$

**Maxima [A]** time = 1.18673, size = 38, normalized size = 1.12

$$\frac{1}{3} f^a x^4 \left( -\frac{b \log(f)}{x^3} \right)^{\frac{4}{3}} \Gamma \left( -\frac{4}{3}, -\frac{b \log(f)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}f^a x^4 (-b \log(f)/x^3)^{4/3} \text{gamma}(-4/3, -b \log(f)/x^3)$

**Fricas [A]** time = 2.04674, size = 149, normalized size = 4.38

$$-\frac{3}{4} (-b \log(f))^{\frac{1}{3}} b f^a \Gamma \left( \frac{2}{3}, -\frac{b \log(f)}{x^3} \right) \log(f) + \frac{1}{4} (x^4 + 3 b x \log(f)) f^{\frac{ax^3+b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)\*x^3,x, algorithm="fricas")

[Out]  $-3/4 * (-b \log(f))^{1/3} * b * f^a * \text{gamma}(2/3, -b \log(f)/x^3) * \log(f) + 1/4 * (x^4 + 3 * b * x * \log(f)) * f^{(a * x^3 + b)/x^3}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a + \frac{b}{x^3}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)\*x\*\*3,x)

[Out] Integral(f\*\*(a + b/x\*\*3)\*x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a + \frac{b}{x^3}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x^3, x)
```

### 3.169 $\int f^{a+\frac{b}{x^3}} x dx$

**Optimal.** Leaf size=34

$$\frac{1}{3} x^2 f^a \left( -\frac{b \log(f)}{x^3} \right)^{2/3} \text{Gamma} \left( -\frac{2}{3}, -\frac{b \log(f)}{x^3} \right)$$

[Out]  $(f^a x^2 \text{Gamma}[-2/3, -(b \text{Log}[f])/x^3]) * (-((b \text{Log}[f])/x^3))^{(2/3)}/3$

**Rubi [A]** time = 0.0142921, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2218}

$$\frac{1}{3} x^2 f^a \left( -\frac{b \log(f)}{x^3} \right)^{2/3} \text{Gamma} \left( -\frac{2}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)\*x,x]

[Out]  $(f^a x^2 \text{Gamma}[-2/3, -(b \text{Log}[f])/x^3]) * (-((b \text{Log}[f])/x^3))^{(2/3)}/3$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+\frac{b}{x^3}} x dx = \frac{1}{3} f^a x^2 \Gamma \left( -\frac{2}{3}, -\frac{b \log(f)}{x^3} \right) \left( -\frac{b \log(f)}{x^3} \right)^{2/3}$$

**Mathematica [A]** time = 0.0035972, size = 34, normalized size = 1.

$$\frac{1}{3} x^2 f^a \left( -\frac{b \log(f)}{x^3} \right)^{2/3} \text{Gamma} \left( -\frac{2}{3}, -\frac{b \log(f)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)\*x,x]

[Out]  $(f^a x^2 \text{Gamma}[-2/3, -(b \text{Log}[f])/x^3]) * (-((b \text{Log}[f])/x^3))^{(2/3)}/3$

**Maple [B]** time = 0.029, size = 105, normalized size = 3.1

$$-\frac{f^a}{3} (-b)^{\frac{2}{3}} (\ln(f))^{\frac{2}{3}} \left( \frac{b\pi\sqrt{3}}{x\Gamma\left(\frac{2}{3}\right)} \sqrt[3]{\ln(f)} (-b)^{-\frac{2}{3}} \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} - \frac{3x^2}{2} e^{\frac{b\ln(f)}{x^3}} (-b)^{-\frac{2}{3}} (\ln(f))^{-\frac{2}{3}} - \frac{3b}{2x} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -\frac{b\ln(f)}{x^3}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3)*x,x)`

[Out]  $-1/3*f^a*(-b)^{(2/3)}*\ln(f)^{(2/3)}*(1/x/(-b)^{(2/3)}*\ln(f)^{(1/3)}*b*\text{Pi}*3^{(1/2)}/\text{Gamma}(2/3)/(-b*\ln(f)/x^3)^{(1/3)}-3/2*x^2/(-b)^{(2/3)}/\ln(f)^{(2/3)}*\exp(b*\ln(f)/x^3)-3/2/x/(-b)^{(2/3)}*\ln(f)^{(1/3)}*b/(-b*\ln(f)/x^3)^{(1/3)}*\text{Gamma}(1/3,-b*\ln(f)/x^3))$

**Maxima [A]** time = 1.1695, size = 38, normalized size = 1.12

$$\frac{1}{3} f^a x^2 \left( -\frac{b \log(f)}{x^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{2}{3}, -\frac{b \log(f)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x,x, algorithm="maxima")`

[Out]  $1/3*f^a*x^2*(-b*\log(f)/x^3)^{(2/3)}*\text{gamma}(-2/3, -b*\log(f)/x^3)$

**Fricas [A]** time = 2.13962, size = 112, normalized size = 3.29

$$\frac{1}{2} f^{\frac{ax^3+b}{x^3}} x^2 - \frac{1}{2} (-b \log(f))^{\frac{2}{3}} f^a \Gamma \left( \frac{1}{3}, -\frac{b \log(f)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3)*x,x, algorithm="fricas")`

[Out]  $1/2*f^{(a*x^3 + b)/x^3}*x^2 - 1/2*(-b*\log(f))^{(2/3)}*f^a*\text{gamma}(1/3, -b*\log(f)/x^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3)*x,x)`

[Out] `Integral(f**(a + b/x**3)*x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)*x,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)*x, x)
```

$$3.170 \quad \int f^{a+\frac{b}{x^3}} dx$$

**Optimal.** Leaf size=32

$$\frac{1}{3} x f^a \sqrt[3]{-\frac{b \log(f)}{x^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)$$

[Out] (f^a\*x\*Gamma[-1/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^(1/3))/3

**Rubi [A]** time = 0.0047191, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2208}

$$\frac{1}{3} x f^a \sqrt[3]{-\frac{b \log(f)}{x^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3), x]

[Out] (f^a\*x\*Gamma[-1/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^(1/3))/3

**Rule 2208**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

**Rubi steps**

$$\int f^{a+\frac{b}{x^3}} dx = \frac{1}{3} f^a x \Gamma\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right) \sqrt[3]{-\frac{b \log(f)}{x^3}}$$

**Mathematica [A]** time = 0.0029922, size = 32, normalized size = 1.

$$\frac{1}{3} x f^a \sqrt[3]{-\frac{b \log(f)}{x^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3), x]

[Out] (f^a\*x\*Gamma[-1/3, -((b\*Log[f])/x^3)]\*(-((b\*Log[f])/x^3))^(1/3))/3

**Maple [B]** time = 0.029, size = 98, normalized size = 3.1

$$-\frac{f^a}{3} \sqrt[3]{-b} \sqrt[3]{\ln(f)} \left( 3 \frac{(\ln(f))^{2/3} b \Gamma(2/3)}{x^2 \sqrt[3]{-b}} \left(-\frac{b \ln(f)}{x^3}\right)^{-2/3} - 3 \frac{x}{\sqrt[3]{-b} \sqrt[3]{\ln(f)}} e^{\frac{b \ln(f)}{x^3}} - 3 \frac{(\ln(f))^{2/3} b}{x^2 \sqrt[3]{-b}} \Gamma\left(2/3, -\frac{b \ln(f)}{x^3}\right) \right) \left(-\frac{b \ln(f)}{x^3}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b/x^3),x)`

[Out] 
$$-1/3*f^a*(-b)^{(1/3)}*\ln(f)^{(1/3)}*(3/x^2/(-b)^{(1/3)}*\ln(f)^{(2/3)}*b*\text{GAMMA}(2/3)/(-b*\ln(f)/x^3)^{(2/3)}-3*x/(-b)^{(1/3)}/\ln(f)^{(1/3)}*\exp(b*\ln(f)/x^3)-3/x^2/(-b)^{(1/3)}*\ln(f)^{(2/3)}*b/(-b*\ln(f)/x^3)^{(2/3)}*\text{GAMMA}(2/3,-b*\ln(f)/x^3))$$

**Maxima [A]** time = 1.24753, size = 35, normalized size = 1.09

$$\frac{1}{3} f^a x \left( -\frac{b \log(f)}{x^3} \right)^{\frac{1}{3}} \Gamma \left( -\frac{1}{3}, -\frac{b \log(f)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3),x, algorithm="maxima")`

[Out] 
$$1/3*f^a*x*(-b*\log(f)/x^3)^{(1/3)}*\text{gamma}(-1/3, -b*\log(f)/x^3)$$

**Fricas [A]** time = 1.8089, size = 100, normalized size = 3.12

$$-(-b \log(f))^{\frac{1}{3}} f^a \Gamma \left( \frac{2}{3}, -\frac{b \log(f)}{x^3} \right) + f^{\frac{ax^3+b}{x^3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b/x^3),x, algorithm="fricas")`

[Out] 
$$-(-b*\log(f))^{(1/3)}*f^a*\text{gamma}(2/3, -b*\log(f)/x^3) + f^{((a*x^3 + b)/x^3)}*x$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b/x**3),x)`

[Out] `Integral(f**(a + b/x**3), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+\frac{b}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3),x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3), x)
```

$$3.171 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3x \sqrt[3]{-\frac{b \log(f)}{x^3}}}$$

[Out] (f^a\*Gamma[1/3, -((b\*Log[f])/x^3))]/(3\*x\*(-((b\*Log[f])/x^3))^(1/3))

**Rubi [A]** time = 0.0236808, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3x \sqrt[3]{-\frac{b \log(f)}{x^3}}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^2, x]

[Out] (f^a\*Gamma[1/3, -((b\*Log[f])/x^3))]/(3\*x\*(-((b\*Log[f])/x^3))^(1/3))

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx = \frac{f^a \Gamma\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3x \sqrt[3]{-\frac{b \log(f)}{x^3}}}$$

**Mathematica [A]** time = 0.0052051, size = 34, normalized size = 1.

$$\frac{f^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3x \sqrt[3]{-\frac{b \log(f)}{x^3}}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^2, x]

[Out] (f^a\*Gamma[1/3, -((b\*Log[f])/x^3))]/(3\*x\*(-((b\*Log[f])/x^3))^(1/3))

**Maple [B]** time = 0.03, size = 82, normalized size = 2.4

$$-\frac{f^a}{3} \left( \frac{2\pi\sqrt{3}}{3x\Gamma(2/3)} \sqrt[3]{-b} \sqrt[3]{\ln(f)} \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} - \frac{1}{x} \sqrt[3]{-b} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -\frac{b\ln(f)}{x^3}\right) \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} \right) \frac{1}{\sqrt[3]{-b}} \frac{1}{\sqrt[3]{\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^2,x)

[Out] -1/3\*f^a/(-b)^(1/3)/ln(f)^(1/3)\*(2/3/x\*(-b)^(1/3)\*ln(f)^(1/3)\*Pi\*3^(1/2)/GA  
MMA(2/3)/(-b\*ln(f)/x^3)^(1/3)-1/x\*(-b)^(1/3)\*ln(f)^(1/3)/(-b\*ln(f)/x^3)^(1/  
3)\*GAMMA(1/3,-b\*ln(f)/x^3))

**Maxima [A]** time = 1.20205, size = 38, normalized size = 1.12

$$\frac{f^a \Gamma\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3x \left(-\frac{b \log(f)}{x^3}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^2,x, algorithm="maxima")

[Out] 1/3\*f^a\*gamma(1/3, -b\*log(f)/x^3)/(x\*(-b\*log(f)/x^3)^(1/3))

**Fricas [A]** time = 1.59362, size = 88, normalized size = 2.59

$$-\frac{(-b \log(f))^{\frac{2}{3}} f^a \Gamma\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right)}{3b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^2,x, algorithm="fricas")

[Out] -1/3\*(-b\*log(f))^(2/3)\*f^a\*gamma(1/3, -b\*log(f)/x^3)/(b\*log(f))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*2,x)

```
[Out] Integral(f**(a + b/x**3)/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b/x^3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(f^(a + b/x^3)/x^2, x)
```

$$3.172 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^2 \left(-\frac{b \log(f)}{x^3}\right)^{2/3}}$$

[Out] (f^a\*Gamma[2/3, -((b\*Log[f])/x^3)])/(3\*x^2\*(-((b\*Log[f])/x^3))^(2/3))

**Rubi [A]** time = 0.0222683, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^2 \left(-\frac{b \log(f)}{x^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^3, x]

[Out] (f^a\*Gamma[2/3, -((b\*Log[f])/x^3)])/(3\*x^2\*(-((b\*Log[f])/x^3))^(2/3))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx = \frac{f^a \Gamma\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^2 \left(-\frac{b \log(f)}{x^3}\right)^{2/3}}$$

**Mathematica [A]** time = 0.0046829, size = 34, normalized size = 1.

$$\frac{f^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^2 \left(-\frac{b \log(f)}{x^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^3, x]

[Out] (f^a\*Gamma[2/3, -((b\*Log[f])/x^3)])/(3\*x^2\*(-((b\*Log[f])/x^3))^(2/3))

---

**Maple [B]** time = 0.03, size = 78, normalized size = 2.3

$$\frac{f^a}{3b} \sqrt[3]{-b} \left( \frac{\Gamma\left(\frac{2}{3}\right)}{x^2} (-b)^{\frac{2}{3}} (\ln(f))^{\frac{2}{3}} \left(-\frac{b \ln(f)}{x^3}\right)^{-\frac{2}{3}} - \frac{1}{x^2} (-b)^{\frac{2}{3}} (\ln(f))^{\frac{2}{3}} \Gamma\left(\frac{2}{3}, -\frac{b \ln(f)}{x^3}\right) \left(-\frac{b \ln(f)}{x^3}\right)^{-\frac{2}{3}} \right) (\ln(f))^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^3,x)

[Out] 1/3\*f^a/b/ln(f)^(2/3)\*(-b)^(1/3)\*(1/x^2\*(-b)^(2/3)\*ln(f)^(2/3)\*GAMMA(2/3)/(-b\*ln(f)/x^3)^(2/3)-1/x^2\*(-b)^(2/3)\*ln(f)^(2/3)/(-b\*ln(f)/x^3)^(2/3)\*GAMMA(2/3,-b\*ln(f)/x^3))

---

**Maxima [A]** time = 1.2261, size = 38, normalized size = 1.12

$$\frac{f^a \Gamma\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3 x^2 \left(-\frac{b \log(f)}{x^3}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^3,x, algorithm="maxima")

[Out] 1/3\*f^a\*gamma(2/3, -b\*log(f)/x^3)/(x^2\*(-b\*log(f)/x^3)^(2/3))

---

**Fricas [A]** time = 1.59512, size = 88, normalized size = 2.59

$$-\frac{(-b \log(f))^{\frac{1}{3}} f^a \Gamma\left(\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)}{3 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^3,x, algorithm="fricas")

[Out] -1/3\*(-b\*log(f))^(1/3)\*f^a\*gamma(2/3, -b\*log(f)/x^3)/(b\*log(f))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*3,x)

[Out] Integral(f\*\*(a + b/x\*\*3)/x\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^3,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)/x^3, x)



$$3.173 \quad \int \frac{f^{a+\frac{b}{x^3}}}{x^5} dx$$

**Optimal.** Leaf size=34

$$\frac{f^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{4/3}}$$

[Out] (f^a\*Gamma[4/3, -((b\*Log[f])/x^3)))/(3\*x^4\*(-((b\*Log[f])/x^3))^(4/3))

**Rubi [A]** time = 0.0230954, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b/x^3)/x^5, x]

[Out] (f^a\*Gamma[4/3, -((b\*Log[f])/x^3)))/(3\*x^4\*(-((b\*Log[f])/x^3))^(4/3))

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5} dx = \frac{f^a \Gamma\left(\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{4/3}}$$

**Mathematica [A]** time = 0.005421, size = 34, normalized size = 1.

$$\frac{f^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b/x^3)/x^5, x]

[Out] (f^a\*Gamma[4/3, -((b\*Log[f])/x^3)))/(3\*x^4\*(-((b\*Log[f])/x^3))^(4/3))

---

**Maple [B]** time = 0.037, size = 112, normalized size = 3.3

$$-\frac{f^a}{3} \left( -\frac{2\pi\sqrt{3}}{9bx\Gamma(2/3)} (-b)^{4/3} \sqrt[3]{\ln(f)} \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} + \frac{1}{bx} (-b)^{4/3} \sqrt[3]{\ln(f)} e^{\frac{b\ln(f)}{x^3}} + \frac{1}{3bx} (-b)^{4/3} \sqrt[3]{\ln(f)} \Gamma\left(\frac{1}{3}, -\frac{b\ln(f)}{x^3}\right) \frac{1}{\sqrt[3]{-\frac{b\ln(f)}{x^3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b/x^3)/x^5,x)

[Out] -1/3\*f^a/(-b)^(4/3)/ln(f)^(4/3)\*(-2/9/x\*(-b)^(4/3)\*ln(f)^(1/3)/b\*Pi\*3^(1/2)/GAMMA(2/3)/(-b\*ln(f)/x^3)^(1/3)+1/x\*(-b)^(4/3)\*ln(f)^(1/3)/b\*exp(b\*ln(f)/x^3)+1/3/x\*(-b)^(4/3)\*ln(f)^(1/3)/b/(-b\*ln(f)/x^3)^(1/3)\*GAMMA(1/3,-b\*ln(f)/x^3))

---

**Maxima [A]** time = 1.22927, size = 38, normalized size = 1.12

$$\frac{f^a \Gamma\left(\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^5,x, algorithm="maxima")

[Out] 1/3\*f^a\*gamma(4/3, -b\*log(f)/x^3)/(x^4\*(-b\*log(f)/x^3)^(4/3))

---

**Fricas [A]** time = 1.63414, size = 144, normalized size = 4.24

$$\frac{(-b \log(f))^{\frac{2}{3}} f^a x \Gamma\left(\frac{1}{3}, -\frac{b \log(f)}{x^3}\right) - 3 b f^{\frac{ax^3+b}{x^3}} \log(f)}{9 b^2 x \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^5,x, algorithm="fricas")

[Out] 1/9\*((-b\*log(f))^(2/3)\*f^a\*x\*gamma(1/3, -b\*log(f)/x^3) - 3\*b\*f^((a\*x^3 + b)/x^3)\*log(f))/(b^2\*x\*log(f)^2)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b/x\*\*3)/x\*\*5,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b/x^3)/x^5,x, algorithm="giac")

[Out] integrate(f^(a + b/x^3)/x^5, x)

### 3.174 $\int f^{a+bx^n} x^m dx$

**Optimal.** Leaf size=46

$$\frac{f^a x^{m+1} (-b \log(f) x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b \log(f) x^n\right)}{n}$$

[Out] -((f^a\*x^(1 + m)\*Gamma[(1 + m)/n, -(b\*x^n\*Log[f])])/(n\*(-(b\*x^n\*Log[f]))^(1 + m)/n)))

**Rubi [A]** time = 0.0264467, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a x^{m+1} (-b \log(f) x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^m,x]

[Out] -((f^a\*x^(1 + m)\*Gamma[(1 + m)/n, -(b\*x^n\*Log[f])])/(n\*(-(b\*x^n\*Log[f]))^(1 + m)/n)))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^n} x^m dx = -\frac{f^a x^{1+m} \Gamma\left(\frac{1+m}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-\frac{1+m}{n}}}{n}$$

**Mathematica [A]** time = 0.0123183, size = 46, normalized size = 1.

$$\frac{f^a x^{m+1} (-b \log(f) x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^m,x]

[Out] -((f^a\*x^(1 + m)\*Gamma[(1 + m)/n, -(b\*x^n\*Log[f])])/(n\*(-(b\*x^n\*Log[f]))^(1 + m)/n)))

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int f^{a+bx^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^m,x)

[Out] int(f^(a+b\*x^n)\*x^m,x)

**Maxima [A]** time = 1.22754, size = 63, normalized size = 1.37

$$-\frac{f^a x^{m+1} \Gamma\left(\frac{m+1}{n}, -bx^n \log(f)\right)}{(-bx^n \log(f))^{\frac{m+1}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^m,x, algorithm="maxima")

[Out] -f^a\*x^(m + 1)\*gamma((m + 1)/n, -b\*x^n\*log(f))/((-b\*x^n\*log(f))^(m + 1)/n)\*n)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{bx^n+a}x^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^m,x, algorithm="fricas")

[Out] integral(f^(b\*x^n + a)\*x^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b*x^n)*x^m,x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^n + a)*x^m, x)
```

### 3.175 $\int f^{a+bx^n} x^3 dx$

**Optimal.** Leaf size=39

$$\frac{x^4 f^a (-b \log(f) x^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(f) x^n\right)}{n}$$

[Out]  $-\left(\frac{f^a x^4 \Gamma\left[\frac{4}{n}, -(b x^n \text{Log}[f])\right]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{4/n}}\right)$

**Rubi [A]** time = 0.0253167, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{x^4 f^a (-b \log(f) x^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^3, x]

[Out]  $-\left(\frac{f^a x^4 \Gamma\left[\frac{4}{n}, -(b x^n \text{Log}[f])\right]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{4/n}}\right)$

#### Rule 2218

Int[(F\_)^(a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^n} x^3 dx = -\frac{f^a x^4 \Gamma\left(\frac{4}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-4/n}}{n}$$

**Mathematica [A]** time = 0.0061174, size = 39, normalized size = 1.

$$\frac{x^4 f^a (-b \log(f) x^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^3, x]

[Out]  $-\left(\frac{f^a x^4 \Gamma\left[\frac{4}{n}, -(b x^n \text{Log}[f])\right]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{4/n}}\right)$

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int f^{a+bx^n} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x^3,x)`

[Out] `int(f^(a+b*x^n)*x^3,x)`

**Maxima [A]** time = 1.22863, size = 55, normalized size = 1.41

$$\frac{f^a x^4 \Gamma\left(\frac{4}{n}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{4}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^3,x, algorithm="maxima")`

[Out] `-f^a*x^4*gamma(4/n, -b*x^n*log(f))/((-b*x^n*log(f))^(4/n)*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{bx^n+a}x^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^3,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)*x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^3,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)*x^3, x)`



### 3.176 $\int f^{a+bx^n} x^2 dx$

**Optimal.** Leaf size=39

$$\frac{x^3 f^a (-b \log(f) x^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(f) x^n\right)}{n}$$

[Out]  $-\left(\left(f^a x^3 \Gamma\left[\frac{3}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{\left(3/n\right)}\right)$

**Rubi [A]** time = 0.0252264, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{x^3 f^a (-b \log(f) x^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^2, x]

[Out]  $-\left(\left(f^a x^3 \Gamma\left[\frac{3}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{\left(3/n\right)}\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^n} x^2 dx = -\frac{f^a x^3 \Gamma\left(\frac{3}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-3/n}}{n}$$

**Mathematica [A]** time = 0.005968, size = 39, normalized size = 1.

$$\frac{x^3 f^a (-b \log(f) x^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^2, x]

[Out]  $-\left(\left(f^a x^3 \Gamma\left[\frac{3}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{\left(3/n\right)}\right)$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int f^{a+bx^n} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x^2,x)`

[Out] `int(f^(a+b*x^n)*x^2,x)`

**Maxima [A]** time = 1.24579, size = 55, normalized size = 1.41

$$\frac{f^a x^3 \Gamma\left(\frac{3}{n}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{3}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^2,x, algorithm="maxima")`

[Out] `-f^a*x^3*gamma(3/n, -b*x^n*log(f))/((-b*x^n*log(f))^(3/n)*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{bx^n+a}x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^2,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)*x^2, x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x**2,x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a}x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^2,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)*x^2, x)`

### 3.177 $\int f^{a+bx^n} x dx$

**Optimal.** Leaf size=39

$$\frac{x^2 f^a (-b \log(f) x^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b \log(f) x^n\right)}{n}$$

[Out]  $-\left(\left(f^a x^2 \Gamma\left[\frac{2}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{(2/n)}\right)$

**Rubi [A]** time = 0.0145723, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2218}

$$\frac{x^2 f^a (-b \log(f) x^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x, x]

[Out]  $-\left(\left(f^a x^2 \Gamma\left[\frac{2}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{(2/n)}\right)$

#### Rule 2218

Int[(F\_)^(a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{a+bx^n} x dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-2/n}}{n}$$

**Mathematica [A]** time = 0.0057416, size = 39, normalized size = 1.

$$\frac{x^2 f^a (-b \log(f) x^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x, x]

[Out]  $-\left(\left(f^a x^2 \Gamma\left[\frac{2}{n}, -(b x^n \text{Log}[f])\right]\right)\right) / \left(n \left(-\left(b x^n \text{Log}[f]\right)\right)^{(2/n)}\right)$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int f^{a+bx^n} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x,x)`

[Out] `int(f^(a+b*x^n)*x,x)`

**Maxima [A]** time = 1.26472, size = 55, normalized size = 1.41

$$-\frac{f^a x^2 \Gamma\left(\frac{2}{n}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{2}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x,x, algorithm="maxima")`

[Out] `-f^a*x^2*gamma(2/n, -b*x^n*log(f))/((-b*x^n*log(f))^(2/n)*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{bx^n+a}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)*x, x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x,x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)*x, x)`

### 3.178 $\int f^{a+bx^n} dx$

**Optimal.** Leaf size=35

$$\frac{xf^a(-b\log(f)x^n)^{-1/n}\Gamma\left(\frac{1}{n}, -b\log(f)x^n\right)}{n}$$

[Out]  $-\left(\frac{f^a x \Gamma[n^{(-1)}, -(b x^n \text{Log}[f])]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{n^{(-1)}}}\right)$

**Rubi [A]** time = 0.0043448, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2208}

$$\frac{xf^a(-b\log(f)x^n)^{-1/n}\Gamma\left(\frac{1}{n}, -b\log(f)x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n), x]

[Out]  $-\left(\frac{f^a x \Gamma[n^{(-1)}, -(b x^n \text{Log}[f])]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{n^{(-1)}}}\right)$

**Rule 2208**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^n)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

**Rubi steps**

$$\int f^{a+bx^n} dx = -\frac{f^a x \Gamma\left(\frac{1}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-1/n}}{n}$$

**Mathematica [A]** time = 0.0045952, size = 35, normalized size = 1.

$$\frac{xf^a(-b\log(f)x^n)^{-1/n}\Gamma\left(\frac{1}{n}, -b\log(f)x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n), x]

[Out]  $-\left(\frac{f^a x \Gamma[n^{(-1)}, -(b x^n \text{Log}[f])]}{n \left(-\left(b x^n \text{Log}[f]\right)\right)^{n^{(-1)}}}\right)$

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int f^{a+bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n),x)`

[Out] `int(f^(a+b*x^n),x)`

**Maxima [A]** time = 1.24017, size = 47, normalized size = 1.34

$$\frac{f^a x \Gamma\left(\frac{1}{n}, -bx^n \log(f)\right)}{(-bx^n \log(f))^{\left(\frac{1}{n}\right)} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n),x, algorithm="maxima")`

[Out] `-f^a*x*gamma(1/n, -b*x^n*log(f))/((-b*x^n*log(f))^(1/n)*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{bx^n+a}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n),x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a), x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n),x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n),x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a), x)`

$$3.179 \quad \int \frac{f^{a+bx^n}}{x} dx$$

**Optimal.** Leaf size=15

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

**Rubi [A]** time = 0.0231667, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)/x,x]

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{f^{a+bx^n}}{x} dx = \frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

**Mathematica [A]** time = 0.0024586, size = 15, normalized size = 1.

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)/x,x]

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

**Maple [A]** time = 0.157, size = 19, normalized size = 1.3

$$\frac{f^a \operatorname{Ei}(1, -bx^n \ln(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)/x,x)`

[Out] `-1/n*f^a*Ei(1,-b*x^n*ln(f))`

**Maxima [A]** time = 1.14101, size = 20, normalized size = 1.33

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="maxima")`

[Out] `f^a*Ei(b*x^n*log(f))/n`

**Fricas [A]** time = 1.51449, size = 32, normalized size = 2.13

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="fricas")`

[Out] `f^a*Ei(b*x^n*log(f))/n`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)/x,x)`

[Out] `Integral(f**(a + b*x**n)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^n+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)/x, x)`



$$3.180 \quad \int \frac{f^{a+bx^n}}{x^2} dx$$

**Optimal.** Leaf size=37

$$\frac{f^a \left(-b \log(f)x^n\right)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(f)x^n\right)}{nx}$$

[Out]  $-\left(\left(f^a \Gamma\left[-n^{-1}\right], -\left(b x^n \operatorname{Log}[f]\right)\right) \cdot \left(-\left(b x^n \operatorname{Log}[f]\right)\right)^{-n^{-1}}\right) / \left(n x\right)$

**Rubi [A]** time = 0.0238832, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a \left(-b \log(f)x^n\right)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(f)x^n\right)}{nx}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)/x^2, x]

[Out]  $-\left(\left(f^a \Gamma\left[-n^{-1}\right], -\left(b x^n \operatorname{Log}[f]\right)\right) \cdot \left(-\left(b x^n \operatorname{Log}[f]\right)\right)^{-n^{-1}}\right) / \left(n x\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^n}}{x^2} dx = -\frac{f^a \Gamma\left(-\frac{1}{n}, -bx^n \log(f)\right) \left(-bx^n \log(f)\right)^{\frac{1}{n}}}{nx}$$

**Mathematica [A]** time = 0.0038872, size = 37, normalized size = 1.

$$\frac{f^a \left(-b \log(f)x^n\right)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(f)x^n\right)}{nx}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)/x^2, x]

[Out]  $-\left(\left(f^a \Gamma\left[-n^{-1}\right], -\left(b x^n \operatorname{Log}[f]\right)\right) \cdot \left(-\left(b x^n \operatorname{Log}[f]\right)\right)^{-n^{-1}}\right) / \left(n x\right)$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^n}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)/x^2,x)`

[Out] `int(f^(a+b*x^n)/x^2,x)`

**Maxima [A]** time = 1.20209, size = 50, normalized size = 1.35

$$\frac{(-bx^n \log(f))^{\left(\frac{1}{n}\right)} f^a \Gamma\left(-\frac{1}{n}, -bx^n \log(f)\right)}{nx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^2,x, algorithm="maxima")`

[Out] `-(-b*x^n*log(f))^(1/n)*f^a*gamma(-1/n, -b*x^n*log(f))/(n*x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{bx^n+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^2,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)/x^2, x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)/x**2,x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^n+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^2,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)/x^2, x)`

$$3.181 \quad \int \frac{f^{a+bx^n}}{x^3} dx$$

**Optimal.** Leaf size=39

$$\frac{f^a (-b \log(f)x^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(f)x^n\right)}{nx^2}$$

[Out]  $-\left(\left(f^a \text{Gamma}\left[-\frac{2}{n}, -(b*x^n*\text{Log}[f])\right]\right)*\left(-\left(b*x^n*\text{Log}[f]\right)\right)^{(2/n)}\right)/(n*x^2)$

**Rubi [A]** time = 0.0246527, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a (-b \log(f)x^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(f)x^n\right)}{nx^2}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)/x^3, x]

[Out]  $-\left(\left(f^a \text{Gamma}\left[-\frac{2}{n}, -(b*x^n*\text{Log}[f])\right]\right)*\left(-\left(b*x^n*\text{Log}[f]\right)\right)^{(2/n)}\right)/(n*x^2)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^n}}{x^3} dx = -\frac{f^a \Gamma\left(-\frac{2}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{2/n}}{nx^2}$$

**Mathematica [A]** time = 0.0040575, size = 39, normalized size = 1.

$$\frac{f^a (-b \log(f)x^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(f)x^n\right)}{nx^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)/x^3, x]

[Out]  $-\left(\left(f^a \text{Gamma}\left[-\frac{2}{n}, -(b*x^n*\text{Log}[f])\right]\right)*\left(-\left(b*x^n*\text{Log}[f]\right)\right)^{(2/n)}\right)/(n*x^2)$

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^n}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)/x^3,x)`

[Out] `int(f^(a+b*x^n)/x^3,x)`

**Maxima [A]** time = 1.21543, size = 53, normalized size = 1.36

$$\frac{(-bx^n \log(f))^{\frac{2}{n}} f^a \Gamma\left(-\frac{2}{n}, -bx^n \log(f)\right)}{nx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^3,x, algorithm="maxima")`

[Out] `-(-b*x^n*log(f))^(2/n)*f^a*gamma(-2/n, -b*x^n*log(f))/(n*x^2)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{bx^n+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^3,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)/x^3, x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)/x**3,x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^n+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^3,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)/x^3, x)`

$$3.182 \quad \int \frac{f^{a+bx^n}}{x^4} dx$$

**Optimal.** Leaf size=39

$$\frac{f^a (-b \log(f)x^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(f)x^n\right)}{nx^3}$$

[Out]  $-\left((f^a \text{Gamma}\left[-\frac{3}{n}, -(b*x^n \text{Log}[f])\right]) * \left(-\left(b*x^n \text{Log}[f]\right)\right)^{\left(\frac{3}{n}\right)}\right) / \left(n*x^3\right)$

**Rubi [A]** time = 0.0241303, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218}

$$\frac{f^a (-b \log(f)x^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(f)x^n\right)}{nx^3}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)/x^4, x]

[Out]  $-\left((f^a \text{Gamma}\left[-\frac{3}{n}, -(b*x^n \text{Log}[f])\right]) * \left(-\left(b*x^n \text{Log}[f]\right)\right)^{\left(\frac{3}{n}\right)}\right) / \left(n*x^3\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{f^{a+bx^n}}{x^4} dx = -\frac{f^a \Gamma\left(-\frac{3}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{3/n}}{nx^3}$$

**Mathematica [A]** time = 0.0041876, size = 39, normalized size = 1.

$$\frac{f^a (-b \log(f)x^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(f)x^n\right)}{nx^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)/x^4, x]

[Out]  $-\left((f^a \text{Gamma}\left[-\frac{3}{n}, -(b*x^n \text{Log}[f])\right]) * \left(-\left(b*x^n \text{Log}[f]\right)\right)^{\left(\frac{3}{n}\right)}\right) / \left(n*x^3\right)$

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^n}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)/x^4,x)`

[Out] `int(f^(a+b*x^n)/x^4,x)`

**Maxima [A]** time = 1.19786, size = 53, normalized size = 1.36

$$\frac{(-bx^n \log(f))^{\frac{3}{n}} f^a \Gamma\left(-\frac{3}{n}, -bx^n \log(f)\right)}{nx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^4,x, algorithm="maxima")`

[Out] `-(-b*x^n*log(f))^(3/n)*f^a*gamma(-3/n, -b*x^n*log(f))/(n*x^3)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{bx^n+a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^4,x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)/x^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)/x**4,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^n+a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x^4,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)/x^4, x)`

### 3.183 $\int f^{a+bx^n} x^{-1+3n} dx$

**Optimal.** Leaf size=71

$$-\frac{2x^n f^{a+bx^n}}{b^2 n \log^2(f)} + \frac{2f^{a+bx^n}}{b^3 n \log^3(f)} + \frac{x^{2n} f^{a+bx^n}}{bn \log(f)}$$

[Out]  $(2*f^{(a + b*x^n)})/(b^3*n*\text{Log}[f]^3) - (2*f^{(a + b*x^n)}*x^n)/(b^2*n*\text{Log}[f]^2) + (f^{(a + b*x^n)}*x^{(2*n)})/(b*n*\text{Log}[f])$

**Rubi [A]** time = 0.075849, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2213, 2209}

$$-\frac{2x^n f^{a+bx^n}}{b^2 n \log^2(f)} + \frac{2f^{a+bx^n}}{b^3 n \log^3(f)} + \frac{x^{2n} f^{a+bx^n}}{bn \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^(-1 + 3\*n),x]

[Out]  $(2*f^{(a + b*x^n)})/(b^3*n*\text{Log}[f]^3) - (2*f^{(a + b*x^n)}*x^n)/(b^2*n*\text{Log}[f]^2) + (f^{(a + b*x^n)}*x^{(2*n)})/(b*n*\text{Log}[f])$

#### Rule 2213

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^Simplify[m - n]\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1+3n} dx &= \frac{f^{a+bx^n} x^{2n}}{bn \log(f)} - \frac{2 \int f^{a+bx^n} x^{-1+2n} dx}{b \log(f)} \\ &= -\frac{2f^{a+bx^n} x^n}{b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^{2n}}{bn \log(f)} + \frac{2 \int f^{a+bx^n} x^{-1+n} dx}{b^2 \log^2(f)} \\ &= \frac{2f^{a+bx^n}}{b^3 n \log^3(f)} - \frac{2f^{a+bx^n} x^n}{b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^{2n}}{bn \log(f)} \end{aligned}$$

**Mathematica [C]** time = 0.0040479, size = 24, normalized size = 0.34

$$\frac{f^a \text{Gamma}(3, -b \log(f) x^n)}{b^3 n \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + 3\*n),x]

[Out] (f^a\*Gamma[3, -(b\*x^n\*Log[f])])/(b^3\*n\*Log[f]^3)

**Maple [A]** time = 0.02, size = 44, normalized size = 0.6

$$\frac{\left((x^n)^2 b^2 (\ln(f))^2 - 2 b x^n \ln(f) + 2\right) f^{a+b x^n}}{(\ln(f))^3 b^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1+3\*n),x)

[Out] ((x^n)^2\*b^2\*ln(f)^2-2\*b\*x^n\*ln(f)+2)/b^3/ln(f)^3/n\*f^(a+b\*x^n)

**Maxima [A]** time = 1.1279, size = 69, normalized size = 0.97

$$\frac{\left(b^2 f^a x^{2n} \log(f)^2 - 2 b f^a x^n \log(f) + 2 f^a\right) f^{b x^n}}{b^3 n \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+3\*n),x, algorithm="maxima")

[Out] (b^2\*f^a\*x^(2\*n)\*log(f)^2 - 2\*b\*f^a\*x^n\*log(f) + 2\*f^a)\*f^(b\*x^n)/(b^3\*n\*log(f)^3)

**Fricas [A]** time = 1.53393, size = 122, normalized size = 1.72

$$\frac{\left(b^2 x^{2n} \log(f)^2 - 2 b x^n \log(f) + 2\right) e^{(b x^n \log(f) + a \log(f))}}{b^3 n \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+3\*n),x, algorithm="fricas")

[Out] (b^2\*x^(2\*n)\*log(f)^2 - 2\*b\*x^n\*log(f) + 2)\*e^(b\*x^n\*log(f) + a\*log(f))/(b^3\*n\*log(f)^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(f**(a+b*x**n)*x**(-1+3*n),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b*x^n)*x^(-1+3*n),x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^n + a)*x^(3*n - 1), x)
```

### 3.184 $\int f^{a+bx^n} x^{-1+2n} dx$

**Optimal.** Leaf size=45

$$\frac{x^n f^{a+bx^n}}{bn \log(f)} - \frac{f^{a+bx^n}}{b^2 n \log^2(f)}$$

[Out]  $-(f^{(a + b*x^n)/(b^2*n*Log[f]^2)}) + (f^{(a + b*x^n)*x^n}/(b*n*Log[f]))$

**Rubi [A]** time = 0.048403, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2213, 2209}

$$\frac{x^n f^{a+bx^n}}{bn \log(f)} - \frac{f^{a+bx^n}}{b^2 n \log^2(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*x^n)*x^{(-1 + 2*n)}, x]$

[Out]  $-(f^{(a + b*x^n)/(b^2*n*Log[f]^2)}) + (f^{(a + b*x^n)*x^n}/(b*n*Log[f]))$

#### Rule 2213

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^Simplify[m - n]*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((e_.) + (f_.)*(x_)^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1+2n} dx &= \frac{f^{a+bx^n} x^n}{bn \log(f)} - \frac{\int f^{a+bx^n} x^{-1+n} dx}{b \log(f)} \\ &= -\frac{f^{a+bx^n}}{b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^n}{bn \log(f)} \end{aligned}$$

**Mathematica [C]** time = 0.0042438, size = 25, normalized size = 0.56

$$-\frac{f^a \text{Gamma}(2, -b \log(f) x^n)}{b^2 n \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + 2\*n), x]

[Out] -((f^a\*Gamma[2, -(b\*x^n\*Log[f])])/(b^2\*n\*Log[f]^2))

**Maple [A]** time = 0.031, size = 56, normalized size = 1.2

$$\frac{e^{n \ln(x)} e^{(be^{n \ln(x)} + a) \ln(f)}}{\ln(f) b n} - \frac{e^{(be^{n \ln(x)} + a) \ln(f)}}{b^2 n (\ln(f))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1+2\*n), x)

[Out] 1/ln(f)/b/n\*exp(n\*ln(x))\*exp((b\*exp(n\*ln(x))+a)\*ln(f))-1/ln(f)^2/b^2/n\*exp((b\*exp(n\*ln(x))+a)\*ln(f))

**Maxima [A]** time = 1.14659, size = 46, normalized size = 1.02

$$\frac{(b f^a x^n \log(f) - f^a) f^{b x^n}}{b^2 n \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+2\*n), x, algorithm="maxima")

[Out] (b\*f^a\*x^n\*log(f) - f^a)\*f^(b\*x^n)/(b^2\*n\*log(f)^2)

**Fricas [A]** time = 1.55595, size = 88, normalized size = 1.96

$$\frac{(b x^n \log(f) - 1) e^{(b x^n \log(f) + a \log(f))}}{b^2 n \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+2\*n), x, algorithm="fricas")

[Out] (b\*x^n\*log(f) - 1)\*e^(b\*x^n\*log(f) + a\*log(f))/(b^2\*n\*log(f)^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1+2\*n), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(2\*n - 1), x)

$$3.185 \quad \int f^{a+bx^n} x^{-1+n} dx$$

**Optimal.** Leaf size=20

$$\frac{f^{a+bx^n}}{bn \log(f)}$$

[Out]  $f^{(a + b*x^n)/(b*n*Log[f])}$

**Rubi [A]** time = 0.0231736, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2209}

$$\frac{f^{a+bx^n}}{bn \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^(-1 + n), x]

[Out]  $f^{(a + b*x^n)/(b*n*Log[f])}$

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int f^{a+bx^n} x^{-1+n} dx = \frac{f^{a+bx^n}}{bn \log(f)}$$

**Mathematica [A]** time = 0.0042465, size = 20, normalized size = 1.

$$\frac{f^{a+bx^n}}{bn \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + n), x]

[Out]  $f^{(a + b*x^n)/(b*n*Log[f])}$

**Maple [A]** time = 0.022, size = 25, normalized size = 1.3

$$\frac{e^{(be^n \ln(x) + a) \ln(f)}}{\ln(f) bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x^(-1+n),x)`

[Out] `1/ln(f)/b/n*exp((b*exp(n*ln(x))+a)*ln(f))`

**Maxima [A]** time = 1.16261, size = 27, normalized size = 1.35

$$\frac{f^{bx^n+a}}{bn \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="maxima")`

[Out] `f^(b*x^n + a)/(b*n*log(f))`

**Fricas [A]** time = 1.56783, size = 57, normalized size = 2.85

$$\frac{e^{(bx^n \log(f)+a \log(f))}}{bn \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="fricas")`

[Out] `e^(b*x^n*log(f) + a*log(f))/(b*n*log(f))`

**Sympy [A]** time = 161.134, size = 39, normalized size = 1.95

$$\begin{cases} \log(x) & \text{for } b = 0 \wedge f = 1 \wedge n = 0 \\ f^{a+b} \log(x) & \text{for } n = 0 \\ \frac{f^a x^n}{n} & \text{for } b = 0 \\ \frac{x^n}{n} & \text{for } f = 1 \\ \frac{f^a f^{bx^n}}{bn \log(f)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x**(-1+n),x)`

[Out] `Piecewise((log(x), Eq(b, 0) & Eq(f, 1) & Eq(n, 0)), (f**(a + b)*log(x), Eq(n, 0)), (f**a*x**n/n, Eq(b, 0)), (x**n/n, Eq(f, 1)), (f**a*f**(b*x**n)/(b*n*log(f)), True))`

**Giac [A]** time = 1.25439, size = 27, normalized size = 1.35

$$\frac{f^{bx^n+a}}{bn \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="giac")
```

```
[Out] f^(b*x^n + a)/(b*n*log(f))
```

$$3.186 \quad \int \frac{f^{a+bx^n}}{x} dx$$

**Optimal.** Leaf size=15

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

**Rubi [A]** time = 0.0229328, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2210}

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)/x, x]

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^n\_))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{f^{a+bx^n}}{x} dx = \frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

**Mathematica [A]** time = 0.0017915, size = 15, normalized size = 1.

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)/x, x]

[Out] (f^a\*ExpIntegralEi[b\*x^n\*Log[f]])/n

**Maple [A]** time = 0., size = 19, normalized size = 1.3

$$\frac{f^a \operatorname{Ei}(1, -bx^n \ln(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(f^(a+b*x^n)/x,x)`

[Out] `-1/n*f^a*Ei(1,-b*x^n*ln(f))`

**Maxima [A]** time = 1.26914, size = 20, normalized size = 1.33

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="maxima")`

[Out] `f^a*Ei(b*x^n*log(f))/n`

**Fricas [A]** time = 1.55444, size = 32, normalized size = 2.13

$$\frac{f^a \operatorname{Ei}(bx^n \log(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="fricas")`

[Out] `f^a*Ei(b*x^n*log(f))/n`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)/x,x)`

[Out] `Integral(f**(a + b*x**n)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx^n+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)/x,x, algorithm="giac")`

[Out] `integrate(f^(b*x^n + a)/x, x)`

$$3.187 \quad \int f^{a+bx^n} x^{-1-n} dx$$

**Optimal.** Leaf size=38

$$\frac{bf^a \log(f) \operatorname{Ei}(bx^n \log(f))}{n} - \frac{x^{-n} f^{a+bx^n}}{n}$$

[Out]  $-(f^{(a + b*x^n)/(n*x^n)}) + (b*f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f])/n$

**Rubi [A]** time = 0.0486405, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2215, 2210}

$$\frac{bf^a \log(f) \operatorname{Ei}(bx^n \log(f))}{n} - \frac{x^{-n} f^{a+bx^n}}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^(-1 - n),x]

[Out]  $-(f^{(a + b*x^n)/(n*x^n)}) + (b*f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f])/n$

#### Rule 2215

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^Simplify[m + n]*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simplify[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]
```

#### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1-n} dx &= -\frac{f^{a+bx^n} x^{-n}}{n} + (b \log(f)) \int \frac{f^{a+bx^n}}{x} dx \\ &= -\frac{f^{a+bx^n} x^{-n}}{n} + \frac{bf^a \operatorname{Ei}(bx^n \log(f)) \log(f)}{n} \end{aligned}$$

**Mathematica [A]** time = 0.0035926, size = 20, normalized size = 0.53

$$\frac{bf^a \log(f) \operatorname{Gamma}(-1, -b \log(f) x^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 - n),x]

[Out]  $(b*f^a*\text{Gamma}[-1, -(b*x^n*\text{Log}[f])]*\text{Log}[f])/n$

**Maple [A]** time = 0.09, size = 43, normalized size = 1.1

$$\frac{f^{bx^n} f^a}{nx^n} - \frac{b \ln(f) f^a \text{Ei}(1, -bx^n \ln(f))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x^(-1-n), x)`

[Out]  $-1/n*f^{(b*x^n)}*f^a/(x^n)-1/n*\ln(f)*b*f^a*\text{Ei}(1, -b*x^n*\ln(f))$

**Maxima [A]** time = 1.22318, size = 27, normalized size = 0.71

$$\frac{bf^a\Gamma(-1, -bx^n \log(f)) \log(f)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1-n), x, algorithm="maxima")`

[Out]  $b*f^a*\text{gamma}(-1, -b*x^n*\log(f))*\log(f)/n$

**Fricas [A]** time = 1.57912, size = 101, normalized size = 2.66

$$\frac{bf^ax^n\text{Ei}(bx^n \log(f)) \log(f) - e^{(bx^n \log(f) + a \log(f))}}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1-n), x, algorithm="fricas")`

[Out]  $(b*f^a*x^n*\text{Ei}(b*x^n*\log(f))*\log(f) - e^{(b*x^n*\log(f) + a*\log(f))})/(n*x^n)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x**(-1-n), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b*x^n)*x^(-1-n),x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^n + a)*x^(-n - 1), x)
```

### 3.188 $\int f^{a+bx^n} x^{-1-2n} dx$

**Optimal.** Leaf size=71

$$\frac{b^2 f^a \log^2(f) \operatorname{Ei}(bx^n \log(f))}{2n} - \frac{x^{-2n} f^{a+bx^n}}{2n} - \frac{b \log(f) x^{-n} f^{a+bx^n}}{2n}$$

[Out]  $-f^{a + b*x^n}/(2*n*x^{(2*n)}) - (b*f^{a + b*x^n}*Log[f])/(2*n*x^n) + (b^2*f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f]^2)/(2*n)$

**Rubi [A]** time = 0.0752343, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2215, 2210}

$$\frac{b^2 f^a \log^2(f) \operatorname{Ei}(bx^n \log(f))}{2n} - \frac{x^{-2n} f^{a+bx^n}}{2n} - \frac{b \log(f) x^{-n} f^{a+bx^n}}{2n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*x^n)}*x^{(-1 - 2*n)}, x]$

[Out]  $-f^{a + b*x^n}/(2*n*x^{(2*n)}) - (b*f^{a + b*x^n}*Log[f])/(2*n*x^n) + (b^2*f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f]^2)/(2*n)$

#### Rule 2215

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \text{Dist}[(b*n*Log[F])/(m + 1), \text{Int}[(c + d*x)^{\text{Simplify}[m + n]}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]

#### Rule 2210

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})/((e_.) + (f_.)*(x_.))}, x\_Symbol] :> \text{Simp}[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1-2n} dx &= -\frac{f^{a+bx^n} x^{-2n}}{2n} + \frac{1}{2}(b \log(f)) \int f^{a+bx^n} x^{-1-n} dx \\ &= -\frac{f^{a+bx^n} x^{-2n}}{2n} - \frac{b f^{a+bx^n} x^{-n} \log(f)}{2n} + \frac{1}{2}(b^2 \log^2(f)) \int \frac{f^{a+bx^n}}{x} dx \\ &= -\frac{f^{a+bx^n} x^{-2n}}{2n} - \frac{b f^{a+bx^n} x^{-n} \log(f)}{2n} + \frac{b^2 f^a \operatorname{Ei}(bx^n \log(f)) \log^2(f)}{2n} \end{aligned}$$

**Mathematica [A]** time = 0.0042671, size = 25, normalized size = 0.35

$$-\frac{b^2 f^a \log^2(f) \operatorname{Gamma}(-2, -b \log(f) x^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 - 2\*n),x]

[Out] -((b^2\*f^a\*Gamma[-2, -(b\*x^n\*Log[f])])\*Log[f]^2)/n

**Maple [A]** time = 0.098, size = 70, normalized size = 1.

$$-\frac{f^{bx^n} f^a}{2n(x^n)^2} - \frac{b \ln(f) f^{bx^n} f^a}{2nx^n} - \frac{(\ln(f))^2 b^2 f^a \text{Ei}(1, -bx^n \ln(f))}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1-2\*n),x)

[Out] -1/2/n\*f^(b\*x^n)\*f^a/(x^n)^2-1/2/n\*ln(f)\*b\*f^(b\*x^n)\*f^a/(x^n)-1/2/n\*ln(f)^2\*b^2\*f^a\*Ei(1,-b\*x^n\*ln(f))

**Maxima [A]** time = 1.23282, size = 34, normalized size = 0.48

$$\frac{b^2 f^a \Gamma(-2, -bx^n \log(f)) \log(f)^2}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-2\*n),x, algorithm="maxima")

[Out] -b^2\*f^a\*gamma(-2, -b\*x^n\*log(f))\*log(f)^2/n

**Fricas [A]** time = 1.49937, size = 149, normalized size = 2.1

$$\frac{b^2 f^a x^{2n} \text{Ei}(bx^n \log(f)) \log(f)^2 - (bx^n \log(f) + 1) e^{(bx^n \log(f) + a \log(f))}}{2nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-2\*n),x, algorithm="fricas")

[Out] 1/2\*(b^2\*f^a\*x^(2\*n)\*Ei(b\*x^n\*log(f))\*log(f)^2 - (b\*x^n\*log(f) + 1)\*e^(b\*x^n\*log(f) + a\*log(f)))/(n\*x^(2\*n))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1-2\*n),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{-2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(-2\*n - 1), x)

### 3.189 $\int f^{a+bx^n} x^{-1+\frac{5n}{2}} dx$

**Optimal.** Leaf size=104

$$\frac{3\sqrt{\pi}f^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{4b^{5/2}n \log^2(f)} - \frac{3x^{n/2}f^{a+bx^n}}{2b^2n \log^2(f)} + \frac{x^{3n/2}f^{a+bx^n}}{bn \log(f)}$$

[Out] (3\*f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x^(n/2)\*Sqrt[Log[f]]])/(4\*b^(5/2)\*n\*Log[f]^(5/2)) - (3\*f^(a + b\*x^n)\*x^(n/2))/(2\*b^2\*n\*Log[f]^2) + (f^(a + b\*x^n)\*x^((3\*n)/2))/(b\*n\*Log[f])

**Rubi [A]** time = 0.110245, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2213, 2211, 2204}

$$\frac{3\sqrt{\pi}f^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{4b^{5/2}n \log^2(f)} - \frac{3x^{n/2}f^{a+bx^n}}{2b^2n \log^2(f)} + \frac{x^{3n/2}f^{a+bx^n}}{bn \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^(-1 + (5\*n)/2), x]

[Out] (3\*f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x^(n/2)\*Sqrt[Log[f]]])/(4\*b^(5/2)\*n\*Log[f]^(5/2)) - (3\*f^(a + b\*x^n)\*x^(n/2))/(2\*b^2\*n\*Log[f]^2) + (f^(a + b\*x^n)\*x^((3\*n)/2))/(b\*n\*Log[f])

#### Rule 2213

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^Simplify[m - n]*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]
```

#### Rule 2211

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps



$$\begin{aligned}
\int f^{a+bx^n} x^{-1+\frac{5n}{2}} dx &= \frac{f^{a+bx^n} x^{3n/2}}{bn \log(f)} - \frac{3 \int f^{a+bx^n} x^{-1+\frac{3n}{2}} dx}{2b \log(f)} \\
&= -\frac{3 f^{a+bx^n} x^{n/2}}{2b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^{3n/2}}{bn \log(f)} + \frac{3 \int f^{a+bx^n} x^{\frac{1}{2}(-2+n)} dx}{4b^2 \log^2(f)} \\
&= -\frac{3 f^{a+bx^n} x^{n/2}}{2b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^{3n/2}}{bn \log(f)} + \frac{3 \text{Subst}\left(\int f^{a+bx^2} dx, x, x^{1+\frac{1}{2}(-2+n)}\right)}{2b^2 n \log^2(f)} \\
&= \frac{3 f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{bx^{n/2}} \sqrt{\log(f)}\right)}{4b^{5/2} n \log^2(f)} - \frac{3 f^{a+bx^n} x^{n/2}}{2b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^{3n/2}}{bn \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0105505, size = 39, normalized size = 0.38

$$-\frac{f^a x^{5n/2} \operatorname{Gamma}\left(\frac{5}{2}, -b \log(f) x^n\right)}{n (-b \log(f) x^n)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + (5\*n)/2), x]

[Out] -((f^a\*x^((5\*n)/2)\*Gamma[5/2, -(b\*x^n\*Log[f])])/(n\*(-(b\*x^n\*Log[f]))^(5/2)))

**Maple [A]** time = 0.084, size = 96, normalized size = 0.9

$$\frac{f^a f^{bx^n}}{\ln(f) bn} \left(x^{\frac{n}{2}}\right)^3 - \frac{3 f^a f^{bx^n}}{2 n (\ln(f))^2 b^2} x^{\frac{n}{2}} + \frac{3 f^a \sqrt{\pi}}{4 n (\ln(f))^2 b^2} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x^{\frac{n}{2}}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1+5/2\*n), x)

[Out] 1/n\*f^a\*f^(b\*x^n)\*(x^(1/2\*n))^3/b/ln(f)-3/2/n\*f^a/ln(f)^2/b^2\*x^(1/2\*n)\*f^(b\*x^n)+3/4/n\*f^a/ln(f)^2/b^2\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x^(1/2\*n))

**Maxima [A]** time = 1.20442, size = 45, normalized size = 0.43

$$-\frac{f^a x^{\frac{5}{2}n} \Gamma\left(\frac{5}{2}, -bx^n \log(f)\right)}{(-bx^n \log(f))^{\frac{5}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+5/2\*n), x, algorithm="maxima")

[Out] -f^a\*x^(5/2\*n)\*gamma(5/2, -b\*x^n\*log(f))/((-b\*x^n\*log(f))^(5/2)\*n)

---

**Fricas [A]** time = 1.64528, size = 228, normalized size = 2.19

$$\frac{3\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x^{\frac{1}{2}n}\right)-2\left(2b^2x^{\frac{3}{2}n}\log(f)^2-3bx^{\frac{1}{2}n}\log(f)\right)e^{(bx^n\log(f)+a\log(f))}}{4b^3n\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+5/2\*n),x, algorithm="fricas")

[Out] -1/4\*(3\*sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))\*x^(1/2\*n)) - 2\*(2\*b^2\*x^(3/2\*n)\*log(f)^2 - 3\*b\*x^(1/2\*n)\*log(f))\*e^(b\*x^n\*log(f) + a\*log(f)))/(b^3\*n\*log(f)^3)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1+5/2\*n),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a}x^{\frac{5}{2}n-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+5/2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(5/2\*n - 1), x)

$$3.190 \quad \int f^{a+bx^n} x^{-1+\frac{3n}{2}} dx$$

**Optimal.** Leaf size=74

$$\frac{x^{n/2} f^{a+bx^n}}{bn \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{\log(f)} x^{n/2}\right)}{2b^{3/2} n \log^{\frac{3}{2}}(f)}$$

[Out]  $-(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x^{(n/2)} \sqrt{\log[f]}]) / (2 b^{(3/2)} n \log[f]^{(3/2)}) + (f^{(a + b x^n)} x^{(n/2)}) / (b n \log[f])$

**Rubi [A]** time = 0.065467, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2213, 2211, 2204}

$$\frac{x^{n/2} f^{a+bx^n}}{bn \log(f)} - \frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{\log(f)} x^{n/2}\right)}{2b^{3/2} n \log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b x^n)} x^{(-1 + (3 n) / 2)}, x]$

[Out]  $-(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x^{(n/2)} \sqrt{\log[f]}]) / (2 b^{(3/2)} n \log[f]^{(3/2)}) + (f^{(a + b x^n)} x^{(n/2)}) / (b n \log[f])$

#### Rule 2213

$\text{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_.))^{(n_.)}) * ((c_.) + (d_.) * (x_.))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m - n + 1)} * F^{(a + b*(c + d*x)^n)} / (b*d*n * \log[F]), x] - \text{Dist}[(m - n + 1) / (b*n * \log[F]), \text{Int}[(c + d*x)^{\text{Simplify}[m - n]} * F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]

#### Rule 2211

$\text{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_.))^{(n_.)}) * ((c_.) + (d_.) * (x_.))^{(m_.)}], x\_Symbol] \rightarrow \text{Dist}[1 / (d*(m + 1)), \text{Subst}[\text{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

$\text{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_.))^{(2)}), x\_Symbol] \rightarrow \text{Simp}[(F^a \sqrt{\pi} \operatorname{Erfi}[(c + d*x) \operatorname{Rt}[b * \log[F], 2]]) / (2*d * \operatorname{Rt}[b * \log[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1+\frac{3n}{2}} dx &= \frac{f^{a+bx^n} x^{n/2}}{bn \log(f)} - \frac{\int f^{a+bx^n} x^{\frac{1}{2}(-2+n)} dx}{2b \log(f)} \\ &= \frac{f^{a+bx^n} x^{n/2}}{bn \log(f)} - \frac{\text{Subst}\left(\int f^{a+bx^2} dx, x, x^{1+\frac{1}{2}(-2+n)}\right)}{bn \log(f)} \\ &= -\frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{bx^{n/2}} \sqrt{\log(f)}\right)}{2b^{3/2} n \log^{\frac{3}{2}}(f)} + \frac{f^{a+bx^n} x^{n/2}}{bn \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0087414, size = 39, normalized size = 0.53

$$-\frac{f^a x^{3n/2} \operatorname{Gamma}\left(\frac{3}{2}, -b \log(f) x^n\right)}{n (-b \log(f) x^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + (3\*n)/2), x]

[Out] -((f^a\*x^((3\*n)/2)\*Gamma[3/2, -(b\*x^n\*Log[f])])/(n\*(-(b\*x^n\*Log[f]))^(3/2)))

**Maple [A]** time = 0.043, size = 67, normalized size = 0.9

$$\frac{f^a f^{bx^n}}{\ln(f) bn} x^{\frac{n}{2}} - \frac{f^a \sqrt{\pi}}{2 \ln(f) bn} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x^{\frac{n}{2}}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1+3/2\*n), x)

[Out] 1/n\*f^a/ln(f)/b\*x^(1/2\*n)\*f^(b\*x^n)-1/2/n\*f^a/ln(f)/b\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x^(1/2\*n))

**Maxima [A]** time = 1.28449, size = 45, normalized size = 0.61

$$-\frac{f^a x^{\frac{3}{2}n} \Gamma\left(\frac{3}{2}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{3}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+3/2\*n), x, algorithm="maxima")

[Out] -f^a\*x^(3/2\*n)\*gamma(3/2, -b\*x^n\*log(f))/((-b\*x^n\*log(f))^(3/2)\*n)

**Fricas [A]** time = 1.60275, size = 182, normalized size = 2.46

$$\frac{2bx^{\frac{1}{2}n}e^{(bx^n \log(f) + a \log(f))} \log(f) + \sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x^{\frac{1}{2}n}\right)}{2b^2n \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+3/2\*n),x, algorithm="fricas")

[Out] 1/2\*(2\*b\*x^(1/2\*n)\*e^(b\*x^n\*log(f) + a\*log(f))\*log(f) + sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))\*x^(1/2\*n)))/(b^2\*n\*log(f)^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1+3/2\*n),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{\frac{3}{2}n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+3/2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(3/2\*n - 1), x)

$$3.191 \quad \int f^{a+bx^n} x^{-1+\frac{n}{2}} dx$$

**Optimal.** Leaf size=43

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{b} \sqrt{\log(f)} x^{n/2})}{\sqrt{bn} \sqrt{\log(f)}}$$

[Out] (f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x^(n/2)\*Sqrt[Log[f]]])/(Sqrt[b]\*n\*Sqrt[Log[f]])

**Rubi [A]** time = 0.0363298, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2211, 2204}

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{b} \sqrt{\log(f)} x^{n/2})}{\sqrt{bn} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*x^(-1 + n/2), x]

[Out] (f^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*x^(n/2)\*Sqrt[Log[f]]])/(Sqrt[b]\*n\*Sqrt[Log[f]])

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1+\frac{n}{2}} dx &= \frac{2 \operatorname{Subst}\left(\int f^{a+bx^2} dx, x, x^{n/2}\right)}{n} \\ &= \frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x^{n/2} \sqrt{\log(f)}\right)}{\sqrt{bn} \sqrt{\log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.008519, size = 43, normalized size = 1.

$$\frac{\sqrt{\pi} f^a \operatorname{Erfi}(\sqrt{b} \sqrt{\log(f)} x^{n/2})}{\sqrt{bn} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 + n/2), x]

[Out]  $(f^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} x^{n/2} \sqrt{\log[f]}]) / (\sqrt{b} n \sqrt{\log[f]})$

**Maple [A]** time = 0.05, size = 32, normalized size = 0.7

$$\frac{f^a \sqrt{\pi}}{n} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x^{\frac{n}{2}}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*x^(-1+1/2*n),x)`

[Out] `1/n*f^a*Pi^(1/2)/(-b*ln(f))^(1/2)*erf((-b*ln(f))^(1/2)*x^(1/2*n))`

**Maxima [A]** time = 1.14225, size = 51, normalized size = 1.19

$$\frac{\sqrt{\pi} f^a x^{\frac{1}{2}n} \left( \operatorname{erf}\left(\sqrt{-bx^n \log(f)}\right) - 1 \right)}{\sqrt{-bx^n \log(f)} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1+1/2*n),x, algorithm="maxima")`

[Out] `sqrt(pi)*f^a*x^(1/2*n)*(erf(sqrt(-b*x^n*log(f))) - 1)/(sqrt(-b*x^n*log(f))*n)`

**Fricas [A]** time = 1.52508, size = 109, normalized size = 2.53

$$\frac{\sqrt{\pi} \sqrt{-b \log(f)} f^a \operatorname{erf}\left(\sqrt{-b \log(f)} x x^{\frac{1}{2}n-1}\right)}{bn \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*x^(-1+1/2*n),x, algorithm="fricas")`

[Out] `-sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x*x^(1/2*n - 1))/(b*n*log(f))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*x**(-1+1/2*n),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{\frac{1}{2}n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1+1/2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(1/2\*n - 1), x)



### 3.192 $\int f^{a+bx^n} x^{-1-\frac{n}{2}} dx$

**Optimal.** Leaf size=66

$$\frac{2\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{n} - \frac{2x^{-n/2}f^{a+bx^n}}{n}$$

[Out]  $(-2*f^{(a + b*x^n)})/(n*x^{(n/2)}) + (2*\operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x^{(n/2)}]*\operatorname{Sqrt}[\operatorname{Log}[f]])*\operatorname{Sqrt}[\operatorname{Log}[f]])/n$

**Rubi [A]** time = 0.0652999, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2215, 2211, 2204}

$$\frac{2\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{n} - \frac{2x^{-n/2}f^{a+bx^n}}{n}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x^n)}*x^{(-1 - n/2)}, x]$

[Out]  $(-2*f^{(a + b*x^n)})/(n*x^{(n/2)}) + (2*\operatorname{Sqrt}[b]*f^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x^{(n/2)}]*\operatorname{Sqrt}[\operatorname{Log}[f]])*\operatorname{Sqrt}[\operatorname{Log}[f]])/n$

#### Rule 2215

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)})) * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m+1)}*F^{(a + b*(c + d*x)^n)} / (d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F]) / (m+1), \operatorname{Int}[(c + d*x)^{\operatorname{Simplify}[m+n]}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m+1)/n]] && LtQ[-4, Simplify[(m+1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)})) * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m+1)}], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m+1)]

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)})) * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]) / (2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} x^{-1-\frac{n}{2}} dx &= -\frac{2f^{a+bx^n} x^{-n/2}}{n} + (2b \log(f)) \int f^{a+bx^n} x^{\frac{1}{2}(-2+n)} dx \\ &= -\frac{2f^{a+bx^n} x^{-n/2}}{n} + \frac{(4b \log(f)) \operatorname{Subst}\left(\int f^{a+bx^2} dx, x, x^{1+\frac{1}{2}(-2+n)}\right)}{n} \\ &= -\frac{2f^{a+bx^n} x^{-n/2}}{n} + \frac{2\sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)\sqrt{\log(f)}}{n} \end{aligned}$$

**Mathematica [A]** time = 0.00714, size = 39, normalized size = 0.59

$$\frac{f^a x^{-n/2} \sqrt{-b \log(f) x^n} \text{Gamma}\left(-\frac{1}{2}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 - n/2), x]

[Out] -((f^a\*Gamma[-1/2, -(b\*x^n\*Log[f])])\*Sqrt[-(b\*x^n\*Log[f])])/(n\*x^(n/2))

**Maple [A]** time = 0.061, size = 59, normalized size = 0.9

$$-2 \frac{f^a f^{bx^n}}{nx^{n/2}} + 2 \frac{f^a \ln(f) b \sqrt{\pi} \text{Erf}\left(\sqrt{-b \ln(f) x^{n/2}}\right)}{n \sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1-1/2\*n), x)

[Out] -2/n\*f^a/(x^(1/2\*n))\*f^(b\*x^n)+2/n\*f^a\*ln(f)\*b\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x^(1/2\*n))

**Maxima [A]** time = 1.20819, size = 47, normalized size = 0.71

$$\frac{\sqrt{-bx^n \log(f)} f^a \Gamma\left(-\frac{1}{2}, -bx^n \log(f)\right)}{nx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-1/2\*n), x, algorithm="maxima")

[Out] -sqrt(-b\*x^n\*log(f))\*f^a\*gamma(-1/2, -b\*x^n\*log(f))/(n\*x^(1/2\*n))

**Fricas [A]** time = 1.60989, size = 208, normalized size = 3.15

$$\frac{2 \left( \sqrt{\pi} \sqrt{-b \log(f)} f^a \text{erf}\left(\frac{\sqrt{-b \log(f)}}{xx^{-\frac{1}{2}n-1}}\right) + xx^{-\frac{1}{2}n-1} e^{\left(\frac{ax^2x^{-n-2} \log(f) + b \log(f)}{x^2x^{-n-2}}\right)} \right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-1/2\*n), x, algorithm="fricas")

[Out] -2\*(sqrt(pi)\*sqrt(-b\*log(f))\*f^a\*erf(sqrt(-b\*log(f))/(x\*x^(-1/2\*n - 1)))) + x\*x^(-1/2\*n - 1)\*e^((a\*x^2\*x^(-n - 2)\*log(f) + b\*log(f))/(x^2\*x^(-n - 2))))/n

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1-1/2\*n), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} x^{-\frac{1}{2}n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-1/2\*n), x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(-1/2\*n - 1), x)

### 3.193 $\int f^{a+bx^n} x^{-1-\frac{3n}{2}} dx$

**Optimal.** Leaf size=96

$$\frac{4\sqrt{\pi}b^{3/2}f^a \log^3(f)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{3n} - \frac{2x^{-3n/2}f^{a+bx^n}}{3n} - \frac{4b \log(f)x^{-n/2}f^{a+bx^n}}{3n}$$

[Out]  $(-2*f^{(a + b*x^n)})/(3*n*x^{((3*n)/2)}) - (4*b*f^{(a + b*x^n)}*Log[f])/(3*n*x^{(n/2)}) + (4*b^{(3/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x^{(n/2)}*sqrt[Log[f]]]*Log[f]^{(3/2)})/(3*n)$

**Rubi [A]** time = 0.0955575, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2215, 2211, 2204}

$$\frac{4\sqrt{\pi}b^{3/2}f^a \log^3(f)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{3n} - \frac{2x^{-3n/2}f^{a+bx^n}}{3n} - \frac{4b \log(f)x^{-n/2}f^{a+bx^n}}{3n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*x^n)}*x^{(-1 - (3*n)/2)}, x]$

[Out]  $(-2*f^{(a + b*x^n)})/(3*n*x^{((3*n)/2)}) - (4*b*f^{(a + b*x^n)}*Log[f])/(3*n*x^{(n/2)}) + (4*b^{(3/2)}*f^a*sqrt[Pi]*Erfi[Sqrt[b]*x^{(n/2)}*sqrt[Log[f]]]*Log[f]^{(3/2)})/(3*n)$

#### Rule 2215

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^Simplify[m + n]*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simplify[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]
```

#### Rule 2211

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int f^{a+bx^n} x^{-1-\frac{3n}{2}} dx &= -\frac{2f^{a+bx^n} x^{-3n/2}}{3n} + \frac{1}{3}(2b \log(f)) \int f^{a+bx^n} x^{-1-\frac{n}{2}} dx \\
&= -\frac{2f^{a+bx^n} x^{-3n/2}}{3n} - \frac{4bf^{a+bx^n} x^{-n/2} \log(f)}{3n} + \frac{1}{3} (4b^2 \log^2(f)) \int f^{a+bx^n} x^{\frac{1}{2}(-2+n)} dx \\
&= -\frac{2f^{a+bx^n} x^{-3n/2}}{3n} - \frac{4bf^{a+bx^n} x^{-n/2} \log(f)}{3n} + \frac{(8b^2 \log^2(f)) \text{Subst}\left(\int f^{a+bx^2} dx, x, x^{1+\frac{1}{2}(-2+n)}\right)}{3n} \\
&= -\frac{2f^{a+bx^n} x^{-3n/2}}{3n} - \frac{4bf^{a+bx^n} x^{-n/2} \log(f)}{3n} + \frac{4b^{3/2} f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x^{n/2} \sqrt{\log(f)}\right) \log^{\frac{3}{2}}(f)}{3n}
\end{aligned}$$

**Mathematica [A]** time = 0.007734, size = 39, normalized size = 0.41

$$-\frac{f^a x^{-3n/2} (-b \log(f) x^n)^{3/2} \operatorname{Gamma}\left(-\frac{3}{2}, -b \log(f) x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*x^(-1 - (3\*n)/2), x]

[Out] -((f^a\*Gamma[-3/2, -(b\*x^n\*Log[f])])\*(-(b\*x^n\*Log[f]))^(3/2))/(n\*x^((3\*n)/2))

**Maple [A]** time = 0.054, size = 88, normalized size = 0.9

$$-\frac{2 f^a f^{bx^n}}{3 n} \left(x^{\frac{n}{2}}\right)^{-3} - \frac{4 f^a \ln(f) b f^{bx^n}}{3 n} \left(x^{\frac{n}{2}}\right)^{-1} + \frac{4 f^a (\ln(f))^2 b^2 \sqrt{\pi}}{3 n} \operatorname{Erf}\left(\sqrt{-b \ln(f)} x^{\frac{n}{2}}\right) \frac{1}{\sqrt{-b \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*x^n)\*x^(-1-3/2\*n), x)

[Out] -2/3/n\*f^a/(x^(1/2\*n))^3\*f^(b\*x^n)-4/3/n\*f^a\*ln(f)\*b/(x^(1/2\*n))\*f^(b\*x^n)+4/3/n\*f^a\*ln(f)^2\*b^2\*Pi^(1/2)/(-b\*ln(f))^(1/2)\*erf((-b\*ln(f))^(1/2)\*x^(1/2\*n))

**Maxima [A]** time = 1.33736, size = 47, normalized size = 0.49

$$-\frac{(-bx^n \log(f))^{\frac{3}{2}} f^a \Gamma\left(-\frac{3}{2}, -bx^n \log(f)\right)}{n x^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-3/2\*n), x, algorithm="maxima")

[Out] -(-b\*x^n\*log(f))^(3/2)\*f^a\*gamma(-3/2, -b\*x^n\*log(f))/(n\*x^(3/2\*n))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{bx^n+a}x^{-\frac{3}{2}n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-3/2\*n),x, algorithm="fricas")

[Out] integral(f^(b\*x^n + a)\*x^(-3/2\*n - 1), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*x\*\*n)\*x\*\*(-1-3/2\*n),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a}x^{-\frac{3}{2}n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*x^n)\*x^(-1-3/2\*n),x, algorithm="giac")

[Out] integrate(f^(b\*x^n + a)\*x^(-3/2\*n - 1), x)

### 3.194 $\int e^{-0.1x} x dx$

**Optimal.** Leaf size=16

$$-10.e^{-0.1x}x - 100.e^{-0.1x}$$

[Out]  $-100./E^{(0.1*x)} - (10.*x)/E^{(0.1*x)}$

**Rubi [A]** time = 0.0075144, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2176, 2194}

$$-10.e^{-0.1x}x - 100.e^{-0.1x}$$

Antiderivative was successfully verified.

[In] Int[x/E^(0.1\*x), x]

[Out]  $-100./E^{(0.1*x)} - (10.*x)/E^{(0.1*x)}$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma == True
```

#### Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned} \int e^{-0.1x} x dx &= -10.e^{-0.1x}x + 10. \int e^{-0.1x} dx \\ &= -100.e^{-0.1x} - 10.e^{-0.1x}x \end{aligned}$$

**Mathematica [A]** time = 0.0049634, size = 11, normalized size = 0.69

$$e^{-0.1x}(-10.x - 100.)$$

Antiderivative was successfully verified.

[In] Integrate[x/E^(0.1\*x), x]

[Out]  $(-99.99999999999999 - 10.*x)/E^{(0.1*x)}$

**Maple [A]** time = 0.003, size = 10, normalized size = 0.6

$$-10.0 (x + 10.0) e^{-0.1000000000x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-.1*x)*x,x)`

[Out] `-10.*(x+10.)*exp(-.1000000000*x)`

**Maxima [A]** time = 1.04478, size = 12, normalized size = 0.75

$$-10(x + 10)e^{\left(-\frac{1}{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-.1*x)*x,x, algorithm="maxima")`

[Out] `-10*(x + 10)*e^(-1/10*x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-.1*x)*x,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0.090221, size = 10, normalized size = 0.62

$$1.0(-10.0x - 100.0)e^{-0.1x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-.1*x)*x,x)`

[Out] `1.0*(-10.0*x - 100.0)*exp(-0.1*x)`

**Giac [A]** time = 1.21675, size = 14, normalized size = 0.88

$$(-10.0x - 100.0)e^{(-0.1x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-.1*x)*x,x, algorithm="giac")`

[Out] `(-10.0*x - 100.0)*e^(-0.1*x)`



### 3.195 $\int f^{c(a+bx)^2} x^3 dx$

**Optimal.** Leaf size=203

$$-\frac{\sqrt{\pi}a^3 \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^4\sqrt{c}\sqrt{\log(f)}} + \frac{3a^2 f^{c(a+bx)^2}}{2b^4c \log(f)} + \frac{3\sqrt{\pi}a \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^4c^{3/2} \log^{\frac{3}{2}}(f)} - \frac{f^{c(a+bx)^2}}{2b^4c^2 \log^2(f)} + \frac{(a+bx)^2 f^{c(a+bx)^2}}{2b^4c \log(f)}$$

[Out]  $-f^{(c*(a + b*x)^2)/(2*b^4*c^2*\operatorname{Log}[f]^2)} + (3*a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c]*(a + b*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]])/(4*b^4*c^{(3/2)*\operatorname{Log}[f]^{(3/2)}}) + (3*a^2*f^{(c*(a + b*x)^2)})/(2*b^4*c*\operatorname{Log}[f]) - (3*a*f^{(c*(a + b*x)^2)*(a + b*x)})/(2*b^4*c*\operatorname{Log}[f]) + (f^{(c*(a + b*x)^2)*(a + b*x)^2})/(2*b^4*c*\operatorname{Log}[f]) - (a^3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c]*(a + b*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]])/(2*b^4*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Log}[f]])$

**Rubi [A]** time = 0.22895, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2226, 2204, 2209, 2212}

$$-\frac{\sqrt{\pi}a^3 \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^4\sqrt{c}\sqrt{\log(f)}} + \frac{3a^2 f^{c(a+bx)^2}}{2b^4c \log(f)} + \frac{3\sqrt{\pi}a \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^4c^{3/2} \log^{\frac{3}{2}}(f)} - \frac{f^{c(a+bx)^2}}{2b^4c^2 \log^2(f)} + \frac{(a+bx)^2 f^{c(a+bx)^2}}{2b^4c \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^2)\*x^3,x]

[Out]  $-f^{(c*(a + b*x)^2)/(2*b^4*c^2*\operatorname{Log}[f]^2)} + (3*a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c]*(a + b*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]])/(4*b^4*c^{(3/2)*\operatorname{Log}[f]^{(3/2)}}) + (3*a^2*f^{(c*(a + b*x)^2)})/(2*b^4*c*\operatorname{Log}[f]) - (3*a*f^{(c*(a + b*x)^2)*(a + b*x)})/(2*b^4*c*\operatorname{Log}[f]) + (f^{(c*(a + b*x)^2)*(a + b*x)^2})/(2*b^4*c*\operatorname{Log}[f]) - (a^3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c]*(a + b*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]])/(2*b^4*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Log}[f]])$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1) \* F^(a + b\*(c + d\*x)^n)) / (b\*d\*n \* Log[F]), x] - Dist[(m - n + 1) / (b\*n \* Log[F]), Int[(c + d\*x)^(m - n) \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1)) /

n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

### Rubi steps

$$\begin{aligned} \int f^{c(a+bx)^2} x^3 dx &= \int \left( -\frac{a^3 f^{c(a+bx)^2}}{b^3} + \frac{3a^2 f^{c(a+bx)^2} (a+bx)}{b^3} - \frac{3a f^{c(a+bx)^2} (a+bx)^2}{b^3} + \frac{f^{c(a+bx)^2} (a+bx)^3}{b^3} \right) dx \\ &= \frac{\int f^{c(a+bx)^2} (a+bx)^3 dx}{b^3} - \frac{(3a) \int f^{c(a+bx)^2} (a+bx)^2 dx}{b^3} + \frac{(3a^2) \int f^{c(a+bx)^2} (a+bx) dx}{b^3} - \frac{a^3 \int f^{c(a+bx)^2} dx}{b^3} \\ &= \frac{3a^2 f^{c(a+bx)^2}}{2b^4 c \log(f)} - \frac{3a f^{c(a+bx)^2} (a+bx)}{2b^4 c \log(f)} + \frac{f^{c(a+bx)^2} (a+bx)^2}{2b^4 c \log(f)} - \frac{a^3 \sqrt{\pi} \operatorname{erfi}(\sqrt{c}(a+bx)\sqrt{\log(f)})}{2b^4 \sqrt{c} \sqrt{\log(f)}} - \frac{\int f^{c(a+bx)^2} dx}{b^3 c \log(f)} \\ &= -\frac{f^{c(a+bx)^2}}{2b^4 c^2 \log^2(f)} + \frac{3a \sqrt{\pi} \operatorname{erfi}(\sqrt{c}(a+bx)\sqrt{\log(f)})}{4b^4 c^{3/2} \log^3(f)} + \frac{3a^2 f^{c(a+bx)^2}}{2b^4 c \log(f)} - \frac{3a f^{c(a+bx)^2} (a+bx)}{2b^4 c \log(f)} + \frac{f^{c(a+bx)^2} (a+bx)^3}{2b^4 c \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.106217, size = 96, normalized size = 0.47

$$\frac{2f^{c(a+bx)^2} (c \log(f) (a^2 - abx + b^2 x^2) - 1) + \sqrt{\pi} a \sqrt{c} \sqrt{\log(f)} (3 - 2a^2 c \log(f)) \operatorname{Erfi}(\sqrt{c} \sqrt{\log(f)} (a + bx))}{4b^4 c^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^2)\*x^3,x]

[Out] (a\*Sqrt[c]\*Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]]\*Sqrt[Log[f]]\*(3 - 2\*a^2\*c\*Log[f]) + 2\*f^(c\*(a + b\*x)^2)\*(-1 + c\*(a^2 - a\*b\*x + b^2\*x^2)\*Log[f]))/(4\*b^4\*c^2\*Log[f]^2)

**Maple [A]** time = 0.073, size = 249, normalized size = 1.2

$$\frac{x^2 f^{cx^2 b^2} f^{2abcx} f^{a^2 c}}{2b^2 c \ln(f)} - \frac{ax f^{cx^2 b^2} f^{2abcx} f^{a^2 c}}{2b^3 c \ln(f)} + \frac{a^2 f^{cx^2 b^2} f^{2abcx} f^{a^2 c}}{2b^4 c \ln(f)} + \frac{a^3 \sqrt{\pi}}{2b^4} \operatorname{Erf} \left( -b \sqrt{-c \ln(f)} x + ac \ln(f) \frac{1}{\sqrt{-c \ln(f)}} \right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)\*x^3,x)

[Out] 1/2/b^2/c/ln(f)\*x^2\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)-1/2\*a/b^3/c/ln(f)\*x\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)+1/2\*a^2/b^4/c/ln(f)\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)+1/2\*a^3/b^4\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf(-b\*(-c\*ln(f))^(1/2)\*x+a\*c\*ln(f)/(-c\*ln(f))^(1/2))-3/4\*a/b^4/c/ln(f)\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf(-b\*(-c\*ln(f))^(1/2)\*x+a\*c\*ln(f)/(-c\*ln(f))^(1/2))-1/2/b^4/c^2/ln(f)^2\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)

**Maxima [A]** time = 1.32804, size = 370, normalized size = 1.82

$$\frac{\sqrt{\pi}(b^2cx+abc)a^3b^3c^3\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}\right)-1\right)\log(f)^4}{(b^2c\log(f))^{\frac{7}{2}}\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}}-\frac{3a^2b^4c^3f\frac{(b^2cx+abc)^2}{b^2c}\log(f)^3}{(b^2c\log(f))^{\frac{7}{2}}}-\frac{3(b^2cx+abc)^3abc\Gamma\left(\frac{3}{2},-\frac{(b^2cx+abc)^2\log(f)}{b^2c}\right)\log(f)^4}{(b^2c\log(f))^{\frac{7}{2}}\left(-\frac{(b^2cx+abc)^2\log(f)}{b^2c}\right)^{\frac{3}{2}}}+ \dots$$


---


$$2\sqrt{b^2c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^3,x, algorithm="maxima")

[Out]  $-1/2*(\sqrt{\pi}*(b^2*c*x + a*b*c)*a^3*b^3*c^3*(\operatorname{erf}(\sqrt{-(b^2*c*x + a*b*c)^2*\log(f)/(b^2*c)}) - 1)*\log(f)^4/((b^2*c*\log(f))^{(7/2)}*\sqrt{-(b^2*c*x + a*b*c)^2*\log(f)/(b^2*c)}) - 3*a^2*b^4*c^3*f^((b^2*c*x + a*b*c)^2/(b^2*c))*\log(f)^3/(b^2*c*\log(f))^{(7/2)} - 3*(b^2*c*x + a*b*c)^3*a*b*c*\gamma(3/2, -(b^2*c*x + a*b*c)^2*\log(f)/(b^2*c))*\log(f)^4/((b^2*c*\log(f))^{(7/2)}*(-(b^2*c*x + a*b*c)^2*\log(f)/(b^2*c))^{(3/2)}) + b^4*c^2*\gamma(2, -(b^2*c*x + a*b*c)^2*\log(f)/(b^2*c))*\log(f)^2/(b^2*c*\log(f))^{(7/2)})/\sqrt{b^2*c*\log(f)}$

**Fricas [A]** time = 1.60004, size = 270, normalized size = 1.33

$$\frac{\sqrt{\pi}(2a^3c\log(f) - 3a)\sqrt{-b^2c\log(f)}\operatorname{erf}\left(\frac{\sqrt{-b^2c\log(f)}(bx+a)}{b}\right) + 2((b^3cx^2 - ab^2cx + a^2bc)\log(f) - b)f^{b^2cx^2+2abcx+a^2c}}{4b^5c^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^3,x, algorithm="fricas")

[Out]  $1/4*(\sqrt{\pi}*(2*a^3*c*\log(f) - 3*a)*\sqrt{-b^2*c*\log(f)}*\operatorname{erf}(\sqrt{-b^2*c*\log(f)}*(b*x + a)/b) + 2*((b^3*c*x^2 - a*b^2*c*x + a^2*b*c)*\log(f) - b)*f^{(b^2*c*x^2 + 2*a*b*c*x + a^2*c)})/(b^5*c^2*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*2)\*x\*\*3,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*2)\*x\*\*3, x)

**Giac [A]** time = 1.24564, size = 184, normalized size = 0.91

$$\frac{\sqrt{\pi}(2a^3c\log(f)-3a)\operatorname{erf}\left(-\sqrt{-c\log(f)}b\left(x+\frac{a}{b}\right)\right)}{\sqrt{-c\log(f)}bc\log(f)} + \frac{2\left(b^2c\left(x+\frac{a}{b}\right)^2\log(f)-3abc\left(x+\frac{a}{b}\right)\log(f)+3a^2c\log(f)-1\right)e^{(b^2cx^2\log(f)+2abcx\log(f)+a^2c\log(f))}}{bc^2\log(f)^2}$$

$4b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*(2*a^3*c*log(f) - 3*a)*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqrt(-c*log(f))*b*c*log(f)) + 2*(b^2*c*(x + a/b)^2*log(f) - 3*a*b*c*(x + a/b)*log(f) + 3*a^2*c*log(f) - 1)*e^(b^2*c*x^2*log(f) + 2*a*b*c*x*log(f) + a^2*c*log(f))/(b*c^2*log(f)^2))/b^3
```

### 3.196 $\int f^{c(a+bx)^2} x^2 dx$

**Optimal.** Leaf size=140

$$\frac{\sqrt{\pi}a^2 \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^3\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^3c^{3/2}\log^{\frac{3}{2}}(f)} + \frac{(a+bx)f^{c(a+bx)^2}}{2b^3c\log(f)} - \frac{af^{c(a+bx)^2}}{b^3c\log(f)}$$

```
[Out] -(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(4*b^3*c^(3/2)*Log[f]^(3/2)) - (a*f^(c*(a + b*x)^2))/(b^3*c*Log[f]) + (f^(c*(a + b*x)^2)*(a + b*x))/(2*b^3*c*Log[f]) + (a^2*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(2*b^3*Sqrt[c]*Sqrt[Log[f]])
```

**Rubi [A]** time = 0.129426, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2226, 2204, 2209, 2212}

$$\frac{\sqrt{\pi}a^2 \operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^3\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^3c^{3/2}\log^{\frac{3}{2}}(f)} + \frac{(a+bx)f^{c(a+bx)^2}}{2b^3c\log(f)} - \frac{af^{c(a+bx)^2}}{b^3c\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(c*(a + b*x)^2)*x^2,x]
```

```
[Out] -(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(4*b^3*c^(3/2)*Log[f]^(3/2)) - (a*f^(c*(a + b*x)^2))/(b^3*c*Log[f]) + (f^(c*(a + b*x)^2)*(a + b*x))/(2*b^3*c*Log[f]) + (a^2*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(2*b^3*Sqrt[c]*Sqrt[Log[f]])
```

#### Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] := Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n)) / (b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1) * F^(a + b*(c + d*x)^n)) / (b*d*n * Log[F]), x] - Dist[(m - n + 1) / (b*n * Log[F]), Int[(c + d*x)^(m - n) * F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

Rubi steps

$$\begin{aligned}
\int f^{c(a+bx)^2} x^2 dx &= \int \left( \frac{a^2 f^{c(a+bx)^2}}{b^2} - \frac{2a f^{c(a+bx)^2} (a+bx)}{b^2} + \frac{f^{c(a+bx)^2} (a+bx)^2}{b^2} \right) dx \\
&= \frac{\int f^{c(a+bx)^2} (a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{c(a+bx)^2} (a+bx) dx}{b^2} + \frac{a^2 \int f^{c(a+bx)^2} dx}{b^2} \\
&= -\frac{a f^{c(a+bx)^2}}{b^3 c \log(f)} + \frac{f^{c(a+bx)^2} (a+bx)}{2b^3 c \log(f)} + \frac{a^2 \sqrt{\pi} \operatorname{erfi}(\sqrt{c}(a+bx) \sqrt{\log(f)})}{2b^3 \sqrt{c} \sqrt{\log(f)}} - \frac{\int f^{c(a+bx)^2} dx}{2b^2 c \log(f)} \\
&= -\frac{\sqrt{\pi} \operatorname{erfi}(\sqrt{c}(a+bx) \sqrt{\log(f)})}{4b^3 c^{3/2} \log^2(f)} - \frac{a f^{c(a+bx)^2}}{b^3 c \log(f)} + \frac{f^{c(a+bx)^2} (a+bx)}{2b^3 c \log(f)} + \frac{a^2 \sqrt{\pi} \operatorname{erfi}(\sqrt{c}(a+bx) \sqrt{\log(f)})}{2b^3 \sqrt{c} \sqrt{\log(f)}}
\end{aligned}$$

**Mathematica [A]** time = 0.066396, size = 83, normalized size = 0.59

$$\frac{\sqrt{\pi} (2a^2 c \log(f) - 1) \operatorname{Erfi}(\sqrt{c} \sqrt{\log(f)} (a+bx)) - 2\sqrt{c} \sqrt{\log(f)} (a-bx) f^{c(a+bx)^2}}{4b^3 c^{3/2} \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^2)\*x^2,x]

[Out] (-2\*Sqrt[c]\*f^(c\*(a + b\*x)^2)\*(a - b\*x)\*Sqrt[Log[f]] + Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]]\*(-1 + 2\*a^2\*c\*Log[f]))/(4\*b^3\*c^(3/2)\*Log[f]^(3/2))

**Maple [A]** time = 0.033, size = 168, normalized size = 1.2

$$\frac{x f^{c x^2 b^2} f^{2 a b c x} f^{a^2 c}}{2 b^2 c \ln(f)} - \frac{a f^{c x^2 b^2} f^{2 a b c x} f^{a^2 c}}{2 b^3 c \ln(f)} - \frac{a^2 \sqrt{\pi}}{2 b^3} \operatorname{Erf}\left(-b \sqrt{-c \ln(f)} x + a c \ln(f) \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}} + \frac{\sqrt{\pi}}{4 b^3 c \ln(f)} \operatorname{Erf}\left(\frac{1}{\sqrt{-c \ln(f)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)\*x^2,x)

[Out] 1/2/b^2/c/ln(f)\*x\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)-1/2\*a/b^3/c/ln(f)\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)-1/2\*a^2/b^3\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf(-b\*(-c\*ln(f))^(1/2)\*x+a\*c\*ln(f)/(-c\*ln(f))^(1/2))+1/4/b^3/c/ln(f)\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf(-b\*(-c\*ln(f))^(1/2)\*x+a\*c\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [A]** time = 1.40084, size = 302, normalized size = 2.16

$$\frac{\sqrt{\pi} (b^2 c x + a b c) a^2 b^2 c^2 \left( \operatorname{erf}\left(\sqrt{-\frac{(b^2 c x + a b c)^2 \log(f)}{b^2 c}}\right) - 1 \right) \log(f)^3}{(b^2 c \log(f))^{\frac{5}{2}} \sqrt{-\frac{(b^2 c x + a b c)^2 \log(f)}{b^2 c}}} - \frac{2 a b^3 c^2 f \frac{(b^2 c x + a b c)^2}{b^2 c} \log(f)^2}{(b^2 c \log(f))^{\frac{5}{2}}} - \frac{(b^2 c x + a b c)^3 \Gamma\left(\frac{3}{2}, -\frac{(b^2 c x + a b c)^2 \log(f)}{b^2 c}\right) \log(f)^3}{(b^2 c \log(f))^{\frac{5}{2}} \left(-\frac{(b^2 c x + a b c)^2 \log(f)}{b^2 c}\right)^{\frac{3}{2}}}$$

$$2 \sqrt{b^2 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}(\sqrt{\pi})(b^2cx + a*bc)*a^2b^2c^2(\operatorname{erf}(\sqrt{-(b^2cx + a*bc)^2 \log(f)/(b^2c)}) - 1) \log(f)^3 / ((b^2c \log(f))^{5/2}) \sqrt{-(b^2cx + a*bc)^2 \log(f)/(b^2c)} - 2*a*b^3*c^2*f^{((b^2cx + a*bc)^2/(b^2c))} \log(f)^2 / (b^2c \log(f))^{5/2} - (b^2cx + a*bc)^3 \operatorname{gamma}(3/2, -(b^2cx + a*bc)^2 \log(f)/(b^2c)) \log(f)^3 / ((b^2c \log(f))^{5/2}) * (-(b^2cx + a*bc)^2 \log(f)/(b^2c))^{3/2}) / \sqrt{b^2c \log(f)}$

**Fricas [A]** time = 1.52419, size = 239, normalized size = 1.71

$$\frac{\sqrt{\pi}(2a^2c \log(f) - 1) \sqrt{-b^2c \log(f)} \operatorname{erf}\left(\frac{\sqrt{-b^2c \log(f)}(bx+a)}{b}\right) - 2(b^2cx - abc) f^{b^2cx^2 + 2abcx + a^2c} \log(f)}{4b^4c^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^2,x, algorithm="fricas")

[Out]  $-1/4(\sqrt{\pi})(2*a^2*c*\log(f) - 1)*\sqrt{-b^2*c*\log(f)}*\operatorname{erf}(\sqrt{-b^2*c*\log(f)}*(b*x + a)/b) - 2*(b^2*c*x - a*b*c)*f^{(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*\log(f)} / (b^4*c^2*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*2)\*x\*\*2,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*2)\*x\*\*2, x)

**Giac [A]** time = 1.24154, size = 144, normalized size = 1.03

$$\frac{\sqrt{\pi}(2a^2c \log(f) - 1) \operatorname{erf}\left(-\sqrt{-c \log(f)}b\left(x + \frac{a}{b}\right)\right)}{\sqrt{-c \log(f)}bc \log(f)} - \frac{2\left(b\left(x + \frac{a}{b}\right) - 2a\right)e^{(b^2cx^2 \log(f) + 2abcx \log(f) + a^2c \log(f))}}{bc \log(f)}$$

$$\frac{1}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^2,x, algorithm="giac")

[Out]  $-1/4(\sqrt{\pi})(2*a^2*c*\log(f) - 1)*\operatorname{erf}(-\sqrt{-c*\log(f)}*b*(x + a/b)) / (\sqrt{-c*\log(f)}*b*c*\log(f)) - 2*(b*(x + a/b) - 2*a)*e^{(b^2*c*x^2*\log(f) + 2*a*b*c*x*\log(f) + a^2*c*\log(f))} / (b*c*\log(f)) / b^2$

### 3.197 $\int f^{c(a+bx)^2} x dx$

**Optimal.** Leaf size=68

$$\frac{f^{c(a+bx)^2}}{2b^2c \log(f)} - \frac{\sqrt{\pi}a \operatorname{Erfi}(\sqrt{c}\sqrt{\log(f)}(a+bx))}{2b^2\sqrt{c}\sqrt{\log(f)}}$$

[Out]  $f^{c(a+bx)^2}/(2b^2c \log(f)) - (a\sqrt{\pi}\operatorname{Erfi}[\sqrt{c}(a+bx)\sqrt{\log(f)}])/(2b^2\sqrt{c}\sqrt{\log(f)})$

**Rubi [A]** time = 0.0579059, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2226, 2204, 2209}

$$\frac{f^{c(a+bx)^2}}{2b^2c \log(f)} - \frac{\sqrt{\pi}a \operatorname{Erfi}(\sqrt{c}\sqrt{\log(f)}(a+bx))}{2b^2\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^2)\*x,x]

[Out]  $f^{c(a+bx)^2}/(2b^2c \log(f)) - (a\sqrt{\pi}\operatorname{Erfi}[\sqrt{c}(a+bx)\sqrt{\log(f)}])/(2b^2\sqrt{c}\sqrt{\log(f)})$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{c(a+bx)^2} x dx &= \int \left( -\frac{a f^{c(a+bx)^2}}{b} + \frac{f^{c(a+bx)^2} (a+bx)}{b} \right) dx \\ &= \frac{\int f^{c(a+bx)^2} (a+bx) dx}{b} - \frac{a \int f^{c(a+bx)^2} dx}{b} \\ &= \frac{f^{c(a+bx)^2}}{2b^2c \log(f)} - \frac{a\sqrt{\pi}\operatorname{erfi}(\sqrt{c}(a+bx)\sqrt{\log(f)})}{2b^2\sqrt{c}\sqrt{\log(f)}} \end{aligned}$$



**Mathematica [A]** time = 0.0278973, size = 63, normalized size = 0.93

$$\frac{f^{c(a+bx)^2} - \sqrt{\pi}a\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^2c\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^2)\*x,x]

[Out] (f^(c\*(a + b\*x)^2) - a\*Sqrt[c]\*Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]]\*Sqrt[Log[f]])/(2\*b^2\*c\*Log[f])

**Maple [A]** time = 0.027, size = 80, normalized size = 1.2

$$\frac{fcx^2b^2f^2abcxfa^2c}{2b^2c\ln(f)} + \frac{a\sqrt{\pi}}{2b^2}\operatorname{Erf}\left(-b\sqrt{-c\ln(f)}x + ac\ln(f)\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)\*x,x)

[Out] 1/2/b^2/c/ln(f)\*f^(c\*x^2\*b^2)\*f^(2\*a\*b\*c\*x)\*f^(a^2\*c)+1/2\*a/b^2\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf(-b\*(-c\*ln(f))^(1/2)\*x+a\*c\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [B]** time = 1.34412, size = 182, normalized size = 2.68

$$\frac{\sqrt{\pi}(b^2cx+abc)abc\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}\right)-1\right)\log(f)^2}{(b^2c\log(f))^{\frac{3}{2}}\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}}-\frac{(b^2cx+abc)^2}{b^2cf\frac{\log(f)}{b^2c}}}{2\sqrt{b^2c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x,x, algorithm="maxima")

[Out] -1/2\*(sqrt(pi)\*(b^2\*c\*x + a\*b\*c)\*a\*b\*c\*(erf(sqrt(-(b^2\*c\*x + a\*b\*c)^2\*log(f)/(b^2\*c))) - 1)\*log(f)^2/((b^2\*c\*log(f))^(3/2)\*sqrt(-(b^2\*c\*x + a\*b\*c)^2\*log(f)/(b^2\*c))) - b^2\*c\*f^((b^2\*c\*x + a\*b\*c)^2/(b^2\*c))\*log(f)/(b^2\*c\*log(f))^(3/2))/sqrt(b^2\*c\*log(f))

**Fricas [A]** time = 1.51366, size = 173, normalized size = 2.54

$$\frac{\sqrt{\pi}\sqrt{-b^2c\log(f)}a\operatorname{erf}\left(\frac{\sqrt{-b^2c\log(f)}(bx+a)}{b}\right) + bf^{b^2cx^2+2abcx+a^2c}}{2b^3c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(-b^2*c*log(f))*a*erf(sqrt(-b^2*c*log(f))*(b*x + a)/b) +
b*f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c))/(b^3*c*log(f))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*(b*x+a)**2)*x,x)
```

```
[Out] Integral(f**(c*(a + b*x)**2)*x, x)
```

**Giac [A]** time = 1.22716, size = 104, normalized size = 1.53

$$\frac{\frac{\sqrt{\pi}a \operatorname{erf}\left(-\sqrt{-c \log(f)}b\left(x+\frac{a}{b}\right)\right)}{\sqrt{-c \log(f)}b} + \frac{e^{(b^2cx^2 \log(f)+2abcx \log(f)+a^2c \log(f))}}{bc \log(f)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*a*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqrt(-c*log(f))*b) + e^(
b^2*c*x^2*log(f) + 2*a*b*c*x*log(f) + a^2*c*log(f))/(b*c*log(f)))/b
```

$$3.198 \quad \int f^{c(a+bx)^2} dx$$

**Optimal.** Leaf size=41

$$\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]])/(2\*b\*Sqrt[c]\*Sqrt[Log[f]])

**Rubi [A]** time = 0.0078993, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2204}

$$\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^2), x]

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]])/(2\*b\*Sqrt[c]\*Sqrt[Log[f]])

**Rule 2204**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)) ^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

**Rubi steps**

$$\int f^{c(a+bx)^2} dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

**Mathematica [A]** time = 0.0038041, size = 41, normalized size = 1.

$$\frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^2), x]

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)\*Sqrt[Log[f]]])/(2\*b\*Sqrt[c]\*Sqrt[Log[f]])

**Maple [A]** time = 0.026, size = 41, normalized size = 1.

$$-\frac{\sqrt{\pi}}{2b}\operatorname{Erf}\left(-b\sqrt{-c\ln(f)}x + ac\ln(f)\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*(b*x+a)^2),x)`

[Out]  $-1/2*\text{Pi}^{(1/2)}/b/(-c*\ln(f))^{(1/2)}*\text{erf}(-b*(-c*\ln(f))^{(1/2)}*x+a*c*\ln(f)/(-c*\ln(f))^{(1/2)})$

**Maxima [A]** time = 1.14094, size = 54, normalized size = 1.32

$$\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-c \log(f)} bx - \frac{ac \log(f)}{\sqrt{-c \log(f)}}\right)}{2 \sqrt{-c \log(f)} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^2),x, algorithm="maxima")`

[Out]  $1/2*\text{sqrt}(\text{pi})*\text{erf}(\text{sqrt}(-c*\log(f))*b*x - a*c*\log(f)/\text{sqrt}(-c*\log(f)))/(\text{sqrt}(-c*\log(f))*b)$

**Fricas [A]** time = 1.55447, size = 117, normalized size = 2.85

$$\frac{\sqrt{\pi} \sqrt{-b^2 c \log(f)} \operatorname{erf}\left(\frac{\sqrt{-b^2 c \log(f)}(bx+a)}{b}\right)}{2 b^2 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^2),x, algorithm="fricas")`

[Out]  $-1/2*\text{sqrt}(\text{pi})*\text{sqrt}(-b^2*c*\log(f))*\text{erf}(\text{sqrt}(-b^2*c*\log(f))*(b*x + a)/b)/(-b^2*c*\log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*(b*x+a)**2),x)`

[Out] `Integral(f**(c*(a + b*x)**2), x)`

**Giac [A]** time = 1.20551, size = 45, normalized size = 1.1

$$-\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log(f)} b \left(x + \frac{a}{b}\right)\right)}{2 \sqrt{-c \log(f)} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqrt(-c*log(f))*b)
```

$$3.199 \quad \int \frac{f^{c(a+bx)^2}}{x} dx$$

**Optimal.** Leaf size=17

$$\text{Unintegrable}\left(\frac{f^{c(a+bx)^2}}{x}, x\right)$$

[Out] Unintegrable[f^(c\*(a + b\*x)^2)/x, x]

**Rubi [A]** time = 0.0257867, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^2)/x, x]

[Out] Defer[Int][f^(c\*(a + b\*x)^2)/x, x]

Rubi steps

$$\int \frac{f^{c(a+bx)^2}}{x} dx = \int \frac{f^{c(a+bx)^2}}{x} dx$$

**Mathematica [A]** time = 0.114495, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^2)/x, x]

[Out] Integrate[f^(c\*(a + b\*x)^2)/x, x]

**Maple [A]** time = 0.016, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)/x, x)

[Out] int(f^(c\*(b\*x+a)^2)/x, x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)/x,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^2\*c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^2cx^2+2abcx+a^2c}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)/x,x, algorithm="fricas")

[Out] integral(f^(b^2\*c\*x^2 + 2\*a\*b\*c\*x + a^2\*c)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*2)/x,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*2)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)/x,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^2\*c)/x, x)

$$3.200 \quad \int \frac{f^{c(a+bx)^2}}{x^2} dx$$

**Optimal.** Leaf size=77

$$2abc \log(f) \text{Unintegrable}\left(\frac{f^{c(a+bx)^2}}{x}, x\right) + \sqrt{\pi} b \sqrt{c} \sqrt{\log(f)} \text{Erfi}\left(\sqrt{c} \sqrt{\log(f)}(a+bx)\right) - \frac{f^{c(a+bx)^2}}{x}$$

[Out]  $-(f^{(c*(a + b*x)^2})/x) + b*\text{Sqrt}[c]*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[c]*(a + b*x)*\text{Sqrt}[\text{Log}[f]]]*\text{Sqrt}[\text{Log}[f]] + 2*a*b*c*\text{Log}[f]*\text{Unintegrable}[f^{(c*(a + b*x)^2})/x, x]$

**Rubi [A]** time = 0.0579473, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[f^{(c*(a + b*x)^2})/x^2, x]$

[Out]  $-(f^{(c*(a + b*x)^2})/x) + b*\text{Sqrt}[c]*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[c]*(a + b*x)*\text{Sqrt}[\text{Log}[f]]]*\text{Sqrt}[\text{Log}[f]] + 2*a*b*c*\text{Log}[f]*\text{Defer}[\text{Int}][f^{(c*(a + b*x)^2})/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{f^{c(a+bx)^2}}{x^2} dx &= -\frac{f^{c(a+bx)^2}}{x} + (2abc \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx + (2b^2c \log(f)) \int f^{c(a+bx)^2} dx \\ &= -\frac{f^{c(a+bx)^2}}{x} + b\sqrt{c}\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)\sqrt{\log(f)} + (2abc \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.307016, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[f^{(c*(a + b*x)^2})/x^2, x]$

[Out]  $\text{Integrate}[f^{(c*(a + b*x)^2})/x^2, x]$

**Maple [A]** time = 0.023, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(f^{(c*(b*x+a)^2})/x^2, x)$



[Out]  $\text{int}(f^{(c*(b*x+a)^2})/x^2, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c*(b*x+a)^2})/x^2, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(f^{((b*x + a)^2*c})/x^2, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^2cx^2+2abcx+a^2c}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c*(b*x+a)^2})/x^2, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(f^{(b^2*c*x^2 + 2*a*b*c*x + a^2*c)})/x^2, x)$

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c*(b*x+a)**2})/x**2, x)$

[Out]  $\text{Integral}(f^{(c*(a + b*x)**2})/x**2, x)$

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c*(b*x+a)^2})/x^2, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(f^{((b*x + a)^2*c})/x^2, x)$

$$3.201 \quad \int \frac{f^{c(a+bx)^2}}{x^3} dx$$

**Optimal.** Leaf size=135

$$2a^2b^2c^2 \log^2(f) \text{Unintegrable} \left( \frac{f^{c(a+bx)^2}}{x}, x \right) + b^2c \log(f) \text{Unintegrable} \left( \frac{f^{c(a+bx)^2}}{x}, x \right) + \sqrt{\pi} ab^2c^{3/2} \log^{\frac{3}{2}}(f) \text{Erfi} \left( \sqrt{c} \sqrt{\log(f)} \right)$$

[Out]  $-f^{c(a+bx)^2}/(2x^2) - (a*b*c*f^{c(a+bx)^2}*\text{Log}[f])/x + a*b^2*c^{3/2}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[c]*(a+bx)*\text{Sqrt}[\text{Log}[f]]]*\text{Log}[f]^{3/2} + b^2*c*\text{Log}[f]*\text{Unintegrable}[f^{c(a+bx)^2}/x, x] + 2*a^2*b^2*c^2*\text{Log}[f]^2*\text{Unintegrable}[f^{c(a+bx)^2}/x, x]$

**Rubi [A]** time = 0.0971859, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^2}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^2)/x^3, x]

[Out]  $-f^{c(a+bx)^2}/(2x^2) - (a*b*c*f^{c(a+bx)^2}*\text{Log}[f])/x + a*b^2*c^{3/2}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[c]*(a+bx)*\text{Sqrt}[\text{Log}[f]]]*\text{Log}[f]^{3/2} + b^2*c*\text{Log}[f]*\text{Defer}[\text{Int}[f^{c(a+bx)^2}/x, x] + 2*a^2*b^2*c^2*\text{Log}[f]^2*\text{Defer}[\text{Int}[f^{c(a+bx)^2}/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{f^{c(a+bx)^2}}{x^3} dx &= -\frac{f^{c(a+bx)^2}}{2x^2} + (abc \log(f)) \int \frac{f^{c(a+bx)^2}}{x^2} dx + (b^2c \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx \\ &= -\frac{f^{c(a+bx)^2}}{2x^2} - \frac{abc f^{c(a+bx)^2} \log(f)}{x} + (b^2c \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx + (2a^2b^2c^2 \log^2(f)) \int \frac{f^{c(a+bx)^2}}{x} dx + (2ab^2c^2 \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx \\ &= -\frac{f^{c(a+bx)^2}}{2x^2} - \frac{abc f^{c(a+bx)^2} \log(f)}{x} + ab^2c^{3/2} \sqrt{\pi} \text{erfi}(\sqrt{c}(a+bx)\sqrt{\log(f)}) \log^{\frac{3}{2}}(f) + (b^2c \log(f)) \int \frac{f^{c(a+bx)^2}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.408973, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^2)/x^3, x]

[Out] Integrate[f^(c\*(a + b\*x)^2)/x^3, x]

**Maple [A]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)/x^3,x)

[Out] int(f^(c\*(b\*x+a)^2)/x^3,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)/x^3,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^2\*c)/x^3, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^2cx^2+2abcx+a^2c}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)/x^3,x, algorithm="fricas")

[Out] integral(f^(b^2\*c\*x^2 + 2\*a\*b\*c\*x + a^2\*c)/x^3, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*2)/x\*\*3,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*2)/x\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^2c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^2*c)/x^3, x)
```

### 3.202 $\int f^{c(a+bx)^3} x^2 dx$

**Optimal.** Leaf size=120

$$\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b^3\sqrt[3]{-c\log(f)(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -c\log(f)(a+bx)^3\right)}{3b^3(-c\log(f)(a+bx)^3)^{2/3}} + \frac{f^{c(a+bx)^3}}{3b^3c\log(f)}$$

[Out]  $f^{c*(a + b*x)^3}/(3*b^3*c*\text{Log}[f]) + (2*a*(a + b*x)^2*\text{Gamma}[2/3, -(c*(a + b*x)^3*\text{Log}[f])])/(3*b^3*(-(c*(a + b*x)^3*\text{Log}[f]))^{(2/3)}) - (a^2*(a + b*x)*\text{Gamma}[1/3, -(c*(a + b*x)^3*\text{Log}[f])])/(3*b^3*(-(c*(a + b*x)^3*\text{Log}[f]))^{(1/3)})$

**Rubi [A]** time = 0.0889143, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2226, 2208, 2218, 2209}

$$\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b^3\sqrt[3]{-c\log(f)(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -c\log(f)(a+bx)^3\right)}{3b^3(-c\log(f)(a+bx)^3)^{2/3}} + \frac{f^{c(a+bx)^3}}{3b^3c\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^3)\*x^2,x]

[Out]  $f^{c*(a + b*x)^3}/(3*b^3*c*\text{Log}[f]) + (2*a*(a + b*x)^2*\text{Gamma}[2/3, -(c*(a + b*x)^3*\text{Log}[f])])/(3*b^3*(-(c*(a + b*x)^3*\text{Log}[f]))^{(2/3)}) - (a^2*(a + b*x)*\text{Gamma}[1/3, -(c*(a + b*x)^3*\text{Log}[f])])/(3*b^3*(-(c*(a + b*x)^3*\text{Log}[f]))^{(1/3)})$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int f^{c(a+bx)^3} x^2 dx &= \int \left( \frac{a^2 f^{c(a+bx)^3}}{b^2} - \frac{2a f^{c(a+bx)^3} (a+bx)}{b^2} + \frac{f^{c(a+bx)^3} (a+bx)^2}{b^2} \right) dx \\
&= \frac{\int f^{c(a+bx)^3} (a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{c(a+bx)^3} (a+bx) dx}{b^2} + \frac{a^2 \int f^{c(a+bx)^3} dx}{b^2} \\
&= \frac{f^{c(a+bx)^3}}{3b^3 c \log(f)} + \frac{2a(a+bx)^2 \Gamma\left(\frac{2}{3}, -c(a+bx)^3 \log(f)\right)}{3b^3 (-c(a+bx)^3 \log(f))^{2/3}} - \frac{a^2(a+bx) \Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right)}{3b^3 \sqrt[3]{-c(a+bx)^3 \log(f)}}
\end{aligned}$$

**Mathematica [A]** time = 0.199875, size = 111, normalized size = 0.92

$$\frac{-\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -c \log(f)(a+bx)^3\right)}{\sqrt[3]{-c \log(f)(a+bx)^3}} + \frac{2a(a+bx)^2 \Gamma\left(\frac{2}{3}, -c \log(f)(a+bx)^3\right)}{(-c \log(f)(a+bx)^3)^{2/3}} + \frac{f^{c(a+bx)^3}}{c \log(f)}}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^3)\*x^2,x]

[Out] (f^(c\*(a + b\*x)^3)/(c\*Log[f]) + (2\*a\*(a + b\*x)^2\*Gamma[2/3, -(c\*(a + b\*x)^3\*Log[f])])/(-c\*(a + b\*x)^3\*Log[f])^(2/3) - (a^2\*(a + b\*x)\*Gamma[1/3, -(c\*(a + b\*x)^3\*Log[f])])/(-c\*(a + b\*x)^3\*Log[f])^(1/3))/(3\*b^3)

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^3} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)\*x^2,x)

[Out] int(f^(c\*(b\*x+a)^3)\*x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3 c} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^2,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)\*x^2, x)

**Fricas [A]** time = 1.55489, size = 373, normalized size = 3.11

$$\frac{(-b^3 c \log(f))^{\frac{2}{3}} a^2 \Gamma\left(\frac{1}{3}, -(b^3 c x^3 + 3 a b^2 c x^2 + 3 a^2 b c x + a^3 c) \log(f)\right) - 2 (-b^3 c \log(f))^{\frac{1}{3}} a b \Gamma\left(\frac{2}{3}, -(b^3 c x^3 + 3 a b^2 c x^2 + 3 a^2 b c x + a^3 c) \log(f)\right)}{3 b^5 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^2,x, algorithm="fricas")

[Out]  $\frac{1}{3} * ((-b^3 * c * \log(f))^{2/3} * a^2 * \text{gamma}(1/3, -(b^3 * c * x^3 + 3 * a * b^2 * c * x^2 + 3 * a^2 * b * c * x + a^3 * c) * \log(f)) - 2 * (-b^3 * c * \log(f))^{1/3} * a * b * \text{gamma}(2/3, -(b^3 * c * x^3 + 3 * a * b^2 * c * x^2 + 3 * a^2 * b * c * x + a^3 * c) * \log(f)) + b^2 * f^{(b^3 * c * x^3 + 3 * a * b^2 * c * x^2 + 3 * a^2 * b * c * x + a^3 * c)}) / (b^5 * c * \log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^3} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3)\*x\*\*2,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*3)\*x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3} c x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^2,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^3\*c)\*x^2, x)

### 3.203 $\int f^{c(a+bx)^3} x dx$

**Optimal.** Leaf size=92

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b^2\sqrt[3]{-c\log(f)(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -c\log(f)(a+bx)^3\right)}{3b^2(-c\log(f)(a+bx)^3)^{2/3}}$$

[Out]  $-\left((a+bx)^2\Gamma\left[\frac{2}{3}, -(c(a+bx)^3\log[f])\right]\right)/\left(3b^2(-(c(a+bx)^3\log[f])^{2/3})\right) + (a(a+bx)\Gamma\left[\frac{1}{3}, -(c(a+bx)^3\log[f])\right])/3b^2(-(c(a+bx)^3\log[f])^{1/3})$

**Rubi [A]** time = 0.0494645, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2226, 2208, 2218}

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b^2\sqrt[3]{-c\log(f)(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -c\log(f)(a+bx)^3\right)}{3b^2(-c\log(f)(a+bx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^3)\*x, x]

[Out]  $-\left((a+bx)^2\Gamma\left[\frac{2}{3}, -(c(a+bx)^3\log[f])\right]\right)/\left(3b^2(-(c(a+bx)^3\log[f])^{2/3})\right) + (a(a+bx)\Gamma\left[\frac{1}{3}, -(c(a+bx)^3\log[f])\right])/3b^2(-(c(a+bx)^3\log[f])^{1/3})$

#### Rule 2226

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps



$$\begin{aligned} \int f^{c(a+bx)^3} x dx &= \int \left( -\frac{a f^{c(a+bx)^3}}{b} + \frac{f^{c(a+bx)^3} (a+bx)}{b} \right) dx \\ &= \frac{\int f^{c(a+bx)^3} (a+bx) dx}{b} - \frac{a \int f^{c(a+bx)^3} dx}{b} \\ &= -\frac{(a+bx)^2 \Gamma\left(\frac{2}{3}, -c(a+bx)^3 \log(f)\right)}{3b^2 \left(-c(a+bx)^3 \log(f)\right)^{2/3}} + \frac{a(a+bx) \Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right)}{3b^2 \sqrt[3]{-c(a+bx)^3 \log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.0471472, size = 86, normalized size = 0.93

$$\frac{(a+bx) \left( (a+bx) \text{Gamma}\left(\frac{2}{3}, -c \log(f)(a+bx)^3\right) - a \sqrt[3]{-c \log(f)(a+bx)^3} \text{Gamma}\left(\frac{1}{3}, -c \log(f)(a+bx)^3\right) \right)}{3b^2 \left(-c \log(f)(a+bx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^3)\*x,x]

[Out] -((a + b\*x)\*((a + b\*x)\*Gamma[2/3, -(c\*(a + b\*x)^3\*Log[f])]) - a\*Gamma[1/3, -(c\*(a + b\*x)^3\*Log[f])])\*(-(c\*(a + b\*x)^3\*Log[f]))^(1/3))/(3\*b^2\*(-(c\*(a + b\*x)^3\*Log[f]))^(2/3))

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^3} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)\*x,x)

[Out] int(f^(c\*(b\*x+a)^3)\*x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3 c} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)\*x, x)

**Fricas [A]** time = 1.56721, size = 288, normalized size = 3.13

$$\frac{\left(-b^3 c \log(f)\right)^{\frac{2}{3}} a \Gamma\left(\frac{1}{3}, -(b^3 c x^3 + 3 a b^2 c x^2 + 3 a^2 b c x + a^3 c) \log(f)\right) - \left(-b^3 c \log(f)\right)^{\frac{1}{3}} b \Gamma\left(\frac{2}{3}, -(b^3 c x^3 + 3 a b^2 c x^2 + 3 a^2 b c x + a^3 c) \log(f)\right)}{3 b^4 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^3)*x,x, algorithm="fricas")
```

```
[Out] -1/3*((-b^3*c*log(f))^(2/3)*a*gamma(1/3, -(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*log(f)) - (-b^3*c*log(f))^(1/3)*b*gamma(2/3, -(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*log(f)))/(b^4*c*log(f))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^3} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*(b*x+a)**3)*x,x)
```

```
[Out] Integral(f**(c*(a + b*x)**3)*x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3 c} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^3)*x,x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^3*c)*x, x)
```

### 3.204 $\int f^{c(a+bx)^3} dx$

**Optimal.** Leaf size=44

$$\frac{(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b\sqrt[3]{-c\log(f)(a+bx)^3}}$$

[Out]  $-\left((a + b*x)*\Gamma\left[\frac{1}{3}, -(c*(a + b*x)^3*\text{Log}[f])\right]\right)/(3*b*(-(c*(a + b*x)^3*\text{Log}[f]))^{(1/3)})$

**Rubi [A]** time = 0.0055393, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2208}

$$\frac{(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b\sqrt[3]{-c\log(f)(a+bx)^3}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^3), x]

[Out]  $-\left((a + b*x)*\Gamma\left[\frac{1}{3}, -(c*(a + b*x)^3*\text{Log}[f])\right]\right)/(3*b*(-(c*(a + b*x)^3*\text{Log}[f]))^{(1/3)})$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int f^{c(a+bx)^3} dx = -\frac{(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3\log(f)\right)}{3b\sqrt[3]{-c(a+bx)^3\log(f)}}$$

**Mathematica [A]** time = 0.0090916, size = 44, normalized size = 1.

$$\frac{(a+bx)\Gamma\left(\frac{1}{3}, -c\log(f)(a+bx)^3\right)}{3b\sqrt[3]{-c\log(f)(a+bx)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^3), x]

[Out]  $-\left((a + b*x)*\Gamma\left[\frac{1}{3}, -(c*(a + b*x)^3*\text{Log}[f])\right]\right)/(3*b*(-(c*(a + b*x)^3*\text{Log}[f]))^{(1/3)})$

**Maple [F]** time = 0.016, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3),x)

[Out] int(f^(c\*(b\*x+a)^3),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3),x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c), x)

---

**Fricas [A]** time = 1.52329, size = 151, normalized size = 3.43

$$\frac{(-b^3c \log(f))^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -(b^3cx^3 + 3ab^2cx^2 + 3a^2bcx + a^3c) \log(f)\right)}{3b^3c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3),x, algorithm="fricas")

[Out] 1/3\*(-b^3\*c\*log(f))^(2/3)\*gamma(1/3, -(b^3\*c\*x^3 + 3\*a\*b^2\*c\*x^2 + 3\*a^2\*b\*c\*x + a^3\*c)\*log(f))/(b^3\*c\*log(f))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3),x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^3),x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^3*c), x)
```

$$3.205 \quad \int \frac{f^{c(a+bx)^3}}{x} dx$$

**Optimal.** Leaf size=17

$$\text{Unintegrable}\left(\frac{f^{c(a+bx)^3}}{x}, x\right)$$

[Out] Unintegrable[f^(c\*(a + b\*x)^3)/x, x]

**Rubi [A]** time = 0.0168283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^3)/x, x]

[Out] Defer[Int][f^(c\*(a + b\*x)^3)/x, x]

Rubi steps

$$\int \frac{f^{c(a+bx)^3}}{x} dx = \int \frac{f^{c(a+bx)^3}}{x} dx$$

**Mathematica [A]** time = 0.295997, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^3)/x, x]

[Out] Integrate[f^(c\*(a + b\*x)^3)/x, x]

**Maple [A]** time = 0.016, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)/x, x)

[Out] int(f^(c\*(b\*x+a)^3)/x, x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3 c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x,x, algorithm="fricas")

[Out] integral(f^(b^3\*c\*x^3 + 3\*a\*b^2\*c\*x^2 + 3\*a^2\*b\*c\*x + a^3\*c)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3)/x,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*3)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3 c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^3\*c)/x, x)

$$3.206 \quad \int \frac{f^{c(a+bx)^3}}{x^2} dx$$

**Optimal.** Leaf size=132

$$3a^2bc \log(f) \text{Unintegrable} \left( \frac{f^{c(a+bx)^3}}{x}, x \right) - \frac{abc \log(f)(a+bx) \Gamma\left(\frac{1}{3}, -c \log(f)(a+bx)^3\right)}{\sqrt[3]{-c \log(f)(a+bx)^3}} - \frac{bc \log(f)(a+bx)^2 \Gamma\left(\frac{2}{3}, -c \log(f)(a+bx)^3\right)}{(-c \log(f)(a+bx)^3)^{2/3}}$$

[Out]  $-(f^{c(a+bx)^3}/x) - (b*c*(a+bx)^2*\Gamma[2/3, -(c*(a+bx)^3*\text{Log}[f])]*\text{Log}[f])/(-(c*(a+bx)^3*\text{Log}[f]))^{(2/3)} - (a*b*c*(a+bx)*\Gamma[1/3, -(c*(a+bx)^3*\text{Log}[f])]*\text{Log}[f])/(-(c*(a+bx)^3*\text{Log}[f]))^{(1/3)} + 3*a^2*b*c*\text{Log}[f]*\text{Unintegrable}[f^{c(a+bx)^3}/x, x]$

**Rubi [A]** time = 0.302344, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^3}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^3)/x^2, x]

[Out]  $-(f^{c(a+bx)^3}/x) - (b*c*(a+bx)^2*\Gamma[2/3, -(c*(a+bx)^3*\text{Log}[f])]*\text{Log}[f])/(-(c*(a+bx)^3*\text{Log}[f]))^{(2/3)} - (a*b*c*(a+bx)*\Gamma[1/3, -(c*(a+bx)^3*\text{Log}[f])]*\text{Log}[f])/(-(c*(a+bx)^3*\text{Log}[f]))^{(1/3)} + 3*a^2*b*c*\text{Log}[f]*\text{Defer}[\text{Int}[f^{c(a+bx)^3}/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{f^{c(a+bx)^3}}{x^2} dx &= -\frac{f^{c(a+bx)^3}}{x} + (3bc \log(f)) \int \frac{f^{c(a+bx)^3} (a+bx)^2}{x} dx \\ &= -\frac{f^{c(a+bx)^3}}{x} + (3bc \log(f)) \int \left( ab f^{c(a+bx)^3} + \frac{a^2 f^{c(a+bx)^3}}{x} + b f^{c(a+bx)^3} (a+bx) \right) dx \\ &= -\frac{f^{c(a+bx)^3}}{x} + (3a^2bc \log(f)) \int \frac{f^{c(a+bx)^3}}{x} dx + (3b^2c \log(f)) \int f^{c(a+bx)^3} (a+bx) dx + (3ab^2c \log(f)) \int f^{c(a+bx)^3} dx \\ &= -\frac{f^{c(a+bx)^3}}{x} - \frac{bc(a+bx)^2 \Gamma\left(\frac{2}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{(-c(a+bx)^3 \log(f))^{2/3}} - \frac{abc(a+bx) \Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{\sqrt[3]{-c(a+bx)^3 \log(f)}} \end{aligned}$$

**Mathematica [A]** time = 1.08802, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^3)/x^2, x]

[Out] Integrate[f^(c\*(a + b\*x)^3)/x^2, x]



---

**Maple [A]** time = 0.023, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^3}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)/x^2,x)

[Out] int(f^(c\*(b\*x+a)^3)/x^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x^2,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)/x^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x^2,x, algorithm="fricas")

[Out] integral(f^(b^3\*c\*x^3 + 3\*a\*b^2\*c\*x^2 + 3\*a^2\*b\*c\*x + a^3\*c)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3)/x\*\*2,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*3)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^3*c)/x^2, x)
```

$$3.207 \quad \int \frac{f^{c(a+bx)^3}}{x^3} dx$$

**Optimal.** Leaf size=261

$$\frac{9}{2}a^4b^2c^2 \log^2(f) \text{Unintegrable}\left(\frac{f^{c(a+bx)^3}}{x}, x\right) - \frac{3a^3b^2c^2 \log^2(f)(a+bx) \text{Gamma}\left(\frac{1}{3}, -c \log(f)(a+bx)^3\right)}{2\sqrt[3]{-c \log(f)(a+bx)^3}} - \frac{3a^2b^2c^2 \log^2(f)}{2}$$

[Out]  $-f^{(c(a+bx)^3)}/(2x^2) - (3a^2b^2c^2f^{(c(a+bx)^3)}\text{Log}[f])/(2x) - (3a^2b^2c^2(a+bx)^2\text{Gamma}[2/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f]^2)/(2(-c(a+bx)^3\text{Log}[f])^{(2/3)}) - (b^2c^2(a+bx)\text{Gamma}[1/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f])/(2(-c(a+bx)^3\text{Log}[f])^{(1/3)}) - (3a^3b^2c^2(a+bx)\text{Gamma}[1/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f]^2)/(2(-c(a+bx)^3\text{Log}[f])^{(1/3)}) + 3a^2b^2c^2\text{Log}[f]\text{Unintegrable}[f^{(c(a+bx)^3)}/x, x] + (9a^4b^2c^2\text{Log}[f]^2\text{Unintegrable}[f^{(c(a+bx)^3)}/x, x])/2$

**Rubi [A]** time = 0.448682, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^3}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^3)/x^3, x]

[Out]  $-f^{(c(a+bx)^3)}/(2x^2) - (3a^2b^2c^2f^{(c(a+bx)^3)}\text{Log}[f])/(2x) - (3a^2b^2c^2(a+bx)^2\text{Gamma}[2/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f]^2)/(2(-c(a+bx)^3\text{Log}[f])^{(2/3)}) - (b^2c^2(a+bx)\text{Gamma}[1/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f])/(2(-c(a+bx)^3\text{Log}[f])^{(1/3)}) - (3a^3b^2c^2(a+bx)\text{Gamma}[1/3, -(c(a+bx)^3\text{Log}[f])]\text{Log}[f]^2)/(2(-c(a+bx)^3\text{Log}[f])^{(1/3)}) + 3a^2b^2c^2\text{Log}[f]\text{Defer}[\text{Int}[f^{(c(a+bx)^3)}/x, x] + (9a^4b^2c^2\text{Log}[f]^2\text{Defer}[\text{Int}[f^{(c(a+bx)^3)}/x, x])/2$

Rubi steps

$$\begin{aligned}
\int \frac{f^{c(a+bx)^3}}{x^3} dx &= -\frac{f^{c(a+bx)^3}}{2x^2} + \frac{1}{2}(3bc \log(f)) \int \frac{f^{c(a+bx)^3}(a+bx)^2}{x^2} dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} + \frac{1}{2}(3bc \log(f)) \int \left( b^2 f^{c(a+bx)^3} + \frac{a^2 f^{c(a+bx)^3}}{x^2} + \frac{2ab f^{c(a+bx)^3}}{x} \right) dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} + \frac{1}{2}(3a^2bc \log(f)) \int \frac{f^{c(a+bx)^3}}{x^2} dx + (3ab^2c \log(f)) \int \frac{f^{c(a+bx)^3}}{x} dx + \frac{1}{2}(3b^3c \log(f)) \int f^{c(a+bx)^3} dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} - \frac{3a^2bc f^{c(a+bx)^3} \log(f)}{2x} - \frac{b^2c(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{2\sqrt[3]{-c(a+bx)^3 \log(f)}} + (3ab^2c \log(f)) \int \frac{f^{c(a+bx)^3}}{x} dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} - \frac{3a^2bc f^{c(a+bx)^3} \log(f)}{2x} - \frac{b^2c(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{2\sqrt[3]{-c(a+bx)^3 \log(f)}} + (3ab^2c \log(f)) \int \frac{f^{c(a+bx)^3}}{x} dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} - \frac{3a^2bc f^{c(a+bx)^3} \log(f)}{2x} - \frac{b^2c(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{2\sqrt[3]{-c(a+bx)^3 \log(f)}} + (3ab^2c \log(f)) \int \frac{f^{c(a+bx)^3}}{x} dx \\
&= -\frac{f^{c(a+bx)^3}}{2x^2} - \frac{3a^2bc f^{c(a+bx)^3} \log(f)}{2x} - \frac{3a^2b^2c^2(a+bx)^2\Gamma\left(\frac{2}{3}, -c(a+bx)^3 \log(f)\right) \log^2(f)}{2(-c(a+bx)^3 \log(f))^{2/3}} - \frac{b^2c(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right) \log(f)}{2\sqrt[3]{-c(a+bx)^3 \log(f)}}
\end{aligned}$$

**Mathematica [A]** time = 1.38929, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^3)/x^3,x]

[Out] Integrate[f^(c\*(a + b\*x)^3)/x^3, x]

**Maple [A]** time = 0.03, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^3}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)/x^3,x)

[Out] int(f^(c\*(b\*x+a)^3)/x^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x^3,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)/x^3, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x^3,x, algorithm="fricas")

[Out] integral(f^(b^3\*c\*x^3 + 3\*a\*b^2\*c\*x^2 + 3\*a^2\*b\*c\*x + a^3\*c)/x^3, x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^3}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3)/x\*\*3,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*3)/x\*\*3, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^3c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)/x^3,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^3\*c)/x^3, x)

### 3.208 $\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^4 dx$

**Optimal.** Leaf size=183

$$\frac{4a^3(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^5(-(a+bx)^3)^{2/3}} - \frac{a^4(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^5\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^5\Gamma\left(\frac{5}{3}, -(a+bx)^3\right)}{3b^5(-(a+bx)^3)^{5/3}} + \frac{4a(a+bx)^4}{3b^5}$$

[Out] (2\*a^2\*E^(a + b\*x)^3)/b^5 - (a^4\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(1/3)) + (4\*a^3\*(a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(2/3)) + (4\*a\*(a + b\*x)^4\*Gamma[4/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(4/3)) - ((a + b\*x)^5\*Gamma[5/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(5/3))

**Rubi [A]** time = 0.183204, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {2227, 2226, 2208, 2218, 2209}

$$\frac{4a^3(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^5(-(a+bx)^3)^{2/3}} - \frac{a^4(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^5\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^5\Gamma\left(\frac{5}{3}, -(a+bx)^3\right)}{3b^5(-(a+bx)^3)^{5/3}} + \frac{4a(a+bx)^4}{3b^5}$$

Antiderivative was successfully verified.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^4, x]

[Out] (2\*a^2\*E^(a + b\*x)^3)/b^5 - (a^4\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(1/3)) + (4\*a^3\*(a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(2/3)) + (4\*a\*(a + b\*x)^4\*Gamma[4/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(4/3)) - ((a + b\*x)^5\*Gamma[5/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(5/3))

#### Rule 2227

Int[(u\_.)\*(F\_)^((a\_.) + (b\_.)\*(v\_)), x\_Symbol] := Int[u\*F^(a + b\*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^4 dx &= \int e^{(a+bx)^3} x^4 dx \\ &= \int \left( \frac{a^4 e^{(a+bx)^3}}{b^4} - \frac{4a^3 e^{(a+bx)^3} (a+bx)}{b^4} + \frac{6a^2 e^{(a+bx)^3} (a+bx)^2}{b^4} - \frac{4a e^{(a+bx)^3} (a+bx)^3}{b^4} + \frac{e^{(a+bx)^3}}{b^4} \right) dx \\ &= \frac{\int e^{(a+bx)^3} (a+bx)^4 dx}{b^4} - \frac{(4a) \int e^{(a+bx)^3} (a+bx)^3 dx}{b^4} + \frac{(6a^2) \int e^{(a+bx)^3} (a+bx)^2 dx}{b^4} - \frac{(4a) \int e^{(a+bx)^3} (a+bx) dx}{b^4} + \frac{\int e^{(a+bx)^3} dx}{b^4} \\ &= \frac{2a^2 e^{(a+bx)^3}}{b^5} - \frac{a^4 (a+bx) \Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^5 \sqrt[3]{-(a+bx)^3}} + \frac{4a^3 (a+bx)^2 \Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^5 \left(-(a+bx)^3\right)^{2/3}} + \frac{4a (a+bx) \Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^5 \left(-(a+bx)^3\right)^{2/3}} + \frac{e^{(a+bx)^3}}{3b^5 \left(-(a+bx)^3\right)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.179857, size = 164, normalized size = 0.9

$$\frac{a^4(-a+bx)\sqrt[3]{-(a+bx)^3}\Gamma\left(\frac{1}{3}, -(a+bx)^3\right) + 4a^3(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right) - 4a(a+bx)\sqrt[3]{-(a+bx)^3}\Gamma\left(\frac{1}{3}, -(a+bx)^3\right) + e^{(a+bx)^3}}{3b^5 \left(-(a+bx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^4, x]

[Out] (6\*a^2\*E^(a + b\*x)^3\*(-(a + b\*x)^3)^(2/3) - a^4\*(a + b\*x)\*(-(a + b\*x)^3)^(1/3)\*Gamma[1/3, -(a + b\*x)^3] + 4\*a^3\*(a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3] - 4\*a\*(a + b\*x)\*(-(a + b\*x)^3)^(1/3)\*Gamma[4/3, -(a + b\*x)^3] + (a + b\*x)^2\*Gamma[5/3, -(a + b\*x)^3])/(3\*b^5\*(-(a + b\*x)^3)^(2/3))

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^4, x)

[Out] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^4,x, algorithm="maxima")

[Out] integrate(x^4\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

**Fricas [A]** time = 1.51148, size = 348, normalized size = 1.9

$$\frac{2(6a^3 + 1)(-b^3)^{\frac{1}{3}} b \Gamma\left(\frac{2}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right) - (3a^4 + 4a)(-b^3)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right) - 3}{9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^4,x, algorithm="fricas")

[Out] -1/9\*(2\*(6\*a^3 + 1)\*(-b^3)^(1/3)\*b\*gamma(2/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) - (3\*a^4 + 4\*a)\*(-b^3)^(2/3)\*gamma(1/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) - 3\*(b^4\*x^2 - 2\*a\*b^3\*x + 3\*a^2\*b^2)\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))/b^7

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)\*x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^4,x, algorithm="giac")

[Out] integrate(x^4\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)



### 3.209 $\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^3 dx$

**Optimal.** Leaf size=138

$$\frac{a^2(a+bx)^2 \Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{b^4 \left(-(a+bx)^3\right)^{2/3}} + \frac{a^3(a+bx) \Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^4 \sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^4 \Gamma\left(\frac{4}{3}, -(a+bx)^3\right)}{3b^4 \left(-(a+bx)^3\right)^{4/3}} - \frac{ae^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{3b^4 \sqrt[3]{-(a+bx)^3}}$$

[Out]  $-\left(\frac{a^2 e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{b^4} + \frac{a^3(a+bx) \Gamma\left[\frac{1}{3}, -(a+bx)^3\right]}{3b^4 \sqrt[3]{-(a+bx)^3}}\right) / \left(3b^4 \sqrt[3]{-(a+bx)^3} \left(-\frac{1}{3} \log\left[-(a+bx)^3\right] - \frac{2}{3} \log\left[-(a+bx)^3\right] - \frac{4}{3} \log\left[-(a+bx)^3\right]\right) - \frac{a^2(a+bx)^2 \Gamma\left[\frac{2}{3}, -(a+bx)^3\right]}{b^4 \left(-\frac{2}{3} \log\left[-(a+bx)^3\right]\right)} - \frac{(a+bx)^4 \Gamma\left[\frac{4}{3}, -(a+bx)^3\right]}{3b^4 \left(-\frac{4}{3} \log\left[-(a+bx)^3\right]\right)}\right)$

**Rubi [A]** time = 0.149339, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {2227, 2226, 2208, 2218, 2209}

$$\frac{a^2(a+bx)^2 \Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{b^4 \left(-(a+bx)^3\right)^{2/3}} + \frac{a^3(a+bx) \Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^4 \sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^4 \Gamma\left(\frac{4}{3}, -(a+bx)^3\right)}{3b^4 \left(-(a+bx)^3\right)^{4/3}} - \frac{ae^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{3b^4 \sqrt[3]{-(a+bx)^3}}$$

Antiderivative was successfully verified.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^3,x]

[Out]  $-\left(\frac{a^2 e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{b^4} + \frac{a^3(a+bx) \Gamma\left[\frac{1}{3}, -(a+bx)^3\right]}{3b^4 \sqrt[3]{-(a+bx)^3}}\right) / \left(3b^4 \sqrt[3]{-(a+bx)^3} \left(-\frac{1}{3} \log\left[-(a+bx)^3\right] - \frac{2}{3} \log\left[-(a+bx)^3\right] - \frac{4}{3} \log\left[-(a+bx)^3\right]\right) - \frac{a^2(a+bx)^2 \Gamma\left[\frac{2}{3}, -(a+bx)^3\right]}{b^4 \left(-\frac{2}{3} \log\left[-(a+bx)^3\right]\right)} - \frac{(a+bx)^4 \Gamma\left[\frac{4}{3}, -(a+bx)^3\right]}{3b^4 \left(-\frac{4}{3} \log\left[-(a+bx)^3\right]\right)}\right)$

#### Rule 2227

Int[(u\_)\*(F\_)^((a\_.) + (b\_.)\*(v\_)), x\_Symbol] := Int[u\*F^(a + b\*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* (u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m+1)\*Gamma[(m+1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m+1/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

### Rubi steps

$$\begin{aligned} \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^3 dx &= \int e^{(a+bx)^3} x^3 dx \\ &= \int \left( -\frac{a^3 e^{(a+bx)^3}}{b^3} + \frac{3a^2 e^{(a+bx)^3} (a+bx)}{b^3} - \frac{3ae^{(a+bx)^3} (a+bx)^2}{b^3} + \frac{e^{(a+bx)^3} (a+bx)^3}{b^3} \right) dx \\ &= \frac{\int e^{(a+bx)^3} (a+bx)^3 dx}{b^3} - \frac{(3a) \int e^{(a+bx)^3} (a+bx)^2 dx}{b^3} + \frac{(3a^2) \int e^{(a+bx)^3} (a+bx) dx}{b^3} - \frac{a^3 \int e^{(a+bx)^3} dx}{b^3} \\ &= -\frac{ae^{(a+bx)^3}}{b^4} + \frac{a^3(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^4\sqrt[3]{-(a+bx)^3}} - \frac{a^2(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{b^4(-(a+bx)^3)^{2/3}} - \frac{(a+bx)^4\Gamma\left(\frac{4}{3}, -(a+bx)^3\right)}{3b^4(-(a+bx)^3)^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.134847, size = 138, normalized size = 1.

$$-\frac{a^2(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{b^4(-(a+bx)^3)^{2/3}} + \frac{a^3(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^4\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^4\Gamma\left(\frac{4}{3}, -(a+bx)^3\right)}{3b^4(-(a+bx)^3)^{4/3}} - \frac{ae^{(a+bx)^3}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*x^3, x]
```

```
[Out] -((a*E^(a + b*x)^3)/b^4) + (a^3*(a + b*x)*Gamma[1/3, -(a + b*x)^3])/(3*b^4*(-(a + b*x)^3)^(1/3)) - (a^2*(a + b*x)^2*Gamma[2/3, -(a + b*x)^3])/(b^4*(-(a + b*x)^3)^(2/3)) - ((a + b*x)^4*Gamma[4/3, -(a + b*x)^3])/(3*b^4*(-(a + b*x)^3)^(4/3))
```

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^3, x)
```

```
[Out] int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^3, x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^3, x, algorithm="maxima")
```

[Out] integrate(x^3\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

**Fricas [A]** time = 1.51519, size = 312, normalized size = 2.26

$$\frac{9(-b^3)^{\frac{1}{3}}a^2b\Gamma\left(\frac{2}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right) - (3a^3 + 1)(-b^3)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right) + 3(b^3x - 2a^3)}{9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^3,x, algorithm="fricas")

[Out] 1/9\*(9\*(-b^3)^(1/3)\*a^2\*b\*gamma(2/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) - (3\*a^3 + 1)\*(-b^3)^(2/3)\*gamma(1/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) + 3\*(b^3\*x - 2\*a\*b^2)\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))/b^6

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)\*x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^3,x, algorithm="giac")

[Out] integrate(x^3\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

### 3.210 $\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^2 dx$

**Optimal.** Leaf size=99

$$-\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^3\sqrt[3]{-(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^3(-(a+bx)^3)^{2/3}} + \frac{e^{(a+bx)^3}}{3b^3}$$

[Out]  $E^{(a+b*x)^3}/(3*b^3) - (a^2*(a+b*x)*\Gamma[1/3, -(a+b*x)^3])/(3*b^3*(-(a+b*x)^3)^{(1/3)}) + (2*a*(a+b*x)^2*\Gamma[2/3, -(a+b*x)^3])/(3*b^3*(-(a+b*x)^3)^{(2/3)})$

**Rubi [A]** time = 0.120953, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {2227, 2226, 2208, 2218, 2209}

$$-\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^3\sqrt[3]{-(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^3(-(a+bx)^3)^{2/3}} + \frac{e^{(a+bx)^3}}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^2,x]

[Out]  $E^{(a+b*x)^3}/(3*b^3) - (a^2*(a+b*x)*\Gamma[1/3, -(a+b*x)^3])/(3*b^3*(-(a+b*x)^3)^{(1/3)}) + (2*a*(a+b*x)^2*\Gamma[2/3, -(a+b*x)^3])/(3*b^3*(-(a+b*x)^3)^{(2/3)})$

#### Rule 2227

Int[(u\_)\*(F\_)^((a\_.) + (b\_.)\*(v\_)), x\_Symbol] := Int[u\*F^(a + b\*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(e + f\*x)^n\*F^(a + b\*(c + d\*x)^n)/(b\*f\*n\*(c + d\*x)^n)

$n \cdot \text{Log}[F], x] / ; \text{FreeQ}[\{F, a, b, c, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{EqQ}[d \cdot e - c \cdot f, 0]$

### Rubi steps

$$\begin{aligned} \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^2 dx &= \int e^{(a+bx)^3} x^2 dx \\ &= \int \left( \frac{a^2 e^{(a+bx)^3}}{b^2} - \frac{2ae^{(a+bx)^3}(a+bx)}{b^2} + \frac{e^{(a+bx)^3}(a+bx)^2}{b^2} \right) dx \\ &= \frac{\int e^{(a+bx)^3}(a+bx)^2 dx}{b^2} - \frac{(2a) \int e^{(a+bx)^3}(a+bx) dx}{b^2} + \frac{a^2 \int e^{(a+bx)^3} dx}{b^2} \\ &= \frac{e^{(a+bx)^3}}{3b^3} - \frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^3\sqrt[3]{-(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^3(-(a+bx)^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.0809934, size = 89, normalized size = 0.9

$$\frac{-\frac{a^2(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{\sqrt[3]{-(a+bx)^3}} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{(-(a+bx)^3)^{2/3}} + e^{(a+bx)^3}}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^2, x]

[Out] (E^(a + b\*x)^3 - (a^2\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(-(a + b\*x)^3)^(1/3) + (2\*a\*(a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(-(a + b\*x)^3)^(2/3)/(3\*b^3)

**Maple [F]** time = 0.017, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^2, x)

[Out] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^2, x, algorithm="maxima")

[Out] integrate(x^2\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

---

**Fricas [A]** time = 1.55053, size = 277, normalized size = 2.8

$$\frac{(-b^3)^{\frac{2}{3}} a^2 \Gamma\left(\frac{1}{3}, -b^3 x^3 - 3 ab^2 x^2 - 3 a^2 b x - a^3\right) - 2 (-b^3)^{\frac{1}{3}} ab \Gamma\left(\frac{2}{3}, -b^3 x^3 - 3 ab^2 x^2 - 3 a^2 b x - a^3\right) + b^2 e^{(b^3 x^3 + 3 ab^2 x^2 + 3 a^2 b x + a^3)}}{3 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^2,x, algorithm="fricas")

[Out] 1/3\*((-b^3)^(2/3)\*a^2\*gamma(1/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) - 2\*(-b^3)^(1/3)\*a\*b\*gamma(2/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) + b^2\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))/b^5

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)\*x\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{(b^3 x^3 + 3 ab^2 x^2 + 3 a^2 b x + a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^2,x, algorithm="giac")

[Out] integrate(x^2\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

### 3.211 $\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x dx$

**Optimal.** Leaf size=80

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2(-(a+bx)^3)^{2/3}}$$

[Out] (a\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(1/3)) - ((a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

**Rubi [A]** time = 0.0684645, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {2227, 2226, 2208, 2218}

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2(-(a+bx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x, x]

[Out] (a\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(1/3)) - ((a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

#### Rule 2227

Int[(u\_.)\*(F\_)^((a\_.) + (b\_.)\*(v\_)), x\_Symbol] :> Int[u\*F^(a + b\*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*u\_, x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x dx &= \int e^{(a+bx)^3} x dx \\
&= \int \left( -\frac{ae^{(a+bx)^3}}{b} + \frac{e^{(a+bx)^3}(a+bx)}{b} \right) dx \\
&= \frac{\int e^{(a+bx)^3} (a+bx) dx}{b} - \frac{a \int e^{(a+bx)^3} dx}{b} \\
&= \frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2\left(-(a+bx)^3\right)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.0309232, size = 74, normalized size = 0.92

$$\frac{(a+bx)\left(a\sqrt[3]{-(a+bx)^3}\Gamma\left(\frac{1}{3}, -(a+bx)^3\right) - (a+bx)\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)\right)}{3b^2\left(-(a+bx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x, x]

[Out] ((a + b\*x)\*(a\*(-(a + b\*x)^3)^(1/3)\*Gamma[1/3, -(a + b\*x)^3] - (a + b\*x)\*Gamma[2/3, -(a + b\*x)^3]))/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

**Maple [F]** time = 0.015, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x, x)

[Out] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int xe^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x, x, algorithm="maxima")

[Out] integrate(x\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

**Fricas [A]** time = 1.46661, size = 203, normalized size = 2.54

$$-\frac{(-b^3)^{\frac{2}{3}} a \Gamma\left(\frac{1}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right) - (-b^3)^{\frac{1}{3}} b \Gamma\left(\frac{2}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right)}{3b^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x,x, algorithm="fricas")

[Out]  $-1/3*((-b^3)^{(2/3)}*a*\text{gamma}(1/3, -b^3*x^3 - 3*a*b^2*x^2 - 3*a^2*b*x - a^3) - (-b^3)^{(1/3)}*b*\text{gamma}(2/3, -b^3*x^3 - 3*a*b^2*x^2 - 3*a^2*b*x - a^3))/b^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{a^3} \int x e^{b^3 x^3} e^{3ab^2 x^2} e^{3a^2 b x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)\*x,x)

[Out] exp(a\*\*3)\*Integral(x\*exp(b\*\*3\*x\*\*3)\*exp(3\*a\*b\*\*2\*x\*\*2)\*exp(3\*a\*\*2\*b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x e^{(b^3 x^3 + 3ab^2 x^2 + 3a^2 b x + a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x,x, algorithm="giac")

[Out] integrate(x\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

$$3.212 \quad \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} dx$$

**Optimal.** Leaf size=38

$$-\frac{(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b\sqrt[3]{-(a+bx)^3}}$$

[Out]  $-\left((a + b*x)*\text{Gamma}[1/3, -(a + b*x)^3]\right)/(3*b*(-(a + b*x)^3)^{(1/3)})$

**Rubi [A]** time = 0.0085331, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2227, 2208}

$$-\frac{(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b\sqrt[3]{-(a+bx)^3}}$$

Antiderivative was successfully verified.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3), x]

[Out]  $-\left((a + b*x)*\text{Gamma}[1/3, -(a + b*x)^3]\right)/(3*b*(-(a + b*x)^3)^{(1/3)})$

#### Rule 2227

Int[(u\_)\*(F\_)^((a\_.) + (b\_.)\*(v\_)), x\_Symbol] :> Int[u\*F^(a + b\*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\begin{aligned} \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} dx &= \int e^{(a+bx)^3} dx \\ &= -\frac{(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b\sqrt[3]{-(a+bx)^3}} \end{aligned}$$

**Mathematica [A]** time = 0.0054023, size = 38, normalized size = 1.

$$-\frac{(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b\sqrt[3]{-(a+bx)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3), x]

[Out]  $-\left((a + b*x)*\text{Gamma}\left[\frac{1}{3}, -(a + b*x)^3\right]\right)/\left(3*b*(-(a + b*x)^3)^{\frac{1}{3}}\right)$

**Maple [F]** time = 0.011, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)`

[Out] `int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")`

[Out] `integrate(e^(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)`

**Fricas [A]** time = 1.47753, size = 101, normalized size = 2.66

$$\frac{(-b^3)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -b^3x^3 - 3ab^2x^2 - 3a^2bx - a^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")`

[Out] `1/3*(-b^3)^(2/3)*gamma(1/3, -b^3*x^3 - 3*a*b^2*x^2 - 3*a^2*b*x - a^3)/b^3`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{a^3} \int e^{b^3x^3} e^{3ab^2x^2} e^{3a^2bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)`

[Out] `exp(a**3)*Integral(exp(b**3*x**3)*exp(3*a*b**2*x**2)*exp(3*a**2*b*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
```

```
[Out] integrate(e^(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

$$3.213 \quad \int \frac{e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{x} dx$$

**Optimal.** Leaf size=35

$$\text{CannotIntegrate}\left(\frac{e^{3a^2bx+a^3+3ab^2x^2+b^3x^3}}{x}, x\right)$$

[Out] CannotIntegrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)/x, x]

**Rubi [A]** time = 0.0931069, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)/x, x]

[Out] Defer[Int][E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)/x, x]

Rubi steps

$$\int \frac{e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{x} dx = \int \frac{e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{x} dx$$

**Mathematica [A]** time = 0.240218, size = 0, normalized size = 0.

$$\int \frac{e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)/x, x]

[Out] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)/x, x]

**Maple [A]** time = 0.013, size = 0, normalized size = 0.

$$\int \frac{e^{b^3x^3+3ab^2x^2+3a^2bx+a^3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)/x, x)

[Out] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)/x, x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)/x,x, algorithm="maxima")

[Out] integrate(e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)/x,x, algorithm="fricas")

[Out] integral(e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^{a^3} \int \frac{e^{b^3x^3} e^{3ab^2x^2} e^{3a^2bx}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)/x,x)

[Out] exp(a\*\*3)\*Integral(exp(b\*\*3\*x\*\*3)\*exp(3\*a\*b\*\*2\*x\*\*2)\*exp(3\*a\*\*2\*b\*x)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)/x,x, algorithm="giac")

[Out] integrate(e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)/x, x)

$$3.214 \quad \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^m dx$$

**Optimal.** Leaf size=35

$$\text{CannotIntegrate}(x^m e^{3a^2bx+a^3+3ab^2x^2+b^3x^3}, x)$$

[Out] CannotIntegrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^m, x]

**Rubi [A]** time = 0.0923404, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^m dx$$

Verification is Not applicable to the result.

[In] Int[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^m, x]

[Out] Defer[Int][E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^m, x]

Rubi steps

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^m dx = \int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^m dx$$

**Mathematica [A]** time = 0.181297, size = 0, normalized size = 0.

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^m, x]

[Out] Integrate[E^(a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*x^m, x]

**Maple [A]** time = 0.027, size = 0, normalized size = 0.

$$\int e^{b^3x^3+3ab^2x^2+3a^2bx+a^3} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^m, x)

[Out] int(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(b^3x^3+3ab^2x^2+3a^2bx+a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^m,x, algorithm="maxima")

[Out] integrate(x^m\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m e^{(b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^m,x, algorithm="fricas")

[Out] integral(x^m\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*\*3\*x\*\*3+3\*a\*b\*\*2\*x\*\*2+3\*a\*\*2\*b\*x+a\*\*3)\*x\*\*m,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int x^m e^{(b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+a^3)\*x^m,x, algorithm="giac")

[Out] integrate(x^m\*e^(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)



### 3.215 $\int e^{\sqrt{5+3x}} dx$

**Optimal.** Leaf size=40

$$\frac{2}{3}e^{\sqrt{3x+5}}\sqrt{3x+5} - \frac{2}{3}e^{\sqrt{3x+5}}$$

[Out]  $(-2E^{\text{Sqrt}[5 + 3*x]})/3 + (2E^{\text{Sqrt}[5 + 3*x]}*\text{Sqrt}[5 + 3*x])/3$

**Rubi [A]** time = 0.0123375, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2207, 2176, 2194}

$$\frac{2}{3}e^{\sqrt{3x+5}}\sqrt{3x+5} - \frac{2}{3}e^{\sqrt{3x+5}}$$

Antiderivative was successfully verified.

[In] Int[E^Sqrt[5 + 3\*x], x]

[Out]  $(-2E^{\text{Sqrt}[5 + 3*x]})/3 + (2E^{\text{Sqrt}[5 + 3*x]}*\text{Sqrt}[5 + 3*x])/3$

#### Rule 2207

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := With[{k = Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)\*F^(a + b\*x^(k\*n)), x], x, (c + d\*x)^(1/k)], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && !IntegerQ[n]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int e^{\sqrt{5+3x}} dx &= \frac{2}{3} \text{Subst} \left( \int e^x dx, x, \sqrt{5+3x} \right) \\ &= \frac{2}{3} e^{\sqrt{5+3x}} \sqrt{5+3x} - \frac{2}{3} \text{Subst} \left( \int e^x dx, x, \sqrt{5+3x} \right) \\ &= -\frac{2}{3} e^{\sqrt{5+3x}} + \frac{2}{3} e^{\sqrt{5+3x}} \sqrt{5+3x} \end{aligned}$$

**Mathematica [A]** time = 0.0117546, size = 26, normalized size = 0.65

$$\frac{2}{3}e^{\sqrt{3x+5}}(\sqrt{3x+5}-1)$$

Antiderivative was successfully verified.

[In] Integrate[E^Sqrt[5 + 3\*x],x]

[Out] (2\*E^Sqrt[5 + 3\*x]\*(-1 + Sqrt[5 + 3\*x]))/3

**Maple [A]** time = 0.003, size = 29, normalized size = 0.7

$$-\frac{2}{3}e^{\sqrt{5+3x}} + \frac{2}{3}e^{\sqrt{5+3x}}\sqrt{5+3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((5+3\*x)^(1/2)),x)

[Out] -2/3\*exp((5+3\*x)^(1/2))+2/3\*exp((5+3\*x)^(1/2))\*(5+3\*x)^(1/2)

**Maxima [A]** time = 1.05085, size = 26, normalized size = 0.65

$$\frac{2}{3}(\sqrt{3x+5}-1)e^{(\sqrt{3x+5})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((5+3\*x)^(1/2)),x, algorithm="maxima")

[Out] 2/3\*(sqrt(3\*x + 5) - 1)\*e^(sqrt(3\*x + 5))

**Fricas [A]** time = 1.45192, size = 58, normalized size = 1.45

$$\frac{2}{3}(\sqrt{3x+5}-1)e^{(\sqrt{3x+5})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((5+3\*x)^(1/2)),x, algorithm="fricas")

[Out] 2/3\*(sqrt(3\*x + 5) - 1)\*e^(sqrt(3\*x + 5))

**Sympy [A]** time = 0.181676, size = 34, normalized size = 0.85

$$\frac{2\sqrt{3x+5}e^{\sqrt{3x+5}}}{3} - \frac{2e^{\sqrt{3x+5}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((5+3\*x)\*\*(1/2)),x)

[Out] 2\*sqrt(3\*x + 5)\*exp(sqrt(3\*x + 5))/3 - 2\*exp(sqrt(3\*x + 5))/3

---

**Giac [A]** time = 1.26458, size = 28, normalized size = 0.7

$$\frac{2}{3}(\sqrt{3x+5}-1)e^{\sqrt{3x+5}} + \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((5+3\*x)^(1/2)),x, algorithm="giac")

[Out] 2/3\*(sqrt(3\*x + 5) - 1)\*e^(sqrt(3\*x + 5)) + 2/3

### 3.216 $\int f^{\frac{c}{a+bx}} x^4 dx$

**Optimal.** Leaf size=291

$$-\frac{c^5 \log^5(f) \Gamma\left(-5, -\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{4ac^4 \log^4(f) \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{a^2 c^3 \log^3(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^5} + \frac{2a^3 c^2 \log^2(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^5}$$

[Out]  $(a^4 f^{c/(a+bx)} (a+bx))/b^5 - (2a^3 f^{c/(a+bx)} (a+bx)^2)/b^5 + (2a^2 f^{c/(a+bx)} (a+bx)^3)/b^5 - (2a^3 c f^{c/(a+bx)} (a+bx) \log[f])/b^5 + (a^2 c f^{c/(a+bx)} (a+bx)^2 \log[f])/b^5 - (a^4 c \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f])/b^5 + (a^2 c^2 f^{c/(a+bx)} (a+bx) \log[f]^2)/b^5 + (2a^3 c^2 \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f]^2)/b^5 - (a^2 c^3 \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f]^3)/b^5 - (4a^4 c \Gamma[-4, -(c \log[f])/(a+bx)]) \log[f]^4/b^5 - (c^5 \Gamma[-5, -(c \log[f])/(a+bx)]) \log[f]^5/b^5$

**Rubi [A]** time = 0.288491, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2226, 2206, 2210, 2214, 2218}

$$-\frac{c^5 \log^5(f) \Gamma\left(-5, -\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{4ac^4 \log^4(f) \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{a^2 c^3 \log^3(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^5} + \frac{2a^3 c^2 \log^2(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x))\*x^4, x]

[Out]  $(a^4 f^{c/(a+bx)} (a+bx))/b^5 - (2a^3 f^{c/(a+bx)} (a+bx)^2)/b^5 + (2a^2 f^{c/(a+bx)} (a+bx)^3)/b^5 - (2a^3 c f^{c/(a+bx)} (a+bx) \log[f])/b^5 + (a^2 c f^{c/(a+bx)} (a+bx)^2 \log[f])/b^5 - (a^4 c \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f])/b^5 + (a^2 c^2 f^{c/(a+bx)} (a+bx) \log[f]^2)/b^5 + (2a^3 c^2 \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f]^2)/b^5 - (a^2 c^3 \text{ExpIntegralEi}[(c \log[f])/(a+bx)] \log[f]^3)/b^5 - (4a^4 c \Gamma[-4, -(c \log[f])/(a+bx)]) \log[f]^4/b^5 - (c^5 \Gamma[-5, -(c \log[f])/(a+bx)]) \log[f]^5/b^5$

#### Rule 2226

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2210

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a \* ExpIntegralEi[b\*(c + d\*x)^n \* Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int f^{\frac{c}{a+bx}} x^4 dx &= \int \left( \frac{a^4 f^{\frac{c}{a+bx}}}{b^4} - \frac{4a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4} + \frac{6a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{b^4} - \frac{4a f^{\frac{c}{a+bx}}(a+bx)^3}{b^4} + \frac{f^{\frac{c}{a+bx}}(a+bx)^4}{b^4} \right) dx \\ &= \frac{\int f^{\frac{c}{a+bx}}(a+bx)^4 dx}{b^4} - \frac{(4a) \int f^{\frac{c}{a+bx}}(a+bx)^3 dx}{b^4} + \frac{(6a^2) \int f^{\frac{c}{a+bx}}(a+bx)^2 dx}{b^4} - \frac{(4a^3) \int f^{\frac{c}{a+bx}}(a+bx) dx}{b^4} \\ &= \frac{a^4 f^{\frac{c}{a+bx}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{a+bx}}(a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{a+bx}}(a+bx)^3}{b^5} - \frac{4ac^4 \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right) \log^4(f)}{b^5} - \frac{c^5 \Gamma(-5, -\frac{c \log(f)}{a+bx})}{b^5} \\ &= \frac{a^4 f^{\frac{c}{a+bx}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{a+bx}}(a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{a+bx}}(a+bx)^3}{b^5} - \frac{2a^3 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^5} + \frac{a^2 c f^{\frac{c}{a+bx}}(a+bx)^2 \log^2(f)}{b^5} \\ &= \frac{a^4 f^{\frac{c}{a+bx}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{a+bx}}(a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{a+bx}}(a+bx)^3}{b^5} - \frac{2a^3 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^5} + \frac{a^2 c f^{\frac{c}{a+bx}}(a+bx)^2 \log^2(f)}{b^5} \\ &= \frac{a^4 f^{\frac{c}{a+bx}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{a+bx}}(a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{a+bx}}(a+bx)^3}{b^5} - \frac{2a^3 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^5} + \frac{a^2 c f^{\frac{c}{a+bx}}(a+bx)^2 \log^2(f)}{b^5} \end{aligned}$$

**Mathematica [A]** time = 0.18964, size = 241, normalized size = 0.83

$$\frac{bx f^{\frac{c}{a+bx}} \left( 2c^2 \log^2(f) (43a^2 - 7abx + b^2x^2) + 2c \log(f) (18a^2bx - 48a^3 - 8ab^2x^2 + 3b^3x^3) + c^3 \log^3(f) (bx - 18a) + 24b^4 \right)}{120b^5}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))\*x^4,x]

[Out] (a\*f^(c/(a + b\*x))\*(24\*a^4 - 154\*a^3\*c\*Log[f] + 102\*a^2\*c^2\*Log[f]^2 - 19\*a\*c^3\*Log[f]^3 + c^4\*Log[f]^4))/(120\*b^5) + (-c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)]\*Log[f]\*(120\*a^4 - 240\*a^3\*c\*Log[f] + 120\*a^2\*c^2\*Log[f]^2 - 20\*a\*c^3\*Log[f]^3 + c^4\*Log[f]^4) + b\*f^(c/(a + b\*x))\*x\*(24\*b^4\*x^4 + 2\*c\*(-48\*a^3 + 18\*a^2\*b\*x - 8\*a\*b^2\*x^2 + 3\*b^3\*x^3)\*Log[f] + 2\*c^2\*(43\*a^2 - 7\*a\*b\*x + b^2\*x^2)\*Log[f]^2 + c^3\*(-18\*a + b\*x)\*Log[f]^3 + c^4\*Log[f]^4))/(120\*b^5)

**Maple [A]** time = 0.112, size = 517, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c/(b*x+a))*x^4,x)`

[Out]  $\frac{1}{b^5 \ln(f)^3 c^3 a^2 \operatorname{Ei}(1, -c \ln(f)/(b*x+a)) - \frac{77}{60} b^5 \ln(f) c f^{c/(b*x+a)} a^4 + \frac{1}{b^5 \ln(f) c a^4 \operatorname{Ei}(1, -c \ln(f)/(b*x+a))} + \frac{1}{5} f^{c/(b*x+a)} x^5 + \frac{1}{60} b^2 \ln(f)^2 c^2 f^{c/(b*x+a)} x^3 + \frac{1}{120} b^3 \ln(f)^3 c^3 f^{c/(b*x+a)} x^2 + \frac{1}{120} b^4 \ln(f)^4 c^4 f^{c/(b*x+a)} x - \frac{2}{b^5 \ln(f)^2 c^2 a^3 \operatorname{Ei}(1, -c \ln(f)/(b*x+a))} + \frac{1}{20} b \ln(f) c f^{c/(b*x+a)} x^4 + \frac{1}{120} b^5 \ln(f)^5 c^5 \operatorname{Ei}(1, -c \ln(f)/(b*x+a)) + \frac{43}{60} b^4 \ln(f)^2 c^2 f^{c/(b*x+a)} a^2 x - \frac{1}{6} b^5 \ln(f)^4 c^4 a \operatorname{Ei}(1, -c \ln(f)/(b*x+a)) - \frac{2}{15} b^2 \ln(f) c f^{c/(b*x+a)} a x^3 + \frac{3}{10} b^3 \ln(f) c f^{c/(b*x+a)} a^2 x^2 - \frac{4}{5} b^4 \ln(f) c f^{c/(b*x+a)} a^3 x + \frac{17}{20} b^5 \ln(f)^2 c^2 f^{c/(b*x+a)} a^3 - \frac{19}{120} b^5 \ln(f)^3 c^3 f^{c/(b*x+a)} a^2 + \frac{1}{120} b^5 \ln(f)^4 c^4 f^{c/(b*x+a)} a - \frac{3}{20} b^4 \ln(f)^3 c^3 f^{c/(b*x+a)} a x + \frac{1}{5} b^5 \ln(f)^5 f^{c/(b*x+a)} - \frac{7}{60} b^3 \ln(f)^2 c^2 f^{c/(b*x+a)} a x^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(24b^4x^5 + 6b^3cx^4 \log(f) + 2(b^2c^2 \log(f)^2 - 8ab^2c \log(f)))x^3 + (bc^3 \log(f)^3 - 14abc^2 \log(f)^2 + 36a^2bc \log(f))x^2 + 120b^4}{120b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))*x^4,x, algorithm="maxima")`

[Out]  $\frac{1}{120} (24b^4x^5 + 6b^3cx^4 \log(f) + 2(b^2c^2 \log(f)^2 - 8ab^2c \log(f)))x^3 + (bc^3 \log(f)^3 - 14abc^2 \log(f)^2 + 36a^2bc \log(f))x^2 + (c^4 \log(f)^4 - 18a^3c^3 \log(f)^3 + 86a^2c^2 \log(f)^2 - 96a^3c \log(f))x) f^{c/(b*x+a)} / b^4 + \operatorname{integrate}(-\frac{1}{120} (a^2c^4 \log(f)^4 - 18a^3c^3 \log(f)^3 + 86a^4c^2 \log(f)^2 - 96a^5c \log(f) - (bc^5 \log(f)^5 - 20a^4bc^4 \log(f)^4 + 120a^2b^3c^3 \log(f)^3 - 240a^3b^2c^2 \log(f)^2 + 120a^4b^2c \log(f)))x) f^{c/(b*x+a)} / (b^6x^2 + 2ab^5x + a^2b^4), x)$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))*x^4,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a))*x**4,x)`

[Out] Integral(f\*\*(c/(a + b\*x))\*x\*\*4, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^4,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a))\*x^4, x)

### 3.217 $\int f^{\frac{c}{a+bx}} x^3 dx$

**Optimal.** Leaf size=269

$$\frac{c^4 \log^4(f) \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right)}{b^4} - \frac{3a^2 c^2 \log^2(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{2b^4} + \frac{a^3 c \log(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^4} + \frac{3a^2 (a+bx)^2 f^{\frac{c}{a+bx}}}{2b^4} - \frac{a^3 (a+bx)}{b^4}$$

[Out]  $-\left(\frac{a^3 f^{c/(a+bx)} (a+bx)}{b^4}\right) + \left(\frac{3a^2 f^{c/(a+bx)} (a+bx)^2}{2b^4}\right) - \left(\frac{a f^{c/(a+bx)} (a+bx)^3}{b^4}\right) + \left(\frac{3a^2 c f^{c/(a+bx)} (a+bx) \log[f]}{2b^4}\right) - \left(\frac{a c f^{c/(a+bx)} (a+bx)^2 \log[f]}{2b^4}\right) + \left(\frac{a^3 c \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]}{b^4}\right) - \left(\frac{a c^2 f^{c/(a+bx)} (a+bx) \log[f]^2}{2b^4}\right) - \left(\frac{3a^2 c^2 \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]^2}{2b^4}\right) + \left(\frac{a c^3 \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]^3}{2b^4}\right) + \left(\frac{c^4 \Gamma\left[-4, -\left(\frac{c \log[f]}{a+bx}\right)\right] \log[f]^4}{b^4}\right)$

**Rubi [A]** time = 0.2543, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2226, 2206, 2210, 2214, 2218}

$$\frac{c^4 \log^4(f) \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right)}{b^4} - \frac{3a^2 c^2 \log^2(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{2b^4} + \frac{a^3 c \log(f) \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^4} + \frac{3a^2 (a+bx)^2 f^{\frac{c}{a+bx}}}{2b^4} - \frac{a^3 (a+bx)}{b^4}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x))\*x^3, x]

[Out]  $-\left(\frac{a^3 f^{c/(a+bx)} (a+bx)}{b^4}\right) + \left(\frac{3a^2 f^{c/(a+bx)} (a+bx)^2}{2b^4}\right) - \left(\frac{a f^{c/(a+bx)} (a+bx)^3}{b^4}\right) + \left(\frac{3a^2 c f^{c/(a+bx)} (a+bx) \log[f]}{2b^4}\right) - \left(\frac{a c f^{c/(a+bx)} (a+bx)^2 \log[f]}{2b^4}\right) + \left(\frac{a^3 c \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]}{b^4}\right) - \left(\frac{a c^2 f^{c/(a+bx)} (a+bx) \log[f]^2}{2b^4}\right) - \left(\frac{3a^2 c^2 \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]^2}{2b^4}\right) + \left(\frac{a c^3 \text{ExpIntegralEi}\left[\frac{c \log[f]}{a+bx}\right] \log[f]^3}{2b^4}\right) + \left(\frac{c^4 \Gamma\left[-4, -\left(\frac{c \log[f]}{a+bx}\right)\right] \log[f]^4}{b^4}\right)$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a \* ExpIntegralEi[b\*(c + d\*x)^n \* Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214



```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n)/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

### Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{a+bx}} x^3 dx &= \int \left( -\frac{a^3 f^{\frac{c}{a+bx}}}{b^3} + \frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{3af^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{b^3} \right) dx \\ &= \frac{\int f^{\frac{c}{a+bx}}(a+bx)^3 dx}{b^3} - \frac{(3a) \int f^{\frac{c}{a+bx}}(a+bx)^2 dx}{b^3} + \frac{(3a^2) \int f^{\frac{c}{a+bx}}(a+bx) dx}{b^3} - \frac{a^3 \int f^{\frac{c}{a+bx}} dx}{b^3} \\ &= -\frac{a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{2b^4} - \frac{af^{\frac{c}{a+bx}}(a+bx)^3}{b^4} + \frac{c^4 \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right) \log^4(f)}{b^4} - \frac{(ac \log(f)) \int f^{\frac{c}{a+bx}} dx}{2b^4} \\ &= -\frac{a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{2b^4} - \frac{af^{\frac{c}{a+bx}}(a+bx)^3}{b^4} + \frac{3a^2 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{2b^4} - \frac{ac f^{\frac{c}{a+bx}}(a+bx)}{2b^4} \\ &= -\frac{a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{2b^4} - \frac{af^{\frac{c}{a+bx}}(a+bx)^3}{b^4} + \frac{3a^2 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{2b^4} - \frac{ac f^{\frac{c}{a+bx}}(a+bx)}{2b^4} \\ &= -\frac{a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{2b^4} - \frac{af^{\frac{c}{a+bx}}(a+bx)^3}{b^4} + \frac{3a^2 c f^{\frac{c}{a+bx}}(a+bx) \log(f)}{2b^4} - \frac{ac f^{\frac{c}{a+bx}}(a+bx)}{2b^4} \end{aligned}$$

**Mathematica [A]** time = 0.154007, size = 179, normalized size = 0.67

$$\frac{b x f^{\frac{c}{a+bx}} \left( 2c \log(f) (9a^2 - 3abx + b^2 x^2) + c^2 \log^2(f) (bx - 10a) + 6b^3 x^3 + c^3 \log^3(f) \right) + c \log(f) (-36a^2 c \log(f) + 24a^3)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(c/(a + b*x))*x^3,x]
```

```
[Out] -(a*f^(c/(a + b*x))*(6*a^3 - 26*a^2*c*Log[f] + 11*a*c^2*Log[f]^2 - c^3*Log[f]^3))/(24*b^4) + (c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]*(24*a^3 - 36*a^2*c*Log[f] + 12*a*c^2*Log[f]^2 - c^3*Log[f]^3) + b*f^(c/(a + b*x))*x*(6*b^3*x^3 + 2*c*(9*a^2 - 3*a*b*x + b^2*x^2)*Log[f] + c^2*(-10*a + b*x)*Log[f]^2 + c^3*Log[f]^3))/(24*b^4)
```

**Maple [A]** time = 0.081, size = 359, normalized size = 1.3

$$-\frac{\ln(f) a^3 c}{b^4} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{bx+a}\right) + \frac{13 \ln(f) a^3 c}{12 b^4} f^{\frac{c}{bx+a}} - \frac{5 (\ln(f))^2 a c^2 x}{12 b^3} f^{\frac{c}{bx+a}} + \frac{(\ln(f))^3 c^3 x}{24 b^3} f^{\frac{c}{bx+a}} + \frac{x^4}{4} f^{\frac{c}{bx+a}} + \frac{(\ln(f))^2}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c/(b*x+a))*x^3,x)`

[Out]  $-1/b^4 \ln(f) * c * a^3 * \text{Ei}(1, -c \ln(f)/(b*x+a)) + 13/12/b^4 \ln(f) * c * f^{c/(b*x+a)} * a^3 - 5/12/b^3 \ln(f)^2 * c^2 * f^{c/(b*x+a)} * a * x + 1/24/b^3 \ln(f)^3 * c^3 * f^{c/(b*x+a)} * x + 1/4 * f^{c/(b*x+a)} * x^4 + 1/24/b^2 \ln(f)^2 * c^2 * f^{c/(b*x+a)} * x^2 - 1/2/b^4 \ln(f)^3 * c^3 * a * \text{Ei}(1, -c \ln(f)/(b*x+a)) + 3/2/b^4 \ln(f)^2 * c^2 * a^2 * \text{Ei}(1, -c \ln(f)/(b*x+a)) + 3/4/b^3 \ln(f) * c * f^{c/(b*x+a)} * a^2 * x - 11/24/b^4 \ln(f)^2 * c^2 * f^{c/(b*x+a)} * a^2 + 1/24/b^4 \ln(f)^3 * c^3 * f^{c/(b*x+a)} * a - 1/4/b^4 * f^{c/(b*x+a)} * a^4 - 1/4/b^2 \ln(f) * c * f^{c/(b*x+a)} * a * x^2 + 1/12/b * \ln(f) * c * f^{c/(b*x+a)} * x^3 + 1/24/b^4 \ln(f)^4 * c^4 * \text{Ei}(1, -c \ln(f)/(b*x+a))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left(6b^3x^4 + 2b^2cx^3 \log(f) + \left(bc^2 \log(f)^2 - 6abc \log(f)\right)x^2 + \left(c^3 \log(f)^3 - 10ac^2 \log(f)^2 + 18a^2c \log(f)\right)x\right) f^{\frac{c}{bx+a}}}{24b^3} - \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))*x^3,x, algorithm="maxima")`

[Out]  $1/24 * (6b^3x^4 + 2b^2cx^3 \log(f) + (bc^2 \log(f)^2 - 6a*b*c \log(f)) * x^2 + (c^3 \log(f)^3 - 10a*c^2 \log(f)^2 + 18a^2*c \log(f)) * x) * f^{c/(b*x+a)} / b^3 - \text{integrate}(1/24 * (a^2 * c^3 \log(f)^3 - 10a^3 * c^2 \log(f)^2 + 18a^4 * c \log(f) - (b*c^4 \log(f)^4 - 12a*b*c^3 \log(f)^3 + 36a^2*b*c^2 \log(f)^2 - 24a^3*b*c \log(f)) * x) * f^{c/(b*x+a)} / (b^5 * x^2 + 2a*b^4 * x + a^2 * b^3), x)$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))*x^3,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a))*x**3,x)`

[Out] `Integral(f**(c/(a + b*x))*x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a))*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a))*x^3, x)
```

### 3.218 $\int f^{\frac{c}{a+bx}} x^2 dx$

**Optimal.** Leaf size=229

$$-\frac{a^2 c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^3} + \frac{a^2 (a+bx) f^{\frac{c}{a+bx}}}{b^3} - \frac{c^3 \log^3(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{6b^3} + \frac{ac^2 \log^2(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^3} + \frac{c^2 \log^2(f) (a+bx) f^{\frac{c}{a+bx}}}{6b^3} + \dots$$

[Out]  $(a^2 f^{c/(a+bx)} (a+bx))/b^3 - (a f^{c/(a+bx)} (a+bx)^2)/b^3 + (f^{c/(a+bx)} (a+bx)^3)/(3b^3) - (a c f^{c/(a+bx)} (a+bx) \operatorname{Log}[f])/b^3 + (c f^{c/(a+bx)} (a+bx)^2 \operatorname{Log}[f])/(6b^3) - (a^2 c \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f])/b^3 + (c^2 f^{c/(a+bx)} (a+bx) \operatorname{Log}[f]^2)/(6b^3) + (a c^2 \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f]^2)/b^3 - (c^3 \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f]^3)/(6b^3)$

**Rubi [A]** time = 0.22341, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2226, 2206, 2210, 2214}

$$-\frac{a^2 c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^3} + \frac{a^2 (a+bx) f^{\frac{c}{a+bx}}}{b^3} - \frac{c^3 \log^3(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{6b^3} + \frac{ac^2 \log^2(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^3} + \frac{c^2 \log^2(f) (a+bx) f^{\frac{c}{a+bx}}}{6b^3} + \dots$$

Antiderivative was successfully verified.

[In] `Int[f^(c/(a + b*x))*x^2,x]`

[Out]  $(a^2 f^{c/(a+bx)} (a+bx))/b^3 - (a f^{c/(a+bx)} (a+bx)^2)/b^3 + (f^{c/(a+bx)} (a+bx)^3)/(3b^3) - (a c f^{c/(a+bx)} (a+bx) \operatorname{Log}[f])/b^3 + (c f^{c/(a+bx)} (a+bx)^2 \operatorname{Log}[f])/(6b^3) - (a^2 c \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f])/b^3 + (c^2 f^{c/(a+bx)} (a+bx) \operatorname{Log}[f]^2)/(6b^3) + (a c^2 \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f]^2)/b^3 - (c^3 \operatorname{ExpIntegralEi}[(c \operatorname{Log}[f])/(a+bx)] \operatorname{Log}[f]^3)/(6b^3)$

#### Rule 2226

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] := Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]`

#### Rule 2206

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] := Simp[((c + d*x)*F^(a + b*(c + d*x)^n))/d, x] - Dist[b*n*Log[F], Int[(c + d*x)^n * F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]`

#### Rule 2210

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a * ExpIntegralEi[b*(c + d*x)^n * Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]`

#### Rule 2214

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^(m + 1) * F^(a + b*(c + d*x)^n) / (d*(m + 1))`

, x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^(n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rubi steps

$$\begin{aligned}
 \int f^{\frac{c}{a+bx}} x^2 dx &= \int \left( \frac{a^2 f^{\frac{c}{a+bx}}}{b^2} - \frac{2af^{\frac{c}{a+bx}}(a+bx)}{b^2} + \frac{f^{\frac{c}{a+bx}}(a+bx)^2}{b^2} \right) dx \\
 &= \frac{\int f^{\frac{c}{a+bx}}(a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{\frac{c}{a+bx}}(a+bx) dx}{b^2} + \frac{a^2 \int f^{\frac{c}{a+bx}} dx}{b^2} \\
 &= \frac{a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{af^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{3b^3} + \frac{(c \log(f)) \int f^{\frac{c}{a+bx}}(a+bx) dx}{3b^2} - \frac{(ac \log(f)) \int f^{\frac{c}{a+bx}} dx}{b^2} \\
 &= \frac{a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{af^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{3b^3} - \frac{acf^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^3} + \frac{cf^{\frac{c}{a+bx}}(a+bx)^2 \log(f)}{6b^3} \\
 &= \frac{a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{af^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{3b^3} - \frac{acf^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^3} + \frac{cf^{\frac{c}{a+bx}}(a+bx)^2 \log(f)}{6b^3} \\
 &= \frac{a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{af^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{3b^3} - \frac{acf^{\frac{c}{a+bx}}(a+bx) \log(f)}{b^3} + \frac{cf^{\frac{c}{a+bx}}(a+bx)^2 \log(f)}{6b^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.113264, size = 128, normalized size = 0.56

$$\frac{bxf^{\frac{c}{a+bx}} \left( \log(f)(bcx - 4ac) + 2b^2x^2 + c^2 \log^2(f) \right) - c \log(f) \left( 6a^2 - 6ac \log(f) + c^2 \log^2(f) \right) \operatorname{Ei} \left( \frac{c \log(f)}{a+bx} \right)}{6b^3} + \frac{a(2a^2 - 5ac \log(f))}{6b^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))\*x^2,x]

[Out] (a\*f^(c/(a + b\*x))\*(2\*a^2 - 5\*a\*c\*Log[f] + c^2\*Log[f]^2))/(6\*b^3) + (-c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)]\*Log[f]\*(6\*a^2 - 6\*a\*c\*Log[f] + c^2\*Log[f]^2) + b\*f^(c/(a + b\*x))\*x\*(2\*b^2\*x^2 + (-4\*a\*c + b\*c\*x)\*Log[f] + c^2\*Log[f]^2))/(6\*b^3)

**Maple [A]** time = 0.072, size = 227, normalized size = 1.

$$\frac{a^3}{3b^3} f^{\frac{c}{bx+a}} + \frac{a^2 c \ln(f)}{b^3} \operatorname{Ei} \left( 1, -\frac{c \ln(f)}{bx+a} \right) + \frac{x^3}{3} f^{\frac{c}{bx+a}} + \frac{c \ln(f) x^2}{6b} f^{\frac{c}{bx+a}} - \frac{2ac \ln(f) x}{3b^2} f^{\frac{c}{bx+a}} - \frac{5a^2 c \ln(f)}{6b^3} f^{\frac{c}{bx+a}} + \frac{(\ln(f))^2}{6} f^{\frac{c}{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a))\*x^2,x)

[Out] 1/3/b^3\*a^3\*f^(c/(b\*x+a))+1/b^3\*ln(f)\*c\*a^2\*Ei(1,-c\*ln(f)/(b\*x+a))+1/3\*f^(c/(b\*x+a))\*x^3+1/6/b\*ln(f)\*c\*f^(c/(b\*x+a))\*x^2-2/3/b^2\*ln(f)\*c\*f^(c/(b\*x+a))\*a\*x-5/6/b^3\*ln(f)\*c\*f^(c/(b\*x+a))\*a^2+1/6/b^2\*ln(f)^2\*c^2\*f^(c/(b\*x+a))\*x+1/6/b^3\*ln(f)^2\*c^2\*f^(c/(b\*x+a))\*a+1/6/b^3\*ln(f)^3\*c^3\*Ei(1,-c\*ln(f)/(b\*x+a))

a)) $-1/b^3*\ln(f)^2*c^2*a*Ei(1,-c*\ln(f)/(b*x+a))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left(2b^2x^3 + bcx^2 \log(f) + \left(c^2 \log(f)^2 - 4ac \log(f)\right)x\right)f^{\frac{c}{bx+a}}}{6b^2} + \int -\frac{\left(a^2c^2 \log(f)^2 - 4a^3c \log(f) - \left(bc^3 \log(f)^3 - 6abc^2 \log(f)^2 + 6a^2b^2c \log(f)\right)x\right)f^{\frac{c}{bx+a}}}{6(b^4x^2 + 2ab^3x + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^2,x, algorithm="maxima")

[Out] 1/6\*(2\*b^2\*x^3 + b\*c\*x^2\*log(f) + (c^2\*log(f)^2 - 4\*a\*c\*log(f))\*x)\*f^(c/(b\*x + a))/b^2 + integrate(-1/6\*(a^2\*c^2\*log(f)^2 - 4\*a^3\*c\*log(f) - (b\*c^3\*log(f)^3 - 6\*a\*b\*c^2\*log(f)^2 + 6\*a^2\*b\*c\*log(f))\*x)\*f^(c/(b\*x + a))/(b^4\*x^2 + 2\*a\*b^3\*x + a^2\*b^2), x)

**Fricas [A]** time = 1.546, size = 263, normalized size = 1.15

$$\frac{\left(2b^3x^3 + 2a^3 + (bc^2x + ac^2) \log(f)^2 + (b^2cx^2 - 4abcx - 5a^2c) \log(f)\right)f^{\frac{c}{bx+a}} - \left(c^3 \log(f)^3 - 6ac^2 \log(f)^2 + 6a^2c \log(f)\right)x}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^2,x, algorithm="fricas")

[Out] 1/6\*((2\*b^3\*x^3 + 2\*a^3 + (b\*c^2\*x + a\*c^2)\*log(f)^2 + (b^2\*c\*x^2 - 4\*a\*b\*c\*x - 5\*a^2\*c)\*log(f))\*f^(c/(b\*x + a)) - (c^3\*log(f)^3 - 6\*a\*c^2\*log(f)^2 + 6\*a^2\*c\*log(f))\*Ei(c\*log(f)/(b\*x + a)))/b^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a))\*x\*\*2,x)

[Out] Integral(f\*\*(c/(a + b\*x))\*x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^2,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a))\*x^2, x)

### 3.219 $\int f^{\frac{c}{a+bx}} x dx$

**Optimal.** Leaf size=120

$$-\frac{c^2 \log^2(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{2b^2} + \frac{ac \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^2} + \frac{(a+bx)^2 f^{\frac{c}{a+bx}}}{2b^2} - \frac{a(a+bx) f^{\frac{c}{a+bx}}}{b^2} + \frac{c \log(f)(a+bx) f^{\frac{c}{a+bx}}}{2b^2}$$

[Out]  $-\left((a*f^{(c/(a+b*x))}*(a+b*x))/b^2\right) + (f^{(c/(a+b*x))}*(a+b*x)^2)/(2*b^2) + (c*f^{(c/(a+b*x))}*(a+b*x)*\operatorname{Log}[f])/(2*b^2) + (a*c*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f])/b^2 - (c^2*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f]^2)/(2*b^2)$

**Rubi [A]** time = 0.116233, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2226, 2206, 2210, 2214}

$$-\frac{c^2 \log^2(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{2b^2} + \frac{ac \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b^2} + \frac{(a+bx)^2 f^{\frac{c}{a+bx}}}{2b^2} - \frac{a(a+bx) f^{\frac{c}{a+bx}}}{b^2} + \frac{c \log(f)(a+bx) f^{\frac{c}{a+bx}}}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(c/(a+b*x))}*x, x]$

[Out]  $-\left((a*f^{(c/(a+b*x))}*(a+b*x))/b^2\right) + (f^{(c/(a+b*x))}*(a+b*x)^2)/(2*b^2) + (c*f^{(c/(a+b*x))}*(a+b*x)*\operatorname{Log}[f])/(2*b^2) + (a*c*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f])/b^2 - (c^2*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f]^2)/(2*b^2)$

#### Rule 2226

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}*(u_), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandLinearProduct}[F^{(a + b*(c + d*x)^n)}, u, c, d, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, n\}, x$  &&  $\operatorname{PolynomialQ}[u, x]$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d*x)*F^{(a + b*(c + d*x)^n)}}{d}, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{IntegerQ}[2/n]$  &&  $\operatorname{LtQ}[n, 0]$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\frac{F^a * \operatorname{ExpIntegralEi}[b*(c + d*x)^n * \operatorname{Log}[F]]}{(f*n)}, x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x$  &&  $\operatorname{EqQ}[d*e - c*f, 0]$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d*x)^{(m+1)} * F^{(a + b*(c + d*x)^n)}}{(d*(m+1))}, x] - \operatorname{Dist}[\frac{b*n*\operatorname{Log}[F]}{(m+1)}, \operatorname{Int}[(c + d*x)^{(m+n)} * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{IntegerQ}[(2*(m+1))/n]$  &&  $\operatorname{LtQ}[-4, (m+1)/n, 5]$  &&  $\operatorname{IntegerQ}[n]$  &&  $(\operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1]) \mid\mid (\operatorname{GtQ}[-n, 0] \&\& \operatorname{LeQ}[-n, m+1])$

Rubi steps

$$\begin{aligned}
\int f^{\frac{c}{a+bx}} x dx &= \int \left( -\frac{af^{\frac{c}{a+bx}}}{b} + \frac{f^{\frac{c}{a+bx}}(a+bx)}{b} \right) dx \\
&= \frac{\int f^{\frac{c}{a+bx}}(a+bx) dx}{b} - \frac{a \int f^{\frac{c}{a+bx}} dx}{b} \\
&= -\frac{af^{\frac{c}{a+bx}}(a+bx)}{b^2} + \frac{f^{\frac{c}{a+bx}}(a+bx)^2}{2b^2} + \frac{(c \log(f)) \int f^{\frac{c}{a+bx}} dx}{2b} - \frac{(ac \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx}{b} \\
&= -\frac{af^{\frac{c}{a+bx}}(a+bx)}{b^2} + \frac{f^{\frac{c}{a+bx}}(a+bx)^2}{2b^2} + \frac{cf^{\frac{c}{a+bx}}(a+bx) \log(f)}{2b^2} + \frac{ac \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log(f)}{b^2} + \frac{(c^2 \log^2(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx}}{2b} \\
&= -\frac{af^{\frac{c}{a+bx}}(a+bx)}{b^2} + \frac{f^{\frac{c}{a+bx}}(a+bx)^2}{2b^2} + \frac{cf^{\frac{c}{a+bx}}(a+bx) \log(f)}{2b^2} + \frac{ac \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log(f)}{b^2} - \frac{c^2 \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log^2(f)}{2b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0655748, size = 82, normalized size = 0.68

$$\frac{c \log(f)(2a - c \log(f)) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) + b x f^{\frac{c}{a+bx}}(bx + c \log(f))}{2b^2} - \frac{a(a - c \log(f)) f^{\frac{c}{a+bx}}}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))\*x,x]

[Out] -(a\*f^(c/(a + b\*x))\*(a - c\*Log[f]))/(2\*b^2) + (c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)]\*Log[f]\*(2\*a - c\*Log[f]) + b\*f^(c/(a + b\*x))\*x\*(b\*x + c\*Log[f]))/(2\*b^2)

**Maple [A]** time = 0.068, size = 126, normalized size = 1.1

$$\frac{x^2 f^{\frac{c}{bx+a}}}{2} - \frac{a^2 f^{\frac{c}{bx+a}}}{2b^2} + \frac{c \ln(f) x f^{\frac{c}{bx+a}}}{2b} + \frac{ac \ln(f) f^{\frac{c}{bx+a}}}{2b^2} + \frac{(\ln(f))^2 c^2}{2b^2} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{bx+a}\right) - \frac{ac \ln(f)}{b^2} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a))\*x,x)

[Out] 1/2\*f^(c/(b\*x+a))\*x^2-1/2/b^2\*f^(c/(b\*x+a))\*a^2+1/2/b\*ln(f)\*c\*f^(c/(b\*x+a))\*x+1/2/b^2\*ln(f)\*c\*f^(c/(b\*x+a))\*a+1/2/b^2\*ln(f)^2\*c^2\*Ei(1,-c\*ln(f)/(b\*x+a))-1/b^2\*ln(f)\*c\*a\*Ei(1,-c\*ln(f)/(b\*x+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bx^2 + cx \log(f)) f^{\frac{c}{bx+a}}}{2b} - \int \frac{(a^2 c \log(f) - (bc^2 \log(f)^2 - 2abc \log(f)) x) f^{\frac{c}{bx+a}}}{2(b^3 x^2 + 2ab^2 x + a^2 b)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(f^(c/(b\*x+a))\*x,x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + c\*x\*log(f))\*f^(c/(b\*x + a))/b - integrate(1/2\*(a^2\*c\*log(f) - (b\*c^2\*log(f)^2 - 2\*a\*b\*c\*log(f))\*x)\*f^(c/(b\*x + a))/(b^3\*x^2 + 2\*a\*b^2\*x + a^2\*b), x)

**Fricas [A]** time = 1.55185, size = 163, normalized size = 1.36

$$\frac{(b^2x^2 - a^2 + (bcx + ac) \log(f))f^{\frac{c}{bx+a}} - \left(c^2 \log(f)^2 - 2ac \log(f)\right) \operatorname{Ei}\left(\frac{c \log(f)}{bx+a}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x,x, algorithm="fricas")

[Out] 1/2\*((b^2\*x^2 - a^2 + (b\*c\*x + a\*c)\*log(f))\*f^(c/(b\*x + a)) - (c^2\*log(f)^2 - 2\*a\*c\*log(f))\*Ei(c\*log(f)/(b\*x + a)))/b^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a))\*x,x)

[Out] Integral(f\*\*(c/(a + b\*x))\*x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a))\*x, x)

### 3.220 $\int f^{\frac{c}{a+bx}} dx$

**Optimal.** Leaf size=41

$$\frac{(a+bx)f^{\frac{c}{a+bx}}}{b} - \frac{c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b}$$

[Out]  $(f^{(c/(a+b*x))}*(a+b*x))/b - (c*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f])/b$

**Rubi [A]** time = 0.0292191, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2206, 2210}

$$\frac{(a+bx)f^{\frac{c}{a+bx}}}{b} - \frac{c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(c/(a+b*x))}, x]$

[Out]  $(f^{(c/(a+b*x))}*(a+b*x))/b - (c*\operatorname{ExpIntegralEi}[(c*\operatorname{Log}[f])/(a+b*x)]*\operatorname{Log}[f])/b$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[2/n] \ \&\& \ \operatorname{LtQ}[n, 0]$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})/((e_.) + (f_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a * \operatorname{ExpIntegralEi}[b*(c + d*x)^n * \operatorname{Log}[F]])/(f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{a+bx}} dx &= \frac{f^{\frac{c}{a+bx}}(a+bx)}{b} + (c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx \\ &= \frac{f^{\frac{c}{a+bx}}(a+bx)}{b} - \frac{c \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log(f)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0152805, size = 41, normalized size = 1.

$$\frac{(a+bx)f^{\frac{c}{a+bx}}}{b} - \frac{c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)),x]

[Out] (f^(c/(a + b\*x))\*(a + b\*x))/b - (c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)]\*Log[f])/b

**Maple [A]** time = 0.06, size = 52, normalized size = 1.3

$$f^{\frac{c}{bx+a}}x + \frac{a}{b}f^{\frac{c}{bx+a}} + \frac{c \ln(f)}{b} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)),x)

[Out] f^(c/(b\*x+a))\*x+1/b\*f^(c/(b\*x+a))\*a+c/b\*ln(f)\*Ei(1,-c\*ln(f)/(b\*x+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$bc \int \frac{f^{\frac{c}{bx+a}}x}{b^2x^2 + 2abx + a^2} dx \log(f) + f^{\frac{c}{bx+a}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)),x, algorithm="maxima")

[Out] b\*c\*integrate(f^(c/(b\*x + a))\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2), x)\*log(f) + f^(c/(b\*x + a))\*x

**Fricas [A]** time = 1.51698, size = 89, normalized size = 2.17

$$\frac{c \operatorname{Ei}\left(\frac{c \log(f)}{bx+a}\right) \log(f) - (bx+a) f^{\frac{c}{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)),x, algorithm="fricas")

[Out] -(c\*Ei(c\*log(f)/(b\*x + a))\*log(f) - (b\*x + a)\*f^(c/(b\*x + a)))/b

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)),x)

[Out] Integral(f\*\*(c/(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)),x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)), x)

$$3.221 \quad \int \frac{f^{\frac{c}{a+bx}}}{x} dx$$

**Optimal.** Leaf size=41

$$f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) - \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)$$

[Out] -ExpIntegralEi[(c\*Log[f])/(a + b\*x)] + f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a\*(a + b\*x)))]

**Rubi [A]** time = 0.131148, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2222, 2210, 2228, 2178}

$$f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) - \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right)$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x))/x,x]

[Out] -ExpIntegralEi[(c\*Log[f])/(a + b\*x)] + f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a\*(a + b\*x)))]

#### Rule 2222

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Dist[d/f, Int[F^(a + b/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*e - c\*f)/f, Int[F^(a + b/(c + d\*x))/((c + d\*x)\*(e + f\*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2228

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/(((e\_.) + (f\_.)\*(x\_))\*((g\_.) + (h\_.)\*(x\_))), x\_Symbol] :> -Dist[d/(f\*(d\*g - c\*h)), Subst[Int[F^(a - (b\*h)/(d\*g - c\*h) + (d\*b\*x)/(d\*g - c\*h))/x, x], (g + h\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{f^{\frac{c}{a+bx}}}{x} dx &= a \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)} dx + b \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx \\
&= -\text{Ei}\left(\frac{c \log(f)}{a+bx}\right) + \text{Subst}\left(\int \frac{f^{\frac{c}{a}-\frac{bcx}{a}}}{x} dx, x, \frac{x}{a+bx}\right) \\
&= -\text{Ei}\left(\frac{c \log(f)}{a+bx}\right) + f^{\frac{c}{a}} \text{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0309776, size = 41, normalized size = 1.

$$f^{\frac{c}{a}} \text{Ei}\left(-\frac{bcx \log(f)}{a^2 + bxa}\right) - \text{Ei}\left(\frac{c \log(f)}{a+bx}\right)$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))/x,x]

[Out] -ExpIntegralEi[(c\*Log[f])/(a + b\*x)] + f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a^2 + a\*b\*x))]

**Maple [A]** time = 0.112, size = 47, normalized size = 1.2

$$-f^{\frac{c}{a}} \text{Ei}\left(1, -\frac{c \ln(f)}{bx+a} + \frac{c \ln(f)}{a}\right) + \text{Ei}\left(1, -\frac{c \ln(f)}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a))/x,x)

[Out] -f^(1/a\*c)\*Ei(1,-c\*ln(f)/(b\*x+a)+c\*ln(f)/a)+Ei(1,-c\*ln(f)/(b\*x+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(f^(c/(b\*x + a))/x, x)

**Fricas [A]** time = 1.56656, size = 89, normalized size = 2.17

$$f^{\frac{c}{a}} \text{Ei}\left(-\frac{bcx \log(f)}{abx + a^2}\right) - \text{Ei}\left(\frac{c \log(f)}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] f^(c/a)*Ei(-b*c*x*log(f)/(a*b*x + a^2)) - Ei(c*log(f)/(b*x + a))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{a+bx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c/(b*x+a))/x,x)
```

```
[Out] Integral(f**(c/(a + b*x))/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a))/x, x)
```

$$3.222 \quad \int \frac{f^{\frac{c}{a+bx}}}{x^2} dx$$

**Optimal.** Leaf size=68

$$-\frac{bc \log(f) f^{\frac{c}{a}} \text{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{a^2} - \frac{bf^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x}$$

[Out] -((b\*f^(c/(a + b\*x)))/a) - f^(c/(a + b\*x))/x - (b\*c\*f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a\*(a + b\*x)))]\*Log[f])/a^2

**Rubi [A]** time = 0.39663, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {2223, 6742, 2222, 2210, 2228, 2178, 2209}

$$-\frac{bc \log(f) f^{\frac{c}{a}} \text{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{a^2} - \frac{bf^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x))/x^2,x]

[Out] -((b\*f^(c/(a + b\*x)))/a) - f^(c/(a + b\*x))/x - (b\*c\*f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a\*(a + b\*x)))]\*Log[f])/a^2

#### Rule 2223

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] := Simp[((e + f*x)^(m + 1)*F^(a + b/(c + d*x)))/(f*(m + 1)), x] + D
ist[(b*d*Log[F])/(f*(m + 1)), Int[((e + f*x)^(m + 1)*F^(a + b/(c + d*x)))/(
c + d*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& ILtQ[m, -1]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2222

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbo
l] := Dist[d/f, Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Dist[(d*e - c*f
)/f, Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a,
b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

#### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

#### Rule 2228

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] := -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h
))/(d*g - c*h) + (d*b*x)/(d*g - c*h)/x, x], x, (g + h*x)/(c + d*x)], x] /;
```



FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{f^{\frac{c}{a+bx}}}{x^2} dx &= -\frac{f^{\frac{c}{a+bx}}}{x} - (bc \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)^2} dx \\
 &= -\frac{f^{\frac{c}{a+bx}}}{x} - (bc \log(f)) \int \left( \frac{f^{\frac{c}{a+bx}}}{a^2 x} - \frac{b f^{\frac{c}{a+bx}}}{a(a+bx)^2} - \frac{b f^{\frac{c}{a+bx}}}{a^2(a+bx)} \right) dx \\
 &= -\frac{f^{\frac{c}{a+bx}}}{x} - \frac{(bc \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x} dx}{a^2} + \frac{(b^2 c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx}{a^2} + \frac{(b^2 c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{(a+bx)^2} dx}{a} \\
 &= -\frac{b f^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x} - \frac{bc \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log(f)}{a^2} - \frac{(bc \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)} dx}{a} - \frac{(b^2 c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx}{a^2} \\
 &= -\frac{b f^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x} - \frac{(bc \log(f)) \operatorname{Subst}\left(\int \frac{f^{\frac{c}{a} - \frac{bcx}{a}}}{x} dx, x, \frac{x}{a+bx}\right)}{a^2} \\
 &= -\frac{b f^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x} - \frac{bc f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) \log(f)}{a^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.102447, size = 68, normalized size = 1.

$$-\frac{bc \log(f) f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a^2 + bxa}\right)}{a^2} - \frac{b f^{\frac{c}{a+bx}}}{a} - \frac{f^{\frac{c}{a+bx}}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))/x^2,x]

[Out] -((b\*f^(c/(a + b\*x)))/a) - f^(c/(a + b\*x))/x - (b\*c\*f^(c/a)\*ExpIntegralEi[-((b\*c\*x\*Log[f])/(a^2 + a\*b\*x))]\*Log[f])/a^2

**Maple [A]** time = 0.093, size = 80, normalized size = 1.2

$$\frac{cb \ln(f)}{a^2} f^{\frac{c}{bx+a}} \left( \frac{c \ln(f)}{bx+a} - \frac{c \ln(f)}{a} \right)^{-1} + \frac{cb \ln(f)}{a^2} f^{\frac{c}{a}} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{bx+a} + \frac{c \ln(f)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c/(b*x+a))/x^2,x)`

[Out]  $1/a^2*\ln(f)*b*c*f^{c/(b*x+a)}/(c*\ln(f)/(b*x+a)-c*\ln(f)/a)+1/a^2*\ln(f)*b*c*f^{(1/a*c)*Ei(1,-c*\ln(f)/(b*x+a)+c*\ln(f)/a)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))/x^2,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a))/x^2, x)`

**Fricas [A]** time = 1.54048, size = 131, normalized size = 1.93

$$\frac{bcf^{\frac{c}{a}}xEi\left(-\frac{bcx\log(f)}{abx+a^2}\right)\log(f) + (abx + a^2)f^{\frac{c}{bx+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))/x^2,x, algorithm="fricas")`

[Out]  $-(b*c*f^{c/a}*x*Ei(-b*c*x*\log(f)/(a*b*x + a^2))*\log(f) + (a*b*x + a^2)*f^{c/(b*x + a)})/(a^2*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a))/x**2,x)`

[Out] `Integral(f**(c/(a + b*x))/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a))/x^2,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a))/x^2, x)`

### 3.223 $\int \frac{f^{\frac{c}{a+bx}}}{x^3} dx$

**Optimal.** Leaf size=166

$$\frac{b^2 c^2 \log^2(f) f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{2a^4} + \frac{b^2 c \log(f) f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{a^3} + \frac{b^2 f^{\frac{c}{a+bx}}}{2a^2} + \frac{b^2 c \log(f) f^{\frac{c}{a+bx}}}{2a^3} + \frac{bc \log(f) f^{\frac{c}{a+bx}}}{2a^2 x} - \frac{f^{\frac{c}{a+bx}}}{2x^2}$$

[Out]  $(b^2 f^{c/(a+bx)})/(2a^2) - f^{c/(a+bx)}/(2x^2) + (b^2 c f^{c/(a+bx)} \operatorname{Log}[f])/(2a^3) + (b^2 c f^{c/(a+bx)} \operatorname{Log}[f])/(2a^2 x) + (b^2 c f^{c/a} \operatorname{ExpIntegralEi}[-(b^2 c x \operatorname{Log}[f])/(a(a+bx))]) \operatorname{Log}[f]/a^3 + (b^2 c^2 f^{c/a} \operatorname{ExpIntegralEi}[-(b^2 c x \operatorname{Log}[f])/(a(a+bx))]) \operatorname{Log}[f]^2/(2a^4)$

**Rubi [A]** time = 0.718332, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {2223, 6742, 2222, 2210, 2228, 2178, 2209}

$$\frac{b^2 c^2 \log^2(f) f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{2a^4} + \frac{b^2 c \log(f) f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{a^3} + \frac{b^2 f^{\frac{c}{a+bx}}}{2a^2} + \frac{b^2 c \log(f) f^{\frac{c}{a+bx}}}{2a^3} + \frac{bc \log(f) f^{\frac{c}{a+bx}}}{2a^2 x} - \frac{f^{\frac{c}{a+bx}}}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{c/(a+bx)}/x^3, x]$

[Out]  $(b^2 f^{c/(a+bx)})/(2a^2) - f^{c/(a+bx)}/(2x^2) + (b^2 c f^{c/(a+bx)} \operatorname{Log}[f])/(2a^3) + (b^2 c f^{c/(a+bx)} \operatorname{Log}[f])/(2a^2 x) + (b^2 c f^{c/a} \operatorname{ExpIntegralEi}[-(b^2 c x \operatorname{Log}[f])/(a(a+bx))]) \operatorname{Log}[f]/a^3 + (b^2 c^2 f^{c/a} \operatorname{ExpIntegralEi}[-(b^2 c x \operatorname{Log}[f])/(a(a+bx))]) \operatorname{Log}[f]^2/(2a^4)$

#### Rule 2223

$\operatorname{Int}[(F_)^{((a_.) + (b_.)/((c_.) + (d_.)(x_)))} * ((e_.) + (f_.)(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e + f*x)^{(m+1)} * F^{(a + b/(c + d*x))} / (f*(m+1)), x] + \operatorname{Dist}[(b*d*\operatorname{Log}[F]) / (f*(m+1)), \operatorname{Int}[(e + f*x)^{(m+1)} * F^{(a + b/(c + d*x))} / (c + d*x)^2, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[d*e - c*f, 0] \ \&\& \operatorname{ILtQ}[m, -1]$

#### Rule 6742

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{ExpandIntegrand}[u, x]\}, \operatorname{Int}[v, x] /;$   $\operatorname{SumQ}[v]$

#### Rule 2222

$\operatorname{Int}[(F_)^{((a_.) + (b_.)/((c_.) + (d_.)(x_)))} / ((e_.) + (f_.)(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[d/f, \operatorname{Int}[F^{(a + b/(c + d*x))} / (c + d*x), x], x] - \operatorname{Dist}[(d*e - c*f) / f, \operatorname{Int}[F^{(a + b/(c + d*x))} / ((c + d*x)*(e + f*x)), x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[d*e - c*f, 0]$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)((c_.) + (d_.)(x_)))^{(n_.)}} / ((e_.) + (f_.)(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[F^{a*} \operatorname{ExpIntegralEi}[b*(c + d*x)^n * \operatorname{Log}[F]] / (f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x \ \&\& \operatorname{EqQ}[d*e - c*f, 0]$

Rule 2228

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] :> -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h
)/(d*g - c*h) + (d*b*x)/(d*g - c*h))/x, x], x, (g + h*x)/(c + d*x)], x] /;
FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !UseGamma == True
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{f^{\frac{c}{a+bx}}}{x^3} dx &= -\frac{f^{\frac{c}{a+bx}}}{2x^2} - \frac{1}{2}(bc \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x^2(a+bx)^2} dx \\
&= -\frac{f^{\frac{c}{a+bx}}}{2x^2} - \frac{1}{2}(bc \log(f)) \int \left( \frac{f^{\frac{c}{a+bx}}}{a^2x^2} - \frac{2bf^{\frac{c}{a+bx}}}{a^3x} + \frac{b^2f^{\frac{c}{a+bx}}}{a^2(a+bx)^2} + \frac{2b^2f^{\frac{c}{a+bx}}}{a^3(a+bx)} \right) dx \\
&= -\frac{f^{\frac{c}{a+bx}}}{2x^2} - \frac{(bc \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x^2} dx}{2a^2} + \frac{(b^2c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x} dx}{a^3} - \frac{(b^3c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{a+bx} dx}{a^3} - \frac{(b^3c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{(a+bx)^2} dx}{2a^2} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{b^2c \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log(f)}{a^3} + \frac{(b^2c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)} dx}{a^2} + \frac{(b^3c \log(f)) \int \frac{f^{\frac{c}{a+bx}}}{(a+bx)^2} dx}{a^3} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{(b^2c \log(f)) \operatorname{Subst}\left(\int \frac{f^{\frac{c}{a+bx}}}{x} dx, x, \frac{x}{a+bx}\right)}{a^3} + \frac{(b^2c^2 \log^2(f)) \int \left(\frac{f^{\frac{c}{a+bx}}}{a^2x} - \frac{f^{\frac{c}{a+bx}}}{a^2(a+bx)}\right) dx}{2a^2} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{b^2cf^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) \log(f)}{a^3} + \frac{(b^2c^2 \log^2(f)) \int \frac{f^{\frac{c}{a+bx}}}{x} dx}{2a^4} - \frac{(b^3c^2 \log^2(f)) \int \frac{f^{\frac{c}{a+bx}}}{(a+bx)^2} dx}{2a^2} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{b^2cf^{\frac{c}{a+bx}} \log(f)}{2a^3} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{b^2cf^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) \log(f)}{a^3} + \frac{b^2c^2 \operatorname{Ei}\left(\frac{c \log(f)}{a+bx}\right) \log^2(f)}{2a^4} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{b^2cf^{\frac{c}{a+bx}} \log(f)}{2a^3} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{b^2cf^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) \log(f)}{a^3} + \frac{(b^2c^2 \log^2(f)) \operatorname{Subst}\left(\int \frac{f^{\frac{c}{a+bx}}}{x} dx, x, \frac{x}{a+bx}\right)}{2a^2} \\
&= \frac{b^2f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{b^2cf^{\frac{c}{a+bx}} \log(f)}{2a^3} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2x} + \frac{b^2cf^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right) \log(f)}{a^3} + \frac{b^2c^2 f^{\frac{c}{a}} \operatorname{Ei}\left(-\frac{bcx \log(f)}{a(a+bx)}\right)}{2a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.185483, size = 115, normalized size = 0.69

$$\frac{b^2c \log(f) f^{\frac{c}{a}} (2a + c \log(f)) \operatorname{Ei}\left(-\frac{bcx \log(f)}{a^2+bx}\right) - \frac{a^2 f^{\frac{c}{a+bx}} (a^2 + b^2x^2 - bcx \log(f))}{x^2}}{2a^4} + \frac{b^2(2a + c \log(f)) f^{\frac{c}{a+bx}}}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x))/x^3,x]

[Out]  $(b^2*f^{c/(a + b*x)}*(2*a + c*\text{Log}[f]))/(2*a^3) + (b^2*c*f^{c/a}*\text{ExpIntegralEi}[-((b*c*x*\text{Log}[f])/(a^2 + a*b*x))]*\text{Log}[f]*(2*a + c*\text{Log}[f]) - (a^2*f^{c/(a + b*x)}*(a^2 + b^2*x^2 - b*c*x*\text{Log}[f]))/x^2)/(2*a^4)$

**Maple [A]** time = 0.097, size = 226, normalized size = 1.4

$$-\frac{b^2 c \ln(f)}{a^3} f^{\frac{c}{bx+a}} \left( \frac{c \ln(f)}{bx+a} - \frac{c \ln(f)}{a} \right)^{-1} - \frac{b^2 c \ln(f)}{a^3} f^{\frac{c}{a}} \text{Ei} \left( 1, -\frac{c \ln(f)}{bx+a} + \frac{c \ln(f)}{a} \right) - \frac{b^2 c^2 (\ln(f))^2}{2 a^4} f^{\frac{c}{bx+a}} \left( \frac{c \ln(f)}{bx+a} - \frac{c \ln(f)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a))/x^3,x)

[Out]  $-b^2*c*\ln(f)/a^3*f^{c/(b*x+a)}/(c*\ln(f)/(b*x+a)-c*\ln(f)/a)-b^2*c*\ln(f)/a^3*f^{1/a*c}*\text{Ei}(1,-c*\ln(f)/(b*x+a)+c*\ln(f)/a)-1/2*b^2*c^2*\ln(f)^2/a^4*f^{c/(b*x+a)}/(c*\ln(f)/(b*x+a)-c*\ln(f)/a)^2-1/2*b^2*c^2*\ln(f)^2/a^4*f^{c/(b*x+a)}/(c*\ln(f)/(b*x+a)-c*\ln(f)/a)-1/2*b^2*c^2*\ln(f)^2/a^4*f^{1/a*c}*\text{Ei}(1,-c*\ln(f)/(b*x+a)+c*\ln(f)/a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))/x^3,x, algorithm="maxima")

[Out] integrate(f^(c/(b\*x + a))/x^3, x)

**Fricas [A]** time = 1.54697, size = 238, normalized size = 1.43

$$\frac{(b^2 c^2 x^2 \log(f)^2 + 2 a b^2 c x^2 \log(f)) f^{\frac{c}{a}} \text{Ei} \left( -\frac{b c x \log(f)}{a b x + a^2} \right) + (a^2 b^2 x^2 - a^4 + (a b^2 c x^2 + a^2 b c x) \log(f)) f^{\frac{c}{bx+a}}}{2 a^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))/x^3,x, algorithm="fricas")

[Out]  $1/2*((b^2*c^2*x^2*\log(f)^2 + 2*a*b^2*c*x^2*\log(f))*f^{c/a}*\text{Ei}(-b*c*x*\log(f)/(a*b*x + a^2)) + (a^2*b^2*x^2 - a^4 + (a*b^2*c*x^2 + a^2*b*c*x)*\log(f))*f^{c/(b*x + a)})/(a^4*x^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a))/x\*\*3,x)

[Out] Integral(f\*\*(c/(a + b\*x))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{bx+a}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))/x^3,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a))/x^3, x)

### 3.224 $\int f^{\frac{c}{(a+bx)^2}} x^4 dx$

**Optimal.** Leaf size=415

$$\frac{4\sqrt{\pi}a^2c^{3/2}\log^3(f)\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} - \frac{\sqrt{\pi}a^4\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} + \frac{2a^3c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^5} + \frac{2a^2(a+bx)^3f^{\frac{c}{(a+bx)^2}}}{b^5}$$

```
[Out] (a^4*f^(c/(a + b*x)^2)*(a + b*x))/b^5 - (2*a^3*f^(c/(a + b*x)^2)*(a + b*x)^2)/b^5 + (2*a^2*f^(c/(a + b*x)^2)*(a + b*x)^3)/b^5 - (a*f^(c/(a + b*x)^2)*(a + b*x)^4)/b^5 + (f^(c/(a + b*x)^2)*(a + b*x)^5)/(5*b^5) - (a^4*Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]])/b^5 + (4*a^2*c*f^(c/(a + b*x)^2)*(a + b*x)*Log[f])/b^5 - (a*c*f^(c/(a + b*x)^2)*(a + b*x)^2*Log[f])/b^5 + (2*c*f^(c/(a + b*x)^2)*(a + b*x)^3*Log[f])/(15*b^5) + (2*a^3*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f])/b^5 - (4*a^2*c^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(3/2))/b^5 + (4*c^2*f^(c/(a + b*x)^2)*(a + b*x)*Log[f]^2)/(15*b^5) + (a*c^2*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f]^2)/b^5 - (4*c^(5/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(5/2))/(15*b^5)
```

**Rubi [A]** time = 0.435434, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$\frac{4\sqrt{\pi}a^2c^{3/2}\log^3(f)\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} - \frac{\sqrt{\pi}a^4\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} + \frac{2a^3c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^5} + \frac{2a^2(a+bx)^3f^{\frac{c}{(a+bx)^2}}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[f^(c/(a + b*x)^2)*x^4, x]
```

```
[Out] (a^4*f^(c/(a + b*x)^2)*(a + b*x))/b^5 - (2*a^3*f^(c/(a + b*x)^2)*(a + b*x)^2)/b^5 + (2*a^2*f^(c/(a + b*x)^2)*(a + b*x)^3)/b^5 - (a*f^(c/(a + b*x)^2)*(a + b*x)^4)/b^5 + (f^(c/(a + b*x)^2)*(a + b*x)^5)/(5*b^5) - (a^4*Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]])/b^5 + (4*a^2*c*f^(c/(a + b*x)^2)*(a + b*x)*Log[f])/b^5 - (a*c*f^(c/(a + b*x)^2)*(a + b*x)^2*Log[f])/b^5 + (2*c*f^(c/(a + b*x)^2)*(a + b*x)^3*Log[f])/(15*b^5) + (2*a^3*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f])/b^5 - (4*a^2*c^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(3/2))/b^5 + (4*c^2*f^(c/(a + b*x)^2)*(a + b*x)*Log[f]^2)/(15*b^5) + (a*c^2*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f]^2)/b^5 - (4*c^(5/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(5/2))/(15*b^5)
```

#### Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^(n_.))*(u_), x_Symbol] := Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]
```

#### Rule 2206

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^(n_.)), x_Symbol] := Simp[((c + d*x)*F^(a + b*(c + d*x)^n))/d, x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]
```

Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^2}} x^4 dx &= \int \left( \frac{a^4 f^{\frac{c}{(a+bx)^2}}}{b^4} - \frac{4a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} + \frac{6a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^4} - \frac{4a f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^4} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^2}} (a+bx)^4 dx}{b^4} - \frac{(4a) \int f^{\frac{c}{(a+bx)^2}} (a+bx)^3 dx}{b^4} + \frac{(6a^2) \int f^{\frac{c}{(a+bx)^2}} (a+bx)^2 dx}{b^4} - \frac{(4a^3) \int f^{\frac{c}{(a+bx)^2}} (a+bx) dx}{b^4} + \frac{\int f^{\frac{c}{(a+bx)^2}} dx}{b^4} \\ &= \frac{a^4 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^5} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^5}{5b^5} \\ &= \frac{a^4 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^5} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^5}{5b^5} \\ &= \frac{a^4 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^5} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^5}{5b^5} \\ &= \frac{a^4 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^5} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^5}{5b^5} \\ &= \frac{a^4 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{b^5} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^5}{5b^5} \end{aligned}$$

**Mathematica [A]** time = 0.202968, size = 195, normalized size = 0.47

$$\frac{bx f^{\frac{c}{(a+bx)^2}} \left( c \log(f) (36a^2 - 9abx + 2b^2x^2) + 3b^4x^4 + 4c^2 \log^2(f) \right) - \sqrt{\pi} \sqrt{c} \sqrt{\log(f)} (60a^2c \log(f) + 15a^4 + 4c^2 \log^2(f)) E}{5b^5}$$

Antiderivative was successfully verified.



[In] Integrate[f^(c/(a + b\*x)^2)\*x^4,x]

[Out] (a\*f^(c/(a + b\*x)^2)\*(3\*a^4 + 47\*a^2\*c\*Log[f] + 4\*c^2\*Log[f]^2))/(15\*b^5) + (15\*a\*c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^2]\*Log[f]\*(2\*a^2 + c\*Log[f]) - Sqrt[c]\*Sqrt[Pi]\*Erfi[(Sqrt[c]\*Sqrt[Log[f]])/(a + b\*x)]\*Sqrt[Log[f]]\*(15\*a^4 + 60\*a^2\*c\*Log[f] + 4\*c^2\*Log[f]^2) + b\*f^(c/(a + b\*x)^2)\*x\*(3\*b^4\*x^4 + c\*(36\*a^2 - 9\*a\*b\*x + 2\*b^2\*x^2)\*Log[f] + 4\*c^2\*Log[f]^2))/(15\*b^5)

**Maple [A]** time = 0.076, size = 343, normalized size = 0.8

$$-\frac{a^4 c \ln(f) \sqrt{\pi}}{b^5} \operatorname{Erf}\left(\frac{1}{bx+a} \sqrt{-c \ln(f)}\right) \frac{1}{\sqrt{-c \ln(f)}} + \frac{x^5}{5} f^{\frac{c}{(bx+a)^2}} + \frac{a^5}{5 b^5} f^{\frac{c}{(bx+a)^2}} + \frac{12 a^2 c \ln(f) x}{5 b^4} f^{\frac{c}{(bx+a)^2}} - 4 \frac{(\ln(f))^2 a}{b^5 \sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)\*x^4,x)

[Out] -1/b^5\*a^4\*ln(f)\*c\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))+ 1/5\*f^(c/(b\*x+a)^2)\*x^5+1/5/b^5\*a^5\*f^(c/(b\*x+a)^2)+12/5/b^4\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*a^2\*x-4/b^5\*a^2\*ln(f)^2\*c^2\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))-1/b^5\*a\*ln(f)^2\*c^2\*Ei(1,-c\*ln(f)/(b\*x+a)^2)+47/15/b^5\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*a^3-3/5/b^3\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*a\*x^2-2/b^5\*a^3\*ln(f)\*c\*Ei(1,-c\*ln(f)/(b\*x+a)^2)+4/15/b^5\*ln(f)^2\*c^2\*f^(c/(b\*x+a)^2)\*a+2/15/b^2\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*x^3+4/15/b^4\*ln(f)^2\*c^2\*f^(c/(b\*x+a)^2)\*x-4/15/b^5\*ln(f)^3\*c^3\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(3 b^4 x^5 + 2 b^2 c x^3 \log(f) - 9 a b c x^2 \log(f) + 4 (9 a^2 c \log(f) + c^2 \log(f)^2) x) f^{\frac{c}{b^2 x^2 + 2 a b x + a^2}}}{15 b^4} - \int \frac{2 (18 a^5 c \log(f) + 2 a^3 c^2 \log(f)^2)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^4,x, algorithm="maxima")

[Out] 1/15\*(3\*b^4\*x^5 + 2\*b^2\*c\*x^3\*log(f) - 9\*a\*b\*c\*x^2\*log(f) + 4\*(9\*a^2\*c\*log(f) + c^2\*log(f)^2)\*x)\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))/b^4 - integrate(2/15\*(18\*a^5\*c\*log(f) + 2\*a^3\*c^2\*log(f)^2 + 15\*(2\*a^3\*b^2\*c\*log(f) + a\*b^2\*c^2\*log(f)^2)\*x^2 + (45\*a^4\*b\*c\*log(f) - 30\*a^2\*b\*c^2\*log(f)^2 - 4\*b\*c^3\*log(f)^3)\*x)\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))/(b^7\*x^3 + 3\*a\*b^6\*x^2 + 3\*a^2\*b^5\*x + a^3\*b^4), x)

**Fricas [A]** time = 1.85563, size = 481, normalized size = 1.16

$$\sqrt{\pi} \left( 15 a^4 b + 60 a^2 b c \log(f) + 4 b c^2 \log(f)^2 \right) \sqrt{-\frac{c \log(f)}{b^2}} \operatorname{erf}\left(\frac{b \sqrt{-\frac{c \log(f)}{b^2}}}{bx+a}\right) + \left( 3 b^5 x^5 + 3 a^5 + 4 (b c^2 x + a c^2) \log(f)^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^4,x, algorithm="fricas")

[Out] 1/15\*(sqrt(pi)\*(15\*a^4\*b + 60\*a^2\*b\*c\*log(f) + 4\*b\*c^2\*log(f)^2)\*sqrt(-c\*log(f)/b^2)\*erf(b\*sqrt(-c\*log(f)/b^2)/(b\*x + a)) + (3\*b^5\*x^5 + 3\*a^5 + 4\*(b\*c^2\*x + a\*c^2)\*log(f)^2 + (2\*b^3\*c\*x^3 - 9\*a\*b^2\*c\*x^2 + 36\*a^2\*b\*c\*x + 47\*a^3\*c)\*log(f))\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + 15\*(2\*a^3\*c\*log(f) + a\*c^2\*log(f)^2)\*Ei(c\*log(f)/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/b^5

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*2)\*x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^4,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^2)\*x^4, x)

### 3.225 $\int f^{\frac{c}{(a+bx)^2}} x^3 dx$

**Optimal.** Leaf size=291

$$\frac{\sqrt{\pi}a^3\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^4} - \frac{3a^2c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{2b^4} + \frac{3a^2(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{2b^4} - \frac{a^3(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^4} + \frac{2\sqrt{\pi}ac^{3/2}\log(f)}{b^4}$$

[Out]  $-\left(\frac{a^3f^{c/(a+bx)^2}(a+bx)}{b^4}\right) + \frac{3a^2f^{c/(a+bx)^2}(a+bx)^2}{2b^4} - \frac{af^{c/(a+bx)^2}(a+bx)^3}{b^4} + \frac{f^{c/(a+bx)^2}(a+bx)^4}{4b^4} + \frac{a^3\sqrt{c}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^4} - \frac{2acf^{c/(a+bx)^2}(a+bx)\log(f)}{b^4} + \frac{cf^{c/(a+bx)^2}(a+bx)^2\log(f)}{4b^4} - \frac{3a^2c\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{(a+bx)^2} + \frac{2ac^{3/2}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log(f)^{3/2}}{(a+bx)} - \frac{c^2\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)^2}{4b^4}$

**Rubi [A]** time = 0.303768, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$\frac{\sqrt{\pi}a^3\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^4} - \frac{3a^2c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{2b^4} + \frac{3a^2(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{2b^4} - \frac{a^3(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^4} + \frac{2\sqrt{\pi}ac^{3/2}\log(f)}{b^4}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^2)\*x^3,x]

[Out]  $-\left(\frac{a^3f^{c/(a+bx)^2}(a+bx)}{b^4}\right) + \frac{3a^2f^{c/(a+bx)^2}(a+bx)^2}{2b^4} - \frac{af^{c/(a+bx)^2}(a+bx)^3}{b^4} + \frac{f^{c/(a+bx)^2}(a+bx)^4}{4b^4} + \frac{a^3\sqrt{c}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^4} - \frac{2acf^{c/(a+bx)^2}(a+bx)\log(f)}{b^4} + \frac{cf^{c/(a+bx)^2}(a+bx)^2\log(f)}{4b^4} - \frac{3a^2c\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{(a+bx)^2} + \frac{2ac^{3/2}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log(f)^{3/2}}{(a+bx)} - \frac{c^2\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)^2}{4b^4}$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>n</sup>))\*((c\_.) + (d\_.)\*(x\_))<sup>m</sup>), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>n</sup>))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^2}} x^3 dx &= \int \left( -\frac{a^3 f^{\frac{c}{(a+bx)^2}}}{b^3} + \frac{3a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^3} - \frac{3af^{\frac{c}{(a+bx)^2}} (a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^3} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^2}} (a+bx)^3 dx}{b^3} - \frac{(3a) \int f^{\frac{c}{(a+bx)^2}} (a+bx)^2 dx}{b^3} + \frac{(3a^2) \int f^{\frac{c}{(a+bx)^2}} (a+bx) dx}{b^3} - \frac{a^3 \int f^{\frac{c}{(a+bx)^2}} dx}{b^3} \\ &= -\frac{a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{2b^4} - \frac{af^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{4b^4} + \frac{(c \log(f)) \int f^{\frac{c}{(a+bx)^2}} dx}{2b^3} \\ &= -\frac{a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{2b^4} - \frac{af^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{4b^4} - \frac{2acf^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} \\ &= -\frac{a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{2b^4} - \frac{af^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{4b^4} + \frac{a^3 \sqrt{c} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{c} \sqrt{\log(f)}}{a+bx} \right)}{b^4} \\ &= -\frac{a^3 f^{\frac{c}{(a+bx)^2}} (a+bx)}{b^4} + \frac{3a^2 f^{\frac{c}{(a+bx)^2}} (a+bx)^2}{2b^4} - \frac{af^{\frac{c}{(a+bx)^2}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^2}} (a+bx)^4}{4b^4} + \frac{a^3 \sqrt{c} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{c} \sqrt{\log(f)}}{a+bx} \right)}{b^4} \end{aligned}$$

**Mathematica [A]** time = 0.136661, size = 148, normalized size = 0.51

$$\frac{4\sqrt{\pi}a\sqrt{c}\sqrt{\log(f)}(a^2 + 2c\log(f))\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right) - c\log(f)(6a^2 + c\log(f))\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right) + bxf^{\frac{c}{(a+bx)^2}}(-6ac\log(f) + b^3x^2)}{4b^4}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^2)\*x^3,x]

[Out] -(a^2\*f^(c/(a + b\*x)^2)\*(a^2 + 7\*c\*Log[f]))/(4\*b^4) + (-c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^2]\*Log[f]\*(6\*a^2 + c\*Log[f])) + 4\*a\*Sqrt[c]\*Sqrt[Pi]\*Erfi[(Sqrt[c]\*Sqrt[Log[f]])/(a + b\*x)]\*Sqrt[Log[f]]\*(a^2 + 2\*c\*Log[f]) + b\*f^(c/(a + b\*x)^2)\*x\*(b^3\*x^3 - 6\*a\*c\*Log[f] + b\*c\*x\*Log[f])/ (4\*b^4)

**Maple [A]** time = 0.046, size = 228, normalized size = 0.8

$$\frac{x^4}{4} f^{\frac{c}{(bx+a)^2}} - \frac{a^4}{4b^4} f^{\frac{c}{(bx+a)^2}} + \frac{c \ln(f) x^2}{4b^2} f^{\frac{c}{(bx+a)^2}} - \frac{3ac \ln(f) x}{2b^3} f^{\frac{c}{(bx+a)^2}} - \frac{7a^2 c \ln(f)}{4b^4} f^{\frac{c}{(bx+a)^2}} + \frac{(\ln(f))^2 c^2}{4b^4} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{(bx+a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)\*x^3,x)

[Out]  $\frac{1}{4} f^{\frac{c}{(bx+a)^2}} x^4 - \frac{1}{4} \frac{a^4}{b^4} f^{\frac{c}{(bx+a)^2}} + \frac{1}{4} \frac{c \ln(f) x^2}{b^2} f^{\frac{c}{(bx+a)^2}} - \frac{3}{2} \frac{ac \ln(f) x}{b^3} f^{\frac{c}{(bx+a)^2}} - \frac{7}{4} \frac{a^2 c \ln(f)}{b^4} f^{\frac{c}{(bx+a)^2}} + \frac{(\ln(f))^2 c^2}{4b^4} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{(bx+a)^2}\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 x^4 + bcx^2 \log(f) - 6acx \log(f)) f^{\frac{c}{b^2 x^2 + 2abx + a^2}}}{4b^3} + \int \frac{(3a^4 c \log(f) + (6a^2 b^2 c \log(f) + b^2 c^2 \log(f)^2) x^2 + 2(4a^3 bc \log(f) + b^2 c^2 \log(f)^2) x)}{2(b^6 x^3 + 3ab^5 x^2 + 3a^2 b^4 x + a^3 b^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^3,x, algorithm="maxima")

[Out]  $\frac{1}{4} (b^3 x^4 + bcx^2 \log(f) - 6acx \log(f)) f^{\frac{c}{b^2 x^2 + 2abx + a^2}} + \int \frac{(3a^4 c \log(f) + (6a^2 b^2 c \log(f) + b^2 c^2 \log(f)^2) x^2 + 2(4a^3 bc \log(f) + b^2 c^2 \log(f)^2) x)}{2(b^6 x^3 + 3ab^5 x^2 + 3a^2 b^4 x + a^3 b^3)} dx$

**Fricas [A]** time = 1.935, size = 366, normalized size = 1.26

$$\frac{4 \sqrt{\pi} (a^3 b + 2abc \log(f)) \sqrt{-\frac{c \log(f)}{b^2}} \operatorname{erf}\left(\frac{b \sqrt{-\frac{c \log(f)}{b^2}}}{bx+a}\right) - (b^4 x^4 - a^4 + (b^2 c x^2 - 6abcx - 7a^2 c) \log(f)) f^{\frac{c}{b^2 x^2 + 2abx + a^2}}}{4b^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^3,x, algorithm="fricas")

[Out]  $-\frac{1}{4} (4 \sqrt{\pi} (a^3 b + 2abc \log(f)) \sqrt{-\frac{c \log(f)}{b^2}} \operatorname{erf}\left(\frac{b \sqrt{-\frac{c \log(f)}{b^2}}}{bx+a}\right) - (b^4 x^4 - a^4 + (b^2 c x^2 - 6abcx - 7a^2 c) \log(f)) f^{\frac{c}{b^2 x^2 + 2abx + a^2}}) / b^4$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c/(b*x+a)**2)*x**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a)^2)*x^3, x)
```

### 3.226 $\int f^{\frac{c}{(a+bx)^2}} x^2 dx$

**Optimal.** Leaf size=206

$$-\frac{\sqrt{\pi}a^2\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^3} + \frac{a^2(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^3} - \frac{2\sqrt{\pi}c^{3/2}\log^2(f)\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{3b^3} + \frac{ac\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^3} + \dots$$

[Out]  $(a^2f^{(c/(a+bx)^2)}(a+bx))/b^3 - (af^{(c/(a+bx)^2)}(a+bx)^2)/b^3 + (f^{(c/(a+bx)^2)}(a+bx)^3)/(3b^3) - (a^2\sqrt{c}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{c}\sqrt{\log(f)})/(a+bx)]\sqrt{\log(f)})/b^3 + (2cf^{(c/(a+bx)^2)}(a+bx)\log(f))/(3b^3) + (ac\operatorname{ExpIntegralEi}[(c\log(f))/(a+bx)^2]\log(f))/b^3 - (2c^{3/2}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{c}\sqrt{\log(f)})/(a+bx)]\log(f)^{(3/2)})/(3b^3)$

**Rubi [A]** time = 0.214925, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$-\frac{\sqrt{\pi}a^2\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^3} + \frac{a^2(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^3} - \frac{2\sqrt{\pi}c^{3/2}\log^2(f)\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{3b^3} + \frac{ac\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^3} + \dots$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a+bx)^2)\*x^2,x]

[Out]  $(a^2f^{(c/(a+bx)^2)}(a+bx))/b^3 - (af^{(c/(a+bx)^2)}(a+bx)^2)/b^3 + (f^{(c/(a+bx)^2)}(a+bx)^3)/(3b^3) - (a^2\sqrt{c}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{c}\sqrt{\log(f)})/(a+bx)]\sqrt{\log(f)})/b^3 + (2cf^{(c/(a+bx)^2)}(a+bx)\log(f))/(3b^3) + (ac\operatorname{ExpIntegralEi}[(c\log(f))/(a+bx)^2]\log(f))/b^3 - (2c^{3/2}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{c}\sqrt{\log(f)})/(a+bx)]\log(f)^{(3/2)})/(3b^3)$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F\*a\*Sqrt[\pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{

F, a, b, c, d}, x] && PosQ[b]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^2}} x^2 dx &= \int \left( \frac{a^2 f^{\frac{c}{(a+bx)^2}}}{b^2} - \frac{2af^{\frac{c}{(a+bx)^2}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^2} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^2}}(a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{\frac{c}{(a+bx)^2}}(a+bx) dx}{b^2} + \frac{a^2 \int f^{\frac{c}{(a+bx)^2}} dx}{b^2} \\ &= \frac{a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^3} - \frac{af^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{3b^3} + \frac{(2c \log(f)) \int f^{\frac{c}{(a+bx)^2}} dx}{3b^2} - \frac{(2ac \log(f)) \int f^{\frac{c}{(a+bx)^2}} dx}{b^2} \\ &= \frac{a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^3} - \frac{af^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{3b^3} + \frac{2cf^{\frac{c}{(a+bx)^2}}(a+bx) \log(f)}{3b^3} + \frac{ac \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{b^3} \\ &= \frac{a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^3} - \frac{af^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{3b^3} - \frac{a^2 \sqrt{c} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{c} \sqrt{\log(f)}}{a+bx}\right) \sqrt{\log(f)}}{b^3} + \frac{2cf^{\frac{c}{(a+bx)^2}} \log(f)}{3b^3} \\ &= \frac{a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^3} - \frac{af^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{3b^3} - \frac{a^2 \sqrt{c} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{c} \sqrt{\log(f)}}{a+bx}\right) \sqrt{\log(f)}}{b^3} + \frac{2cf^{\frac{c}{(a+bx)^2}} \log(f)}{3b^3} \end{aligned}$$

**Mathematica [A]** time = 0.103422, size = 131, normalized size = 0.64

$$\frac{-\sqrt{\pi} \sqrt{c} \sqrt{\log(f)} (3a^2 + 2c \log(f)) \operatorname{Erfi}\left(\frac{\sqrt{c} \sqrt{\log(f)}}{a+bx}\right) + b x f^{\frac{c}{(a+bx)^2}} (b^2 x^2 + 2c \log(f)) + 3ac \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{3b^3} + \frac{a (a^2 + 2c \log(f))}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^2)\*x^2,x]

[Out] (a\*f^(c/(a + b\*x)^2)\*(a^2 + 2\*c\*Log[f]))/(3\*b^3) + (3\*a\*c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^2]\*Log[f] - Sqrt[c]\*Sqrt[Pi]\*Erfi[(Sqrt[c]\*Sqrt[Log[f]])/(a + b\*x)]\*Sqrt[Log[f]]\*(3\*a^2 + 2\*c\*Log[f]) + b\*f^(c/(a + b\*x)^2)\*x\*(b^2\*x^2 + 2\*c\*Log[f]))/(3\*b^3)



**Maple [A]** time = 0.04, size = 175, normalized size = 0.9

$$\frac{x^3}{3} f^{\frac{c}{(bx+a)^2}} + \frac{a^3}{3b^3} f^{\frac{c}{(bx+a)^2}} + \frac{2c \ln(f)x}{3b^2} f^{\frac{c}{(bx+a)^2}} + \frac{2ac \ln(f)}{3b^3} f^{\frac{c}{(bx+a)^2}} - \frac{2(\ln(f))^2 c^2 \sqrt{\pi}}{3b^3} \operatorname{Erf}\left(\frac{1}{bx+a} \sqrt{-c \ln(f)}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)\*x^2,x)

[Out] 1/3\*f^(c/(b\*x+a)^2)\*x^3+1/3/b^3\*a^3\*f^(c/(b\*x+a)^2)+2/3/b^2\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*x+2/3/b^3\*ln(f)\*c\*f^(c/(b\*x+a)^2)\*a-2/3/b^3\*ln(f)^2\*c^2\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))-1/b^3\*a^2\*ln(f)\*c\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))-1/b^3\*a\*ln(f)\*c\*Ei(1,-c\*ln(f)/(b\*x+a)^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2x^3 + 2cx \log(f)) f^{\frac{c}{b^2x^2 + 2abx + a^2}}}{3b^2} - \int \frac{2(3ab^2cx^2 \log(f) + a^3c \log(f) + (3a^2bc \log(f) - 2bc^2 \log(f)^2)x) f^{\frac{c}{b^2x^2 + 2abx + a^2}}}{3(b^5x^3 + 3ab^4x^2 + 3a^2b^3x + a^3b^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^2,x, algorithm="maxima")

[Out] 1/3\*(b^2\*x^3 + 2\*c\*x\*log(f))\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))/b^2 - integrate(2/3\*(3\*a\*b^2\*c\*x^2\*log(f) + a^3\*c\*log(f) + (3\*a^2\*b\*c\*log(f) - 2\*b\*c^2\*log(f)^2)\*x)\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))/(b^5\*x^3 + 3\*a\*b^4\*x^2 + 3\*a^2\*b^3\*x + a^3\*b^2), x)

**Fricas [A]** time = 1.8878, size = 312, normalized size = 1.51

$$\frac{3ac \operatorname{Ei}\left(\frac{c \log(f)}{b^2x^2 + 2abx + a^2}\right) \log(f) + \sqrt{\pi}(3a^2b + 2bc \log(f)) \sqrt{-\frac{c \log(f)}{b^2}} \operatorname{erf}\left(\frac{b \sqrt{-\frac{c \log(f)}{b^2}}}{bx+a}\right) + (b^3x^3 + a^3 + 2(bc x + ac) \log(f))}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^2,x, algorithm="fricas")

[Out] 1/3\*(3\*a\*c\*Ei(c\*log(f)/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*log(f) + sqrt(pi)\*(3\*a^2\*b + 2\*b\*c\*log(f))\*sqrt(-c\*log(f)/b^2)\*erf(b\*sqrt(-c\*log(f)/b^2)/(b\*x + a)) + (b^3\*x^3 + a^3 + 2\*(b\*c\*x + a\*c)\*log(f))\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/b^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c/(b*x+a)**2)*x**2,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a)^2)*x^2, x)
```

### 3.227 $\int f^{\frac{c}{(a+bx)^2}} x dx$

**Optimal.** Leaf size=111

$$\frac{\sqrt{\pi}a\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^2} - \frac{c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{2b^2} + \frac{(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{2b^2} - \frac{a(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^2}$$

[Out]  $-\left(\frac{a f^{c/(a+bx)^2} (a+bx)}{b^2}\right) + \frac{f^{c/(a+bx)^2} (a+bx)^2}{2b^2} + \frac{a \sqrt{c} \sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{c} \sqrt{\log(f)}}{a+bx}\right)}{b^2} - \frac{c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{2b^2} - \frac{c \operatorname{ExpIntegralEi}\left(\frac{c \log(f)}{(a+bx)^2}\right) \log(f)}{2b^2}$

**Rubi [A]** time = 0.118964, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$\frac{\sqrt{\pi}a\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^2} - \frac{c\log(f)\operatorname{Ei}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{2b^2} + \frac{(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{2b^2} - \frac{a(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^2)\*x,x]

[Out]  $-\left(\frac{a f^{c/(a+bx)^2} (a+bx)}{b^2}\right) + \frac{f^{c/(a+bx)^2} (a+bx)^2}{2b^2} + \frac{a \sqrt{c} \sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{c} \sqrt{\log(f)}}{a+bx}\right)}{b^2} - \frac{c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{2b^2} - \frac{c \operatorname{ExpIntegralEi}\left(\frac{c \log(f)}{(a+bx)^2}\right) \log(f)}{2b^2}$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1))

, x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^2}} x dx &= \int \left( -\frac{af^{\frac{c}{(a+bx)^2}}}{b} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^2}}(a+bx) dx}{b} - \frac{a \int f^{\frac{c}{(a+bx)^2}} dx}{b} \\ &= -\frac{af^{\frac{c}{(a+bx)^2}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{2b^2} + \frac{(c \log(f)) \int \frac{f^{\frac{c}{(a+bx)^2}}}{a+bx} dx}{b} - \frac{(2ac \log(f)) \int \frac{f^{\frac{c}{(a+bx)^2}}}{(a+bx)^2} dx}{b} \\ &= -\frac{af^{\frac{c}{(a+bx)^2}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{2b^2} - \frac{c \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right) \log(f)}{2b^2} + \frac{(2ac \log(f)) \operatorname{Subst}\left(\int f^{cx^2} dx, x, \frac{1}{a+bx}\right)}{b^2} \\ &= -\frac{af^{\frac{c}{(a+bx)^2}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{2b^2} + \frac{a\sqrt{c}\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right) \sqrt{\log(f)}}{b^2} - \frac{c \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right) \log(f)}{2b^2} \end{aligned}$$

**Mathematica [A]** time = 0.053062, size = 89, normalized size = 0.8

$$\frac{(b^2x^2 - a^2) f^{\frac{c}{(a+bx)^2}} + 2\sqrt{\pi}a\sqrt{c}\sqrt{\log(f)} \operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right) - c \log(f) \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^2)\*x,x]

[Out] (f^(c/(a + b\*x)^2)\*(-a^2 + b^2\*x^2) + 2\*a\*Sqrt[c]\*Sqrt[Pi]\*Erfi[(Sqrt[c]\*Sqrt[Log[f]])/(a + b\*x)]\*Sqrt[Log[f]] - c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^2]\*Log[f])/(2\*b^2)

**Maple [A]** time = 0.032, size = 93, normalized size = 0.8

$$\frac{x^2}{2} f^{\frac{c}{(bx+a)^2}} - \frac{a^2}{2b^2} f^{\frac{c}{(bx+a)^2}} + \frac{c \ln(f)}{2b^2} \operatorname{Ei}\left(1, -\frac{c \ln(f)}{(bx+a)^2}\right) + \frac{ac \ln(f) \sqrt{\pi}}{b^2} \operatorname{Erf}\left(\frac{1}{bx+a} \sqrt{-c \ln(f)}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)\*x,x)

[Out] 1/2\*f^(c/(b\*x+a)^2)\*x^2-1/2/b^2\*f^(c/(b\*x+a)^2)\*a^2+1/2/b^2\*ln(f)\*c\*Ei(1,-c\*ln(f)/(b\*x+a)^2)+1/b^2\*a\*ln(f)\*c\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/sqrt(-c\*ln(f)))

$(1/2)/(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$bc \int \frac{f^{\frac{c}{b^2x^2+2abx+a^2}} x^2}{b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3} dx \log(f) + \frac{1}{2} f^{\frac{c}{b^2x^2+2abx+a^2}} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x,x, algorithm="maxima")

[Out] b\*c\*integrate(f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*x^2/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)\*log(f) + 1/2\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*x^2

**Fricas [A]** time = 1.82717, size = 248, normalized size = 2.23

$$\frac{2\sqrt{\pi}ab\sqrt{-\frac{c\log(f)}{b^2}} \operatorname{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right) + c\operatorname{Ei}\left(\frac{c\log(f)}{b^2x^2+2abx+a^2}\right)\log(f) - (b^2x^2 - a^2)f^{\frac{c}{b^2x^2+2abx+a^2}}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x,x, algorithm="fricas")

[Out] -1/2\*(2\*sqrt(pi)\*a\*b\*sqrt(-c\*log(f)/b^2)\*erf(b\*sqrt(-c\*log(f)/b^2)/(b\*x + a)) + c\*Ei(c\*log(f)/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*log(f) - (b^2\*x^2 - a^2)\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/b^2

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*2)\*x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^2)\*x, x)

$$3.228 \quad \int f^{\frac{c}{(a+bx)^2}} dx$$

**Optimal.** Leaf size=62

$$\frac{(a+bx)f^{\frac{c}{(a+bx)^2}}}{b} - \frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b}$$

[Out]  $(f^{(c/(a+b*x)^2)*(a+b*x)})/b - (\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Log}[f]])/(a+b*x)]*\operatorname{Sqrt}[\operatorname{Log}[f]])/b$

**Rubi [A]** time = 0.0393502, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2206, 2211, 2204}

$$\frac{(a+bx)f^{\frac{c}{(a+bx)^2}}}{b} - \frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(c/(a+b*x)^2)}, x]$

[Out]  $(f^{(c/(a+b*x)^2)*(a+b*x)})/b - (\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[\operatorname{Log}[f]])/(a+b*x)]*\operatorname{Sqrt}[\operatorname{Log}[f]])/b$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_.))^n)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)*F^{(a+b*(c+d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c+d*x)^n]*F^{(a+b*(c+d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{IntegerQ}[2/n] \ \&\& \ \operatorname{LtQ}[n, 0]$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_.))^n)*((c_.)+(d_.)*(x_.))^m}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{(a+b*x^2)}, x], x, (c+d*x)^{(m+1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n, x\} \ \&\& \ \operatorname{EqQ}[n, 2*(m+1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^2}} dx &= \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} + (2c \log(f)) \int \frac{f^{\frac{c}{(a+bx)^2}}}{(a+bx)^2} dx \\ &= \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} - \frac{(2c \log(f)) \operatorname{Subst}\left(\int f^{cx^2} dx, x, \frac{1}{a+bx}\right)}{b} \\ &= \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} - \frac{\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0202576, size = 62, normalized size = 1.

$$\frac{(a + bx)f^{\frac{c}{(a+bx)^2}}}{b} - \frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}\operatorname{Erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^2), x]

[Out] (f^(c/(a + b\*x)^2)\*(a + b\*x))/b - (Sqrt[c]\*Sqrt[Pi]\*Erfi[(Sqrt[c]\*Sqrt[Log[f]])/(a + b\*x)]\*Sqrt[Log[f]])/b

**Maple [A]** time = 0.025, size = 65, normalized size = 1.1

$$f^{\frac{c}{(bx+a)^2}}x + \frac{a}{b}f^{\frac{c}{(bx+a)^2}} - \frac{c \ln(f) \sqrt{\pi}}{b} \operatorname{Erf}\left(\frac{1}{bx+a} \sqrt{-c \ln(f)}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2), x)

[Out] f^(c/(b\*x+a)^2)\*x+1/b\*f^(c/(b\*x+a)^2)\*a-1/b\*ln(f)\*c\*Pi^(1/2)/(-c\*ln(f))^(1/2)\*erf((-c\*ln(f))^(1/2)/(b\*x+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2bc \int \frac{f^{\frac{c}{b^2x^2+2abx+a^2}}x}{b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3} dx \log(f) + f^{\frac{c}{b^2x^2+2abx+a^2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2), x, algorithm="maxima")

[Out] 2\*b\*c\*integrate(f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*x/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3), x)\*log(f) + f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*x

**Fricas [A]** time = 1.56506, size = 158, normalized size = 2.55

$$\frac{\sqrt{\pi}b\sqrt{-\frac{c \log(f)}{b^2}} \operatorname{erf}\left(\frac{b\sqrt{-\frac{c \log(f)}{b^2}}}{bx+a}\right) + (bx+a)f^{\frac{c}{b^2x^2+2abx+a^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2), x, algorithm="fricas")

[Out] (sqrt(pi)\*b\*sqrt(-c\*log(f)/b^2)\*erf(b\*sqrt(-c\*log(f)/b^2)/(b\*x + a)) + (b\*x + a)\*f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/b

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(a+bx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*2),x)

[Out] Integral(f\*\*(c/(a + b\*x)\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2),x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^2), x)



$$3.229 \quad \int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx$$

**Optimal.** Leaf size=17

$$\text{Unintegrable}\left(\frac{f^{\frac{c}{(a+bx)^2}}}{x}, x\right)$$

[Out] Unintegrable[f^(c/(a + b\*x)^2)/x, x]

**Rubi [A]** time = 0.0180934, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^2)/x,x]

[Out] Defer[Int][f^(c/(a + b\*x)^2)/x, x]

Rubi steps

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx$$

**Mathematica [A]** time = 0.069978, size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^2)/x,x]

[Out] Integrate[f^(c/(a + b\*x)^2)/x, x]

**Maple [A]** time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{x} f^{\frac{c}{(bx+a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)/x,x)

[Out]  $\text{int}(f^{(c/(b*x+a)^2)}/x,x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c/(b*x+a)^2)}/x,x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(f^{(c/(b*x + a)^2)}/x, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^2x^2+2abx+a^2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c/(b*x+a)^2)}/x,x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(f^{(c/(b^2*x^2 + 2*a*b*x + a^2))}/x, x)$

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c/(b*x+a)^2)}/x,x)$

[Out]  $\text{Integral}(f^{(c/(a + b*x)^2)}/x, x)$

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(f^{(c/(b*x+a)^2)}/x,x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(f^{(c/(b*x + a)^2)}/x, x)$

$$3.230 \quad \int \frac{f^{(a+bx)^2}}{x^2} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate} \left( \frac{f^{(a+bx)^2}}{x^2}, x \right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^2)/x^2, x]

**Rubi [A]** time = 0.0425695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^2)/x^2,x]

[Out] Defer[Int][f^(c/(a + b\*x)^2)/x^2, x]

Rubi steps

$$\int \frac{f^{(a+bx)^2}}{x^2} dx = \int \frac{f^{(a+bx)^2}}{x^2} dx$$

**Mathematica [A]** time = 0.214236, size = 0, normalized size = 0.

$$\int \frac{f^{(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^2)/x^2,x]

[Out] Integrate[f^(c/(a + b\*x)^2)/x^2, x]

**Maple [A]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} f^{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)/x^2,x)

[Out] `int(f^(c/(b*x+a)^2)/x^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a)^2)/x^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^2x^2+2abx+a^2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="fricas")`

[Out] `integral(f^(c/(b^2*x^2 + 2*a*b*x + a^2)))/x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a)**2)/x**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a)^2)/x^2, x)`

$$3.231 \quad \int \frac{f^{(a+bx)^2}}{x^3} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate} \left( \frac{f^{(a+bx)^2}}{x^3}, x \right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^2)/x^3, x]

**Rubi [A]** time = 0.0380722, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{(a+bx)^2}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^2)/x^3,x]

[Out] Defer[Int][f^(c/(a + b\*x)^2)/x^3, x]

Rubi steps

$$\int \frac{f^{(a+bx)^2}}{x^3} dx = \int \frac{f^{(a+bx)^2}}{x^3} dx$$

**Mathematica [A]** time = 0.576102, size = 0, normalized size = 0.

$$\int \frac{f^{(a+bx)^2}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^2)/x^3,x]

[Out] Integrate[f^(c/(a + b\*x)^2)/x^3, x]

**Maple [A]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} f^{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)/x^3,x)

[Out] `int(f^(c/(b*x+a)^2)/x^3,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a)^2)/x^3, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^2x^2+2abx+a^2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="fricas")`

[Out] `integral(f^(c/(b^2*x^2 + 2*a*b*x + a^2))/x^3, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a)**2)/x**3,x)`

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a)^2)/x^3, x)`

### 3.232 $\int f^{\frac{c}{(a+bx)^3}} x^4 dx$

**Optimal.** Leaf size=239

$$\frac{4a^3(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5} + \frac{a^4(a+bx)^3 \sqrt{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5} + \frac{(a+bx)^5 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{5/3} \Gamma\left(-\frac{5}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5}$$

[Out] (2\*a^2\*f^(c/(a + b\*x)^3)\*(a + b\*x)^3)/b^5 - (2\*a^2\*c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^3]\*Log[f])/b^5 + (a^4\*(a + b\*x)\*Gamma[-1/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(1/3)/(3\*b^5) - (4\*a^3\*(a + b\*x)^2\*Gamma[-2/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(2/3)/(3\*b^5) - (4\*a\*(a + b\*x)^4\*Gamma[-4/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(4/3)/(3\*b^5) + ((a + b\*x)^5\*Gamma[-5/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(5/3)/(3\*b^5)

**Rubi [A]** time = 0.200715, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2226, 2208, 2218, 2214, 2210}

$$\frac{4a^3(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5} + \frac{a^4(a+bx)^3 \sqrt{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5} + \frac{(a+bx)^5 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{5/3} \Gamma\left(-\frac{5}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^5}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^3)\*x^4,x]

[Out] (2\*a^2\*f^(c/(a + b\*x)^3)\*(a + b\*x)^3)/b^5 - (2\*a^2\*c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^3]\*Log[f])/b^5 + (a^4\*(a + b\*x)\*Gamma[-1/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(1/3)/(3\*b^5) - (4\*a^3\*(a + b\*x)^2\*Gamma[-2/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(2/3)/(3\*b^5) - (4\*a\*(a + b\*x)^4\*Gamma[-4/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(4/3)/(3\*b^5) + ((a + b\*x)^5\*Gamma[-5/3, -(c\*Log[f])/(a + b\*x)^3])\*(-(c\*Log[f])/(a + b\*x)^3)^(5/3)/(3\*b^5)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^3}} x^4 dx &= \int \left( \frac{a^4 f^{\frac{c}{(a+bx)^3}}}{b^4} - \frac{4a^3 f^{\frac{c}{(a+bx)^3}} (a+bx)}{b^4} + \frac{6a^2 f^{\frac{c}{(a+bx)^3}} (a+bx)^2}{b^4} - \frac{4a f^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^4} + \frac{f^{\frac{c}{(a+bx)^3}} (a+bx)^4}{b^4} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^3}} (a+bx)^4 dx}{b^4} - \frac{(4a) \int f^{\frac{c}{(a+bx)^3}} (a+bx)^3 dx}{b^4} + \frac{(6a^2) \int f^{\frac{c}{(a+bx)^3}} (a+bx)^2 dx}{b^4} - \frac{(4a^3) \int f^{\frac{c}{(a+bx)^3}} (a+bx) dx}{b^4} + \frac{\int f^{\frac{c}{(a+bx)^3}} dx}{b^4} \\ &= \frac{2a^2 f^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^5} + \frac{a^4 (a+bx) \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^5} - \frac{4a^3 (a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^2}{3b^5} \\ &= \frac{2a^2 f^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^5} - \frac{2a^2 c \operatorname{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right) \log(f)}{b^5} + \frac{a^4 (a+bx) \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^5} - \frac{4a^3 (a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^2}{3b^5} \end{aligned}$$

**Mathematica [A]** time = 0.193196, size = 219, normalized size = 0.92

$$-4a^3(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \operatorname{Gamma}\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) + a^4(a+bx) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \operatorname{Gamma}\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) + (a+bx)^5 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(c/(a + b*x)^3)*x^4, x]
```

```
[Out] (6*a^2*f^(c/(a + b*x)^3)*(a + b*x)^3 - 6*a^2*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^3]*Log[f] + a^4*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(1/3) + 4*a*c*(a + b*x)*Gamma[-4/3, -((c*Log[f])/(a + b*x)^3)]*Log[f]*(-((c*Log[f])/(a + b*x)^3))^(1/3) - 4*a^3*(a + b*x)^2*Gamma[-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3) + (a + b*x)^5*Gamma[-5/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(5/3))/(3*b^5)
```

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c/(b*x+a)^3)*x^4, x)
```





Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a)^3)*x^4,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a)^3)*x^4, x)
```

### 3.233 $\int f^{\frac{c}{(a+bx)^3}} x^3 dx$

**Optimal.** Leaf size=184

$$\frac{a^2(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} - \frac{a^3(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^4 \left(-\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}$$

[Out]  $-\left(\frac{a^2 f^{c/(a+bx)^3} (a+bx)^3}{b^4} + \frac{a^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^4 \left(-\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}\right) \log(f) - \frac{a^3 (a+bx)^3 \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{a^2 (a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} + \frac{(a+bx)^4 \Gamma\left(-\frac{4}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^3 \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} + \frac{(a+bx) \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{\Gamma\left(-\frac{4}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}$

**Rubi [A]** time = 0.149328, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2226, 2208, 2218, 2214, 2210}

$$\frac{a^2(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} - \frac{a^3(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^4 \left(-\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^3)\*x^3,x]

[Out]  $-\left(\frac{a^2 f^{c/(a+bx)^3} (a+bx)^3}{b^4} + \frac{a^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^4 \left(-\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}\right) \log(f) - \frac{a^3 (a+bx)^3 \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{a^2 (a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} + \frac{(a+bx)^4 \Gamma\left(-\frac{4}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^3 \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{(a+bx)^2 \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{b^4} + \frac{(a+bx) \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4} + \frac{\Gamma\left(-\frac{4}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^4}$

#### Rule 2226

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m+1)\*Gamma[(m+1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m+1/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(c + d\*x)^(m+1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m+1))

, x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^(n)), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^3}} x^3 dx &= \int \left( -\frac{a^3 f^{\frac{c}{(a+bx)^3}}}{b^3} + \frac{3a^2 f^{\frac{c}{(a+bx)^3}} (a+bx)}{b^3} - \frac{3af^{\frac{c}{(a+bx)^3}} (a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^3} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^3}} (a+bx)^3 dx}{b^3} - \frac{(3a) \int f^{\frac{c}{(a+bx)^3}} (a+bx)^2 dx}{b^3} + \frac{(3a^2) \int f^{\frac{c}{(a+bx)^3}} (a+bx) dx}{b^3} - \frac{a^3 \int f^{\frac{c}{(a+bx)^3}} dx}{b^3} \\ &= -\frac{af^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^4} - \frac{a^3(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c\log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b^4} + \frac{a^2(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c\log(f)}{(a+bx)^3}\right) \left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}}{b^4} \\ &= -\frac{af^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^4} + \frac{ac\text{Ei}\left(\frac{c\log(f)}{(a+bx)^3}\right) \log(f)}{b^4} - \frac{a^3(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c\log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b^4} + \frac{a^2(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c\log(f)}{(a+bx)^3}\right) \left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}}{b^4} \end{aligned}$$

**Mathematica [A]** time = 0.362696, size = 167, normalized size = 0.91

$$\frac{3ac \log(f) \text{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right) - (a+bx) \left( a^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) + 3a(a+bx) \left( (a+bx) f^{\frac{c}{(a+bx)^3}} - a \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \text{Gamma}\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \right) \right)}{3b^4}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^3)\*x^3,x]

[Out] (3\*a\*c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^3]\*Log[f] - (a + b\*x)\*(a^3\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3) + c\*Gamma[-4/3, -((c\*Log[f])/(a + b\*x)^3)]\*Log[f]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3) + 3\*a\*(a + b\*x)\*(f^(c/(a + b\*x)^3)\*(a + b\*x) - a\*Gamma[-2/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(2/3)))/(3\*b^4)

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)\*x^3,x)

[Out] int(f^(c/(b\*x+a)^3)\*x^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^3x^4 + 3cx \log(f))f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{4b^3} - \int \frac{3 \left( 4ab^3cx^3 \log(f) + 6a^2b^2cx^2 \log(f) + a^4c \log(f) + (4a^3bc \log(f) - 3b^2c^2 \log(f)^2) \right) f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{4(b^7x^4 + 4ab^6x^3 + 6a^2b^5x^2 + 4a^3b^4x + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^3,x, algorithm="maxima")

[Out] 1/4\*(b^3\*x^4 + 3\*c\*x\*log(f))\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))/b^3 - integrate(3/4\*(4\*a\*b^3\*c\*x^3\*log(f) + 6\*a^2\*b^2\*c\*x^2\*log(f) + a^4\*c\*log(f) + (4\*a^3\*b\*c\*log(f) - 3\*b\*c^2\*log(f)^2)\*x)\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))/(b^7\*x^4 + 4\*a\*b^6\*x^3 + 6\*a^2\*b^5\*x^2 + 4\*a^3\*b^4\*x + a^4\*b^3), x)

---

**Fricas [A]** time = 1.63792, size = 516, normalized size = 2.8

$$\frac{6a^2b^2 \left( -\frac{c \log(f)}{b^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{1}{3}, -\frac{c \log(f)}{b^3x^3+3ab^2x^2+3a^2bx+a^3} \right) - 4ac \operatorname{Ei} \left( \frac{c \log(f)}{b^3x^3+3ab^2x^2+3a^2bx+a^3} \right) \log(f) - (4a^3b - 3bc \log(f)) \left( -\frac{c \log(f)}{b^3} \right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^3,x, algorithm="fricas")

[Out] -1/4\*(6\*a^2\*b^2\*(-c\*log(f)/b^3)^(2/3)\*gamma(1/3, -c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)) - 4\*a\*c\*Ei(c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*log(f) - (4\*a^3\*b - 3\*b\*c\*log(f))\*(-c\*log(f)/b^3)^(1/3)\*gamma(a(2/3, -c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)) - (b^4\*x^4 - a^4 + 3\*(b\*c\*x + a\*c)\*log(f))\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)))/b^4

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*3)\*x\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a)^3)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a)^3)*x^3, x)
```

### 3.234 $\int f^{\frac{c}{(a+bx)^3}} x^2 dx$

**Optimal.** Leaf size=142

$$\frac{a^2(a+bx)^3 \sqrt{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3} - \frac{2a(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3} - \frac{c \log(f) \text{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3}$$

[Out]  $(f^{(c/(a+bx)^3)}*(a+bx)^3)/(3*b^3) - (c*\text{ExpIntegralEi}[(c*\text{Log}[f])/(a+bx)^3]*\text{Log}[f])/(3*b^3) + (a^2*(a+bx)*\Gamma[-1/3, -((c*\text{Log}[f])/(a+bx)^3)]*(-((c*\text{Log}[f])/(a+bx)^3))^{(1/3)})/(3*b^3) - (2*a*(a+bx)^2*\Gamma[-2/3, -((c*\text{Log}[f])/(a+bx)^3)]*(-((c*\text{Log}[f])/(a+bx)^3))^{(2/3)})/(3*b^3)$

**Rubi [A]** time = 0.117823, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2226, 2208, 2218, 2214, 2210}

$$\frac{a^2(a+bx)^3 \sqrt{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3} - \frac{2a(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3} - \frac{c \log(f) \text{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right)}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a+bx)^3)\*x^2,x]

[Out]  $(f^{(c/(a+bx)^3)}*(a+bx)^3)/(3*b^3) - (c*\text{ExpIntegralEi}[(c*\text{Log}[f])/(a+bx)^3]*\text{Log}[f])/(3*b^3) + (a^2*(a+bx)*\Gamma[-1/3, -((c*\text{Log}[f])/(a+bx)^3)]*(-((c*\text{Log}[f])/(a+bx)^3))^{(1/3)})/(3*b^3) - (2*a*(a+bx)^2*\Gamma[-2/3, -((c*\text{Log}[f])/(a+bx)^3)]*(-((c*\text{Log}[f])/(a+bx)^3))^{(2/3)})/(3*b^3)$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0

] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_  
Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free  
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^3}} x^2 dx &= \int \left( \frac{a^2 f^{\frac{c}{(a+bx)^3}}}{b^2} - \frac{2af^{\frac{c}{(a+bx)^3}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)^2}{b^2} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^3}}(a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{\frac{c}{(a+bx)^3}}(a+bx) dx}{b^2} + \frac{a^2 \int f^{\frac{c}{(a+bx)^3}} dx}{b^2} \\ &= \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)^3}{3b^3} + \frac{a^2(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^3} - \frac{2a(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3}}{3b^3} + \\ &= \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)^3}{3b^3} - \frac{c \text{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right) \log(f)}{3b^3} + \frac{a^2(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^3} - \frac{2a(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3}}{3b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0635233, size = 127, normalized size = 0.89

$$\frac{a^2(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) - 2a(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \text{Gamma}\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) - c \log(f) \text{Ei}\left(\frac{c \log(f)}{(a+bx)^3}\right) + (c \log(f))^3}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^3)\*x^2,x]

[Out] (f^(c/(a + b\*x)^3)\*(a + b\*x)^3 - c\*ExpIntegralEi[(c\*Log[f])/(a + b\*x)^3]\*Log[f] + a^2\*(a + b\*x)\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3) - 2\*a\*(a + b\*x)^2\*Gamma[-2/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(2/3))/(3\*b^3)

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)\*x^2,x)

[Out] int(f^(c/(b\*x+a)^3)\*x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x^3 + bc \int \frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x^3}{b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4} dx \log(f)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^2,x, algorithm="maxima")

[Out] 1/3\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x^3 + b\*c\*integrate(f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x^3/(b^4\*x^4 + 4\*a\*b^3\*x^3 + 6\*a^2\*b^2\*x^2 + 4\*a^3\*b\*x + a^4), x)\*log(f)

**Fricas [A]** time = 1.59886, size = 450, normalized size = 3.17

$$\frac{3ab^2 \left( -\frac{c \log(f)}{b^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{1}{3}, -\frac{c \log(f)}{b^3 x^3 + 3ab^2 x^2 + 3a^2 bx + a^3} \right) - 3a^2 b \left( -\frac{c \log(f)}{b^3} \right)^{\frac{1}{3}} \Gamma \left( \frac{2}{3}, -\frac{c \log(f)}{b^3 x^3 + 3ab^2 x^2 + 3a^2 bx + a^3} \right) - c \operatorname{Ei} \left( \frac{c \log(f)}{b^3 x^3 + 3ab^2 x^2 + 3a^2 bx + a^3} \right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^2,x, algorithm="fricas")

[Out] 1/3\*(3\*a\*b^2\*(-c\*log(f)/b^3)^(2/3)\*gamma(1/3, -c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)) - 3\*a^2\*b\*(-c\*log(f)/b^3)^(1/3)\*gamma(2/3, -c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)) - c\*Ei(c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*log(f) + (b^3\*x^3 + a^3)\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)))/b^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*3)\*x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^2,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^3)\*x^2, x)

### 3.235 $\int f^{\frac{c}{(a+bx)^3}} x dx$

**Optimal.** Leaf size=92

$$\frac{(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^2} - \frac{a(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^2}$$

[Out]  $-(a*(a + b*x)*\Gamma[-1/3, -((c*\text{Log}[f])/(a + b*x)^3)]*(-((c*\text{Log}[f])/(a + b*x)^3))^{(1/3)})/(3*b^2) + ((a + b*x)^2*\Gamma[-2/3, -((c*\text{Log}[f])/(a + b*x)^3)]*(-((c*\text{Log}[f])/(a + b*x)^3))^{(2/3)})/(3*b^2)$

**Rubi [A]** time = 0.0518568, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2226, 2208, 2218}

$$\frac{(a+bx)^2 \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^2} - \frac{a(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^3)\*x,x]

[Out]  $-(a*(a + b*x)*\Gamma[-1/3, -((c*\text{Log}[f])/(a + b*x)^3)]*(-((c*\text{Log}[f])/(a + b*x)^3))^{(1/3)})/(3*b^2) + ((a + b*x)^2*\Gamma[-2/3, -((c*\text{Log}[f])/(a + b*x)^3)]*(-((c*\text{Log}[f])/(a + b*x)^3))^{(2/3)})/(3*b^2)$

#### Rule 2226

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)))\*(e\_. + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{\frac{c}{(a+bx)^3}} x dx &= \int \left( -\frac{af^{\frac{c}{(a+bx)^3}}}{b} + \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)}{b} \right) dx \\ &= \frac{\int f^{\frac{c}{(a+bx)^3}}(a+bx) dx}{b} - \frac{a \int f^{\frac{c}{(a+bx)^3}} dx}{b} \\ &= -\frac{a(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^2} + \frac{(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3}}{3b^2} \end{aligned}$$

**Mathematica [A]** time = 0.067011, size = 86, normalized size = 0.93

$$\frac{(a+bx)\left(a+bx\right)\left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3} \text{Gamma}\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) - a^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \text{Gamma}\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^3)\*x, x]

[Out] ((a + b\*x)\*(-(a\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3)) + (a + b\*x)\*Gamma[-2/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(2/3)))/(3\*b^2)

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)\*x, x)

[Out] int(f^(c/(b\*x+a)^3)\*x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3bc \int \frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x^2}{2(b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4)} dx \log(f) + \frac{1}{2} f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x, x, algorithm="maxima")

[Out] 3\*b\*c\*integrate(1/2\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x^2/(b^4\*x^4 + 4\*a\*b^3\*x^3 + 6\*a^2\*b^2\*x^2 + 4\*a^3\*b\*x + a^4), x)\*log(f) + 1/2\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x^2

**Fricas [A]** time = 1.5793, size = 354, normalized size = 3.85

$$\frac{b^2 \left( -\frac{c \log(f)}{b^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{1}{3}, -\frac{c \log(f)}{b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3} \right) - 2 a b \left( -\frac{c \log(f)}{b^3} \right)^{\frac{1}{3}} \Gamma \left( \frac{2}{3}, -\frac{c \log(f)}{b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3} \right) - (b^2 x^2 - a^2) f^{\frac{c}{b^3 x^3 + 3 a b^2 x^2 + 3 a^2 b x + a^3}}}{2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x,x, algorithm="fricas")

[Out]  $-1/2*(b^2*(-c*\log(f)/b^3)^{(2/3)}*\gamma(1/3, -c*\log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) - 2*a*b*(-c*\log(f)/b^3)^{(1/3)}*\gamma(2/3, -c*\log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) - (b^2*x^2 - a^2)*f^{(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))})/b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*3)\*x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^3)\*x, x)

$$3.236 \quad \int f^{\frac{c}{(a+bx)^3}} dx$$

**Optimal.** Leaf size=44

$$\frac{(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b}$$

[Out] ((a + b\*x)\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3))/(3\*b)

**Rubi [A]** time = 0.0054403, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2208}

$$\frac{(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[f^(c/(a + b\*x)^3), x]

[Out] ((a + b\*x)\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3))/(3\*b)

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a \*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int f^{\frac{c}{(a+bx)^3}} dx = \frac{(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b}$$

**Mathematica [A]** time = 0.0083877, size = 44, normalized size = 1.

$$\frac{(a+bx)^3 \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c/(a + b\*x)^3), x]

[Out] ((a + b\*x)\*Gamma[-1/3, -((c\*Log[f])/(a + b\*x)^3)]\*(-((c\*Log[f])/(a + b\*x)^3))^(1/3))/(3\*b)

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3),x)

[Out] int(f^(c/(b\*x+a)^3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3bc \int \frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x}{b^4x^4 + 4ab^3x^3 + 6a^2b^2x^2 + 4a^3bx + a^4} dx \log(f) + f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3),x, algorithm="maxima")

[Out] 3\*b\*c\*integrate(f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x/(b^4\*x^4 + 4\*a\*b^3\*x^3 + 6\*a^2\*b^2\*x^2 + 4\*a^3\*b\*x + a^4), x)\*log(f) + f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x

**Fricas [B]** time = 1.5492, size = 208, normalized size = 4.73

$$\frac{b \left( -\frac{c \log(f)}{b^3} \right)^{\frac{1}{3}} \Gamma \left( \frac{2}{3}, -\frac{c \log(f)}{b^3x^3+3ab^2x^2+3a^2bx+a^3} \right) - (bx+a) f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3),x, algorithm="fricas")

[Out] -(b\*(-c\*log(f)/b^3)^(1/3)\*gamma(2/3, -c\*log(f)/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)) - (b\*x + a)\*f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3)))/b

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*3),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c/(b*x+a)^3),x, algorithm="giac")
```

```
[Out] integrate(f^(c/(b*x + a)^3), x)
```

$$3.237 \quad \int \frac{f^{(a+bx)^3}}{x} dx$$

**Optimal.** Leaf size=17

$$\text{Unintegrable} \left( \frac{f^{(a+bx)^3}}{x}, x \right)$$

[Out] Unintegrable[f^(c/(a + b\*x)^3)/x, x]

**Rubi [A]** time = 0.0179304, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{(a+bx)^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^3)/x, x]

[Out] Defer[Int][f^(c/(a + b\*x)^3)/x, x]

Rubi steps

$$\int \frac{f^{(a+bx)^3}}{x} dx = \int \frac{f^{(a+bx)^3}}{x} dx$$

**Mathematica [A]** time = 0.0611491, size = 0, normalized size = 0.

$$\int \frac{f^{(a+bx)^3}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^3)/x, x]

[Out] Integrate[f^(c/(a + b\*x)^3)/x, x]

**Maple [A]** time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x} f^{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)/x, x)



[Out] `int(f^(c/(b*x+a)^3)/x,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a)^3)/x, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x,x, algorithm="fricas")`

[Out] `integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)))/x, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a)**3)/x,x)`

[Out] `Integral(f**(c/(a + b*x)**3)/x, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a)^3)/x, x)`

$$3.238 \quad \int \frac{f^{(a+bx)^3}}{x^2} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{f^{(a+bx)^3}}{x^2}, x\right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^3)/x^2, x]

**Rubi [A]** time = 0.0433816, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{(a+bx)^3}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^3)/x^2, x]

[Out] Defer[Int][f^(c/(a + b\*x)^3)/x^2, x]

Rubi steps

$$\int \frac{f^{(a+bx)^3}}{x^2} dx = \int \frac{f^{(a+bx)^3}}{x^2} dx$$

**Mathematica [A]** time = 0.290216, size = 0, normalized size = 0.

$$\int \frac{f^{(a+bx)^3}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^3)/x^2, x]

[Out] Integrate[f^(c/(a + b\*x)^3)/x^2, x]

**Maple [A]** time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} f^{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)/x^2, x)

[Out] `int(f^(c/(b*x+a)^3)/x^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a)^3)/x^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="fricas")`

[Out] `integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a)**3)/x**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a)^3)/x^2, x)`

$$3.239 \quad \int \frac{f^{(a+bx)^3}}{x^3} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{f^{(a+bx)^3}}{x^3}, x\right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^3)/x^3, x]

**Rubi [A]** time = 0.0383083, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{(a+bx)^3}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^3)/x^3, x]

[Out] Defer[Int][f^(c/(a + b\*x)^3)/x^3, x]

Rubi steps

$$\int \frac{f^{(a+bx)^3}}{x^3} dx = \int \frac{f^{(a+bx)^3}}{x^3} dx$$

**Mathematica [A]** time = 0.0360148, size = 0, normalized size = 0.

$$\int \frac{f^{(a+bx)^3}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^3)/x^3, x]

[Out] Integrate[f^(c/(a + b\*x)^3)/x^3, x]

**Maple [A]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} f^{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)/x^3, x)

[Out] `int(f^(c/(b*x+a)^3)/x^3,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="maxima")`

[Out] `integrate(f^(c/(b*x + a)^3)/x^3, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="fricas")`

[Out] `integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c/(b*x+a)**3)/x**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="giac")`

[Out] `integrate(f^(c/(b*x + a)^3)/x^3, x)`

$$3.240 \quad \int f^{c(a+bx)^3} x^m dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}(x^m f^{c(a+bx)^3}, x)$$

[Out] CannotIntegrate[f^(c\*(a + b\*x)^3)\*x^m, x]

**Rubi [A]** time = 0.0468174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{c(a+bx)^3} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^3)\*x^m, x]

[Out] Defer[Int][f^(c\*(a + b\*x)^3)\*x^m, x]

Rubi steps

$$\int f^{c(a+bx)^3} x^m dx = \int f^{c(a+bx)^3} x^m dx$$

**Mathematica [A]** time = 0.241196, size = 0, normalized size = 0.

$$\int f^{c(a+bx)^3} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^3)\*x^m, x]

[Out] Integrate[f^(c\*(a + b\*x)^3)\*x^m, x]

**Maple [A]** time = 0.036, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^3} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^3)\*x^m, x)

[Out] int(f^(c\*(b\*x+a)^3)\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(bx+a)^3} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^m,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^3\*c)\*x^m, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^m,x, algorithm="fricas")

[Out] integral(f^(b^3\*c\*x^3 + 3\*a\*b^2\*c\*x^2 + 3\*a^2\*b\*c\*x + a^3\*c)\*x^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*3)\*x\*\*m,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^3c}x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^3)\*x^m,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^3\*c)\*x^m, x)

$$3.241 \quad \int f^{c(a+bx)^2} x^m dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}(x^m f^{a^2c+2abcx+b^2cx^2}, x)$$

[Out] Unintegrable[f^(a^2\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)\*x^m, x]

**Rubi [A]** time = 0.0556472, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{c(a+bx)^2} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^2)\*x^m, x]

[Out] Defer[Int][f^(a^2\*c + 2\*a\*b\*c\*x + b^2\*c\*x^2)\*x^m, x]

Rubi steps

$$\int f^{c(a+bx)^2} x^m dx = \int f^{a^2c+2abcx+b^2cx^2} x^m dx$$

**Mathematica [A]** time = 0.140308, size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^2)\*x^m, x]

[Out] Integrate[f^(c\*(a + b\*x)^2)\*x^m, x]

**Maple [A]** time = 0.033, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^2} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^2)\*x^m, x)

[Out] int(f^(c\*(b\*x+a)^2)\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^2c} x^m dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^m,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^2\*c)\*x^m, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{b^2cx^2+2abcx+a^2c}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^m,x, algorithm="fricas")

[Out] integral(f^(b^2\*c\*x^2 + 2\*a\*b\*c\*x + a^2\*c)\*x^m, x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^2} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*2)\*x\*\*m,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*2)\*x\*\*m, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^2} c x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^2)\*x^m,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^2\*c)\*x^m, x)

### 3.242 $\int f^{c(a+bx)} x^m dx$

**Optimal.** Leaf size=41

$$\frac{x^m f^{ac} (-bcx \log(f))^{-m} \Gamma(m+1, -bcx \log(f))}{bc \log(f)}$$

[Out] (f^(a\*c)\*x^m\*Gamma[1 + m, -(b\*c\*x\*Log[f])])/(b\*c\*Log[f]\*(-(b\*c\*x\*Log[f]))^m)

**Rubi [A]** time = 0.0217245, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2181}

$$\frac{x^m f^{ac} (-bcx \log(f))^{-m} \Gamma(m+1, -bcx \log(f))}{bc \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x))\*x^m, x]

[Out] (f^(a\*c)\*x^m\*Gamma[1 + m, -(b\*c\*x\*Log[f])])/(b\*c\*Log[f]\*(-(b\*c\*x\*Log[f]))^m)

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rubi steps

$$\int f^{c(a+bx)} x^m dx = \frac{f^{ac} x^m \Gamma(1+m, -bcx \log(f)) (-bcx \log(f))^{-m}}{bc \log(f)}$$

**Mathematica [A]** time = 0.0078306, size = 36, normalized size = 0.88

$$x^{m+1} (-f^{ac}) (-bcx \log(f))^{-m-1} \Gamma(m+1, -bcx \log(f))$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x))\*x^m, x]

[Out] -(f^(a\*c)\*x^(1 + m)\*Gamma[1 + m, -(b\*c\*x\*Log[f])]\*(-(b\*c\*x\*Log[f]))^(-1 - m))

**Maple [B]** time = 0.036, size = 117, normalized size = 2.9

$$\frac{f^{ac} (-bc)^{-m} (\ln(f))^{-m-1} \left( x^m (-bc)^m (\ln(f))^m m \Gamma(m) (-bcx \ln(f))^{-m} - x^m (-bc)^m (\ln(f))^m e^{bcx \ln(f)} - x^m (-bc)^m (\ln(f)) \right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*(b*x+a))*x^m,x)`

[Out]  $-f^{(a*c)*(-b*c)^{-m}*\ln(f)^{-m-1}/b/c*(x^m*(-b*c)^m*\ln(f)^m*\text{GAMMA}(m)*(-b*c*x*\ln(f))^{-m}-x^m*(-b*c)^m*\ln(f)^m*\exp(b*c*x*\ln(f))-x^m*(-b*c)^m*\ln(f)^m*m*(-b*c*x*\ln(f))^{-m}*\text{GAMMA}(m,-b*c*x*\ln(f)))$

**Maxima [A]** time = 1.26006, size = 49, normalized size = 1.2

$$-(-bcx \log(f))^{-m-1} f^{ac} x^{m+1} \Gamma(m+1, -bcx \log(f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a))*x^m,x, algorithm="maxima")`

[Out]  $-(-b*c*x*\log(f))^{-m-1}*f^{(a*c)}*x^{(m+1)}*\text{gamma}(m+1, -b*c*x*\log(f))$

**Fricas [A]** time = 1.5318, size = 105, normalized size = 2.56

$$\frac{e^{(ac \log(f) - m \log(-bc \log(f)))} \Gamma(m+1, -bcx \log(f))}{bc \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a))*x^m,x, algorithm="fricas")`

[Out]  $e^{(a*c*\log(f) - m*\log(-b*c*\log(f)))*\text{gamma}(m+1, -b*c*x*\log(f))/(b*c*\log(f))}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*(b*x+a))*x**m,x)`

[Out] `Integral(f**(c*(a + b*x))*x**m, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)c} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a))*x^m,x, algorithm="giac")`

[Out] `integrate(f^((b*x + a)*c))*x^m, x)`

$$3.243 \quad \int f^{\frac{c}{a+bx}} x^m dx$$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(x^m f^{\frac{c}{a+bx}}, x\right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x))\*x^m, x]

**Rubi [A]** time = 0.0391837, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{\frac{c}{a+bx}} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x))\*x^m, x]

[Out] Defer[Int][f^(c/(a + b\*x))\*x^m, x]

Rubi steps

$$\int f^{\frac{c}{a+bx}} x^m dx = \int f^{\frac{c}{a+bx}} x^m dx$$

**Mathematica [A]** time = 0.0420825, size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x))\*x^m, x]

[Out] Integrate[f^(c/(a + b\*x))\*x^m, x]

**Maple [A]** time = 0.039, size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a))\*x^m, x)

[Out] int(f^(c/(b\*x+a))\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^m,x, algorithm="maxima")

[Out] integrate(f^(c/(b\*x + a))\*x^m, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{\frac{c}{bx+a}} x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^m,x, algorithm="fricas")

[Out] integral(f^(c/(b\*x + a))\*x^m, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{a+bx}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a))\*x\*\*m,x)

[Out] Integral(f\*\*(c/(a + b\*x))\*x\*\*m, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{bx+a}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a))\*x^m,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a))\*x^m, x)

$$3.244 \quad \int f^{\frac{c}{(a+bx)^2}} x^m dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(x^m f^{\frac{c}{(a+bx)^2}}, x\right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^2)\*x^m, x]

**Rubi [A]** time = 0.0415169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{\frac{c}{(a+bx)^2}} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^2)\*x^m, x]

[Out] Defer[Int][f^(c/(a + b\*x)^2)\*x^m, x]

Rubi steps

$$\int f^{\frac{c}{(a+bx)^2}} x^m dx = \int f^{\frac{c}{(a+bx)^2}} x^m dx$$

**Mathematica [A]** time = 0.0823226, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(a+bx)^2}} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^2)\*x^m, x]

[Out] Integrate[f^(c/(a + b\*x)^2)\*x^m, x]

**Maple [A]** time = 0.043, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^2)\*x^m, x)

[Out] int(f^(c/(b\*x+a)^2)\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^m,x, algorithm="maxima")

[Out] integrate(f^(c/(b\*x + a)^2)\*x^m, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{\frac{c}{b^2x^2+2abx+a^2}} x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^m,x, algorithm="fricas")

[Out] integral(f^(c/(b^2\*x^2 + 2\*a\*b\*x + a^2))\*x^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*2)\*x\*\*m,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^2)\*x^m,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^2)\*x^m, x)

$$3.245 \quad \int f^{\frac{c}{(a+bx)^3}} x^m dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(x^m f^{\frac{c}{(a+bx)^3}}, x\right)$$

[Out] CannotIntegrate[f^(c/(a + b\*x)^3)\*x^m, x]

**Rubi [A]** time = 0.0427128, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{\frac{c}{(a+bx)^3}} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c/(a + b\*x)^3)\*x^m, x]

[Out] Defer[Int][f^(c/(a + b\*x)^3)\*x^m, x]

Rubi steps

$$\int f^{\frac{c}{(a+bx)^3}} x^m dx = \int f^{\frac{c}{(a+bx)^3}} x^m dx$$

**Mathematica [A]** time = 0.0769496, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(a+bx)^3}} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c/(a + b\*x)^3)\*x^m, x]

[Out] Integrate[f^(c/(a + b\*x)^3)\*x^m, x]

**Maple [A]** time = 0.051, size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c/(b\*x+a)^3)\*x^m, x)

[Out] int(f^(c/(b\*x+a)^3)\*x^m, x)



**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^m,x, algorithm="maxima")

[Out] integrate(f^(c/(b\*x + a)^3)\*x^m, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}} x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^m,x, algorithm="fricas")

[Out] integral(f^(c/(b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x + a^3))\*x^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c/(b\*x+a)\*\*3)\*x\*\*m,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{\frac{c}{(bx+a)^3}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c/(b\*x+a)^3)\*x^m,x, algorithm="giac")

[Out] integrate(f^(c/(b\*x + a)^3)\*x^m, x)

$$3.246 \quad \int f^{c(a+bx)^n} x^m dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}(x^m f^{c(a+bx)^n}, x)$$

[Out] CannotIntegrate[f^(c\*(a + b\*x)^n)\*x^m, x]

**Rubi [A]** time = 0.0520335, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int f^{c(a+bx)^n} x^m dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^n)\*x^m, x]

[Out] Defer[Int][f^(c\*(a + b\*x)^n)\*x^m, x]

Rubi steps

$$\int f^{c(a+bx)^n} x^m dx = \int f^{c(a+bx)^n} x^m dx$$

**Mathematica [A]** time = 0.0522545, size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} x^m dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^n)\*x^m, x]

[Out] Integrate[f^(c\*(a + b\*x)^n)\*x^m, x]

**Maple [A]** time = 0.027, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)\*x^m, x)

[Out] int(f^(c\*(b\*x+a)^n)\*x^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x^m,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)\*x^m, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{(bx+a)^n}c x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x^m,x, algorithm="fricas")

[Out] integral(f^((b\*x + a)^n\*c)\*x^m, x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*n)\*x\*\*m,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*n)\*x\*\*m, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n}c x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x^m,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^n\*c)\*x^m, x)

### 3.247 $\int f^{c(a+bx)^n} x^3 dx$

**Optimal.** Leaf size=207

$$\frac{3a^2(a+bx)^2(-c\log(f)(a+bx)^n)^{-2/n}\Gamma\left(\frac{2}{n}, -c\log(f)(a+bx)^n\right)}{b^{4n}} + \frac{a^3(a+bx)(-c\log(f)(a+bx)^n)^{-1/n}\Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^{4n}}$$

[Out] -(((a + b\*x)^4\*Gamma[4/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(4/n))) + (3\*a\*(a + b\*x)^3\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(3/n)) - (3\*a^2\*(a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n)) + (a^3\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(n^(-1)))

**Rubi [A]** time = 0.157204, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2226, 2208, 2218}

$$\frac{3a^2(a+bx)^2(-c\log(f)(a+bx)^n)^{-2/n}\Gamma\left(\frac{2}{n}, -c\log(f)(a+bx)^n\right)}{b^{4n}} + \frac{a^3(a+bx)(-c\log(f)(a+bx)^n)^{-1/n}\Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^{4n}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^n)\*x^3,x]

[Out] -(((a + b\*x)^4\*Gamma[4/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(4/n))) + (3\*a\*(a + b\*x)^3\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(3/n)) - (3\*a^2\*(a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n)) + (a^3\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(n^(-1)))

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{c(a+bx)^n} x^3 dx &= \int \left( -\frac{a^3 f^{c(a+bx)^n}}{b^3} + \frac{3a^2 f^{c(a+bx)^n} (a+bx)}{b^3} - \frac{3a f^{c(a+bx)^n} (a+bx)^2}{b^3} + \frac{f^{c(a+bx)^n} (a+bx)^3}{b^3} \right) dx \\ &= \frac{\int f^{c(a+bx)^n} (a+bx)^3 dx}{b^3} - \frac{(3a) \int f^{c(a+bx)^n} (a+bx)^2 dx}{b^3} + \frac{(3a^2) \int f^{c(a+bx)^n} (a+bx) dx}{b^3} - \frac{a^3 \int f^{c(a+bx)^n} dx}{b^3} \\ &= -\frac{(a+bx)^4 \Gamma\left(\frac{4}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-4/n}}{b^4 n} + \frac{3a(a+bx)^3 \Gamma\left(\frac{3}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-3/n}}{b^4 n} \end{aligned}$$

**Mathematica [A]** time = 0.14956, size = 183, normalized size = 0.88

$$\frac{(a+bx) (-c \log(f) (a+bx)^n)^{-4/n} \left( (a+bx)^3 \text{Gamma}\left(\frac{4}{n}, -c \log(f) (a+bx)^n\right) - a (-c \log(f) (a+bx)^n)^{\frac{1}{n}} \left( a (-c \log(f) (a+bx)^n)^{\frac{1}{n}} \right) \right)}{b^4 n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^n)\*x^3,x]

[Out] -(((a + b\*x)\*((a + b\*x)^3\*Gamma[4/n, -(c\*(a + b\*x)^n\*Log[f])]) - a\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1)\*(3\*(a + b\*x)^2\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])]) + a\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1)\*(-3\*(a + b\*x)\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])]) + a\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])]\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1))))/(b^4\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(4/n))

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^n} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)\*x^3,x)

[Out] int(f^(c\*(b\*x+a)^n)\*x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x^3,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)\*x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{(bx+a)^n} x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^n)*x^3,x, algorithm="fricas")
```

```
[Out] integral(f^((b*x + a)^n*c)*x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*(b*x+a)**n)*x**3,x)
```

```
[Out] Integral(f**(c*(a + b*x)**n)*x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} c x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^n)*x^3,x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^n*c)*x^3, x)
```

### 3.248 $\int f^{c(a+bx)^n} x^2 dx$

**Optimal.** Leaf size=154

$$\frac{a^2(a+bx)(-c\log(f)(a+bx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^3n} - \frac{(a+bx)^3(-c\log(f)(a+bx)^n)^{-3/n} \Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^3n}$$

[Out] -(((a + b\*x)^3\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(3/n))) + (2\*a\*(a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n)) - (a^2\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1))

**Rubi [A]** time = 0.108436, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2226, 2208, 2218}

$$\frac{a^2(a+bx)(-c\log(f)(a+bx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^3n} - \frac{(a+bx)^3(-c\log(f)(a+bx)^n)^{-3/n} \Gamma\left(\frac{1}{n}, -c\log(f)(a+bx)^n\right)}{b^3n}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^n)\*x^2,x]

[Out] -(((a + b\*x)^3\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(3/n))) + (2\*a\*(a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n)) - (a^2\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1))

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{c(a+bx)^n} x^2 dx &= \int \left( \frac{a^2 f^{c(a+bx)^n}}{b^2} - \frac{2a f^{c(a+bx)^n} (a+bx)}{b^2} + \frac{f^{c(a+bx)^n} (a+bx)^2}{b^2} \right) dx \\ &= \frac{\int f^{c(a+bx)^n} (a+bx)^2 dx}{b^2} - \frac{(2a) \int f^{c(a+bx)^n} (a+bx) dx}{b^2} + \frac{a^2 \int f^{c(a+bx)^n} dx}{b^2} \\ &= -\frac{(a+bx)^3 \Gamma\left(\frac{3}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-3/n}}{b^3 n} + \frac{2a(a+bx)^2 \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-2/n}}{b^3 n} \end{aligned}$$

**Mathematica [A]** time = 0.0689383, size = 136, normalized size = 0.88

$$\frac{(a+bx) (-c \log(f)(a+bx)^n)^{-3/n} \left( a (-c \log(f)(a+bx)^n)^{1/n} \left( a (-c \log(f)(a+bx)^n)^{1/n} \Gamma\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right) - 2 \right) \right)}{b^3 n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^n)\*x^2,x]

[Out] -(((a + b\*x)\*((a + b\*x)^2\*Gamma[3/n, -(c\*(a + b\*x)^n\*Log[f])]) + a\*(-(c\*(a + b\*x)^n\*Log[f]))^n^(-1)\*(-2\*(a + b\*x)\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])]) + a\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])\*(-(c\*(a + b\*x)^n\*Log[f]))^n^(-1)))/(b^3\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(3/n))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^n} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)\*x^2,x)

[Out] int(f^(c\*(b\*x+a)^n)\*x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} c x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x^2,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{(bx+a)^n} c x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(f^(c*(b*x+a)^n)*x^2,x, algorithm="fricas")
```

```
[Out] integral(f^((b*x + a)^n*c)*x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*(b*x+a)**n)*x**2,x)
```

```
[Out] Integral(f**(c*(a + b*x)**n)*x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} c x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*(b*x+a)^n)*x^2,x, algorithm="giac")
```

```
[Out] integrate(f^((b*x + a)^n*c)*x^2, x)
```

### 3.249 $\int f^{c(a+bx)^n} x dx$

**Optimal.** Leaf size=99

$$\frac{a(a+bx)(-c \log(f)(a+bx)^n)^{-1/n} \operatorname{Gamma}\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right)}{b^{2n}} - \frac{(a+bx)^2(-c \log(f)(a+bx)^n)^{-2/n} \operatorname{Gamma}\left(\frac{2}{n}, -c \log(f)(a+bx)^n\right)}{b^{2n}}$$

[Out] -(((a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^2\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n))) + (a\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^2\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1))

**Rubi [A]** time = 0.0584529, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2226, 2208, 2218}

$$\frac{a(a+bx)(-c \log(f)(a+bx)^n)^{-1/n} \operatorname{Gamma}\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right)}{b^{2n}} - \frac{(a+bx)^2(-c \log(f)(a+bx)^n)^{-2/n} \operatorname{Gamma}\left(\frac{2}{n}, -c \log(f)(a+bx)^n\right)}{b^{2n}}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^n)\*x, x]

[Out] -(((a + b\*x)^2\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])])/(b^2\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(2/n))) + (a\*(a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b^2\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1))

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* (u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int f^{c(a+bx)^n} x dx &= \int \left( -\frac{a f^{c(a+bx)^n}}{b} + \frac{f^{c(a+bx)^n} (a+bx)}{b} \right) dx \\ &= \frac{\int f^{c(a+bx)^n} (a+bx) dx}{b} - \frac{a \int f^{c(a+bx)^n} dx}{b} \\ &= -\frac{(a+bx)^2 \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-2/n}}{b^{2n}} + \frac{a(a+bx) \Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-1/n}}{b^{2n}} \end{aligned}$$

**Mathematica [A]** time = 0.0330934, size = 91, normalized size = 0.92

$$\frac{(a + bx)(-c \log(f)(a + bx)^n)^{-2/n} \left( (a + bx) \text{Gamma}\left(\frac{2}{n}, -c \log(f)(a + bx)^n\right) - a(-c \log(f)(a + bx)^n)^{\frac{1}{n}} \text{Gamma}\left(\frac{1}{n}, -c \log(f)(a + bx)^n\right) \right)}{b^{2n}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^n)\*x, x]

[Out] -(((a + b\*x)\*((a + b\*x)\*Gamma[2/n, -(c\*(a + b\*x)^n\*Log[f])]) - a\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])\*(-(c\*(a + b\*x)^n\*Log[f])^(-1)))/(b^2\*n\*(-(c\*(a + b\*x)^n\*Log[f])^(2/n)))

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^n} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)\*x, x)

[Out] int(f^(c\*(b\*x+a)^n)\*x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n c} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x, x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)\*x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{(bx+a)^n c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)\*x, x, algorithm="fricas")

[Out] integral(f^((b\*x + a)^n\*c)\*x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*(b*x+a)**n)*x,x)`

[Out] `Integral(f**(c*(a + b*x)**n)*x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} c x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^n)*x,x, algorithm="giac")`

[Out] `integrate(f^((b*x + a)^n*c)*x, x)`

### 3.250 $\int f^{c(a+bx)^n} dx$

**Optimal.** Leaf size=47

$$\frac{(a+bx)(-c \log(f)(a+bx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right)}{bn}$$

[Out] -(((a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1)))

**Rubi [A]** time = 0.0057907, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2208}

$$\frac{(a+bx)(-c \log(f)(a+bx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[f^(c\*(a + b\*x)^n), x]

[Out] -(((a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1)))

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :- Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int f^{c(a+bx)^n} dx = \frac{(a+bx)\Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(f)\right) (-c(a+bx)^n \log(f))^{-1/n}}{bn}$$

**Mathematica [A]** time = 0.0063458, size = 47, normalized size = 1.

$$\frac{(a+bx)(-c \log(f)(a+bx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -c \log(f)(a+bx)^n\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[f^(c\*(a + b\*x)^n), x]

[Out] -(((a + b\*x)\*Gamma[n^(-1), -(c\*(a + b\*x)^n\*Log[f])])/(b\*n\*(-(c\*(a + b\*x)^n\*Log[f]))^(-1)))

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int f^{c(bx+a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*(b*x+a)^n),x)`

[Out] `int(f^(c*(b*x+a)^n),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^n),x, algorithm="maxima")`

[Out] `integrate(f^((b*x + a)^n*c), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{(bx+a)^n c}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^n),x, algorithm="fricas")`

[Out] `integral(f^((b*x + a)^n*c), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{c(a+bx)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*(b*x+a)**n),x)`

[Out] `Integral(f**(c*(a + b*x)**n), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(bx+a)^n c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*(b*x+a)^n),x, algorithm="giac")`

[Out] `integrate(f^((b*x + a)^n*c), x)`

$$3.251 \quad \int \frac{f^{c(a+bx)^n}}{x} dx$$

**Optimal.** Leaf size=17

$$\text{Unintegrable}\left(\frac{f^{c(a+bx)^n}}{x}, x\right)$$

[Out] Unintegrable[f^(c\*(a + b\*x)^n)/x, x]

**Rubi [A]** time = 0.0224096, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^n}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^n)/x, x]

[Out] Defer[Int][f^(c\*(a + b\*x)^n)/x, x]

Rubi steps

$$\int \frac{f^{c(a+bx)^n}}{x} dx = \int \frac{f^{c(a+bx)^n}}{x} dx$$

**Mathematica [A]** time = 0.0348841, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^n)/x, x]

[Out] Integrate[f^(c\*(a + b\*x)^n)/x, x]

**Maple [A]** time = 0.015, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)/x, x)

[Out] int(f^(c\*(b\*x+a)^n)/x, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{(bx+a)^n c}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x,x, algorithm="fricas")

[Out] integral(f^((b\*x + a)^n\*c)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*n)/x,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*n)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^n\*c)/x, x)



$$3.252 \quad \int \frac{f^{c(a+bx)^n}}{x^2} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{f^{c(a+bx)^n}}{x^2}, x\right)$$

[Out] CannotIntegrate[f^(c\*(a + b\*x)^n)/x^2, x]

**Rubi [A]** time = 0.0457516, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^n}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^n)/x^2,x]

[Out] Defer[Int][f^(c\*(a + b\*x)^n)/x^2, x]

Rubi steps

$$\int \frac{f^{c(a+bx)^n}}{x^2} dx = \int \frac{f^{c(a+bx)^n}}{x^2} dx$$

**Mathematica [A]** time = 0.0317066, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^n)/x^2,x]

[Out] Integrate[f^(c\*(a + b\*x)^n)/x^2, x]

**Maple [A]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^n}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)/x^2,x)

[Out] int(f^(c\*(b\*x+a)^n)/x^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^2,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)/x^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{(bx+a)^n c}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^2,x, algorithm="fricas")

[Out] integral(f^((b\*x + a)^n\*c)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*n)/x\*\*2,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*n)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^2,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^n\*c)/x^2, x)

$$3.253 \quad \int \frac{f^{c(a+bx)^n}}{x^3} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{f^{c(a+bx)^n}}{x^3}, x\right)$$

[Out] CannotIntegrate[f^(c\*(a + b\*x)^n)/x^3, x]

**Rubi [A]** time = 0.0449138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{c(a+bx)^n}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[f^(c\*(a + b\*x)^n)/x^3,x]

[Out] Defer[Int][f^(c\*(a + b\*x)^n)/x^3, x]

Rubi steps

$$\int \frac{f^{c(a+bx)^n}}{x^3} dx = \int \frac{f^{c(a+bx)^n}}{x^3} dx$$

**Mathematica [A]** time = 0.0347925, size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(c\*(a + b\*x)^n)/x^3,x]

[Out] Integrate[f^(c\*(a + b\*x)^n)/x^3, x]

**Maple [A]** time = 0.013, size = 0, normalized size = 0.

$$\int \frac{f^{c(bx+a)^n}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*(b\*x+a)^n)/x^3,x)

[Out] int(f^(c\*(b\*x+a)^n)/x^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^3,x, algorithm="maxima")

[Out] integrate(f^((b\*x + a)^n\*c)/x^3, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{(bx+a)^n c}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^3,x, algorithm="fricas")

[Out] integral(f^((b\*x + a)^n\*c)/x^3, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{c(a+bx)^n}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*(b\*x+a)\*\*n)/x\*\*3,x)

[Out] Integral(f\*\*(c\*(a + b\*x)\*\*n)/x\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{(bx+a)^n c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*(b\*x+a)^n)/x^3,x, algorithm="giac")

[Out] integrate(f^((b\*x + a)^n\*c)/x^3, x)

### 3.254 $\int F^{a+b(c+dx)^2} (c+dx)^m dx$

**Optimal.** Leaf size=61

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^2)^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -b \log(F)(c+dx)^2\right)}{2d}$$

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/2, -(b(c+dx)^2 \text{Log}[F])]) * (-(b(c+dx)^2 \text{Log}[F]))^{((-1-m)/2)} / (2*d)$

**Rubi [A]** time = 0.0650038, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^2)^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -b \log(F)(c+dx)^2\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^m, x]

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/2, -(b(c+dx)^2 \text{Log}[F])]) * (-(b(c+dx)^2 \text{Log}[F]))^{((-1-m)/2)} / (2*d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} (c+dx)^m dx = -\frac{F^a(c+dx)^{1+m} \Gamma\left(\frac{1+m}{2}, -b(c+dx)^2 \log(F)\right) (-b(c+dx)^2 \log(F))^{\frac{1}{2}(-1-m)}}{2d}$$

**Mathematica [A]** time = 0.0357247, size = 61, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^2)^{\frac{1}{2}(-m-1)} \text{Gamma}\left(\frac{m+1}{2}, -b \log(F)(c+dx)^2\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^m, x]

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/2, -(b(c+dx)^2 \text{Log}[F])]) * (-(b(c+dx)^2 \text{Log}[F]))^{((-1-m)/2)} / (2*d)$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^2} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^m,x)

[Out] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^m,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^2 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^((d\*x + c)^2 \* b + a), x)

**Fricas [A]** time = 1.58731, size = 163, normalized size = 2.67

$$\frac{e^{\left(-\frac{1}{2}(m-1)\log(-b\log(F))+a\log(F)\right)} \Gamma\left(\frac{1}{2}m + \frac{1}{2}, -(bd^2x^2 + 2bcdx + bc^2)\log(F)\right)}{2bd\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] 1/2\*e^(-1/2\*(m - 1)\*log(-b\*log(F)) + a\*log(F))\*gamma(1/2\*m + 1/2, -(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))/(b\*d\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^2 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*F^((d*x + c)^2*b + a), x)
```

### 3.255 $\int F^{a+b(c+dx)^2} (c+dx)^{11} dx$

**Optimal.** Leaf size=105

$$\frac{F^{a+b(c+dx)^2} \left( -b^5 \log^5(F)(c+dx)^{10} + 5b^4 \log^4(F)(c+dx)^8 - 20b^3 \log^3(F)(c+dx)^6 + 60b^2 \log^2(F)(c+dx)^4 - 120b \log(F)(c+dx)^2 + 120 \right)}{2b^6 d \log^6(F)}$$

[Out]  $-(F^{a+b(c+dx)^2} (120 - 120b(c+dx)^2 \log[F] + 60b^2(c+dx)^4 \log[F]^2 - 20b^3(c+dx)^6 \log[F]^3 + 5b^4(c+dx)^8 \log[F]^4 - b^5(c+dx)^{10} \log[F]^5)) / (2b^6 d \log[F]^6)$

**Rubi [C]** time = 0.0726111, antiderivative size = 31, normalized size of antiderivative = 0.3, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^2)}{2b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^11, x]

[Out]  $-(F^a \text{Gamma}[6, -(b*(c + d*x)^2 \log[F])]) / (2*b^6*d*\log[F]^6)$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} (c+dx)^{11} dx = -\frac{F^a \Gamma(6, -b(c+dx)^2 \log(F))}{2b^6 d \log^6(F)}$$

**Mathematica [C]** time = 0.0083566, size = 31, normalized size = 0.3

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^2)}{2b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^11, x]

[Out]  $-(F^a \text{Gamma}[6, -(b*(c + d*x)^2 \log[F])]) / (2*b^6*d*\log[F]^6)$

**Maple [B]** time = 0.021, size = 579, normalized size = 5.5

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^11,x)

[Out]  $\frac{1}{2}(-120+120*\ln(F)*b*d^2*x^2+20*\ln(F)^3*b^3*c^6-60*\ln(F)^2*b^2*c^4+120*\ln(F)*b*c^2+240*\ln(F)*b*c*d*x-360*\ln(F)^2*b^2*c^2*d^2*x^2-240*\ln(F)^2*b^2*c^3*d*x-40*c*d^7*x^7*b^4*\ln(F)^4+45*\ln(F)^5*b^5*c^8*d^2*x^2-140*\ln(F)^4*b^4*c^2*d^6*x^6+10*\ln(F)^5*b^5*c^9*d*x-280*\ln(F)^4*b^4*c^3*d^5*x^5-350*\ln(F)^4*b^4*c^4*d^4*x^4-280*\ln(F)^4*b^4*c^5*d^3*x^3-140*\ln(F)^4*b^4*c^6*d^2*x^2-40*\ln(F)^4*b^4*c^7*d*x+120*c*d^5*x^5*b^3*\ln(F)^3+300*\ln(F)^3*b^3*c^2*d^4*x^4+400*\ln(F)^3*b^3*c^3*d^3*x^3+300*\ln(F)^3*b^3*c^4*d^2*x^2+120*\ln(F)^3*b^3*c^5*d*x-240*d^3*c*x^3*b^2*\ln(F)^2+10*d^9*c*x^9*b^5*\ln(F)^5+45*\ln(F)^5*b^5*c^2*d^8*x^8+120*\ln(F)^5*b^5*c^3*d^7*x^7+210*\ln(F)^5*b^5*c^4*d^6*x^6+252*\ln(F)^5*b^5*c^5*d^5*x^5+210*\ln(F)^5*b^5*c^6*d^4*x^4+120*\ln(F)^5*b^5*c^7*d^3*x^3-5*\ln(F)^4*b^4*c^8+\ln(F)^5*b^5*c^{10}+d^{10}*x^{10}*b^5*\ln(F)^5-5*d^8*x^8*b^4*\ln(F)^4+20*d^6*x^6*b^3*\ln(F)^3-60*d^4*x^4*b^2*\ln(F)^2)*F^(b*d^2*x^2+2*b*c*d*x+b*c^2+a)/b^6/\ln(F)^6/d$

**Maxima [C]** time = 3.57992, size = 7426, normalized size = 70.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^11,x, algorithm="maxima")

[Out]  $-11/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*\log(F)/(b*d^2*\log(F))^{3/2})*F^a*c^{10}*d/\sqrt{b*d^2*\log(F)} + 55/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{5/2} - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{5/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}))*F^a*c^9*d^2/\sqrt{b*d^2*\log(F)} - 165/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^4/((b*d^2*\log(F))^{7/2})*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*d^4*\log(F)^3/(b*d^2*\log(F))^{7/2} - 3*(b*d^2*x + b*c*d)^3*b*c*d*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/((b*d^2*\log(F))^{7/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + b^2*d^4*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^2/(b*d^2*\log(F))^{7/2})*F^a*c^8*d^3/\sqrt{b*d^2*\log(F)} + 165*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^5/((b*d^2*\log(F))^{9/2})*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*d^5*\log(F)^4/(b*d^2*\log(F))^{9/2} - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{9/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + 4*b^3*c*d^5*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{9/2} - (b*d^2*x + b*c*d)^5*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{9/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{5/2}))*F^a*c^7*d^4/\sqrt{b*d^2*\log(F)} - 231*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^6/((b*d^2*\log(F))^{11/2})*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{11/2} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{11/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + 10*b^4*c^2$

$$\begin{aligned}
& *d^6 \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^4 / (b*d^2 \log(F))^{11/2} \\
& - b^3 d^6 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^3 / (b*d^2 \log(F))^{11/2} \\
& - 5*(b*d^2*x + b*c*d)^5 b*c*d \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / ((b*d^2 \log(F))^{11/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& * F^a c^6 d^5 / \sqrt{b*d^2 \log(F)} + 231 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^6 c^6 d^6 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) \\
& - 1) * \log(F)^7 / ((b*d^2 \log(F))^{13/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 6 * F^((b*d^2*x + b*c*d)^2 / (b*d^2)) * b^6 c^5 d^7 * \log(F)^6 / (b*d^2 \log(F))^{13/2} \\
& - 15 * (b*d^2*x + b*c*d)^3 b^4 c^4 d^4 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / ((b*d^2 \log(F))^{13/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 20 * b^5 c^3 d^7 \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / (b*d^2 \log(F))^{13/2} \\
& - 6 * b^4 c^4 d^7 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^4 / (b*d^2 \log(F))^{13/2} \\
& - 15 * (b*d^2*x + b*c*d)^5 b^2 c^2 d^2 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / ((b*d^2 \log(F))^{13/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& - (b*d^2*x + b*c*d)^7 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / ((b*d^2 \log(F))^{13/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& * F^a c^5 d^6 / \sqrt{b*d^2 \log(F)} - 165 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^7 c^7 d^7 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) \\
& - 1) * \log(F)^8 / ((b*d^2 \log(F))^{15/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 7 * F^((b*d^2*x + b*c*d)^2 / (b*d^2)) * b^7 c^6 d^8 * \log(F)^7 / (b*d^2 \log(F))^{15/2} \\
& - 21 * (b*d^2*x + b*c*d)^3 b^5 c^5 d^5 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / ((b*d^2 \log(F))^{15/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 35 * b^6 c^4 d^8 \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / (b*d^2 \log(F))^{15/2} \\
& - 21 * b^5 c^2 d^8 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / (b*d^2 \log(F))^{15/2} \\
& - 35 * (b*d^2*x + b*c*d)^5 b^3 c^3 d^3 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / ((b*d^2 \log(F))^{15/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + b^4 d^8 \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^4 / (b*d^2 \log(F))^{15/2} \\
& - 7 * (b*d^2*x + b*c*d)^7 b*c*d \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / ((b*d^2 \log(F))^{15/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& * F^a c^4 d^7 / \sqrt{b*d^2 \log(F)} + 165/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^8 c^8 d^8 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) \\
& - 1) * \log(F)^9 / ((b*d^2 \log(F))^{17/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 8 * F^((b*d^2*x + b*c*d)^2 / (b*d^2)) * b^8 c^7 d^9 * \log(F)^8 / (b*d^2 \log(F))^{17/2} \\
& - 28 * (b*d^2*x + b*c*d)^3 b^6 c^6 d^6 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 56 * b^7 c^5 d^9 \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / (b*d^2 \log(F))^{17/2} \\
& - 56 * b^6 c^3 d^9 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / (b*d^2 \log(F))^{17/2} \\
& - 70 * (b*d^2*x + b*c*d)^5 b^4 c^4 d^4 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + 8 * b^5 c^4 d^9 \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / (b*d^2 \log(F))^{17/2} \\
& - 28 * (b*d^2*x + b*c*d)^7 b^2 c^2 d^2 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& - (b*d^2*x + b*c*d)^9 \gamma(9/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{9/2}) \\
& * F^a c^3 d^8 / \sqrt{b*d^2 \log(F)} - 55/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^9 c^9 d^9 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) \\
& - 1) * \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 9 * F^((b*d^2*x + b*c*d)^2 / (b*d^2)) * b^9 c^8 d^{10} * \log(F)^9 / (b*d^2 \log(F))^{19/2} \\
& - 36 * (b*d^2*x + b*c*d)^3 b^7 c^7 d^7 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 84 * b^8 c^6 d^{10} \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / (b*d^2 \log(F))^{19/2} \\
& - 126 * b^7 c^4 d^{10} \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / (b*d^2 \log(F))^{19/2} \\
& - 126 * (b*d^2*x + b*c*d)^5 b^5 c^5 d^5 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + 36 * b^6 c^2 d^{10} \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / (b*d^2 \log(F))^{19/2} \\
& - 84 * (b*d^2*x + b*c*d)
\end{aligned}$$

$$\begin{aligned}
& )^7 b^3 c^3 d^3 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{10} / \\
& (b*d^2 \log(F))^{(19/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(7/2)} - b^5 d^{10} \gamma(5, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / (b*d^2 \log(F))^{(19/2)} - 9 * (b*d^2*x + b*c*d)^9 b^9 c^9 d^9 \gamma(9/2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{10} / ((b*d^2 \log(F))^{(19/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(9/2)}) \\
& ) * F^a c^2 d^9 / \sqrt{b*d^2 \log(F)} + 11/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^{10} c^{10} d^{10} * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) - 1) * \log(F)^{11} / \\
& ((b*d^2 \log(F))^{(21/2)} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) - 10 * F^{(b*d^2*x + b*c*d)^2 / (b*d^2)} * b^{10} c^9 d^{11} \log(F)^{10} / (b*d^2 \log(F))^{(21/2)} - 45 * (b*d^2*x + b*c*d)^3 b^8 c^8 d^8 \gamma(3/2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{11} / ((b*d^2 \log(F))^{(21/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(3/2)}) + 120 * b^9 c^7 d^{11} \gamma(2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / (b*d^2 \log(F))^{(21/2)} - 252 * b^8 c^5 d^{11} \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / \\
& (b*d^2 \log(F))^{(21/2)} - 210 * (b*d^2*x + b*c*d)^5 b^6 c^6 d^6 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{11} / ((b*d^2 \log(F))^{(21/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(5/2)}) + \\
& 120 * b^7 c^3 d^{11} \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / (b*d^2 \log(F))^{(21/2)} - 210 * (b*d^2*x + b*c*d)^7 b^4 c^4 d^4 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{11} / \\
& ((b*d^2 \log(F))^{(21/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(7/2)}) - 10 * b^6 c^4 d^{11} \gamma(5, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / (b*d^2 \log(F))^{(21/2)} - 45 * (b*d^2*x + b*c*d)^9 b^2 c^2 d^2 \gamma(9/2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{11} / ((b*d^2 \log(F))^{(21/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(9/2)}) - (b*d^2*x + b*c*d)^{11} \gamma(11/2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{11} / ((b*d^2 \log(F))^{(21/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(11/2)}) * F^a c^2 d^{10} / \sqrt{b*d^2 \log(F)} - 1/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^{11} c^{11} d^{11} * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) - 1) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) - 11 * F^{(b*d^2*x + b*c*d)^2 / (b*d^2)} * b^{11} c^{10} d^{12} \log(F)^{11} / (b*d^2 \log(F))^{(23/2)} - 55 * (b*d^2*x + b*c*d)^3 b^9 c^9 d^9 \gamma(3/2, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(3/2)}) + 165 * b^{10} c^8 d^{12} \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{10} / (b*d^2 \log(F))^{(23/2)} - 462 * b^9 c^6 d^{12} \gamma(3, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^9 / (b*d^2 \log(F))^{(23/2)} - 330 * (b*d^2*x + b*c*d)^5 b^7 c^7 d^7 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(5/2)}) + 330 * b^8 c^4 d^{12} \gamma(4, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^8 / (b*d^2 \log(F))^{(23/2)} - 462 * (b*d^2*x + b*c*d)^7 b^5 c^5 d^5 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(7/2)}) - 55 * b^7 c^2 d^{12} \gamma(5, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^7 / (b*d^2 \log(F))^{(23/2)} - 165 * (b*d^2*x + b*c*d)^9 b^3 c^3 d^3 \gamma(9/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(9/2)}) + b^6 d^{12} \gamma(6, \\
& -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^6 / (b*d^2 \log(F))^{(23/2)} - 11 * (b*d^2*x + b*c*d)^{11} b^3 c^3 d^3 \gamma(11/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^{12} / ((b*d^2 \log(F))^{(23/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(11/2)}) * F^a d^{11} / \sqrt{b*d^2 \log(F)} + 1/2 * \sqrt{\pi} * F^{(b*c^2 + a) * c^{11} \operatorname{erf}(\sqrt{-b \log(F)}) * d * x - b * c \log(F) / \sqrt{-b \log(F)}} / (\sqrt{-b \log(F)} * F^{(b*c^2) * d})
\end{aligned}$$

**Fricas [B]** time = 1.54839, size = 999, normalized size = 9.51

$$((b^5 d^{10} x^{10} + 10 b^5 c d^9 x^9 + 45 b^5 c^2 d^8 x^8 + 120 b^5 c^3 d^7 x^7 + 210 b^5 c^4 d^6 x^6 + 252 b^5 c^5 d^5 x^5 + 210 b^5 c^6 d^4 x^4 + 120 b^5 c^7 d^3 x^3 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^11,x, algorithm="fricas")

```
[Out] 1/2*((b^5*d^10*x^10 + 10*b^5*c*d^9*x^9 + 45*b^5*c^2*d^8*x^8 + 120*b^5*c^3*d^7*x^7 + 210*b^5*c^4*d^6*x^6 + 252*b^5*c^5*d^5*x^5 + 210*b^5*c^6*d^4*x^4 + 120*b^5*c^7*d^3*x^3 + 45*b^5*c^8*d^2*x^2 + 10*b^5*c^9*d*x + b^5*c^10)*log(F)^5 - 5*(b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^3*d^5*x^5 + 70*b^4*c^4*d^4*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 + 8*b^4*c^7*d*x + b^4*c^8)*log(F)^4 + 20*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 - 60*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 + 120*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) - 120)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^6*d*log(F)^6)
```

**Sympy [A]** time = 0.398578, size = 796, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**11,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x)**2)*(b**5*c**10*log(F)**5 + 10*b**5*c**9*d*x*log(F)**5 + 45*b**5*c**8*d**2*x**2*log(F)**5 + 120*b**5*c**7*d**3*x**3*log(F)**5 + 210*b**5*c**6*d**4*x**4*log(F)**5 + 252*b**5*c**5*d**5*x**5*log(F)**5 + 210*b**5*c**4*d**6*x**6*log(F)**5 + 120*b**5*c**3*d**7*x**7*log(F)**5 + 45*b**5*c**2*d**8*x**8*log(F)**5 + 10*b**5*c*d**9*x**9*log(F)**5 + b**5*d**10*x**10*log(F)**5 - 5*b**4*c**8*log(F)**4 - 40*b**4*c**7*d*x*log(F)**4 - 140*b**4*c**6*d**2*x**2*log(F)**4 - 280*b**4*c**5*d**3*x**3*log(F)**4 - 350*b**4*c**4*d**4*x**4*log(F)**4 - 280*b**4*c**3*d**5*x**5*log(F)**4 - 140*b**4*c**2*d**6*x**6*log(F)**4 - 40*b**4*c*d**7*x**7*log(F)**4 - 5*b**4*d**8*x**8*log(F)**4 + 20*b**3*c**6*log(F)**3 + 120*b**3*c**5*d*x*log(F)**3 + 300*b**3*c**4*d**2*x**2*log(F)**3 + 400*b**3*c**3*d**3*x**3*log(F)**3 + 300*b**3*c**2*d**4*x**4*log(F)**3 + 120*b**3*c*d**5*x**5*log(F)**3 + 20*b**3*d**6*x**6*log(F)**3 - 60*b**2*c**4*log(F)**2 - 240*b**2*c**3*d*x*log(F)**2 - 360*b**2*c**2*d**2*x**2*log(F)**2 - 240*b**2*c*d**3*x**3*log(F)**2 - 60*b**2*d**4*x**4*log(F)**2 + 120*b*c**2*log(F) + 240*b*c*d*x*log(F) + 120*b*d**2*x**2*log(F) - 120)/(2*b**6*d*log(F)**6), Ne(2*b**6*d*log(F)**6, 0)), (c**11*x + 11*c**10*d*x**2/2 + 55*c**9*d**2*x**3/3 + 165*c**8*d**3*x**4/4 + 66*c**7*d**4*x**5 + 77*c**6*d**5*x**6 + 66*c**5*d**6*x**7 + 165*c**4*d**7*x**8/4 + 55*c**3*d**8*x**9/3 + 11*c**2*d**9*x**10/2 + c*d**10*x**11 + d**11*x**12/12, True))
```

**Giac [A]** time = 1.33087, size = 196, normalized size = 1.87

$$\frac{\left(b^5 d^{10} \left(x + \frac{c}{d}\right)^{10} \log(F)^5 - 5 b^4 d^8 \left(x + \frac{c}{d}\right)^8 \log(F)^4 + 20 b^3 d^6 \left(x + \frac{c}{d}\right)^6 \log(F)^3 - 60 b^2 d^4 \left(x + \frac{c}{d}\right)^4 \log(F)^2 + 120 b d^2 \left(x + \frac{c}{d}\right)^2 \log(F) - 120\right) e^{(b d^2 x^2 \log(F) + 2 b c d x \log(F) + b c^2 \log(F) + a \log(F))}}{2 b^6 d \log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*d^10*(x + c/d)^10*log(F)^5 - 5*b^4*d^8*(x + c/d)^8*log(F)^4 + 20*b^3*d^6*(x + c/d)^6*log(F)^3 - 60*b^2*d^4*(x + c/d)^4*log(F)^2 + 120*b*d^2*(x + c/d)^2*log(F) - 120)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^6*d*log(F)^6)
```

### 3.256 $\int F^{a+b(c+dx)^2} (c+dx)^9 dx$

**Optimal.** Leaf size=88

$$\frac{F^{a+b(c+dx)^2} (b^4 \log^4(F)(c+dx)^8 - 4b^3 \log^3(F)(c+dx)^6 + 12b^2 \log^2(F)(c+dx)^4 - 24b \log(F)(c+dx)^2 + 24)}{2b^5 d \log^5(F)}$$

[Out]  $(F^{(a + b*(c + d*x)^2})*(24 - 24*b*(c + d*x)^2*\text{Log}[F] + 12*b^2*(c + d*x)^4*\text{Log}[F]^2 - 4*b^3*(c + d*x)^6*\text{Log}[F]^3 + b^4*(c + d*x)^8*\text{Log}[F]^4))/(2*b^5*d*\text{Log}[F]^5)$

**Rubi [C]** time = 0.0693239, antiderivative size = 31, normalized size of antiderivative = 0.35, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^2)}{2b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^9, x]

[Out]  $(F^a*\text{Gamma}[5, -(b*(c + d*x)^2*\text{Log}[F])])/(2*b^5*d*\text{Log}[F]^5)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} (c+dx)^9 dx = \frac{F^a \Gamma(5, -b(c+dx)^2 \log(F))}{2b^5 d \log^5(F)}$$

**Mathematica [C]** time = 0.0077248, size = 31, normalized size = 0.35

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^2)}{2b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^9, x]

[Out]  $(F^a*\text{Gamma}[5, -(b*(c + d*x)^2*\text{Log}[F])])/(2*b^5*d*\text{Log}[F]^5)$

**Maple [B]** time = 0.015, size = 396, normalized size = 4.5

$$(24 - 24 \ln(F) b d^2 x^2 - 4 (\ln(F))^3 b^3 c^6 + 12 (\ln(F))^2 b^2 c^4 - 24 \ln(F) b c^2 - 48 \ln(F) b c d x + 72 (\ln(F))^2 b^2 c^2 d^2 x^2 + 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x)
```

```
[Out] 1/2*(24-24*ln(F)*b*d^2*x^2-4*ln(F)^3*b^3*c^6+12*ln(F)^2*b^2*c^4-24*ln(F)*b*c^2-48*ln(F)*b*c*d*x+72*ln(F)^2*b^2*c^2*d^2*x^2+48*ln(F)^2*b^2*c^3*d*x+8*c*d^7*x^7*b^4*ln(F)^4+28*ln(F)^4*b^4*c^2*d^6*x^6+56*ln(F)^4*b^4*c^3*d^5*x^5+70*ln(F)^4*b^4*c^4*d^4*x^4+56*ln(F)^4*b^4*c^5*d^3*x^3+28*ln(F)^4*b^4*c^6*d^2*x^2+8*ln(F)^4*b^4*c^7*d*x-24*c*d^5*x^5*b^3*ln(F)^3-60*ln(F)^3*b^3*c^2*d^4*x^4-80*ln(F)^3*b^3*c^3*d^3*x^3-60*ln(F)^3*b^3*c^4*d^2*x^2-24*ln(F)^3*b^3*c^5*d*x+48*d^3*c*x^3*b^2*ln(F)^2+ln(F)^4*b^4*c^8+d^8*x^8*b^4*ln(F)^4-4*d^6*x^6*b^3*ln(F)^3+12*d^4*x^4*b^2*ln(F)^2)*F^(b*d^2*x^2+2*b*c*d*x+b*c^2+a)/b^5/ln(F)^5/d
```

**Maxima [C]** time = 3.09527, size = 5261, normalized size = 59.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -9/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*d*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^2/((b*d^2*log(F))^(3/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*log(F)/(b*d^2*log(F))^(3/2))*F^a*c^8*d/sqrt(b*d^2*log(F)) + 18*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*d^2*log(F))^(5/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*log(F)^2/(b*d^2*log(F))^(5/2) - (b*d^2*x + b*c*d)^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*d^2*log(F))^(5/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2))*F^a*c^7*d^2/sqrt(b*d^2*log(F)) - 42*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*d^2*log(F))^(7/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*d^4*log(F)^3/(b*d^2*log(F))^(7/2) - 3*(b*d^2*x + b*c*d)^3*b*c*d*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*d^2*log(F))^(7/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*d^4*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^2/(b*d^2*log(F))^(7/2))*F^a*c^6*d^3/sqrt(b*d^2*log(F)) + 63*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^5/((b*d^2*log(F))^(9/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*d^5*log(F)^4/(b*d^2*log(F))^(9/2) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 4*b^3*c*d^5*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))^(9/2) - (b*d^2*x + b*c*d)^5*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(5/2))*F^a*c^5*d^4/sqrt(b*d^2*log(F)) - 63*(sqrt(pi)*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^6/((b*d^2*log(F))^(11/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 5*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^5*c^4*d^6*log(F)^5/(b*d^2*log(F))^(11/2) - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^6/((b*d^2*log(F))^(11/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 10*b^4*c^2*d^6*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/(b*d^2*log(F))^(11/2) - b^3*d^6*gamma(3, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))^(11/2) - 5*(b*d^2*x + b*c*d)^5*b*c*d*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^6/((b*d^2*log(F))^(11/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(
```

$$\begin{aligned}
& b*d^2)^{(5/2)}) * F^a * c^4 * d^5 / \sqrt{b*d^2 * \log(F)} + 42 * (\sqrt{\pi}) * (b*d^2 * x + b * \\
& c*d) * b^6 * c^6 * d^6 * (\operatorname{erf}(\sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)})) - 1) * \log(F) \\
& )^7 / ((b*d^2 * \log(F))^{(13/2)} * \sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)}) - 6 * F \\
& ^((b*d^2 * x + b*c*d)^2 / (b*d^2)) * b^6 * c^5 * d^7 * \log(F)^6 / (b*d^2 * \log(F))^{(13/2)} - \\
& 15 * (b*d^2 * x + b*c*d)^3 * b^4 * c^4 * d^4 * \gamma(3/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / \\
& (b*d^2)) * \log(F)^7 / ((b*d^2 * \log(F))^{(13/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2 \\
& ))^{(3/2)}) + 20 * b^5 * c^3 * d^7 * \gamma(2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^5 / (b*d^2 * \log(F))^{(13/2)} - 6 * b^4 * c * d^7 * \gamma(3, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^4 / (b*d^2 * \log(F))^{(13/2)} - 15 * (b*d^2 * x + b*c*d)^5 * b^2 * c^2 * d^2 * \gamma(5/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^7 / ((b*d^2 * \log(F))^{(13/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)}) - (b*d^2 * x + b*c*d) \\
& ^7 * \gamma(7/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^7 / ((b*d^2 * \log(F))^{(13/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) \\
& ) * F^a * c^3 * d^6 / \sqrt{b*d^2 * \log(F)} - 18 * (\sqrt{\pi}) * (b*d^2 * x + b*c*d) * b^7 * c^7 * d^7 * (\operatorname{erf}(\sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)})) - 1) * \log(F) \\
& ^8 / ((b*d^2 * \log(F))^{(15/2)} * \sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)}) - 7 * F^((b*d^2 * x + b*c*d)^2 / (b*d^2)) * b^7 * c^6 * d^8 * \log(F) \\
& ^7 / (b*d^2 * \log(F))^{(15/2)} - 21 * (b*d^2 * x + b*c*d)^3 * b^5 * c^5 * d^5 * \gamma(3/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^8 / ((b*d^2 * \log(F))^{(15/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(3/2)}) + 35 * b^6 * c^4 * d^8 * \gamma(2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^6 / (b*d^2 * \log(F))^{(15/2)} - 21 * b^5 * c^2 * d^8 * \gamma(3, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^5 / (b*d^2 * \log(F))^{(15/2)} - 35 * (b*d^2 * x + b*c*d)^5 * b^3 * c^3 * d^3 * \gamma(5/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^8 / ((b*d^2 * \log(F))^{(15/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)}) + b^4 * d^8 * \gamma(4, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^4 / (b*d^2 * \log(F))^{(15/2)} - 7 * (b*d^2 * x + b*c*d)^7 * b * c * d * \gamma(7/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^8 / ((b*d^2 * \log(F))^{(15/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) * F^a * c^2 * d^7 / \sqrt{b*d^2 * \log(F)} \\
& + 9/2 * (\sqrt{\pi}) * (b*d^2 * x + b*c*d) * b^8 * c^8 * d^8 * (\operatorname{erf}(\sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)})) - 1) * \log(F) \\
& ^9 / ((b*d^2 * \log(F))^{(17/2)} * \sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)}) - 8 * F^((b*d^2 * x + b*c*d)^2 / (b*d^2)) * b^8 * c^7 * d^9 * \log(F) \\
& ^8 / (b*d^2 * \log(F))^{(17/2)} - 28 * (b*d^2 * x + b*c*d)^3 * b^6 * c^6 * d^6 * \gamma(3/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^9 / ((b*d^2 * \log(F))^{(17/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(3/2)}) + 56 * b^7 * c^5 * d^9 * \gamma(2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^7 / (b*d^2 * \log(F))^{(17/2)} - 56 * b^6 * c^3 * d^9 * \gamma(3, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^6 / (b*d^2 * \log(F))^{(17/2)} - 70 * (b*d^2 * x + b*c*d)^5 * b^4 * c^4 * d^4 * \gamma(5/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^9 / ((b*d^2 * \log(F))^{(17/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)}) + 8 * b^5 * c * d^9 * \gamma(4, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^5 / (b*d^2 * \log(F))^{(17/2)} - 28 * (b*d^2 * x + b*c*d)^7 * b^2 * c^2 * d^2 * \gamma(7/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^9 / ((b*d^2 * \log(F))^{(17/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) - (b*d^2 * x + b*c*d)^9 * \gamma(9/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^9 / ((b*d^2 * \log(F))^{(17/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(9/2)}) * F^a * c * d^8 / \sqrt{b*d^2 * \log(F)} - 1/2 * (\sqrt{\pi}) * (b*d^2 * x + b*c*d) * b^9 * c^9 * d^9 * (\operatorname{erf}(\sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)})) - 1) * \log(F) \\
& ^10 / ((b*d^2 * \log(F))^{(19/2)} * \sqrt{-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)}) - 9 * F^((b*d^2 * x + b*c*d)^2 / (b*d^2)) * b^9 * c^8 * d^10 * \log(F) \\
& ^9 / (b*d^2 * \log(F))^{(19/2)} - 36 * (b*d^2 * x + b*c*d)^3 * b^7 * c^7 * d^7 * \gamma(3/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^10 / ((b*d^2 * \log(F))^{(19/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(3/2)}) + 84 * b^8 * c^6 * d^10 * \gamma(2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^8 / (b*d^2 * \log(F))^{(19/2)} - 126 * b^7 * c^4 * d^10 * \gamma(3, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^7 / (b*d^2 * \log(F))^{(19/2)} - 126 * (b*d^2 * x + b*c*d)^5 * b^5 * c^5 * d^5 * \gamma(5/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^10 / ((b*d^2 * \log(F))^{(19/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)}) + 36 * b^6 * c^2 * d^10 * \gamma(4, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^6 / (b*d^2 * \log(F))^{(19/2)} - 84 * (b*d^2 * x + b*c*d)^7 * b^3 * c^3 * d^3 * \gamma(7/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^10 / ((b*d^2 * \log(F))^{(19/2)} * (-(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) - b^5 * d^10 * \gamma(5, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) \\
& ^5 / (b*d^2 * \log(F))^{(19/2)} - 9 * (b*d^2 * x + b*c*d)^9 * b * c * d * \gamma(9/2, -(b*d^2 * x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F) ^1
\end{aligned}$$

$$\frac{0/((b*d^2*\log(F))^{(19/2)*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(9/2))})*F^a*d^9/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^9*\operatorname{erf}(\sqrt{-b*\log(F)})}*d*x - b*c*\log(F)/\sqrt{-b*\log(F)})/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d}$$

**Fricas [B]** time = 1.5642, size = 687, normalized size = 7.81

$$\left( (b^4 d^8 x^8 + 8 b^4 c d^7 x^7 + 28 b^4 c^2 d^6 x^6 + 56 b^4 c^3 d^5 x^5 + 70 b^4 c^4 d^4 x^4 + 56 b^4 c^5 d^3 x^3 + 28 b^4 c^6 d^2 x^2 + 8 b^4 c^7 d x + b^4 c^8) \log(F)^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] 1/2*((b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^3*d^5*x^5 + 70*b^4*c^4*d^4*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 + 8*b^4*c^7*d*x + b^4*c^8)*log(F)^4 - 4*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 + 12*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 - 24*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) + 24)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^5*d*log(F)^5)
```

**Sympy [A]** time = 0.318528, size = 558, normalized size = 6.34

$$\left\{ \frac{F^{a+b(c+dx)^2} (b^4 c^8 \log(F)^4 + 8 b^4 c^7 d x \log(F)^4 + 28 b^4 c^6 d^2 x^2 \log(F)^4 + 56 b^4 c^5 d^3 x^3 \log(F)^4 + 70 b^4 c^4 d^4 x^4 \log(F)^4 + 56 b^4 c^3 d^5 x^5 \log(F)^4 + 28 b^4 c^2 d^6 x^6 \log(F)^4 + 8 b^4 c d^7 x^7 \log(F)^4 + b^4 c^8 \log(F)^4)}{c^9 x + \frac{9 c^8 d x^2}{2} + 12 c^7 d^2 x^3 + 21 c^6 d^3 x^4 + \frac{126 c^5 d^4 x^5}{5} + 21 c^4 d^5 x^6 + 12 c^3 d^6 x^7 + \frac{9 c^2 d^7 x^8}{2} + c d^8 x^9 + \frac{d^9 x^{10}}{10}} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**9,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x)**2)*(b**4*c**8*log(F)**4 + 8*b**4*c**7*d*x*log(F)**4 + 28*b**4*c**6*d**2*x**2*log(F)**4 + 56*b**4*c**5*d**3*x**3*log(F)**4 + 70*b**4*c**4*d**4*x**4*log(F)**4 + 56*b**4*c**3*d**5*x**5*log(F)**4 + 28*b**4*c**2*d**6*x**6*log(F)**4 + 8*b**4*c*d**7*x**7*log(F)**4 + b**4*d**8*x**8*log(F)**4 - 4*b**3*c**6*log(F)**3 - 24*b**3*c**5*d*x*log(F)**3 - 60*b**3*c**4*d**2*x**2*log(F)**3 - 80*b**3*c**3*d**3*x**3*log(F)**3 - 60*b**3*c**2*d**4*x**4*log(F)**3 - 24*b**3*c*d**5*x**5*log(F)**3 - 4*b**3*d**6*x**6*log(F)**3 + 12*b**2*c**4*log(F)**2 + 48*b**2*c**3*d*x*log(F)**2 + 72*b**2*c**2*d**2*x**2*log(F)**2 + 48*b**2*c*d**3*x**3*log(F)**2 + 12*b**2*d**4*x**4*log(F)**2 - 24*b*c**2*log(F) - 48*b*c*d*x*log(F) - 24*b*d**2*x**2*log(F) + 24)/(2*b**5*d*log(F)**5), Ne(2*b**5*d*log(F)**5, 0)), (c**9*x + 9*c**8*d*x**2/2 + 12*c**7*d**2*x**3 + 21*c**6*d**3*x**4 + 126*c**5*d**4*x**5/5 + 21*c**4*d**5*x**6 + 12*c**3*d**6*x**7 + 9*c**2*d**7*x**8/2 + c*d**8*x**9 + d**9*x**10/10, True))
```

**Giac [A]** time = 1.31809, size = 167, normalized size = 1.9

$$\frac{\left( b^4 d^8 \left( x + \frac{c}{d} \right)^8 \log(F)^4 - 4 b^3 d^6 \left( x + \frac{c}{d} \right)^6 \log(F)^3 + 12 b^2 d^4 \left( x + \frac{c}{d} \right)^4 \log(F)^2 - 24 b d^2 \left( x + \frac{c}{d} \right)^2 \log(F) + 24 \right) e^{(b d^2 x^2 \log(F) + 2 b c d x + b^2 c^2) \log(F)}}{2 b^5 d \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^8*(x + c/d)^8*log(F)^4 - 4*b^3*d^6*(x + c/d)^6*log(F)^3 + 12*b^2*d^4*(x + c/d)^4*log(F)^2 - 24*b*d^2*(x + c/d)^2*log(F) + 24)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^5*d*log(F)^5)
```

### 3.257 $\int F^{a+b(c+dx)^2} (c+dx)^7 dx$

**Optimal.** Leaf size=126

$$-\frac{3(c+dx)^4 F^{a+b(c+dx)^2}}{2b^2 d \log^2(F)} + \frac{3(c+dx)^2 F^{a+b(c+dx)^2}}{b^3 d \log^3(F)} - \frac{3F^{a+b(c+dx)^2}}{b^4 d \log^4(F)} + \frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

[Out]  $(-3F^{a+b(c+dx)^2})/(b^4 d \log^4(F)) + (3F^{a+b(c+dx)^2} (c+dx)^6)/(2bd \log(F)) - (3F^{a+b(c+dx)^2} (c+dx)^2)/(b^3 d \log^3(F)) - (3F^{a+b(c+dx)^2} (c+dx)^4)/(2b^2 d \log^2(F)) + (F^{a+b(c+dx)^2} (c+dx)^6)/(2b d \log(F))$

**Rubi [A]** time = 0.25554, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$-\frac{3(c+dx)^4 F^{a+b(c+dx)^2}}{2b^2 d \log^2(F)} + \frac{3(c+dx)^2 F^{a+b(c+dx)^2}}{b^3 d \log^3(F)} - \frac{3F^{a+b(c+dx)^2}}{b^4 d \log^4(F)} + \frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^7, x]

[Out]  $(-3F^{a+b(c+dx)^2})/(b^4 d \log^4(F)) + (3F^{a+b(c+dx)^2} (c+dx)^6)/(2bd \log(F)) - (3F^{a+b(c+dx)^2} (c+dx)^2)/(b^3 d \log^3(F)) - (3F^{a+b(c+dx)^2} (c+dx)^4)/(2b^2 d \log^2(F)) + (F^{a+b(c+dx)^2} (c+dx)^6)/(2b d \log(F))$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (c+dx)^7 dx &= \frac{F^{a+b(c+dx)^2} (c+dx)^6}{2bd \log(F)} - \frac{3 \int F^{a+b(c+dx)^2} (c+dx)^5 dx}{b \log(F)} \\ &= -\frac{3F^{a+b(c+dx)^2} (c+dx)^4}{2b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^6}{2bd \log(F)} + \frac{6 \int F^{a+b(c+dx)^2} (c+dx)^3 dx}{b^2 \log^2(F)} \\ &= \frac{3F^{a+b(c+dx)^2} (c+dx)^2}{b^3 d \log^3(F)} - \frac{3F^{a+b(c+dx)^2} (c+dx)^4}{2b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^6}{2bd \log(F)} - \frac{6 \int F^{a+b(c+dx)^2} (c+dx) dx}{b^3 \log^3(F)} \\ &= -\frac{3F^{a+b(c+dx)^2}}{b^4 d \log^4(F)} + \frac{3F^{a+b(c+dx)^2} (c+dx)^2}{b^3 d \log^3(F)} - \frac{3F^{a+b(c+dx)^2} (c+dx)^4}{2b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^6}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0411262, size = 72, normalized size = 0.57

$$\frac{F^{a+b(c+dx)^2} (b^3 \log^3(F)(c+dx)^6 - 3b^2 \log^2(F)(c+dx)^4 + 6b \log(F)(c+dx)^2 - 6)}{2b^4 d \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^7, x]

[Out] (F^(a + b\*(c + d\*x)^2)\*(-6 + 6\*b\*(c + d\*x)^2\*Log[F] - 3\*b^2\*(c + d\*x)^4\*Log[F]^2 + b^3\*(c + d\*x)^6\*Log[F]^3))/(2\*b^4\*d\*Log[F]^4)

**Maple [B]** time = 0.009, size = 249, normalized size = 2.

$$\frac{(d^6 x^6 b^3 (\ln(F))^3 + 6 c d^5 x^5 b^3 (\ln(F))^3 + 15 (\ln(F))^3 b^3 c^2 d^4 x^4 + 20 (\ln(F))^3 b^3 c^3 d^3 x^3 + 15 (\ln(F))^3 b^3 c^4 d^2 x^2 + 6 (\ln(F))^3 b^3 c^5 d x + 6 (\ln(F))^3 b^3 c^6)}{2 b^4 d \log^4(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^7, x)

[Out] 1/2\*(d^6\*x^6\*b^3\*ln(F)^3+6\*c\*d^5\*x^5\*b^3\*ln(F)^3+15\*ln(F)^3\*b^3\*c^2\*d^4\*x^4+20\*ln(F)^3\*b^3\*c^3\*d^3\*x^3+15\*ln(F)^3\*b^3\*c^4\*d^2\*x^2+6\*ln(F)^3\*b^3\*c^5\*d\*x+ln(F)^3\*b^3\*c^6-3\*d^4\*x^4\*b^2\*ln(F)^2-12\*d^3\*c\*x^3\*b^2\*ln(F)^2-18\*ln(F)^2\*b^2\*c^2\*d^2\*x^2-12\*ln(F)^2\*b^2\*c^3\*d\*x-3\*ln(F)^2\*b^2\*c^4+6\*ln(F)\*b\*d^2\*x^2+12\*ln(F)\*b\*c\*d\*x+6\*ln(F)\*b\*c^2-6)\*F^(b\*d^2\*x^2+2\*b\*c\*d\*x+b\*c^2+a)/ln(F)^4/b^4/d

**Maxima [C]** time = 2.7074, size = 3461, normalized size = 27.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^7, x, algorithm="maxima")

[Out] -7/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b\*c\*d\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^2/((b\*d^2\*log(F))^(3/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b\*d^2\*log(F)/(b\*d^2\*log(F))^(3/2))\*F^a\*c^6\*d/sqrt(b\*d^2\*log(F)) + 21/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^2\*c^2\*d^2\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^3/((b\*d^2\*log(F))^(5/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 2\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^2\*c\*d^3\*log(F)^2/(b\*d^2\*log(F))^(5/2) - (b\*d^2\*x + b\*c\*d)^3\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^3/((b\*d^2\*log(F))^(5/2)\*(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2))\*F^a\*c^5\*d^2/sqrt(b\*d^2\*log(F)) - 35/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^3\*c^3\*d^3\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^4/((b\*d^2\*log(F))^(7/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 3\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^3\*c^2\*d^4\*log(F)^3/(b\*d^2\*log(F))^(7/2) - 3\*(b\*d^2\*x + b\*c\*d)^3\*b\*c\*d\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^4/((b\*d^2\*log(F))^(7/2))\*(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2) + b^2\*d^4\*gamma(2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^2/(b\*d^2\*log(F))^(7/2))\*F^a\*c^4\*d^3/sqrt(b\*d^2\*log(F)) + 35/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^4\*c^4\*d^4\*(erf(sqrt(-

$$\begin{aligned}
& b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)) - 1)*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*\sqrt{- (b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))} - 4*F^{((b*d^2*x + b*c*d)^2/(b*d^2))} \\
& *b^4*c^3*d^5*\log(F)^4/(b*d^2*\log(F))^{(9/2)} - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 4*b^3*c*d^5*\gamma(2, \\
& -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(9/2)} - (b*d^2*x + b*c*d)^5*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) *F^a*c^3*d^4/\sqrt{b*d^2*\log(F)} - 21/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^{((b*d^2*x + b*c*d)^2/(b*d^2))} *b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{(11/2)} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 10*b^4*c^2*d^6*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(11/2)} - b^3*d^6*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(11/2)} - 5*(b*d^2*x + b*c*d)^5*b*c*d*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) *F^a*c^2*d^5/\sqrt{b*d^2*\log(F)} + 7/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^6*c^6*d^6*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 6*F^{((b*d^2*x + b*c*d)^2/(b*d^2))} *b^6*c^5*d^7*\log(F)^6/(b*d^2*\log(F))^{(13/2)} - 15*(b*d^2*x + b*c*d)^3*b^4*c^4*d^4*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 20*b^5*c^3*d^7*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(13/2)} - 6*b^4*c*d^7*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(13/2)} - 15*(b*d^2*x + b*c*d)^5*b^2*c^2*d^2*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) - (b*d^2*x + b*c*d)^7*\gamma(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)}) *F^a*c*d^6/\sqrt{b*d^2*\log(F)} - 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^7*c^7*d^7*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 7*F^{((b*d^2*x + b*c*d)^2/(b*d^2))} *b^7*c^6*d^8*\log(F)^7/(b*d^2*\log(F))^{(15/2)} - 21*(b*d^2*x + b*c*d)^3*b^5*c^5*d^5*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 35*b^6*c^4*d^8*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/(b*d^2*\log(F))^{(15/2)} - 21*b^5*c^2*d^8*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(15/2)} - 35*(b*d^2*x + b*c*d)^5*b^3*c^3*d^3*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) + b^4*d^8*\gamma(4, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(15/2)} - 7*(b*d^2*x + b*c*d)^7*b*c*d*\gamma(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)}) *F^a*d^7/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^7*\operatorname{erf}(\sqrt{-b*\log(F)})}*d*x - b*c*\log(F)/\sqrt{-b*\log(F)})/(\sqrt{-b*\log(F)}*F^{(b*c^2)*d})
\end{aligned}$$

**Fricas [A]** time = 1.57978, size = 446, normalized size = 3.54

$$\frac{\left( (b^3 d^6 x^6 + 6 b^3 c d^5 x^5 + 15 b^3 c^2 d^4 x^4 + 20 b^3 c^3 d^3 x^3 + 15 b^3 c^4 d^2 x^2 + 6 b^3 c^5 d x + b^3 c^6) \log(F)^3 - 3 (b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 6 b^2 c^3 d x + b^2 c^4) \log(F)^2 - 3 (b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 6 b^2 c^3 d x + b^2 c^4) \log(F) - 3 b^2 d^4 \log(F)^4 \right)}{2 b^4 d \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^7,x, algorithm="fricas")

[Out]  $1/2*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*\log(F)^3 - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\log(F)^2 + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\log(F) - 6)*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^4*d*\log(F)^4)}$

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**Sympy [A]** time = 0.262146, size = 366, normalized size = 2.9

$$\left\{ \frac{F^{a+b(c+dx)^2} \left( b^3 c^6 \log(F)^3 + 6 b^3 c^5 d x \log(F)^3 + 15 b^3 c^4 d^2 x^2 \log(F)^3 + 20 b^3 c^3 d^3 x^3 \log(F)^3 + 15 b^3 c^2 d^4 x^4 \log(F)^3 + 6 b^3 c d^5 x^5 \log(F)^3 + b^3 d^6 x^6 \log(F)^3 - 3 b^2 c^4 \log(F)^3 \right)}{2 b^4 d \log(F)^4}, \right. \\ \left. c^7 x + \frac{7 c^6 d x^2}{2} + 7 c^5 d^2 x^3 + \frac{35 c^4 d^3 x^4}{4} + 7 c^3 d^4 x^5 + \frac{7 c^2 d^5 x^6}{2} + c d^6 x^7 + \frac{d^7 x^8}{8} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**7, x)`

[Out] `Piecewise((F**(a + b*(c + d*x)**2)*(b**3*c**6*log(F)**3 + 6*b**3*c**5*d*x*log(F)**3 + 15*b**3*c**4*d**2*x**2*log(F)**3 + 20*b**3*c**3*d**3*x**3*log(F)**3 + 15*b**3*c**2*d**4*x**4*log(F)**3 + 6*b**3*c*d**5*x**5*log(F)**3 + b**3*d**6*x**6*log(F)**3 - 3*b**2*c**4*log(F)**2 - 12*b**2*c**3*d*x*log(F)**2 - 18*b**2*c**2*d**2*x**2*log(F)**2 - 12*b**2*c*d**3*x**3*log(F)**2 - 3*b**2*d**4*x**4*log(F)**2 + 6*b*c**2*log(F) + 12*b*c*d*x*log(F) + 6*b*d**2*x**2*log(F) - 6)/(2*b**4*d*log(F)**4), Ne(2*b**4*d*log(F)**4, 0)), (c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8, True))`

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**Giac [A]** time = 1.67317, size = 139, normalized size = 1.1

$$\left( b^3 d^6 \left( x + \frac{c}{d} \right)^6 \log(F)^3 - 3 b^2 d^4 \left( x + \frac{c}{d} \right)^4 \log(F)^2 + 6 b d^2 \left( x + \frac{c}{d} \right)^2 \log(F) - 6 \right) e^{(bd^2x^2 \log(F) + 2bcdx \log(F) + bc^2 \log(F) + a \log(F))} \\ \frac{}{2 b^4 d \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^7, x, algorithm="giac")`

[Out]  $1/2*(b^3*d^6*(x + c/d)^6*\log(F)^3 - 3*b^2*d^4*(x + c/d)^4*\log(F)^2 + 6*b*d^2*(x + c/d)^2*\log(F) - 6)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F))/(b^4*d*\log(F)^4)}$

### 3.258 $\int F^{a+b(c+dx)^2} (c+dx)^5 dx$

**Optimal.** Leaf size=91

$$-\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2}}{b^3 d \log^3(F)} + \frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

[Out]  $F^{(a+b(c+dx)^2)}/(b^3 d \log[F]^3) - (F^{(a+b(c+dx)^2)}(c+dx)^2)/(b^2 d \log[F]^2) + (F^{(a+b(c+dx)^2)}(c+dx)^4)/(2 b d \log[F])$

**Rubi [A]** time = 0.175959, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$-\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2}}{b^3 d \log^3(F)} + \frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^5, x]

[Out]  $F^{(a+b(c+dx)^2)}/(b^3 d \log[F]^3) - (F^{(a+b(c+dx)^2)}(c+dx)^2)/(b^2 d \log[F]^2) + (F^{(a+b(c+dx)^2)}(c+dx)^4)/(2 b d \log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (c+dx)^5 dx &= \frac{F^{a+b(c+dx)^2} (c+dx)^4}{2bd \log(F)} - \frac{2 \int F^{a+b(c+dx)^2} (c+dx)^3 dx}{b \log(F)} \\ &= -\frac{F^{a+b(c+dx)^2} (c+dx)^2}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^4}{2bd \log(F)} + \frac{2 \int F^{a+b(c+dx)^2} (c+dx) dx}{b^2 \log^2(F)} \\ &= \frac{F^{a+b(c+dx)^2}}{b^3 d \log^3(F)} - \frac{F^{a+b(c+dx)^2} (c+dx)^2}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^4}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0328125, size = 56, normalized size = 0.62

$$\frac{F^{a+b(c+dx)^2} (b^2 \log^2(F)(c+dx)^4 - 2b \log(F)(c+dx)^2 + 2)}{2b^3 d \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^5,x]

[Out] (F^(a + b\*(c + d\*x)^2)\*(2 - 2\*b\*(c + d\*x)^2\*Log[F] + b^2\*(c + d\*x)^4\*Log[F]^2))/(2\*b^3\*d\*Log[F]^3)

**Maple [A]** time = 0.005, size = 138, normalized size = 1.5

$$\frac{(d^4 x^4 b^2 (\ln(F))^2 + 4 d^3 c x^3 b^2 (\ln(F))^2 + 6 (\ln(F))^2 b^2 c^2 d^2 x^2 + 4 (\ln(F))^2 b^2 c^3 d x + (\ln(F))^2 b^2 c^4 - 2 \ln(F) b d^2 x^2 - 4 \ln(F) b^2 c^2 d x - 4 \ln(F) b^2 c^3 d)}{2 (\ln(F))^3 b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^5,x)

[Out] 1/2\*(d^4\*x^4\*b^2\*ln(F)^2+4\*d^3\*c\*x^3\*b^2\*ln(F)^2+6\*ln(F)^2\*b^2\*c^2\*d^2\*x^2+4\*ln(F)^2\*b^2\*c^3\*d\*x+ln(F)^2\*b^2\*c^4-2\*ln(F)\*b\*d^2\*x^2-4\*ln(F)\*b\*c\*d\*x-2\*ln(F)\*b\*c^2+2)\*F^(b\*d^2\*x^2+2\*b\*c\*d\*x+b\*c^2+a)/ln(F)^3/b^3/d

**Maxima [C]** time = 2.15168, size = 2030, normalized size = 22.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="maxima")

[Out] -5/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b\*c\*d\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^2/((b\*d^2\*log(F))^(3/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b\*d^2\*log(F)/(b\*d^2\*log(F))^(3/2))\*F^a\*c^4\*d/sqrt(b\*d^2\*log(F)) + 5\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^2\*c^2\*d^2\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^3/((b\*d^2\*log(F))^(5/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 2\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^2\*c\*d^3\*log(F)^2/(b\*d^2\*log(F))^(5/2) - (b\*d^2\*x + b\*c\*d)^3\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^3/((b\*d^2\*log(F))^(5/2)\*(- (b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2)))\*F^a\*c^3\*d^2/sqrt(b\*d^2\*log(F)) - 5\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^3\*c^3\*d^3\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^4/((b\*d^2\*log(F))^(7/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 3\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^3\*c^2\*d^4\*log(F)^3/(b\*d^2\*log(F))^(7/2) - 3\*(b\*d^2\*x + b\*c\*d)^3\*b\*c\*d\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^4/((b\*d^2\*log(F))^(7/2)\*(- (b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2)) + b^2\*d^4\*gamma(2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^2/(b\*d^2\*log(F))^(7/2))\*F^a\*c^2\*d^3/sqrt(b\*d^2\*log(F)) + 5/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^4\*c^4\*d^4\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^5/((b\*d^2\*log(F))^(9/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 4\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^4\*c^3\*d^5\*log(F)^4/(b\*d^2\*log(F))^(9/2) - 6\*(b\*d^2\*x + b\*c\*d)^3\*b^2\*c^2\*d^2\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^5/((b\*d^2\*log(F))^(9/2)\*(- (b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2)) + 4\*b^3\*c\*d^5\*gamma(2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^3/(b\*d^2\*log(F))^(9/2) - (b\*d^2\*x + b\*c\*d)^5\*gamma(5/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^5/((b\*d^2\*log(F))^(9/2)\*(- (b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(5/2)))\*F^a\*c\*d^4/sqrt(b\*d^2\*log(F)) - 1/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^5\*c^5\*d^5\*(erf(sqrt(-(b\*d^2

$*x + b*c*d)^2*\log(F)/(b*d^2))) - 1)*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{(11/2)} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 10*b^4*c^2*d^6*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(11/2)} - b^3*d^6*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(11/2)} - 5*(b*d^2*x + b*c*d)^5*b*c*d*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)})}*F^a*d^5/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^5}*erf(\sqrt{-b*\log(F)})*d*x - b*c*\log(F)/\sqrt{-b*\log(F)})/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d}$

**Fricas [A]** time = 1.54344, size = 265, normalized size = 2.91

$$\frac{\left( (b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 d x + b^2 c^4) \log(F)^2 - 2 (b d^2 x^2 + 2 b c d x + b c^2) \log(F) + 2 \right) F^{b d^2 x^2 + 2 b c d x + b c^2 + a}}{2 b^3 d \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="fricas")

[Out] 1/2\*((b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4)\*log(F)^2 - 2\*(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F) + 2)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)/(b^3\*d\*log(F)^3)

**Sympy [A]** time = 0.212551, size = 214, normalized size = 2.35

$$\left\{ \begin{array}{l} \frac{F^{a+b(c+dx)^2} (b^2 c^4 \log(F)^2 + 4 b^2 c^3 d x \log(F)^2 + 6 b^2 c^2 d^2 x^2 \log(F)^2 + 4 b^2 c d^3 x^3 \log(F)^2 + b^2 d^4 x^4 \log(F)^2 - 2 b c^2 \log(F) - 4 b c d x \log(F) - 2 b d^2 x^2 \log(F) + 2)}{2 b^3 d \log(F)^3} \\ c^5 x + \frac{5 c^4 d x^2}{2} + \frac{10 c^3 d^2 x^3}{3} + \frac{5 c^2 d^3 x^4}{2} + c d^4 x^5 + \frac{d^5 x^6}{6} \end{array} \right. \begin{array}{l} \text{for } 2b^3 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*5,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x)\*\*2)\*(b\*\*2\*c\*\*4\*log(F)\*\*2 + 4\*b\*\*2\*c\*\*3\*d\*x\*log(F)\*\*2 + 6\*b\*\*2\*c\*\*2\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 4\*b\*\*2\*c\*d\*\*3\*x\*\*3\*log(F)\*\*2 + b\*\*2\*d\*\*4\*x\*\*4\*log(F)\*\*2 - 2\*b\*c\*\*2\*log(F) - 4\*b\*c\*d\*x\*log(F) - 2\*b\*d\*\*2\*x\*\*2\*log(F) + 2)/(2\*b\*\*3\*d\*log(F)\*\*3), Ne(2\*b\*\*3\*d\*log(F)\*\*3, 0)), (c\*\*5\*x + 5\*c\*\*4\*d\*x\*\*2/2 + 10\*c\*\*3\*d\*\*2\*x\*\*3/3 + 5\*c\*\*2\*d\*\*3\*x\*\*4/2 + c\*d\*\*4\*x\*\*5 + d\*\*5\*x\*\*6/6, True))

**Giac [A]** time = 1.27021, size = 111, normalized size = 1.22

$$\frac{\left( b^2 d^4 \left( x + \frac{c}{d} \right)^4 \log(F)^2 - 2 b d^2 \left( x + \frac{c}{d} \right)^2 \log(F) + 2 \right) e^{(b d^2 x^2 \log(F) + 2 b c d x \log(F) + b c^2 \log(F) + a \log(F))}}{2 b^3 d \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="giac")



```
[Out] 1/2*(b^2*d^4*(x + c/d)^4*log(F)^2 - 2*b*d^2*(x + c/d)^2*log(F) + 2)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^3*d*log(F)^3)
```

### 3.259 $\int F^{a+b(c+dx)^2} (c+dx)^3 dx$

**Optimal.** Leaf size=62

$$\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd \log(F)} - \frac{F^{a+b(c+dx)^2}}{2b^2 d \log^2(F)}$$

[Out]  $-F^{(a+b(c+dx)^2)}/(2*b^2*d*Log[F]^2) + (F^{(a+b(c+dx)^2)}*(c+dx)^2)/(2*b*d*Log[F])$

**Rubi [A]** time = 0.104733, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd \log(F)} - \frac{F^{a+b(c+dx)^2}}{2b^2 d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^3,x]

[Out]  $-F^{(a+b(c+dx)^2)}/(2*b^2*d*Log[F]^2) + (F^{(a+b(c+dx)^2)}*(c+dx)^2)/(2*b*d*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (c+dx)^3 dx &= \frac{F^{a+b(c+dx)^2} (c+dx)^2}{2bd \log(F)} - \frac{\int F^{a+b(c+dx)^2} (c+dx) dx}{b \log(F)} \\ &= -\frac{F^{a+b(c+dx)^2}}{2b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^2}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0218805, size = 40, normalized size = 0.65

$$\frac{F^{a+b(c+dx)^2} (b \log(F) (c+dx)^2 - 1)}{2b^2 d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^3,x]

[Out] (F^(a + b\*(c + d\*x)^2)\*(-1 + b\*(c + d\*x)^2\*Log[F]))/(2\*b^2\*d\*Log[F]^2)

**Maple [A]** time = 0.007, size = 63, normalized size = 1.

$$\frac{(\ln(F)bd^2x^2 + 2\ln(F)bcdx + \ln(F)bc^2 - 1)F^{bd^2x^2 + 2bcdx + c^2b + a}}{2(\ln(F))^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^3,x)

[Out] 1/2\*(ln(F)\*b\*d^2\*x^2+2\*ln(F)\*b\*c\*d\*x+ln(F)\*b\*c^2-1)\*F^(b\*d^2\*x^2+2\*b\*c\*d\*x+b\*c^2+a)/ln(F)^2/b^2/d

**Maxima [C]** time = 1.68766, size = 965, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -3/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) \\ & - F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b*d^2*\log(F)/(b*d^2*\log(F))^{3/2}*F^a*c^2*d/\sqrt{b*d^2*\log(F)} + 3/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) \\ & - 2*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{5/2} - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{5/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) \\ & *F^a*c*d^2/\sqrt{b*d^2*\log(F)} - 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^4/((b*d^2*\log(F))^{7/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) \\ & - 3*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^3*c^2*d^4*\log(F)^3/(b*d^2*\log(F))^{7/2} - 3*(b*d^2*x + b*c*d)^3*b*c*d*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/((b*d^2*\log(F))^{7/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) \\ & + b^2*d^4*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^2/(b*d^2*\log(F))^{7/2}*F^a*d^3/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^3*\operatorname{erf}(\sqrt{-b*\log(F)})*d*x - b*c*\log(F)/\sqrt{-b*\log(F)}}/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d} \end{aligned}$$

**Fricas [A]** time = 1.49666, size = 142, normalized size = 2.29

$$\frac{((bd^2x^2 + 2bcdx + bc^2)\log(F) - 1)F^{bd^2x^2 + 2bcdx + bc^2 + a}}{2b^2d\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="fricas")

[Out] 1/2\*((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F) - 1)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)/(b^2\*d\*log(F)^2)

**Sympy [A]** time = 0.171906, size = 100, normalized size = 1.61

$$\begin{cases} \frac{F^{a+b(c+dx)^2} (bc^2 \log(F) + 2bcdx \log(F) + bd^2x^2 \log(F) - 1)}{2b^2d \log(F)^2} & \text{for } 2b^2d \log(F)^2 \neq 0 \\ c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*3,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x)\*\*2)\*(b\*c\*\*2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*d\*\*2\*x\*\*2\*log(F) - 1)/(2\*b\*\*2\*d\*log(F)\*\*2), Ne(2\*b\*\*2\*d\*log(F)\*\*2, 0)), (c\*\*3\*x + 3\*c\*\*2\*d\*x\*\*2/2 + c\*d\*\*2\*x\*\*3 + d\*\*3\*x\*\*4/4, True))

**Giac [A]** time = 1.27073, size = 82, normalized size = 1.32

$$\frac{\left(bd^2\left(x + \frac{c}{d}\right)^2 \log(F) - 1\right) e^{(bd^2x^2 \log(F) + 2bcdx \log(F) + bc^2 \log(F) + a \log(F))}}{2b^2d \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="giac")

[Out] 1/2\*(b\*d^2\*(x + c/d)^2\*log(F) - 1)\*e^(b\*d^2\*x^2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*c^2\*log(F) + a\*log(F))/(b^2\*d\*log(F)^2)

$$3.260 \quad \int F^{a+b(c+dx)^2} (c + dx) dx$$

**Optimal.** Leaf size=27

$$\frac{F^{a+b(c+dx)^2}}{2bd \log(F)}$$

[Out]  $F^{(a + b*(c + d*x)^2)/(2*b*d*\text{Log}[F])}$

**Rubi [A]** time = 0.0362265, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2209}

$$\frac{F^{a+b(c+dx)^2}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x), x]

[Out]  $F^{(a + b*(c + d*x)^2)/(2*b*d*\text{Log}[F])}$

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+b(c+dx)^2} (c + dx) dx = \frac{F^{a+b(c+dx)^2}}{2bd \log(F)}$$

**Mathematica [A]** time = 0.0073765, size = 27, normalized size = 1.

$$\frac{F^{a+b(c+dx)^2}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x), x]

[Out]  $F^{(a + b*(c + d*x)^2)/(2*b*d*\text{Log}[F])}$

**Maple [A]** time = 0.003, size = 36, normalized size = 1.3

$$\frac{F^{bd^2x^2+2bcdx+c^2b+a}}{2bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2)*(d*x+c),x)`

[Out]  $1/2 * F^{(b*d^2*x^2+2*b*c*d*x+b*c^2+a)}/b/d/\ln(F)$

**Maxima [A]** time = 1.01561, size = 34, normalized size = 1.26

$$\frac{F^{(dx+c)^2 b+a}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="maxima")`

[Out]  $1/2 * F^{((d*x + c)^2 * b + a)}/(b*d*\log(F))$

**Fricas [A]** time = 1.53574, size = 76, normalized size = 2.81

$$\frac{F^{bd^2x^2+2bcdx+bc^2+a}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="fricas")`

[Out]  $1/2 * F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)}/(b*d*\log(F))$

**Sympy [A]** time = 0.136847, size = 36, normalized size = 1.33

$$\begin{cases} \frac{F^{a+b(c+dx)^2}}{2bd \log(F)} & \text{for } 2bd \log(F) \neq 0 \\ cx + \frac{dx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)*(d*x+c),x)`

[Out] `Piecewise((F**(a + b*(c + d*x)**2)/(2*b*d*log(F)), Ne(2*b*d*log(F), 0)), (c*x + d*x**2/2, True))`

**Giac [A]** time = 1.22253, size = 34, normalized size = 1.26

$$\frac{F^{(dx+c)^2 b+a}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="giac")`

[Out]  $1/2 * F^{((d*x + c)^2 * b + a)}/(b*d*\log(F))$

$$3.261 \quad \int \frac{F^{a+b(c+dx)^2}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$\frac{F^a \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d}$$

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]])/(2\*d)

**Rubi [A]** time = 0.0676199, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$\frac{F^a \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]])/(2\*d)

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx} dx = \frac{F^a \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d}$$

**Mathematica [A]** time = 0.0062464, size = 22, normalized size = 1.

$$\frac{F^a \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]])/(2\*d)

**Maple [A]** time = 0.021, size = 23, normalized size = 1.1

$$-\frac{F^a \operatorname{Ei}(1, -b(dx+c)^2 \ln(F))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2)/(d*x+c),x)`

[Out] `-1/2/d*F^a*Ei(1,-b*(d*x+c)^2*ln(F))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c), x)`

**Fricas [A]** time = 1.5086, size = 73, normalized size = 3.32

$$\frac{F^a \operatorname{Ei}\left(\left(bd^2x^2 + 2bcdx + bc^2\right) \log(F)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="fricas")`

[Out] `1/2*F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))/d`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)/(d*x+c),x)`

[Out] `Integral(F**(a + b*(c + d*x)**2)/(c + d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c), x)`



$$3.262 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=53

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d} - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}$$

[Out]  $-F^{a+b(c+dx)^2}/(2d(c+dx)^2) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^2 \log(F)] \log(F))/(2d)$

**Rubi [A]** time = 0.132028, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{2d} - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b(c+dx)^2}/(c+dx)^3, x]$

[Out]  $-F^{a+b(c+dx)^2}/(2d(c+dx)^2) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^2 \log(F)] \log(F))/(2d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m], x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{m+1} F^{a+b(c+dx)^n}/(d(m+1)), x] - \operatorname{Dist}[(b \log(F))/(m+1), \operatorname{Int}[(c+dx)^{m+n} F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[F^a \operatorname{ExpIntegralEi}[b(c+dx)^n \log(F)]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} dx &= -\frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2} + (b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2} + \frac{bF^a \operatorname{Ei}(b(c+dx)^2 \log(F)) \log(F)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0393825, size = 47, normalized size = 0.89

$$\frac{F^a \left( b \log(F) \operatorname{Ei}(b(c+dx)^2 \log(F)) - \frac{F^{b(c+dx)^2}}{(c+dx)^2} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^3,x]

[Out] (F^a\*(-(F^(b\*(c + d\*x)^2)/(c + d\*x)^2) + b\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]]\*Log[F]))/(2\*d)

**Maple [A]** time = 0.034, size = 53, normalized size = 1.

$$\frac{F^{b(dx+c)^2} F^a}{2d(dx+c)^2} - \frac{b \ln(F) F^a \operatorname{Ei}\left(1, -b(dx+c)^2 \ln(F)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^3,x)

[Out] -1/2/d/(d\*x+c)^2\*F^(b\*(d\*x+c)^2)\*F^a-1/2/d\*b\*ln(F)\*F^a\*Ei(1,-b\*(d\*x+c)^2\*ln(F))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^3, x)

**Fricas [B]** time = 1.53328, size = 220, normalized size = 4.15

$$\frac{(bd^2x^2 + 2bcdx + bc^2)F^a \operatorname{Ei}\left(\left(bd^2x^2 + 2bcdx + bc^2\right) \log(F)\right) \log(F) - F^{bd^2x^2 + 2bcdx + bc^2 + a}}{2(d^3x^2 + 2cd^2x + c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^3,x, algorithm="fricas")

[Out] 1/2\*((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*log(F) - F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a))/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*3,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)/(c + d\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^3,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^3, x)

$$3.263 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5} dx$$

**Optimal.** Leaf size=87

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{4d} - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} - \frac{b \log(F) F^{a+b(c+dx)^2}}{4d(c+dx)^2}$$

[Out]  $-F^{a+b(c+dx)^2}/(4*d*(c+dx)^4) - (b*F^{a+b(c+dx)^2}*Log[F])/(4*d*(c+dx)^2) + (b^2*F^a*ExpIntegralEi[b*(c+dx)^2*Log[F]]*Log[F]^2)/(4*d)$

**Rubi [A]** time = 0.198022, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{4d} - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} - \frac{b \log(F) F^{a+b(c+dx)^2}}{4d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^5, x]

[Out]  $-F^{a+b(c+dx)^2}/(4*d*(c+dx)^4) - (b*F^{a+b(c+dx)^2}*Log[F])/(4*d*(c+dx)^2) + (b^2*F^a*ExpIntegralEi[b*(c+dx)^2*Log[F]]*Log[F]^2)/(4*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5} dx &= -\frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} + \frac{1}{2}(b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} - \frac{bF^{a+b(c+dx)^2} \log(F)}{4d(c+dx)^2} + \frac{1}{2}(b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^2}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} - \frac{bF^{a+b(c+dx)^2} \log(F)}{4d(c+dx)^2} + \frac{b^2 F^a \operatorname{Ei}(b(c+dx)^2 \log(F)) \log^2(F)}{4d} \end{aligned}$$

**Mathematica [A]** time = 0.0767754, size = 64, normalized size = 0.74

$$\frac{F^a \left( b^2 \log^2(F) \operatorname{Ei} \left( b(c+dx)^2 \log(F) \right) - \frac{F^{b(c+dx)^2} (b \log(F)(c+dx)^2 + 1)}{(c+dx)^4} \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^5, x]

[Out] (F^a\*(b^2\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]]\*Log[F]^2 - (F^(b\*(c + d\*x)^2)\*(1 + b\*(c + d\*x)^2\*Log[F]))/(c + d\*x)^4)/(4\*d)

**Maple [A]** time = 0.048, size = 86, normalized size = 1.

$$\frac{F^{b(dx+c)^2} F^a}{4d(dx+c)^4} - \frac{b \ln(F) F^{b(dx+c)^2} F^a}{4d(dx+c)^2} - \frac{b^2 (\ln(F))^2 F^a \operatorname{Ei} \left( 1, -b(dx+c)^2 \ln(F) \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^5, x)

[Out] -1/4/d/(d\*x+c)^4\*F^(b\*(d\*x+c)^2)\*F^a-1/4/d\*b\*ln(F)/(d\*x+c)^2\*F^(b\*(d\*x+c)^2)\*F^a-1/4/d\*b^2\*ln(F)^2\*F^a\*Ei(1,-b\*(d\*x+c)^2\*ln(F))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^5, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^5, x)

**Fricas [B]** time = 1.54994, size = 389, normalized size = 4.47

$$\frac{(b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 d x + b^2 c^4) F^a \operatorname{Ei} \left( (bd^2 x^2 + 2 bcdx + bc^2) \log(F) \right) \log(F)^2 - ((bd^2 x^2 + 2 bcdx + bc^2) \log(F) + 1) F^{(bd^2 x^2 + 2 bcdx + bc^2) \log(F) + a}}{4(d^5 x^4 + 4 cd^4 x^3 + 6 c^2 d^3 x^2 + 4 c^3 d^2 x + c^4 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^5, x, algorithm="fricas")

[Out] 1/4\*((b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*log(F)^2 - ((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F) + 1)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a))/(d^5\*x^4 + 4\*c\*d^4\*x^3 + 6\*c^2\*d^3\*x^2 + 4\*c^3\*d^2\*x + c^4\*d)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*5,x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^5,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^5, x)

### 3.264 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} dx$

**Optimal.** Leaf size=121

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{12d} - \frac{b^2 \log^2(F) F^{a+b(c+dx)^2}}{12d(c+dx)^2} - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{b \log(F) F^{a+b(c+dx)^2}}{12d(c+dx)^4}$$

[Out]  $-F^{a+b(c+dx)^2}/(6*d*(c+dx)^6) - (b*F^{a+b(c+dx)^2}*Log[F])/(12*d*(c+dx)^4) - (b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(12*d*(c+dx)^2) + (b^3*F^a*ExpIntegralEi[b*(c+dx)^2*Log[F]]*Log[F]^3)/(12*d)$

**Rubi [A]** time = 0.25758, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^2 \log(F))}{12d} - \frac{b^2 \log^2(F) F^{a+b(c+dx)^2}}{12d(c+dx)^2} - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{b \log(F) F^{a+b(c+dx)^2}}{12d(c+dx)^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b(c+dx)^2}/(c+dx)^7, x]$

[Out]  $-F^{a+b(c+dx)^2}/(6*d*(c+dx)^6) - (b*F^{a+b(c+dx)^2}*Log[F])/(12*d*(c+dx)^4) - (b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(12*d*(c+dx)^2) + (b^3*F^a*ExpIntegralEi[b*(c+dx)^2*Log[F]]*Log[F]^3)/(12*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)})) * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(c+dx)^{(m+1)} * F^{a+b(c+dx)^2} / (d*(m+1)), x] - \operatorname{Dist}[(b*n*Log[F]) / (m+1), \operatorname{Int}[(c+dx)^{(m+n)} * F^{a+b(c+dx)^2}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)})) / ((e_.) + (f_.)*(x_)), x\_Symbol] :> \operatorname{Simp}[F^a * \operatorname{ExpIntegralEi}[b*(c+dx)^2 * n * Log[F]] / (f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} dx &= -\frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} + \frac{1}{3}(b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{b F^{a+b(c+dx)^2} \log(F)}{12d(c+dx)^4} + \frac{1}{6}(b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{b F^{a+b(c+dx)^2} \log(F)}{12d(c+dx)^4} - \frac{b^2 F^{a+b(c+dx)^2} \log^2(F)}{12d(c+dx)^2} + \frac{1}{6}(b^3 \log^3(F)) \int \frac{F^{a+b(c+dx)^2}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{b F^{a+b(c+dx)^2} \log(F)}{12d(c+dx)^4} - \frac{b^2 F^{a+b(c+dx)^2} \log^2(F)}{12d(c+dx)^2} + \frac{b^3 F^a \operatorname{Ei}(b(c+dx)^2 \log(F)) \log^3(F)}{12d} \end{aligned}$$

**Mathematica [A]** time = 0.0942973, size = 79, normalized size = 0.65

$$\frac{F^a \left( b^3 \log^3(F) \operatorname{Ei} \left( b(c+dx)^2 \log(F) \right) - \frac{F^{b(c+dx)^2} (b^2 \log^2(F)(c+dx)^4 + b \log(F)(c+dx)^2 + 2)}{(c+dx)^6} \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^7, x]

[Out] (F^a\*(b^3\*ExpIntegralEi[b\*(c + d\*x)^2\*Log[F]]\*Log[F]^3 - (F^(b\*(c + d\*x)^2)\*(2 + b\*(c + d\*x)^2\*Log[F] + b^2\*(c + d\*x)^4\*Log[F]^2))/(c + d\*x)^6))/(12\*d)

**Maple [A]** time = 0.064, size = 119, normalized size = 1.

$$-\frac{F^{b(dx+c)^2} F^a}{6d(dx+c)^6} - \frac{b \ln(F) F^{b(dx+c)^2} F^a}{12d(dx+c)^4} - \frac{b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{12d(dx+c)^2} - \frac{b^3 (\ln(F))^3 F^a \operatorname{Ei}(1, -b(dx+c)^2 \ln(F))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^7, x)

[Out] -1/6/d/(d\*x+c)^6\*F^(b\*(d\*x+c)^2)\*F^a-1/12/d\*b\*ln(F)/(d\*x+c)^4\*F^(b\*(d\*x+c)^2)\*F^a-1/12/d\*b^2\*ln(F)^2/(d\*x+c)^2\*F^(b\*(d\*x+c)^2)\*F^a-1/12/d\*b^3\*ln(F)^3\*F^a\*Ei(1, -b\*(d\*x+c)^2\*ln(F))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^7, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^7, x)

**Fricas [B]** time = 1.52452, size = 616, normalized size = 5.09

$$\frac{(b^3 d^6 x^6 + 6 b^3 c d^5 x^5 + 15 b^3 c^2 d^4 x^4 + 20 b^3 c^3 d^3 x^3 + 15 b^3 c^4 d^2 x^2 + 6 b^3 c^5 d x + b^3 c^6) F^a \operatorname{Ei} \left( (b d^2 x^2 + 2 b c d x + b c^2) \log(F) \right)}{12 (d^7 x^6 + 6 c d^6 x^5 + 15 c^2 d^5 x^4 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^7, x, algorithm="fricas")

[Out] 1/12\*((b^3\*d^6\*x^6 + 6\*b^3\*c\*d^5\*x^5 + 15\*b^3\*c^2\*d^4\*x^4 + 20\*b^3\*c^3\*d^3\*x^3 + 15\*b^3\*c^4\*d^2\*x^2 + 6\*b^3\*c^5\*d\*x + b^3\*c^6)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*log(F)^3 - ((b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*log(F)^2 - ((b\*d^3\*x^3 + 3\*b\*d^2\*c\*x^2 + 3\*b\*d\*c^2\*x + b\*c^3)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*log(F) - (b\*d^4\*x^4 + 4\*b\*d^3\*c\*x^3 + 6\*b\*d^2\*c^2\*x^2 + 4\*b\*d\*c^3\*x + b\*c^4)\*F^a\*Ei((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F)))



$$c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\log(F)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\log(F) + 2)*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)}/(d^7*x^6 + 6*c*d^6*x^5 + 15*c^2*d^5*x^4 + 20*c^3*d^4*x^3 + 15*c^4*d^3*x^2 + 6*c^5*d^2*x + c^6*d)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*7,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^7,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^7, x)

$$3.265 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} dx$$

**Optimal.** Leaf size=31

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^2)}{2d}$$

[Out]  $-(b^4 F^a \Gamma[-4, -(b*(c + d*x)^2 * \text{Log}[F])]) * \text{Log}[F]^4 / (2*d)$

**Rubi [A]** time = 0.062995, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^2)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^9, x]

[Out]  $-(b^4 F^a \Gamma[-4, -(b*(c + d*x)^2 * \text{Log}[F])]) * \text{Log}[F]^4 / (2*d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} dx = -\frac{b^4 F^a \Gamma(-4, -b(c+dx)^2 \log(F)) \log^4(F)}{2d}$$

**Mathematica [A]** time = 0.0067544, size = 31, normalized size = 1.

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^2)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^9, x]

[Out]  $-(b^4 F^a \Gamma[-4, -(b*(c + d*x)^2 * \text{Log}[F])]) * \text{Log}[F]^4 / (2*d)$

**Maple [B]** time = 0.091, size = 152, normalized size = 4.9

$$\frac{F^{b(dx+c)^2} F^a}{8d(dx+c)^8} - \frac{b \ln(F) F^{b(dx+c)^2} F^a}{24d(dx+c)^6} - \frac{b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{48d(dx+c)^4} - \frac{b^3 (\ln(F))^3 F^{b(dx+c)^2} F^a}{48d(dx+c)^2} - \frac{b^4 (\ln(F))^4 F^a \text{Ei}(1, -b(dx+c)^2)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x)`

[Out] 
$$-1/8/d/(d*x+c)^8*F^{(b*(d*x+c)^2)*F^{a-1}/24/d*b*\ln(F)/(d*x+c)^6*F^{(b*(d*x+c)^2)*F^{a-1}/48/d*b^2*\ln(F)^2/(d*x+c)^4*F^{(b*(d*x+c)^2)*F^{a-1}/48/d*b^3*\ln(F)^3/(d*x+c)^2*F^{(b*(d*x+c)^2)*F^{a-1}/48/d*b^4*\ln(F)^4*F^a*Ei(1,-b*(d*x+c)^2*\ln(F))}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c)^9, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**9,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c)^9, x)`

$$3.266 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} dx$$

**Optimal.** Leaf size=31

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^2)}{2d}$$

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^2\*Log[F])]\*Log[F]^5)/(2\*d)

**Rubi [A]** time = 0.0642407, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^2)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^11,x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^2\*Log[F])]\*Log[F]^5)/(2\*d)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} dx = \frac{b^5 F^a \Gamma(-5, -b(c+dx)^2 \log(F)) \log^5(F)}{2d}$$

**Mathematica [A]** time = 0.0068067, size = 31, normalized size = 1.

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^2)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^11,x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^2\*Log[F])]\*Log[F]^5)/(2\*d)

**Maple [B]** time = 0.137, size = 185, normalized size = 6.

$$\frac{F^{b(dx+c)^2} F^a}{10 d (dx+c)^{10}} - \frac{b \ln(F) F^{b(dx+c)^2} F^a}{40 d (dx+c)^8} - \frac{b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{120 d (dx+c)^6} - \frac{b^3 (\ln(F))^3 F^{b(dx+c)^2} F^a}{240 d (dx+c)^4} - \frac{b^4 (\ln(F))^4 F^{b(dx+c)^2} F^a}{240 d (dx+c)^2} - \frac{b^5 (\ln(F))^5 F^{b(dx+c)^2} F^a}{240 d (dx+c)^0}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x)`

[Out] 
$$-1/10/d/(d*x+c)^{10}*F^{(b*(d*x+c)^2)}*F^{a-1/40}/d*b*\ln(F)/(d*x+c)^8*F^{(b*(d*x+c)^2)}*F^{a-1/120}/d*b^2*\ln(F)^2/(d*x+c)^6*F^{(b*(d*x+c)^2)}*F^{a-1/240}/d*b^3*\ln(F)^3/(d*x+c)^4*F^{(b*(d*x+c)^2)}*F^{a-1/240}/d*b^4*\ln(F)^4/(d*x+c)^2*F^{(b*(d*x+c)^2)}*F^{a-1/240}/d*b^5*\ln(F)^5*F^a*Ei(1,-b*(d*x+c)^2*\ln(F))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c)^11, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**11,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(d*x + c)^11, x)`

### 3.267 $\int F^{a+b(c+dx)^2} (c+dx)^{12} dx$

**Optimal.** Leaf size=49

$$-\frac{F^a(c+dx)^{13} \text{Gamma}\left(\frac{13}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{13/2}}$$

[Out]  $-(F^a(c+dx)^{13} \text{Gamma}[13/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(13/2)})$

**Rubi [A]** time = 0.0656325, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{F^a(c+dx)^{13} \text{Gamma}\left(\frac{13}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^12, x]

[Out]  $-(F^a(c+dx)^{13} \text{Gamma}[13/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(13/2)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} (c+dx)^{12} dx = -\frac{F^a(c+dx)^{13} \Gamma\left(\frac{13}{2}, -b(c+dx)^2 \log(F)\right)}{2d \left(-b(c+dx)^2 \log(F)\right)^{13/2}}$$

**Mathematica [A]** time = 0.0276344, size = 49, normalized size = 1.

$$-\frac{F^a(c+dx)^{13} \text{Gamma}\left(\frac{13}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^12, x]

[Out]  $-(F^a(c+dx)^{13} \text{Gamma}[13/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(13/2)})$

---

**Maple [B]** time = 0.504, size = 1896, normalized size = 38.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{a+b(dx+c)^2})(dx+c)^{12}, x$

[Out]  $165/2*d^7*c^3/\ln(F)/b*x^8*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+3465/8*d^2*c^4/\ln(F)^3/b^3*x^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-693/2*d^4*c^4/\ln(F)^2/b^2*x^5*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-231*d^5*c^3/\ln(F)^2/b^2*x^6*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+3465/8*d^3*c^3/\ln(F)^3/b^3*x^4*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-3465/8*d*c^3/\ln(F)^4/b^4*x^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-99*d^6*c^2/\ln(F)^2/b^2*x^7*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-3465/8*d^2*c^2/\ln(F)^4/b^4*x^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+2079/8*d^4*c^2/\ln(F)^3/b^3*x^5*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-99/4*d^7*c/\ln(F)^2/b^2*x^8*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+693/8*d^5*c/\ln(F)^3/b^3*x^6*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-3465/16*d^3*c/\ln(F)^4/b^4*x^4*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+10395/32*d*c/\ln(F)^5/b^5*x^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+55/2*d^8*c^2/\ln(F)/b*x^9*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+231*d^4*c^6/\ln(F)/b*x^5*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+165*d^3*c^7/\ln(F)/b*x^4*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+165/2*d^2*c^8/\ln(F)/b*x^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+55/2*d*c^9/\ln(F)/b*x^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-99*d*c^7/\ln(F)^2/b^2*x^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+11/2*d^9*c/\ln(F)/b*x^10*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-231*d^2*c^6/\ln(F)^2/b^2*x^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+231*d^5*c^5/\ln(F)/b*x^6*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-693/2*d^3*c^5/\ln(F)^2/b^2*x^4*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+2079/8*d*c^5/\ln(F)^3/b^3*x^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+165*d^6*c^4/\ln(F)/b*x^7*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-10395/64/d*c/\ln(F)^6/b^6*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-11/4/d*c^9/\ln(F)^2/b^2*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+99/8/d*c^7/\ln(F)^3/b^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-693/16/d*c^5/\ln(F)^4/b^4*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+3465/32/d*c^3/\ln(F)^5/b^5*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+1/2/d*c^11/\ln(F)/b*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-3465/16*c^4/\ln(F)^4/b^4*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+10395/32*c^2/\ln(F)^5/b^5*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+11/2*c^10/\ln(F)/b*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-99/4*c^8/\ln(F)^2/b^2*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+693/8*c^6/\ln(F)^3/b^3*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+1/2*d^10/\ln(F)/b*x^11*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-11/4*d^8/\ln(F)^2/b^2*x^9*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+99/8*d^6/\ln(F)^3/b^3*x^7*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+3465/32*d^2/\ln(F)^5/b^5*x^3*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-693/16*d^4/\ln(F)^4/b^4*x^5*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-10395/64/\ln(F)^6/b^6*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-10395/128/d/\ln(F)^6/b^6*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))}$

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**Maxima [B]** time = 3.66944, size = 8659, normalized size = 176.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{a+b(dx+c)^2})(dx+c)^{12}, x, \text{algorithm}=\text{"maxima"})$

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[Out] -6*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*d*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/
(b*d^2)))) - 1)*log(F)^2/((b*d^2*log(F))^(3/2)*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*log(F)/(b*d^2*log(F))
^(3/2))*F^a*c^11*d/sqrt(b*d^2*log(F)) + 33*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*
c^2*d^2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)*log(F)^3/((b*d
^2*log(F))^(5/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x
+ b*c*d)^2/(b*d^2))*b^2*c*d^3*log(F)^2/(b*d^2*log(F))^(5/2) - (b*d^2*x + b
*c*d)^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*d^2*lo
g(F))^(5/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2))*F^a*c^10*d^2/sqrt
(b*d^2*log(F)) - 110*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(erf(sqrt(-(b*
d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)*log(F)^4/((b*d^2*log(F))^(7/2)*sqrt(
-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b
^3*c^2*d^4*log(F)^3/(b*d^2*log(F))^(7/2) - 3*(b*d^2*x + b*c*d)^3*b*c*d*gamma
a(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*d^2*log(F))^(7/2)*
(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*d^4*gamma(2, -(b*d^2*x +
b*c*d)^2*log(F)/(b*d^2))*log(F)^2/(b*d^2*log(F))^(7/2))*F^a*c^9*d^3/sqrt(b
*d^2*log(F)) + 495/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(erf(sqrt(-(b*
d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)*log(F)^5/((b*d^2*log(F))^(9/2)*sqrt(
-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b
^4*c^3*d^5*log(F)^4/(b*d^2*log(F))^(9/2) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^
2*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(
9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 4*b^3*c*d^5*gamma(2, -
(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))^(9/2) - (b*d^2*x
+ b*c*d)^5*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d
^2*log(F))^(9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(5/2))*F^a*c^8*d^4/
sqrt(b*d^2*log(F)) - 396*(sqrt(pi)*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(erf(sqrt(
-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)*log(F)^6/((b*d^2*log(F))^(11/2)*
sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 5*F^((b*d^2*x + b*c*d)^2/(b*d^
2))*b^5*c^4*d^6*log(F)^5/(b*d^2*log(F))^(11/2) - 10*(b*d^2*x + b*c*d)^3*b^3
*c^3*d^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^6/((b*d^2*lo
g(F))^(11/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 10*b^4*c^2*d^6
*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/(b*d^2*log(F))^(11/
2) - b^3*d^6*gamma(3, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*
log(F))^(11/2) - 5*(b*d^2*x + b*c*d)^5*b*c*d*gamma(5/2, -(b*d^2*x + b*c*d)^
2*log(F)/(b*d^2))*log(F)^6/((b*d^2*log(F))^(11/2))*(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))^(5/2))*F^a*c^7*d^5/sqrt(b*d^2*log(F)) + 462*(sqrt(pi)*(b*d^2*x
+ b*c*d)*b^6*c^6*d^6*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)
*log(F)^7/((b*d^2*log(F))^(13/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))
- 6*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^6*c^5*d^7*log(F)^6/(b*d^2*log(F))^(1
3/2) - 15*(b*d^2*x + b*c*d)^3*b^4*c^4*d^4*gamma(3/2, -(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))*log(F)^7/((b*d^2*log(F))^(13/2))*(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))^(3/2)) + 20*b^5*c^3*d^7*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d
^2))*log(F)^5/(b*d^2*log(F))^(13/2) - 6*b^4*c*d^7*gamma(3, -(b*d^2*x + b*c*
d)^2*log(F)/(b*d^2))*log(F)^4/(b*d^2*log(F))^(13/2) - 15*(b*d^2*x + b*c*d)^
5*b^2*c^2*d^2*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^7/((b*
d^2*log(F))^(13/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(5/2)) - (b*d^2*x
+ b*c*d)^7*gamma(7/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^7/((b*d^2
*log(F))^(13/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(7/2))*F^a*c^6*d^6/s
qrt(b*d^2*log(F)) - 396*(sqrt(pi)*(b*d^2*x + b*c*d)*b^7*c^7*d^7*(erf(sqrt(
-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))) - 1)*log(F)^8/((b*d^2*log(F))^(15/2)*s
qrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 7*F^((b*d^2*x + b*c*d)^2/(b*d^2
))*b^7*c^6*d^8*log(F)^7/(b*d^2*log(F))^(15/2) - 21*(b*d^2*x + b*c*d)^3*b^5*
c^5*d^5*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^8/((b*d^2*lo
g(F))^(15/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 35*b^6*c^4*d^8*
gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^6/(b*d^2*log(F))^(15/2
) - 21*b^5*c^2*d^8*gamma(3, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/(
b*d^2*log(F))^(15/2) - 35*(b*d^2*x + b*c*d)^5*b^3*c^3*d^3*gamma(5/2, -(b*d^
2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^8/((b*d^2*log(F))^(15/2))*(-(b*d^2*x +
b*c*d)^2*log(F)/(b*d^2))^(5/2)) + b^4*d^8*gamma(4, -(b*d^2*x + b*c*d)^2*lo

```



$$\begin{aligned}
& g(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(15/2)} - 7*(b*d^2*x + b*c*d)^7*b*c*d* \\
& \text{gamma}(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}* \\
& (-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)})*F^a*c^5*d^7/\text{sqrt}(b*d^2*\log(F)) + 495/2*(\text{sqrt}(\pi)*(b*d^2*x + b*c*d)*b^8*c^8*d^8*(\text{erf}(\text{sqrt}(-(b*d^2*x \\
& + b*c*d)^2*\log(F)/(b*d^2))) - 1)*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*\text{sqrt}(-(b*d^2*x \\
& + b*c*d)^2*\log(F)/(b*d^2))) - 8*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^8*c^7*d^9*\log(F)^8/(b*d^2*\log(F))^{(17/2)} - 28*(b*d^2*x + b*c*d)^3*b^6*c^6*d^6*\text{gamma}(3/2, \\
& -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)} + 56*b^7*c^5*d^9*\text{gamma}(2, \\
& -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/(b*d^2*\log(F))^{(17/2)} - 56*b^6*c^3*d^9*\text{gamma}(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/(b*d^2*\log(F))^{(17/2)} - 70*(b*d^2*x + b*c*d)^5*b^4*c^4*d^4*\text{gamma}(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)} + 8*b^5*c*d^9*\text{gamma}(4, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(17/2)} - 28*(b*d^2*x + b*c*d)^7*b^2*c^2*d^2*\text{gamma}(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)} - (b*d^2*x + b*c*d)^9*\text{gamma}(9/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(9/2)})*F^a*c^4*d^8/\text{sqrt}(b*d^2*\log(F)) - 110*(\text{sqrt}(\pi)*(b*d^2*x + b*c*d)*b^9*c^9*d^9*(\text{erf}(\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 1)*\log(F)^10/((b*d^2*\log(F))^{(19/2)}*\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 9*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^9*c^8*d^10*\log(F)^9/(b*d^2*\log(F))^{(19/2)} - 36*(b*d^2*x + b*c*d)^3*b^7*c^7*d^7*\text{gamma}(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^10/((b*d^2*\log(F))^{(19/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)} + 84*b^8*c^6*d^10*\text{gamma}(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/(b*d^2*\log(F))^{(19/2)} - 126*b^7*c^4*d^10*\text{gamma}(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/(b*d^2*\log(F))^{(19/2)} - 126*(b*d^2*x + b*c*d)^5*b^5*c^5*d^5*\text{gamma}(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^10/((b*d^2*\log(F))^{(19/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)} + 36*b^6*c^2*d^10*\text{gamma}(4, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/(b*d^2*\log(F))^{(19/2)} - 84*(b*d^2*x + b*c*d)^7*b^3*c^3*d^3*\text{gamma}(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^10/((b*d^2*\log(F))^{(19/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)} - b^5*d^10*\text{gamma}(5, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(19/2)} - 9*(b*d^2*x + b*c*d)^9*b*c*d*\text{gamma}(9/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^10/((b*d^2*\log(F))^{(19/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(9/2)})*F^a*c^3*d^9/\text{sqrt}(b*d^2*\log(F)) + 33*(\text{sqrt}(\pi)*(b*d^2*x + b*c*d)*b^10*c^10*d^10*(\text{erf}(\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 1)*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 10*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^10*c^9*d^11*\log(F)^10/(b*d^2*\log(F))^{(21/2)} - 45*(b*d^2*x + b*c*d)^3*b^8*c^8*d^8*\text{gamma}(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)} + 120*b^9*c^7*d^11*\text{gamma}(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/(b*d^2*\log(F))^{(21/2)} - 252*b^8*c^5*d^11*\text{gamma}(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/(b*d^2*\log(F))^{(21/2)} - 210*(b*d^2*x + b*c*d)^5*b^6*c^6*d^6*\text{gamma}(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)} + 120*b^7*c^3*d^11*\text{gamma}(4, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/(b*d^2*\log(F))^{(21/2)} - 210*(b*d^2*x + b*c*d)^7*b^4*c^4*d^4*\text{gamma}(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)} - 10*b^6*c*d^11*\text{gamma}(5, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/(b*d^2*\log(F))^{(21/2)} - 45*(b*d^2*x + b*c*d)^9*b^2*c^2*d^2*\text{gamma}(9/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(9/2)} - (b*d^2*x + b*c*d)^11*\text{gamma}(11/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^11/((b*d^2*\log(F))^{(21/2)}*(-b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(11/2)})*F^a*c^2*d^10/\text{sqrt}(b*d^2*\log(F)) - 6*(\text{sqrt}(\pi)*(b*d^2*x + b*c*d)*b^11*c^11*d^11*(\text{erf}(\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 1)*\log(F)^12/((b*d^2*\log(F))^{(23/2)}*\text{sqrt}(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))) - 11*F^((b*d^2*x + b*c*d)^2/(b*d^2))
\end{aligned}$$

$$\begin{aligned}
& ^2)) * b^{11} c^{10} d^{12} \log(F)^{11} / (b d^2 \log(F))^{(23/2)} - 55 (b d^2 x + b c d)^{10} \\
& 3 b^9 c^9 d^9 \gamma(3/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{12} / ((b \\
& d^2 \log(F))^{(23/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(3/2)}) + 165 b^{10} \\
& c^8 d^{12} \gamma(2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{10} / (b d^2 \log \\
& (F))^{(23/2)} - 462 b^9 c^6 d^{12} \gamma(3, -(b d^2 x + b c d)^2 \log(F) / (b d^2 \\
& )) \log(F)^9 / (b d^2 \log(F))^{(23/2)} - 330 (b d^2 x + b c d)^5 b^7 c^7 d^7 \gamma \\
& (5/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{12} / ((b d^2 \log(F))^{(23/ \\
& 2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(5/2)}) + 330 b^8 c^4 d^{12} \gamma(4, \\
& -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^8 / (b d^2 \log(F))^{(23/2)} - 462 * \\
& (b d^2 x + b c d)^7 b^5 c^5 d^5 \gamma(7/2, -(b d^2 x + b c d)^2 \log(F) / (b d \\
& ^2)) \log(F)^{12} / ((b d^2 \log(F))^{(23/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2)) \\
& ^{(7/2)}) - 55 b^7 c^2 d^{12} \gamma(5, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log \\
& (F)^7 / (b d^2 \log(F))^{(23/2)} - 165 (b d^2 x + b c d)^9 b^3 c^3 d^3 \gamma(9/2 \\
& , -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{12} / ((b d^2 \log(F))^{(23/2)} * (-( \\
& b d^2 x + b c d)^2 \log(F) / (b d^2))^{(9/2)}) + b^6 d^{12} \gamma(6, -(b d^2 x + b \\
& c d)^2 \log(F) / (b d^2)) \log(F)^6 / (b d^2 \log(F))^{(23/2)} - 11 (b d^2 x + b c * \\
& d)^{11} b c d \gamma(11/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{12} / ((b * \\
& d^2 \log(F))^{(23/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(11/2)}) * F^a c d^1 \\
& 1 / \sqrt{b d^2 \log(F)} + 1/2 * (\sqrt{\pi}) * (b d^2 x + b c d) b^{12} c^{12} d^{12} * (\operatorname{erf} \\
& (\sqrt{-(b d^2 x + b c d)^2 \log(F) / (b d^2)}) - 1) \log(F)^{13} / ((b d^2 \log(F))^{( \\
& 25/2)} * \sqrt{-(b d^2 x + b c d)^2 \log(F) / (b d^2)}) - 12 F^((b d^2 x + b c d)^ \\
& 2 / (b d^2)) b^{12} c^{11} d^{13} \log(F)^{12} / (b d^2 \log(F))^{(25/2)} - 66 (b d^2 x + b \\
& c d)^3 b^{10} c^{10} d^{10} \gamma(3/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F) \\
& ^{13} / ((b d^2 \log(F))^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(3/2)}) + \\
& 220 b^{11} c^9 d^{13} \gamma(2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{11} / \\
& (b d^2 \log(F))^{(25/2)} - 792 b^{10} c^7 d^{13} \gamma(3, -(b d^2 x + b c d)^2 \log \\
& (F) / (b d^2)) \log(F)^{10} / (b d^2 \log(F))^{(25/2)} - 495 (b d^2 x + b c d)^5 b^8 * \\
& c^8 d^8 \gamma(5/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{13} / ((b d^2 * \log \\
& (F))^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(5/2)}) + 792 b^9 c^5 d^ \\
& 13 \gamma(4, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^9 / (b d^2 \log(F))^{(2 \\
& 5/2)} - 924 (b d^2 x + b c d)^7 b^6 c^6 d^6 \gamma(7/2, -(b d^2 x + b c d)^2 * \\
& \log(F) / (b d^2)) \log(F)^{13} / ((b d^2 \log(F))^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2)) \\
& ^{(7/2)}) - 220 b^8 c^3 d^{13} \gamma(5, -(b d^2 x + b c d)^2 \log(F) / \\
& (b d^2)) \log(F)^8 / (b d^2 \log(F))^{(25/2)} - 495 (b d^2 x + b c d)^9 b^4 c^4 d \\
& ^4 \gamma(9/2, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^{13} / ((b d^2 \log(F) \\
& )^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b d^2))^{(9/2)}) + 12 b^7 c d^{13} \gamma \\
& (6, -(b d^2 x + b c d)^2 \log(F) / (b d^2)) \log(F)^7 / (b d^2 \log(F))^{(25/2)} - 6 \\
& 6 (b d^2 x + b c d)^{11} b^2 c^2 d^2 \gamma(11/2, -(b d^2 x + b c d)^2 \log(F) / \\
& (b d^2)) \log(F)^{13} / ((b d^2 \log(F))^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b * \\
& d^2))^{(11/2)}) - (b d^2 x + b c d)^{13} \gamma(13/2, -(b d^2 x + b c d)^2 \log(F) / \\
& (b d^2)) \log(F)^{13} / ((b d^2 \log(F))^{(25/2)} * (-(b d^2 x + b c d)^2 \log(F) / (b * \\
& d^2))^{(13/2)}) * F^a d^{12} / \sqrt{b d^2 \log(F)} + 1/2 * \sqrt{\pi} * F^{(b c^2 + a) c^1} \\
& 2 * \operatorname{erf}(\sqrt{-b \log(F)} * d * x - b c \log(F) / \sqrt{-b \log(F)}) / (\sqrt{-b \log(F)} * F^{ \\
& (b c^2) * d}
\end{aligned}$$

**Fricas [A]** time = 1.63886, size = 1355, normalized size = 27.65

$$10395 \sqrt{\pi} \sqrt{-b d^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-b d^2 \log(F)}(d x+c)}{d}\right) - 2\left(32\left(b^6 d^{12} x^{11} + 11 b^6 c d^{11} x^{10} + 55 b^6 c^2 d^{10} x^9 + 165 b^6 c^3 d^9 x^8 + 330 b^6 c^4 d^8 x^7 + 330 b^6 c^5 d^7 x^6 + 165 b^6 c^6 d^6 x^5 + 462 b^6 c^7 d^5 x^4 + 165 b^6 c^8 d^4 x^3 + 462 b^6 c^9 d^3 x^2 + 165 b^6 c^{10} d^2 x + 55 b^6 c^{11} d x + b^6 c^{12}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^12,x, algorithm="fricas")

[Out] -1/128\*(10395\*sqrt(pi)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F)))\*(d\*x + c)/d) - 2\*(32\*(b^6\*d^12\*x^11 + 11\*b^6\*c\*d^11\*x^10 + 55\*b^6\*c^2\*d^10\*x^9

$$\begin{aligned}
& + 165b^6c^3d^9x^8 + 330b^6c^4d^8x^7 + 462b^6c^5d^7x^6 + 462b^6 \\
& c^6d^6x^5 + 330b^6c^7d^5x^4 + 165b^6c^8d^4x^3 + 55b^6c^9d^3x^2 \\
& + 11b^6c^{10}d^2x + b^6c^{11}d) \log(F)^6 - 176(b^5d^{10}x^9 + 9b^5c \\
& d^9x^8 + 36b^5c^2d^8x^7 + 84b^5c^3d^7x^6 + 126b^5c^4d^6x^5 + \\
& 126b^5c^5d^5x^4 + 84b^5c^6d^4x^3 + 36b^5c^7d^3x^2 + 9b^5c^8d^2x \\
& + b^5c^9d) \log(F)^5 + 792(b^4d^8x^7 + 7b^4c^7d^7x^6 + 21b^4c^2 \\
& d^6x^5 + 35b^4c^3d^5x^4 + 35b^4c^4d^4x^3 + 21b^4c^5d^3x^2 + \\
& 7b^4c^6d^2x + b^4c^7d) \log(F)^4 - 2772(b^3d^6x^5 + 5b^3c^5d^5x^4 \\
& + 10b^3c^2d^4x^3 + 10b^3c^3d^3x^2 + 5b^3c^4d^2x + b^3c^5d) \log(F)^3 \\
& + 6930(b^2d^4x^3 + 3b^2c^3d^3x^2 + 3b^2c^2d^2x + b^2c^3d) \log(F)^2 \\
& - 10395(bd^2x + bcd) \log(F) * F^{(bd^2x^2 + 2b^2cdx + b^2c^2 + a)} / (b^7d^2 \log(F)^7)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*12,x)

[Out] Timed out

**Giac [A]** time = 1.29933, size = 263, normalized size = 5.37

$$\frac{\left(32b^5d^{10}\left(x + \frac{c}{d}\right)^{11} \log(F)^5 - 176b^4d^8\left(x + \frac{c}{d}\right)^9 \log(F)^4 + 792b^3d^6\left(x + \frac{c}{d}\right)^7 \log(F)^3 - 2772b^2d^4\left(x + \frac{c}{d}\right)^5 \log(F)^2 + 6930bd^2\left(x + \frac{c}{d}\right)^3 \log(F) - 10395x - 10395c/d\right) e^{(bd^2x^2 \log(F) + 2b^2cdx \log(F) + b^2c^2 \log(F) + a \log(F))}}{64b^6 \log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^12,x, algorithm="giac")

[Out] 1/64\*(32\*b^5\*d^10\*(x + c/d)^11\*log(F)^5 - 176\*b^4\*d^8\*(x + c/d)^9\*log(F)^4 + 792\*b^3\*d^6\*(x + c/d)^7\*log(F)^3 - 2772\*b^2\*d^4\*(x + c/d)^5\*log(F)^2 + 6930\*b\*d^2\*(x + c/d)^3\*log(F) - 10395\*x - 10395\*c/d)\*e^(b\*d^2\*x^2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*c^2\*log(F) + a\*log(F))/(b^6\*log(F)^6) - 10395/128\*sqrt(pi)\*F^a\*erf(-sqrt(-b\*log(F))\*d\*(x + c/d))/(sqrt(-b\*log(F))\*b^6\*d\*log(F)^6)

### 3.268 $\int F^{a+b(c+dx)^2} (c+dx)^{10} dx$

**Optimal.** Leaf size=49

$$\frac{F^a(c+dx)^{11} \text{Gamma}\left(\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{11/2}}$$

[Out]  $-(F^a(c+dx)^{11} \text{Gamma}[11/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(11/2)})$

**Rubi [A]** time = 0.0669821, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{11} \text{Gamma}\left(\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^10, x]

[Out]  $-(F^a(c+dx)^{11} \text{Gamma}[11/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(11/2)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} (c+dx)^{10} dx = -\frac{F^a(c+dx)^{11} \Gamma\left(\frac{11}{2}, -b(c+dx)^2 \log(F)\right)}{2d \left(-b(c+dx)^2 \log(F)\right)^{11/2}}$$

**Mathematica [A]** time = 0.0249053, size = 49, normalized size = 1.

$$\frac{F^a(c+dx)^{11} \text{Gamma}\left(\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d \left(-b \log(F)(c+dx)^2\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^10, x]

[Out]  $-(F^a(c+dx)^{11} \text{Gamma}[11/2, -(b*(c+dx)^2 \text{Log}[F])]) / (2*d*(-(b*(c+dx)^2 \text{Log}[F]))^{(11/2)})$

**Maple [B]** time = 0.222, size = 1359, normalized size = 27.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^10,x)

[Out] 
$$42*d^2*c^6/\ln(F)/b*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+18*d*c^7/\ln(F)/b*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-189/4*d*c^5/\ln(F)^2/b^2*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-315/4*d^2*c^4/\ln(F)^2/b^2*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+315/8*d^3*c/\ln(F)^3/b^3*x^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-945/16*d*c/\ln(F)^4/b^4*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-63/4*d^5*c/\ln(F)^2/b^2*x^6*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+9/2*d^7*c/\ln(F)/b*x^8*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+18*d^6*c^2/\ln(F)/b*x^7*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-189/4*d^4*c^2/\ln(F)^2/b^2*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+315/4*d^2*c^2/\ln(F)^3/b^3*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+315/4*d*c^3/\ln(F)^3/b^3*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+42*d^5*c^3/\ln(F)/b*x^6*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-315/4*d^3*c^3/\ln(F)^2/b^2*x^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+63*d^4*c^4/\ln(F)/b*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+63*d^3*c^5/\ln(F)/b*x^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+9/2*c^8/\ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-63/4*c^6/\ln(F)^2/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+315/8*c^4/\ln(F)^3/b^3*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-945/16*c^2/\ln(F)^4/b^4*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+63/8*d^4/\ln(F)^3/b^3*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-315/16*d^2/\ln(F)^4/b^4*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+1/2*d^8/\ln(F)/b*x^9*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+1/2/d*c^9/\ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-9/4/d*c^7/\ln(F)^2/b^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+63/8/d*c^5/\ln(F)^3/b^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-315/16/d*c^3/\ln(F)^4/b^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+945/32/d*c/\ln(F)^5/b^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a-9/4*d^6/\ln(F)^2/b^2*x^7*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+945/32/\ln(F)^5/b^5*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+945/64/d/\ln(F)^5/b^5*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))$$

**Maxima [B]** time = 3.16986, size = 6310, normalized size = 128.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^10,x, algorithm="maxima")

[Out] 
$$-5*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - F^{a*c^9*d/\sqrt{b*d^2*\log(F)}} + 45/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^{a*c^8*d^2/\sqrt{b*d^2*\log(F)}} - 60*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^4/((b*d^2*\log(F))^{7/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})$$

$$\begin{aligned}
& ^2*x + b*c*d)^2*\log(F)/(b*d^2)) - 1)*\log(F)^4/((b*d^2*\log(F))^{(7/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 3*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*d^4*\log(F)^3/(b*d^2*\log(F))^{(7/2)} - 3*(b*d^2*x + b*c*d)^3*b*c*d*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/((b*d^2*\log(F))^{(7/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + b^2*d^4*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^2/(b*d^2*\log(F))^{(7/2)})*F^a*c^7*d^3/\sqrt{b*d^2*\log(F)} + 105*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 4*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*d^5*\log(F)^4/(b*d^2*\log(F))^{(9/2)} - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 4*b^3*c*d^5*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(9/2)} - (b*d^2*x + b*c*d)^5*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)})}*F^a*c^6*d^4/\sqrt{b*d^2*\log(F)} - 126*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{(11/2)} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 10*b^4*c^2*d^6*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(11/2)} - b^3*d^6*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(11/2)} - 5*(b*d^2*x + b*c*d)^5*b*c*d*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)})}*F^a*c^5*d^5/\sqrt{b*d^2*\log(F)} + 105*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b^6*c^6*d^6*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 6*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^6*c^5*d^7*\log(F)^6/(b*d^2*\log(F))^{(13/2)} - 15*(b*d^2*x + b*c*d)^3*b^4*c^4*d^4*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 20*b^5*c^3*d^7*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(13/2)} - 6*b^4*c*d^7*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(13/2)} - 15*(b*d^2*x + b*c*d)^5*b^2*c^2*d^2*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) - (b*d^2*x + b*c*d)^7*\gamma(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{(13/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)})}*F^a*c^4*d^6/\sqrt{b*d^2*\log(F)} - 60*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b^7*c^7*d^7*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 7*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^7*c^6*d^8*\log(F)^7/(b*d^2*\log(F))^{(15/2)} - 21*(b*d^2*x + b*c*d)^3*b^5*c^5*d^5*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 35*b^6*c^4*d^8*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/(b*d^2*\log(F))^{(15/2)} - 21*b^5*c^2*d^8*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{(15/2)} - 35*(b*d^2*x + b*c*d)^5*b^3*c^3*d^3*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}) + b^4*d^8*\gamma(4, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(15/2)} - 7*(b*d^2*x + b*c*d)^7*b*c*d*\gamma(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^8/((b*d^2*\log(F))^{(15/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(7/2)})}*F^a*c^3*d^7/\sqrt{b*d^2*\log(F)} + 45/2*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b^8*c^8*d^8*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 8*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^8*c^7*d^9*\log(F)^8/(b*d^2*\log(F))^{(17/2)} - 28*(b*d^2*x + b*c*d)^3*b^6*c^6*d^6*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^9/((b*d^2*\log(F))^{(17/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 56*b^7*c^5*d^9*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/(b*d^2*\log(F))^{(17/2)} - 56*b^6*c^3
\end{aligned}$$

$$\begin{aligned}
& *d^9 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^6 / (b*d^2 \log(F))^{17/2} \\
& - 70*(b*d^2*x + b*c*d)^5 b^4 c^4 d^4 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + 8*b^5*c*d^9 \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^5 / (b*d^2 \log(F))^{17/2} \\
& - 28*(b*d^2*x + b*c*d)^7 b^2 c^2 d^2 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& - (b*d^2*x + b*c*d)^9 \gamma(9/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^9 / ((b*d^2 \log(F))^{17/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{9/2}) \\
& * F^a * c^2 * d^8 / \sqrt{b*d^2 \log(F)} - 5*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^9*c^9*d^9*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) - 1) \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 9*F^((b*d^2*x + b*c*d)^2/(b*d^2)) * b^9*c^8*d^10 \log(F)^9 / (b*d^2 \log(F))^{19/2} \\
& - 36*(b*d^2*x + b*c*d)^3 b^7*c^7*d^7 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 84*b^8*c^6*d^10 \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^8 / (b*d^2 \log(F))^{19/2} \\
& - 126*b^7*c^4*d^10 \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^7 / (b*d^2 \log(F))^{19/2} \\
& - 126*(b*d^2*x + b*c*d)^5 b^5*c^5*d^5 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + 36*b^6*c^2*d^10 \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^6 / (b*d^2 \log(F))^{19/2} \\
& - 84*(b*d^2*x + b*c*d)^7 b^3*c^3*d^3 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& - b^5*d^10 \gamma(5, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^5 / (b*d^2 \log(F))^{19/2} \\
& - 9*(b*d^2*x + b*c*d)^9 b*c*d \gamma(9/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{10} / ((b*d^2 \log(F))^{19/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{9/2}) \\
& * F^a * c*d^9 / \sqrt{b*d^2 \log(F)} + 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^{10}*c^{10}*d^{10}*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) - 1) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) \\
& - 10*F^((b*d^2*x + b*c*d)^2/(b*d^2)) * b^{10}*c^9*d^{11} \log(F)^{10} / (b*d^2 \log(F))^{21/2} \\
& - 45*(b*d^2*x + b*c*d)^3 b^8*c^8*d^8 \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{3/2}) \\
& + 120*b^9*c^7*d^{11} \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^9 / (b*d^2 \log(F))^{21/2} \\
& - 252*b^8*c^5*d^{11} \gamma(3, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^8 / (b*d^2 \log(F))^{21/2} \\
& - 210*(b*d^2*x + b*c*d)^5 b^6*c^6*d^6 \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{5/2}) \\
& + 120*b^7*c^3*d^{11} \gamma(4, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^7 / (b*d^2 \log(F))^{21/2} \\
& - 210*(b*d^2*x + b*c*d)^7 b^4*c^4*d^4 \gamma(7/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{7/2}) \\
& - 10*b^6*c*d^{11} \gamma(5, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^6 / (b*d^2 \log(F))^{21/2} \\
& - 45*(b*d^2*x + b*c*d)^9 b^2*c^2*d^2 \gamma(9/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{9/2}) \\
& - (b*d^2*x + b*c*d)^{11} \gamma(11/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) \log(F)^{11} / ((b*d^2 \log(F))^{21/2} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{11/2}) \\
& * F^a * d^{10} / \sqrt{b*d^2 \log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^{10}}*\operatorname{erf}(\sqrt{-b*\log(F)})*d*x - b*c*\log(F) / \sqrt{-b*\log(F)}) / (\sqrt{-b*\log(F)})*F^{(b*c^2)*d}
\end{aligned}$$

**Fricas [A]** time = 1.59994, size = 992, normalized size = 20.24

$$945 \sqrt{\pi} \sqrt{-bd^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)(dx+c)}}{d}\right) + 2 \left(16 \left(b^5 d^{10} x^9 + 9 b^5 c d^9 x^8 + 36 b^5 c^2 d^8 x^7 + 84 b^5 c^3 d^7 x^6 + 126 b^5 c^4 d^6 x^5 + \dots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^10,x, algorithm="fricas")

```
[Out] 1/64*(945*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d) + 2*(16*(b^5*d^10*x^9 + 9*b^5*c*d^9*x^8 + 36*b^5*c^2*d^8*x^7 + 84*b^5*c^3*d^7*x^6 + 126*b^5*c^4*d^6*x^5 + 126*b^5*c^5*d^5*x^4 + 84*b^5*c^6*d^4*x^3 + 36*b^5*c^7*d^3*x^2 + 9*b^5*c^8*d^2*x + b^5*c^9*d)*log(F)^5 - 72*(b^4*d^8*x^7 + 7*b^4*c*d^7*x^6 + 21*b^4*c^2*d^6*x^5 + 35*b^4*c^3*d^5*x^4 + 35*b^4*c^4*d^4*x^3 + 21*b^4*c^5*d^3*x^2 + 7*b^4*c^6*d^2*x + b^4*c^7*d)*log(F)^4 + 252*(b^3*d^6*x^5 + 5*b^3*c*d^5*x^4 + 10*b^3*c^2*d^4*x^3 + 10*b^3*c^3*d^3*x^2 + 5*b^3*c^4*d^2*x + b^3*c^5*d)*log(F)^3 - 630*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*log(F)^2 + 945*(b*d^2*x + b*c*d)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^6*d^2*log(F)^6)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**10,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.22319, size = 235, normalized size = 4.8

$$\frac{\left(16b^4d^8\left(x + \frac{c}{d}\right)^9 \log(F)^4 - 72b^3d^6\left(x + \frac{c}{d}\right)^7 \log(F)^3 + 252b^2d^4\left(x + \frac{c}{d}\right)^5 \log(F)^2 - 630bd^2\left(x + \frac{c}{d}\right)^3 \log(F) + 945x + \frac{945c}{d}\right)}{32b^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^4*d^8*(x + c/d)^9*log(F)^4 - 72*b^3*d^6*(x + c/d)^7*log(F)^3 + 252*b^2*d^4*(x + c/d)^5*log(F)^2 - 630*b*d^2*(x + c/d)^3*log(F) + 945*x + 945*c/d)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^5*log(F)^5) + 945/64*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b^5*d*log(F)^5)
```



### 3.269 $\int F^{a+b(c+dx)^2} (c+dx)^8 dx$

**Optimal.** Leaf size=179

$$\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{32b^{9/2}d\log^2(F)} - \frac{7(c+dx)^5F^{a+b(c+dx)^2}}{4b^2d\log^2(F)} + \frac{35(c+dx)^3F^{a+b(c+dx)^2}}{8b^3d\log^3(F)} - \frac{105(c+dx)F^{a+b(c+dx)^2}}{16b^4d\log^4(F)} + \frac{(c+dx)^8}{2}$$

[Out] (105\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(32\*b^(9/2)\*d\*Log[F]^(9/2)) - (105\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x))/(16\*b^4\*d\*Log[F]^4) + (35\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^3)/(8\*b^3\*d\*Log[F]^3) - (7\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^5)/(4\*b^2\*d\*Log[F]^2) + (F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^7)/(2\*b\*d\*Log[F])

**Rubi [A]** time = 0.331287, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2204}

$$\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{32b^{9/2}d\log^2(F)} - \frac{7(c+dx)^5F^{a+b(c+dx)^2}}{4b^2d\log^2(F)} + \frac{35(c+dx)^3F^{a+b(c+dx)^2}}{8b^3d\log^3(F)} - \frac{105(c+dx)F^{a+b(c+dx)^2}}{16b^4d\log^4(F)} + \frac{(c+dx)^8}{2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^8,x]

[Out] (105\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(32\*b^(9/2)\*d\*Log[F]^(9/2)) - (105\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x))/(16\*b^4\*d\*Log[F]^4) + (35\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^3)/(8\*b^3\*d\*Log[F]^3) - (7\*F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^5)/(4\*b^2\*d\*Log[F]^2) + (F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^7)/(2\*b\*d\*Log[F])

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2}(c+dx)^8 dx &= \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd \log(F)} - \frac{7 \int F^{a+b(c+dx)^2}(c+dx)^6 dx}{2b \log(F)} \\ &= -\frac{7F^{a+b(c+dx)^2}(c+dx)^5}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd \log(F)} + \frac{35 \int F^{a+b(c+dx)^2}(c+dx)^4 dx}{4b^2 \log^2(F)} \\ &= \frac{35F^{a+b(c+dx)^2}(c+dx)^3}{8b^3d \log^3(F)} - \frac{7F^{a+b(c+dx)^2}(c+dx)^5}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd \log(F)} - \frac{105 \int F^{a+b(c+dx)^2}(c+dx)^2 dx}{8b^3 \log^3(F)} \\ &= -\frac{105F^{a+b(c+dx)^2}(c+dx)}{16b^4d \log^4(F)} + \frac{35F^{a+b(c+dx)^2}(c+dx)^3}{8b^3d \log^3(F)} - \frac{7F^{a+b(c+dx)^2}(c+dx)^5}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd \log(F)} \\ &= \frac{105F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{32b^{9/2}d \log^{\frac{9}{2}}(F)} - \frac{105F^{a+b(c+dx)^2}(c+dx)}{16b^4d \log^4(F)} + \frac{35F^{a+b(c+dx)^2}(c+dx)^3}{8b^3d \log^3(F)} - \frac{7F^{a+b(c+dx)^2}(c+dx)^5}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.37086, size = 153, normalized size = 0.85

$$\frac{F^a \left( \frac{105\sqrt{\pi}\operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx))}{b^{7/2} \log^{\frac{7}{2}}(F)} + \frac{140(c+dx)^3 F^{b(c+dx)^2}}{b^2 \log^2(F)} - \frac{210(c+dx) F^{b(c+dx)^2}}{b^3 \log^3(F)} + 16(c+dx)^7 F^{b(c+dx)^2} - \frac{56(c+dx)^5 F^{b(c+dx)^2}}{b \log(F)} \right)}{32bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^8, x]

[Out] (F^a\*(16\*F^(b\*(c + d\*x)^2)\*(c + d\*x)^7 + (105\*sqrt(Pi)\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]))/(b^(7/2)\*Log[F]^(7/2)) - (210\*F^(b\*(c + d\*x)^2)\*(c + d\*x)^5)/(b^3\*Log[F]^3) + (140\*F^(b\*(c + d\*x)^2)\*(c + d\*x)^3)/(b^2\*Log[F]^2) - (56\*F^(b\*(c + d\*x)^2)\*(c + d\*x)^5)/(b\*Log[F]))/(32\*b\*d\*Log[F])

**Maple [B]** time = 0.123, size = 914, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^8, x)

[Out] 1/2\*d^6/ln(F)/b\*x^7\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-7/4\*d^4/ln(F)^2/b^2\*x^5\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+35/2\*d^2\*c^4/ln(F)/b\*x^3\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+21/2\*d\*c^5/ln(F)/b\*x^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-35/2\*d\*c^3/ln(F)^2/b^2\*x^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-35/2\*d^2\*c^2/ln(F)^2/b^2\*x^3\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+7/2\*d^5\*c/ln(F)/b\*x^6\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-35/4\*d^3\*c/ln(F)^2/b^2\*x^4\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+105/8\*d\*c/ln(F)^3/b^3\*x^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+21/2\*d^4\*c^2/ln(F)/b\*x^5\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+35/2\*d^3\*c^3/ln(F)/b\*x^4\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-105/32/d/ln(F)^4/b^4\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))-105/16/ln(F)^4/b^4\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+7/2\*c^6/ln(F)/b\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-35/4\*c^4/ln(F)^2/b^2\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+105/8\*c^2/ln(F)^3/b^3\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+1/2/d\*c^7/ln(F)/b\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-7/4/d\*c^5/ln(F)^2/b^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d

```
*x)*F^(c^2*b)*F^a+35/8/d*c^3/ln(F)^3/b^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2
*b)*F^a-105/16/d*c/ln(F)^4/b^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a+35
/8*d^2/ln(F)^3/b^3*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)*F^(c^2*b)*F^a
```

**Maxima [B]** time = 2.63602, size = 4327, normalized size = 24.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -4*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*d*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/
(b*d^2))) - 1)*log(F)^2/((b*d^2*log(F))^(3/2)*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*log(F)/(b*d^2*log(F))
^(3/2))*F^a*c^7*d/sqrt(b*d^2*log(F)) + 14*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c
^2*d^2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*d^
2*log(F))^(5/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x
+ b*c*d)^2/(b*d^2))*b^2*c*d^3*log(F)^2/(b*d^2*log(F))^(5/2) - (b*d^2*x + b*
c*d)^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*d^2*log
(F))^(5/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^6*d^2/sqrt(b
*d^2*log(F)) - 28*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(erf(sqrt(-(b*d^2
*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*d^2*log(F))^(7/2)*sqrt(-(b
*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*
c^2*d^4*log(F)^3/(b*d^2*log(F))^(7/2) - 3*(b*d^2*x + b*c*d)^3*b*c*d*gamma(3
/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*d^2*log(F))^(7/2)*(-(
b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*d^4*gamma(2, -(b*d^2*x + b*
c*d)^2*log(F)/(b*d^2))*log(F)^2/(b*d^2*log(F))^(7/2))*F^a*c^5*d^3/sqrt(b*d^
2*log(F)) + 35*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(erf(sqrt(-(b*d^2*x
+ b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^5/((b*d^2*log(F))^(9/2)*sqrt(-(b*d^
2*x + b*c*d)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3
*d^5*log(F)^4/(b*d^2*log(F))^(9/2) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*gamma
a(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(9/2)*
(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 4*b^3*c*d^5*gamma(2, -(b*d^2
*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))^(9/2) - (b*d^2*x + b*
c*d)^5*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log
(F))^(9/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(5/2)))*F^a*c^4*d^4/sqrt(b
*d^2*log(F)) - 28*(sqrt(pi)*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(erf(sqrt(-(b*d^2
*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^6/((b*d^2*log(F))^(11/2)*sqrt(-(
b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 5*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^5
*c^4*d^6*log(F)^5/(b*d^2*log(F))^(11/2) - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^
3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^6/((b*d^2*log(F))^(
11/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 10*b^4*c^2*d^6*gamma(
2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/(b*d^2*log(F))^(11/2) - b^
3*d^6*gamma(3, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))
^(11/2) - 5*(b*d^2*x + b*c*d)^5*b*c*d*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))*log(F)^6/((b*d^2*log(F))^(11/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b
*d^2))^(5/2)))*F^a*c^3*d^5/sqrt(b*d^2*log(F)) + 14*(sqrt(pi)*(b*d^2*x + b*c*
d)*b^6*c^6*d^6*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^
7/((b*d^2*log(F))^(13/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 6*F^
((b*d^2*x + b*c*d)^2/(b*d^2))*b^6*c^5*d^7*log(F)^6/(b*d^2*log(F))^(13/2) - 1
5*(b*d^2*x + b*c*d)^3*b^4*c^4*d^4*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b
*d^2))*log(F)^7/((b*d^2*log(F))^(13/2)*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)
)^(3/2)) + 20*b^5*c^3*d^7*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log
(F)^5/(b*d^2*log(F))^(13/2) - 6*b^4*c*d^7*gamma(3, -(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))*log(F)^4/(b*d^2*log(F))^(13/2) - 15*(b*d^2*x + b*c*d)^5*b^2*c^
2*d^2*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^7/((b*d^2*log(
```

$$\begin{aligned}
 & F)^{(13/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)} - (b*d^2*x + b*c*d) \\
 & ^7 * \text{gamma}(7/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^7 / ((b*d^2 * \log(F)) \\
 & ^{(13/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) * F^a * c^2 * d^6 / \text{sqrt}(b*d^2 \\
 & * \log(F)) - 4 * (\text{sqrt}(\pi) * (b*d^2*x + b*c*d) * b^7 * c^7 * d^7 * (\text{erf}(\text{sqrt}(- (b*d^2*x + \\
 & b*c*d)^2 * \log(F) / (b*d^2)))) - 1) * \log(F)^8 / ((b*d^2 * \log(F))^{(15/2)} * \text{sqrt}(- (b*d^2 \\
 & * x + b*c*d)^2 * \log(F) / (b*d^2))) - 7 * F^{((b*d^2*x + b*c*d)^2 / (b*d^2))} * b^7 * c^6 \\
 & * d^8 * \log(F)^7 / (b*d^2 * \log(F))^{(15/2)} - 21 * (b*d^2*x + b*c*d)^3 * b^5 * c^5 * d^5 * \text{ga} \\
 & \text{mma}(3/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^8 / ((b*d^2 * \log(F))^{(15/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(3/2)}) + 35 * b^6 * c^4 * d^8 * \text{gamma}(2, - \\
 & (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^6 / (b*d^2 * \log(F))^{(15/2)} - 21 * b^5 \\
 & * c^2 * d^8 * \text{gamma}(3, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^5 / (b*d^2 * \log(F) \\
 & )^{(15/2)} - 35 * (b*d^2*x + b*c*d)^5 * b^3 * c^3 * d^3 * \text{gamma}(5/2, - (b*d^2*x + b*c*d) \\
 & ^2 * \log(F) / (b*d^2)) * \log(F)^8 / ((b*d^2 * \log(F))^{(15/2)} * (- (b*d^2*x + b*c*d)^2 * \\
 & \log(F) / (b*d^2))^{(5/2)}) + b^4 * d^8 * \text{gamma}(4, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2 \\
 & )) * \log(F)^4 / (b*d^2 * \log(F))^{(15/2)} - 7 * (b*d^2*x + b*c*d)^7 * b * c * d * \text{gamma}(7/2, \\
 & - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^8 / ((b*d^2 * \log(F))^{(15/2)} * (- (b*d^2 \\
 & * x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) * F^a * c^d^7 / \text{sqrt}(b*d^2 * \log(F)) + 1/2 \\
 & * (\text{sqrt}(\pi) * (b*d^2*x + b*c*d) * b^8 * c^8 * d^8 * (\text{erf}(\text{sqrt}(- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)))) - 1) * \log(F)^9 / ((b*d^2 * \log(F))^{(17/2)} * \text{sqrt}(- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))) - 8 * F^{((b*d^2*x + b*c*d)^2 / (b*d^2))} * b^8 * c^7 * d^9 * \log(F)^8 / (b*d^2 * \log(F))^{(17/2)} - 28 * (b*d^2*x + b*c*d)^3 * b^6 * c^6 * d^6 * \text{gamma}(3/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^9 / ((b*d^2 * \log(F))^{(17/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(3/2)}) + 56 * b^7 * c^5 * d^9 * \text{gamma}(2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^7 / (b*d^2 * \log(F))^{(17/2)} - 56 * b^6 * c^3 * d^9 * \text{gamma}(3, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^6 / (b*d^2 * \log(F))^{(17/2)} - 70 * (b*d^2*x + b*c*d)^5 * b^4 * c^4 * d^4 * \text{gamma}(5/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^9 / ((b*d^2 * \log(F))^{(17/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(5/2)}) + 8 * b^5 * c^d^9 * \text{gamma}(4, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^5 / (b*d^2 * \log(F))^{(17/2)} - 28 * (b*d^2*x + b*c*d)^7 * b^2 * c^2 * d^2 * \text{gamma}(7/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^9 / ((b*d^2 * \log(F))^{(17/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(7/2)}) - (b*d^2*x + b*c*d)^9 * \text{gamma}(9/2, - (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^9 / ((b*d^2 * \log(F))^{(17/2)} * (- (b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{(9/2)}) * F^a * d^8 / \text{sqrt}(b*d^2 * \log(F)) + 1/2 * \text{sqrt}(\pi) * F^{(b*c^2 + a)} * c^8 * \text{erf}(\text{sqrt}(-b * \log(F)) * d * x - b * c * \log(F) / \text{sqrt}(-b * \log(F))) / (\text{sqrt}(-b * \log(F)) * F^{(b*c^2)} * d)
 \end{aligned}$$

**Fricas [B]** time = 1.59617, size = 713, normalized size = 3.98

$$105 \sqrt{\pi} \sqrt{-bd^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) - 2 \left(8 \left(b^4 d^8 x^7 + 7 b^4 c d^7 x^6 + 21 b^4 c^2 d^6 x^5 + 35 b^4 c^3 d^5 x^4 + 35 b^4 c^4 d^4 x^3 + 28 b^4 c^5 d^3 x^2 + 7 b^4 c^6 d^2 x + b^4 c^7 d\right) \log(F)^4 - 28 \left(b^3 d^6 x^5 + 5 b^3 c d^5 x^4 + 10 b^3 c^2 d^4 x^3 + 10 b^3 c^3 d^3 x^2 + 5 b^3 c^4 d^2 x + b^3 c^5 d\right) \log(F)^3 + 70 \left(b^2 d^4 x^3 + 3 b^2 c d^3 x^2 + 3 b^2 c^2 d^2 x + b^2 c^3 d\right) \log(F)^2 - 105 \left(b d^2 x + b c d\right) \log(F) \right) F^{(b d^2 x^2 + 2 b c d x + b c^2 + a)} / (b^5 d^2 \log(F)^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^8,x, algorithm="fricas")

[Out] -1/32\*(105\*sqrt(pi)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d) - 2\*(8\*(b^4\*d^8\*x^7 + 7\*b^4\*c\*d^7\*x^6 + 21\*b^4\*c^2\*d^6\*x^5 + 35\*b^4\*c^3\*d^5\*x^4 + 35\*b^4\*c^4\*d^4\*x^3 + 21\*b^4\*c^5\*d^3\*x^2 + 7\*b^4\*c^6\*d^2\*x + b^4\*c^7\*d)\*log(F)^4 - 28\*(b^3\*d^6\*x^5 + 5\*b^3\*c\*d^5\*x^4 + 10\*b^3\*c^2\*d^4\*x^3 + 10\*b^3\*c^3\*d^3\*x^2 + 5\*b^3\*c^4\*d^2\*x + b^3\*c^5\*d)\*log(F)^3 + 70\*(b^2\*d^4\*x^3 + 3\*b^2\*c\*d^3\*x^2 + 3\*b^2\*c^2\*d^2\*x + b^2\*c^3\*d)\*log(F)^2 - 105\*(b\*d^2\*x + b\*c\*d)\*log(F))\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a))/(b^5\*d^2\*log(F)^5)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*8,x)

[Out] Timed out

**Giac [A]** time = 1.30266, size = 207, normalized size = 1.16

$$\frac{\left(8b^3d^6\left(x + \frac{c}{d}\right)^7 \log(F)^3 - 28b^2d^4\left(x + \frac{c}{d}\right)^5 \log(F)^2 + 70bd^2\left(x + \frac{c}{d}\right)^3 \log(F) - 105x - \frac{105c}{d}\right)e^{(bd^2x^2 \log(F) + 2bcdx \log(F) + bc^2)}}{16b^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^8,x, algorithm="giac")

[Out] 1/16\*(8\*b^3\*d^6\*(x + c/d)^7\*log(F)^3 - 28\*b^2\*d^4\*(x + c/d)^5\*log(F)^2 + 70\*b\*d^2\*(x + c/d)^3\*log(F) - 105\*x - 105\*c/d)\*e^(b\*d^2\*x^2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*c^2\*log(F) + a\*log(F))/(b^4\*log(F)^4) - 105/32\*sqrt(pi)\*F^a\*erf(-sqrt(-b\*log(F))\*d\*(x + c/d))/(sqrt(-b\*log(F))\*b^4\*d\*log(F)^4)

### 3.270 $\int F^{a+b(c+dx)^2} (c+dx)^6 dx$

**Optimal.** Leaf size=145

$$\frac{15\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{16b^{7/2}d\log^2(F)} - \frac{5(c+dx)^3F^{a+b(c+dx)^2}}{4b^2d\log^2(F)} + \frac{15(c+dx)F^{a+b(c+dx)^2}}{8b^3d\log^3(F)} + \frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}$$

[Out]  $(-15F^a\sqrt{\pi}\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(16b^{(7/2)}*d*\operatorname{Log}[F]^{(7/2)}) + (15F^{(a+b*(c+d*x)^2)}*(c+d*x))/(8b^3*d*\operatorname{Log}[F]^3) - (5F^{(a+b*(c+d*x)^2)}*(c+d*x)^3)/(4b^2*d*\operatorname{Log}[F]^2) + (F^{(a+b*(c+d*x)^2)}*(c+d*x)^5)/(2*b*d*\operatorname{Log}[F])$

**Rubi [A]** time = 0.230178, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2204}

$$\frac{15\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{16b^{7/2}d\log^2(F)} - \frac{5(c+dx)^3F^{a+b(c+dx)^2}}{4b^2d\log^2(F)} + \frac{15(c+dx)F^{a+b(c+dx)^2}}{8b^3d\log^3(F)} + \frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a+b*(c+d*x)^2)}*(c+d*x)^6, x]$

[Out]  $(-15F^a\sqrt{\pi}\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(16b^{(7/2)}*d*\operatorname{Log}[F]^{(7/2)}) + (15F^{(a+b*(c+d*x)^2)}*(c+d*x))/(8b^3*d*\operatorname{Log}[F]^3) - (5F^{(a+b*(c+d*x)^2)}*(c+d*x)^3)/(4b^2*d*\operatorname{Log}[F]^2) + (F^{(a+b*(c+d*x)^2)}*(c+d*x)^5)/(2*b*d*\operatorname{Log}[F])$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{(n_.)})*((c_.)+(d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^{(m-n+1)}*F^{(a+b*(c+d*x)^n)}/(b*d*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(m-n+1)/(b*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-n)}*F^{(a+b*(c+d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{IntegerQ}[(2*(m+1))/n] \&\& \operatorname{LtQ}[0, (m+1)/n, 5] \&\& \operatorname{IntegerQ}[n] \&\& (\operatorname{LtQ}[0, n, m+1] \mid\mid \operatorname{LtQ}[m, n, 0])$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)^2}(c+dx)^6 dx &= \frac{F^{a+b(c+dx)^2}(c+dx)^5}{2bd \log(F)} - \frac{5 \int F^{a+b(c+dx)^2}(c+dx)^4 dx}{2b \log(F)} \\
&= -\frac{5F^{a+b(c+dx)^2}(c+dx)^3}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^5}{2bd \log(F)} + \frac{15 \int F^{a+b(c+dx)^2}(c+dx)^2 dx}{4b^2 \log^2(F)} \\
&= \frac{15F^{a+b(c+dx)^2}(c+dx)}{8b^3d \log^3(F)} - \frac{5F^{a+b(c+dx)^2}(c+dx)^3}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^5}{2bd \log(F)} - \frac{15 \int F^{a+b(c+dx)^2} dx}{8b^3 \log^3(F)} \\
&= -\frac{15F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{16b^{7/2}d \log^{7/2}(F)} + \frac{15F^{a+b(c+dx)^2}(c+dx)}{8b^3d \log^3(F)} - \frac{5F^{a+b(c+dx)^2}(c+dx)^3}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^5}{2bd \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.144107, size = 126, normalized size = 0.87

$$\frac{F^a \left( -\frac{15\sqrt{\pi}\operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx))}{b^{5/2}\log^{5/2}(F)} + \frac{30(c+dx)F^{b(c+dx)^2}}{b^2\log^2(F)} + 8(c+dx)^5F^{b(c+dx)^2} - \frac{20(c+dx)^3F^{b(c+dx)^2}}{b\log(F)} \right)}{16bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^6, x]

[Out] (F^a\*(8\*(F^(b\*(c + d\*x)^2)\*(c + d\*x)^5 - (15\*sqrt(Pi)\*Erfi[sqrt(b)\*(c + d\*x)\*sqrt(Log[F]])/(b^(5/2)\*Log[F]^(5/2))) + (30\*(F^(b\*(c + d\*x)^2)\*(c + d\*x))/(b^2\*Log[F]^2) - (20\*(F^(b\*(c + d\*x)^2)\*(c + d\*x)^3)/(b\*Log[F])))/(16\*b\*d\*Log[F])

**Maple [B]** time = 0.081, size = 561, normalized size = 3.9

$$\frac{5cd^3x^4F^{bd^2x^2}F^{2bcdx}F^{c^2b}F^a}{2b \ln(F)} + 5 \frac{d^2c^2x^3F^{bd^2x^2}F^{2bcdx}F^{c^2b}F^a}{b \ln(F)} + 5 \frac{dc^3x^2F^{bd^2x^2}F^{2bcdx}F^{c^2b}F^a}{b \ln(F)} - \frac{15cdx^2F^{bd^2x^2}F^{2bcdx}F^{c^2b}F^a}{4(\ln(F))^2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^6, x)

[Out] 5/2\*d^3\*c/ln(F)/b\*x^4\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+5\*d^2\*c^2/ln(F)/b\*x^3\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+5\*d\*c^3/ln(F)/b\*x^2\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-15/4\*d\*c/ln(F)^2/b^2\*x^2\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+15/8/ln(F)^3/b^3\*x\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+15/16/d/ln(F)^3/b^3\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+1/2\*d^4/ln(F)/b\*x^5\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+5/2\*c^4/ln(F)/b\*x\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+1/2/d\*c^5/ln(F)/b\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-5/4/d\*c^3/ln(F)^2/b^2\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-15/4\*c^2/ln(F)^2/b^2\*x\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+15/8/d\*c/ln(F)^3/b^3\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-5/4\*d^2/ln(F)^2/b^2\*x^3\*(F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a

**Maxima [B]** time = 2.16137, size = 2712, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^6,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -3*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b*d^2*\log(F)/(b*d^2*\log(F))^{3/2} \\ & *F^a*c^5*d/\sqrt{b*d^2*\log(F)} + 15/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{5/2} \\ & - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{5/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}))*F^a*c^4*d^2/\sqrt{b*d^2*\log(F)} - 10*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^4/((b*d^2*\log(F))^{7/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 3*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^3*c^2*d^4*\log(F)^3/(b*d^2*\log(F))^{7/2} \\ & - 3*(b*d^2*x + b*c*d)^3*b*c*d*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/((b*d^2*\log(F))^{7/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + b^2*d^4*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^2/(b*d^2*\log(F))^{7/2})*F^a*c^3*d^3/\sqrt{b*d^2*\log(F)} + 15/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^5/((b*d^2*\log(F))^{9/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 4*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^4*c^3*d^5*\log(F)^4/(b*d^2*\log(F))^{9/2} \\ & - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{9/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + 4*b^3*c*d^5*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{9/2} - (b*d^2*x + b*c*d)^5*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{9/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{5/2}))*F^a*c^2*d^4/\sqrt{b*d^2*\log(F)} \\ & - 3*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^6/((b*d^2*\log(F))^{11/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{11/2} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{11/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) \\ & + 10*b^4*c^2*d^6*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{11/2} - b^3*d^6*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{11/2} - 5*(b*d^2*x + b*c*d)^5*b*c*d*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{11/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{5/2}))*F^a*c*d^5/\sqrt{b*d^2*\log(F)} + 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^6*c^6*d^6*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^7/((b*d^2*\log(F))^{13/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 6*F^{(b*d^2*x + b*c*d)^2/(b*d^2)}*b^6*c^5*d^7*\log(F)^6/(b*d^2*\log(F))^{13/2} \\ & - 15*(b*d^2*x + b*c*d)^3*b^4*c^4*d^4*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{13/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}) + 20*b^5*c^3*d^7*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/(b*d^2*\log(F))^{13/2} - 6*b^4*c*d^7*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{13/2} \\ & - 15*(b*d^2*x + b*c*d)^5*b^2*c^2*d^2*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{13/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{5/2}) - (b*d^2*x + b*c*d)^7*\gamma(7/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^7/((b*d^2*\log(F))^{13/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{7/2}))*F^a*d^6/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^6*\operatorname{erf}(\sqrt{-b*\log(F)})}*d*x - b*c*\log(F)/\sqrt{-b*\log(F)})/(\sqrt{-b*\log(F)}*F^{(b*c^2)*d}) \end{aligned}$$



**Fricas [A]** time = 1.52384, size = 493, normalized size = 3.4

$$15 \sqrt{\pi} \sqrt{-bd^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) + 2 \left(4 \left(b^3 d^6 x^5 + 5 b^3 c d^5 x^4 + 10 b^3 c^2 d^4 x^3 + 10 b^3 c^3 d^3 x^2 + 5 b^3 c^4 d^2 x + b^3 c^5\right)\right. \\ \left.16 b^4 d^2 \log(F)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^6,x, algorithm="fricas")

[Out] 1/16\*(15\*sqrt(pi)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d) + 2\*(4\*(b^3\*d^6\*x^5 + 5\*b^3\*c\*d^5\*x^4 + 10\*b^3\*c^2\*d^4\*x^3 + 10\*b^3\*c^3\*d^3\*x^2 + 5\*b^3\*c^4\*d^2\*x + b^3\*c^5\*d)\*log(F)^3 - 10\*(b^2\*d^4\*x^3 + 3\*b^2\*c\*d^3\*x^2 + 3\*b^2\*c^2\*d^2\*x + b^2\*c^3\*d)\*log(F)^2 + 15\*(b\*d^2\*x + b\*c\*d)\*log(F))\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a))/(b^4\*d^2\*log(F)^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*6,x)

[Out] Timed out

**Giac [A]** time = 1.27098, size = 178, normalized size = 1.23

$$\frac{\left(4 b^2 d^4 \left(x + \frac{c}{d}\right)^5 \log(F)^2 - 10 b d^2 \left(x + \frac{c}{d}\right)^3 \log(F) + 15 x + \frac{15 c}{d}\right) e^{\left(b d^2 x^2 \log(F) + 2 b c d x \log(F) + b c^2 \log(F) + a \log(F)\right)}}{8 b^3 \log(F)^3} + \frac{15 \sqrt{\pi} F^a \operatorname{erf}\left(\frac{\sqrt{-b \log(F)}(d x + c)}{d}\right)}{16 \sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^6,x, algorithm="giac")

[Out] 1/8\*(4\*b^2\*d^4\*(x + c/d)^5\*log(F)^2 - 10\*b\*d^2\*(x + c/d)^3\*log(F) + 15\*x + 15\*c/d)\*e^(b\*d^2\*x^2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*c^2\*log(F) + a\*log(F))/(b^3\*log(F)^3) + 15/16\*sqrt(pi)\*F^a\*erf(-sqrt(-b\*log(F))\*d\*(x + c/d))/(sqrt(-b\*log(F))\*b^3\*d\*log(F)^3)

### 3.271 $\int F^{a+b(c+dx)^2} (c+dx)^4 dx$

**Optimal.** Leaf size=111

$$\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d \log^2(F)} - \frac{3(c+dx)F^{a+b(c+dx)^2}}{4b^2d \log^2(F)} + \frac{(c+dx)^3 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

[Out]  $(3F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}]) / (8b^{5/2}d \log[F]^{5/2}) - (3F^{a+b(c+dx)^2}(c+dx)) / (4b^2d \log[F]^2) + (F^{a+b(c+dx)^2}(c+dx)^3) / (2bd \log[F])$

**Rubi [A]** time = 0.153824, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2204}

$$\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d \log^2(F)} - \frac{3(c+dx)F^{a+b(c+dx)^2}}{4b^2d \log^2(F)} + \frac{(c+dx)^3 F^{a+b(c+dx)^2}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b(c+dx)^2}(c+dx)^4, x]$

[Out]  $(3F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}]) / (8b^{5/2}d \log[F]^{5/2}) - (3F^{a+b(c+dx)^2}(c+dx)) / (4b^2d \log[F]^2) + (F^{a+b(c+dx)^2}(c+dx)^3) / (2bd \log[F])$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)((c_.) + (d_.)(x_))^{(n_.)})}((c_.) + (d_.)(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m-n+1)}F^{a+b(c+dx)^n} / (b*d*n*\log[F]), x] - \operatorname{Dist}[(m-n+1) / (b*n*\log[F]), \operatorname{Int}[(c+dx)^{(m-n)}F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[0, (m+1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m+1] || LtQ[m, n, 0])

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)((c_.) + (d_.)(x_))^{(2)})}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a \sqrt{\pi} \operatorname{Erfi}[(c+dx)*\operatorname{Rt}[b*\log[F], 2]] / (2*d*\operatorname{Rt}[b*\log[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (c+dx)^4 dx &= \frac{F^{a+b(c+dx)^2} (c+dx)^3}{2bd \log(F)} - \frac{3 \int F^{a+b(c+dx)^2} (c+dx)^2 dx}{2b \log(F)} \\ &= -\frac{3F^{a+b(c+dx)^2} (c+dx)}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^3}{2bd \log(F)} + \frac{3 \int F^{a+b(c+dx)^2} dx}{4b^2 \log^2(F)} \\ &= \frac{3F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{8b^{5/2}d \log^2(F)} - \frac{3F^{a+b(c+dx)^2} (c+dx)}{4b^2d \log^2(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)^3}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0939819, size = 90, normalized size = 0.81

$$\frac{F^a \left( 3\sqrt{\pi} \operatorname{Erfi} \left( \sqrt{b} \sqrt{\log(F)} (c + dx) \right) + 2\sqrt{b} \sqrt{\log(F)} (c + dx) F^{b(c+dx)^2} \left( 2b \log(F) (c + dx)^2 - 3 \right) \right)}{8b^{5/2} d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^4,x]

[Out] (F^a\*(3\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]] + 2\*Sqrt[b]\*F^(b\*(c + d\*x)^2)\*(c + d\*x)\*Sqrt[Log[F]]\*(-3 + 2\*b\*(c + d\*x)^2\*Log[F]))/(8\*b^(5/2)\*d\*Log[F]^(5/2))

**Maple [B]** time = 0.056, size = 300, normalized size = 2.7

$$\frac{d^2 x^3 F^{bd^2 x^2} F^{2bcdx} F^{c^2 b F^a}}{2b \ln(F)} + \frac{3cdx^2 F^{bd^2 x^2} F^{2bcdx} F^{c^2 b F^a}}{2b \ln(F)} + \frac{3c^2 x F^{bd^2 x^2} F^{2bcdx} F^{c^2 b F^a}}{2b \ln(F)} + \frac{c^3 F^{bd^2 x^2} F^{2bcdx} F^{c^2 b F^a}}{2d \ln(F)b} - \frac{3c F^{bd^2 x^2} F^{2bcdx}}{4(\ln(F))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^4,x)

[Out] 1/2\*d^2/ln(F)/b\*x^3\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+3/2\*d\*c/ln(F)/b\*x^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+3/2\*c^2/ln(F)/b\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+1/2/d\*c^3/ln(F)/b\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-3/4/d\*c/ln(F)^2/b^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-3/4/ln(F)^2/b^2\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-3/8/d/ln(F)^2/b^2\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))

**Maxima [B]** time = 1.78637, size = 1463, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="maxima")

[Out] -2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b\*c\*d\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^2/((b\*d^2\*log(F))^(3/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b\*d^2\*log(F)/(b\*d^2\*log(F))^(3/2))\*F^a\*c^3\*d/sqrt(b\*d^2\*log(F)) + 3\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^2\*c^2\*d^2\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^3/((b\*d^2\*log(F))^(5/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 2\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^2\*c\*d^3\*log(F)^2/(b\*d^2\*log(F))^(5/2) - (b\*d^2\*x + b\*c\*d)^3\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^3/((b\*d^2\*log(F))^(5/2))\*(-b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2))\*F^a\*c^2\*d^2/sqrt(b\*d^2\*log(F)) - 2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b^3\*c^3\*d^3\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^4/((b\*d^2\*log(F))^(7/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 3\*F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b^3\*c^2\*d^4\*log(F)^3/(b\*d^2\*log(F))^(7/2) - 3\*(b\*d^2\*x + b\*c\*d)^3\*b\*c\*d\*gamma(3/2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))\*log(F)^4/((b\*d^2\*log(F))^(7/2))\*(-b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2) + b^2\*d^4\*gamma(2, -(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))^(3/2))

$d^2 \log(F)/(b*d^2) * \log(F)^2 / (b*d^2 \log(F))^{(7/2)} * F^a * c*d^3 / \sqrt{b*d^2 \log(F)} + 1/2 * (\sqrt{\pi} * (b*d^2*x + b*c*d) * b^4 * c^4 * d^4 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)})) - 1) * \log(F)^5 / ((b*d^2 \log(F))^{(9/2)} * \sqrt{-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)}) - 4 * F^{((b*d^2*x + b*c*d)^2 / (b*d^2))} * b^4 * c^3 * d^5 * \log(F)^4 / (b*d^2 \log(F))^{(9/2)} - 6 * (b*d^2*x + b*c*d)^3 * b^2 * c^2 * d^2 * \gamma(3/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / ((b*d^2 \log(F))^{(9/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(3/2)}) + 4 * b^3 * c * d^5 * \gamma(2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^3 / (b*d^2 \log(F))^{(9/2)} - (b*d^2*x + b*c*d)^5 * \gamma(5/2, -(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2)) * \log(F)^5 / ((b*d^2 \log(F))^{(9/2)} * (-(b*d^2*x + b*c*d)^2 \log(F)/(b*d^2))^{(5/2)}) * F^a * d^4 / \sqrt{b*d^2 \log(F)} + 1/2 * \sqrt{\pi} * F^{(b*c^2 + a)} * c^4 * \operatorname{erf}(\sqrt{-b \log(F)}) * d * x - b * c * \log(F) / \sqrt{-b \log(F)}) / (\sqrt{-b \log(F)}) * F^{(b*c^2)} * d$

**Fricas [A]** time = 1.53707, size = 331, normalized size = 2.98

$$\frac{3 \sqrt{\pi} \sqrt{-bd^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) - 2 \left(2(b^2 d^4 x^3 + 3 b^2 c d^3 x^2 + 3 b^2 c^2 d^2 x + b^2 c^3 d) \log(F)^2 - 3(bd^2 x + bcd) \log(F)\right)}{8 b^3 d^2 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="fricas")

[Out]  $-1/8 * (3 * \sqrt{\pi} * \sqrt{-b*d^2 \log(F)} * F^a * \operatorname{erf}(\sqrt{-b*d^2 \log(F)} * (d*x + c) / d) - 2 * (2 * (b^2 * d^4 * x^3 + 3 * b^2 * c * d^3 * x^2 + 3 * b^2 * c^2 * d^2 * x + b^2 * c^3 * d) * \log(F)^2 - 3 * (b*d^2*x + b*c*d) * \log(F)) * F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)}) / (b^3 * d^2 * \log(F)^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (c + dx)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*4,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(c + d\*x)\*\*4, x)

**Giac [A]** time = 1.33989, size = 150, normalized size = 1.35

$$\frac{\left(2 b d^2 \left(x + \frac{c}{d}\right)^3 \log(F) - 3 x - \frac{3 c}{d}\right) e^{(b d^2 x^2 \log(F) + 2 b c d x \log(F) + b c^2 \log(F) + a \log(F))}}{4 b^2 \log(F)^2} - \frac{3 \sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right)}{8 \sqrt{-b \log(F)} b^2 d \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="giac")

[Out]  $1/4 * (2 * b * d^2 * (x + c/d)^3 * \log(F) - 3 * x - 3 * c/d) * e^{(b*d^2*x^2 \log(F) + 2*b*c*d*x \log(F) + b*c^2 \log(F) + a \log(F)) / (b^2 \log(F)^2)} - 3/8 * \sqrt{\pi} * F^a * \operatorname{erf}(-\sqrt{-b \log(F)} * d * (x + c/d)) / (\sqrt{-b \log(F)} * b^2 * d * \log(F)^2)$

### 3.272 $\int F^{a+b(c+dx)^2} (c+dx)^2 dx$

**Optimal.** Leaf size=77

$$\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd \log(F)} - \frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d \log^{\frac{3}{2}}(F)}$$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}]) / (4b^{3/2}d \log[F]^{3/2}) + (F^{a+b(c+dx)^2}(c+dx)) / (2b d \log[F])$

**Rubi [A]** time = 0.0817664, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2204}

$$\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd \log(F)} - \frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d \log^{\frac{3}{2}}(F)}$$

Antiderivative was successfully verified.

[In]  $\int F^{a+b(c+dx)^2}(c+dx)^2 dx$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}]) / (4b^{3/2}d \log[F]^{3/2}) + (F^{a+b(c+dx)^2}(c+dx)) / (2b d \log[F])$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_) + (b_)*((c_) + (d_)*(x_))^{(n_)})*((c_) + (d_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m-n+1)}F^{a+b(c+dx)^2} / (b d n \log[F]), x] - \operatorname{Dist}[(m-n+1)/(b n \log[F]), \operatorname{Int}[(c+dx)^{(m-n)}F^{a+b(c+dx)^2}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{IntegerQ}[(2(m+1))/n] \ \&\& \ \operatorname{LtQ}[0, (m+1)/n, 5] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ (\operatorname{LtQ}[0, n, m+1] \ || \ \operatorname{LtQ}[m, n, 0])$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_) + (b_)*((c_) + (d_)*(x_))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a \sqrt{\pi} \operatorname{Erfi}[(c+dx)\operatorname{Rt}[b \log[F], 2]]) / (2d \operatorname{Rt}[b \log[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (c+dx)^2 dx &= \frac{F^{a+b(c+dx)^2} (c+dx)}{2bd \log(F)} - \frac{\int F^{a+b(c+dx)^2} dx}{2b \log(F)} \\ &= -\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d \log^{\frac{3}{2}}(F)} + \frac{F^{a+b(c+dx)^2} (c+dx)}{2bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0431065, size = 77, normalized size = 1.

$$\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd \log(F)} - \frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d \log^{\frac{3}{2}}(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(c + d\*x)^2,x]

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}(c + d*x) \sqrt{\log(F)}]) / (4*b^{3/2}*d*\log(F)^{(3/2)}) + (F^{(a + b*(c + d*x)^2)*(c + d*x)}) / (2*b*d*\log(F))$

**Maple [B]** time = 0.043, size = 131, normalized size = 1.7

$$\frac{x F^{bd^2 x^2} F^{2bcdx} F^{c^2 b} F^a}{2b \ln(F)} + \frac{c F^{bd^2 x^2} F^{2bcdx} F^{c^2 b} F^a}{2d \ln(F) b} + \frac{\sqrt{\pi} F^a}{4d \ln(F) b} \operatorname{Erf}\left(-d\sqrt{-b \ln(F)} x + bc \ln(F) \frac{1}{\sqrt{-b \ln(F)}}\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^2,x)

[Out]  $1/2/\ln(F)/b*x*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^{a+1/2/d*c/\ln(F)/b}*F^{(b*d^2*x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^{a+1/4/d/\ln(F)/b*\pi^{1/2}*F^a/(-b*\ln(F))^{1/2}}*erf(-d*(-b*\ln(F))^{1/2}*x+b*c*\ln(F)/(-b*\ln(F))^{1/2})}$

**Maxima [B]** time = 1.4017, size = 583, normalized size = 7.57

$$\frac{\left( \frac{\sqrt{\pi}(bd^2x+bcd)bcd \left( \operatorname{erf}\left( \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}} \right) - 1 \right) \log(F)^2 - \frac{(bd^2x+bcd)^2}{F} \frac{bd^2 \log(F)}{bd^2}}{(bd^2 \log(F))^{\frac{3}{2}} \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}}} - \frac{F^a cd}{(bd^2 \log(F))^{\frac{3}{2}}} \right)}{\sqrt{bd^2 \log(F)}} + \frac{\left( \frac{\sqrt{\pi}(bd^2x+bcd)b^2 c^2 d^2 \left( \operatorname{erf}\left( \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}} \right) - 1 \right) \log(F)}{(bd^2 \log(F))^{\frac{5}{2}} \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}}} \right)}{\sqrt{bd^2 \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^2,x, algorithm="maxima")

[Out]  $-(\sqrt{\pi}*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*\log(F)/(b*d^2*\log(F))^{3/2}}*F^a*c*d/\sqrt{b*d^2*\log(F)} + 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{5/2}} - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{5/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}))*F^a*d^2/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*c^2*\operatorname{erf}(\sqrt{-b*\log(F)})*d*x - b*c*\log(F)/\sqrt{-b*\log(F)}}/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d}$

**Fricas [A]** time = 1.57817, size = 220, normalized size = 2.86

$$\frac{\sqrt{\pi} \sqrt{-bd^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) + 2(bd^2x + bcd) F^{bd^2x^2 + 2bcdx + bc^2 + a} \log(F)}{4b^2d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^2,x, algorithm="fricas")

[Out] 1/4\*(sqrt(pi)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d) + 2\*(b\*d^2\*x + b\*c\*d)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)\*log(F))/(b^2\*d^2\*log(F)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (c+dx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(d\*x+c)\*\*2,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(c + d\*x)\*\*2, x)

**Giac [A]** time = 1.21137, size = 123, normalized size = 1.6

$$\frac{\left(x + \frac{c}{d}\right) e^{(bd^2x^2 \log(F) + 2bcdx \log(F) + bc^2 \log(F) + a \log(F))}}{2b \log(F)} + \frac{\sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right)}{4 \sqrt{-b \log(F)} b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(d\*x+c)^2,x, algorithm="giac")

[Out] 1/2\*(x + c/d)\*e^(b\*d^2\*x^2\*log(F) + 2\*b\*c\*d\*x\*log(F) + b\*c^2\*log(F) + a\*log(F))/(b\*log(F)) + 1/4\*sqrt(pi)\*F^a\*erf(-sqrt(-b\*log(F))\*d\*(x + c/d))/(sqrt(-b\*log(F))\*b\*d\*log(F))

### 3.273 $\int F^{a+b(c+dx)^2} dx$

**Optimal.** Leaf size=44

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd} \sqrt{\log(F)}}$$

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0106032, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2204}

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2), x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} dx = \frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{bd} \sqrt{\log(F)}}$$

**Mathematica [A]** time = 0.0055194, size = 44, normalized size = 1.

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2), x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

**Maple [A]** time = 0.028, size = 58, normalized size = 1.3

$$-\frac{\sqrt{\pi} F^{c^2 b+a} F^{-c^2 b}}{2d} \operatorname{Erf}\left(-d\sqrt{-b \ln(F)}x + bc \ln(F) \frac{1}{\sqrt{-b \ln(F)}}\right) \frac{1}{\sqrt{-b \ln(F)}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2),x)`

[Out] 
$$-1/2*\text{Pi}^{(1/2)}*F^{(b*c^2+a)}*F^{(-c^2*b)/d}/(-b*\ln(F))^{(1/2)}*\text{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})$$

**Maxima [A]** time = 1.02789, size = 78, normalized size = 1.77

$$\frac{\sqrt{\pi}F^{bc^2+a} \operatorname{erf}\left(\sqrt{-b \log(F)}dx - \frac{bc \log(F)}{\sqrt{-b \log(F)}}\right)}{2 \sqrt{-b \log(F)}F^{bc^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2),x, algorithm="maxima")`

[Out] 
$$1/2*\text{sqrt}(\text{pi})*F^{(b*c^2 + a)}*\text{erf}(\text{sqrt}(-b*\log(F))*d*x - b*c*\log(F)/\text{sqrt}(-b*\log(F)))/(\text{sqrt}(-b*\log(F))*F^{(b*c^2)*d})$$

**Fricas [A]** time = 1.54634, size = 123, normalized size = 2.8

$$\frac{\sqrt{\pi}\sqrt{-bd^2 \log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right)}{2bd^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2),x, algorithm="fricas")`

[Out] 
$$-1/2*\text{sqrt}(\text{pi})*\text{sqrt}(-b*d^2*\log(F))*F^a*\text{erf}(\text{sqrt}(-b*d^2*\log(F))*(d*x + c)/d)/(b*d^2*\log(F))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2),x)`

[Out] `Integral(F**(a + b*(c + d*x)**2), x)`

**Giac [A]** time = 1.22694, size = 49, normalized size = 1.11

$$\frac{\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b \log(F)}d\left(x + \frac{c}{d}\right)\right)}{2 \sqrt{-b \log(F)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*d)
```

$$3.274 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=67

$$\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d} - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}$$

[Out]  $-(F^{(a + b*(c + d*x)^2})/(d*(c + d*x))) + (\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c + d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Sqrt}[\operatorname{Log}[F]])/d$

**Rubi [A]** time = 0.0792093, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2204}

$$\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d} - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*(c + d*x)^2)/(c + d*x)^2}, x]$

[Out]  $-(F^{(a + b*(c + d*x)^2})/(d*(c + d*x))) + (\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c + d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Sqrt}[\operatorname{Log}[F]])/d$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*F^{(a + b*(c + d*x)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c + d*x)^{(m+n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx &= -\frac{F^{a+b(c+dx)^2}}{d(c+dx)} + (2b \log(F)) \int F^{a+b(c+dx)^2} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{d(c+dx)} + \frac{\sqrt{b}F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\sqrt{\log(F)}}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0447884, size = 63, normalized size = 0.94

$$\frac{F^a \left( \sqrt{\pi}\sqrt{b}\sqrt{\log(F)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right) - \frac{F^{b(c+dx)^2}}{c+dx} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^2,x]

[Out] (F^a\*(-(F^(b\*(c + d\*x)^2)/(c + d\*x)) + Sqrt[b]\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*Sqrt[Log[F]]))/d

**Maple [A]** time = 0.049, size = 62, normalized size = 0.9

$$-\frac{F^{b(dx+c)^2}F^a}{(dx+c)d} + \frac{b \ln(F) \sqrt{\pi} F^a}{d} \operatorname{Erf}\left(\sqrt{-b \ln(F)}(dx+c)\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^2,x)

[Out] -1/d/(d\*x+c)\*F^(b\*(d\*x+c)^2)\*F^a+1/d\*b\*ln(F)\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)\*(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^2, x)

**Fricas [A]** time = 1.52966, size = 192, normalized size = 2.87

$$\frac{\sqrt{\pi} \sqrt{-bd^2 \log(F)}(dx+c)F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) + F^{bd^2x^2+2bcdx+bc^2+a}d}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^2,x, algorithm="fricas")

[Out] -(sqrt(pi)\*sqrt(-b\*d^2\*log(F))\*(d\*x + c)\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d) + F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)\*d)/(d^3\*x + c\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**2,x)
```

```
[Out] Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^2*b + a)/(d*x + c)^2, x)
```

$$3.275 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=102

$$\frac{2\sqrt{\pi}b^{3/2}F^a \log^3(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{3d} - \frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} - \frac{2b \log(F)F^{a+b(c+dx)^2}}{3d(c+dx)}$$

[Out]  $-F^{(a + b*(c + d*x)^2)/(3*d*(c + d*x)^3) - (2*b*F^{(a + b*(c + d*x)^2)*\operatorname{Log}[F]})/(3*d*(c + d*x)) + (2*b^{(3/2)*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c + d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Log}[F]^{(3/2)})/(3*d)$

**Rubi [A]** time = 0.145172, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2204}

$$\frac{2\sqrt{\pi}b^{3/2}F^a \log^3(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{3d} - \frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} - \frac{2b \log(F)F^{a+b(c+dx)^2}}{3d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*(c + d*x)^2)/(c + d*x)^4}, x]$

[Out]  $-F^{(a + b*(c + d*x)^2)/(3*d*(c + d*x)^3) - (2*b*F^{(a + b*(c + d*x)^2)*\operatorname{Log}[F]})/(3*d*(c + d*x)) + (2*b^{(3/2)*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c + d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Log}[F]^{(3/2)})/(3*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*F^{(a + b*(c + d*x)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c + d*x)^{(m+n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[(2*(m+1))/n] \ \&\& \ \operatorname{LtQ}[-4, (m+1)/n, 5] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ ((\operatorname{GtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m, -1]) \ || \ (\operatorname{GtQ}[-n, 0] \ \&\& \ \operatorname{LeQ}[-n, m+1]))$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4} dx &= -\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} + \frac{1}{3}(2b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{3d(c+dx)} + \frac{1}{3}(4b^2 \log^2(F)) \int F^{a+b(c+dx)^2} dx \\ &= -\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{3d(c+dx)} + \frac{2b^{3/2}F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right) \log^3(F)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.0972484, size = 81, normalized size = 0.79

$$\frac{F^a \left( 2\sqrt{\pi} b^{3/2} \log^3(F) \operatorname{Erfi} \left( \sqrt{b} \sqrt{\log(F)} (c + dx) \right) - \frac{F^{b(c+dx)^2} (2b \log(F)(c+dx)^2 + 1)}{(c+dx)^3} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^4, x]

[Out] (F^a\*(2\*b^(3/2)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*Log[F]^(3/2) - (F^(b\*(c + d\*x)^2)\*(1 + 2\*b\*(c + d\*x)^2\*Log[F]))/(c + d\*x)^3)/(3\*d)

**Maple [A]** time = 0.049, size = 96, normalized size = 0.9

$$\frac{F^{b(dx+c)^2} F^a}{3d(dx+c)^3} - \frac{2b \ln(F) F^{b(dx+c)^2} F^a}{3(dx+c)d} + \frac{2b^2 (\ln(F))^2 \sqrt{\pi} F^a}{3d} \operatorname{Erf} \left( \sqrt{-b \ln(F)} (dx+c) \right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^4, x)

[Out] -1/3/d/(d\*x+c)^3\*F^(b\*(d\*x+c)^2)\*F^a-2/3/d\*b\*ln(F)/(d\*x+c)\*F^(b\*(d\*x+c)^2)\*F^a+2/3/d\*b^2\*ln(F)^2\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)\*(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^4, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^4, x)

**Fricas [A]** time = 1.56351, size = 373, normalized size = 3.66

$$\frac{2\sqrt{\pi}(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3)\sqrt{-bd^2 \log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) \log(F) + (2(bd^3x^2 + 2bcd^2x + bc^2d))}{3(d^5x^3 + 3cd^4x^2 + 3c^2d^3x + c^3d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^4, x, algorithm="fricas")

[Out] -1/3\*(2\*sqrt(pi)\*(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d)\*log(F) + (2\*(b\*d^3\*x^2 + 2\*b\*c\*d^2\*x + b\*c^2\*d)\*log(F) + d)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)

)/(d<sup>5</sup>\*x<sup>3</sup> + 3\*c\*d<sup>4</sup>\*x<sup>2</sup> + 3\*c<sup>2</sup>\*d<sup>3</sup>\*x + c<sup>3</sup>\*d<sup>2</sup>)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^4,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^4, x)



$$3.276 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6} dx$$

**Optimal.** Leaf size=136

$$\frac{4\sqrt{\pi}b^{5/2}F^a \log^2(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{15d} - \frac{4b^2 \log^2(F)F^{a+b(c+dx)^2}}{15d(c+dx)} - \frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2b \log(F)F^{a+b(c+dx)^2}}{15d(c+dx)^3}$$

[Out]  $-F^{a+b(c+dx)^2}/(5*d*(c+dx)^5) - (2*b*F^{a+b(c+dx)^2}*Log[F])/((15*d*(c+dx)^3) - (4*b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(15*d*(c+dx))) + (4*b^{5/2}*F^a*sqrt[Pi]*Erfi[sqrt[b]*(c+dx)*sqrt[Log[F]]]*Log[F]^{5/2})/(15*d)$

**Rubi [A]** time = 0.218594, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2204}

$$\frac{4\sqrt{\pi}b^{5/2}F^a \log^2(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{15d} - \frac{4b^2 \log^2(F)F^{a+b(c+dx)^2}}{15d(c+dx)} - \frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2b \log(F)F^{a+b(c+dx)^2}}{15d(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^6, x]

[Out]  $-F^{a+b(c+dx)^2}/(5*d*(c+dx)^5) - (2*b*F^{a+b(c+dx)^2}*Log[F])/((15*d*(c+dx)^3) - (4*b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(15*d*(c+dx))) + (4*b^{5/2}*F^a*sqrt[Pi]*Erfi[sqrt[b]*(c+dx)*sqrt[Log[F]]]*Log[F]^{5/2})/(15*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6} dx &= -\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} + \frac{1}{5}(2b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{15d(c+dx)^3} + \frac{1}{15} (4b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{15d(c+dx)^3} - \frac{4b^2F^{a+b(c+dx)^2} \log^2(F)}{15d(c+dx)} + \frac{1}{15} (8b^3 \log^3(F)) \int F^{a+b(c+dx)^2} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{15d(c+dx)^3} - \frac{4b^2F^{a+b(c+dx)^2} \log^2(F)}{15d(c+dx)} + \frac{4b^{5/2}F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{15d}
\end{aligned}$$

**Mathematica [A]** time = 0.119602, size = 97, normalized size = 0.71

$$\frac{F^a \left( 4\sqrt{\pi}b^{5/2} \log^5(F) \operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx)) - \frac{F^{b(c+dx)^2} (4b^2 \log^2(F)(c+dx)^4 + 2b \log(F)(c+dx)^2 + 3)}{(c+dx)^5} \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^6, x]

[Out] (F^a\*(4\*b^(5/2)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*Log[F]^(5/2) - (F^(b\*(c + d\*x)^2)\*(3 + 2\*b\*(c + d\*x)^2\*Log[F] + 4\*b^2\*(c + d\*x)^4\*Log[F]^2))/(c + d\*x)^5))/(15\*d)

**Maple [A]** time = 0.064, size = 129, normalized size = 1.

$$-\frac{F^{b(dx+c)^2} F^a}{5d(dx+c)^5} - \frac{2b \ln(F) F^{b(dx+c)^2} F^a}{15d(dx+c)^3} - \frac{4b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{15(dx+c)d} + \frac{4b^3 (\ln(F))^3 \sqrt{\pi} F^a}{15d} \operatorname{Erf}\left(\sqrt{-b \ln(F)}(dx+c)\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^6, x)

[Out] -1/5/d/(d\*x+c)^5\*F^(b\*(d\*x+c)^2)\*F^a-2/15/d\*b\*ln(F)/(d\*x+c)^3\*F^(b\*(d\*x+c)^2)\*F^a-4/15/d\*b^2\*ln(F)^2/(d\*x+c)\*F^(b\*(d\*x+c)^2)\*F^a+4/15/d\*b^3\*ln(F)^3\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)\*(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^6, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^6, x)

**Fricas [B]** time = 1.5253, size = 621, normalized size = 4.57

$$\frac{4\sqrt{\pi}(b^2d^5x^5 + 5b^2cd^4x^4 + 10b^2c^2d^3x^3 + 10b^2c^3d^2x^2 + 5b^2c^4dx + b^2c^5)\sqrt{-bd^2\log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)\log(F)}{15(d^7x^5 + 5cd^6x^4 + 10c^2d^5x^3 + 10c^3d^4x^2 + 5c^4d^3x + c^5d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^6,x, algorithm="fricas")

[Out] -1/15\*(4\*sqrt(pi)\*(b^2\*d^5\*x^5 + 5\*b^2\*c\*d^4\*x^4 + 10\*b^2\*c^2\*d^3\*x^3 + 10\*b^2\*c^3\*d^2\*x^2 + 5\*b^2\*c^4\*d\*x + b^2\*c^5)\*sqrt(-b\*d^2\*log(F))\*F^a\*erf(sqrt(-b\*d^2\*log(F))\*(d\*x + c)/d)\*log(F)^2 + (4\*(b^2\*d^5\*x^4 + 4\*b^2\*c\*d^4\*x^3 + 6\*b^2\*c^2\*d^3\*x^2 + 4\*b^2\*c^3\*d^2\*x + b^2\*c^4\*d)\*log(F)^2 + 2\*(b\*d^3\*x^2 + 2\*b\*c\*d^2\*x + b\*c^2\*d)\*log(F) + 3\*d)\*F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a))/(d^7\*x^5 + 5\*c\*d^6\*x^4 + 10\*c^2\*d^5\*x^3 + 10\*c^3\*d^4\*x^2 + 5\*c^4\*d^3\*x + c^5\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2b+a}}{(dx+c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^6,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^6, x)

$$3.277 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx$$

**Optimal.** Leaf size=170

$$\frac{8\sqrt{\pi}b^{7/2}F^a \log^{7/2}(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{105d} - \frac{8b^3 \log^3(F)F^{a+b(c+dx)^2}}{105d(c+dx)} - \frac{4b^2 \log^2(F)F^{a+b(c+dx)^2}}{105d(c+dx)^3} - \frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2b \log(F)}{35d(c+dx)}$$

[Out]  $-F^{a+b(c+dx)^2}/(7*d*(c+dx)^7) - (2*b*F^{a+b(c+dx)^2}*Log[F])/ (35*d*(c+dx)^5) - (4*b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(105*d*(c+dx)^3) - (8*b^3*F^{a+b(c+dx)^2}*Log[F]^3)/(105*d*(c+dx)) + (8*b^{7/2}*F^a*sqrt[Pi]*Erfi[Sqrt[b]*(c+dx)*Sqrt[Log[F]]]*Log[F]^{7/2})/(105*d)$

**Rubi [A]** time = 0.285971, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2204}

$$\frac{8\sqrt{\pi}b^{7/2}F^a \log^{7/2}(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{105d} - \frac{8b^3 \log^3(F)F^{a+b(c+dx)^2}}{105d(c+dx)} - \frac{4b^2 \log^2(F)F^{a+b(c+dx)^2}}{105d(c+dx)^3} - \frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2b \log(F)}{35d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^8, x]

[Out]  $-F^{a+b(c+dx)^2}/(7*d*(c+dx)^7) - (2*b*F^{a+b(c+dx)^2}*Log[F])/ (35*d*(c+dx)^5) - (4*b^2*F^{a+b(c+dx)^2}*Log[F]^2)/(105*d*(c+dx)^3) - (8*b^3*F^{a+b(c+dx)^2}*Log[F]^3)/(105*d*(c+dx)) + (8*b^{7/2}*F^a*sqrt[Pi]*Erfi[Sqrt[b]*(c+dx)*Sqrt[Log[F]]]*Log[F]^{7/2})/(105*d)$

#### Rule 2214

Int[(F\_)^(a\_ + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2204

Int[(F\_)^(a\_ + (b\_)\*((c\_) + (d\_)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx &= -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} + \frac{1}{7}(2b \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{35d(c+dx)^5} + \frac{1}{35} (4b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{35d(c+dx)^5} - \frac{4b^2 F^{a+b(c+dx)^2} \log^2(F)}{105d(c+dx)^3} + \frac{1}{105} (8b^3 \log^3(F)) \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{35d(c+dx)^5} - \frac{4b^2 F^{a+b(c+dx)^2} \log^2(F)}{105d(c+dx)^3} - \frac{8b^3 F^{a+b(c+dx)^2} \log^3(F)}{105d(c+dx)} + \frac{1}{105} (16b^4 \log^4(F)) \int \frac{F^{a+b(c+dx)^2}}{c+dx} dx \\
&= -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2bF^{a+b(c+dx)^2} \log(F)}{35d(c+dx)^5} - \frac{4b^2 F^{a+b(c+dx)^2} \log^2(F)}{105d(c+dx)^3} - \frac{8b^3 F^{a+b(c+dx)^2} \log^3(F)}{105d(c+dx)} + \frac{8b^{7/2} F^a \sqrt{\pi} \operatorname{Erfi}(\sqrt{b} \sqrt{\log(F)}(c+dx))}{105d}
\end{aligned}$$

**Mathematica [A]** time = 0.152442, size = 112, normalized size = 0.66

$$\frac{F^a \left( 8\sqrt{\pi} b^{7/2} \log^2(F) \operatorname{Erfi}(\sqrt{b} \sqrt{\log(F)}(c+dx)) + \frac{F^{b(c+dx)^2} (-8b^3 \log^3(F)(c+dx)^6 - 4b^2 \log^2(F)(c+dx)^4 - 6b \log(F)(c+dx)^2 - 15)}{(c+dx)^7} \right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^8, x]

[Out] (F^a\*(8\*b^(7/2)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])\*Log[F]^(7/2) + (F^(b\*(c + d\*x)^2)\*(-15 - 6\*b\*(c + d\*x)^2\*Log[F] - 4\*b^2\*(c + d\*x)^4\*Log[F]^2 - 8\*b^3\*(c + d\*x)^6\*Log[F]^3))/(c + d\*x)^7)/(105\*d)

**Maple [A]** time = 0.09, size = 162, normalized size = 1.

$$\frac{F^{b(dx+c)^2} F^a}{7d(dx+c)^7} - \frac{2b \ln(F) F^{b(dx+c)^2} F^a}{35d(dx+c)^5} - \frac{4b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{105d(dx+c)^3} - \frac{8b^3 (\ln(F))^3 F^{b(dx+c)^2} F^a}{105(dx+c)d} + \frac{8b^4 (\ln(F))^4 \sqrt{\pi} F^a}{105d} \operatorname{Erfi}\left(\sqrt{b} \sqrt{\ln(F)}(c+dx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^8, x)

[Out] -1/7/d/(d\*x+c)^7\*F^(b\*(d\*x+c)^2)\*F^a-2/35/d\*b\*ln(F)/(d\*x+c)^5\*F^(b\*(d\*x+c)^2)\*F^a-4/105/d\*b^2\*ln(F)^2/(d\*x+c)^3\*F^(b\*(d\*x+c)^2)\*F^a-8/105/d\*b^3\*ln(F)^3/(d\*x+c)\*F^(b\*(d\*x+c)^2)\*F^a+8/105/d\*b^4\*ln(F)^4\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)\*(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^8, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^8, x)

**Fricas [B]** time = 1.60913, size = 913, normalized size = 5.37

$$8\sqrt{\pi}(b^3d^7x^7 + 7b^3cd^6x^6 + 21b^3c^2d^5x^5 + 35b^3c^3d^4x^4 + 35b^3c^4d^3x^3 + 21b^3c^5d^2x^2 + 7b^3c^6dx + b^3c^7)\sqrt{-bd^2\log(F)F^a}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^8,x, algorithm="fricas")

[Out] 
$$-1/105*(8*\sqrt{\pi}*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 21*b^3*c^2*d^5*x^5 + 35*b^3*c^3*d^4*x^4 + 35*b^3*c^4*d^3*x^3 + 21*b^3*c^5*d^2*x^2 + 7*b^3*c^6*d*x + b^3*c^7)*\sqrt{-b*d^2*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*d^2*\log(F)}*(d*x + c)/d)*\log(F)^3 + (8*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*\log(F)^3 + 4*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x + b^2*c^4*d)*\log(F)^2 + 6*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*\log(F) + 15*d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*8,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2b+a}}{(dx+c)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^8,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^8, x)

$$3.278 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{9/2} \Gamma\left(-\frac{9}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^9}$$

[Out]  $-(F^a \Gamma[-9/2, -(b*(c+d*x)^2 \text{Log}[F])]) * (-(b*(c+d*x)^2 \text{Log}[F]))^{(9/2)} / (2*d*(c+d*x)^9)$

**Rubi [A]** time = 0.0613516, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{9/2} \Gamma\left(-\frac{9}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^9}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^10, x]

[Out]  $-(F^a \Gamma[-9/2, -(b*(c+d*x)^2 \text{Log}[F])]) * (-(b*(c+d*x)^2 \text{Log}[F]))^{(9/2)} / (2*d*(c+d*x)^9)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx = -\frac{F^a \Gamma\left(-\frac{9}{2}, -b(c+dx)^2 \log(F)\right) \left(-b(c+dx)^2 \log(F)\right)^{9/2}}{2d(c+dx)^9}$$

**Mathematica [A]** time = 0.0299586, size = 49, normalized size = 1.

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{9/2} \Gamma\left(-\frac{9}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^9}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^10, x]

[Out]  $-(F^a \Gamma[-9/2, -(b*(c+d*x)^2 \text{Log}[F])]) * (-(b*(c+d*x)^2 \text{Log}[F]))^{(9/2)} / (2*d*(c+d*x)^9)$

**Maple [A]** time = 0.122, size = 195, normalized size = 4.

$$\frac{F^{b(dx+c)^2} F^a}{9 d (dx+c)^9} - \frac{2 b \ln(F) F^{b(dx+c)^2} F^a}{63 d (dx+c)^7} - \frac{4 b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{315 d (dx+c)^5} - \frac{8 b^3 (\ln(F))^3 F^{b(dx+c)^2} F^a}{945 d (dx+c)^3} - \frac{16 b^4 (\ln(F))^4 F^{b(dx+c)^2} F^a}{945 (dx+c) d} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^10,x)

[Out]  $-1/9/d/(d*x+c)^9 * F^{(b*(d*x+c)^2)} * F^{a-2}/63/d*b*\ln(F)/(d*x+c)^7 * F^{(b*(d*x+c)^2)} * F^{a-4}/315/d*b^2*\ln(F)^2/(d*x+c)^5 * F^{(b*(d*x+c)^2)} * F^{a-8}/945/d*b^3*\ln(F)^3/(d*x+c)^3 * F^{(b*(d*x+c)^2)} * F^{a-16}/945/d*b^4*\ln(F)^4/(d*x+c) * F^{(b*(d*x+c)^2)} * F^{a+16}/945/d*b^5*\ln(F)^5 * \text{Pi}^{(1/2)} * F^a / (-b*\ln(F))^{(1/2)} * \text{erf}((-b*\ln(F))^{(1/2)}) * (d*x+c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^10,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^10, x)

**Fricas [B]** time = 1.77045, size = 1273, normalized size = 25.98

$$16 \sqrt{\pi} (b^4 d^9 x^9 + 9 b^4 c d^8 x^8 + 36 b^4 c^2 d^7 x^7 + 84 b^4 c^3 d^6 x^6 + 126 b^4 c^4 d^5 x^5 + 126 b^4 c^5 d^4 x^4 + 84 b^4 c^6 d^3 x^3 + 36 b^4 c^7 d^2 x^2 + 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^10,x, algorithm="fricas")

[Out]  $-1/945*(16*\text{sqrt}(\text{pi})*(b^4*d^9*x^9 + 9*b^4*c*d^8*x^8 + 36*b^4*c^2*d^7*x^7 + 84*b^4*c^3*d^6*x^6 + 126*b^4*c^4*d^5*x^5 + 126*b^4*c^5*d^4*x^4 + 84*b^4*c^6*d^3*x^3 + 36*b^4*c^7*d^2*x^2 + 9*b^4*c^8*d*x + b^4*c^9)*\text{sqrt}(-b*d^2*\log(F)) * F^a * \text{erf}(\text{sqrt}(-b*d^2*\log(F)) * (d*x + c)/d) * \log(F)^4 + (16*(b^4*d^9*x^8 + 8*b^4*c*d^8*x^7 + 28*b^4*c^2*d^7*x^6 + 56*b^4*c^3*d^6*x^5 + 70*b^4*c^4*d^5*x^4 + 56*b^4*c^5*d^4*x^3 + 28*b^4*c^6*d^3*x^2 + 8*b^4*c^7*d^2*x + b^4*c^8*d) * \log(F)^4 + 8*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d) * \log(F)^3 + 12*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x + b^2*c^4*d) * \log(F)^2 + 30*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d) * \log(F) + 105*d) * F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)} / (d^{11}*x^9 + 9*c*d^{10}*x^8 + 36*c^2*d^9*x^7 + 84*c^3*d^8*x^6 + 126*c^4*d^7*x^5 + 126*c^5*d^6*x^4 + 84*c^6*d^5*x^3 + 36*c^7*d^4*x^2 + 9*c^8*d^3*x + c^9*d^2)$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^10,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^10, x)

$$3.279 \quad \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{11/2} \Gamma\left(-\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^{11}}$$

[Out]  $-(F^a \Gamma[-11/2, -(b*(c+d*x)^2 * \text{Log}[F])]) * (-(b*(c+d*x)^2 * \text{Log}[F]))^{(11/2)} / (2*d*(c+d*x)^{11})$

**Rubi [A]** time = 0.0619664, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{11/2} \Gamma\left(-\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^{11}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^12,x]

[Out]  $-(F^a \Gamma[-11/2, -(b*(c+d*x)^2 * \text{Log}[F])]) * (-(b*(c+d*x)^2 * \text{Log}[F]))^{(11/2)} / (2*d*(c+d*x)^{11})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n \* Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n \* Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx = -\frac{F^a \Gamma\left(-\frac{11}{2}, -b(c+dx)^2 \log(F)\right) \left(-b(c+dx)^2 \log(F)\right)^{11/2}}{2d(c+dx)^{11}}$$

**Mathematica [A]** time = 0.0299479, size = 49, normalized size = 1.

$$\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{11/2} \Gamma\left(-\frac{11}{2}, -b \log(F)(c+dx)^2\right)}{2d(c+dx)^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(c + d\*x)^12,x]

[Out]  $-(F^a \Gamma[-11/2, -(b*(c+d*x)^2 * \text{Log}[F])]) * (-(b*(c+d*x)^2 * \text{Log}[F]))^{(11/2)} / (2*d*(c+d*x)^{11})$

**Maple [A]** time = 0.182, size = 228, normalized size = 4.7

$$\frac{F^{b(dx+c)^2} F^a}{11 d (dx + c)^{11}} - \frac{2 b \ln(F) F^{b(dx+c)^2} F^a}{99 d (dx + c)^9} - \frac{4 b^2 (\ln(F))^2 F^{b(dx+c)^2} F^a}{693 d (dx + c)^7} - \frac{8 b^3 (\ln(F))^3 F^{b(dx+c)^2} F^a}{3465 d (dx + c)^5} - \frac{16 b^4 (\ln(F))^4 F^{b(dx+c)^2} F^a}{10395 d (dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^12,x)

[Out]  $-1/11/d/(d*x+c)^{11}*F^{(b*(d*x+c)^2)*F^{a-2}/99/d*b*\ln(F)/(d*x+c)^9*F^{(b*(d*x+c)^2)*F^{a-4}/693/d*b^2*\ln(F)^2/(d*x+c)^7*F^{(b*(d*x+c)^2)*F^{a-8}/3465/d*b^3*\ln(F)^3/(d*x+c)^5*F^{(b*(d*x+c)^2)*F^{a-16}/10395/d*b^4*\ln(F)^4/(d*x+c)^3*F^{(b*(d*x+c)^2)*F^{a-32}/10395/d*b^5*\ln(F)^5/(d*x+c)*F^{(b*(d*x+c)^2)*F^{a+32}/10395/d*b^6*\ln(F)^6*\pi^{(1/2)*F^a/(-b*\ln(F))^{(1/2)}*erf((-b*\ln(F))^{(1/2)}*(d*x+c))}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^12,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^12, x)

**Fricas [B]** time = 2.05855, size = 1729, normalized size = 35.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^12,x, algorithm="fricas")

[Out]  $-1/10395*(32*\sqrt{\pi}*(b^5*d^{11}*x^{11} + 11*b^5*c*d^{10}*x^{10} + 55*b^5*c^2*d^9*x^9 + 165*b^5*c^3*d^8*x^8 + 330*b^5*c^4*d^7*x^7 + 462*b^5*c^5*d^6*x^6 + 462*b^5*c^6*d^5*x^5 + 330*b^5*c^7*d^4*x^4 + 165*b^5*c^8*d^3*x^3 + 55*b^5*c^9*d^2*x^2 + 11*b^5*c^{10}*d*x + b^5*c^{11})*\sqrt{-b*d^2*\log(F)}*F^a*erf(\sqrt{-b*d^2*\log(F)}*(d*x + c)/d)*\log(F)^5 + (32*(b^5*d^{11}*x^{10} + 10*b^5*c*d^{10}*x^9 + 45*b^5*c^2*d^9*x^8 + 120*b^5*c^3*d^8*x^7 + 210*b^5*c^4*d^7*x^6 + 252*b^5*c^5*d^6*x^5 + 210*b^5*c^6*d^5*x^4 + 120*b^5*c^7*d^4*x^3 + 45*b^5*c^8*d^3*x^2 + 10*b^5*c^9*d^2*x + b^5*c^{10}*d)*\log(F)^5 + 16*(b^4*d^9*x^8 + 8*b^4*c*d^8*x^7 + 28*b^4*c^2*d^7*x^6 + 56*b^4*c^3*d^6*x^5 + 70*b^4*c^4*d^5*x^4 + 56*b^4*c^5*d^4*x^3 + 28*b^4*c^6*d^3*x^2 + 8*b^4*c^7*d^2*x + b^4*c^8*d)*\log(F)^4 + 24*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*\log(F)^3 + 60*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x + b^2*c^4*d)*\log(F)^2 + 210*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*\log(F) + 945*d)*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)}/(d^{13}*x^{11} + 11*c*d^{12}*x^{10} + 55*c^2*d^{11}*x^9 + 165*c^3*d^{10}*x^8 + 330*c^4*d^9*x^7 + 462*c^5*d^8*x^6 + 462*c^6*d^7*x^5 + 330*c^7*d^6*x^4 + 165*c^8*d^5*x^3 + 55*c^9*d^4*x^2 + 11*c^{10}*d^3*x + c^{11}*d^2)$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(d\*x+c)\*\*12,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(dx+c)^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(d\*x+c)^12,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(d\*x + c)^12, x)

### 3.280 $\int F^{a+b(c+dx)^3} (c+dx)^m dx$

**Optimal.** Leaf size=61

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^3)^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -b \log(F)(c+dx)^3\right)}{3d}$$

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/3, -(b(c+dx)^3 \text{Log}[F])]) * (-(b(c+dx)^3 \text{Log}[F]))^{((-1-m)/3)} / (3*d)$

**Rubi [A]** time = 0.061417, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^3)^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -b \log(F)(c+dx)^3\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^m,x]

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/3, -(b(c+dx)^3 \text{Log}[F])]) * (-(b(c+dx)^3 \text{Log}[F]))^{((-1-m)/3)} / (3*d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^3} (c+dx)^m dx = -\frac{F^a(c+dx)^{1+m} \Gamma\left(\frac{1+m}{3}, -b(c+dx)^3 \log(F)\right) (-b(c+dx)^3 \log(F))^{\frac{1}{3}(-1-m)}}{3d}$$

**Mathematica [A]** time = 0.0401902, size = 61, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1} (-b \log(F)(c+dx)^3)^{\frac{1}{3}(-m-1)} \text{Gamma}\left(\frac{m+1}{3}, -b \log(F)(c+dx)^3\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^m,x]

[Out]  $-(F^a(c+dx)^{(1+m)} \text{Gamma}[(1+m)/3, -(b(c+dx)^3 \text{Log}[F])]) * (-(b(c+dx)^3 \text{Log}[F]))^{((-1-m)/3)} / (3*d)$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^3} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^m,x)

[Out] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^m,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^((d\*x + c)^3 \* b + a), x)

**Fricas [A]** time = 1.90598, size = 188, normalized size = 3.08

$$\frac{e^{\left(-\frac{1}{3}(m-2)\log(-b\log(F))+a\log(F)\right)} \Gamma\left(\frac{1}{3}m + \frac{1}{3}, -(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3)\log(F)\right)}{3bd\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] 1/3\*e^(-1/3\*(m - 2)\*log(-b\*log(F)) + a\*log(F))\*gamma(1/3\*m + 1/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))/(b\*d\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*F^((d*x + c)^3*b + a), x)
```

$$3.281 \quad \int F^{a+b(c+dx)^3} (c+dx)^{17} dx$$

**Optimal.** Leaf size=105

$$\frac{F^{a+b(c+dx)^3} (-b^5 \log^5(F)(c+dx)^{15} + 5b^4 \log^4(F)(c+dx)^{12} - 20b^3 \log^3(F)(c+dx)^9 + 60b^2 \log^2(F)(c+dx)^6 - 120b \log(F)(c+dx)^3 + 120) }{3b^6 d \log^6(F)}$$

[Out]  $-(F^{a+b(c+dx)^3} (120 - 120*b*(c+dx)^3*\text{Log}[F] + 60*b^2*(c+dx)^6*\text{Log}[F]^2 - 20*b^3*(c+dx)^9*\text{Log}[F]^3 + 5*b^4*(c+dx)^{12}*\text{Log}[F]^4 - b^5*(c+dx)^{15}*\text{Log}[F]^5)) / (3*b^6*d*\text{Log}[F]^6)$

**Rubi [C]** time = 0.0669175, antiderivative size = 31, normalized size of antiderivative = 0.3, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^3)}{3b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^17, x]

[Out]  $-(F^a*\text{Gamma}[6, -(b*(c + d*x)^3*\text{Log}[F])]) / (3*b^6*d*\text{Log}[F]^6)$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+b(c+dx)^3} (c+dx)^{17} dx = -\frac{F^a \Gamma(6, -b(c+dx)^3 \log(F))}{3b^6 d \log^6(F)}$$

**Mathematica [C]** time = 0.0101806, size = 31, normalized size = 0.3

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^3)}{3b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^17, x]

[Out]  $-(F^a*\text{Gamma}[6, -(b*(c + d*x)^3*\text{Log}[F])]) / (3*b^6*d*\text{Log}[F]^6)$

**Maple [B]** time = 0.023, size = 857, normalized size = 8.2

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^17,x)

[Out]  $\frac{1}{3}(-120+120*\ln(F)*b*c^3+360*\ln(F)*b*c*d^2*x^2+360*\ln(F)*b*c^2*d*x+120*\ln(F)*b*d^3*x^3-3960*\ln(F)^4*b^4*c^7*d^5*x^5-2475*\ln(F)^4*b^4*c^8*d^4*x^4-1100*\ln(F)^4*b^4*c^9*d^3*x^3+180*c*d^8*x^8*\ln(F)^3*b^3-330*\ln(F)^4*b^4*c^10*d^2*x^2+720*c^2*d^7*x^7*\ln(F)^3*b^3-60*\ln(F)^4*b^4*c^11*d*x+1680*\ln(F)^3*b^3*c^3*d^6*x^6+2520*\ln(F)^3*b^3*c^4*d^5*x^5+2520*\ln(F)^3*b^3*c^5*d^4*x^4+1680*\ln(F)^3*b^3*c^6*d^3*x^3+720*\ln(F)^3*b^3*c^7*d^2*x^2+180*\ln(F)^3*b^3*c^8*d*x-360*c*d^5*x^5*\ln(F)^2*b^2-900*c^2*d^4*x^4*\ln(F)^2*b^2-1200*\ln(F)^2*b^2*c^3*d^3*x^3-900*\ln(F)^2*b^2*c^4*d^2*x^2-360*\ln(F)^2*b^2*c^5*d*x+15*d^14*c*x^14*\ln(F)^5*b^5+105*d^13*c^2*x^13*\ln(F)^5*b^5+455*\ln(F)^5*b^5*c^3*d^12*x^12+1365*\ln(F)^5*b^5*c^4*d^11*x^11+3003*\ln(F)^5*b^5*c^5*d^10*x^10+5005*\ln(F)^5*b^5*c^6*d^9*x^9+6435*\ln(F)^5*b^5*c^7*d^8*x^8+6435*\ln(F)^5*b^5*c^8*d^7*x^7+5005*\ln(F)^5*b^5*c^9*d^6*x^6-60*c*d^11*x^11*\ln(F)^4*b^4+3003*\ln(F)^5*b^5*c^10*d^5*x^5-330*c^2*d^10*x^10*\ln(F)^4*b^4+1365*\ln(F)^5*b^5*c^11*d^4*x^4-1100*\ln(F)^4*b^4*c^3*d^9*x^9+455*\ln(F)^5*b^5*c^12*d^3*x^3-2475*\ln(F)^4*b^4*c^4*d^8*x^8+105*\ln(F)^5*b^5*c^13*d^2*x^2-3960*\ln(F)^4*b^4*c^5*d^7*x^7+15*\ln(F)^5*b^5*c^14*d*x-4620*\ln(F)^4*b^4*c^6*d^6*x^6+d^15*x^15*\ln(F)^5*b^5-5*d^12*x^12*\ln(F)^4*b^4+20*d^9*x^9*\ln(F)^3*b^3-60*d^6*x^6*\ln(F)^2*b^2+\ln(F)^5*b^5*c^15-5*\ln(F)^4*b^4*c^12+20*\ln(F)^3*b^3*c^9-60*\ln(F)^2*b^2*c^6)*F^(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/d/\ln(F)^6/b^6$

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**Maxima [B]** time = 1.67115, size = 1712, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^17,x, algorithm="maxima")

[Out]  $\frac{1}{3}(F^(b*c^3 + a)*b^5*d^15*x^15*\log(F)^5 + 15*F^(b*c^3 + a)*b^5*c*d^14*x^14*\log(F)^5 + 105*F^(b*c^3 + a)*b^5*c^2*d^13*x^13*\log(F)^5 + F^(b*c^3 + a)*b^5*c^15*\log(F)^5 - 5*F^(b*c^3 + a)*b^4*c^12*\log(F)^4 + 20*F^(b*c^3 + a)*b^3*c^9*\log(F)^3 + 5*(91*F^(b*c^3 + a)*b^5*c^3*d^12*\log(F)^5 - F^(b*c^3 + a)*b^4*d^12*\log(F)^4)*x^12 + 15*(91*F^(b*c^3 + a)*b^5*c^4*d^11*\log(F)^5 - 4*F^(b*c^3 + a)*b^4*c*d^11*\log(F)^4)*x^11 + 33*(91*F^(b*c^3 + a)*b^5*c^5*d^10*\log(F)^5 - 10*F^(b*c^3 + a)*b^4*c^2*d^10*\log(F)^4)*x^10 - 60*F^(b*c^3 + a)*b^2*c^6*\log(F)^2 + 5*(1001*F^(b*c^3 + a)*b^5*c^6*d^9*\log(F)^5 - 220*F^(b*c^3 + a)*b^4*c^3*d^9*\log(F)^4 + 4*F^(b*c^3 + a)*b^3*d^9*\log(F)^3)*x^9 + 45*(143*F^(b*c^3 + a)*b^5*c^7*d^8*\log(F)^5 - 55*F^(b*c^3 + a)*b^4*c^4*d^8*\log(F)^4 + 4*F^(b*c^3 + a)*b^3*c*d^8*\log(F)^3)*x^8 + 45*(143*F^(b*c^3 + a)*b^5*c^8*d^7*\log(F)^5 - 88*F^(b*c^3 + a)*b^4*c^5*d^7*\log(F)^4 + 16*F^(b*c^3 + a)*b^3*c^2*d^7*\log(F)^3)*x^7 + 5*(1001*F^(b*c^3 + a)*b^5*c^9*d^6*\log(F)^5 - 924*F^(b*c^3 + a)*b^4*c^6*d^6*\log(F)^4 + 336*F^(b*c^3 + a)*b^3*c^3*d^6*\log(F)^3 - 12*F^(b*c^3 + a)*b^2*d^6*\log(F)^2)*x^6 + 3*(1001*F^(b*c^3 + a)*b^5*c^10*d^5*\log(F)^5 - 1320*F^(b*c^3 + a)*b^4*c^7*d^5*\log(F)^4 + 840*F^(b*c^3 + a)*b^3*c^4*d^5*\log(F)^3 - 120*F^(b*c^3 + a)*b^2*c*d^5*\log(F)^2)*x^5 + 120*F^(b*c^3 + a)*b*c^3*\log(F) + 15*(91*F^(b*c^3 + a)*b^5*c^11*d^4*\log(F)^5 - 165*F^(b*c^3 + a)*b^4*c^8*d^4*\log(F)^4 + 168*F^(b*c^3 + a)*b^3*c^5*d^4*\log(F)^3 - 60*F^(b*c^3 + a)*b^2*c^2*d^4*\log(F)^2)*x^4 + 5*(91*F^(b*c^3 + a)*b^5*c^12*d^3*\log(F)^5 - 220*F^(b*c^3 + a)*b^4*c^9*d^3*\log(F)^4 + 336*F^(b*c^3 + a)*b^3*c^6*d^3*\log(F)^3 - 240*F^(b*c^3 + a)*b^2*c^3*d^3*\log(F)^2 + 24*F^(b*c^3 + a)*b*d^3*\log(F))*x^3 + 15*(7*F^(b*c^3 + a)*b^5*c^13*d^2*\log(F)^5 - 22*F^(b*c^3 + a)*b^4*c^10*d^2*\log(F)^4 + 48*F^(b*c^3 + a)*b^3*c^7*d^2*\log(F)^3 - 60*F^(b*c^3 + a)*b^2*c^4*d^2*\log(F)^2 + 24*F^(b*c^3 + a)*b*c*d^2*\log(F))*x^2$

$$2 + 15*(F^{(b*c^3 + a)*b^5*c^{14}*d*\log(F)^5} - 4*F^{(b*c^3 + a)*b^4*c^{11}*d*\log(F)^4} + 12*F^{(b*c^3 + a)*b^3*c^8*d*\log(F)^3} - 24*F^{(b*c^3 + a)*b^2*c^5*d*\log(F)^2} + 24*F^{(b*c^3 + a)*b*c^2*d*\log(F)}*x - 120*F^{(b*c^3 + a)}*e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F))}/(b^6*d*\log(F)^6)$$

**Fricas [B]** time = 1.89123, size = 1513, normalized size = 14.41

$$\left( (b^5 d^{15} x^{15} + 15 b^5 c d^{14} x^{14} + 105 b^5 c^2 d^{13} x^{13} + 455 b^5 c^3 d^{12} x^{12} + 1365 b^5 c^4 d^{11} x^{11} + 3003 b^5 c^5 d^{10} x^{10} + 5005 b^5 c^6 d^9 x^9 + 6435 b^5 c^7 d^8 x^8 + 6435 b^5 c^8 d^7 x^7 + 5005 b^5 c^9 d^6 x^6 + 3003 b^5 c^{10} d^5 x^5 + 1365 b^5 c^{11} d^4 x^4 + 455 b^5 c^{12} d^3 x^3 + 105 b^5 c^{13} d^2 x^2 + 15 b^5 c^{14} d x + b^5 c^{15}) \log(F)^5 - 5 (b^4 d^{12} x^{12} + 12 b^4 c d^{11} x^{11} + 66 b^4 c^2 d^{10} x^{10} + 220 b^4 c^3 d^9 x^9 + 495 b^4 c^4 d^8 x^8 + 792 b^4 c^5 d^7 x^7 + 924 b^4 c^6 d^6 x^6 + 792 b^4 c^7 d^5 x^5 + 495 b^4 c^8 d^4 x^4 + 220 b^4 c^9 d^3 x^3 + 66 b^4 c^{10} d^2 x^2 + 12 b^4 c^{11} d x + b^4 c^{12}) \log(F)^4 + 20 (b^3 d^9 x^9 + 9 b^3 c d^8 x^8 + 36 b^3 c^2 d^7 x^7 + 84 b^3 c^3 d^6 x^6 + 126 b^3 c^4 d^5 x^5 + 126 b^3 c^5 d^4 x^4 + 84 b^3 c^6 d^3 x^3 + 36 b^3 c^7 d^2 x^2 + 9 b^3 c^8 d x + b^3 c^9) \log(F)^3 - 60 (b^2 d^6 x^6 + 6 b^2 c d^5 x^5 + 15 b^2 c^2 d^4 x^4 + 20 b^2 c^3 d^3 x^3 + 15 b^2 c^4 d^2 x^2 + 6 b^2 c^5 d x + b^2 c^6) \log(F)^2 + 120 (b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F) - 120 \right) * F^{(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)}/(b^6*d*\log(F)^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^17,x, algorithm="fricas")

[Out] 1/3\*((b^5\*d^15\*x^15 + 15\*b^5\*c\*d^14\*x^14 + 105\*b^5\*c^2\*d^13\*x^13 + 455\*b^5\*c^3\*d^12\*x^12 + 1365\*b^5\*c^4\*d^11\*x^11 + 3003\*b^5\*c^5\*d^10\*x^10 + 5005\*b^5\*c^6\*d^9\*x^9 + 6435\*b^5\*c^7\*d^8\*x^8 + 6435\*b^5\*c^8\*d^7\*x^7 + 5005\*b^5\*c^9\*d^6\*x^6 + 3003\*b^5\*c^10\*d^5\*x^5 + 1365\*b^5\*c^11\*d^4\*x^4 + 455\*b^5\*c^12\*d^3\*x^3 + 105\*b^5\*c^13\*d^2\*x^2 + 15\*b^5\*c^14\*d\*x + b^5\*c^15)\*log(F)^5 - 5\*(b^4\*d^12\*x^12 + 12\*b^4\*c\*d^11\*x^11 + 66\*b^4\*c^2\*d^10\*x^10 + 220\*b^4\*c^3\*d^9\*x^9 + 495\*b^4\*c^4\*d^8\*x^8 + 792\*b^4\*c^5\*d^7\*x^7 + 924\*b^4\*c^6\*d^6\*x^6 + 792\*b^4\*c^7\*d^5\*x^5 + 495\*b^4\*c^8\*d^4\*x^4 + 220\*b^4\*c^9\*d^3\*x^3 + 66\*b^4\*c^10\*d^2\*x^2 + 12\*b^4\*c^11\*d\*x + b^4\*c^12)\*log(F)^4 + 20\*(b^3\*d^9\*x^9 + 9\*b^3\*c\*d^8\*x^8 + 36\*b^3\*c^2\*d^7\*x^7 + 84\*b^3\*c^3\*d^6\*x^6 + 126\*b^3\*c^4\*d^5\*x^5 + 126\*b^3\*c^5\*d^4\*x^4 + 84\*b^3\*c^6\*d^3\*x^3 + 36\*b^3\*c^7\*d^2\*x^2 + 9\*b^3\*c^8\*d\*x + b^3\*c^9)\*log(F)^3 - 60\*(b^2\*d^6\*x^6 + 6\*b^2\*c\*d^5\*x^5 + 15\*b^2\*c^2\*d^4\*x^4 + 20\*b^2\*c^3\*d^3\*x^3 + 15\*b^2\*c^4\*d^2\*x^2 + 6\*b^2\*c^5\*d\*x + b^2\*c^6)\*log(F)^2 + 120\*(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F) - 120)\*F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a)/(b^6\*d\*log(F)^6)

**Sympy [A]** time = 0.549061, size = 1171, normalized size = 11.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c)\*\*17,x)

[Out] Piecewise(((F\*\*(a + b\*(c + d\*x)\*\*3)\*(b\*\*5\*c\*\*15\*log(F)\*\*5 + 15\*b\*\*5\*c\*\*14\*d\*x\*log(F)\*\*5 + 105\*b\*\*5\*c\*\*13\*d\*\*2\*x\*\*2\*log(F)\*\*5 + 455\*b\*\*5\*c\*\*12\*d\*\*3\*x\*\*3\*log(F)\*\*5 + 1365\*b\*\*5\*c\*\*11\*d\*\*4\*x\*\*4\*log(F)\*\*5 + 3003\*b\*\*5\*c\*\*10\*d\*\*5\*x\*\*5\*log(F)\*\*5 + 5005\*b\*\*5\*c\*\*9\*d\*\*6\*x\*\*6\*log(F)\*\*5 + 6435\*b\*\*5\*c\*\*8\*d\*\*7\*x\*\*7\*log(F)\*\*5 + 6435\*b\*\*5\*c\*\*7\*d\*\*8\*x\*\*8\*log(F)\*\*5 + 5005\*b\*\*5\*c\*\*6\*d\*\*9\*x\*\*9\*log(F)\*\*5 + 3003\*b\*\*5\*c\*\*5\*d\*\*10\*x\*\*10\*log(F)\*\*5 + 1365\*b\*\*5\*c\*\*4\*d\*\*11\*x\*\*11\*log(F)\*\*5 + 455\*b\*\*5\*c\*\*3\*d\*\*12\*x\*\*12\*log(F)\*\*5 + 105\*b\*\*5\*c\*\*2\*d\*\*13\*x\*\*13\*log(F)\*\*5 + 15\*b\*\*5\*c\*d\*\*14\*x\*\*14\*log(F)\*\*5 + b\*\*5\*d\*\*15\*x\*\*15\*log(F)\*\*5 - 5\*b\*\*4\*c\*\*12\*log(F)\*\*4 - 60\*b\*\*4\*c\*\*11\*d\*x\*log(F)\*\*4 - 330\*b\*\*4\*c\*\*10\*d\*\*2\*x\*\*2\*log(F)\*\*4 - 1100\*b\*\*4\*c\*\*9\*d\*\*3\*x\*\*3\*log(F)\*\*4 - 2475\*b\*\*4\*c\*\*8\*d\*\*4\*x\*\*4\*log(F)\*\*4 - 3960\*b\*\*4\*c\*\*7\*d\*\*5\*x\*\*5\*log(F)\*\*4 - 4620\*b\*\*4\*c\*\*6\*d\*\*6\*x\*\*6\*log(F)\*\*4 - 3960\*b\*\*4\*c\*\*5\*d\*\*7\*x\*\*7\*log(F)\*\*4 - 2475\*b\*\*4\*c\*\*4\*d\*\*8\*x\*\*8\*log(F)\*\*4 - 1100\*b\*\*4\*c\*\*3\*d\*\*9\*x\*\*9\*log(F)\*\*4 - 330\*b\*\*4\*c\*\*2\*d\*\*10\*x\*\*10\*log(F)\*\*4 - 60\*b\*\*4\*c\*d\*\*11\*x\*\*11\*log(F)\*\*4 - 5\*b\*\*4\*d\*\*12\*x\*\*12\*log(F)\*\*4 + 20\*b\*\*3\*c\*\*9\*log(F)\*\*3 + 180\*b\*\*3\*c\*\*8\*d\*x\*log(F)\*\*3 + 720\*b\*\*3\*c\*\*7\*d\*\*2\*x\*\*2\*log(F)\*\*3 + 1680\*b\*\*3\*c\*\*6\*d\*\*3\*x\*\*3\*log(F)\*\*3 + 2520\*b\*\*3\*c\*\*5

```

*d**4*x**4*log(F)**3 + 2520*b**3*c**4*d**5*x**5*log(F)**3 + 1680*b**3*c**3*
d**6*x**6*log(F)**3 + 720*b**3*c**2*d**7*x**7*log(F)**3 + 180*b**3*c*d**8*x
**8*log(F)**3 + 20*b**3*d**9*x**9*log(F)**3 - 60*b**2*c**6*log(F)**2 - 360*
b**2*c**5*d*x*log(F)**2 - 900*b**2*c**4*d**2*x**2*log(F)**2 - 1200*b**2*c**
3*d**3*x**3*log(F)**2 - 900*b**2*c**2*d**4*x**4*log(F)**2 - 360*b**2*c*d**5
*x**5*log(F)**2 - 60*b**2*d**6*x**6*log(F)**2 + 120*b*c**3*log(F) + 360*b*c
**2*d*x*log(F) + 360*b*c*d**2*x**2*log(F) + 120*b*d**3*x**3*log(F) - 120)/(
3*b**6*d*log(F)**6), Ne(3*b**6*d*log(F)**6, 0)), (c**17*x + 17*c**16*d*x**2
/2 + 136*c**15*d**2*x**3/3 + 170*c**14*d**3*x**4 + 476*c**13*d**4*x**5 + 30
94*c**12*d**5*x**6/3 + 1768*c**11*d**6*x**7 + 2431*c**10*d**7*x**8 + 24310*
c**9*d**8*x**9/9 + 2431*c**8*d**9*x**10 + 1768*c**7*d**10*x**11 + 3094*c**6
*d**11*x**12/3 + 476*c**5*d**12*x**13 + 170*c**4*d**13*x**14 + 136*c**3*d**
14*x**15/3 + 17*c**2*d**15*x**16/2 + c*d**16*x**17 + d**17*x**18/18, True))

```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.282 \quad \int F^{a+b(c+dx)^3} (c+dx)^{14} dx$$

**Optimal.** Leaf size=88

$$\frac{F^{a+b(c+dx)^3} (b^4 \log^4(F)(c+dx)^{12} - 4b^3 \log^3(F)(c+dx)^9 + 12b^2 \log^2(F)(c+dx)^6 - 24b \log(F)(c+dx)^3 + 24)}{3b^5 d \log^5(F)}$$

[Out] (F^(a + b\*(c + d\*x)^3)\*(24 - 24\*b\*(c + d\*x)^3\*Log[F] + 12\*b^2\*(c + d\*x)^6\*Log[F]^2 - 4\*b^3\*(c + d\*x)^9\*Log[F]^3 + b^4\*(c + d\*x)^12\*Log[F]^4))/(3\*b^5\*d\*Log[F]^5)

**Rubi [C]** time = 0.0670609, antiderivative size = 31, normalized size of antiderivative = 0.35, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^3)}{3b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^14,x]

[Out] (F^a\*Gamma[5, -(b\*(c + d\*x)^3\*Log[F])])/(3\*b^5\*d\*Log[F]^5)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^3} (c+dx)^{14} dx = \frac{F^a \Gamma(5, -b(c+dx)^3 \log(F))}{3b^5 d \log^5(F)}$$

**Mathematica [C]** time = 0.0089296, size = 31, normalized size = 0.35

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^3)}{3b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^14,x]

[Out] (F^a\*Gamma[5, -(b\*(c + d\*x)^3\*Log[F])])/(3\*b^5\*d\*Log[F]^5)

**Maple [B]** time = 0.019, size = 584, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^14,x)

[Out]  $\frac{1}{3}*(24-24*\ln(F)*b*c^3-72*\ln(F)*b*c*d^2*x^2-72*\ln(F)*b*c^2*d*x-24*\ln(F)*b*d^3*x^3+792*\ln(F)^4*b^4*c^7*d^5*x^5+495*\ln(F)^4*b^4*c^8*d^4*x^4+220*\ln(F)^4*b^4*c^9*d^3*x^3-36*c*d^8*x^8*\ln(F)^3*b^3+66*\ln(F)^4*b^4*c^10*d^2*x^2-144*c^2*d^7*x^7*\ln(F)^3*b^3+12*\ln(F)^4*b^4*c^11*d*x-336*\ln(F)^3*b^3*c^3*d^6*x^6-504*\ln(F)^3*b^3*c^4*d^5*x^5-504*\ln(F)^3*b^3*c^5*d^4*x^4-336*\ln(F)^3*b^3*c^6*d^3*x^3-144*\ln(F)^3*b^3*c^7*d^2*x^2-36*\ln(F)^3*b^3*c^8*d*x+72*c*d^5*x^5*\ln(F)^2*b^2+180*c^2*d^4*x^4*\ln(F)^2*b^2+240*\ln(F)^2*b^2*c^3*d^3*x^3+180*\ln(F)^2*b^2*c^4*d^2*x^2+72*\ln(F)^2*b^2*c^5*d*x+12*c*d^11*x^11*\ln(F)^4*b^4+66*c^2*d^10*x^10*\ln(F)^4*b^4+220*\ln(F)^4*b^4*c^3*d^9*x^9+495*\ln(F)^4*b^4*c^4*d^8*x^8+792*\ln(F)^4*b^4*c^5*d^7*x^7+924*\ln(F)^4*b^4*c^6*d^6*x^6+d^12*x^12*\ln(F)^4*b^4-4*d^9*x^9*\ln(F)^3*b^3+12*d^6*x^6*\ln(F)^2*b^2+\ln(F)^4*b^4*c^12-4*\ln(F)^3*b^3*c^9+12*\ln(F)^2*b^2*c^6)*F^(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/d/\ln(F)^5/b^5$

**Maxima [B]** time = 1.71363, size = 1180, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^14,x, algorithm="maxima")

[Out]  $\frac{1}{3}*(F^(b*c^3 + a)*b^4*d^12*x^12*\log(F)^4 + 12*F^(b*c^3 + a)*b^4*c*d^11*x^11*\log(F)^4 + 66*F^(b*c^3 + a)*b^4*c^2*d^10*x^10*\log(F)^4 + F^(b*c^3 + a)*b^4*c^3*d^9*x^9*\log(F)^4 - 4*F^(b*c^3 + a)*b^3*c^9*\log(F)^3 + 12*F^(b*c^3 + a)*b^2*c^6*\log(F)^2 + 4*(55*F^(b*c^3 + a)*b^4*c^3*d^9*\log(F)^4 - F^(b*c^3 + a)*b^3*d^9*\log(F)^3)*x^9 + 9*(55*F^(b*c^3 + a)*b^4*c^4*d^8*\log(F)^4 - 4*F^(b*c^3 + a)*b^3*c*d^8*\log(F)^3)*x^8 + 72*(11*F^(b*c^3 + a)*b^4*c^5*d^7*\log(F)^4 - 2*F^(b*c^3 + a)*b^3*c^2*d^7*\log(F)^3)*x^7 + 12*(77*F^(b*c^3 + a)*b^4*c^6*d^6*\log(F)^4 - 28*F^(b*c^3 + a)*b^3*c^3*d^6*\log(F)^3 + F^(b*c^3 + a)*b^2*d^6*\log(F)^2)*x^6 + 72*(11*F^(b*c^3 + a)*b^4*c^7*d^5*\log(F)^4 - 7*F^(b*c^3 + a)*b^3*c^4*d^5*\log(F)^3 + F^(b*c^3 + a)*b^2*c*d^5*\log(F)^2)*x^5 - 24*F^(b*c^3 + a)*b*c^3*\log(F) + 9*(55*F^(b*c^3 + a)*b^4*c^8*d^4*\log(F)^4 - 56*F^(b*c^3 + a)*b^3*c^5*d^4*\log(F)^3 + 20*F^(b*c^3 + a)*b^2*c^2*d^4*\log(F)^2)*x^4 + 4*(55*F^(b*c^3 + a)*b^4*c^9*d^3*\log(F)^4 - 84*F^(b*c^3 + a)*b^3*c^6*d^3*\log(F)^3 + 60*F^(b*c^3 + a)*b^2*c^3*d^3*\log(F)^2 - 6*F^(b*c^3 + a)*b*d^3*\log(F))*x^3 + 6*(11*F^(b*c^3 + a)*b^4*c^10*d^2*\log(F)^4 - 24*F^(b*c^3 + a)*b^3*c^7*d^2*\log(F)^3 + 30*F^(b*c^3 + a)*b^2*c^4*d^2*\log(F)^2 - 12*F^(b*c^3 + a)*b*c*d^2*\log(F))*x^2 + 12*(F^(b*c^3 + a)*b^4*c^11*d*\log(F)^4 - 3*F^(b*c^3 + a)*b^3*c^8*d*\log(F)^3 + 6*F^(b*c^3 + a)*b^2*c^5*d*\log(F)^2 - 6*F^(b*c^3 + a)*b*c^2*d*\log(F))*x + 24*F^(b*c^3 + a)*e^(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F))/(b^5*d*\log(F)^5)$

**Fricas [B]** time = 1.8485, size = 1019, normalized size = 11.58

$\frac{((b^4 d^{12} x^{12} + 12 b^4 c d^{11} x^{11} + 66 b^4 c^2 d^{10} x^{10} + 220 b^4 c^3 d^9 x^9 + 495 b^4 c^4 d^8 x^8 + 792 b^4 c^5 d^7 x^7 + 924 b^4 c^6 d^6 x^6 + 792 b^4 c^7 d^5 x^5 + 495 b^4 c^8 d^4 x^4 + 220 b^4 c^9 d^3 x^3 + 66 b^4 c^{10} d^2 x^2 + 12 b^4 c^{11} d x + b^4 c^{12}))}{(b^5 d \log(F)^5)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x, algorithm="fricas")
```

```
[Out] 1/3*((b^4*d^12*x^12 + 12*b^4*c*d^11*x^11 + 66*b^4*c^2*d^10*x^10 + 220*b^4*c^3*d^9*x^9 + 495*b^4*c^4*d^8*x^8 + 792*b^4*c^5*d^7*x^7 + 924*b^4*c^6*d^6*x^6 + 792*b^4*c^7*d^5*x^5 + 495*b^4*c^8*d^4*x^4 + 220*b^4*c^9*d^3*x^3 + 66*b^4*c^10*d^2*x^2 + 12*b^4*c^11*d*x + b^4*c^12)*log(F)^4 - 4*(b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*x^6 + 126*b^3*c^4*d^5*x^5 + 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*b^3*c^7*d^2*x^2 + 9*b^3*c^8*d*x + b^3*c^9)*log(F)^3 + 12*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)^2 - 24*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) + 24)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b^5*d*log(F)^5)
```

---

**Sympy [A]** time = 0.439723, size = 823, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**14,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x)**3)*(b**4*c**12*log(F)**4 + 12*b**4*c**11*d*x*log(F)**4 + 66*b**4*c**10*d**2*x**2*log(F)**4 + 220*b**4*c**9*d**3*x**3*log(F)**4 + 495*b**4*c**8*d**4*x**4*log(F)**4 + 792*b**4*c**7*d**5*x**5*log(F)**4 + 924*b**4*c**6*d**6*x**6*log(F)**4 + 792*b**4*c**5*d**7*x**7*log(F)**4 + 495*b**4*c**4*d**8*x**8*log(F)**4 + 220*b**4*c**3*d**9*x**9*log(F)**4 + 66*b**4*c**2*d**10*x**10*log(F)**4 + 12*b**4*c*d**11*x**11*log(F)**4 + b**4*d**12*x**12*log(F)**4 - 4*b**3*c**9*log(F)**3 - 36*b**3*c**8*d*x*log(F)**3 - 144*b**3*c**7*d**2*x**2*log(F)**3 - 336*b**3*c**6*d**3*x**3*log(F)**3 - 504*b**3*c**5*d**4*x**4*log(F)**3 - 504*b**3*c**4*d**5*x**5*log(F)**3 - 36*b**3*c**3*d**6*x**6*log(F)**3 - 144*b**3*c**2*d**7*x**7*log(F)**3 - 36*b**3*c*d**8*x**8*log(F)**3 - 4*b**3*d**9*x**9*log(F)**3 + 12*b**2*c**6*log(F)**2 + 72*b**2*c**5*d*x*log(F)**2 + 180*b**2*c**4*d**2*x**2*log(F)**2 + 240*b**2*c**3*d**3*x**3*log(F)**2 + 180*b**2*c**2*d**4*x**4*log(F)**2 + 72*b**2*c*d**5*x**5*log(F)**2 + 12*b**2*d**6*x**6*log(F)**2 - 24*b*c**3*log(F) - 72*b*c**2*d*x*log(F) - 72*b*c*d**2*x**2*log(F) - 24*b*d**3*x**3*log(F) + 24)/(3*b**5*d*log(F)**5), Ne(3*b**5*d*log(F)**5, 0)), (c**14*x + 7*c**13*d*x**2 + 91*c**12*d**2*x**3/3 + 91*c**11*d**3*x**4 + 1001*c**10*d**4*x**5/5 + 1001*c**9*d**5*x**6/3 + 429*c**8*d**6*x**7 + 429*c**7*d**7*x**8 + 1001*c**6*d**8*x**9/3 + 1001*c**5*d**9*x**10/5 + 91*c**4*d**10*x**11 + 91*c**3*d**11*x**12/3 + 7*c**2*d**12*x**13 + c*d**13*x**14 + d**14*x**15/15, True))
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.283 $\int F^{a+b(c+dx)^3} (c+dx)^{11} dx$

**Optimal.** Leaf size=124

$$-\frac{(c+dx)^6 F^{a+b(c+dx)^3}}{b^2 d \log^2(F)} + \frac{2(c+dx)^3 F^{a+b(c+dx)^3}}{b^3 d \log^3(F)} - \frac{2 F^{a+b(c+dx)^3}}{b^4 d \log^4(F)} + \frac{(c+dx)^9 F^{a+b(c+dx)^3}}{3bd \log(F)}$$

[Out]  $(-2F^{a+b(c+dx)^3})/(b^4 d \log^4(F)) + (2F^{a+b(c+dx)^3})(c+dx)^3/(b^3 d \log^3(F)) - (F^{a+b(c+dx)^3})(c+dx)^6/(b^2 d \log^2(F)) + (F^{a+b(c+dx)^3})(c+dx)^9/(3bd \log(F))$

**Rubi [A]** time = 0.283723, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$-\frac{(c+dx)^6 F^{a+b(c+dx)^3}}{b^2 d \log^2(F)} + \frac{2(c+dx)^3 F^{a+b(c+dx)^3}}{b^3 d \log^3(F)} - \frac{2 F^{a+b(c+dx)^3}}{b^4 d \log^4(F)} + \frac{(c+dx)^9 F^{a+b(c+dx)^3}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^11, x]

[Out]  $(-2F^{a+b(c+dx)^3})/(b^4 d \log^4(F)) + (2F^{a+b(c+dx)^3})(c+dx)^3/(b^3 d \log^3(F)) - (F^{a+b(c+dx)^3})(c+dx)^6/(b^2 d \log^2(F)) + (F^{a+b(c+dx)^3})(c+dx)^9/(3bd \log(F))$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^3} (c+dx)^{11} dx &= \frac{F^{a+b(c+dx)^3} (c+dx)^9}{3bd \log(F)} - \frac{3 \int F^{a+b(c+dx)^3} (c+dx)^8 dx}{b \log(F)} \\ &= -\frac{F^{a+b(c+dx)^3} (c+dx)^6}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^9}{3bd \log(F)} + \frac{6 \int F^{a+b(c+dx)^3} (c+dx)^5 dx}{b^2 \log^2(F)} \\ &= \frac{2F^{a+b(c+dx)^3} (c+dx)^3}{b^3 d \log^3(F)} - \frac{F^{a+b(c+dx)^3} (c+dx)^6}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^9}{3bd \log(F)} - \frac{6 \int F^{a+b(c+dx)^3} (c+dx)^2 dx}{b^3 \log^3(F)} \\ &= -\frac{2F^{a+b(c+dx)^3}}{b^4 d \log^4(F)} + \frac{2F^{a+b(c+dx)^3} (c+dx)^3}{b^3 d \log^3(F)} - \frac{F^{a+b(c+dx)^3} (c+dx)^6}{b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^9}{3bd \log(F)} \end{aligned}$$





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**Fricas [B]** time = 1.73513, size = 640, normalized size = 5.16

$$\left( (b^3 d^9 x^9 + 9 b^3 c d^8 x^8 + 36 b^3 c^2 d^7 x^7 + 84 b^3 c^3 d^6 x^6 + 126 b^3 c^4 d^5 x^5 + 126 b^3 c^5 d^4 x^4 + 84 b^3 c^6 d^3 x^3 + 36 b^3 c^7 d^2 x^2 + 9 b^3 c^8 d x + b^3 c^9) \log(F)^3 - 3(b^2 d^6 x^6 + 6 b^2 c d^5 x^5 + 15 b^2 c^2 d^4 x^4 + 20 b^2 c^3 d^3 x^3 + 15 b^2 c^4 d^2 x^2 + 6 b^2 c^5 d x + b^2 c^6) \log(F)^2 + 6(b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F) - 6 \right) F^{(b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3 + a) / (b^4 d \log(F)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^11,x, algorithm="fricas")

[Out]  $\frac{1}{3} \left( (b^3 d^9 x^9 + 9 b^3 c d^8 x^8 + 36 b^3 c^2 d^7 x^7 + 84 b^3 c^3 d^6 x^6 + 126 b^3 c^4 d^5 x^5 + 126 b^3 c^5 d^4 x^4 + 84 b^3 c^6 d^3 x^3 + 36 b^3 c^7 d^2 x^2 + 9 b^3 c^8 d x + b^3 c^9) \log(F)^3 - 3(b^2 d^6 x^6 + 6 b^2 c d^5 x^5 + 15 b^2 c^2 d^4 x^4 + 20 b^2 c^3 d^3 x^3 + 15 b^2 c^4 d^2 x^2 + 6 b^2 c^5 d x + b^2 c^6) \log(F)^2 + 6(b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F) - 6 \right) F^{(b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3 + a) / (b^4 d \log(F)^4)}$

---

**Sympy [A]** time = 0.348558, size = 537, normalized size = 4.33

$$\left\{ \frac{F^{a+b(c+dx)^3} (b^3 c^9 \log(F)^3 + 9 b^3 c^8 dx \log(F)^3 + 36 b^3 c^7 d^2 x^2 \log(F)^3 + 84 b^3 c^6 d^3 x^3 \log(F)^3 + 126 b^3 c^5 d^4 x^4 \log(F)^3 + 126 b^3 c^4 d^5 x^5 \log(F)^3 + 84 b^3 c^3 d^6 x^6 \log(F)^3 + 36 b^3 c^2 d^7 x^7 \log(F)^3 + 9 b^3 c d^8 x^8 \log(F)^3 + b^3 c^9 \log(F)^3)}{c^{11} x + \frac{11 c^{10} dx^2}{2} + \frac{55 c^9 d^2 x^3}{3} + \frac{165 c^8 d^3 x^4}{4} + 66 c^7 d^4 x^5 + 77 c^6 d^5 x^6 + 66 c^5 d^6 x^7 + \frac{165 c^4 d^7 x^8}{4} + \frac{55 c^3 d^8 x^9}{3} + \frac{11 c^2 d^9 x^{10}}{2} + c d^{10} x^{11} + d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c)\*\*11,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x)\*\*3)\*(b\*\*3\*c\*\*9\*log(F)\*\*3 + 9\*b\*\*3\*c\*\*8\*d\*x\*log(F)\*\*3 + 36\*b\*\*3\*c\*\*7\*d\*\*2\*x\*\*2\*log(F)\*\*3 + 84\*b\*\*3\*c\*\*6\*d\*\*3\*x\*\*3\*log(F)\*\*3 + 126\*b\*\*3\*c\*\*5\*d\*\*4\*x\*\*4\*log(F)\*\*3 + 126\*b\*\*3\*c\*\*4\*d\*\*5\*x\*\*5\*log(F)\*\*3 + 84\*b\*\*3\*c\*\*3\*d\*\*6\*x\*\*6\*log(F)\*\*3 + 36\*b\*\*3\*c\*\*2\*d\*\*7\*x\*\*7\*log(F)\*\*3 + 9\*b\*\*3\*c\*d\*\*8\*x\*\*8\*log(F)\*\*3 + b\*\*3\*d\*\*9\*x\*\*9\*log(F)\*\*3 - 3\*b\*\*2\*c\*\*6\*log(F)\*\*2 - 18\*b\*\*2\*c\*\*5\*d\*x\*log(F)\*\*2 - 45\*b\*\*2\*c\*\*4\*d\*\*2\*x\*\*2\*log(F)\*\*2 - 60\*b\*\*2\*c\*\*3\*d\*\*3\*x\*\*3\*log(F)\*\*2 - 45\*b\*\*2\*c\*\*2\*d\*\*4\*x\*\*4\*log(F)\*\*2 - 18\*b\*\*2\*c\*d\*\*5\*x\*\*5\*log(F)\*\*2 - 3\*b\*\*2\*d\*\*6\*x\*\*6\*log(F)\*\*2 + 6\*b\*c\*\*3\*log(F) + 18\*b\*c\*\*2\*d\*x\*log(F) + 18\*b\*c\*d\*\*2\*x\*\*2\*log(F) + 6\*b\*d\*\*3\*x\*\*3\*log(F) - 6)/(3\*b\*\*4\*d\*log(F)\*\*4), Ne(3\*b\*\*4\*d\*log(F)\*\*4, 0)), (c\*\*11\*x + 11\*c\*\*10\*d\*x\*\*2/2 + 55\*c\*\*9\*d\*\*2\*x\*\*3/3 + 165\*c\*\*8\*d\*\*3\*x\*\*4/4 + 66\*c\*\*7\*d\*\*4\*x\*\*5 + 77\*c\*\*6\*d\*\*5\*x\*\*6 + 66\*c\*\*5\*d\*\*6\*x\*\*7 + 165\*c\*\*4\*d\*\*7\*x\*\*8/4 + 55\*c\*\*3\*d\*\*8\*x\*\*9/3 + 11\*c\*\*2\*d\*\*9\*x\*\*10/2 + c\*d\*\*10\*x\*\*11 + d\*\*11\*x\*\*12/12, True))

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**Giac [B]** time = 1.32446, size = 1782, normalized size = 14.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^11,x, algorithm="giac")

[Out]  $\frac{1}{3} (b^3 d^9 x^9 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F)) \log(F)^3} + 9 b^3 c d^8 x^8 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F)) \log(F)^2} + 36 b^3 c^2 d^7 x^7 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F)) \log(F)} + 84 b^3 c^3 d^6 x^6 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + 126 b^3 c^4 d^5 x^5 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + 126 b^3 c^5 d^4 x^4 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + 84 b^3 c^6 d^3 x^3 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + 36 b^3 c^7 d^2 x^2 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + 9 b^3 c^8 d x e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))} + b^3 c^9 e^{(b d^3 x^3 \log(F) + 3 b c d^2 x^2 \log(F) + 3 b c^2 d x \log(F) + b c^3 \log(F) + a \log(F))})$

$$\begin{aligned}
& * \log(F)^3 + 36*b^3*c^2*d^7*x^7*e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 + 84*b^3*c^3*d^6*x^6} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 + 126*b^3*c^4*d^5*x^5} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 + 126*b^3*c^5*d^4*x^4} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 + 84*b^3*c^6*d^3*x^3} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 + 36*b^3*c^7*d^2*x^2} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 - 3*b^2*d^6*x^6} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 + 9*b^3*c^8*d*x} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 - 18*b^2*c^d^5*x^5} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 + b^3*c^9} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^3 - 45*b^2*c^2*d^4*x^4} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 - 60*b^2*c^3*d^3*x^3} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 - 45*b^2*c^4*d^2*x^2} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 - 18*b^2*c^5*d*x} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 - 3*b^2*c^6} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F)^2 + 6*b*d^3*x^3} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F) + 18*b*c*d^2*x^2} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F) + 18*b*c^2*d*x} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F) + 6*b*c^3} \\
& e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))*\log(F) - 6*e^{(b*d^3*x^3*\log(F) + 3*b*c*d^2*x^2*\log(F) + 3*b*c^2*d*x*\log(F) + b*c^3*\log(F) + a*\log(F))}} \\
& / (b^4*d*\log(F)^4)
\end{aligned}$$

### 3.284 $\int F^{a+b(c+dx)^3} (c+dx)^8 dx$

**Optimal.** Leaf size=96

$$-\frac{2(c+dx)^3 F^{a+b(c+dx)^3}}{3b^2 d \log^2(F)} + \frac{2F^{a+b(c+dx)^3}}{3b^3 d \log^3(F)} + \frac{(c+dx)^6 F^{a+b(c+dx)^3}}{3bd \log(F)}$$

[Out]  $(2F^{a+b(c+dx)^3})/(3b^3 d \log[F]^3) - (2F^{a+b(c+dx)^3}*(c+dx)^3)/(3b^2 d \log[F]^2) + (F^{a+b(c+dx)^3}*(c+dx)^6)/(3b*d \log[F])$

**Rubi [A]** time = 0.208719, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$-\frac{2(c+dx)^3 F^{a+b(c+dx)^3}}{3b^2 d \log^2(F)} + \frac{2F^{a+b(c+dx)^3}}{3b^3 d \log^3(F)} + \frac{(c+dx)^6 F^{a+b(c+dx)^3}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^8,x]

[Out]  $(2F^{a+b(c+dx)^3})/(3b^3 d \log[F]^3) - (2F^{a+b(c+dx)^3}*(c+dx)^3)/(3b^2 d \log[F]^2) + (F^{a+b(c+dx)^3}*(c+dx)^6)/(3b*d \log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^3} (c+dx)^8 dx &= \frac{F^{a+b(c+dx)^3} (c+dx)^6}{3bd \log(F)} - \frac{2 \int F^{a+b(c+dx)^3} (c+dx)^5 dx}{b \log(F)} \\ &= -\frac{2F^{a+b(c+dx)^3} (c+dx)^3}{3b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^6}{3bd \log(F)} + \frac{2 \int F^{a+b(c+dx)^3} (c+dx)^2 dx}{b^2 \log^2(F)} \\ &= \frac{2F^{a+b(c+dx)^3}}{3b^3 d \log^3(F)} - \frac{2F^{a+b(c+dx)^3} (c+dx)^3}{3b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^6}{3bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0421616, size = 56, normalized size = 0.58

$$\frac{F^{a+b(c+dx)^3} (b^2 \log^2(F)(c+dx)^6 - 2b \log(F)(c+dx)^3 + 2)}{3b^3 d \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^8,x]

[Out] (F^(a + b\*(c + d\*x)^3)\*(2 - 2\*b\*(c + d\*x)^3\*Log[F] + b^2\*(c + d\*x)^6\*Log[F]^2))/(3\*b^3\*d\*Log[F]^3)

**Maple [B]** time = 0.008, size = 200, normalized size = 2.1

$$\frac{(d^6 x^6 (\ln(F))^2 b^2 + 6 c d^5 x^5 (\ln(F))^2 b^2 + 15 c^2 d^4 x^4 (\ln(F))^2 b^2 + 20 (\ln(F))^2 b^2 c^3 d^3 x^3 + 15 (\ln(F))^2 b^2 c^4 d^2 x^2 + 6 (\ln(F))^2 b^2 c^5 d x + b^2 c^6) \log(F)^2 - 2 F^{bc^3+a} b c^3 \log(F) + 3 (\ln(F))^3}{3 (\ln(F))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^8,x)

[Out] 1/3\*(d^6\*x^6\*ln(F)^2\*b^2+6\*c\*d^5\*x^5\*ln(F)^2\*b^2+15\*c^2\*d^4\*x^4\*ln(F)^2\*b^2+20\*ln(F)^2\*b^2\*c^3\*d^3\*x^3+15\*ln(F)^2\*b^2\*c^4\*d^2\*x^2+6\*ln(F)^2\*b^2\*c^5\*d\*x+ln(F)^2\*b^2\*c^6-2\*ln(F)\*b\*d^3\*x^3-6\*ln(F)\*b\*c\*d^2\*x^2-6\*ln(F)\*b\*c^2\*d\*x-2\*ln(F)\*b\*c^3+2)\*F^(b\*d^3\*x^3+3\*b\*c\*d^2\*x^2+3\*b\*c^2\*d\*x+b\*c^3+a)/ln(F)^3/b^3/d

**Maxima [B]** time = 1.58906, size = 416, normalized size = 4.33

$$\frac{(F^{bc^3+a} b^2 d^6 x^6 \log(F)^2 + 6 F^{bc^3+a} b^2 c d^5 x^5 \log(F)^2 + 15 F^{bc^3+a} b^2 c^2 d^4 x^4 \log(F)^2 + F^{bc^3+a} b^2 c^6 \log(F)^2 - 2 F^{bc^3+a} b c^3 \log(F) + 3 (\ln(F))^3)}{3 (\ln(F))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^8,x, algorithm="maxima")

[Out] 1/3\*(F^(b\*c^3 + a)\*b^2\*d^6\*x^6\*log(F)^2 + 6\*F^(b\*c^3 + a)\*b^2\*c\*d^5\*x^5\*log(F)^2 + 15\*F^(b\*c^3 + a)\*b^2\*c^2\*d^4\*x^4\*log(F)^2 + F^(b\*c^3 + a)\*b^2\*c^6\*log(F)^2 - 2\*F^(b\*c^3 + a)\*b\*c^3\*log(F) + 2\*(10\*F^(b\*c^3 + a)\*b^2\*c^3\*d^3\*log(F)^2 - F^(b\*c^3 + a)\*b\*d^3\*log(F))\*x^3 + 3\*(5\*F^(b\*c^3 + a)\*b^2\*c^4\*d^2\*log(F)^2 - 2\*F^(b\*c^3 + a)\*b\*c\*d^2\*log(F))\*x^2 + 6\*(F^(b\*c^3 + a)\*b^2\*c^5\*d\*log(F)^2 - F^(b\*c^3 + a)\*b\*c^2\*d\*log(F))\*x + 2\*F^(b\*c^3 + a)\*e^(b\*d^3\*x^3\*log(F) + 3\*b\*c\*d^2\*x^2\*log(F) + 3\*b\*c^2\*d\*x\*log(F))/(b^3\*d\*log(F)^3)

**Fricas [A]** time = 1.53499, size = 371, normalized size = 3.86

$$\frac{((b^2 d^6 x^6 + 6 b^2 c d^5 x^5 + 15 b^2 c^2 d^4 x^4 + 20 b^2 c^3 d^3 x^3 + 15 b^2 c^4 d^2 x^2 + 6 b^2 c^5 d x + b^2 c^6) \log(F)^2 - 2 (b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F) + 3 (\ln(F))^3)}{3 b^3 d \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^8,x, algorithm="fricas")

[Out]  $\frac{1}{3} * ((b^2 * d^6 * x^6 + 6 * b^2 * c * d^5 * x^5 + 15 * b^2 * c^2 * d^4 * x^4 + 20 * b^2 * c^3 * d^3 * x^3 + 15 * b^2 * c^4 * d^2 * x^2 + 6 * b^2 * c^5 * d * x + b^2 * c^6) * \log(F)^2 - 2 * (b * d^3 * x^3 + 3 * b * c * d^2 * x^2 + 3 * b * c^2 * d * x + b * c^3) * \log(F) + 2) * F^{(b * d^3 * x^3 + 3 * b * c * d^2 * x^2 + 3 * b * c^2 * d * x + b * c^3 + a)} / (b^3 * d * \log(F)^3)$

**Sympy [A]** time = 0.275429, size = 306, normalized size = 3.19

$$\left\{ \frac{F^{a+b(c+dx)^3} (b^2 c^6 \log(F)^2 + 6 b^2 c^5 d x \log(F)^2 + 15 b^2 c^4 d^2 x^2 \log(F)^2 + 20 b^2 c^3 d^3 x^3 \log(F)^2 + 15 b^2 c^2 d^4 x^4 \log(F)^2 + 6 b^2 c d^5 x^5 \log(F)^2 + b^2 d^6 x^6 \log(F)^2 - 2 b c^3 \log(F)^2 - 2 b c^3 \log(F) + 2) (c^8 x + 4 c^7 d x^2 + \frac{28 c^6 d^2 x^3}{3} + 14 c^5 d^3 x^4 + 14 c^4 d^4 x^5 + \frac{28 c^3 d^5 x^6}{3} + 4 c^2 d^6 x^7 + c d^7 x^8 + \frac{d^8 x^9}{9})}{3 b^3 d \log(F)^3} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c)\*\*8,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x)\*\*3)\*(b\*\*2\*c\*\*6\*log(F)\*\*2 + 6\*b\*\*2\*c\*\*5\*d\*x\*log(F)\*\*2 + 15\*b\*\*2\*c\*\*4\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 20\*b\*\*2\*c\*\*3\*d\*\*3\*x\*\*3\*log(F)\*\*2 + 15\*b\*\*2\*c\*\*2\*d\*\*4\*x\*\*4\*log(F)\*\*2 + 6\*b\*\*2\*c\*d\*\*5\*x\*\*5\*log(F)\*\*2 + b\*\*2\*d\*\*6\*x\*\*6\*log(F)\*\*2 - 2\*b\*c\*\*3\*log(F) - 6\*b\*c\*\*2\*d\*x\*log(F) - 6\*b\*c\*d\*\*2\*x\*\*2\*log(F) - 2\*b\*d\*\*3\*x\*\*3\*log(F) + 2)/(3\*b\*\*3\*d\*log(F)\*\*3), Ne(3\*b\*\*3\*d\*log(F)\*\*3, 0)), (c\*\*8\*x + 4\*c\*\*7\*d\*x\*\*2 + 28\*c\*\*6\*d\*\*2\*x\*\*3/3 + 14\*c\*\*5\*d\*\*3\*x\*\*4 + 14\*c\*\*4\*d\*\*4\*x\*\*5 + 28\*c\*\*3\*d\*\*5\*x\*\*6/3 + 4\*c\*\*2\*d\*\*6\*x\*\*7 + c\*d\*\*7\*x\*\*8 + d\*\*8\*x\*\*9/9, True))

**Giac [B]** time = 1.28072, size = 952, normalized size = 9.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^8,x, algorithm="giac")

[Out]  $\frac{1}{3} * (b^2 * d^6 * x^6 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + 6 * b^2 * c * d^5 * x^5 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + 15 * b^2 * c^2 * d^4 * x^4 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + 20 * b^2 * c^3 * d^3 * x^3 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + 15 * b^2 * c^4 * d^2 * x^2 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + 6 * b^2 * c^5 * d * x * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 + b^2 * c^6 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F)^2 - 2 * b * d^3 * x^3 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F) - 6 * b * c * d^2 * x^2 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F) - 6 * b * c^2 * d * x * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F) - 2 * b * c^3 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} * \log(F) + 2 * e^{(b * d^3 * x^3 * \log(F) + 3 * b * c * d^2 * x^2 * \log(F) + 3 * b * c^2 * d * x * \log(F) + b * c^3 * \log(F) + a * \log(F))} / (b^3 * d * \log(F)^3)$

### 3.285 $\int F^{a+b(c+dx)^3} (c+dx)^5 dx$

**Optimal.** Leaf size=62

$$\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd \log(F)} - \frac{F^{a+b(c+dx)^3}}{3b^2 d \log^2(F)}$$

[Out]  $-F^{a+b(c+dx)^3}/(3b^2 d \log(F)^2) + (F^{a+b(c+dx)^3})(c+dx)^3/(3b d \log(F))$

**Rubi [A]** time = 0.13814, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd \log(F)} - \frac{F^{a+b(c+dx)^3}}{3b^2 d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^5, x]

[Out]  $-F^{a+b(c+dx)^3}/(3b^2 d \log(F)^2) + (F^{a+b(c+dx)^3})(c+dx)^3/(3b d \log(F))$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^3} (c+dx)^5 dx &= \frac{F^{a+b(c+dx)^3} (c+dx)^3}{3bd \log(F)} - \frac{\int F^{a+b(c+dx)^3} (c+dx)^2 dx}{b \log(F)} \\ &= -\frac{F^{a+b(c+dx)^3}}{3b^2 d \log^2(F)} + \frac{F^{a+b(c+dx)^3} (c+dx)^3}{3bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0248922, size = 40, normalized size = 0.65

$$\frac{F^{a+b(c+dx)^3} (b \log(F) (c+dx)^3 - 1)}{3b^2 d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^5,x]

[Out] (F^(a + b\*(c + d\*x)^3)\*(-1 + b\*(c + d\*x)^3\*Log[F]))/(3\*b^2\*d\*Log[F]^2)

**Maple [A]** time = 0.007, size = 89, normalized size = 1.4

$$\frac{(\ln(F)bd^3x^3 + 3\ln(F)bcd^2x^2 + 3\ln(F)bc^2dx + \ln(F)bc^3 - 1)F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}}{3(\ln(F))^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^5,x)

[Out] 1/3\*(ln(F)\*b\*d^3\*x^3+3\*ln(F)\*b\*c\*d^2\*x^2+3\*ln(F)\*b\*c^2\*d\*x+ln(F)\*b\*c^3-1)\*F^(b\*d^3\*x^3+3\*b\*c\*d^2\*x^2+3\*b\*c^2\*d\*x+b\*c^3+a)/ln(F)^2/b^2/d

**Maxima [B]** time = 1.56596, size = 180, normalized size = 2.9

$$\frac{(F^{bc^3+a}bd^3x^3\log(F) + 3F^{bc^3+a}bcd^2x^2\log(F) + 3F^{bc^3+a}bc^2dx\log(F) + F^{bc^3+a}bc^3\log(F) - F^{bc^3+a})e^{(bd^3x^3\log(F)+3bcd^2x^2\log(F)+3bc^2dx\log(F)+bc^3\log(F))}}{3b^2d\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^5,x, algorithm="maxima")

[Out] 1/3\*(F^(b\*c^3 + a)\*b\*d^3\*x^3\*log(F) + 3F^(b\*c^3 + a)\*b\*c\*d^2\*x^2\*log(F) + 3F^(b\*c^3 + a)\*b\*c^2\*d\*x\*log(F) + F^(b\*c^3 + a)\*b\*c^3\*log(F) - F^(b\*c^3 + a))\*e^(b\*d^3\*x^3\*log(F) + 3\*b\*c\*d^2\*x^2\*log(F) + 3\*b\*c^2\*d\*x\*log(F))/(b^2\*d\*log(F)^2)

**Fricas [A]** time = 1.56277, size = 190, normalized size = 3.06

$$\frac{((bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3)\log(F) - 1)F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}}{3b^2d\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^5,x, algorithm="fricas")

[Out] 1/3\*((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F) - 1)\*F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a)/(b^2\*d\*log(F)^2)

**Sympy [A]** time = 0.209934, size = 144, normalized size = 2.32

$$\begin{cases} \frac{F^{a+b(c+dx)^3}(bc^3\log(F)+3bc^2dx\log(F)+3bcd^2x^2\log(F)+bd^3x^3\log(F)-1)}{3b^2d\log(F)^2} & \text{for } 3b^2d\log(F)^2 \neq 0 \\ c^5x + \frac{5c^4dx^2}{2} + \frac{10c^3d^2x^3}{3} + \frac{5c^2d^3x^4}{2} + cd^4x^5 + \frac{d^5x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**5,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x)**3)*(b*c**3*log(F) + 3*b*c**2*d*x*log(F) + 3*
b*c*d**2*x**2*log(F) + b*d**3*x**3*log(F) - 1)/(3*b**2*d*log(F)**2), Ne(3*
b**2*d*log(F)**2, 0)), (c**5*x + 5*c**4*d*x**2/2 + 10*c**3*d**2*x**3/3 + 5*
c**2*d**3*x**4/2 + c*d**4*x**5 + d**5*x**6/6, True))
```

**Giac [B]** time = 1.42148, size = 1203, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(2*(2*(((d*x + c)^3*b*log(abs(F)) - 1)*(pi^2*b^2*sgn(F) - pi^2*b^2 + 2*
b^2*log(abs(F))^2)/((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)^2 +
4*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))^2) + (pi*(d*x + c)^3*b*
sgn(F) - pi*(d*x + c)^3*b)*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))/
((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)^2 + 4*(pi*b^2*log(abs(F)
))*sgn(F) - pi*b^2*log(abs(F)))^2))*cos(-1/2*pi*b*d^3*x^3*sgn(F) + 1/2*pi*b
*d^3*x^3 - 3/2*pi*b*c*d^2*x^2*sgn(F) + 3/2*pi*b*c*d^2*x^2 - 3/2*pi*b*c^2*d*
x*sgn(F) + 3/2*pi*b*c^2*d*x - 1/2*pi*b*c^3*sgn(F) + 1/2*pi*b*c^3 - 1/2*pi*a
*sgn(F) + 1/2*pi*a) + ((pi*(d*x + c)^3*b*sgn(F) - pi*(d*x + c)^3*b)*(pi^2*b
^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)/((pi^2*b^2*sgn(F) - pi^2*b^2 +
2*b^2*log(abs(F))^2)^2 + 4*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))
^2) - 4*((d*x + c)^3*b*log(abs(F)) - 1)*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2
*log(abs(F)))/((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)^2 + 4*(pi
*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))^2))*sin(-1/2*pi*b*d^3*x^3*sgn
(F) + 1/2*pi*b*d^3*x^3 - 3/2*pi*b*c*d^2*x^2*sgn(F) + 3/2*pi*b*c*d^2*x^2 - 3
/2*pi*b*c^2*d*x*sgn(F) + 3/2*pi*b*c^2*d*x - 1/2*pi*b*c^3*sgn(F) + 1/2*pi*b*
c^3 - 1/2*pi*a*sgn(F) + 1/2*pi*a)*e^((d*x + c)^3*b*log(abs(F)) + a*log(abs
(F))) - ((2*(d*x + c)^3*b*i*log(abs(F)) - pi*(d*x + c)^3*b*sgn(F) + pi*(d*x
+ c)^3*b - 2*i)*e^(1/2*(pi*(d*x + c)^3*b*(sgn(F) - 1) + pi*a*(sgn(F) - 1))
*i)/(2*pi*b^2*i*log(abs(F))*sgn(F) - 2*pi*b^2*i*log(abs(F)) + pi^2*b^2*sgn(
F) - pi^2*b^2 + 2*b^2*log(abs(F))^2) + (2*(d*x + c)^3*b*i*log(abs(F)) + pi*
(d*x + c)^3*b*sgn(F) - pi*(d*x + c)^3*b - 2*i)*e^(-1/2*(pi*(d*x + c)^3*b*(s
gn(F) - 1) + pi*a*(sgn(F) - 1))*i)/(2*pi*b^2*i*log(abs(F))*sgn(F) - 2*pi*b^
2*i*log(abs(F)) - pi^2*b^2*sgn(F) + pi^2*b^2 - 2*b^2*log(abs(F))^2))*e^((d*
x + c)^3*b*log(abs(F)) + a*log(abs(F)))/i)/d
```



$$3.286 \quad \int F^{a+b(c+dx)^3} (c+dx)^2 dx$$

**Optimal.** Leaf size=27

$$\frac{F^{a+b(c+dx)^3}}{3bd \log(F)}$$

[Out] F^(a + b\*(c + d\*x)^3)/(3\*b\*d\*Log[F])

**Rubi [A]** time = 0.0672355, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2209}

$$\frac{F^{a+b(c+dx)^3}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^2,x]

[Out] F^(a + b\*(c + d\*x)^3)/(3\*b\*d\*Log[F])

Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int F^{a+b(c+dx)^3} (c+dx)^2 dx = \frac{F^{a+b(c+dx)^3}}{3bd \log(F)}$$

**Mathematica [A]** time = 0.0084016, size = 27, normalized size = 1.

$$\frac{F^{a+b(c+dx)^3}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^2,x]

[Out] F^(a + b\*(c + d\*x)^3)/(3\*b\*d\*Log[F])

**Maple [A]** time = 0.005, size = 48, normalized size = 1.8

$$\frac{F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}}{3bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x)`

[Out]  $\frac{1}{3}F^{(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)}/b/d/\ln(F)$

**Maxima [A]** time = 1.02495, size = 34, normalized size = 1.26

$$\frac{F^{(dx+c)^3 b+a}}{3bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{3}F^{((d*x + c)^3*b + a)}/(b*d*\log(F))$

**Fricas [A]** time = 1.54858, size = 100, normalized size = 3.7

$$\frac{F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}}{3bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{3}F^{(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)}/(b*d*\log(F))$

**Sympy [A]** time = 0.159267, size = 46, normalized size = 1.7

$$\begin{cases} \frac{F^{a+b(c+dx)^3}}{3bd \log(F)} & \text{for } 3bd \log(F) \neq 0 \\ c^2x + cdx^2 + \frac{d^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**2,x)`

[Out] `Piecewise((F**(a + b*(c + d*x)**3)/(3*b*d*log(F)), Ne(3*b*d*log(F), 0)), (c**2*x + c*d*x**2 + d**2*x**3/3, True))`

**Giac [A]** time = 1.22812, size = 34, normalized size = 1.26

$$\frac{F^{(dx+c)^3 b+a}}{3bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="giac")`

[Out]  $\frac{1}{3}F^{((d*x + c)^3*b + a)}/(b*d*\log(F))$

$$3.287 \quad \int \frac{F^{a+b(c+dx)^3}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$\frac{F^a \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d}$$

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]])/(3\*d)

**Rubi [A]** time = 0.0658895, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$\frac{F^a \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]])/(3\*d)

**Rule 2210**

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))/((e\_) + (f\_)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx} dx = \frac{F^a \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d}$$

**Mathematica [A]** time = 0.0064917, size = 22, normalized size = 1.

$$\frac{F^a \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]])/(3\*d)

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^3)/(d*x+c),x)`

[Out] `int(F^(a+b*(d*x+c)^3)/(d*x+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c), x)`

**Fricas [B]** time = 1.6311, size = 97, normalized size = 4.41

$$\frac{F^a \text{Ei}\left(\left(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3\right) \log(F)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="fricas")`

[Out] `1/3*F^a*Ei((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))/d`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**3)/(d*x+c),x)`

[Out] `Integral(F**(a + b*(c + d*x)**3)/(c + d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c), x)`

$$3.288 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=53

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d} - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}$$

[Out]  $-F^{a+b(c+dx)^3}/(3d(c+dx)^3) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^3 \log(F)] \log(F))/(3d)$

**Rubi [A]** time = 0.125406, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{3d} - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b(c+dx)^3}/(c+dx)^4, x]$

[Out]  $-F^{a+b(c+dx)^3}/(3d(c+dx)^3) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^3 \log(F)] \log(F))/(3d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m+1)} F^{a+b(c+dx)^n} / (d(m+1)), x] - \operatorname{Dist}[(b n \log(F)) / (m+1), \operatorname{Int}[(c+dx)^{(m+n)} F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})) / ((e_.) + (f_.)*(x_))], x\_Symbol] \rightarrow \operatorname{Simp}[F^a \operatorname{ExpIntegralEi}[b(c+dx)^n \log(F)] / (f n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d e - c f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx &= -\frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3} + (b \log(F)) \int \frac{F^{a+b(c+dx)^3}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3} + \frac{bF^a \operatorname{Ei}(b(c+dx)^3 \log(F)) \log(F)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.0316843, size = 47, normalized size = 0.89

$$\frac{F^a \left( b \log(F) \operatorname{Ei}(b(c+dx)^3 \log(F)) - \frac{F^{b(c+dx)^3}}{(c+dx)^3} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^4,x]

[Out] (F^a\*(-(F^(b\*(c + d\*x)^3)/(c + d\*x)^3) + b\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]]\*Log[F]))/(3\*d)

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^4,x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^4,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^4, x)

**Fricas [B]** time = 1.49807, size = 315, normalized size = 5.94

$$\frac{(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3)F^a \operatorname{Ei}\left(\left(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3\right)\log(F)\right)\log(F) - F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}}{3(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^4,x, algorithm="fricas")

[Out] 1/3\*((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*F^a\*Ei((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))\*log(F) - F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a))/(d^4\*x^3 + 3\*c\*d^3\*x^2 + 3\*c^2\*d^2\*x + c^3\*d)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^3*b + a)/(d*x + c)^4, x)
```

$$3.289 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} dx$$

**Optimal.** Leaf size=87

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{6d} - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} - \frac{b \log(F) F^{a+b(c+dx)^3}}{6d(c+dx)^3}$$

[Out]  $-F^{(a + b*(c + d*x)^3)}/(6*d*(c + d*x)^6) - (b*F^{(a + b*(c + d*x)^3)}*Log[F]) / (6*d*(c + d*x)^3) + (b^2*F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[F]^2) / (6*d)$

**Rubi [A]** time = 0.192794, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{6d} - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} - \frac{b \log(F) F^{a+b(c+dx)^3}}{6d(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^7, x]

[Out]  $-F^{(a + b*(c + d*x)^3)}/(6*d*(c + d*x)^6) - (b*F^{(a + b*(c + d*x)^3)}*Log[F]) / (6*d*(c + d*x)^3) + (b^2*F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[F]^2) / (6*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} dx &= -\frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} + \frac{1}{2}(b \log(F)) \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} - \frac{bF^{a+b(c+dx)^3} \log(F)}{6d(c+dx)^3} + \frac{1}{2}(b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^3}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} - \frac{bF^{a+b(c+dx)^3} \log(F)}{6d(c+dx)^3} + \frac{b^2 F^a \operatorname{Ei}(b(c+dx)^3 \log(F)) \log^2(F)}{6d} \end{aligned}$$



**Mathematica [A]** time = 0.0756531, size = 64, normalized size = 0.74

$$\frac{F^a \left( b^2 \log^2(F) \operatorname{Ei} \left( b(c+dx)^3 \log(F) \right) - \frac{F^{b(c+dx)^3} (b \log(F)(c+dx)^3 + 1)}{(c+dx)^6} \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^7, x]

[Out] (F^a\*(b^2\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]]\*Log[F]^2 - (F^(b\*(c + d\*x)^3)\*(1 + b\*(c + d\*x)^3\*Log[F]))/(c + d\*x)^6))/(6\*d)

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^7, x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^7, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^7, x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^7, x)

**Fricas [B]** time = 1.58922, size = 567, normalized size = 6.52

$$\frac{(b^2 d^6 x^6 + 6 b^2 c d^5 x^5 + 15 b^2 c^2 d^4 x^4 + 20 b^2 c^3 d^3 x^3 + 15 b^2 c^4 d^2 x^2 + 6 b^2 c^5 d x + b^2 c^6) F^a \operatorname{Ei} \left( (b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F) \right)}{6 (d^7 x^6 + 6 c d^6 x^5 + 15 c^2 d^5 x^4 + 20 c^3 d^4 x^3 + 15 c^4 d^3 x^2 + 6 c^5 d^2 x + c^6 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^7, x, algorithm="fricas")

[Out] 1/6\*((b^2\*d^6\*x^6 + 6\*b^2\*c\*d^5\*x^5 + 15\*b^2\*c^2\*d^4\*x^4 + 20\*b^2\*c^3\*d^3\*x^3 + 15\*b^2\*c^4\*d^2\*x^2 + 6\*b^2\*c^5\*d\*x + b^2\*c^6)\*F^a\*Ei((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))\*log(F)^2 - ((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F) + 1)\*F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a))/(d^7\*x^6 + 6\*c\*d^6\*x^5 + 15\*c^2\*d^5\*x^4 + 20\*c^3\*d^4\*x^3 + 15\*c^4\*d^3\*x^2 + 6\*c^5\*d^2\*x + c^6\*d)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)/(d\*x+c)\*\*7,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^7,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^7, x)

$$3.290 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}} dx$$

**Optimal.** Leaf size=121

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{18d} - \frac{b^2 \log^2(F) F^{a+b(c+dx)^3}}{18d(c+dx)^3} - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} - \frac{b \log(F) F^{a+b(c+dx)^3}}{18d(c+dx)^6}$$

[Out]  $-F^{(a + b*(c + d*x)^3)/(9*d*(c + d*x)^9) - (b*F^{(a + b*(c + d*x)^3)*\operatorname{Log}[F])}/(18*d*(c + d*x)^6) - (b^2*F^{(a + b*(c + d*x)^3)*\operatorname{Log}[F]^2)/(18*d*(c + d*x)^3) + (b^3*F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^3*\operatorname{Log}[F]]*\operatorname{Log}[F]^3)/(18*d)$

**Rubi [A]** time = 0.259571, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^3 \log(F))}{18d} - \frac{b^2 \log^2(F) F^{a+b(c+dx)^3}}{18d(c+dx)^3} - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} - \frac{b \log(F) F^{a+b(c+dx)^3}}{18d(c+dx)^6}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*(c + d*x)^3)/(c + d*x)^{10}}, x]$

[Out]  $-F^{(a + b*(c + d*x)^3)/(9*d*(c + d*x)^9) - (b*F^{(a + b*(c + d*x)^3)*\operatorname{Log}[F])}/(18*d*(c + d*x)^6) - (b^2*F^{(a + b*(c + d*x)^3)*\operatorname{Log}[F]^2)/(18*d*(c + d*x)^3) + (b^3*F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^3*\operatorname{Log}[F]]*\operatorname{Log}[F]^3)/(18*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}/((e_.) + (f_.)*(x_))], x\_Symbol] :> \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}} dx &= -\frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} + \frac{1}{3}(b \log(F)) \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} - \frac{bF^{a+b(c+dx)^3} \log(F)}{18d(c+dx)^6} + \frac{1}{6}(b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} - \frac{bF^{a+b(c+dx)^3} \log(F)}{18d(c+dx)^6} - \frac{b^2 F^{a+b(c+dx)^3} \log^2(F)}{18d(c+dx)^3} + \frac{1}{6}(b^3 \log^3(F)) \int \frac{F^{a+b(c+dx)^3}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9} - \frac{bF^{a+b(c+dx)^3} \log(F)}{18d(c+dx)^6} - \frac{b^2 F^{a+b(c+dx)^3} \log^2(F)}{18d(c+dx)^3} + \frac{b^3 F^a \operatorname{Ei}(b(c+dx)^3 \log(F)) \log^3(F)}{18d} \end{aligned}$$

**Mathematica [A]** time = 0.099647, size = 80, normalized size = 0.66

$$\frac{F^a \left( b^3 \log^3(F) \operatorname{Ei} \left( b(c+dx)^3 \log(F) \right) + \frac{F^{b(c+dx)^3} \left( -b^2 \log^2(F)(c+dx)^6 - b \log(F)(c+dx)^3 - 2 \right)}{(c+dx)^9} \right)}{18d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^10,x]

[Out] (F^a\*(b^3\*ExpIntegralEi[b\*(c + d\*x)^3\*Log[F]]\*Log[F]^3 + (F^(b\*(c + d\*x)^3)\*(-2 - b\*(c + d\*x)^3\*Log[F] - b^2\*(c + d\*x)^6\*Log[F]^2))/(c + d\*x)^9)/(18\*d)

**Maple [F]** time = 0.101, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^10,x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^10,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^10,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^10, x)

**Fricas [B]** time = 1.55468, size = 907, normalized size = 7.5

$$\frac{(b^3 d^9 x^9 + 9 b^3 c d^8 x^8 + 36 b^3 c^2 d^7 x^7 + 84 b^3 c^3 d^6 x^6 + 126 b^3 c^4 d^5 x^5 + 126 b^3 c^5 d^4 x^4 + 84 b^3 c^6 d^3 x^3 + 36 b^3 c^7 d^2 x^2 + 9 b^3 c^8 d x + b^3 c^9)}{18 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^10,x, algorithm="fricas")

[Out] 1/18\*((b^3\*d^9\*x^9 + 9\*b^3\*c\*d^8\*x^8 + 36\*b^3\*c^2\*d^7\*x^7 + 84\*b^3\*c^3\*d^6\*x^6 + 126\*b^3\*c^4\*d^5\*x^5 + 126\*b^3\*c^5\*d^4\*x^4 + 84\*b^3\*c^6\*d^3\*x^3 + 36\*b^3\*c^7\*d^2\*x^2 + 9\*b^3\*c^8\*d\*x + b^3\*c^9)\*F^a\*Ei((b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))\*log(F)^3 - ((b^2\*d^6\*x^6 + 6\*b^2\*c\*d^5\*x^5 + 15\*b^2\*c^2\*d^4\*x^4 + 20\*b^2\*c^3\*d^3\*x^3 + 15\*b^2\*c^4\*d^2\*x^2 + 6\*b^2\*c^5\*d\*x + b^2\*c^6)\*log(F)^2 + (b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F)))/18\*d^9

$$d*x + b^2*c^6)*\log(F)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3) * \log(F) + 2)*F^{(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)} / (d^{10} * x^9 + 9*c*d^9*x^8 + 36*c^2*d^8*x^7 + 84*c^3*d^7*x^6 + 126*c^4*d^6*x^5 + 126*c^5*d^5*x^4 + 84*c^6*d^4*x^3 + 36*c^7*d^3*x^2 + 9*c^8*d^2*x + c^9*d)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)/(d\*x+c)\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^10,x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^10, x)

$$3.291 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}} dx$$

**Optimal.** Leaf size=31

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^3)}{3d}$$

[Out]  $-(b^4 F^a \Gamma(-4, -(b(c+dx)^3 \log(F))) \log(F)^4)/(3d)$

**Rubi [A]** time = 0.0614399, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^3)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^13, x]

[Out]  $-(b^4 F^a \Gamma(-4, -(b(c+dx)^3 \log(F))) \log(F)^4)/(3d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}} dx = -\frac{b^4 F^a \Gamma(-4, -b(c+dx)^3 \log(F)) \log^4(F)}{3d}$$

**Mathematica [A]** time = 0.007538, size = 31, normalized size = 1.

$$-\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^3)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^13, x]

[Out]  $-(b^4 F^a \Gamma(-4, -(b(c+dx)^3 \log(F))) \log(F)^4)/(3d)$

**Maple [F]** time = 0.154, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x)`

[Out] `int(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c)^13, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**13,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c)^13, x)`

$$3.292 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} dx$$

**Optimal.** Leaf size=31

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^3)}{3d}$$

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^3\*Log[F])]\*Log[F]^5)/(3\*d)

**Rubi [A]** time = 0.0615456, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^3)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^16,x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^3\*Log[F])]\*Log[F]^5)/(3\*d)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} dx = \frac{b^5 F^a \Gamma(-5, -b(c+dx)^3 \log(F)) \log^5(F)}{3d}$$

**Mathematica [A]** time = 0.0075347, size = 31, normalized size = 1.

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^3)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^16,x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^3\*Log[F])]\*Log[F]^5)/(3\*d)

**Maple [F]** time = 0.187, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{16}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x)`

[Out] `int(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c)^16, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**16,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^3*b + a)/(d*x + c)^16, x)`

### 3.293 $\int F^{a+b(c+dx)^3} (c+dx)^3 dx$

**Optimal.** Leaf size=49

$$-\frac{F^a(c+dx)^4 \text{Gamma}\left(\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{4/3}}$$

[Out]  $-(F^a(c+dx)^4 \text{Gamma}[4/3, -(b(c+dx)^3 \text{Log}[F])]) / (3d(-(b(c+dx)^3 \text{Log}[F]))^{(4/3)})$

**Rubi [A]** time = 0.0638079, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{F^a(c+dx)^4 \text{Gamma}\left(\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^3, x]

[Out]  $-(F^a(c+dx)^4 \text{Gamma}[4/3, -(b(c+dx)^3 \text{Log}[F])]) / (3d(-(b(c+dx)^3 \text{Log}[F]))^{(4/3)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^3} (c+dx)^3 dx = -\frac{F^a(c+dx)^4 \Gamma\left(\frac{4}{3}, -b(c+dx)^3 \log(F)\right)}{3d(-b(c+dx)^3 \log(F))^{4/3}}$$

**Mathematica [A]** time = 0.0253807, size = 49, normalized size = 1.

$$-\frac{F^a(c+dx)^4 \text{Gamma}\left(\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x)^3, x]

[Out]  $-(F^a(c+dx)^4 \text{Gamma}[4/3, -(b(c+dx)^3 \text{Log}[F])]) / (3d(-(b(c+dx)^3 \text{Log}[F]))^{(4/3)})$

---

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^3} (dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^3,x)

[Out] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate((d\*x + c)^3 \* F^((d\*x + c)^3 \* b + a), x)

---

**Fricas [B]** time = 1.62648, size = 282, normalized size = 5.76

$$\frac{3(bd^3x + bcd^2)F^{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a} \log(F) - (-bd^3 \log(F))^{\frac{2}{3}} F^a \Gamma\left(\frac{1}{3}, -(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3) \log(F)\right) \log(F)}{9b^2d^3 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c)^3,x, algorithm="fricas")

[Out] 1/9\*(3\*(b\*d^3\*x + b\*c\*d^2)\*F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a)\*log(F) - (-b\*d^3\*log(F))^(2/3)\*F^a\*gamma(1/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F)))/(b^2\*d^3\*log(F)^2)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^3} (c+dx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c)\*\*3,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*3)\*(c + d\*x)\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*F^((d*x + c)^3*b + a), x)
```

### 3.294 $\int F^{a+b(c+dx)^3} (c + dx) dx$

**Optimal.** Leaf size=49

$$-\frac{F^a(c+dx)^2 \text{Gamma}\left(\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{2/3}}$$

[Out]  $-(F^{a*(c+d*x)^2} \text{Gamma}[2/3, -(b*(c+d*x)^3 \text{Log}[F])]) / (3*d*(-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)})$

**Rubi [A]** time = 0.0411224, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2218}

$$-\frac{F^a(c+dx)^2 \text{Gamma}\left(\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)\*(c + d\*x), x]

[Out]  $-(F^{a*(c+d*x)^2} \text{Gamma}[2/3, -(b*(c+d*x)^3 \text{Log}[F])]) / (3*d*(-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)})$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^3} (c + dx) dx = -\frac{F^a(c+dx)^2 \Gamma\left(\frac{2}{3}, -b(c+dx)^3 \log(F)\right)}{3d(-b(c+dx)^3 \log(F))^{2/3}}$$

**Mathematica [A]** time = 0.0231367, size = 49, normalized size = 1.

$$-\frac{F^a(c+dx)^2 \text{Gamma}\left(\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(-b \log(F)(c+dx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)\*(c + d\*x), x]

[Out]  $-(F^{a*(c+d*x)^2} \text{Gamma}[2/3, -(b*(c+d*x)^3 \text{Log}[F])]) / (3*d*(-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)})$

---

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^3} (dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c), x)

[Out] int(F^(a+b\*(d\*x+c)^3)\*(d\*x+c), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)F^{(dx+c)^3b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c), x, algorithm="maxima")

[Out] integrate((d\*x + c)\*F^((d\*x + c)^3\*b + a), x)

---

**Fricas [A]** time = 1.58064, size = 157, normalized size = 3.2

$$\frac{(-bd^3 \log(F))^{\frac{1}{3}} F^a \Gamma\left(\frac{2}{3}, -(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3) \log(F)\right)}{3bd^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)\*(d\*x+c), x, algorithm="fricas")

[Out] 1/3\*(-b\*d^3\*log(F))^(1/3)\*F^a\*gamma(2/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))/(b\*d^2\*log(F))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^3} (c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)\*(d\*x+c), x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*3)\*(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)F^{(dx+c)^3b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)*(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*F^((d*x + c)^3*b + a), x)
```

### 3.295 $\int F^{a+b(c+dx)^3} dx$

**Optimal.** Leaf size=47

$$\frac{F^a(c+dx)\Gamma\left(\frac{1}{3}, -b\log(F)(c+dx)^3\right)}{3d\sqrt[3]{-b\log(F)(c+dx)^3}}$$

[Out]  $-(F^{a*(c+d*x)}*\Gamma[1/3, -(b*(c+d*x)^3*\text{Log}[F])])/(3*d*(-(b*(c+d*x)^3*\text{Log}[F]))^{(1/3)})$

**Rubi [A]** time = 0.0070726, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2208}

$$\frac{F^a(c+dx)\Gamma\left(\frac{1}{3}, -b\log(F)(c+dx)^3\right)}{3d\sqrt[3]{-b\log(F)(c+dx)^3}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3), x]

[Out]  $-(F^{a*(c+d*x)}*\Gamma[1/3, -(b*(c+d*x)^3*\text{Log}[F])])/(3*d*(-(b*(c+d*x)^3*\text{Log}[F]))^{(1/3)})$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int F^{a+b(c+dx)^3} dx = -\frac{F^a(c+dx)\Gamma\left(\frac{1}{3}, -b(c+dx)^3\log(F)\right)}{3d\sqrt[3]{-b(c+dx)^3\log(F)}}$$

**Mathematica [A]** time = 0.0160868, size = 47, normalized size = 1.

$$\frac{F^a(c+dx)\Gamma\left(\frac{1}{3}, -b\log(F)(c+dx)^3\right)}{3d\sqrt[3]{-b\log(F)(c+dx)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3), x]

[Out]  $-(F^{a*(c+d*x)}*\Gamma[1/3, -(b*(c+d*x)^3*\text{Log}[F])])/(3*d*(-(b*(c+d*x)^3*\text{Log}[F]))^{(1/3)})$



**Maple [F]** time = 0.017, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3), x)

[Out] int(F^(a+b\*(d\*x+c)^3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3), x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a), x)

---

**Fricas [A]** time = 1.5441, size = 157, normalized size = 3.34

$$\frac{(-bd^3 \log(F))^{\frac{2}{3}} F^a \Gamma\left(\frac{1}{3}, -(bd^3 x^3 + 3bcd^2 x^2 + 3bc^2 dx + bc^3) \log(F)\right)}{3bd^3 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3), x, algorithm="fricas")

[Out] 1/3\*(-b\*d^3\*log(F))^(2/3)\*F^a\*gamma(1/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))/(b\*d^3\*log(F))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3), x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^3*b + a), x)
```

$$3.296 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \sqrt[3]{-b \log(F)(c+dx)^3} \Gamma\left(-\frac{1}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)}$$

[Out]  $-(F^a \Gamma[-1/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(1/3)} / (3*d*(c + d*x))$

**Rubi [A]** time = 0.0643983, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \sqrt[3]{-b \log(F)(c+dx)^3} \Gamma\left(-\frac{1}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^2, x]

[Out]  $-(F^a \Gamma[-1/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(1/3)} / (3*d*(c + d*x))$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx = -\frac{F^a \Gamma\left(-\frac{1}{3}, -b(c+dx)^3 \log(F)\right) \sqrt[3]{-b(c+dx)^3 \log(F)}}{3d(c+dx)}$$

**Mathematica [A]** time = 0.0159363, size = 49, normalized size = 1.

$$\frac{F^a \sqrt[3]{-b \log(F)(c+dx)^3} \Gamma\left(-\frac{1}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^2, x]

[Out]  $-(F^a \Gamma[-1/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(1/3)} / (3*d*(c + d*x))$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^2,x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^2, x)

**Fricas [B]** time = 1.54923, size = 250, normalized size = 5.1

$$\frac{(-bd^3 \log(F))^{\frac{1}{3}} (dx+c) F^a \Gamma\left(\frac{2}{3}, -(bd^3 x^3 + 3bcd^2 x^2 + 3bc^2 dx + bc^3) \log(F)\right) - F^{bd^3 x^3 + 3bcd^2 x^2 + 3bc^2 dx + bc^3 + a} d}{d^3 x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^2,x, algorithm="fricas")

[Out] ((-b\*d^3\*log(F))^(1/3)\*(d\*x + c)\*F^a\*gamma(2/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F)) - F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a)\*d)/(d^3\*x + c\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)/(d\*x+c)\*\*2,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*3)/(c + d\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F(dx+c)^{b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^3*b + a)/(d*x + c)^2, x)
```

$$3.297 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a (-b \log(F)(c+dx)^3)^{2/3} \Gamma\left(-\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^2}$$

[Out]  $-(F^a \Gamma[-2/3, -(b*(c+d*x)^3 \text{Log}[F])]) * (-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)} / (3*d*(c+d*x)^2)$

**Rubi [A]** time = 0.0629146, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a (-b \log(F)(c+dx)^3)^{2/3} \Gamma\left(-\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^3, x]

[Out]  $-(F^a \Gamma[-2/3, -(b*(c+d*x)^3 \text{Log}[F])]) * (-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)} / (3*d*(c+d*x)^2)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3} dx = -\frac{F^a \Gamma\left(-\frac{2}{3}, -b(c+dx)^3 \log(F)\right) (-b(c+dx)^3 \log(F))^{2/3}}{3d(c+dx)^2}$$

**Mathematica [A]** time = 0.0165271, size = 49, normalized size = 1.

$$\frac{F^a (-b \log(F)(c+dx)^3)^{2/3} \Gamma\left(-\frac{2}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^3, x]

[Out]  $-(F^a \Gamma[-2/3, -(b*(c+d*x)^3 \text{Log}[F])]) * (-(b*(c+d*x)^3 \text{Log}[F]))^{(2/3)} / (3*d*(c+d*x)^2)$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^3,x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^3, x)

**Fricas [B]** time = 1.5409, size = 301, normalized size = 6.14

$$\frac{(-bd^3 \log(F))^{\frac{2}{3}} (d^2x^2 + 2cdx + c^2) F^a \Gamma\left(\frac{1}{3}, -(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3) \log(F)\right) - F^{bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + a} d^{\frac{2}{3}}}{2(d^5x^2 + 2cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^3,x, algorithm="fricas")

[Out] 1/2\*((-b\*d^3\*log(F))^(2/3)\*(d^2\*x^2 + 2\*c\*d\*x + c^2)\*F^a\*gamma(1/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F)) - F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a)\*d^(2/3))/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)/(d\*x+c)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^3*b + a)/(d*x + c)^3, x)
```



$$3.298 \quad \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \left(-b \log(F)(c+dx)^3\right)^{4/3} \Gamma\left(-\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^4}$$

[Out]  $-(F^a \Gamma[-4/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(4/3)} / (3*d*(c + d*x)^4)$

**Rubi [A]** time = 0.0626988, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \left(-b \log(F)(c+dx)^3\right)^{4/3} \Gamma\left(-\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^5, x]

[Out]  $-(F^a \Gamma[-4/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(4/3)} / (3*d*(c + d*x)^4)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx = -\frac{F^a \Gamma\left(-\frac{4}{3}, -b(c+dx)^3 \log(F)\right) \left(-b(c+dx)^3 \log(F)\right)^{4/3}}{3d(c+dx)^4}$$

**Mathematica [A]** time = 0.0231699, size = 49, normalized size = 1.

$$\frac{F^a \left(-b \log(F)(c+dx)^3\right)^{4/3} \Gamma\left(-\frac{4}{3}, -b \log(F)(c+dx)^3\right)}{3d(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^3)/(c + d\*x)^5, x]

[Out]  $-(F^a \Gamma[-4/3, -(b*(c + d*x)^3 \text{Log}[F])]) * (-(b*(c + d*x)^3 \text{Log}[F]))^{(4/3)} / (3*d*(c + d*x)^4)$

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^5,x)

[Out] int(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^3 b+a}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^5,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^3\*b + a)/(d\*x + c)^5, x)

**Fricas [B]** time = 1.618, size = 501, normalized size = 10.22

$$\frac{3 \left( b d^4 x^4 + 4 b c d^3 x^3 + 6 b c^2 d^2 x^2 + 4 b c^3 d x + b c^4 \right) \left( -b d^3 \log(F) \right)^{\frac{1}{3}} F^a \Gamma\left(\frac{2}{3}, -(b d^3 x^3 + 3 b c d^2 x^2 + 3 b c^2 d x + b c^3) \log(F)\right) \log(F)}{4 \left( d^6 x^4 + 4 c d^5 x^3 + 6 c^2 d^4 x^2 + 4 c^3 d^3 x + c^4 d^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^3)/(d\*x+c)^5,x, algorithm="fricas")

[Out] 1/4\*(3\*(b\*d^4\*x^4 + 4\*b\*c\*d^3\*x^3 + 6\*b\*c^2\*d^2\*x^2 + 4\*b\*c^3\*d\*x + b\*c^4)\*(-b\*d^3\*log(F))^(1/3)\*F^a\*gamma(2/3, -(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))\*log(F) - (3\*(b\*d^4\*x^3 + 3\*b\*c\*d^3\*x^2 + 3\*b\*c^2\*d^2\*x + b\*c^3\*d)\*log(F) + d)\*F^(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + a))/(d^6\*x^4 + 4\*c\*d^5\*x^3 + 6\*c^2\*d^4\*x^2 + 4\*c^3\*d^3\*x + c^4\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*3)/(d\*x+c)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F(dx+c)^{b+a}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^3*b + a)/(d*x + c)^5, x)
```

### 3.299 $\int f^{a+b\sqrt{c+dx}} dx$

**Optimal.** Leaf size=64

$$\frac{2\sqrt{c+dx}f^{a+b\sqrt{c+dx}}}{bd\log(f)} - \frac{2f^{a+b\sqrt{c+dx}}}{b^2d\log^2(f)}$$

[Out]  $(-2*f^{(a + b*\text{Sqrt}[c + d*x])})/(b^2*d*\text{Log}[f]^2) + (2*f^{(a + b*\text{Sqrt}[c + d*x])}*\text{Sqrt}[c + d*x])/(b*d*\text{Log}[f])$

**Rubi [A]** time = 0.0324213, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2207, 2176, 2194}

$$\frac{2\sqrt{c+dx}f^{a+b\sqrt{c+dx}}}{bd\log(f)} - \frac{2f^{a+b\sqrt{c+dx}}}{b^2d\log^2(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(a + b*\text{Sqrt}[c + d*x])}, x]$

[Out]  $(-2*f^{(a + b*\text{Sqrt}[c + d*x])})/(b^2*d*\text{Log}[f]^2) + (2*f^{(a + b*\text{Sqrt}[c + d*x])}*\text{Sqrt}[c + d*x])/(b*d*\text{Log}[f])$

#### Rule 2207

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)}), x\_Symbol] := \text{With}[\{k = \text{Denominator}[n]\}, \text{Dist}[k/d, \text{Subst}[\text{Int}[x^{(k-1)}*F^{(a + b*x^{(k*n)})}, x], x, (c + d*x)^{(1/k)}], x]] /; \text{FreeQ}[\{F, a, b, c, d\}, x] \&\& \text{IntegerQ}[2/n] \&\& !\text{IntegerQ}[n]$

#### Rule 2176

$\text{Int}[(b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}), x\_Symbol] := \text{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\$UseGamma == True$

#### Rule 2194

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}), x\_Symbol] := \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int f^{a+b\sqrt{c+dx}} dx &= \frac{2 \text{Subst}\left(\int f^{a+bx} x dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2f^{a+b\sqrt{c+dx}}\sqrt{c+dx}}{bd\log(f)} - \frac{2 \text{Subst}\left(\int f^{a+bx} dx, x, \sqrt{c+dx}\right)}{bd\log(f)} \\ &= -\frac{2f^{a+b\sqrt{c+dx}}}{b^2d\log^2(f)} + \frac{2f^{a+b\sqrt{c+dx}}\sqrt{c+dx}}{bd\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0340865, size = 42, normalized size = 0.66

$$\frac{2f^{a+b\sqrt{c+dx}}(b\log(f)\sqrt{c+dx}-1)}{b^2d\log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*Sqrt[c + d\*x]), x]

[Out] (2\*f^(a + b\*Sqrt[c + d\*x])\*(-1 + b\*Sqrt[c + d\*x]\*Log[f]))/(b^2\*d\*Log[f]^2)

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int f^{a+b\sqrt{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*(d\*x+c)^(1/2)), x)

[Out] int(f^(a+b\*(d\*x+c)^(1/2)), x)

**Maxima [A]** time = 1.01327, size = 58, normalized size = 0.91

$$\frac{2(\sqrt{dx+cb}f^a\log(f)-f^a)f^{\sqrt{dx+cb}}}{b^2d\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/2)), x, algorithm="maxima")

[Out] 2\*(sqrt(d\*x + c)\*b\*f^a\*log(f) - f^a)\*f^(sqrt(d\*x + c)\*b)/(b^2\*d\*log(f)^2)

**Fricas [A]** time = 1.52162, size = 117, normalized size = 1.83

$$\frac{2(\sqrt{dx+cb}\log(f)-1)e^{(\sqrt{dx+cb}\log(f)+a\log(f))}}{b^2d\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/2)), x, algorithm="fricas")

[Out] 2\*(sqrt(d\*x + c)\*b\*log(f) - 1)\*e^(sqrt(d\*x + c)\*b\*log(f) + a\*log(f))/(b^2\*d\*log(f)^2)

**Sympy [A]** time = 0.75721, size = 76, normalized size = 1.19

$$\begin{cases} x & \text{for } b = 0 \wedge d = 0 \wedge f = 1 \\ f^a x & \text{for } b = 0 \\ x & \text{for } f = 1 \\ f^{a+b\sqrt{c}} x & \text{for } d = 0 \\ \frac{2f^a f^{b\sqrt{c+dx}} \sqrt{c+dx}}{bd \log(f)} - \frac{2f^a f^{b\sqrt{c+dx}}}{b^2 d \log(f)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*(d\*x+c)\*\*(1/2)),x)

[Out] Piecewise((x, Eq(b, 0) & Eq(d, 0) & Eq(f, 1)), (f\*\*a\*x, Eq(b, 0)), (x, Eq(f, 1)), (f\*\*(a + b\*sqrt(c))\*x, Eq(d, 0)), (2\*f\*\*a\*f\*\*(b\*sqrt(c + d\*x))\*sqrt(c + d\*x)/(b\*d\*log(f)) - 2\*f\*\*a\*f\*\*(b\*sqrt(c + d\*x))/(b\*\*2\*d\*log(f)\*\*2), True))

**Giac [B]** time = 2.13246, size = 1436, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/2)),x, algorithm="giac")

[Out] (2\*(2\*((pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))\*(pi\*sqrt(d\*x + c)\*b\*sgn(f) - pi\*sqrt(d\*x + c)\*b)/((pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)^2 + 4\*(pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))^2) + (pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)\*(sqrt(d\*x + c)\*b\*log(abs(f)) - 1)/((pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)^2 + 4\*(pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))^2))\*cos(-1/2\*pi\*sqrt(d\*x + c)\*b\*sgn(f) - 1/2\*pi\*a\*sgn(f) + 1/2\*pi\*sqrt(d\*x + c)\*b + 1/2\*pi\*a) + ((pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)\*(pi\*sqrt(d\*x + c)\*b\*sgn(f) - pi\*sqrt(d\*x + c)\*b)/((pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)^2 + 4\*(pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))^2) - 4\*(pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))\*(sqrt(d\*x + c)\*b\*log(abs(f)) - 1)/((pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2)^2 + 4\*(pi\*b^2\*log(abs(f))\*sgn(f) - pi\*b^2\*log(abs(f)))^2))\*sin(-1/2\*pi\*sqrt(d\*x + c)\*b\*sgn(f) - 1/2\*pi\*a\*sgn(f) + 1/2\*pi\*sqrt(d\*x + c)\*b + 1/2\*pi\*a))\*e^(sqrt(d\*x + c)\*b\*log(abs(f)) + a\*log(abs(f))) - ((2\*sqrt(d\*x + c)\*b\*i\*log(abs(f)) - pi\*sqrt(d\*x + c)\*b\*sgn(f) + pi\*sqrt(d\*x + c)\*b - 2\*i)\*e^(1/2\*(pi\*sqrt(d\*x + c)\*b\*(sgn(f) - 1) + pi\*a\*(sgn(f) - 1))\*i)/(2\*pi\*b^2\*i\*log(abs(f))\*sgn(f) - 2\*pi\*b^2\*i\*log(abs(f)) + pi^2\*b^2\*sgn(f) - pi^2\*b^2 + 2\*b^2\*log(abs(f))^2) + (2\*sqrt(d\*x + c)\*b\*i\*log(abs(f)) + pi\*sqrt(d\*x + c)\*b\*sgn(f) - pi\*sqrt(d\*x + c)\*b - 2\*i)\*e^(-1/2\*(pi\*sqrt(d\*x + c)\*b\*(sgn(f) - 1) + pi\*a\*(sgn(f) - 1))\*i)/(2\*pi\*b^2\*i\*log(abs(f))\*sgn(f) - 2\*pi\*b^2\*i\*log(abs(f)) - pi^2\*b^2\*sgn(f) + pi^2\*b^2 - 2\*b^2\*log(abs(f))^2))\*e^(sqrt(d\*x + c)\*b\*log(abs(f)) + a\*log(abs(f)))/i - 2\*(2\*pi\*i\*abs(f)^a\*cos(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a)\*log(abs(f))\*sgn(f) - pi^2\*i\*abs(f)^a\*sgn(f)\*sin(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a) - 2\*pi\*i\*abs(f)^a\*cos(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a)\*log(abs(f)) + pi^2\*i\*abs(f)^a\*cos(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a)\*sgn(f) + pi^2\*i\*abs(f)^a\*sin(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a) - 2\*i\*abs(f)^a\*log(abs(f))^2\*sin(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a) + 2\*pi\*abs(f)^a\*log(abs(f))\*sgn(f)\*sin(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a) - pi^2\*abs(f)^a\*cos(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a) + 2\*abs(f)^a\*cos(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a)\*log(abs(f))^2 - 2\*pi\*abs(f)^a\*log(abs(f))\*sin(1/2\*pi\*a\*sgn(f) - 1/2\*pi\*a))/(pi^4\*b^2\*sgn(f) + 2\*pi^2\*

$$\frac{b^2 \log(\operatorname{abs}(f))^2 \operatorname{sgn}(f) - \pi^4 b^2 - 2\pi^2 b^2 \log(\operatorname{abs}(f))^2 - 2b^2 \log(\operatorname{abs}(f))^4}{d}$$

### 3.300 $\int f^{a+b} \sqrt[3]{c+dx} dx$

**Optimal.** Leaf size=100

$$-\frac{6\sqrt[3]{c+dx} f^{a+b} \sqrt[3]{c+dx}}{b^2 d \log^2(f)} + \frac{6 f^{a+b} \sqrt[3]{c+dx}}{b^3 d \log^3(f)} + \frac{3(c+dx)^{2/3} f^{a+b} \sqrt[3]{c+dx}}{bd \log(f)}$$

[Out]  $(6*f^{(a + b*(c + d*x)^{(1/3))})/(b^3*d*\text{Log}[f]^3) - (6*f^{(a + b*(c + d*x)^{(1/3))})*(c + d*x)^{(1/3)})/(b^2*d*\text{Log}[f]^2) + (3*f^{(a + b*(c + d*x)^{(1/3))})*(c + d*x)^{(2/3)})/(b*d*\text{Log}[f])$

**Rubi [A]** time = 0.0640819, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2207, 2176, 2194}

$$-\frac{6\sqrt[3]{c+dx} f^{a+b} \sqrt[3]{c+dx}}{b^2 d \log^2(f)} + \frac{6 f^{a+b} \sqrt[3]{c+dx}}{b^3 d \log^3(f)} + \frac{3(c+dx)^{2/3} f^{a+b} \sqrt[3]{c+dx}}{bd \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*(c + d\*x)^(1/3)), x]

[Out]  $(6*f^{(a + b*(c + d*x)^{(1/3))})/(b^3*d*\text{Log}[f]^3) - (6*f^{(a + b*(c + d*x)^{(1/3))})*(c + d*x)^{(1/3)})/(b^2*d*\text{Log}[f]^2) + (3*f^{(a + b*(c + d*x)^{(1/3))})*(c + d*x)^{(2/3)})/(b*d*\text{Log}[f])$

#### Rule 2207

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_), x\_Symbol] := With[{k = Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)\*F^(a + b\*x^(k\*n)), x], x, (c + d\*x)^(1/k)], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && !IntegerQ[n]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps



$$\begin{aligned}
\int f^{a+b\sqrt[3]{c+dx}} dx &= \frac{3 \operatorname{Subst}\left(\int f^{a+bx} x^2 dx, x, \sqrt[3]{c+dx}\right)}{d} \\
&= \frac{3 f^{a+b\sqrt[3]{c+dx}} (c+dx)^{2/3}}{bd \log(f)} - \frac{6 \operatorname{Subst}\left(\int f^{a+bx} x dx, x, \sqrt[3]{c+dx}\right)}{bd \log(f)} \\
&= -\frac{6 f^{a+b\sqrt[3]{c+dx}} \sqrt[3]{c+dx}}{b^2 d \log^2(f)} + \frac{3 f^{a+b\sqrt[3]{c+dx}} (c+dx)^{2/3}}{bd \log(f)} + \frac{6 \operatorname{Subst}\left(\int f^{a+bx} dx, x, \sqrt[3]{c+dx}\right)}{b^2 d \log^2(f)} \\
&= \frac{6 f^{a+b\sqrt[3]{c+dx}}}{b^3 d \log^3(f)} - \frac{6 f^{a+b\sqrt[3]{c+dx}} \sqrt[3]{c+dx}}{b^2 d \log^2(f)} + \frac{3 f^{a+b\sqrt[3]{c+dx}} (c+dx)^{2/3}}{bd \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0412831, size = 60, normalized size = 0.6

$$\frac{3 f^{a+b\sqrt[3]{c+dx}} (b^2 \log^2(f) (c+dx)^{2/3} - 2b \log(f) \sqrt[3]{c+dx} + 2)}{b^3 d \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*(c + d\*x)^(1/3)), x]

[Out] (3\*f^(a + b\*(c + d\*x)^(1/3))\*(2 - 2\*b\*(c + d\*x)^(1/3)\*Log[f] + b^2\*(c + d\*x)^(2/3)\*Log[f]^2)/(b^3\*d\*Log[f]^3)

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int f^{a+b\sqrt[3]{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a+b\*(d\*x+c)^(1/3)), x)

[Out] int(f^(a+b\*(d\*x+c)^(1/3)), x)

**Maxima [A]** time = 1.01445, size = 84, normalized size = 0.84

$$\frac{3 \left( (dx+c)^{\frac{2}{3}} b^2 f^a \log(f)^2 - 2(dx+c)^{\frac{1}{3}} b f^a \log(f) + 2 f^a \right) f^{(dx+c)^{\frac{1}{3}} b}}{b^3 d \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/3)), x, algorithm="maxima")

[Out] 3\*((d\*x + c)^(2/3)\*b^2\*f^a\*log(f)^2 - 2\*(d\*x + c)^(1/3)\*b\*f^a\*log(f) + 2\*f^a)\*f^((d\*x + c)^(1/3)\*b)/(b^3\*d\*log(f)^3)

**Fricas [A]** time = 1.57271, size = 167, normalized size = 1.67

$$\frac{3 \left( (dx+c)^{\frac{2}{3}} b^2 \log(f)^2 - 2 (dx+c)^{\frac{1}{3}} b \log(f) + 2 \right) e^{\left( (dx+c)^{\frac{1}{3}} b \log(f) + a \log(f) \right)}}{b^3 d \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/3)),x, algorithm="fricas")

[Out] 3\*((d\*x + c)^(2/3)\*b^2\*log(f)^2 - 2\*(d\*x + c)^(1/3)\*b\*log(f) + 2)\*e^((d\*x + c)^(1/3)\*b\*log(f) + a\*log(f))/(b^3\*d\*log(f)^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+b\sqrt[3]{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(a+b\*(d\*x+c)\*\*(1/3)),x)

[Out] Integral(f\*\*(a + b\*(c + d\*x)\*\*(1/3)), x)

**Giac [B]** time = 1.52837, size = 1804, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(a+b\*(d\*x+c)^(1/3)),x, algorithm="giac")

[Out] 
$$\frac{3}{2} * (2 * ((3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f))) + 2 * b^3 * \log(\text{abs}(f))^3) * (\pi^2 * (d * x + c)^{(2/3)} * b^2 * \text{sgn}(f) - \pi^2 * (d * x + c)^{(2/3)} * b^2 + 2 * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f))^2 - 4 * (d * x + c)^{(1/3)} * b * \log(\text{abs}(f)) + 4) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2) - 2 * (\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2) * (\pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) * \text{sgn}(f) - \pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) - \pi * (d * x + c)^{(1/3)} * b * \text{sgn}(f) + \pi * (d * x + c)^{(1/3)} * b) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2) * \cos(-1/2 * \pi * (d * x + c)^{(1/3)} * b * \text{sgn}(f) - 1/2 * \pi * a * \text{sgn}(f) + 1/2 * \pi * (d * x + c)^{(1/3)} * b + 1/2 * \pi * a) + ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2) * (\pi^2 * (d * x + c)^{(2/3)} * b^2 * \text{sgn}(f) - \pi^2 * (d * x + c)^{(2/3)} * b^2 + 2 * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f))^2 - 4 * (d * x + c)^{(1/3)} * b * \log(\text{abs}(f)) + 4) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2) + 2 * (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3) * (\pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) * \text{sgn}(f) - \pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) - \pi * (d * x + c)^{(1/3)} * b * \text{sgn}(f) + \pi * (d * x + c)^{(1/3)} * b) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2) + 2 * (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3) * (\pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) * \text{sgn}(f) - \pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) - \pi * (d * x + c)^{(1/3)} * b * \text{sgn}(f) + \pi * (d * x + c)^{(1/3)} * b) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2) + 2 * (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3) * (\pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) * \text{sgn}(f) - \pi * (d * x + c)^{(2/3)} * b^2 * \log(\text{abs}(f)) - \pi * (d * x + c)^{(1/3)} * b * \text{sgn}(f) + \pi * (d * x + c)^{(1/3)} * b) / ((\pi^3 * b^3 * \text{sgn}(f) - 3 * \pi * b^3 * \log(\text{abs}(f))^2 * \text{sgn}(f) - \pi^3 * b^3 + 3 * \pi * b^3 * \log(\text{abs}(f))^2)^2 + (3 * \pi^2 * b^3 * \log(\text{abs}(f)) * \text{sgn}(f) - 3 * \pi^2 * b^3 * \log(\text{abs}(f)) + 2 * b^3 * \log(\text{abs}(f))^3)^2)$$

$$\begin{aligned}
&)) + 2*b^3*\log(\text{abs}(f))^3)^2)*\sin(-1/2*\pi*(d*x + c)^{1/3}*b*\text{sgn}(f) - 1/2*\pi \\
&*a*\text{sgn}(f) + 1/2*\pi*(d*x + c)^{1/3}*b + 1/2*\pi*a))*e^{((d*x + c)^{1/3}*b*\log(\text{abs}(f)) + a*\log(\text{abs}(f)))} + ((\pi^2*(d*x + c)^{2/3}*b^2*i*\text{sgn}(f) - \pi^2*(d*x \\
&+ c)^{2/3}*b^2*i + 2*(d*x + c)^{2/3}*b^2*i*\log(\text{abs}(f))^2 - 2*\pi*(d*x + c)^{2/3}*b^2*\log(\text{abs}(f))*\text{sgn}(f) + 2*\pi*(d*x + c)^{2/3}*b^2*\log(\text{abs}(f)) - 4*(d*x \\
&+ c)^{1/3}*b*i*\log(\text{abs}(f)) + 2*\pi*(d*x + c)^{1/3}*b*\text{sgn}(f) - 2*\pi*(d*x + c)^{1/3}*b + 4*i)*e^{1/2*(\pi*(d*x + c)^{1/3}*b*(\text{sgn}(f) - 1) + \pi*a*(\text{sgn}(f) - \\
&1))*i}/(\pi^3*b^3*i*\text{sgn}(f) - 3*\pi*b^3*i*\log(\text{abs}(f))^2*\text{sgn}(f) - \pi^3*b^3*i + \\
&3*\pi*b^3*i*\log(\text{abs}(f))^2 - 3*\pi^2*b^3*\log(\text{abs}(f))*\text{sgn}(f) + 3*\pi^2*b^3*\log(\text{abs}(f)) - 2*b^3*\log(\text{abs}(f))^3) + (\pi^2*(d*x + c)^{2/3}*b^2*i*\text{sgn}(f) - \pi^2* \\
&(d*x + c)^{2/3}*b^2*i + 2*(d*x + c)^{2/3}*b^2*i*\log(\text{abs}(f))^2 + 2*\pi*(d*x + c)^{2/3}*b^2*\log(\text{abs}(f))*\text{sgn}(f) - 2*\pi*(d*x + c)^{2/3}*b^2*\log(\text{abs}(f)) - 4 \\
&)*(d*x + c)^{1/3}*b*i*\log(\text{abs}(f)) - 2*\pi*(d*x + c)^{1/3}*b*\text{sgn}(f) + 2*\pi*(d*x + c)^{1/3}*b + 4*i)*e^{-1/2*(\pi*(d*x + c)^{1/3}*b*(\text{sgn}(f) - 1) + \pi*a*(\text{sgn}(f) - \\
&1))*i}/(\pi^3*b^3*i*\text{sgn}(f) - 3*\pi*b^3*i*\log(\text{abs}(f))^2*\text{sgn}(f) - \pi^3*b^3*i + 3*\pi*b^3*i*\log(\text{abs}(f))^2 + 3*\pi^2*b^3*\log(\text{abs}(f))*\text{sgn}(f) - 3*\pi^2*b^3* \\
&3*\log(\text{abs}(f)) + 2*b^3*\log(\text{abs}(f))^3))*e^{((d*x + c)^{1/3}*b*\log(\text{abs}(f)) + a*\log(\text{abs}(f)))/i)/d
\end{aligned}$$

### 3.301 $\int F^{a+\frac{b}{c+dx}}(c+dx)^m dx$

**Optimal.** Leaf size=50

$$\frac{F^a(c+dx)^{m+1}\left(-\frac{b\log(F)}{c+dx}\right)^{m+1}\Gamma\left(-m-1,-\frac{b\log(F)}{c+dx}\right)}{d}$$

[Out]  $(F^a(c+dx)^{(1+m)}\Gamma[-1-m, -((b*\text{Log}[F])/(c+dx))]*(-((b*\text{Log}[F])/(c+dx)))^{(1+m)})/d$

**Rubi [A]** time = 0.0437885, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1}\left(-\frac{b\log(F)}{c+dx}\right)^{m+1}\Gamma\left(-m-1,-\frac{b\log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))\*(c + d\*x)^m, x]

[Out]  $(F^a(c+dx)^{(1+m)}\Gamma[-1-m, -((b*\text{Log}[F])/(c+dx))]*(-((b*\text{Log}[F])/(c+dx)))^{(1+m)})/d$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n\*Log[F]))]/(f\*n\*(-(b\*(c + d\*x)^(n\*Log[F])))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m dx = \frac{F^a(c+dx)^{1+m}\Gamma\left(-1-m,-\frac{b\log(F)}{c+dx}\right)\left(-\frac{b\log(F)}{c+dx}\right)^{1+m}}{d}$$

**Mathematica [A]** time = 0.0181963, size = 50, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1}\left(-\frac{b\log(F)}{c+dx}\right)^{m+1}\Gamma\left(-m-1,-\frac{b\log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))\*(c + d\*x)^m, x]

[Out]  $(F^a(c+dx)^{(1+m)}\Gamma[-1-m, -((b*\text{Log}[F])/(c+dx))]*(-((b*\text{Log}[F])/(c+dx)))^{(1+m)})/d$

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{dx+c}} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))\*(d\*x+c)^m,x)

[Out] int(F^(a+b/(d\*x+c))\*(d\*x+c)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^(a + b/(d\*x + c)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m F^{\frac{adx+ac+b}{dx+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^m,x, algorithm="fricas")

[Out] integral((d\*x + c)^m \* F^((a\*d\*x + a\*c + b)/(d\*x + c)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))\*(d\*x+c)\*\*m,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate((d\*x + c)^m \* F^(a + b/(d\*x + c)), x)

### 3.302 $\int F^{a+\frac{b}{c+dx}}(c+dx)^4 dx$

**Optimal.** Leaf size=29

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{c+dx}\right)}{d}$$

[Out]  $-\left(b^5 F^a \Gamma[-5, -\frac{b \log(F)}{c+dx}]\right) \log[F]^5 / d$

**Rubi [A]** time = 0.0446677, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))\*(c + d\*x)^4, x]

[Out]  $-\left(b^5 F^a \Gamma[-5, -\frac{b \log(F)}{c+dx}]\right) \log[F]^5 / d$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4 dx = -\frac{b^5 F^a \Gamma\left(-5, -\frac{b \log(F)}{c+dx}\right) \log^5(F)}{d}$$

**Mathematica [A]** time = 0.0060069, size = 29, normalized size = 1.

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))\*(c + d\*x)^4, x]

[Out]  $-\left(b^5 F^a \Gamma[-5, -\frac{b \log(F)}{c+dx}]\right) \log[F]^5 / d$

**Maple [B]** time = 0.122, size = 534, normalized size = 18.4

$$\frac{d^4 F^a x^5}{5} F^{\frac{b}{dx+c}} + d^3 F^a F^{\frac{b}{dx+c}} c x^4 + 2 d^2 F^a F^{\frac{b}{dx+c}} c^2 x^3 + 2 d F^a F^{\frac{b}{dx+c}} c^3 x^2 + F^a F^{\frac{b}{dx+c}} c^4 x + \frac{F^a c^5}{5 d} F^{\frac{b}{dx+c}} + \frac{\ln(F) b d^3 F^a x^4}{20} F^{\frac{b}{dx+c}} + \frac{\ln(F)}{20} F^{\frac{b}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c))*(d*x+c)^4,x)`

[Out]  $\frac{1}{5}d^4F^aF^{b/(d*x+c)}x^5+d^3F^aF^{b/(d*x+c)}c*x^4+2d^2F^aF^{b/(d*x+c)}c^2*x^3+2dF^aF^{b/(d*x+c)}c^3*x^2+F^aF^{b/(d*x+c)}c^4*x+1/5/dF^aF^{b/(d*x+c)}c^5+1/20*d^3*b*\ln(F)*F^aF^{b/(d*x+c)}x^4+1/5*d^2*b*\ln(F)*F^aF^{b/(d*x+c)}c*x^3+3/10*d*b*\ln(F)*F^aF^{b/(d*x+c)}c^2*x^2+1/5*b*\ln(F)*F^aF^{b/(d*x+c)}c^3*x+1/20/d*b*\ln(F)*F^aF^{b/(d*x+c)}c^4+1/60*d^2*b^2*\ln(F)^2F^aF^{b/(d*x+c)}x^3+1/20*d*b^2*\ln(F)^2F^aF^{b/(d*x+c)}c*x^2+1/20*b^2*\ln(F)^2F^aF^{b/(d*x+c)}c^2*x+1/60/d*b^2*\ln(F)^2F^aF^{b/(d*x+c)}c^3+1/120*d*b^3*\ln(F)^3F^aF^{b/(d*x+c)}x^2+1/60*b^3*\ln(F)^3F^aF^{b/(d*x+c)}c*x+1/120/d*b^3*\ln(F)^3F^aF^{b/(d*x+c)}c^2+1/120*b^4*\ln(F)^4F^aF^{b/(d*x+c)}x+1/120/d*b^4*\ln(F)^4F^aF^{b/(d*x+c)}c+1/120/d*b^5*\ln(F)^5F^a*Ei(1,-b*\ln(F)/(d*x+c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{120} \left( 24 F^a d^4 x^5 + 6 (F^a b d^3 \log(F) + 20 F^a c d^3) x^4 + 2 (F^a b^2 d^2 \log(F)^2 + 12 F^a b c d^2 \log(F) + 120 F^a c^2 d^2) x^3 + (F^a b^3 d \log(F)^3 + 6 F^a b^2 c d \log(F)^2 + 36 F^a b c^2 d \log(F) + 240 F^a c^3 d) x^2 + (F^a b^4 \log(F)^4 + 2 F^a b^3 c \log(F)^3 + 6 F^a b^2 c^2 \log(F)^2 + 24 F^a b c^3 \log(F) + 120 F^a c^4) x \right) F^{b/(d*x+c)} + \text{integrate}(1/120*(F^a*b^5*d*x*\log(F)^5 - F^a*b^4*c^2*\log(F)^4 - 2*F^a*b^3*c^3*\log(F)^3 - 6*F^a*b^2*c^4*\log(F)^2 - 24*F^a*b*c^5*\log(F))*F^{b/(d*x+c)})/(d^2*x^2 + 2*c*d*x + c^2), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))*(d*x+c)^4,x, algorithm="maxima")`

[Out]  $\frac{1}{120}*(24*F^a*d^4*x^5 + 6*(F^a*b*d^3*\log(F) + 20*F^a*c*d^3)*x^4 + 2*(F^a*b^2*d^2*\log(F)^2 + 12*F^a*b*c*d^2*\log(F) + 120*F^a*c^2*d^2)*x^3 + (F^a*b^3*d*\log(F)^3 + 6*F^a*b^2*c*d*\log(F)^2 + 36*F^a*b*c^2*d*\log(F) + 240*F^a*c^3*d)*x^2 + (F^a*b^4*\log(F)^4 + 2*F^a*b^3*c*\log(F)^3 + 6*F^a*b^2*c^2*\log(F)^2 + 24*F^a*b*c^3*\log(F) + 120*F^a*c^4)*x)*F^{b/(d*x+c)} + \text{integrate}(1/120*(F^a*b^5*d*x*\log(F)^5 - F^a*b^4*c^2*\log(F)^4 - 2*F^a*b^3*c^3*\log(F)^3 - 6*F^a*b^2*c^4*\log(F)^2 - 24*F^a*b*c^5*\log(F))*F^{b/(d*x+c)})/(d^2*x^2 + 2*c*d*x + c^2), x)$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))*(d*x+c)^4,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c))*(d*x+c)**4,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 F^{a + \frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^4,x, algorithm="giac")

[Out] integrate((d\*x + c)^4\*F^(a + b/(d\*x + c)), x)



### 3.303 $\int F^{a+\frac{b}{c+dx}}(c+dx)^3 dx$

**Optimal.** Leaf size=28

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{c+dx}\right)}{d}$$

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x))])\*Log[F]^4/d

**Rubi [A]** time = 0.0441052, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))\*(c + d\*x)^3, x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x))])\*Log[F]^4/d

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3 dx = \frac{b^4 F^a \Gamma\left(-4, -\frac{b \log(F)}{c+dx}\right) \log^4(F)}{d}$$

**Mathematica [A]** time = 0.0057956, size = 28, normalized size = 1.

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))\*(c + d\*x)^3, x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x))])\*Log[F]^4/d

**Maple [B]** time = 0.086, size = 368, normalized size = 13.1

$$\frac{d^3 F^a x^4}{4} F^{\frac{b}{dx+c}} + d^2 F^a F^{\frac{b}{dx+c}} c x^3 + \frac{3 d F^a c^2 x^2}{2} F^{\frac{b}{dx+c}} + F^a F^{\frac{b}{dx+c}} c^3 x + \frac{F^a c^4}{4 d} F^{\frac{b}{dx+c}} + \frac{\ln(F) b d^2 F^a x^3}{12} F^{\frac{b}{dx+c}} + \frac{\ln(F) b d F^a c x^2}{4} F^{\frac{b}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c))*(d*x+c)^3,x)`

[Out]  $\frac{1}{4}d^3F^aF^{b/(d*x+c)}x^4 + d^2F^aF^{b/(d*x+c)}c^3x^3 + \frac{3}{2}dF^aF^{b/(d*x+c)}c^2x^2 + F^aF^{b/(d*x+c)}c^3x + \frac{1}{4}dF^aF^{b/(d*x+c)}c^4 + \frac{1}{12}d^2*b*\ln(F)*F^aF^{b/(d*x+c)}x^3 + \frac{1}{4}d*b*\ln(F)*F^aF^{b/(d*x+c)}c^2x^2 + \frac{1}{4}b*\ln(F)*F^aF^{b/(d*x+c)}c^2x + \frac{1}{12}d*b*\ln(F)*F^aF^{b/(d*x+c)}c^3 + \frac{1}{24}d*b^2*\ln(F)^2F^aF^{b/(d*x+c)}x^2 + \frac{1}{12}b^2*\ln(F)^2F^aF^{b/(d*x+c)}c^2x + \frac{1}{4}d*b^2*\ln(F)^2F^aF^{b/(d*x+c)}c^2 + \frac{1}{24}b^3*\ln(F)^3F^aF^{b/(d*x+c)}x + \frac{1}{24}d*b^3*\ln(F)^3F^aF^{b/(d*x+c)}c + \frac{1}{24}d*b^4*\ln(F)^4F^aEi(1, -b*\ln(F)/(d*x+c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{24} \left( 6F^a d^3 x^4 + 2(F^a b d^2 \log(F) + 12F^a c d^2) x^3 + (F^a b^2 d \log(F)^2 + 6F^a b c d \log(F) + 36F^a c^2 d) x^2 + (F^a b^3 \log(F)^3 + 2F^a b^2 d \log(F)^2 + 6F^a b^2 c d \log(F) + 36F^a b^2 c^2 d) x + F^a b^3 \log(F)^3 + 2F^a b^2 d \log(F)^2 + 6F^a b^2 c d \log(F) + 24F^a b^2 c^2 d) F^{b/(d*x+c)} + \int \frac{1}{24} (F^a b^4 d^4 x \log(F)^4 - F^a b^3 c^2 \log(F)^3 - 2F^a b^2 c^3 \log(F)^2 - 6F^a b^2 c^4 \log(F)) F^{b/(d*x+c)} / (d^2 x^2 + 2c d x + c^2), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{24} (6F^a d^3 x^4 + 2(F^a b d^2 \log(F) + 12F^a c d^2) x^3 + (F^a b^2 d \log(F)^2 + 6F^a b c d \log(F) + 36F^a c^2 d) x^2 + (F^a b^3 \log(F)^3 + 2F^a b^2 d \log(F)^2 + 6F^a b^2 c d \log(F) + 24F^a b^2 c^2 d) x) F^{b/(d*x+c)} + \int \frac{1}{24} (F^a b^4 d^4 x \log(F)^4 - F^a b^3 c^2 \log(F)^3 - 2F^a b^2 c^3 \log(F)^2 - 6F^a b^2 c^4 \log(F)) F^{b/(d*x+c)} / (d^2 x^2 + 2c d x + c^2), x$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c))*(d*x+c)**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 F^{a + \frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*F^(a + b/(d*x + c)), x)
```

### 3.304 $\int F^{a+\frac{b}{c+dx}}(c+dx)^2 dx$

**Optimal.** Leaf size=119

$$-\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{6d} + \frac{b^2 \log^2(F)(c+dx) F^{a+\frac{b}{c+dx}}}{6d} + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d} + \frac{b \log(F)(c+dx)^2 F^{a+\frac{b}{c+dx}}}{6d}$$

[Out]  $(F^{(a + b/(c + d*x))}*(c + d*x)^3)/(3*d) + (b*F^{(a + b/(c + d*x))}*(c + d*x)^2*\operatorname{Log}[F])/(6*d) + (b^2*F^{(a + b/(c + d*x))}*(c + d*x)*\operatorname{Log}[F]^2)/(6*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)]*\operatorname{Log}[F]^3)/(6*d)$

**Rubi [A]** time = 0.132549, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2214, 2206, 2210}

$$-\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{6d} + \frac{b^2 \log^2(F)(c+dx) F^{a+\frac{b}{c+dx}}}{6d} + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d} + \frac{b \log(F)(c+dx)^2 F^{a+\frac{b}{c+dx}}}{6d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x))}*(c + d*x)^2, x]$

[Out]  $(F^{(a + b/(c + d*x))}*(c + d*x)^3)/(3*d) + (b*F^{(a + b/(c + d*x))}*(c + d*x)^2*\operatorname{Log}[F])/(6*d) + (b^2*F^{(a + b/(c + d*x))}*(c + d*x)*\operatorname{Log}[F]^2)/(6*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)]*\operatorname{Log}[F]^3)/(6*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})} * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(c + d*x)^{(m + 1)} * F^{(a + b*(c + d*x)^n)} / (d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F]) / (m + 1), \operatorname{Int}[(c + d*x)^{(m + n)} * F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}, x\_Symbol] := \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)} / d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})} / ((e_.) + (f_.)*(x_)), x\_Symbol] := \operatorname{Simp}[F^a * \operatorname{ExpIntegralEi}[b*(c + d*x)^n * \operatorname{Log}[F]] / (f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int F^{a+\frac{b}{c+dx}}(c+dx)^2 dx &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^3}{3d} + \frac{1}{3}(b \log(F)) \int F^{a+\frac{b}{c+dx}}(c+dx) dx \\
&= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^3}{3d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx)^2 \log(F)}{6d} + \frac{1}{6}(b^2 \log^2(F)) \int F^{a+\frac{b}{c+dx}} dx \\
&= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^3}{3d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx)^2 \log(F)}{6d} + \frac{b^2 F^{a+\frac{b}{c+dx}}(c+dx) \log^2(F)}{6d} + \frac{1}{6}(b^3 \log^3(F)) \int F^{a+\frac{b}{c+dx}} dx \\
&= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^3}{3d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx)^2 \log(F)}{6d} + \frac{b^2 F^{a+\frac{b}{c+dx}}(c+dx) \log^2(F)}{6d} - \frac{b^3 F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{6d}
\end{aligned}$$

**Mathematica [A]** time = 0.0683396, size = 76, normalized size = 0.64

$$\frac{F^a \left( (c+dx) F^{\frac{b}{c+dx}} \left( b^2 \log^2(F) + b \log(F)(c+dx) + 2(c+dx)^2 \right) - b^3 \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))\*(c + d\*x)^2, x]

[Out] (F^a\*(-(b^3\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)]\*Log[F]^3) + F^(b/(c + d\*x))\*((c + d\*x)\*(2\*(c + d\*x)^2 + b\*(c + d\*x)\*Log[F] + b^2\*Log[F]^2)))/(6\*d)

**Maple [B]** time = 0.085, size = 234, normalized size = 2.

$$\frac{d^2 F^a x^3}{3} F^{\frac{b}{dx+c}} + d F^a F^{\frac{b}{dx+c}} c x^2 + F^a F^{\frac{b}{dx+c}} c^2 x + \frac{F^a c^3}{3d} F^{\frac{b}{dx+c}} + \frac{\ln(F) b d F^a x^2}{6} F^{\frac{b}{dx+c}} + \frac{b \ln(F) F^a c x}{3} F^{\frac{b}{dx+c}} + \frac{b \ln(F) F^a c^2}{6d} F^{\frac{b}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))\*(d\*x+c)^2, x)

[Out] 1/3\*d^2\*F^a\*F^(b/(d\*x+c))\*x^3+d\*F^a\*F^(b/(d\*x+c))\*c\*x^2+F^a\*F^(b/(d\*x+c))\*c^2\*x+1/3/d\*F^a\*F^(b/(d\*x+c))\*c^3+1/6\*d\*b\*ln(F)\*F^a\*F^(b/(d\*x+c))\*x^2+1/3\*b\*ln(F)\*F^a\*F^(b/(d\*x+c))\*c\*x+1/6/d\*b\*ln(F)\*F^a\*F^(b/(d\*x+c))\*c^2+1/6\*b^2\*ln(F)^2\*F^a\*F^(b/(d\*x+c))\*x+1/6/d\*b^2\*ln(F)^2\*F^a\*F^(b/(d\*x+c))\*c+1/6/d\*b^3\*ln(F)^3\*F^a\*Ei(1, -b\*ln(F)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} \left( 2 F^a d^2 x^3 + (F^a b d \log(F) + 6 F^a c d) x^2 + (F^a b^2 \log(F)^2 + 2 F^a b c \log(F) + 6 F^a c^2) x \right) F^{\frac{b}{dx+c}} + \int \frac{(F^a b^3 dx \log(F))^3 - F^a b^3}{6(d^3 x^3 + \dots)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^2, x, algorithm="maxima")

[Out] 1/6\*(2\*F^a\*d^2\*x^3 + (F^a\*b\*d\*log(F) + 6\*F^a\*c\*d)\*x^2 + (F^a\*b^2\*log(F)^2 + 2\*F^a\*b\*c\*log(F) + 6\*F^a\*c^2)\*x)\*F^(b/(d\*x + c)) + integrate(1/6\*(F^a\*b^3\*

$d*x*\log(F)^3 - F^a*b^2*c^2*\log(F)^2 - 2*F^a*b*c^3*\log(F))*F^{(b/(d*x + c))}/(d^2*x^2 + 2*c*d*x + c^2), x)$

**Fricas [A]** time = 1.56683, size = 270, normalized size = 2.27

$$\frac{F^a b^3 \operatorname{Ei}\left(\frac{b \log(F)}{d x + c}\right) \log(F)^3 - (2 d^3 x^3 + 6 c d^2 x^2 + 6 c^2 d x + 2 c^3 + (b^2 d x + b^2 c) \log(F)^2 + (b d^2 x^2 + 2 b c d x + b c^2) \log(F)) F^{\frac{a}{d}}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-1/6*(F^a*b^3*\operatorname{Ei}(b*\log(F)/(d*x + c))*\log(F)^3 - (2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x + 2*c^3 + (b^2*d*x + b^2*c)*\log(F)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\log(F))*F^{(a*d*x + a*c + b)/(d*x + c)})/d$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))\*(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d x + c)^2 F^{a + \frac{b}{d x + c}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2\*F^(a + b/(d\*x + c)), x)

### 3.305 $\int F^{a+\frac{b}{c+dx}}(c+dx) dx$

**Optimal.** Leaf size=85

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{2d} + \frac{(c+dx)^2 F^{a+\frac{b}{c+dx}}}{2d} + \frac{b \log(F)(c+dx) F^{a+\frac{b}{c+dx}}}{2d}$$

[Out] (F^(a + b/(c + d\*x))\*(c + d\*x)^2)/(2\*d) + (b\*F^(a + b/(c + d\*x))\*(c + d\*x)\*Log[F])/(2\*d) - (b^2\*F^a\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)]\*Log[F]^2)/(2\*d)

**Rubi [A]** time = 0.0798134, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2214, 2206, 2210}

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{2d} + \frac{(c+dx)^2 F^{a+\frac{b}{c+dx}}}{2d} + \frac{b \log(F)(c+dx) F^{a+\frac{b}{c+dx}}}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))\*(c + d\*x), x]

[Out] (F^(a + b/(c + d\*x))\*(c + d\*x)^2)/(2\*d) + (b\*F^(a + b/(c + d\*x))\*(c + d\*x)\*Log[F])/(2\*d) - (b^2\*F^a\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)]\*Log[F]^2)/(2\*d)

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n] && LtQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{c+dx}}(c+dx) dx &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^2}{2d} + \frac{1}{2}(b \log(F)) \int F^{a+\frac{b}{c+dx}} dx \\ &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^2}{2d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx) \log(F)}{2d} + \frac{1}{2}(b^2 \log^2(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)^2}{2d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx) \log(F)}{2d} - \frac{b^2 F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \log^2(F)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0456404, size = 58, normalized size = 0.68

$$\frac{F^a \left( (c+dx) F^{\frac{b}{c+dx}} (b \log(F) + c+dx) - b^2 \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))\*(c + d\*x), x]

[Out] (F^a\*(-(b^2\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)]\*Log[F]^2) + F^(b/(c + d\*x))\* (c + d\*x)\*(c + d\*x + b\*Log[F]))) / (2\*d)

**Maple [A]** time = 0.083, size = 133, normalized size = 1.6

$$\frac{dF^a x^2}{2} F^{\frac{b}{dx+c}} + F^a F^{\frac{b}{dx+c}} c x + \frac{F^a c^2}{2d} F^{\frac{b}{dx+c}} + \frac{b \ln(F) F^a x}{2} F^{\frac{b}{dx+c}} + \frac{b \ln(F) F^a c}{2d} F^{\frac{b}{dx+c}} + \frac{b^2 (\ln(F))^2 F^a}{2d} \operatorname{Ei}\left(1, -\frac{b \ln(F)}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))\*(d\*x+c), x)

[Out] 1/2\*d\*F^a\*F^(b/(d\*x+c))\*x^2+F^a\*F^(b/(d\*x+c))\*c\*x+1/2/d\*F^a\*F^(b/(d\*x+c))\*c^2+1/2\*b\*ln(F)\*F^a\*F^(b/(d\*x+c))\*x+1/2/d\*b\*ln(F)\*F^a\*F^(b/(d\*x+c))\*c+1/2/d\*b^2\*ln(F)^2\*F^a\*Ei(1, -b\*ln(F)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \left( F^a dx^2 + (F^a b \log(F) + 2 F^a c) x \right) F^{\frac{b}{dx+c}} + \int \frac{(F^a b^2 dx \log(F)^2 - F^a b c^2 \log(F)) F^{\frac{b}{dx+c}}}{2(d^2 x^2 + 2 c dx + c^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c), x, algorithm="maxima")

[Out] 1/2\*(F^a\*d\*x^2 + (F^a\*b\*log(F) + 2\*F^a\*c)\*x)\*F^(b/(d\*x + c)) + integrate(1/2\*(F^a\*b^2\*d\*x\*log(F)^2 - F^a\*b\*c^2\*log(F))\*F^(b/(d\*x + c))/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)



**Fricas [A]** time = 1.59668, size = 180, normalized size = 2.12

$$\frac{F^a b^2 \operatorname{Ei}\left(\frac{b \log(F)}{dx+c}\right) \log(F)^2 - (d^2 x^2 + 2cdx + c^2 + (bdx + bc) \log(F)) F^{\frac{adx+ac+b}{dx+c}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c), x, algorithm="fricas")

[Out]  $-1/2*(F^a*b^2*\operatorname{Ei}(b*\log(F)/(d*x + c))*\log(F)^2 - (d^2*x^2 + 2*c*d*x + c^2 + (b*d*x + b*c)*\log(F))*F^{(a*d*x + a*c + b)/(d*x + c)})/d$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{c+dx}} (c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))\*(d\*x+c), x)

[Out] Integral(F\*\*(a + b/(c + d\*x))\*(c + d\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c) F^{a+\frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))\*(d\*x+c), x, algorithm="giac")

[Out] integrate((d\*x + c)\*F^(a + b/(d\*x + c)), x)

### 3.306 $\int F^{a+\frac{b}{c+dx}} dx$

**Optimal.** Leaf size=46

$$\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

[Out]  $(F^{(a + b/(c + d*x))*(c + d*x)})/d - (bF^a * \operatorname{ExpIntegralEi}[(b * \operatorname{Log}[F])/(c + d*x)] * \operatorname{Log}[F])/d$

**Rubi [A]** time = 0.0519797, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2206, 2210}

$$\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x))}, x]$

[Out]  $(F^{(a + b/(c + d*x))*(c + d*x)})/d - (bF^a * \operatorname{ExpIntegralEi}[(b * \operatorname{Log}[F])/(c + d*x)] * \operatorname{Log}[F])/d$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{IntegerQ}[2/n]$  &&  $\operatorname{LtQ}[n, 0]$

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}/((e_.) + (f_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a * \operatorname{ExpIntegralEi}[b*(c + d*x)^n * \operatorname{Log}[F]]/(f*n), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x$  &&  $\operatorname{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{c+dx}} dx &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)}{d} + (b \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{c+dx}}(c+dx)}{d} - \frac{bF^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \log(F)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0245606, size = 42, normalized size = 0.91

$$\frac{F^a \left( (c+dx)F^{\frac{b}{c+dx}} - b \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)),x]

[Out] (F^a\*(F^(b/(c + d\*x))\*(c + d\*x) - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)]\*Log[F]))/d

**Maple [A]** time = 0.076, size = 61, normalized size = 1.3

$$F^a F^{\frac{b}{dx+c}} x + \frac{F^a c}{d} F^{\frac{b}{dx+c}} + \frac{b \ln(F) F^a}{d} \text{Ei}\left(1, -\frac{b \ln(F)}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)),x)

[Out] F^a \* F^(b/(d\*x+c)) \* x + 1/d \* F^a \* F^(b/(d\*x+c)) \* c + b/d \* ln(F) \* F^a \* Ei(1, -b\*ln(F)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$F^a b d \int \frac{F^{\frac{b}{dx+c}} x}{d^2 x^2 + 2 c d x + c^2} dx \log(F) + F^a F^{\frac{b}{dx+c}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)),x, algorithm="maxima")

[Out] F^a \* b \* d \* integrate(F^(b/(d\*x + c)) \* x / (d^2 \* x^2 + 2 \* c \* d \* x + c^2), x) \* log(F) + F^a \* F^(b/(d\*x + c)) \* x

**Fricas [A]** time = 1.53923, size = 116, normalized size = 2.52

$$\frac{F^a b \text{Ei}\left(\frac{b \log(F)}{dx+c}\right) \log(F) - (dx+c) F^{\frac{adx+ac+b}{dx+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)),x, algorithm="fricas")

[Out] -(F^a \* b \* Ei(b\*log(F)/(d\*x + c)) \* log(F) - (d\*x + c) \* F^((a\*d\*x + a\*c + b)/(d\*x + c)))/d

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)),x)

[Out] Integral(F\*\*(a + b/(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)),x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)), x)

$$3.307 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx$$

**Optimal.** Leaf size=20

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

[Out]  $-\left((F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)])\right)/d$

**Rubi [A]** time = 0.0438938, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x), x]

[Out]  $-\left((F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)])\right)/d$

**Rule 2210**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx = -\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

**Mathematica [A]** time = 0.0048202, size = 20, normalized size = 1.

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x), x]

[Out]  $-\left((F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)])\right)/d$

**Maple [A]** time = 0.079, size = 22, normalized size = 1.1

$$\frac{F^a}{d} \operatorname{Ei}\left(1, -\frac{b \ln(F)}{dx + c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c))/(d*x+c),x)`

[Out] `1/d*F^a*Ei(1,-b*ln(F)/(d*x+c))`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c))/(d*x + c), x)`

---

**Fricas [A]** time = 1.60219, size = 42, normalized size = 2.1

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="fricas")`

[Out] `-F^a*Ei(b*log(F)/(d*x + c))/d`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c))/(d*x+c),x)`

[Out] `Integral(F**(a + b/(c + d*x))/(c + d*x), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^(a + b/(d*x + c))/(d*x + c), x)`

$$3.308 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=25

$$-\frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)}$$

[Out]  $-(F^{(a + b/(c + d*x))}/(b*d*\text{Log}[F]))$

**Rubi [A]** time = 0.0423626, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2209}

$$-\frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^2,x]

[Out]  $-(F^{(a + b/(c + d*x))}/(b*d*\text{Log}[F]))$

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx = -\frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)}$$

**Mathematica [A]** time = 0.0065504, size = 25, normalized size = 1.

$$-\frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^2,x]

[Out]  $-(F^{(a + b/(c + d*x))}/(b*d*\text{Log}[F]))$

**Maple [A]** time = 0.003, size = 26, normalized size = 1.

$$-\frac{1}{\ln(F) bd} F^{a+\frac{b}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c))/(d*x+c)^2,x)`

[Out] `-F^(a+b/(d*x+c))/b/d/ln(F)`

**Maxima [A]** time = 0.999679, size = 34, normalized size = 1.36

$$-\frac{F^{a+\frac{b}{dx+c}}}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="maxima")`

[Out] `-F^(a + b/(d*x + c))/(b*d*log(F))`

**Fricas [A]** time = 1.54189, size = 63, normalized size = 2.52

$$-\frac{F^{\frac{adx+ac+b}{dx+c}}}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="fricas")`

[Out] `-F^((a*d*x + a*c + b)/(d*x + c))/(b*d*log(F))`

**Sympy [A]** time = 0.371855, size = 34, normalized size = 1.36

$$\begin{cases} -\frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)} & \text{for } bd \log(F) \neq 0 \\ -\frac{1}{cd+d^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c))/(d*x+c)**2,x)`

[Out] `Piecewise((-F**(a + b/(c + d*x))/(b*d*log(F)), Ne(b*d*log(F), 0)), (-1/(c*d + d**2*x), True))`

**Giac [A]** time = 1.34049, size = 34, normalized size = 1.36

$$-\frac{F^{a+\frac{b}{dx+c}}}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -F^(a + b/(d*x + c))/(b*d*log(F))
```

$$3.309 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=57

$$\frac{F^{a+\frac{b}{c+dx}}}{b^2 d \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)}$$

[Out]  $F^{(a + b/(c + d*x))}/(b^2*d*\text{Log}[F]^2) - F^{(a + b/(c + d*x))}/(b*d*(c + d*x)*\text{Log}[F])$

**Rubi [A]** time = 0.0849565, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{F^{a+\frac{b}{c+dx}}}{b^2 d \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^3,x]

[Out]  $F^{(a + b/(c + d*x))}/(b^2*d*\text{Log}[F]^2) - F^{(a + b/(c + d*x))}/(b*d*(c + d*x)*\text{Log}[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx &= -\frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)\log(F)} - \frac{\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{b\log(F)} \\ &= \frac{F^{a+\frac{b}{c+dx}}}{b^2 d \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)\log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0174742, size = 41, normalized size = 0.72

$$\frac{F^{a+\frac{b}{c+dx}}(-b\log(F) + c + dx)}{b^2 d \log^2(F)(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^3,x]

[Out] (F^(a + b/(c + d\*x))\*(c + d\*x - b\*Log[F]))/(b^2\*d\*(c + d\*x)\*Log[F]^2)

**Maple [A]** time = 0.021, size = 106, normalized size = 1.9

$$\frac{1}{(dx+c)^2} \left( \frac{dx^2}{(\ln(F))^2 b^2} e^{\left(a+\frac{b}{dx+c}\right)\ln(F)} - \frac{(b \ln(F) - 2c)x}{(\ln(F))^2 b^2} e^{\left(a+\frac{b}{dx+c}\right)\ln(F)} - \frac{c(b \ln(F) - c)}{(\ln(F))^2 b^2 d} e^{\left(a+\frac{b}{dx+c}\right)\ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(d\*x+c)^3,x)

[Out] (1/ln(F)^2/b^2\*d\*x^2\*exp((a+b/(d\*x+c))\*ln(F))-(b\*ln(F)-2\*c)/ln(F)^2/b^2\*x\*exp((a+b/(d\*x+c))\*ln(F))-c\*(b\*ln(F)-c)/d/ln(F)^2/b^2\*exp((a+b/(d\*x+c))\*ln(F)))/(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^3, x)

**Fricas [A]** time = 1.59182, size = 117, normalized size = 2.05

$$\frac{(dx - b \log(F) + c)F^{\frac{adx+ac+b}{dx+c}}}{(b^2d^2x + b^2cd) \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^3,x, algorithm="fricas")

[Out] (d\*x - b\*log(F) + c)\*F^((a\*d\*x + a\*c + b)/(d\*x + c))/((b^2\*d^2\*x + b^2\*c\*d)\*log(F)^2)

**Sympy [A]** time = 0.198009, size = 44, normalized size = 0.77

$$\frac{F^{a+\frac{b}{c+dx}}(-b \log(F) + c + dx)}{b^2cd \log(F)^2 + b^2d^2x \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c))/(d*x+c)**3,x)
```

```
[Out] F**(a + b/(c + d*x))*(-b*log(F) + c + d*x)/(b**2*c*d*log(F)**2 + b**2*d**2*x*log(F)**2)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c))/(d*x + c)^3, x)
```

$$3.310 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=90

$$\frac{2F^{a+\frac{b}{c+dx}}}{b^2d \log^2(F)(c+dx)} - \frac{2F^{a+\frac{b}{c+dx}}}{b^3d \log^3(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)^2}$$

[Out]  $(-2F^{a+b/(c+d*x)})/(b^3*d*Log[F]^3) + (2F^{a+b/(c+d*x)})/(b^2*d*(c+d*x)*Log[F]^2) - F^{a+b/(c+d*x)}/(b*d*(c+d*x)^2*Log[F])$

**Rubi [A]** time = 0.133095, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{2F^{a+\frac{b}{c+dx}}}{b^2d \log^2(F)(c+dx)} - \frac{2F^{a+\frac{b}{c+dx}}}{b^3d \log^3(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^4, x]

[Out]  $(-2F^{a+b/(c+d*x)})/(b^3*d*Log[F]^3) + (2F^{a+b/(c+d*x)})/(b^2*d*(c+d*x)*Log[F]^2) - F^{a+b/(c+d*x)}/(b*d*(c+d*x)^2*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4} dx &= -\frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^2 \log(F)} - \frac{2 \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx}{b \log(F)} \\ &= \frac{2F^{a+\frac{b}{c+dx}}}{b^2d(c+dx) \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^2 \log(F)} + \frac{2 \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{b^2 \log^2(F)} \\ &= -\frac{2F^{a+\frac{b}{c+dx}}}{b^3d \log^3(F)} + \frac{2F^{a+\frac{b}{c+dx}}}{b^2d(c+dx) \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^2 \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0247779, size = 60, normalized size = 0.67

$$\frac{F^{a+\frac{b}{c+dx}} \left( b^2 \log^2(F) - 2b \log(F)(c+dx) + 2(c+dx)^2 \right)}{b^3 d \log^3(F)(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^4,x]

[Out] -((F^(a + b/(c + d\*x))\*(2\*(c + d\*x)^2 - 2\*b\*(c + d\*x)\*Log[F] + b^2\*Log[F]^2))/(b^3\*d\*(c + d\*x)^2\*Log[F]^3))

**Maple [A]** time = 0.028, size = 169, normalized size = 1.9

$$\frac{1}{(dx+c)^3} \left( -2 \frac{d^2 x^3}{(\ln(F))^3 b^3} e^{\left(a+\frac{b}{dx+c}\right) \ln(F)} - \frac{((\ln(F))^2 b^2 - 4bc \ln(F) + 6c^2) x}{(\ln(F))^3 b^3} e^{\left(a+\frac{b}{dx+c}\right) \ln(F)} + 2 \frac{d(b \ln(F) - 3c) x^2}{(\ln(F))^3 b^3} e^{\left(a+\frac{b}{dx+c}\right) \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(d\*x+c)^4,x)

[Out] (-2\*d^2/ln(F)^3/b^3\*x^3\*exp((a+b/(d\*x+c))\*ln(F))-(ln(F)^2\*b^2-4\*b\*c\*ln(F)+6\*c^2)/ln(F)^3/b^3\*x\*exp((a+b/(d\*x+c))\*ln(F))+2\*d\*(b\*ln(F)-3\*c)/ln(F)^3/b^3\*x^2\*exp((a+b/(d\*x+c))\*ln(F))-(ln(F)^2\*b^2-2\*b\*c\*ln(F)+2\*c^2)\*c/b^3/ln(F)^3/d\*exp((a+b/(d\*x+c))\*ln(F)))/(d\*x+c)^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^4,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^4, x)

**Fricas [A]** time = 1.62484, size = 212, normalized size = 2.36

$$\frac{(2d^2x^2 + b^2 \log(F)^2 + 4cdx + 2c^2 - 2(bdx + bc) \log(F)) F^{\frac{adx+ac+b}{dx+c}}}{(b^3d^3x^2 + 2b^3cd^2x + b^3c^2d) \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^4,x, algorithm="fricas")

[Out] -(2\*d^2\*x^2 + b^2\*log(F)^2 + 4\*c\*d\*x + 2\*c^2 - 2\*(b\*d\*x + b\*c)\*log(F))\*F^((a\*d\*x + a\*c + b)/(d\*x + c))/((b^3\*d^3\*x^2 + 2\*b^3\*c\*d^2\*x + b^3\*c^2\*d)\*log(F)^3)

$F)^3)$

---

**Sympy [A]** time = 0.236293, size = 102, normalized size = 1.13

$$\frac{F^{a+\frac{b}{c+dx}} \left( -b^2 \log(F)^2 + 2bc \log(F) + 2bdx \log(F) - 2c^2 - 4cdx - 2d^2x^2 \right)}{b^3c^2d \log(F)^3 + 2b^3cd^2x \log(F)^3 + b^3d^3x^2 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(d\*x+c)\*\*4,x)

[Out] F\*\*(a + b/(c + d\*x))\*(-b\*\*2\*log(F)\*\*2 + 2\*b\*c\*log(F) + 2\*b\*d\*x\*log(F) - 2\*c\*\*2 - 4\*c\*d\*x - 2\*d\*\*2\*x\*\*2)/(b\*\*3\*c\*\*2\*d\*log(F)\*\*3 + 2\*b\*\*3\*c\*d\*\*2\*x\*log(F)\*\*3 + b\*\*3\*d\*\*3\*x\*\*2\*log(F)\*\*3)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^4,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^4, x)

$$3.311 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx$$

**Optimal.** Leaf size=122

$$\frac{3F^{a+\frac{b}{c+dx}}}{b^2d \log^2(F)(c+dx)^2} - \frac{6F^{a+\frac{b}{c+dx}}}{b^3d \log^3(F)(c+dx)} + \frac{6F^{a+\frac{b}{c+dx}}}{b^4d \log^4(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)^3}$$

[Out]  $(6F^{a+b/(c+dx)})/(b^4d \log[F]^4) - (6F^{a+b/(c+dx)})/(b^3d(c+dx) \log[F]^3) + (3F^{a+b/(c+dx)})/(b^2d(c+dx)^2 \log[F]^2) - F^{a+b/(c+dx)}/(b*d*(c+dx)^3 \log[F])$

**Rubi [A]** time = 0.18513, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{3F^{a+\frac{b}{c+dx}}}{b^2d \log^2(F)(c+dx)^2} - \frac{6F^{a+\frac{b}{c+dx}}}{b^3d \log^3(F)(c+dx)} + \frac{6F^{a+\frac{b}{c+dx}}}{b^4d \log^4(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd \log(F)(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^5, x]

[Out]  $(6F^{a+b/(c+dx)})/(b^4d \log[F]^4) - (6F^{a+b/(c+dx)})/(b^3d(c+dx) \log[F]^3) + (3F^{a+b/(c+dx)})/(b^2d(c+dx)^2 \log[F]^2) - F^{a+b/(c+dx)}/(b*d*(c+dx)^3 \log[F])$

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx &= -\frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^3 \log(F)} - \frac{3 \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4} dx}{b \log(F)} \\
&= \frac{3F^{a+\frac{b}{c+dx}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^3 \log(F)} + \frac{6 \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx}{b^2 \log^2(F)} \\
&= -\frac{6F^{a+\frac{b}{c+dx}}}{b^3 d(c+dx) \log^3(F)} + \frac{3F^{a+\frac{b}{c+dx}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^3 \log(F)} - \frac{6 \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{b^3 \log^3(F)} \\
&= \frac{6F^{a+\frac{b}{c+dx}}}{b^4 d \log^4(F)} - \frac{6F^{a+\frac{b}{c+dx}}}{b^3 d(c+dx) \log^3(F)} + \frac{3F^{a+\frac{b}{c+dx}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^3 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0341799, size = 76, normalized size = 0.62

$$\frac{F^{a+\frac{b}{c+dx}} (3b^2 \log^2(F)(c+dx) - b^3 \log^3(F) - 6b \log(F)(c+dx)^2 + 6(c+dx)^3)}{b^4 d \log^4(F)(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^5,x]

[Out] (F^(a + b/(c + d\*x))\*(6\*(c + d\*x)^3 - 6\*b\*(c + d\*x)^2\*Log[F] + 3\*b^2\*(c + d\*x)\*Log[F]^2 - b^3\*Log[F]^3))/(b^4\*d\*(c + d\*x)^3\*Log[F]^4)

**Maple [A]** time = 0.038, size = 243, normalized size = 2.

$$\frac{1}{(dx+c)^4} \left( -\frac{((\ln(F))^3 b^3 - 6(\ln(F))^2 b^2 c + 18 \ln(F) b c^2 - 24 c^3) x e^{\left(a+\frac{b}{dx+c}\right) \ln(F)}}{b^4 (\ln(F))^4} + 6 \frac{d^3 x^4}{b^4 (\ln(F))^4} e^{\left(a+\frac{b}{dx+c}\right) \ln(F)} + 3 \frac{d \left( \ln(F) \right)}{b^4 (\ln(F))^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(d\*x+c)^5,x)

[Out] (-ln(F)^3\*b^3-6\*ln(F)^2\*b^2\*c+18\*ln(F)\*b\*c^2-24\*c^3)/ln(F)^4/b^4\*x\*exp((a+b/(d\*x+c))\*ln(F))+6/ln(F)^4/b^4\*d^3\*x^4\*exp((a+b/(d\*x+c))\*ln(F))+3\*d\*(ln(F)^2\*b^2-6\*b\*c\*ln(F)+12\*c^2)/ln(F)^4/b^4\*x^2\*exp((a+b/(d\*x+c))\*ln(F))-6\*d^2\*(b\*ln(F)-4\*c)/ln(F)^4/b^4\*x^3\*exp((a+b/(d\*x+c))\*ln(F))-ln(F)^3\*b^3-3\*ln(F)^2\*b^2\*c+6\*ln(F)\*b\*c^2-6\*c^3)\*c/b^4/ln(F)^4/d\*exp((a+b/(d\*x+c))\*ln(F))/(d\*x+c)^4

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^5,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^5, x)

**Fricas [A]** time = 1.57381, size = 328, normalized size = 2.69

$$\frac{(6d^3x^3 - b^3 \log(F)^3 + 18cd^2x^2 + 18c^2dx + 6c^3 + 3(b^2dx + b^2c) \log(F)^2 - 6(bd^2x^2 + 2bcdx + bc^2) \log(F)) F^{\frac{adx+ac+b}{dx+c}}}{(b^4d^4x^3 + 3b^4cd^3x^2 + 3b^4c^2d^2x + b^4c^3d) \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^5,x, algorithm="fricas")

[Out] (6\*d^3\*x^3 - b^3\*log(F)^3 + 18\*c\*d^2\*x^2 + 18\*c^2\*d\*x + 6\*c^3 + 3\*(b^2\*d\*x + b^2\*c)\*log(F)^2 - 6\*(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*F^((a\*d\*x + a\*c + b)/(d\*x + c))/((b^4\*d^4\*x^3 + 3\*b^4\*c\*d^3\*x^2 + 3\*b^4\*c^2\*d^2\*x + b^4\*c^3\*d)\*log(F)^4)

**Sympy [A]** time = 0.275247, size = 177, normalized size = 1.45

$$\frac{F^{a+\frac{b}{c+dx}} \left( -b^3 \log(F)^3 + 3b^2c \log(F)^2 + 3b^2dx \log(F)^2 - 6bc^2 \log(F) - 12bcdx \log(F) - 6bd^2x^2 \log(F) + 6c^3 + 18c^2dx + 18cd^2x^2 \right)}{b^4c^3d \log(F)^4 + 3b^4c^2d^2x \log(F)^4 + 3b^4cd^3x^2 \log(F)^4 + b^4d^4x^3 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(d\*x+c)\*\*5,x)

[Out] F\*\*(a + b/(c + d\*x))\*(-b\*\*3\*log(F)\*\*3 + 3\*b\*\*2\*c\*log(F)\*\*2 + 3\*b\*\*2\*d\*x\*log(F)\*\*2 - 6\*b\*c\*\*2\*log(F) - 12\*b\*c\*d\*x\*log(F) - 6\*b\*d\*\*2\*x\*\*2\*log(F) + 6\*c\*\*3 + 18\*c\*\*2\*d\*x + 18\*c\*d\*\*2\*x\*\*2 + 6\*d\*\*3\*x\*\*3)/(b\*\*4\*c\*\*3\*d\*log(F)\*\*4 + 3\*b\*\*4\*c\*\*2\*d\*\*2\*x\*log(F)\*\*4 + 3\*b\*\*4\*c\*d\*\*3\*x\*\*2\*log(F)\*\*4 + b\*\*4\*d\*\*4\*x\*\*3\*log(F)\*\*4)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^5,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^5, x)

$$3.312 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6} dx$$

**Optimal.** Leaf size=92

$$\frac{F^{a+\frac{b}{c+dx}} (12b^2 \log^2(F)(c+dx)^2 - 4b^3 \log^3(F)(c+dx) + b^4 \log^4(F) - 24b \log(F)(c+dx)^3 + 24(c+dx)^4)}{b^5 d \log^5(F)(c+dx)^4}$$

[Out] -((F^(a + b/(c + d\*x)))\*(24\*(c + d\*x)^4 - 24\*b\*(c + d\*x)^3\*Log[F] + 12\*b^2\*(c + d\*x)^2\*Log[F]^2 - 4\*b^3\*(c + d\*x)\*Log[F]^3 + b^4\*Log[F]^4))/(b^5\*d\*(c + d\*x)^4\*Log[F]^5))

**Rubi [C]** time = 0.0485709, antiderivative size = 29, normalized size of antiderivative = 0.32, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{c+dx}\right)}{b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^6, x]

[Out] -((F^a\*Gamma[5, -(b\*Log[F])/(c + d\*x)]))/(b^5\*d\*Log[F]^5))

**Rule 2218**

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6} dx = -\frac{F^a \Gamma\left(5, -\frac{b \log(F)}{c+dx}\right)}{b^5 d \log^5(F)}$$

**Mathematica [C]** time = 0.006196, size = 29, normalized size = 0.32

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{c+dx}\right)}{b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^6, x]

[Out] -((F^a\*Gamma[5, -(b\*Log[F])/(c + d\*x)]))/(b^5\*d\*Log[F]^5))

**Maple [B]** time = 0.048, size = 329, normalized size = 3.6

$$\frac{1}{(dx + c)^5} \left( -24 \frac{d^4 x^5}{b^5 (\ln(F))^5} e^{\left(a + \frac{b}{dx+c}\right) \ln(F)} - \frac{(b^4 (\ln(F))^4 - 8 (\ln(F))^3 b^3 c + 36 (\ln(F))^2 b^2 c^2 - 96 \ln(F) b c^3 + 120 c^4) x}{b^5 (\ln(F))^5} e^{\left(a + \frac{b}{dx+c}\right) \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a+b/(d*x+c))/(d*x+c)^6,x)
```

```
[Out] (-24*d^4/ln(F)^5/b^5*x^5*exp((a+b/(d*x+c))*ln(F))-(b^4*ln(F)^4-8*ln(F)^3*b^3*c+36*ln(F)^2*b^2*c^2-96*ln(F)*b*c^3+120*c^4)/ln(F)^5/b^5*x*exp((a+b/(d*x+c))*ln(F))+4*d*(ln(F)^3*b^3-9*ln(F)^2*b^2*c+36*ln(F)*b*c^2-60*c^3)/ln(F)^5/b^5*x^2*exp((a+b/(d*x+c))*ln(F))-12*d^2*(ln(F)^2*b^2-8*b*c*ln(F)+20*c^2)/ln(F)^5/b^5*x^3*exp((a+b/(d*x+c))*ln(F))+24*d^3*(b*ln(F)-5*c)/ln(F)^5/b^5*x^4*exp((a+b/(d*x+c))*ln(F))-(b^4*ln(F)^4-4*ln(F)^3*b^3*c+12*ln(F)^2*b^2*c^2-24*ln(F)*b*c^3+24*c^4)*c/b^5/ln(F)^5/d*exp((a+b/(d*x+c))*ln(F)))/(d*x+c)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx + c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))/(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate(F^(a + b/(d*x + c))/(d*x + c)^6, x)
```

**Fricas [B]** time = 1.66196, size = 479, normalized size = 5.21

$$\frac{(24 d^4 x^4 + b^4 \log(F)^4 + 96 c d^3 x^3 + 144 c^2 d^2 x^2 + 96 c^3 d x + 24 c^4 - 4 (b^3 d x + b^3 c) \log(F)^3 + 12 (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) \log(F)^2 - 24 (b d^3 x^3 + 3 b^2 c d^2 x^2 + 3 b^2 c^2 d x + b^2 c^3) \log(F) + 24 c^4 d \log(F)^4 - 4 (b^3 d x + b^3 c) \log(F)^3 + 12 (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) \log(F)^2 - 24 (b d^3 x^3 + 3 b^2 c d^2 x^2 + 3 b^2 c^2 d x + b^2 c^3) \log(F) + 24 c^4 d \log(F)^4}{(b^5 d^5 x^4 + 4 b^5 c d^4 x^3 + 6 b^5 c^2 d^3 x^2 + 4 b^5 c^3 d^2 x + b^5 c^4 d) \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))/(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -(24*d^4*x^4 + b^4*log(F)^4 + 96*c*d^3*x^3 + 144*c^2*d^2*x^2 + 96*c^3*d*x + 24*c^4 - 4*(b^3*d*x + b^3*c)*log(F)^3 + 12*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(F)^2 - 24*(b*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*log(F))*F^((a*d*x + a*c + b)/(d*x + c))/((b^5*d^5*x^4 + 4*b^5*c*d^4*x^3 + 6*b^5*c^2*d^3*x^2 + 4*b^5*c^3*d^2*x + b^5*c^4*d)*log(F)^5)
```

**Sympy [B]** time = 0.318188, size = 272, normalized size = 2.96

$$\frac{F^{a+\frac{b}{c+dx}} \left( -b^4 \log(F)^4 + 4b^3 c \log(F)^3 + 4b^3 dx \log(F)^3 - 12b^2 c^2 \log(F)^2 - 24b^2 c dx \log(F)^2 - 12b^2 d^2 x^2 \log(F)^2 + 24bc^3 \log(F) + 24c^4 d \log(F)^4 - 4(b^3 dx + b^3 c) \log(F)^3 + 12(b^2 d^2 x^2 + 2b^2 c dx + b^2 c^2) \log(F)^2 - 24(b d^3 x^3 + 3b^2 c d^2 x^2 + 3b^2 c^2 dx + b^2 c^3) \log(F) + 24c^4 d \log(F)^4 \right)}{b^5 c^4 d \log(F)^5 + 4b^5 c^3 d^2 x \log(F)^5 + 6b^5 c^2 d^3 x^2 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(d\*x+c)\*\*6,x)

[Out] F\*\*(a + b/(c + d\*x))\*(-b\*\*4\*log(F)\*\*4 + 4\*b\*\*3\*c\*log(F)\*\*3 + 4\*b\*\*3\*d\*x\*log(F)\*\*3 - 12\*b\*\*2\*c\*\*2\*log(F)\*\*2 - 24\*b\*\*2\*c\*d\*x\*log(F)\*\*2 - 12\*b\*\*2\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 24\*b\*c\*\*3\*log(F) + 72\*b\*c\*\*2\*d\*x\*log(F) + 72\*b\*c\*d\*\*2\*x\*\*2\*log(F) + 24\*b\*d\*\*3\*x\*\*3\*log(F) - 24\*c\*\*4 - 96\*c\*\*3\*d\*x - 144\*c\*\*2\*d\*\*2\*x\*\*2 - 96\*c\*d\*\*3\*x\*\*3 - 24\*d\*\*4\*x\*\*4)/(b\*\*5\*c\*\*4\*d\*log(F)\*\*5 + 4\*b\*\*5\*c\*\*3\*d\*\*2\*x\*log(F)\*\*5 + 6\*b\*\*5\*c\*\*2\*d\*\*3\*x\*\*2\*log(F)\*\*5 + 4\*b\*\*5\*c\*d\*\*4\*x\*\*3\*log(F)\*\*5 + b\*\*5\*d\*\*5\*x\*\*4\*log(F)\*\*5)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^6,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^6, x)

$$3.313 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} dx$$

**Optimal.** Leaf size=108

$$\frac{F^{a+\frac{b}{c+dx}} (60b^2 \log^2(F)(c+dx)^3 - 20b^3 \log^3(F)(c+dx)^2 + 5b^4 \log^4(F)(c+dx) - b^5 \log^5(F) - 120b \log(F)(c+dx)^4 + 120(c+dx)^5)}{b^6 d \log^6(F)(c+dx)^5}$$

[Out]  $(F^{a+\frac{b}{c+dx}}) \cdot (120(c+dx)^5 - 120b(c+dx)^4 \text{Log}[F] + 60b^2(c+dx)^3 \text{Log}[F]^2 - 20b^3(c+dx)^2 \text{Log}[F]^3 + 5b^4(c+dx) \text{Log}[F]^4 - b^5 \text{Log}[F]^5) / (b^6 d (c+dx)^5 \text{Log}[F]^6)$

**Rubi [C]** time = 0.0533513, antiderivative size = 28, normalized size of antiderivative = 0.26, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(6, -\frac{b \log(F)}{c+dx}\right)}{b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(c + d\*x)^7, x]

[Out] (F^a\*Gamma[6, -((b\*Log[F])/(c + d\*x))])/(b^6\*d\*Log[F]^6)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n, x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} dx = \frac{F^a \Gamma\left(6, -\frac{b \log(F)}{c+dx}\right)}{b^6 d \log^6(F)}$$

**Mathematica [C]** time = 0.0060418, size = 28, normalized size = 0.26

$$\frac{F^a \text{Gamma}\left(6, -\frac{b \log(F)}{c+dx}\right)}{b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(c + d\*x)^7, x]

[Out] (F^a\*Gamma[6, -((b\*Log[F])/(c + d\*x))])/(b^6\*d\*Log[F]^6)



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(d\*x+c)\*\*7,x)

[Out] F\*\*(a + b/(c + d\*x))\*(-b\*\*5\*log(F)\*\*5 + 5\*b\*\*4\*c\*log(F)\*\*4 + 5\*b\*\*4\*d\*x\*log(F)\*\*4 - 20\*b\*\*3\*c\*\*2\*log(F)\*\*3 - 40\*b\*\*3\*c\*d\*x\*log(F)\*\*3 - 20\*b\*\*3\*d\*\*2\*x\*\*2\*log(F)\*\*3 + 60\*b\*\*2\*c\*\*3\*log(F)\*\*2 + 180\*b\*\*2\*c\*\*2\*d\*x\*log(F)\*\*2 + 180\*b\*\*2\*c\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 60\*b\*\*2\*d\*\*3\*x\*\*3\*log(F)\*\*2 - 120\*b\*c\*\*4\*log(F) - 480\*b\*c\*\*3\*d\*x\*log(F) - 720\*b\*c\*\*2\*d\*\*2\*x\*\*2\*log(F) - 480\*b\*c\*d\*\*3\*x\*\*3\*log(F) - 120\*b\*d\*\*4\*x\*\*4\*log(F) + 120\*c\*\*5 + 600\*c\*\*4\*d\*x + 1200\*c\*\*3\*d\*\*2\*x\*\*2 + 1200\*c\*\*2\*d\*\*3\*x\*\*3 + 600\*c\*d\*\*4\*x\*\*4 + 120\*d\*\*5\*x\*\*5)/(b\*\*6\*c\*\*5\*d\*log(F)\*\*6 + 5\*b\*\*6\*c\*\*4\*d\*\*2\*x\*log(F)\*\*6 + 10\*b\*\*6\*c\*\*3\*d\*\*3\*x\*\*2\*log(F)\*\*6 + 10\*b\*\*6\*c\*\*2\*d\*\*4\*x\*\*3\*log(F)\*\*6 + 5\*b\*\*6\*c\*d\*\*5\*x\*\*4\*log(F)\*\*6 + b\*\*6\*d\*\*6\*x\*\*5\*log(F)\*\*6)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(d\*x+c)^7,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(d\*x + c)^7, x)



$$3.314 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^m dx$$

**Optimal.** Leaf size=61

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{\frac{m+1}{2}} \text{Gamma}\left(\frac{1}{2}(-m-1), -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(1 + m)/2)/(2\*d)

**Rubi [A]** time = 0.0536999, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{\frac{m+1}{2}} \text{Gamma}\left(\frac{1}{2}(-m-1), -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^m, x]

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(1 + m)/2)/(2\*d)

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1 + m)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^m dx = \frac{F^a(c+dx)^{1+m} \Gamma\left(\frac{1}{2}(-1-m), -\frac{b \log(F)}{(c+dx)^2}\right) \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{\frac{1+m}{2}}}{2d}$$

**Mathematica [A]** time = 0.034179, size = 61, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{\frac{m+1}{2}} \text{Gamma}\left(\frac{1}{2}(-m-1), -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^m, x]

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(1 + m)/2)/(2\*d)

---

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^2}} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^m,x)

[Out] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^(a + b/(d\*x + c)^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m F^{\frac{ad^2x^2+2acdx+ac^2+b}{d^2x^2+2cdx+c^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] integral((d\*x + c)^m \* F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*m,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*F^(a + b/(d*x + c)^2), x)
```

$$3.315 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^9 dx$$

**Optimal.** Leaf size=31

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out]  $-(b^5 F^a \Gamma[-5, -(b \log[F]) / (c + d*x)^2]) * \log[F]^5 / (2*d)$

**Rubi [A]** time = 0.0519668, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^9, x]

[Out]  $-(b^5 F^a \Gamma[-5, -(b \log[F]) / (c + d*x)^2]) * \log[F]^5 / (2*d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^9 dx = -\frac{b^5 F^a \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^2}\right) \log^5(F)}{2d}$$

**Mathematica [A]** time = 0.008138, size = 31, normalized size = 1.

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^9, x]

[Out]  $-(b^5 F^a \Gamma[-5, -(b \log[F]) / (c + d*x)^2]) * \log[F]^5 / (2*d)$

**Maple [B]** time = 0.096, size = 961, normalized size = 31.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(a+b/(d*x+c)^2)}*(d*x+c)^9, x)$

[Out]  $\frac{1}{240}d^4F^{ab^4}\ln(F)^4F^{(b/(d*x+c)^2)}x^2 + \frac{1}{40}d^7F^{ab}\ln(F)F^{(b/(d*x+c)^2)}x^8 + \frac{1}{40}d^4F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^8 + \frac{1}{120}d^4F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}c^6 + \frac{1}{240}d^4F^{ab^3}\ln(F)^3F^{(b/(d*x+c)^2)}c^4 + \frac{1}{240}d^4F^{ab^4}\ln(F)^4F^{(b/(d*x+c)^2)}c^2 + \frac{1}{120}d^5F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}x^6 + \frac{1}{240}d^3F^{ab^3}\ln(F)^3F^{(b/(d*x+c)^2)}x^4 + \frac{1}{5}F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^7x + \frac{1}{20}F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}c^5x + \frac{1}{60}F^{ab^3}\ln(F)^3F^{(b/(d*x+c)^2)}c^3x + \frac{1}{120}F^{ab^4}\ln(F)^4F^{(b/(d*x+c)^2)}cx + \frac{7}{10}d^5F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^2x^6 + \frac{1}{5}d^6F^{ab}\ln(F)F^{(b/(d*x+c)^2)}cx^7 + \frac{1}{10}d^9F^{ab}F^{(b/(d*x+c)^2)}x^{10} + \frac{1}{10}d^4F^{ab}F^{(b/(d*x+c)^2)}c^{10} + F^{ab}F^{(b/(d*x+c)^2)}c^9x^7 + \frac{7}{5}d^4F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^3x^5 + \frac{7}{4}d^3F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^4x^4 + \frac{7}{5}d^2F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^5x^3 + \frac{7}{10}d^4F^{ab}\ln(F)F^{(b/(d*x+c)^2)}c^6x^2 + \frac{1}{20}d^4F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}c^2x^4 + \frac{1}{6}d^2F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}c^3x^3 + \frac{1}{8}d^4F^{ab^2}\ln(F)^2F^{(b/(d*x+c)^2)}c^4x^2 + \frac{1}{60}d^2F^{ab^3}\ln(F)^3F^{(b/(d*x+c)^2)}c^2x^2 + \frac{1}{40}d^4F^{ab^3}\ln(F)^3F^{(b/(d*x+c)^2)}c^6x^4 + \frac{12}{d^2}F^{ab}F^{(b/(d*x+c)^2)}c^7x^3 + \frac{9}{2}d^2F^{ab}F^{(b/(d*x+c)^2)}c^8x^2 + \frac{9}{2}d^7F^{ab}F^{(b/(d*x+c)^2)}c^2x^8 + \frac{12}{d^6}F^{ab}F^{(b/(d*x+c)^2)}c^3x^7 + \frac{21}{d^5}F^{ab}F^{(b/(d*x+c)^2)}c^4x^6 + \frac{126}{5}d^4F^{ab}F^{(b/(d*x+c)^2)}c^5x^5 + \frac{1}{240}d^4F^{ab^5}\ln(F)^5Ei(1, -b*\ln(F)/(d*x+c)^2) + d^8F^{ab}F^{(b/(d*x+c)^2)}c^2x^9$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{240} \left( 24 F^a d^9 x^{10} + 240 F^a c d^8 x^9 + 6 \left( 180 F^a c^2 d^7 + F^a b d^7 \log(F) \right) x^8 + 48 \left( 60 F^a c^3 d^6 + F^a b c d^6 \log(F) \right) x^7 + 2 \left( 2520 F^a c^4 d^5 + 84 F^a b c^2 d^5 \log(F) + F^a b^2 c d^5 \log(F)^2 \right) x^6 + 12 \left( 504 F^a c^5 d^4 + 28 F^a b c^3 d^4 \log(F) + F^a b^2 c d^4 \log(F)^2 \right) x^5 + \left( 5040 F^a c^6 d^3 + 420 F^a b c^4 d^3 \log(F) + 30 F^a b^2 c^2 d^3 \log(F)^2 + F^a b^3 d^3 \log(F)^3 \right) x^4 + 4 \left( 720 F^a c^7 d^2 + 84 F^a b c^5 d^2 \log(F) + 10 F^a b^2 c^3 d^2 \log(F)^2 + F^a b^3 c d^2 \log(F)^3 \right) x^3 + \left( 1080 F^a c^8 d + 168 F^a b c^6 d \log(F) + 30 F^a b^2 c^4 d \log(F)^2 + 6 F^a b^3 c^2 d \log(F)^3 + F^a b^4 c d \log(F)^4 \right) x^2 + 2 \left( 120 F^a c^9 + 24 F^a b c^7 \log(F) + 6 F^a b^2 c^5 \log(F)^2 + 2 F^a b^3 c^3 \log(F)^3 + F^a b^4 c \log(F)^4 \right) x \right) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} + \text{integrate}(1/120*(F^{ab^5}d^2x^2\log(F)^5 + 2F^{ab^5}c*d*x*\log(F)^5 - 24F^{ab}c^{10}\log(F) - 6F^{ab}c^8\log(F)^2 - 2F^{ab}c^3c^6\log(F)^3 - F^{ab}c^4c^4\log(F)^4)F^{(b/(d^2*x^2 + 2*c*d*x + c^2))}/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)$$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**9,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^9 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^9 * F^(a + b/(d*x + c)^2), x)
```

$$3.316 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^7 dx$$

**Optimal.** Leaf size=31

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x)^2)]\*Log[F]^4)/(2\*d)

**Rubi [A]** time = 0.0505083, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^7, x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x)^2)]\*Log[F]^4)/(2\*d)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^7 dx = \frac{b^4 F^a \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^2}\right) \log^4(F)}{2d}$$

**Mathematica [A]** time = 0.0075925, size = 31, normalized size = 1.

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^7, x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F])/(c + d\*x)^2)]\*Log[F]^4)/(2\*d)

**Maple [B]** time = 0.064, size = 646, normalized size = 20.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x)`

[Out]  $\frac{1}{48}d^3F^a b^2 \ln(F)^2 F^{b/(d*x+c)^2} x^4 + \frac{1}{48}d^3 F^a b^3 \ln(F)^3 F^{b/(d*x+c)^2} x^2 + \frac{1}{24}d^5 F^a b \ln(F) F^{b/(d*x+c)^2} x^6 + \frac{1}{24}d^4 F^a b \ln(F) F^{b/(d*x+c)^2} c^6 + \frac{1}{48}d^4 F^a b^2 \ln(F)^2 F^{b/(d*x+c)^2} c^4 + \frac{1}{48}d^4 F^a b^3 \ln(F)^3 F^{b/(d*x+c)^2} c^2 + \frac{1}{4}F^a b \ln(F) F^{b/(d*x+c)^2} c^5 x + \frac{1}{12}F^a b^2 \ln(F)^2 F^{b/(d*x+c)^2} c^3 x + \frac{1}{24}F^a b^3 \ln(F)^3 F^{b/(d*x+c)^2} c x + \frac{1}{12}d^2 F^a b^2 \ln(F)^2 F^{b/(d*x+c)^2} c^2 x^3 + \frac{1}{8}d^2 F^a b^2 \ln(F)^2 F^{b/(d*x+c)^2} c^2 x^2 + \frac{1}{4}d^4 F^a b \ln(F) F^{b/(d*x+c)^2} c^2 x^5 + \frac{5}{8}d^3 F^a b \ln(F) F^{b/(d*x+c)^2} c^2 x^4 + \frac{5}{6}d^2 F^a b \ln(F) F^{b/(d*x+c)^2} c^3 x^3 + \frac{5}{8}d^2 F^a b \ln(F) F^{b/(d*x+c)^2} c^4 x^2 + \frac{1}{48}d^4 F^a b^4 \ln(F)^4 \text{Ei}(1, -b \ln(F)/(d*x+c)^2) + d^6 F^a F^{b/(d*x+c)^2} c^2 x^7 + \frac{7}{2}d^5 F^a F^{b/(d*x+c)^2} c^2 x^6 + 7d^4 F^a F^{b/(d*x+c)^2} c^3 x^5 + \frac{35}{4}d^3 F^a F^{b/(d*x+c)^2} c^4 x^4 + 7d^2 F^a F^{b/(d*x+c)^2} c^5 x^3 + \frac{7}{2}d^2 F^a F^{b/(d*x+c)^2} c^6 x^2 + F^a F^{b/(d*x+c)^2} c^7 x + \frac{1}{8}d^7 F^a F^{b/(d*x+c)^2} c^8 + \frac{1}{8}d^7 F^a F^{b/(d*x+c)^2} x^8$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{48} \left( 6F^a d^7 x^8 + 48F^a c d^6 x^7 + 2 \left( 84F^a c^2 d^5 + F^a b d^5 \log(F) \right) x^6 + 12 \left( 28F^a c^3 d^4 + F^a b c d^4 \log(F) \right) x^5 + \left( 420F^a c^4 d^3 + 30F^a b c^4 d^3 \log(F) + F^a b^2 c^4 d^3 \log(F)^2 \right) x^4 + 4 \left( 84F^a c^5 d^2 + 10F^a b c^5 d^2 \log(F) + F^a b^2 c^5 d^2 \log(F)^2 \right) x^3 + \left( 168F^a c^6 d + 30F^a b c^6 d \log(F) + 6F^a b^2 c^6 d \log(F)^2 + F^a b^3 c^6 d \log(F)^3 \right) x^2 + 2 \left( 24F^a c^7 + 6F^a b c^7 \log(F) + 2F^a b^2 c^7 \log(F)^2 + F^a b^3 c^7 \log(F)^3 \right) x \right) F^{b/(d^2 x^2 + 2c d x + c^2)} + \text{integrate}\left(\frac{1}{24} F^a b^4 d^2 x^2 \log(F)^4 + 2F^a b^4 c d x \log(F)^4 - 6F^a b^4 c^2 \log(F) - 2F^a b^2 c^6 \log(F)^2 - F^a b^3 c^4 \log(F)^3\right) F^{b/(d^2 x^2 + 2c d x + c^2)} / (d^3 x^3 + 3c d^2 x^2 + 3c^2 d x + c^3), x$$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*7,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^7 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^7,x, algorithm="giac")

[Out] integrate((d\*x + c)^7 \* F^(a + b/(d\*x + c)^2), x)

$$3.317 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^5 dx$$

**Optimal.** Leaf size=121

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{12d} + \frac{b^2 \log^2(F) (c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{12d} + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^2}}}{6d} + \frac{b \log(F) (c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{12d}$$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^6)/(6*d) + (b*F^{(a + b/(c + d*x)^2)}*(c + d*x)^4*\operatorname{Log}[F])/(12*d) + (b^2*F^{(a + b/(c + d*x)^2)}*(c + d*x)^2*\operatorname{Log}[F]^2)/(12*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^2]*\operatorname{Log}[F]^3)/(12*d)$

**Rubi [A]** time = 0.185648, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{12d} + \frac{b^2 \log^2(F) (c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{12d} + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^2}}}{6d} + \frac{b \log(F) (c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{12d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}*(c + d*x)^5, x]$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^6)/(6*d) + (b*F^{(a + b/(c + d*x)^2)}*(c + d*x)^4*\operatorname{Log}[F])/(12*d) + (b^2*F^{(a + b/(c + d*x)^2)}*(c + d*x)^2*\operatorname{Log}[F]^2)/(12*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^2]*\operatorname{Log}[F]^3)/(12*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}]/((e_.) + (f_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^5 dx &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^6}{6d} + \frac{1}{3} (b \log(F)) \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^3 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^6}{6d} + \frac{b F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4 \log(F)}{12d} + \frac{1}{6} (b^2 \log^2(F)) \int F^{a+\frac{b}{(c+dx)^2}} (c+dx) dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^6}{6d} + \frac{b F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4 \log(F)}{12d} + \frac{b^2 F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2 \log^2(F)}{12d} + \frac{1}{6} (b^3 \log^3(F)) \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^6}{6d} + \frac{b F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4 \log(F)}{12d} + \frac{b^2 F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2 \log^2(F)}{12d} - \frac{b^3 F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{12d} \end{aligned}$$

**Mathematica [A]** time = 0.166385, size = 96, normalized size = 0.79

$$\frac{F^a \left( b \log(F) \left( b \log(F) \left( (c + dx)^2 F^{\frac{b}{(c+dx)^2}} - b \log(F) \operatorname{Ei} \left( \frac{b \log(F)}{(c+dx)^2} \right) \right) + (c + dx)^4 F^{\frac{b}{(c+dx)^2}} \right) + 2(c + dx)^6 F^{\frac{b}{(c+dx)^2}} \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^5,x]

[Out] (F^a\*(2\*F^(b/(c + d\*x)^2)\*(c + d\*x)^6 + b\*Log[F]\*(F^(b/(c + d\*x)^2)\*(c + d\*x)^4 + b\*Log[F]\*(F^(b/(c + d\*x)^2)\*(c + d\*x)^2 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^2]\*Log[F]))))/(12\*d)

**Maple [B]** time = 0.05, size = 395, normalized size = 3.3

$$\frac{d^5 F^a x^6}{6} F^{\frac{b}{(dx+c)^2}} + d^4 F^a F^{\frac{b}{(dx+c)^2}} c x^5 + \frac{5 d^3 F^a c^2 x^4}{2} F^{\frac{b}{(dx+c)^2}} + \frac{10 d^2 F^a c^3 x^3}{3} F^{\frac{b}{(dx+c)^2}} + \frac{5 d F^a c^4 x^2}{2} F^{\frac{b}{(dx+c)^2}} + F^a F^{\frac{b}{(dx+c)^2}} c^5 x + \frac{F^a c^6}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^5,x)

[Out] 1/6\*d^5\*F^a\*F^(b/(d\*x+c)^2)\*x^6+d^4\*F^a\*F^(b/(d\*x+c)^2)\*c\*x^5+5/2\*d^3\*F^a\*F^(b/(d\*x+c)^2)\*c^2\*x^4+10/3\*d^2\*F^a\*F^(b/(d\*x+c)^2)\*c^3\*x^3+5/2\*d\*F^a\*F^(b/(d\*x+c)^2)\*c^4\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c^5\*x+1/6/d\*F^a\*F^(b/(d\*x+c)^2)\*c^6+1/12\*d^3\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*x^4+1/3\*d^2\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c\*x^3+1/2\*d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^2\*x^2+1/3\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^3\*x+1/12/d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^4+1/12\*d\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*x^2+1/6\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c\*x+1/12/d\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c^2+1/12/d\*F^a\*b^3\*ln(F)^3\*Ei(1,-b\*ln(F)/(d\*x+c)^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{12} \left( 2 F^a d^5 x^6 + 12 F^a c d^4 x^5 + (30 F^a c^2 d^3 + F^a b d^3 \log(F)) x^4 + 4 (10 F^a c^3 d^2 + F^a b c d^2 \log(F)) x^3 + (30 F^a c^4 d + 6 F^a b c^2 d \log(F) + F^a b^2 c^2 d \log(F)^2) x^2 + 2 (6 F^a c^5 + 2 F^a b c^3 \log(F) + F^a b^2 c \log(F)^2) x \right) F^{\frac{b}{(d^2 x^2 + 2 c d x + c^2)}} + \operatorname{integrate} \left( \frac{1}{6} (F^a b^3 d^2 x^2 \log(F)^3 + 2 F^a b^3 c d x \log(F)^3 - 2 F^a b c^6 \log(F) - F^a b^2 c^4 \log(F)^2) F^{\frac{b}{(d^2 x^2 + 2 c d x + c^2)}} \right) / (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="maxima")

[Out] 1/12\*(2\*F^a\*d^5\*x^6 + 12\*F^a\*c\*d^4\*x^5 + (30\*F^a\*c^2\*d^3 + F^a\*b\*d^3\*log(F))\*x^4 + 4\*(10\*F^a\*c^3\*d^2 + F^a\*b\*c\*d^2\*log(F))\*x^3 + (30\*F^a\*c^4\*d + 6\*F^a\*b\*c^2\*d\*log(F) + F^a\*b^2\*c^2\*d\*log(F)^2)\*x^2 + 2\*(6\*F^a\*c^5 + 2\*F^a\*b\*c^3\*log(F) + F^a\*b^2\*c\*log(F)^2)\*x)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + integrate(1/6\*(F^a\*b^3\*d^2\*x^2\*log(F)^3 + 2\*F^a\*b^3\*c\*d\*x\*log(F)^3 - 2\*F^a\*b\*c^6\*log(F) - F^a\*b^2\*c^4\*log(F)^2)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

**Fricas [A]** time = 1.66616, size = 486, normalized size = 4.02

$$\frac{F^a b^3 \operatorname{Ei}\left(\frac{b \log(F)}{d^2 x^2 + 2cdx + c^2}\right) \log(F)^3 - (2d^6 x^6 + 12cd^5 x^5 + 30c^2 d^4 x^4 + 40c^3 d^3 x^3 + 30c^4 d^2 x^2 + 12c^5 dx + 2c^6 + (b^2 d^2 x^2 + 2b^2 d^2 x^2 + 2b^2 d^2 x^2))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="fricas")

[Out] -1/12\*(F^a\*b^3\*Ei(b\*log(F)/(d^2\*x^2 + 2\*c\*d\*x + c^2))\*log(F)^3 - (2\*d^6\*x^6 + 12\*c\*d^5\*x^5 + 30\*c^2\*d^4\*x^4 + 40\*c^3\*d^3\*x^3 + 30\*c^4\*d^2\*x^2 + 12\*c^5\*d\*x + 2\*c^6 + (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*log(F)^2 + (b\*d^4\*x^4 + 4\*b\*c\*d^3\*x^3 + 6\*b\*c^2\*d^2\*x^2 + 4\*b\*c^3\*d\*x + b\*c^4)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^5 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^5,x, algorithm="giac")

[Out] integrate((d\*x + c)^5\*F^(a + b/(d\*x + c)^2), x)

$$3.318 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^3 dx$$

**Optimal.** Leaf size=87

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{4d} + \frac{(c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{4d} + \frac{b \log(F) (c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{4d}$$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^4)/(4*d) + (b*F^{(a + b/(c + d*x)^2)}*(c + d*x)^2*\operatorname{Log}[F])/(4*d) - (b^2*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^2]*\operatorname{Log}[F]^2)/(4*d)$

**Rubi [A]** time = 0.124042, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{4d} + \frac{(c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{4d} + \frac{b \log(F) (c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{4d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}*(c + d*x)^3, x]$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^4)/(4*d) + (b*F^{(a + b/(c + d*x)^2)}*(c + d*x)^2*\operatorname{Log}[F])/(4*d) - (b^2*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^2]*\operatorname{Log}[F]^2)/(4*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})/((e_.) + (f_.)*(x_))}, x\_Symbol] :> \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^3 dx &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4}{4d} + \frac{1}{2} (b \log(F)) \int F^{a+\frac{b}{(c+dx)^2}} (c+dx) dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4}{4d} + \frac{b F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2 \log(F)}{4d} + \frac{1}{2} (b^2 \log^2(F)) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4}{4d} + \frac{b F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2 \log(F)}{4d} - \frac{b^2 F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right) \log^2(F)}{4d} \end{aligned}$$

**Mathematica [A]** time = 0.04246, size = 71, normalized size = 0.82

$$\frac{F^a \left( b \log(F) \left( (c + dx)^2 F^{\frac{b}{(c+dx)^2}} - b \log(F) \operatorname{Ei} \left( \frac{b \log(F)}{(c+dx)^2} \right) \right) + (c + dx)^4 F^{\frac{b}{(c+dx)^2}} \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^3,x]

[Out] (F^a\*(F^(b/(c + d\*x)^2)\*(c + d\*x)^4 + b\*Log[F]\*(F^(b/(c + d\*x)^2)\*(c + d\*x)^2 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^2]\*Log[F])))/(4\*d)

**Maple [B]** time = 0.04, size = 208, normalized size = 2.4

$$\frac{d^3 F^a x^4}{4 F^{(dx+c)^2}} + d^2 F^a F^{\frac{b}{(dx+c)^2}} c x^3 + \frac{3 d F^a c^2 x^2}{2 F^{(dx+c)^2}} + F^a F^{\frac{b}{(dx+c)^2}} c^3 x + \frac{F^a c^4}{4 d} F^{\frac{b}{(dx+c)^2}} + \frac{d F^a b \ln(F) x^2}{4 F^{(dx+c)^2}} + \frac{F^a b \ln(F) c x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^3,x)

[Out] 1/4\*d^3\*F^a\*F^(b/(d\*x+c)^2)\*x^4+d^2\*F^a\*F^(b/(d\*x+c)^2)\*c\*x^3+3/2\*d\*F^a\*F^(b/(d\*x+c)^2)\*c^2\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c^3\*x+1/4/d\*F^a\*F^(b/(d\*x+c)^2)\*c^4+1/4\*d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*x^2+1/2\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c\*x+1/4/d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^2+1/4/d\*F^a\*b^2\*ln(F)^2\*Ei(1,-b\*ln(F)/(d\*x+c)^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( F^a d^3 x^4 + 4 F^a c d^2 x^3 + (6 F^a c^2 d + F^a b d \log(F)) x^2 + 2 (2 F^a c^3 + F^a b c \log(F)) x \right) F^{\frac{b}{d^2 x^2 + 2 c d x + c^2}} + \int \frac{(F^a b^2 d^2 x^2 \log(F)^2 + 2 F^a b^2 d^2 x \log(F) \log(F) + F^a b^2 c^2 \log(F)^2)}{2 (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] 1/4\*(F^a\*d^3\*x^4 + 4\*F^a\*c\*d^2\*x^3 + (6\*F^a\*c^2\*d + F^a\*b\*d\*log(F))\*x^2 + 2\*(2\*F^a\*c^3 + F^a\*b\*c\*log(F))\*x)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + integrate(1/2\*(F^a\*b^2\*d^2\*x^2\*log(F)^2 + 2\*F^a\*b^2\*c\*d\*x\*log(F)^2 - F^a\*b\*c^4\*log(F))\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

**Fricas [A]** time = 1.62082, size = 315, normalized size = 3.62

$$\frac{F^a b^2 \operatorname{Ei} \left( \frac{b \log(F)}{d^2 x^2 + 2 c d x + c^2} \right) \log(F)^2 - (d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d x + c^4 + (b d^2 x^2 + 2 b c d x + b c^2) \log(F)) F^{\frac{a d^2 x^2 + 2 a c d x + a c^2}{d^2 x^2 + 2 c d x + c^2}}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="fricas")

[Out] 
$$-1/4*(F^a*b^2*Ei(b*\log(F)/(d^2*x^2 + 2*c*d*x + c^2))*\log(F)^2 - (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*F^(a + b/(d\*x + c)^2), x)

$$3.319 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx) dx$$

**Optimal.** Leaf size=53

$$\frac{(c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{2d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^2)/(2*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2]*Log[F])/(2*d)$

**Rubi [A]** time = 0.0713583, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2214, 2210}

$$\frac{(c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{2d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x), x]

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^2)/(2*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2]*Log[F])/(2*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}} (c+dx) dx &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2}{2d} + (b \log(F)) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2}{2d} - \frac{bF^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right) \log(F)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0228826, size = 47, normalized size = 0.89

$$\frac{F^a \left( (c+dx)^2 F^{\frac{b}{(c+dx)^2}} - b \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right) \right)}{2d}$$



Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x), x]

[Out] (F^a\*(F^(b/(c + d\*x)^2)\*(c + d\*x)^2 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^2]\*Log[F]))/(2\*d)

**Maple [A]** time = 0.03, size = 86, normalized size = 1.6

$$\frac{dF^a x^2}{2} F^{\frac{b}{(dx+c)^2}} + F^a F^{\frac{b}{(dx+c)^2}} cx + \frac{F^a c^2}{2d} F^{\frac{b}{(dx+c)^2}} + \frac{F^a b \ln(F)}{2d} \text{Ei}\left(1, -\frac{b \ln(F)}{(dx+c)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c), x)

[Out] 1/2\*d\*F^a\*F^(b/(d\*x+c)^2)\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c\*x+1/2/d\*F^a\*F^(b/(d\*x+c)^2)\*c^2+1/2/d\*F^a\*b\*ln(F)\*Ei(1, -b\*ln(F)/(d\*x+c)^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} (F^a dx^2 + 2F^a cx) F^{\frac{b}{d^2x^2+2cdx+c^2}} + \int \frac{(F^a b d^2 x^2 \log(F) + 2F^a b c d x \log(F)) F^{\frac{b}{d^2x^2+2cdx+c^2}}}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c), x, algorithm="maxima")

[Out] 1/2\*(F^a\*d\*x^2 + 2\*F^a\*c\*x)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + integrate((F^a\*b\*d^2\*x^2\*log(F) + 2\*F^a\*b\*c\*d\*x\*log(F))\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

**Fricas [A]** time = 1.59978, size = 211, normalized size = 3.98

$$\frac{F^a b \text{Ei}\left(\frac{b \log(F)}{d^2x^2+2cdx+c^2}\right) \log(F) - (d^2x^2 + 2cdx + c^2) F^{\frac{ad^2x^2+2acdx+ac^2+b}{d^2x^2+2cdx+c^2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c), x, algorithm="fricas")

[Out] -1/2\*(F^a\*b\*Ei(b\*log(F)/(d^2\*x^2 + 2\*c\*d\*x + c^2))\*log(F) - (d^2\*x^2 + 2\*c\*d\*x + c^2)\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c) F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c),x, algorithm="giac")

[Out] integrate((d\*x + c)\*F^(a + b/(d\*x + c)^2), x)

$$3.320 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^2])/(2*d)$

**Rubi [A]** time = 0.0447805, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}/(c + d*x), x]$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^2])/(2*d)$

**Rule 2210**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}/((e_.) + (f_.)*(x_)), x_ \text{ Symbol}] :> \operatorname{Simp}[(F^a \operatorname{ExpIntegralEi}[b*(c + d*x)^n \operatorname{Log}[F]])/(f*n), x] /;$  Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx = -\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

**Mathematica [A]** time = 0.0058154, size = 22, normalized size = 1.

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[F^{(a + b/(c + d*x)^2)}/(c + d*x), x]$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^2])/(2*d)$

**Maple [A]** time = 0.026, size = 23, normalized size = 1.1

$$\frac{F^a}{2d} \operatorname{Ei}\left(1, -\frac{b \ln(F)}{(dx + c)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^2)/(d*x+c),x)`

[Out] `1/2/d*F^a*Ei(1,-b*ln(F)/(d*x+c)^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^2)/(d*x + c), x)`

**Fricas [A]** time = 1.61045, size = 69, normalized size = 3.14

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{d^2 x^2 + 2 c d x + c^2}\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="fricas")`

[Out] `-1/2*F^a*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))/d`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**2)/(d*x+c),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^(a + b/(d*x + c)^2)/(d*x + c), x)`

$$3.321 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=27

$$-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)}$$

[Out]  $-F^{(a + b/(c + d*x)^2)/(2*b*d*Log[F])}$

**Rubi [A]** time = 0.042856, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2209}

$$-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^3,x]

[Out]  $-F^{(a + b/(c + d*x)^2)/(2*b*d*Log[F])}$

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx = -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)}$$

**Mathematica [A]** time = 0.0093709, size = 27, normalized size = 1.

$$-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^3,x]

[Out]  $-F^{(a + b/(c + d*x)^2)/(2*b*d*Log[F])}$

**Maple [A]** time = 0.005, size = 26, normalized size = 1.

$$-\frac{1}{2 \ln(F) bd} F^{a+\frac{b}{(dx+c)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x)`

[Out]  $-1/2 * F^{(a+b/(d*x+c)^2)} / b/d / \ln(F)$

**Maxima [A]** time = 0.991408, size = 34, normalized size = 1.26

$$-\frac{F^{a+\frac{b}{(dx+c)^2}}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="maxima")`

[Out]  $-1/2 * F^{(a + b/(d*x + c)^2)} / (b*d*log(F))$

**Fricas [B]** time = 1.57732, size = 115, normalized size = 4.26

$$-\frac{F^{\frac{ad^2x^2+2acdx+ac^2+b}{d^2x^2+2cdx+c^2}}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="fricas")`

[Out]  $-1/2 * F^{((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))} / (b*d * \log(F))$

**Sympy [A]** time = 0.440417, size = 54, normalized size = 2.

$$\begin{cases} -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)} & \text{for } 2bd \log(F) \neq 0 \\ -\frac{1}{2c^2d+4cd^2x+2d^3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**3,x)`

[Out] `Piecewise((-F**(a + b/(c + d*x)**2)/(2*b*d*log(F)), Ne(2*b*d*log(F), 0)), (-1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2), True))`

**Giac [A]** time = 1.183, size = 34, normalized size = 1.26

$$-\frac{F^{a+\frac{b}{(dx+c)^2}}}{2bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*F^(a + b/(d*x + c)^2)/(b*d*log(F))
```

$$3.322 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5} dx$$

**Optimal.** Leaf size=62

$$\frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^2}$$

[Out]  $F^{a+b/(c+d*x)^2}/(2*b^2*d*Log[F]^2) - F^{a+b/(c+d*x)^2}/(2*b*d*(c+d*x)^2*Log[F])$

**Rubi [A]** time = 0.0866581, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^5, x]

[Out]  $F^{a+b/(c+d*x)^2}/(2*b^2*d*Log[F]^2) - F^{a+b/(c+d*x)^2}/(2*b*d*(c+d*x)^2*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^2 \log(F)} - \frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx}{b \log(F)} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^2 \log(F)} \end{aligned}$$



**Mathematica [A]** time = 0.0248331, size = 47, normalized size = 0.76

$$\frac{F^{a+\frac{b}{(c+dx)^2}} \left( (c+dx)^2 - b \log(F) \right)}{2b^2 d \log^2(F) (c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^5,x]

[Out] (F^(a + b/(c + d\*x)^2)\*((c + d\*x)^2 - b\*Log[F]))/(2\*b^2\*d\*(c + d\*x)^2\*Log[F]^2)

**Maple [B]** time = 0.037, size = 185, normalized size = 3.

$$\frac{1}{(dx+c)^4} \left( \frac{d^3 x^4}{2 (\ln(F))^2 b^2} e^{\left(a+\frac{b}{(dx+c)^2}\right) \ln(F)} - \frac{c (b \ln(F) - 2c^2) x}{(\ln(F))^2 b^2} e^{\left(a+\frac{b}{(dx+c)^2}\right) \ln(F)} - \frac{c^2 (b \ln(F) - c^2)}{2 (\ln(F))^2 b^2 d} e^{\left(a+\frac{b}{(dx+c)^2}\right) \ln(F)} - \frac{d (b \ln(F) - c^2)}{2 (\ln(F))^2 b^2} e^{\left(a+\frac{b}{(dx+c)^2}\right) \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^5,x)

[Out] (1/2/ln(F)^2/b^2\*d^3\*x^4\*exp((a+b/(d\*x+c)^2)\*ln(F))-c\*(b\*ln(F)-2\*c^2)/ln(F)^2/b^2\*x\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*c^2\*(b\*ln(F)-c^2)/d/ln(F)^2/b^2\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*d\*(b\*ln(F)-6\*c^2)/ln(F)^2/b^2\*x^2\*exp((a+b/(d\*x+c)^2)\*ln(F))+2\*d^2\*c/ln(F)^2/b^2\*x^3\*exp((a+b/(d\*x+c)^2)\*ln(F)))/(d\*x+c)^4

**Maxima [A]** time = 1.07444, size = 136, normalized size = 2.19

$$\frac{(F^a d^2 x^2 + 2 F^a c d x + F^a c^2 - F^a b \log(F)) F^{\frac{b}{d^2 x^2 + 2 c d x + c^2}}}{2 (b^2 d^3 x^2 \log(F)^2 + 2 b^2 c d^2 x \log(F)^2 + b^2 c^2 d \log(F)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^5,x, algorithm="maxima")

[Out] 1/2\*(F^a\*d^2\*x^2 + 2\*F^a\*c\*d\*x + F^a\*c^2 - F^a\*b\*log(F))\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(b^2\*d^3\*x^2\*log(F)^2 + 2\*b^2\*c\*d^2\*x\*log(F)^2 + b^2\*c^2\*d\*log(F)^2)

**Fricas [A]** time = 1.54418, size = 217, normalized size = 3.5

$$\frac{(d^2 x^2 + 2 c d x + c^2 - b \log(F)) F^{\frac{a d^2 x^2 + 2 a c d x + a c^2 + b}{d^2 x^2 + 2 c d x + c^2}}}{2 (b^2 d^3 x^2 + 2 b^2 c d^2 x + b^2 c^2 d) \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^5,x, algorithm="fricas")

[Out]  $\frac{1}{2}(d^2x^2 + 2cdx + c^2 - b\log(F))F^{\frac{a+d^2x^2 + 2acd + ac^2}{d^2x^2 + 2cdx + c^2}} / ((b^2d^3x^2 + 2b^2cd^2x + b^2c^2d)\log(F)^2)$

**Sympy [A]** time = 0.269897, size = 82, normalized size = 1.32

$$\frac{F^{\frac{a+\frac{b}{(c+dx)^2}}}{(-b\log(F) + c^2 + 2cdx + d^2x^2)}}{2b^2c^2d\log(F)^2 + 4b^2cd^2x\log(F)^2 + 2b^2d^3x^2\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*5,x)

[Out]  $F^{a + b/(c + dx)^2}(-b\log(F) + c^2 + 2cdx + d^2x^2) / (2b^2c^2d^2\log(F)^2 + 4b^2cd^2x\log(F)^2 + 2b^2d^3x^2\log(F)^2)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+\frac{b}{(dx+c)^2}}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^5,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^5, x)

$$3.323 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx$$

**Optimal.** Leaf size=91

$$\frac{F^{a+\frac{b}{(c+dx)^2}}}{b^2 d \log^2(F)(c+dx)^2} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{b^3 d \log^3(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^4}$$

[Out]  $-(F^{a+b/(c+d*x)^2}/(b^3*d*Log[F]^3)) + F^{a+b/(c+d*x)^2}/(b^2*d*(c+d*x)^2*Log[F]^2) - F^{a+b/(c+d*x)^2}/(2*b*d*(c+d*x)^4*Log[F])$

**Rubi [A]** time = 0.137092, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{F^{a+\frac{b}{(c+dx)^2}}}{b^2 d \log^2(F)(c+dx)^2} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{b^3 d \log^3(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^7, x]

[Out]  $-(F^{a+b/(c+d*x)^2}/(b^3*d*Log[F]^3)) + F^{a+b/(c+d*x)^2}/(b^2*d*(c+d*x)^2*Log[F]^2) - F^{a+b/(c+d*x)^2}/(2*b*d*(c+d*x)^4*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^4 \log(F)} - \frac{2 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5} dx}{b \log(F)} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^4 \log(F)} + \frac{2 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx}{b^2 \log^2(F)} \\ &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{b^3 d \log^3(F)} + \frac{F^{a+\frac{b}{(c+dx)^2}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^4 \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0325184, size = 64, normalized size = 0.7

$$\frac{F^{a+\frac{b}{(c+dx)^2}} \left( b^2 \log^2(F) - 2b \log(F)(c+dx)^2 + 2(c+dx)^4 \right)}{2b^3 d \log^3(F)(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^7, x]

[Out] -(F^(a + b/(c + d\*x)^2)\*(2\*(c + d\*x)^4 - 2\*b\*(c + d\*x)^2\*Log[F] + b^2\*Log[F]^2))/(2\*b^3\*d\*(c + d\*x)^4\*Log[F]^3)

**Maple [B]** time = 0.061, size = 301, normalized size = 3.3

$$\frac{1}{(dx+c)^6} \left( \frac{d^3 (b \ln(F) - 15c^2) x^4}{(\ln(F))^3 b^3} e^{\left(a + \frac{b}{(dx+c)^2}\right) \ln(F)} - \frac{d^5 x^6}{(\ln(F))^3 b^3} e^{\left(a + \frac{b}{(dx+c)^2}\right) \ln(F)} - \frac{c \left( (\ln(F))^2 b^2 - 4 \ln(F) bc^2 + 6c^4 \right) x}{(\ln(F))^3 b^3} e^{\left(a + \frac{b}{(dx+c)^2}\right) \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^7, x)

[Out] (d^3\*(b\*ln(F)-15\*c^2)/ln(F)^3/b^3\*x^4\*exp((a+b/(d\*x+c)^2)\*ln(F))-d^5/ln(F)^3/b^3\*x^6\*exp((a+b/(d\*x+c)^2)\*ln(F))-c\*(ln(F)^2\*b^2-4\*ln(F)\*b\*c^2+6\*c^4)/b^3/ln(F)^3\*x\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*d\*(ln(F)^2\*b^2-12\*ln(F)\*b\*c^2+30\*c^4)/ln(F)^3/b^3\*x^2\*exp((a+b/(d\*x+c)^2)\*ln(F))-6\*d^4\*c/ln(F)^3/b^3\*x^5\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*(ln(F)^2\*b^2-2\*ln(F)\*b\*c^2+2\*c^4)\*c^2/b^3/ln(F)^3/d\*exp((a+b/(d\*x+c)^2)\*ln(F))+4\*c\*d^2\*(b\*ln(F)-5\*c^2)/ln(F)^3/b^3\*x^3\*exp((a+b/(d\*x+c)^2)\*ln(F)))/(d\*x+c)^6

**Maxima [B]** time = 1.02802, size = 281, normalized size = 3.09

$$\frac{(2F^a d^4 x^4 + 8F^a c d^3 x^3 + 2F^a c^4 - 2F^a b c^2 \log(F) + F^a b^2 \log(F)^2 + 2(6F^a c^2 d^2 - F^a b d^2 \log(F))x^2 + 4(2F^a c^3 d - F^a b c d \log(F)))}{2(b^3 d^5 x^4 \log(F)^3 + 4b^3 c d^4 x^3 \log(F)^3 + 6b^3 c^2 d^3 x^2 \log(F)^3 + 4b^3 c^3 d^2 x \log(F)^3 + b^3 c^4 d \log(F)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^7, x, algorithm="maxima")

[Out] -1/2\*(2\*F^a\*d^4\*x^4 + 8\*F^a\*c\*d^3\*x^3 + 2\*F^a\*c^4 - 2\*F^a\*b\*c^2\*log(F) + F^a\*b^2\*log(F)^2 + 2\*(6\*F^a\*c^2\*d^2 - F^a\*b\*d^2\*log(F))\*x^2 + 4\*(2\*F^a\*c^3\*d - F^a\*b\*c\*d\*log(F))\*x)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(b^3\*d^5\*x^4\*log(F)^3 + 4\*b^3\*c\*d^4\*x^3\*log(F)^3 + 6\*b^3\*c^2\*d^3\*x^2\*log(F)^3 + 4\*b^3\*c^3\*d^2\*x\*log(F)^3 + b^3\*c^4\*d\*log(F)^3)

**Fricas [B]** time = 1.81084, size = 386, normalized size = 4.24

$$\frac{(2d^4x^4 + 8cd^3x^3 + 12c^2d^2x^2 + 8c^3dx + 2c^4 + b^2 \log(F)^2 - 2(bd^2x^2 + 2bcdx + bc^2) \log(F)) F^{\frac{ad^2x^2 + 2acdx + ac^2 + b}{d^2x^2 + 2cdx + c^2}}}{2(b^3d^5x^4 + 4b^3cd^4x^3 + 6b^3c^2d^3x^2 + 4b^3c^3d^2x + b^3c^4d) \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^7,x, algorithm="fricas")

[Out] 
$$-1/2*(2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 + 8*c^3*d*x + 2*c^4 + b^2*\log(F)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^3*d^5*x^4 + 4*b^3*c*d^4*x^3 + 6*b^3*c^2*d^3*x^2 + 4*b^3*c^3*d^2*x + b^3*c^4*d)*\log(F)^3)$$

**Sympy [B]** time = 0.351453, size = 189, normalized size = 2.08

$$F^{a+\frac{b}{(c+dx)^2}} \frac{(-b^2 \log(F)^2 + 2bc^2 \log(F) + 4bcdx \log(F) + 2bd^2x^2 \log(F) - 2c^4 - 8c^3dx - 12c^2d^2x^2 - 8cd^3x^3 - 2d^4x^4)}{2b^3c^4d \log(F)^3 + 8b^3c^3d^2x \log(F)^3 + 12b^3c^2d^3x^2 \log(F)^3 + 8b^3cd^4x^3 \log(F)^3 + 2b^3d^5x^4 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*7,x)

[Out] 
$$F^{(a + b/(c + dx))^2} * (-b^{**2} * \log(F)^{**2} + 2*b*c^{**2} * \log(F) + 4*b*c*d*x*\log(F) + 2*b*d^{**2}*x^{**2}*\log(F) - 2*c^{**4} - 8*c^{**3}*d*x - 12*c^{**2}*d^{**2}*x^{**2} - 8*c*d^{**3}*x^{**3} - 2*d^{**4}*x^{**4}) / (2*b^{**3}*c^{**4}*d*\log(F)^{**3} + 8*b^{**3}*c^{**3}*d^{**2}*x*\log(F)^{**3} + 12*b^{**3}*c^{**2}*d^{**3}*x^{**2}*\log(F)^{**3} + 8*b^{**3}*c*d^{**4}*x^{**3}*\log(F)^{**3} + 2*b^{**3}*d^{**5}*x^{**4}*\log(F)^{**3})$$

**Giac [B]** time = 1.34182, size = 2236, normalized size = 24.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^7,x, algorithm="giac")

[Out] 
$$-1/2*((2*(\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*(\pi*b*d^2*\operatorname{sgn}(F)/(d*x + c)^2 - \pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F)/(d*x + c)^4 - \pi*b*d^2/(d*x + c)^2 + \pi*b^2*d^2*\log(\operatorname{abs}(F))/(d*x + c)^4)/((\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2 + (3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3)^2 + (3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3*(\pi^2*b^2*d^2*\operatorname{sgn}(F)/(d*x + c)^4 - \pi^2*b^2*d^2/(d*x + c)^4 + 4*d^2 - 4*b*d^2*\log(\operatorname{abs}(F))/(d*x + c)^2 + 2*b^2*d^2*\log(\operatorname{abs}(F))^2/(d*x + c)^4)/((\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2 + (3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3)^2)*\cos(-1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a - 1/2*\pi*b*\operatorname{sgn}(F)/(d^2*x^2 + 2*c*d*x + c^2) + 1/2*\pi*b/(d^2*x^2 + 2*c*d*x + c^2)) - (2*(3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3*(\pi*b*d^2*\operatorname{sgn}(F)/(d*x + c)^2 - \pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F)/(d*x + c)^4 - \pi*b*d^2/(d*x + c)^2 + \pi*b^2*d^2*\log(\operatorname{abs}(F))/(d*x + c)^4)/((\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2 + (3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3)^2) - (\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F)))^2*(\pi^2*b^2*d^2*\operatorname{sgn}(F)/(d*x + c)^4 - \pi^2*b^2*d^2*$$

$$\begin{aligned}
& d^2/(d*x + c)^4 + 4*d^2 - 4*b*d^2*\log(\text{abs}(F))/(d*x + c)^2 + 2*b^2*d^2*\log(\text{abs}(F))^2/(d*x + c)^4 / ((\pi^3*b^3*d^3*\text{sgn}(F) - 3*\pi*b^3*d^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*d^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\text{abs}(F)) + 2*b^3*d^3*\log(\text{abs}(F))^3)^2) * \\
& \sin(-1/2*\pi*a*\text{sgn}(F) + 1/2*\pi*a - 1/2*\pi*b*\text{sgn}(F)/(d^2*x^2 + 2*c*d*x + c^2) + 1/2*\pi*b/(d^2*x^2 + 2*c*d*x + c^2)) * e^{(a*\log(\text{abs}(F)) + b*\log(\text{abs}(F)))/(d*x + c)^2} - 1/4*((\pi^2*b^2*d^2*i*\text{sgn}(F)/(d*x + c)^4 - \pi^2*b^2*d^2*i/(d*x + c)^4 + 4*d^2*i - 4*b*d^2*i*\log(\text{abs}(F))/(d*x + c)^2 + 2*b^2*d^2*i*\log(\text{abs}(F)))^2/(d*x + c)^4 + 2*\pi*b*d^2*\text{sgn}(F)/(d*x + c)^2 - 2*\pi*b^2*d^2*\log(\text{abs}(F))*\text{sgn}(F)/(d*x + c)^4 - 2*\pi*b*d^2/(d*x + c)^2 + 2*\pi*b^2*d^2*\log(\text{abs}(F))/(d*x + c)^4) * e^{(1/2*(\pi*a*(\text{sgn}(F) - 1) + \pi*b*(\text{sgn}(F) - 1)/(d*x + c)^2)*i) / (\pi^3*b^3*d^3*i*\text{sgn}(F) - 3*\pi*b^3*d^3*i*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*d^3*i + 3*\pi*b^3*d^3*i*\log(\text{abs}(F))^2 - 3*\pi^2*b^3*d^3*\log(\text{abs}(F))*\text{sgn}(F) + 3*\pi^2*b^3*d^3*\log(\text{abs}(F)) - 2*b^3*d^3*\log(\text{abs}(F))^3) + (\pi^2*b^2*d^2*i*\text{sgn}(F)/(d*x + c)^4 - \pi^2*b^2*d^2*i/(d*x + c)^4 + 4*d^2*i - 4*b*d^2*i*\log(\text{abs}(F))/(d*x + c)^2 + 2*b^2*d^2*i*\log(\text{abs}(F))^2/(d*x + c)^4 - 2*\pi*b*d^2*\text{sgn}(F)/(d*x + c)^2 + 2*\pi*b^2*d^2*\log(\text{abs}(F))*\text{sgn}(F)/(d*x + c)^4 + 2*\pi*b*d^2/(d*x + c)^2 - 2*\pi*b^2*d^2*\log(\text{abs}(F))/(d*x + c)^4) * e^{(-1/2*(\pi*a*(\text{sgn}(F) - 1) + \pi*b*(\text{sgn}(F) - 1)/(d*x + c)^2)*i) / (\pi^3*b^3*d^3*i*\text{sgn}(F) - 3*\pi*b^3*d^3*i*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*d^3*i + 3*\pi*b^3*d^3*i*\log(\text{abs}(F))^2 + 3*\pi^2*b^3*d^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\text{abs}(F)) + 2*b^3*d^3*\log(\text{abs}(F))^3) * e^{(a*\log(\text{abs}(F)) + b*\log(\text{abs}(F)))/(d*x + c)^2} / i
\end{aligned}$$

$$3.324 \quad \int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^9} dx$$

**Optimal.** Leaf size=126

$$\frac{3F^{a + \frac{b}{(c+dx)^2}}}{2b^2d \log^2(F)(c+dx)^4} - \frac{3F^{a + \frac{b}{(c+dx)^2}}}{b^3d \log^3(F)(c+dx)^2} + \frac{3F^{a + \frac{b}{(c+dx)^2}}}{b^4d \log^4(F)} - \frac{F^{a + \frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^6}$$

[Out] (3\*F^(a + b/(c + d\*x)^2))/(b^4\*d\*Log[F]^4) - (3\*F^(a + b/(c + d\*x)^2))/(b^3\*d\*(c + d\*x)^2\*Log[F]^3) + (3\*F^(a + b/(c + d\*x)^2))/(2\*b^2\*d\*(c + d\*x)^4\*Log[F]^2) - F^(a + b/(c + d\*x)^2)/(2\*b\*d\*(c + d\*x)^6\*Log[F])

**Rubi [A]** time = 0.191042, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{3F^{a + \frac{b}{(c+dx)^2}}}{2b^2d \log^2(F)(c+dx)^4} - \frac{3F^{a + \frac{b}{(c+dx)^2}}}{b^3d \log^3(F)(c+dx)^2} + \frac{3F^{a + \frac{b}{(c+dx)^2}}}{b^4d \log^4(F)} - \frac{F^{a + \frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^6}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^9,x]

[Out] (3\*F^(a + b/(c + d\*x)^2))/(b^4\*d\*Log[F]^4) - (3\*F^(a + b/(c + d\*x)^2))/(b^3\*d\*(c + d\*x)^2\*Log[F]^3) + (3\*F^(a + b/(c + d\*x)^2))/(2\*b^2\*d\*(c + d\*x)^4\*Log[F]^2) - F^(a + b/(c + d\*x)^2)/(2\*b\*d\*(c + d\*x)^6\*Log[F])

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^6 \log(F)} - \frac{3 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx}{b \log(F)} \\
&= \frac{3F^{a+\frac{b}{(c+dx)^2}}}{2b^2d(c+dx)^4 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^6 \log(F)} + \frac{6 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5} dx}{b^2 \log^2(F)} \\
&= -\frac{3F^{a+\frac{b}{(c+dx)^2}}}{b^3d(c+dx)^2 \log^3(F)} + \frac{3F^{a+\frac{b}{(c+dx)^2}}}{2b^2d(c+dx)^4 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^6 \log(F)} - \frac{6 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} dx}{b^3 \log^3(F)} \\
&= \frac{3F^{a+\frac{b}{(c+dx)^2}}}{b^4d \log^4(F)} - \frac{3F^{a+\frac{b}{(c+dx)^2}}}{b^3d(c+dx)^2 \log^3(F)} + \frac{3F^{a+\frac{b}{(c+dx)^2}}}{2b^2d(c+dx)^4 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^6 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0421996, size = 81, normalized size = 0.64

$$\frac{F^{a+\frac{b}{(c+dx)^2}} \left( 3b^2 \log^2(F)(c+dx)^2 - b^3 \log^3(F) - 6b \log(F)(c+dx)^4 + 6(c+dx)^6 \right)}{2b^4d \log^4(F)(c+dx)^6}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^9,x]

[Out] (F^(a + b/(c + d\*x)^2)\*(6\*(c + d\*x)^6 - 6\*b\*(c + d\*x)^4\*Log[F] + 3\*b^2\*(c + d\*x)^2\*Log[F]^2 - b^3\*Log[F]^3))/(2\*b^4\*d\*(c + d\*x)^6\*Log[F]^4)

**Maple [B]** time = 0.093, size = 444, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^9,x)

[Out] (3\*d^7/ln(F)^4/b^4\*x^8\*exp((a+b/(d\*x+c)^2)\*ln(F))-c\*(ln(F)^3\*b^3-6\*ln(F)^2\*b^2\*c^2+18\*ln(F)\*b\*c^4-24\*c^6)/b^4/ln(F)^4\*x\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*d\*(ln(F)^3\*b^3-18\*ln(F)^2\*b^2\*c^2+90\*ln(F)\*b\*c^4-168\*c^6)/ln(F)^4/b^4\*x^2\*exp((a+b/(d\*x+c)^2)\*ln(F))+3/2\*d^3\*(ln(F)^2\*b^2-30\*ln(F)\*b\*c^2+140\*c^4)/ln(F)^4/b^4\*x^4\*exp((a+b/(d\*x+c)^2)\*ln(F))-3\*d^5\*(b\*ln(F)-28\*c^2)/ln(F)^4/b^4\*x^6\*exp((a+b/(d\*x+c)^2)\*ln(F))+24\*d^6\*c/ln(F)^4/b^4\*x^7\*exp((a+b/(d\*x+c)^2)\*ln(F))-1/2\*(ln(F)^3\*b^3-3\*ln(F)^2\*b^2\*c^2+6\*ln(F)\*b\*c^4-6\*c^6)\*c^2/b^4/ln(F)^4/d\*exp((a+b/(d\*x+c)^2)\*ln(F))+6\*c\*d^2\*(ln(F)^2\*b^2-10\*ln(F)\*b\*c^2+28\*c^4)/ln(F)^4/b^4\*x^3\*exp((a+b/(d\*x+c)^2)\*ln(F))-6\*c\*d^4\*(3\*b\*ln(F)-28\*c^2)/ln(F)^4/b^4\*x^5\*exp((a+b/(d\*x+c)^2)\*ln(F)))/(d\*x+c)^8

**Maxima [B]** time = 1.04967, size = 471, normalized size = 3.74

$$\frac{(6F^ad^6x^6 + 36F^acd^5x^5 + 6F^ac^6 - 6F^abc^4 \log(F) + 3F^ab^2c^2 \log(F)^2 - F^ab^3 \log(F)^3 + 6(15F^ac^2d^4 - F^abd^4 \log(F))x^4 + 2(b^4d^7x^6 \log(F)^4 + 6b^4cd^6x^5 \log(F)^4 + 15b^4c^2d^5x^4 \log(F)^4 + \dots)}{2(b^4d^7x^6 \log(F)^4 + 6b^4cd^6x^5 \log(F)^4 + 15b^4c^2d^5x^4 \log(F)^4 + \dots)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^9,x, algorithm="maxima")

[Out]  $\frac{1}{2}(6F^a d^6 x^6 + 36F^a c d^5 x^5 + 6F^a c^6 - 6F^a b c^4 \log(F) + 3F^a b^2 c^2 \log(F)^2 - F^a b^3 \log(F)^3 + 6(15F^a c^2 d^4 - F^a b d^4 \log(F))x^4 + 24(5F^a c^3 d^3 - F^a b c d^3 \log(F))x^3 + 3(30F^a c^4 d^2 - 12F^a b c^2 d^2 \log(F) + F^a b^2 d^2 \log(F)^2)x^2 + 6(6F^a c^5 d - 4F^a b c^3 d \log(F) + F^a b^2 c d \log(F)^2)x)F^{\frac{b}{d^2 x^2 + 2c d x + c^2}} / (b^4 d^7 x^6 \log(F)^4 + 6b^4 c d^6 x^5 \log(F)^4 + 15b^4 c^2 d^5 x^4 \log(F)^4 + 20b^4 c^3 d^4 x^3 \log(F)^4 + 15b^4 c^4 d^3 x^2 \log(F)^4 + 6b^4 c^5 d^2 x \log(F)^4 + b^4 c^6 d \log(F)^4)$

**Fricas [B]** time = 2.00736, size = 610, normalized size = 4.84

$$\frac{(6d^6x^6 + 36cd^5x^5 + 90c^2d^4x^4 + 120c^3d^3x^3 + 90c^4d^2x^2 + 36c^5dx + 6c^6 - b^3 \log(F)^3 + 3(b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(F)^2 - 6b^3 \log(F)^3 + 3(b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(F)^2)}{2(b^4d^7x^6 + 6b^4cd^6x^5 + 15b^4c^2d^5x^4 + 20b^4c^3d^4x^3 + 15b^4c^4d^3x^2 + 6b^4c^5d^2x + b^4c^6d) \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^9,x, algorithm="fricas")

[Out]  $\frac{1}{2}(6d^6x^6 + 36c^5d^5x^5 + 90c^2d^4x^4 + 120c^3d^3x^3 + 90c^4d^2x^2 + 36c^5d^2x + 6c^6 - b^3 \log(F)^3 + 3(b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(F)^2 - 6(b^3d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2cd^3x + b^2c^4) \log(F))F^{\frac{b}{d^2 x^2 + 2c d x + c^2}} / ((b^4d^7x^6 + 6b^4cd^6x^5 + 15b^4c^2d^5x^4 + 20b^4c^3d^4x^3 + 15b^4c^4d^3x^2 + 6b^4c^5d^2x + b^4c^6d) \log(F)^4)$

**Sympy [B]** time = 0.431121, size = 333, normalized size = 2.64

$$\frac{F^{\frac{a+b}{(c+dx)^2}} \left( -b^3 \log(F)^3 + 3b^2c^2 \log(F)^2 + 6b^2cdx \log(F)^2 + 3b^2d^2x^2 \log(F)^2 - 6bc^4 \log(F) - 24bc^3dx \log(F) - 36bc^2d^2x^2 \log(F) + 6c^5d^5 + 36c^5d^5dx + 90c^4d^4x^2 + 120c^3d^3x^3 + 90c^4d^2x^2 + 36c^5d^2x + b^4c^6d \log(F)^4 + 12b^4c^5d^2x \log(F)^4 + 30b^4c^4d^3x^2 \log(F)^4 + 40b^4c^3d^4x \log(F)^4 - 6b^3 \log(F)^3 + 3(b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(F)^2 - 6(b^3d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2cd^3x + b^2c^4) \log(F) \right)}{2(b^4d^7x^6 + 6b^4cd^6x^5 + 15b^4c^2d^5x^4 + 20b^4c^3d^4x^3 + 15b^4c^4d^3x^2 + 6b^4c^5d^2x + b^4c^6d) \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*9,x)

[Out]  $F^{a + \frac{b}{(c + dx)^2}} \left( -b^3 \log(F)^3 + 3b^2c^2 \log(F)^2 + 6b^2cdx \log(F)^2 + 3b^2d^2x^2 \log(F)^2 - 6bc^4 \log(F) - 24bc^3dx \log(F) - 36bc^2d^2x^2 \log(F) + 6c^5d^5 + 36c^5d^5dx + 90c^4d^4x^2 + 120c^3d^3x^3 + 90c^4d^2x^2 + 36c^5d^2x + b^4c^6d \log(F)^4 + 12b^4c^5d^2x \log(F)^4 + 30b^4c^4d^3x^2 \log(F)^4 + 40b^4c^3d^4x \log(F)^4 - 6b^3 \log(F)^3 + 3(b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(F)^2 - 6(b^3d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2cd^3x + b^2c^4) \log(F) \right)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^2}}}{(dx+c)^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^9, x)
```

$$3.325 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}} dx$$

**Optimal.** Leaf size=96

$$\frac{F^{a+\frac{b}{(c+dx)^2}} \left( 12b^2 \log^2(F)(c+dx)^4 - 4b^3 \log^3(F)(c+dx)^2 + b^4 \log^4(F) - 24b \log(F)(c+dx)^6 + 24(c+dx)^8 \right)}{2b^5 d \log^5(F)(c+dx)^8}$$

[Out]  $-(F^{a+b/(c+d*x)^2}*(24*(c+d*x)^8 - 24*b*(c+d*x)^6*\text{Log}[F] + 12*b^2*(c+d*x)^4*\text{Log}[F]^2 - 4*b^3*(c+d*x)^2*\text{Log}[F]^3 + b^4*\text{Log}[F]^4))/(2*b^5*d*(c+d*x)^8*\text{Log}[F]^5)$

**Rubi [C]** time = 0.0449171, antiderivative size = 31, normalized size of antiderivative = 0.32, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^11, x]

[Out]  $-(F^a*\text{Gamma}[5, -((b*\text{Log}[F])/(c+d*x)^2)))/(2*b^5*d*\text{Log}[F]^5)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}} dx = -\frac{F^a \Gamma\left(5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^5 d \log^5(F)}$$

**Mathematica [C]** time = 0.0068502, size = 31, normalized size = 0.32

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^11, x]

[Out]  $-(F^a*\text{Gamma}[5, -((b*\text{Log}[F])/(c+d*x)^2)))/(2*b^5*d*\text{Log}[F]^5)$

**Maple [B]** time = 0.145, size = 609, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(a+b/(d*x+c)^2)})/(d*x+c)^{11}, x$

[Out]  $(-12*d^9/\ln(F)^5/b^5*x^{10}*exp((a+b/(d*x+c)^2)*\ln(F))-c*(b^4*\ln(F)^4-8*\ln(F)^3*b^3*c^2+36*\ln(F)^2*b^2*c^4-96*\ln(F)*b*c^6+120*c^8)/b^5/\ln(F)^5*x*exp((a+b/(d*x+c)^2)*\ln(F))-1/2*d*(b^4*\ln(F)^4-24*\ln(F)^3*b^3*c^2+180*\ln(F)^2*b^2*c^4-672*\ln(F)*b*c^6+1080*c^8)/\ln(F)^5/b^5*x^2*exp((a+b/(d*x+c)^2)*\ln(F))+2*d^3*(\ln(F)^3*b^3-45*\ln(F)^2*b^2*c^2+420*\ln(F)*b*c^4-1260*c^6)/\ln(F)^5/b^5*x^4*exp((a+b/(d*x+c)^2)*\ln(F))-6*d^5*(\ln(F)^2*b^2-56*\ln(F)*b*c^2+420*c^4)/\ln(F)^5/b^5*x^6*exp((a+b/(d*x+c)^2)*\ln(F))+12*d^7*(b*\ln(F)-45*c^2)/\ln(F)^5/b^5*x^8*exp((a+b/(d*x+c)^2)*\ln(F))-120*d^8*c/\ln(F)^5/b^5*x^9*exp((a+b/(d*x+c)^2)*\ln(F))-1/2*(b^4*\ln(F)^4-4*\ln(F)^3*b^3*c^2+12*\ln(F)^2*b^2*c^4-24*\ln(F)*b*c^6+24*c^8)*c^2/b^5/\ln(F)^5/d*exp((a+b/(d*x+c)^2)*\ln(F))+8*c*d^2*(\ln(F)^3*b^3-15*\ln(F)^2*b^2*c^2+84*\ln(F)*b*c^4-180*c^6)/\ln(F)^5/b^5*x^3*exp((a+b/(d*x+c)^2)*\ln(F))-12*c*d^4*(3*\ln(F)^2*b^2-56*\ln(F)*b*c^2+252*c^4)/\ln(F)^5/b^5*x^5*exp((a+b/(d*x+c)^2)*\ln(F))+96*c*d^6*(b*\ln(F)-15*c^2)/\ln(F)^5/b^5*x^7*exp((a+b/(d*x+c)^2)*\ln(F)))/(d*x+c)^{10}$

**Maxima [B]** time = 1.08379, size = 710, normalized size = 7.4

$(24 F^a d^8 x^8 + 192 F^a c d^7 x^7 + 24 F^a c^8 - 24 F^a b c^6 \log(F) + 12 F^a b^2 c^4 \log(F)^2 - 4 F^a b^3 c^2 \log(F)^3 + F^a b^4 \log(F)^4 + 24 (28 F^a b^2 c^2 d^6 - F^a b^3 c^2 d^6 \log(F)) x^6 + 48 (28 F^a c^3 d^5 - 3 F^a b^2 c^2 d^5 \log(F)) x^5 + 12 (140 F^a c^4 d^4 - 30 F^a b^2 c^2 d^4 \log(F) + F^a b^2 c^2 d^4 \log(F)^2) x^4 + 48 (28 F^a c^5 d^3 - 10 F^a b^2 c^3 d^3 \log(F) + F^a b^2 c^3 d^3 \log(F)^2) x^3 + 4 (168 F^a c^6 d^2 - 90 F^a b^2 c^4 d^2 \log(F) + 18 F^a b^2 c^2 d^2 \log(F)^2 - F^a b^3 d^2 \log(F)^3) x^2 + 8 (24 F^a c^7 d - 18 F^a b^2 c^5 d \log(F) + 6 F^a b^2 c^3 d \log(F)^2 - F^a b^3 c d \log(F)^3) x) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} / (b^5*d^9*x^8*\log(F)^5 + 8*b^5*c*d^8*x^7*\log(F)^5 + 28*b^5*c^2*d^7*x^6*\log(F)^5 + 56*b^5*c^3*d^6*x^5*\log(F)^5 + 70*b^5*c^4*d^5*x^4*\log(F)^5 + 56*b^5*c^5*d^4*x^3*\log(F)^5 + 28*b^5*c^6*d^3*x^2*\log(F)^5 + 8*b^5*c^7*d^2*x*\log(F)^5 + b^5*c^8*d*\log(F)^5)$

**Fricas [B]** time = 1.97487, size = 907, normalized size = 9.45

$(24 d^8 x^8 + 192 c d^7 x^7 + 672 c^2 d^6 x^6 + 1344 c^3 d^5 x^5 + 1680 c^4 d^4 x^4 + 1344 c^5 d^3 x^3 + 672 c^6 d^2 x^2 + 192 c^7 d x + 24 c^8 + b^4 \log(F)) / (b^5 d^9 x^8 + 8 b^5 c d^8 x^7 + 28 b^5 c^2 d^7 x^6 + 56 b^5 c^3 d^6 x^5 + 70 b^5 c^4 d^5 x^4 + 56 b^5 c^5 d^4 x^3 + 28 b^5 c^6 d^3 x^2 + 8 b^5 c^7 d^2 x + b^5 c^8 d)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^11,x, algorithm="fricas")

[Out]  $-1/2*(24*d^8*x^8 + 192*c*d^7*x^7 + 672*c^2*d^6*x^6 + 1344*c^3*d^5*x^5 + 1680*c^4*d^4*x^4 + 1344*c^5*d^3*x^3 + 672*c^6*d^2*x^2 + 192*c^7*d*x + 24*c^8 + b^4*\log(F)^4 - 4*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*\log(F)^3 + 12*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\log(F)^2 - 24*(b*d^6*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d^3*x^3 + 15*b*c^4*d^2*x^2 + 6*b*c^5*d*x + b*c^6)*\log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^5*d^9*x^8 + 8*b^5*c*d^8*x^7 + 28*b^5*c^2*d^7*x^6 + 56*b^5*c^3*d^6*x^5 + 70*b^5*c^4*d^5*x^4 + 56*b^5*c^5*d^4*x^3 + 28*b^5*c^6*d^3*x^2 + 8*b^5*c^7*d^2*x + b^5*c^8*d)*\log(F)^5)$

**Sympy [B]** time = 0.537117, size = 518, normalized size = 5.4

$$F^{\frac{a+b}{(c+dx)^2}} \left( -b^4 \log(F)^4 + 4b^3 c^2 \log(F)^3 + 8b^3 c dx \log(F)^3 + 4b^3 d^2 x^2 \log(F)^3 - 12b^2 c^4 \log(F)^2 - 48b^2 c^3 dx \log(F)^2 - 72b^2 c^2 d^2 x^2 \log(F)^2 + 24b^2 c^2 d^2 x^2 \log(F)^2 - 48b^2 c^2 d^3 x^3 \log(F)^2 - 12b^2 c^2 d^4 x^4 \log(F)^2 + 24b^2 c^2 d^4 x^4 \log(F)^2 + 144b^2 c^2 d^4 x^4 \log(F)^2 + 360b^2 c^2 d^4 x^4 \log(F)^2 + 480b^2 c^2 d^4 x^4 \log(F)^2 + 360b^2 c^2 d^4 x^4 \log(F)^2 + 144b^2 c^2 d^5 x^5 \log(F)^2 + 24b^2 c^2 d^6 x^6 \log(F)^2 - 24c^8 - 192c^7 d x - 672c^6 d^2 x^2 - 1344c^5 d^3 x^3 - 1680c^4 d^4 x^4 - 1344c^3 d^5 x^5 - 672c^2 d^6 x^6 - 192c d^7 x^7 - 24d^8 x^8 \right) / (2b^5 c^8 d \log(F)^5 + 16b^5 c^7 d^2 x \log(F)^5 + 56b^5 c^6 d^3 x^2 \log(F)^5 + 112b^5 c^5 d^4 x^3 \log(F)^5 + 140b^5 c^4 d^5 x^4 \log(F)^5 + 112b^5 c^3 d^6 x^5 \log(F)^5 + 56b^5 c^2 d^7 x^6 \log(F)^5 + 16b^5 c d^8 x^7 \log(F)^5 + 2b^5 d^9 x^8 \log(F)^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*11,x)

[Out]  $F^{(a + b/(c + dx)^2)} * (-b^4 * \log(F)^4 + 4*b^3*c^2*\log(F)^3 + 8*b^3*c*d*x*\log(F)^3 + 4*b^3*d^2*x^2*\log(F)^3 - 12*b^2*c^4*\log(F)^2 - 48*b^2*c^3*d*x*\log(F)^2 - 72*b^2*c^2*d^2*x^2*\log(F)^2 - 48*b^2*c^2*d^3*x^3*\log(F)^2 - 12*b^2*c^2*d^4*x^4*\log(F)^2 + 24*b^2*c^2*d^4*x^4*\log(F)^2 + 144*b^2*c^2*d^4*x^4*\log(F)^2 + 360*b^2*c^2*d^4*x^4*\log(F)^2 + 480*b^2*c^2*d^4*x^4*\log(F)^2 + 360*b^2*c^2*d^4*x^4*\log(F)^2 + 144*b^2*c^2*d^5*x^5*\log(F)^2 + 24*b^2*c^2*d^6*x^6*\log(F)^2 - 24*c^8 - 192*c^7*d*x - 672*c^6*d^2*x^2 - 1344*c^5*d^3*x^3 - 1680*c^4*d^4*x^4 - 1344*c^3*d^5*x^5 - 672*c^2*d^6*x^6 - 192*c*d^7*x^7 - 24*d^8*x^8) / (2*b^5*c^8*d*\log(F)^5 + 16*b^5*c^7*d^2*x*\log(F)^5 + 56*b^5*c^6*d^3*x^2*\log(F)^5 + 112*b^5*c^5*d^4*x^3*\log(F)^5 + 140*b^5*c^4*d^5*x^4*\log(F)^5 + 112*b^5*c^3*d^6*x^5*\log(F)^5 + 56*b^5*c^2*d^7*x^6*\log(F)^5 + 16*b^5*c*d^8*x^7*\log(F)^5 + 2*b^5*d^9*x^8*\log(F)^5)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^2}}}{(dx+c)^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^11,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^11, x)

$$3.326 \quad \int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{13}} dx$$

**Optimal.** Leaf size=113

$$\frac{F^{a + \frac{b}{(c+dx)^2}} \left( 60b^2 \log^2(F)(c+dx)^6 - 20b^3 \log^3(F)(c+dx)^4 + 5b^4 \log^4(F)(c+dx)^2 - b^5 \log^5(F) - 120b \log(F)(c+dx)^8 + 120b^2 \log^2(F)(c+dx)^6 - 20b^3 \log^3(F)(c+dx)^4 + 5b^4 \log^4(F)(c+dx)^2 - b^5 \log^5(F) - 120b \log(F)(c+dx)^8 + 120b^2 \log^2(F)(c+dx)^6 \right)}{2b^6 d \log^6(F)(c+dx)^{10}}$$

[Out] (F^(a + b/(c + d\*x)^2))\*(120\*(c + d\*x)^10 - 120\*b\*(c + d\*x)^8\*Log[F] + 60\*b^2\*(c + d\*x)^6\*Log[F]^2 - 20\*b^3\*(c + d\*x)^4\*Log[F]^3 + 5\*b^4\*(c + d\*x)^2\*Log[F]^4 - b^5\*Log[F]^5)/(2\*b^6\*d\*(c + d\*x)^10\*Log[F]^6)

**Rubi [C]** time = 0.0456741, antiderivative size = 31, normalized size of antiderivative = 0.27, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \Gamma\left(6, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^13, x]

[Out] (F^a\*Gamma[6, -(b\*Log[F])/(c + d\*x)^2])/(2\*b^6\*d\*Log[F]^6)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :- Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{13}} dx = \frac{F^a \Gamma\left(6, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^6 d \log^6(F)}$$

**Mathematica [C]** time = 0.0073282, size = 31, normalized size = 0.27

$$\frac{F^a \Gamma\left(6, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^13, x]

[Out] (F^a\*Gamma[6, -(b\*Log[F])/(c + d\*x)^2])/(2\*b^6\*d\*Log[F]^6)

**Maple [B]** time = 0.189, size = 797, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(a+b/(d*x+c)^2)})/(d*x+c)^{13}, x$

[Out]  $(720*d^{10}*c/\ln(F)^6/b^6*x^{11}*exp((a+b/(d*x+c)^2)*\ln(F))-1/2*(b^5*\ln(F)^5-5*\ln(F)^4*b^4*c^2+20*\ln(F)^3*b^3*c^4-60*\ln(F)^2*b^2*c^6+120*\ln(F)*b*c^8-120*c^{10})*c^2/b^6/\ln(F)^6/d*exp((a+b/(d*x+c)^2)*\ln(F))-c*(b^5*\ln(F)^5-10*\ln(F)^4*b^4*c^2+60*\ln(F)^3*b^3*c^4-240*\ln(F)^2*b^2*c^6+600*\ln(F)*b*c^8-720*c^{10})/b^6/\ln(F)^6*x*exp((a+b/(d*x+c)^2)*\ln(F))-1/2*d*(b^5*\ln(F)^5-30*\ln(F)^4*b^4*c^2+300*\ln(F)^3*b^3*c^4-1680*\ln(F)^2*b^2*c^6+5400*\ln(F)*b*c^8-7920*c^{10})/\ln(F)^6/b^6*x^2*exp((a+b/(d*x+c)^2)*\ln(F))+5/2*d^3*(b^4*\ln(F)^4-60*\ln(F)^3*b^3*c^2+840*\ln(F)^2*b^2*c^4-5040*\ln(F)*b*c^6+11880*c^8)/\ln(F)^6/b^6*x^4*exp((a+b/(d*x+c)^2)*\ln(F))-10*d^5*(\ln(F)^3*b^3-84*\ln(F)^2*b^2*c^2+1260*\ln(F)*b*c^4-5544*c^6)/\ln(F)^6/b^6*x^6*exp((a+b/(d*x+c)^2)*\ln(F))+30*d^7*(\ln(F)^2*b^2-90*\ln(F)*b*c^2+990*c^4)/\ln(F)^6/b^6*x^8*exp((a+b/(d*x+c)^2)*\ln(F))-60*d^9*(b*\ln(F)-66*c^2)/\ln(F)^6/b^6*x^{10}*exp((a+b/(d*x+c)^2)*\ln(F))+10*c*d^2*(b^4*\ln(F)^4-20*\ln(F)^3*b^3*c^2+168*\ln(F)^2*b^2*c^4-720*\ln(F)*b*c^6+1320*c^8)/\ln(F)^6/b^6*x^3*exp((a+b/(d*x+c)^2)*\ln(F))-60*c*d^4*(\ln(F)^3*b^3-28*\ln(F)^2*b^2*c^2+252*\ln(F)*b*c^4-792*c^6)/\ln(F)^6/b^6*x^5*exp((a+b/(d*x+c)^2)*\ln(F))+240*c*d^6*(\ln(F)^2*b^2-30*\ln(F)*b*c^2+198*c^4)/\ln(F)^6/b^6*x^7*exp((a+b/(d*x+c)^2)*\ln(F))-600*c*d^8*(b*\ln(F)-22*c^2)/\ln(F)^6/b^6*x^9*exp((a+b/(d*x+c)^2)*\ln(F))+60*d^{11}/\ln(F)^6/b^6*x^{12}*exp((a+b/(d*x+c)^2)*\ln(F)))/(d*x+c)^{12}$

**Maxima [B]** time = 1.07428, size = 999, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^2)})/(d*x+c)^{13}, x, \text{algorithm}=\text{"maxima"})$

[Out]  $1/2*(120*F^a*d^{10}*x^{10} + 1200*F^a*c*d^9*x^9 + 120*F^a*c^{10} - 120*F^a*b*c^8*\log(F) + 60*F^a*b^2*c^6*\log(F)^2 - 20*F^a*b^3*c^4*\log(F)^3 + 5*F^a*b^4*c^2*\log(F)^4 - F^a*b^5*\log(F)^5 + 120*(45*F^a*c^2*d^8 - F^a*b*d^8*\log(F))*x^8 + 960*(15*F^a*c^3*d^7 - F^a*b*c*d^7*\log(F))*x^7 + 60*(420*F^a*c^4*d^6 - 56*F^a*b*c^2*d^6*\log(F) + F^a*b^2*d^6*\log(F)^2)*x^6 + 120*(252*F^a*c^5*d^5 - 56*F^a*b*c^3*d^5*\log(F) + 3*F^a*b^2*c*d^5*\log(F)^2)*x^5 + 20*(1260*F^a*c^6*d^4 - 420*F^a*b*c^4*d^4*\log(F) + 45*F^a*b^2*c^2*d^4*\log(F)^2 - F^a*b^3*d^4*\log(F)^3)*x^4 + 80*(180*F^a*c^7*d^3 - 84*F^a*b*c^5*d^3*\log(F) + 15*F^a*b^2*c^3*d^3*\log(F)^2 - F^a*b^3*c*d^3*\log(F)^3)*x^3 + 5*(1080*F^a*c^8*d^2 - 672*F^a*b*c^6*d^2*\log(F) + 180*F^a*b^2*c^4*d^2*\log(F)^2 - 24*F^a*b^3*c^2*d^2*\log(F)^3 + F^a*b^4*d^2*\log(F)^4)*x^2 + 10*(120*F^a*c^9*d - 96*F^a*b*c^7*d*\log(F) + 36*F^a*b^2*c^5*d*\log(F)^2 - 8*F^a*b^3*c^3*d*\log(F)^3 + F^a*b^4*c*d*\log(F)^4)*x)*F^{(b/(d^2*x^2 + 2*c*d*x + c^2))}/(b^6*d^{11}*x^{10}*\log(F)^6 + 10*b^6*c*d^{10}*x^9*\log(F)^6 + 45*b^6*c^2*d^9*x^8*\log(F)^6 + 120*b^6*c^3*d^8*x^7*\log(F)^6 + 210*b^6*c^4*d^7*x^6*\log(F)^6 + 252*b^6*c^5*d^6*x^5*\log(F)^6 + 210*b^6*c^6*d^5*x^4*\log(F)^6 + 120*b^6*c^7*d^4*x^3*\log(F)^6 + 45*b^6*c^8*d^3*x^2*\log(F)^6 + 10*b^6*c^9*d^2*x*\log(F)^6 + b^6*c^{10}*d*\log(F)^6)$

**Fricas [B]** time = 2.17798, size = 1283, normalized size = 11.35

$$(120 d^{10} x^{10} + 1200 c d^9 x^9 + 5400 c^2 d^8 x^8 + 14400 c^3 d^7 x^7 + 25200 c^4 d^6 x^6 + 30240 c^5 d^5 x^5 + 25200 c^6 d^4 x^4 + 14400 c^7 d^3 x^3 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^13,x, algorithm="fricas")

[Out] 1/2\*(120\*d^10\*x^10 + 1200\*c\*d^9\*x^9 + 5400\*c^2\*d^8\*x^8 + 14400\*c^3\*d^7\*x^7 + 25200\*c^4\*d^6\*x^6 + 30240\*c^5\*d^5\*x^5 + 25200\*c^6\*d^4\*x^4 + 14400\*c^7\*d^3\*x^3 + 5400\*c^8\*d^2\*x^2 + 1200\*c^9\*d\*x + 120\*c^10 - b^5\*log(F)^5 + 5\*(b^4\*d^2\*x^2 + 2\*b^4\*c\*d\*x + b^4\*c^2)\*log(F)^4 - 20\*(b^3\*d^4\*x^4 + 4\*b^3\*c\*d^3\*x^3 + 6\*b^3\*c^2\*d^2\*x^2 + 4\*b^3\*c^3\*d\*x + b^3\*c^4)\*log(F)^3 + 60\*(b^2\*d^6\*x^6 + 6\*b^2\*c\*d^5\*x^5 + 15\*b^2\*c^2\*d^4\*x^4 + 20\*b^2\*c^3\*d^3\*x^3 + 15\*b^2\*c^4\*d^2\*x^2 + 6\*b^2\*c^5\*d\*x + b^2\*c^6)\*log(F)^2 - 120\*(b\*d^8\*x^8 + 8\*b\*c\*d^7\*x^7 + 28\*b\*c^2\*d^6\*x^6 + 56\*b\*c^3\*d^5\*x^5 + 70\*b\*c^4\*d^4\*x^4 + 56\*b\*c^5\*d^3\*x^3 + 28\*b\*c^6\*d^2\*x^2 + 8\*b\*c^7\*d\*x + b\*c^8)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2))/((b^6\*d^11\*x^10 + 10\*b^6\*c\*d^10\*x^9 + 45\*b^6\*c^2\*d^9\*x^8 + 120\*b^6\*c^3\*d^8\*x^7 + 210\*b^6\*c^4\*d^7\*x^6 + 252\*b^6\*c^5\*d^6\*x^5 + 210\*b^6\*c^6\*d^5\*x^4 + 120\*b^6\*c^7\*d^4\*x^3 + 45\*b^6\*c^8\*d^3\*x^2 + 10\*b^6\*c^9\*d^2\*x + b^6\*c^10\*d)\*log(F)^6)

**Sympy [B]** time = 0.763467, size = 745, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*13,x)

[Out] F\*\*(a + b/(c + d\*x)\*\*2)\*(-b\*\*5\*log(F)\*\*5 + 5\*b\*\*4\*c\*\*2\*log(F)\*\*4 + 10\*b\*\*4\*c\*d\*x\*log(F)\*\*4 + 5\*b\*\*4\*d\*\*2\*x\*\*2\*log(F)\*\*4 - 20\*b\*\*3\*c\*\*4\*log(F)\*\*3 - 80\*b\*\*3\*c\*\*3\*d\*x\*log(F)\*\*3 - 120\*b\*\*3\*c\*\*2\*d\*\*2\*x\*\*2\*log(F)\*\*3 - 80\*b\*\*3\*c\*d\*\*3\*x\*\*3\*log(F)\*\*3 - 20\*b\*\*3\*d\*\*4\*x\*\*4\*log(F)\*\*3 + 60\*b\*\*2\*c\*\*6\*log(F)\*\*2 + 360\*b\*\*2\*c\*\*5\*d\*x\*log(F)\*\*2 + 900\*b\*\*2\*c\*\*4\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 1200\*b\*\*2\*c\*\*3\*d\*\*3\*x\*\*3\*log(F)\*\*2 + 900\*b\*\*2\*c\*\*2\*d\*\*4\*x\*\*4\*log(F)\*\*2 + 360\*b\*\*2\*c\*d\*\*5\*x\*\*5\*log(F)\*\*2 + 60\*b\*\*2\*d\*\*6\*x\*\*6\*log(F)\*\*2 - 120\*b\*c\*\*8\*log(F) - 960\*b\*c\*\*7\*d\*x\*log(F) - 3360\*b\*c\*\*6\*d\*\*2\*x\*\*2\*log(F) - 6720\*b\*c\*\*5\*d\*\*3\*x\*\*3\*log(F) - 8400\*b\*c\*\*4\*d\*\*4\*x\*\*4\*log(F) - 6720\*b\*c\*\*3\*d\*\*5\*x\*\*5\*log(F) - 3360\*b\*c\*\*2\*d\*\*6\*x\*\*6\*log(F) - 960\*b\*c\*d\*\*7\*x\*\*7\*log(F) - 120\*b\*d\*\*8\*x\*\*8\*log(F) + 120\*c\*\*10 + 1200\*c\*\*9\*d\*x + 5400\*c\*\*8\*d\*\*2\*x\*\*2 + 14400\*c\*\*7\*d\*\*3\*x\*\*3 + 25200\*c\*\*6\*d\*\*4\*x\*\*4 + 30240\*c\*\*5\*d\*\*5\*x\*\*5 + 25200\*c\*\*4\*d\*\*6\*x\*\*6 + 14400\*c\*\*3\*d\*\*7\*x\*\*7 + 5400\*c\*\*2\*d\*\*8\*x\*\*8 + 1200\*c\*d\*\*9\*x\*\*9 + 120\*d\*\*10\*x\*\*10)/(2\*b\*\*6\*c\*\*10\*d\*log(F)\*\*6 + 20\*b\*\*6\*c\*\*9\*d\*\*2\*x\*log(F)\*\*6 + 90\*b\*\*6\*c\*\*8\*d\*\*3\*x\*\*2\*log(F)\*\*6 + 240\*b\*\*6\*c\*\*7\*d\*\*4\*x\*\*3\*log(F)\*\*6 + 420\*b\*\*6\*c\*\*6\*d\*\*5\*x\*\*4\*log(F)\*\*6 + 504\*b\*\*6\*c\*\*5\*d\*\*6\*x\*\*5\*log(F)\*\*6 + 420\*b\*\*6\*c\*\*4\*d\*\*7\*x\*\*6\*log(F)\*\*6 + 240\*b\*\*6\*c\*\*3\*d\*\*8\*x\*\*7\*log(F)\*\*6 + 90\*b\*\*6\*c\*\*2\*d\*\*9\*x\*\*8\*log(F)\*\*6 + 20\*b\*\*6\*c\*d\*\*10\*x\*\*9\*log(F)\*\*6 + 2\*b\*\*6\*d\*\*11\*x\*\*10\*log(F)\*\*6)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{13}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^13, x)
```

$$3.327 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^{10} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2} \text{Gamma}\left(-\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out] (F^a\*(c + d\*x)^11\*Gamma[-11/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(11/2))/(2\*d)

**Rubi [A]** time = 0.0484642, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2} \text{Gamma}\left(-\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^10,x]

[Out] (F^a\*(c + d\*x)^11\*Gamma[-11/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(11/2))/(2\*d)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^{10} dx = \frac{F^a(c+dx)^{11} \Gamma\left(-\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right) \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}{2d}$$

**Mathematica [A]** time = 0.0293848, size = 49, normalized size = 1.

$$\frac{F^a(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2} \text{Gamma}\left(-\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^10,x]

[Out] (F^a\*(c + d\*x)^11\*Gamma[-11/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(11/2))/(2\*d)

**Maple [B]** time = 0.14, size = 1173, normalized size = 23.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(a+b/(d*x+c)^2)}*(d*x+c)^{10}, x)$

[Out] 
$$\begin{aligned} & 2/11*d^7*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c*x^8+8/11*d^6*F^{a*b*\ln(F)}*F^{(b/(d*x+c)} \\ & )^2)*c^2*x^7+56/33*d^5*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^3*x^6+30*d^3*F^{a*b*\ln(F)} \\ & (b/(d*x+c)^2)*c^7*x^4+15*d^2*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^8*x^3+5*d*F^{a*b*\ln(F)} \\ & (b/(d*x+c)^2)*c^9*x^2+28/11*d^4*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^4*x^5+28/11*d^3*F^{a*b*\ln} \\ & (F)*F^{(b/(d*x+c)^2)*c^5*x^4+56/33*d^2*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^6*x^3+8 \\ & /11*d*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^7*x^2+4/99*d^5*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)} \\ & )^2)*c*x^6+4/33*d^4*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^2*x^5+20/99*d^3*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)} \\ & )^2)*c^3*x^4+20/99*d^2*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^4*x^3+4/33*d*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)} \\ & )^2)*c^5*x^2-32/10395/d*F^{a*b^6*\ln(F)^6*\ln(F)^{1/2}/(-b*\ln(F))^{1/2}*erf((-b*\ln(F))^{1/2}/(d*x+c))+8/693*d \\ & ^3*F^{a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2)*c*x^4+16/693*d^2*F^{a*b^3*\ln(F)^3*F^{(b/(d} \\ & *x+c)^2)*c^2*x^3+16/693*d*F^{a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2)*c^3*x^2+16/3465*d \\ & *F^{a*b^4*\ln(F)^4*F^{(b/(d*x+c)^2)*c*x^2+2/11*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^8 \\ & *x+4/99*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^6*x+8/693*F^{a*b^3*\ln(F)^3*F^{(b/(d} \\ & *x+c)^2)*c^4*x+16/3465*F^{a*b^4*\ln(F)^4*F^{(b/(d*x+c)^2)*c^2*x+2/99/d*F^{a*b*\ln} \\ & (F)*F^{(b/(d*x+c)^2)*c^9+1/11/d*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^11+32/10395*F^{a*b^5*\ln} \\ & (F)^5*F^{(b/(d*x+c)^2)*x+d^9*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c*x^10+5*d^8*F^{a*b*\ln(F)}*F^{(b/(d*x} \\ & +c)^2)*c^2*x^9+15*d^7*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^3*x^8+30*d^6*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2} \\ & )*c^4*x^7+42*d^5*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^5*x^6+42*d^4*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^6 \\ & *x^5+1/11*d^10*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*x^11+F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*c^10*x+4/693/d* \\ & F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^7+8/3465/d*F^{a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2} \\ & )*c^5+16/10395/d*F^{a*b^4*\ln(F)^4*F^{(b/(d*x+c)^2)*c^3+32/10395/d*F^{a*b^5*\ln(F)} \\ & )^5*F^{(b/(d*x+c)^2)*c+4/693*d^6*F^{a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*x^7+8/3465 \\ & *d^4*F^{a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2)*x^5+16/10395*d^2*F^{a*b^4*\ln(F)^4*F^{(b/} \\ & (d*x+c)^2)*x^3+2/99*d^8*F^{a*b*\ln(F)}*F^{(b/(d*x+c)^2)*x^9} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^2)}*(d*x+c)^{10}, x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & 1/10395*(945*F^{a*d^{10}*x^{11}} + 10395*F^{a*c*d^9*x^{10}} + 105*(495*F^{a*c^2*d^8} + \\ & 2*F^{a*b*d^8*\log(F)})*x^9 + 945*(165*F^{a*c^3*d^7} + 2*F^{a*b*c*d^7*\log(F)})*x^8 \\ & + 30*(10395*F^{a*c^4*d^6} + 252*F^{a*b*c^2*d^6*\log(F)} + 2*F^{a*b^2*d^6*\log(F)^2} \\ & )*x^7 + 210*(2079*F^{a*c^5*d^5} + 84*F^{a*b*c^3*d^5*\log(F)} + 2*F^{a*b^2*c*d^5*\log} \\ & (F)^2)*x^6 + 6*(72765*F^{a*c^6*d^4} + 4410*F^{a*b*c^4*d^4*\log(F)} + 210*F^{a*b} \\ & ^2*c^2*d^4*\log(F)^2 + 4*F^{a*b^3*d^4*\log(F)^3})*x^5 + 30*(10395*F^{a*c^7*d^3} + \\ & 882*F^{a*b*c^5*d^3*\log(F)} + 70*F^{a*b^2*c^3*d^3*\log(F)^2} + 4*F^{a*b^3*c*d^3*\log} \\ & (F)^3)*x^4 + (155925*F^{a*c^8*d^2} + 17640*F^{a*b*c^6*d^2*\log(F)} + 2100*F^{a*b} \\ & ^2*c^4*d^2*\log(F)^2 + 240*F^{a*b^3*c^2*d^2*\log(F)^3} + 16*F^{a*b^4*d^2*\log(F)} \\ & ^4)*x^3 + 3*(17325*F^{a*c^9*d} + 2520*F^{a*b*c^7*d*\log(F)} + 420*F^{a*b^2*c^5*d* \\ & \log(F)^2} + 80*F^{a*b^3*c^3*d*\log(F)^3} + 16*F^{a*b^4*c*d*\log(F)^4})*x^2 + (1039 \\ & 5*F^{a*c^{10}} + 1890*F^{a*b*c^8*\log(F)} + 420*F^{a*b^2*c^6*\log(F)^2} + 120*F^{a*b^3} \\ & *c^4*\log(F)^3 + 48*F^{a*b^4*c^2*\log(F)^4} + 32*F^{a*b^5*\log(F)^5})*x)*F^{(b/(d^2} \\ & *x^2 + 2*c*d*x + c^2)) + \text{integrate}(2/10395*(32*F^{a*b^6*d*x*\log(F)^6} - 945*F \end{aligned}$$

$$^a b^c^{11} \log(F) - 210 F^a b^2 c^9 \log(F)^2 - 60 F^a b^3 c^7 \log(F)^3 - 24 F^a b^4 c^5 \log(F)^4 - 16 F^a b^5 c^3 \log(F)^5) F^{(b/(d^2 x^2 + 2 c d x + c^2)) / (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3)}, x)$$

**Fricas [B]** time = 1.68314, size = 1285, normalized size = 26.22

$$32 \sqrt{\pi} F^a b^5 d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) \log(F)^5 + (945 d^{11} x^{11} + 10395 c d^{10} x^{10} + 51975 c^2 d^9 x^9 + 155925 c^3 d^8 x^8 + 311850$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^10,x, algorithm="fricas")

[Out] 1/10395\*(32\*sqrt(pi)\*F^a\*b^5\*d\*sqrt(-b\*log(F)/d^2)\*erf(d\*sqrt(-b\*log(F)/d^2)/(d\*x + c))\*log(F)^5 + (945\*d^11\*x^11 + 10395\*c\*d^10\*x^10 + 51975\*c^2\*d^9\*x^9 + 155925\*c^3\*d^8\*x^8 + 311850\*c^4\*d^7\*x^7 + 436590\*c^5\*d^6\*x^6 + 436590\*c^6\*d^5\*x^5 + 311850\*c^7\*d^4\*x^4 + 155925\*c^8\*d^3\*x^3 + 51975\*c^9\*d^2\*x^2 + 10395\*c^10\*d\*x + 945\*c^11 + 32\*(b^5\*d\*x + b^5\*c)\*log(F)^5 + 16\*(b^4\*d^3\*x^3 + 3\*b^4\*c\*d^2\*x^2 + 3\*b^4\*c^2\*d\*x + b^4\*c^3)\*log(F)^4 + 24\*(b^3\*d^5\*x^5 + 5\*b^3\*c\*d^4\*x^4 + 10\*b^3\*c^2\*d^3\*x^3 + 10\*b^3\*c^3\*d^2\*x^2 + 5\*b^3\*c^4\*d\*x + b^3\*c^5)\*log(F)^3 + 60\*(b^2\*d^7\*x^7 + 7\*b^2\*c\*d^6\*x^6 + 21\*b^2\*c^2\*d^5\*x^5 + 35\*b^2\*c^3\*d^4\*x^4 + 35\*b^2\*c^4\*d^3\*x^3 + 21\*b^2\*c^5\*d^2\*x^2 + 7\*b^2\*c^6\*d\*x + b^2\*c^7)\*log(F)^2 + 210\*(b\*d^9\*x^9 + 9\*b\*c\*d^8\*x^8 + 36\*b\*c^2\*d^7\*x^7 + 84\*b\*c^3\*d^6\*x^6 + 126\*b\*c^4\*d^5\*x^5 + 126\*b\*c^5\*d^4\*x^4 + 84\*b\*c^6\*d^3\*x^3 + 36\*b\*c^7\*d^2\*x^2 + 9\*b\*c^8\*d\*x + b\*c^9)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{10} F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^10,x, algorithm="giac")

[Out] integrate((d\*x + c)^10\*F^(a + b/(d\*x + c)^2), x)

$$3.328 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^8 dx$$

**Optimal.** Leaf size=49

$$\frac{F^a(c+dx)^9 \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{9/2} \Gamma\left(-\frac{9}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

[Out] (F^a\*(c + d\*x)^9\*Gamma[-9/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(9/2))/(2\*d)

**Rubi [A]** time = 0.0493251, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^9 \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{9/2} \Gamma\left(-\frac{9}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^8, x]

[Out] (F^a\*(c + d\*x)^9\*Gamma[-9/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(9/2))/(2\*d)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^8 dx = \frac{F^a(c+dx)^9 \Gamma\left(-\frac{9}{2}, -\frac{b \log(F)}{(c+dx)^2}\right) \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{9/2}}{2d}$$

**Mathematica [A]** time = 0.025662, size = 49, normalized size = 1.

$$\frac{F^a(c+dx)^9 \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{9/2} \Gamma\left(-\frac{9}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^8, x]

[Out] (F^a\*(c + d\*x)^9\*Gamma[-9/2, -((b\*Log[F])/(c + d\*x)^2)]\*(-((b\*Log[F])/(c + d\*x)^2))^(9/2))/(2\*d)

**Maple [B]** time = 0.079, size = 826, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(a+b/(d*x+c)^2)}*(d*x+c)^8,x)$

[Out]  $2/63/d^2 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^7 + 4/315/d^2 F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} c^5 + 8/945/d^2 F^{a*b^3} \ln(F)^3 F^{(b/(d*x+c)^2)} c^3 + 16/945/d^2 F^{a*b^4} \ln(F)^4 F^{(b/(d*x+c)^2)} c + 2/63*d^6 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} x^7 + 4/315*d^4 F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} x^5 + 8/945*d^2 F^{a*b^3} \ln(F)^3 F^{(b/(d*x+c)^2)} x^3 + 8/315 F^{a*b^3} \ln(F)^3 F^{(b/(d*x+c)^2)} c^2 x + 2/9 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^6 x + 4/63 F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} c^4 x + 16/945 F^{a*b^4} \ln(F)^4 F^{(b/(d*x+c)^2)} x + d^7 F^{a*b} F^{(b/(d*x+c)^2)} c^2 x^8 + 4*d^6 F^{a*b} F^{(b/(d*x+c)^2)} c^2 x^7 + 28/3*d^5 F^{a*b} F^{(b/(d*x+c)^2)} c^3 x^6 + 14*d^4 F^{a*b} F^{(b/(d*x+c)^2)} c^4 x^5 + 14*d^3 F^{a*b} F^{(b/(d*x+c)^2)} c^5 x^4 + 28/3*d^2 F^{a*b} F^{(b/(d*x+c)^2)} c^6 x^3 + 4*d F^{a*b} F^{(b/(d*x+c)^2)} c^7 x^2 - 16/945/d^2 F^{a*b^5} \ln(F)^5 \pi^{(1/2)} / (-b*\ln(F))^{(1/2)} * \text{erf}((-b*\ln(F))^{(1/2)} / (d*x+c)) + 2/9*d^5 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^2 x^6 + 2/3*d^4 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^2 x^5 + 10/9*d^3 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^3 x^4 + 10/9*d^2 F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^4 x^3 + 2/3*d F^{a*b} \ln(F) F^{(b/(d*x+c)^2)} c^5 x^2 + 4/63*d^3 F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} c^2 x^4 + 8/63*d^2 F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} c^2 x^3 + 8/63*d F^{a*b^2} \ln(F)^2 F^{(b/(d*x+c)^2)} c^3 x^2 + 8/315*d F^{a*b^3} \ln(F)^3 F^{(b/(d*x+c)^2)} c^2 x^2 + F^{a*b} F^{(b/(d*x+c)^2)} c^8 x + 1/9/d F^{a*b} F^{(b/(d*x+c)^2)} c^9 + 1/9*d^8 F^{a*b} F^{(b/(d*x+c)^2)} c^2 x^9$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{945} (105 F^a d^8 x^9 + 945 F^a c d^7 x^8 + 30 (126 F^a c^2 d^6 + F^a b d^6 \log(F)) x^7 + 210 (42 F^a c^3 d^5 + F^a b c d^5 \log(F)) x^6 + 6 (2205 F^a c^4 d^4 + 105 F^a b c^2 d^4 \log(F) + 2 F^a b^2 d^4 \log(F)^2) x^5 + 30 (441 F^a c^5 d^3 + 35 F^a b c^3 d^3 \log(F) + 2 F^a b^2 c d^3 \log(F)^2) x^4 + 2 (4410 F^a c^6 d^2 + 525 F^a b c^4 d^2 \log(F) + 60 F^a b^2 c^2 d^2 \log(F)^2 + 4 F^a b^3 d^2 \log(F)^3) x^3 + 6 (630 F^a c^7 d + 105 F^a b c^5 d \log(F) + 20 F^a b^2 c^3 d \log(F)^2 + 4 F^a b^3 c d \log(F)^3) x^2 + (945 F^a c^8 + 210 F^a b c^6 \log(F) + 60 F^a b^2 c^4 \log(F)^2 + 24 F^a b^3 c^2 \log(F)^3 + 16 F^a b^4 \log(F)^4) x) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} + \text{integrate}(2/945*(16 F^{a*b^5} d*x*\log(F)^5 - 105 F^{a*b} c^9 \log(F) - 30 F^{a*b^2} c^7 \log(F)^2 - 12 F^{a*b^3} c^5 \log(F)^3 - 8 F^{a*b^4} c^3 \log(F)^4) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} / (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^2)}*(d*x+c)^8,x, \text{algorithm}="maxima")$

[Out]  $1/945*(105 F^{a*d^8*x^9} + 945 F^{a*c*d^7*x^8} + 30*(126 F^{a*c^2*d^6} + F^{a*b*d^6} \log(F)) x^7 + 210*(42 F^{a*c^3*d^5} + F^{a*b*c*d^5} \log(F)) x^6 + 6*(2205 F^{a*c^4*d^4} + 105 F^{a*b*c^2*d^4} \log(F) + 2 F^{a*b^2*d^4} \log(F)^2) x^5 + 30*(441 F^{a*c^5*d^3} + 35 F^{a*b*c^3*d^3} \log(F) + 2 F^{a*b^2*c*d^3} \log(F)^2) x^4 + 2*(4410 F^{a*c^6*d^2} + 525 F^{a*b*c^4*d^2} \log(F) + 60 F^{a*b^2*c^2*d^2} \log(F)^2 + 4 F^{a*b^3*d^2} \log(F)^3) x^3 + 6*(630 F^{a*c^7*d} + 105 F^{a*b*c^5*d} \log(F) + 20 F^{a*b^2*c^3*d} \log(F)^2 + 4 F^{a*b^3*c*d} \log(F)^3) x^2 + (945 F^{a*c^8} + 210 F^{a*b*c^6} \log(F) + 60 F^{a*b^2*c^4} \log(F)^2 + 24 F^{a*b^3*c^2} \log(F)^3 + 16 F^{a*b^4} \log(F)^4) x) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} + \text{integrate}(2/945*(16 F^{a*b^5} d*x*\log(F)^5 - 105 F^{a*b} c^9 \log(F) - 30 F^{a*b^2} c^7 \log(F)^2 - 12 F^{a*b^3} c^5 \log(F)^3 - 8 F^{a*b^4} c^3 \log(F)^4) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))} / (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)$

**Fricas [B]** time = 1.6409, size = 934, normalized size = 19.06

$$16 \sqrt{\pi} F^a b^4 d \sqrt{-\frac{b \log(F)}{d^2}} \text{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) \log(F)^4 + (105 d^9 x^9 + 945 c d^8 x^8 + 3780 c^2 d^7 x^7 + 8820 c^3 d^6 x^6 + 13230 c^4 d^5 x^5 + 10500 c^5 d^4 x^4 + 3780 c^6 d^3 x^3 + 882 c^7 d^2 x^2 + 105 c^8 d x + c^9) F^{(b/(d^2*x^2 + 2*c*d*x + c^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/945*(16*sqrt(pi)*F^a*b^4*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/
(d*x + c))*log(F)^4 + (105*d^9*x^9 + 945*c*d^8*x^8 + 3780*c^2*d^7*x^7 + 882
0*c^3*d^6*x^6 + 13230*c^4*d^5*x^5 + 13230*c^5*d^4*x^4 + 8820*c^6*d^3*x^3 +
3780*c^7*d^2*x^2 + 945*c^8*d*x + 105*c^9 + 16*(b^4*d*x + b^4*c)*log(F)^4 +
8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(F)^3 + 12*(
b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5
*b^2*c^4*d*x + b^2*c^5)*log(F)^2 + 30*(b*d^7*x^7 + 7*b*c*d^6*x^6 + 21*b*c^2
*d^5*x^5 + 35*b*c^3*d^4*x^4 + 35*b*c^4*d^3*x^3 + 21*b*c^5*d^2*x^2 + 7*b*c^6
*d*x + b*c^7)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)))/d
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**8,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^8 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^8*F^(a + b/(d*x + c)^2), x)
```

$$3.329 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^6 dx$$

**Optimal.** Leaf size=170

$$-\frac{8\sqrt{\pi}b^{7/2}F^a \log^2(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{105d} + \frac{8b^3 \log^3(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{105d} + \frac{4b^2 \log^2(F)(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{105d} + \frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

[Out]  $(F^{(a+b/(c+dx)^2)}(c+dx)^7)/(7*d) + (2*b*F^{(a+b/(c+dx)^2)}(c+dx)^5*\operatorname{Log}[F])/(35*d) + (4*b^2*F^{(a+b/(c+dx)^2)}(c+dx)^3*\operatorname{Log}[F]^2)/(105*d) + (8*b^3*F^{(a+b/(c+dx)^2)}(c+dx)*\operatorname{Log}[F]^3)/(105*d) - (8*b^{(7/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c+dx)]*\operatorname{Log}[F]^{(7/2)})/(105*d)$

**Rubi [A]** time = 0.233666, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{8\sqrt{\pi}b^{7/2}F^a \log^2(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{105d} + \frac{8b^3 \log^3(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{105d} + \frac{4b^2 \log^2(F)(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{105d} + \frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a+b/(c+dx)^2)}(c+dx)^6, x]$

[Out]  $(F^{(a+b/(c+dx)^2)}(c+dx)^7)/(7*d) + (2*b*F^{(a+b/(c+dx)^2)}(c+dx)^5*\operatorname{Log}[F])/(35*d) + (4*b^2*F^{(a+b/(c+dx)^2)}(c+dx)^3*\operatorname{Log}[F]^2)/(105*d) + (8*b^3*F^{(a+b/(c+dx)^2)}(c+dx)*\operatorname{Log}[F]^3)/(105*d) - (8*b^{(7/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c+dx)]*\operatorname{Log}[F]^{(7/2)})/(105*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})} * ((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m+1)}*F^{(a+b*(c+dx)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c+dx)^{(m+n)}*F^{(a+b*(c+dx)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{IntegerQ}[(2*(m+1))/n]$  &&  $\operatorname{LtQ}[-4, (m+1)/n, 5]$  &&  $\operatorname{IntegerQ}[n]$  &&  $(\operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1]) \mid\mid (\operatorname{GtQ}[-n, 0] \&\& \operatorname{LeQ}[-n, m+1])$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)*F^{(a+b*(c+dx)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c+dx)^n*F^{(a+b*(c+dx)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{IntegerQ}[2/n]$  &&  $\operatorname{LtQ}[n, 0]$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})} * ((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{(a+b*x^2)}, x], x, (c+dx)^{(m+1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n\}, x$  &&  $\operatorname{EqQ}[n, 2*(m+1)]$

#### Rule 2204



Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6 dx &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{1}{7}(2b \log(F)) \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5 \log(F)}{35d} + \frac{1}{35}(4b^2 \log^2(F)) \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5 \log(F)}{35d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log^2(F)}{105d} + \frac{1}{105} \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^0 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5 \log(F)}{35d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log^2(F)}{105d} + \frac{8b^3F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{8b^3F^{a+\frac{b}{(c+dx)^2}}} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5 \log(F)}{35d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log^2(F)}{105d} + \frac{8b^3F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{8b^3F^{a+\frac{b}{(c+dx)^2}}} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5 \log(F)}{35d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log^2(F)}{105d} + \frac{8b^3F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{8b^3F^{a+\frac{b}{(c+dx)^2}}} \end{aligned}$$

**Mathematica [A]** time = 0.127847, size = 113, normalized size = 0.66

$$\frac{F^a \left( (c+dx)F^{\frac{b}{(c+dx)^2}} \left( 4b^2 \log^2(F)(c+dx)^2 + 8b^3 \log^3(F) + 6b \log(F)(c+dx)^4 + 15(c+dx)^6 \right) - 8\sqrt{\pi}b^{7/2} \log^2(F) \operatorname{Erfi} \left( \frac{\sqrt{b}}{\sqrt{c+dx}} \right) \right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^6,x]

[Out] (F^a\*(-8\*b^(7/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]\*Log[F]^(7/2) + F^(b/(c + d\*x)^2)\*(c + d\*x)\*(15\*(c + d\*x)^6 + 6\*b\*(c + d\*x)^4\*Log[F] + 4\*b^2\*(c + d\*x)^2\*Log[F]^2 + 8\*b^3\*Log[F]^3))/(105\*d)

**Maple [B]** time = 0.059, size = 543, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^6,x)

[Out] 1/7\*d^6\*F^a\*F^(b/(d\*x+c)^2)\*x^7+d^5\*F^a\*F^(b/(d\*x+c)^2)\*c\*x^6+3\*d^4\*F^a\*F^(b/(d\*x+c)^2)\*c^2\*x^5+5\*d^3\*F^a\*F^(b/(d\*x+c)^2)\*c^3\*x^4+5\*d^2\*F^a\*F^(b/(d\*x+c)^2)\*c^4\*x^3+3\*d\*F^a\*F^(b/(d\*x+c)^2)\*c^5\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c^6\*x+1/7/d\*F^a\*F^(b/(d\*x+c)^2)\*c^7+2/35\*d^4\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*x^5+2/7\*d^3\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c\*x^4+4/7\*d^2\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^2\*x^3+4/7\*d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^3\*x^2+2/7\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^4\*x+2/35/d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^5+4/105\*d^2\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*x^3+4/35\*d\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c\*x^2+4/35\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c^2\*x+2/35/d\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c^3+4/105\*d^2\*F^a\*b^3\*ln(F)^3\*F^(b/(d\*x+c)^2)\*x+2/105\*d\*F^a\*b^3\*ln(F)^3\*F^(b/(d\*x+c)^2)\*c+2/105\*F^a\*b^3\*ln(F)^3\*F^(b/(d\*x+c)^2)\*c^2

$$2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^2*x+4/105/d*F^a*b^2*\ln(F)^2*F^{(b/(d*x+c)^2)*c^3+8/105*F^a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2)*x+8/105/d*F^a*b^3*\ln(F)^3*F^{(b/(d*x+c)^2)*c-8/105/d*F^a*b^4*\ln(F)^4*\pi^{(1/2)/(-b*\ln(F))^{(1/2)}*erf((-b*\ln(F))^{(1/2)/(d*x+c))}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{105} (15 F^a d^6 x^7 + 105 F^a c d^5 x^6 + 3 (105 F^a c^2 d^4 + 2 F^a b d^4 \log(F)) x^5 + 15 (35 F^a c^3 d^3 + 2 F^a b c d^3 \log(F)) x^4 + (525 F^a c^4 d^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/105*(15*F^a*d^6*x^7 + 105*F^a*c*d^5*x^6 + 3*(105*F^a*c^2*d^4 + 2*F^a*b*d^4*log(F))*x^5 + 15*(35*F^a*c^3*d^3 + 2*F^a*b*c*d^3*log(F))*x^4 + (525*F^a*c^4*d^2 + 60*F^a*b*c^2*d^2*log(F) + 4*F^a*b^2*d^2*log(F)^2)*x^3 + 3*(105*F^a*c^5*d + 20*F^a*b*c^3*d*log(F) + 4*F^a*b^2*c*d*log(F)^2)*x^2 + (105*F^a*c^6 + 30*F^a*b*c^4*log(F) + 12*F^a*b^2*c^2*log(F)^2 + 8*F^a*b^3*log(F)^3)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(2/105*(8*F^a*b^4*d*x*log(F)^4 - 15*F^a*b*c^7*log(F) - 6*F^a*b^2*c^5*log(F)^2 - 4*F^a*b^3*c^3*log(F)^3)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

**Fricas [A]** time = 1.61024, size = 663, normalized size = 3.9

$$8 \sqrt{\pi} F^a b^3 d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{d x+c}\right) \log(F)^3 + (15 d^7 x^7 + 105 c d^6 x^6 + 315 c^2 d^5 x^5 + 525 c^3 d^4 x^4 + 525 c^4 d^3 x^3 + 315 c^5 d^2 x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/105*(8*sqrt(pi)*F^a*b^3*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c))*log(F)^3 + (15*d^7*x^7 + 105*c*d^6*x^6 + 315*c^2*d^5*x^5 + 525*c^3*d^4*x^4 + 525*c^4*d^3*x^3 + 315*c^5*d^2*x^2 + 105*c^6*d*x + 15*c^7 + 8*(b^3*d*x + b^3*c)*log(F)^3 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*log(F)^2 + 6*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/d
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**6,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^6 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^6,x, algorithm="giac")

[Out] integrate((d\*x + c)^6\*F^(a + b/(d\*x + c)^2), x)

$$3.330 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^4 dx$$

**Optimal.** Leaf size=136

$$-\frac{4\sqrt{\pi}b^{5/2}F^a \log^5(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{15d} + \frac{4b^2 \log^2(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{15d} + \frac{(c+dx)^5 F^{a+\frac{b}{(c+dx)^2}}}{5d} + \frac{2b \log(F)(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{15d}$$

[Out]  $(F^{(a+b/(c+d*x)^2)}*(c+d*x)^5)/(5*d) + (2*b*F^{(a+b/(c+d*x)^2)}*(c+d*x)^3*\operatorname{Log}[F])/(15*d) + (4*b^2*F^{(a+b/(c+d*x)^2)}*(c+d*x)*\operatorname{Log}[F]^2)/(15*d) - (4*b^{(5/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c+d*x)])*\operatorname{Log}[F]^{(5/2)})/(15*d)$

**Rubi [A]** time = 0.168491, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{4\sqrt{\pi}b^{5/2}F^a \log^5(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{15d} + \frac{4b^2 \log^2(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{15d} + \frac{(c+dx)^5 F^{a+\frac{b}{(c+dx)^2}}}{5d} + \frac{2b \log(F)(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{15d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^4, x]

[Out]  $(F^{(a+b/(c+d*x)^2)}*(c+d*x)^5)/(5*d) + (2*b*F^{(a+b/(c+d*x)^2)}*(c+d*x)^3*\operatorname{Log}[F])/(15*d) + (4*b^2*F^{(a+b/(c+d*x)^2)}*(c+d*x)*\operatorname{Log}[F]^2)/(15*d) - (4*b^{(5/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c+d*x)])*\operatorname{Log}[F]^{(5/2)})/(15*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[\pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4 dx &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{1}{5}(2b \log(F)) \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log(F)}{15d} + \frac{1}{15}(4b^2 \log^2(F)) \int F^{a+\frac{b}{(c+dx)^2}} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log(F)}{15d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx) \log^2(F)}{15d} + \frac{1}{15}(8b^3 \log^3(F)) \int F^{a+\frac{b}{(c+dx)^2}} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log(F)}{15d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx) \log^2(F)}{15d} - \frac{(8b^3 \log^3(F)) \int F^{a+\frac{b}{(c+dx)^2}} dx}{15d} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^3 \log(F)}{15d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx) \log^2(F)}{15d} - \frac{4b^5/2F^a}{15d} \end{aligned}$$

**Mathematica [A]** time = 0.0963702, size = 97, normalized size = 0.71

$$\frac{F^a \left( (c+dx)F^{\frac{b}{(c+dx)^2}} \left( 4b^2 \log^2(F) + 2b \log(F)(c+dx)^2 + 3(c+dx)^4 \right) - 4\sqrt{\pi}b^{5/2} \log^{\frac{5}{2}}(F) \operatorname{Erfi} \left( \frac{\sqrt{b}\sqrt{\log(F)}}{c+dx} \right) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^4,x]

[Out] (F^a\*(-4\*b^(5/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]\*Log[F]^(5/2) + F^(b/(c + d\*x)^2)\*(c + d\*x)\*(3\*(c + d\*x)^4 + 2\*b\*(c + d\*x)^2\*Log[F] + 4\*b^2\*Log[F]^2)))/(15\*d)

**Maple [B]** time = 0.046, size = 324, normalized size = 2.4

$$\frac{d^4 F^a x^5}{5} F^{\frac{b}{(dx+c)^2}} + d^3 F^a F^{\frac{b}{(dx+c)^2}} c x^4 + 2 d^2 F^a F^{\frac{b}{(dx+c)^2}} c^2 x^3 + 2 d F^a F^{\frac{b}{(dx+c)^2}} c^3 x^2 + F^a F^{\frac{b}{(dx+c)^2}} c^4 x + \frac{F^a c^5}{5 d} F^{\frac{b}{(dx+c)^2}} + \frac{2 d^2 F^a b \ln(F)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^4,x)

[Out] 1/5\*d^4\*F^a\*F^(b/(d\*x+c)^2)\*x^5+d^3\*F^a\*F^(b/(d\*x+c)^2)\*c\*x^4+2\*d^2\*F^a\*F^(b/(d\*x+c)^2)\*c^2\*x^3+2\*d\*F^a\*F^(b/(d\*x+c)^2)\*c^3\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c^4\*x+1/5/d\*F^a\*F^(b/(d\*x+c)^2)\*c^5+2/15\*d^2\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*x^3+2/5\*d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c\*x^2+2/5\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^2\*x+2/15/d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c^3+4/15\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*x+4/15/d\*F^a\*b^2\*ln(F)^2\*F^(b/(d\*x+c)^2)\*c-4/15/d\*F^a\*b^3\*ln(F)^3\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{15} \left( 3F^a d^4 x^5 + 15F^a c d^3 x^4 + 2(15F^a c^2 d^2 + F^a b d^2 \log(F)) x^3 + 6(5F^a c^3 d + F^a b c d \log(F)) x^2 + (15F^a c^4 + 6F^a b c^2 \log(F)) x + 3F^a c^5 + 4F^a b c^3 \log(F) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="maxima")

[Out] 1/15\*(3F^a\*d^4\*x^5 + 15F^a\*c\*d^3\*x^4 + 2\*(15F^a\*c^2\*d^2 + F^a\*b\*d^2\*log(F))\*x^3 + 6\*(5F^a\*c^3\*d + F^a\*b\*c\*d\*log(F))\*x^2 + (15F^a\*c^4 + 6F^a\*b\*c^2\*log(F) + 4F^a\*b^2\*log(F)^2)\*x)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + integrate(2/15\*(4F^a\*b^3\*d\*x\*log(F)^3 - 3F^a\*b\*c^5\*log(F) - 2F^a\*b^2\*c^3\*log(F)^2)\*F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

**Fricas [A]** time = 1.6879, size = 458, normalized size = 3.37

$$4\sqrt{\pi}F^a b^2 d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) \log(F)^2 + (3d^5 x^5 + 15cd^4 x^4 + 30c^2 d^3 x^3 + 30c^3 d^2 x^2 + 15c^4 dx + 3c^5 + 4(b^2 dx + b^2 c)) \log(F)$$

15d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="fricas")

[Out] 1/15\*(4\*sqrt(pi)\*F^a\*b^2\*d\*sqrt(-b\*log(F)/d^2)\*erf(d\*sqrt(-b\*log(F)/d^2)/(d\*x + c))\*log(F)^2 + (3\*d^5\*x^5 + 15\*c\*d^4\*x^4 + 30\*c^2\*d^3\*x^3 + 30\*c^3\*d^2\*x^2 + 15\*c^4\*d\*x + 3\*c^5 + 4\*(b^2\*d\*x + b^2\*c))\*log(F)^2 + 2\*(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)\*(d\*x+c)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^4,x, algorithm="giac")

[Out] integrate((d\*x + c)^4 \* F^(a + b/(d\*x + c)^2), x)

$$3.331 \quad \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)^2 dx$$

**Optimal.** Leaf size=102

$$-\frac{2\sqrt{\pi}b^{3/2}F^a \log^{\frac{3}{2}}(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{3d} + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d} + \frac{2b \log(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^3)/(3*d) + (2*b*F^{(a + b/(c + d*x)^2)}*(c + d*x)*\operatorname{Log}[F])/(3*d) - (2*b^{(3/2)}*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c + d*x)])*\operatorname{Log}[F]^{(3/2)})/(3*d)$

**Rubi [A]** time = 0.116043, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2214, 2206, 2211, 2204}

$$-\frac{2\sqrt{\pi}b^{3/2}F^a \log^{\frac{3}{2}}(F)\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{3d} + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d} + \frac{2b \log(F)(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}*(c + d*x)^2, x]$

[Out]  $(F^{(a + b/(c + d*x)^2)}*(c + d*x)^3)/(3*d) + (2*b*F^{(a + b/(c + d*x)^2)}*(c + d*x)*\operatorname{Log}[F])/(3*d) - (2*b^{(3/2)}*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c + d*x)])*\operatorname{Log}[F]^{(3/2)})/(3*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*F^{(a + b*(c + d*x)^n)}/(d*(m+1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m+1), \operatorname{Int}[(c + d*x)^{(m+n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m+1))/n] && LtQ[-4, (m+1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m+1]))

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m, x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m+1)}], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m+1)]

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2 dx &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3}{3d} + \frac{1}{3}(2b \log(F)) \int F^{a+\frac{b}{(c+dx)^2}} dx \\
&= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3}{3d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx) \log(F)}{3d} + \frac{1}{3}(4b^2 \log^2(F)) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx \\
&= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3}{3d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx) \log(F)}{3d} - \frac{(4b^2 \log^2(F)) \operatorname{Subst}\left(\int F^{a+bx^2} dx, x, \frac{1}{c+dx}\right)}{3d} \\
&= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3}{3d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx) \log(F)}{3d} - \frac{2b^{3/2}F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) \log^{\frac{3}{2}}(F)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.0759613, size = 79, normalized size = 0.77

$$\frac{F^a \left( (c+dx) F^{\frac{b}{(c+dx)^2}} (2b \log(F) + (c+dx)^2) - 2\sqrt{\pi} b^{3/2} \log^{\frac{3}{2}}(F) \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)\*(c + d\*x)^2, x]

[Out] (F^a\*(-2\*b^(3/2)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]\*Log[F]^(3/2) + F^(b/(c + d\*x)^2)\*(c + d\*x)\*((c + d\*x)^2 + 2\*b\*Log[F]))/(3\*d)

**Maple [A]** time = 0.036, size = 169, normalized size = 1.7

$$\frac{d^2 F^a x^3}{3 F^{(dx+c)^2}} + d F^a F^{\frac{b}{(dx+c)^2}} c x^2 + F^a F^{\frac{b}{(dx+c)^2}} c^2 x + \frac{F^a c^3}{3d} F^{\frac{b}{(dx+c)^2}} + \frac{2 F^a b \ln(F) x}{3} F^{\frac{b}{(dx+c)^2}} + \frac{2 F^a b \ln(F) c}{3d} F^{\frac{b}{(dx+c)^2}} - \frac{2 F^a b^2 (\ln(F))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^2, x)

[Out] 1/3\*d^2\*F^a\*F^(b/(d\*x+c)^2)\*x^3+d\*F^a\*F^(b/(d\*x+c)^2)\*c\*x^2+F^a\*F^(b/(d\*x+c)^2)\*c^2\*x+1/3/d\*F^a\*F^(b/(d\*x+c)^2)\*c^3+2/3\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*x+2/3/d\*F^a\*b\*ln(F)\*F^(b/(d\*x+c)^2)\*c-2/3/d\*F^a\*b^2\*ln(F)^2\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} \left( F^a d^2 x^3 + 3 F^a c d x^2 + (3 F^a c^2 + 2 F^a b \log(F)) x \right) F^{\frac{b}{d^2 x^2 + 2 c d x + c^2}} + \int \frac{2 (2 F^a b^2 d x \log(F)^2 - F^a b c^3 \log(F)) F^{\frac{b}{d^2 x^2 + 2 c d x + c^2}}}{3 (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)\*(d\*x+c)^2, x, algorithm="maxima")



[Out]  $\frac{1}{3}(F^a d^2 x^3 + 3F^a c d x^2 + (3F^a c^2 + 2F^a b \log(F))x) F^{b/(d^2 x^2 + 2c d x + c^2)} + \int \frac{2/3(2F^a b^2 d x \log(F)^2 - F^a b c^3 \log(F)) F^{b/(d^2 x^2 + 2c d x + c^2)}}{(d^3 x^3 + 3c d^2 x^2 + 3c^2 d x + c^3), x}$

**Fricas [A]** time = 1.55369, size = 306, normalized size = 3.

$$\frac{2\sqrt{\pi}F^a b d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) \log(F) + (d^3 x^3 + 3c d^2 x^2 + 3c^2 d x + c^3 + 2(b d x + b c) \log(F)) F^{\frac{a d^2 x^2 + 2a c d x + a c^2 + b}{d^2 x^2 + 2c d x + c^2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{3}(2\sqrt{\pi}F^a b d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) \log(F) + (d^3 x^3 + 3c d^2 x^2 + 3c^2 d x + c^3 + 2(b d x + b c) \log(F)) F^{(a d^2 x^2 + 2a c d x + a c^2 + b)/(d^2 x^2 + 2c d x + c^2)})/d$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 F^{a + \frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^2 * F^(a + b/(d*x + c)^2), x)`

$$3.332 \quad \int F^{a+\frac{b}{(c+dx)^2}} dx$$

**Optimal.** Leaf size=67

$$\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}$$

[Out]  $(F^{(a + b/(c + d*x)^2)*(c + d*x)})/d - (\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c + d*x)]*\operatorname{Sqrt}[\operatorname{Log}[F]])/d$

**Rubi [A]** time = 0.0635389, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2206, 2211, 2204}

$$\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}, x]$

[Out]  $(F^{(a + b/(c + d*x)^2)*(c + d*x)})/d - (\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Log}[F]])/(c + d*x)]*\operatorname{Sqrt}[\operatorname{Log}[F]])/d$

#### Rule 2206

$\operatorname{Int}[(F_{-})^{((a_{-}) + (b_{-})*((c_{-}) + (d_{-})*(x_{-}))^{(n_{-}))}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c + d*x)*F^{(a + b*(c + d*x)^n)}/d, x] - \operatorname{Dist}[b*n*\operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\}$  &&  $\operatorname{IntegerQ}[2/n]$  &&  $\operatorname{LtQ}[n, 0]$

#### Rule 2211

$\operatorname{Int}[(F_{-})^{((a_{-}) + (b_{-})*((c_{-}) + (d_{-})*(x_{-}))^{(n_{-}))} * ((c_{-}) + (d_{-})*(x_{-}))^{(m_{-})}), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/(d*(m + 1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n, x\}$  &&  $\operatorname{EqQ}[n, 2*(m + 1)]$

#### Rule 2204

$\operatorname{Int}[(F_{-})^{((a_{-}) + (b_{-})*((c_{-}) + (d_{-})*(x_{-}))^2)}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\}$  &&  $\operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^2}} dx &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{d} + (2b \log(F)) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{d} - \frac{(2b \log(F)) \operatorname{Subst}\left(\int F^{a+bx^2} dx, x, \frac{1}{c+dx}\right)}{d} \\ &= \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{d} - \frac{\sqrt{b}F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\sqrt{\log(F)}}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0339963, size = 63, normalized size = 0.94

$$\frac{F^a \left( (c + dx) F^{\frac{b}{(c+dx)^2}} - \sqrt{\pi} \sqrt{b} \sqrt{\log(F)} \operatorname{Erfi} \left( \frac{\sqrt{b} \sqrt{\log(F)}}{c+dx} \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2), x]

[Out] (F^a\*(F^(b/(c + d\*x)^2)\*(c + d\*x) - Sqrt[b]\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]\*Sqrt[Log[F]]))/d

**Maple [A]** time = 0.029, size = 74, normalized size = 1.1

$$F^a F^{\frac{b}{(dx+c)^2}} x + \frac{F^a c}{d} F^{\frac{b}{(dx+c)^2}} - \frac{F^a b \ln(F) \sqrt{\pi}}{d} \operatorname{Erf} \left( \frac{1}{dx+c} \sqrt{-b \ln(F)} \right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2), x)

[Out] F^a \* F^(b/(d\*x+c)^2) \* x + 1/d \* F^a \* F^(b/(d\*x+c)^2) \* c - 1/d \* F^a \* b \* ln(F) \* Pi^(1/2) / (-b \* ln(F))^(1/2) \* erf((-b \* ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2F^a b d \int \frac{F^{\frac{b}{d^2 x^2 + 2cdx + c^2}} x}{d^3 x^3 + 3cd^2 x^2 + 3c^2 dx + c^3} dx \log(F) + F^a F^{\frac{b}{d^2 x^2 + 2cdx + c^2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2), x, algorithm="maxima")

[Out] 2 \* F^a \* b \* d \* integrate(F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) \* x / (d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) \* log(F) + F^a \* F^(b/(d^2\*x^2 + 2\*c\*d\*x + c^2)) \* x

**Fricas [A]** time = 1.54225, size = 209, normalized size = 3.12

$$\frac{\sqrt{\pi} F^a d \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf} \left( \frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c} \right) + (dx+c) F^{\frac{ad^2 x^2 + 2acdx + ac^2 + b}{d^2 x^2 + 2cdx + c^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2), x, algorithm="fricas")

[Out] (sqrt(pi) \* F^a \* d \* sqrt(-b \* log(F) / d^2) \* erf(d \* sqrt(-b \* log(F) / d^2) / (d \* x + c)) + (d \* x + c) \* F^((a \* d^2 \* x^2 + 2 \* a \* c \* d \* x + a \* c^2 + b) / (d^2 \* x^2 + 2 \* c \* d \* x + c^2)))

)/d

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2),x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2), x)

$$3.333 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=46

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(F)})/(c + d*x)])/(2 \sqrt{b} * d \sqrt{\log(F)})$

**Rubi [A]** time = 0.054265, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2211, 2204}

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^2)}/(c + d*x)^2, x]$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(F)})/(c + d*x)])/(2 \sqrt{b} * d \sqrt{\log(F)})$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m, x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m + 1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n, x\} \ \&\& \ \operatorname{EqQ}[n, 2*(m + 1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a \sqrt{\pi} \operatorname{Erfi}[(c + d*x) \operatorname{Rt}[b \operatorname{Log}[F], 2]])/(2*d \operatorname{Rt}[b \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx &= -\frac{\operatorname{Subst}\left(\int F^{a+bx^2} dx, x, \frac{1}{c+dx}\right)}{d} \\ &= -\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{bd}\sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.0093136, size = 46, normalized size = 1.

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^2,x]

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(F)})/(c + d*x)])/(2 \sqrt{b} d \sqrt{\log(F)})$

**Maple [A]** time = 0.031, size = 35, normalized size = 0.8

$$-\frac{F^a \sqrt{\pi}}{2d} \operatorname{Erf}\left(\frac{1}{dx+c} \sqrt{-b \ln(F)}\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^2,x)

[Out]  $-1/2/d * F^a * \pi^{(1/2)} / (-b * \ln(F))^{(1/2)} * \operatorname{erf}((-b * \ln(F))^{(1/2)} / (d * x + c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^2}}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^2, x)

**Fricas [A]** time = 1.53979, size = 116, normalized size = 2.52

$$\frac{\sqrt{\pi} F^a \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right)}{2 b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $1/2 * \sqrt{\pi} * F^a * \sqrt{-b * \log(F) / d^2} * \operatorname{erf}(d * \sqrt{-b * \log(F) / d^2} / (d * x + c)) / (b * \log(F))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^2}}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^2, x)
```

$$3.334 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=81

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)}$$

[Out] (F^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]/(4\*b^(3/2)\*d\*Log[F]^(3/2)) - F^(a + b/(c + d\*x)^2)/(2\*b\*d\*(c + d\*x)\*Log[F])

**Rubi [A]** time = 0.102135, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2212, 2211, 2204}

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^4, x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]/(4\*b^(3/2)\*d\*Log[F]^(3/2)) - F^(a + b/(c + d\*x)^2)/(2\*b\*d\*(c + d\*x)\*Log[F])

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2211

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps



$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)\log(F)} - \frac{\int \frac{F^{\frac{a+\frac{b}{(c+dx)^2}}{(c+dx)^2}} dx}{2b\log(F)}}{2bd\log(F)} \\ &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)\log(F)} + \frac{\text{Subst}\left(\int F^{a+bx^2} dx, x, \frac{1}{c+dx}\right)}{2bd\log(F)} \\ &= \frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)\log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0461247, size = 81, normalized size = 1.

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^4, x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)]/(4\*b^(3/2)\*d\*Log[F]^(3/2)) - F^(a + b/(c + d\*x)^2)/(2\*b\*d\*(c + d\*x)\*Log[F])

**Maple [A]** time = 0.054, size = 76, normalized size = 0.9

$$-\frac{F^a}{2(dx+c)db\ln(F)} F^{\frac{b}{(dx+c)^2}} + \frac{F^a\sqrt{\pi}}{4\ln(F)bd} \operatorname{Erf}\left(\frac{1}{dx+c}\sqrt{-b\ln(F)}\right) \frac{1}{\sqrt{-b\ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^4, x)

[Out] -1/2/d\*F^a\*F^(b/(d\*x+c)^2)/(d\*x+c)/b/ln(F)+1/4/d\*F^a/b/ln(F)\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^4, x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^4, x)

**Fricas [A]** time = 1.54413, size = 275, normalized size = 3.4

$$\frac{\sqrt{\pi}(d^2x + cd)F^a \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf}\left(\frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c}\right) + 2F^{\frac{ad^2x^2+2acdx+ac^2+b}{d^2x^2+2cdx+c^2}} b \log(F)}{4(b^2d^2x + b^2cd) \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^4,x, algorithm="fricas")

[Out] -1/4\*(sqrt(pi)\*(d^2\*x + c\*d)\*F^a\*sqrt(-b\*log(F)/d^2)\*erf(d\*sqrt(-b\*log(F)/d^2)/(d\*x + c)) + 2\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2))\*b\*log(F))/((b^2\*d^2\*x + b^2\*c\*d)\*log(F)^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^4,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^4, x)

$$3.335 \quad \int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^6} dx$$

**Optimal.** Leaf size=115

$$-\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{8b^{5/2}d \log^2(F)} + \frac{3F^{a + \frac{b}{(c+dx)^2}}}{4b^2d \log^2(F)(c+dx)} - \frac{F^{a + \frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^3}$$

[Out]  $(-3F^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(F)})/(c + dx)])/(8b^{5/2} d \log[F]^{5/2}) + (3F^{a + b/(c + dx)^2})/(4b^2 d (c + dx) \log[F]^2) - F^{a + b/(c + dx)^2}/(2b d (c + dx)^3 \log[F])$

**Rubi [A]** time = 0.150726, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2212, 2211, 2204}

$$-\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{8b^{5/2}d \log^2(F)} + \frac{3F^{a + \frac{b}{(c+dx)^2}}}{4b^2d \log^2(F)(c+dx)} - \frac{F^{a + \frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + dx)^2)/(c + dx)^6, x]

[Out]  $(-3F^a \sqrt{\pi} \operatorname{Erfi}[(\sqrt{b} \sqrt{\log(F)})/(c + dx)])/(8b^{5/2} d \log[F]^{5/2}) + (3F^{a + b/(c + dx)^2})/(4b^2 d (c + dx) \log[F]^2) - F^{a + b/(c + dx)^2}/(2b d (c + dx)^3 \log[F])$

#### Rule 2212

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + dx)^(m - n + 1)\*F^(a + b\*(c + dx)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + dx)^(m - n)\*F^(a + b\*(c + dx)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2211

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + dx)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^3 \log(F)} - \frac{3 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx}{2b \log(F)} \\
&= \frac{3F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx) \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^3 \log(F)} + \frac{3 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx}{4b^2 \log^2(F)} \\
&= \frac{3F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx) \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^3 \log(F)} - \frac{3 \operatorname{Subst}\left(\int F^{a+bx^2} dx, x, \frac{1}{c+dx}\right)}{4b^2d \log^2(F)} \\
&= -\frac{3F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{\log(F)}}{c+dx}\right)}{8b^{5/2}d \log^{\frac{5}{2}}(F)} + \frac{3F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx) \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^3 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0930658, size = 95, normalized size = 0.83

$$\frac{F^a \left( -3\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) - \frac{2\sqrt{b}\sqrt{\log(F)}F^{\frac{b}{(c+dx)^2}}(2b \log(F) - 3(c+dx)^2)}{(c+dx)^3} \right)}{8b^{5/2}d \log^{\frac{5}{2}}(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^6, x]

[Out] (F^a\*(-3\*sqrt(Pi)\*Erfi[(sqrt(b)\*sqrt(Log[F])]/(c + d\*x))] - (2\*sqrt(b)\*F^(b/(c + d\*x)^2)\*sqrt(Log[F])\*(-3\*(c + d\*x)^2 + 2\*b\*Log[F]))/(c + d\*x)^3))/(8\*b^(5/2)\*d\*Log[F]^(5/2))

**Maple [A]** time = 0.078, size = 109, normalized size = 1.

$$-\frac{F^a}{2d(dx+c)^3 b \ln(F)} F^{\frac{b}{(dx+c)^2}} + \frac{3F^a}{4(\ln(F))^2 b^2 d(dx+c)} F^{\frac{b}{(dx+c)^2}} - \frac{3F^a \sqrt{\pi}}{8(\ln(F))^2 b^2 d} \operatorname{Erf}\left(\frac{1}{dx+c} \sqrt{-b \ln(F)}\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^6, x)

[Out] -1/2/d\*F^a\*F^(b/(d\*x+c)^2)/(d\*x+c)^3/b/ln(F)+3/4/d\*F^a/b^2/ln(F)^2\*F^(b/(d\*x+c)^2)/(d\*x+c)-3/8/d\*F^a/b^2/ln(F)^2\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^6,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^6, x)

**Fricas [B]** time = 1.65095, size = 443, normalized size = 3.85

$$\frac{3\sqrt{\pi}(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d)F^a \sqrt{-\frac{b\log(F)}{d^2}} \operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right) - 2(2b^2\log(F)^2 - 3(bd^2x^2 + 2bcdx + bc^2)\log(F)) \log(F)}{8(b^3d^4x^3 + 3b^3cd^3x^2 + 3b^3c^2d^2x + b^3c^3d)\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^6,x, algorithm="fricas")

[Out] 1/8\*(3\*sqrt(pi)\*(d^4\*x^3 + 3\*c\*d^3\*x^2 + 3\*c^2\*d^2\*x + c^3\*d)\*F^a\*sqrt(-b\*log(F)/d^2)\*erf(d\*sqrt(-b\*log(F)/d^2)/(d\*x + c)) - 2\*(2\*b^2\*log(F)^2 - 3\*(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/((b^3\*d^4\*x^3 + 3\*b^3\*c\*d^3\*x^2 + 3\*b^3\*c^2\*d^2\*x + b^3\*c^3\*d)\*log(F)^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^6,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^6, x)

$$3.336 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8} dx$$

**Optimal.** Leaf size=149

$$\frac{15\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{16b^{7/2}d \log^2(F)} + \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d \log^2(F)(c+dx)^3} - \frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d \log^3(F)(c+dx)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^5}$$

[Out] (15\*F^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c+d\*x)]/(16\*b^(7/2)\*d\*Log[F]^(7/2)) - (15\*F^(a+b/(c+d\*x)^2))/(8\*b^3\*d\*(c+d\*x)\*Log[F]^3) + (5\*F^(a+b/(c+d\*x)^2))/(4\*b^2\*d\*(c+d\*x)^3\*Log[F]^2) - F^(a+b/(c+d\*x)^2)/(2\*b\*d\*(c+d\*x)^5\*Log[F])

**Rubi [A]** time = 0.207721, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2212, 2211, 2204}

$$\frac{15\sqrt{\pi}F^a \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{16b^{7/2}d \log^2(F)} + \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d \log^2(F)(c+dx)^3} - \frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d \log^3(F)(c+dx)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd \log(F)(c+dx)^5}$$

Antiderivative was successfully verified.

[In] Int[F^(a+b/(c+d\*x)^2)/(c+d\*x)^8,x]

[Out] (15\*F^a\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c+d\*x)]/(16\*b^(7/2)\*d\*Log[F]^(7/2)) - (15\*F^(a+b/(c+d\*x)^2))/(8\*b^3\*d\*(c+d\*x)\*Log[F]^3) + (5\*F^(a+b/(c+d\*x)^2))/(4\*b^2\*d\*(c+d\*x)^3\*Log[F]^2) - F^(a+b/(c+d\*x)^2)/(2\*b\*d\*(c+d\*x)^5\*Log[F])

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5 \log(F)} - \frac{5 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx}{2b \log(F)} \\
&= \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5 \log(F)} + \frac{15 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx}{4b^2 \log^2(F)} \\
&= -\frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx) \log^3(F)} + \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5 \log(F)} - \frac{15 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx}{8b^3 \log^3(F)} \\
&= -\frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx) \log^3(F)} + \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5 \log(F)} + \frac{15 \text{Subst}\left(\int F^{a+bx^2} dx, x, \frac{c+dx}{\sqrt{b}}\right)}{8b^3d \log^3(F)} \\
&= \frac{15F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{16b^{7/2}d \log^{\frac{7}{2}}(F)} - \frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx) \log^3(F)} + \frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.128488, size = 111, normalized size = 0.74

$$\frac{F^a \left( 15\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) - \frac{2\sqrt{b}\sqrt{\log(F)}F^{\frac{b}{(c+dx)^2}}(4b^2 \log^2(F) - 10b \log(F)(c+dx)^2 + 15(c+dx)^4)}{(c+dx)^5} \right)}{16b^{7/2}d \log^{\frac{7}{2}}(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^8,x]

[Out] (F^a\*(15\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[Log[F]])/(c + d\*x)] - (2\*Sqrt[b]\*F^(b/(c + d\*x)^2)\*Sqrt[Log[F]]\*(15\*(c + d\*x)^4 - 10\*b\*(c + d\*x)^2\*Log[F] + 4\*b^2\*Log[F]^2))/(c + d\*x)^5))/(16\*b^(7/2)\*d\*Log[F]^(7/2))

**Maple [A]** time = 0.113, size = 142, normalized size = 1.

$$-\frac{F^a}{2d(dx+c)^5 b \ln(F)} F^{\frac{b}{(dx+c)^2}} + \frac{5F^a}{4(\ln(F))^2 b^2 d(dx+c)^3} F^{\frac{b}{(dx+c)^2}} - \frac{15F^a}{8db^3(\ln(F))^3(dx+c)} F^{\frac{b}{(dx+c)^2}} + \frac{15F^a \sqrt{\pi}}{16db^3(\ln(F))^3} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^8,x)

[Out] -1/2/d\*F^a\*F^(b/(d\*x+c)^2)/(d\*x+c)^5/b/ln(F)+5/4/d\*F^a/b^2/ln(F)^2\*F^(b/(d\*x+c)^2)/(d\*x+c)^3-15/8/d\*F^a/b^3/ln(F)^3\*F^(b/(d\*x+c)^2)/(d\*x+c)+15/16/d\*F^a/b^3/ln(F)^3\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^8,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^8, x)

**Fricas [B]** time = 1.68478, size = 670, normalized size = 4.5

$$\frac{15\sqrt{\pi}(d^6x^5 + 5cd^5x^4 + 10c^2d^4x^3 + 10c^3d^3x^2 + 5c^4d^2x + c^5d)F^a\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right) + 2(4b^3\log(F)^3 - 10(b^2d^2x^2 + 2b^2cdx + b^2c^2)\log(F)^2 + 15(bd^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2c^3dx + b^2c^4)\log(F))F^{\frac{a+d^2x^2+2acd^2x+ac^2+b}{d^2x^2+2cdx+c^2}}}{16(b^4d^6x^5 + 5b^4cd^5x^4 + 10b^4c^2d^4x^3 + 10b^4c^3d^3x^2 + 5b^4c^4d^2x + b^4c^5d)\log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^8,x, algorithm="fricas")

[Out] 
$$-1/16*(15*\sqrt{\pi}*(d^6*x^5 + 5*c*d^5*x^4 + 10*c^2*d^4*x^3 + 10*c^3*d^3*x^2 + 5*c^4*d^2*x + c^5*d)*F^a*\sqrt{-b*\log(F)/d^2}*\operatorname{erf}(d*\sqrt{-b*\log(F)/d^2}/(d*x + c)) + 2*(4*b^3*\log(F)^3 - 10*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(F)^2 + 15*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*\log(F))*F^{\frac{a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b}{d^2*x^2 + 2*c*d*x + c^2}})/((b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 10*b^4*c^2*d^4*x^3 + 10*b^4*c^3*d^3*x^2 + 5*b^4*c^4*d^2*x + b^4*c^5*d)*\log(F)^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*8,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^8,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^8, x)



$$3.337 \quad \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}} dx$$

**Optimal.** Leaf size=183

$$-\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{32b^{9/2}d\log^2(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d\log^2(F)(c+dx)^5} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d\log^3(F)(c+dx)^3} + \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d\log^4(F)(c+dx)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

[Out]  $(-105F^a\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{\log(F)})/(c+dx)]/(32b^{9/2}d\log^2(F)) + (105F^{a+b/(c+dx)^2})/(16b^4d(c+dx)\log^4(F)) - (35F^{a+b/(c+dx)^2})/(8b^3d(c+dx)^3\log^3(F)) + (7F^{a+b/(c+dx)^2})/(4b^2d(c+dx)^5\log^2(F)) - F^{a+b/(c+dx)^2}/(2bdc(c+dx)\log(F)))$

**Rubi [A]** time = 0.264987, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2212, 2211, 2204}

$$-\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{32b^{9/2}d\log^2(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d\log^2(F)(c+dx)^5} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d\log^3(F)(c+dx)^3} + \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d\log^4(F)(c+dx)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b/(c+dx)^2}/(c+dx)^{10}, x]$

[Out]  $(-105F^a\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{\log(F)})/(c+dx)]/(32b^{9/2}d\log^2(F)) + (105F^{a+b/(c+dx)^2})/(16b^4d(c+dx)\log^4(F)) - (35F^{a+b/(c+dx)^2})/(8b^3d(c+dx)^3\log^3(F)) + (7F^{a+b/(c+dx)^2})/(4b^2d(c+dx)^5\log^2(F)) - F^{a+b/(c+dx)^2}/(2bdc(c+dx)\log(F)))$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m-n+1)}F^{a+b(c+dx)^n}/(b*d*n*\log(F)), x] - \operatorname{Dist}[(m-n+1)/(b*n*\log(F)), \operatorname{Int}[(c+dx)^{(m-n)}F^{a+b(c+dx)^n}, x], x] /;$   $\text{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \text{IntegerQ}[(2*(m+1))/n] \ \&\& \ \text{LtQ}[0, (m+1)/n, 5] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{LtQ}[0, n, m+1] \ || \ \text{LtQ}[m, n, 0])$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m+1)), \operatorname{Subst}[\operatorname{Int}[F^{a+b*x^2}, x], x, (c+dx)^{(m+1)}], x] /;$   $\text{FreeQ}\{F, a, b, c, d, m, n, x\} \ \&\& \ \text{EqQ}[n, 2*(m+1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)})}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\sqrt{\pi}\operatorname{Erfi}[(c+dx)*\operatorname{Rt}[b*\log(F), 2]]/(2*d*\operatorname{Rt}[b*\log(F), 2]), x] /;$   $\text{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \text{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}} dx &= -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7 \log(F)} - \frac{7 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8} dx}{2b \log(F)} \\
&= \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7 \log(F)} + \frac{35 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx}{4b^2 \log^2(F)} \\
&= -\frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)^3 \log^3(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7 \log(F)} - \frac{105 \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx}{8b^3 \log^3(F)} \\
&= \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d(c+dx) \log^4(F)} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)^3 \log^3(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7 \log(F)} + \frac{105}{16} \\
&= \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d(c+dx) \log^4(F)} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)^3 \log^3(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7 \log(F)} - \frac{105}{16} \\
&= -\frac{105F^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{\log(F)}}{c+dx}\right)}{32b^{9/2}d \log^2(F)} + \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d(c+dx) \log^4(F)} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)^3 \log^3(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5 \log^2(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.154358, size = 127, normalized size = 0.69

$$\frac{F^a \left( \frac{2\sqrt{b}\sqrt{\log(F)}F^{\frac{b}{(c+dx)^2}} (28b^2 \log^2(F)(c+dx)^2 - 8b^3 \log^3(F) - 70b \log(F)(c+dx)^4 + 105(c+dx)^6)}{(c+dx)^7} - 105\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) \right)}{32b^{9/2}d \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^10,x]

[Out] (F^a\*(-105\*sqrt(Pi)\*Erfi[(sqrt(b)\*sqrt(Log[F])]/(c + d\*x))] + (2\*sqrt(b)\*F^(b/(c + d\*x)^2)\*sqrt(Log[F])\*(105\*(c + d\*x)^6 - 70\*b\*(c + d\*x)^4\*Log[F] + 28\*b^2\*(c + d\*x)^2\*Log[F]^2 - 8\*b^3\*Log[F]^3))/(c + d\*x)^7)/(32\*b^(9/2)\*d\*Log[F]^(9/2))

**Maple [A]** time = 0.155, size = 175, normalized size = 1.

$$-\frac{F^a}{2d(dx+c)^7} \frac{F^{\frac{b}{(dx+c)^2}}}{b \ln(F)} + \frac{7F^a}{4(\ln(F))^2} \frac{F^{\frac{b}{(dx+c)^2}}}{b^2d(dx+c)^5} - \frac{35F^a}{8db^3(\ln(F))^3} \frac{F^{\frac{b}{(dx+c)^2}}}{(dx+c)^3} + \frac{105F^a}{16db^4(\ln(F))^4} (dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^2)/(d\*x+c)^10,x)

[Out] -1/2/d\*F^a\*F^(b/(d\*x+c)^2)/(d\*x+c)^7/b/ln(F)+7/4/d\*F^a/b^2/ln(F)^2\*F^(b/(d\*x+c)^2)/(d\*x+c)^5-35/8/d\*F^a/b^3/ln(F)^3\*F^(b/(d\*x+c)^2)/(d\*x+c)^3+105/16/d\*F^a/b^4/ln(F)^4\*F^(b/(d\*x+c)^2)/(d\*x+c)-105/32/d\*F^a/b^4/ln(F)^4\*Pi^(1/2)/(-b\*ln(F))^(1/2)\*erf((-b\*ln(F))^(1/2)/(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^2}}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^10,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^10, x)

**Fricas [B]** time = 1.79248, size = 950, normalized size = 5.19

$$105 \sqrt{\pi} (d^8 x^7 + 7 c d^7 x^6 + 21 c^2 d^6 x^5 + 35 c^3 d^5 x^4 + 35 c^4 d^4 x^3 + 21 c^5 d^3 x^2 + 7 c^6 d^2 x + c^7 d) F^a \sqrt{-\frac{b \log(F)}{d^2}} \operatorname{erf} \left( \frac{d \sqrt{-\frac{b \log(F)}{d^2}}}{dx+c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^10,x, algorithm="fricas")

[Out] 1/32\*(105\*sqrt(pi)\*(d^8\*x^7 + 7\*c\*d^7\*x^6 + 21\*c^2\*d^6\*x^5 + 35\*c^3\*d^5\*x^4 + 35\*c^4\*d^4\*x^3 + 21\*c^5\*d^3\*x^2 + 7\*c^6\*d^2\*x + c^7\*d)\*F^a\*sqrt(-b\*log(F)/d^2)\*erf(d\*sqrt(-b\*log(F)/d^2)/(d\*x + c)) - 2\*(8\*b^4\*log(F)^4 - 28\*(b^3\*d^2\*x^2 + 2\*b^3\*c\*d\*x + b^3\*c^2)\*log(F)^3 + 70\*(b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4)\*log(F)^2 - 105\*(b\*d^6\*x^6 + 6\*b\*c\*d^5\*x^5 + 15\*b\*c^2\*d^4\*x^4 + 20\*b\*c^3\*d^3\*x^3 + 15\*b\*c^4\*d^2\*x^2 + 6\*b\*c^5\*d\*x + b\*c^6)\*log(F))\*F^((a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + b)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/((b^5\*d^8\*x^7 + 7\*b^5\*c\*d^7\*x^6 + 21\*b^5\*c^2\*d^6\*x^5 + 35\*b^5\*c^3\*d^5\*x^4 + 35\*b^5\*c^4\*d^4\*x^3 + 21\*b^5\*c^5\*d^3\*x^2 + 7\*b^5\*c^6\*d^2\*x + b^5\*c^7\*d)\*log(F)^5)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^2}}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^10, x)
```

$$3.338 \quad \int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{12}} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}$$

[Out] (F^a\*Gamma[11/2, -((b\*Log[F])/(c + d\*x)^2)]/(2\*d\*(c + d\*x)^11\*(-((b\*Log[F])/(c + d\*x)^2)))^(11/2))

**Rubi [A]** time = 0.0453887, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^12, x]

[Out] (F^a\*Gamma[11/2, -((b\*Log[F])/(c + d\*x)^2)]/(2\*d\*(c + d\*x)^11\*(-((b\*Log[F])/(c + d\*x)^2)))^(11/2))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{12}} dx = \frac{F^a \Gamma\left(\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}$$

**Mathematica [A]** time = 0.0406678, size = 49, normalized size = 1.

$$\frac{F^a \text{Gamma}\left(\frac{11}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^12, x]

[Out]  $(F^a \Gamma[11/2, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^2)]) / (2 \cdot d \cdot (c + d \cdot x)^{11} \cdot (-((b \cdot \text{Log}[F]) / (c + d \cdot x)^2))^{(11/2)})$

**Maple [A]** time = 0.254, size = 208, normalized size = 4.2

$$-\frac{F^a}{2 d (d x + c)^9 b \ln(F)} F^{\frac{b}{(d x + c)^2}} + \frac{9 F^a}{4 (\ln(F))^2 b^2 d (d x + c)^7} F^{\frac{b}{(d x + c)^2}} - \frac{63 F^a}{8 d b^3 (\ln(F))^3 (d x + c)^5} F^{\frac{b}{(d x + c)^2}} + \frac{315 F^a}{16 d b^4 (\ln(F))^4 (d x + c)^3} F^{\frac{b}{(d x + c)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x)`

[Out]  $-1/2/d \cdot F^a \cdot F^{(b/(d \cdot x + c)^2)} / (d \cdot x + c)^9 / b / \ln(F) + 9/4/d \cdot F^a / b^2 / \ln(F)^2 \cdot F^{(b/(d \cdot x + c)^2)} / (d \cdot x + c)^7 - 63/8/d \cdot F^a / b^3 / \ln(F)^3 \cdot F^{(b/(d \cdot x + c)^2)} / (d \cdot x + c)^5 + 315/16/d \cdot F^a / b^4 / \ln(F)^4 \cdot F^{(b/(d \cdot x + c)^2)} / (d \cdot x + c)^3 - 945/32/d \cdot F^a / b^5 / \ln(F)^5 \cdot F^{(b/(d \cdot x + c)^2)} / (d \cdot x + c) + 945/64/d \cdot F^a / b^5 / \ln(F)^5 \cdot \text{Pi}^{(1/2)} / (-b \cdot \ln(F))^{(1/2)} \cdot \text{erf}((-b \cdot \ln(F))^{(1/2)} / (d \cdot x + c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^2}}}{(dx+c)^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^12, x)`

**Fricas [A]** time = 2.0105, size = 1300, normalized size = 26.53

$$945 \sqrt{\pi} (d^{10} x^9 + 9 c d^9 x^8 + 36 c^2 d^8 x^7 + 84 c^3 d^7 x^6 + 126 c^4 d^6 x^5 + 126 c^5 d^5 x^4 + 84 c^6 d^4 x^3 + 36 c^7 d^3 x^2 + 9 c^8 d^2 x + c^9 d) F^a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x, algorithm="fricas")`

[Out]  $-1/64 \cdot (945 \cdot \text{sqrt}(\pi) \cdot (d^{10} x^9 + 9 c d^9 x^8 + 36 c^2 d^8 x^7 + 84 c^3 d^7 x^6 + 126 c^4 d^6 x^5 + 126 c^5 d^5 x^4 + 84 c^6 d^4 x^3 + 36 c^7 d^3 x^2 + 9 c^8 d^2 x + c^9 d) \cdot F^a \cdot \text{sqrt}(-b \cdot \log(F) / d^2) \cdot \text{erf}(d \cdot \text{sqrt}(-b \cdot \log(F) / d^2) / (d \cdot x + c)) + 2 \cdot (16 \cdot b^5 \cdot \log(F)^5 - 72 \cdot (b^4 \cdot d^2 \cdot x^2 + 2 \cdot b^4 \cdot c \cdot d \cdot x + b^4 \cdot c^2) \cdot \log(F)^4 + 252 \cdot (b^3 \cdot d^4 \cdot x^4 + 4 \cdot b^3 \cdot c \cdot d^3 \cdot x^3 + 6 \cdot b^3 \cdot c^2 \cdot d^2 \cdot x^2 + 4 \cdot b^3 \cdot c^3 \cdot d \cdot x + b^3 \cdot c^4) \cdot \log(F)^3 - 630 \cdot (b^2 \cdot d^6 \cdot x^6 + 6 \cdot b^2 \cdot c \cdot d^5 \cdot x^5 + 15 \cdot b^2 \cdot c^2 \cdot d^4 \cdot x^4 + 20 \cdot b^2 \cdot c^3 \cdot d^3 \cdot x^3 + 15 \cdot b^2 \cdot c^4 \cdot d^2 \cdot x^2 + 6 \cdot b^2 \cdot c^5 \cdot d \cdot x + b^2 \cdot c^6) \cdot \log(F)^2 + 945 \cdot (b \cdot d^8 \cdot x^8 + 8 \cdot b \cdot c \cdot d^7 \cdot x^7 + 28 \cdot b \cdot c^2 \cdot d^6 \cdot x^6 + 56 \cdot b \cdot c^3 \cdot d^5 \cdot x^5 + 70 \cdot b \cdot c^4 \cdot d^4 \cdot x^4 + 56 \cdot b \cdot c^5 \cdot d^3 \cdot x^3 + 28 \cdot b \cdot c^6 \cdot d^2 \cdot x^2 + 8 \cdot b \cdot c^7 \cdot d \cdot x + b \cdot c^8) \cdot \log(F)) \cdot F^{((a \cdot d^2 \cdot x^2 + 2 \cdot a \cdot c \cdot d \cdot x + a \cdot c^2 + b) / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2))}$

$$\frac{+ c^2))}{(b^6 d^{10} x^9 + 9 b^6 c d^9 x^8 + 36 b^6 c^2 d^8 x^7 + 84 b^6 c^3 d^7 x^6 + 126 b^6 c^4 d^6 x^5 + 126 b^6 c^5 d^5 x^4 + 84 b^6 c^6 d^4 x^3 + 36 b^6 c^7 d^3 x^2 + 9 b^6 c^8 d^2 x + b^6 c^9 d) \log(F)^6}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*12,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^2}}}{(dx+c)^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^12,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^12, x)

$$3.339 \quad \int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{14}} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{13/2}}$$

[Out] (F^a\*Gamma[13/2, -((b\*Log[F])/(c + d\*x)^2)])/(2\*d\*(c + d\*x)^13\*(-((b\*Log[F])/(c + d\*x)^2))^(13/2))

**Rubi [A]** time = 0.0453299, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^2)/(c + d\*x)^14,x]

[Out] (F^a\*Gamma[13/2, -((b\*Log[F])/(c + d\*x)^2)])/(2\*d\*(c + d\*x)^13\*(-((b\*Log[F])/(c + d\*x)^2))^(13/2))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a + \frac{b}{(c+dx)^2}}}{(c+dx)^{14}} dx = \frac{F^a \Gamma\left(\frac{13}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{13/2}}$$

**Mathematica [A]** time = 0.026041, size = 49, normalized size = 1.

$$\frac{F^a \text{Gamma}\left(\frac{13}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^2)/(c + d\*x)^14,x]



[Out]  $(F^a \Gamma[13/2, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^2)]) / (2 \cdot d \cdot (c + d \cdot x)^{13} \cdot (-((b \cdot \text{Log}[F]) / (c + d \cdot x)^2))^{13/2})$

**Maple [A]** time = 0.341, size = 241, normalized size = 4.9

$$-\frac{F^a}{2 d (d x + c)^{11} b \ln(F)} F^{\frac{b}{(d x + c)^2}} + \frac{11 F^a}{4 (\ln(F))^2 b^2 d (d x + c)^9} F^{\frac{b}{(d x + c)^2}} - \frac{99 F^a}{8 d b^3 (\ln(F))^3 (d x + c)^7} F^{\frac{b}{(d x + c)^2}} + \frac{693 F^a}{16 d b^4 (\ln(F))^4 (d x + c)^5} F^{\frac{b}{(d x + c)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(a+b/(d*x+c)^2)} / (d*x+c)^{14}, x)$

[Out]  $-1/2/d \cdot F^a \cdot F^{(b/(d*x+c)^2)} / (d*x+c)^{11} / b / \ln(F) + 11/4/d \cdot F^a / b^2 / \ln(F)^2 \cdot F^{(b/(d*x+c)^2)} / (d*x+c)^9 - 99/8/d \cdot F^a / b^3 / \ln(F)^3 \cdot F^{(b/(d*x+c)^2)} / (d*x+c)^7 + 693/16/d \cdot F^a / b^4 / \ln(F)^4 \cdot F^{(b/(d*x+c)^2)} / (d*x+c)^5 - 3465/32/d \cdot F^a / b^5 / \ln(F)^5 \cdot F^{(b/(d*x+c)^2)} / (d*x+c)^3 + 10395/64/d \cdot F^a / b^6 / \ln(F)^6 \cdot F^{(b/(d*x+c)^2)} / (d*x+c) - 10395/128/d \cdot F^a / b^6 / \ln(F)^6 \cdot \text{Pi}^{(1/2)} / (-b \cdot \ln(F))^{(1/2)} \cdot \text{erf}((-b \cdot \ln(F))^{(1/2)} / (d \cdot x + c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^2}}}{(dx+c)^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^2)} / (d*x+c)^{14}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{(a + b/(d*x + c)^2)} / (d*x + c)^{14}, x)$

**Fricas [A]** time = 2.0817, size = 1750, normalized size = 35.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^2)} / (d*x+c)^{14}, x, \text{algorithm}="fricas")$

[Out]  $1/128 \cdot (10395 \cdot \text{sqrt}(\text{pi})) \cdot (d^{12} \cdot x^{11} + 11 \cdot c \cdot d^{11} \cdot x^{10} + 55 \cdot c^2 \cdot d^{10} \cdot x^9 + 165 \cdot c^3 \cdot d^9 \cdot x^8 + 330 \cdot c^4 \cdot d^8 \cdot x^7 + 462 \cdot c^5 \cdot d^7 \cdot x^6 + 462 \cdot c^6 \cdot d^6 \cdot x^5 + 330 \cdot c^7 \cdot d^5 \cdot x^4 + 165 \cdot c^8 \cdot d^4 \cdot x^3 + 55 \cdot c^9 \cdot d^3 \cdot x^2 + 11 \cdot c^{10} \cdot d^2 \cdot x + c^{11} \cdot d) \cdot F^a \cdot \text{sqrt}(-b \cdot \log(F) / d^2) \cdot \text{erf}(d \cdot \text{sqrt}(-b \cdot \log(F) / d^2) / (d \cdot x + c)) - 2 \cdot (32 \cdot b^6 \cdot \log(F)^6 - 176 \cdot (b^5 \cdot d^2 \cdot x^2 + 2 \cdot b^5 \cdot c \cdot d \cdot x + b^5 \cdot c^2) \cdot \log(F)^5 + 792 \cdot (b^4 \cdot d^4 \cdot x^4 + 4 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 6 \cdot b^4 \cdot c^2 \cdot d^2 \cdot x^2 + 4 \cdot b^4 \cdot c^3 \cdot d \cdot x + b^4 \cdot c^4) \cdot \log(F)^4 - 2772 \cdot (b^3 \cdot d^6 \cdot x^6 + 6 \cdot b^3 \cdot c \cdot d^5 \cdot x^5 + 15 \cdot b^3 \cdot c^2 \cdot d^4 \cdot x^4 + 20 \cdot b^3 \cdot c^3 \cdot d^3 \cdot x^3 + 15 \cdot b^3 \cdot c^4 \cdot d^2 \cdot x^2 + 6 \cdot b^3 \cdot c^5 \cdot d \cdot x + b^3 \cdot c^6) \cdot \log(F)^3 + 6930 \cdot (b^2 \cdot d^8 \cdot x^8 + 8 \cdot b^2 \cdot c \cdot d^7 \cdot x^7 + 28 \cdot b^2 \cdot c^2 \cdot d^6 \cdot x^6 + 56 \cdot b^2 \cdot c^3 \cdot d^5 \cdot x^5 + 70 \cdot b^2 \cdot c^4 \cdot d^4 \cdot x^4 + 56 \cdot b^2 \cdot c^5 \cdot d^3 \cdot x^3 + 28 \cdot b^2 \cdot c^6 \cdot d^2 \cdot x^2 + 8 \cdot b^2 \cdot c^7 \cdot d \cdot x + b^2 \cdot c^8) \cdot \log(F)^2 - 10395 \cdot (b \cdot d^{10} \cdot x^{10} + 10 \cdot b \cdot c \cdot d^9 \cdot x^9 + 45 \cdot b \cdot c^2 \cdot d^8 \cdot x^8 + 120 \cdot b \cdot c^3 \cdot d^7 \cdot x^7 + 210 \cdot b \cdot c^4 \cdot d^6 \cdot x^6 + 252 \cdot b \cdot c^5 \cdot d^5 \cdot x^5 + 210 \cdot b \cdot c^6 \cdot d^4 \cdot x^4 + 120 \cdot b \cdot c^7 \cdot d^3 \cdot x^3 + 45 \cdot b \cdot c^8 \cdot d^2 \cdot x^2 + 10 \cdot b \cdot c^9 \cdot d \cdot x + b \cdot c^{10}) \cdot \log(F)) \cdot F^{($

$$\frac{(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)}{(b^7*d^12*x^{11} + 11*b^7*c*d^{11}*x^{10} + 55*b^7*c^2*d^{10}*x^9 + 165*b^7*c^3*d^9*x^8 + 330*b^7*c^4*d^8*x^7 + 462*b^7*c^5*d^7*x^6 + 462*b^7*c^6*d^6*x^5 + 330*b^7*c^7*d^5*x^4 + 165*b^7*c^8*d^4*x^3 + 55*b^7*c^9*d^3*x^2 + 11*b^7*c^{10}*d^2*x + b^7*c^{11}*d)*\log(F)^7}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*2)/(d\*x+c)\*\*14,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^2)/(d\*x+c)^14,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^2)/(d\*x + c)^14, x)

$$3.340 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^m dx$$

**Optimal.** Leaf size=61

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{\frac{m+1}{3}} \Gamma\left(\frac{1}{3}(-m-1), -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1 + m)/3)/(3\*d)

**Rubi [A]** time = 0.0446864, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{\frac{m+1}{3}} \Gamma\left(\frac{1}{3}(-m-1), -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^m, x]

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1 + m)/3)/(3\*d)

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1 + m/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^m dx = \frac{F^a(c+dx)^{1+m} \Gamma\left(\frac{1}{3}(-1-m), -\frac{b \log(F)}{(c+dx)^3}\right) \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{\frac{1+m}{3}}}{3d}$$

**Mathematica [A]** time = 0.0381834, size = 61, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1} \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{\frac{m+1}{3}} \Gamma\left(\frac{1}{3}(-m-1), -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^m, x]

[Out] (F^a\*(c + d\*x)^(1 + m)\*Gamma[(-1 - m)/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1 + m)/3)/(3\*d)

---

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^m,x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^(a + b/(d\*x + c)^3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m F^{\frac{ad^3x^3+3acd^2x^2+3ac^2dx+ac^3+b}{d^3x^3+3cd^2x^2+3c^2dx+c^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] integral((d\*x + c)^m \* F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)\*(d\*x+c)\*\*m,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m * F^(a + b/(d*x + c)^3), x)
```

$$3.341 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^{14} dx$$

Optimal. Leaf size=31

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out]  $-(b^5 F^a \Gamma[-5, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)] \cdot \text{Log}[F]^5) / (3 \cdot d)$

**Rubi [A]** time = 0.0463526, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^14,x]

[Out]  $-(b^5 F^a \Gamma[-5, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)] \cdot \text{Log}[F]^5) / (3 \cdot d)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n)\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^(n)\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^{14} dx = -\frac{b^5 F^a \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^3}\right) \log^5(F)}{3d}$$

**Mathematica [A]** time = 0.0099077, size = 31, normalized size = 1.

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^14,x]

[Out]  $-(b^5 F^a \Gamma[-5, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)] \cdot \text{Log}[F]^5) / (3 \cdot d)$

**Maple [F]** time = 0.146, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c)^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x)
```

```
[Out] int(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="maxima")
```

```
[Out] 1/360*(24*F^a*d^14*x^15 + 360*F^a*c*d^13*x^14 + 2520*F^a*c^2*d^12*x^13 + 6*
(1820*F^a*c^3*d^11 + F^a*b*d^11*log(F))*x^12 + 72*(455*F^a*c^4*d^10 + F^a*b
*c*d^10*log(F))*x^11 + 396*(182*F^a*c^5*d^9 + F^a*b*c^2*d^9*log(F))*x^10 +
2*(60060*F^a*c^6*d^8 + 660*F^a*b*c^3*d^8*log(F) + F^a*b^2*d^8*log(F)^2)*x^9
+ 18*(8580*F^a*c^7*d^7 + 165*F^a*b*c^4*d^7*log(F) + F^a*b^2*c*d^7*log(F)^2
)*x^8 + 72*(2145*F^a*c^8*d^6 + 66*F^a*b*c^5*d^6*log(F) + F^a*b^2*c^2*d^6*lo
g(F)^2)*x^7 + (120120*F^a*c^9*d^5 + 5544*F^a*b*c^6*d^5*log(F) + 168*F^a*b^2
*c^3*d^5*log(F)^2 + F^a*b^3*d^5*log(F)^3)*x^6 + 6*(12012*F^a*c^10*d^4 + 792
*F^a*b*c^7*d^4*log(F) + 42*F^a*b^2*c^4*d^4*log(F)^2 + F^a*b^3*c*d^4*log(F)^
3)*x^5 + 3*(10920*F^a*c^11*d^3 + 990*F^a*b*c^8*d^3*log(F) + 84*F^a*b^2*c^5*
d^3*log(F)^2 + 5*F^a*b^3*c^2*d^3*log(F)^3)*x^4 + (10920*F^a*c^12*d^2 + 1320
*F^a*b*c^9*d^2*log(F) + 168*F^a*b^2*c^6*d^2*log(F)^2 + 20*F^a*b^3*c^3*d^2*1
og(F)^3 + F^a*b^4*d^2*log(F)^4)*x^3 + 3*(840*F^a*c^13*d + 132*F^a*b*c^10*d*
log(F) + 24*F^a*b^2*c^7*d*log(F)^2 + 5*F^a*b^3*c^4*d*log(F)^3 + F^a*b^4*c*d
*log(F)^4)*x^2 + 3*(120*F^a*c^14 + 24*F^a*b*c^11*log(F) + 6*F^a*b^2*c^8*log
(F)^2 + 2*F^a*b^3*c^5*log(F)^3 + F^a*b^4*c^2*log(F)^4)*x)*F^(b/(d^3*x^3 + 3
*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(-1/120*(24*F^a*b*c^15*log(F) + 6
*F^a*b^2*c^12*log(F)^2 - F^a*b^5*d^3*x^3*log(F)^5 + 2*F^a*b^3*c^9*log(F)^3
- 3*F^a*b^5*c*d^2*x^2*log(F)^5 + F^a*b^4*c^6*log(F)^4 - 3*F^a*b^5*c^2*d*x*1
og(F)^5)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3
*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**14,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{14} F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^14 * F^(a + b/(d*x + c)^3), x)
```



$$3.342 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^{11} dx$$

**Optimal.** Leaf size=31

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F]))/(c + d\*x)^3])\*Log[F]^4/(3\*d)

**Rubi [A]** time = 0.0463236, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^11,x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F]))/(c + d\*x)^3])\*Log[F]^4/(3\*d)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^{11} dx = \frac{b^4 F^a \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^3}\right) \log^4(F)}{3d}$$

**Mathematica [A]** time = 0.0092728, size = 31, normalized size = 1.

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^11,x]

[Out] (b^4\*F^a\*Gamma[-4, -((b\*Log[F]))/(c + d\*x)^3])\*Log[F]^4/(3\*d)

**Maple [F]** time = 0.114, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c)^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x)`

[Out] `int(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{72} \left( 6F^a d^{11} x^{12} + 72F^a c d^{10} x^{11} + 396F^a c^2 d^9 x^{10} + 2 \left( 660F^a c^3 d^8 + F^a b d^8 \log(F) \right) x^9 + 18 \left( 165F^a c^4 d^7 + F^a b c d^7 \log(F) \right) x^8 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x, algorithm="maxima")`

[Out] `1/72*(6*F^a*d^11*x^12 + 72*F^a*c*d^10*x^11 + 396*F^a*c^2*d^9*x^10 + 2*(660*F^a*c^3*d^8 + F^a*b*d^8*log(F))*x^9 + 18*(165*F^a*c^4*d^7 + F^a*b*c*d^7*log(F))*x^8 + 72*(66*F^a*c^5*d^6 + F^a*b*c^2*d^6*log(F))*x^7 + (5544*F^a*c^6*d^5 + 168*F^a*b*c^3*d^5*log(F) + F^a*b^2*d^5*log(F)^2)*x^6 + 6*(792*F^a*c^7*d^4 + 42*F^a*b*c^4*d^4*log(F) + F^a*b^2*c*d^4*log(F)^2)*x^5 + 3*(990*F^a*c^8*d^3 + 84*F^a*b*c^5*d^3*log(F) + 5*F^a*b^2*c^2*d^3*log(F)^2)*x^4 + (1320*F^a*c^9*d^2 + 168*F^a*b*c^6*d^2*log(F) + 20*F^a*b^2*c^3*d^2*log(F)^2 + F^a*b^3*d^2*log(F)^3)*x^3 + 3*(132*F^a*c^10*d + 24*F^a*b*c^7*d*log(F) + 5*F^a*b^2*c^4*d*log(F)^2 + F^a*b^3*c*d*log(F)^3)*x^2 + 3*(24*F^a*c^11 + 6*F^a*b*c^8*log(F) + 2*F^a*b^2*c^5*log(F)^2 + F^a*b^3*c^2*log(F)^3)*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(-1/24*(6*F^a*b*c^12*log(F) - F^a*b^4*d^3*x^3*log(F)^4 + 2*F^a*b^2*c^9*log(F)^2 - 3*F^a*b^4*c*d^2*x^2*log(F)^4 + F^a*b^3*c^6*log(F)^3 - 3*F^a*b^4*c^2*d*x*log(F)^4)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**11,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{11} F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^11,x, algorithm="giac")

[Out] integrate((d\*x + c)^11\*F^(a + b/(d\*x + c)^3), x)

### 3.343 $\int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^8 dx$

**Optimal.** Leaf size=121

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{18d} + \frac{b^2 \log^2(F) (c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{18d} + \frac{(c+dx)^9 F^{a+\frac{b}{(c+dx)^3}}}{9d} + \frac{b \log(F) (c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{18d}$$

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^9)/(9*d) + (b*F^{(a + b/(c + d*x)^3)}*(c + d*x)^6*\operatorname{Log}[F])/(18*d) + (b^2*F^{(a + b/(c + d*x)^3)}*(c + d*x)^3*\operatorname{Log}[F]^2)/(18*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^3]*\operatorname{Log}[F]^3)/(18*d)$

**Rubi [A]** time = 0.191009, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{18d} + \frac{b^2 \log^2(F) (c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{18d} + \frac{(c+dx)^9 F^{a+\frac{b}{(c+dx)^3}}}{9d} + \frac{b \log(F) (c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{18d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^3)}*(c + d*x)^8, x]$

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^9)/(9*d) + (b*F^{(a + b/(c + d*x)^3)}*(c + d*x)^6*\operatorname{Log}[F])/(18*d) + (b^2*F^{(a + b/(c + d*x)^3)}*(c + d*x)^3*\operatorname{Log}[F]^2)/(18*d) - (b^3*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^3]*\operatorname{Log}[F]^3)/(18*d)$

#### Rule 2214

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{(m + n)}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}/((e_.) + (f_.)*(x_))], x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^8 dx &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^9}{9d} + \frac{1}{3} (b \log(F)) \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^5 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^9}{9d} + \frac{b F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6 \log(F)}{18d} + \frac{1}{6} (b^2 \log^2(F)) \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^2 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^9}{9d} + \frac{b F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6 \log(F)}{18d} + \frac{b^2 F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 \log^2(F)}{18d} + \frac{1}{6} (b^3 \log^3(F)) \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^9}{9d} + \frac{b F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6 \log(F)}{18d} + \frac{b^2 F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 \log^2(F)}{18d} - \frac{b^3 F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{18d} \end{aligned}$$

**Mathematica [A]** time = 0.206696, size = 96, normalized size = 0.79

$$\frac{F^a \left( b \log(F) \left( b \log(F) \left( (c + dx)^3 F^{\frac{b}{(c+dx)^3}} - b \log(F) \operatorname{Ei} \left( \frac{b \log(F)}{(c+dx)^3} \right) \right) + (c + dx)^6 F^{\frac{b}{(c+dx)^3}} \right) + 2(c + dx)^9 F^{\frac{b}{(c+dx)^3}} \right)}{18d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^8,x]

[Out] (F^a\*(2\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^9 + b\*Log[F]\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^6 + b\*Log[F]\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^3 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^3]\*Log[F]))))/(18\*d)

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int F^{a + \frac{b}{(dx+c)^3}} (dx + c)^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^8,x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^8,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{18} (2 F^a d^8 x^9 + 18 F^a c d^7 x^8 + 72 F^a c^2 d^6 x^7 + (168 F^a c^3 d^5 + F^a b d^5 \log(F)) x^6 + 6 (42 F^a c^4 d^4 + F^a b c d^4 \log(F)) x^5 + 3 (84$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^8,x, algorithm="maxima")

[Out] 1/18\*(2\*(F^a\*d^8\*x^9 + 18\*(F^a\*c\*d^7\*x^8 + 72\*(F^a\*c^2\*d^6\*x^7 + (168\*(F^a\*c^3\*d^5 + F^a\*b\*d^5\*log(F))\*x^6 + 6\*(42\*(F^a\*c^4\*d^4 + F^a\*b\*c\*d^4\*log(F))\*x^5 + 3\*(84\*(F^a\*c^5\*d^3 + 5\*(F^a\*b\*c^2\*d^3\*log(F))\*x^4 + (168\*(F^a\*c^6\*d^2 + 20\*(F^a\*b\*c^3\*d^2\*log(F) + F^a\*b^2\*d^2\*log(F)^2)\*x^3 + 3\*(24\*(F^a\*c^7\*d + 5\*(F^a\*b\*c^4\*d\*log(F) + F^a\*b^2\*c\*d\*log(F)^2)\*x^2 + 3\*(6\*(F^a\*c^8 + 2\*(F^a\*b\*c^5\*log(F) + F^a\*b^2\*c^2\*log(F)^2)\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))) + integrate(1/6\*(F^a\*b^3\*d^3\*x^3\*log(F)^3 - 2\*(F^a\*b\*c^9\*log(F) + 3\*(F^a\*b^3\*c\*d^2\*x^2\*log(F)^3 - F^a\*b^2\*c^6\*log(F)^2 + 3\*(F^a\*b^3\*c^2\*d\*x\*log(F)^3)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.68754, size = 707, normalized size = 5.84

$$\frac{F^a b^3 \operatorname{Ei} \left( \frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right) \log(F)^3 - (2 d^9 x^9 + 18 c d^8 x^8 + 72 c^2 d^7 x^7 + 168 c^3 d^6 x^6 + 252 c^4 d^5 x^5 + 252 c^5 d^4 x^4 + 168$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/18*(F^a*b^3*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(F)
)^3 - (2*d^9*x^9 + 18*c*d^8*x^8 + 72*c^2*d^7*x^7 + 168*c^3*d^6*x^6 + 252*c^
4*d^5*x^5 + 252*c^5*d^4*x^4 + 168*c^6*d^3*x^3 + 72*c^7*d^2*x^2 + 18*c^8*d*x
+ 2*c^9 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*log(F)
^2 + (b*d^6*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d^3*x^3 + 15*
b*c^4*d^2*x^2 + 6*b*c^5*d*x + b*c^6)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2
+ 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**8,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^8 F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^8*F^(a + b/(d*x + c)^3), x)
```

$$3.344 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^5 dx$$

**Optimal.** Leaf size=87

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{6d} + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{6d} + \frac{b \log(F) (c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{6d}$$

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^6)/(6*d) + (b*F^{(a + b/(c + d*x)^3)}*(c + d*x)^3*\operatorname{Log}[F])/(6*d) - (b^2*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^3]*\operatorname{Log}[F]^2)/(6*d)$

**Rubi [A]** time = 0.139198, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$-\frac{b^2 F^a \log^2(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{6d} + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{6d} + \frac{b \log(F) (c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{6d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^5,x]

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^6)/(6*d) + (b*F^{(a + b/(c + d*x)^3)}*(c + d*x)^3*\operatorname{Log}[F])/(6*d) - (b^2*F^a*\operatorname{ExpIntegralEi}[(b*\operatorname{Log}[F])/(c + d*x)^3]*\operatorname{Log}[F]^2)/(6*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^5 dx &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6}{6d} + \frac{1}{2} (b \log(F)) \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^2 dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6}{6d} + \frac{b F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 \log(F)}{6d} + \frac{1}{2} (b^2 \log^2(F)) \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^6}{6d} + \frac{b F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 \log(F)}{6d} - \frac{b^2 F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right) \log^2(F)}{6d} \end{aligned}$$

**Mathematica [A]** time = 0.0599291, size = 71, normalized size = 0.82

$$\frac{F^a \left( b \log(F) \left( (c + dx)^3 F^{\frac{b}{(c+dx)^3}} - b \log(F) \operatorname{Ei} \left( \frac{b \log(F)}{(c+dx)^3} \right) \right) + (c + dx)^6 F^{\frac{b}{(c+dx)^3}} \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^5, x]

[Out] (F^a\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^6 + b\*Log[F]\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^3 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^3]\*Log[F])))/(6\*d)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int F^{a + \frac{b}{(dx+c)^3}} (dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^5, x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^5, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} (F^a d^5 x^6 + 6 F^a c d^4 x^5 + 15 F^a c^2 d^3 x^4 + (20 F^a c^3 d^2 + F^a b d^2 \log(F)) x^3 + 3 (5 F^a c^4 d + F^a b c d \log(F)) x^2 + 3 (2 F^a c^5 + F^a b c^2 \log(F)) x + \operatorname{Ei} \left( \frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right) (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^5, x, algorithm="maxima")

[Out] 1/6\*(F^a\*d^5\*x^6 + 6\*F^a\*c\*d^4\*x^5 + 15\*F^a\*c^2\*d^3\*x^4 + (20\*F^a\*c^3\*d^2 + F^a\*b\*d^2\*log(F))\*x^3 + 3\*(5\*F^a\*c^4\*d + F^a\*b\*c\*d\*log(F))\*x^2 + 3\*(2\*F^a\*c^5 + F^a\*b\*c^2\*log(F))\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate(1/2\*(F^a\*b^2\*d^3\*x^3\*log(F)^2 + 3\*F^a\*b^2\*c\*d^2\*x^2\*log(F)^2 - F^a\*b\*c^6\*log(F) + 3\*F^a\*b^2\*c^2\*d\*x\*log(F)^2)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.69202, size = 454, normalized size = 5.22

$$\frac{F^a b^2 \operatorname{Ei} \left( \frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right) \log(F)^2 - (d^6 x^6 + 6 c d^5 x^5 + 15 c^2 d^4 x^4 + 20 c^3 d^3 x^3 + 15 c^4 d^2 x^2 + 6 c^5 d x + c^6 + (b d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3)) \log(F)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^5, x, algorithm="fricas")



```
[Out] -1/6*(F^a*b^2*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(F)
^2 - (d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*
x^2 + 6*c^5*d*x + c^6 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*l
og(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**5,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^5 F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^5*F^(a + b/(d*x + c)^3), x)
```

$$3.345 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^2 dx$$

**Optimal.** Leaf size=53

$$\frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^3)/(3*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3]*Log[F])/(3*d)$

**Rubi [A]** time = 0.0910202, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2214, 2210}

$$\frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d} - \frac{bF^a \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^2,x]

[Out]  $(F^{(a + b/(c + d*x)^3)}*(c + d*x)^3)/(3*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3]*Log[F])/(3*d)$

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^2 dx &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3}{3d} + (b \log(F)) \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3}{3d} - \frac{bF^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right) \log(F)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.0356271, size = 47, normalized size = 0.89

$$\frac{F^a \left( (c+dx)^3 F^{\frac{b}{(c+dx)^3}} - b \log(F) \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^2,x]

[Out] (F^a\*(F^(b/(c + d\*x)^3)\*(c + d\*x)^3 - b\*ExpIntegralEi[(b\*Log[F])/(c + d\*x)^3]\*Log[F]))/(3\*d)

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^2,x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} (F^a d^2 x^3 + 3 F^a c d x^2 + 3 F^a c^2 x) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}} + \int \frac{(F^a b d^3 x^3 \log(F) + 3 F^a b c d^2 x^2 \log(F) + 3 F^a b c^2 d x \log(F)) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d x + c^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^2,x, algorithm="maxima")

[Out] 1/3\*(F^a\*d^2\*x^3 + 3\*F^a\*c\*d\*x^2 + 3\*F^a\*c^2\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate((F^a\*b\*d^3\*x^3\*log(F) + 3\*F^a\*b\*c\*d^2\*x^2\*log(F) + 3\*F^a\*b\*c^2\*d\*x\*log(F))\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.67173, size = 300, normalized size = 5.66

$$\frac{F^a b \operatorname{Ei}\left(\frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right) \log(F) - (d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3) F^{\frac{a d^3 x^3 + 3 a c d^2 x^2 + 3 a c^2 d x + a c^3 + b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^2,x, algorithm="fricas")

[Out] -1/3\*(F^a\*b\*Ei(b\*log(F)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))\*log(F) - (d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)\*(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2 \* F^(a + b/(d\*x + c)^3), x)

$$3.346 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^3])/(3*d)$

**Rubi [A]** time = 0.0435177, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b/(c + d*x)^3)}/(c + d*x), x]$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^3])/(3*d)$

**Rule 2210**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}/((e_.) + (f_.)*(x_.)), x]$   
 Symbol] :>  $\operatorname{Simp}[(F^a \operatorname{ExpIntegralEi}[b*(c + d*x)^n \operatorname{Log}[F]])/(f*n), x]$  /;  $\operatorname{FreeQ}\{F, a, b, c, d, e, f, n, x\}$  &&  $\operatorname{EqQ}[d*e - c*f, 0]$

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx = -\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

**Mathematica [A]** time = 0.0068292, size = 22, normalized size = 1.

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[F^{(a + b/(c + d*x)^3)}/(c + d*x), x]$

[Out]  $-(F^a \operatorname{ExpIntegralEi}[(b \operatorname{Log}[F])/(c + d*x)^3])/(3*d)$

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{dx + c} F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)/(d*x+c),x)`

[Out] `int(F^(a+b/(d*x+c)^3)/(d*x+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^3)/(d*x + c), x)`

**Fricas [B]** time = 1.62323, size = 90, normalized size = 4.09

$$-\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="fricas")`

[Out] `-1/3*F^a*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/d`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)/(d*x+c),x)`

[Out] `Integral(F**(a + b/(c + d*x)**3)/(c + d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^(a + b/(d*x + c)^3)/(d*x + c), x)`

$$3.347 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=27

$$-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)}$$

[Out]  $-F^{(a + b/(c + d*x)^3)}/(3*b*d*Log[F])$

**Rubi [A]** time = 0.0425225, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2209}

$$-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^4,x]

[Out]  $-F^{(a + b/(c + d*x)^3)}/(3*b*d*Log[F])$

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx = -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)}$$

**Mathematica [A]** time = 0.0119075, size = 27, normalized size = 1.

$$-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^4,x]

[Out]  $-F^{(a + b/(c + d*x)^3)}/(3*b*d*Log[F])$

**Maple [A]** time = 0.003, size = 26, normalized size = 1.

$$-\frac{1}{3 \ln(F) bd} F^{a+\frac{b}{(dx+c)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x)`

[Out]  $-1/3 * F^{(a+b/(d*x+c)^3)} / b/d / \ln(F)$

**Maxima [A]** time = 0.974407, size = 34, normalized size = 1.26

$$-\frac{F^{a + \frac{b}{(dx+c)^3}}}{3bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="maxima")`

[Out]  $-1/3 * F^{(a + b/(d*x + c)^3)} / (b*d*log(F))$

**Fricas [B]** time = 1.57842, size = 161, normalized size = 5.96

$$-\frac{F^{\frac{ad^3x^3+3acd^2x^2+3ac^2dx+ac^3+b}{d^3x^3+3cd^2x^2+3c^2dx+c^3}}}{3bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="fricas")`

[Out]  $-1/3 * F^{((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))} / (b*d*log(F))$

**Sympy [A]** time = 0.521485, size = 66, normalized size = 2.44

$$\begin{cases} \frac{F^{a + \frac{b}{(c+dx)^3}}}{3bd \log(F)} & \text{for } 3bd \log(F) \neq 0 \\ \frac{1}{3c^3d+9c^2d^2x+9cd^3x^2+3d^4x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**4,x)`

[Out] `Piecewise((-F**(a + b/(c + d*x)**3)/(3*b*d*log(F)), Ne(3*b*d*log(F), 0)), (-1/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3), True))`

**Giac [A]** time = 1.38827, size = 34, normalized size = 1.26

$$-\frac{F^{a + \frac{b}{(dx+c)^3}}}{3bd \log(F)}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/3*F^(a + b/(d*x + c)^3)/(b*d*log(F))
```

$$3.348 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx$$

**Optimal.** Leaf size=62

$$\frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^3}$$

[Out]  $F^{a+b/(c+d*x)^3}/(3*b^2*d*Log[F]^2) - F^{a+b/(c+d*x)^3}/(3*b*d*(c+d*x)^3*Log[F])$

**Rubi [A]** time = 0.0867742, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^7, x]

[Out]  $F^{a+b/(c+d*x)^3}/(3*b^2*d*Log[F]^2) - F^{a+b/(c+d*x)^3}/(3*b*d*(c+d*x)^3*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx &= -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^3 \log(F)} - \frac{\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx}{b \log(F)} \\ &= \frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^3 \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0289271, size = 47, normalized size = 0.76

$$\frac{F^{a+\frac{b}{(c+dx)^3}} \left( (c+dx)^3 - b \log(F) \right)}{3b^2 d \log^2(F) (c+dx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^7,x]

[Out] (F^(a + b/(c + d\*x)^3)\*((c + d\*x)^3 - b\*Log[F]))/(3\*b^2\*d\*(c + d\*x)^3\*Log[F]^2)

**Maple [B]** time = 0.061, size = 261, normalized size = 4.2

$$\frac{1}{(dx+c)^6} \left( \frac{d^5 x^6}{3 (\ln(F))^2 b^2} e^{\left(a+\frac{b}{(dx+c)^3}\right) \ln(F)} - \frac{c^2 (-2c^3 + b \ln(F)) x}{(\ln(F))^2 b^2} e^{\left(a+\frac{b}{(dx+c)^3}\right) \ln(F)} - \frac{c^3 (-c^3 + b \ln(F))}{3 (\ln(F))^2 b^2 d} e^{\left(a+\frac{b}{(dx+c)^3}\right) \ln(F)} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)/(d\*x+c)^7,x)

[Out] (1/3/ln(F)^2/b^2\*d^5\*x^6\*exp((a+b/(d\*x+c)^3)\*ln(F))-c^2\*(-2\*c^3+b\*ln(F))/ln(F)^2/b^2\*x\*exp((a+b/(d\*x+c)^3)\*ln(F))-1/3\*c^3\*(-c^3+b\*ln(F))/d/ln(F)^2/b^2\*exp((a+b/(d\*x+c)^3)\*ln(F))-1/3\*d^2\*(-20\*c^3+b\*ln(F))/ln(F)^2/b^2\*x^3\*exp((a+b/(d\*x+c)^3)\*ln(F))+5\*d^3\*c^2/ln(F)^2/b^2\*x^4\*exp((a+b/(d\*x+c)^3)\*ln(F))+2\*d^4\*c/ln(F)^2/b^2\*x^5\*exp((a+b/(d\*x+c)^3)\*ln(F))-c\*d\*(-5\*c^3+b\*ln(F))/ln(F)^2/b^2\*x^2\*exp((a+b/(d\*x+c)^3)\*ln(F)))/(d\*x+c)^6

**Maxima [B]** time = 1.03431, size = 194, normalized size = 3.13

$$\frac{(F^a d^3 x^3 + 3 F^a c d^2 x^2 + 3 F^a c^2 d x + F^a c^3 - F^a b \log(F)) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{3 (b^2 d^4 x^3 \log(F)^2 + 3 b^2 c d^3 x^2 \log(F)^2 + 3 b^2 c^2 d^2 x \log(F)^2 + b^2 c^3 d \log(F)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^7,x, algorithm="maxima")

[Out] 1/3\*(F^a\*d^3\*x^3 + 3\*F^a\*c\*d^2\*x^2 + 3\*F^a\*c^2\*d\*x + F^a\*c^3 - F^a\*b\*log(F))\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(b^2\*d^4\*x^3\*log(F)^2 + 3\*b^2\*c\*d^3\*x^2\*log(F)^2 + 3\*b^2\*c^2\*d^2\*x\*log(F)^2 + b^2\*c^3\*d\*log(F)^2)

**Fricas [B]** time = 1.59322, size = 312, normalized size = 5.03

$$\frac{(d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3 - b \log(F)) F^{\frac{a d^3 x^3 + 3 a c d^2 x^2 + 3 a c^2 d x + a c^3 + b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{3 (b^2 d^4 x^3 + 3 b^2 c d^3 x^2 + 3 b^2 c^2 d^2 x + b^2 c^3 d) \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^7,x, algorithm="fricas")

[Out] 1/3\*(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3 - b\*log(F))\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/((b^2\*d^4\*x^3 + 3\*b^2\*c\*d^3\*x^2 + 3\*b^2\*c^2\*d^2\*x + b^2\*c^3\*d)\*log(F)^2)

**Sympy [B]** time = 0.339172, size = 114, normalized size = 1.84

$$\frac{F^{a+\frac{b}{(c+dx)^3}} \left( -b \log(F) + c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3 \right)}{3b^2c^3d \log(F)^2 + 9b^2c^2d^2x \log(F)^2 + 9b^2cd^3x^2 \log(F)^2 + 3b^2d^4x^3 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)/(d\*x+c)\*\*7,x)

[Out] F\*\*(a + b/(c + d\*x)\*\*3)\*(-b\*log(F) + c\*\*3 + 3\*c\*\*2\*d\*x + 3\*c\*d\*\*2\*x\*\*2 + d\*\*3\*x\*\*3)/(3\*b\*\*2\*c\*\*3\*d\*log(F)\*\*2 + 9\*b\*\*2\*c\*\*2\*d\*\*2\*x\*log(F)\*\*2 + 9\*b\*\*2\*c\*d\*\*3\*x\*\*2\*log(F)\*\*2 + 3\*b\*\*2\*d\*\*4\*x\*\*3\*log(F)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^7,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^3)/(d\*x + c)^7, x)

$$3.349 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx$$

**Optimal.** Leaf size=96

$$\frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^2d \log^2(F)(c+dx)^3} - \frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^3d \log^3(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^6}$$

[Out]  $(-2F^{a+b/(c+d*x)^3})/(3*b^3*d*Log[F]^3) + (2F^{a+b/(c+d*x)^3})/(3*b^2*d*(c+d*x)^3*Log[F]^2) - F^{a+b/(c+d*x)^3}/(3*b*d*(c+d*x)^6*Log[F])$

**Rubi [A]** time = 0.134824, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^2d \log^2(F)(c+dx)^3} - \frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^3d \log^3(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^6}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^10,x]

[Out]  $(-2F^{a+b/(c+d*x)^3})/(3*b^3*d*Log[F]^3) + (2F^{a+b/(c+d*x)^3})/(3*b^2*d*(c+d*x)^3*Log[F]^2) - F^{a+b/(c+d*x)^3}/(3*b*d*(c+d*x)^6*Log[F])$

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx &= -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^6 \log(F)} - \frac{2 \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx}{b \log(F)} \\ &= \frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^6 \log(F)} + \frac{2 \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx}{b^2 \log^2(F)} \\ &= -\frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^3d \log^3(F)} + \frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^2d(c+dx)^3 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^6 \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0438258, size = 64, normalized size = 0.67

$$-\frac{F^{a+\frac{b}{(c+dx)^3}} \left( b^2 \log^2(F) - 2b \log(F)(c+dx)^3 + 2(c+dx)^6 \right)}{3b^3d \log^3(F)(c+dx)^6}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^10,x]

[Out] -(F^(a + b/(c + d\*x)^3)\*(2\*(c + d\*x)^6 - 2\*b\*(c + d\*x)^3\*Log[F] + b^2\*Log[F]^2))/(3\*b^3\*d\*(c + d\*x)^6\*Log[F]^3)

**Maple [B]** time = 0.112, size = 434, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)/(d\*x+c)^10,x)

[Out] 
$$\begin{aligned} &(-2/3*d^8/\ln(F)^3/b^3*x^9*\exp((a+b/(d*x+c)^3)*\ln(F))-c^2*(6*c^6-4*\ln(F)*b*c^3+\ln(F)^2*b^2)/b^3/\ln(F)^3*x*\exp((a+b/(d*x+c)^3)*\ln(F))-1/3*d^2*(168*c^6-40*\ln(F)*b*c^3+\ln(F)^2*b^2)/\ln(F)^3/b^3*x^3*\exp((a+b/(d*x+c)^3)*\ln(F))+2/3*d^5*(-84*c^3+b*\ln(F))/\ln(F)^3/b^3*x^6*\exp((a+b/(d*x+c)^3)*\ln(F))-24*d^6*c^2/\ln(F)^3/b^3*x^7*\exp((a+b/(d*x+c)^3)*\ln(F))-6*d^7*c/\ln(F)^3/b^3*x^8*\exp((a+b/(d*x+c)^3)*\ln(F))-1/3*(2*c^6-2*\ln(F)*b*c^3+\ln(F)^2*b^2)*c^3/b^3/\ln(F)^3/d*\exp((a+b/(d*x+c)^3)*\ln(F))-c*d*(24*c^6-10*\ln(F)*b*c^3+\ln(F)^2*b^2)/\ln(F)^3/b^3*x^2*\exp((a+b/(d*x+c)^3)*\ln(F))+4*c*d^4*(-21*c^3+b*\ln(F))/\ln(F)^3/b^3*x^5*\exp((a+b/(d*x+c)^3)*\ln(F))+2*c^2*d^3*(-42*c^3+5*b*\ln(F))/\ln(F)^3/b^3*x^4*\exp((a+b/(d*x+c)^3)*\ln(F)))/(d*x+c)^9 \end{aligned}$$

**Maxima [B]** time = 1.05068, size = 405, normalized size = 4.22

$$\frac{(2F^a d^6 x^6 + 12F^a c d^5 x^5 + 30F^a c^2 d^4 x^4 + 2F^a c^6 - 2F^a b c^3 \log(F) + F^a b^2 \log(F)^2 + 2(20F^a c^3 d^3 - F^a b d^3 \log(F))x^3 + 6(5b^3 d^7 x^6 \log(F)^3 + 6b^3 c d^6 x^5 \log(F)^3 + 15b^3 c^2 d^5 x^4 \log(F)^3 + 20b^3 c^3 d^4 x^3 \log(F)^3 + 15b^3 c^4 x^2 \log(F)^3 + 6b^3 c^5 x \log(F)^3 + 6b^3 c^6 \log(F)^3))}{3(b^3 d^7 x^6 \log(F)^3 + 6b^3 c d^6 x^5 \log(F)^3 + 15b^3 c^2 d^5 x^4 \log(F)^3 + 20b^3 c^3 d^4 x^3 \log(F)^3 + 15b^3 c^4 x^2 \log(F)^3 + 6b^3 c^5 x \log(F)^3 + 6b^3 c^6 \log(F)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^10,x, algorithm="maxima")

```
[Out] -1/3*(2*F^a*d^6*x^6 + 12*F^a*c*d^5*x^5 + 30*F^a*c^2*d^4*x^4 + 2*F^a*c^6 - 2
*F^a*b*c^3*log(F) + F^a*b^2*log(F)^2 + 2*(20*F^a*c^3*d^3 - F^a*b*d^3*log(F)
)*x^3 + 6*(5*F^a*c^4*d^2 - F^a*b*c*d^2*log(F))*x^2 + 6*(2*F^a*c^5*d - F^a*b
*c^2*d*log(F))*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^3*d^7*
x^6*log(F)^3 + 6*b^3*c*d^6*x^5*log(F)^3 + 15*b^3*c^2*d^5*x^4*log(F)^3 + 20*
b^3*c^3*d^4*x^3*log(F)^3 + 15*b^3*c^4*d^3*x^2*log(F)^3 + 6*b^3*c^5*d^2*x*lo
g(F)^3 + b^3*c^6*d*log(F)^3)
```

---

**Fricas [B]** time = 1.62525, size = 563, normalized size = 5.86

$$\frac{(2d^6x^6 + 12cd^5x^5 + 30c^2d^4x^4 + 40c^3d^3x^3 + 30c^4d^2x^2 + 12c^5dx + 2c^6 + b^2 \log(F)^2 - 2(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + b^3c^3)) \log(F)^3}{3(b^3d^7x^6 + 6b^3cd^6x^5 + 15b^3c^2d^5x^4 + 20b^3c^3d^4x^3 + 15b^3c^4d^3x^2 + 6b^3c^5d^2x + b^3c^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] -1/3*(2*d^6*x^6 + 12*c*d^5*x^5 + 30*c^2*d^4*x^4 + 40*c^3*d^3*x^3 + 30*c^4*d
^2*x^2 + 12*c^5*d*x + 2*c^6 + b^2*log(F)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x +
a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b^3*d^7*x^6 + 6*b^
3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2
+ 6*b^3*c^5*d^2*x + b^3*c^6*d)*log(F)^3)
```

---

**Sympy [B]** time = 0.469879, size = 270, normalized size = 2.81

$$\frac{F^{\frac{a+b}{c+dx}} \left( -b^2 \log(F)^2 + 2bc^3 \log(F) + 6bc^2 dx \log(F) + 6bcd^2 x^2 \log(F) + 2bd^3 x^3 \log(F) - 2c^6 - 12c^5 dx - 30c^4 d^2 x^2 - 3b^3 c^6 d \log(F)^3 + 18b^3 c^5 d^2 x \log(F)^3 + 45b^3 c^4 d^3 x^2 \log(F)^3 + 60b^3 c^3 d^4 x^3 \log(F)^3 + 45b^3 c^2 d^5 x^4 \log(F)^3 + 18b^3 c d^6 x^5 \log(F)^3 \right)}{3b^3 c^6 d \log(F)^3 + 18b^3 c^5 d^2 x \log(F)^3 + 45b^3 c^4 d^3 x^2 \log(F)^3 + 60b^3 c^3 d^4 x^3 \log(F)^3 + 45b^3 c^2 d^5 x^4 \log(F)^3 + 18b^3 c d^6 x^5 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**10,x)
```

```
[Out] F**(a + b/(c + d*x)**3)*(-b**2*log(F)**2 + 2*b*c**3*log(F) + 6*b*c**2*d*x*1
og(F) + 6*b*c*d**2*x**2*log(F) + 2*b*d**3*x**3*log(F) - 2*c**6 - 12*c**5*d*
x - 30*c**4*d**2*x**2 - 40*c**3*d**3*x**3 - 30*c**2*d**4*x**4 - 12*c*d**5*x
**5 - 2*d**6*x**6)/(3*b**3*c**6*d*log(F)**3 + 18*b**3*c**5*d**2*x*log(F)**3
+ 45*b**3*c**4*d**3*x**2*log(F)**3 + 60*b**3*c**3*d**4*x**3*log(F)**3 + 45
*b**3*c**2*d**5*x**4*log(F)**3 + 18*b**3*c*d**6*x**5*log(F)**3 + 3*b**3*d**
7*x**6*log(F)**3)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^3}}}{(dx+c)^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^10, x)
```

$$3.350 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}} dx$$

**Optimal.** Leaf size=123

$$\frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2 d \log^2(F)(c+dx)^6} - \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^3 d \log^3(F)(c+dx)^3} + \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^4 d \log^4(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^9}$$

[Out] (2\*F^(a + b/(c + d\*x)^3))/(b^4\*d\*Log[F]^4) - (2\*F^(a + b/(c + d\*x)^3))/(b^3\*d\*(c + d\*x)^3\*Log[F]^3) + F^(a + b/(c + d\*x)^3)/(b^2\*d\*(c + d\*x)^6\*Log[F]^2) - F^(a + b/(c + d\*x)^3)/(3\*b\*d\*(c + d\*x)^9\*Log[F])

**Rubi [A]** time = 0.185693, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2212, 2209}

$$\frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2 d \log^2(F)(c+dx)^6} - \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^3 d \log^3(F)(c+dx)^3} + \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^4 d \log^4(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd \log(F)(c+dx)^9}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^13,x]

[Out] (2\*F^(a + b/(c + d\*x)^3))/(b^4\*d\*Log[F]^4) - (2\*F^(a + b/(c + d\*x)^3))/(b^3\*d\*(c + d\*x)^3\*Log[F]^3) + F^(a + b/(c + d\*x)^3)/(b^2\*d\*(c + d\*x)^6\*Log[F]^2) - F^(a + b/(c + d\*x)^3)/(3\*b\*d\*(c + d\*x)^9\*Log[F])

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}} dx &= -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^9 \log(F)} - \frac{3 \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx}{b \log(F)} \\
&= \frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2 d(c+dx)^6 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^9 \log(F)} + \frac{6 \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx}{b^2 \log^2(F)} \\
&= -\frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^3 d(c+dx)^3 \log^3(F)} + \frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2 d(c+dx)^6 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^9 \log(F)} - \frac{6 \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx}{b^3 \log^3(F)} \\
&= \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^4 d \log^4(F)} - \frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^3 d(c+dx)^3 \log^3(F)} + \frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2 d(c+dx)^6 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^9 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0333669, size = 73, normalized size = 0.59

$$\frac{F^{a+\frac{b}{(c+dx)^3}} \left( -\frac{b^3 \log^3(F)}{(c+dx)^9} + \frac{3b^2 \log^2(F)}{(c+dx)^6} - \frac{6b \log(F)}{(c+dx)^3} + 6 \right)}{3b^4 d \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^13, x]

[Out] (F^(a + b/(c + d\*x)^3)\*(6 - (6\*b\*Log[F]))/(c + d\*x)^3 + (3\*b^2\*Log[F]^2)/(c + d\*x)^6 - (b^3\*Log[F]^3)/(c + d\*x)^9))/(3\*b^4\*d\*Log[F]^4)

**Maple [B]** time = 0.185, size = 641, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)/(d\*x+c)^13, x)

[Out] (-1/3\*(-6\*c^9+6\*ln(F)\*b\*c^6-3\*ln(F)^2\*b^2\*c^3+ln(F)^3\*b^3)\*c^3/b^4/ln(F)^4/d\*exp((a+b/(d\*x+c)^3)\*ln(F))-c^2\*(-24\*c^9+18\*ln(F)\*b\*c^6-6\*ln(F)^2\*b^2\*c^3+ln(F)^3\*b^3)/b^4/ln(F)^4\*x\*exp((a+b/(d\*x+c)^3)\*ln(F))-1/3\*d^2\*(-1320\*c^9+504\*ln(F)\*b\*c^6-60\*ln(F)^2\*b^2\*c^3+ln(F)^3\*b^3)/ln(F)^4/b^4\*x^3\*exp((a+b/(d\*x+c)^3)\*ln(F))+d^5\*(1848\*c^6-168\*ln(F)\*b\*c^3+ln(F)^2\*b^2)/ln(F)^4/b^4\*x^6\*exp((a+b/(d\*x+c)^3)\*ln(F))-2\*d^8\*(-220\*c^3+b\*ln(F))/ln(F)^4/b^4\*x^9\*exp((a+b/(d\*x+c)^3)\*ln(F))+132\*d^9\*c^2/ln(F)^4/b^4\*x^10\*exp((a+b/(d\*x+c)^3)\*ln(F))+24\*d^10\*c/ln(F)^4/b^4\*x^11\*exp((a+b/(d\*x+c)^3)\*ln(F))-c\*d\*(-132\*c^9+72\*ln(F)\*b\*c^6-15\*ln(F)^2\*b^2\*c^3+ln(F)^3\*b^3)/ln(F)^4/b^4\*x^2\*exp((a+b/(d\*x+c)^3)\*ln(F))+3\*c^2\*d^3\*(330\*c^6-84\*ln(F)\*b\*c^3+5\*ln(F)^2\*b^2)/ln(F)^4/b^4\*x^4\*exp((a+b/(d\*x+c)^3)\*ln(F))+6\*c\*d^4\*(264\*c^6-42\*ln(F)\*b\*c^3+ln(F)^2\*b^2)/ln(F)^4/b^4\*x^5\*exp((a+b/(d\*x+c)^3)\*ln(F))-72\*c^2\*d^6\*(-22\*c^3+b\*ln(F))/ln(F)^4/b^4\*x^7\*exp((a+b/(d\*x+c)^3)\*ln(F))-18\*c\*d^7\*(-55\*c^3+b\*ln(F))/ln(F)^4/b^4\*x^8\*exp((a+b/(d\*x+c)^3)\*ln(F))+2\*d^11/ln(F)^4/b^4\*x^12\*exp((a+b/(d\*x+c)^3)\*ln(F)))/(d\*x+c)^12

**Maxima [B]** time = 1.08542, size = 684, normalized size = 5.56

$$\frac{(6F^a d^9 x^9 + 54F^a c d^8 x^8 + 216F^a c^2 d^7 x^7 + 6F^a c^9 - 6F^a b c^6 \log(F) + 3F^a b^2 c^3 \log(F)^2 + 6(84F^a c^3 d^6 - F^a b d^6 \log(F))x^6 - 3(b^4 d^{10} x^9 \log(F)^4 + 9b^4 c d^9 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^13,x, algorithm="maxima")

[Out] 1/3\*(6F^a\*d^9\*x^9 + 54F^a\*c\*d^8\*x^8 + 216F^a\*c^2\*d^7\*x^7 + 6F^a\*c^9 - 6F^a\*b\*c^6\*log(F) + 3F^a\*b^2\*c^3\*log(F)^2 + 6\*(84F^a\*c^3\*d^6 - F^a\*b\*d^6\*log(F))\*x^6 - F^a\*b^3\*log(F)^3 + 36\*(21F^a\*c^4\*d^5 - F^a\*b\*c\*d^5\*log(F))\*x^5 + 18\*(42F^a\*c^5\*d^4 - 5F^a\*b\*c^2\*d^4\*log(F))\*x^4 + 3\*(168F^a\*c^6\*d^3 - 40F^a\*b\*c^3\*d^3\*log(F) + F^a\*b^2\*d^3\*log(F)^2)\*x^3 + 9\*(24F^a\*c^7\*d^2 - 10F^a\*b\*c^4\*d^2\*log(F) + F^a\*b^2\*c\*d^2\*log(F)^2)\*x^2 + 9\*(6F^a\*c^8\*d - 4F^a\*b\*c^5\*d\*log(F) + F^a\*b^2\*c^2\*d\*log(F)^2)\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(b^4\*d^10\*x^9\*log(F)^4 + 9\*b^4\*c\*d^9\*x^8\*log(F)^4 + 36\*b^4\*c^2\*d^8\*x^7\*log(F)^4 + 84\*b^4\*c^3\*d^7\*x^6\*log(F)^4 + 126\*b^4\*c^4\*d^6\*x^5\*log(F)^4 + 126\*b^4\*c^5\*d^5\*x^4\*log(F)^4 + 84\*b^4\*c^6\*d^4\*x^3\*log(F)^4 + 36\*b^4\*c^7\*d^3\*x^2\*log(F)^4 + 9\*b^4\*c^8\*d^2\*x\*log(F)^4 + b^4\*c^9\*d\*log(F)^4)

**Fricas [B]** time = 1.81621, size = 900, normalized size = 7.32

$$\frac{(6d^9x^9 + 54cd^8x^8 + 216c^2d^7x^7 + 504c^3d^6x^6 + 756c^4d^5x^5 + 756c^5d^4x^4 + 504c^6d^3x^3 + 216c^7d^2x^2 + 54c^8dx + 6c^9 - b^3)}{3(b^4d^{10}x^9 + 9b^4cd^9x^8 + 36b^4c^2d^8x^7 + 84b^4c^3d^7x^6 + 126b^4c^4d^6x^5 + 126b^4c^5d^5x^4 + 84b^4c^6d^4x^3 + 36b^4c^7d^3x^2 + 9b^4c^8d^2x + b^4c^9d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^13,x, algorithm="fricas")

[Out] 1/3\*(6\*d^9\*x^9 + 54\*c\*d^8\*x^8 + 216\*c^2\*d^7\*x^7 + 504\*c^3\*d^6\*x^6 + 756\*c^4\*d^5\*x^5 + 756\*c^5\*d^4\*x^4 + 504\*c^6\*d^3\*x^3 + 216\*c^7\*d^2\*x^2 + 54\*c^8\*d\*x + 6\*c^9 - b^3\*log(F)^3 + 3\*(b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*log(F)^2 - 6\*(b\*d^6\*x^6 + 6\*b\*c\*d^5\*x^5 + 15\*b\*c^2\*d^4\*x^4 + 20\*b\*c^3\*d^3\*x^3 + 15\*b\*c^4\*d^2\*x^2 + 6\*b\*c^5\*d\*x + b\*c^6)\*log(F))\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/((b^4\*d^10\*x^9 + 9\*b^4\*c\*d^9\*x^8 + 36\*b^4\*c^2\*d^8\*x^7 + 84\*b^4\*c^3\*d^7\*x^6 + 126\*b^4\*c^4\*d^6\*x^5 + 126\*b^4\*c^5\*d^5\*x^4 + 84\*b^4\*c^6\*d^4\*x^3 + 36\*b^4\*c^7\*d^3\*x^2 + 9\*b^4\*c^8\*d^2\*x + b^4\*c^9\*d)\*log(F)^4)

**Sympy [B]** time = 0.62872, size = 484, normalized size = 3.93

$$\frac{F^{\frac{a+b}{c+dx}} (-b^3 \log(F)^3 + 3b^2c^3 \log(F)^2 + 9b^2c^2dx \log(F)^2 + 9b^2cd^2x^2 \log(F)^2 + 3b^2d^3x^3 \log(F)^2 - 6bc^6 \log(F) - 36bc^5d)}{3b^4c^9d \log(F)^4 + 27b^4c^8d^2x \log(F)^4 + 108b^4c^7d^3x^2 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)/(d\*x+c)\*\*13,x)

[Out] F\*\*(a + b/(c + d\*x)\*\*3)\*(-b\*\*3\*log(F)\*\*3 + 3\*b\*\*2\*c\*\*3\*log(F)\*\*2 + 9\*b\*\*2\*c\*\*2\*d\*x\*log(F)\*\*2 + 9\*b\*\*2\*c\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 3\*b\*\*2\*d\*\*3\*x\*\*3\*log(F)\*

```

*2 - 6*b*c**6*log(F) - 36*b*c**5*d*x*log(F) - 90*b*c**4*d**2*x**2*log(F) -
120*b*c**3*d**3*x**3*log(F) - 90*b*c**2*d**4*x**4*log(F) - 36*b*c*d**5*x**5
*log(F) - 6*b*d**6*x**6*log(F) + 6*c**9 + 54*c**8*d*x + 216*c**7*d**2*x**2
+ 504*c**6*d**3*x**3 + 756*c**5*d**4*x**4 + 756*c**4*d**5*x**5 + 504*c**3*d
**6*x**6 + 216*c**2*d**7*x**7 + 54*c*d**8*x**8 + 6*d**9*x**9)/(3*b**4*c**9*
d*log(F)**4 + 27*b**4*c**8*d**2*x*log(F)**4 + 108*b**4*c**7*d**3*x**2*log(F)
)**4 + 252*b**4*c**6*d**4*x**3*log(F)**4 + 378*b**4*c**5*d**5*x**4*log(F)**
4 + 378*b**4*c**4*d**6*x**5*log(F)**4 + 252*b**4*c**3*d**7*x**6*log(F)**4 +
108*b**4*c**2*d**8*x**7*log(F)**4 + 27*b**4*c*d**9*x**8*log(F)**4 + 3*b**4
*d**10*x**9*log(F)**4)

```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^13,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^3)/(d\*x + c)^13, x)

$$3.351 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}} dx$$

**Optimal.** Leaf size=96

$$\frac{F^{a+\frac{b}{(c+dx)^3}} \left( 12b^2 \log^2(F)(c+dx)^6 - 4b^3 \log^3(F)(c+dx)^3 + b^4 \log^4(F) - 24b \log(F)(c+dx)^9 + 24(c+dx)^{12} \right)}{3b^5 d \log^5(F)(c+dx)^{12}}$$

[Out]  $-(F^{a+b/(c+d*x)^3}*(24*(c+d*x)^{12}-24*b*(c+d*x)^9*\text{Log}[F]+12*b^2*(c+d*x)^6*\text{Log}[F]^2-4*b^3*(c+d*x)^3*\text{Log}[F]^3+b^4*\text{Log}[F]^4))/(3*b^5*d*(c+d*x)^{12}*\text{Log}[F]^5)$

**Rubi [C]** time = 0.0440779, antiderivative size = 31, normalized size of antiderivative = 0.32, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^16, x]

[Out]  $-(F^a*\text{Gamma}[5, -((b*\text{Log}[F])/(c+d*x)^3)])/(3*b^5*d*\text{Log}[F]^5)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}} dx = -\frac{F^a \Gamma\left(5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^5 d \log^5(F)}$$

**Mathematica [C]** time = 0.0086928, size = 31, normalized size = 0.32

$$\frac{F^a \text{Gamma}\left(5, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^5 d \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^16, x]

[Out]  $-(F^a*\text{Gamma}[5, -((b*\text{Log}[F])/(c+d*x)^3)])/(3*b^5*d*\text{Log}[F]^5)$

**Maple [B]** time = 0.29, size = 889, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(a+b/(d*x+c)^3})/(d*x+c)^{16}, x)$

[Out]  $(-d*c*(840*c^{12}-528*\ln(F)*b*c^9+144*\ln(F)^2*b^2*c^6-20*\ln(F)^3*b^3*c^3+b^4*\ln(F)^4)/b^5/\ln(F)^5*x^2*\exp((a+b/(d*x+c)^3)*\ln(F))+4*c^2*d^3*(-2730*c^9+990*\ln(F)*b*c^6-126*\ln(F)^2*b^2*c^3+5*\ln(F)^3*b^3)/\ln(F)^5/b^5*x^4*\exp((a+b/(d*x+c)^3)*\ln(F))+8*c*d^4*(-3003*c^9+792*\ln(F)*b*c^6-63*\ln(F)^2*b^2*c^3+\ln(F)^3*b^3)/\ln(F)^5/b^5*x^5*\exp((a+b/(d*x+c)^3)*\ln(F))-72*c^2*d^6*(715*c^6-88*\ln(F)*b*c^3+2*\ln(F)^2*b^2)/\ln(F)^5/b^5*x^7*\exp((a+b/(d*x+c)^3)*\ln(F))-36*c*d^7*(1430*c^6-110*\ln(F)*b*c^3+\ln(F)^2*b^2)/\ln(F)^5/b^5*x^8*\exp((a+b/(d*x+c)^3)*\ln(F))+264*c^2*d^9*(-91*c^3+2*b*\ln(F))/\ln(F)^5/b^5*x^10*\exp((a+b/(d*x+c)^3)*\ln(F))+24*c*d^10*(-455*c^3+4*b*\ln(F))/\ln(F)^5/b^5*x^11*\exp((a+b/(d*x+c)^3)*\ln(F))-8*d^14/\ln(F)^5/b^5*x^15*\exp((a+b/(d*x+c)^3)*\ln(F))-c^2*(120*c^12-96*\ln(F)*b*c^9+36*\ln(F)^2*b^2*c^6-8*\ln(F)^3*b^3*c^3+b^4*\ln(F)^4)/b^5/\ln(F)^5*x*\exp((a+b/(d*x+c)^3)*\ln(F))-1/3*d^2*(10920*c^12-5280*\ln(F)*b*c^9+1008*\ln(F)^2*b^2*c^6-80*\ln(F)^3*b^3*c^3+b^4*\ln(F)^4)/\ln(F)^5/b^5*x^3*\exp((a+b/(d*x+c)^3)*\ln(F))+4/3*d^5*(-30030*c^9+5544*\ln(F)*b*c^6-252*\ln(F)^2*b^2*c^3+\ln(F)^3*b^3)/\ln(F)^5/b^5*x^6*\exp((a+b/(d*x+c)^3)*\ln(F))-4*d^8*(10010*c^6-440*\ln(F)*b*c^3+\ln(F)^2*b^2)/\ln(F)^5/b^5*x^9*\exp((a+b/(d*x+c)^3)*\ln(F))+8*d^11*(-455*c^3+b*\ln(F))/\ln(F)^5/b^5*x^12*\exp((a+b/(d*x+c)^3)*\ln(F))-840*d^12*c^2/\ln(F)^5/b^5*x^13*\exp((a+b/(d*x+c)^3)*\ln(F))-120*d^13*c/\ln(F)^5/b^5*x^14*\exp((a+b/(d*x+c)^3)*\ln(F))-1/3*(24*c^12-24*\ln(F)*b*c^9+12*\ln(F)^2*b^2*c^6-4*\ln(F)^3*b^3*c^3+b^4*\ln(F)^4)*c^3/b^5/\ln(F)^5/d*\exp((a+b/(d*x+c)^3)*\ln(F)))/(d*x+c)^{15}$

**Maxima [B]** time = 1.11036, size = 1040, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^3})/(d*x+c)^{16}, x, \text{algorithm}="maxima")$

[Out]  $-1/3*(24*F^a*d^{12}*x^{12} + 288*F^a*c*d^{11}*x^{11} + 1584*F^a*c^2*d^{10}*x^{10} + 24*F^a*c^{12} - 24*F^a*b*c^9*\log(F) + 12*F^a*b^2*c^6*\log(F)^2 + 24*(220*F^a*c^3*d^9 - F^a*b*d^9*\log(F))*x^9 - 4*F^a*b^3*c^3*\log(F)^3 + 216*(55*F^a*c^4*d^8 - F^a*b*c*d^8*\log(F))*x^8 + F^a*b^4*\log(F)^4 + 864*(22*F^a*c^5*d^7 - F^a*b*c^2*d^7*\log(F))*x^7 + 12*(1848*F^a*c^6*d^6 - 168*F^a*b*c^3*d^6*\log(F) + F^a*b^2*d^6*\log(F)^2)*x^6 + 72*(264*F^a*c^7*d^5 - 42*F^a*b*c^4*d^5*\log(F) + F^a*b^2*c*d^5*\log(F)^2)*x^5 + 36*(330*F^a*c^8*d^4 - 84*F^a*b*c^5*d^4*\log(F) + 5*F^a*b^2*c^2*d^4*\log(F)^2)*x^4 + 4*(1320*F^a*c^9*d^3 - 504*F^a*b*c^6*d^3*\log(F) + 60*F^a*b^2*c^3*d^3*\log(F)^2 - F^a*b^3*d^3*\log(F)^3)*x^3 + 12*(132*F^a*c^{10}*d^2 - 72*F^a*b*c^7*d^2*\log(F) + 15*F^a*b^2*c^4*d^2*\log(F)^2 - F^a*b^3*c*d^2*\log(F)^3)*x^2 + 12*(24*F^a*c^{11}*d - 18*F^a*b*c^8*d*\log(F) + 6*F^a*b^2*c^5*d*\log(F)^2 - F^a*b^3*c^2*d*\log(F)^3)*x)*F^{(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))}/(b^5*d^{13}*x^{12}*\log(F)^5 + 12*b^5*c*d^{12}*x^{11}*\log(F)^5 + 66*b^5*c^2*d^{11}*x^{10}*\log(F)^5 + 220*b^5*c^3*d^{10}*x^9*\log(F)^5 + 495*b^5*c^4*d^9*x^8*\log(F)^5 + 792*b^5*c^5*d^8*x^7*\log(F)^5 + 924*b^5*c^6*d^7*x^6*\log(F)^5 + 792*b^5*c^7*d^6*x^5*\log(F)^5 + 495*b^5*c^8*d^5*x^4*\log(F)^5 + 220*b^5*c^9*d^4*x^3*\log(F)^5 + 66*b^5*c^{10}*d^3*x^2*\log(F)^5 + 12*b^5*c^{11}*d^2*x*\log(F)^5 + b^5*c^{12}*d*\log(F)^5)$

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**Fricas [B]** time = 2.0205, size = 1381, normalized size = 14.39

$$(24d^{12}x^{12} + 288cd^{11}x^{11} + 1584c^2d^{10}x^{10} + 5280c^3d^9x^9 + 11880c^4d^8x^8 + 19008c^5d^7x^7 + 22176c^6d^6x^6 + 19008c^7d^5x^5$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^16,x, algorithm="fricas")

[Out] 
$$-1/3*(24*d^{12}*x^{12} + 288*c*d^{11}*x^{11} + 1584*c^2*d^{10}*x^{10} + 5280*c^3*d^9*x^9 + 11880*c^4*d^8*x^8 + 19008*c^5*d^7*x^7 + 22176*c^6*d^6*x^6 + 19008*c^7*d^5*x^5 + 11880*c^8*d^4*x^4 + 5280*c^9*d^3*x^3 + 1584*c^{10}*d^2*x^2 + 288*c^{11}*d*x + 24*c^{12} + b^4*\log(F)^4 - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(F)^3 + 12*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*\log(F)^2 - 24*(b*d^9*x^9 + 9*b*c*d^8*x^8 + 36*b*c^2*d^7*x^7 + 84*b*c^3*d^6*x^6 + 126*b*c^4*d^5*x^5 + 126*b*c^5*d^4*x^4 + 84*b*c^6*d^3*x^3 + 36*b*c^7*d^2*x^2 + 9*b*c^8*d*x + b*c^9)*\log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b^5*d^13*x^{12} + 12*b^5*c*d^{12}*x^{11} + 66*b^5*c^2*d^{11}*x^{10} + 220*b^5*c^3*d^{10}*x^9 + 495*b^5*c^4*d^9*x^8 + 792*b^5*c^5*d^8*x^7 + 924*b^5*c^6*d^7*x^6 + 792*b^5*c^7*d^6*x^5 + 495*b^5*c^8*d^5*x^4 + 220*b^5*c^9*d^4*x^3 + 66*b^5*c^{10}*d^3*x^2 + 12*b^5*c^{11}*d^2*x + b^5*c^{12}*d)*\log(F)^5)$$

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**Sympy [B]** time = 1.06645, size = 760, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)/(d\*x+c)\*\*16,x)

[Out] 
$$F^{(a + b/(c + d*x)^3)*(-b^{**4}*\log(F)^{**4} + 4*b^{**3}*c^{**3}*\log(F)^{**3} + 12*b^{**3}*c^{**2}*d*x*\log(F)^{**3} + 12*b^{**3}*c*d^{**2}*x^{**2}*\log(F)^{**3} + 4*b^{**3}*d^{**3}*x^{**3}*\log(F)^{**3} - 12*b^{**2}*c^{**6}*\log(F)^{**2} - 72*b^{**2}*c^{**5}*d*x*\log(F)^{**2} - 180*b^{**2}*c^{**4}*d^{**2}*x^{**2}*\log(F)^{**2} - 240*b^{**2}*c^{**3}*d^{**3}*x^{**3}*\log(F)^{**2} - 180*b^{**2}*c^{**2}*d^{**4}*x^{**4}*\log(F)^{**2} - 72*b^{**2}*c*d^{**5}*x^{**5}*\log(F)^{**2} - 12*b^{**2}*d^{**6}*x^{**6}*\log(F)^{**2} + 24*b*c^{**9}*\log(F) + 216*b*c^{**8}*d*x*\log(F) + 864*b*c^{**7}*d^{**2}*x^{**2}*\log(F) + 2016*b*c^{**6}*d^{**3}*x^{**3}*\log(F) + 3024*b*c^{**5}*d^{**4}*x^{**4}*\log(F) + 3024*b*c^{**4}*d^{**5}*x^{**5}*\log(F) + 2016*b*c^{**3}*d^{**6}*x^{**6}*\log(F) + 864*b*c^{**2}*d^{**7}*x^{**7}*\log(F) + 216*b*c*d^{**8}*x^{**8}*\log(F) + 24*b*d^{**9}*x^{**9}*\log(F) - 24*c^{**12} - 288*c^{**11}*d*x - 1584*c^{**10}*d^{**2}*x^{**2} - 5280*c^{**9}*d^{**3}*x^{**3} - 11880*c^{**8}*d^{**4}*x^{**4} - 19008*c^{**7}*d^{**5}*x^{**5} - 22176*c^{**6}*d^{**6}*x^{**6} - 19008*c^{**5}*d^{**7}*x^{**7} - 11880*c^{**4}*d^{**8}*x^{**8} - 5280*c^{**3}*d^{**9}*x^{**9} - 1584*c^{**2}*d^{**10}*x^{**10} - 288*c*d^{**11}*x^{**11} - 24*d^{**12}*x^{**12})/(3*b^{**5}*c^{**12}*d*\log(F)^{**5} + 36*b^{**5}*c^{**11}*d^{**2}*x*\log(F)^{**5} + 198*b^{**5}*c^{**10}*d^{**3}*x^{**2}*\log(F)^{**5} + 660*b^{**5}*c^{**9}*d^{**4}*x^{**3}*\log(F)^{**5} + 1485*b^{**5}*c^{**8}*d^{**5}*x^{**4}*\log(F)^{**5} + 2376*b^{**5}*c^{**7}*d^{**6}*x^{**5}*\log(F)^{**5} + 2772*b^{**5}*c^{**6}*d^{**7}*x^{**6}*\log(F)^{**5} + 2376*b^{**5}*c^{**5}*d^{**8}*x^{**7}*\log(F)^{**5} + 1485*b^{**5}*c^{**4}*d^{**9}*x^{**8}*\log(F)^{**5} + 660*b^{**5}*c^{**3}*d^{**10}*x^{**9}*\log(F)^{**5} + 198*b^{**5}*c^{**2}*d^{**11}*x^{**10}*\log(F)^{**5} + 36*b^{**5}*c*d^{**12}*x^{**11}*\log(F)^{**5} + 3*b^{**5}*d^{**13}*x^{**12}*\log(F)^{**5})$$

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^3}}}{(dx+c)^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^16, x)
```

$$3.352 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}} dx$$

**Optimal.** Leaf size=113

$$\frac{F^{a+\frac{b}{(c+dx)^3}} \left( 60b^2 \log^2(F)(c+dx)^9 - 20b^3 \log^3(F)(c+dx)^6 + 5b^4 \log^4(F)(c+dx)^3 - b^5 \log^5(F) - 120b \log(F)(c+dx)^{12} + 120b^2 \log^2(F)(c+dx)^9 \right)}{3b^6 d \log^6(F)(c+dx)^{15}}$$

[Out] (F^(a + b/(c + d\*x)^3))\*(120\*(c + d\*x)^15 - 120\*b\*(c + d\*x)^12\*Log[F] + 60\*b^2\*(c + d\*x)^9\*Log[F]^2 - 20\*b^3\*(c + d\*x)^6\*Log[F]^3 + 5\*b^4\*(c + d\*x)^3\*Log[F]^4 - b^5\*Log[F]^5)/(3\*b^6\*d\*(c + d\*x)^15\*Log[F]^6)

**Rubi [C]** time = 0.0449279, antiderivative size = 31, normalized size of antiderivative = 0.27, number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(6, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^19, x]

[Out] (F^a\*Gamma[6, -((b\*Log[F])/(c + d\*x)^3))]/(3\*b^6\*d\*Log[F]^6)

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}} dx = \frac{F^a \Gamma\left(6, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^6 d \log^6(F)}$$

**Mathematica [C]** time = 0.0083544, size = 31, normalized size = 0.27

$$\frac{F^a \text{Gamma}\left(6, -\frac{b \log(F)}{(c+dx)^3}\right)}{3b^6 d \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^19, x]

[Out] (F^a\*Gamma[6, -((b\*Log[F])/(c + d\*x)^3))]/(3\*b^6\*d\*Log[F]^6)



**Maple [B]** time = 0.041, size = 733, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(a+b/(d*x+c)^3)})/(d*x+c)^{19}, x$

[Out] 
$$-1/3*(20*\ln(F)^3*b^3*c^6-120*d^15*x^15-120*c^15+120*\ln(F)*b*d^12*x^12-60*\ln(F)^2*b^2*d^9*x^9-5*\ln(F)^4*b^4*d^3*x^3-163800*c^11*d^4*x^4-54600*c^12*d^3*x^3-12600*c^13*d^2*x^2-1800*c^14*d*x+120*\ln(F)*b*c^12-60*\ln(F)^2*b^2*c^9-5*\ln(F)^4*b^4*c^3-1800*c*d^14*x^14-12600*c^2*d^13*x^13-54600*c^3*d^12*x^12-163800*c^4*d^11*x^11-360360*c^5*d^10*x^10-600600*c^6*d^9*x^9-772200*c^7*d^8*x^8-772200*c^8*d^7*x^7-600600*c^9*d^6*x^6-360360*c^10*d^5*x^5-2160*\ln(F)^2*b^2*c^7*d^2*x^2+1440*\ln(F)*b*c^11*d*x-540*\ln(F)^2*b^2*c^8*d*x-15*\ln(F)^4*b^4*c*d^2*x^2-15*\ln(F)^4*b^4*c^2*d*x+120*c*d^5*x^5*b^3*\ln(F)^3+300*\ln(F)^3*b^3*c^2*d^4*x^4+400*\ln(F)^3*b^3*c^3*d^3*x^3+300*\ln(F)^3*b^3*c^4*d^2*x^2+120*\ln(F)^3*b^3*c^5*d*x+1440*\ln(F)*b*c*d^11*x^11+7920*\ln(F)*b*c^2*d^10*x^10+26400*\ln(F)*b*c^3*d^9*x^9+59400*\ln(F)*b*c^4*d^8*x^8-540*\ln(F)^2*b^2*c*d^8*x^8+95040*\ln(F)*b*c^5*d^7*x^7-2160*\ln(F)^2*b^2*c^2*d^7*x^7+110880*\ln(F)*b*c^6*d^6*x^6-5040*\ln(F)^2*b^2*c^3*d^6*x^6+95040*\ln(F)*b*c^7*d^5*x^5-7560*\ln(F)^2*b^2*c^4*d^5*x^5+59400*\ln(F)*b*c^8*d^4*x^4-7560*\ln(F)^2*b^2*c^5*d^4*x^4+26400*\ln(F)*b*c^9*d^3*x^3-5040*\ln(F)^2*b^2*c^6*d^3*x^3+7920*\ln(F)*b*c^10*d^2*x^2+b^5*\ln(F)^5+20*d^6*x^6*b^3*\ln(F)^3)/\ln(F)^6/b^6/d/(d*x+c)^15*F^((a*d^3*x^3+3*a*c*d^2*x^2+3*a*c^2*d*x+a*c^3+b)/(d*x+c)^3)$$

**Maxima [B]** time = 1.17461, size = 1465, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b/(d*x+c)^3)})/(d*x+c)^{19}, x, \text{algorithm}="maxima")$

[Out] 
$$1/3*(120*F^a*d^15*x^15 + 1800*F^a*c*d^14*x^14 + 12600*F^a*c^2*d^13*x^13 + 120*F^a*c^15 - 120*F^a*b*c^12*\log(F) + 60*F^a*b^2*c^9*\log(F)^2 + 120*(455*F^a*c^3*d^12 - F^a*b*d^12*\log(F))*x^12 - 20*F^a*b^3*c^6*\log(F)^3 + 360*(455*F^a*c^4*d^11 - 4*F^a*b*c*d^11*\log(F))*x^11 + 5*F^a*b^4*c^3*\log(F)^4 + 3960*(91*F^a*c^5*d^10 - 2*F^a*b*c^2*d^10*\log(F))*x^10 - F^a*b^5*\log(F)^5 + 60*(10010*F^a*c^6*d^9 - 440*F^a*b*c^3*d^9*\log(F) + F^a*b^2*d^9*\log(F)^2)*x^9 + 540*(1430*F^a*c^7*d^8 - 110*F^a*b*c^4*d^8*\log(F) + F^a*b^2*c*d^8*\log(F)^2)*x^8 + 1080*(715*F^a*c^8*d^7 - 88*F^a*b*c^5*d^7*\log(F) + 2*F^a*b^2*c^2*d^7*\log(F)^2)*x^7 + 20*(30030*F^a*c^9*d^6 - 5544*F^a*b*c^6*d^6*\log(F) + 252*F^a*b^2*c^3*d^6*\log(F)^2 - F^a*b^3*d^6*\log(F)^3)*x^6 + 120*(3003*F^a*c^10*d^5 - 792*F^a*b*c^7*d^5*\log(F) + 63*F^a*b^2*c^4*d^5*\log(F)^2 - F^a*b^3*c*d^5*\log(F)^3)*x^5 + 60*(2730*F^a*c^11*d^4 - 990*F^a*b*c^8*d^4*\log(F) + 126*F^a*b^2*c^5*d^4*\log(F)^2 - 5*F^a*b^3*c^2*d^4*\log(F)^3)*x^4 + 5*(10920*F^a*c^12*d^3 - 5280*F^a*b*c^9*d^3*\log(F) + 1008*F^a*b^2*c^6*d^3*\log(F)^2 - 80*F^a*b^3*c^3*d^3*\log(F)^3 + F^a*b^4*d^3*\log(F)^4)*x^3 + 15*(840*F^a*c^13*d^2 - 528*F^a*b*c^10*d^2*\log(F) + 144*F^a*b^2*c^7*d^2*\log(F)^2 - 20*F^a*b^3*c^4*d^2*\log(F)^3 + F^a*b^4*c*d^2*\log(F)^4)*x^2 + 15*(120*F^a*c^14*d - 96*F^a*b*c^11*d*\log(F) + 36*F^a*b^2*c^8*d*\log(F)^2 - 8*F^a*b^3*c^5*d*\log(F)^3 + F^a*b^4*c^2*d*\log(F)^4)*x*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^6*d^16*x^15*\log(F)^6 + 15*b^6*c*d^15*x^14*\log(F)^6 + 105*b^6*c^2*d^14*x^13*\log(F)^6 + 455*b^6*c^3*d^13*x^12*\log(F)^6 + 1365*b^6*c^4*d^12*x^11*\log(F)^6 + 3003*b^6*c^5*d^11*x^10*\log(F)^6 + 5005*b^6*c^6*d^10*x^9*\log(F)^6 + 6435*b^6*c^7*d^$$

$$9x^8 \log(F)^6 + 6435b^6c^8d^8x^7 \log(F)^6 + 5005b^6c^9d^7x^6 \log(F)^6 + 3003b^6c^{10}d^6x^5 \log(F)^6 + 1365b^6c^{11}d^5x^4 \log(F)^6 + 455b^6c^{12}d^4x^3 \log(F)^6 + 105b^6c^{13}d^3x^2 \log(F)^6 + 15b^6c^{14}d^2x \log(F)^6 + b^6c^{15}d \log(F)^6$$

**Fricas [B]** time = 2.42029, size = 1979, normalized size = 17.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^19,x, algorithm="fricas")

[Out]  $\frac{1}{3} \cdot (120d^{15}x^{15} + 1800c^4d^{14}x^{14} + 12600c^2d^{13}x^{13} + 54600c^3d^{12}x^{12} + 163800c^4d^{11}x^{11} + 360360c^5d^{10}x^{10} + 600600c^6d^9x^9 + 772200c^7d^8x^8 + 772200c^8d^7x^7 + 600600c^9d^6x^6 + 360360c^{10}d^5x^5 + 163800c^{11}d^4x^4 + 54600c^{12}d^3x^3 + 12600c^{13}d^2x^2 + 1800c^{14}dx + 120c^{15} - b^5 \log(F)^5 + 5(b^4d^3x^3 + 3b^4c^2d^2x^2 + 3b^4c^2dx + b^4c^3) \log(F)^4 - 20(b^3d^6x^6 + 6b^3c^5d^5x^5 + 15b^3c^2d^4x^4 + 20b^3c^3d^3x^3 + 15b^3c^4d^2x^2 + 6b^3c^5dx + b^3c^6) \log(F)^3 + 60(b^2d^9x^9 + 9b^2c^8d^8x^8 + 36b^2c^2d^7x^7 + 84b^2c^3d^6x^6 + 126b^2c^4d^5x^5 + 126b^2c^5d^4x^4 + 84b^2c^6d^3x^3 + 36b^2c^7d^2x^2 + 9b^2c^8dx + b^2c^9) \log(F)^2 - 120(bd^{12}x^{12} + 12b^2cd^{11}x^{11} + 66b^2c^2d^{10}x^{10} + 220b^2c^3d^9x^9 + 495b^2c^4d^8x^8 + 792b^2c^5d^7x^7 + 924b^2c^6d^6x^6 + 792b^2c^7d^5x^5 + 495b^2c^8d^4x^4 + 220b^2c^9d^3x^3 + 66b^2c^{10}d^2x^2 + 12b^2c^{11}dx + b^2c^{12}) \log(F)) \cdot F^{\frac{(a^3d^3x^3 + 3a^2cd^2x^2 + 3acd^2dx + a^3c^3 + b^3)/(d^3x^3 + 3c^2d^2x^2 + 3cd^2dx + c^3)}{(b^6d^{16}x^{15} + 15b^6cd^{15}x^{14} + 105b^6c^2d^{14}x^{13} + 455b^6c^3d^{13}x^{12} + 1365b^6c^4d^{12}x^{11} + 3003b^6c^5d^{11}x^{10} + 5005b^6c^6d^{10}x^9 + 6435b^6c^7d^9x^8 + 6435b^6c^8d^8x^7 + 5005b^6c^9d^7x^6 + 3003b^6c^{10}d^6x^5 + 1365b^6c^{11}d^5x^4 + 455b^6c^{12}d^4x^3 + 105b^6c^{13}d^3x^2 + 15b^6c^{14}d^2x + b^6c^{15}d) \log(F)^6}$

**Sympy [B]** time = 3.4134, size = 1096, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)/(d\*x+c)\*\*19,x)

[Out]  $F^{(a + b/(c + dx))^3} \cdot (-b^5 \log(F)^5 + 5b^4c^3 \log(F)^4 + 15b^4c^2d^2x \log(F)^4 + 15b^4c^2d^2x^2 \log(F)^4 + 5b^4d^3x^3 \log(F)^4 - 20b^3c^6 \log(F)^3 - 120b^3c^5d^2x \log(F)^3 - 300b^3c^4d^2x^2 \log(F)^3 - 400b^3c^3d^3x^3 \log(F)^3 - 300b^3c^2d^4x^4 \log(F)^3 - 120b^3c^2d^5x^5 \log(F)^3 - 20b^3d^6x^6 \log(F)^3 + 60b^2c^9 \log(F)^2 + 540b^2c^8d^2x \log(F)^2 + 2160b^2c^7d^2x^2 \log(F)^2 + 5040b^2c^6d^3x^3 \log(F)^2 + 7560b^2c^5d^4x^4 \log(F)^2 + 7560b^2c^4d^5x^5 \log(F)^2 + 5040b^2c^3d^6x^6 \log(F)^2 + 2160b^2c^2d^7x^7 \log(F)^2 + 540b^2c^2d^8x^8 \log(F)^2 + 60b^2d^9x^9 \log(F)^2 - 120b^2c^{12} \log(F) - 1440b^2c^{11}d^2x \log(F) - 7920b^2c^{10}d^2x^2 \log(F) - 26400b^2c^9d^3x^3 \log(F) - 59400b^2c^8d^4x^4 \log(F) - 95040b^2c^7d^5x^5 \log(F) - 11$

0880\*b\*c\*\*6\*d\*\*6\*x\*\*6\*log(F) - 95040\*b\*c\*\*5\*d\*\*7\*x\*\*7\*log(F) - 59400\*b\*c\*\*4  
 \*d\*\*8\*x\*\*8\*log(F) - 26400\*b\*c\*\*3\*d\*\*9\*x\*\*9\*log(F) - 7920\*b\*c\*\*2\*d\*\*10\*x\*\*10  
 \*log(F) - 1440\*b\*c\*d\*\*11\*x\*\*11\*log(F) - 120\*b\*d\*\*12\*x\*\*12\*log(F) + 120\*c\*\*1  
 5 + 1800\*c\*\*14\*d\*x + 12600\*c\*\*13\*d\*\*2\*x\*\*2 + 54600\*c\*\*12\*d\*\*3\*x\*\*3 + 163800  
 \*c\*\*11\*d\*\*4\*x\*\*4 + 360360\*c\*\*10\*d\*\*5\*x\*\*5 + 600600\*c\*\*9\*d\*\*6\*x\*\*6 + 772200\*  
 c\*\*8\*d\*\*7\*x\*\*7 + 772200\*c\*\*7\*d\*\*8\*x\*\*8 + 600600\*c\*\*6\*d\*\*9\*x\*\*9 + 360360\*c\*\*  
 5\*d\*\*10\*x\*\*10 + 163800\*c\*\*4\*d\*\*11\*x\*\*11 + 54600\*c\*\*3\*d\*\*12\*x\*\*12 + 12600\*c\*\*  
 2\*d\*\*13\*x\*\*13 + 1800\*c\*d\*\*14\*x\*\*14 + 120\*d\*\*15\*x\*\*15)/(3\*b\*\*6\*c\*\*15\*d\*log(  
 F)\*\*6 + 45\*b\*\*6\*c\*\*14\*d\*\*2\*x\*log(F)\*\*6 + 315\*b\*\*6\*c\*\*13\*d\*\*3\*x\*\*2\*log(F)\*\*6  
 + 1365\*b\*\*6\*c\*\*12\*d\*\*4\*x\*\*3\*log(F)\*\*6 + 4095\*b\*\*6\*c\*\*11\*d\*\*5\*x\*\*4\*log(F)\*\*  
 6 + 9009\*b\*\*6\*c\*\*10\*d\*\*6\*x\*\*5\*log(F)\*\*6 + 15015\*b\*\*6\*c\*\*9\*d\*\*7\*x\*\*6\*log(F)\*\*  
 6 + 19305\*b\*\*6\*c\*\*8\*d\*\*8\*x\*\*7\*log(F)\*\*6 + 19305\*b\*\*6\*c\*\*7\*d\*\*9\*x\*\*8\*log(F)  
 \*\*6 + 15015\*b\*\*6\*c\*\*6\*d\*\*10\*x\*\*9\*log(F)\*\*6 + 9009\*b\*\*6\*c\*\*5\*d\*\*11\*x\*\*10\*log  
 (F)\*\*6 + 4095\*b\*\*6\*c\*\*4\*d\*\*12\*x\*\*11\*log(F)\*\*6 + 1365\*b\*\*6\*c\*\*3\*d\*\*13\*x\*\*12\*  
 log(F)\*\*6 + 315\*b\*\*6\*c\*\*2\*d\*\*14\*x\*\*13\*log(F)\*\*6 + 45\*b\*\*6\*c\*d\*\*15\*x\*\*14\*log  
 (F)\*\*6 + 3\*b\*\*6\*d\*\*16\*x\*\*15\*log(F)\*\*6)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^19,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^3)/(d\*x + c)^19, x)

$$3.353 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 dx$$

**Optimal.** Leaf size=49

$$\frac{F^a(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3} \text{Gamma}\left(-\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out] (F^a\*(c + d\*x)^4\*Gamma[-4/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(4/3))/(3\*d)

**Rubi [A]** time = 0.0479236, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3} \text{Gamma}\left(-\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^3,x]

[Out] (F^a\*(c + d\*x)^4\*Gamma[-4/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(4/3))/(3\*d)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^3 dx = \frac{F^a(c+dx)^4 \Gamma\left(-\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right) \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3}}{3d}$$

**Mathematica [A]** time = 0.0280762, size = 49, normalized size = 1.

$$\frac{F^a(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3} \text{Gamma}\left(-\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x)^3,x]

[Out] (F^a\*(c + d\*x)^4\*Gamma[-4/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(4/3))/(3\*d)

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^3,x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( F^a d^3 x^4 + 4 F^a c d^2 x^3 + 6 F^a c^2 d x^2 + (4 F^a c^3 + 3 F^a b \log(F)) x \right) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}} + \int -\frac{3(F^a b c^4 \log(F) - 3 F^a b^2 dx \log(F))}{4(d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d x + c^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] 1/4\*(F^a\*d^3\*x^4 + 4\*F^a\*c\*d^2\*x^3 + 6\*F^a\*c^2\*d\*x^2 + (4\*F^a\*c^3 + 3\*F^a\*b\*log(F))\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate(-3/4\*(F^a\*b\*c^4\*log(F) - 3\*F^a\*b^2\*d\*x\*log(F)^2)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.58905, size = 402, normalized size = 8.2

$$\frac{3 F^a b d \left( -\frac{b \log(F)}{d^3} \right)^{\frac{1}{3}} \Gamma \left( \frac{2}{3} - \frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right) \log(F) - (d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d x + c^4 + 3 (b d x + b c) \log(F)) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^3,x, algorithm="fricas")

[Out] -1/4\*(3\*F^a\*b\*d\*(-b\*log(F)/d^3)^(1/3)\*gamma(2/3, -b\*log(F)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))\*log(F) - (d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4 + 3\*(b\*d\*x + b\*c)\*log(F))\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)\*(d\*x+c)\*\*3,x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*F^(a + b/(d\*x + c)^3), x)

$$3.354 \quad \int F^{a+\frac{b}{(c+dx)^3}} (c+dx) dx$$

**Optimal.** Leaf size=49

$$\frac{F^a(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out] (F^a\*(c + d\*x)^2\*Gamma[-2/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(2/3))/(3\*d)

**Rubi [A]** time = 0.0273341, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2218}

$$\frac{F^a(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)\*(c + d\*x), x]

[Out] (F^a\*(c + d\*x)^2\*Gamma[-2/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(2/3))/(3\*d)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^3}} (c+dx) dx = \frac{F^a(c+dx)^2 \Gamma\left(-\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right) \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3}}{3d}$$

**Mathematica [A]** time = 0.0211379, size = 49, normalized size = 1.

$$\frac{F^a(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)\*(c + d\*x), x]

[Out] (F^a\*(c + d\*x)^2\*Gamma[-2/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(2/3))/(3\*d)

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} (dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c), x)

[Out] int(F^(a+b/(d\*x+c)^3)\*(d\*x+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} (F^a dx^2 + 2 F^a c x) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 dx + c^3}} + \int \frac{3 (F^a b d^2 x^2 \log(F) + 2 F^a b c d x \log(F)) F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 dx + c^3}}}{2 (d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 dx + c^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c), x, algorithm="maxima")

[Out] 1/2\*(F^a\*d\*x^2 + 2\*F^a\*c\*x)\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate(3/2\*(F^a\*b\*d^2\*x^2\*log(F) + 2\*F^a\*b\*c\*d\*x\*log(F))\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.63266, size = 313, normalized size = 6.39

$$\frac{F^a d^2 \left( -\frac{b \log(F)}{d^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{1}{3}, -\frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 dx + c^3} \right) - (d^2 x^2 + 2 c dx + c^2) F^{\frac{ad^3 x^3 + 3 acd^2 x^2 + 3 ac^2 dx + ac^3 + b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 dx + c^3}}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c), x, algorithm="fricas")

[Out] -1/2\*(F^a\*d^2\*(-b\*log(F)/d^3)^(2/3)\*gamma(1/3, -b\*log(F)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) - (d^2\*x^2 + 2\*c\*d\*x + c^2)\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3)\*(d\*x+c), x)

[Out] Timed out



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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c) F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)\*(d\*x+c),x, algorithm="giac")

[Out] integrate((d\*x + c)\*F^(a + b/(d\*x + c)^3), x)

$$3.355 \quad \int F^{a+\frac{b}{(c+dx)^3}} dx$$

**Optimal.** Leaf size=47

$$\frac{F^a(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

[Out] (F^a\*(c + d\*x)\*Gamma[-1/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1/3))/(3\*d)

**Rubi [A]** time = 0.0068177, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2208}

$$\frac{F^a(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3), x]

[Out] (F^a\*(c + d\*x)\*Gamma[-1/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1/3))/(3\*d)

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F])]]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int F^{a+\frac{b}{(c+dx)^3}} dx = \frac{F^a(c+dx) \Gamma\left(-\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}}}{3d}$$

**Mathematica [A]** time = 0.0119261, size = 47, normalized size = 1.

$$\frac{F^a(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3), x]

[Out] (F^a\*(c + d\*x)\*Gamma[-1/3, -((b\*Log[F])/(c + d\*x)^3)]\*(-((b\*Log[F])/(c + d\*x)^3))^(1/3))/(3\*d)

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c)^3), x)

[Out] int(F^(a+b/(d\*x+c)^3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3F^a b d \int \frac{F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}} x}{d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d x + c^4} dx \log(F) + F^a F^{\frac{b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3), x, algorithm="maxima")

[Out] 3\*F^a\*b\*d\*integrate(F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))\*x/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)\*log(F) + F^a\*F^(b/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))\*x

**Fricas [B]** time = 1.62543, size = 284, normalized size = 6.04

$$\frac{F^a d \left( -\frac{b \log(F)}{d^3} \right)^{\frac{1}{3}} \Gamma\left( \frac{2}{3}, -\frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right) - (d x + c) F^{\frac{a d^3 x^3 + 3 a c d^2 x^2 + 3 a c^2 d x + a c^3 + b}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3), x, algorithm="fricas")

[Out] -(F^a\*d\*(-b\*log(F)/d^3)^(1/3)\*gamma(2/3, -b\*log(F)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) - (d\*x + c)\*F^((a\*d^3\*x^3 + 3\*a\*c\*d^2\*x^2 + 3\*a\*c^2\*d\*x + a\*c^3 + b)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c)\*\*3), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^3), x)
```

$$3.356 \quad \int \frac{F^{a + \frac{b}{(c+dx)^3}}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}}}$$

[Out] (F^a\*Gamma[1/3, -((b\*Log[F])/(c + d\*x)^3))]/(3\*d\*(c + d\*x)\*(-((b\*Log[F])/(c + d\*x)^3))^(1/3))

**Rubi [A]** time = 0.0456179, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^2, x]

[Out] (F^a\*Gamma[1/3, -((b\*Log[F])/(c + d\*x)^3))]/(3\*d\*(c + d\*x)\*(-((b\*Log[F])/(c + d\*x)^3))^(1/3))

### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\int \frac{F^{a + \frac{b}{(c+dx)^3}}}{(c+dx)^2} dx = \frac{F^a \Gamma\left(\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}}}$$

**Mathematica [A]** time = 0.0283035, size = 49, normalized size = 1.

$$\frac{F^a \text{Gamma}\left(\frac{1}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx) \sqrt[3]{-\frac{b \log(F)}{(c+dx)^3}}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^2, x]

[Out]  $(F^a \Gamma[1/3, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)]) / (3 \cdot d \cdot (c + d \cdot x) \cdot (-((b \cdot \text{Log}[F]) / (c + d \cdot x)^3))^{(1/3)})$

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2} F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x)`

[Out] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^3}}}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^2, x)`

**Fricas [A]** time = 1.62422, size = 147, normalized size = 3.

$$\frac{F^a d \left( -\frac{b \log(F)}{d^3} \right)^{\frac{2}{3}} \Gamma \left( \frac{1}{3}, -\frac{b \log(F)}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} \right)}{3 b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="fricas")`

[Out]  $-1/3 \cdot F^a \cdot d \cdot (-b \cdot \log(F) / d^3)^{(2/3)} \cdot \text{gamma}(1/3, -b \cdot \log(F) / (d^3 \cdot x^3 + 3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot c^2 \cdot d \cdot x + c^3)) / (b \cdot \log(F))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^3}}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^2, x)
```

$$3.357 \quad \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3}}$$

[Out] (F^a\*Gamma[2/3, -((b\*Log[F])/(c + d\*x)^3))]/(3\*d\*(c + d\*x)^2\*(-((b\*Log[F])/(c + d\*x)^3))^(2/3))

**Rubi [A]** time = 0.0438045, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^3,x]

[Out] (F^a\*Gamma[2/3, -((b\*Log[F])/(c + d\*x)^3))]/(3\*d\*(c + d\*x)^2\*(-((b\*Log[F])/(c + d\*x)^3))^(2/3))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx = \frac{F^a \Gamma\left(\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3}}$$

**Mathematica [A]** time = 0.0275629, size = 49, normalized size = 1.

$$\frac{F^a \text{Gamma}\left(\frac{2}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^3,x]



[Out]  $(F^a \Gamma[2/3, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)]) / (3 \cdot d \cdot (c + d \cdot x)^2 \cdot (-((b \cdot \text{Log}[F]) / (c + d \cdot x)^3))^{(2/3)})$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3} F^{a+\frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x)`

[Out] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^3, x)`

**Fricas [A]** time = 1.59104, size = 144, normalized size = 2.94

$$-\frac{F^a \left( -\frac{b \log(F)}{d^3} \right)^{\frac{1}{3}} \Gamma \left( \frac{2}{3}, -\frac{b \log(F)}{d^3 x^3 + 3cd^2 x^2 + 3c^2 dx + c^3} \right)}{3b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x, algorithm="fricas")`

[Out]  $-1/3 \cdot F^a \cdot (-b \cdot \log(F) / d^3)^{(1/3)} \cdot \text{gamma}(2/3, -b \cdot \log(F) / (d^3 \cdot x^3 + 3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot c^2 \cdot d \cdot x + c^3)) / (b \cdot \log(F))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**3,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{a+b}{(dx+c)^3}}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^3,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^3)/(d\*x + c)^3, x)

$$3.358 \quad \int \frac{F^{a + \frac{b}{(c+dx)^3}}}{(c+dx)^5} dx$$

**Optimal.** Leaf size=49

$$\frac{F^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3}}$$

[Out] (F^a\*Gamma[4/3, -((b\*Log[F])/(c + d\*x)^3)])/(3\*d\*(c + d\*x)^4\*(-((b\*Log[F])/(c + d\*x)^3))^(4/3))

**Rubi [A]** time = 0.0435577, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x)^3)/(c + d\*x)^5, x]

[Out] (F^a\*Gamma[4/3, -((b\*Log[F])/(c + d\*x)^3)])/(3\*d\*(c + d\*x)^4\*(-((b\*Log[F])/(c + d\*x)^3))^(4/3))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a + \frac{b}{(c+dx)^3}}}{(c+dx)^5} dx = \frac{F^a \Gamma\left(\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3}}$$

**Mathematica [A]** time = 0.0358109, size = 49, normalized size = 1.

$$\frac{F^a \text{Gamma}\left(\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x)^3)/(c + d\*x)^5, x]

[Out]  $(F^a \Gamma[4/3, -((b \cdot \text{Log}[F]) / (c + d \cdot x)^3)]) / (3 \cdot d \cdot (c + d \cdot x)^4 \cdot (-((b \cdot \text{Log}[F]) / (c + d \cdot x)^3))^{(4/3)})$

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^5} F^{a + \frac{b}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x)`

[Out] `int(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^3}}}{(dx + c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^5, x)`

**Fricas [B]** time = 1.68297, size = 350, normalized size = 7.14

$$\frac{(d^3x + cd^2)F^a \left(-\frac{b \log(F)}{d^3}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -\frac{b \log(F)}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3}\right) - 3F^{\frac{ad^3x^3 + 3acd^2x^2 + 3ac^2dx + ac^3 + b}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3}} b \log(F)}{9(b^2d^2x + b^2cd) \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x, algorithm="fricas")`

[Out]  $\frac{1}{9} \cdot ((d^3x + c \cdot d^2) \cdot F^a \cdot (-b \cdot \log(F) / d^3)^{(2/3)} \cdot \text{gamma}(1/3, -b \cdot \log(F) / (d^3x^3 + 3 \cdot c \cdot d^2x^2 + 3 \cdot c^2 \cdot dx + c^3))) - 3 \cdot F^{((a \cdot d^3x^3 + 3 \cdot a \cdot c \cdot d^2x^2 + 3 \cdot a \cdot c^2 \cdot dx + a \cdot c^3 + b) / (d^3x^3 + 3 \cdot c \cdot d^2x^2 + 3 \cdot c^2 \cdot dx + c^3)) \cdot b \cdot \log(F)} / ((b^2 \cdot d^2x + b^2 \cdot cd) \cdot \log(F)^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**5,x)`

[Out] Timed out

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{(dx+c)^3}}}{(dx+c)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c)^3)/(d\*x+c)^5,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c)^3)/(d\*x + c)^5, x)

### 3.359 $\int F^{a+b(c+dx)^n} (c+dx)^m dx$

**Optimal.** Leaf size=61

$$\frac{F^a(c+dx)^{m+1}(-b\log(F)(c+dx)^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

[Out]  $-\left(\left(F^a(c+dx)^{1+m}\right)\text{Gamma}\left[\frac{1+m}{n}, -b(c+dx)^n\text{Log}[F]\right]\right)/\left(d^n\left(-b(c+dx)^n\text{Log}[F]\right)^{\frac{1+m}{n}}\right)$

**Rubi [A]** time = 0.034471, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^{m+1}(-b\log(F)(c+dx)^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^m, x]

[Out]  $-\left(\left(F^a(c+dx)^{1+m}\right)\text{Gamma}\left[\frac{1+m}{n}, -b(c+dx)^n\text{Log}[F]\right]\right)/\left(d^n\left(-b(c+dx)^n\text{Log}[F]\right)^{\frac{1+m}{n}}\right)$

#### Rule 2218

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx)^m dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{n}, -b(c+dx)^n\log(F)\right)(-b(c+dx)^n\log(F))^{-\frac{1+m}{n}}}{dn}$$

**Mathematica [A]** time = 0.0209261, size = 61, normalized size = 1.

$$\frac{F^a(c+dx)^{m+1}(-b\log(F)(c+dx)^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^m, x]

[Out]  $-\left(\left(F^a(c+dx)^{1+m}\right)\text{Gamma}\left[\frac{1+m}{n}, -b(c+dx)^n\text{Log}[F]\right]\right)/\left(d^n\left(-b(c+dx)^n\text{Log}[F]\right)^{\frac{1+m}{n}}\right)$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^n} (dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^m,x)

[Out] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m \* F^((d\*x + c)^n \* b + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx+c)^m F^{(dx+c)^n b+a}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] integral((d\*x + c)^m \* F^((d\*x + c)^n \* b + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*m,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate((d\*x + c)^m \* F^((d\*x + c)^n \* b + a), x)

### 3.360 $\int F^{a+b(c+dx)^n} (c+dx)^3 dx$

**Optimal.** Leaf size=54

$$\frac{F^a(c+dx)^4 (-b \log(F)(c+dx)^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

[Out]  $-\left(\left(F^{a*(c+d*x)^4} \Gamma\left[\frac{4}{n}, -(b*(c+d*x)^n \log[F])\right]\right) / \left(d*n*(-(b*(c+d*x)^n \log[F])\right)^{(4/n)}\right)$

**Rubi [A]** time = 0.0361046, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^4 (-b \log(F)(c+dx)^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^3, x]

[Out]  $-\left(\left(F^{a*(c+d*x)^4} \Gamma\left[\frac{4}{n}, -(b*(c+d*x)^n \log[F])\right]\right) / \left(d*n*(-(b*(c+d*x)^n \log[F])\right)^{(4/n)}\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx)^3 dx = -\frac{F^a(c+dx)^4 \Gamma\left(\frac{4}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{-4/n}}{dn}$$

**Mathematica [A]** time = 0.0147712, size = 54, normalized size = 1.

$$\frac{F^a(c+dx)^4 (-b \log(F)(c+dx)^n)^{-4/n} \Gamma\left(\frac{4}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^3, x]

[Out]  $-\left(\left(F^{a*(c+d*x)^4} \Gamma\left[\frac{4}{n}, -(b*(c+d*x)^n \log[F])\right]\right) / \left(d*n*(-(b*(c+d*x)^n \log[F])\right)^{(4/n)}\right)$



**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^n} (dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^3,x)

[Out] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate((d\*x + c)^3\*F^((d\*x + c)^n\*b + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3\right)F^{(dx+c)^n b+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^3,x, algorithm="fricas")

[Out] integral((d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)\*F^((d\*x + c)^n\*b + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*F^((d\*x + c)^n\*b + a), x)

### 3.361 $\int F^{a+b(c+dx)^n} (c+dx)^2 dx$

**Optimal.** Leaf size=54

$$\frac{F^a(c+dx)^3 (-b \log(F)(c+dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

[Out]  $-\left(\left(F^{a*(c+d*x)^3*\Gamma[3/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(3/n)}\right)\right)$

**Rubi [A]** time = 0.0357216, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a(c+dx)^3 (-b \log(F)(c+dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^2, x]

[Out]  $-\left(\left(F^{a*(c+d*x)^3*\Gamma[3/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(3/n)}\right)\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx)^2 dx = -\frac{F^a(c+dx)^3 \Gamma\left(\frac{3}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{-3/n}}{dn}$$

**Mathematica [A]** time = 0.0132768, size = 54, normalized size = 1.

$$\frac{F^a(c+dx)^3 (-b \log(F)(c+dx)^n)^{-3/n} \Gamma\left(\frac{3}{n}, -b \log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^2, x]

[Out]  $-\left(\left(F^{a*(c+d*x)^3*\Gamma[3/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(3/n)}\right)\right)$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^n} (dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^2,x)

[Out] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^2 \* F^((d\*x + c)^n \* b + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2 + 2cdx + c^2\right)F^{(dx+c)^n b+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^2,x, algorithm="fricas")

[Out] integral((d^2\*x^2 + 2\*c\*d\*x + c^2)\*F^((d\*x + c)^n\*b + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2 \* F^((d\*x + c)^n \* b + a), x)

### 3.362 $\int F^{a+b(c+dx)^n} (c+dx) dx$

**Optimal.** Leaf size=54

$$\frac{F^a(c+dx)^2(-b\log(F)(c+dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

[Out]  $-\left(\left(F^{a*(c+d*x)^2*\Gamma[2/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(2/n)}\right)$

**Rubi [A]** time = 0.0207883, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2218}

$$\frac{F^a(c+dx)^2(-b\log(F)(c+dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x), x]

[Out]  $-\left(\left(F^{a*(c+d*x)^2*\Gamma[2/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(2/n)}\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx) dx = -\frac{F^a(c+dx)^2 \Gamma\left(\frac{2}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{-2/n}}{dn}$$

**Mathematica [A]** time = 0.0116291, size = 54, normalized size = 1.

$$\frac{F^a(c+dx)^2(-b\log(F)(c+dx)^n)^{-2/n} \Gamma\left(\frac{2}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x), x]

[Out]  $-\left(\left(F^{a*(c+d*x)^2*\Gamma[2/n, -(b*(c+d*x)^n*\text{Log}[F])]\right)\right)/\left(d*n*(-(b*(c+d*x)^n*\text{Log}[F]))^{(2/n)}\right)$

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^n} (dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c),x)

[Out] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)F^{(dx+c)^nb+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c),x, algorithm="maxima")

[Out] integrate((d\*x + c)\*F^((d\*x + c)^n\*b + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx+c)F^{(dx+c)^nb+a},x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c),x, algorithm="fricas")

[Out] integral((d\*x + c)\*F^((d\*x + c)^n\*b + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)F^{(dx+c)^nb+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c),x, algorithm="giac")

[Out] integrate((d\*x + c)\*F^((d\*x + c)^n\*b + a), x)

### 3.363 $\int F^{a+b(c+dx)^n} dx$

**Optimal.** Leaf size=50

$$\frac{F^a(c+dx)(-b\log(F)(c+dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

[Out]  $-\left(\left(F^{a*(c+d*x)}*\Gamma[n^{(-1)}, -(b*(c+d*x)^n*\log[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\log[F]))^{(-1)}\right)\right)$

**Rubi [A]** time = 0.0065579, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2208}

$$\frac{F^a(c+dx)(-b\log(F)(c+dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n), x]

[Out]  $-\left(\left(F^{a*(c+d*x)}*\Gamma[n^{(-1)}, -(b*(c+d*x)^n*\log[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\log[F]))^{(-1)}\right)\right)$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} dx = -\frac{F^a(c+dx)\Gamma\left(\frac{1}{n}, -b(c+dx)^n \log(F)\right)(-b(c+dx)^n \log(F))^{-1/n}}{dn}$$

**Mathematica [A]** time = 0.0090424, size = 50, normalized size = 1.

$$\frac{F^a(c+dx)(-b\log(F)(c+dx)^n)^{-1/n} \Gamma\left(\frac{1}{n}, -b\log(F)(c+dx)^n\right)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n), x]

[Out]  $-\left(\left(F^{a*(c+d*x)}*\Gamma[n^{(-1)}, -(b*(c+d*x)^n*\log[F])]\right)/\left(d*n*(-(b*(c+d*x)^n*\log[F]))^{(-1)}\right)\right)$

**Maple [F]** time = 0.024, size = 0, normalized size = 0.

$$\int F^{a+b(dx+c)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n),x)`

[Out] `int(F^(a+b*(d*x+c)^n),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(dx+c)^{n+b+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n),x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^n*b + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(F^{(dx+c)^{n+b+a}}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n),x, algorithm="fricas")`

[Out] `integral(F^((d*x + c)^n*b + a), x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n),x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(dx+c)^{n+b+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n),x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^n*b + a), x)`

$$3.364 \quad \int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

**Rubi [A]** time = 0.0349404, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx = \frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

**Mathematica [A]** time = 0.0040453, size = 22, normalized size = 1.

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

**Maple [A]** time = 0.31, size = 26, normalized size = 1.2

$$-\frac{F^a \operatorname{Ei}(1, -b(dx+c)^n \ln(F))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(F^(a+b*(d*x+c)^n)/(d*x+c),x)`

[Out] `-1/d/n*F^a*Ei(1,-b*(d*x+c)^n*ln(F))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)`

**Fricas [A]** time = 1.57146, size = 49, normalized size = 2.23

$$\frac{F^a \operatorname{Ei}((dx+c)^n b \log(F))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="fricas")`

[Out] `F^a*Ei((d*x + c)^n*b*log(F))/(d*n)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)/(d*x+c),x)`

[Out] `Integral(F**(a + b*(c + d*x)**n)/(c + d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)`

$$3.365 \quad \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx$$

**Optimal.** Leaf size=52

$$\frac{F^a (-b \log(F)(c + dx)^n)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)}$$

[Out]  $-\left(\left(F^a \Gamma[-n^(-1), -(b*(c + d*x)^n * \text{Log}[F])]\right) * \left(-\left(b*(c + d*x)^n * \text{Log}[F]\right)\right)^n (-1)\right) / (d*n*(c + d*x))$

**Rubi [A]** time = 0.0345223, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a (-b \log(F)(c + dx)^n)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^2, x]

[Out]  $-\left(\left(F^a \Gamma[-n^(-1), -(b*(c + d*x)^n * \text{Log}[F])]\right) * \left(-\left(b*(c + d*x)^n * \text{Log}[F]\right)\right)^n (-1)\right) / (d*n*(c + d*x))$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n \* Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n \* Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx = -\frac{F^a \Gamma\left(-\frac{1}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{\frac{1}{n}}}{dn(c+dx)}$$

**Mathematica [A]** time = 0.0120298, size = 52, normalized size = 1.

$$\frac{F^a (-b \log(F)(c + dx)^n)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^2, x]

[Out]  $-\left(\left(F^a \Gamma[-n^(-1), -(b*(c + d*x)^n * \text{Log}[F])]\right) * \left(-\left(b*(c + d*x)^n * \text{Log}[F]\right)\right)^n (-1)\right) / (d*n*(c + d*x))$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^2,x)

[Out] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^n\*b + a)/(d\*x + c)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{(dx+c)^n b+a}}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(F^((d\*x + c)^n\*b + a)/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)/(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^n*b + a)/(d*x + c)^2, x)
```

$$3.366 \quad \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} dx$$

**Optimal.** Leaf size=54

$$\frac{F^a (-b \log(F)(c + dx)^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^2}$$

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{2}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{2}{n}}\right) / \left(d*n*(c + d*x)^2\right)$

**Rubi [A]** time = 0.0347216, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a (-b \log(F)(c + dx)^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^3, x]

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{2}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{2}{n}}\right) / \left(d*n*(c + d*x)^2\right)$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} dx = -\frac{F^a \Gamma\left(-\frac{2}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{2/n}}{dn(c+dx)^2}$$

**Mathematica [A]** time = 0.009541, size = 54, normalized size = 1.

$$\frac{F^a (-b \log(F)(c + dx)^n)^{2/n} \text{Gamma}\left(-\frac{2}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^3, x]

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{2}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{2}{n}}\right) / \left(d*n*(c + d*x)^2\right)$

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^3,x)

[Out] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^3,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^n\*b + a)/(d\*x + c)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{(dx+c)^n b+a}}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^3,x, algorithm="fricas")

[Out] integral(F^((d\*x + c)^n\*b + a)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)/(d\*x+c)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^n*b + a)/(d*x + c)^3, x)
```

$$3.367 \quad \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx$$

**Optimal.** Leaf size=54

$$\frac{F^a (-b \log(F)(c + dx)^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^3}$$

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{3}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{3}{n}}\right) / (d*n*(c + d*x)^3)$

**Rubi [A]** time = 0.0367762, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2218}

$$\frac{F^a (-b \log(F)(c + dx)^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^4, x]

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{3}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{3}{n}}\right) / (d*n*(c + d*x)^3)$

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx = -\frac{F^a \Gamma\left(-\frac{3}{n}, -b(c+dx)^n \log(F)\right) (-b(c+dx)^n \log(F))^{3/n}}{dn(c+dx)^3}$$

**Mathematica [A]** time = 0.0104566, size = 54, normalized size = 1.

$$\frac{F^a (-b \log(F)(c + dx)^n)^{3/n} \text{Gamma}\left(-\frac{3}{n}, -b \log(F)(c + dx)^n\right)}{dn(c + dx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)/(c + d\*x)^4, x]

[Out]  $-\left(\left(F^a \text{Gamma}\left[-\frac{3}{n}, -(b*(c + d*x)^n \text{Log}[F])\right]\right) * \left(-\left(b*(c + d*x)^n \text{Log}[F]\right)\right)^{\frac{3}{n}}\right) / (d*n*(c + d*x)^3)$



**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^4,x)

[Out] int(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^4,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^4,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^n\*b + a)/(d\*x + c)^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{(dx+c)^n b+a}}{d^4 x^4 + 4 c d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 dx + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)/(d\*x+c)^4,x, algorithm="fricas")

[Out] integral(F^((d\*x + c)^n\*b + a)/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)/(d\*x+c)\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{(dx+c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^n*b + a)/(d*x + c)^4, x)
```

### 3.368 $\int F^{a+b(c+dx)^n} (c+dx)^{-1+6n} dx$

**Optimal.** Leaf size=114

$$\frac{F^{a+b(c+dx)^n} (60b^2 \log^2(F)(c+dx)^{2n} - 20b^3 \log^3(F)(c+dx)^{3n} + 5b^4 \log^4(F)(c+dx)^{4n} - b^5 \log^5(F)(c+dx)^{5n} - 120b \log(F)(c+dx)^{6n})}{b^6 dn \log^6(F)}$$

[Out]  $-\left(\frac{F^{a+b(c+dx)^n} (120 - 120b(c+dx)^n \text{Log}[F] + 60b^2(c+dx)^{2n} \text{Log}[F]^2 - 20b^3(c+dx)^{3n} \text{Log}[F]^3 + 5b^4(c+dx)^{4n} \text{Log}[F]^4 - b^5(c+dx)^{5n} \text{Log}[F]^5)}{b^6 d^n \text{Log}[F]^6}\right)$

**Rubi [C]** time = 0.0375697, antiderivative size = 32, normalized size of antiderivative = 0.28, number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^n)}{b^6 dn \log^6(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 6\*n), x]

[Out]  $-\left(\frac{F^a \text{Gamma}[6, -(b*(c + d*x)^n \text{Log}[F])]}{b^6 d^n \text{Log}[F]^6}\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1+6n} dx = -\frac{F^a \Gamma(6, -b(c+dx)^n \log(F))}{b^6 dn \log^6(F)}$$

**Mathematica [C]** time = 0.0066138, size = 32, normalized size = 0.28

$$\frac{F^a \text{Gamma}(6, -b \log(F)(c+dx)^n)}{b^6 dn \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 6\*n), x]

[Out]  $-\left(\frac{F^a \text{Gamma}[6, -(b*(c + d*x)^n \text{Log}[F])]}{b^6 d^n \text{Log}[F]^6}\right)$

**Maple [A]** time = 0.027, size = 113, normalized size = 1.

$$\frac{\left(\left((dx+c)^n\right)^5 b^5 (\ln(F))^5 - 5 \left((dx+c)^n\right)^4 b^4 (\ln(F))^4 + 20 \left((dx+c)^n\right)^3 b^3 (\ln(F))^3 - 60 \left((dx+c)^n\right)^2 b^2 (\ln(F))^2 + 120 \left((dx+c)^n\right) b (\ln(F)) - 120 b \ln(F)\right)}{b^6 (\ln(F))^6 nd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x)`

[Out] 
$$\frac{((d*x+c)^n)^5*b^5*\ln(F)^5-5*((d*x+c)^n)^4*b^4*\ln(F)^4+20*((d*x+c)^n)^3*b^3*\ln(F)^3-60*((d*x+c)^n)^2*b^2*\ln(F)^2+120*b*(d*x+c)^n*\ln(F)-120)/b^6/\ln(F)^6/n/d*F^(a+b*(d*x+c)^n)}$$

**Maxima [A]** time = 1.02355, size = 174, normalized size = 1.53

$$\frac{((dx+c)^{5n}F^ab^5\log(F)^5-5(dx+c)^{4n}F^ab^4\log(F)^4+20(dx+c)^{3n}F^ab^3\log(F)^3-60(dx+c)^{2n}F^ab^2\log(F)^2+120(dx+c)^nF^ab\log(F)-120)/b^6dn\log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="maxima")`

[Out] 
$$\frac{((d*x+c)^{5n})*F^a*b^5*\log(F)^5-5*(d*x+c)^{4n})*F^a*b^4*\log(F)^4+20*(d*x+c)^{3n})*F^a*b^3*\log(F)^3-60*(d*x+c)^{2n})*F^a*b^2*\log(F)^2+120*(d*x+c)^n)*F^a*b*\log(F)-120*F^a)*F^{((d*x+c)^n*b)/(b^6*d*n*\log(F)^6)}$$

**Fricas [A]** time = 1.62008, size = 298, normalized size = 2.61

$$\frac{((dx+c)^{5n}b^5\log(F)^5-5(dx+c)^{4n}b^4\log(F)^4+20(dx+c)^{3n}b^3\log(F)^3-60(dx+c)^{2n}b^2\log(F)^2+120(dx+c)^nb\log(F)-120)/b^6dn\log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="fricas")`

[Out] 
$$\frac{((d*x+c)^{5n})*b^5*\log(F)^5-5*(d*x+c)^{4n})*b^4*\log(F)^4+20*(d*x+c)^{3n})*b^3*\log(F)^3-60*(d*x+c)^{2n})*b^2*\log(F)^2+120*(d*x+c)^n)*b*\log(F)-120)*e^{((d*x+c)^n*b*\log(F)+a*\log(F))/(b^6*d*n*\log(F)^6)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+6*n),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{6n-1}F^{(dx+c)^nb+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(6*n - 1)*F^((d*x + c)^n*b + a), x)
```

### 3.369 $\int F^{a+b(c+dx)^n} (c+dx)^{-1+5n} dx$

**Optimal.** Leaf size=94

$$\frac{F^{a+b(c+dx)^n} (12b^2 \log^2(F)(c+dx)^{2n} - 4b^3 \log^3(F)(c+dx)^{3n} + b^4 \log^4(F)(c+dx)^{4n} - 24b \log(F)(c+dx)^n + 24)}{b^5 dn \log^5(F)}$$

[Out]  $(F^{(a + b*(c + d*x)^n})*(24 - 24*b*(c + d*x)^n*\text{Log}[F] + 12*b^2*(c + d*x)^{(2*n)}*\text{Log}[F]^2 - 4*b^3*(c + d*x)^{(3*n)}*\text{Log}[F]^3 + b^4*(c + d*x)^{(4*n)}*\text{Log}[F]^4)) / (b^5*d*n*\text{Log}[F]^5)$

**Rubi [C]** time = 0.0396707, antiderivative size = 31, normalized size of antiderivative = 0.33, number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2218}

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^n)}{b^5 dn \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 5\*n), x]

[Out] (F^a\*Gamma[5, -(b\*(c + d\*x)^n\*Log[F])]) / (b^5\*d\*n\*Log[F]^5)

#### Rule 2218

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))\*((e\_) + (f\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1+5n} dx = \frac{F^a \Gamma(5, -b(c+dx)^n \log(F))}{b^5 dn \log^5(F)}$$

**Mathematica [C]** time = 0.0068639, size = 31, normalized size = 0.33

$$\frac{F^a \text{Gamma}(5, -b \log(F)(c+dx)^n)}{b^5 dn \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 5\*n), x]

[Out] (F^a\*Gamma[5, -(b\*(c + d\*x)^n\*Log[F])]) / (b^5\*d\*n\*Log[F]^5)

**Maple [A]** time = 0.022, size = 95, normalized size = 1.

$$\frac{\left( (dx+c)^n \right)^4 b^4 (\ln(F))^4 - 4 \left( (dx+c)^n \right)^3 b^3 (\ln(F))^3 + 12 \left( (dx+c)^n \right)^2 b^2 (\ln(F))^2 - 24 b (dx+c)^n \ln(F) + 24 \right) F^{a+b(dx+c)^n}}{b^5 (\ln(F))^5 nd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x)`

[Out]  $((d*x+c)^n)^4*b^4*\ln(F)^4-4*((d*x+c)^n)^3*b^3*\ln(F)^3+12*((d*x+c)^n)^2*b^2*\ln(F)^2-24*b*(d*x+c)^n*\ln(F)+24)/b^5/\ln(F)^5/n/d*F^(a+b*(d*x+c)^n)$

**Maxima [A]** time = 1.03157, size = 146, normalized size = 1.55

$$\frac{((dx+c)^{4n}F^a b^4 \log(F)^4 - 4(dx+c)^{3n}F^a b^3 \log(F)^3 + 12(dx+c)^{2n}F^a b^2 \log(F)^2 - 24(dx+c)^n F^a b \log(F) + 24F^a)F^{(d*x+c)^n}}{b^5 d n \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="maxima")`

[Out]  $((d*x+c)^{4*n}*F^a*b^4*\log(F)^4 - 4*(d*x+c)^{3*n}*F^a*b^3*\log(F)^3 + 12*(d*x+c)^{2*n}*F^a*b^2*\log(F)^2 - 24*(d*x+c)^n*F^a*b*\log(F) + 24*F^a)*F^{((d*x+c)^n*b)}/(b^5*d*n*\log(F)^5)$

**Fricas [A]** time = 1.57409, size = 250, normalized size = 2.66

$$\frac{((dx+c)^{4n}b^4 \log(F)^4 - 4(dx+c)^{3n}b^3 \log(F)^3 + 12(dx+c)^{2n}b^2 \log(F)^2 - 24(dx+c)^n b \log(F) + 24)e^{((dx+c)^n b \log(F)+a)}}{b^5 d n \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="fricas")`

[Out]  $((d*x+c)^{4*n}*b^4*\log(F)^4 - 4*(d*x+c)^{3*n}*b^3*\log(F)^3 + 12*(d*x+c)^{2*n}*b^2*\log(F)^2 - 24*(d*x+c)^n*b*\log(F) + 24)*e^{((d*x+c)^n*b*\log(F) + a*\log(F))}/(b^5*d*n*\log(F)^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+5*n),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{5n-1}F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5*n - 1)*F^((d*x + c)^n*b + a), x)
```



### 3.370 $\int F^{a+b(c+dx)^n} (c+dx)^{-1+4n} dx$

**Optimal.** Leaf size=137

$$\frac{6(c+dx)^n F^{a+b(c+dx)^n}}{b^3 dn \log^3(F)} - \frac{3(c+dx)^{2n} F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)} - \frac{6F^{a+b(c+dx)^n}}{b^4 dn \log^4(F)} + \frac{(c+dx)^{3n} F^{a+b(c+dx)^n}}{bdn \log(F)}$$

[Out]  $(-6F^{a+b(c+dx)^n})/(b^4 d n \log[F]^4) + (6F^{a+b(c+dx)^n} (c+dx)^n)/(b^3 d n \log[F]^3) - (3F^{a+b(c+dx)^n} (c+dx)^{2n})/(b^2 d n \log[F]^2) + (F^{a+b(c+dx)^n} (c+dx)^{3n})/(b d n \log[F])$

**Rubi [A]** time = 0.167854, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2213, 2209}

$$\frac{6(c+dx)^n F^{a+b(c+dx)^n}}{b^3 dn \log^3(F)} - \frac{3(c+dx)^{2n} F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)} - \frac{6F^{a+b(c+dx)^n}}{b^4 dn \log^4(F)} + \frac{(c+dx)^{3n} F^{a+b(c+dx)^n}}{bdn \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{a+b(c+dx)^n} (c+dx)^{-1+4n}, x]$

[Out]  $(-6F^{a+b(c+dx)^n})/(b^4 d n \log[F]^4) + (6F^{a+b(c+dx)^n} (c+dx)^n)/(b^3 d n \log[F]^3) - (3F^{a+b(c+dx)^n} (c+dx)^{2n})/(b^2 d n \log[F]^2) + (F^{a+b(c+dx)^n} (c+dx)^{3n})/(b d n \log[F])$

#### Rule 2213

$\text{Int}[(F_.)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_)))^n)} * ((c_.) + (d_.) * (x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[(c+dx)^{(m-n+1)} F^{a+b(c+dx)^n} / (b d n \log[F]), x] - \text{Dist}[(m-n+1) / (b n \log[F]), \text{Int}[(c+dx)^{\text{Simplify}[m-n]} F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m+1)/n]] && LtQ[0, Simplify[(m+1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]

#### Rule 2209

$\text{Int}[(F_.)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_)))^n)} * ((e_.) + (f_.) * (x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[(e+fx)^n F^{a+b(c+dx)^n} / (b f n (c+dx)^n \log[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n-1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1+4n} dx &= \frac{F^{a+b(c+dx)^n} (c+dx)^{3n}}{bdn \log(F)} - \frac{3 \int F^{a+b(c+dx)^n} (c+dx)^{-1+3n} dx}{b \log(F)} \\ &= -\frac{3F^{a+b(c+dx)^n} (c+dx)^{2n}}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^{3n}}{bdn \log(F)} + \frac{6 \int F^{a+b(c+dx)^n} (c+dx)^{-1+2n} dx}{b^2 \log^2(F)} \\ &= \frac{6F^{a+b(c+dx)^n} (c+dx)^n}{b^3 dn \log^3(F)} - \frac{3F^{a+b(c+dx)^n} (c+dx)^{2n}}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^{3n}}{bdn \log(F)} - \frac{6 \int F^{a+b(c+dx)^n} (c+dx)^{-1+2n} dx}{b^3 \log^3(F)} \\ &= -\frac{6F^{a+b(c+dx)^n}}{b^4 dn \log^4(F)} + \frac{6F^{a+b(c+dx)^n} (c+dx)^n}{b^3 dn \log^3(F)} - \frac{3F^{a+b(c+dx)^n} (c+dx)^{2n}}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^{3n}}{bdn \log(F)} \end{aligned}$$

**Mathematica [C]** time = 0.0061688, size = 32, normalized size = 0.23

$$\frac{F^a \text{Gamma}(4, -b \log(F)(c + dx)^n)}{b^4 d n \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 4\*n), x]

[Out] -((F^a\*Gamma[4, -(b\*(c + d\*x)^n\*Log[F])])/(b^4\*d\*n\*Log[F]^4))

**Maple [A]** time = 0.019, size = 77, normalized size = 0.6

$$\frac{\left( (dx + c)^n \right)^3 b^3 (\ln(F))^3 - 3 \left( (dx + c)^n \right)^2 b^2 (\ln(F))^2 + 6 b (dx + c)^n \ln(F) - 6 \right) F^{a+b(dx+c)^n}}{b^4 (\ln(F))^4 n d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+4\*n), x)

[Out] (((d\*x+c)^n)^3\*b^3\*ln(F)^3-3\*((d\*x+c)^n)^2\*b^2\*ln(F)^2+6\*b\*(d\*x+c)^n\*ln(F)-6)/b^4/ln(F)^4/n/d\*F^(a+b\*(d\*x+c)^n)

**Maxima [A]** time = 1.04289, size = 117, normalized size = 0.85

$$\frac{\left( (dx + c)^{3n} F^a b^3 \log(F)^3 - 3 (dx + c)^{2n} F^a b^2 \log(F)^2 + 6 (dx + c)^n F^a b \log(F) - 6 F^a \right) F^{(dx+c)^n b}}{b^4 d n \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+4\*n), x, algorithm="maxima")

[Out] ((d\*x + c)^(3\*n)\*F^a\*b^3\*log(F)^3 - 3\*(d\*x + c)^(2\*n)\*F^a\*b^2\*log(F)^2 + 6\*(d\*x + c)^n\*F^a\*b\*log(F) - 6\*F^a)\*F^((d\*x + c)^n\*b)/(b^4\*d\*n\*log(F)^4)

**Fricas [A]** time = 1.60181, size = 201, normalized size = 1.47

$$\frac{\left( (dx + c)^{3n} b^3 \log(F)^3 - 3 (dx + c)^{2n} b^2 \log(F)^2 + 6 (dx + c)^n b \log(F) - 6 \right) e^{(dx+c)^n b \log(F) + a \log(F)}}{b^4 d n \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+4\*n), x, algorithm="fricas")

[Out] ((d\*x + c)^(3\*n)\*b^3\*log(F)^3 - 3\*(d\*x + c)^(2\*n)\*b^2\*log(F)^2 + 6\*(d\*x + c)^n\*b\*log(F) - 6)\*e^((d\*x + c)^n\*b\*log(F) + a\*log(F))/(b^4\*d\*n\*log(F)^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*(-1+4\*n), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{4n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+4\*n), x, algorithm="giac")

[Out] integrate((d\*x + c)^(4\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

### 3.371 $\int F^{a+b(c+dx)^n} (c+dx)^{-1+3n} dx$

**Optimal.** Leaf size=100

$$-\frac{2(c+dx)^n F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)} + \frac{2F^{a+b(c+dx)^n}}{b^3 dn \log^3(F)} + \frac{(c+dx)^{2n} F^{a+b(c+dx)^n}}{bdn \log(F)}$$

[Out]  $(2F^{a+b(c+dx)^n})/(b^3 d n \text{Log}[F]^3) - (2F^{a+b(c+dx)^n}*(c+dx)^n)/(b^2 d n \text{Log}[F]^2) + (F^{a+b(c+dx)^n}*(c+dx)^{(2n)})/(b*d n \text{Log}[F])$

**Rubi [A]** time = 0.119527, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2213, 2209}

$$-\frac{2(c+dx)^n F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)} + \frac{2F^{a+b(c+dx)^n}}{b^3 dn \log^3(F)} + \frac{(c+dx)^{2n} F^{a+b(c+dx)^n}}{bdn \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{a+b(c+dx)^n}*(c+dx)^{-1+3n}, x]$

[Out]  $(2F^{a+b(c+dx)^n})/(b^3 d n \text{Log}[F]^3) - (2F^{a+b(c+dx)^n}*(c+dx)^n)/(b^2 d n \text{Log}[F]^2) + (F^{a+b(c+dx)^n}*(c+dx)^{(2n)})/(b*d n \text{Log}[F])$

#### Rule 2213

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(c+dx)^{(m-n+1)}*F^{a+b(c+dx)^n})/(b*d*n*\text{Log}[F]), x] - \text{Dist}[(m-n+1)/(b*n*\text{Log}[F]), \text{Int}[(c+dx)^{\text{Simplify}[m-n]}*F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m+1)/n]] && LtQ[0, Simplify[(m+1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]

#### Rule 2209

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(e+fx)^n * F^{a+b(c+dx)^n})/(b*f*n*(c+dx)^n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n-1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1+3n} dx &= \frac{F^{a+b(c+dx)^n} (c+dx)^{2n}}{bdn \log(F)} - \frac{2 \int F^{a+b(c+dx)^n} (c+dx)^{-1+2n} dx}{b \log(F)} \\ &= -\frac{2F^{a+b(c+dx)^n} (c+dx)^n}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^{2n}}{bdn \log(F)} + \frac{2 \int F^{a+b(c+dx)^n} (c+dx)^{-1+n} dx}{b^2 \log^2(F)} \\ &= \frac{2F^{a+b(c+dx)^n}}{b^3 dn \log^3(F)} - \frac{2F^{a+b(c+dx)^n} (c+dx)^n}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^{2n}}{bdn \log(F)} \end{aligned}$$

**Mathematica [C]** time = 0.0065167, size = 31, normalized size = 0.31

$$\frac{F^a \text{Gamma}(3, -b \log(F)(c+dx)^n)}{b^3 dn \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 3\*n), x]

[Out] (F^a\*Gamma[3, -(b\*(c + d\*x)^n\*Log[F])])/(b^3\*d\*n\*Log[F]^3)

**Maple [A]** time = 0.02, size = 59, normalized size = 0.6

$$\frac{\left(\left(dx + c\right)^n\right)^2 b^2 (\ln(F))^2 - 2 b (dx + c)^n \ln(F) + 2}{(\ln(F))^3 b^3 n d} F^{a+b(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+3\*n), x)

[Out] (((d\*x+c)^n)^2\*b^2\*ln(F)^2-2\*b\*(d\*x+c)^n\*ln(F)+2)/b^3/ln(F)^3/n/d\*F^(a+b\*(d\*x+c)^n)

**Maxima [A]** time = 1.04473, size = 89, normalized size = 0.89

$$\frac{\left((dx + c)^{2n} F^a b^2 \log(F)^2 - 2(dx + c)^n F^a b \log(F) + 2 F^a\right) F^{(dx+c)^n b}}{b^3 d n \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+3\*n), x, algorithm="maxima")

[Out] ((d\*x + c)^(2\*n)\*F^a\*b^2\*log(F)^2 - 2\*(d\*x + c)^n\*F^a\*b\*log(F) + 2\*F^a)\*F^(d\*x + c)^n\*b)/(b^3\*d\*n\*log(F)^3)

**Fricas [A]** time = 1.56674, size = 157, normalized size = 1.57

$$\frac{\left((dx + c)^{2n} b^2 \log(F)^2 - 2(dx + c)^n b \log(F) + 2\right) e^{(dx+c)^n b \log(F) + a \log(F)}}{b^3 d n \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+3\*n), x, algorithm="fricas")

[Out] ((d\*x + c)^(2\*n)\*b^2\*log(F)^2 - 2\*(d\*x + c)^n\*b\*log(F) + 2)\*e^((d\*x + c)^n\*b\*log(F) + a\*log(F))/(b^3\*d\*n\*log(F)^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+3*n),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{3n-1} F^{(dx+c)^{n*b+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3*n - 1)*F^((d*x + c)^n*b + a), x)
```

$$3.372 \quad \int F^{a+b(c+dx)^n} (c+dx)^{-1+2n} dx$$

**Optimal.** Leaf size=63

$$\frac{(c+dx)^n F^{a+b(c+dx)^n}}{bdn \log(F)} - \frac{F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)}$$

[Out]  $-(F^{a+b(c+dx)^n})/(b^2 d n \log(F)^2) + (F^{a+b(c+dx)^n}) * (c+dx)^n / (b d n \log(F))$

**Rubi [A]** time = 0.074351, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2213, 2209}

$$\frac{(c+dx)^n F^{a+b(c+dx)^n}}{bdn \log(F)} - \frac{F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 2\*n), x]

[Out]  $-(F^{a+b(c+dx)^n})/(b^2 d n \log(F)^2) + (F^{a+b(c+dx)^n}) * (c+dx)^n / (b d n \log(F))$

#### Rule 2213

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)) \* ((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1) \* F^(a + b\*(c + d\*x)^n)) / (b\*d\*n\*Log[F]), x] - Dist[(m - n + 1) / (b\*n\*Log[F]), Int[(c + d\*x)^Simplify[m - n] \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, -n]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)) \* ((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1+2n} dx &= \frac{F^{a+b(c+dx)^n} (c+dx)^n}{bdn \log(F)} - \frac{\int F^{a+b(c+dx)^n} (c+dx)^{-1+n} dx}{b \log(F)} \\ &= -\frac{F^{a+b(c+dx)^n}}{b^2 dn \log^2(F)} + \frac{F^{a+b(c+dx)^n} (c+dx)^n}{bdn \log(F)} \end{aligned}$$

**Mathematica [C]** time = 0.0063835, size = 32, normalized size = 0.51

$$-\frac{F^a \text{Gamma}(2, -b \log(F)(c+dx)^n)}{b^2 dn \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 + 2\*n), x]

[Out]  $-\left(\frac{F^a \Gamma\left[2, -(b(c + dx)^n \log(F))\right]}{b^2 d n \log(F)^2}\right)$

**Maple [A]** time = 0.056, size = 74, normalized size = 1.2

$$\frac{e^{n \ln(dx+c)} e^{(a+be^n \ln(dx+c)) \ln(F)}}{\ln(F) b d n} - \frac{e^{(a+be^n \ln(dx+c)) \ln(F)}}{(\ln(F))^2 b^2 d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+2\*n), x)

[Out]  $\frac{1}{d} \frac{1}{b} \frac{1}{n} \frac{1}{\ln(F)} \exp(n \ln(d*x+c)) \exp((a+b \exp(n \ln(d*x+c))) \ln(F)) - \frac{1}{b^2} \frac{1}{n} \frac{1}{d} \frac{1}{\ln(F)^2} \exp((a+b \exp(n \ln(d*x+c))) \ln(F))$

**Maxima [A]** time = 1.0147, size = 61, normalized size = 0.97

$$\frac{\left((dx + c)^n F^a b \log(F) - F^a\right) F^{(dx+c)^n b}}{b^2 d n \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+2\*n), x, algorithm="maxima")

[Out]  $\left((d*x + c)^n * F^a * b * \log(F) - F^a\right) * F^{((d*x + c)^n * b)} / (b^2 * d * n * \log(F)^2)$

**Fricas [A]** time = 1.57775, size = 112, normalized size = 1.78

$$\frac{\left((dx + c)^n b \log(F) - 1\right) e^{(dx+c)^n b \log(F) + a \log(F)}}{b^2 d n \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+2\*n), x, algorithm="fricas")

[Out]  $\left((d*x + c)^n * b * \log(F) - 1\right) * e^{((d*x + c)^n * b * \log(F) + a * \log(F))} / (b^2 * d * n * \log(F)^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*(-1+2\*n), x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{2n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1+2\*n),x, algorithm="giac")

[Out] integrate((d\*x + c)^(2\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

$$3.373 \quad \int F^{a+b(c+dx)^n} (c+dx)^{-1+n} dx$$

**Optimal.** Leaf size=27

$$\frac{F^{a+b(c+dx)^n}}{bdn \log(F)}$$

[Out]  $F^{(a + b*(c + d*x)^n)/(b*d*n*\text{Log}[F])}$

**Rubi [A]** time = 0.037157, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {2209}

$$\frac{F^{a+b(c+dx)^n}}{bdn \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(a + b*(c + d*x)^n)*(c + d*x)^{-1 + n}}, x]$

[Out]  $F^{(a + b*(c + d*x)^n)/(b*d*n*\text{Log}[F])}$

**Rule 2209**

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^n * F^{(a + b*(c + d*x)^n)} / (b*f*n*(c + d*x)^n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1+n} dx = \frac{F^{a+b(c+dx)^n}}{bdn \log(F)}$$

**Mathematica [A]** time = 0.0072523, size = 27, normalized size = 1.

$$\frac{F^{a+b(c+dx)^n}}{bdn \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[F^{(a + b*(c + d*x)^n)*(c + d*x)^{-1 + n}}, x]$

[Out]  $F^{(a + b*(c + d*x)^n)/(b*d*n*\text{Log}[F])}$

**Maple [A]** time = 0.034, size = 32, normalized size = 1.2

$$\frac{e^{(a+be^n \ln(dx+c)) \ln(F)}}{\ln(F) bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x)`

[Out] `1/d/b/n/ln(F)*exp((a+b*exp(n*ln(d*x+c)))*ln(F))`

**Maxima [A]** time = 0.968427, size = 36, normalized size = 1.33

$$\frac{F^{(dx+c)^n b+a}}{bdn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="maxima")`

[Out] `F^((d*x + c)^n*b + a)/(b*d*n*log(F))`

**Fricas [A]** time = 1.53083, size = 70, normalized size = 2.59

$$\frac{e^{((dx+c)^n b \log(F)+a \log(F))}}{bdn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="fricas")`

[Out] `e^((d*x + c)^n*b*log(F) + a*log(F))/(b*d*n*log(F))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+n),x)`

[Out] Timed out

**Giac [A]** time = 1.40768, size = 36, normalized size = 1.33

$$\frac{F^{(dx+c)^n b+a}}{bdn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="giac")`

[Out] `F^((d*x + c)^n*b + a)/(b*d*n*log(F))`

$$3.374 \quad \int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$$

**Optimal.** Leaf size=22

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

**Rubi [A]** time = 0.0358306, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2210}

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx = \frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

**Mathematica [A]** time = 0.0018404, size = 22, normalized size = 1.

$$\frac{F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)/(c + d\*x), x]

[Out] (F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(d\*n)

**Maple [A]** time = 0., size = 26, normalized size = 1.2

$$-\frac{F^a \operatorname{Ei}(1, -b(dx+c)^n \ln(F))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)/(d*x+c),x)`

[Out] `-1/d/n*F^a*Ei(1,-b*(d*x+c)^n*ln(F))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)`

**Fricas [A]** time = 1.57954, size = 49, normalized size = 2.23

$$\frac{F^a \operatorname{Ei}((dx+c)^n b \log(F))}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="fricas")`

[Out] `F^a*Ei((d*x + c)^n*b*log(F))/(d*n)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)/(d*x+c),x)`

[Out] `Integral(F**(a + b*(c + d*x)**n)/(c + d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^n b+a}}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)`

$$3.375 \quad \int F^{a+b(c+dx)^n} (c+dx)^{-1-n} dx$$

**Optimal.** Leaf size=56

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{dn} - \frac{(c+dx)^{-n} F^{a+b(c+dx)^n}}{dn}$$

[Out]  $-(F^{a+b(c+dx)^n}/(d^n(c+dx)^n)) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^n \log(F)] \log(F))/(d^n)$

**Rubi [A]** time = 0.0725984, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2215, 2210}

$$\frac{bF^a \log(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{dn} - \frac{(c+dx)^{-n} F^{a+b(c+dx)^n}}{dn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+b(c+dx)^n} (c+dx)^{-1-n}, x]$

[Out]  $-(F^{a+b(c+dx)^n}/(d^n(c+dx)^n)) + (bF^a \operatorname{ExpIntegralEi}[b(c+dx)^n \log(F)] \log(F))/(d^n)$

#### Rule 2215

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})) * ((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+dx)^{(m+1)} F^{a+b(c+dx)^n} / (d(m+1)), x] - \operatorname{Dist}[(b^n \log(F)) / (m+1), \operatorname{Int}[(c+dx)^{\operatorname{Simplify}[m+n]} F^{a+b(c+dx)^n}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m+1)/n]] && LtQ[-4, Simplify[(m+1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})) / ((e_.) + (f_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[F^a \operatorname{ExpIntegralEi}[b(c+dx)^n \log(F)] / (f^n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1-n} dx &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-n}}{dn} + (b \log(F)) \int \frac{F^{a+b(c+dx)^n}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-n}}{dn} + \frac{bF^a \operatorname{Ei}(b(c+dx)^n \log(F)) \log(F)}{dn} \end{aligned}$$

**Mathematica [A]** time = 0.0063399, size = 27, normalized size = 0.48

$$\frac{bF^a \log(F) \operatorname{Gamma}(-1, -b \log(F)(c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[F^{a+b(c+dx)^n} (c+dx)^{-1-n}, x]$

[Out]  $(b \cdot F^a \cdot \text{Gamma}[-1, -(b \cdot (c + d \cdot x)^n \cdot \text{Log}[F])]) \cdot \text{Log}[F] / (d \cdot n)$

**Maple [A]** time = 0.12, size = 61, normalized size = 1.1

$$\frac{F^{b(dx+c)^n} F^a}{dn(dx+c)^n} - \frac{b \ln(F) F^a \text{Ei}\left(1, -b(dx+c)^n \ln(F)\right)}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n), x)`

[Out]  $-1/n/d \cdot F^{b(dx+c)^n} \cdot F^a / ((dx+c)^n) - 1/n/d \cdot \ln(F) \cdot b \cdot F^a \cdot \text{Ei}\left(1, -b(dx+c)^n \ln(F)\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{-n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n), x, algorithm="maxima")`

[Out] `integrate((d*x + c)^(-n - 1)*F^((d*x + c)^n*b + a), x)`

**Fricas [A]** time = 1.50631, size = 147, normalized size = 2.62

$$\frac{(dx+c)^n F^a b \text{Ei}\left((dx+c)^n b \log(F)\right) \log(F) - e^{((dx+c)^n b \log(F) + a \log(F))}}{(dx+c)^n dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n), x, algorithm="fricas")`

[Out]  $((dx+c)^n F^a b \text{Ei}((dx+c)^n b \log(F)) \log(F) - e^{((dx+c)^n b \log(F) + a \log(F))}) / ((dx+c)^n d n)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-n), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(-n - 1)*F^((d*x + c)^n*b + a), x)
```



### 3.376 $\int F^{a+b(c+dx)^n} (c+dx)^{-1-2n} dx$

**Optimal.** Leaf size=100

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{2dn} - \frac{(c+dx)^{-2n} F^{a+b(c+dx)^n}}{2dn} - \frac{b \log(F) (c+dx)^{-n} F^{a+b(c+dx)^n}}{2dn}$$

[Out]  $-F^{a+b(c+dx)^n}/(2*d*n*(c+dx)^{(2*n)}) - (b*F^{a+b(c+dx)^n}*Log[F])/(2*d*n*(c+dx)^n) + (b^2*F^a*ExpIntegralEi[b*(c+dx)^n*Log[F]])*Log[F]^2/(2*d*n)$

**Rubi [A]** time = 0.114021, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2215, 2210}

$$\frac{b^2 F^a \log^2(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{2dn} - \frac{(c+dx)^{-2n} F^{a+b(c+dx)^n}}{2dn} - \frac{b \log(F) (c+dx)^{-n} F^{a+b(c+dx)^n}}{2dn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{a+b(c+dx)^n}*(c+dx)^{-1-2n}, x]$

[Out]  $-F^{a+b(c+dx)^n}/(2*d*n*(c+dx)^{(2*n)}) - (b*F^{a+b(c+dx)^n}*Log[F])/(2*d*n*(c+dx)^n) + (b^2*F^a*ExpIntegralEi[b*(c+dx)^n*Log[F]])*Log[F]^2/(2*d*n)$

#### Rule 2215

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m], x\_Symbol] :> \text{Simp}[(c+dx)^{m+1} F^{a+b(c+dx)^n} / (d*(m+1)), x] - \text{Dist}[(b*n*Log[F]) / (m+1), \text{Int}[(c+dx)^{Simplify[m+n]} F^{a+b(c+dx)^n}], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m+1)/n]] && LtQ[-4, Simplify[(m+1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]

#### Rule 2210

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)} / ((e_.) + (f_.)*(x_.))], x\_Symbol] :> \text{Simp}[F^a * \text{ExpIntegralEi}[b*(c+dx)^n * Log[F]] / (f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1-2n} dx &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-2n}}{2dn} + \frac{1}{2} (b \log(F)) \int F^{a+b(c+dx)^n} (c+dx)^{-1-n} dx \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-2n}}{2dn} - \frac{b F^{a+b(c+dx)^n} (c+dx)^{-n} \log(F)}{2dn} + \frac{1}{2} (b^2 \log^2(F)) \int \frac{F^{a+b(c+dx)^n}}{c+dx} dx \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-2n}}{2dn} - \frac{b F^{a+b(c+dx)^n} (c+dx)^{-n} \log(F)}{2dn} + \frac{b^2 F^a \operatorname{Ei}(b(c+dx)^n \log(F))}{2dn} \end{aligned}$$

**Mathematica [A]** time = 0.0067658, size = 32, normalized size = 0.32

$$\frac{b^2 F^a \log^2(F) \operatorname{Gamma}(-2, -b \log(F) (c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 2\*n), x]

[Out] -((b^2\*F^a\*Gamma[-2, -(b\*(c + d\*x)^n\*Log[F])])\*Log[F]^2)/(d\*n))

**Maple [A]** time = 0.129, size = 99, normalized size = 1.

$$-\frac{F^{b(dx+c)^n} F^a}{2 dn ((dx+c)^n)^2} - \frac{b \ln(F) F^{b(dx+c)^n} F^a}{2 dn (dx+c)^n} - \frac{(\ln(F))^2 b^2 F^a \text{Ei}(1, -b(dx+c)^n \ln(F))}{2 dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-2\*n), x)

[Out] -1/2/n/d\*F^(b\*(d\*x+c)^n)\*F^a/((d\*x+c)^n)^2-1/2/n/d\*ln(F)\*b\*F^(b\*(d\*x+c)^n)\*F^a/((d\*x+c)^n)-1/2/n/d\*ln(F)^2\*b^2\*F^a\*Ei(1, -b\*(d\*x+c)^n\*ln(F))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{-2n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-2\*n), x, algorithm="maxima")

[Out] integrate((d\*x + c)^(-2\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

**Fricas [A]** time = 1.58313, size = 205, normalized size = 2.05

$$\frac{(dx+c)^{2n} F^a b^2 \text{Ei}((dx+c)^n b \log(F)) \log(F)^2 - ((dx+c)^n b \log(F) + 1) e^{((dx+c)^n b \log(F) + a \log(F))}}{2(dx+c)^{2n} dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-2\*n), x, algorithm="fricas")

[Out] 1/2\*((d\*x + c)^(2\*n)\*F^a\*b^2\*Ei((d\*x + c)^n\*b\*log(F))\*log(F)^2 - ((d\*x + c)^n\*b\*log(F) + 1)\*e^((d\*x + c)^n\*b\*log(F) + a\*log(F)))/((d\*x + c)^(2\*n)\*d\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*(-1-2\*n), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-2n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-2\*n),x, algorithm="giac")

[Out] integrate((d\*x + c)^(-2\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

### 3.377 $\int F^{a+b(c+dx)^n} (c+dx)^{-1-3n} dx$

**Optimal.** Leaf size=139

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{6dn} - \frac{b^2 \log^2(F)(c+dx)^{-n} F^{a+b(c+dx)^n}}{6dn} - \frac{(c+dx)^{-3n} F^{a+b(c+dx)^n}}{3dn} - \frac{b \log(F)(c+dx)^{-2n} F^{a+b(c+dx)^n}}{6dn}$$

[Out]  $-F^{(a + b*(c + d*x)^n)/(3*d*n*(c + d*x)^{(3*n))} - (b*F^{(a + b*(c + d*x)^n)*\operatorname{Log}[F]})/(6*d*n*(c + d*x)^{(2*n))} - (b^2*F^{(a + b*(c + d*x)^n)*\operatorname{Log}[F]^2})/(6*d*n*(c + d*x)^n) + (b^3*F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]])*\operatorname{Log}[F]^3)/(6*d*n)$

**Rubi [A]** time = 0.164464, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2215, 2210}

$$\frac{b^3 F^a \log^3(F) \operatorname{Ei}(b(c+dx)^n \log(F))}{6dn} - \frac{b^2 \log^2(F)(c+dx)^{-n} F^{a+b(c+dx)^n}}{6dn} - \frac{(c+dx)^{-3n} F^{a+b(c+dx)^n}}{3dn} - \frac{b \log(F)(c+dx)^{-2n} F^{a+b(c+dx)^n}}{6dn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*(c + d*x)^n)*(c + d*x)^{-1 - 3*n}], x]$

[Out]  $-F^{(a + b*(c + d*x)^n)/(3*d*n*(c + d*x)^{(3*n))} - (b*F^{(a + b*(c + d*x)^n)*\operatorname{Log}[F]})/(6*d*n*(c + d*x)^{(2*n))} - (b^2*F^{(a + b*(c + d*x)^n)*\operatorname{Log}[F]^2})/(6*d*n*(c + d*x)^n) + (b^3*F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]])*\operatorname{Log}[F]^3)/(6*d*n)$

#### Rule 2215

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*F^{(a + b*(c + d*x)^n)}/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{\operatorname{Simplify}[m + n]}*F^{(a + b*(c + d*x)^n)}, x], x] /;$  FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2\*Simplify[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumSimplerQ[m, n]

#### Rule 2210

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})}/((e_.) + (f_.)*(x_.))], x\_Symbol] :> \operatorname{Simp}[F^a*\operatorname{ExpIntegralEi}[b*(c + d*x)^n*\operatorname{Log}[F]]/(f*n), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^n} (c+dx)^{-1-3n} dx &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-3n}}{3dn} + \frac{1}{3}(b \log(F)) \int F^{a+b(c+dx)^n} (c+dx)^{-1-2n} dx \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-3n}}{3dn} - \frac{b F^{a+b(c+dx)^n} (c+dx)^{-2n} \log(F)}{6dn} + \frac{1}{6}(b^2 \log^2(F)) \int F^{a+b(c+dx)^n} (c+dx)^{-1-2n} dx \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-3n}}{3dn} - \frac{b F^{a+b(c+dx)^n} (c+dx)^{-2n} \log(F)}{6dn} - \frac{b^2 F^{a+b(c+dx)^n} (c+dx)^{-n} \log^2(F)}{6dn} \\ &= -\frac{F^{a+b(c+dx)^n} (c+dx)^{-3n}}{3dn} - \frac{b F^{a+b(c+dx)^n} (c+dx)^{-2n} \log(F)}{6dn} - \frac{b^2 F^{a+b(c+dx)^n} (c+dx)^{-n} \log^2(F)}{6dn} \end{aligned}$$

**Mathematica [A]** time = 0.0068938, size = 31, normalized size = 0.22

$$\frac{b^3 F^a \log^3(F) \Gamma(-3, -b \log(F)(c + dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 3\*n), x]

[Out] (b^3 F^a Gamma[-3, -(b\*(c + d\*x)^n\*Log[F])]\*Log[F]^3)/(d\*n)

**Maple [A]** time = 0.089, size = 137, normalized size = 1.

$$\frac{F^{b(dx+c)^n} F^a}{3 dn ((dx+c)^n)^3} - \frac{b \ln(F) F^{b(dx+c)^n} F^a}{6 dn ((dx+c)^n)^2} - \frac{(\ln(F))^2 b^2 F^{b(dx+c)^n} F^a}{6 dn (dx+c)^n} - \frac{(\ln(F))^3 b^3 F^a \text{Ei}(1, -b(dx+c)^n \ln(F))}{6 dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-3\*n), x)

[Out] -1/3/n/d\*F^(b\*(d\*x+c)^n)\*F^a/((d\*x+c)^n)^3-1/6/n/d\*ln(F)\*b\*F^(b\*(d\*x+c)^n)\*F^a/((d\*x+c)^n)^2-1/6/n/d\*ln(F)^2\*b^2\*F^(b\*(d\*x+c)^n)\*F^a/((d\*x+c)^n)-1/6/n/d\*ln(F)^3\*b^3\*F^a\*Ei(1, -b\*(d\*x+c)^n\*ln(F))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-3n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-3\*n), x, algorithm="maxima")

[Out] integrate((d\*x + c)^(-3\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

**Fricas [A]** time = 1.55027, size = 247, normalized size = 1.78

$$\frac{(dx + c)^{3n} F^a b^3 \text{Ei}((dx + c)^n b \log(F)) \log(F)^3 - ((dx + c)^{2n} b^2 \log(F)^2 + (dx + c)^n b \log(F) + 2) e^{(dx+c)^n b \log(F) + a \log(F)}}{6 (dx + c)^{3n} dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-3\*n), x, algorithm="fricas")

[Out] 1/6\*((d\*x + c)^(3\*n)\*F^a\*b^3\*Ei((d\*x + c)^n\*b\*log(F))\*log(F)^3 - ((d\*x + c)^(2\*n)\*b^2\*log(F)^2 + (d\*x + c)^n\*b\*log(F) + 2)\*e^((d\*x + c)^n\*b\*log(F) + a\*log(F)))/((d\*x + c)^(3\*n)\*d\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*n)\*(d\*x+c)\*\*(-1-3\*n), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-3n-1} F^{(dx+c)^{n*b+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^n)\*(d\*x+c)^(-1-3\*n), x, algorithm="giac")

[Out] integrate((d\*x + c)^(-3\*n - 1)\*F^((d\*x + c)^n\*b + a), x)

$$3.378 \quad \int F^{a+b(c+dx)^n} (c+dx)^{-1-4n} dx$$

**Optimal.** Leaf size=32

$$\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^n)}{dn}$$

[Out]  $-\left((b^4 F^a \Gamma(-4, -(b*(c+d*x)^n * \text{Log}[F])) * \text{Log}[F]^4) / (d*n)\right)$

**Rubi [A]** time = 0.0427772, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2218}

$$\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 4\*n), x]

[Out]  $-\left((b^4 F^a \Gamma(-4, -(b*(c+d*x)^n * \text{Log}[F])) * \text{Log}[F]^4) / (d*n)\right)$

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])^(m + 1)/n)]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1-4n} dx = -\frac{b^4 F^a \Gamma(-4, -b(c+dx)^n \log(F)) \log^4(F)}{dn}$$

**Mathematica [A]** time = 0.0068897, size = 32, normalized size = 1.

$$\frac{b^4 F^a \log^4(F) \Gamma(-4, -b \log(F)(c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 4\*n), x]

[Out]  $-\left((b^4 F^a \Gamma(-4, -(b*(c+d*x)^n * \text{Log}[F])) * \text{Log}[F]^4) / (d*n)\right)$

**Maple [B]** time = 0.089, size = 175, normalized size = 5.5

$$\frac{F^{b(dx+c)^n} F^a}{4 dn ((dx+c)^n)^4} - \frac{b \ln(F) F^{b(dx+c)^n} F^a}{12 dn ((dx+c)^n)^3} - \frac{(\ln(F))^2 b^2 F^{b(dx+c)^n} F^a}{24 dn ((dx+c)^n)^2} - \frac{(\ln(F))^3 b^3 F^{b(dx+c)^n} F^a}{24 dn (dx+c)^n} - \frac{b^4 (\ln(F))^4 F^a \text{Ei}(1, -b(dx+c)^n)}{24 dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x)`

[Out] 
$$-1/4/n/d * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^4 - 1/12/n/d * \ln(F) * b * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^3 - 1/24/n/d * \ln(F)^2 * b^2 * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^2 - 1/24/n/d * \ln(F)^3 * b^3 * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n) - 1/24/n/d * \ln(F)^4 * b^4 * F^a * Ei(1, -b*(d*x+c)^n * \ln(F))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-4n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="maxima")`

[Out] `integrate((d*x + c)^(-4*n - 1)*F^((d*x + c)^n*b + a), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-4*n),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-4n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="giac")`

[Out] `integrate((d*x + c)^(-4*n - 1)*F^((d*x + c)^n*b + a), x)`



$$3.379 \quad \int F^{a+b(c+dx)^n} (c+dx)^{-1-5n} dx$$

**Optimal.** Leaf size=31

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^n)}{dn}$$

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^n\*Log[F])]\*Log[F]^5)/(d\*n)

**Rubi [A]** time = 0.0392378, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2218}

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 5\*n), x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^n\*Log[F])]\*Log[F]^5)/(d\*n)

**Rule 2218**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])^(m + 1)/n, x] / (f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1-5n} dx = \frac{b^5 F^a \Gamma(-5, -b(c+dx)^n \log(F)) \log^5(F)}{dn}$$

**Mathematica [A]** time = 0.0072041, size = 31, normalized size = 1.

$$\frac{b^5 F^a \log^5(F) \Gamma(-5, -b \log(F)(c+dx)^n)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^n)\*(c + d\*x)^(-1 - 5\*n), x]

[Out] (b^5\*F^a\*Gamma[-5, -(b\*(c + d\*x)^n\*Log[F])]\*Log[F]^5)/(d\*n)

**Maple [B]** time = 0.095, size = 213, normalized size = 6.9

$$\frac{F^{b(dx+c)^n} F^a}{5 dn ((dx+c)^n)^5} - \frac{b \ln(F) F^{b(dx+c)^n} F^a}{20 dn ((dx+c)^n)^4} - \frac{(\ln(F))^2 b^2 F^{b(dx+c)^n} F^a}{60 dn ((dx+c)^n)^3} - \frac{(\ln(F))^3 b^3 F^{b(dx+c)^n} F^a}{120 dn ((dx+c)^n)^2} - \frac{b^4 (\ln(F))^4 F^{b(dx+c)^n} F^a}{120 dn (dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x)`

[Out] 
$$-1/5/n/d * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^5 - 1/20/n/d * \ln(F) * b * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^4 - 1/60/n/d * \ln(F)^2 * b^2 * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^3 - 1/120/n/d * \ln(F)^3 * b^3 * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n)^2 - 1/120/n/d * \ln(F)^4 * b^4 * F^{(b*(d*x+c)^n)} * F^a / ((d*x+c)^n) - 1/120/n/d * \ln(F)^5 * b^5 * F^a * Ei(1, -b*(d*x+c)^n * \ln(F))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-5n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="maxima")`

[Out] `integrate((d*x + c)^(-5*n - 1)*F^((d*x + c)^n*b + a), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-5*n),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{-5n-1} F^{(dx+c)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="giac")`

[Out] `integrate((d*x + c)^(-5*n - 1)*F^((d*x + c)^n*b + a), x)`

$$3.380 \quad \int F^{c(a+bx)^n} (a+bx)^{-1+\frac{n}{2}} dx$$

**Optimal.** Leaf size=47

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)^(n/2)\*Sqrt[Log[F]]])/(b\*Sqrt[c]\*n\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0651868, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2211, 2204}

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x)^n)\*(a + b\*x)^(-1 + n/2), x]

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)^(n/2)\*Sqrt[Log[F]]])/(b\*Sqrt[c]\*n\*Sqrt[Log[F]])

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int F^{c(a+bx)^n} (a+bx)^{-1+\frac{n}{2}} dx &= \frac{2 \operatorname{Subst}\left(\int F^{cx^2} dx, x, (a+bx)^{n/2}\right)}{bn} \\ &= \frac{\sqrt{\pi} \operatorname{erfi}\left(\sqrt{c} (a+bx)^{n/2} \sqrt{\log(F)}\right)}{b \sqrt{cn} \sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.0117368, size = 47, normalized size = 1.

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x)^n)\*(a + b\*x)^(-1 + n/2), x]

[Out] (Sqrt[Pi]\*Erfi[Sqrt[c]\*(a + b\*x)^(n/2)\*Sqrt[Log[F]]])/(b\*Sqrt[c]\*n\*Sqrt[Log[F]])

**Maple [A]** time = 0.177, size = 36, normalized size = 0.8

$$\frac{\sqrt{\pi}}{bn} \operatorname{Erf}\left(\sqrt{-c \ln(F)} (bx + a)^{\frac{n}{2}}\right) \frac{1}{\sqrt{-c \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a)^n)\*(b\*x+a)^(-1+1/2\*n), x)

[Out] 1/n/b\*Pi^(1/2)/(-c\*ln(F))^(1/2)\*erf((-c\*ln(F))^(1/2)\*(b\*x+a)^(1/2\*n))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{1}{2}n-1} F^{(bx+a)^n c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)^n)\*(b\*x+a)^(-1+1/2\*n), x, algorithm="maxima")

[Out] integrate((b\*x + a)^(1/2\*n - 1)\*F^((b\*x + a)^n\*c), x)

**Fricas [A]** time = 1.65197, size = 128, normalized size = 2.72

$$\frac{\sqrt{\pi} \sqrt{-c \log(F)} \operatorname{erf}\left((bx + a) \sqrt{-c \log(F)} (bx + a)^{\frac{1}{2}n-1}\right)}{bcn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)^n)\*(b\*x+a)^(-1+1/2\*n), x, algorithm="fricas")

[Out] -sqrt(pi)\*sqrt(-c\*log(F))\*erf((b\*x + a)\*sqrt(-c\*log(F))\*(b\*x + a)^(1/2\*n - 1))/(b\*c\*n\*log(F))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)^n} (a + bx)^{\frac{n}{2}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a)\*\*n)\*(b\*x+a)\*\*(-1+1/2\*n), x)

[Out] Integral(F\*\*(c\*(a + b\*x)\*\*n)\*(a + b\*x)\*\*(n/2 - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^{\frac{1}{2}n-1} F^{(bx+a)^n c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/2*n - 1)*F^((b*x + a)^n*c), x)
```

$$3.381 \quad \int F^{-c(a+bx)^n} (a+bx)^{-1+\frac{n}{2}} dx$$

**Optimal.** Leaf size=47

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

[Out] (Sqrt[Pi]\*Erf[Sqrt[c]\*(a + b\*x)^(n/2)\*Sqrt[Log[F]]])/(b\*Sqrt[c]\*n\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0484519, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2211, 2205}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^(-1 + n/2)/F^(c\*(a + b\*x)^n), x]

[Out] (Sqrt[Pi]\*Erf[Sqrt[c]\*(a + b\*x)^(n/2)\*Sqrt[Log[F]]])/(b\*Sqrt[c]\*n\*Sqrt[Log[F]])

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int F^{-c(a+bx)^n} (a+bx)^{-1+\frac{n}{2}} dx &= \frac{2 \operatorname{Subst}\left(\int F^{-cx^2} dx, x, (a+bx)^{n/2}\right)}{bn} \\ &= \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{c} (a+bx)^{n/2} \sqrt{\log(F)}\right)}{b \sqrt{cn} \sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.00731, size = 47, normalized size = 1.

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{c} \sqrt{\log(F)} (a+bx)^{n/2}\right)}{b \sqrt{cn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^(-1 + n/2)/F^(c\*(a + b\*x)^n), x]

[Out]  $(\sqrt{\pi} \operatorname{Erf}[\sqrt{c} (a + bx)^{n/2} \sqrt{\log(F)}]) / (b \sqrt{c} n \sqrt{\log(F)})$

**Maple [A]** time = 0.079, size = 34, normalized size = 0.7

$$\frac{\sqrt{\pi}}{bn} \operatorname{Erf}\left(\sqrt{c \ln(F)} (bx + a)^{\frac{n}{2}}\right) \frac{1}{\sqrt{c \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x)`

[Out]  $1/n/b\pi^{1/2}/(c\ln(F))^{1/2}*\operatorname{erf}((c\ln(F))^{1/2}*(b*x+a)^{1/2*n})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)^{\frac{1}{2}n-1}}{F^{(bx+a)^n c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x, algorithm="maxima")`

[Out] `integrate((b*x + a)^(1/2*n - 1)/F^((b*x + a)^n*c), x)`

**Fricas [A]** time = 1.60274, size = 124, normalized size = 2.64

$$\frac{\sqrt{\pi} \sqrt{c \log(F)} \operatorname{erf}\left((bx + a) \sqrt{c \log(F)} (bx + a)^{\frac{1}{2}n-1}\right)}{bcn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x, algorithm="fricas")`

[Out]  $\sqrt{\pi} \sqrt{c \log(F)} \operatorname{erf}((b*x + a) \sqrt{c \log(F)} (b*x + a)^{1/2*n - 1}) / (b*c*n*\log(F))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**(-1+1/2*n)/(F**(c*(b*x+a)**n)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)^{\frac{1}{2}n-1}}{F^{(bx+a)^n c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^(-1+1/2\*n)/(F^(c\*(b\*x+a)^n)),x, algorithm="giac")

[Out] integrate((b\*x + a)^(1/2\*n - 1)/F^((b\*x + a)^n\*c), x)



### 3.382 $\int F^{a+b(c+dx)^2} (e + fx)^5 dx$

**Optimal.** Leaf size=518

$$\frac{5\sqrt{\pi}f^2F^a(de - cf)^3\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2b^{3/2}d^6\log^3(F)} + \frac{15\sqrt{\pi}f^4F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{8b^{5/2}d^6\log^5(F)} - \frac{5f^3(de - cf)^2F^{a+b}}{b^2d^6\log^2(F)}$$

```
[Out] (f^5F^(a + b*(c + d*x)^2))/(b^3*d^6*Log[F]^3) + (15*f^4*(d*e - c*f)*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(8*b^(5/2)*d^6*Log[F]^(5/2)) - (5*f^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2))/(b^2*d^6*Log[F]^2) - (15*f^4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x))/(4*b^2*d^6*Log[F]^2) - (f^5*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b^2*d^6*Log[F]^2) - (5*f^2*(d*e - c*f)^3*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*b^(3/2)*d^6*Log[F]^(3/2)) + (5*f*(d*e - c*f)^4*F^(a + b*(c + d*x)^2))/(2*b*d^6*Log[F]) + (5*f^2*(d*e - c*f)^3*F^(a + b*(c + d*x)^2)*(c + d*x))/(b*d^6*Log[F]) + (5*f^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b*d^6*Log[F]) + (5*f^4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x)^3)/(2*b*d^6*Log[F]) + (f^5*F^(a + b*(c + d*x)^2)*(c + d*x)^4)/(2*b*d^6*Log[F]) + ((d*e - c*f)^5*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^6*Sqrt[Log[F]])
```

**Rubi [A]** time = 0.943333, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2226, 2204, 2209, 2212}

$$\frac{5\sqrt{\pi}f^2F^a(de - cf)^3\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2b^{3/2}d^6\log^3(F)} + \frac{15\sqrt{\pi}f^4F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{8b^{5/2}d^6\log^5(F)} - \frac{5f^3(de - cf)^2F^{a+b}}{b^2d^6\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*(c + d*x)^2)*(e + f*x)^5, x]
```

```
[Out] (f^5*F^(a + b*(c + d*x)^2))/(b^3*d^6*Log[F]^3) + (15*f^4*(d*e - c*f)*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(8*b^(5/2)*d^6*Log[F]^(5/2)) - (5*f^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2))/(b^2*d^6*Log[F]^2) - (15*f^4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x))/(4*b^2*d^6*Log[F]^2) - (f^5*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b^2*d^6*Log[F]^2) - (5*f^2*(d*e - c*f)^3*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*b^(3/2)*d^6*Log[F]^(3/2)) + (5*f*(d*e - c*f)^4*F^(a + b*(c + d*x)^2))/(2*b*d^6*Log[F]) + (5*f^2*(d*e - c*f)^3*F^(a + b*(c + d*x)^2)*(c + d*x))/(b*d^6*Log[F]) + (5*f^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b*d^6*Log[F]) + (5*f^4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x)^3)/(2*b*d^6*Log[F]) + (f^5*F^(a + b*(c + d*x)^2)*(c + d*x)^4)/(2*b*d^6*Log[F]) + ((d*e - c*f)^5*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^6*Sqrt[Log[F]])
```

#### Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] := Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (e+fx)^5 dx &= \int \left( \frac{(de-cf)^5 F^{a+b(c+dx)^2}}{d^5} + \frac{5f(de-cf)^4 F^{a+b(c+dx)^2} (c+dx)}{d^5} + \frac{10f^2(de-cf)^3 F^{a+b(c+dx)^2} (c+dx)^2}{d^5} \right. \\ &= \frac{f^5 \int F^{a+b(c+dx)^2} (c+dx)^5 dx}{d^5} + \frac{(5f^4(de-cf)) \int F^{a+b(c+dx)^2} (c+dx)^4 dx}{d^5} + \frac{(10f^3(de-cf)^2) \int F^{a+b(c+dx)^2} (c+dx)^3 dx}{d^5} \\ &= \frac{5f(de-cf)^4 F^{a+b(c+dx)^2}}{2bd^6 \log(F)} + \frac{5f^2(de-cf)^3 F^{a+b(c+dx)^2} (c+dx)}{bd^6 \log(F)} + \frac{5f^3(de-cf)^2 F^{a+b(c+dx)^2} (c+dx)^2}{bd^6 \log(F)} \\ &= -\frac{5f^3(de-cf)^2 F^{a+b(c+dx)^2}}{b^2 d^6 \log^2(F)} - \frac{15f^4(de-cf) F^{a+b(c+dx)^2} (c+dx)}{4b^2 d^6 \log^2(F)} - \frac{f^5 F^{a+b(c+dx)^2} (c+dx)^2}{b^2 d^6 \log^2(F)} - \frac{5f^2}{b^2 d^6 \log^2(F)} \\ &= \frac{f^5 F^{a+b(c+dx)^2}}{b^3 d^6 \log^3(F)} + \frac{15f^4(de-cf) F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{8b^{5/2} d^6 \log^{5/2}(F)} - \frac{5f^3(de-cf)^2 F^{a+b(c+dx)^2}}{b^2 d^6 \log^2(F)} - \frac{5f^2}{b^2 d^6 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.606487, size = 412, normalized size = 0.8

$$F^a \left( 4\sqrt{\pi} b^{3/2} \log^{3/2}(F) (de-cf)^5 \operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx)) + \frac{15f^4(cf-de)(2\sqrt{b}\sqrt{\log(F)}(c+dx)F^{b(c+dx)^2} - \sqrt{\pi} \operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx)))}{\sqrt{b}\sqrt{\log(F)}} + 20\sqrt{\pi} b^{3/2} \log^{3/2}(F) (de-cf)^5 \operatorname{Erfi}(\sqrt{b}\sqrt{\log(F)}(c+dx)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*(c + d*x)^2)*(e + f*x)^5,x]
```

```
[Out] (F^a*(-40*f^3*(d*e - c*f)^2*F^(b*(c + d*x)^2) + (15*f^4*(-(d*e) + c*f)*(-(Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]) + 2*Sqrt[b]*F^(b*(c + d*x)^2)*(c + d*x)*Sqrt[Log[F]]))/(Sqrt[b]*Sqrt[Log[F]]) + 20*Sqrt[b]*f^2*(-(d*e) + c*f)^3*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]] + 20*b*f*(d*e - c*f)^4*F^(b*(c + d*x)^2)*Log[F] + 40*b*f^2*(d*e - c*f)^3*F^(b*(c + d*x)^2)*(c + d*x)*Log[F] + 40*b*f^3*(d*e - c*f)^2*F^(b*(c + d*x)^2)*(c + d*x)^2*Log[F] + 20*b*f^4*(d*e - c*f)*F^(b*(c + d*x)^2)*(c + d*x)^3*Log[F] + 4*b*f^5*F^(b*(c + d*x)^2)*(c + d*x)^4*Log[F] + 4*b^(3/2)*(d*e - c*f)^5*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Log[F]^(3/2) + (8*f^5*F^(b*(c + d*x)^2)*(1 - b*(c + d*x)^2*Log[F]))/(b*Log[F]))/(8*b^2*d^6*Log[F]^2)
```

**Maple [B]** time = 0.089, size = 1657, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{a+b(dx+c)^2})(f*x+e)^5, x$

[Out]  $\frac{1}{2}f^5c^5/d^6\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-1/2e^5\pi^{1/2}F^a/d/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})+1/2f^5/\ln(F)/b/d^2x^4F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+1/2f^5c^4/d^6/\ln(F)/b*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-5/2f^5c^3/d^6/\ln(F)/b*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-9/4f^5c^2/d^6/\ln(F)^2/b^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+15/8f^5c/d^6/\ln(F)^2/b^2*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-f^5/\ln(F)^2/b^2/d^4x^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-15/8e*f^4/\ln(F)^2/b^2/d^5*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-5/2e*f^4c^4/d^5*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})+5e^2*f^3c^2/d^4/\ln(F)/b*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-15/2e^2f^3c/d^4/\ln(F)/b*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})+5e^3f^2/\ln(F)/b/d^2x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-5e^3f^2c/d^3/\ln(F)/b*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5/2e^4f*c/d^2*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-5e^3f^2c^2/d^3*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-5e^2f^3/\ln(F)^2/b^2/d^4*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5/2e^3f^2/\ln(F)/b/d^3*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})+5/2e^4f/\ln(F)/b/d^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5e^2f^3c^3/d^4*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})-1/2f^5c/d^3/\ln(F)/b*x^3*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+1/2f^5c^2/d^4/\ln(F)/b*x^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-1/2f^5c^3/d^5/\ln(F)/b*x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+7/4f^5c/d^5/\ln(F)^2/b^2*x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5/2e*f^4/\ln(F)/b/d^2*x^3*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-5/2e*f^4c^3/d^5/\ln(F)/b*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+15/2e*f^4c^2/d^5/\ln(F)/b*\pi^{1/2}F^a/(-b\ln(F))^{1/2}\operatorname{erf}(-d*(-b\ln(F))^{1/2}x+b*c*\ln(F)/(-b\ln(F))^{1/2})+25/4e*f^4c/d^5/\ln(F)^2/b^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-15/4e*f^4/\ln(F)^2/b^2/d^4*x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5e^2f^3/\ln(F)/b/d^2*x^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-5/2e*f^4c/d^3/\ln(F)/b*x^2*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+5/2e*f^4c^2/d^4/\ln(F)/b*x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a-5e^2f^3c/d^3/\ln(F)/b*x*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a+f^5/\ln(F)^3/b^3/d^6*F^{(b*d^2x^2)*F^{(2*b*c*d*x)*F^{(c^2*b)*F^a}}$

**Maxima [B]** time = 1.90643, size = 2030, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(F^{a+b(dx+c)^2})(f*x+e)^5, x, \text{algorithm}="maxima")$

[Out]  $-5/2*(\sqrt{\pi})*(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*1$

$\log(F)/(b*d^2)) - F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*\log(F)/(b*d^2*\log(F))^{(3/2)}}*F^a*e^{4*f/\sqrt{b*d^2*\log(F)}} + 5*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^3/((b*d^2*\log(F))^{(5/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{(5/2)}} - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{(5/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}))*F^a*e^{3*f^2/\sqrt{b*d^2*\log(F)}} - 5*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^4/((b*d^2*\log(F))^{(7/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 3*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*d^4*\log(F)^3/(b*d^2*\log(F))^{(7/2)}} - 3*(b*d^2*x + b*c*d)^3*b*c*d*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/((b*d^2*\log(F))^{(7/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + b^2*d^4*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^2/(b*d^2*\log(F))^{(7/2)})*F^a*e^{2*f^3/\sqrt{b*d^2*\log(F)}} + 5/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 4*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*d^5*\log(F)^4/(b*d^2*\log(F))^{(9/2)}} - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 4*b^3*c*d^5*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(9/2)}} - (b*d^2*x + b*c*d)^5*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^5/((b*d^2*\log(F))^{(9/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}))*F^a*e^{f^4/\sqrt{b*d^2*\log(F)}} - 1/2*(\sqrt{\pi}*(b*d^2*x + b*c*d)*b^5*c^5*d^5*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)})) - 1)*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 5*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^5*c^4*d^6*\log(F)^5/(b*d^2*\log(F))^{(11/2)}} - 10*(b*d^2*x + b*c*d)^3*b^3*c^3*d^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(3/2)}) + 10*b^4*c^2*d^6*\gamma(2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^4/(b*d^2*\log(F))^{(11/2)}} - b^3*d^6*\gamma(3, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/(b*d^2*\log(F))^{(11/2)}} - 5*(b*d^2*x + b*c*d)^5*b*c*d*\gamma(5/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^6/((b*d^2*\log(F))^{(11/2)}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{(5/2)}))*F^a*f^5/\sqrt{b*d^2*\log(F)}} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*e^{5*\operatorname{erf}(\sqrt{-b*\log(F)})}*d*x - b*c*\log(F)/\sqrt{-b*\log(F)}}/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d}$

**Fricas [A]** time = 1.66688, size = 1118, normalized size = 2.16

$$\sqrt{\pi}(15 def^4 - 15 cf^5 + 4(b^2 d^5 e^5 - 5 b^2 cd^4 e^4 f + 10 b^2 c^2 d^3 e^3 f^2 - 10 b^2 c^3 d^2 e^2 f^3 + 5 b^2 c^4 d e f^4 - b^2 c^5 f^5) \log(F)^2 - 20(bd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^5,x, algorithm="fricas")

[Out]  $-1/8*(\sqrt{\pi}*(15*d*e*f^4 - 15*c*f^5 + 4*(b^2*d^5*e^5 - 5*b^2*c*d^4*e^4*f + 10*b^2*c^2*d^3*e^3*f^2 - 10*b^2*c^3*d^2*e^2*f^3 + 5*b^2*c^4*d*e*f^4 - b^2*c^5*f^5)*\log(F)^2 - 20*(b*d^3*e^3*f^2 - 3*b*c*d^2*e^2*f^3 + 3*b*c^2*d*e*f^4 - b*c^3*f^5)*\log(F))*\sqrt{-b*d^2*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*d^2*\log(F)})*(d*x + c)/d - 2*(4*d*f^5 + 2*(b^2*d^5*f^5*x^4 + 5*b^2*d^5*e^4*f - 10*b^2*c*d^4*e^3*f^2 + 10*b^2*c^2*d^3*e^2*f^3 - 5*b^2*c^3*d^2*e*f^4 + b^2*c^4*d*f^5 + (5*b^2*d^5*e*f^4 - b^2*c*d^4*f^5)*x^3 + (10*b^2*d^5*e^2*f^3 - 5*b^2*c*d^4*e*f^4 + b^2*c^2*d^3*f^5)*x^2 + (10*b^2*d^5*e^3*f^2 - 10*b^2*c*d^4*e^2*f^3 + 5*b^2*c^2*d^3*e*f^4 - b^2*c^3*d^2*f^5)*x)*\log(F)^2 - (4*b*d^3*f^5*x^2 + 20*b*d^3*e^2*f^3 - 25*b*c*d^2*e*f^4 + 9*b*c^2*d*f^5 + (15*b*d^3*e*f^4 - 7*b*c*d$

$$^2*f^5)*x)*\log(F))*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)} / (b^3*d^7*\log(F)^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(f\*x+e)\*\*5,x)

[Out] Timed out

**Giac [A]** time = 1.29744, size = 1272, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d))*e^{(a*\log(F) + 5)} / (\sqrt{-b*\log(F)}*d + 5/2*(\sqrt{\pi}*c*f*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d))*e^{(a*\log(F) + 4)} / (\sqrt{-b*\log(F)}*d) + f*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 4)} / (b*d*\log(F))) / d - 5/2*(\sqrt{\pi}*(2*b*c^2*f^2*\log(F) - f^2)*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d))*e^{(a*\log(F) + 3)} / (\sqrt{-b*\log(F)})*b*d*\log(F) - 2*(d*f^2*(x + c/d) - 2*c*f^2)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 3)} / (b*d*\log(F))) / d^2 + 5/2*(\sqrt{\pi}*(2*b*c^3*f^3*\log(F) - 3*c*f^3)*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d))*e^{(a*\log(F) + 2)} / (\sqrt{-b*\log(F)})*b*d*\log(F) + 2*(b*d^2*f^3*(x + c/d)^2*\log(F) - 3*b*c*d*f^3*(x + c/d)*\log(F) + 3*b*c^2*f^3*\log(F) - f^3)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 2)} / (b^2*d*\log(F)^2)) / d^3 - 5/8*(\sqrt{\pi}*(4*b^2*c^4*f^4*\log(F)^2 - 12*b*c^2*f^4*\log(F) + 3*f^4)*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d))*e^{(a*\log(F) + 1)} / (\sqrt{-b*\log(F)})*b^2*d*\log(F)^2) - 2*(2*b*d^3*f^4*(x + c/d)^3*\log(F) - 8*b*c*d^2*f^4*(x + c/d)^2*\log(F) + 12*b*c^2*d*f^4*(x + c/d)*\log(F) - 8*b*c^3*f^4*\log(F) - 3*d*f^4*(x + c/d) + 8*c*f^4)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 1)} / (b^2*d*\log(F)^2)) / d^4 + 1/8*(\sqrt{\pi}*(4*b^2*c^5*f^5*\log(F)^2 - 20*b*c^3*f^5*\log(F) + 15*c*f^5)*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)})*d*(x + c/d)) / (\sqrt{-b*\log(F)})*b^2*d*\log(F)^2 + 2*(2*b^2*d^4*f^5*(x + c/d)^4*\log(F)^2 - 10*b^2*c*d^3*f^5*(x + c/d)^3*\log(F)^2 + 20*b^2*c^2*d^2*f^5*(x + c/d)^2*\log(F)^2 - 20*b^2*c^3*d*f^5*(x + c/d)*\log(F)^2 + 10*b^2*c^4*f^5*\log(F)^2 - 4*b*d^2*f^5*(x + c/d)^2*\log(F) + 15*b*c*d*f^5*(x + c/d)*\log(F) - 20*b*c^2*f^5*\log(F) + 4*f^5)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F))} / (b^3*d*\log(F)^3)) / d^5 \end{aligned}$$

### 3.383 $\int F^{a+b(c+dx)^2} (e+fx)^4 dx$

**Optimal.** Leaf size=389

$$\frac{3\sqrt{\pi}f^2F^a(de-cf)^2\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2b^{3/2}d^5\log^3(F)} - \frac{2f^3(de-cf)F^{a+b(c+dx)^2}}{b^2d^5\log^2(F)} + \frac{3\sqrt{\pi}f^4F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d^5\log^5(F)} - \frac{3f^4(c+dx)^4}{4b^2d^5\log^2(F)}$$

[Out]  $(3*f^4*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(8*b^{(5/2)}*d^5*\log[F]^{(5/2)}) - (2*f^3*(d*e-c*f)*F^{(a+b*(c+d*x)^2)})/(b^2*d^5*\log[F]^2) - (3*f^4*F^{(a+b*(c+d*x)^2)}*(c+d*x))/(4*b^2*d^5*\log[F]^2) - (3*f^2*(d*e-c*f)^2*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(2*b^{(3/2)}*d^5*\log[F]^{(3/2)}) + (2*f*(d*e-c*f)^3*F^{(a+b*(c+d*x)^2)})/(b*d^5*\log[F]) + (3*f^2*(d*e-c*f)^2*F^{(a+b*(c+d*x)^2)}*(c+d*x))/(b*d^5*\log[F]) + (2*f^3*(d*e-c*f)*F^{(a+b*(c+d*x)^2)}*(c+d*x)^2)/(b*d^5*\log[F]) + (f^4*F^{(a+b*(c+d*x)^2)}*(c+d*x)^3)/(2*b*d^5*\log[F]) + ((d*e-c*f)^4*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(2*\sqrt{b}*d^5*\sqrt{\log[F]})$

**Rubi [A]** time = 0.653645, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2226, 2204, 2209, 2212}

$$\frac{3\sqrt{\pi}f^2F^a(de-cf)^2\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2b^{3/2}d^5\log^3(F)} - \frac{2f^3(de-cf)F^{a+b(c+dx)^2}}{b^2d^5\log^2(F)} + \frac{3\sqrt{\pi}f^4F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d^5\log^5(F)} - \frac{3f^4(c+dx)^4}{4b^2d^5\log^2(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a+b*(c+d*x)^2)}*(e+fx)^4, x]$

[Out]  $(3*f^4*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(8*b^{(5/2)}*d^5*\log[F]^{(5/2)}) - (2*f^3*(d*e-c*f)*F^{(a+b*(c+d*x)^2)})/(b^2*d^5*\log[F]^2) - (3*f^4*F^{(a+b*(c+d*x)^2)}*(c+d*x))/(4*b^2*d^5*\log[F]^2) - (3*f^2*(d*e-c*f)^2*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(2*b^{(3/2)}*d^5*\log[F]^{(3/2)}) + (2*f*(d*e-c*f)^3*F^{(a+b*(c+d*x)^2)})/(b*d^5*\log[F]) + (3*f^2*(d*e-c*f)^2*F^{(a+b*(c+d*x)^2)}*(c+d*x))/(b*d^5*\log[F]) + (2*f^3*(d*e-c*f)*F^{(a+b*(c+d*x)^2)}*(c+d*x)^2)/(b*d^5*\log[F]) + (f^4*F^{(a+b*(c+d*x)^2)}*(c+d*x)^3)/(2*b*d^5*\log[F]) + ((d*e-c*f)^4*F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*(c+d*x)*\sqrt{\log[F]}])/(2*\sqrt{b}*d^5*\sqrt{\log[F]})$

#### Rule 2226

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*(u_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandLinearProduct}[F^{(a+b*(c+d*x)^n)}, u, c, d, x], x] /;$  FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\sqrt{\pi}*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\log[F], 2]])/(2*d*\operatorname{Rt}[b*\log[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2209

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(e+fx)^n*F^{(a+b*(c+d*x)^n)}/(b*f^n*(c+d*x)^n*\log[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n-1] && EqQ

[d\*e - c\*f, 0]

### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (e+fx)^4 dx &= \int \left( \frac{(de-cf)^4 F^{a+b(c+dx)^2}}{d^4} + \frac{4f(de-cf)^3 F^{a+b(c+dx)^2} (c+dx)}{d^4} + \frac{6f^2(de-cf)^2 F^{a+b(c+dx)^2} (c+dx)^2}{d^4} \right. \\ &+ \frac{4f^3(de-cf) F^{a+b(c+dx)^2} (c+dx)^3}{d^4} + \left. \frac{f^4 F^{a+b(c+dx)^2} (c+dx)^4}{d^4} \right) dx \\ &= \frac{2f(de-cf)^3 F^{a+b(c+dx)^2}}{bd^5 \log(F)} + \frac{3f^2(de-cf)^2 F^{a+b(c+dx)^2} (c+dx)}{bd^5 \log(F)} + \frac{2f^3(de-cf) F^{a+b(c+dx)^2} (c+dx)^2}{bd^5 \log(F)} \\ &+ \frac{2f^3(de-cf) F^{a+b(c+dx)^2}}{b^2 d^5 \log^2(F)} - \frac{3f^4 F^{a+b(c+dx)^2} (c+dx)}{4b^2 d^5 \log^2(F)} - \frac{3f^2(de-cf)^2 F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{2b^{3/2} d^5 \log^{3/2}(F)} \\ &= \frac{3f^4 F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)})}{8b^{5/2} d^5 \log^{5/2}(F)} - \frac{2f^3(de-cf) F^{a+b(c+dx)^2}}{b^2 d^5 \log^2(F)} - \frac{3f^4 F^{a+b(c+dx)^2} (c+dx)}{4b^2 d^5 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.41881, size = 220, normalized size = 0.57

$$\frac{F^a \left( \sqrt{\pi} \left( 4b^2 \log^2(F) (de-cf)^4 - 12bf^2 \log(F) (de-cf)^2 + 3f^4 \right) \operatorname{Erfi} \left( \sqrt{b} \sqrt{\log(F)} (c+dx) \right) + 2\sqrt{b} f \sqrt{\log(F)} F^{b(c+dx)^2} \right) (2b^{5/2} \log^{5/2}(F) - \dots)}{8b^{5/2} d^5 \log^{5/2}(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(e + f\*x)^4,x]

[Out] (F^a\*(2\*sqrt[b]\*f\*F^(b\*(c + d\*x)^2)\*sqrt[Log[F]]\*(f^2\*(-8\*d\*e + 5\*c\*f - 3\*d\*f\*x) + 2\*b\*(-(c^3\*f^3) + c^2\*d\*f^2\*(4\*e + f\*x) - c\*d^2\*f\*(6\*e^2 + 4\*e\*f\*x + f^2\*x^2) + d^3\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3))\*Log[F]) + sqrt[Pi]\*Erfi[sqrt[b]\*(c + d\*x)\*sqrt[Log[F]]]\*(3\*f^4 - 12\*b\*f^2\*(d\*e - c\*f)^2\*Log[F] + 4\*b^2\*(d\*e - c\*f)^4\*Log[F]^2))/(8\*b^(5/2)\*d^5\*Log[F]^(5/2))

**Maple [B]** time = 0.075, size = 1063, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^4,x)

[Out] -1/2\*e^4\*Pi^(1/2)\*F^a/d/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+1/2\*f^4/ln(F)/b/d^2\*x^3\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(

$$\begin{aligned}
& c^2 b) * F^{a-1/2} f^4 c / d^3 / \ln(F) / b x^2 F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} * \\
& F^{a+1/2} f^4 c^2 / d^4 / \ln(F) / b x F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-1/2} \\
& f^4 c^3 / d^5 / \ln(F) / b F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-1/2} f^4 c^4 / \\
& d^5 \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& + 3/2 f^4 c^2 / d^5 / \ln(F) / b \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& + 5/4 f^4 c / d^5 / \ln(F)^2 / b^2 F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-3/4} f^4 / \ln(F)^2 / b^2 / d^4 x F^{(b d^2 x^2)} \\
& F^{(2 b c d x)} F^{(c^2 b)} F^{a-3/8} f^4 / \ln(F)^2 / b^2 / d^5 \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& + 3 e^2 f^2 / \ln(F) / b / d^2 x F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-3} e^2 f^2 c / d^3 / \ln(F) / b F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-3} e^2 f^2 c^2 / d^3 \text{Pi}^{(1/2)} F^a \\
& / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) + 3/2 e^2 f^2 / \ln(F) / b / d^3 \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& + 2 e^3 f / \ln(F) / b / d^2 F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a+2} e^3 f c / d^2 \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& + 2 f^3 e / \ln(F) / b / d^2 x^2 F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a-2} f^3 e c / d^3 / \ln(F) / b x F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a+2} f^3 e c^2 / d^4 / \ln(F) / b F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^{a+2} f^3 e c^3 / d^4 \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) \\
& - 3 f^3 e c / d^4 / \ln(F) / b \text{Pi}^{(1/2)} F^a / (-b \ln(F))^{(1/2)} \text{erf}(-d * (-b \ln(F))^{(1/2)} x + b c \ln(F) / (-b \ln(F))^{(1/2)}) - 2 f^3 e / \ln(F)^2 / b^2 / d^4 F^{(b d^2 x^2)} F^{(2 b c d x)} F^{(c^2 b)} F^a
\end{aligned}$$

**Maxima [B]** time = 1.71729, size = 1463, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] -2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*d*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^2/((b*d^2*log(F))^(3/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*log(F)/(b*d^2*log(F))^(3/2))*F^a*e^3*f/sqrt(b*d^2*log(F)) + 3*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*d^2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*d^2*log(F))^(5/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*log(F)^2/(b*d^2*log(F))^(5/2) - (b*d^2*x + b*c*d)^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*d^2*log(F))^(5/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2))*F^a*e^2*f^2/sqrt(b*d^2*log(F)) - 2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*d^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*d^2*log(F))^(7/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*d^4*log(F)^3/(b*d^2*log(F))^(7/2) - 3*(b*d^2*x + b*c*d)^3*b*c*d*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*d^2*log(F))^(7/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*d^4*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^2/(b*d^2*log(F))^(7/2))*F^a*e*f^3/sqrt(b*d^2*log(F)) + 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*d^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^5/((b*d^2*log(F))^(9/2)*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*d^5*log(F)^4/(b*d^2*log(F))^(9/2) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*d^2*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + 4*b^3*c*d^5*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/(b*d^2*log(F))^(9/2) - (b*d^2*x + b*c*d)^5*gamma(5/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^5/((b*d^2*log(F))^(9/2))*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(5/2))*F^a*f^4/sqrt(b*d^2*log(F)) + 1/2*sqrt(pi)*F^(b*c^2 + a)*e^4*erf(sqrt(-b*log(F))*d*x - b*c*log(F)
```



$/\sqrt{-b\log(F)})/(\sqrt{-b\log(F)})*F^{(b*c^2)*d}$

**Fricas [A]** time = 1.6664, size = 775, normalized size = 1.99

$$\frac{\sqrt{\pi}(3f^4 + 4(b^2d^4e^4 - 4b^2cd^3e^3f + 6b^2c^2d^2e^2f^2 - 4b^2c^3def^3 + b^2c^4f^4)\log(F)^2 - 12(bd^2e^2f^2 - 2bcdef^3 + bc^2f^4))}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^4,x, algorithm="fricas")

[Out]  $-1/8*(\sqrt{\pi}*(3f^4 + 4*(b^2d^4e^4 - 4b^2c*d^3e^3f + 6b^2c^2d^2e^2f^2 - 4b^2c^3d*e*f^3 + b^2c^4f^4)*\log(F)^2 - 12*(b*d^2e^2f^2 - 2*b*c*d*e*f^3 + b*c^2f^4)*\log(F))*\sqrt{-b*d^2*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*d^2*\log(F)}*(d*x + c)/d) - 2*(2*(b^2*d^4*f^4*x^3 + 4*b^2*d^4*e^3*f - 6*b^2*c*d^3*e^2*f^2 + 4*b^2*c^2*d^2*e*f^3 - b^2*c^3*d*f^4 + (4*b^2*d^4*e^3*f - b^2*c*d^3*f^4)*x^2 + (6*b^2*d^4*e^2*f^2 - 4*b^2*c*d^3*e*f^3 + b^2*c^2*d^2*f^4)*x)*\log(F)^2 - (3*b*d^2*f^4*x + 8*b*d^2*e*f^3 - 5*b*c*d*f^4)*\log(F))*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)}/(b^3*d^6*\log(F)^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (e + fx)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(f\*x+e)\*\*4,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(e + f\*x)\*\*4, x)

**Giac [A]** time = 1.24738, size = 869, normalized size = 2.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^4,x, algorithm="giac")

[Out]  $-1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 4)}/(\sqrt{-b*\log(F)}*d) + 2*(\sqrt{\pi}*c*f*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 3)}/(\sqrt{-b*\log(F)}*d) + f*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 3)}/(b*d*\log(F)))/d - 3/2*(\sqrt{\pi}*(2*b*c^2*f^2*\log(F) - f^2)*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 2)}/(\sqrt{-b*\log(F)}*b*d*\log(F)) - 2*(d*f^2*(x + c/d) - 2*c*f^2)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 2)}/(b*d*\log(F)))/d^2 + (\sqrt{\pi}*(2*b*c^3*f^3*\log(F) - 3*c*f^3)*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 1)}/(\sqrt{-b*\log(F)}*b*d*\log(F)) + 2*(b*d^2*f^3*(x + c/d)^2*\log(F) - 3*b*c*d*f^3*(x + c/d)*\log(F) + 3*b*c^2*f^3*\log(F) - f^3)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 1)}/(b^2*d*\log(F)^2))/d^3 - 1/8*(\sqrt{\pi}*(4*b^2*c^4*f^4*\log(F)^2 - 12*b*c^2*f^4*\log(F) + 3*f^4)*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))/(\sqrt{-b*\log(F)}*b^2*d*\log(F)^2) - 2*(2*b*d^3*f^4*($

$$\frac{(x + c/d)^3 \log(F) - 8*b*c*d^2*f^4*(x + c/d)^2*\log(F) + 12*b*c^2*d*f^4*(x + c/d)*\log(F) - 8*b*c^3*f^4*\log(F) - 3*d*f^4*(x + c/d) + 8*c*f^4)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F))/(b^2*d*\log(F)^2)}}{d^4}$$

### 3.384 $\int F^{a+b(c+dx)^2} (e + fx)^3 dx$

**Optimal.** Leaf size=258

$$\frac{3\sqrt{\pi}f^2F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{4b^{3/2}d^4\log^3(F)} - \frac{f^3F^{a+b(c+dx)^2}}{2b^2d^4\log^2(F)} + \frac{\sqrt{\pi}F^a(de - cf)^3\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2\sqrt{b}d^4\sqrt{\log(F)}} + \frac{3f^2(c + dx)^3}{2d^4}$$

[Out]  $-(f^3F^{a+b(c+dx)^2})/(2b^2d^4\log[F]^2) - (3f^2(de - cf)F^a\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}])/(4b^{3/2}d^4\log[F]^{3/2}) + (3f^2(de - cf)^2F^{a+b(c+dx)^2})/(2b^2d^4\log[F]) + (3f^2(de - cf)F^{a+b(c+dx)^2}(c+dx))/(2b^2d^4\log[F]) + (f^3F^{a+b(c+dx)^2}(c+dx)^2)/(2b^2d^4\log[F]) + ((de - cf)^3F^a\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}])/(2\sqrt{b}d^4\sqrt{\log[F]})$

**Rubi [A]** time = 0.435618, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2226, 2204, 2209, 2212}

$$\frac{3\sqrt{\pi}f^2F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{4b^{3/2}d^4\log^3(F)} - \frac{f^3F^{a+b(c+dx)^2}}{2b^2d^4\log^2(F)} + \frac{\sqrt{\pi}F^a(de - cf)^3\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2\sqrt{b}d^4\sqrt{\log(F)}} + \frac{3f^2(c + dx)^3}{2d^4}$$

Antiderivative was successfully verified.

[In]  $\int F^{a+b(c+dx)^2}(e+fx)^3 dx$

[Out]  $-(f^3F^{a+b(c+dx)^2})/(2b^2d^4\log[F]^2) - (3f^2(de - cf)F^a\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}])/(4b^{3/2}d^4\log[F]^{3/2}) + (3f^2(de - cf)^2F^{a+b(c+dx)^2})/(2b^2d^4\log[F]) + (3f^2(de - cf)F^{a+b(c+dx)^2}(c+dx))/(2b^2d^4\log[F]) + (f^3F^{a+b(c+dx)^2}(c+dx)^2)/(2b^2d^4\log[F]) + ((de - cf)^3F^a\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}(c+dx)\sqrt{\log[F]}])/(2\sqrt{b}d^4\sqrt{\log[F]})$

#### Rule 2226

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)}))*(u_), x\_Symbol] := \operatorname{Int}[\operatorname{ExpandLinearProduct}[F^{a+b(c+dx)^n}, u, c, d, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, n\}, x] \&\& \operatorname{PolynomialQ}[u, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)}))*(e_. + (f_.)*(x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(F^a\sqrt{\pi}\operatorname{Erfi}[(c+dx)\operatorname{Rt}[b\log[F], 2]])/(2d\operatorname{Rt}[b\log[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rule 2209

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)}))*(e_. + (f_.)*(x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(e+fx)^n F^{a+b(c+dx)^n} / (b^n f^n (c+dx)^n \log[F]), x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x] \&\& \operatorname{EqQ}[m, n-1] \&\& \operatorname{EqQ}[d*e - c*f, 0]$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_)}))*(c_. + (d_.)*(x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(c+dx)^{m-n+1} F^{a+b(c+dx)^n} / (b^n d^n)$

Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2}(e+fx)^3 dx &= \int \left( \frac{(de-cf)^3 F^{a+b(c+dx)^2}}{d^3} + \frac{3f(de-cf)^2 F^{a+b(c+dx)^2}(c+dx)}{d^3} + \frac{3f^2(de-cf) F^{a+b(c+dx)^2}(c+dx)}{d^3} \right. \\ &= \frac{f^3 \int F^{a+b(c+dx)^2}(c+dx)^3 dx}{d^3} + \frac{(3f^2(de-cf)) \int F^{a+b(c+dx)^2}(c+dx)^2 dx}{d^3} + \frac{(3f(de-cf)^2) \int F^{a+b(c+dx)^2}(c+dx) dx}{d^3} \\ &= \frac{3f(de-cf)^2 F^{a+b(c+dx)^2}}{2bd^4 \log(F)} + \frac{3f^2(de-cf) F^{a+b(c+dx)^2}(c+dx)}{2bd^4 \log(F)} + \frac{f^3 F^{a+b(c+dx)^2}(c+dx)^2}{2bd^4 \log(F)} + \frac{(de-cf)^3 F^{a+b(c+dx)^2}}{2bd^4 \log(F)} \\ &= -\frac{f^3 F^{a+b(c+dx)^2}}{2b^2 d^4 \log^2(F)} - \frac{3f^2(de-cf) F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2} d^4 \log^2(F)} + \frac{3f(de-cf)^2 F^{a+b(c+dx)^2}}{2bd^4 \log(F)} + \frac{(de-cf)^3 F^{a+b(c+dx)^2}}{2bd^4 \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.226596, size = 148, normalized size = 0.57

$$\frac{F^a \left( 2f F^{b(c+dx)^2} \left( b \log(F) (c^2 f^2 - cdf(3e+fx) + d^2(3e^2 + 3efx + f^2 x^2)) - f^2 \right) + \sqrt{\pi} \sqrt{b} \sqrt{\log(F)} (de-cf) (2b \log(F)(de-cf) - f^2) \right)}{4b^2 d^4 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(e + f\*x)^3, x]

[Out] (F^a\*(Sqrt[b]\*(d\*e - c\*f)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*Sqrt[Log[F]]\*(-3\*f^2 + 2\*b\*(d\*e - c\*f)^2\*Log[F]) + 2\*f\*F^(b\*(c + d\*x)^2)\*(-f^2 + b\*(c^2\*f^2 - c\*d\*f\*(3\*e + f\*x) + d^2\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2))\*Log[F]))/(4\*b^2\*d^4\*Log[F]^2)

**Maple [B]** time = 0.063, size = 617, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^3, x)

[Out] 
$$-1/2*e^3*\Pi^{(1/2)}*F^a/d/(-b*\ln(F))^{(1/2)}*\operatorname{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})+1/2*f^3/\ln(F)/b/d^2*x^2*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a-1/2}*f^3*c/d^3/\ln(F)/b*x*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a+1/2}*f^3*c^2/d^4/\ln(F)/b*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a+1/2}*f^3*c^3/d^4*\Pi^{(1/2)}*F^a/(-b*\ln(F))^{(1/2)}*\operatorname{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})-3/4*f^3*c/d^4/\ln(F)/b*\Pi^{(1/2)}*F^a/(-b*\ln(F))^{(1/2)}*\operatorname{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})-1/2*f^3/\ln(F)^2/b^2/d^4*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a+3/2}*e*f^2/\ln(F)/b/d^2*x*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a-3/2}*e*f^2*c/d^3/\ln(F)/b*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a-3/2}*e*f^2*c^2/d^3*\Pi^{(1/2)}*F^a/(-b*\ln(F))^{(1/2)}*\operatorname{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})+3/4*e*f^2/\ln(F)/b/d^3*\Pi^{(1/2)}*F^a/(-b*\ln(F))^{(1/2)}*\operatorname{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})+3/2*e^2*f/\ln(F)/b/d^2*F^{(b*d^2*x^2)}*F^{(2*b*c*d*x)}*F^{(c^2*b)}*F^{a+3/2}*e$$

$$\frac{2* f * c / d^2 * \pi^{1/2} * F^a / (-b * \ln(F))^{1/2} * \operatorname{erf}(-d * (-b * \ln(F))^{1/2} * x + b * c * \ln(F) / (-b * \ln(F))^{1/2})}{(-b * \ln(F))^{1/2}}$$

**Maxima [B]** time = 1.53804, size = 965, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -3/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b*c*d * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) - 1) * \log(F)^2 / ((b*d^2 * \log(F))^{3/2} * \sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) \\ & - F^{(b*d^2*x + b*c*d)^2 / (b*d^2)} * b*d^2 * \log(F) / (b*d^2 * \log(F))^{3/2} * F^a * e^{2*f} / \sqrt{b*d^2 * \log(F)} + 3/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^2 * c^2 * d^2 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) - 1) * \log(F)^3 / ((b*d^2 * \log(F))^{5/2} * \sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) \\ & - 2 * F^{(b*d^2*x + b*c*d)^2 / (b*d^2)} * b^2 * c * d^3 * \log(F)^2 / (b*d^2 * \log(F))^{5/2} - (b*d^2*x + b*c*d)^3 * \gamma(3/2, -(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^3 / ((b*d^2 * \log(F))^{5/2} * (-b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{3/2}) \\ & * F^a * e^{f^2} / \sqrt{b*d^2 * \log(F)} - 1/2 * (\sqrt{\pi}) * (b*d^2*x + b*c*d) * b^3 * c^3 * d^3 * (\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) - 1) * \log(F)^4 / ((b*d^2 * \log(F))^{7/2} * \sqrt{-(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)}) \\ & - 3 * F^{(b*d^2*x + b*c*d)^2 / (b*d^2)} * b^3 * c^2 * d^4 * \log(F)^3 / (b*d^2 * \log(F))^{7/2} - 3 * (b*d^2*x + b*c*d)^3 * b * c * d * \gamma(3/2, -(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^4 / ((b*d^2 * \log(F))^{7/2} * (-b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2))^{3/2}) \\ & + b^2 * d^4 * \gamma(2, -(b*d^2*x + b*c*d)^2 * \log(F) / (b*d^2)) * \log(F)^2 / (b*d^2 * \log(F))^{7/2} * F^a * f^3 / \sqrt{b*d^2 * \log(F)} + 1/2 * \sqrt{\pi} * F^{(b*c^2 + a)} * e^3 * \operatorname{erf}(\sqrt{-b * \log(F)}) * d * x - b * c * \log(F) / \sqrt{-b * \log(F)}) / (\sqrt{-b * \log(F)}) * F^{(b*c^2) * d} \end{aligned}$$

**Fricas [A]** time = 1.55833, size = 467, normalized size = 1.81

$$\frac{\sqrt{\pi} (3 d e f^2 - 3 c f^3 - 2 (b d^3 e^3 - 3 b c d^2 e^2 f + 3 b c^2 d e f^2 - b c^3 f^3) \log(F)) \sqrt{-b d^2 \log(F)} F^a \operatorname{erf}\left(\frac{\sqrt{-b d^2 \log(F)} (d x + c)}{d}\right) - 2 (d f^3 - (b d^3 f^3 x^2 + 3 b d^3 e^2 f - 3 b^2 c d^2 e f^2 + b^2 c^2 d f^3 + (3 b d^3 e f^2 - b^2 c d^2 f^3) x) \log(F)) F^{(b d^2 x^2 + 2 b c d x + b c^2 + a)}}{4 b^2 d^5 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/4 * (\sqrt{\pi}) * (3*d*e*f^2 - 3*c*f^3 - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \log(F)) * \sqrt{-b*d^2 * \log(F)} * F^a * \operatorname{erf}(\sqrt{-b*d^2 * \log(F)}) * (d*x + c) / d \\ & - 2 * (d*f^3 - (b*d^3*f^3*x^2 + 3*b*d^3*e^2*f - 3*b*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2*f^3)*x) * \log(F)) * F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)} / (b^2*d^5 * \log(F)^2) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (e + fx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(f\*x+e)\*\*3,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(e + f\*x)\*\*3, x)

**Giac [A]** time = 1.30584, size = 575, normalized size = 2.23

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right) e^{(a \log(F)+3)}}{2 \sqrt{-b \log(F)} d} + \frac{3 \left( \frac{\sqrt{\pi} c f \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right) e^{(a \log(F)+2)}}{\sqrt{-b \log(F)} d} + \frac{f e^{(b d^2 x^2 \log(F)+2 b c d x \log(F)+b c^2 \log(F)+a \log(F)+2)}}{b d \log(F)} \right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 3)}/(\sqrt{-b*\log(F)}*d) + 3/2*(\sqrt{\pi}*c*f*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 2)}/(\sqrt{-b*\log(F)}*d) + f*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 2)/(b*d*\log(F))})/d - 3/4*(\sqrt{\pi}*(2*b*c^2*f^2*\log(F) - f^2)*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 1)}/(\sqrt{-b*\log(F)}*b*d*\log(F)) - 2*(d*f^2*(x + c/d) - 2*c*f^2)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F) + 1)/(b*d*\log(F))})/d^2 + 1/4*(\sqrt{\pi}*(2*b*c^3*f^3*\log(F) - 3*c*f^3)*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))/(\sqrt{-b*\log(F)}*b*d*\log(F)) + 2*(b*d^2*f^3*(x + c/d)^2*\log(F) - 3*b*c*d*f^3*(x + c/d)*\log(F) + 3*b*c^2*f^3*\log(F) - f^3)*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F))}/(b^2*d*\log(F)^2))/d^3 \end{aligned}$$

### 3.385 $\int F^{a+b(c+dx)^2} (e + fx)^2 dx$

**Optimal.** Leaf size=170

$$-\frac{\sqrt{\pi}f^2F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d^3\log^{\frac{3}{2}}(F)} + \frac{\sqrt{\pi}F^a(de-cf)^2\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^3\sqrt{\log(F)}} + \frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3\log(F)} + \frac{f^2(c+dx)^2}{2bd^3}$$

[Out]  $-(f^2F^a\sqrt{\pi}\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(4*b^{(3/2)}*d^3*\operatorname{Log}[F]^{(3/2)}) + (f*(d*e - c*f)*F^{(a+b*(c+d*x)^2)})/(b*d^3*\operatorname{Log}[F]) + (f^2F^{(a+b*(c+d*x)^2)}*(c+d*x))/(2*b*d^3*\operatorname{Log}[F]) + ((d*e - c*f)^2F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(2*\operatorname{Sqrt}[b]*d^3*\operatorname{Sqrt}[\operatorname{Log}[F]])$

**Rubi [A]** time = 0.314288, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2226, 2204, 2209, 2212}

$$-\frac{\sqrt{\pi}f^2F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d^3\log^{\frac{3}{2}}(F)} + \frac{\sqrt{\pi}F^a(de-cf)^2\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^3\sqrt{\log(F)}} + \frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3\log(F)} + \frac{f^2(c+dx)^2}{2bd^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a+b*(c+d*x)^2)}*(e+f*x)^2,x]$

[Out]  $-(f^2F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(4*b^{(3/2)}*d^3*\operatorname{Log}[F]^{(3/2)}) + (f*(d*e - c*f)*F^{(a+b*(c+d*x)^2)})/(b*d^3*\operatorname{Log}[F]) + (f^2F^{(a+b*(c+d*x)^2)}*(c+d*x))/(2*b*d^3*\operatorname{Log}[F]) + ((d*e - c*f)^2F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*(c+d*x)*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(2*\operatorname{Sqrt}[b]*d^3*\operatorname{Sqrt}[\operatorname{Log}[F]])$

#### Rule 2226

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*(u_), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandLinearProduct}[F^{(a+b*(c+d*x)^n)}, u, c, d, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, n\}, x] \&\& \operatorname{PolynomialQ}[u, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rule 2209

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((e_.) + (f_.)*(x_.))^m], x\_Symbol] \rightarrow \operatorname{Simp}[(e+f*x)^nF^{(a+b*(c+d*x)^n)}/(b*f*n*(c+d*x)^n*\operatorname{Log}[F]), x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, n\}, x] \&\& \operatorname{EqQ}[m, n-1] \&\& \operatorname{EqQ}[d*e - c*f, 0]$

#### Rule 2212

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^n)}*((c_.) + (d_.)*(x_.))^m], x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^{(m-n+1)}F^{(a+b*(c+d*x)^n)}/(b*d*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(m-n+1)/(b*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-n)}F^{(a+b*(c+d*x)^n)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{IntegerQ}[(2*(m+1))/n] \&\& \operatorname{LtQ}[0, (m+1)/n, 5] \&\& \operatorname{IntegerQ}[n] \&\& (\operatorname{LtQ}[0, n, m+1] \mid \mid \operatorname{LtQ}[m, n, 0])$

Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)^2}(e+fx)^2 dx &= \int \left( \frac{(de-cf)^2 F^{a+b(c+dx)^2}}{d^2} + \frac{2f(de-cf)F^{a+b(c+dx)^2}(c+dx)}{d^2} + \frac{f^2 F^{a+b(c+dx)^2}(c+dx)^2}{d^2} \right) dx \\
&= \frac{f^2 \int F^{a+b(c+dx)^2}(c+dx)^2 dx}{d^2} + \frac{(2f(de-cf)) \int F^{a+b(c+dx)^2}(c+dx) dx}{d^2} + \frac{(de-cf)^2 \int F^{a+b(c+dx)^2} dx}{d^2} \\
&= \frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3 \log(F)} + \frac{f^2 F^{a+b(c+dx)^2}(c+dx)}{2bd^3 \log(F)} + \frac{(de-cf)^2 F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^3 \sqrt{\log(F)}} \\
&= -\frac{f^2 F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d^3 \log^3(F)} + \frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3 \log(F)} + \frac{f^2 F^{a+b(c+dx)^2}(c+dx)}{2bd^3 \log(F)} + \frac{(de-cf)^2 F^a}{bd^3 \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.141903, size = 105, normalized size = 0.62

$$\frac{F^a \left( \sqrt{\pi} (2b \log(F)(de-cf)^2 - f^2) \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right) + 2\sqrt{b}f\sqrt{\log(F)}F^{b(c+dx)^2}(-cf+2de+dfx) \right)}{4b^{3/2}d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(e + f\*x)^2,x]

[Out] (F^a\*(2\*Sqrt[b]\*f\*F^(b\*(c + d\*x)^2)\*(2\*d\*e - c\*f + d\*f\*x)\*Sqrt[Log[F]] + Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*(-f^2 + 2\*b\*(d\*e - c\*f)^2\*Log[F]))/(4\*b^(3/2)\*d^3\*Log[F]^(3/2))

**Maple [B]** time = 0.045, size = 324, normalized size = 1.9

$$-\frac{e^2 \sqrt{\pi} F^a}{2d} \operatorname{Erf}\left(-d\sqrt{-b \ln(F)}x + bc \ln(F) \frac{1}{\sqrt{-b \ln(F)}}\right) \frac{1}{\sqrt{-b \ln(F)}} + \frac{f^2 x F^{bd^2 x^2} F^{2bcdx} F^{c^2 b} F^a}{2 \ln(F) bd^2} - \frac{cf^2 F^{bd^2 x^2} F^{2bcdx} F^{c^2 b} F^a}{2 \ln(F) bd^3} - \frac{f^2 c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^2,x)

[Out] -1/2\*e^2\*Pi^(1/2)\*F^a/d/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+1/2\*f^2/ln(F)/b/d^2\*x\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-1/2\*f^2\*c/d^3/ln(F)/b\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a-1/2\*f^2\*c^2/d^3\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+1/4\*f^2/ln(F)/b/d^3\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+f\*e/ln(F)/b/d^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+f\*e\*c/d^2\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))

**Maxima [B]** time = 1.37659, size = 583, normalized size = 3.43

$$\frac{\left( \frac{\sqrt{\pi}(bd^2x+bcd)bcd \left( \operatorname{erf}\left( \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}} \right) - 1 \right) \log(F)^2}{(bd^2 \log(F))^{\frac{3}{2}} \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}}} - \frac{(bd^2x+bcd)^2}{F \frac{bd^2}{bd^2} bd^2 \log(F)} \right) F^a e f}{\sqrt{bd^2 \log(F)}} + \frac{\left( \frac{\sqrt{\pi}(bd^2x+bcd)b^2 c^2 d^2 \left( \operatorname{erf}\left( \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}} \right) - 1 \right) \log(F)}{(bd^2 \log(F))^{\frac{5}{2}} \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}}} \right)}{\sqrt{bd^2 \log(F)}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^2,x, algorithm="maxima")

[Out]  $-(\sqrt{\pi})(b*d^2*x + b*c*d)*b*c*d*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^2/((b*d^2*\log(F))^{3/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b*d^2*\log(F)/(b*d^2*\log(F))^{3/2}}*F^a*e*f/\sqrt{b*d^2*\log(F)} + 1/2*(\sqrt{\pi})(b*d^2*x + b*c*d)*b^2*c^2*d^2*(\operatorname{erf}(\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 1)*\log(F)^3/((b*d^2*\log(F))^{5/2}*\sqrt{-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2)}) - 2*F^{((b*d^2*x + b*c*d)^2/(b*d^2))*b^2*c*d^3*\log(F)^2/(b*d^2*\log(F))^{5/2}} - (b*d^2*x + b*c*d)^3*\gamma(3/2, -(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))*\log(F)^3/((b*d^2*\log(F))^{5/2}*(-(b*d^2*x + b*c*d)^2*\log(F)/(b*d^2))^{3/2}))*F^a*f^2/\sqrt{b*d^2*\log(F)} + 1/2*\sqrt{\pi}*F^{(b*c^2 + a)*e^2*\operatorname{erf}(\sqrt{-b*\log(F)})*d*x - b*c*\log(F)/\sqrt{-b*\log(F)})}/(\sqrt{-b*\log(F)})*F^{(b*c^2)*d}$

**Fricas [A]** time = 1.57884, size = 324, normalized size = 1.91

$$\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}(f^2 - 2(bd^2e^2 - 2bcdef + bc^2f^2)\log(F))F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right) + 2(bd^2f^2x + 2bd^2ef - bcd^2f^2)F^a}{4b^2d^4\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^2,x, algorithm="fricas")

[Out]  $1/4*(\sqrt{\pi})*\sqrt{-b*d^2*\log(F)}*(f^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\log(F))*F^a*\operatorname{erf}(\sqrt{-b*d^2*\log(F)}*(d*x + c)/d) + 2*(b*d^2*f^2*x + 2*b*d^2*e*f - b*c*d*f^2)*F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*\log(F)}/(b^2*d^4*\log(F)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(f\*x+e)\*\*2,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(e + f\*x)\*\*2, x)

**Giac [A]** time = 1.26918, size = 348, normalized size = 2.05

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x + \frac{c}{d}\right)\right)e^{(a\log(F)+2)}}{2\sqrt{-b\log(F)}d} + \frac{\sqrt{\pi}cf\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x + \frac{c}{d}\right)\right)e^{(a\log(F)+1)}}{\sqrt{-b\log(F)}d} + \frac{fe^{(bd^2x^2\log(F)+2bcdx\log(F)+bc^2\log(F)+a\log(F))}}{bd\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e)^2,x, algorithm="giac")

```
[Out] -1/2*sqrt(pi)*erf(-sqrt(-b*log(F))*d*(x + c/d))*e^(a*log(F) + 2)/(sqrt(-b*log(F))*d) + (sqrt(pi)*c*f*erf(-sqrt(-b*log(F))*d*(x + c/d))*e^(a*log(F) + 1)/(sqrt(-b*log(F))*d) + f*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F) + 1)/(b*d*log(F)))/d - 1/4*(sqrt(pi)*(2*b*c^2*f^2*log(F) - f^2)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b*d*log(F)) - 2*(d*f^2*(x + c/d) - 2*c*f^2)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b*d*log(F)))/d^2
```

### 3.386 $\int F^{a+b(c+dx)^2} (e + fx) dx$

**Optimal.** Leaf size=81

$$\frac{\sqrt{\pi}F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2\sqrt{bd^2}\sqrt{\log(F)}} + \frac{fF^{a+b(c+dx)^2}}{2bd^2\log(F)}$$

[Out] (f\*F^(a + b\*(c + d\*x)^2))/(2\*b\*d^2\*Log[F]) + ((d\*e - c\*f)\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d^2\*Sqrt[Log[F]])

**Rubi [A]** time = 0.146504, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2226, 2204, 2209}

$$\frac{\sqrt{\pi}F^a(de - cf)\operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c + dx)\right)}{2\sqrt{bd^2}\sqrt{\log(F)}} + \frac{fF^{a+b(c+dx)^2}}{2bd^2\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2)\*(e + f\*x), x]

[Out] (f\*F^(a + b\*(c + d\*x)^2))/(2\*b\*d^2\*Log[F]) + ((d\*e - c\*f)\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d^2\*Sqrt[Log[F]])

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)^2} (e + fx) dx &= \int \left( \frac{(de - cf)F^{a+b(c+dx)^2}}{d} + \frac{fF^{a+b(c+dx)^2}(c + dx)}{d} \right) dx \\ &= \frac{f \int F^{a+b(c+dx)^2}(c + dx) dx}{d} + \frac{(de - cf) \int F^{a+b(c+dx)^2} dx}{d} \\ &= \frac{fF^{a+b(c+dx)^2}}{2bd^2\log(F)} + \frac{(de - cf)F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c + dx)\sqrt{\log(F)}\right)}{2\sqrt{bd^2}\sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.0671947, size = 74, normalized size = 0.91

$$\frac{F^a \left( \sqrt{\pi} \sqrt{b} \sqrt{\log(F)} (de - cf) \operatorname{Erfi} \left( \sqrt{b} \sqrt{\log(F)} (c + dx) \right) + f F^{b(c+dx)^2} \right)}{2bd^2 \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2)\*(e + f\*x),x]

[Out] (F^a\*(f\*F^(b\*(c + d\*x)^2) + Sqrt[b]\*(d\*e - c\*f)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]]\*Sqrt[Log[F]])/(2\*b\*d^2\*Log[F])

**Maple [A]** time = 0.035, size = 132, normalized size = 1.6

$$-\frac{e\sqrt{\pi}F^a}{2d}\operatorname{Erf}\left(-d\sqrt{-b\ln(F)}x + bc\ln(F)\frac{1}{\sqrt{-b\ln(F)}}\right)\frac{1}{\sqrt{-b\ln(F)}} + \frac{fF^{bd^2x^2}F^{2bcdx}F^{c^2b}F^a}{2\ln(F)bd^2} + \frac{cf\sqrt{\pi}F^a}{2d^2}\operatorname{Erf}\left(-d\sqrt{-b\ln(F)}x + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)\*(f\*x+e),x)

[Out] -1/2\*e\*Pi^(1/2)\*F^a/d/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))+1/2\*f/ln(F)/b/d^2\*F^(b\*d^2\*x^2)\*F^(2\*b\*c\*d\*x)\*F^(c^2\*b)\*F^a+1/2\*f\*c/d^2\*Pi^(1/2)\*F^a/(-b\*ln(F))^(1/2)\*erf(-d\*(-b\*ln(F))^(1/2)\*x+b\*c\*ln(F)/(-b\*ln(F))^(1/2))

**Maxima [B]** time = 1.217, size = 269, normalized size = 3.32

$$\frac{\left( \frac{\sqrt{\pi}(bd^2x+bcd)bcd \left( \operatorname{erf} \left( \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}} \right) - 1 \right) \log(F)^2 - \frac{(bd^2x+bcd)^2}{bd^2} \log(F)}{(bd^2 \log(F))^{\frac{3}{2}} \sqrt{-\frac{(bd^2x+bcd)^2 \log(F)}{bd^2}}} - \frac{(bd^2 \log(F))^{\frac{3}{2}}}{(bd^2 \log(F))^{\frac{3}{2}}} \right) F^a f}{2\sqrt{bd^2 \log(F)}} + \frac{\sqrt{\pi}F^{bc^2+a}e \operatorname{erf}\left(\sqrt{-b \log(F)}dx - \frac{bc \log(F)}{\sqrt{-b \log(F)}}\right)}{2\sqrt{-b \log(F)}F^{bc^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e),x, algorithm="maxima")

[Out] -1/2\*(sqrt(pi)\*(b\*d^2\*x + b\*c\*d)\*b\*c\*d\*(erf(sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - 1)\*log(F)^2/((b\*d^2\*log(F))^(3/2)\*sqrt(-(b\*d^2\*x + b\*c\*d)^2\*log(F)/(b\*d^2))) - F^((b\*d^2\*x + b\*c\*d)^2/(b\*d^2))\*b\*d^2\*log(F)/(b\*d^2\*log(F))^(3/2))\*F^a\*f/sqrt(b\*d^2\*log(F)) + 1/2\*sqrt(pi)\*F^(b\*c^2 + a)\*e\*erf(sqrt(-b\*log(F))\*d\*x - b\*c\*log(F)/sqrt(-b\*log(F)))/(sqrt(-b\*log(F))\*F^(b\*c^2)\*d)

**Fricas [A]** time = 1.54248, size = 201, normalized size = 2.48

$$\frac{\sqrt{\pi}\sqrt{-bd^2 \log(F)}(de - cf)F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right) - F^{bd^2x^2+2bcdx+bc^2+a}df}{2bd^3 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e),x, algorithm="fricas")

[Out]  $-1/2*(\sqrt{\pi})*\sqrt{-b*d^2*\log(F)}*(d*e - c*f)*F^a*\operatorname{erf}(\sqrt{-b*d^2*\log(F)}*(d*x + c)/d) - F^{(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*d*f}/(b*d^3*\log(F))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)\*(f\*x+e),x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)\*(e + f\*x), x)

**Giac [A]** time = 1.27421, size = 171, normalized size = 2.11

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right) e^{(a \log(F)+1)}}{2 \sqrt{-b \log(F)} d} + \frac{\frac{\sqrt{\pi} F^a c f \operatorname{erf}\left(-\sqrt{-b \log(F)} d \left(x + \frac{c}{d}\right)\right)}{\sqrt{-b \log(F)} d} + \frac{f e^{(bd^2 x^2 \log(F)+2bcdx \log(F)+bc^2 \log(F)+a \log(F))}}{bd \log(F)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)\*(f\*x+e),x, algorithm="giac")

[Out]  $-1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))*e^{(a*\log(F) + 1)}/(\sqrt{-b*\log(F)}*d) + 1/2*(\sqrt{\pi})*F^a*c*f*\operatorname{erf}(-\sqrt{-b*\log(F)}*d*(x + c/d))/(\sqrt{-b*\log(F)}*d) + f*e^{(b*d^2*x^2*\log(F) + 2*b*c*d*x*\log(F) + b*c^2*\log(F) + a*\log(F))}/(b*d*\log(F))/d$

### 3.387 $\int F^{a+b(c+dx)^2} dx$

**Optimal.** Leaf size=44

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0122826, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2204}

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x)^2), x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\int F^{a+b(c+dx)^2} dx = \frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

**Mathematica [A]** time = 0.0058501, size = 44, normalized size = 1.

$$\frac{\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{bd}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x)^2), x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*(c + d\*x)\*Sqrt[Log[F]]])/(2\*Sqrt[b]\*d\*Sqrt[Log[F]])

**Maple [A]** time = 0.002, size = 58, normalized size = 1.3

$$-\frac{\sqrt{\pi}F^{c^2b+a}F^{-c^2b}}{2d} \operatorname{Erf}\left(-d\sqrt{-b \ln(F)}x + bc \ln(F) \frac{1}{\sqrt{-b \ln(F)}}\right) \frac{1}{\sqrt{-b \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2),x)`

[Out] 
$$-1/2*\text{Pi}^{(1/2)}*F^{(b*c^2+a)}*F^{(-c^2*b)/d}/(-b*\ln(F))^{(1/2)}*\text{erf}(-d*(-b*\ln(F))^{(1/2)}*x+b*c*\ln(F)/(-b*\ln(F))^{(1/2)})$$

**Maxima [A]** time = 1.01588, size = 78, normalized size = 1.77

$$\frac{\sqrt{\pi}F^{bc^2+a} \operatorname{erf}\left(\sqrt{-b \log(F)}dx - \frac{bc \log(F)}{\sqrt{-b \log(F)}}\right)}{2 \sqrt{-b \log(F)}F^{bc^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2),x, algorithm="maxima")`

[Out] 
$$1/2*\text{sqrt}(\text{pi})*F^{(b*c^2 + a)}*\text{erf}(\text{sqrt}(-b*\log(F))*d*x - b*c*\log(F)/\text{sqrt}(-b*\log(F)))/(\text{sqrt}(-b*\log(F))*F^{(b*c^2)*d})$$

**Fricas [A]** time = 1.58251, size = 123, normalized size = 2.8

$$\frac{\sqrt{\pi}\sqrt{-bd^2 \log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2 \log(F)}(dx+c)}{d}\right)}{2bd^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2),x, algorithm="fricas")`

[Out] 
$$-1/2*\text{sqrt}(\text{pi})*\text{sqrt}(-b*d^2*\log(F))*F^a*\text{erf}(\text{sqrt}(-b*d^2*\log(F))*(d*x + c)/d)/(b*d^2*\log(F))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2),x)`

[Out] `Integral(F**(a + b*(c + d*x)**2), x)`

**Giac [A]** time = 1.25157, size = 49, normalized size = 1.11

$$\frac{\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b \log(F)}d\left(x + \frac{c}{d}\right)\right)}{2 \sqrt{-b \log(F)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*d)
```



$$3.388 \quad \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{F^{a+b(c+dx)^2}}{e+fx}, x\right)$$

[Out] Unintegrable[F^(a + b\*(c + d\*x)^2)/(e + f\*x), x]

**Rubi [A]** time = 0.0685714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Int[F^(a + b\*(c + d\*x)^2)/(e + f\*x), x]

[Out] Defer[Int][F^(a + b\*(c + d\*x)^2)/(e + f\*x), x]

Rubi steps

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx} dx = \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$$

**Mathematica [A]** time = 0.288026, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x), x]

[Out] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x), x]

**Maple [A]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^2}}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(f\*x+e), x)

[Out] int(F^(a+b\*(d\*x+c)^2)/(f\*x+e), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(f\*x+e),x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(f\*x + e), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bd^2x^2+2bcdx+bc^2+a}}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(f\*x+e),x, algorithm="fricas")

[Out] integral(F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)/(f\*x + e), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(f\*x+e),x)

[Out] Integral(F\*\*(a + b\*(c + d\*x)\*\*2)/(e + f\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(f\*x+e),x, algorithm="giac")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(f\*x + e), x)

$$3.389 \quad \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx$$

**Optimal.** Leaf size=108

$$\frac{2bd \log(F)(de - cf) \text{Unintegrable}\left(\frac{F^{a+b(c+dx)^2}}{e+fx}, x\right)}{f^2} - \frac{F^{a+b(c+dx)^2}}{f(e+fx)} + \frac{\sqrt{\pi} \sqrt{bd} F^a \sqrt{\log(F)} \text{Erfi}\left(\sqrt{b} \sqrt{\log(F)}(c+dx)\right)}{f^2}$$

[Out]  $-(F^{(a + b*(c + d*x)^2})/(f*(e + f*x))) + (\text{Sqrt}[b]*d*F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*(c + d*x)*\text{Sqrt}[\text{Log}[F]]]*\text{Sqrt}[\text{Log}[F]])/f^2 - (2*b*d*(d*e - c*f)*\text{Log}[F]*\text{Unintegrable}[F^{(a + b*(c + d*x)^2})/(e + f*x), x])/f^2$

**Rubi [A]** time = 0.156402, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^2, x]

[Out]  $-(F^{(a + b*(c + d*x)^2})/(f*(e + f*x))) + (\text{Sqrt}[b]*d*F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*(c + d*x)*\text{Sqrt}[\text{Log}[F]]]*\text{Sqrt}[\text{Log}[F]])/f^2 - (2*b*d*(d*e - c*f)*\text{Log}[F]*\text{Defer}[\text{Int}[F^{(a + b*(c + d*x)^2})/(e + f*x), x])/f^2$

Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx &= -\frac{F^{a+b(c+dx)^2}}{f(e+fx)} + \frac{(2bd^2 \log(F)) \int F^{a+b(c+dx)^2} dx}{f^2} - \frac{(2bd(de - cf) \log(F)) \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx}{f^2} \\ &= -\frac{F^{a+b(c+dx)^2}}{f(e+fx)} + \frac{\sqrt{bd} F^a \sqrt{\pi} \text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right) \sqrt{\log(F)}}{f^2} - \frac{(2bd(de - cf) \log(F)) \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx}{f^2} \end{aligned}$$

**Mathematica [A]** time = 0.747803, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^2, x]

[Out] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^2, x]

**Maple [A]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^2}}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)`

[Out] `int(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")`

[Out] `integrate(F^((d*x + c)^2*b + a)/(f*x + e)^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bd^2x^2+2bcdx+bc^2+a}}{f^2x^2+2efx+e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")`

[Out] `integral(F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*e*f*x + e^2), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c)**2)/(f*x+e)**2,x)`

[Out] `Integral(F**(a + b*(c + d*x)**2)/(e + f*x)**2, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

$$3.390 \quad \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$$

**Optimal.** Leaf size=198

$$\frac{2b^2d^2 \log^2(F)(de - cf)^2 \text{Unintegrable}\left(\frac{F^{a+b(c+dx)^2}}{e+fx}, x\right)}{f^4} + \frac{bd^2 \log(F) \text{Unintegrable}\left(\frac{F^{a+b(c+dx)^2}}{e+fx}, x\right)}{f^2} - \frac{\sqrt{\pi}b^{3/2}d^2F^a \log^{\frac{3}{2}}(F)(de - cf)^2}{f^4}$$

[Out]  $-F^{a+b(c+dx)^2}/(2f(e+fx)^2) + (bd(d*de - c*f)*F^{a+b(c+dx)^2}*\text{Log}[F])/(f^3*(e+fx)) - (b^{3/2}*d^2*(d*de - c*f)*F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*(c+dx)*\text{Sqrt}[\text{Log}[F]]]*\text{Log}[F]^{3/2})/f^4 + (bd^2*\text{Log}[F]*\text{Unintegrable}[F^{a+b(c+dx)^2}/(e+fx), x])/f^2 + (2*b^2*d^2*(d*de - c*f)^2*\text{Log}[F]^2*\text{Unintegrable}[F^{a+b(c+dx)^2}/(e+fx), x])/f^4$

**Rubi [A]** time = 0.313875, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^3, x]

[Out]  $-F^{a+b(c+dx)^2}/(2f(e+fx)^2) + (bd(d*de - c*f)*F^{a+b(c+dx)^2}*\text{Log}[F])/(f^3*(e+fx)) - (b^{3/2}*d^2*(d*de - c*f)*F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*(c+dx)*\text{Sqrt}[\text{Log}[F]]]*\text{Log}[F]^{3/2})/f^4 + (bd^2*\text{Log}[F]*\text{Defer}[\text{Int}[F^{a+b(c+dx)^2}/(e+fx), x])/f^2 + (2*b^2*d^2*(d*de - c*f)^2*\text{Log}[F]^2*\text{Defer}[\text{Int}[F^{a+b(c+dx)^2}/(e+fx), x])/f^4$

Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx &= -\frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2} + \frac{(bd^2 \log(F)) \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx}{f^2} - \frac{(bd(de - cf) \log(F)) \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} dx}{f^2} \\ &= -\frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2} + \frac{bd(de - cf)F^{a+b(c+dx)^2} \log(F)}{f^3(e+fx)} + \frac{(bd^2 \log(F)) \int \frac{F^{a+b(c+dx)^2}}{e+fx} dx}{f^2} - \frac{(2b^2d^3(de - cf) \log^2(F))}{f^4} \\ &= -\frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2} + \frac{bd(de - cf)F^{a+b(c+dx)^2} \log(F)}{f^3(e+fx)} - \frac{b^{3/2}d^2(de - cf)F^a \sqrt{\pi} \text{erfi}(\sqrt{b}(c+dx)\sqrt{\log(F)}) \log^{\frac{3}{2}}(F)}{f^4} \end{aligned}$$

**Mathematica [A]** time = 1.12085, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^3, x]

[Out] Integrate[F^(a + b\*(c + d\*x)^2)/(e + f\*x)^3, x]

**Maple [A]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{F^{a+b(dx+c)^2}}{(fx+e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c)^2)/(f\*x+e)^3,x)

[Out] int(F^(a+b\*(d\*x+c)^2)/(f\*x+e)^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(dx+c)^2 b+a}}{(fx+e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(f\*x+e)^3,x, algorithm="maxima")

[Out] integrate(F^((d\*x + c)^2\*b + a)/(f\*x + e)^3, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bd^2x^2+2bcdx+bc^2+a}}{f^3x^3 + 3ef^2x^2 + 3e^2fx + e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c)^2)/(f\*x+e)^3,x, algorithm="fricas")

[Out] integral(F^(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + a)/(f^3\*x^3 + 3\*e\*f^2\*x^2 + 3\*e^2\*f\*x + e^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c)\*\*2)/(f\*x+e)\*\*3,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F(dx+c)^{2b+a}}{(fx+e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((d*x + c)^2*b + a)/(f*x + e)^3, x)
```



### 3.391 $\int e^{e(c+dx)^3} (a + bx)^3 dx$

**Optimal.** Leaf size=177

$$\frac{b(c+dx)^2(bc-ad)^2\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{d^4(-e(c+dx)^3)^{2/3}} + \frac{(c+dx)(bc-ad)^3\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^4\sqrt[3]{-e(c+dx)^3}} - \frac{b^3(c+dx)^4\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^4(-e(c+dx)^3)^{1/3}}$$

[Out]  $-\left(\frac{b^2(b*c - a*d)*E^{(e*(c + d*x)^3)}}{d^4*e}\right) + \left(\frac{(b*c - a*d)^3*(c + d*x)*\Gamma\left[\frac{1}{3}, -(e*(c + d*x)^3)\right]}{3*d^4*(-(e*(c + d*x)^3))^{(1/3)}} - \frac{b*(b*c - a*d)^2*(c + d*x)^2*\Gamma\left[\frac{2}{3}, -(e*(c + d*x)^3)\right]}{d^4*(-(e*(c + d*x)^3))^{(2/3)}} - \frac{b^3*(c + d*x)^4*\Gamma\left[\frac{4}{3}, -(e*(c + d*x)^3)\right]}{3*d^4*(-(e*(c + d*x)^3))^{(4/3)}}\right)$

**Rubi [A]** time = 0.155975, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2226, 2208, 2218, 2209}

$$\frac{b(c+dx)^2(bc-ad)^2\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{d^4(-e(c+dx)^3)^{2/3}} + \frac{(c+dx)(bc-ad)^3\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^4\sqrt[3]{-e(c+dx)^3}} - \frac{b^3(c+dx)^4\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^4(-e(c+dx)^3)^{1/3}}$$

Antiderivative was successfully verified.

[In] Int[E^(e\*(c + d\*x)^3)\*(a + b\*x)^3,x]

[Out]  $-\left(\frac{b^2(b*c - a*d)*E^{(e*(c + d*x)^3)}}{d^4*e}\right) + \left(\frac{(b*c - a*d)^3*(c + d*x)*\Gamma\left[\frac{1}{3}, -(e*(c + d*x)^3)\right]}{3*d^4*(-(e*(c + d*x)^3))^{(1/3)}} - \frac{b*(b*c - a*d)^2*(c + d*x)^2*\Gamma\left[\frac{2}{3}, -(e*(c + d*x)^3)\right]}{d^4*(-(e*(c + d*x)^3))^{(2/3)}} - \frac{b^3*(c + d*x)^4*\Gamma\left[\frac{4}{3}, -(e*(c + d*x)^3)\right]}{3*d^4*(-(e*(c + d*x)^3))^{(4/3)}}\right)$

#### Rule 2226

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2209

Int[(F\_)^(a\_. + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(e + f\*x)^m\*(F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ

[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int e^{e(c+dx)^3} (a+bx)^3 dx &= \int \left( \frac{(-bc+ad)^3 e^{e(c+dx)^3}}{d^3} + \frac{3b(bc-ad)^2 e^{e(c+dx)^3} (c+dx)}{d^3} - \frac{3b^2(bc-ad) e^{e(c+dx)^3} (c+dx)^2}{d^3} + \frac{b^3 e^{e(c+dx)^3} (c+dx)^3}{d^3} \right) dx \\ &= \frac{b^3 \int e^{e(c+dx)^3} (c+dx)^3 dx}{d^3} - \frac{(3b^2(bc-ad)) \int e^{e(c+dx)^3} (c+dx)^2 dx}{d^3} + \frac{(3b(bc-ad)^2) \int e^{e(c+dx)^3} (c+dx) dx}{d^3} - \frac{\int e^{e(c+dx)^3} dx}{d^3} \\ &= -\frac{b^2(bc-ad) e^{e(c+dx)^3}}{d^4 e} + \frac{(bc-ad)^3 (c+dx) \Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^4 \sqrt[3]{-e(c+dx)^3}} - \frac{b(bc-ad)^2 (c+dx)^2 \Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{d^4 (-e(c+dx)^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.213214, size = 167, normalized size = 0.94

$$\frac{3b(c+dx)^2(bc-ad)^2 \Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{(-e(c+dx)^3)^{2/3}} + \frac{(c+dx)(bc-ad)^3 \Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{\sqrt[3]{-e(c+dx)^3}} + \frac{b^3(c+dx) \Gamma\left(\frac{4}{3}, -e(c+dx)^3\right)}{e \sqrt[3]{-e(c+dx)^3}} - \frac{3b^2(bc-ad) e^{e(c+dx)^3}}{e}$$


---


$$3d^4$$

Antiderivative was successfully verified.

[In] Integrate[E^(e\*(c + d\*x)^3)\*(a + b\*x)^3,x]

[Out] ((-3\*b^2\*(b\*c - a\*d)\*E^(e\*(c + d\*x)^3))/e + ((b\*c - a\*d)^3\*(c + d\*x)\*Gamma[1/3, -(e\*(c + d\*x)^3)]/(-(e\*(c + d\*x)^3)^(1/3) - (3\*b\*(b\*c - a\*d)^2\*(c + d\*x)^2\*Gamma[2/3, -(e\*(c + d\*x)^3)]/(-(e\*(c + d\*x)^3)^(2/3) + (b^3\*(c + d\*x)\*Gamma[4/3, -(e\*(c + d\*x)^3)]/(e\*(-(e\*(c + d\*x)^3)^(1/3)))/(3\*d^4)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int e^{e(dx+c)^3} (bx+a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*(d\*x+c)^3)\*(b\*x+a)^3,x)

[Out] int(exp(e\*(d\*x+c)^3)\*(b\*x+a)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^3 e^{(dx+c)^3 e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^3,x, algorithm="maxima")

[Out] integrate((b\*x + a)^3\*e^((d\*x + c)^3\*e), x)

**Fricas [A]** time = 1.53813, size = 514, normalized size = 2.9

$$\frac{9(b^3c^2d - 2ab^2cd^2 + a^2bd^3)(-d^3e)^{\frac{1}{3}}e\Gamma\left(\frac{2}{3}, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e\right) - (-d^3e)^{\frac{2}{3}}(b^3 + 3(b^3c^3 - 3ab^2c^2d + 3a^2bd^3))}{9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^3,x, algorithm="fricas")

[Out] 1/9\*(9\*(b^3\*c^2\*d - 2\*a\*b^2\*c\*d^2 + a^2\*b\*d^3)\*(-d^3\*e)^(1/3)\*e\*gamma(2/3, -d^3\*e\*x^3 - 3\*c\*d^2\*e\*x^2 - 3\*c^2\*d\*e\*x - c^3\*e) - (-d^3\*e)^(2/3)\*(b^3 + 3\*(b^3\*c^3 - 3\*a\*b^2\*c\*d^2 + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*e)\*gamma(1/3, -d^3\*e\*x^3 - 3\*c\*d^2\*e\*x^2 - 3\*c^2\*d\*e\*x - c^3\*e) + 3\*(b^3\*d^3\*e\*x - (2\*b^3\*c\*d^2 - 3\*a\*b^2\*d^3)\*e)\*e^(d^3\*e\*x^3 + 3\*c\*d^2\*e\*x^2 + 3\*c^2\*d\*e\*x + c^3\*e))/(d^6\*e^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)\*\*3)\*(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 e^{(dx+c)^3e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((b\*x + a)^3\*e^((d\*x + c)^3\*e), x)

### 3.392 $\int e^{e(c+dx)^3} (a+bx)^2 dx$

**Optimal.** Leaf size=126

$$\frac{2b(c+dx)^2(bc-ad)\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{3d^3(-e(c+dx)^3)^{2/3}} - \frac{(c+dx)(bc-ad)^2\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^3\sqrt[3]{-e(c+dx)^3}} + \frac{b^2e^{e(c+dx)^3}}{3d^3e}$$

[Out]  $(b^2E^{(e*(c+d*x)^3)})/(3*d^3*e) - ((b*c - a*d)^2*(c+d*x)*Gamma[1/3, -(e*(c+d*x)^3)])/(3*d^3*(-(e*(c+d*x)^3))^{(1/3)}) + (2*b*(b*c - a*d)*(c+d*x)^2*Gamma[2/3, -(e*(c+d*x)^3)])/(3*d^3*(-(e*(c+d*x)^3))^{(2/3)})$

**Rubi [A]** time = 0.106701, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2226, 2208, 2218, 2209}

$$\frac{2b(c+dx)^2(bc-ad)\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{3d^3(-e(c+dx)^3)^{2/3}} - \frac{(c+dx)(bc-ad)^2\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^3\sqrt[3]{-e(c+dx)^3}} + \frac{b^2e^{e(c+dx)^3}}{3d^3e}$$

Antiderivative was successfully verified.

[In] Int[E^(e\*(c+d\*x)^3)\*(a+b\*x)^2,x]

[Out]  $(b^2E^{(e*(c+d*x)^3)})/(3*d^3*e) - ((b*c - a*d)^2*(c+d*x)*Gamma[1/3, -(e*(c+d*x)^3)])/(3*d^3*(-(e*(c+d*x)^3))^{(1/3)}) + (2*b*(b*c - a*d)*(c+d*x)^2*Gamma[2/3, -(e*(c+d*x)^3)])/(3*d^3*(-(e*(c+d*x)^3))^{(2/3)})$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int e^{e(c+dx)^3} (a+bx)^2 dx &= \int \left( \frac{(-bc+ad)^2 e^{e(c+dx)^3}}{d^2} - \frac{2b(bc-ad)e^{e(c+dx)^3}(c+dx)}{d^2} + \frac{b^2 e^{e(c+dx)^3}(c+dx)^2}{d^2} \right) dx \\ &= \frac{b^2 \int e^{e(c+dx)^3} (c+dx)^2 dx}{d^2} - \frac{(2b(bc-ad)) \int e^{e(c+dx)^3} (c+dx) dx}{d^2} + \frac{(bc-ad)^2 \int e^{e(c+dx)^3} dx}{d^2} \\ &= \frac{b^2 e^{e(c+dx)^3}}{3d^3 e} - \frac{(bc-ad)^2 (c+dx) \Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^3 \sqrt[3]{-e(c+dx)^3}} + \frac{2b(bc-ad)(c+dx)^2 \Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{3d^3 (-e(c+dx)^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.0854033, size = 117, normalized size = 0.93

$$\frac{2b(c+dx)^2(bc-ad)\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{(-e(c+dx)^3)^{2/3}} - \frac{(c+dx)(bc-ad)^2\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{\sqrt[3]{-e(c+dx)^3}} + \frac{b^2 e^{e(c+dx)^3}}{e}$$


---


$$3d^3$$

Antiderivative was successfully verified.

[In] Integrate[E^(e\*(c + d\*x)^3)\*(a + b\*x)^2, x]

[Out] ((b^2\*E^(e\*(c + d\*x)^3))/e - ((b\*c - a\*d)^2\*(c + d\*x)\*Gamma[1/3, -(e\*(c + d\*x)^3)]/(-(e\*(c + d\*x)^3))^(1/3) + (2\*b\*(b\*c - a\*d)\*(c + d\*x)^2\*Gamma[2/3, -(e\*(c + d\*x)^3)]/(-(e\*(c + d\*x)^3))^(2/3))/(3\*d^3))

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int e^{e(dx+c)^3} (bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*(d\*x+c)^3)\*(b\*x+a)^2, x)

[Out] int(exp(e\*(d\*x+c)^3)\*(b\*x+a)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2 e^{(dx+c)^3 e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^2, x, algorithm="maxima")

[Out] integrate((b\*x + a)^2\*e^((d\*x + c)^3\*e), x)

**Fricas [A]** time = 1.53158, size = 385, normalized size = 3.06

$$\frac{b^2 d^2 e^{(d^3 e x^3 + 3 c d^2 e x^2 + 3 c^2 d e x + c^3 e)} + (b^2 c^2 - 2 a b c d + a^2 d^2) (-d^3 e)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -d^3 e x^3 - 3 c d^2 e x^2 - 3 c^2 d e x - c^3 e\right) - 2 (b^2 c d - a b c d)}{3 d^5 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{3}(b^2d^2e^{(d^3ex^3 + 3cd^2ex^2 + 3c^2dex + c^3e)} + (b^2c^2 - 2ab*cd + a^2d^2)(-d^3e)^{2/3}\gamma(1/3, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e) - 2(b^2cd - ab*d^2)(-d^3e)^{1/3}\gamma(2/3, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e))/(d^5e)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)\*\*3)\*(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 e^{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*e^((d\*x + c)^3\*e), x)

### 3.393 $\int e^{e(c+dx)^3} (a + bx) dx$

**Optimal.** Leaf size=92

$$\frac{(c + dx)(bc - ad)\Gamma\left(\frac{1}{3}, -e(c + dx)^3\right)}{3d^2\sqrt[3]{-e(c + dx)^3}} - \frac{b(c + dx)^2\Gamma\left(\frac{2}{3}, -e(c + dx)^3\right)}{3d^2(-e(c + dx)^3)^{2/3}}$$

[Out]  $((b*c - a*d)*(c + d*x)*\Gamma[1/3, -(e*(c + d*x)^3)]/(3*d^2*(-(e*(c + d*x)^3))^{(1/3)}) - (b*(c + d*x)^2*\Gamma[2/3, -(e*(c + d*x)^3)]/(3*d^2*(-(e*(c + d*x)^3))^{(2/3)}))$

**Rubi [A]** time = 0.0602533, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2226, 2208, 2218}

$$\frac{(c + dx)(bc - ad)\Gamma\left(\frac{1}{3}, -e(c + dx)^3\right)}{3d^2\sqrt[3]{-e(c + dx)^3}} - \frac{b(c + dx)^2\Gamma\left(\frac{2}{3}, -e(c + dx)^3\right)}{3d^2(-e(c + dx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[E^(e\*(c + d\*x)^3)\*(a + b\*x), x]

[Out]  $((b*c - a*d)*(c + d*x)*\Gamma[1/3, -(e*(c + d*x)^3)]/(3*d^2*(-(e*(c + d*x)^3))^{(1/3)}) - (b*(c + d*x)^2*\Gamma[2/3, -(e*(c + d*x)^3)]/(3*d^2*(-(e*(c + d*x)^3))^{(2/3)}))$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int e^{e(c+dx)^3} (a+bx) dx &= \int \left( \frac{(-bc+ad)e^{e(c+dx)^3}}{d} + \frac{be^{e(c+dx)^3}(c+dx)}{d} \right) dx \\ &= \frac{b \int e^{e(c+dx)^3} (c+dx) dx}{d} + \frac{(-bc+ad) \int e^{e(c+dx)^3} dx}{d} \\ &= \frac{(bc-ad)(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d^2\sqrt[3]{-e(c+dx)^3}} - \frac{b(c+dx)^2\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{3d^2(-e(c+dx)^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.0598877, size = 86, normalized size = 0.93

$$\frac{(c+dx)\left(b(c+dx)\text{Gamma}\left(\frac{2}{3}, -e(c+dx)^3\right) - (bc-ad)\sqrt[3]{-e(c+dx)^3}\text{Gamma}\left(\frac{1}{3}, -e(c+dx)^3\right)\right)}{3d^2(-e(c+dx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e\*(c + d\*x)^3)\*(a + b\*x), x]

[Out] -((c + d\*x)\*(-(b\*c - a\*d)\*(-(e\*(c + d\*x)^3))^(1/3)\*Gamma[1/3, -(e\*(c + d\*x)^3)]) + b\*(c + d\*x)\*Gamma[2/3, -(e\*(c + d\*x)^3)])/(3\*d^2\*(-(e\*(c + d\*x)^3))^(2/3))

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int e^{e(dx+c)^3} (bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*(d\*x+c)^3)\*(b\*x+a), x)

[Out] int(exp(e\*(d\*x+c)^3)\*(b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)e^{((dx+c)^3e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)\*(b\*x+a), x, algorithm="maxima")

[Out] integrate((b\*x + a)\*e^((d\*x + c)^3\*e), x)

**Fricas [A]** time = 1.49636, size = 250, normalized size = 2.72

$$\frac{(-d^3e)^{\frac{1}{3}}bd\Gamma\left(\frac{2}{3}, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e\right) - (-d^3e)^{\frac{2}{3}}(bc-ad)\Gamma\left(\frac{1}{3}, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e\right)}{3d^4e}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*(d*x+c)^3)*(b*x+a), x, algorithm="fricas")
```

```
[Out] 1/3*((-d^3*e)^(1/3)*b*d*gamma(2/3, -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x
- c^3*e) - (-d^3*e)^(2/3)*(b*c - a*d)*gamma(1/3, -d^3*e*x^3 - 3*c*d^2*e*x^
2 - 3*c^2*d*e*x - c^3*e))/(d^4*e)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\left( \int a e^{d^3 e x^3} e^{3 c d^2 e x^2} e^{3 c^2 d e x} dx + \int b x e^{d^3 e x^3} e^{3 c d^2 e x^2} e^{3 c^2 d e x} dx \right) e^{c^3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*(d*x+c)**3)*(b*x+a), x)
```

```
[Out] (Integral(a*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x) + I
ntegral(b*x*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x))*ex
p(c**3*e)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)e^{(dx+c)^3 e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*(d*x+c)^3)*(b*x+a), x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*e^((d*x + c)^3*e), x)
```

### 3.394 $\int e^{e(c+dx)^3} dx$

**Optimal.** Leaf size=40

$$\frac{(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

[Out]  $-\left((c+d*x)*\Gamma\left[\frac{1}{3}, -(e*(c+d*x)^3)\right]\right)/\left(3*d*(-(e*(c+d*x)^3))^{(1/3)}\right)$

**Rubi [A]** time = 0.0051903, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2208}

$$\frac{(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

Antiderivative was successfully verified.

[In] Int[E^(e\*(c + d\*x)^3), x]

[Out]  $-\left((c+d*x)*\Gamma\left[\frac{1}{3}, -(e*(c+d*x)^3)\right]\right)/\left(3*d*(-(e*(c+d*x)^3))^{(1/3)}\right)$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int e^{e(c+dx)^3} dx = -\frac{(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

**Mathematica [A]** time = 0.0061005, size = 40, normalized size = 1.

$$\frac{(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e\*(c + d\*x)^3), x]

[Out]  $-\left((c+d*x)*\Gamma\left[\frac{1}{3}, -(e*(c+d*x)^3)\right]\right)/\left(3*d*(-(e*(c+d*x)^3))^{(1/3)}\right)$

**Maple [F]** time = 0.012, size = 0, normalized size = 0.

$$\int e^{e(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(e*(d*x+c)^3),x)`

[Out] `int(exp(e*(d*x+c)^3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e*(d*x+c)^3),x, algorithm="maxima")`

[Out] `integrate(e^((d*x + c)^3*e), x)`

**Fricas [A]** time = 1.53867, size = 120, normalized size = 3.

$$\frac{(-d^3e)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -d^3ex^3 - 3cd^2ex^2 - 3c^2dex - c^3e\right)}{3d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e*(d*x+c)^3),x, algorithm="fricas")`

[Out] `1/3*(-d^3*e)^(2/3)*gamma(1/3, -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x - c^3*e)/(d^3*e)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{c^3e} \int e^{d^3ex^3} e^{3cd^2ex^2} e^{3c^2dex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e*(d*x+c)**3),x)`

[Out] `exp(c**3*e)*Integral(exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(dx+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e*(d*x+c)^3),x, algorithm="giac")`

[Out] `integrate(e^((d*x + c)^3*e), x)`

$$3.395 \quad \int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

**Optimal.** Leaf size=21

$$\text{Unintegrable} \left( \frac{e^{e(c+dx)^3}}{a+bx}, x \right)$$

[Out] Unintegrable[E^(e\*(c + d\*x)^3)/(a + b\*x), x]

**Rubi [A]** time = 0.0226154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e\*(c + d\*x)^3)/(a + b\*x), x]

[Out] Defer[Int][E^(e\*(c + d\*x)^3)/(a + b\*x), x]

Rubi steps

$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx = \int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

**Mathematica [A]** time = 0.384807, size = 0, normalized size = 0.

$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e\*(c + d\*x)^3)/(a + b\*x), x]

[Out] Integrate[E^(e\*(c + d\*x)^3)/(a + b\*x), x]

**Maple [A]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{e^{e(dx+c)^3}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*(d\*x+c)^3)/(b\*x+a), x)

[Out] int(exp(e\*(d\*x+c)^3)/(b\*x+a), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{((dx+c)^3e)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a),x, algorithm="maxima")

[Out] integrate(e^((d\*x + c)^3\*e)/(b\*x + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(d^3ex^3+3cd^2ex^2+3c^2dex+c^3e)}}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a),x, algorithm="fricas")

[Out] integral(e^(d^3\*e\*x^3 + 3\*c\*d^2\*e\*x^2 + 3\*c^2\*d\*e\*x + c^3\*e)/(b\*x + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^{c^3e} \int \frac{e^{d^3ex^3} e^{3cd^2ex^2} e^{3c^2dex}}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)\*\*3)/(b\*x+a), x)

[Out] exp(c\*\*3\*e)\*Integral(exp(d\*\*3\*e\*x\*\*3)\*exp(3\*c\*d\*\*2\*e\*x\*\*2)\*exp(3\*c\*\*2\*d\*e\*x)/(a + b\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{((dx+c)^3e)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a),x, algorithm="giac")

[Out] integrate(e^((d\*x + c)^3\*e)/(b\*x + a), x)

$$3.396 \quad \int \frac{e^{e(c+dx)^3}}{(a+bx)^2} dx$$

**Optimal.** Leaf size=152

$$\frac{3de(bc-ad)^2 \text{Unintegrable}\left(\frac{e^{e(c+dx)^3}}{a+bx}, x\right)}{b^3} - \frac{de(c+dx)(bc-ad)\text{Gamma}\left(\frac{1}{3}, -e(c+dx)^3\right)}{b^3 \sqrt[3]{-e(c+dx)^3}} - \frac{de(c+dx)^2 \text{Gamma}\left(\frac{2}{3}, -e(c+dx)^3\right)}{b^2 (-e(c+dx)^3)^{2/3}}$$

[Out]  $-(E^{(e*(c+d*x)^3)/(b*(a+b*x))}) - (d*(b*c-a*d)*e*(c+d*x)*\text{Gamma}[1/3, -(e*(c+d*x)^3)]/(b^3*(-(e*(c+d*x)^3))^{(1/3)}) - (d*e*(c+d*x)^2*\text{Gamma}[2/3, -(e*(c+d*x)^3)]/(b^2*(-(e*(c+d*x)^3))^{(2/3)}) + (3*d*(b*c-a*d)^2*e*\text{Unintegrable}[E^{(e*(c+d*x)^3)/(a+b*x)}, x])/b^3$

**Rubi [A]** time = 0.350316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e\*(c+d\*x)^3)/(a+b\*x)^2,x]

[Out]  $-(E^{(e*(c+d*x)^3)/(b*(a+b*x))}) - (d*(b*c-a*d)*e*(c+d*x)*\text{Gamma}[1/3, -(e*(c+d*x)^3)]/(b^3*(-(e*(c+d*x)^3))^{(1/3)}) - (d*e*(c+d*x)^2*\text{Gamma}[2/3, -(e*(c+d*x)^3)]/(b^2*(-(e*(c+d*x)^3))^{(2/3)}) + (3*d*(b*c-a*d)^2*e*\text{Defer}[\text{Int}[E^{(e*(c+d*x)^3)/(a+b*x)}, x])/b^3$

**Rubi steps**

$$\begin{aligned} \int \frac{e^{e(c+dx)^3}}{(a+bx)^2} dx &= -\frac{e^{e(c+dx)^3}}{b(a+bx)} + \frac{(3de) \int \frac{e^{e(c+dx)^3}(c+dx)^2}{a+bx} dx}{b} \\ &= -\frac{e^{e(c+dx)^3}}{b(a+bx)} + \frac{(3de) \int \left( \frac{d(bc-ad)e^{e(c+dx)^3}}{b^2} + \frac{(bc-ad)^2 e^{e(c+dx)^3}}{b^2(a+bx)} + \frac{de^{e(c+dx)^3}(c+dx)}{b} \right) dx}{b} \\ &= -\frac{e^{e(c+dx)^3}}{b(a+bx)} + \frac{(3d^2e) \int e^{e(c+dx)^3}(c+dx) dx}{b^2} + \frac{(3d^2(bc-ad)e) \int e^{e(c+dx)^3} dx}{b^3} + \frac{(3d(bc-ad)^2e) \int \frac{e^{e(c+dx)^3}}{a+bx} dx}{b^3} \\ &= -\frac{e^{e(c+dx)^3}}{b(a+bx)} - \frac{d(bc-ad)e(c+dx)\Gamma\left(\frac{1}{3}, -e(c+dx)^3\right)}{b^3 \sqrt[3]{-e(c+dx)^3}} - \frac{de(c+dx)^2\Gamma\left(\frac{2}{3}, -e(c+dx)^3\right)}{b^2 (-e(c+dx)^3)^{2/3}} + \frac{(3d(bc-ad)^2e)}{b^3} \end{aligned}$$

**Mathematica [A]** time = 2.05486, size = 0, normalized size = 0.

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e\*(c+d\*x)^3)/(a+b\*x)^2,x]

[Out] Integrate[E^(e\*(c + d\*x)^3)/(a + b\*x)^2, x]

**Maple [A]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{e^{e(dx+c)^3}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*(d\*x+c)^3)/(b\*x+a)^2, x)

[Out] int(exp(e\*(d\*x+c)^3)/(b\*x+a)^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{((dx+c)^3 e)}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a)^2, x, algorithm="maxima")

[Out] integrate(e^((d\*x + c)^3\*e)/(b\*x + a)^2, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(d^3 ex^3 + 3 cd^2 ex^2 + 3 c^2 dex + c^3 e)}}{b^2 x^2 + 2 abx + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a)^2, x, algorithm="fricas")

[Out] integral(e^(d^3\*e\*x^3 + 3\*c\*d^2\*e\*x^2 + 3\*c^2\*d\*e\*x + c^3\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)\*\*3)/(b\*x+a)\*\*2, x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*(d\*x+c)^3)/(b\*x+a)^2,x, algorithm="giac")

[Out] undef



$$3.397 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx$$

**Optimal.** Leaf size=71

$$\frac{F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{f} - \frac{F^a \operatorname{Ei}\left(\frac{b\log(F)}{c+dx}\right)}{f}$$

[Out]  $-\left(\frac{F^a \operatorname{ExpIntegralEi}[(b \cdot \log(F))/(c + d \cdot x)]}{f}\right) + \frac{F^{a - (b \cdot f)/(d \cdot e - c \cdot f)}}{f} \operatorname{ExpIntegralEi}[(b \cdot d \cdot (e + f \cdot x) \cdot \log(F))/((d \cdot e - c \cdot f) \cdot (c + d \cdot x))]/f$

**Rubi [A]** time = 0.405028, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2222, 2210, 2228, 2178}

$$\frac{F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{f} - \frac{F^a \operatorname{Ei}\left(\frac{b\log(F)}{c+dx}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(e + f\*x),x]

[Out]  $-\left(\frac{F^a \operatorname{ExpIntegralEi}[(b \cdot \log(F))/(c + d \cdot x)]}{f}\right) + \frac{F^{a - (b \cdot f)/(d \cdot e - c \cdot f)}}{f} \operatorname{ExpIntegralEi}[(b \cdot d \cdot (e + f \cdot x) \cdot \log(F))/((d \cdot e - c \cdot f) \cdot (c + d \cdot x))]/f$

#### Rule 2222

Int[(F\_)^((a\_) + (b\_)/((c\_) + (d\_)\*(x\_)))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> Dist[d/f, Int[F^(a + b/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*e - c\*f)/f, Int[F^(a + b/(c + d\*x))/((c + d\*x)\*(e + f\*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^(n\_))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2228

Int[(F\_)^((a\_) + (b\_)/((c\_) + (d\_)\*(x\_)))/(((e\_) + (f\_)\*(x\_))\*((g\_) + (h\_)\*(x\_))), x\_Symbol] :> -Dist[d/(f\*(d\*g - c\*h)), Subst[Int[F^(a - (b\*h)/(d\*g - c\*h) + (d\*b\*x)/(d\*g - c\*h))/x, x], x, (g + h\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx = \frac{d \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx}{f} - \frac{(de-cf) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)(e+fx)} dx}{f}$$

$$= -\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{f} + \frac{\operatorname{Subst}\left(\int \frac{F^{a-\frac{bf}{de-cf}+\frac{bdx}{de-cf}}}{x} dx, x, \frac{e+fx}{c+dx}\right)}{f}$$

$$= -\frac{F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right)}{f} + \frac{F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right)}{f}$$

**Mathematica [A]** time = 0.123814, size = 66, normalized size = 0.93

$$\frac{F^a \left( F^{\frac{bf}{cf-de}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right) - \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b/(c + d\*x))/(e + f\*x),x]

[Out] (F^a\*(-ExpIntegralEi[(b\*Log[F])/(c + d\*x)] + F^((b\*f)/(-(d\*e) + c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]))/f

**Maple [A]** time = 0.162, size = 106, normalized size = 1.5

$$\frac{F^a}{f} \operatorname{Ei}\left(1, -\frac{b \ln(F)}{dx+c}\right) - \frac{1}{f} F^{\frac{acf-ade+bf}{cf-de}} \operatorname{Ei}\left(1, -\frac{b \ln(F)}{dx+c} - \ln(F) a - \frac{-\ln(F) acf + \ln(F) ade - \ln(F) bf}{cf-de}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(f\*x+e), x)

[Out] 1/f\*F^a\*Ei(1, -b\*ln(F)/(d\*x+c))-1/f\*F^((a\*c\*f-a\*d\*e+b\*f)/(c\*f-d\*e))\*Ei(1, -b\*ln(F)/(d\*x+c)-ln(F)\*a-(-ln(F)\*a\*c\*f+ln(F)\*a\*d\*e-ln(F)\*b\*f)/(c\*f-d\*e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e),x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e), x)

**Fricas [A]** time = 1.57847, size = 185, normalized size = 2.61

$$\frac{F^{\frac{ade-(ac+b)f}{de-cf}} \operatorname{Ei}\left(\frac{(bdfx+bde)\log(F)}{cde-c^2f+(d^2e-cdf)x}\right) - F^a \operatorname{Ei}\left(\frac{b\log(F)}{dx+c}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e), x, algorithm="fricas")

[Out] (F^((a\*d\*e - (a\*c + b)\*f)/(d\*e - c\*f))\*Ei((b\*d\*f\*x + b\*d\*e)\*log(F)/(c\*d\*e - c^2\*f + (d^2\*e - c\*d\*f)\*x)) - F^a\*Ei(b\*log(F)/(d\*x + c)))/f

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(f\*x+e), x)

[Out] Integral(F\*\*(a + b/(c + d\*x))/(e + f\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e), x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e), x)

$$3.398 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2} dx$$

**Optimal.** Leaf size=116

$$-\frac{bd \log(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^2} + \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

[Out] (d\*F^(a + b/(c + d\*x)))/(f\*(d\*e - c\*f)) - F^(a + b/(c + d\*x))/(f\*(e + f\*x)) - (b\*d\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F])/(d\*e - c\*f)^2

**Rubi [A]** time = 1.01075, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2223, 6742, 2209, 2210, 2222, 2228, 2178}

$$-\frac{bd \log(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^2} + \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(e + f\*x)^2,x]

[Out] (d\*F^(a + b/(c + d\*x)))/(f\*(d\*e - c\*f)) - F^(a + b/(c + d\*x))/(f\*(e + f\*x)) - (b\*d\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F])/(d\*e - c\*f)^2

#### Rule 2223

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(f\*(m + 1)), x] + Dist[(b\*d\*Log[F])/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(c + d\*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && ILtQ[m, -1]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2222

```
Int[(F_)^((a_) + (b_)/((c_) + (d_)*(x_)))/((e_) + (f_)*(x_)), x_Symbol]
:> Dist[d/f, Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Dist[(d*e - c*f)
)/f, Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a,
b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

**Rule 2228**

```
Int[(F_)^((a_) + (b_)/((c_) + (d_)*(x_)))/(((e_) + (f_)*(x_))*((g_)
+ (h_)*(x_))), x_Symbol] :> -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h)
)/(d*g - c*h) + (d*b*x)/(d*g - c*h)/x, x], x, (g + h*x)/(c + d*x)], x] /;
FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

**Rule 2178**

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

**Rubi steps**

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2} dx = -\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} - \frac{(bd \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)} dx}{f}$$

$$= -\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} - \frac{(bd \log(F)) \int \left( \frac{dF^{a+\frac{b}{c+dx}}}{(de-cf)(c+dx)^2} - \frac{dfF^{a+\frac{b}{c+dx}}}{(de-cf)^2(c+dx)} + \frac{f^2F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)} \right) dx}{f}$$

$$= -\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} + \frac{(bd^2 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx}{(de-cf)^2} - \frac{(bdf \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx}{(de-cf)^2} - \frac{(bd^2 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{f(de-cf)}$$

$$= \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} - \frac{bdF^a \text{Ei}\left(\frac{b \log(F)}{c+dx}\right) \log(F)}{(de-cf)^2} - \frac{(bd^2 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx}{(de-cf)^2} + \frac{(bd \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{de-cf}$$

$$= \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} - \frac{(bd \log(F)) \text{Subst}\left(\int \frac{F^{a-\frac{bf}{de-cf} + \frac{bdx}{de-cf}}}{x} dx, x, \frac{e+fx}{c+dx}\right)}{(de-cf)^2}$$

$$= \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)} - \frac{bdF^{a-\frac{bf}{de-cf}} \text{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right) \log(F)}{(de-cf)^2}$$

**Mathematica [A]** time = 0.339725, size = 116, normalized size = 1.

$$-\frac{bd \log(F) F^{a+\frac{bf}{cf-de}} \text{Ei}\left(\frac{b \log(F)}{c+dx} - \frac{bf \log(F)}{cf-de}\right)}{(de-cf)^2} + \frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)} - \frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b/(c + d*x))/(e + f*x)^2,x]
```

```
[Out] (d*F^(a + b/(c + d*x)))/(f*(d*e - c*f)) - F^(a + b/(c + d*x))/(f*(e + f*x))
- (b*d*F^(a + (b*f)/(-d*e) + c*f))/(-d*e) + c*f))*ExpIntegralEi[-((b*f*Log[F])/(-d*e) +
c*f)) + (b*Log[F])/(c + d*x))*Log[F])/(d*e - c*f)^2
```

---

**Maple [A]** time = 0.128, size = 191, normalized size = 1.7

$$\frac{\ln(F) b d F^a}{(c f - d e)^2} F^{\frac{b}{d x + c}} \left( \frac{b \ln(F)}{d x + c} + \ln(F) a - \frac{\ln(F) a c f}{c f - d e} + \frac{\ln(F) a d e}{c f - d e} - \frac{\ln(F) b f}{c f - d e} \right)^{-1} + \frac{\ln(F) b d}{(c f - d e)^2} F^{\frac{a c f - a d e + b f}{c f - d e}} \operatorname{Ei} \left( 1, -\frac{b \ln(F)}{d x + c} - \ln(F) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(f\*x+e)^2,x)

[Out] d\*ln(F)\*b/(c\*f-d\*e)^2\*F^a\*F^(b/(d\*x+c))/(b\*ln(F)/(d\*x+c)+ln(F)\*a-1/(c\*f-d\*e))\*ln(F)\*a\*c\*f+1/(c\*f-d\*e)\*ln(F)\*a\*d\*e-1/(c\*f-d\*e)\*ln(F)\*b\*f)+d\*ln(F)\*b/(c\*f-d\*e)^2\*F^((a\*c\*f-a\*d\*e+b\*f)/(c\*f-d\*e))\*Ei(1,-b\*ln(F)/(d\*x+c)-ln(F)\*a-(-ln(F)\*a\*c\*f+ln(F)\*a\*d\*e-ln(F)\*b\*f)/(c\*f-d\*e))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e)^2,x, algorithm="maxima")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e)^2, x)

---

**Fricas [A]** time = 1.6438, size = 375, normalized size = 3.23

$$\frac{(b d f x + b d e) F^{\frac{a d e - (a c + b) f}{d e - c f}} \operatorname{Ei} \left( \frac{(b d f x + b d e) \log(F)}{c d e - c^2 f + (d^2 e - c d f) x} \right) \log(F) - (c d e - c^2 f + (d^2 e - c d f) x) F^{\frac{a d x + a c + b}{d x + c}}}{d^2 e^3 - 2 c d e^2 f + c^2 e f^2 + (d^2 e^2 f - 2 c d e f^2 + c^2 f^3) x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e)^2,x, algorithm="fricas")

[Out] -((b\*d\*f\*x + b\*d\*e)\*F^((a\*d\*e - (a\*c + b)\*f)/(d\*e - c\*f))\*Ei((b\*d\*f\*x + b\*d\*e)\*log(F)/(c\*d\*e - c^2\*f + (d^2\*e - c\*d\*f)\*x))\*log(F) - (c\*d\*e - c^2\*f + (d^2\*e - c\*d\*f)\*x)\*F^((a\*d\*x + a\*c + b)/(d\*x + c))/(d^2\*e^3 - 2\*c\*d\*e^2\*f + c^2\*e\*f^2 + (d^2\*e^2\*f - 2\*c\*d\*e\*f^2 + c^2\*f^3)\*x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(f\*x+e)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e)^2, x)

$$3.399 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} dx$$

**Optimal.** Leaf size=267

$$\frac{b^2 d^2 f \log^2(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{2(de-cf)^4} - \frac{bd^2 \log(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3} + \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{bd^2 \log(F) F^{a+\frac{b}{c+dx}}}{2(de-cf)^3} - \frac{F}{2f}$$

[Out] (d^2\*F^(a + b/(c + d\*x)))/(2\*f\*(d\*e - c\*f)^2) - F^(a + b/(c + d\*x))/(2\*f\*(e + f\*x)^2) - (b\*d^2\*F^(a + b/(c + d\*x))\*Log[F])/(2\*(d\*e - c\*f)^3) + (b\*d\*F^(a + b/(c + d\*x))\*Log[F])/(2\*(d\*e - c\*f)^2\*(e + f\*x)) - (b\*d^2\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F])/(d\*e - c\*f)^3 + (b^2\*d^2\*f\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F]^2)/(2\*(d\*e - c\*f)^4)

**Rubi [A]** time = 1.91399, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2223, 6742, 2209, 2210, 2222, 2228, 2178}

$$\frac{b^2 d^2 f \log^2(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{2(de-cf)^4} - \frac{bd^2 \log(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3} + \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{bd^2 \log(F) F^{a+\frac{b}{c+dx}}}{2(de-cf)^3} - \frac{F}{2f}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b/(c + d\*x))/(e + f\*x)^3, x]

[Out] (d^2\*F^(a + b/(c + d\*x)))/(2\*f\*(d\*e - c\*f)^2) - F^(a + b/(c + d\*x))/(2\*f\*(e + f\*x)^2) - (b\*d^2\*F^(a + b/(c + d\*x))\*Log[F])/(2\*(d\*e - c\*f)^3) + (b\*d\*F^(a + b/(c + d\*x))\*Log[F])/(2\*(d\*e - c\*f)^2\*(e + f\*x)) - (b\*d^2\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F])/(d\*e - c\*f)^3 + (b^2\*d^2\*f\*F^(a - (b\*f)/(d\*e - c\*f))\*ExpIntegralEi[(b\*d\*(e + f\*x)\*Log[F])/((d\*e - c\*f)\*(c + d\*x))]\*Log[F]^2)/(2\*(d\*e - c\*f)^4)

#### Rule 2223

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(f\*(m + 1)), x] + Dist[(b\*d\*Log[F])/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(c + d\*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && ILtQ[m, -1]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]



Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2222

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbol]
:> Dist[d/f, Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Dist[(d*e - c*f
)/f, Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a,
b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 2228

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*(g_.)
+ (h_.)*(x_)), x_Symbol] :> -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h
)/(d*g - c*h) + (d*b*x)/(d*g - c*h))/x, x], x, (g + h*x)/(c + d*x)], x] /;
FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} dx &= -\frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} - \frac{(bd \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)^2} dx}{2f} \\
&= -\frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} - \frac{(bd \log(F)) \int \left( \frac{d^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^2(c+dx)^2} - \frac{2d^2 f F^{a+\frac{b}{c+dx}}}{(de-cf)^3(c+dx)} + \frac{f^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)^2} + \frac{2df^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^3(e+fx)} \right) dx}{2f} \\
&= -\frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} + \frac{(bd^3 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx}{(de-cf)^3} - \frac{(bd^2 f \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx}{(de-cf)^3} - \frac{(bd^3 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{2f(de-cf)^2} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{bd^2 F^a \operatorname{Ei}\left(\frac{b \log(F)}{c+dx}\right) \log(F)}{(de-cf)^3} - \frac{(bd^3 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}}{(de-cf)^3} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{(bd^2 \log(F)) \operatorname{Subst}\left(\int \frac{F^{a-\frac{bf}{de-cf} + \frac{bdx}{de-cf}}}{x} dx, x, \frac{e+fx}{c+dx}\right)}{(de-cf)^3} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{bd^2 F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right) \log(F)}{(de-cf)^3} - \frac{(b^2 d^3 f \log(F))}{2f(de-cf)^3} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} - \frac{bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{bd^2 F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} - \frac{bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{bd^2 F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3} \\
&= \frac{d^2 F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} - \frac{bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{2(de-cf)^2(e+fx)} - \frac{bd^2 F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx) \log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3}
\end{aligned}$$

**Mathematica [F]** time = 0.65279, size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(a + b/(c + d\*x))/(e + f\*x)^3,x]

[Out] Integrate[F^(a + b/(c + d\*x))/(e + f\*x)^3, x]

**Maple [A]** time = 0.15, size = 506, normalized size = 1.9

$$-\frac{(\ln(F))^2 b^2 d^2 f F^a}{2 (cf - de)^4} F^{\frac{b}{dx+c}} \left( \frac{b \ln(F)}{dx+c} + \ln(F) a - \frac{\ln(F) acf}{cf - de} + \frac{\ln(F) ade}{cf - de} - \frac{\ln(F) b f}{cf - de} \right)^{-2} - \frac{(\ln(F))^2 b^2 d^2 f F^a}{2 (cf - de)^4} F^{\frac{b}{dx+c}} \left( \frac{b \ln(F)}{dx+c} + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b/(d\*x+c))/(f\*x+e)^3,x)

```
[Out] -1/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)
*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)^2
-1/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)
*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)-1
/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln
(F)/(d*x+c)-ln(F)*a-(-ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))-b*d^2*ln
(F)/(c*f-d*e)^3*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)*a-1/(c*f-d*e)*ln(
F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)-b*d^2*ln(F)/(c*f-d*
e)^3*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a-(-ln(F)*
a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(F^(a + b/(d*x + c))/(f*x + e)^3, x)
```

**Fricas [B]** time = 1.63569, size = 1116, normalized size = 4.18

$$\frac{\left(\left(b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f\right) \log (F)^2 - 2\left(b d^3 e^3 - b c d^2 e^2 f + \left(b d^3 e f^2 - b c d^2 f^3\right) x^2 + 2\left(b d^3 e^2 f - b c d^2 e f^2\right) x\right) \log (F)}{2\left(d^4 e^6 - 4 c d^3 e^5 f + 6 c^2 d^2 e^4 f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b/(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*log(F)^2 - 2*(b
*d^3*e^3 - b*c*d^2*e^2*f + (b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 2*(b*d^3*e^2*f
- b*c*d^2*e*f^2)*x)*log(F))*F^((a*d*e - (a*c + b)*f)/(d*e - c*f))*Ei((b*d*
f*x + b*d*e)*log(F)/(c*d*e - c^2*f + (d^2*e - c*d*f)*x)) + (2*c*d^3*e^3 - 5
*c^2*d^2*e^2*f + 4*c^3*d*e*f^2 - c^4*f^3 + (d^4*e^2*f - 2*c*d^3*e*f^2 + c^2
*d^2*f^3)*x^2 + 2*(d^4*e^3 - 2*c*d^3*e^2*f + c^2*d^2*e*f^2)*x - (b*c*d^2*e^
2*f - b*c^2*d*e*f^2 + (b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + (b*d^3*e^2*f - b*c^
2*d*f^3)*x)*log(F))*F^((a*d*x + a*c + b)/(d*x + c))/(d^4*e^6 - 4*c*d^3*e^5
*f + 6*c^2*d^2*e^4*f^2 - 4*c^3*d*e*f^3 + c^4*e^2*f^4 + (d^4*e^4*f^2 - 4*c
*d^3*e^3*f^3 + 6*c^2*d^2*e^2*f^4 - 4*c^3*d*e*f^5 + c^4*f^6)*x^2 + 2*(d^4*e^
5*f - 4*c*d^3*e^4*f^2 + 6*c^2*d^2*e^3*f^3 - 4*c^3*d*e^2*f^4 + c^4*e*f^5)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b/(d*x+c))/(f*x+e)**3,x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e)^3,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e)^3, x)

$$3.400 \quad \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4} dx$$

**Optimal.** Leaf size=460

$$\frac{b^3 d^3 f^2 \log^3(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{6(de-cf)^6} + \frac{b^2 d^3 f \log^2(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^5} - \frac{b^2 d^2 f \log^2(F) F^{a+\frac{b}{c+dx}}}{6(e+fx)(de-cf)^4} + \frac{b^2 d^3 f \log^3(F) F^{a+\frac{b}{c+dx}}}{6(de-cf)^4}$$

```
[Out] (d^3*F^(a + b/(c + d*x)))/(3*f*(d*e - c*f)^3) - F^(a + b/(c + d*x))/(3*f*(e
+ f*x)^3) - (5*b*d^3*F^(a + b/(c + d*x))*Log[F])/(6*(d*e - c*f)^4) + (b*d*
F^(a + b/(c + d*x))*Log[F])/(6*(d*e - c*f)^2*(e + f*x)^2) + (2*b*d^2*F^(a +
b/(c + d*x))*Log[F])/(3*(d*e - c*f)^3*(e + f*x)) - (b*d^3*F^(a - (b*f)/(d*
e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))]*Log
[F])/(d*e - c*f)^4 + (b^2*d^3*f*F^(a + b/(c + d*x))*Log[F]^2)/(6*(d*e - c*f
)^5) - (b^2*d^2*f*F^(a + b/(c + d*x))*Log[F]^2)/(6*(d*e - c*f)^4*(e + f*x))
+ (b^2*d^3*f*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F]
)/((d*e - c*f)*(c + d*x))]*Log[F]^2)/(d*e - c*f)^5 - (b^3*d^3*f^2*F^(a - (b
*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x
))]*Log[F]^3)/(6*(d*e - c*f)^6)
```

**Rubi [A]** time = 3.75106, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2223, 6742, 2209, 2210, 2222, 2228, 2178}

$$\frac{b^3 d^3 f^2 \log^3(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{6(de-cf)^6} + \frac{b^2 d^3 f \log^2(F) F^{a-\frac{bf}{de-cf}} \operatorname{Ei}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^5} - \frac{b^2 d^2 f \log^2(F) F^{a+\frac{b}{c+dx}}}{6(e+fx)(de-cf)^4} + \frac{b^2 d^3 f \log^3(F) F^{a+\frac{b}{c+dx}}}{6(de-cf)^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b/(c + d*x))/(e + f*x)^4, x]
```

```
[Out] (d^3*F^(a + b/(c + d*x)))/(3*f*(d*e - c*f)^3) - F^(a + b/(c + d*x))/(3*f*(e
+ f*x)^3) - (5*b*d^3*F^(a + b/(c + d*x))*Log[F])/(6*(d*e - c*f)^4) + (b*d*
F^(a + b/(c + d*x))*Log[F])/(6*(d*e - c*f)^2*(e + f*x)^2) + (2*b*d^2*F^(a +
b/(c + d*x))*Log[F])/(3*(d*e - c*f)^3*(e + f*x)) - (b*d^3*F^(a - (b*f)/(d*
e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))]*Log
[F])/(d*e - c*f)^4 + (b^2*d^3*f*F^(a + b/(c + d*x))*Log[F]^2)/(6*(d*e - c*f
)^5) - (b^2*d^2*f*F^(a + b/(c + d*x))*Log[F]^2)/(6*(d*e - c*f)^4*(e + f*x))
+ (b^2*d^3*f*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F]
)/((d*e - c*f)*(c + d*x))]*Log[F]^2)/(d*e - c*f)^5 - (b^3*d^3*f^2*F^(a - (b
*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x
))]*Log[F]^3)/(6*(d*e - c*f)^6)
```

#### Rule 2223

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] := Simp[((e + f*x)^(m + 1)*F^(a + b/(c + d*x)))/(f*(m + 1)), x] + D
ist[(b*d*Log[F])/(f*(m + 1)), Int[((e + f*x)^(m + 1)*F^(a + b/(c + d*x)))/(
c + d*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& ILtQ[m, -1]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2222

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbol] := Dist[d/f, Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Dist[(d*e - c*f)/f, Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 2228

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.) + (h_.)*(x_))), x_Symbol] := -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h)/(d*g - c*h) + (d*b*x)/(d*g - c*h))/x, x], x, (g + h*x)/(c + d*x)], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4} dx &= -\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{(bd \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)^3} dx}{3f} \\
&= -\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{(bd \log(F)) \int \left( \frac{d^3 F^{a+\frac{b}{c+dx}}}{(de-cf)^3(c+dx)^2} - \frac{3d^3 f F^{a+\frac{b}{c+dx}}}{(de-cf)^4(c+dx)} + \frac{f^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)^3} + \frac{2df^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^3(e+fx)^2} + \frac{3d^2 f^2 F^{a+\frac{b}{c+dx}}}{(de-cf)^4} \right) dx}{3f} \\
&= -\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} + \frac{(bd^4 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx}{(de-cf)^4} - \frac{(bd^3 f \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{e+fx} dx}{(de-cf)^4} - \frac{(bd^4 \log(F)) \int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx}{3f(de-cf)^3} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} - \frac{bd^3 F^{a+\frac{b}{c+dx}} \text{Ei}\left(\frac{b \log(F)}{c+dx}\right) \log(F)}{(de-cf)^4} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} - \frac{(bd^3 \log(F)) \text{Subst}\left(\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx, \frac{b \log(F)}{c+dx}\right)}{(de-cf)^4} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} - \frac{bd^3 F^{a+\frac{b}{c+dx}} \text{Ei}\left(\frac{bd(e+fx)}{(de-cf)^2}\right)}{(de-cf)^4} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)} \\
&= \frac{d^3 F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3 F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^4} + \frac{bd F^{a+\frac{b}{c+dx}} \log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2 F^{a+\frac{b}{c+dx}} \log(F)}{3(de-cf)^3(e+fx)}
\end{aligned}$$

**Mathematica [F]** time = 0.627395, size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(a + b/(c + d\*x))/(e + f\*x)^4, x]

[Out] Integrate[F^(a + b/(c + d\*x))/(e + f\*x)^4, x]

**Maple [B]** time = 0.2, size = 922, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b/(d*x+c))/(f*x+e)^4,x)`

[Out] 
$$\begin{aligned} & b^2 d^3 \ln(F)^2 f / (c f - d e)^5 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / \\ & (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f)^2 + b^2 d^3 \ln(F)^2 f / (c f - d e)^5 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f + b^2 d^3 \ln(F)^2 f / (c f - d e)^5 F^{((a c f - a d e + b f) / (c f - d e))} * E i(1, -b \ln(F) / (d x + c) - \ln(F) a - (-\ln(F) a c f + \ln(F) a d e - \ln(F) b f) / (c f - d e)) + b d^3 \ln(F) / (c f - d e)^4 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f + b d^3 \ln(F) / (c f - d e)^4 F^{((a c f - a d e + b f) / (c f - d e))} * E i(1, -b \ln(F) / (d x + c) - \ln(F) a - (-\ln(F) a c f + \ln(F) a d e - \ln(F) b f) / (c f - d e)) + 1/3 b^3 d^3 \ln(F)^3 f^2 / (c f - d e)^6 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f)^3 + 1/6 b^3 d^3 \ln(F)^3 f^2 / (c f - d e)^6 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f)^2 + 1/6 b^3 d^3 \ln(F)^3 f^2 / (c f - d e)^6 F^a F^{b/(d x + c)} / (b \ln(F) / (d x + c) + \ln(F))^{a-1} / (c f - d e) \ln(F) a c f + 1 / (c f - d e) \ln(F) a d e - 1 / (c f - d e) \ln(F) b f + 1/6 b^3 d^3 \ln(F)^3 f^2 / (c f - d e)^6 F^{((a c f - a d e + b f) / (c f - d e))} * E i(1, -b \ln(F) / (d x + c) - \ln(F) a - (-\ln(F) a c f + \ln(F) a d e - \ln(F) b f) / (c f - d e)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a + \frac{b}{dx+c}}}{(fx + e)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(f*x+e)^4,x, algorithm="maxima")`

[Out] `integrate(F^(a + b/(d*x + c))/(f*x + e)^4, x)`

**Fricas [B]** time = 1.81402, size = 2736, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b/(d*x+c))/(f*x+e)^4,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/6 * (((b^3 d^3 f^5 x^3 + 3 b^3 d^3 e f^4 x^2 + 3 b^3 d^3 e^2 f^3 x + b^3 d^3 e^3 f^2) * \log(F)^3 - 6 (b^2 d^4 e^4 f - b^2 c d^3 e^3 f^2 + (b^2 d^4 e f^4 - b^2 c d^3 f^5) x^3 + 3 (b^2 d^4 e^2 f^3 - b^2 c d^3 e f^4) x^2 + 3 (b^2 d^4 e^3 f^2 - b^2 c d^3 e^2 f^3) x) * \log(F)^2 + 6 (b d^5 e^5 - 2 b c d^4 e^4 f + b c^2 d^3 e^3 f^2 + (b d^5 e^2 f^3 - 2 b c d^4 e f^4 + b c^2 d^3 f^5) x^3 + 3 (b d^5 e^3 f^2 - 2 b c d^4 e^2 f^3 + b c^2 d^3 e f^4) x^2 + 3 (b d^5 e^4 f - 2 b c d^4 e^3 f^2 + b c^2 d^3 e^2 f^3) x) * \log(F)) * F^{(a d e - (a \end{aligned}$$



```

*c + b)*f)/(d*e - c*f))*Ei((b*d*f*x + b*d*e)*log(F)/(c*d*e - c^2*f + (d^2*e
- c*d*f)*x)) - (6*c*d^5*e^5 - 24*c^2*d^4*e^4*f + 38*c^3*d^3*e^3*f^2 - 30*c
^4*d^2*e^2*f^3 + 12*c^5*d*e*f^4 - 2*c^6*f^5 + 2*(d^6*e^3*f^2 - 3*c*d^5*e^2*
f^3 + 3*c^2*d^4*e*f^4 - c^3*d^3*f^5)*x^3 + 6*(d^6*e^4*f - 3*c*d^5*e^3*f^2 +
3*c^2*d^4*e^2*f^3 - c^3*d^3*e*f^4)*x^2 + (b^2*c*d^3*e^3*f^2 - b^2*c^2*d^2*
e^2*f^3 + (b^2*d^4*e*f^4 - b^2*c*d^3*f^5)*x^3 + (2*b^2*d^4*e^2*f^3 - b^2*c*
d^3*e*f^4 - b^2*c^2*d^2*f^5)*x^2 + (b^2*d^4*e^3*f^2 + b^2*c*d^3*e^2*f^3 - 2
*b^2*c^2*d^2*e*f^4)*x)*log(F)^2 + 6*(d^6*e^5 - 3*c*d^5*e^4*f + 3*c^2*d^4*e^
3*f^2 - c^3*d^3*e^2*f^3)*x - (6*b*c*d^4*e^4*f - 13*b*c^2*d^3*e^3*f^2 + 8*b*
c^3*d^2*e^2*f^3 - b*c^4*d*e*f^4 + 5*(b*d^5*e^2*f^3 - 2*b*c*d^4*e*f^4 + b*c^
2*d^3*f^5)*x^3 + (11*b*d^5*e^3*f^2 - 18*b*c*d^4*e^2*f^3 + 3*b*c^2*d^3*e*f^4
+ 4*b*c^3*d^2*f^5)*x^2 + (6*b*d^5*e^4*f - 2*b*c*d^4*e^3*f^2 - 15*b*c^2*d^3
*e^2*f^3 + 12*b*c^3*d^2*e*f^4 - b*c^4*d*f^5)*x)*log(F))*F^((a*d*x + a*c + b
)/(d*x + c)))/(d^6*e^9 - 6*c*d^5*e^8*f + 15*c^2*d^4*e^7*f^2 - 20*c^3*d^3*e^
6*f^3 + 15*c^4*d^2*e^5*f^4 - 6*c^5*d*e^4*f^5 + c^6*e^3*f^6 + (d^6*e^6*f^3 -
6*c*d^5*e^5*f^4 + 15*c^2*d^4*e^4*f^5 - 20*c^3*d^3*e^3*f^6 + 15*c^4*d^2*e^2
*f^7 - 6*c^5*d*e*f^8 + c^6*f^9)*x^3 + 3*(d^6*e^7*f^2 - 6*c*d^5*e^6*f^3 + 15
*c^2*d^4*e^5*f^4 - 20*c^3*d^3*e^4*f^5 + 15*c^4*d^2*e^3*f^6 - 6*c^5*d*e^2*f^
7 + c^6*e*f^8)*x^2 + 3*(d^6*e^8*f - 6*c*d^5*e^7*f^2 + 15*c^2*d^4*e^6*f^3 -
20*c^3*d^3*e^5*f^4 + 15*c^4*d^2*e^4*f^5 - 6*c^5*d*e^3*f^6 + c^6*e^2*f^7)*x)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b/(d\*x+c))/(f\*x+e)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b/(d\*x+c))/(f\*x+e)^4,x, algorithm="giac")

[Out] integrate(F^(a + b/(d\*x + c))/(f\*x + e)^4, x)

### 3.401 $\int e^{\frac{e}{c+dx}}(a+bx)^4 dx$

**Optimal.** Leaf size=346

$$\frac{4b^3e^4(bc-ad)\Gamma\left(-4, -\frac{e}{c+dx}\right)}{d^5} - \frac{b^4e^5\Gamma\left(-5, -\frac{e}{c+dx}\right)}{d^5} - \frac{b^2e^3(bc-ad)^2\text{Ei}\left(\frac{e}{c+dx}\right)}{d^5} + \frac{b^2e^2(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^5}$$

[Out]  $((b*c - a*d)^4 * E^{(e/(c + d*x))} * (c + d*x)) / d^5 - (2*b*(b*c - a*d)^3 * e * E^{(e/(c + d*x))} * (c + d*x)) / d^5 + (b^2*(b*c - a*d)^2 * e^2 * E^{(e/(c + d*x))} * (c + d*x)) / d^5 - (2*b*(b*c - a*d)^3 * E^{(e/(c + d*x))} * (c + d*x)^2) / d^5 + (b^2*(b*c - a*d)^2 * e * E^{(e/(c + d*x))} * (c + d*x)^2) / d^5 + (2*b^2*(b*c - a*d)^2 * E^{(e/(c + d*x))} * (c + d*x)^3) / d^5 - ((b*c - a*d)^4 * e * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 + (2*b*(b*c - a*d)^3 * e^2 * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 - (b^2*(b*c - a*d)^2 * e^3 * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 - (b^4 * e^5 * \Gamma[-5, -(e/(c + d*x))]) / d^5 - (4*b^3*(b*c - a*d) * e^4 * \Gamma[-4, -(e/(c + d*x))]) / d^5$

**Rubi [A]** time = 0.361779, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2226, 2206, 2210, 2214, 2218}

$$\frac{4b^3e^4(bc-ad)\Gamma\left(-4, -\frac{e}{c+dx}\right)}{d^5} - \frac{b^4e^5\Gamma\left(-5, -\frac{e}{c+dx}\right)}{d^5} - \frac{b^2e^3(bc-ad)^2\text{Ei}\left(\frac{e}{c+dx}\right)}{d^5} + \frac{b^2e^2(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^5}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))\*(a + b\*x)^4, x]

[Out]  $((b*c - a*d)^4 * E^{(e/(c + d*x))} * (c + d*x)) / d^5 - (2*b*(b*c - a*d)^3 * e * E^{(e/(c + d*x))} * (c + d*x)) / d^5 + (b^2*(b*c - a*d)^2 * e^2 * E^{(e/(c + d*x))} * (c + d*x)) / d^5 - (2*b*(b*c - a*d)^3 * E^{(e/(c + d*x))} * (c + d*x)^2) / d^5 + (b^2*(b*c - a*d)^2 * e * E^{(e/(c + d*x))} * (c + d*x)^2) / d^5 + (2*b^2*(b*c - a*d)^2 * E^{(e/(c + d*x))} * (c + d*x)^3) / d^5 - ((b*c - a*d)^4 * e * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 + (2*b*(b*c - a*d)^3 * e^2 * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 - (b^2*(b*c - a*d)^2 * e^3 * \text{ExpIntegralEi}[e/(c + d*x)]) / d^5 - (b^4 * e^5 * \Gamma[-5, -(e/(c + d*x))]) / d^5 - (4*b^3*(b*c - a*d) * e^4 * \Gamma[-4, -(e/(c + d*x))]) / d^5$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a \* ExpIntegralEi[b\*(c + d\*x)^n \* Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n))/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

### Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

### Rubi steps

$$\begin{aligned}
 \int e^{\frac{e}{c+dx}}(a+bx)^4 dx &= \int \left( \frac{(-bc+ad)^4 e^{\frac{e}{c+dx}}}{d^4} - \frac{4b(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)}{d^4} + \frac{6b^2(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)^2}{d^4} - \frac{4b^3(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^3}{d^4} + \frac{b^4 e^{\frac{e}{c+dx}}(c+dx)^4}{d^4} \right) dx \\
 &= \frac{b^4 \int e^{\frac{e}{c+dx}}(c+dx)^4 dx}{d^4} - \frac{(4b^3(bc-ad)) \int e^{\frac{e}{c+dx}}(c+dx)^3 dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int e^{\frac{e}{c+dx}}(c+dx)^2 dx}{d^4} - \frac{4b^3(bc-ad) \int e^{\frac{e}{c+dx}}(c+dx) dx}{d^4} + \frac{b^4 \int e^{\frac{e}{c+dx}} dx}{d^4} \\
 &= \frac{(bc-ad)^4 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)^2}{d^5} + \frac{2b^2(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)^3}{d^5} - \frac{b^4 e^5 \Gamma(-5, -\frac{e}{c+dx})}{d^5} \\
 &= \frac{(bc-ad)^4 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)^2}{d^5} + \frac{b^2(bc-ad)^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} \\
 &= \frac{(bc-ad)^4 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e e^{\frac{e}{c+dx}}(c+dx)}{d^5} + \frac{b^2(bc-ad)^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} \\
 &= \frac{(bc-ad)^4 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e e^{\frac{e}{c+dx}}(c+dx)}{d^5} + \frac{b^2(bc-ad)^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.487314, size = 468, normalized size = 1.35

$$\frac{dxe^{\frac{e}{c+dx}}(120a^2b^2d^2(-4ce+2d^2x^2+dex+e^2)+240a^3bd^3(dx+e)+120a^4d^4+20ab^3d(18c^2e-2ce(3dx+5e)+2d^2ex^2+6d^3x^3-2c^2e(5e+3d*x))+b^4(-96c^3e+e^4+d^3e^3x+2d^2e^2*x^2+6d^3e*x^3+24d^4*x^4+2c^2e(43e+18d*x)-2c^2e(9e^2+7d^2e*x+8d^2*x^2)))-e(120a^4d^4-240a^3bd^3(2c-e)+120a^2b^2d^2(6c^2-6c^2e+e^2)-20a^3bd^3(24c^3-36c^2e+12c^2e^2-e^3)+b^4(120c^4-240c^3e+120c^2e^2-20c^2e^3+e^4))\text{ExpIntegralEi}[e/(c+d*x)]}{(120d^5)}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x))\*(a + b\*x)^4, x]

[Out] (c\*(120\*a^4\*d^4 - 240\*a^3\*b\*d^3\*(c - e) + 120\*a^2\*b^2\*d^2\*(2\*c^2 - 5\*c\*e + e^2) - 20\*a\*b^3\*d\*(6\*c^3 - 26\*c^2\*e + 11\*c\*e^2 - e^3) + b^4\*(24\*c^4 - 154\*c^3\*e + 102\*c^2\*e^2 - 19\*c\*e^3 + e^4))\*E^(e/(c + d\*x)))/(120\*d^5) + (d\*E^(e/(c + d\*x))\*x\*(120\*a^4\*d^4 + 240\*a^3\*b\*d^3\*(e + d\*x) + 120\*a^2\*b^2\*d^2\*(-4\*c\*e + e^2 + d\*e\*x + 2\*d^2\*x^2) + 20\*a\*b^3\*d\*(18\*c^2\*e + e^3 + d\*e^2\*x + 2\*d^2\*e\*x^2 + 6\*d^3\*x^3 - 2\*c\*e\*(5\*e + 3\*d\*x)) + b^4\*(-96\*c^3\*e + e^4 + d\*e^3\*x + 2\*d^2\*e^2\*x^2 + 6\*d^3\*e\*x^3 + 24\*d^4\*x^4 + 2\*c^2\*e\*(43\*e + 18\*d\*x) - 2\*c^2\*e\*(9\*e^2 + 7\*d^2\*e\*x + 8\*d^2\*x^2))) - e\*(120\*a^4\*d^4 - 240\*a^3\*b\*d^3\*(2\*c - e) + 120\*a^2\*b^2\*d^2\*(6\*c^2 - 6\*c^2\*e + e^2) - 20\*a^3\*b\*d^3\*(24\*c^3 - 36\*c^2\*e + 12\*c^2\*e^2 - e^3) + b^4\*(120\*c^4 - 240\*c^3\*e + 120\*c^2\*e^2 - 20\*c^2\*e^3 + e^4))\*ExpIntegralEi[e/(c + d\*x)]/(120\*d^5)

**Maple [B]** time = 0.013, size = 1146, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(e/(d*x+c))*(b*x+a)^4,x)`

[Out] 
$$\begin{aligned} & -1/d*e*(a^4*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b^4/d^4*e^4*(-1/5*(d*x+c)^5/e^5*\exp(e/(d*x+c))-1/20*(d*x+c)^4/e^4*\exp(e/(d*x+c))-1/60*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/120*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/120*(d*x+c)/e*\exp(e/(d*x+c))-1/120*Ei(1,-e/(d*x+c)))+b^4/d^4*c^4*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+4*b^3/d^3*e^3*a*(-1/4*(d*x+c)^4/e^4*\exp(e/(d*x+c))-1/12*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/24*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/24*(d*x+c)/e*\exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+c)))-4*b^4/d^4*e^3*c*(-1/4*(d*x+c)^4/e^4*\exp(e/(d*x+c))-1/12*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/24*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/24*(d*x+c)/e*\exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+c)))+6*b^2/d^2*e^2*a^2*(-1/3*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/6*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/6*(d*x+c)/e*\exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+6*b^4/d^4*e^2*c^2*(-1/3*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/6*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/6*(d*x+c)/e*\exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+4*b/d*e*a^3*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))-4*b^4/d^4*e*c^3*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))-4*b/d*c*a^3*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+6*b^2/d^2*c^2*a^2*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))-4*b^3/d^3*c^3*a*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))-12*b^3/d^3*e^2*c*a*(-1/3*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/6*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/6*(d*x+c)/e*\exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))-12*b^2/d^2*e*c*a^2*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))+12*b^3/d^3*e*c^2*a*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c))) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(24b^4d^4x^5 + 6(20ab^3d^4 + b^4d^3e)x^4 + 2(120a^2b^2d^4 + 20ab^3d^3e - (8cd^2e - d^2e^2)b^4)x^3 + (240a^3bd^4 + 120a^2b^2d^3e - 20$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a)^4,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/120*(24*b^4*d^4*x^5 + 6*(20*a*b^3*d^4 + b^4*d^3*e)*x^4 + 2*(120*a^2*b^2*d^4 + 20*a*b^3*d^3*e - (8*c*d^2*e - d^2*e^2)*b^4)*x^3 + (240*a^3*b*d^4 + 120*a^2*b^2*d^3*e - 20*(6*c*d^2*e - d^2*e^2)*a*b^3 + (36*c^2*d*e - 14*c*d*e^2 + d*e^3)*b^4)*x^2 + (120*a^4*d^4 + 240*a^3*b*d^3*e - 120*(4*c*d^2*e - d^2*e^2)*a^2*b^2 + 20*(18*c^2*d*e - 10*c*d*e^2 + d*e^3)*a*b^3 - (96*c^3*e - 86*c^2*e^2 + 18*c*e^3 - e^4)*b^4)*x)*e^(e/(d*x + c))/d^4 + integrate(-1/120*(240*a^3*b*c^2*d^3*e - 120*(4*c^3*d^2*e - c^2*d^2*e^2)*a^2*b^2 + 20*(18*c^4*d*e - 10*c^3*d*e^2 + c^2*d*e^3)*a*b^3 - (96*c^5*e - 86*c^4*e^2 + 18*c^3*e^3 - c^2*e^4)*b^4 - (120*a^4*d^5*e - 240*(2*c*d^4*e - d^4*e^2)*a^3*b + 120*(6*c^2*d^3*e - 6*c*d^3*e^2 + d^3*e^3)*a^2*b^2 - 20*(24*c^3*d^2*e - 36*c^2*d^2*e^2 + 12*c*d^2*e^3 - d^2*e^4)*a*b^3 + (120*c^4*d*e - 240*c^3*d*e^2 + 120*c^2*d*e^3 - 20*c*d*e^4 + d*e^5)*b^4)*x)*e^(e/(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4), x) \end{aligned}$$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))\*(b\*x+a)^4,x, algorithm="fricas")

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx)^4 e^{\frac{e}{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))\*(b\*x+a)\*\*4,x)

[Out] Integral((a + b\*x)\*\*4\*exp(e/(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^4 e^{\left(\frac{e}{dx+c}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))\*(b\*x+a)^4,x, algorithm="giac")

[Out] integrate((b\*x + a)^4\*e^(e/(d\*x + c)), x)

### 3.402 $\int e^{\frac{e}{c+dx}}(a+bx)^3 dx$

**Optimal.** Leaf size=320

$$\frac{b^3 e^4 \Gamma(-4, -\frac{e}{c+dx})}{d^4} + \frac{b^2 e^3 (bc - ad) \text{Ei}(\frac{e}{c+dx})}{2d^4} - \frac{b^2 e^2 (c + dx)(bc - ad) e^{\frac{e}{c+dx}}}{2d^4} - \frac{b^2 e (c + dx)^2 (bc - ad) e^{\frac{e}{c+dx}}}{2d^4} - \frac{b^2 (c + dx)^3 e^{\frac{e}{c+dx}}}{2d^4}$$

[Out] -(((b\*c - a\*d)^3\*E^(e/(c + d\*x))\*(c + d\*x))/d^4) + (3\*b\*(b\*c - a\*d)^2\*e\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^4) - (b^2\*(b\*c - a\*d)\*e^2\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^4) + (3\*b\*(b\*c - a\*d)^2\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^4) - (b^2\*(b\*c - a\*d)\*e\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^4) - (b^2\*(b\*c - a\*d)\*E^(e/(c + d\*x))\*(c + d\*x)^3)/d^4 + ((b\*c - a\*d)^3\*e\*ExpIntegralEi[e/(c + d\*x)])/d^4 - (3\*b\*(b\*c - a\*d)^2\*e^2\*ExpIntegralEi[e/(c + d\*x)])/d^4 + (b^2\*(b\*c - a\*d)\*e^3\*ExpIntegralEi[e/(c + d\*x)])/d^4 + (b^3\*e^4\*Gamma[-4, -(e/(c + d\*x))])/d^4

**Rubi [A]** time = 0.31705, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2226, 2206, 2210, 2214, 2218}

$$\frac{b^3 e^4 \Gamma(-4, -\frac{e}{c+dx})}{d^4} + \frac{b^2 e^3 (bc - ad) \text{Ei}(\frac{e}{c+dx})}{2d^4} - \frac{b^2 e^2 (c + dx)(bc - ad) e^{\frac{e}{c+dx}}}{2d^4} - \frac{b^2 e (c + dx)^2 (bc - ad) e^{\frac{e}{c+dx}}}{2d^4} - \frac{b^2 (c + dx)^3 e^{\frac{e}{c+dx}}}{2d^4}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))\*(a + b\*x)^3, x]

[Out] -(((b\*c - a\*d)^3\*E^(e/(c + d\*x))\*(c + d\*x))/d^4) + (3\*b\*(b\*c - a\*d)^2\*e\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^4) - (b^2\*(b\*c - a\*d)\*e^2\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^4) + (3\*b\*(b\*c - a\*d)^2\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^4) - (b^2\*(b\*c - a\*d)\*e\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^4) - (b^2\*(b\*c - a\*d)\*E^(e/(c + d\*x))\*(c + d\*x)^3)/d^4 + ((b\*c - a\*d)^3\*e\*ExpIntegralEi[e/(c + d\*x)])/d^4 - (3\*b\*(b\*c - a\*d)^2\*e^2\*ExpIntegralEi[e/(c + d\*x)])/d^4 + (b^2\*(b\*c - a\*d)\*e^3\*ExpIntegralEi[e/(c + d\*x)])/d^4 + (b^3\*e^4\*Gamma[-4, -(e/(c + d\*x))])/d^4

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*F^(a + b*(c + d*x)^n)/(d*(m + 1)), x] - Dist[(b*n*Log[F])/(m + 1), Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))
```

**Rule 2218**

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int e^{\frac{e}{c+dx}}(a+bx)^3 dx &= \int \left( \frac{(-bc+ad)^3 e^{\frac{e}{c+dx}}}{d^3} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{3b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{d^3} + \frac{b^3 e^{\frac{e}{c+dx}}(c+dx)^3}{d^3} \right) dx \\ &= \frac{b^3 \int e^{\frac{e}{c+dx}}(c+dx)^3 dx}{d^3} - \frac{(3b^2(bc-ad) \int e^{\frac{e}{c+dx}}(c+dx)^2 dx)}{d^3} + \frac{(3b(bc-ad)^2 \int e^{\frac{e}{c+dx}}(c+dx) dx)}{d^3} \\ &= -\frac{(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)^2}{2d^4} - \frac{b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^3}{d^4} + \frac{b^3 e^4 \Gamma(-4, -\frac{e}{c+dx})}{d^4} \\ &= -\frac{(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{2d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)^2}{2d^4} - \frac{b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^3}{2d^4} \\ &= -\frac{(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{2d^4} - \frac{b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{2d^4} + \frac{3b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{2d^4} \\ &= -\frac{(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{2d^4} - \frac{b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{2d^4} + \frac{3b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{2d^4} \end{aligned}$$

**Mathematica [A]** time = 0.316403, size = 292, normalized size = 0.91

$$\frac{dxe^{\frac{e}{c+dx}}(36a^2bd^2(dx+e)+24a^3d^3+12ab^2d(-4ce+2d^2x^2+dex+e^2))+b^3(18c^2e-2ce(3dx+5e)+2d^2ex^2+6d^3x^3)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(e/(c+d*x))*(a+b*x)^3,x]
```

```
[Out] -(c*(-24*a^3*d^3+36*a^2*b*d^2*(c-e)-12*a*b^2*d*(2*c^2-5*c*e+e^2))+b^3*(6*c^3-26*c^2*e+11*c*e^2-e^3))*E^(e/(c+d*x))/(24*d^4)+(d*E^(e/(c+d*x))*x*(24*a^3*d^3+36*a^2*b*d^2*(e+d*x)+12*a*b^2*d*(-4*c*e+e^2+d*e*x+2*d^2*x^2))+b^3*(18*c^2*e+e^3+d*e^2*x+2*d^2*e*x^2+6*d^3*x^3-2*c*e*(5*e+3*d*x)))-e*(24*a^3*d^3+36*a^2*b*d^2*(-2*c+e)+12*a*b^2*d*(6*c^2-6*c*e+e^2))+b^3*(-24*c^3+36*c^2*e-12*c*e^2+e^3))*ExpIntegralEi[e/(c+d*x)]/(24*d^4)
```

**Maple [B]** time = 0.01, size = 682, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(e/(d*x+c))*(b*x+a)^3,x)`

[Out] 
$$\begin{aligned} & -1/d*e*(a^3*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b^3/d^3*e^3*(-1/4*(d*x+c)^4/e^4*\exp(e/(d*x+c))-1/12*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/24*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/24*(d*x+c)/e*\exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+c)))- \\ & b^3/d^3*c^3*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+3*b^2/d^2*e^2*a*(-1/3*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/6*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/6*(d*x+c)/e*\exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))-3*b^3/d^3*e^2*c*(-1/3*(d*x+c)^3/e^3*\exp(e/(d*x+c))-1/6*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/6*(d*x+c)/e*\exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+3*b/d*e*a^2*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))+3*b^3/d^3*e*c^2*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))-3*b/d*c*a^2*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+3*b^2/d^2*c^2*a*(-(d*x+c)/e*\exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))-6*b^2/d^2*e*c*a*(-1/2*\exp(e/(d*x+c))*(d*x+c)^2/e^2-1/2*(d*x+c)/e*\exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c))) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(6b^3d^3x^4 + 2(12ab^2d^3 + b^3d^2e)x^3 + (36a^2bd^3 + 12ab^2d^2e - (6cde - de^2)b^3)x^2 + (24a^3d^3 + 36a^2bd^2e - 12(4cde - de^2))x + 24d^3)}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/24*(6*b^3*d^3*x^4 + 2*(12*a*b^2*d^3 + b^3*d^2*e)*x^3 + (36*a^2*b*d^3 + 12*a*b^2*d^2*e - (6*c*d*e - d*e^2)*b^3)*x^2 + (24*a^3*d^3 + 36*a^2*b*d^2*e - 12*(4*c*d*e - d*e^2)*a*b^2 + (18*c^2*e - 10*c*e^2 + e^3)*b^3)*x)*e^{e/(d*x+c)}/d^3 + \\ & \text{integrate}(-1/24*(36*a^2*b*c^2*d^2*e - 12*(4*c^3*d*e - c^2*d*e^2)*a*b^2 + (18*c^4*e - 10*c^3*e^2 + c^2*e^3)*b^3 - (24*a^3*d^4*e - 36*(2*c*d^3*e - d^3*e^2)*a^2*b + 12*(6*c^2*d^2*e - 6*c*d^2*e^2 + d^2*e^3)*a*b^2 - (24*c^3*d*e - 36*c^2*d*e^2 + 12*c*d*e^3 - d*e^4)*b^3)*x)*e^{e/(d*x+c)}/(d^5*x^2 + 2*c*d^4*x + c^2*d^3), x) \end{aligned}$$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx)^3 e^{\frac{e}{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(exp(e/(d*x+c))*(b*x+a)**3,x)
```

```
[Out] Integral((a + b*x)**3*exp(e/(c + d*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 e^{\left(\frac{e}{dx+c}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*e^(e/(d*x + c)), x)
```

### 3.403 $\int e^{\frac{e}{c+dx}}(a+bx)^2 dx$

**Optimal.** Leaf size=255

$$\frac{be^2(bc-ad)\text{Ei}\left(\frac{e}{c+dx}\right)}{d^3} - \frac{e(bc-ad)^2\text{Ei}\left(\frac{e}{c+dx}\right)}{d^3} - \frac{be(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^3} - \frac{b(c+dx)^2(bc-ad)e^{\frac{e}{c+dx}}}{d^3} + \frac{(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^3}$$

[Out]  $((b*c - a*d)^2 * E^{(e/(c + d*x))} * (c + d*x))/d^3 - (b*(b*c - a*d) * e * E^{(e/(c + d*x))} * (c + d*x))/d^3 + (b^2 * e^2 * E^{(e/(c + d*x))} * (c + d*x))/(6*d^3) - (b*(b*c - a*d) * E^{(e/(c + d*x))} * (c + d*x)^2)/d^3 + (b^2 * e * E^{(e/(c + d*x))} * (c + d*x)^2)/(6*d^3) + (b^2 * E^{(e/(c + d*x))} * (c + d*x)^3)/(3*d^3) - ((b*c - a*d)^2 * e * \text{ExpIntegralEi}[e/(c + d*x)])/d^3 + (b*(b*c - a*d) * e^2 * \text{ExpIntegralEi}[e/(c + d*x)])/d^3 - (b^2 * e^3 * \text{ExpIntegralEi}[e/(c + d*x)])/(6*d^3)$

**Rubi [A]** time = 0.256185, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2226, 2206, 2210, 2214}

$$\frac{be^2(bc-ad)\text{Ei}\left(\frac{e}{c+dx}\right)}{d^3} - \frac{e(bc-ad)^2\text{Ei}\left(\frac{e}{c+dx}\right)}{d^3} - \frac{be(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^3} - \frac{b(c+dx)^2(bc-ad)e^{\frac{e}{c+dx}}}{d^3} + \frac{(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^3}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))\*(a + b\*x)^2, x]

[Out]  $((b*c - a*d)^2 * E^{(e/(c + d*x))} * (c + d*x))/d^3 - (b*(b*c - a*d) * e * E^{(e/(c + d*x))} * (c + d*x))/d^3 + (b^2 * e^2 * E^{(e/(c + d*x))} * (c + d*x))/(6*d^3) - (b*(b*c - a*d) * E^{(e/(c + d*x))} * (c + d*x)^2)/d^3 + (b^2 * e * E^{(e/(c + d*x))} * (c + d*x)^2)/(6*d^3) + (b^2 * E^{(e/(c + d*x))} * (c + d*x)^3)/(3*d^3) - ((b*c - a*d)^2 * e * \text{ExpIntegralEi}[e/(c + d*x)])/d^3 + (b*(b*c - a*d) * e^2 * \text{ExpIntegralEi}[e/(c + d*x)])/d^3 - (b^2 * e^3 * \text{ExpIntegralEi}[e/(c + d*x)])/(6*d^3)$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a \* ExpIntegralEi[b\*(c + d\*x)^n \* Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1) \* F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n) \* F^(a + b\*(c + d\*x)^n), x], x]

n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{c+dx}}(a+bx)^2 dx &= \int \left( \frac{(-bc+ad)^2 e^{\frac{e}{c+dx}}}{d^2} - \frac{2b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^2} + \frac{b^2 e^{\frac{e}{c+dx}}(c+dx)^2}{d^2} \right) dx \\ &= \frac{b^2 \int e^{\frac{e}{c+dx}}(c+dx)^2 dx}{d^2} - \frac{(2b(bc-ad)) \int e^{\frac{e}{c+dx}}(c+dx) dx}{d^2} + \frac{(bc-ad)^2 \int e^{\frac{e}{c+dx}} dx}{d^2} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{d^3} + \frac{b^2 e^{\frac{e}{c+dx}}(c+dx)^3}{3d^3} + \frac{(b^2 e) \int e^{\frac{e}{c+dx}}(c+dx) dx}{3d^2} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)e e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{d^3} + \frac{b^2 e e^{\frac{e}{c+dx}}(c+dx)}{6d^3} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)e e^{\frac{e}{c+dx}}(c+dx)}{d^3} + \frac{b^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{6d^3} - \frac{b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^3} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)e e^{\frac{e}{c+dx}}(c+dx)}{d^3} + \frac{b^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{6d^3} - \frac{b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^3} \end{aligned}$$

**Mathematica [A]** time = 0.199511, size = 170, normalized size = 0.67

$$\frac{dx e^{\frac{e}{c+dx}} (6a^2 d^2 + 6abd(dx+e) + b^2(-4ce + 2d^2 x^2 + dex + e^2)) - e(6a^2 d^2 + 6abd(e-2c) + b^2(6c^2 - 6ce + e^2)) \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c+d\*x))\*(a+b\*x)^2,x]

[Out] (c\*(6\*a^2\*d^2 + 6\*a\*b\*d\*(-c + e) + b^2\*(2\*c^2 - 5\*c\*e + e^2))\*E^(e/(c+d\*x)))/(6\*d^3) + (d\*E^(e/(c+d\*x))\*x\*(6\*a^2\*d^2 + 6\*a\*b\*d\*(e + d\*x) + b^2\*(-4\*c\*e + e^2 + d\*e\*x + 2\*d^2\*x^2)) - e\*(6\*a^2\*d^2 + 6\*a\*b\*d\*(-2\*c + e) + b^2\*(6\*c^2 - 6\*c\*e + e^2))\*ExpIntegralEi[e/(c+d\*x)]/(6\*d^3)

**Maple [A]** time = 0.007, size = 356, normalized size = 1.4

$$-\frac{e}{d} \left( a^2 \left( -\frac{dx+c}{e} e^{\frac{e}{dx+c}} - \operatorname{Ei}\left(1, -\frac{e}{dx+c}\right) \right) + \frac{b^2 e^2}{d^2} \left( -\frac{(dx+c)^3}{3e^3} e^{\frac{e}{dx+c}} - \frac{(dx+c)^2}{6e^2} e^{\frac{e}{dx+c}} - \frac{dx+c}{6e} e^{\frac{e}{dx+c}} - \frac{1}{6} \operatorname{Ei}\left(1, -\frac{e}{dx+c}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c))\*(b\*x+a)^2,x)

[Out] -1/d\*e\*(a^2\*(-(d\*x+c)/e\*exp(e/(d\*x+c))-Ei(1,-e/(d\*x+c)))+b^2/d^2\*e^2\*(-1/3\*(d\*x+c)^3/e^3\*exp(e/(d\*x+c))-1/6\*exp(e/(d\*x+c))\*(d\*x+c)^2/e^2-1/6\*(d\*x+c)/e\*exp(e/(d\*x+c))-1/6\*Ei(1,-e/(d\*x+c)))+b^2/d^2\*c^2\*(-(d\*x+c)/e\*exp(e/(d\*x+c))-Ei(1,-e/(d\*x+c)))+2\*b/d\*e\*a\*(-1/2\*exp(e/(d\*x+c))\*(d\*x+c)^2/e^2-1/2\*(d\*x+c)/e\*exp(e/(d\*x+c))-1/2\*Ei(1,-e/(d\*x+c)))-2\*b^2/d^2\*e\*c\*(-1/2\*exp(e/(d\*x+c))\*(d\*x+c)^2/e^2-1/2\*(d\*x+c)/e\*exp(e/(d\*x+c))-1/2\*Ei(1,-e/(d\*x+c)))-2\*b/d\*c\*a



### 3.404 $\int e^{\frac{e}{c+dx}}(a+bx) dx$

**Optimal.** Leaf size=125

$$\frac{e(bc-ad)\text{Ei}\left(\frac{e}{c+dx}\right)}{d^2} - \frac{(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^2} - \frac{be^2\text{Ei}\left(\frac{e}{c+dx}\right)}{2d^2} + \frac{be(c+dx)e^{\frac{e}{c+dx}}}{2d^2} + \frac{b(c+dx)^2e^{\frac{e}{c+dx}}}{2d^2}$$

[Out] -(((b\*c - a\*d)\*E^(e/(c + d\*x))\*(c + d\*x))/d^2) + (b\*e\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^2) + (b\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^2) + ((b\*c - a\*d)\*e\*ExpIntegralEi[e/(c + d\*x)])/d^2 - (b\*e^2\*ExpIntegralEi[e/(c + d\*x)])/(2\*d^2)

**Rubi [A]** time = 0.128417, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2226, 2206, 2210, 2214}

$$\frac{e(bc-ad)\text{Ei}\left(\frac{e}{c+dx}\right)}{d^2} - \frac{(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^2} - \frac{be^2\text{Ei}\left(\frac{e}{c+dx}\right)}{2d^2} + \frac{be(c+dx)e^{\frac{e}{c+dx}}}{2d^2} + \frac{b(c+dx)^2e^{\frac{e}{c+dx}}}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))\*(a + b\*x), x]

[Out] -(((b\*c - a\*d)\*E^(e/(c + d\*x))\*(c + d\*x))/d^2) + (b\*e\*E^(e/(c + d\*x))\*(c + d\*x))/(2\*d^2) + (b\*E^(e/(c + d\*x))\*(c + d\*x)^2)/(2\*d^2) + ((b\*c - a\*d)\*e\*ExpIntegralEi[e/(c + d\*x)])/d^2 - (b\*e^2\*ExpIntegralEi[e/(c + d\*x)])/(2\*d^2)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rubi steps

$$\begin{aligned}
\int e^{\frac{e}{c+dx}}(a+bx) dx &= \int \left( \frac{(-bc+ad)e^{\frac{e}{c+dx}}}{d} + \frac{be^{\frac{e}{c+dx}}(c+dx)}{d} \right) dx \\
&= \frac{b \int e^{\frac{e}{c+dx}}(c+dx) dx}{d} + \frac{(-bc+ad) \int e^{\frac{e}{c+dx}} dx}{d} \\
&= -\frac{(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^2} + \frac{be^{\frac{e}{c+dx}}(c+dx)^2}{2d^2} + \frac{(be) \int e^{\frac{e}{c+dx}} dx}{2d} + \frac{((-bc+ad)e) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{d} \\
&= -\frac{(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^2} + \frac{bee^{\frac{e}{c+dx}}(c+dx)}{2d^2} + \frac{be^{\frac{e}{c+dx}}(c+dx)^2}{2d^2} + \frac{(bc-ad)e\text{Ei}\left(\frac{e}{c+dx}\right)}{d^2} + \frac{(be^2) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{2d} \\
&= -\frac{(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^2} + \frac{bee^{\frac{e}{c+dx}}(c+dx)}{2d^2} + \frac{be^{\frac{e}{c+dx}}(c+dx)^2}{2d^2} + \frac{(bc-ad)e\text{Ei}\left(\frac{e}{c+dx}\right)}{d^2} - \frac{be^2\text{Ei}\left(\frac{e}{c+dx}\right)}{2d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0935075, size = 91, normalized size = 0.73

$$\frac{dxe^{\frac{e}{c+dx}}(2ad+b(dx+e)) - e(2ad+b(e-2c))\text{Ei}\left(\frac{e}{c+dx}\right)}{2d^2} + \frac{ce^{\frac{e}{c+dx}}(2ad+b(e-c))}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c+d\*x))\*(a+b\*x),x]

[Out] (c\*(2\*a\*d + b\*(-c + e))\*E^(e/(c + d\*x)))/(2\*d^2) + (d\*E^(e/(c + d\*x))\*x\*(2\*a\*d + b\*(e + d\*x)) - e\*(2\*a\*d + b\*(-2\*c + e))\*ExpIntegralEi[e/(c + d\*x)])/(2\*d^2)

**Maple [A]** time = 0.006, size = 150, normalized size = 1.2

$$-\frac{e}{d} \left( a \left( -\frac{dx+c}{e} e^{\frac{e}{dx+c}} - \text{Ei} \left( 1, -\frac{e}{dx+c} \right) \right) + \frac{be}{d} \left( -\frac{(dx+c)^2}{2e^2} e^{\frac{e}{dx+c}} - \frac{dx+c}{2e} e^{\frac{e}{dx+c}} - \frac{1}{2} \text{Ei} \left( 1, -\frac{e}{dx+c} \right) \right) - \frac{bc}{d} \left( -\frac{dx+c}{e} e^{\frac{e}{dx+c}} - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c))\*(b\*x+a),x)

[Out] -1/d\*e\*(a\*(-(d\*x+c)/e\*exp(e/(d\*x+c))-Ei(1,-e/(d\*x+c)))+b/d\*e\*(-1/2\*exp(e/(d\*x+c))\*(d\*x+c)^2/e^2-1/2\*(d\*x+c)/e\*exp(e/(d\*x+c))-1/2\*Ei(1,-e/(d\*x+c)))-b/d\*c\*(-(d\*x+c)/e\*exp(e/(d\*x+c))-Ei(1,-e/(d\*x+c))))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bdx^2 + (2ad + be)x)e^{\left(\frac{e}{dx+c}\right)}}{2d} + \int -\frac{(bc^2e - (2ad^2e - (2cde - de^2)b)x)e^{\left(\frac{e}{dx+c}\right)}}{2(d^3x^2 + 2cd^2x + c^2d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))\*(b\*x+a),x, algorithm="maxima")

[Out]  $1/2*(b*d*x^2 + (2*a*d + b*e)*x)*e^{(e/(d*x + c))/d} + \text{integrate}(-1/2*(b*c^2*e - (2*a*d^2*e - (2*c*d*e - d*e^2)*b)*x)*e^{(e/(d*x + c))}/(d^3*x^2 + 2*c*d^2*x + c^2*d), x)$

**Fricas [A]** time = 1.55061, size = 178, normalized size = 1.42

$$\frac{(be^2 - 2(bc - ad)e)\text{Ei}\left(\frac{e}{dx+c}\right) - (bd^2x^2 - bc^2 + 2acd + bce + (2ad^2 + bde)x)e^{\left(\frac{e}{dx+c}\right)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a), x, algorithm="fricas")`

[Out]  $-1/2*((b*e^2 - 2*(b*c - a*d)*e)*\text{Ei}(e/(d*x + c)) - (b*d^2*x^2 - b*c^2 + 2*a*c*d + b*c*e + (2*a*d^2 + b*d*e)*x)*e^{(e/(d*x + c))})/d^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx) e^{\frac{e}{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a), x)`

[Out] `Integral((a + b*x)*exp(e/(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) e^{\left(\frac{e}{dx+c}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c))*(b*x+a), x, algorithm="giac")`

[Out] `integrate((b*x + a)*e^{(e/(d*x + c))}, x)`

### 3.405 $\int e^{\frac{e}{c+dx}} dx$

**Optimal.** Leaf size=37

$$\frac{(c+dx)e^{\frac{e}{c+dx}}}{d} - \frac{e\text{Ei}\left(\frac{e}{c+dx}\right)}{d}$$

[Out]  $(E^{(e/(c+d*x))}*(c+d*x))/d - (e*ExpIntegralEi[e/(c+d*x)])/d$

**Rubi [A]** time = 0.0301463, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2206, 2210}

$$\frac{(c+dx)e^{\frac{e}{c+dx}}}{d} - \frac{e\text{Ei}\left(\frac{e}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c+d\*x)),x]

[Out]  $(E^{(e/(c+d*x))}*(c+d*x))/d - (e*ExpIntegralEi[e/(c+d*x)])/d$

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && I LtQ[n, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{c+dx}} dx &= \frac{e^{\frac{e}{c+dx}}(c+dx)}{d} + e \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx \\ &= \frac{e^{\frac{e}{c+dx}}(c+dx)}{d} - \frac{e\text{Ei}\left(\frac{e}{c+dx}\right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0129464, size = 37, normalized size = 1.

$$\frac{(c+dx)e^{\frac{e}{c+dx}}}{d} - \frac{e\text{Ei}\left(\frac{e}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c+d\*x)),x]

[Out]  $(E^{(e/(c+d*x))}*(c+d*x))/d - (e*ExpIntegralEi[e/(c+d*x)])/d$



---

**Maple [A]** time = 0.002, size = 42, normalized size = 1.1

$$-\frac{e}{d} \left( -\frac{dx+c}{e} e^{\frac{e}{dx+c}} - \text{Ei} \left( 1, -\frac{e}{dx+c} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)), x)

[Out] -1/d\*e\*(-(d\*x+c)/e\*exp(e/(d\*x+c))-Ei(1,-e/(d\*x+c)))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$de \int \frac{x e^{\left(\frac{e}{dx+c}\right)}}{d^2 x^2 + 2 c dx + c^2} dx + x e^{\left(\frac{e}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)), x, algorithm="maxima")

[Out] d\*e\*integrate(x\*e^(e/(d\*x + c))/(d^2\*x^2 + 2\*c\*d\*x + c^2), x) + x\*e^(e/(d\*x + c))

---

**Fricas [A]** time = 1.52595, size = 70, normalized size = 1.89

$$\frac{e \text{Ei} \left( \frac{e}{dx+c} \right) - (dx+c) e^{\left(\frac{e}{dx+c}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)), x, algorithm="fricas")

[Out] -(e\*Ei(e/(d\*x + c)) - (d\*x + c)\*e^(e/(d\*x + c)))/d

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{\frac{e}{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)), x)

[Out] Integral(exp(e/(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{e}{dx+c}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(e/(d*x + c)), x)
```

$$3.406 \quad \int \frac{e^{c+dx}}{a+bx} dx$$

**Optimal.** Leaf size=62

$$\frac{e^{\frac{be}{bc-ad}} \text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{b} - \frac{\text{Ei}\left(\frac{e}{c+dx}\right)}{b}$$

[Out]  $-(\text{ExpIntegralEi}[e/(c + d*x)]/b) + (E^{((b*e)/(b*c - a*d))}*\text{ExpIntegralEi}[-(d*e*(a + b*x))/((b*c - a*d)*(c + d*x))])/b$

**Rubi [A]** time = 0.202567, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2222, 2210, 2228, 2178}

$$\frac{e^{\frac{be}{bc-ad}} \text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{b} - \frac{\text{Ei}\left(\frac{e}{c+dx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))/(a + b\*x), x]

[Out]  $-(\text{ExpIntegralEi}[e/(c + d*x)]/b) + (E^{((b*e)/(b*c - a*d))}*\text{ExpIntegralEi}[-(d*e*(a + b*x))/((b*c - a*d)*(c + d*x))])/b$

#### Rule 2222

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Dist[d/f, Int[F^(a + b/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*e - c\*f)/f, Int[F^(a + b/(c + d\*x))/((c + d\*x)\*(e + f\*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]]/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2228

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/(((e\_.) + (f\_.)\*(x\_))\*((g\_.) + (h\_.)\*(x\_))), x\_Symbol] :> -Dist[d/(f\*(d\*g - c\*h)), Subst[Int[F^(a - (b\*h)/(d\*g - c\*h) + (d\*b\*x)/(d\*g - c\*h))/x, x], x, (g + h\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\begin{aligned} \int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx &= \frac{d \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{b} - \frac{(-bc+ad) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)} dx}{b} \\ &= -\frac{\text{Ei}\left(\frac{e}{c+dx}\right)}{b} + \frac{\text{Subst}\left(\int \frac{\exp\left(-\frac{be}{-bc+ad} + \frac{dex}{-bc+ad}\right)}{x} dx, x, \frac{a+bx}{c+dx}\right)}{b} \\ &= -\frac{\text{Ei}\left(\frac{e}{c+dx}\right)}{b} + \frac{e^{\frac{be}{bc-ad}} \text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0625747, size = 56, normalized size = 0.9

$$\frac{e^{\frac{be}{bc-ad}} \text{Ei}\left(e\left(\frac{b}{ad-bc} + \frac{1}{c+dx}\right)\right) - \text{Ei}\left(\frac{e}{c+dx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x))/(a + b\*x), x]

[Out] (-ExpIntegralEi[e/(c + d\*x)] + E^((b\*e)/(b\*c - a\*d))\*ExpIntegralEi[e\*(b/(-(b\*c) + a\*d) + (c + d\*x)^(-1))])/b

**Maple [A]** time = 0.011, size = 79, normalized size = 1.3

$$-\frac{e}{d} \left( -\frac{d}{be} \text{Ei}\left(1, -\frac{e}{dx+c}\right) + \frac{d}{be} e^{-\frac{be}{ad-bc}} \text{Ei}\left(1, -\frac{e}{dx+c} - \frac{be}{ad-bc}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c))/(b\*x+a), x)

[Out] -1/d\*e\*(-d/b/e\*Ei(1, -e/(d\*x+c))+1/b/e\*d\*exp(-b\*e/(a\*d-b\*c))\*Ei(1, -e/(d\*x+c)-b\*e/(a\*d-b\*c)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a), x, algorithm="maxima")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a), x)

**Fricas [A]** time = 1.57333, size = 138, normalized size = 2.23

$$\frac{\text{Ei}\left(-\frac{bdex+ade}{bc^2-acd+(bcd-ad^2)x}\right) e^{\left(\frac{be}{bc-ad}\right)} - \text{Ei}\left(\frac{e}{dx+c}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a),x, algorithm="fricas")

[Out] (Ei(-(b\*d\*e\*x + a\*d\*e)/(b\*c^2 - a\*c\*d + (b\*c\*d - a\*d^2)\*x))\*e^(b\*e/(b\*c - a\*d)) - Ei(e/(d\*x + c)))/b

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a),x)

[Out] Integral(exp(e/(c + d\*x))/(a + b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a),x, algorithm="giac")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a), x)

$$3.407 \quad \int \frac{e^{c+dx}}{(a+bx)^2} dx$$

**Optimal.** Leaf size=107

$$\frac{d e e^{\frac{be}{bc-ad}} \text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^2} - \frac{d e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

[Out]  $-\left(\frac{d * E^{(e/(c + d*x))}}{(b*(b*c - a*d))} - \frac{E^{(e/(c + d*x))}}{(b*(a + b*x))} - (d * e * E^{((b*e)/(b*c - a*d))} * \text{ExpIntegralEi}\left[-\frac{(d*e*(a + b*x))}{(b*c - a*d)*(c + d*x)}\right]\right) / (b*c - a*d)^2$

**Rubi [A]** time = 0.544528, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {2223, 6742, 2222, 2210, 2228, 2178, 2209}

$$\frac{d e e^{\frac{be}{bc-ad}} \text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^2} - \frac{d e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))/(a + b\*x)^2, x]

[Out]  $-\left(\frac{d * E^{(e/(c + d*x))}}{(b*(b*c - a*d))} - \frac{E^{(e/(c + d*x))}}{(b*(a + b*x))} - (d * e * E^{((b*e)/(b*c - a*d))} * \text{ExpIntegralEi}\left[-\frac{(d*e*(a + b*x))}{(b*c - a*d)*(c + d*x)}\right]\right) / (b*c - a*d)^2$

#### Rule 2223

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(f\*(m + 1)), x] + Dist[(b\*d\*Log[F])/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(c + d\*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && ILtQ[m, -1]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2222

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Dist[d/f, Int[F^(a + b/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*e - c\*f)/f, Int[F^(a + b/(c + d\*x))/((c + d\*x)\*(e + f\*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2228

```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] := -Dist[d/(f*(d*g - c*h)), Subst[Int[F^(a - (b*h)
)/(d*g - c*h) + (d*b*x)/(d*g - c*h))/x, x], x, (g + h*x)/(c + d*x)], x] /;
FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} dx &= -\frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{(de) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)^2} dx}{b} \\
&= -\frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{(de) \int \left( \frac{b^2 e^{\frac{e}{c+dx}}}{(bc-ad)^2(a+bx)} - \frac{de e^{\frac{e}{c+dx}}}{(bc-ad)(c+dx)^2} - \frac{bde e^{\frac{e}{c+dx}}}{(bc-ad)^2(c+dx)} \right) dx}{b} \\
&= -\frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{(bde) \int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx}{(bc-ad)^2} + \frac{(d^2 e) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{(bc-ad)^2} + \frac{(d^2 e) \int \frac{e^{\frac{e}{c+dx}}}{(c+dx)^2} dx}{b(bc-ad)} \\
&= -\frac{de e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{de \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{(bc-ad)^2} - \frac{(d^2 e) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{(bc-ad)^2} - \frac{(de) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)} dx}{bc-ad} \\
&= -\frac{de e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{(de) \operatorname{Subst}\left(\int \frac{\exp\left(-\frac{be}{-bc+ad} + \frac{dex}{-bc+ad}\right)}{x} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2} \\
&= -\frac{de e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)} - \frac{dee^{\frac{be}{bc-ad}} \operatorname{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.162542, size = 105, normalized size = 0.98

$$-\frac{dee^{\frac{be}{bc-ad}} \operatorname{Ei}\left(\frac{e}{c+dx} - \frac{be}{bc-ad}\right)}{(ad-bc)^2} - \frac{de e^{\frac{e}{c+dx}}}{b(bc-ad)} - \frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(e/(c + d*x))/(a + b*x)^2, x]
```

```
[Out] -((d * E^(e/(c + d*x)))/(b*(b*c - a*d))) - E^(e/(c + d*x))/(b*(a + b*x)) - (d
 * e * E^((b*e)/(b*c - a*d)) * ExpIntegralEi[-((b*e)/(b*c - a*d)) + e/(c + d*x)])
 /(-b*c + a*d)^2
```

**Maple [A]** time = 0.01, size = 97, normalized size = 0.9

$$-\frac{de}{(ad-bc)^2} \left( -e^{\frac{e}{dx+c}} \left( \frac{e}{dx+c} + \frac{be}{ad-bc} \right)^{-1} - e^{-\frac{be}{ad-bc}} \operatorname{Ei} \left( 1, -\frac{e}{dx+c} - \frac{be}{ad-bc} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c))/(b\*x+a)^2,x)

[Out] -d\*e/(a\*d-b\*c)^2\*(-exp(e/(d\*x+c))/(e/(d\*x+c)+b\*e/(a\*d-b\*c))-exp(-b\*e/(a\*d-b\*c))\*Ei(1,-e/(d\*x+c)-b\*e/(a\*d-b\*c)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^2,x, algorithm="maxima")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a)^2, x)

**Fricas [A]** time = 1.59035, size = 312, normalized size = 2.92

$$-\frac{(bdex + ade) \operatorname{Ei} \left( -\frac{bdex+ade}{bc^2-acd+(bcd-ad^2)x} \right) e^{\left(\frac{be}{bc-ad}\right)} + (bc^2 - acd + (bcd - ad^2)x) e^{\left(\frac{e}{dx+c}\right)}}{ab^2c^2 - 2a^2bcd + a^3d^2 + (b^3c^2 - 2ab^2cd + a^2bd^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^2,x, algorithm="fricas")

[Out] -((b\*d\*e\*x + a\*d\*e)\*Ei(-(b\*d\*e\*x + a\*d\*e)/(b\*c^2 - a\*c\*d + (b\*c\*d - a\*d^2)\*x))\*e^(b\*e/(b\*c - a\*d)) + (b\*c^2 - a\*c\*d + (b\*c\*d - a\*d^2)\*x)\*e^(e/(d\*x + c)))/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2 + (b^3\*c^2 - 2\*a\*b^2\*c\*d + a^2\*b\*d^2)\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)\*\*2,x)

[Out] Integral(exp(e/(c + d\*x))/(a + b\*x)\*\*2, x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a)^2, x)

$$3.408 \quad \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3} dx$$

**Optimal.** Leaf size=240

$$\frac{bd^2e^2e^{\frac{be}{bc-ad}}\text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{2(bc-ad)^4} + \frac{d^2ee^{\frac{be}{bc-ad}}\text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{d^2ee^{\frac{e}{c+dx}}}{2(bc-ad)^3} + \frac{d^2e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(a+bx)(bc-ad)^2} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)}$$

[Out] (d^2\*E^(e/(c + d\*x)))/(2\*b\*(b\*c - a\*d)^2) + (d^2\*e\*E^(e/(c + d\*x)))/(2\*(b\*c - a\*d)^3) - E^(e/(c + d\*x))/(2\*b\*(a + b\*x)^2) + (d\*e\*E^(e/(c + d\*x)))/(2\*(b\*c - a\*d)^2\*(a + b\*x)) + (d^2\*e\*E^((b\*e)/(b\*c - a\*d))\*ExpIntegralEi[-((d\*e\*(a + b\*x))/((b\*c - a\*d)\*(c + d\*x)))])/(b\*c - a\*d)^3 + (b\*d^2\*e^2\*E^((b\*e)/(b\*c - a\*d))\*ExpIntegralEi[-((d\*e\*(a + b\*x))/((b\*c - a\*d)\*(c + d\*x)))])/(2\*(b\*c - a\*d)^4)

**Rubi [A]** time = 1.03656, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.368, Rules used = {2223, 6742, 2222, 2210, 2228, 2178, 2209}

$$\frac{bd^2e^2e^{\frac{be}{bc-ad}}\text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{2(bc-ad)^4} + \frac{d^2ee^{\frac{be}{bc-ad}}\text{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{d^2ee^{\frac{e}{c+dx}}}{2(bc-ad)^3} + \frac{d^2e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(a+bx)(bc-ad)^2} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x))/(a + b\*x)^3, x]

[Out] (d^2\*E^(e/(c + d\*x)))/(2\*b\*(b\*c - a\*d)^2) + (d^2\*e\*E^(e/(c + d\*x)))/(2\*(b\*c - a\*d)^3) - E^(e/(c + d\*x))/(2\*b\*(a + b\*x)^2) + (d\*e\*E^(e/(c + d\*x)))/(2\*(b\*c - a\*d)^2\*(a + b\*x)) + (d^2\*e\*E^((b\*e)/(b\*c - a\*d))\*ExpIntegralEi[-((d\*e\*(a + b\*x))/((b\*c - a\*d)\*(c + d\*x)))])/(b\*c - a\*d)^3 + (b\*d^2\*e^2\*E^((b\*e)/(b\*c - a\*d))\*ExpIntegralEi[-((d\*e\*(a + b\*x))/((b\*c - a\*d)\*(c + d\*x)))])/(2\*(b\*c - a\*d)^4)

#### Rule 2223

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(f\*(m + 1)), x] + Dist[(b\*d\*Log[F])/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*F^(a + b/(c + d\*x)))/(c + d\*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && ILtQ[m, -1]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2222

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Dist[d/f, Int[F^(a + b/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*e - c\*f)/f, Int[F^(a + b/(c + d\*x))/((c + d\*x)\*(e + f\*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rule 2228

Int[(F\_)^((a\_.) + (b\_.)/((c\_.) + (d\_.)\*(x\_)))/(((e\_.) + (f\_.)\*(x\_))\*((g\_.) + (h\_.)\*(x\_))), x\_Symbol] := -Dist[d/(f\*(d\*g - c\*h)), Subst[Int[F^(a - (b\*h)/(d\*g - c\*h) + (d\*b\*x)/(d\*g - c\*h))/x, x], x, (g + h\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*(F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3} dx &= -\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} - \frac{(de) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2(c+dx)^2} dx}{2b} \\
 &= -\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} - \frac{(de) \int \left( \frac{b^2 e^{\frac{e}{c+dx}}}{(bc-ad)^2(a+bx)^2} - \frac{2b^2 d e^{\frac{e}{c+dx}}}{(bc-ad)^3(a+bx)} + \frac{d^2 e^{\frac{e}{c+dx}}}{(bc-ad)^2(c+dx)^2} + \frac{2bd^2 e^{\frac{e}{c+dx}}}{(bc-ad)^3(c+dx)} \right) dx}{2b} \\
 &= -\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{(bd^2 e) \int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx}{(bc-ad)^3} - \frac{(d^3 e) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{(bc-ad)^3} - \frac{(bde) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} dx}{2(bc-ad)^2} - \frac{(d^3 e) \int \frac{e^{\frac{e}{c+dx}}}{(c+dx)^2} dx}{2b(bc-ad)^2} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{d^2 e \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{(bc-ad)^3} + \frac{(d^3 e) \int \frac{e^{\frac{e}{c+dx}}}{c+dx} dx}{(bc-ad)^3} + \frac{(d^2 e) \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)} dx}{(bc-ad)^3} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{(d^2 e) \operatorname{Subst}\left(\int \frac{\exp\left(-\frac{be}{-bc+ad} + \frac{dex}{-bc+ad}\right)}{x} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{d^2 e e^{\frac{be}{bc-ad}} \operatorname{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{(b^2 d^2 e^2) \int \frac{e^{\frac{e}{c+dx}}}{a+bx} dx}{2(bc-ad)^4} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{d^2 e e^{\frac{e}{c+dx}}}{2(bc-ad)^3} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{bd^2 e^2 \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{2(bc-ad)^4} + \frac{d^2 e e^{\frac{be}{bc-ad}} \operatorname{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{d^2 e e^{\frac{e}{c+dx}}}{2(bc-ad)^3} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{d^2 e e^{\frac{be}{bc-ad}} \operatorname{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{bd^2 e^2 \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{2(bc-ad)^4} \\
 &= \frac{d^2 e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{d^2 e e^{\frac{e}{c+dx}}}{2(bc-ad)^3} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{d^2 e e^{\frac{be}{bc-ad}} \operatorname{Ei}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{bd^2 e^2 \operatorname{Ei}\left(\frac{e}{c+dx}\right)}{2(bc-ad)^4}
 \end{aligned}$$

**Mathematica [F]** time = 0.401324, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x))/(a + b\*x)^3,x]

[Out] Integrate[E^(e/(c + d\*x))/(a + b\*x)^3, x]

**Maple [A]** time = 0.01, size = 240, normalized size = 1.

$$-\frac{e}{d} \left( -\frac{bed^3}{(ad-bc)^4} \left( -\frac{1}{2} e^{\frac{e}{dx+c}} \left( \frac{e}{dx+c} + \frac{be}{ad-bc} \right)^{-2} - \frac{1}{2} e^{\frac{e}{dx+c}} \left( \frac{e}{dx+c} + \frac{be}{ad-bc} \right)^{-1} - \frac{1}{2} e^{-\frac{be}{ad-bc}} \text{Ei} \left( 1, -\frac{e}{dx+c} - \frac{be}{ad-bc} \right) \right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c))/(b\*x+a)^3,x)

[Out]  $-\frac{1}{d} e^{e/(dx+c)} (-b/(a*d-b*c))^{-4} d^3 (-\frac{1}{2} \exp(e/(d*x+c)) / (e/(d*x+c) + b*e/(a*d-b*c))^{-2} - \frac{1}{2} \exp(e/(d*x+c)) / (e/(d*x+c) + b*e/(a*d-b*c)) - \frac{1}{2} \exp(-b*e/(a*d-b*c)) * \text{Ei}(1, -e/(d*x+c) - b*e/(a*d-b*c)) + d^3 / (a*d-b*c)^3 (-\exp(e/(d*x+c)) / (e/(d*x+c) + b*e/(a*d-b*c)) - \exp(-b*e/(a*d-b*c)) * \text{Ei}(1, -e/(d*x+c) - b*e/(a*d-b*c)))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^3,x, algorithm="maxima")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a)^3, x)

**Fricas [B]** time = 1.6449, size = 1007, normalized size = 4.2

$$\frac{(a^2bd^2e^2 + (b^3d^2e^2 + 2(b^3cd^2 - ab^2d^3)e)x^2 + 2(a^2bcd^2 - a^3d^3)e + 2(ab^2d^2e^2 + 2(ab^2cd^2 - a^2bd^3)e)x)\text{Ei}\left(-\frac{bdex+ade}{bc^2-acd+(bcd-a^2d^2)}\right)}{2(a^2b^4c^4 - 4a^3b^3c^3d + 6a^4b^2c^2d^2 - 4a^5bcd^3 + a^6d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} * ((a^2*b*d^2*e^2 + (b^3*d^2*e^2 + 2*(b^3*c*d^2 - a*b^2*d^3)*e)*x^2 + 2*(a^2*b*c*d^2 - a^3*d^3)*e + 2*(a*b^2*d^2*e^2 + 2*(a*b^2*c*d^2 - a^2*b*d^3)*e)*x)*\text{Ei}\left(-\frac{(b*d*e*x + a*d*e)}{(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)}\right)*e^{(b*e/(b*c - a*d))} - (b^3*c^4 - 4*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 - 2*a^3*c*d^3 - (b^3*$

$$c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4 + (b^3*c*d^2 - a*b^2*d^3)*e)*x^2 - (a*b^2*c^2*d - a^2*b*c*d^2)*e - (2*a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 2*a^3*d^4 + (b^3*c^2*d - a^2*b*d^3)*e)*x)*e^{(e/(d*x + c))}/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 2*(a*b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)\*\*3,x)

[Out] Integral(exp(e/(c + d\*x))/(a + b\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c))/(b\*x+a)^3,x, algorithm="giac")

[Out] integrate(e^(e/(d\*x + c))/(b\*x + a)^3, x)

### 3.409 $\int e^{\frac{e}{(c+dx)^2}} (a+bx)^3 dx$

**Optimal.** Leaf size=322

$$\frac{2\sqrt{\pi}b^2e^{3/2}(bc-ad)\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4} - \frac{b^2(c+dx)^3(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} - \frac{2b^2e(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} + \frac{\sqrt{\pi}\sqrt{e}(bc-ad)^3\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4}$$

[Out] -(((b\*c - a\*d)^3\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^4) - (2\*b^2\*(b\*c - a\*d)\*e\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^4 + (3\*b\*(b\*c - a\*d)^2\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(2\*d^4) + (b^3\*e\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(4\*d^4) - (b^2\*(b\*c - a\*d)\*E^(e/(c + d\*x)^2)\*(c + d\*x)^3)/d^4 + (b^3\*E^(e/(c + d\*x)^2)\*(c + d\*x)^4)/(4\*d^4) + ((b\*c - a\*d)^3\*Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)])/d^4 + (2\*b^2\*(b\*c - a\*d)\*e^(3/2)\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)]/d^4 - (3\*b\*(b\*c - a\*d)^2\*e\*ExpIntegralEi[e/(c + d\*x)^2])/(2\*d^4) - (b^3\*e^2\*ExpIntegralEi[e/(c + d\*x)^2])/(4\*d^4)

**Rubi [A]** time = 0.338777, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$\frac{2\sqrt{\pi}b^2e^{3/2}(bc-ad)\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4} - \frac{b^2(c+dx)^3(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} - \frac{2b^2e(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} + \frac{\sqrt{\pi}\sqrt{e}(bc-ad)^3\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x)^2)\*(a + b\*x)^3,x]

[Out] -(((b\*c - a\*d)^3\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^4) - (2\*b^2\*(b\*c - a\*d)\*e\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^4 + (3\*b\*(b\*c - a\*d)^2\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(2\*d^4) + (b^3\*e\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(4\*d^4) - (b^2\*(b\*c - a\*d)\*E^(e/(c + d\*x)^2)\*(c + d\*x)^3)/d^4 + (b^3\*E^(e/(c + d\*x)^2)\*(c + d\*x)^4)/(4\*d^4) + ((b\*c - a\*d)^3\*Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)])/d^4 + (2\*b^2\*(b\*c - a\*d)\*e^(3/2)\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)]/d^4 - (3\*b\*(b\*c - a\*d)^2\*e\*ExpIntegralEi[e/(c + d\*x)^2])/(2\*d^4) - (b^3\*e^2\*ExpIntegralEi[e/(c + d\*x)^2])/(4\*d^4)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>n</sup>)\*((c\_.) + (d\_.)\*(x\_))<sup>m</sup>, x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>n</sup>)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int e^{\frac{e}{(c+dx)^2}} (a+bx)^3 dx = \int \left( \frac{(-bc+ad)^3 e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^2}} (c+dx)}{d^3} - \frac{3b^2(bc-ad) e^{\frac{e}{(c+dx)^2}} (c+dx)^2}{d^3} + \frac{b^3 e^{\frac{e}{(c+dx)^2}} (c+dx)^3}{d^3} \right) dx$$

$$= \frac{b^3 \int e^{\frac{e}{(c+dx)^2}} (c+dx)^3 dx}{d^3} - \frac{(3b^2(bc-ad)) \int e^{\frac{e}{(c+dx)^2}} (c+dx)^2 dx}{d^3} + \frac{(3b(bc-ad)^2) \int e^{\frac{e}{(c+dx)^2}} (c+dx) dx}{d^3} - \frac{b^3 \int e^{\frac{e}{(c+dx)^2}} dx}{d^3}$$

$$= -\frac{(bc-ad)^3 e^{\frac{e}{(c+dx)^2}} (c+dx)}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^2}} (c+dx)^2}{2d^4} - \frac{b^2(bc-ad) e^{\frac{e}{(c+dx)^2}} (c+dx)^3}{d^4} + \frac{b^3 e^{\frac{e}{(c+dx)^2}} (c+dx)^4}{4d^4}$$

$$= -\frac{(bc-ad)^3 e^{\frac{e}{(c+dx)^2}} (c+dx)}{d^4} - \frac{2b^2(bc-ad) e^{\frac{e}{(c+dx)^2}} (c+dx)^2}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^2}} (c+dx)^3}{2d^4} + \frac{b^3 e^{\frac{e}{(c+dx)^2}} (c+dx)^4}{4d^4}$$

$$= -\frac{(bc-ad)^3 e^{\frac{e}{(c+dx)^2}} (c+dx)}{d^4} - \frac{2b^2(bc-ad) e^{\frac{e}{(c+dx)^2}} (c+dx)^2}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^2}} (c+dx)^3}{2d^4} + \frac{b^3 e^{\frac{e}{(c+dx)^2}} (c+dx)^4}{4d^4}$$

$$= -\frac{(bc-ad)^3 e^{\frac{e}{(c+dx)^2}} (c+dx)}{d^4} - \frac{2b^2(bc-ad) e^{\frac{e}{(c+dx)^2}} (c+dx)^2}{d^4} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^2}} (c+dx)^3}{2d^4} + \frac{b^3 e^{\frac{e}{(c+dx)^2}} (c+dx)^4}{4d^4}$$

**Mathematica [A]** time = 0.307828, size = 243, normalized size = 0.75

$$\frac{4\sqrt{\pi}\sqrt{e}(bc-ad)(a^2d^2 - 2abcd + b^2(c^2 + 2e)) \operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right) - be(6a^2d^2 - 12abcd + b^2(6c^2 + e)) \operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right) + dx e^{\frac{e}{(c+dx)^2}}}{4d^4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^2)\*(a + b\*x)^3,x]

[Out] -(c\*(6\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3 - 4\*a\*b^2\*d\*(c^2 + 2\*e) + b^3\*(c^3 + 7\*c\*e)) \*E^(e/(c + d\*x)^2))/(4\*d^4) + (d\*E^(e/(c + d\*x)^2)\*x\*(4\*a^3\*d^3 + 6\*a^2\*b\*d^3\*x + 4\*a\*b^2\*d\*(2\*e + d^2\*x^2) + b^3\*(-6\*c\*e + d\*e\*x + d^3\*x^3)) + 4\*(b\*c - a\*d)\*Sqrt[e]\*(-2\*a\*b\*c\*d + a^2\*d^2 + b^2\*(c^2 + 2\*e))\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)] - b\*e\*(-12\*a\*b\*c\*d + 6\*a^2\*d^2 + b^2\*(6\*c^2 + e))\*ExpIntegra

$\text{liEi}[e/(c + d*x)^2]/(4*d^4)$

**Maple [A]** time = 0.018, size = 560, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\exp(e/(d*x+c)^2)*(b*x+a)^3, x)$

[Out]  $-1/d*(a^3*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c)))+b^3/d^3*(-1/4*(d*x+c)^4*\exp(e/(d*x+c)^2)+1/2*e*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2)))+3*b^2/d^2*a*(-1/3*(d*x+c)^3*\exp(e/(d*x+c)^2)+2/3*e*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c))))-3*b^3/d^3*c*(-1/3*(d*x+c)^3*\exp(e/(d*x+c)^2)+2/3*e*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c))))+3*b/d*a^2*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2))+3*b^3/d^3*c^2*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2))-b^3/d^3*c^3*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c)))-6*b^2/d^2*c*a*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2))-3*b/d*c*a^2*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c)))+3*b^2/d^2*c^2*a*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)}/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)}/(d*x+c)))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 d^3 x^4 + 4 a b^2 d^3 x^3 + (6 a^2 b d^3 + b^3 d e) x^2 + 2 (2 a^3 d^3 - 3 b^3 c e + 4 a b^2 d e) x) e^{\left(\frac{e}{d^2 x^2 + 2 c d x + c^2}\right)}}{4 d^3} + \int \frac{(3 b^3 c^4 e - 4 a b^2 c^3 d e - (12 a^3 d^3 - 3 b^3 c e + 4 a b^2 d e) x) e^{\left(\frac{e}{d^2 x^2 + 2 c d x + c^2}\right)}}{4 d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\exp(e/(d*x+c)^2)*(b*x+a)^3, x, \text{algorithm}="maxima")$

[Out]  $1/4*(b^3*d^3*x^4 + 4*a*b^2*d^3*x^3 + (6*a^2*b*d^3 + b^3*d*e)*x^2 + 2*(2*a^3*d^3 - 3*b^3*c*e + 4*a*b^2*d*e)*x)*e^{(e/(d^2*x^2 + 2*c*d*x + c^2))/d^3} + \text{integrate}(1/2*(3*b^3*c^4*e - 4*a*b^2*c^3*d*e - (12*a*b^2*c*d^3*e - 6*a^2*b*d^4*e - (6*c^2*d^2*e + d^2*e^2)*b^3)*x^2 + 2*(2*a^3*d^4*e - 2*(3*c^2*d^2*e - 2*d^2*e^2)*a*b^2 + (4*c^3*d*e - 3*c*d*e^2)*b^3)*x)*e^{(e/(d^2*x^2 + 2*c*d*x + c^2))}/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x)$

**Fricas [A]** time = 1.64628, size = 635, normalized size = 1.97

$$4 \sqrt{\pi} (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 - a^3 d^4 + 2 (b^3 c d - a b^2 d^2) e) \sqrt{-\frac{e}{d^2}} \text{erf}\left(\frac{d \sqrt{-\frac{e}{d^2}}}{d x + c}\right) + (b^3 e^2 + 6 (b^3 c^2 - 2 a b^2 c d + a^2 b d^2) e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\exp(e/(d*x+c)^2)*(b*x+a)^3, x, \text{algorithm}="fricas")$



```
[Out] -1/4*(4*sqrt(pi)*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4 + 2
*(b^3*c*d - a*b^2*d^2)*e)*sqrt(-e/d^2)*erf(d*sqrt(-e/d^2)/(d*x + c)) + (b^3
*e^2 + 6*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*e)*Ei(e/(d^2*x^2 + 2*c*d*x + c
^2)) - (b^3*d^4*x^4 + 4*a*b^2*d^4*x^3 - b^3*c^4 + 4*a*b^2*c^3*d - 6*a^2*b*c
^2*d^2 + 4*a^3*c*d^3 + (6*a^2*b*d^4 + b^3*d^2*e)*x^2 - (7*b^3*c^2 - 8*a*b^2
*c*d)*e + 2*(2*a^3*d^4 - (3*b^3*c*d - 4*a*b^2*d^2)*e)*x)*e^(e/(d^2*x^2 + 2*
c*d*x + c^2)))/d^4
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c)**2)*(b*x+a)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 e^{\left(\frac{e}{(dx+c)^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c)^2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*e^(e/(d*x + c)^2), x)
```

### 3.410 $\int e^{\frac{e}{(c+dx)^2}} (a+bx)^2 dx$

**Optimal.** Leaf size=215

$$-\frac{\sqrt{\pi}\sqrt{e}(bc-ad)^2\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^3} + \frac{be(bc-ad)\operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right)}{d^3} - \frac{b(c+dx)^2(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{(c+dx)(bc-ad)^2e^{\frac{e}{(c+dx)^2}}}{d^3} - \frac{2\sqrt{\pi}b^2}{d^3}$$

[Out]  $((b*c - a*d)^2 * E^{(e/(c + d*x)^2)} * (c + d*x)) / d^3 + (2*b^2 * e * E^{(e/(c + d*x)^2)} * (c + d*x)) / (3*d^3) - (b*(b*c - a*d) * E^{(e/(c + d*x)^2)} * (c + d*x)^2) / d^3 + (b^2 * E^{(e/(c + d*x)^2)} * (c + d*x)^3) / (3*d^3) - ((b*c - a*d)^2 * \operatorname{Sqrt}[e] * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[e]/(c + d*x)]) / d^3 - (2*b^2 * e^{(3/2)} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[e]/(c + d*x)]) / (3*d^3) + (b*(b*c - a*d) * e * \operatorname{ExpIntegralEi}[e/(c + d*x)^2]) / d^3$

**Rubi [A]** time = 0.233229, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$-\frac{\sqrt{\pi}\sqrt{e}(bc-ad)^2\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^3} + \frac{be(bc-ad)\operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right)}{d^3} - \frac{b(c+dx)^2(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{(c+dx)(bc-ad)^2e^{\frac{e}{(c+dx)^2}}}{d^3} - \frac{2\sqrt{\pi}b^2}{d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(e/(c + d*x)^2)} * (a + b*x)^2, x]$

[Out]  $((b*c - a*d)^2 * E^{(e/(c + d*x)^2)} * (c + d*x)) / d^3 + (2*b^2 * e * E^{(e/(c + d*x)^2)} * (c + d*x)) / (3*d^3) - (b*(b*c - a*d) * E^{(e/(c + d*x)^2)} * (c + d*x)^2) / d^3 + (b^2 * E^{(e/(c + d*x)^2)} * (c + d*x)^3) / (3*d^3) - ((b*c - a*d)^2 * \operatorname{Sqrt}[e] * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[e]/(c + d*x)]) / d^3 - (2*b^2 * e^{(3/2)} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[e]/(c + d*x)]) / (3*d^3) + (b*(b*c - a*d) * e * \operatorname{ExpIntegralEi}[e/(c + d*x)^2]) / d^3$

#### Rule 2226

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{(n_.)})} * (u_), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandLinearProduct}[F^{(a + b*(c + d*x)^n)}, u, c, d, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, n\}, x \ \&\& \ \operatorname{PolynomialQ}[u, x]$

#### Rule 2206

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x) * F^{(a + b*(c + d*x)^n)} / d, x] - \operatorname{Dist}[b * n * \operatorname{Log}[F], \operatorname{Int}[(c + d*x)^n * F^{(a + b*(c + d*x)^n)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[2/n] \ \&\& \ \operatorname{LtQ}[n, 0]$

#### Rule 2211

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{(n_.)})} * ((c_.) + (d_.) * (x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*(m + 1)), \operatorname{Subst}[\operatorname{Int}[F^{(a + b*x^2)}, x], x, (c + d*x)^{(m + 1)}], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, m, n\}, x \ \&\& \ \operatorname{EqQ}[n, 2*(m + 1)]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{(2)})}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2])])} / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^2}} (a+bx)^2 dx &= \int \left( \frac{(-bc+ad)^2 e^{\frac{e}{(c+dx)^2}}}{d^2} - \frac{2b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^2} + \frac{b^2 e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^2} \right) dx \\ &= \frac{b^2 \int e^{\frac{e}{(c+dx)^2}}(c+dx)^2 dx}{d^2} - \frac{(2b(bc-ad)) \int e^{\frac{e}{(c+dx)^2}}(c+dx) dx}{d^2} + \frac{(bc-ad)^2 \int e^{\frac{e}{(c+dx)^2}} dx}{d^2} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^3} - \frac{b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^3} + \frac{b^2 e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{3d^3} + \frac{(2b^2 e) \int e^{\frac{e}{(c+dx)^2}} dx}{3d^2} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^3} + \frac{2b^2 e e^{\frac{e}{(c+dx)^2}}(c+dx)}{3d^3} - \frac{b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^3} + \frac{b^2 e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{3d^3} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^3} + \frac{2b^2 e e^{\frac{e}{(c+dx)^2}}(c+dx)}{3d^3} - \frac{b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^3} + \frac{b^2 e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{3d^3} \\ &= \frac{(bc-ad)^2 e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^3} + \frac{2b^2 e e^{\frac{e}{(c+dx)^2}}(c+dx)}{3d^3} - \frac{b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^3} + \frac{b^2 e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{3d^3} \end{aligned}$$

**Mathematica [A]** time = 0.188064, size = 176, normalized size = 0.82

$$\frac{-\sqrt{\pi}\sqrt{e}(3a^2d^2 - 6abcd + b^2(3c^2 + 2e))\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right) + dx e^{\frac{e}{(c+dx)^2}}(3a^2d^2 + 3abd^2x + b^2(d^2x^2 + 2e)) + 3be(bc - ad)\operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right)}{3d^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^2)\*(a + b\*x)^2,x]

[Out] (c\*(-3\*a\*b\*c\*d + 3\*a^2\*d^2 + b^2\*(c^2 + 2\*e))\*E^(e/(c + d\*x)^2))/(3\*d^3) + (d\*E^(e/(c + d\*x)^2)\*x\*(3\*a^2\*d^2 + 3\*a\*b\*d^2\*x + b^2\*(2\*e + d^2\*x^2)) - Sqrt[e]\*(-6\*a\*b\*c\*d + 3\*a^2\*d^2 + b^2\*(3\*c^2 + 2\*e))\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)] + 3\*b\*(b\*c - a\*d)\*e\*ExpIntegralEi[e/(c + d\*x)^2]/(3\*d^3)

**Maple [A]** time = 0.009, size = 313, normalized size = 1.5

$$-\frac{1}{d} \left( a^2 \left( -(dx+c) e^{\frac{e}{(dx+c)^2}} + e\sqrt{\pi} \operatorname{Erf}\left(\frac{1}{dx+c} \sqrt{-e}\right) \frac{1}{\sqrt{-e}} \right) + \frac{b^2}{d^2} \left( -\frac{(dx+c)^3}{3} e^{\frac{e}{(dx+c)^2}} + \frac{2e}{3} \left( -(dx+c) e^{\frac{e}{(dx+c)^2}} + e\sqrt{\pi} \operatorname{Erf}\left(\frac{1}{dx+c} \sqrt{-e}\right) \frac{1}{\sqrt{-e}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(e/(d*x+c)^2)*(b*x+a)^2,x)`

[Out] 
$$-1/d*(a^2*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)/(d*x+c))})+b^2/d^2*(-1/3*(d*x+c)^3*\exp(e/(d*x+c)^2)+2/3*e*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)/(d*x+c))})+b^2/d^2*c^2*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)/(d*x+c))})+2*b/d*a*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2))-2*b^2/d^2*c*(-1/2*\exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*\text{Ei}(1,-e/(d*x+c)^2))-2*b/d*c*a*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)/(d*x+c))})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2d^2x^3 + 3abd^2x^2 + (3a^2d^2 + 2b^2e)x)e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{3d^2} + \int -\frac{2(b^2c^3e + 3(b^2cd^2e - abd^3e)x^2 - (3a^2d^3e - (3c^2de - 2de^2))x)}{3(d^5x^3 + 3cd^4x^2 + 3c^2d^3x + c^3d^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)*(b*x+a)^2,x, algorithm="maxima")`

[Out] 
$$1/3*(b^2*d^2*x^3 + 3*a*b*d^2*x^2 + (3*a^2*d^2 + 2*b^2*e)*x)*e^{(e/(d^2*x^2 + 2*c*d*x + c^2))/d^2} + \text{integrate}(-2/3*(b^2*c^3*e + 3*(b^2*c*d^2*e - a*b*d^3*e)*x^2 - (3*a^2*d^3*e - (3*c^2*d*e - 2*d*e^2)*b^2)*x)*e^{(e/(d^2*x^2 + 2*c*d*x + c^2))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)}, x)$$

**Fricas [A]** time = 1.57501, size = 420, normalized size = 1.95

$$\frac{3(b^2c - abd)e\text{Ei}\left(\frac{e}{d^2x^2+2cdx+c^2}\right) + \sqrt{\pi}(3b^2c^2d - 6abcd^2 + 3a^2d^3 + 2b^2de)\sqrt{\frac{-e}{d^2}}\text{erf}\left(\frac{d\sqrt{\frac{-e}{d^2}}}{dx+c}\right) + (b^2d^3x^3 + 3abd^3x^2 + b^2c^3)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)*(b*x+a)^2,x, algorithm="fricas")`

[Out] 
$$1/3*(3*(b^2*c - a*b*d)*e*\text{Ei}(e/(d^2*x^2 + 2*c*d*x + c^2)) + \text{sqrt}(\text{pi})*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3 + 2*b^2*d*e)*\text{sqrt}(-e/d^2)*\text{erf}(d*\text{sqrt}(-e/d^2)/(d*x + c)) + (b^2*d^3*x^3 + 3*a*b*d^3*x^2 + b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2 + 2*b^2*c*e + (3*a^2*d^3 + 2*b^2*d*e)*x)*e^{(e/(d^2*x^2 + 2*c*d*x + c^2)))/d^3$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**2)*(b*x+a)**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 e^{\left(\frac{e}{(dx+c)^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2)\*(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*e^(e/(d\*x + c)^2), x)

### 3.411 $\int e^{\frac{e}{(c+dx)^2}} (a + bx) dx$

**Optimal.** Leaf size=111

$$\frac{\sqrt{\pi}\sqrt{e}(bc-ad)\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^2} - \frac{(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^2} - \frac{be\operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right)}{2d^2} + \frac{b(c+dx)^2e^{\frac{e}{(c+dx)^2}}}{2d^2}$$

[Out] -(((b\*c - a\*d)\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^2) + (b\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(2\*d^2) + ((b\*c - a\*d)\*Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)])/d^2 - (b\*e\*ExpIntegralEi[e/(c + d\*x)^2])/(2\*d^2)

**Rubi [A]** time = 0.129492, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2226, 2206, 2211, 2204, 2214, 2210}

$$\frac{\sqrt{\pi}\sqrt{e}(bc-ad)\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^2} - \frac{(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^2} - \frac{be\operatorname{Ei}\left(\frac{e}{(c+dx)^2}\right)}{2d^2} + \frac{b(c+dx)^2e^{\frac{e}{(c+dx)^2}}}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x)^2)\*(a + b\*x), x]

[Out] -(((b\*c - a\*d)\*E^(e/(c + d\*x)^2)\*(c + d\*x))/d^2) + (b\*E^(e/(c + d\*x)^2)\*(c + d\*x)^2)/(2\*d^2) + ((b\*c - a\*d)\*Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)])/d^2 - (b\*e\*ExpIntegralEi[e/(c + d\*x)^2])/(2\*d^2)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] := Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n \* F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && LtQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1))

, x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^2}} (a + bx) dx &= \int \left( \frac{(-bc + ad)e^{\frac{e}{(c+dx)^2}}}{d} + \frac{be^{\frac{e}{(c+dx)^2}}(c + dx)}{d} \right) dx \\ &= \frac{b \int e^{\frac{e}{(c+dx)^2}} (c + dx) dx}{d} + \frac{(-bc + ad) \int e^{\frac{e}{(c+dx)^2}} dx}{d} \\ &= -\frac{(bc - ad)e^{\frac{e}{(c+dx)^2}}(c + dx)}{d^2} + \frac{be^{\frac{e}{(c+dx)^2}}(c + dx)^2}{2d^2} + \frac{(be) \int \frac{e^{\frac{e}{(c+dx)^2}}}{c+dx} dx}{d} + \frac{(2(-bc + ad)e) \int \frac{e^{\frac{e}{(c+dx)^2}}}{(c+dx)^2} dx}{d} \\ &= -\frac{(bc - ad)e^{\frac{e}{(c+dx)^2}}(c + dx)}{d^2} + \frac{be^{\frac{e}{(c+dx)^2}}(c + dx)^2}{2d^2} - \frac{be\text{Ei}\left(\frac{e}{(c+dx)^2}\right)}{2d^2} + \frac{(2(bc - ad)e) \text{Subst}\left(\int e^{e x^2} dx\right)}{d^2} \\ &= -\frac{(bc - ad)e^{\frac{e}{(c+dx)^2}}(c + dx)}{d^2} + \frac{be^{\frac{e}{(c+dx)^2}}(c + dx)^2}{2d^2} + \frac{(bc - ad)\sqrt{e}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^2} - \frac{be\text{Ei}\left(\frac{e}{(c+dx)^2}\right)}{2d^2} \end{aligned}$$

**Mathematica [A]** time = 0.116447, size = 85, normalized size = 0.77

$$\frac{2\sqrt{\pi}\sqrt{e}(ad - bc)\text{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right) + (c + dx)e^{\frac{e}{(c+dx)^2}}(-2ad + bc - bdx) + be\text{Ei}\left(\frac{e}{(c+dx)^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^2)\*(a + b\*x), x]

[Out] -(E^(e/(c + d\*x)^2)\*(c + d\*x)\*(b\*c - 2\*a\*d - b\*d\*x) + 2\*(-(b\*c) + a\*d)\*Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)] + b\*e\*ExpIntegralEi[e/(c + d\*x)^2])/ (2\*d^2)

**Maple [A]** time = 0.007, size = 140, normalized size = 1.3

$$-\frac{1}{d} \left( a \left( -(dx + c) e^{\frac{e}{(dx+c)^2}} + e\sqrt{\pi}\text{Erf}\left(\frac{1}{dx+c}\sqrt{-e}\right) \frac{1}{\sqrt{-e}} \right) + \frac{b}{d} \left( -\frac{(dx+c)^2}{2} e^{\frac{e}{(dx+c)^2}} - \frac{e}{2} \text{Ei}\left(1, -\frac{e}{(dx+c)^2}\right) \right) \right) - \frac{bc}{d} \left( -(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^2)\*(b\*x+a), x)

[Out] -1/d\*(a\*(-(d\*x+c)\*exp(e/(d\*x+c)^2)+e\*Pi^(1/2)/(-e)^(1/2)\*erf((-e)^(1/2)/(d\*x+c)))+b/d\*(-1/2\*exp(e/(d\*x+c)^2)\*(d\*x+c)^2-1/2\*e\*Ei(1,-e/(d\*x+c)^2))-b/d\*c

$*(-(d*x+c)*\exp(e/(d*x+c)^2)+e*\text{Pi}^{(1/2)/(-e)^{(1/2)}*\text{erf}((-e)^{(1/2)/(d*x+c))})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} (bx^2 + 2ax) e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)} + \int \frac{(bdex^2 + 2adex) e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2)\*(b\*x+a),x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + 2\*a\*x)\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + integrate((b\*d\*e\*x^2 + 2\*a\*d\*e\*x)\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x)

**Fricas [A]** time = 1.47604, size = 265, normalized size = 2.39

$$\frac{be\text{Ei}\left(\frac{e}{d^2x^2+2cdx+c^2}\right) + 2\sqrt{\pi}(bcd - ad^2)\sqrt{-\frac{e}{d^2}} \text{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right) - (bd^2x^2 + 2ad^2x - bc^2 + 2acd)e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2)\*(b\*x+a),x, algorithm="fricas")

[Out] -1/2\*(b\*e\*Ei(e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*sqrt(pi)\*(b\*c\*d - a\*d^2)\*sqrt(-e/d^2)\*erf(d\*sqrt(-e/d^2)/(d\*x + c)) - (b\*d^2\*x^2 + 2\*a\*d^2\*x - b\*c^2 + 2\*a\*c\*d)\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/d^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx) e^{\frac{e}{c^2+2cdx+d^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)\*\*2)\*(b\*x+a),x)

[Out] Integral((a + b\*x)\*exp(e/(c\*\*2 + 2\*c\*d\*x + d\*\*2\*x\*\*2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) e^{\left(\frac{e}{(dx+c)^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2)\*(b\*x+a),x, algorithm="giac")

[Out] integrate((b\*x + a)\*e^(e/(d\*x + c)^2), x)



$$3.412 \quad \int e^{\frac{e}{(c+dx)^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{(c+dx)e^{\frac{e}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{e}\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

[Out] (E^(e/(c + d\*x)^2)\*(c + d\*x))/d - (Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)]/d)

**Rubi [A]** time = 0.0399308, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2206, 2211, 2204}

$$\frac{(c+dx)e^{\frac{e}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{e}\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x)^2), x]

[Out] (E^(e/(c + d\*x)^2)\*(c + d\*x))/d - (Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)]/d)

#### Rule 2206

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[((c + d\*x)\*F^(a + b\*(c + d\*x)^n))/d, x] - Dist[b\*n\*Log[F], Int[(c + d\*x)^n\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && IntegerQ[n, 0]

#### Rule 2211

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/(d\*(m + 1)), Subst[Int[F^(a + b\*x^2), x], x, (c + d\*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2\*(m + 1)]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^2}} dx &= \frac{e^{\frac{e}{(c+dx)^2}}(c+dx)}{d} + (2e) \int \frac{e^{\frac{e}{(c+dx)^2}}}{(c+dx)^2} dx \\ &= \frac{e^{\frac{e}{(c+dx)^2}}(c+dx)}{d} - \frac{(2e) \operatorname{Subst}\left(\int e^{ex^2} dx, x, \frac{1}{c+dx}\right)}{d} \\ &= \frac{e^{\frac{e}{(c+dx)^2}}(c+dx)}{d} - \frac{\sqrt{e}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0163764, size = 50, normalized size = 1.

$$\frac{(c + dx)e^{\frac{e}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{e}\operatorname{Erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^2), x]

[Out] (E^(e/(c + d\*x)^2)\*(c + d\*x))/d - (Sqrt[e]\*Sqrt[Pi]\*Erfi[Sqrt[e]/(c + d\*x)]/d)

**Maple [A]** time = 0.005, size = 48, normalized size = 1.

$$-\frac{1}{d}\left(- (dx + c) e^{\frac{e}{(dx+c)^2}} + e\sqrt{\pi}\operatorname{Erf}\left(\frac{1}{dx + c}\sqrt{-e}\right)\frac{1}{\sqrt{-e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^2), x)

[Out] -1/d\*(-(d\*x+c)\*exp(e/(d\*x+c)^2)+e\*Pi^(1/2)/(-e)^(1/2)\*erf((-e)^(1/2)/(d\*x+c)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2de \int \frac{xe^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx + xe^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2), x, algorithm="maxima")

[Out] 2\*d\*e\*integrate(x\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) + x\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2))

**Fricas [A]** time = 1.54464, size = 139, normalized size = 2.78

$$\frac{\sqrt{\pi}d\sqrt{-\frac{e}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right) + (dx + c)e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2), x, algorithm="fricas")

[Out] (sqrt(pi)\*d\*sqrt(-e/d^2)\*erf(d\*sqrt(-e/d^2)/(d\*x + c)) + (d\*x + c)\*e^(e/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/d

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{\frac{e}{(c+dx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)\*\*2), x)

[Out] Integral(exp(e/(c + d\*x)\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{e}{(dx+c)^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^2), x, algorithm="giac")

[Out] integrate(e^(e/(d\*x + c)^2), x)

$$3.413 \quad \int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx$$

**Optimal.** Leaf size=21

$$\text{Unintegrable} \left( \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}, x \right)$$

[Out] Unintegrable[E^(e/(c + d\*x)^2)/(a + b\*x), x]

**Rubi [A]** time = 0.0230121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e/(c + d\*x)^2)/(a + b\*x), x]

[Out] Defer[Int][E^(e/(c + d\*x)^2)/(a + b\*x), x]

Rubi steps

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx$$

**Mathematica [A]** time = 0.0372044, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x), x]

[Out] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x), x]

**Maple [A]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a} e^{\frac{e}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^2)/(b\*x+a), x)

[Out] `int(exp(e/(d*x+c)^2)/(b*x+a), x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a), x, algorithm="maxima")`

[Out] `integrate(e^(e/(d*x + c)^2)/(b*x + a), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a), x, algorithm="fricas")`

[Out] `integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(b*x + a), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**2)/(b*x+a), x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a), x, algorithm="giac")`

[Out] `integrate(e^(e/(d*x + c)^2)/(b*x + a), x)`

$$3.414 \quad \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate} \left( \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}, x \right)$$

[Out] CannotIntegrate[E^(e/(c + d\*x)^2)/(a + b\*x)^2, x]

**Rubi [A]** time = 0.0575595, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e/(c + d\*x)^2)/(a + b\*x)^2, x]

[Out] Defer[Int][E^(e/(c + d\*x)^2)/(a + b\*x)^2, x]

Rubi steps

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx$$

**Mathematica [A]** time = 0.216188, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x)^2, x]

[Out] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x)^2, x]

**Maple [A]** time = 0.103, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^2} e^{\frac{e}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^2)/(b\*x+a)^2, x)

[Out] `int(exp(e/(d*x+c)^2)/(b*x+a)^2,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate(e^(e/(d*x + c)^2)/(b*x + a)^2, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{b^2x^2 + 2abx + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*x^2 + 2*a*b*x + a^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**2)/(b*x+a)**2,x)`

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="giac")`

[Out] undef

$$3.415 \quad \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate}\left(\frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}, x\right)$$

[Out] CannotIntegrate[E^(e/(c + d\*x)^2)/(a + b\*x)^3, x]

**Rubi [A]** time = 0.0530332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e/(c + d\*x)^2)/(a + b\*x)^3, x]

[Out] Defer[Int][E^(e/(c + d\*x)^2)/(a + b\*x)^3, x]

Rubi steps

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$$

**Mathematica [A]** time = 0.0831338, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x)^3, x]

[Out] Integrate[E^(e/(c + d\*x)^2)/(a + b\*x)^3, x]

**Maple [A]** time = 0.14, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^3} e^{\frac{e}{(dx+c)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^2)/(b\*x+a)^3, x)



[Out] `int(exp(e/(d*x+c)^2)/(b*x+a)^3,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="maxima")`

[Out] `integrate(e^(e/(d*x + c)^2)/(b*x + a)^3, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\frac{e}{d^2x^2+2cdx+c^2}\right)}}{b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="fricas")`

[Out] `integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2)))/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**2)/(b*x+a)**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="giac")`

[Out] `integrate(e^(e/(d*x + c)^2)/(b*x + a)^3, x)`

### 3.416 $\int e^{\frac{e}{(c+dx)^3}} (a + bx)^3 dx$

**Optimal.** Leaf size=206

$$\frac{b(c+dx)^2(bc-ad)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{d^4} - \frac{(c+dx)(bc-ad)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4} + \frac{b^3(c+dx)^4}{d^4}$$

[Out]  $-\left((b^2*(b*c - a*d)*E^{(e/(c + d*x)^3)}*(c + d*x)^3)/d^4 + (b^2*(b*c - a*d)*\text{ExpIntegralEi}[e/(c + d*x)^3])/d^4 + (b^3*(-(e/(c + d*x)^3))^{4/3}*(c + d*x)^4*\Gamma[-4/3, -(e/(c + d*x)^3)])/(3*d^4) + (b*(b*c - a*d)^2*(-(e/(c + d*x)^3))^{2/3}*(c + d*x)^2*\Gamma[-2/3, -(e/(c + d*x)^3)])/d^4 - ((b*c - a*d)^3*(-(e/(c + d*x)^3))^{1/3}*(c + d*x)*\Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d^4)\right)$

**Rubi [A]** time = 0.189264, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2226, 2208, 2218, 2214, 2210}

$$\frac{b(c+dx)^2(bc-ad)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{d^4} - \frac{(c+dx)(bc-ad)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4} + \frac{b^3(c+dx)^4}{d^4}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x)^3)\*(a + b\*x)^3, x]

[Out]  $-\left((b^2*(b*c - a*d)*E^{(e/(c + d*x)^3)}*(c + d*x)^3)/d^4 + (b^2*(b*c - a*d)*\text{ExpIntegralEi}[e/(c + d*x)^3])/d^4 + (b^3*(-(e/(c + d*x)^3))^{4/3}*(c + d*x)^4*\Gamma[-4/3, -(e/(c + d*x)^3)])/(3*d^4) + (b*(b*c - a*d)^2*(-(e/(c + d*x)^3))^{2/3}*(c + d*x)^2*\Gamma[-2/3, -(e/(c + d*x)^3)])/d^4 - ((b*c - a*d)^3*(-(e/(c + d*x)^3))^{1/3}*(c + d*x)*\Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d^4)\right)$

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* (u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-

4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^3}} (a+bx)^3 dx &= \int \left( \frac{(-bc+ad)^3 e^{\frac{e}{(c+dx)^3}}}{d^3} + \frac{3b(bc-ad)^2 e^{\frac{e}{(c+dx)^3}} (c+dx)}{d^3} - \frac{3b^2(bc-ad) e^{\frac{e}{(c+dx)^3}} (c+dx)^2}{d^3} + \frac{b^3 e^{\frac{e}{(c+dx)^3}} (c+dx)^3}{d^3} \right) dx \\ &= \frac{b^3 \int e^{\frac{e}{(c+dx)^3}} (c+dx)^3 dx}{d^3} - \frac{(3b^2(bc-ad)) \int e^{\frac{e}{(c+dx)^3}} (c+dx)^2 dx}{d^3} + \frac{(3b(bc-ad)^2) \int e^{\frac{e}{(c+dx)^3}} (c+dx) dx}{d^3} - \frac{b^3 \int e^{\frac{e}{(c+dx)^3}} (c+dx)^3 dx}{d^3} \\ &= -\frac{b^2(bc-ad) e^{\frac{e}{(c+dx)^3}} (c+dx)^3}{d^4} + \frac{b^3 \left(-\frac{e}{(c+dx)^3}\right)^{4/3} (c+dx)^4 \Gamma\left(-\frac{4}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4} + \frac{b(bc-ad)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} (c+dx)^4 \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4} \\ &= -\frac{b^2(bc-ad) e^{\frac{e}{(c+dx)^3}} (c+dx)^3}{d^4} + \frac{b^2(bc-ad) e \operatorname{Ei}\left(\frac{e}{(c+dx)^3}\right)}{d^4} + \frac{b^3 \left(-\frac{e}{(c+dx)^3}\right)^{4/3} (c+dx)^4 \Gamma\left(-\frac{4}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4} \end{aligned}$$

**Mathematica [A]** time = 0.145976, size = 195, normalized size = 0.95

$$\frac{3b(c+dx)^2(bc-ad)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \operatorname{Gamma}\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right) - (c+dx)(bc-ad)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \operatorname{Gamma}\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right) + b^3 \left(-\frac{e}{(c+dx)^3}\right)^{4/3} (c+dx)^4 \Gamma\left(-\frac{4}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^3)\*(a + b\*x)^3, x]

[Out] (-3\*b^2\*(b\*c - a\*d)\*E^(e/(c + d\*x)^3)\*(c + d\*x)^3 + 3\*b^2\*(b\*c - a\*d)\*e\*ExpIntegralEi[e/(c + d\*x)^3] + b^3\*(-(e/(c + d\*x)^3))^(4/3)\*(c + d\*x)^4\*Gamma[-4/3, -(e/(c + d\*x)^3)] + 3\*b\*(b\*c - a\*d)^2\*(-(e/(c + d\*x)^3))^(2/3)\*(c + d\*x)^2\*Gamma[-2/3, -(e/(c + d\*x)^3)] - (b\*c - a\*d)^3\*(-(e/(c + d\*x)^3))^(1/3)\*(c + d\*x)\*Gamma[-1/3, -(e/(c + d\*x)^3)])/(3\*d^4)

**Maple [F]** time = 0.098, size = 0, normalized size = 0.

$$\int e^{\frac{e}{(dx+c)^3}} (bx+a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^3)\*(b\*x+a)^3, x)

[Out] int(exp(e/(d\*x+c)^3)\*(b\*x+a)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 d^3 x^4 + 4 a b^2 d^3 x^3 + 6 a^2 b d^3 x^2 + (4 a^3 d^3 + 3 b^3 e) x) e^{\left(\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right)}}{4 d^3} + \int -\frac{3 (b^3 c^4 e + 4 (b^3 c d^3 e - a b^2 d^4 e) x^3 + 6 (b^3 c^2 d^2 e - a^2 b d^4 e) x^2 - (4 a^3 d^4 e - (4 c^3 d e - 3 d e^2) b^3) x) e^{\left(\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right)}}{4 (d^7 x^4 + 4 c d^6 x^3 + 6 c^2 d^5 x^2 + 4 c^3 d^4 x + c^4 d^3)}, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a)^3,x, algorithm="maxima")

[Out] 1/4\*(b^3\*d^3\*x^4 + 4\*a\*b^2\*d^3\*x^3 + 6\*a^2\*b\*d^3\*x^2 + (4\*a^3\*d^3 + 3\*b^3\*e)\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/d^3 + integrate(-3/4\*(b^3\*c^4\*e + 4\*(b^3\*c\*d^3\*e - a\*b^2\*d^4\*e)\*x^3 + 6\*(b^3\*c^2\*d^2\*e - a^2\*b\*d^4\*e)\*x^2 - (4\*a^3\*d^4\*e - (4\*c^3\*d\*e - 3\*d\*e^2)\*b^3)\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(d^7\*x^4 + 4\*c\*d^6\*x^3 + 6\*c^2\*d^5\*x^2 + 4\*c^3\*d^4\*x + c^4\*d^3), x)

**Fricas [A]** time = 1.64772, size = 736, normalized size = 3.57

$$4 (b^3 c - a b^2 d) e \operatorname{Ei}\left(\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right) - 6 (b^3 c^2 d^2 - 2 a b^2 c d^3 + a^2 b d^4) \left(-\frac{e}{d^3}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right) + (4 b^3 c^3 d - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a)^3,x, algorithm="fricas")

[Out] 1/4\*(4\*(b^3\*c - a\*b^2\*d)\*e\*Ei(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) - 6\*(b^3\*c^2\*d^2 - 2\*a\*b^2\*c\*d^3 + a^2\*b\*d^4)\*(-e/d^3)^(2/3)\*gamma(1/3, -e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + (4\*b^3\*c^3\*d - 12\*a\*b^2\*c^2\*d^2 + 12\*a^2\*b\*c\*d^3 - 4\*a^3\*d^4 - 3\*b^3\*d\*e)\*(-e/d^3)^(1/3)\*gamma(2/3, -e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + (b^3\*d^4\*x^4 + 4\*a\*b^2\*d^4\*x^3 + 6\*a^2\*b\*d^4\*x^2 - b^3\*c^4 + 4\*a\*b^2\*c^3\*d - 6\*a^2\*b\*c^2\*d^2 + 4\*a^3\*c\*d^3 + 3\*b^3\*c\*e + (4\*a^3\*d^4 + 3\*b^3\*d\*e)\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d^4

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)\*\*3)\*(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b x + a)^3 e^{\left(\frac{e}{(d x + c)^3}\right)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c)^3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*e^(e/(d*x + c)^3), x)
```

### 3.417 $\int e^{\frac{e}{(c+dx)^3}} (a+bx)^2 dx$

**Optimal.** Leaf size=151

$$\frac{2b(c+dx)^2(bc-ad)\left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} + \frac{(c+dx)(bc-ad)^2 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} - \frac{b^2 e^{\frac{e}{(c+dx)^3}}}{3d^3}$$

[Out] (b^2\*E^(e/(c+d\*x)^3)\*(c+d\*x)^3)/(3\*d^3) - (b^2\*e\*ExpIntegralEi[e/(c+d\*x)^3])/(3\*d^3) - (2\*b\*(b\*c-a\*d)\*(-(e/(c+d\*x)^3))^(2/3)\*(c+d\*x)^2\*Gamma[-2/3, -(e/(c+d\*x)^3)])/(3\*d^3) + ((b\*c-a\*d)^2\*(-(e/(c+d\*x)^3))^(1/3)\*(c+d\*x)\*Gamma[-1/3, -(e/(c+d\*x)^3)])/(3\*d^3)

**Rubi [A]** time = 0.133174, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2226, 2208, 2218, 2214, 2210}

$$\frac{2b(c+dx)^2(bc-ad)\left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} + \frac{(c+dx)(bc-ad)^2 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} - \frac{b^2 e^{\frac{e}{(c+dx)^3}}}{3d^3}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c+d\*x)^3)\*(a+b\*x)^2,x]

[Out] (b^2\*E^(e/(c+d\*x)^3)\*(c+d\*x)^3)/(3\*d^3) - (b^2\*e\*ExpIntegralEi[e/(c+d\*x)^3])/(3\*d^3) - (2\*b\*(b\*c-a\*d)\*(-(e/(c+d\*x)^3))^(2/3)\*(c+d\*x)^2\*Gamma[-2/3, -(e/(c+d\*x)^3)])/(3\*d^3) + ((b\*c-a\*d)^2\*(-(e/(c+d\*x)^3))^(1/3)\*(c+d\*x)\*Gamma[-1/3, -(e/(c+d\*x)^3)])/(3\*d^3)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* (u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n)/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0

] && LeQ[-n, m + 1]))

### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_ Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; Free Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^3}} (a+bx)^2 dx &= \int \left( \frac{(-bc+ad)^2 e^{\frac{e}{(c+dx)^3}}}{d^2} - \frac{2b(bc-ad)e^{\frac{e}{(c+dx)^3}}(c+dx)}{d^2} + \frac{b^2 e^{\frac{e}{(c+dx)^3}}(c+dx)^2}{d^2} \right) dx \\ &= \frac{b^2 \int e^{\frac{e}{(c+dx)^3}}(c+dx)^2 dx}{d^2} - \frac{(2b(bc-ad)) \int e^{\frac{e}{(c+dx)^3}}(c+dx) dx}{d^2} + \frac{(bc-ad)^2 \int e^{\frac{e}{(c+dx)^3}} dx}{d^2} \\ &= \frac{b^2 e^{\frac{e}{(c+dx)^3}}(c+dx)^3}{3d^3} - \frac{2b(bc-ad) \left(-\frac{e}{(c+dx)^3}\right)^{2/3} (c+dx)^2 \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} + \frac{(bc-ad)^2 \sqrt[3]{-\frac{e}{(c+dx)^3}}}{3d^3} \\ &= \frac{b^2 e^{\frac{e}{(c+dx)^3}}(c+dx)^3}{3d^3} - \frac{b^2 e \operatorname{Ei}\left(\frac{e}{(c+dx)^3}\right)}{3d^3} - \frac{2b(bc-ad) \left(-\frac{e}{(c+dx)^3}\right)^{2/3} (c+dx)^2 \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^3} + \frac{(bc-ad)^2 \sqrt[3]{-\frac{e}{(c+dx)^3}}}{3d^3} \end{aligned}$$

**Mathematica [A]** time = 0.0759187, size = 136, normalized size = 0.9

$$\frac{-2b(c+dx)^2(bc-ad) \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \operatorname{Gamma}\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right) + (c+dx)(bc-ad)^2 \sqrt[3]{-\frac{e}{(c+dx)^3}} \operatorname{Gamma}\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right) - b^2}{3d^3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c + d\*x)^3)\*(a + b\*x)^2,x]

[Out] (b^2\*E^(e/(c + d\*x)^3)\*(c + d\*x)^3 - b^2\*e\*ExpIntegralEi[e/(c + d\*x)^3] - 2\*b\*(b\*c - a\*d)\*(-(e/(c + d\*x)^3))^(2/3)\*(c + d\*x)^2\*Gamma[-2/3, -(e/(c + d\*x)^3)] + (b\*c - a\*d)^2\*(-(e/(c + d\*x)^3))^(1/3)\*(c + d\*x)\*Gamma[-1/3, -(e/(c + d\*x)^3)))/(3\*d^3)

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int e^{\frac{e}{(dx+c)^3}} (bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^3)\*(b\*x+a)^2,x)

[Out] int(exp(e/(d\*x+c)^3)\*(b\*x+a)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} (b^2 x^3 + 3 abx^2 + 3 a^2 x) e^{\left(\frac{e}{d^3 x^3 + 3 cd^2 x^2 + 3 c^2 dx + c^3}\right)} + \int \frac{(b^2 dex^3 + 3 abdex^2 + 3 a^2 dex) e^{\left(\frac{e}{d^3 x^3 + 3 cd^2 x^2 + 3 c^2 dx + c^3}\right)}}{d^4 x^4 + 4 cd^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 dx + c^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/3\*(b^2\*x^3 + 3\*a\*b\*x^2 + 3\*a^2\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate((b^2\*d\*e\*x^3 + 3\*a\*b\*d\*e\*x^2 + 3\*a^2\*d\*e\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [A]** time = 1.57348, size = 551, normalized size = 3.65

$$\frac{b^2 e \operatorname{Ei}\left(\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right) - 3\left(b^2 c d^2 - a b d^3\right)\left(-\frac{e}{d^3}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right) + 3\left(b^2 c^2 d - 2 a b c d^2 + a^2 d^3\right)\left(-\frac{e}{d^3}\right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{e}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3}\right)}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a)^2,x, algorithm="fricas")

[Out] -1/3\*(b^2\*e\*Ei(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) - 3\*(b^2\*c\*d^2 - a\*b\*d^3)\*(-e/d^3)^(2/3)\*gamma(1/3, -e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + 3\*(b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*(-e/d^3)^(1/3)\*gamma(2/3, -e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) - (b^2\*d^3\*x^3 + 3\*a\*b\*d^3\*x^2 + 3\*a^2\*d^3\*x + b^2\*c^3 - 3\*a\*b\*c^2\*d + 3\*a^2\*c\*d^2)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)))/d^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)\*\*3)\*(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 e^{\left(\frac{e}{(dx+c)^3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*e^(e/(d\*x + c)^3), x)



### 3.418 $\int e^{\frac{e}{(c+dx)^3}} (a + bx) dx$

**Optimal.** Leaf size=92

$$\frac{b(c+dx)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^2} - \frac{(c+dx)(bc-ad) \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^2}$$

[Out] (b\*(-(e/(c + d\*x)^3))^(2/3)\*(c + d\*x)^2\*Gamma[-2/3, -(e/(c + d\*x)^3)])/(3\*d^2) - ((b\*c - a\*d)\*(-(e/(c + d\*x)^3))^(1/3)\*(c + d\*x)\*Gamma[-1/3, -(e/(c + d\*x)^3)])/(3\*d^2)

**Rubi [A]** time = 0.0567096, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2226, 2208, 2218}

$$\frac{b(c+dx)^2 \left(-\frac{e}{(c+dx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^2} - \frac{(c+dx)(bc-ad) \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c + d\*x)^3)\*(a + b\*x), x]

[Out] (b\*(-(e/(c + d\*x)^3))^(2/3)\*(c + d\*x)^2\*Gamma[-2/3, -(e/(c + d\*x)^3)])/(3\*d^2) - ((b\*c - a\*d)\*(-(e/(c + d\*x)^3))^(1/3)\*(c + d\*x)\*Gamma[-1/3, -(e/(c + d\*x)^3)])/(3\*d^2)

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int e^{\frac{e}{(c+dx)^3}} (a+bx) dx &= \int \left( \frac{(-bc+ad)e^{\frac{e}{(c+dx)^3}}}{d} + \frac{be^{\frac{e}{(c+dx)^3}}(c+dx)}{d} \right) dx \\ &= \frac{b \int e^{\frac{e}{(c+dx)^3}} (c+dx) dx}{d} + \frac{(-bc+ad) \int e^{\frac{e}{(c+dx)^3}} dx}{d} \\ &= \frac{b \left( -\frac{e}{(c+dx)^3} \right)^{2/3} (c+dx)^2 \Gamma \left( -\frac{2}{3}, -\frac{e}{(c+dx)^3} \right)}{3d^2} - \frac{(bc-ad) \sqrt[3]{-\frac{e}{(c+dx)^3}} (c+dx) \Gamma \left( -\frac{1}{3}, -\frac{e}{(c+dx)^3} \right)}{3d^2} \end{aligned}$$

**Mathematica [A]** time = 0.0847917, size = 85, normalized size = 0.92

$$\frac{(c+dx) \left( (ad-bc) \sqrt[3]{-\frac{e}{(c+dx)^3}} \text{Gamma} \left( -\frac{1}{3}, -\frac{e}{(c+dx)^3} \right) + b(c+dx) \left( -\frac{e}{(c+dx)^3} \right)^{2/3} \text{Gamma} \left( -\frac{2}{3}, -\frac{e}{(c+dx)^3} \right) \right)}{3d^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c+d\*x)^3)\*(a+b\*x),x]

[Out] ((c+d\*x)\*(b\*(-e/(c+d\*x)^3))^(2/3)\*(c+d\*x)\*Gamma[-2/3,-(e/(c+d\*x)^3)] + (-b\*c)+a\*d)\*(-e/(c+d\*x)^3)^(1/3)\*Gamma[-1/3,-(e/(c+d\*x)^3)])/(3\*d^2)

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int e^{\frac{e}{(dx+c)^3}} (bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^3)\*(b\*x+a),x)

[Out] int(exp(e/(d\*x+c)^3)\*(b\*x+a),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} (bx^2 + 2ax) e^{\left( \frac{e}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} \right)} + \int \frac{3(bdex^2 + 2adex) e^{\left( \frac{e}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} \right)}}{2(d^4x^4 + 4cd^3x^3 + 6c^2d^2x^2 + 4c^3dx + c^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a),x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + 2\*a\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3)) + integrate(3/2\*(b\*d\*e\*x^2 + 2\*a\*d\*e\*x)\*e^(e/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3))/(d^4\*x^4 + 4\*c\*d^3\*x^3 + 6\*c^2\*d^2\*x^2 + 4\*c^3\*d\*x + c^4), x)

**Fricas [B]** time = 1.53108, size = 370, normalized size = 4.02

$$\frac{bd^2 \left(-\frac{e}{d^3}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right) - 2(bcd - ad^2) \left(-\frac{e}{d^3}\right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right) - (bd^2x^2 + 2ad^2x - bc^2 + \dots)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a), x, algorithm="fricas")

[Out]  $-1/2*(b*d^2*(-e/d^3)^{(2/3)}*\text{gamma}(1/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*(b*c*d - a*d^2)*(-e/d^3)^{(1/3)}*\text{gamma}(2/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - (b*d^2*x^2 + 2*a*d^2*x - b*c^2 + 2*a*c*d)*e^{(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))}/d^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)\*\*3)\*(b\*x+a), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)e^{\left(\frac{e}{(dx+c)^3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e/(d\*x+c)^3)\*(b\*x+a), x, algorithm="giac")

[Out] integrate((b\*x + a)\*e^(e/(d\*x + c)^3), x)

$$3.419 \quad \int e^{\frac{e}{(c+dx)^3}} dx$$

**Optimal.** Leaf size=40

$$\frac{(c+dx)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d}$$

[Out]  $((-(e/(c+d*x)^3))^{(1/3)}*(c+d*x)*\Gamma[-1/3, -(e/(c+d*x)^3)])/(3*d)$

**Rubi [A]** time = 0.0055857, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2208}

$$\frac{(c+dx)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[E^(e/(c+d\*x)^3),x]

[Out]  $((-(e/(c+d*x)^3))^{(1/3)}*(c+d*x)*\Gamma[-1/3, -(e/(c+d*x)^3)])/(3*d)$

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a \* (c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\int e^{\frac{e}{(c+dx)^3}} dx = \frac{\sqrt[3]{-\frac{e}{(c+dx)^3}} (c+dx) \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d}$$

**Mathematica [A]** time = 0.0060606, size = 40, normalized size = 1.

$$\frac{(c+dx)^3 \sqrt[3]{-\frac{e}{(c+dx)^3}} \Gamma\left(-\frac{1}{3}, -\frac{e}{(c+dx)^3}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[E^(e/(c+d\*x)^3),x]

[Out]  $((-(e/(c+d*x)^3))^{(1/3)}*(c+d*x)*\Gamma[-1/3, -(e/(c+d*x)^3)])/(3*d)$

**Maple [F]** time = 0.017, size = 0, normalized size = 0.

$$\int e^{\frac{e}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(e/(d*x+c)^3),x)`

[Out] `int(exp(e/(d*x+c)^3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3de \int \frac{xe^{\left(\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right)}}{d^4x^4+4cd^3x^3+6c^2d^2x^2+4c^3dx+c^4} dx + xe^{\left(\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3),x, algorithm="maxima")`

[Out] `3*d*e*integrate(x*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + x*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))`

**Fricas [B]** time = 1.57395, size = 189, normalized size = 4.72

$$\frac{d \left(-\frac{e}{d^3}\right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, -\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right) - (dx+c)e^{\left(\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3),x, algorithm="fricas")`

[Out] `-(d*(-e/d^3)^(1/3)*gamma(2/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - (d*x + c)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**3),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\frac{e}{(dx+c)^3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(e^(e/(d*x + c)^3), x)
```

$$3.420 \quad \int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx$$

**Optimal.** Leaf size=21

$$\text{Unintegrable} \left( \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}, x \right)$$

[Out] Unintegrable[E^(e/(c + d\*x)^3)/(a + b\*x), x]

**Rubi [A]** time = 0.0222072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e/(c + d\*x)^3)/(a + b\*x), x]

[Out] Defer[Int][E^(e/(c + d\*x)^3)/(a + b\*x), x]

Rubi steps

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx = \int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx$$

**Mathematica [A]** time = 0.0314481, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x)^3)/(a + b\*x), x]

[Out] Integrate[E^(e/(c + d\*x)^3)/(a + b\*x), x]

**Maple [A]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a} e^{\frac{e}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^3)/(b\*x+a), x)

[Out] `int(exp(e/(d*x+c)^3)/(b*x+a),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="maxima")`

[Out] `integrate(e^(e/(d*x + c)^3)/(b*x + a), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right)}}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="fricas")`

[Out] `integral(e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/(b*x + a), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**3)/(b*x+a),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="giac")`

[Out] `integrate(e^(e/(d*x + c)^3)/(b*x + a), x)`



$$3.421 \quad \int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate}\left(\frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}, x\right)$$

[Out] CannotIntegrate[E^(e/(c + d\*x)^3)/(a + b\*x)^2, x]

**Rubi [A]** time = 0.0573897, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[E^(e/(c + d\*x)^3)/(a + b\*x)^2, x]

[Out] Defer[Int][E^(e/(c + d\*x)^3)/(a + b\*x)^2, x]

Rubi steps

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx = \int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$$

**Mathematica [A]** time = 0.298304, size = 0, normalized size = 0.

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(e/(c + d\*x)^3)/(a + b\*x)^2, x]

[Out] Integrate[E^(e/(c + d\*x)^3)/(a + b\*x)^2, x]

**Maple [A]** time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)^2} e^{\frac{e}{(dx+c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e/(d\*x+c)^3)/(b\*x+a)^2, x)

[Out] `int(exp(e/(d*x+c)^3)/(b*x+a)^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate(e^(e/(d*x + c)^3)/(b*x + a)^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\frac{e}{d^3x^3+3cd^2x^2+3c^2dx+c^3}\right)}}{b^2x^2+2abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral(e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^2*x^2 + 2*a*b*x + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)**3)/(b*x+a)**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="giac")`

[Out] undef

$$3.422 \quad \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx$$

**Optimal.** Leaf size=104

$$\frac{F^{\frac{f(bg-ah)}{dg-ch}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{h} - \frac{F^{\frac{bf}{d}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h}$$

[Out]  $-\left(\left(F^{e+(b*f)/d}\right)*\operatorname{ExpIntegralEi}\left[-\left(\left(b*c-a*d\right)*f*\operatorname{Log}[F]\right)/\left(d*(c+d*x)\right)\right]\right)/h + \left(F^{e+(f*(b*g-a*h))/(d*g-c*h)}\right)*\operatorname{ExpIntegralEi}\left[-\left(\left(b*c-a*d\right)*f*(g+h*x)*\operatorname{Log}[F]\right)/\left(\left(d*g-c*h\right)*(c+d*x)\right)\right]/h$

**Rubi [A]** time = 1.04559, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {2231, 2230, 2210, 2233, 2178}

$$\frac{F^{\frac{f(bg-ah)}{dg-ch}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{h} - \frac{F^{\frac{bf}{d}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[F^{e+(f*(a+b*x))/(c+d*x)}\right]/(g+h*x), x]$

[Out]  $-\left(\left(F^{e+(b*f)/d}\right)*\operatorname{ExpIntegralEi}\left[-\left(\left(b*c-a*d\right)*f*\operatorname{Log}[F]\right)/\left(d*(c+d*x)\right)\right]\right)/h + \left(F^{e+(f*(b*g-a*h))/(d*g-c*h)}\right)*\operatorname{ExpIntegralEi}\left[-\left(\left(b*c-a*d\right)*f*(g+h*x)*\operatorname{Log}[F]\right)/\left(\left(d*g-c*h\right)*(c+d*x)\right)\right]/h$

#### Rule 2231

$\operatorname{Int}\left[\left(F_{\cdot}\right)^{\left(e_{\cdot}\right)+\left(f_{\cdot}\right)*\left(a_{\cdot}\right)+\left(b_{\cdot}\right)*\left(x_{\cdot}\right)\right]/\left(\left(c_{\cdot}\right)+\left(d_{\cdot}\right)*\left(x_{\cdot}\right)\right)/\left(\left(g_{\cdot}\right)+\left(h_{\cdot}\right)*\left(x_{\cdot}\right)\right), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}\left[d/h, \operatorname{Int}\left[F^{e+(f*(a+b*x))/(c+d*x)}\right]/(c+d*x), x\right] - \operatorname{Dist}\left[(d*g-c*h)/h, \operatorname{Int}\left[F^{e+(f*(a+b*x))/(c+d*x)}\right]/\left((c+d*x)*(g+h*x)\right), x\right] /;$  FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b\*c - a\*d, 0] && NeQ[d\*g - c\*h, 0]

#### Rule 2230

$\operatorname{Int}\left[\left(F_{\cdot}\right)^{\left(e_{\cdot}\right)+\left(f_{\cdot}\right)*\left(a_{\cdot}\right)+\left(b_{\cdot}\right)*\left(x_{\cdot}\right)\right]/\left(\left(c_{\cdot}\right)+\left(d_{\cdot}\right)*\left(x_{\cdot}\right)\right)*\left(\left(g_{\cdot}\right)+\left(h_{\cdot}\right)*\left(x_{\cdot}\right)\right)^{\left(m_{\cdot}\right)}, x_{\text{Symbol}}] \rightarrow \operatorname{Int}\left[\left(g+h*x\right)^m F^{(d*e+b*f)/d-(f*(b*c-a*d))/(d*(c+d*x))}, x\right] /;$  FreeQ[{F, a, b, c, d, e, f, g, h, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[d\*g - c\*h, 0]

#### Rule 2210

$\operatorname{Int}\left[\left(F_{\cdot}\right)^{\left(a_{\cdot}\right)+\left(b_{\cdot}\right)*\left(\left(c_{\cdot}\right)+\left(d_{\cdot}\right)*\left(x_{\cdot}\right)\right)^{\left(n_{\cdot}\right)}}/\left(\left(e_{\cdot}\right)+\left(f_{\cdot}\right)*\left(x_{\cdot}\right)\right), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}\left[\left(F^{a*\operatorname{ExpIntegralEi}\left[b*(c+d*x)^n*\operatorname{Log}[F]\right]}(f*n)\right), x\right] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2233

$\operatorname{Int}\left[\left(F_{\cdot}\right)^{\left(e_{\cdot}\right)+\left(f_{\cdot}\right)*\left(a_{\cdot}\right)+\left(b_{\cdot}\right)*\left(x_{\cdot}\right)\right]/\left(\left(c_{\cdot}\right)+\left(d_{\cdot}\right)*\left(x_{\cdot}\right)\right)/\left(\left(g_{\cdot}\right)+\left(h_{\cdot}\right)*\left(x_{\cdot}\right)\right)*\left(\left(i_{\cdot}\right)+\left(j_{\cdot}\right)*\left(x_{\cdot}\right)\right), x_{\text{Symbol}}] \rightarrow -\operatorname{Dist}\left[d/\left(h*(d*i-c*j)\right), \operatorname{Subst}\left[\operatorname{Int}\left[F^{e+(f*(b*i-a*j))/(d*i-c*j)}\right]-\left((b*c-a*d)*f*x\right)/\left(d*i-c*j\right)\right]/x, x\right], x, (i+j*x)/(c+d*x)] /;$  FreeQ[{F, a, b, c, d, e, f, g, h

}, x] && EqQ[d\*g - c\*h, 0]

### Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

### Rubi steps

$$\begin{aligned} \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx &= \frac{d \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{h} - \frac{(dg-ch) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)(g+hx)} dx}{h} \\ &= \frac{\text{Subst}\left(\int \frac{F^{e+\frac{f(bg-ah)}{dg-ch}-\frac{(bc-ad)fx}{dg-ch}}}{x} dx, x, \frac{g+hx}{c+dx}\right)}{h} + \frac{d \int \frac{F^{\frac{de+bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{c+dx} dx}{h} \\ &= -\frac{F^{e+\frac{bf}{d}} \text{Ei}\left(-\frac{(bc-ad)f \log(F)}{d(c+dx)}\right)}{h} + \frac{F^{e+\frac{f(bg-ah)}{dg-ch}} \text{Ei}\left(-\frac{(bc-ad)f(g+hx) \log(F)}{(dg-ch)(c+dx)}\right)}{h} \end{aligned}$$

**Mathematica [A]** time = 0.316962, size = 103, normalized size = 0.99

$$\frac{F^{\frac{bf}{d}+e} \left( F^{\frac{fh(bc-ad)}{d(dg-ch)}} \text{Ei}\left(\frac{(bc-ad)f(g+hx) \log(F)}{(ch-dg)(c+dx)}\right) - \text{Ei}\left(\frac{(adf-bcf) \log(F)}{d(c+dx)}\right) \right)}{h}$$

Antiderivative was successfully verified.

[In] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x), x]

[Out] (F^(e + (b\*f)/d)\*(-ExpIntegralEi[(-(b\*c\*f) + a\*d\*f)\*Log[F]]/(d\*(c + d\*x))) + F^(((b\*c - a\*d)\*f\*h)/(d\*(d\*g - c\*h))\*ExpIntegralEi[((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F]]/((-d\*g) + c\*h)\*(c + d\*x))))/h

**Maple [B]** time = 0.242, size = 432, normalized size = 4.2

$$\frac{ad}{h(ad-bc)} F^{\frac{bf+de}{d}} \text{Ei}\left(1, -\frac{f(ad-bc) \ln(F)}{(dx+c)d} - \frac{(bf+de) \ln(F)}{d} - \frac{-\ln(F)bf - de \ln(F)}{d}\right) - \frac{bc}{h(ad-bc)} F^{\frac{bf+de}{d}} \text{Ei}\left(1, -\frac{f(ad-bc) \ln(F)}{(dx+c)d} - \frac{(bf+de) \ln(F)}{d} - \frac{-\ln(F)bf - de \ln(F)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g), x)

[Out] d/h/(a\*d-b\*c)\*F^((b\*f+d\*e)/d)\*Ei(1, -f\*(a\*d-b\*c)\*ln(F)/d/(d\*x+c)-(b\*f+d\*e)\*ln(F)/d-(-ln(F)\*b\*f-d\*e\*ln(F))/d)\*a-1/h/(a\*d-b\*c)\*F^((b\*f+d\*e)/d)\*Ei(1, -f\*(a\*d-b\*c)\*ln(F)/d/(d\*x+c)-(b\*f+d\*e)\*ln(F)/d-(-ln(F)\*b\*f-d\*e\*ln(F))/d)\*b\*c-d/h/(a\*d-b\*c)\*F^((a\*f\*h-b\*f\*g+c\*e\*h-d\*e\*g)/(c\*h-d\*g))\*Ei(1, -f\*(a\*d-b\*c)\*ln(F)/d/(d\*x+c)-(b\*f+d\*e)\*ln(F)/d-(-ln(F)\*a\*f\*h+ln(F)\*b\*f\*g-ln(F)\*c\*e\*h+ln(F)\*d\*e\*g)/(c\*h-d\*g))\*a+1/h/(a\*d-b\*c)\*F^((a\*f\*h-b\*f\*g+c\*e\*h-d\*e\*g)/(c\*h-d\*g))\*Ei(1, -f\*(a\*d-b\*c)\*ln(F)/d/(d\*x+c)-(b\*f+d\*e)\*ln(F)/d-(-ln(F)\*a\*f\*h+ln(F)\*b\*f\*g-ln(F)\*c\*e\*h+ln(F)\*d\*e\*g)/(c\*h-d\*g))\*b\*c

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g),x, algorithm="maxima")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g), x)

---

**Fricas [A]** time = 1.60862, size = 270, normalized size = 2.6

$$\frac{F^{\frac{de+bf}{d}} \operatorname{Ei}\left(-\frac{(bc-ad)f \log(F)}{d^2x+cd}\right) - F^{\frac{(de+bf)g-(ce+af)h}{dg-ch}} \operatorname{Ei}\left(-\frac{((bc-ad)fhx+(bc-ad)fg) \log(F)}{cdg-c^2h+(d^2g-cdh)x}\right)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g),x, algorithm="fricas")

[Out]  $-(F^{((d*e + b*f)/d}) * \operatorname{Ei}(-((b*c - a*d)*f*\log(F)/(d^2*x + c*d))) - F^{(((d*e + b*f)*g - (c*e + a*f)*h)/(d*g - c*h)}) * \operatorname{Ei}(-((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*\log(F)/(c*d*g - c^2*h + (d^2*g - c*d*h)*x)))/h$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g),x, algorithm="giac")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g), x)

$$3.423 \quad \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2} dx$$

**Optimal.** Leaf size=159

$$\frac{f \log(F)(bc-ad)F^{\frac{f(bg-ah)}{dg-ch}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^2} + \frac{dF^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{h(dg-ch)} - \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{h(g+hx)}$$

[Out] (d\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x))))/(h\*(d\*g - c\*h)) - F^(e + (f\*(a + b\*x))/(c + d\*x))/(h\*(g + h\*x)) + ((b\*c - a\*d)\*f\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h)))\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]/(d\*g - c\*h)^2

**Rubi [A]** time = 2.55867, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2232, 6742, 2230, 2209, 2210, 2231, 2233, 2178}

$$\frac{f \log(F)(bc-ad)F^{\frac{f(bg-ah)}{dg-ch}+e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^2} + \frac{dF^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{h(dg-ch)} - \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{h(g+hx)}$$

Antiderivative was successfully verified.

[In] Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^2, x]

[Out] (d\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x))))/(h\*(d\*g - c\*h)) - F^(e + (f\*(a + b\*x))/(c + d\*x))/(h\*(g + h\*x)) + ((b\*c - a\*d)\*f\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h)))\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]/(d\*g - c\*h)^2

#### Rule 2232

```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.) + (h_.)*(x_))^(m_), x_Symbol] := Simp[((g + h*x)^(m + 1)*F^(e + (f*(a + b*x))/(c + d*x)))/(h*(m + 1)), x] - Dist[(f*(b*c - a*d)*Log[F])/(h*(m + 1)), Int[((g + h*x)^(m + 1)*F^(e + (f*(a + b*x))/(c + d*x)))/(c + d*x)^2, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b*c - a*d, 0] && NeQ[d*g - c*h, 0] && ILtQ[m, -1]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2230

```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Int[(g + h*x)^m*F^((d*e + b*f)/d - (f*(b*c - a*d))/(d*(c + d*x))), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m}, x] && NeQ[b*c - a*d, 0] && EqQ[d*g - c*h, 0]
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[(e + f*x)^n*F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n)
```

$n \cdot \text{Log}[F], x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 2231

Int[(F\_)^((e\_.) + ((f\_.)\*((a\_.) + (b\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)))/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Dist[d/h, Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b\*c - a\*d, 0] && NeQ[d\*g - c\*h, 0]

Rule 2233

Int[(F\_)^((e\_.) + ((f\_.)\*((a\_.) + (b\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)))/((g\_.) + (h\_.)\*(x\_))\*((i\_.) + (j\_.)\*(x\_)), x\_Symbol] :> -Dist[d/(h\*(d\*i - c\*j)), Subst[Int[F^(e + (f\*(b\*i - a\*j))/(d\*i - c\*j) - ((b\*c - a\*d)\*f\*x)/(d\*i - c\*j)]/x, x], x, (i + j\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && EqQ[d\*g - c\*h, 0]

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2} dx = -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} + \frac{((bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)} dx}{h}$$

$$= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} + \frac{((bc-ad)f \log(F)) \int \left( \frac{dF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)(c+dx)^2} - \frac{dF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(c+dx)} + \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)} \right) dx}{h}$$

$$= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} - \frac{(d(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^2} + \frac{((bc-ad)fh \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^2} + \frac{(d(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^2}$$

$$= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} - \frac{(d(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^2} + \frac{(d(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^2} - \frac{((bc-ad)fh \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^2}$$

$$= \frac{dF^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{h(dg-ch)} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} + \frac{(bc-ad)fF^{e+\frac{bf}{d}} \text{Ei}\left(-\frac{(bc-ad)f \log(F)}{d(c+dx)}\right) \log(F)}{(dg-ch)^2} + \frac{((bc-ad)f \log(F)) \text{Subst}\left[\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx, \frac{g+hx}{c+dx}\right]}{(dg-ch)^2}$$

$$= \frac{dF^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{h(dg-ch)} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} + \frac{(bc-ad)fF^{e+\frac{f(bg-ah)}{dg-ch}} \text{Ei}\left(-\frac{(bc-ad)f(g+hx) \log(F)}{(dg-ch)(c+dx)}\right) \log(F)}{(dg-ch)^2}$$

**Mathematica [F]** time = 0.984719, size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^2,x]

[Out] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^2, x]

**Maple [B]** time = 0.17, size = 580, normalized size = 3.7

$$\frac{\ln(F) adf}{(ch-dg)^2} F^{\frac{bf+de}{d}} F^{\frac{f(ad-bc)}{(dx+c)d}} \left( \frac{f \ln(F) a}{dx+c} - \frac{\ln(F) bcf}{(dx+c)d} + \frac{\ln(F) bf}{d} + \ln(F) e - \frac{\ln(F) afh}{ch-dg} + \frac{\ln(F) bfg}{ch-dg} - \frac{\ln(F) ceh}{ch-dg} + \frac{\ln(F) deg}{ch-dg} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^2,x)

[Out]  $f \ln(F) / (c * h - d * g)^2 * F^{((b * f + d * e) / d)} * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * a * d - f * \ln(F) / (c * h - d * g)^2 * F^{((b * f + d * e) / d)} * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * b * c + f * \ln(F) / (c * h - d * g)^2 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * a * d - f * \ln(F) / (c * h - d * g)^2 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * b * c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^2,x, algorithm="maxima")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g)^2, x)

**Fricas [A]** time = 1.53879, size = 456, normalized size = 2.87

$$\frac{((bc-ad)fhx + (bc-ad)fg)F^{\frac{(de+bf)g-(ce+af)h}{dg-ch}} Ei\left(-\frac{((bc-ad)fhx+(bc-ad)fg)\log(F)}{cdg-c^2h+(d^2g-cdh)x}\right) \log(F) + (cdg-c^2h+(d^2g-cdh)x)F^{\frac{ce+af+(de+bf)g-(ce+af)h}{dx+c}}}{d^2g^3 - 2cdg^2h + c^2gh^2 + (d^2g^2h - 2cdgh^2 + c^2h^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^2,x, algorithm="fricas")

[Out] (((b\*c - a\*d)\*f\*h\*x + (b\*c - a\*d)\*f\*g)\*F^(((d\*e + b\*f)\*g - (c\*e + a\*f)\*h)/(d\*g - c\*h))\*Ei(-((b\*c - a\*d)\*f\*h\*x + (b\*c - a\*d)\*f\*g)\*log(F)/(c\*d\*g - c^2\*h + (d^2\*g - c\*d\*h)\*x))\*log(F) + (c\*d\*g - c^2\*h + (d^2\*g - c\*d\*h)\*x)\*F^((c\*e + a\*f + (d\*e + b\*f)\*x)/(d\*x + c)))/(d^2\*g^3 - 2\*c\*d\*g^2\*h + c^2\*g\*h^2 + (d^2\*g^2\*h - 2\*c\*d\*g\*h^2 + c^2\*h^3)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^2,x, algorithm="giac")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g)^2, x)

$$3.424 \quad \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3} dx$$

**Optimal.** Leaf size=366

$$\frac{d^2 F^{-\frac{f(bc-ad)}{d(c+dx)} + \frac{bf}{d} + e}}{2h(dg-ch)^2} + \frac{f^2 h \log^2(F)(bc-ad)^2 F^{\frac{f(bg-ah)}{dg-ch} + e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{2(dg-ch)^4} + \frac{df \log(F)(bc-ad) F^{\frac{f(bg-ah)}{dg-ch} + e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^3}$$

[Out]  $(d^2 F^{(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))}/(2*h*(d*g - c*h)^2) - F^{(e + (f*(a + b*x))/(c + d*x))}/(2*h*(g + h*x)^2) + (d*(b*c - a*d)*f*F^{(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))}*\operatorname{Log}[F])/ (2*(d*g - c*h)^3) - ((b*c - a*d)*f*F^{(e + (f*(a + b*x))/(c + d*x))}*\operatorname{Log}[F])/ (2*(d*g - c*h)^2*(g + h*x)) + (d*(b*c - a*d)*f*F^{(e + (f*(b*g - a*h))/(d*g - c*h))}*\operatorname{ExpIntegralEi}[-((b*c - a*d)*f*(g + h*x)*\operatorname{Log}[F])/((d*g - c*h)*(c + d*x))]}*\operatorname{Log}[F])/ (d*g - c*h)^3 + ((b*c - a*d)^2*f^2*F^{(e + (f*(b*g - a*h))/(d*g - c*h))}*\operatorname{ExpIntegralEi}[-((b*c - a*d)*f*(g + h*x)*\operatorname{Log}[F])/((d*g - c*h)*(c + d*x))]}*\operatorname{Log}[F]^2)/ (2*(d*g - c*h)^4)$

**Rubi [A]** time = 4.75749, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2232, 6742, 2230, 2209, 2210, 2231, 2233, 2178}

$$\frac{d^2 F^{-\frac{f(bc-ad)}{d(c+dx)} + \frac{bf}{d} + e}}{2h(dg-ch)^2} + \frac{f^2 h \log^2(F)(bc-ad)^2 F^{\frac{f(bg-ah)}{dg-ch} + e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{2(dg-ch)^4} + \frac{df \log(F)(bc-ad) F^{\frac{f(bg-ah)}{dg-ch} + e} \operatorname{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(e + (f*(a + b*x))/(c + d*x))}/(g + h*x)^3, x]$

[Out]  $(d^2 F^{(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))}/(2*h*(d*g - c*h)^2) - F^{(e + (f*(a + b*x))/(c + d*x))}/(2*h*(g + h*x)^2) + (d*(b*c - a*d)*f*F^{(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))}*\operatorname{Log}[F])/ (2*(d*g - c*h)^3) - ((b*c - a*d)*f*F^{(e + (f*(a + b*x))/(c + d*x))}*\operatorname{Log}[F])/ (2*(d*g - c*h)^2*(g + h*x)) + (d*(b*c - a*d)*f*F^{(e + (f*(b*g - a*h))/(d*g - c*h))}*\operatorname{ExpIntegralEi}[-((b*c - a*d)*f*(g + h*x)*\operatorname{Log}[F])/((d*g - c*h)*(c + d*x))]}*\operatorname{Log}[F])/ (d*g - c*h)^3 + ((b*c - a*d)^2*f^2*F^{(e + (f*(b*g - a*h))/(d*g - c*h))}*\operatorname{ExpIntegralEi}[-((b*c - a*d)*f*(g + h*x)*\operatorname{Log}[F])/((d*g - c*h)*(c + d*x))]}*\operatorname{Log}[F]^2)/ (2*(d*g - c*h)^4)$

#### Rule 2232

$\operatorname{Int}[(F_)^{((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/(c_.) + (d_.)*(x_))}*((g_.) + (h_.)*(x_))^{(m_)}], x\_Symbol] \rightarrow \operatorname{Simp}[(g + h*x)^{(m+1)}*F^{(e + (f*(a + b*x))/(c + d*x))}/(h*(m+1)), x] - \operatorname{Dist}[(f*(b*c - a*d)*\operatorname{Log}[F])/ (h*(m+1)), \operatorname{Int}[(g + h*x)^{(m+1)}*F^{(e + (f*(a + b*x))/(c + d*x))}/(c + d*x)^2, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, h\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[d*g - c*h, 0] \&\& \operatorname{ILtQ}[m, -1]$

#### Rule 6742

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{ExpandIntegrand}[u, x]\}, \operatorname{Int}[v, x] /; \operatorname{SumQ}[v]$

#### Rule 2230

Int[(F\_)^((e\_.) + ((f\_.)\*(a\_.) + (b\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)))\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(g + h\*x)^m\*F^((d\*e + b\*f)/d - (f\*(b\*c - a\*d))/(d\*(c + d\*x))), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m}, x] & NeQ[b\*c - a\*d, 0] && EqQ[d\*g - c\*h, 0]

#### Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

#### Rule 2210

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := Simp[(F^a\*ExpIntegralEi[b\*(c + d\*x)^n\*Log[F]])/(f\*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 2231

Int[(F\_)^((e\_.) + ((f\_.)\*(a\_.) + (b\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)))/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Dist[d/h, Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b\*c - a\*d, 0] && NeQ[d\*g - c\*h, 0]

#### Rule 2233

Int[(F\_)^((e\_.) + ((f\_.)\*(a\_.) + (b\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)))/(((g\_.) + (h\_.)\*(x\_))\*((i\_.) + (j\_.)\*(x\_))), x\_Symbol] := -Dist[d/(h\*(d\*i - c\*j)), Subst[Int[F^(e + (f\*(b\*i - a\*j))/(d\*i - c\*j) - ((b\*c - a\*d)\*f\*x)/(d\*i - c\*j))/x, x], x, (i + j\*x)/(c + d\*x)], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && EqQ[d\*g - c\*h, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
 \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3} dx &= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} + \frac{((bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^2} dx}{2h} \\
 &= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} + \frac{((bc-ad)f \log(F)) \int \left( \frac{d^2 F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(c+dx)^2} - \frac{2d^2 F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)} h + \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)^2} h^2 + \frac{2d F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(g+hx)} h^2 \right) dx}{2h} \\
 &= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} - \frac{(d^2(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^3} + \frac{(d(bc-ad)fh \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^3} + \frac{(d^2(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^3} \\
 &= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} - \frac{(d^2(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^3} + \frac{(d^2(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^3} \\
 &= \frac{d^2 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} + \frac{d(bc-ad)f F^{e+\frac{bf}{d}} \text{Ei}\left(-\frac{(bc-ad)f \log(F)}{d(c+dx)}\right) \log(F)}{(dg-ch)^3} \\
 &= \frac{d^2 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} + \frac{d(bc-ad)f F^{e+\frac{f(bg-ah)}{dg-ch}} \text{Ei}\left(-\frac{(bc-ad)f(g+hx)}{(dg-ch)(c+dx)}\right) \log(F)}{(dg-ch)^3} \\
 &= \frac{d^2 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} + \frac{d(bc-ad)f F^{e+\frac{f(bg-ah)}{dg-ch}} \text{Ei}\left(-\frac{(bc-ad)f(g+hx)}{(dg-ch)(c+dx)}\right) \log(F)}{(dg-ch)^3} \\
 &= \frac{d^2 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} + \frac{d(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{2(dg-ch)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} + \frac{d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} \\
 &= \frac{d^2 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2} + \frac{d(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{2(dg-ch)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)} + \frac{d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{2(dg-ch)^2(g+hx)}
 \end{aligned}$$

**Mathematica [F]** time = 0.503373, size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^3, x]

[Out] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^3, x]

**Maple [B]** time = 0.213, size = 2014, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^3, x)

[Out] -1/2\*d^2\*f^2\*ln(F)^2\*h/(c\*h-d\*g)^4\*F^((b\*f+d\*e)/d)\*F^(f\*(a\*d-b\*c)/d/(d\*x+c))/(f\*ln(F)/(d\*x+c)\*a-f\*ln(F)/d/(d\*x+c)\*b\*c+ln(F)/d\*b\*f+ln(F)\*e-1/(c\*h-d\*g)\*

$\ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g)^2 * a^2 + d * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g)^2 * a * b * c - 1 / 2 * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * a^2 + d * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * a * b * c - 1 / 2 * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * b^2 * c^2 - 1 / 2 * d^2 * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * a^2 + d * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * a * b * c - 1 / 2 * f^2 * \ln(F)^2 * h / (c * h - d * g)^4 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * b^2 * c^2 - d^2 * f * \ln(F) / (c * h - d * g)^3 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * a + d * f * \ln(F) / (c * h - d * g)^3 * F^{((b * f + d * e) / d) * F^{(f * (a * d - b * c) / d / (d * x + c))} / (f * \ln(F) / (d * x + c) * a - f * \ln(F) / d / (d * x + c) * b * c + \ln(F) / d * b * f + \ln(F) * e - 1 / (c * h - d * g) * \ln(F) * a * f * h + 1 / (c * h - d * g) * \ln(F) * b * f * g - 1 / (c * h - d * g) * \ln(F) * c * e * h + 1 / (c * h - d * g) * \ln(F) * d * e * g) * b * c - d^2 * f * \ln(F) / (c * h - d * g)^3 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * a + d * f * \ln(F) / (c * h - d * g)^3 * F^{((a * f * h - b * f * g + c * e * h - d * e * g) / (c * h - d * g))} * Ei(1, -f * (a * d - b * c) * \ln(F) / d / (d * x + c) - (b * f + d * e) * \ln(F) / d - (-\ln(F) * a * f * h + \ln(F) * b * f * g - \ln(F) * c * e * h + \ln(F) * d * e * g) / (c * h - d * g)) * b * c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e + \frac{(bx+a)f}{dx+c}}}{(hx + g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^3,x, algorithm="maxima")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g)^3, x)

**Fricas [B]** time = 1.73227, size = 1507, normalized size = 4.12

$$\left(\left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f^2 h^3 x^2 + 2\left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f^2 g h^2 x + \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f^2 g^2 h\right) \log (F)^2 + 2\left(\left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) h^3 x^2 + \left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) g h^2 x + \left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) g^2 h\right) \log (F) + 2\left(\left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) h^3 x^2 + \left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) g h^2 x + \left(b c d^2 - \left(b^2 c^2 - 2 a b c d + a^2 d^2\right) f\right) g^2 h\right) \log (F)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^2*h^3*x^2 + 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*f^2*g*h^2*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^2*g^2*h)*log(F)
^2 + 2*((b*c*d^2 - a*d^3)*f*g^3 - (b*c^2*d - a*c*d^2)*f*g^2*h + ((b*c*d^2 -
a*d^3)*f*g*h^2 - (b*c^2*d - a*c*d^2)*f*h^3)*x^2 + 2*((b*c*d^2 - a*d^3)*f*g
^2*h - (b*c^2*d - a*c*d^2)*f*g*h^2)*x)*log(F))*F^(((d*e + b*f)*g - (c*e +
a*f)*h)/(d*g - c*h))*Ei(-((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*log(F)/(c*d*g
- c^2*h + (d^2*g - c*d*h)*x)) + (2*c*d^3*g^3 - 5*c^2*d^2*g^2*h + 4*c^3*d*g
*h^2 - c^4*h^3 + (d^4*g^2*h - 2*c*d^3*g*h^2 + c^2*d^2*h^3)*x^2 + 2*(d^4*g^3
- 2*c*d^3*g^2*h + c^2*d^2*g*h^2)*x + ((b*c^2*d - a*c*d^2)*f*g^2*h - (b*c^3
- a*c^2*d)*f*g*h^2 + ((b*c*d^2 - a*d^3)*f*g*h^2 - (b*c^2*d - a*c*d^2)*f*h^
3)*x^2 + ((b*c*d^2 - a*d^3)*f*g^2*h - (b*c^3 - a*c^2*d)*f*h^3)*x)*log(F))*F
^(((c*e + a*f + (d*e + b*f)*x)/(d*x + c)))/(d^4*g^6 - 4*c*d^3*g^5*h + 6*c^2*
d^2*g^4*h^2 - 4*c^3*d*g^3*h^3 + c^4*g^2*h^4 + (d^4*g^4*h^2 - 4*c*d^3*g^3*h^
3 + 6*c^2*d^2*g^2*h^4 - 4*c^3*d*g*h^5 + c^4*h^6)*x^2 + 2*(d^4*g^5*h - 4*c*d
^3*g^4*h^2 + 6*c^2*d^2*g^3*h^3 - 4*c^3*d*g^2*h^4 + c^4*g*h^5)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(e+f*(b*x+a)/(d*x+c))/(h*x+g)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e + \frac{(bx+a)f}{dx+c}}}{(hx+g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^3, x)
```

**3.425** 
$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4} dx$$

**Optimal.** Leaf size=634

$$\frac{d^2 f \log(F)(bc - ad)F^{\frac{f(bg-ah)}{dg-ch}+e} \text{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg - ch)^4} + \frac{d^3 F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{3h(dg - ch)^3} + \frac{5d^2 f \log(F)(bc - ad)F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{6(dg - ch)^4} + \dots$$

[Out] (d^3\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x))))/(3\*h\*(d\*g - c\*h)^3) - F^(e + (f\*(a + b\*x))/(c + d\*x))/(3\*h\*(g + h\*x)^3) + (5\*d^2\*(b\*c - a\*d)\*f\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x)))\*Log[F]/(6\*(d\*g - c\*h)^4) - ((b\*c - a\*d)\*f\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*Log[F]/(6\*(d\*g - c\*h)^2\*(g + h\*x)^2) - (2\*d\*(b\*c - a\*d)\*f\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*Log[F]/(3\*(d\*g - c\*h)^3\*(g + h\*x)) + (d^2\*(b\*c - a\*d)\*f\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]/(d\*g - c\*h)^4 + (d\*(b\*c - a\*d)^2\*f^2\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x)))\*h\*Log[F]^2/(6\*(d\*g - c\*h)^5) - ((b\*c - a\*d)^2\*f^2\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*h\*Log[F]^2/(6\*(d\*g - c\*h)^4\*(g + h\*x)) + (d\*(b\*c - a\*d)^2\*f^2\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*h\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]^2/(d\*g - c\*h)^5 + ((b\*c - a\*d)^3\*f^3\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*h^2\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]^3)/(6\*(d\*g - c\*h)^6)

**Rubi [A]** time = 9.40524, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2232, 6742, 2230, 2209, 2210, 2231, 2233, 2178}

$$\frac{d^2 f \log(F)(bc - ad)F^{\frac{f(bg-ah)}{dg-ch}+e} \text{Ei}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg - ch)^4} + \frac{d^3 F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{3h(dg - ch)^3} + \frac{5d^2 f \log(F)(bc - ad)F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{6(dg - ch)^4} + \dots$$

Antiderivative was successfully verified.

[In] Int[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^4, x]

[Out] (d^3\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x))))/(3\*h\*(d\*g - c\*h)^3) - F^(e + (f\*(a + b\*x))/(c + d\*x))/(3\*h\*(g + h\*x)^3) + (5\*d^2\*(b\*c - a\*d)\*f\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x)))\*Log[F]/(6\*(d\*g - c\*h)^4) - ((b\*c - a\*d)\*f\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*Log[F]/(6\*(d\*g - c\*h)^2\*(g + h\*x)^2) - (2\*d\*(b\*c - a\*d)\*f\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*Log[F]/(3\*(d\*g - c\*h)^3\*(g + h\*x)) + (d^2\*(b\*c - a\*d)\*f\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]/(d\*g - c\*h)^4 + (d\*(b\*c - a\*d)^2\*f^2\*F^(e + (b\*f)/d - ((b\*c - a\*d)\*f)/(d\*(c + d\*x)))\*h\*Log[F]^2/(6\*(d\*g - c\*h)^5) - ((b\*c - a\*d)^2\*f^2\*F^(e + (f\*(a + b\*x))/(c + d\*x))\*h\*Log[F]^2/(6\*(d\*g - c\*h)^4\*(g + h\*x)) + (d\*(b\*c - a\*d)^2\*f^2\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*h\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]^2/(d\*g - c\*h)^5 + ((b\*c - a\*d)^3\*f^3\*F^(e + (f\*(b\*g - a\*h))/(d\*g - c\*h))\*h^2\*ExpIntegralEi[-(((b\*c - a\*d)\*f\*(g + h\*x)\*Log[F])/((d\*g - c\*h)\*(c + d\*x)))]\*Log[F]^3)/(6\*(d\*g - c\*h)^6)

**Rule 2232**

Int[(F\_)^((e\_.) + ((f\_.)\*((a\_.) + (b\_.)\*(x\_)))/(c\_.) + (d\_.)\*(x\_)))\*((g\_.) + (h\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*F^(e + (f\*(a + b\*x)/(c + d\*x)))/(g + h\*x)^4, x]

```
x))/(c + d*x))/(h*(m + 1)), x] - Dist[(f*(b*c - a*d)*Log[F])/(h*(m + 1)),
Int[((g + h*x)^(m + 1)*F^(e + (f*(a + b*x))/(c + d*x)))/(c + d*x)^2, x], x]
/; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b*c - a*d, 0] && NeQ[d*g -
c*h, 0] && ILtQ[m, -1]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2230

```
Int[(F_)^((e_.) + ((f_.)*(a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))*((g_.)
+ (h_.)*(x_))^(m_.), x_Symbol] := Int[(g + h*x)^m*F^((d*e + b*f)/d - (f*(b
*c - a*d))/(d*(c + d*x))), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m}, x] &
& NeQ[b*c - a*d, 0] && EqQ[d*g - c*h, 0]
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[(e + f*x)^n*F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

#### Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] := Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

#### Rule 2231

```
Int[(F_)^((e_.) + ((f_.)*(a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))/((g_.)
+ (h_.)*(x_)), x_Symbol] := Dist[d/h, Int[F^(e + (f*(a + b*x))/(c + d*x))/
(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[F^(e + (f*(a + b*x))/(c + d*x))
/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] &&
NeQ[b*c - a*d, 0] && NeQ[d*g - c*h, 0]
```

#### Rule 2233

```
Int[(F_)^((e_.) + ((f_.)*(a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))/(((g_.)
+ (h_.)*(x_))*((i_.) + (j_.)*(x_))), x_Symbol] := -Dist[d/(h*(d*i - c*j))
, Subst[Int[F^(e + (f*(b*i - a*j))/(d*i - c*j) - ((b*c - a*d)*f*x)/(d*i - c
*j))/x, x], x, (i + j*x)/(c + d*x)], x] /; FreeQ[{F, a, b, c, d, e, f, g, h
}, x] && EqQ[d*g - c*h, 0]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps



$$\begin{aligned}
\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4} dx &= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{((bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^3} dx}{3h} \\
&= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{((bc-ad)f \log(F)) \int \left( \frac{d^3 F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)^2} - \frac{3d^3 F^{e+\frac{f(a+bx)}{c+dx}} h}{(dg-ch)^4(c+dx)} + \frac{F^{e+\frac{f(a+bx)}{c+dx}} h^2}{(dg-ch)^2(g+hx)^3} + \frac{2d F^{e+\frac{f(a+bx)}{c+dx}} h^2}{(dg-ch)^3(g+hx)^2} \right) dx}{3h} \\
&= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} - \frac{(d^3(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx} dx}{(dg-ch)^4} + \frac{(d^2(bc-ad)fh \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} + \frac{(d^3(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} \\
&= -\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} - \frac{2d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{3(dg-ch)^3(g+hx)} - \frac{(d^3(bc-ad)f \log(F)) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} - \frac{2d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{3(dg-ch)^3(g+hx)} + \frac{d^2(bc-ad)f \log(F) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} - \frac{2d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{3(dg-ch)^3(g+hx)} + \frac{d^2(bc-ad)f \log(F) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} - \frac{2d(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{3(dg-ch)^3(g+hx)} + \frac{d^2(bc-ad)f \log(F) \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx} dx}{(dg-ch)^4} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{6(dg-ch)^4} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{6(dg-ch)^4} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{6(dg-ch)^4} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{6(dg-ch)^4} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2} \\
&= \frac{d^3 F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)f F^{e+\frac{bf}{d} - \frac{(bc-ad)f}{d(c+dx)}} \log(F)}{6(dg-ch)^4} - \frac{(bc-ad)f F^{e+\frac{f(a+bx)}{c+dx}} \log(F)}{6(dg-ch)^2(g+hx)^2}
\end{aligned}$$

**Mathematica [F]** time = 0.950908, size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^4, x]

[Out] Integrate[F^(e + (f\*(a + b\*x))/(c + d\*x))/(g + h\*x)^4, x]

**Maple [B]** time = 0.27, size = 4671, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (F^{(e+f*(b*x+a)/(d*x+c))}/(h*x+g)^4, x)$

[Out] 
$$-1/2*d^2*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))} * Ei(1, -f*(a*d-b*c)*\ln(F)/d/(d*x+c) - (b*f+d*e)*\ln(F)/d - (-\ln(F)*a*f*h+\ln(F)*b*f*g - \ln(F)*c*e*h+\ln(F)*d*e*g)/(c*h-d*g)) * a^2*b*c+1/2*d*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))} * Ei(1, -f*(a*d-b*c)*\ln(F)/d/(d*x+c) - (b*f+d*e)*\ln(F)/d - (-\ln(F)*a*f*h+\ln(F)*b*f*g - \ln(F)*c*e*h+\ln(F)*d*e*g)/(c*h-d*g)) * a*b^2*c^2+d*f^2*\ln(F)^2*h/(c*h-d*g)^5*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^2*b^2*c^2+d*f^2*\ln(F)^2*h/(c*h-d*g)^5*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^2*b^2*c^2-2*d^2*f^2*\ln(F)^2*h/(c*h-d*g)^5*F^{((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))} * Ei(1, -f*(a*d-b*c)*\ln(F)/d/(d*x+c) - (b*f+d*e)*\ln(F)/d - (-\ln(F)*a*f*h+\ln(F)*b*f*g - \ln(F)*c*e*h+\ln(F)*d*e*g)/(c*h-d*g)) * a*b*c-d^2*f*\ln(F)/(c*h-d*g)^4*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^3*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^3*b^3*c^3-1/6*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^3*a^3+1/6*d^3*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^2*a^3+1/6*d^3*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^2*a^2+d^3*f^2*\ln(F)^2*h/(c*h-d*g)^5*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^2*a^2+d^3*f^2*\ln(F)^2*h/(c*h-d*g)^5*F^{((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))} * Ei(1, -f*(a*d-b*c)*\ln(F)/d/(d*x+c) - (b*f+d*e)*\ln(F)/d - (-\ln(F)*a*f*h+\ln(F)*b*f*g - \ln(F)*c*e*h+\ln(F)*d*e*g)/(c*h-d*g)) * a-d^2*f^3*\ln(F)^3*h^2/(c*h-d*g)^6*F^{((b*f+d*e)/d)} * F^{(f*(a*d-b*c)/d/(d*x+c))}/(f*\ln(F)/(d*x+c)*a-f*\ln(F)/d/(d*x+c)*b*c+\ln(F)/d*b*f+\ln(F)*e-1/(c*h-d*g)*\ln(F)*a*f*h+1/(c*h-d*g)*\ln(F)*b*f*g-1/(c*h-d*g)*\ln(F)*c*e*h+1/(c*h-d*g)*\ln(F)*d*e*g)^3*a^2*b*c+d*f^$$

$$3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^3 a^* b^2 c^2 - 1 / 2 d^2 f^3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a^2 b^* c + 1 / 2 d^2 f^3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a^* b^2 c^2 - 1 / 2 d^2 f^3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a^* b^2 c^2 - 2 d^2 f^2 \ln(F)^2 h / (c h - d g)^5 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a^* b^* c - 2 d^2 f^2 \ln(F)^2 h / (c h - d g)^5 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a^* b^* c + d^3 f^2 \ln(F)^2 h / (c h - d g)^5 F^{((a f^* h - b^* f^* g + c^* e^* h - d^* e^* g) / (c h - d g))} * Ei(1, -f(a d - b c) \ln(F) / d / (d x + c) - (b f + d e) \ln(F) / d - (-\ln(F) a^* f^* h + \ln(F) b^* f^* g - \ln(F) c^* e^* h + \ln(F) d^* e^* g) / (c h - d g)) a^2 + d^3 f \ln(F) / (c h - d g)^4 F^{((b f + d e) / d)} F^{(f(a d - b c) / d / (d x + c))} / (f \ln(F) / (d x + c) a - f \ln(F) / d / (d x + c) b^* c + \ln(F) / d b^* f + \ln(F) e^{-1 / (c h - d g)} \ln(F) a^* f^* h + 1 / (c h - d g) \ln(F) b^* f^* g - 1 / (c h - d g) \ln(F) c^* e^* h + 1 / (c h - d g) \ln(F) d^* e^* g)^2 a - d^2 f \ln(F) / (c h - d g)^4 F^{((a f^* h - b^* f^* g + c^* e^* h - d^* e^* g) / (c h - d g))} * Ei(1, -f(a d - b c) \ln(F) / d / (d x + c) - (b f + d e) \ln(F) / d - (-\ln(F) a^* f^* h + \ln(F) b^* f^* g - \ln(F) c^* e^* h + \ln(F) d^* e^* g) / (c h - d g)) b^3 c^3 + 1 / 6 d^3 f^3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((a f^* h - b^* f^* g + c^* e^* h - d^* e^* g) / (c h - d g))} * Ei(1, -f(a d - b c) \ln(F) / d / (d x + c) - (b f + d e) \ln(F) / d - (-\ln(F) a^* f^* h + \ln(F) b^* f^* g - \ln(F) c^* e^* h + \ln(F) d^* e^* g) / (c h - d g)) b^3 c^3 + 1 / 6 d^3 f^3 \ln(F)^3 h^2 / (c h - d g)^6 F^{((a f^* h - b^* f^* g + c^* e^* h - d^* e^* g) / (c h - d g))} * Ei(1, -f(a d - b c) \ln(F) / d / (d x + c) - (b f + d e) \ln(F) / d - (-\ln(F) a^* f^* h + \ln(F) b^* f^* g - \ln(F) c^* e^* h + \ln(F) d^* e^* g) / (c h - d g)) a^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e + \frac{(bx+a)f}{dx+c}}}{(hx+g)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^4,x, algorithm="maxima")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g)^4, x)

**Fricas [B]** time = 1.9458, size = 4427, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^4,x, algorithm="fricas")

[Out]  $\frac{1}{6} \left( \left( (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) f^3 h^5 x^3 + 3(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) f^3 g h^4 x^2 + 3(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) f^3 g^2 h^3 x + (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) f^3 g^3 h^2 \right) \log(F)^3 + 6 \left( (b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g^4 h - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g^3 h^2 + ((b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g h^4 - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 h^5) x^3 + 3 \left( (b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g^2 h^3 - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g h^4 \right) x^2 + 3 \left( (b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g^3 h^2 - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g^2 h^3 \right) x \right) \log(F)^2 + 6 \left( (b^2cd^4 - a^2d^5) f g^5 - 2(b^2cd^3 - a^2cd^4) f g^4 h + (b^2cd^2 - a^2cd^3) f g^3 h^2 + ((b^2cd^4 - a^2d^5) f g^2 h^3 - 2(b^2cd^3 - a^2cd^4) f g h^4 + (b^2cd^2 - a^2cd^3) f h^5) x^3 + 3 \left( (b^2cd^4 - a^2d^5) f g^3 h^2 - 2(b^2cd^3 - a^2cd^4) f g^2 h^3 + (b^2cd^2 - a^2cd^3) f g h^4 \right) x^2 + 3 \left( (b^2cd^4 - a^2d^5) f g^4 h - 2(b^2cd^3 - a^2cd^4) f g^3 h^2 + (b^2cd^2 - a^2cd^3) f g^2 h^3 \right) x \right) \log(F) \right) F^{\left( \frac{(d^2e + b^2f)g - (c^2e + a^2f)h}{d^2g - c^2h} \right)} \operatorname{Ei} \left( - \left( (b^2c - a^2d) f h x + (b^2c - a^2d) f g \right) \log(F) / (c^2d g - c^2h + (d^2g - c^2h) x) \right) + (6c^2d^5g^5 - 24c^2d^4g^4h + 38c^3d^3g^3h^2 - 30c^4d^2g^2h^3 + 12c^5d^2g^2h^4 - 2c^6d^2h^5 + 2(d^6g^3h^2 - 3c^2d^5g^2h^3 + 3c^2d^4g^2h^4 - c^3d^3g^2h^5) x^3 + 6(d^6g^4h - 3c^2d^5g^3h^2 + 3c^2d^4g^2h^3 - c^3d^3g^2h^4) x^2 + ((b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g^3 h^2 - (b^2c^4 - 2ab^2c^3d + a^2c^2d^2) f^2 g^2 h^3 + ((b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g h^4 - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 h^5) x^3 + (2(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g^2 h^3 - (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g h^4 - (b^2c^4 - 2ab^2c^3d + a^2c^2d^2) f^2 h^5) x^2 + ((b^2c^2d^2 - 2ab^2cd^3 + a^2d^4) f^2 g^3 h^2 + (b^2c^3d - 2ab^2c^2d^2 + a^2cd^3) f^2 g^2 h^3 - 2(b^2c^4 - 2ab^2c^3d + a^2c^2d^2) f^2 g h^4) x) \log(F)^2 + 6(d^6g^5 - 3c^2d^5g^4h + 3c^2d^4g^3h^2 - c^3d^3g^2h^3) x + (6(b^2cd^3 - a^2cd^4) f g^4 h - 13(b^2cd^2 - a^2cd^3) f g^3 h^2 + 8(b^2cd - a^2c^3d^2) f g^2 h^3 - (b^2c^5 - a^2c^4d) f g h^4 + 5((b^2cd^4 - a^2d^5) f g^2 h^3 - 2(b^2cd^3 - a^2cd^4) f g h^4 + (b^2cd^2 - a^2cd^3) f h^5) x^3 + (11(b^2cd^4 - a^2d^5) f g^3 h^2 - 18(b^2cd^3 - a^2cd^4) f g^2 h^3 + 3(b^2cd^2 - a^2cd^3) f g h^4 + 4(b^2cd - a^2c^3d^2) f h^5) x^2 + (6(b^2cd^4 - a^2d^5) f g^4 h - 2(b^2cd^3 - a^2cd^4) f g^3 h^2 - 15(b^2cd^2 - a^2cd^3) f g^2 h^3 + 12(b^2cd - a^2c^3d^2) f g h^4 - (b^2c^5 - a^2c^4d) f h^5) x) \log(F) \right) F^{\left( \frac{c^2e + a^2f + (d^2e + b^2f)x}{d^2x + c} \right)} / (d^6g^9 - 6c^2d^5g^8h + 15c^2d^4g^7h^2 - 20c^3d^3g^6h^3 + 15c^4d^2g^5h^4 - 6c^5d^2g^4h^5 + c^6g^3h^6 + (d^6g^6h^3 - 6c^2d^5g^5h^4 + 15c^2d^4g^4h^5 - 20c^3d^3g^3h^6 + 15c^4d^2g^2h^7 - 6c^5d^2g^2h^8 + c^6h^9) x^3 + 3(d^6g^7h^2 - 6c^2d^5g^6h^3 + 15c^2d^4g^5h^4 - 20c^3d^3g^4h^5 + 15c^4d^2g^3h^6 - 6c^5d^2g^2h^7 + c^6g^2h^8) x^2 + 3(d^6g^8h - 6c^2d^5g^7h^2 + 15c^2d^4g^6h^3 - 20c^3d^3g^5h^4 + 15c^4d^2g^4h^5 - 6c^5d^2g^3h^6 + c^6g^2h^7) x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)\*\*4,x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(e+f\*(b\*x+a)/(d\*x+c))/(h\*x+g)^4,x, algorithm="giac")

[Out] integrate(F^(e + (b\*x + a)\*f/(d\*x + c))/(h\*x + g)^4, x)

### 3.426 $\int f^{a+bx+cx^2} x^3 dx$

**Optimal.** Leaf size=217

$$\frac{3\sqrt{\pi} b f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} - \frac{\sqrt{\pi} b^3 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{16c^{7/2} \sqrt{\log(f)}} + \frac{b^2 f^{a+bx+cx^2}}{8c^3 \log(f)} - \frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} - \frac{bx f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{x^2 f^{a+bx+cx^2}}{2c \log(f)}$$

[Out]  $-f^{(a + b*x + c*x^2)/(2*c^2*\log[f]^2)} + (3*b*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}\left[\frac{(b + 2*c*x)*\sqrt{\log[f]}}{(2*\sqrt{c})}\right])/(8*c^{(5/2)*\log[f]^{(3/2)}}) + (b^2*f^{(a + b*x + c*x^2)})/(8*c^3*\log[f]) - (b*f^{(a + b*x + c*x^2)}*x)/(4*c^2*\log[f]) + (f^{(a + b*x + c*x^2)}*x^2)/(2*c*\log[f]) - (b^3*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}\left[\frac{(b + 2*c*x)*\sqrt{\log[f]}}{(2*\sqrt{c})}\right])/(16*c^{(7/2)*\sqrt{\log[f]}})$

**Rubi [A]** time = 0.230831, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2241, 2240, 2234, 2204}

$$\frac{3\sqrt{\pi} b f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} - \frac{\sqrt{\pi} b^3 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{16c^{7/2} \sqrt{\log(f)}} + \frac{b^2 f^{a+bx+cx^2}}{8c^3 \log(f)} - \frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} - \frac{bx f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{x^2 f^{a+bx+cx^2}}{2c \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*x^3, x]

[Out]  $-f^{(a + b*x + c*x^2)/(2*c^2*\log[f]^2)} + (3*b*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}\left[\frac{(b + 2*c*x)*\sqrt{\log[f]}}{(2*\sqrt{c})}\right])/(8*c^{(5/2)*\log[f]^{(3/2)}}) + (b^2*f^{(a + b*x + c*x^2)})/(8*c^3*\log[f]) - (b*f^{(a + b*x + c*x^2)}*x)/(4*c^2*\log[f]) + (f^{(a + b*x + c*x^2)}*x^2)/(2*c*\log[f]) - (b^3*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}\left[\frac{(b + 2*c*x)*\sqrt{\log[f]}}{(2*\sqrt{c})}\right])/(16*c^{(7/2)*\sqrt{\log[f]}})$

#### Rule 2241

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] + (-Dist[(b\*e - 2\*c\*d)/(2\*c), Int[(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2), x], x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d)/(2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2} x^3 dx &= \frac{f^{a+bx+cx^2} x^2}{2c \log(f)} - \frac{b \int f^{a+bx+cx^2} x^2 dx}{2c} - \frac{\int f^{a+bx+cx^2} x dx}{c \log(f)} \\ &= -\frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} - \frac{b f^{a+bx+cx^2} x}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x^2}{2c \log(f)} + \frac{b^2 \int f^{a+bx+cx^2} x dx}{4c^2} + \frac{b \int f^{a+bx+cx^2} dx}{4c^2 \log(f)} + \frac{b \int f^{a+bx+cx^2}}{2c^2 \log(f)} \\ &= -\frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{b^2 f^{a+bx+cx^2}}{8c^3 \log(f)} - \frac{b f^{a+bx+cx^2} x}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x^2}{2c \log(f)} - \frac{b^3 \int f^{a+bx+cx^2} dx}{8c^3} + \frac{\left(b f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)}{4c}}}{4c^2 \log(f)} \\ &= -\frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{3b f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} + \frac{b^2 f^{a+bx+cx^2}}{8c^3 \log(f)} - \frac{b f^{a+bx+cx^2} x}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x^2}{2c \log(f)} - \frac{\left(b^3 f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)}{4c}}}{4c^2 \log(f)} \\ &= -\frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{3b f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} + \frac{b^2 f^{a+bx+cx^2}}{8c^3 \log(f)} - \frac{b f^{a+bx+cx^2} x}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x^2}{2c \log(f)} - \frac{b^3 f^{a-\frac{b^2}{4c}} \int f^{\frac{(b+2cx)}{4c}}}{4c^2 \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.176507, size = 122, normalized size = 0.56

$$\frac{f^{a-\frac{b^2}{4c}} \left( 2\sqrt{c} f^{\frac{(b+2cx)^2}{4c}} (\log(f) (b^2 - 2bcx + 4c^2x^2) - 4c) + \sqrt{\pi} b \sqrt{\log(f)} (6c - b^2 \log(f)) \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right) \right)}{16c^{7/2} \log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*x^3, x]
```

```
[Out] (f^(a - b^2/(4*c))*(b*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]) *Sqrt[Log[f]]*(6*c - b^2*Log[f]) + 2*Sqrt[c]*f^((b + 2*c*x)^2/(4*c))*(-4*c + (b^2 - 2*b*c*x + 4*c^2*x^2)*Log[f]))/(16*c^(7/2)*Log[f]^2)
```

**Maple [A]** time = 0.086, size = 218, normalized size = 1.

$$\frac{x^2 f c x^2 f b x f a}{2 c \ln(f)} - \frac{b x f c x^2 f b x f a}{4 \ln(f) c^2} + \frac{b^2 f c x^2 f b x f a}{8 c^3 \ln(f)} + \frac{b^3 \sqrt{\pi} f^a}{16 c^3} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}} - \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*x^3, x)
```

```
[Out] 1/2/c/ln(f)*x^2*f^(c*x^2)*f^(b*x)*f^(a-1/4/c^2*b/ln(f)*x*f^(c*x^2)*f^(b*x)*f^(a+1/8/c^3*b^2/ln(f)*f^(c*x^2)*f^(b*x)*f^(a+1/16/c^3*b^3*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))-3/8/c^2*b/ln(f)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))-1/2/c^2/ln(f)^2*f^(c*x^2)*f^(b*x)*f^a
```

**Maxima [A]** time = 1.16182, size = 271, normalized size = 1.25

$$\frac{\left( \frac{\sqrt{\pi}(2cx+b)b^3 \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}}\right) - 1 \right) \log(f)^4}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{7}{2}}} - \frac{12(2cx+b)^3 b \Gamma\left(\frac{3}{2}, -\frac{(2cx+b)^2 \log(f)}{4c}\right) \log(f)^4}{\left(-\frac{(2cx+b)^2 \log(f)}{c}\right)^{\frac{3}{2}} (c \log(f))^{\frac{7}{2}}} - \frac{6b^2 c f^{\frac{(2cx+b)^2}{4c}} \log(f)^3}{(c \log(f))^{\frac{7}{2}}} + \frac{8c^2 \Gamma\left(2, -\frac{(2cx+b)^2 \log(f)}{4c}\right)}{(c \log(f))^{\frac{7}{2}}} \right)}{16 \sqrt{c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x^3,x, algorithm="maxima")

[Out]  $-1/16 * (\sqrt{\pi} * (2 * c * x + b) * b^3 * (\operatorname{erf}(1/2 * \sqrt{- (2 * c * x + b)^2 * \log(f) / c}) - 1) * \log(f)^4 / (\sqrt{- (2 * c * x + b)^2 * \log(f) / c} * (c * \log(f))^{\frac{7}{2}}) - 12 * (2 * c * x + b)^3 * b * \gamma(3/2, -1/4 * (2 * c * x + b)^2 * \log(f) / c) * \log(f)^4 / ((- (2 * c * x + b)^2 * \log(f) / c)^{\frac{3}{2}} * (c * \log(f))^{\frac{7}{2}}) - 6 * b^2 * c * f^{(1/4 * (2 * c * x + b)^2 / c)} * \log(f)^3 / (c * \log(f))^{\frac{7}{2}} + 8 * c^2 * \gamma(2, -1/4 * (2 * c * x + b)^2 * \log(f) / c) * \log(f)^2 / (c * \log(f))^{\frac{7}{2}}) * f^{(a - 1/4 * b^2 / c)} / \sqrt{c * \log(f)}$

**Fricas [A]** time = 1.57129, size = 278, normalized size = 1.28

$$\frac{2 \left( 4c^2 - (4c^3x^2 - 2bc^2x + b^2c) \log(f) \right) f^{cx^2+bx+a} - \frac{\sqrt{\pi}(b^3 \log(f) - 6bc) \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c \log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{16c^4 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x^3,x, algorithm="fricas")

[Out]  $-1/16 * (2 * (4 * c^2 - (4 * c^3 * x^2 - 2 * b * c^2 * x + b^2 * c) * \log(f)) * f^{(c * x^2 + b * x + a)} - \sqrt{\pi} * (b^3 * \log(f) - 6 * b * c) * \sqrt{-c * \log(f)} * \operatorname{erf}(1/2 * (2 * c * x + b) * \sqrt{-c * \log(f) / c}) / f^{(1/4 * (b^2 - 4 * a * c) / c)}) / (c^4 * \log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*x\*\*3,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)\*x\*\*3, x)

**Giac [A]** time = 1.25069, size = 185, normalized size = 0.85

$$\frac{\sqrt{\pi}(b^3 \log(f) - 6bc) \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c \log(f)}\left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2 \log(f) - 4ac \log(f)}{4c}\right)}}{\sqrt{-c \log(f)} \log(f)} + \frac{2 \left( c^2 \left( 2x + \frac{b}{c} \right)^2 \log(f) - 3bc \left( 2x + \frac{b}{c} \right) \log(f) + 3b^2 \log(f) - 4c \right) e^{(cx^2 \log(f) + bx \log(f) + a)}}{\log(f)^2}$$

16c<sup>3</sup>



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*x^3,x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(pi)*(b^3*log(f) - 6*b*c)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e  
^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/(sqrt(-c*log(f))*log(f)) + 2*(c^2*(2*  
x + b/c)^2*log(f) - 3*b*c*(2*x + b/c)*log(f) + 3*b^2*log(f) - 4*c)*e^(c*x^2  
*log(f) + b*x*log(f) + a*log(f))/log(f)^2)/c^3
```

### 3.427 $\int f^{a+bx+cx^2} x^2 dx$

**Optimal.** Leaf size=164

$$-\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \log^2(f)} + \frac{\sqrt{\pi} b^2 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2} \sqrt{\log(f)}} - \frac{b f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{x f^{a+bx+cx^2}}{2c \log(f)}$$

[Out]  $-(f^{(a - b^2/(4*c))} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b + 2*c*x) * \operatorname{Sqrt}[\operatorname{Log}[f]] / (2 * \operatorname{Sqrt}[c])]) / (4*c^{(3/2)} * \operatorname{Log}[f]^{(3/2)}) - (b*f^{(a + b*x + c*x^2)}) / (4*c^2 * \operatorname{Log}[f]) + (f^{(a + b*x + c*x^2)} * x) / (2*c * \operatorname{Log}[f]) + (b^2*f^{(a - b^2/(4*c))} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b + 2*c*x) * \operatorname{Sqrt}[\operatorname{Log}[f]] / (2 * \operatorname{Sqrt}[c])]) / (8*c^{(5/2)} * \operatorname{Sqrt}[\operatorname{Log}[f]])$

**Rubi [A]** time = 0.0925418, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2241, 2240, 2234, 2204}

$$-\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \log^2(f)} + \frac{\sqrt{\pi} b^2 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2} \sqrt{\log(f)}} - \frac{b f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{x f^{a+bx+cx^2}}{2c \log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x + c*x^2)} * x^2, x]$

[Out]  $-(f^{(a - b^2/(4*c))} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b + 2*c*x) * \operatorname{Sqrt}[\operatorname{Log}[f]] / (2 * \operatorname{Sqrt}[c])]) / (4*c^{(3/2)} * \operatorname{Log}[f]^{(3/2)}) - (b*f^{(a + b*x + c*x^2)}) / (4*c^2 * \operatorname{Log}[f]) + (f^{(a + b*x + c*x^2)} * x) / (2*c * \operatorname{Log}[f]) + (b^2*f^{(a - b^2/(4*c))} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b + 2*c*x) * \operatorname{Sqrt}[\operatorname{Log}[f]] / (2 * \operatorname{Sqrt}[c])]) / (8*c^{(5/2)} * \operatorname{Sqrt}[\operatorname{Log}[f]])$

#### Rule 2241

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*(d + e*x)^{(m-1)} * F^{(a + b*x + c*x^2)}) / (2*c * \operatorname{Log}[F]), x] + (-\operatorname{Dist}[(b*e - 2*c*d) / (2*c), \operatorname{Int}[(d + e*x)^{(m-1)} * F^{(a + b*x + c*x^2)}, x], x] - \operatorname{Dist}[(m-1)*e^2 / (2*c * \operatorname{Log}[F]), \operatorname{Int}[(d + e*x)^{(m-2)} * F^{(a + b*x + c*x^2)}, x], x]) /; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x \&\& \operatorname{NeQ}[b*e - 2*c*d, 0] \&\& \operatorname{GtQ}[m, 1]$

#### Rule 2240

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(e * F^{(a + b*x + c*x^2)}) / (2*c * \operatorname{Log}[F]), x] - \operatorname{Dist}[(b*e - 2*c*d) / (2*c), \operatorname{Int}[F^{(a + b*x + c*x^2)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x \&\& \operatorname{NeQ}[b*e - 2*c*d, 0]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /; \operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]]) / (2*d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int f^{a+bx+cx^2} x^2 dx &= \frac{f^{a+bx+cx^2} x}{2c \log(f)} - \frac{b \int f^{a+bx+cx^2} x dx}{2c} - \frac{\int f^{a+bx+cx^2} dx}{2c \log(f)} \\
 &= -\frac{b f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x}{2c \log(f)} + \frac{b^2 \int f^{a+bx+cx^2} dx}{4c^2} - \frac{f^{a-\frac{b^2}{4c}} \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c \log(f)} \\
 &= -\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2} \log^{\frac{3}{2}}(f)} - \frac{b f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x}{2c \log(f)} + \frac{\left(b^2 f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{4c^2} \\
 &= -\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2} \log^{\frac{3}{2}}(f)} - \frac{b f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{f^{a+bx+cx^2} x}{2c \log(f)} + \frac{b^2 f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2} \sqrt{\log(f)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.115661, size = 104, normalized size = 0.63

$$\frac{f^{a-\frac{b^2}{4c}} \left( \sqrt{\pi} (b^2 \log(f) - 2c) \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right) - 2\sqrt{c} \sqrt{\log(f)} (b - 2cx) f^{\frac{(b+2cx)^2}{4c}} \right)}{8c^{5/2} \log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*x^2,x]

[Out] (f^(a - b^2/(4\*c))\*(-2\*Sqrt[c]\*f^((b + 2\*c\*x)^2/(4\*c))\*(b - 2\*c\*x)\*Sqrt[Log[f]] + Sqrt[Pi]\*Erfi[(b + 2\*c\*x)\*Sqrt[Log[f]]]/(2\*Sqrt[c]))\*(-2\*c + b^2\*Log[f]))/(8\*c^(5/2)\*Log[f]^(3/2))

**Maple [A]** time = 0.031, size = 163, normalized size = 1.

$$\frac{x f^{cx^2} f^{bx} f^a}{2c \ln(f)} - \frac{b f^{cx^2} f^{bx} f^a}{4c^2 \ln(f)} - \frac{b^2 \sqrt{\pi} f^a}{8c^2} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}} + \frac{\sqrt{\pi} f^a}{4c \ln(f)} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*x^2,x)

[Out] 1/2/c/ln(f)\*x\*f^(c\*x^2)\*f^(b\*x)\*f^a-1/4/c^2\*b/ln(f)\*f^(c\*x^2)\*f^(b\*x)\*f^a-1/8/c^2\*b^2\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))+1/4/c/ln(f)\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [A]** time = 1.1764, size = 224, normalized size = 1.37

$$\frac{\left( \frac{\sqrt{\pi}(2cx+b)b^2 \left( \operatorname{erf}\left( \frac{1}{2} \sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} \right) - 1 \right) \log(f)^3}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{5}{2}}} - \frac{4(2cx+b)^3 \Gamma\left( \frac{3}{2}, -\frac{(2cx+b)^2 \log(f)}{4c} \right) \log(f)^3}{\left( -\frac{(2cx+b)^2 \log(f)}{c} \right)^{\frac{3}{2}} (c \log(f))^{\frac{5}{2}}} - \frac{4bcf \frac{(2cx+b)^2}{4c} \log(f)^2}{(c \log(f))^{\frac{5}{2}}} \right) f^{a-\frac{b^2}{4c}}}{8 \sqrt{c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x^2,x, algorithm="maxima")

[Out] 1/8\*(sqrt(pi)\*(2\*c\*x + b)\*b^2\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2\*log(f)/c)) - 1)\*log(f)^3/(sqrt(-(2\*c\*x + b)^2\*log(f)/c)\*(c\*log(f))^(5/2)) - 4\*(2\*c\*x + b)^3\*gamma(3/2, -1/4\*(2\*c\*x + b)^2\*log(f)/c)\*log(f)^3/((-2\*c\*x + b)^2\*log(f)/c)^(3/2)\*(c\*log(f))^(5/2)) - 4\*b\*c\*f^(1/4\*(2\*c\*x + b)^2/c)\*log(f)^2/(c\*log(f))^(5/2))\*f^(a - 1/4\*b^2/c)/sqrt(c\*log(f))

**Fricas [A]** time = 1.55679, size = 238, normalized size = 1.45

$$\frac{2(2c^2x - bc) f^{cx^2+bx+a} \log(f) - \frac{\sqrt{\pi}(b^2 \log(f) - 2c) \sqrt{-c \log(f)} \operatorname{erf}\left( \frac{(2cx+b)\sqrt{-c \log(f)}}{2c} \right)}{f^{\frac{b^2-4ac}{4c}}}}{8c^3 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x^2,x, algorithm="fricas")

[Out] 1/8\*(2\*(2\*c^2\*x - b\*c)\*f^(c\*x^2 + b\*x + a)\*log(f) - sqrt(pi)\*(b^2\*log(f) - 2\*c)\*sqrt(-c\*log(f))\*erf(1/2\*(2\*c\*x + b)\*sqrt(-c\*log(f))/c)/f^(1/4\*(b^2 - 4\*a\*c)/c))/(c^3\*log(f)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*x\*\*2,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)\*x\*\*2, x)

**Giac [A]** time = 1.22308, size = 146, normalized size = 0.89

$$\frac{\frac{\sqrt{\pi}(b^2 \log(f) - 2c) \operatorname{erf}\left( -\frac{1}{2} \sqrt{-c \log(f)} \left( 2x + \frac{b}{c} \right) \right) e^{\left( -\frac{b^2 \log(f) - 4ac \log(f)}{4c} \right)}}{\sqrt{-c \log(f)} \log(f)} - \frac{2 \left( c \left( 2x + \frac{b}{c} \right) - 2b \right) e^{(cx^2 \log(f) + bx \log(f) + a \log(f))}}{\log(f)}}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*x^2,x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(pi)*(b^2*log(f) - 2*c)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/(sqrt(-c*log(f))*log(f)) - 2*(c*(2*x + b/c) - 2*b)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log(f))/c^2
```

### 3.428 $\int f^{a+bx+cx^2} x dx$

**Optimal.** Leaf size=81

$$\frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{\sqrt{\pi} b f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \sqrt{\log(f)}}$$

[Out]  $f^{(a + b*x + c*x^2)/(2*c*\log[f])} - (b*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}(((b + 2*c*x)*\sqrt{\log[f]})/(2*\sqrt{c}))))/(4*c^{(3/2)}*\sqrt{\log[f]})$

**Rubi [A]** time = 0.0383484, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2240, 2234, 2204}

$$\frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{\sqrt{\pi} b f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*x, x]

[Out]  $f^{(a + b*x + c*x^2)/(2*c*\log[f])} - (b*f^{(a - b^2/(4*c))}*\sqrt{\pi}*\operatorname{Erfi}(((b + 2*c*x)*\sqrt{\log[f]})/(2*\sqrt{c}))))/(4*c^{(3/2)}*\sqrt{\log[f]})$

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d)/(2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2} x dx &= \frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{b \int f^{a+bx+cx^2} dx}{2c} \\ &= \frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{\left(b f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c} \\ &= \frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{b f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2} \sqrt{\log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.0644649, size = 81, normalized size = 1.

$$\frac{f^{a+bx+cx^2}}{2c \log(f)} - \frac{\sqrt{\pi} b f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*x, x]

[Out]  $f^{a + b*x + c*x^2} / (2*c*\operatorname{Log}[f]) - (b*f^{a - b^2/(4*c)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\frac{(b + 2*c*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]}{2*\operatorname{Sqrt}[c]}]) / (4*c^{3/2}*\operatorname{Sqrt}[\operatorname{Log}[f]])$

**Maple [A]** time = 0.029, size = 79, normalized size = 1.

$$\frac{f^{cx^2} f^{bx} f^a}{2c \ln(f)} + \frac{b\sqrt{\pi} f^a}{4c} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*x, x)

[Out]  $1/2/c/\ln(f)*f^{(c*x^2)}*f^{(b*x)}*f^{a+1/4*b/c*\operatorname{Pi}^{(1/2)}}*f^a*f^{(-1/4*b^2/c)/(-c*\ln(f))^{(1/2)}}*\operatorname{erf}(-(-c*\ln(f))^{(1/2)}*x+1/2*b*\ln(f)/(-c*\ln(f))^{(1/2)})$

**Maxima [A]** time = 1.12411, size = 144, normalized size = 1.78

$$\frac{\left( \frac{\sqrt{\pi}(2cx+b)b \left( \operatorname{erf}\left( \frac{1}{2} \sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} \right) - 1 \right) \log(f)^2}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c}} \log(f)}{(c \log(f))^{\frac{3}{2}}} \right) f^{a-\frac{b^2}{4c}}}{4 \sqrt{c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x, x, algorithm="maxima")

[Out]  $-1/4*(\operatorname{sqrt}(\operatorname{pi})*(2*c*x + b)*b*(\operatorname{erf}(1/2*\operatorname{sqrt}(-(2*c*x + b)^2*\log(f)/c)) - 1)*\log(f)^2/(\operatorname{sqrt}(-(2*c*x + b)^2*\log(f)/c)*(c*\log(f))^{\frac{3}{2}}) - 2*c*f^{(1/4*(2*c*x + b)^2/c)*\log(f)/(c*\log(f))^{\frac{3}{2}}})*f^{(a - 1/4*b^2/c)/\operatorname{sqrt}(c*\log(f))}$

**Fricas [A]** time = 1.53768, size = 184, normalized size = 2.27

$$\frac{2cf^{cx^2+bx+a} + \frac{\sqrt{\pi} \sqrt{-c \log(f)} b \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c \log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{4c^2 \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x,x, algorithm="fricas")

[Out]  $\frac{1}{4} * (2 * c * f^{(c * x^2 + b * x + a)} + \sqrt{\pi} * \sqrt{-c * \log(f)} * b * \operatorname{erf}\left(\frac{1}{2} * (2 * c * x + b) * \sqrt{-c * \log(f)} / c\right) / f^{(1/4 * (b^2 - 4 * a * c) / c)}) / (c^2 * \log(f))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*x,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)\*x, x)

**Giac [A]** time = 1.19864, size = 108, normalized size = 1.33

$$\frac{\frac{\sqrt{\pi} b \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log(f)} \left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2 \log(f) - 4ac \log(f)}{4c}\right)}}{\sqrt{-c \log(f)}} + \frac{2e^{(cx^2 \log(f) + bx \log(f) + a \log(f))}}{\log(f)}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*x,x, algorithm="giac")

[Out]  $\frac{1}{4} * (\sqrt{\pi} * b * \operatorname{erf}\left(-\frac{1}{2} * \sqrt{-c * \log(f)} * (2 * x + b / c)\right) * e^{(-1/4 * (b^2 * \log(f) - 4 * a * c * \log(f)) / c)} / \sqrt{-c * \log(f)} + 2 * e^{(c * x^2 * \log(f) + b * x * \log(f) + a * \log(f))} / \log(f)) / c$



### 3.429 $\int f^{a+bx+cx^2} dx$

**Optimal.** Leaf size=56

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

[Out] (f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c]))/(2\*Sqrt[c]\*Sqrt[Log[f]])

**Rubi [A]** time = 0.0149092, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2234, 2204}

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2), x]

[Out] (f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c]))/(2\*Sqrt[c]\*Sqrt[Log[f]])

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2} dx &= f^{a-\frac{b^2}{4c}} \int f^{\frac{(b+2cx)^2}{4c}} dx \\ &= \frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.0129495, size = 56, normalized size = 1.

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2),x]

[Out] (f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c]))/(2\*Sqrt[c]\*Sqrt[Log[f]])

**Maple [A]** time = 0.025, size = 50, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a}{2}f^{-\frac{b^2}{4c}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}x + \frac{b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a),x)

[Out] -1/2\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [A]** time = 0.984201, size = 68, normalized size = 1.21

$$\frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] 1/2\*sqrt(pi)\*f^a\*erf(sqrt(-c\*log(f))\*x - 1/2\*b\*log(f)/sqrt(-c\*log(f)))/(sqrt(-c\*log(f))\*f^(1/4\*b^2/c))

**Fricas [A]** time = 1.55385, size = 142, normalized size = 2.54

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{2cf^{\frac{b^2-4ac}{4c}}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-c\*log(f))\*erf(1/2\*(2\*c\*x + b)\*sqrt(-c\*log(f))/c)/(c\*f^(1/4\*(b^2 - 4\*a\*c)/c)\*log(f))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a),x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2), x)

**Giac [A]** time = 1.27278, size = 68, normalized size = 1.21

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log(f)} \left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2 \log(f) - 4ac \log(f)}{4c}\right)}}{2 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*erf(-1/2\*sqrt(-c\*log(f))\*(2\*x + b/c))\*e^(-1/4\*(b^2\*log(f) - 4\*a\*c\*log(f))/c)/sqrt(-c\*log(f))

$$3.430 \quad \int \frac{f^{a+bx+cx^2}}{x} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{f^{a+bx+cx^2}}{x}, x\right)$$

[Out] Unintegrable[f^(a + b\*x + c\*x^2)/x, x]

**Rubi [A]** time = 0.0254935, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{a+bx+cx^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[f^(a + b\*x + c\*x^2)/x, x]

[Out] Defer[Int][f^(a + b\*x + c\*x^2)/x, x]

Rubi steps

$$\int \frac{f^{a+bx+cx^2}}{x} dx = \int \frac{f^{a+bx+cx^2}}{x} dx$$

**Mathematica [A]** time = 0.110627, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(a + b\*x + c\*x^2)/x, x]

[Out] Integrate[f^(a + b\*x + c\*x^2)/x, x]

**Maple [A]** time = 0.014, size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/x, x)

[Out] int(f^(c\*x^2+b\*x+a)/x, x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/x,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{cx^2+bx+a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/x,x, algorithm="fricas")

[Out] integral(f^(c\*x^2 + b\*x + a)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)/x,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/x,x, algorithm="giac")

[Out] integrate(f^(c\*x^2 + b\*x + a)/x, x)

$$3.431 \quad \int \frac{f^{a+bx+cx^2}}{x^2} dx$$

**Optimal.** Leaf size=93

$$b \log(f) \text{Unintegrable} \left( \frac{f^{a+bx+cx^2}}{x}, x \right) + \sqrt{\pi} \sqrt{c} \sqrt{\log(f)} f^{a-\frac{b^2}{4c}} \text{Erfi} \left( \frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}} \right) - \frac{f^{a+bx+cx^2}}{x}$$

[Out]  $-(f^{(a + b*x + c*x^2)}/x) + \text{Sqrt}[c]*f^{(a - b^2/(4*c))}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\frac{(b + 2*c*x)*\text{Sqrt}[\text{Log}[f]]}{(2*\text{Sqrt}[c])}]*\text{Sqrt}[\text{Log}[f]] + b*\text{Log}[f]*\text{Unintegrable}[f^{(a + b*x + c*x^2)}/x, x]$

**Rubi [A]** time = 0.0759386, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{a+bx+cx^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[f^{(a + b*x + c*x^2)}/x^2, x]$

[Out]  $-(f^{(a + b*x + c*x^2)}/x) + \text{Sqrt}[c]*f^{(a - b^2/(4*c))}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\frac{(b + 2*c*x)*\text{Sqrt}[\text{Log}[f]]}{(2*\text{Sqrt}[c])}]*\text{Sqrt}[\text{Log}[f]] + b*\text{Log}[f]*\text{Defer}[\text{Int}][f^{(a + b*x + c*x^2)}/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx+cx^2}}{x^2} dx &= -\frac{f^{a+bx+cx^2}}{x} + (b \log(f)) \int \frac{f^{a+bx+cx^2}}{x} dx + (2c \log(f)) \int f^{a+bx+cx^2} dx \\ &= -\frac{f^{a+bx+cx^2}}{x} + (b \log(f)) \int \frac{f^{a+bx+cx^2}}{x} dx + \left( 2c f^{a-\frac{b^2}{4c}} \log(f) \right) \int f^{\frac{(b+2cx)^2}{4c}} dx \\ &= -\frac{f^{a+bx+cx^2}}{x} + \sqrt{c} f^{a-\frac{b^2}{4c}} \sqrt{\pi} \text{erfi} \left( \frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}} \right) \sqrt{\log(f)} + (b \log(f)) \int \frac{f^{a+bx+cx^2}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.257055, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[f^{(a + b*x + c*x^2)}/x^2, x]$

[Out]  $\text{Integrate}[f^{(a + b*x + c*x^2)}/x^2, x]$

**Maple [A]** time = 0.021, size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x+a)/x^2,x)`

[Out] `int(f^(c*x^2+b*x+a)/x^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="maxima")`

[Out] `integrate(f^(c*x^2 + b*x + a)/x^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{cx^2+bx+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="fricas")`

[Out] `integral(f^(c*x^2 + b*x + a)/x^2, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x+a)/x**2,x)`

[Out] `Integral(f**(a + b*x + c*x**2)/x**2, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="giac")`

[Out] `integrate(f^(c*x^2 + b*x + a)/x^2, x)`

### 3.432 $\int e^{a+bx-cx^2} x^3 dx$

**Optimal.** Leaf size=181

$$\frac{\sqrt{\pi} b^3 e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{16c^{7/2}} - \frac{3\sqrt{\pi} b e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{b^2 e^{a+bx-cx^2}}{8c^3} - \frac{bx e^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2}}{2c^2} - \frac{x^2 e^{a+bx-cx^2}}{2c}$$

[Out]  $-(b^2 E^{(a + b*x - c*x^2)})/(8*c^3) - E^{(a + b*x - c*x^2)}/(2*c^2) - (b*E^{(a + b*x - c*x^2)*x})/(4*c^2) - (E^{(a + b*x - c*x^2)*x^2})/(2*c) - (b^3*E^{(a + b^2/(4*c))}*Sqrt[\pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(16*c^{(7/2)}) - (3*b*E^{(a + b^2/(4*c))}*Sqrt[\pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(8*c^{(5/2)})$

**Rubi [A]** time = 0.178745, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2241, 2240, 2234, 2205}

$$\frac{\sqrt{\pi} b^3 e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{16c^{7/2}} - \frac{3\sqrt{\pi} b e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{b^2 e^{a+bx-cx^2}}{8c^3} - \frac{bx e^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2}}{2c^2} - \frac{x^2 e^{a+bx-cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x - c\*x^2)\*x^3,x]

[Out]  $-(b^2 E^{(a + b*x - c*x^2)})/(8*c^3) - E^{(a + b*x - c*x^2)}/(2*c^2) - (b*E^{(a + b*x - c*x^2)*x})/(4*c^2) - (E^{(a + b*x - c*x^2)*x^2})/(2*c) - (b^3*E^{(a + b^2/(4*c))}*Sqrt[\pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(16*c^{(7/2)}) - (3*b*E^{(a + b^2/(4*c))}*Sqrt[\pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(8*c^{(5/2)})$

#### Rule 2241

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] + (-Dist[(b\*e - 2\*c\*d)/(2\*c), Int[(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2), x], x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d)/(2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[\pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]



Rubi steps

$$\begin{aligned}
\int e^{a+bx-cx^2} x^3 dx &= -\frac{e^{a+bx-cx^2} x^2}{2c} + \frac{\int e^{a+bx-cx^2} x dx}{c} + \frac{b \int e^{a+bx-cx^2} x^2 dx}{2c} \\
&= -\frac{e^{a+bx-cx^2}}{2c^2} - \frac{be^{a+bx-cx^2} x}{4c^2} - \frac{e^{a+bx-cx^2} x^2}{2c} + \frac{b \int e^{a+bx-cx^2} dx}{4c^2} + \frac{b \int e^{a+bx-cx^2} dx}{2c^2} + \frac{b^2 \int e^{a+bx-cx^2} x dx}{4c^2} \\
&= -\frac{b^2 e^{a+bx-cx^2}}{8c^3} - \frac{e^{a+bx-cx^2}}{2c^2} - \frac{be^{a+bx-cx^2} x}{4c^2} - \frac{e^{a+bx-cx^2} x^2}{2c} + \frac{b^3 \int e^{a+bx-cx^2} dx}{8c^3} + \frac{\left( be^{a+\frac{b^2}{4c}} \right) \int e^{-\frac{(b-2cx)^2}{4c}} dx}{4c^2} \\
&= -\frac{b^2 e^{a+bx-cx^2}}{8c^3} - \frac{e^{a+bx-cx^2}}{2c^2} - \frac{be^{a+bx-cx^2} x}{4c^2} - \frac{e^{a+bx-cx^2} x^2}{2c} - \frac{3be^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} + \frac{\left( b^3 e^{a+\frac{b^2}{4c}} \right) \int e^{-\frac{(b-2cx)^2}{4c}} dx}{8c^3} \\
&= -\frac{b^2 e^{a+bx-cx^2}}{8c^3} - \frac{e^{a+bx-cx^2}}{2c^2} - \frac{be^{a+bx-cx^2} x}{4c^2} - \frac{e^{a+bx-cx^2} x^2}{2c} - \frac{b^3 e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{16c^{7/2}} - \frac{3be^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.217722, size = 91, normalized size = 0.5

$$\frac{e^a \left( \sqrt{\pi} b (b^2 + 6c) e^{\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right) + 2\sqrt{c} e^{x(b-cx)} (b^2 + 2bcx + 4c(cx^2 + 1)) \right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x - c\*x^2)\*x^3,x]

[Out] -(E^a\*(2\*sqrt[c]\*E^(x\*(b - c\*x))\*(b^2 + 2\*b\*c\*x + 4\*c\*(1 + c\*x^2)) + b\*(b^2 + 6\*c)\*E^(b^2/(4\*c))\*sqrt[Pi]\*Erf[(b - 2\*c\*x)/(2\*sqrt[c])]))/(16\*c^(7/2))

**Maple [A]** time = 0.011, size = 194, normalized size = 1.1

$$-\frac{e^{-cx^2+bx+ax^2}}{2c} + \frac{b}{2c} \left( -\frac{e^{-cx^2+bx+ax}}{2c} + \frac{b}{2c} \left( -\frac{e^{-cx^2+bx+a}}{2c} - \frac{b\sqrt{\pi}}{4} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{cx} + \frac{b}{2\sqrt{c}}\right) c^{-\frac{3}{2}} \right) - \frac{\sqrt{\pi}}{4} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{cx} + \frac{b}{2\sqrt{c}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-c\*x^2+b\*x+a)\*x^3,x)

[Out] -1/2\*exp(-c\*x^2+b\*x+a)\*x^2/c+1/2\*b/c\*(-1/2\*exp(-c\*x^2+b\*x+a)\*x/c+1/2\*b/c\*(-1/2\*exp(-c\*x^2+b\*x+a)/c-1/4\*b/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2)))-1/4/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2))+1/c\*(-1/2\*exp(-c\*x^2+b\*x+a)/c-1/4\*b/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2)))

**Maxima [A]** time = 1.15163, size = 244, normalized size = 1.35

$$\frac{\left( \frac{\sqrt{\pi}(2cx-b)b^3 \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{(2cx-b)^2}{c}}\right) - 1 \right)}{\sqrt{\frac{(2cx-b)^2}{c}}(-c)^{\frac{7}{2}}} - \frac{6b^2ce^{\left(-\frac{(2cx-b)^2}{4c}\right)}}{(-c)^{\frac{7}{2}}} - \frac{12(2cx-b)^3b\Gamma\left(\frac{3}{2}, \frac{(2cx-b)^2}{4c}\right)}{\left(\frac{(2cx-b)^2}{c}\right)^{\frac{3}{2}}(-c)^{\frac{7}{2}}} - \frac{8c^2\Gamma\left(2, \frac{(2cx-b)^2}{4c}\right)}{(-c)^{\frac{7}{2}}} \right) e^{\left(a+\frac{b^2}{4c}\right)}}{16\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x^3,x, algorithm="maxima")

[Out]  $\frac{1}{16}(\sqrt{\pi})(2cx - b)b^3(\operatorname{erf}(\frac{1}{2}\sqrt{c}(2cx - b)) - 1)/(\sqrt{c}(2cx - b)^{7/2}) - 6b^2c e^{-1/4(2cx - b)^2/c}/(-c)^{7/2} - 12(2cx - b)^3 b \operatorname{gamma}(3/2, 1/4(2cx - b)^2/c)/((2cx - b)^2/c)^{3/2} (-c)^{7/2} - 8c^2 \operatorname{gamma}(2, 1/4(2cx - b)^2/c)/(-c)^{7/2} e^{(a + 1/4b^2/c)}/\sqrt{-c}$

**Fricas [A]** time = 1.54274, size = 217, normalized size = 1.2

$$\frac{\sqrt{\pi}(b^3 + 6bc)\sqrt{c} \operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right) e^{\left(\frac{b^2+4ac}{4c}\right)} - 2(4c^3x^2 + 2bc^2x + b^2c + 4c^2)e^{(-cx^2+bx+a)}}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x^3,x, algorithm="fricas")

[Out]  $\frac{1}{16}(\sqrt{\pi})(b^3 + 6b^2c)\sqrt{c} \operatorname{erf}(1/2*(2cx - b)/\sqrt{c})e^{(1/4*(b^2 + 4a*c)/c)} - 2*(4*c^3*x^2 + 2*b*c^2*x + b^2*c + 4*c^2)*e^{(-c*x^2 + b*x + a)}/c^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int x^3 e^{bx} e^{-cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x\*\*2+b\*x+a)\*x\*\*3,x)

[Out] exp(a)\*Integral(x\*\*3\*exp(b\*x)\*exp(-c\*x\*\*2), x)

**Giac [A]** time = 1.26629, size = 140, normalized size = 0.77

$$\frac{\sqrt{\pi}(b^3+6bc) \operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{b}{c}\right)\right) e^{\left(\frac{b^2+4ac}{4c}\right)}}{\sqrt{c}} + 2\left(c^2\left(2x-\frac{b}{c}\right)^2 + 3bc\left(2x-\frac{b}{c}\right) + 3b^2 + 4c\right) e^{(-cx^2+bx+a)}}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x^3,x, algorithm="giac")

[Out]  $-1/16*(\sqrt{\pi})(b^3 + 6b^2c)*\operatorname{erf}(-1/2*\sqrt{c}*(2*x - b/c))*e^{(1/4*(b^2 + 4*a*c)/c)}/\sqrt{c} + 2*(c^2*(2*x - b/c)^2 + 3*b*c*(2*x - b/c) + 3*b^2 + 4*c)*e^{(-c*x^2 + b*x + a)}/c^3$

### 3.433 $\int e^{a+bx-cx^2} x^2 dx$

**Optimal.** Leaf size=134

$$-\frac{\sqrt{\pi} b^2 e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{\pi} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{be^{a+bx-cx^2}}{4c^2} - \frac{xe^{a+bx-cx^2}}{2c}$$

[Out]  $-(bE^{(a + b*x - c*x^2)})/(4*c^2) - (E^{(a + b*x - c*x^2)}*x)/(2*c) - (b^2*E^{(a + b^2/(4*c))*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])]})/(8*c^{(5/2)}) - (E^{(a + b^2/(4*c))*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])]})/(4*c^{(3/2)})$

**Rubi [A]** time = 0.0821288, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2241, 2240, 2234, 2205}

$$-\frac{\sqrt{\pi} b^2 e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{\pi} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{be^{a+bx-cx^2}}{4c^2} - \frac{xe^{a+bx-cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x - c\*x^2)\*x^2,x]

[Out]  $-(bE^{(a + b*x - c*x^2)})/(4*c^2) - (E^{(a + b*x - c*x^2)}*x)/(2*c) - (b^2*E^{(a + b^2/(4*c))*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])]})/(8*c^{(5/2)}) - (E^{(a + b^2/(4*c))*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])]})/(4*c^{(3/2)})$

#### Rule 2241

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] + (-Dist[(b\*e - 2\*c\*d)/(2\*c), Int[(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2), x], x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d)/(2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*\operatorname{Sqrt}[\operatorname{Pi}]\*\operatorname{Erf}[(c + d\*x)\*\operatorname{Rt}[-(b\*\operatorname{Log}[F]), 2]])/(2\*d\*\operatorname{Rt}[-(b\*\operatorname{Log}[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int e^{a+bx-cx^2} x^2 dx &= -\frac{e^{a+bx-cx^2} x}{2c} + \frac{\int e^{a+bx-cx^2} dx}{2c} + \frac{b \int e^{a+bx-cx^2} x dx}{2c} \\
&= -\frac{be^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2} x}{2c} + \frac{b^2 \int e^{a+bx-cx^2} dx}{4c^2} + \frac{e^{a+\frac{b^2}{4c}} \int e^{-\frac{(b-2cx)^2}{4c}} dx}{2c} \\
&= -\frac{be^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2} x}{2c} - \frac{e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} + \frac{\left(b^2 e^{a+\frac{b^2}{4c}}\right) \int e^{-\frac{(b-2cx)^2}{4c}} dx}{4c^2} \\
&= -\frac{be^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2} x}{2c} - \frac{b^2 e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.114031, size = 79, normalized size = 0.59

$$\frac{e^a \left( \sqrt{\pi} (b^2 + 2c) e^{\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{2cx-b}{2\sqrt{c}}\right) - 2\sqrt{c} e^{x(b-cx)} (b + 2cx) \right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x - c\*x^2)\*x^2,x]

[Out] (E^a\*(-2\*sqrt[c]\*E^(x\*(b - c\*x))\*(b + 2\*c\*x) + (b^2 + 2\*c)\*E^(b^2/(4\*c))\*sqrt[Pi]\*Erf[(-b + 2\*c\*x)/(2\*sqrt[c])])/(8\*c^(5/2))

**Maple [A]** time = 0.004, size = 111, normalized size = 0.8

$$-\frac{e^{-cx^2+bx+a} x}{2c} + \frac{b}{2c} \left( -\frac{e^{-cx^2+bx+a}}{2c} - \frac{b\sqrt{\pi}}{4} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{cx} + \frac{b}{2\sqrt{c}}\right) c^{-\frac{3}{2}} \right) - \frac{\sqrt{\pi}}{4} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{cx} + \frac{b}{2\sqrt{c}}\right) c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-c\*x^2+b\*x+a)\*x^2,x)

[Out] -1/2\*exp(-c\*x^2+b\*x+a)\*x/c+1/2\*b/c\*(-1/2\*exp(-c\*x^2+b\*x+a)/c-1/4\*b/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2)))-1/4/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2))

**Maxima [A]** time = 1.16079, size = 204, normalized size = 1.52

$$\frac{\left( \frac{\sqrt{\pi}(2cx-b)b^2 \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{(2cx-b)^2}{c}}\right) - 1 \right)}{\sqrt{\frac{(2cx-b)^2}{c}} (-c)^{\frac{5}{2}}} - \frac{4bce^{\left(-\frac{(2cx-b)^2}{4c}\right)}}{(-c)^{\frac{5}{2}}} - \frac{4(2cx-b)^3 \Gamma\left(\frac{3}{2}, \frac{(2cx-b)^2}{4c}\right)}{\left(\frac{(2cx-b)^2}{c}\right)^{\frac{3}{2}} (-c)^{\frac{5}{2}}} \right) e^{\left(a+\frac{b^2}{4c}\right)}}{8\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x^2,x, algorithm="maxima")

[Out]  $-1/8 * (\sqrt{\pi}) * (2 * c * x - b) * b^2 * (\operatorname{erf}(1/2 * \sqrt{(2 * c * x - b)^2 / c}) - 1) / (\sqrt{(2 * c * x - b)^2 / c} * (-c)^{5/2}) - 4 * b * c * e^{(-1/4 * (2 * c * x - b)^2 / c)} / (-c)^{5/2} - 4 * (2 * c * x - b)^3 * \operatorname{gamma}(3/2, 1/4 * (2 * c * x - b)^2 / c) / (((2 * c * x - b)^2 / c)^{3/2} * (-c)^{5/2}) * e^{(a + 1/4 * b^2 / c)} / \sqrt{-c}$

**Fricas [A]** time = 1.53471, size = 181, normalized size = 1.35

$$\frac{\sqrt{\pi}(b^2 + 2c)\sqrt{c} \operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right) e^{\left(\frac{b^2+4ac}{4c}\right)} - 2(2c^2x + bc)e^{(-cx^2+bx+a)}}{8c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a)*x^2,x, algorithm="fricas")`

[Out]  $1/8 * (\sqrt{\pi}) * (b^2 + 2 * c) * \sqrt{c} * \operatorname{erf}(1/2 * (2 * c * x - b) / \sqrt{c}) * e^{(1/4 * (b^2 + 4 * a * c) / c)} - 2 * (2 * c^2 * x + b * c) * e^{(-c * x^2 + b * x + a)} / c^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int x^2 e^{bx} e^{-cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x**2+b*x+a)*x**2,x)`

[Out] `exp(a)*Integral(x**2*exp(b*x)*exp(-c*x**2), x)`

**Giac [A]** time = 1.26651, size = 108, normalized size = 0.81

$$-\frac{\frac{\sqrt{\pi}(b^2+2c) \operatorname{erf}\left(-\frac{1}{2} \sqrt{c}\left(2x-\frac{b}{c}\right)\right) e^{\left(\frac{b^2+4ac}{4c}\right)}}{\sqrt{c}} + 2\left(c\left(2x-\frac{b}{c}\right) + 2b\right) e^{(-cx^2+bx+a)}}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a)*x^2,x, algorithm="giac")`

[Out]  $-1/8 * (\sqrt{\pi}) * (b^2 + 2 * c) * \operatorname{erf}(-1/2 * \sqrt{c} * (2 * x - b / c)) * e^{(1/4 * (b^2 + 4 * a * c) / c)} / \sqrt{c} + 2 * (c * (2 * x - b / c) + 2 * b) * e^{(-c * x^2 + b * x + a)} / c^2$

### 3.434 $\int e^{a+bx-cx^2} x dx$

**Optimal.** Leaf size=66

$$-\frac{\sqrt{\pi} b e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{e^{a+bx-cx^2}}{2c}$$

[Out]  $-E^{(a + b*x - c*x^2)/(2*c)} - (b*E^{(a + b^2/(4*c))*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])])/(4*c^{(3/2)})$

**Rubi [A]** time = 0.0323039, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2240, 2234, 2205}

$$-\frac{\sqrt{\pi} b e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{e^{a+bx-cx^2}}{2c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a + b*x - c*x^2)*x}, x]$

[Out]  $-E^{(a + b*x - c*x^2)/(2*c)} - (b*E^{(a + b^2/(4*c))*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(b - 2*c*x)/(2*\operatorname{Sqrt}[c])])/(4*c^{(3/2)})$

#### Rule 2240

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*F^{(a + b*x + c*x^2)})/(2*c*\operatorname{Log}[F]), x] - \operatorname{Dist}[(b*e - 2*c*d)/(2*c), \operatorname{Int}[F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x \&\& \operatorname{NeQ}[b*e - 2*c*d, 0]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]]/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{NegQ}[b]$

#### Rubi steps

$$\begin{aligned} \int e^{a+bx-cx^2} x dx &= -\frac{e^{a+bx-cx^2}}{2c} + \frac{b \int e^{a+bx-cx^2} dx}{2c} \\ &= -\frac{e^{a+bx-cx^2}}{2c} + \frac{\left( b e^{a+\frac{b^2}{4c}} \right) \int e^{-\frac{(b-2cx)^2}{4c}} dx}{2c} \\ &= -\frac{e^{a+bx-cx^2}}{2c} - \frac{b e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0464995, size = 68, normalized size = 1.03

$$\frac{\sqrt{\pi} b e^{a + \frac{b^2}{4c}} \operatorname{Erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{e^{a+bx-cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x - c\*x^2)\*x, x]

[Out] -E^(a + b\*x - c\*x^2)/(2\*c) + (b\*E^(a + b^2/(4\*c))\*Sqrt[Pi]\*Erf[(-b + 2\*c\*x)/(2\*Sqrt[c])])/(4\*c^(3/2))

**Maple [A]** time = 0.003, size = 53, normalized size = 0.8

$$-\frac{e^{-cx^2+bx+a}}{2c} - \frac{b\sqrt{\pi}}{4} e^{a+\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{cx} + \frac{b}{2\sqrt{c}}\right) c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-c\*x^2+b\*x+a)\*x, x)

[Out] -1/2\*exp(-c\*x^2+b\*x+a)/c-1/4\*b/c^(3/2)\*Pi^(1/2)\*exp(a+1/4\*b^2/c)\*erf(-c^(1/2)\*x+1/2\*b/c^(1/2))

**Maxima [A]** time = 1.12839, size = 132, normalized size = 2.

$$\frac{\left( \frac{\sqrt{\pi}(2cx-b)\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{(2cx-b)^2}{c}}\right)-1\right)}{\sqrt{\frac{(2cx-b)^2}{c}}(-c)^{\frac{3}{2}}} - \frac{2ce^{\left(-\frac{(2cx-b)^2}{4c}\right)}}{(-c)^{\frac{3}{2}}} \right) e^{\left(a+\frac{b^2}{4c}\right)}}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x, x, algorithm="maxima")

[Out] 1/4\*(sqrt(pi)\*(2\*c\*x - b)\*b\*(erf(1/2\*sqrt((2\*c\*x - b)^2/c)) - 1)/(sqrt((2\*c\*x - b)^2/c)\*(-c)^(3/2)) - 2\*c\*e^(-1/4\*(2\*c\*x - b)^2/c)/(-c)^(3/2))\*e^(a + 1/4\*b^2/c)/sqrt(-c)

**Fricas [A]** time = 1.5284, size = 149, normalized size = 2.26

$$\frac{\sqrt{\pi} b \sqrt{c} \operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right) e^{\left(\frac{b^2+4ac}{4c}\right)} - 2ce^{(-cx^2+bx+a)}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x, x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{\pi} b \sqrt{c} \operatorname{erf}\left(\frac{1}{2} \sqrt{c} (2x - b)\right) e^{\frac{1}{4}(b^2 + 4ac)/c} - 2c e^{(-cx^2 + bx + a)}/c^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int x e^{bx} e^{-cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x\*\*2+b\*x+a)\*x,x)

[Out] exp(a)\*Integral(x\*exp(b\*x)\*exp(-c\*x\*\*2), x)

**Giac [A]** time = 1.3829, size = 78, normalized size = 1.18

$$\frac{\frac{\sqrt{\pi} b \operatorname{erf}\left(-\frac{1}{2} \sqrt{c} \left(2x - \frac{b}{c}\right)\right) e^{\frac{b^2 + 4ac}{4c}}}{\sqrt{c}} + 2e^{(-cx^2 + bx + a)}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)\*x,x, algorithm="giac")

[Out]  $-\frac{1}{4} \sqrt{\pi} b \operatorname{erf}\left(-\frac{1}{2} \sqrt{c} (2x - b/c)\right) e^{\frac{1}{4}(b^2 + 4ac)/c} / \sqrt{c} + 2e^{(-cx^2 + bx + a)}/c$



$$3.435 \quad \int e^{a+bx-cx^2} dx$$

**Optimal.** Leaf size=44

$$-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

[Out]  $-(E^{(a + b^2/(4*c))}*Sqrt[\text{Pi}]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(2*Sqrt[c])$

**Rubi [A]** time = 0.0107908, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2234, 2205}

$$-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x - c\*x^2),x]

[Out]  $-(E^{(a + b^2/(4*c))}*Sqrt[\text{Pi}]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(2*Sqrt[c])$

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[\text{Pi}]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int e^{a+bx-cx^2} dx &= e^{a+\frac{b^2}{4c}} \int e^{-\frac{(b-2cx)^2}{4c}} dx \\ &= -\frac{e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{2\sqrt{c}} \end{aligned}$$

**Mathematica [A]** time = 0.0117936, size = 46, normalized size = 1.05

$$\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{Erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x - c\*x^2),x]

[Out]  $(E^{(a + b^2/(4*c))}*\text{Sqrt}[\text{Pi}]*\text{Erf}[-(b + 2*c*x)/(2*\text{Sqrt}[c])])/(2*\text{Sqrt}[c])$

**Maple [A]** time = 0.003, size = 34, normalized size = 0.8

$$-\frac{\sqrt{\pi}}{2}e^{a+\frac{b^2}{4c}}\text{Erf}\left(-\sqrt{c}x+\frac{b}{2\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-c*x^2+b*x+a), x)`

[Out]  $-1/2*\text{Pi}^{(1/2)}*\exp(a+1/4*b^2/c)/c^{(1/2)}*\text{erf}(-c^{(1/2)}*x+1/2*b/c^{(1/2)})$

**Maxima [A]** time = 0.978502, size = 43, normalized size = 0.98

$$\frac{\sqrt{\pi}\text{erf}\left(\sqrt{c}x-\frac{b}{2\sqrt{c}}\right)e^{\left(a+\frac{b^2}{4c}\right)}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a), x, algorithm="maxima")`

[Out]  $1/2*\text{sqrt}(\text{pi})*\text{erf}(\text{sqrt}(c)*x - 1/2*b/\text{sqrt}(c))*e^{(a + 1/4*b^2/c)}/\text{sqrt}(c)$

**Fricas [A]** time = 1.51209, size = 101, normalized size = 2.3

$$\frac{\sqrt{\pi}\text{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)e^{\left(\frac{b^2+4ac}{4c}\right)}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a), x, algorithm="fricas")`

[Out]  $1/2*\text{sqrt}(\text{pi})*\text{erf}(1/2*(2*c*x - b)/\text{sqrt}(c))*e^{(1/4*(b^2 + 4*a*c)/c)}/\text{sqrt}(c)$

**Sympy [A]** time = 0.789646, size = 41, normalized size = 0.93

$$\frac{\sqrt{\pi}\sqrt{-\frac{1}{c}}e^{a+\frac{b^2}{4c}}\text{erfi}\left(\frac{b-2cx}{2\sqrt{-c}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x**2+b*x+a), x)`

[Out]  $\text{sqrt}(\text{pi})*\text{sqrt}(-1/c)*\exp(a + b**2/(4*c))*\text{erfi}((b - 2*c*x)/(2*\text{sqrt}(-c)))/2$

---

**Giac [A]** time = 1.34232, size = 51, normalized size = 1.16

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{c} \left(2x - \frac{b}{c}\right)\right) e^{\left(\frac{b^2+4ac}{4c}\right)}}{2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a),x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*erf(-1/2\*sqrt(c)\*(2\*x - b/c))\*e^(1/4\*(b^2 + 4\*a\*c)/c)/sqrt(c)

$$3.436 \quad \int \frac{e^{a+bx-cx^2}}{x} dx$$

**Optimal.** Leaf size=19

$$\text{Unintegrable}\left(\frac{e^{a+bx-cx^2}}{x}, x\right)$$

[Out] Unintegrable[E^(a + b\*x - c\*x^2)/x, x]

**Rubi [A]** time = 0.0276418, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{a+bx-cx^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[E^(a + b\*x - c\*x^2)/x, x]

[Out] Defer[Int][E^(a + b\*x - c\*x^2)/x, x]

Rubi steps

$$\int \frac{e^{a+bx-cx^2}}{x} dx = \int \frac{e^{a+bx-cx^2}}{x} dx$$

**Mathematica [A]** time = 0.120651, size = 0, normalized size = 0.

$$\int \frac{e^{a+bx-cx^2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(a + b\*x - c\*x^2)/x, x]

[Out] Integrate[E^(a + b\*x - c\*x^2)/x, x]

**Maple [A]** time = 0.012, size = 0, normalized size = 0.

$$\int \frac{e^{-cx^2+bx+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-c\*x^2+b\*x+a)/x, x)

[Out] int(exp(-c\*x^2+b\*x+a)/x, x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-cx^2+bx+a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)/x,x, algorithm="maxima")

[Out] integrate(e^(-c\*x^2 + b\*x + a)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(-cx^2+bx+a)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)/x,x, algorithm="fricas")

[Out] integral(e^(-c\*x^2 + b\*x + a)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{e^{bx} e^{-cx^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x\*\*2+b\*x+a)/x,x)

[Out] exp(a)\*Integral(exp(b\*x)\*exp(-c\*x\*\*2)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-cx^2+bx+a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-c\*x^2+b\*x+a)/x,x, algorithm="giac")

[Out] integrate(e^(-c\*x^2 + b\*x + a)/x, x)

$$3.437 \quad \int \frac{e^{a+bx-cx^2}}{x^2} dx$$

**Optimal.** Leaf size=81

$$b\text{Unintegrable}\left(\frac{e^{a+bx-cx^2}}{x}, x\right) + \sqrt{\pi}\sqrt{c}e^{a+\frac{b^2}{4c}}\text{Erf}\left(\frac{b-2cx}{2\sqrt{c}}\right) - \frac{e^{a+bx-cx^2}}{x}$$

[Out]  $-(E^{(a + b*x - c*x^2)}/x) + \text{Sqrt}[c]*E^{(a + b^2/(4*c))}*\text{Sqrt}[\text{Pi}]*\text{Erf}[(b - 2*c*x)/(2*\text{Sqrt}[c])] + b*\text{Unintegrable}[E^{(a + b*x - c*x^2)}/x, x]$

**Rubi [A]** time = 0.0705695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[E^{(a + b*x - c*x^2)}/x^2, x]$

[Out]  $-(E^{(a + b*x - c*x^2)}/x) + \text{Sqrt}[c]*E^{(a + b^2/(4*c))}*\text{Sqrt}[\text{Pi}]*\text{Erf}[(b - 2*c*x)/(2*\text{Sqrt}[c])] + b*\text{Defer}[\text{Int}][E^{(a + b*x - c*x^2)}/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx-cx^2}}{x^2} dx &= -\frac{e^{a+bx-cx^2}}{x} + b \int \frac{e^{a+bx-cx^2}}{x} dx - (2c) \int e^{a+bx-cx^2} dx \\ &= -\frac{e^{a+bx-cx^2}}{x} + b \int \frac{e^{a+bx-cx^2}}{x} dx - \left(2ce^{a+\frac{b^2}{4c}}\right) \int e^{-\frac{(b-2cx)^2}{4c}} dx \\ &= -\frac{e^{a+bx-cx^2}}{x} + \sqrt{c}e^{a+\frac{b^2}{4c}}\sqrt{\pi}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right) + b \int \frac{e^{a+bx-cx^2}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.220979, size = 0, normalized size = 0.

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[E^{(a + b*x - c*x^2)}/x^2, x]$

[Out]  $\text{Integrate}[E^{(a + b*x - c*x^2)}/x^2, x]$

**Maple [A]** time = 0.016, size = 0, normalized size = 0.

$$\int \frac{e^{-cx^2+bx+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-c*x^2+b*x+a)/x^2,x)`

[Out] `int(exp(-c*x^2+b*x+a)/x^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-cx^2+bx+a)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="maxima")`

[Out] `integrate(e^(-c*x^2 + b*x + a)/x^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(-cx^2+bx+a)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="fricas")`

[Out] `integral(e^(-c*x^2 + b*x + a)/x^2, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{e^{bx} e^{-cx^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x**2+b*x+a)/x**2,x)`

[Out] `exp(a)*Integral(exp(b*x)*exp(-c*x**2)/x**2, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(-cx^2+bx+a)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="giac")`

[Out] `integrate(e^(-c*x^2 + b*x + a)/x^2, x)`

### 3.438 $\int e^{(a+bx)(c+dx)} x^3 dx$

**Optimal.** Leaf size=297

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc)^3 \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{16b^{7/2}d^{7/2}} + \frac{3\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc) \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}} + \frac{(ad+bc)^2 e^{x(ad+bc)+ac+bdx^2}}{8b^3d^3} - \frac{x(ad+bc)}{8b^3d^3}$$

[Out]  $-E^{(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b^2*d^2)} + ((b*c + a*d)^2 * E^{(a*c + (b*c + a*d)*x + b*d*x^2)}) / (8*b^3*d^3) - ((b*c + a*d) * E^{(a*c + (b*c + a*d)*x + b*d*x^2)}) / (4*b^2*d^2) + (E^{(a*c + (b*c + a*d)*x + b*d*x^2)} * x^2) / (2*b*d) + (3*(b*c + a*d) * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])]) / (8*b^{5/2}*d^{5/2} * E^{((b*c - a*d)^2/(4*b*d))}) - ((b*c + a*d)^3 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])]) / (16*b^{7/2}*d^{7/2} * E^{((b*c - a*d)^2/(4*b*d))})$

**Rubi [A]** time = 0.63586, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2244, 2241, 2240, 2234, 2204}

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc)^3 \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{16b^{7/2}d^{7/2}} + \frac{3\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc) \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}} + \frac{(ad+bc)^2 e^{x(ad+bc)+ac+bdx^2}}{8b^3d^3} - \frac{x(ad+bc)}{8b^3d^3}$$

Antiderivative was successfully verified.

[In] Int[E^{(a + b\*x)\*(c + d\*x)}\*x^3, x]

[Out]  $-E^{(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b^2*d^2)} + ((b*c + a*d)^2 * E^{(a*c + (b*c + a*d)*x + b*d*x^2)}) / (8*b^3*d^3) - ((b*c + a*d) * E^{(a*c + (b*c + a*d)*x + b*d*x^2)}) / (4*b^2*d^2) + (E^{(a*c + (b*c + a*d)*x + b*d*x^2)} * x^2) / (2*b*d) + (3*(b*c + a*d) * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])]) / (8*b^{5/2}*d^{5/2} * E^{((b*c - a*d)^2/(4*b*d))}) - ((b*c + a*d)^3 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])]) / (16*b^{7/2}*d^{7/2} * E^{((b*c - a*d)^2/(4*b*d))})$

#### Rule 2244

Int[(F\_)^(v\_)\*(u\_)^(m\_), x\_Symbol] :> Int[ExpandToSum[u, x]^m \* F^ExpandToSum[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

#### Rule 2241

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1) \* F^(a + b\*x + c\*x^2)) / (2\*c\*Log[F]), x] + (-Dist[(b\*e - 2\*c\*d) / (2\*c), Int[(d + e\*x)^(m - 1) \* F^(a + b\*x + c\*x^2), x], x] - Dist[((m - 1) \* e^2) / (2\*c\*Log[F]), Int[(d + e\*x)^(m - 2) \* F^(a + b\*x + c\*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e \* F^(a + b\*x + c\*x^2)) / (2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d) / (2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]



Rule 2234

$\text{Int}[(F_)^\wedge((a_) + (b_) * (x_) + (c_) * (x_)^\wedge 2), x\_Symbol] := \text{Dist}[F^\wedge(a - b^2 / (4 * c)), \text{Int}[F^\wedge((b + 2 * c * x)^\wedge 2 / (4 * c)), x], x] /; \text{FreeQ}\{F, a, b, c\}, x]$

Rule 2204

$\text{Int}[(F_)^\wedge((a_) + (b_) * ((c_) + (d_) * (x_)^\wedge 2), x\_Symbol] := \text{Simp}[(F^\wedge a * \text{Sqrt}[\text{Pi}] * \text{Erfi}[(c + d * x) * \text{Rt}[b * \text{Log}[F], 2]]) / (2 * d * \text{Rt}[b * \text{Log}[F], 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

Rubi steps

$$\begin{aligned} \int e^{(a+bx)(c+dx)} x^3 dx &= \int e^{ac+(bc+ad)x+bdx^2} x^3 dx \\ &= \frac{e^{ac+(bc+ad)x+bdx^2} x^2}{2bd} - \frac{\int e^{ac+(bc+ad)x+bdx^2} x dx}{bd} - \frac{(bc+ad) \int e^{ac+(bc+ad)x+bdx^2} x^2 dx}{2bd} \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{2b^2d^2} - \frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2} x}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x^2}{2bd} + \frac{(bc+ad) \int e^{ac+(bc+ad)x+bdx^2} x^2 dx}{4b^2d^2} \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{2b^2d^2} + \frac{(bc+ad)^2 e^{ac+(bc+ad)x+bdx^2}}{8b^3d^3} - \frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2} x}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x^2}{2bd} \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{2b^2d^2} + \frac{(bc+ad)^2 e^{ac+(bc+ad)x+bdx^2}}{8b^3d^3} - \frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2} x}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x^2}{2bd} \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{2b^2d^2} + \frac{(bc+ad)^2 e^{ac+(bc+ad)x+bdx^2}}{8b^3d^3} - \frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2} x}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x^2}{2bd} \end{aligned}$$

**Mathematica [A]** time = 0.354298, size = 191, normalized size = 0.64

$$\frac{e^{-\frac{(bc-ad)^2}{4bd}} \left( 2\sqrt{b}\sqrt{d} e^{\frac{(ad+b(c+2dx))^2}{4bd}} (a^2d^2 - 2bd(-ac + adx + 2) + b^2(c^2 - 2cdx + 4d^2x^2)) - \sqrt{\pi} (a^3d^3 + 3b^2cd(ac - 2) + 3ab) \right)}{16b^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^((a + b\*x)\*(c + d\*x))\*x^3,x]

[Out] (2\*Sqrt[b]\*Sqrt[d]\*E^((a\*d + b\*(c + 2\*d\*x))^2/(4\*b\*d))\*(a^2\*d^2 - 2\*b\*d\*(2 - a\*c + a\*d\*x) + b^2\*(c^2 - 2\*c\*d\*x + 4\*d^2\*x^2)) - (b^3\*c^3 + 3\*b^2\*c\*(-2 + a\*c)\*d + 3\*a\*b\*(-2 + a\*c)\*d^2 + a^3\*d^3)\*Sqrt[Pi]\*Erfi[(a\*d + b\*(c + 2\*d\*x))/(2\*Sqrt[b]\*Sqrt[d])])/(16\*b^(7/2)\*d^(7/2)\*E^((b\*c - a\*d)^2/(4\*b\*d))

**Maple [A]** time = 0.011, size = 368, normalized size = 1.2

$$\frac{e^{ac+(ad+bc)x+bdx^2} x^2}{2bd} - \frac{ad+bc}{2bd} \left( \frac{e^{ac+(ad+bc)x+bdx^2} x}{2bd} - \frac{ad+bc}{2bd} \left( \frac{e^{ac+(ad+bc)x+bdx^2}}{2bd} + \frac{(ad+bc)\sqrt{\pi}}{4bd} e^{ac-\frac{(ad+bc)^2}{4bd}} \text{Erf}\left(-\sqrt{-bdx} + \dots\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c))\*x^3,x)

```
[Out] 1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x^2/b/d-1/2*(a*d+b*c)/b/d*(1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x/b/d-1/2*(a*d+b*c)/b/d*(1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b/d+1/4*(a*d+b*c)/b/d*Pi^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(1/2)))+1/4/b/d*Pi^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(1/2)))-1/b/d*(1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b/d+1/4*(a*d+b*c)/b/d*Pi^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(1/2)))/(-b*d)^(1/2))
```

**Maxima [A]** time = 1.19841, size = 360, normalized size = 1.21

$$\frac{\left( \frac{\sqrt{\pi}(2bdx+bc+ad)(bc+ad)^3 \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}\right)-1\right)}{(bd)^{\frac{7}{2}}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}} - \frac{6(bc+ad)^2 b d e^{\left(\frac{(2bdx+bc+ad)^2}{4bd}\right)}}{(bd)^{\frac{7}{2}}} + \frac{8b^2 d^2 \Gamma\left(2, -\frac{(2bdx+bc+ad)^2}{4bd}\right)}{(bd)^{\frac{7}{2}}} - \frac{12(2bdx+bc+ad)^3 (bc+ad) \Gamma\left(\frac{3}{2}, -\frac{(2bdx+bc+ad)^2}{4bd}\right)}{(bd)^{\frac{7}{2}}\left(-\frac{(2bdx+bc+ad)^2}{4bd}\right)} \right)}{16\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((b*x+a)*(d*x+c))*x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*(2*b*d*x + b*c + a*d)*(b*c + a*d)^3*(erf(1/2*sqrt(-(2*b*d*x + b*c + a*d)^2/(b*d))) - 1)/((b*d)^(7/2)*sqrt(-(2*b*d*x + b*c + a*d)^2/(b*d))) - 6*(b*c + a*d)^2*b*d*e^(1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(7/2) + 8*b^2*d^2*gamma(2, -1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(7/2) - 12*(2*b*d*x + b*c + a*d)^3*(b*c + a*d)*gamma(3/2, -1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/((b*d)^(7/2)*(-2*b*d*x + b*c + a*d)^2/(b*d))^(3/2))*e^(a*c - 1/4*(b*c + a*d)^2/(b*d))/sqrt(b*d)
```

**Fricas [A]** time = 1.51075, size = 460, normalized size = 1.55

$$\frac{\sqrt{\pi}(b^3c^3 + a^3d^3 + 3(a^2bc - 2ab)d^2 + 3(ab^2c^2 - 2b^2c)d)\sqrt{-bd} \operatorname{erf}\left(\frac{(2bdx+bc+ad)\sqrt{-bd}}{2bd}\right) e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)} + 2(4b^3d^3x^2 + b^3c^3d + a^2b^2d^3 + 2(a*b^2*c - 2*b^2)*d^2 - 2*(b^3*c*d^2 + a*b^2*d^3)*x)*e^{(b*d*x^2 + a*c + (b*c + a*d)*x)}}{16b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((b*x+a)*(d*x+c))*x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(b^3*c^3 + a^3*d^3 + 3*(a^2*b*c - 2*a*b)*d^2 + 3*(a*b^2*c^2 - 2*b^2*c)*d)*sqrt(-b*d)*erf(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b*d))*e^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d)) + 2*(4*b^3*d^3*x^2 + b^3*c^2*d + a^2*b*d^3 + 2*(a*b^2*c - 2*b^2)*d^2 - 2*(b^3*c*d^2 + a*b^2*d^3)*x)*e^(b*d*x^2 + a*c + (b*c + a*d)*x))/(b^4*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((b*x+a)*(d*x+c))*x**3,x)
```

[Out] Timed out

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**Giac [A]** time = 1.16414, size = 338, normalized size = 1.14

$$\frac{\sqrt{\pi}(b^3c^3+3ab^2c^2d+3a^2bcd^2+a^3d^3-6b^2cd-6abd^2)\operatorname{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}}{\sqrt{-bd}} + \frac{2\left(b^2d^2\left(2x+\frac{bc+ad}{bd}\right)^2 - 3b^2cd\left(2x+\frac{bc+ad}{bd}\right)\right)}{16b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x^3,x, algorithm="giac")

[Out] 1/16\*(sqrt(pi)\*(b^3\*c^3 + 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 + a^3\*d^3 - 6\*b^2\*c\*d - 6\*a\*b\*d^2)\*erf(-1/2\*sqrt(-b\*d)\*(2\*x + (b\*c + a\*d)/(b\*d)))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d))/sqrt(-b\*d) + 2\*(b^2\*d^2\*(2\*x + (b\*c + a\*d)/(b\*d))^2 - 3\*b^2\*c\*d\*(2\*x + (b\*c + a\*d)/(b\*d)) - 3\*a\*b\*d^2\*(2\*x + (b\*c + a\*d)/(b\*d)) + 3\*b^2\*c^2 + 6\*a\*b\*c\*d + 3\*a^2\*d^2 - 4\*b\*d)\*e^(b\*d\*x^2 + b\*c\*x + a\*d\*x + a\*c))/(b^3\*d^3)

### 3.439 $\int e^{(a+bx)(c+dx)} x^2 dx$

**Optimal.** Leaf size=216

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc)^2 \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}} - \frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}} - \frac{(ad+bc)e^{x(ad+bc)+ac+bdx^2}}{4b^2d^2} + \frac{\chi e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

[Out]  $-\left((b*c + a*d)*E^{(a*c + (b*c + a*d)*x + b*d*x^2)}\right)/(4*b^2*d^2) + \left(E^{(a*c + (b*c + a*d)*x + b*d*x^2)}\right)/(2*b*d) - \left(\operatorname{Sqrt}[\pi]*\operatorname{Erfi}\left[\frac{(b*c + a*d + 2*b*d*x)}{2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]}\right]\right)/(4*b^{3/2}*d^{3/2}*E^{((b*c - a*d)^2/(4*b*d))}) + \left((b*c + a*d)^2*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}\left[\frac{(b*c + a*d + 2*b*d*x)}{2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]}\right]\right)/(8*b^{5/2}*d^{5/2}*E^{((b*c - a*d)^2/(4*b*d))})$

**Rubi [A]** time = 0.271386, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2244, 2241, 2240, 2234, 2204}

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} (ad+bc)^2 \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}} - \frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}} - \frac{(ad+bc)e^{x(ad+bc)+ac+bdx^2}}{4b^2d^2} + \frac{\chi e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[E^{(a + b*x)*(c + d*x)} x^2, x\right]$

[Out]  $-\left((b*c + a*d)*E^{(a*c + (b*c + a*d)*x + b*d*x^2)}\right)/(4*b^2*d^2) + \left(E^{(a*c + (b*c + a*d)*x + b*d*x^2)}\right)/(2*b*d) - \left(\operatorname{Sqrt}[\pi]*\operatorname{Erfi}\left[\frac{(b*c + a*d + 2*b*d*x)}{2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]}\right]\right)/(4*b^{3/2}*d^{3/2}*E^{((b*c - a*d)^2/(4*b*d))}) + \left((b*c + a*d)^2*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}\left[\frac{(b*c + a*d + 2*b*d*x)}{2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]}\right]\right)/(8*b^{5/2}*d^{5/2}*E^{((b*c - a*d)^2/(4*b*d))})$

#### Rule 2244

$\operatorname{Int}[(F_)^{(v_*)}(u_)^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandToSum}[u, x]^m F^{\operatorname{ExpandToSum}[v, x], x}] /; \operatorname{FreeQ}[\{F, m\}, x] \ \&\& \operatorname{LinearQ}[u, x] \ \&\& \operatorname{QuadraticQ}[v, x] \ \&\& \operatorname{!}(\operatorname{LinearMatchQ}[u, x] \ \&\& \operatorname{QuadraticMatchQ}[v, x])]$

#### Rule 2241

$\operatorname{Int}[(F_)^{((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2)*((d_*) + (e_*)*(x_*))^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*(d + e*x)^{(m-1)}*F^{(a + b*x + c*x^2)})/(2*c*\operatorname{Log}[F]), x] + (-\operatorname{Dist}[(b*e - 2*c*d)/(2*c), \operatorname{Int}[(d + e*x)^{(m-1)}*F^{(a + b*x + c*x^2)}, x], x] - \operatorname{Dist}[(m-1)*e^2/(2*c*\operatorname{Log}[F]), \operatorname{Int}[(d + e*x)^{(m-2)}*F^{(a + b*x + c*x^2)}, x], x]) /; \operatorname{FreeQ}[\{F, a, b, c, d, e\}, x] \ \&\& \operatorname{NeQ}[b*e - 2*c*d, 0] \ \&\& \operatorname{GtQ}[m, 1]$

#### Rule 2240

$\operatorname{Int}[(F_)^{((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2)*((d_*) + (e_*)*(x_*))}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*F^{(a + b*x + c*x^2)})/(2*c*\operatorname{Log}[F]), x] - \operatorname{Dist}[(b*e - 2*c*d)/(2*c), \operatorname{Int}[F^{(a + b*x + c*x^2)}, x], x] /; \operatorname{FreeQ}[\{F, a, b, c, d, e\}, x] \ \&\& \operatorname{NeQ}[b*e - 2*c*d, 0]$

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

**Rule 2204**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

**Rubi steps**

$$\begin{aligned} \int e^{(a+bx)(c+dx)} x^2 dx &= \int e^{ac+(bc+ad)x+bdx^2} x^2 dx \\ &= \frac{e^{ac+(bc+ad)x+bdx^2} x}{2bd} - \frac{\int e^{ac+(bc+ad)x+bdx^2} dx}{2bd} - \frac{(bc+ad) \int e^{ac+(bc+ad)x+bdx^2} x dx}{2bd} \\ &= -\frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2}}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x}{2bd} + \frac{(bc+ad)^2 \int e^{ac+(bc+ad)x+bdx^2} dx}{4b^2d^2} - \frac{e^{-\frac{(bc-ad)^2}{4bd}} \int e^{\frac{(bc-ad)^2}{4bd}} dx}{2bd} \\ &= -\frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2}}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x}{2bd} - \frac{e^{-\frac{(bc-ad)^2}{4bd}} \sqrt{\pi} \operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}} + \frac{\left((bc+ad)^2 e^{-\frac{(bc-ad)^2}{4bd}} \int e^{\frac{(bc-ad)^2}{4bd}} dx\right)}{4b^2d^2} \\ &= -\frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2}}{4b^2d^2} + \frac{e^{ac+(bc+ad)x+bdx^2} x}{2bd} - \frac{e^{-\frac{(bc-ad)^2}{4bd}} \sqrt{\pi} \operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}} + \frac{(bc+ad)^2 e^{-\frac{(bc-ad)^2}{4bd}} \int e^{\frac{(bc-ad)^2}{4bd}} dx}{4b^2d^2} \end{aligned}$$

**Mathematica [A]** time = 0.177909, size = 144, normalized size = 0.67

$$\frac{e^{-\frac{(bc-ad)^2}{4bd}} \left( \sqrt{\pi} (a^2d^2 + 2bd(ac-1) + b^2c^2) \operatorname{Erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right) - 2\sqrt{b}\sqrt{d} e^{\frac{(ad+b(c+2dx))^2}{4bd}} (ad + b(c-2dx)) \right)}{8b^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^((a + b\*x)\*(c + d\*x))\*x^2, x]

[Out] (-2\*Sqrt[b]\*Sqrt[d]\*E^((a\*d + b\*(c + 2\*d\*x))^2/(4\*b\*d))\*(a\*d + b\*(c - 2\*d\*x)) + (b^2\*c^2 + 2\*b\*(-1 + a\*c)\*d + a^2\*d^2)\*Sqrt[Pi]\*Erfi[(a\*d + b\*(c + 2\*d\*x))/(2\*Sqrt[b]\*Sqrt[d])])/(8\*b^(5/2)\*d^(5/2)\*E^((b\*c - a\*d)^2/(4\*b\*d)))

**Maple [A]** time = 0.006, size = 212, normalized size = 1.

$$\frac{e^{ac+(ad+bc)x+bdx^2} x}{2bd} - \frac{ad+bc}{2bd} \left( \frac{e^{ac+(ad+bc)x+bdx^2}}{2bd} + \frac{(ad+bc)\sqrt{\pi}}{4bd} e^{ac-\frac{(ad+bc)^2}{4bd}} \operatorname{Erf}\left(-\sqrt{-bd}x + \frac{ad+bc}{2} \frac{1}{\sqrt{-bd}}\right) \frac{1}{\sqrt{-bd}} \right) + \frac{\sqrt{\pi}}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c))\*x^2, x)

[Out] 1/2\*exp(a\*c+(a\*d+b\*c)\*x+b\*d\*x^2)\*x/b/d-1/2\*(a\*d+b\*c)/b/d\*(1/2\*exp(a\*c+(a\*d+b\*c)\*x+b\*d\*x^2)/b/d+1/4\*(a\*d+b\*c)/b/d\*Pi^(1/2)\*exp(a\*c-1/4\*(a\*d+b\*c)^2/b/d)/(-b\*d)^(1/2)\*erf(-(-b\*d)^(1/2)\*x+1/2\*(a\*d+b\*c)/(-b\*d)^(1/2))+1/4/b/d\*Pi^(1/2)\*exp(a\*c-1/4\*(a\*d+b\*c)^2/b/d)/(-b\*d)^(1/2)\*erf(-(-b\*d)^(1/2)\*x+1/2\*(a\*d+b\*c)/(-b\*d)^(1/2))

**Maxima [A]** time = 1.17927, size = 298, normalized size = 1.38

$$\frac{\left( \frac{\sqrt{\pi}(2bdx+bc+ad)(bc+ad)^2 \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}\right) - 1\right)}{(bd)^{\frac{5}{2}}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}} - \frac{4(bc+ad)bde^{\left(\frac{(2bdx+bc+ad)^2}{4bd}\right)}}{(bd)^{\frac{5}{2}}} - \frac{4(2bdx+bc+ad)^3\Gamma\left(\frac{3}{2}, -\frac{(2bdx+bc+ad)^2}{4bd}\right)}{(bd)^{\frac{5}{2}}\left(-\frac{(2bdx+bc+ad)^2}{bd}\right)^{\frac{3}{2}}} \right) e^{\left(ac - \frac{(bc+ad)^2}{4bd}\right)}}{8\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x^2,x, algorithm="maxima")

[Out] 1/8\*(sqrt(pi)\*(2\*b\*d\*x + b\*c + a\*d)\*(b\*c + a\*d)^2\*(erf(1/2\*sqrt(-(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))) - 1)/((b\*d)^(5/2)\*sqrt(-(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))) - 4\*(b\*c + a\*d)\*b\*d\*e^(1/4\*(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))/(b\*d)^(5/2) - 4\*(2\*b\*d\*x + b\*c + a\*d)^3\*gamma(3/2, -1/4\*(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))/((b\*d)^(5/2)\*(-(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))^(3/2))\*e^(a\*c - 1/4\*(b\*c + a\*d)^2/(b\*d))/sqrt(b\*d)

**Fricas [A]** time = 1.58228, size = 328, normalized size = 1.52

$$\frac{\sqrt{\pi}(b^2c^2 + a^2d^2 + 2(abc - b)d)\sqrt{-bd} \operatorname{erf}\left(\frac{(2bdx+bc+ad)\sqrt{-bd}}{2bd}\right) e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)} - 2(2b^2d^2x - b^2cd - abd^2)e^{(bdx^2+ac+(bc+ad)x)}}{8b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x^2,x, algorithm="fricas")

[Out] -1/8\*(sqrt(pi)\*(b^2\*c^2 + a^2\*d^2 + 2\*(a\*b\*c - b)\*d)\*sqrt(-b\*d)\*erf(1/2\*(2\*b\*d\*x + b\*c + a\*d)\*sqrt(-b\*d)/(b\*d))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d)) - 2\*(2\*b^2\*d^2\*x - b^2\*c\*d - a\*b\*d^2)\*e^(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)/(b^3\*d^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.23967, size = 205, normalized size = 0.95

$$\frac{\sqrt{\pi}(b^2c^2+2abcd+a^2d^2-2bd) \operatorname{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2x+\frac{bc+ad}{bd}\right)\right) e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}}{\sqrt{-bd}} - 2\left(bd\left(2x+\frac{bc+ad}{bd}\right) - 2bc - 2ad\right) e^{(bdx^2+bcx+adx+ac)}}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x^2,x, algorithm="giac")

[Out] 
$$-1/8*(\sqrt{\pi}*(b^2*c^2 + 2*a*b*c*d + a^2*d^2 - 2*b*d)*\operatorname{erf}(-1/2*\sqrt{-b*d}*(2*x + (b*c + a*d)/(b*d)))*e^{(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))}/\sqrt{-b*d} - 2*(b*d*(2*x + (b*c + a*d)/(b*d)) - 2*b*c - 2*a*d)*e^{(b*d*x^2 + b*c*x + a*d*x + a*c)})/(b^2*d^2)$$

### 3.440 $\int e^{(a+bx)(c+dx)} x dx$

**Optimal.** Leaf size=107

$$\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd} - \frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}$$

[Out]  $E^{(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b*d)} - ((b*c + a*d)*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])])/(4*b^{(3/2)}*d^{(3/2)}*E^{((b*c - a*d)^2/(4*b*d))})$

**Rubi [A]** time = 0.104187, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2244, 2240, 2234, 2204}

$$\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd} - \frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a + b*x)*(c + d*x)}*x, x]$

[Out]  $E^{(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b*d)} - ((b*c + a*d)*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])])/(4*b^{(3/2)}*d^{(3/2)}*E^{((b*c - a*d)^2/(4*b*d))})$

#### Rule 2244

$\operatorname{Int}[(F_)^(v_)*(u_)^(m_), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandToSum}[u, x]^m * F^{\operatorname{ExpandToSum}[v, x]}], x] /;$   $\operatorname{FreeQ}\{F, m\}, x] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{QuadraticQ}[v, x] \&\& !(\operatorname{LinearMatchQ}[u, x] \&\& \operatorname{QuadraticMatchQ}[v, x])$

#### Rule 2240

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*F^{(a + b*x + c*x^2)})/(2*c*\operatorname{Log}[F]), x] - \operatorname{Dist}[(b*e - 2*c*d)/(2*c), \operatorname{Int}[F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{NeQ}[b*e - 2*c*d, 0]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps



$$\begin{aligned}
\int e^{(a+bx)(c+dx)} x dx &= \int e^{ac+(bc+ad)x+bdx^2} x dx \\
&= \frac{e^{ac+(bc+ad)x+bdx^2}}{2bd} - \frac{(bc+ad) \int e^{ac+(bc+ad)x+bdx^2} dx}{2bd} \\
&= \frac{e^{ac+(bc+ad)x+bdx^2}}{2bd} - \frac{\left( (bc+ad)e^{-\frac{(bc-ad)^2}{4bd}} \right) \int e^{\frac{(bc+ad+2bdx)^2}{4bd}} dx}{2bd} \\
&= \frac{e^{ac+(bc+ad)x+bdx^2}}{2bd} - \frac{(bc+ad)e^{-\frac{(bc-ad)^2}{4bd}} \sqrt{\pi} \operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0880173, size = 116, normalized size = 1.08

$$\frac{e^{-\frac{(bc-ad)^2}{4bd}} \left( 2\sqrt{b}\sqrt{d} e^{\frac{(ad+b(c+2dx))^2}{4bd}} - \sqrt{\pi}(ad+bc) \operatorname{Erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right) \right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^((a + b\*x)\*(c + d\*x))\*x, x]

[Out] (2\*sqrt[b]\*sqrt[d]\*E^((a\*d + b\*(c + 2\*d\*x))^2/(4\*b\*d)) - (b\*c + a\*d)\*sqrt[Pi]\*Erfi[(a\*d + b\*(c + 2\*d\*x))/(2\*sqrt[b]\*sqrt[d])])/(4\*b^(3/2)\*d^(3/2)\*E^((b\*c - a\*d)^2/(4\*b\*d)))

**Maple [A]** time = 0.004, size = 102, normalized size = 1.

$$\frac{e^{ac+(ad+bc)x+bdx^2}}{2bd} + \frac{(ad+bc)\sqrt{\pi}}{4bd} e^{ac-\frac{(ad+bc)^2}{4bd}} \operatorname{Erf}\left(-\sqrt{-bd}x + \frac{ad+bc}{2} \frac{1}{\sqrt{-bd}}\right) \frac{1}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c))\*x, x)

[Out] 1/2\*exp(a\*c+(a\*d+b\*c)\*x+b\*d\*x^2)/b/d+1/4\*(a\*d+b\*c)/b/d\*Pi^(1/2)\*exp(a\*c-1/4\*(a\*d+b\*c)^2/b/d)/(-b\*d)^(1/2)\*erf(-(-b\*d)^(1/2)\*x+1/2\*(a\*d+b\*c)/(-b\*d)^(1/2))

**Maxima [A]** time = 1.16907, size = 193, normalized size = 1.8

$$\frac{\left( \frac{\sqrt{\pi}(2bdx+bc+ad)(bc+ad) \left( \operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}\right) - 1 \right)}{(bd)^{\frac{3}{2}}\sqrt{-\frac{(2bdx+bc+ad)^2}{bd}}} - \frac{2bde^{\left(\frac{(2bdx+bc+ad)^2}{4bd}\right)}}{(bd)^{\frac{3}{2}}} \right) e^{ac-\frac{(bc+ad)^2}{4bd}}}{4\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x, x, algorithm="maxima")

[Out] -1/4\*(sqrt(pi)\*(2\*b\*d\*x + b\*c + a\*d)\*(b\*c + a\*d)\*(erf(1/2\*sqrt(-(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))) - 1)/((b\*d)^(3/2)\*sqrt(-(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d)))

) - 2\*b\*d\*e^(1/4\*(2\*b\*d\*x + b\*c + a\*d)^2/(b\*d))/(b\*d)^(3/2))\*e^(a\*c - 1/4\*(b\*c + a\*d)^2/(b\*d))/sqrt(b\*d)

**Fricas [A]** time = 1.52755, size = 251, normalized size = 2.35

$$\frac{\sqrt{\pi}(bc+ad)\sqrt{-bd}\operatorname{erf}\left(\frac{(2bdx+bc+ad)\sqrt{-bd}}{2bd}\right)e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}+2bde^{(bdx^2+ac+(bc+ad)x)}}{4b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x,x, algorithm="fricas")

[Out] 1/4\*(sqrt(pi)\*(b\*c + a\*d)\*sqrt(-b\*d)\*erf(1/2\*(2\*b\*d\*x + b\*c + a\*d)\*sqrt(-b\*d)/(b\*d))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d)) + 2\*b\*d\*e^(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x))/(b^2\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{ac} \int x e^{adx} e^{bcx} e^{bdx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x,x)

[Out] exp(a\*c)\*Integral(x\*exp(a\*d\*x)\*exp(b\*c\*x)\*exp(b\*d\*x\*\*2), x)

**Giac [A]** time = 1.22384, size = 140, normalized size = 1.31

$$\frac{\frac{\sqrt{\pi}(bc+ad)\operatorname{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}}{\sqrt{-bd}}}{4bd}+2e^{(bdx^2+bcx+adx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))\*x,x, algorithm="giac")

[Out] 1/4\*(sqrt(pi)\*(b\*c + a\*d)\*erf(-1/2\*sqrt(-b\*d)\*(2\*x + (b\*c + a\*d)/(b\*d)))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d))/sqrt(-b\*d) + 2\*e^(b\*d\*x^2 + b\*c\*x + a\*d\*x + a\*c))/(b\*d)

### 3.441 $\int e^{(a+bx)(c+dx)} dx$

**Optimal.** Leaf size=68

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

[Out] (Sqrt[Pi]\*Erfi[(b\*c + a\*d + 2\*b\*d\*x)/(2\*Sqrt[b]\*Sqrt[d])])/(2\*Sqrt[b]\*Sqrt[d])\*E^((b\*c - a\*d)^2/(4\*b\*d))

**Rubi [A]** time = 0.0254584, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2235, 2234, 2204}

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[E^((a + b\*x)\*(c + d\*x)),x]

[Out] (Sqrt[Pi]\*Erfi[(b\*c + a\*d + 2\*b\*d\*x)/(2\*Sqrt[b]\*Sqrt[d])])/(2\*Sqrt[b]\*Sqrt[d])\*E^((b\*c - a\*d)^2/(4\*b\*d))

#### Rule 2235

Int[(F\_)^(v\_), x\_Symbol] := Int[F^ExpandToSum[v, x], x] /; FreeQ[F, x] && QuadraticQ[v, x] && !QuadraticMatchQ[v, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int e^{(a+bx)(c+dx)} dx &= \int e^{ac+(bc+ad)x+bdx^2} dx \\ &= e^{-\frac{(bc-ad)^2}{4bd}} \int e^{\frac{(bc+ad+2bdx)^2}{4bd}} dx \\ &= \frac{e^{-\frac{(bc-ad)^2}{4bd}} \sqrt{\pi} \operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} \end{aligned}$$

**Mathematica [A]** time = 0.0164936, size = 68, normalized size = 1.

$$\frac{\sqrt{\pi} e^{-\frac{(bc-ad)^2}{4bd}} \operatorname{Erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[E^((a + b\*x)\*(c + d\*x)), x]

[Out] (Sqrt[Pi]\*Erfi[(a\*d + b\*(c + 2\*d\*x))/(2\*Sqrt[b]\*Sqrt[d]])/(2\*Sqrt[b]\*Sqrt[d])\*E^((b\*c - a\*d)^2/(4\*b\*d))

**Maple [A]** time = 0.003, size = 60, normalized size = 0.9

$$-\frac{\sqrt{\pi}}{2} e^{ac - \frac{(ad+bc)^2}{4bd}} \operatorname{Erf}\left(-\sqrt{-bd}x + \frac{ad+bc}{2} \frac{1}{\sqrt{-bd}}\right) \frac{1}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c)), x)

[Out] -1/2\*Pi^(1/2)\*exp(a\*c-1/4\*(a\*d+b\*c)^2/b/d)/(-b\*d)^(1/2)\*erf(-(-b\*d)^(1/2)\*x+1/2\*(a\*d+b\*c)/(-b\*d)^(1/2))

**Maxima [A]** time = 1.00926, size = 78, normalized size = 1.15

$$\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-bd}x - \frac{bc+ad}{2\sqrt{-bd}}\right) e^{\left(ac - \frac{(bc+ad)^2}{4bd}\right)}}{2\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c)), x, algorithm="maxima")

[Out] 1/2\*sqrt(pi)\*erf(sqrt(-b\*d)\*x - 1/2\*(b\*c + a\*d)/sqrt(-b\*d))\*e^(a\*c - 1/4\*(b\*c + a\*d)^2/(b\*d))/sqrt(-b\*d)

**Fricas [A]** time = 1.53094, size = 171, normalized size = 2.51

$$\frac{\sqrt{\pi}\sqrt{-bd} \operatorname{erf}\left(\frac{(2bdx+bc+ad)\sqrt{-bd}}{2bd}\right) e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c)), x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-b\*d)\*erf(1/2\*(2\*b\*d\*x + b\*c + a\*d)\*sqrt(-b\*d)/(b\*d))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d))/(b\*d)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{ac} \int e^{adx} e^{bcx} e^{bdx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c)),x)

[Out] exp(a\*c)\*Integral(exp(a\*d\*x)\*exp(b\*c\*x)\*exp(b\*d\*x\*\*2), x)

---

**Giac [A]** time = 1.21417, size = 92, normalized size = 1.35

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-bd} \left(2x + \frac{bc+ad}{bd}\right)\right) e^{\left(-\frac{b^2c^2-2abcd+a^2d^2}{4bd}\right)}}{2\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c)),x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*erf(-1/2\*sqrt(-b\*d)\*(2\*x + (b\*c + a\*d)/(b\*d)))\*e^(-1/4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/(b\*d))/sqrt(-b\*d)

$$3.442 \quad \int \frac{e^{(a+bx)(c+dx)}}{x} dx$$

**Optimal.** Leaf size=27

$$\text{Unintegrable} \left( \frac{e^{x(ad+bc)+ac+bdx^2}}{x}, x \right)$$

[Out] Unintegrable[E^(a\*c + (b\*c + a\*d)\*x + b\*d\*x^2)/x, x]

**Rubi [A]** time = 0.138539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{(a+bx)(c+dx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[E^((a + b\*x)\*(c + d\*x))/x, x]

[Out] Defer[Int][E^(a\*c + (b\*c + a\*d)\*x + b\*d\*x^2)/x, x]

Rubi steps

$$\int \frac{e^{(a+bx)(c+dx)}}{x} dx = \int \frac{e^{ac+(bc+ad)x+bdx^2}}{x} dx$$

**Mathematica [A]** time = 0.54493, size = 0, normalized size = 0.

$$\int \frac{e^{(a+bx)(c+dx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^((a + b\*x)\*(c + d\*x))/x, x]

[Out] Integrate[E^((a + b\*x)\*(c + d\*x))/x, x]

**Maple [A]** time = 0.015, size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)(dx+c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c))/x, x)

[Out] int(exp((b\*x+a)\*(d\*x+c))/x, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)(dx+c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x,x, algorithm="maxima")

[Out] integrate(e^((b\*x + a)\*(d\*x + c))/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(bdx^2+ac+(bc+ad)x)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x,x, algorithm="fricas")

[Out] integral(e^(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^{ac} \int \frac{e^{adx} e^{bcx} e^{bdx^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x,x)

[Out] exp(a\*c)\*Integral(exp(a\*d\*x)\*exp(b\*c\*x)\*exp(b\*d\*x\*\*2)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)(dx+c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x,x, algorithm="giac")

[Out] integrate(e^((b\*x + a)\*(d\*x + c))/x, x)

$$3.443 \quad \int \frac{e^{(a+bx)(c+dx)}}{x^2} dx$$

**Optimal.** Leaf size=127

$$(ad + bc)\text{Unintegrable}\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{x}, x\right) + \sqrt{\pi}\sqrt{b}\sqrt{d}e^{-\frac{(bc-ad)^2}{4bd}} \text{Erfi}\left(\frac{ad + bc + 2bdx}{2\sqrt{b}\sqrt{d}}\right) - \frac{e^{x(ad+bc)+ac+bdx^2}}{x}$$

[Out]  $-(E^{(a*c + (b*c + a*d)*x + b*d*x^2)/x}) + (\text{Sqrt}[b]*\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\text{Sqrt}[b]*\text{Sqrt}[d])])/E^{((b*c - a*d)^2/(4*b*d))} + (b*c + a*d)*\text{Unintegrable}[E^{(a*c + (b*c + a*d)*x + b*d*x^2)/x}, x]$

**Rubi [A]** time = 0.264586, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[E^{((a + b*x)*(c + d*x))/x^2}, x]$

[Out]  $-(E^{(a*c + (b*c + a*d)*x + b*d*x^2)/x}) + (\text{Sqrt}[b]*\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(b*c + a*d + 2*b*d*x)/(2*\text{Sqrt}[b]*\text{Sqrt}[d])])/E^{((b*c - a*d)^2/(4*b*d))} + (b*c + a*d)*\text{Defer}[\text{Int}[E^{(a*c + (b*c + a*d)*x + b*d*x^2)/x}, x]$

Rubi steps

$$\begin{aligned} \int \frac{e^{(a+bx)(c+dx)}}{x^2} dx &= \int \frac{e^{ac+(bc+ad)x+bdx^2}}{x^2} dx \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{x} + (2bd) \int e^{ac+(bc+ad)x+bdx^2} dx - (-bc - ad) \int \frac{e^{ac+(bc+ad)x+bdx^2}}{x} dx \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{x} - (-bc - ad) \int \frac{e^{ac+(bc+ad)x+bdx^2}}{x} dx + \left(2bde^{-\frac{(bc-ad)^2}{4bd}}\right) \int e^{\frac{(bc+ad+2bdx)^2}{4bd}} dx \\ &= -\frac{e^{ac+(bc+ad)x+bdx^2}}{x} + \sqrt{b}\sqrt{d}e^{-\frac{(bc-ad)^2}{4bd}} \sqrt{\pi} \text{erfi}\left(\frac{bc + ad + 2bdx}{2\sqrt{b}\sqrt{d}}\right) - (-bc - ad) \int \frac{e^{ac+(bc+ad)x+bdx^2}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.378521, size = 0, normalized size = 0.

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[E^{((a + b*x)*(c + d*x))/x^2}, x]$

[Out]  $\text{Integrate}[E^{((a + b*x)*(c + d*x))/x^2}, x]$



**Maple [A]** time = 0.017, size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)(dx+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)\*(d\*x+c))/x^2,x)

[Out] int(exp((b\*x+a)\*(d\*x+c))/x^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{((bx+a)(dx+c))}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x^2,x, algorithm="maxima")

[Out] integrate(e^((b\*x + a)\*(d\*x + c))/x^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(bdx^2+ac+(bc+ad)x)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x^2,x, algorithm="fricas")

[Out] integral(e^(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$e^{ac} \int \frac{e^{adx} e^{bcx} e^{bdx^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*(d\*x+c))/x\*\*2,x)

[Out] exp(a\*c)\*Integral(exp(a\*d\*x)\*exp(b\*c\*x)\*exp(b\*d\*x\*\*2)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)(dx+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((b*x+a)*(d*x+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*(d*x + c))/x^2, x)
```

### 3.444 $\int f^{a+bx+cx^2} (d+ex)^3 dx$

**Optimal.** Leaf size=266

$$\frac{3\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}(2cd-be)\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\log^3(f)} + \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)^3\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{16c^{7/2}\sqrt{\log(f)}} + \frac{e(2cd-be)^2 f^{a+bx+cx^2}}{8c^3\log(f)} + \frac{e(d+ex)^3 f^{a+bx+cx^2}}{8c^3\log(f)}$$

[Out]  $-(e^3 f^{a+bx+cx^2})/(2c^2 \operatorname{Log}[f]^2) - (3e^2(2cd-be)f^{a-b^2/(4c)})\sqrt{\pi}\operatorname{Erfi}(((b+2cx)\sqrt{\operatorname{Log}[f]})/(2\sqrt{c})))/(8c^{5/2}\operatorname{Log}[f]^{3/2}) + (e(2cd-be)^2 f^{a+bx+cx^2})/(8c^3 \operatorname{Log}[f]) + (e(2cd-be)f^{a+bx+cx^2}(d+ex))/(4c^2 \operatorname{Log}[f]) + (ef^{a+bx+cx^2}(d+ex)^2)/(2c \operatorname{Log}[f]) + ((2cd-be)^3 f^{a-b^2/(4c)})\sqrt{\pi}\operatorname{Erfi}(((b+2cx)\sqrt{\operatorname{Log}[f]})/(2\sqrt{c})))/(16c^{7/2}\sqrt{\operatorname{Log}[f]})$

**Rubi [A]** time = 0.32375, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2241, 2240, 2234, 2204}

$$\frac{3\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}(2cd-be)\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\log^3(f)} + \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)^3\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{16c^{7/2}\sqrt{\log(f)}} + \frac{e(2cd-be)^2 f^{a+bx+cx^2}}{8c^3\log(f)} + \frac{e(d+ex)^3 f^{a+bx+cx^2}}{8c^3\log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{a+bx+cx^2}(d+ex)^3, x]$

[Out]  $-(e^3 f^{a+bx+cx^2})/(2c^2 \operatorname{Log}[f]^2) - (3e^2(2cd-be)f^{a-b^2/(4c)})\sqrt{\pi}\operatorname{Erfi}(((b+2cx)\sqrt{\operatorname{Log}[f]})/(2\sqrt{c})))/(8c^{5/2}\operatorname{Log}[f]^{3/2}) + (e(2cd-be)^2 f^{a+bx+cx^2})/(8c^3 \operatorname{Log}[f]) + (e(2cd-be)f^{a+bx+cx^2}(d+ex))/(4c^2 \operatorname{Log}[f]) + (ef^{a+bx+cx^2}(d+ex)^2)/(2c \operatorname{Log}[f]) + ((2cd-be)^3 f^{a-b^2/(4c)})\sqrt{\pi}\operatorname{Erfi}(((b+2cx)\sqrt{\operatorname{Log}[f]})/(2\sqrt{c})))/(16c^{7/2}\sqrt{\operatorname{Log}[f]})$

#### Rule 2241

$\operatorname{Int}[(F_)^{((a_.) + (b_.)(x_) + (c_.)(x_)^2)((d_.) + (e_.)(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[(e*(d+ex)^{(m-1)}F^{(a+bx+cx^2)})/(2c*\operatorname{Log}[F]), x] + (-\operatorname{Dist}[(b*e-2*c*d)/(2*c), \operatorname{Int}[(d+ex)^{(m-1)}F^{(a+bx+cx^2)}, x], x] - \operatorname{Dist}[(m-1)*e^2/(2*c*\operatorname{Log}[F]), \operatorname{Int}[(d+ex)^{(m-2)}F^{(a+bx+cx^2)}, x], x]) /; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{NeQ}[b*e-2*c*d, 0] \&\& \operatorname{GtQ}[m, 1]$

#### Rule 2240

$\operatorname{Int}[(F_)^{((a_.) + (b_.)(x_) + (c_.)(x_)^2)((d_.) + (e_.)(x_))}, x\_Symbol] := \operatorname{Simp}[(eF^{(a+bx+cx^2)})/(2c*\operatorname{Log}[F]), x] - \operatorname{Dist}[(b*e-2*c*d)/(2*c), \operatorname{Int}[F^{(a+bx+cx^2)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{NeQ}[b*e-2*c*d, 0]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)(x_) + (c_.)(x_)^2)}, x\_Symbol] := \operatorname{Dist}[F^{(a-b^2/(4*c))}, \operatorname{Int}[F^{((b+2*c*x)^2/(4*c))}, x], x] /; \operatorname{FreeQ}\{F, a, b, c\}, x]$

**Rule 2204**

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^-2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

**Rubi steps**

$$\begin{aligned} \int f^{a+bx+cx^2}(d+ex)^3 dx &= \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c \log(f)} - \frac{(-2cd+be) \int f^{a+bx+cx^2}(d+ex)^2 dx}{2c} - \frac{e^2 \int f^{a+bx+cx^2}(d+ex) dx}{c \log(f)} \\ &= -\frac{e^3 f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}(d+ex)}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c \log(f)} + \frac{(2cd-be)^2 \int f^{a+bx+cx^2}(d+ex) dx}{4c^2} \\ &= -\frac{e^3 f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{e(2cd-be)^2 f^{a+bx+cx^2}}{8c^3 \log(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}(d+ex)}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c \log(f)} + \frac{(2cd-be)^2 \int f^{a+bx+cx^2}(d+ex) dx}{4c^2} \\ &= -\frac{e^3 f^{a+bx+cx^2}}{2c^2 \log^2(f)} - \frac{3e^2(2cd-be)f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)^2 f^{a+bx+cx^2}}{8c^3 \log(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}(d+ex)}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c \log(f)} \\ &= -\frac{e^3 f^{a+bx+cx^2}}{2c^2 \log^2(f)} - \frac{3e^2(2cd-be)f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2} \log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)^2 f^{a+bx+cx^2}}{8c^3 \log(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}(d+ex)}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.275, size = 169, normalized size = 0.64

$$\frac{f^{a-\frac{b^2}{4c}} \left( 2\sqrt{c} e f^{\frac{(b+2cx)^2}{4c}} (\log(f) (b^2 e^2 - 2bce(3d+ex) + 4c^2 (3d^2 + 3dex + e^2 x^2)) - 4ce^2) + \sqrt{\pi} \sqrt{\log(f)} (2cd-be) (\log(f) (be^2 + 2cd+ex) + e^2 x^2) \right)}{16c^{7/2} \log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*(d + e*x)^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*((2*c*d - b*e)*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])])*Sqrt[Log[f]]*(-6*c*e^2 + (-2*c*d + b*e)^2*Log[f]) + 2*Sqrt[c]*
e*f^((b + 2*c*x)^2/(4*c))*(-4*c*e^2 + (b^2*e^2 - 2*b*c*e*(3*d + e*x) + 4*c^
2*(3*d^2 + 3*d*e*x + e^2*x^2))*Log[f]))/(16*c^(7/2)*Log[f]^2)
```

**Maple [B]** time = 0.052, size = 550, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*(e*x+d)^3,x)
```

```
[Out] -1/2*d^3*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)
*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+1/2*e^3/c/ln(f)*x^2*f^(c*x^2)*f^(b*x)*f^a-
1/4*e^3/c^2*b/ln(f)*x*f^(c*x^2)*f^(b*x)*f^a+1/8*e^3/c^3*b^2/ln(f)*f^(c*x^2)
*f^(b*x)*f^a+1/16*e^3/c^3*b^3*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*
erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))-3/8*e^3/c^2*b/ln(f)*P
i^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*1
```

$n(f)/(-c*\ln(f))^{(1/2)}-1/2*e^3/c^2/\ln(f)^2*f^{(c*x^2)}*f^{(b*x)}*f^{a+3/2*d*e^2/c/\ln(f)*x*f^{(c*x^2)}*f^{(b*x)}*f^{a-3/4*d*e^2/c^2*b/\ln(f)*f^{(c*x^2)}*f^{(b*x)}*f^{a-3/8*d*e^2/c^2*b^2*Pi^{(1/2)}*f^a*f^{(-1/4*b^2/c)/(-c*\ln(f))^{(1/2)}*erf(-(-c*\ln(f))^{(1/2)}*x+1/2*b*\ln(f)/(-c*\ln(f))^{(1/2)})+3/4*d*e^2/c/\ln(f)*Pi^{(1/2)}*f^a*f^{(-1/4*b^2/c)/(-c*\ln(f))^{(1/2)}*erf(-(-c*\ln(f))^{(1/2)}*x+1/2*b*\ln(f)/(-c*\ln(f))^{(1/2)})+3/2*e*d^2/c/\ln(f)*f^{(c*x^2)}*f^{(b*x)}*f^{a+3/4*e*d^2*b/c*Pi^{(1/2)}*f^a*f^{(-1/4*b^2/c)/(-c*\ln(f))^{(1/2)}*erf(-(-c*\ln(f))^{(1/2)}*x+1/2*b*\ln(f)/(-c*\ln(f))^{(1/2)})}$

**Maxima [B]** time = 1.35878, size = 728, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^3,x, algorithm="maxima")

[Out]  $-3/4*(\sqrt{\pi}*(2*c*x + b)*b*(\operatorname{erf}(1/2*\sqrt{-(2*c*x + b)^2*\log(f)/c}) - 1)*\log(f)^2/(\sqrt{-(2*c*x + b)^2*\log(f)/c}*(c*\log(f))^{(3/2)}) - 2*c*f^{(1/4*(2*c*x + b)^2/c)*\log(f)/(c*\log(f))^{(3/2)}}*d^2*e*f^{(a - 1/4*b^2/c)}/\sqrt{c*\log(f)} + 3/8*(\sqrt{\pi}*(2*c*x + b)*b^2*(\operatorname{erf}(1/2*\sqrt{-(2*c*x + b)^2*\log(f)/c}) - 1)*\log(f)^3/(\sqrt{-(2*c*x + b)^2*\log(f)/c}*(c*\log(f))^{(5/2)}) - 4*(2*c*x + b)^3*\gamma(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^3/((-2*c*x + b)^2*\log(f)/c)^{(3/2)}*(c*\log(f))^{(5/2)}) - 4*b*c*f^{(1/4*(2*c*x + b)^2/c)*\log(f)^2/(c*\log(f))^{(5/2)}}*d*e^2*f^{(a - 1/4*b^2/c)}/\sqrt{c*\log(f)} - 1/16*(\sqrt{\pi}*(2*c*x + b)*b^3*(\operatorname{erf}(1/2*\sqrt{-(2*c*x + b)^2*\log(f)/c}) - 1)*\log(f)^4/(\sqrt{-(2*c*x + b)^2*\log(f)/c}*(c*\log(f))^{(7/2)}) - 12*(2*c*x + b)^3*b*\gamma(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^4/((-2*c*x + b)^2*\log(f)/c)^{(3/2)}*(c*\log(f))^{(7/2)}) - 6*b^2*c*f^{(1/4*(2*c*x + b)^2/c)*\log(f)^3/(c*\log(f))^{(7/2)}} + 8*c^2*\gamma(2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^2/(c*\log(f))^{(7/2)}}*e^3*f^{(a - 1/4*b^2/c)}/\sqrt{c*\log(f)} + 1/2*\sqrt{\pi}*d^3*f^a*\operatorname{erf}(\sqrt{-c*\log(f)})*x - 1/2*b*\log(f)/\sqrt{-c*\log(f)})/(\sqrt{-c*\log(f)})*f^{(1/4*b^2/c)}$

**Fricas [A]** time = 1.56033, size = 458, normalized size = 1.72

$$2(4c^2e^3 - (4c^3e^3x^2 + 12c^3d^2e - 6bc^2de^2 + b^2ce^3 + 2(6c^3de^2 - bc^2e^3)x)\log(f))f^{cx^2+bx+a} - \frac{\sqrt{\pi}(12c^2de^2 - 6bce^3 - (8c^3d^3 - 12b*c^2*d^2*e + 6b^2*c*d*e^2 - b^3*e^3)*\log(f))*\sqrt{-c*\log(f)}*\operatorname{erf}(1/2*(2*c*x + b)*\sqrt{-c*\log(f)})/c}{f^{(1/4*(b^2 - 4*a*c)/c)}}/(c^4*\log(f)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^3,x, algorithm="fricas")

[Out]  $-1/16*(2*(4*c^2*e^3 - (4*c^3*e^3*x^2 + 12*c^3*d^2*e - 6*b*c^2*d*e^2 + b^2*c*e^3 + 2*(6*c^3*d*e^2 - b*c^2*e^3)*x)*\log(f))*f^{(c*x^2 + b*x + a)} - \sqrt{\pi}*(12*c^2*d*e^2 - 6*b*c*e^3 - (8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*\log(f))*\sqrt{-c*\log(f)}*\operatorname{erf}(1/2*(2*c*x + b)*\sqrt{-c*\log(f)})/c)/f^{(1/4*(b^2 - 4*a*c)/c)}}/(c^4*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*(e\*x+d)\*\*3,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)\*(d + e\*x)\*\*3, x)

**Giac [A]** time = 1.36433, size = 541, normalized size = 2.03

$$\frac{\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{2\sqrt{-c\log(f)}} + \frac{3\left(\frac{\sqrt{\pi}bd^2 \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2\log(f)-4ac\log(f)-4c}{4c}\right)}}{\sqrt{-c\log(f)}}\right) + \frac{2d^2e^{(cx^2+bx+a)}}{4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*\sqrt{\pi}*d^3*\operatorname{erf}(-1/2*\sqrt{-c*\log(f)}*(2*x + b/c))*e^{(-1/4*(b^2*\log(f) \\ & - 4*a*c*\log(f))/c)/\sqrt{-c*\log(f)}} + 3/4*(\sqrt{\pi}*b*d^2*\operatorname{erf}(-1/2*\sqrt{-c* \\ \log(f)}*(2*x + b/c))*e^{(-1/4*(b^2*\log(f) - 4*a*c*\log(f) - 4*c)/c)/\sqrt{-c*1 \\ \log(f)}} + 2*d^2*e^{(c*x^2*\log(f) + b*x*\log(f) + a*\log(f) + 1)/\log(f))/c - 3/8 \\ *(\sqrt{\pi}*(b^2*d*\log(f) - 2*c*d)*\operatorname{erf}(-1/2*\sqrt{-c*\log(f)}*(2*x + b/c))*e^{( \\ -1/4*(b^2*\log(f) - 4*a*c*\log(f) - 8*c)/c)/(\sqrt{-c*\log(f)}*\log(f))} - 2*(c*d \\ *(2*x + b/c) - 2*b*d)*e^{(c*x^2*\log(f) + b*x*\log(f) + a*\log(f) + 2)/\log(f)})/ \\ c^2 + 1/16*(\sqrt{\pi}*(b^3*\log(f) - 6*b*c)*\operatorname{erf}(-1/2*\sqrt{-c*\log(f)}*(2*x + b \\ /c))*e^{(-1/4*(b^2*\log(f) - 4*a*c*\log(f) - 12*c)/c)/(\sqrt{-c*\log(f)}*\log(f))} \\ + 2*(c^2*(2*x + b/c)^2*\log(f) - 3*b*c*(2*x + b/c)*\log(f) + 3*b^2*\log(f) - \\ 4*c)*e^{(c*x^2*\log(f) + b*x*\log(f) + a*\log(f) + 3)/\log(f)^2}/c^3 \end{aligned}$$

### 3.445 $\int f^{a+bx+cx^2} (d+ex)^2 dx$

**Optimal.** Leaf size=189

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} (2cd-be)^2 \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(f)}} - \frac{\sqrt{\pi} e^2 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{e(d+ex)f^{a+bx+cx^2}}{2c \log(f)}$$

[Out]  $-(e^2 f^{a-b^2/(4c)}) \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[\frac{(b+2cx) \operatorname{Sqrt}[\operatorname{Log}[f]]}{2 \operatorname{Sqrt}[c]}] / (2 \operatorname{Sqrt}[c]) / (4c^{3/2} \operatorname{Log}[f]^{3/2}) + (e(2cd-be) f^{a+bx+cx^2}) / (4c^2 \operatorname{Log}[f]) + (e f^{a+bx+cx^2} (d+ex)) / (2c \operatorname{Log}[f]) + ((2cd-be)^2 f^{a-b^2/(4c)}) \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[\frac{(b+2cx) \operatorname{Sqrt}[\operatorname{Log}[f]]}{2 \operatorname{Sqrt}[c]}] / (8c^{5/2} \operatorname{Sqrt}[\operatorname{Log}[f]])$

**Rubi [A]** time = 0.108072, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2241, 2240, 2234, 2204}

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} (2cd-be)^2 \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(f)}} - \frac{\sqrt{\pi} e^2 f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2} \log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{e(d+ex)f^{a+bx+cx^2}}{2c \log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{a+bx+cx^2} (d+ex)^2, x]$

[Out]  $-(e^2 f^{a-b^2/(4c)}) \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[\frac{(b+2cx) \operatorname{Sqrt}[\operatorname{Log}[f]]}{2 \operatorname{Sqrt}[c]}] / (2 \operatorname{Sqrt}[c]) / (4c^{3/2} \operatorname{Log}[f]^{3/2}) + (e(2cd-be) f^{a+bx+cx^2}) / (4c^2 \operatorname{Log}[f]) + (e f^{a+bx+cx^2} (d+ex)) / (2c \operatorname{Log}[f]) + ((2cd-be)^2 f^{a-b^2/(4c)}) \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[\frac{(b+2cx) \operatorname{Sqrt}[\operatorname{Log}[f]]}{2 \operatorname{Sqrt}[c]}] / (8c^{5/2} \operatorname{Sqrt}[\operatorname{Log}[f]])$

#### Rule 2241

$\operatorname{Int}[(F_)^{\frac{b}{c}} ((a_) + (b_)(x_) + (c_)(x_)^2) ((d_) + (e_)(x_))^{(m_)} , x\_Symbol] \rightarrow \operatorname{Simp}[(e(d+ex)^{(m-1)} F^{a+bx+cx^2}) / (2c \operatorname{Log}[F]), x] + (-\operatorname{Dist}[(b e - 2c d) / (2c), \operatorname{Int}[(d+ex)^{(m-1)} F^{a+bx+cx^2}], x] - \operatorname{Dist}[(m-1) e^2 / (2c \operatorname{Log}[F]), \operatorname{Int}[(d+ex)^{(m-2)} F^{a+bx+cx^2}], x]) / ; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{NeQ}[b e - 2c d, 0] \&\& \operatorname{GtQ}[m, 1]$

#### Rule 2240

$\operatorname{Int}[(F_)^{\frac{b}{c}} ((a_) + (b_)(x_) + (c_)(x_)^2) ((d_) + (e_)(x_)) , x\_Symbol] \rightarrow \operatorname{Simp}[(e F^{a+bx+cx^2}) / (2c \operatorname{Log}[F]), x] - \operatorname{Dist}[(b e - 2c d) / (2c), \operatorname{Int}[F^{a+bx+cx^2}], x] / ; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{NeQ}[b e - 2c d, 0]$

#### Rule 2234

$\operatorname{Int}[(F_)^{\frac{b}{c}} ((a_) + (b_)(x_) + (c_)(x_)^2) , x\_Symbol] \rightarrow \operatorname{Dist}[F^{a-b^2/(4c)}, \operatorname{Int}[F^{(b+2cx)^2/(4c)}], x] / ; \operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{\frac{b}{c}} ((a_) + (b_)((c_) + (d_)(x_))^{2}) , x\_Symbol] \rightarrow \operatorname{Simp}[(F^a \operatorname{Sqrt}[\operatorname{Pi}] \operatorname{Erfi}[(c+dx) \operatorname{Rt}[b \operatorname{Log}[F], 2]]) / (2d \operatorname{Rt}[b \operatorname{Log}[F], 2]), x] / ; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int f^{a+bx+cx^2}(d+ex)^2 dx &= \frac{ef^{a+bx+cx^2}(d+ex)}{2c \log(f)} - \frac{(-2cd+be) \int f^{a+bx+cx^2}(d+ex) dx}{2c} - \frac{e^2 \int f^{a+bx+cx^2} dx}{2c \log(f)} \\
 &= \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)}{2c \log(f)} + \frac{(2cd-be)^2 \int f^{a+bx+cx^2} dx}{4c^2} - \frac{\left(e^2 f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c \log(f)} \\
 &= -\frac{e^2 f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2} \log^3(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)}{2c \log(f)} + \frac{\left((2cd-be)^2 f^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{4c^2 \log(f)} \\
 &= -\frac{e^2 f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2} \log^3(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2 \log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)}{2c \log(f)} + \frac{(2cd-be)^2 f^{a-\frac{b^2}{4c}} \int f^{\frac{(b+2cx)^2}{4c}} dx}{8c^2 \log(f)}
 \end{aligned}$$

**Mathematica [A]** time = 0.17507, size = 123, normalized size = 0.65

$$\frac{f^{a-\frac{b^2}{4c}} \left( \sqrt{\pi} (\log(f)(be-2cd)^2 - 2ce^2) \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right) + 2\sqrt{ce}\sqrt{\log(f)} f^{\frac{(b+2cx)^2}{4c}} (-be+4cd+2cex) \right)}{8c^{5/2} \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*(d + e\*x)^2,x]

[Out] (f^(a - b^2/(4\*c))\*(2\*Sqrt[c]\*e\*f^((b + 2\*c\*x)^2/(4\*c))\*(4\*c\*d - b\*e + 2\*c\*e\*x)\*Sqrt[Log[f]] + Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])])\*(-2\*c\*e^2 + (-2\*c\*d + b\*e)^2\*Log[f]))/(8\*c^(5/2)\*Log[f]^(3/2))

**Maple [A]** time = 0.043, size = 307, normalized size = 1.6

$$-\frac{d^2 \sqrt{\pi} f^a}{2} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}} + \frac{e^2 x f^{cx^2} f^{bx} f^a}{2c \ln(f)} - \frac{be^2 f^{cx^2} f^{bx} f^a}{4 \ln(f) c^2} - \frac{b^2 e^2 \sqrt{\pi} f^a}{8c^2} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*(e\*x+d)^2,x)

[Out] -1/2\*d^2\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))+1/2\*e^2/c/ln(f)\*x\*f^(c\*x^2)\*f^(b\*x)\*f^a-1/4\*e^2/c^2\*b/ln(f)\*f^(c\*x^2)\*f^(b\*x)\*f^a-1/8\*e^2/c^2\*b^2\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))+1/4\*e^2/c/ln(f)\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))+d\*e/c/ln(f)\*f^(c\*x^2)\*f^(b\*x)\*f^a+1/2\*d\*e\*b/c\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))



**Maxima [B]** time = 1.24732, size = 448, normalized size = 2.37

$$\frac{\left( \frac{\sqrt{\pi}(2cx+b)b \left( \operatorname{erf}\left( \frac{1}{2} \sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} \right) - 1 \right) \log(f)^2}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c}} \log(f)}{(c \log(f))^{\frac{3}{2}}} \right) de f^{a-\frac{b^2}{4c}}}{2\sqrt{c \log(f)}} + \frac{\left( \frac{\sqrt{\pi}(2cx+b)b^2 \left( \operatorname{erf}\left( \frac{1}{2} \sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} \right) - 1 \right) \log(f)^3}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{5}{2}}} - \frac{4cf^{\frac{(2cx+b)^2}{4c}} \log(f)^2}{(c \log(f))^{\frac{5}{2}}} \right) de f^{a-\frac{b^2}{4c}}}{2\sqrt{c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-1/2*(\sqrt{\pi}*(2*c*x + b)*b*(\operatorname{erf}(1/2*\sqrt{-(2*c*x + b)^2*\log(f)/c}) - 1)*\log(f)^2/(\sqrt{-(2*c*x + b)^2*\log(f)/c}*(c*\log(f))^{3/2}) - 2*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)/(c*\log(f))^{3/2})*d*e*f^{(a - 1/4*b^2/c)/\sqrt{c*\log(f)}} + 1/8*(\sqrt{\pi}*(2*c*x + b)*b^2*(\operatorname{erf}(1/2*\sqrt{-(2*c*x + b)^2*\log(f)/c}) - 1)*\log(f)^3/(\sqrt{-(2*c*x + b)^2*\log(f)/c}*(c*\log(f))^{5/2}) - 4*(2*c*x + b)^3*\gamma(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^3/((-2*c*x + b)^2*\log(f)/c)^{3/2}*(c*\log(f))^{5/2}) - 4*b*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)^2/(c*\log(f))^{5/2})*e^2*f^{(a - 1/4*b^2/c)/\sqrt{c*\log(f)}} + 1/2*\sqrt{\pi}*d^2*f^a*\operatorname{erf}(\sqrt{-c*\log(f)}*x - 1/2*b*\log(f)/\sqrt{-c*\log(f)})/(\sqrt{-c*\log(f)}*f^{1/4*b^2/c})$

**Fricas [A]** time = 1.59018, size = 311, normalized size = 1.65

$$\frac{2(2c^2e^2x + 4c^2de - bce^2)f^{cx^2+bx+a} \log(f) + \frac{\sqrt{\pi}(2c^2e^2 - (4c^2d^2 - 4bcde + b^2e^2)\log(f))\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{8c^3 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^2,x, algorithm="fricas")

[Out]  $1/8*(2*(2*c^2*e^2*x + 4*c^2*d*e - b*c*e^2)*f^{(c*x^2 + b*x + a)*\log(f)} + \sqrt{\pi}*(2*c*e^2 - (4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*\log(f))*\sqrt{-c*\log(f)}*\operatorname{erf}(1/2*(2*c*x + b)*\sqrt{-c*\log(f)})/c)/f^{1/4*(b^2 - 4*a*c)/c})/(c^3*\log(f)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*(e\*x+d)\*\*2,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)\*(d + e\*x)\*\*2, x)

**Giac [A]** time = 1.2856, size = 340, normalized size = 1.8

$$\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{2\sqrt{-c\log(f)}} + \frac{\frac{\sqrt{\pi}bd \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2\log(f)-4ac\log(f)-4c}{4c}\right)}}{\sqrt{-c\log(f)}}}{2c} + \frac{2de^{(cx^2\log(f)+1)}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d)^2,x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*d^2\*erf(-1/2\*sqrt(-c\*log(f))\*(2\*x + b/c))\*e^(-1/4\*(b^2\*log(f) - 4\*a\*c\*log(f))/c)/sqrt(-c\*log(f)) + 1/2\*(sqrt(pi)\*b\*d\*erf(-1/2\*sqrt(-c\*log(f))\*(2\*x + b/c))\*e^(-1/4\*(b^2\*log(f) - 4\*a\*c\*log(f) - 4\*c)/c)/sqrt(-c\*log(f)) + 2\*d\*e^(c\*x^2\*log(f) + b\*x\*log(f) + a\*log(f) + 1)/log(f))/c - 1/8\*(sqrt(pi)\*(b^2\*log(f) - 2\*c)\*erf(-1/2\*sqrt(-c\*log(f))\*(2\*x + b/c))\*e^(-1/4\*(b^2\*log(f) - 4\*a\*c\*log(f) - 8\*c)/c)/(sqrt(-c\*log(f))\*log(f)) - 2\*(c\*(2\*x + b/c) - 2\*b)\*e^(c\*x^2\*log(f) + b\*x\*log(f) + a\*log(f) + 2)/log(f))/c^2

### 3.446 $\int f^{a+bx+cx^2} (d + ex) dx$

**Optimal.** Leaf size=90

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} (2cd - be) \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}} + \frac{e f^{a+bx+cx^2}}{2c \log(f)}$$

[Out] (e\*f^(a + b\*x + c\*x^2))/(2\*c\*Log[f]) + ((2\*c\*d - b\*e)\*f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])])/(4\*c^(3/2)\*Sqrt[Log[f]])

**Rubi [A]** time = 0.0411925, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2240, 2234, 2204}

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} (2cd - be) \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}} + \frac{e f^{a+bx+cx^2}}{2c \log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*(d + e\*x), x]

[Out] (e\*f^(a + b\*x + c\*x^2))/(2\*c\*Log[f]) + ((2\*c\*d - b\*e)\*f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])])/(4\*c^(3/2)\*Sqrt[Log[f]])

#### Rule 2240

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[(b\*e - 2\*c\*d)/(2\*c), Int[F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b\*e - 2\*c\*d, 0]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2}(d+ex) dx &= \frac{ef^{a+bx+cx^2}}{2c \log(f)} - \frac{(-2cd+be) \int f^{a+bx+cx^2} dx}{2c} \\ &= \frac{ef^{a+bx+cx^2}}{2c \log(f)} + \frac{\left( (2cd-be)f^{a-\frac{b^2}{4c}} \right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c} \\ &= \frac{ef^{a+bx+cx^2}}{2c \log(f)} + \frac{(2cd-be)f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi} \left( \frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}} \right)}{4c^{3/2}\sqrt{\log(f)}} \end{aligned}$$

**Mathematica [A]** time = 0.1029, size = 96, normalized size = 1.07

$$\frac{f^{a-\frac{b^2}{4c}} \left( \sqrt{\pi} \sqrt{\log(f)} (2cd-be) \operatorname{Erfi} \left( \frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}} \right) + 2\sqrt{c} e f^{\frac{(b+2cx)^2}{4c}} \right)}{4c^{3/2} \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*(d + e\*x),x]

[Out] (f^(a - b^2/(4\*c))\*(2\*Sqrt[c]\*e\*f^((b + 2\*c\*x)^2/(4\*c)) + (2\*c\*d - b\*e)\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])]\*Sqrt[Log[f]]))/(4\*c^(3/2)\*Log[f])

**Maple [A]** time = 0.033, size = 131, normalized size = 1.5

$$-\frac{d\sqrt{\pi}f^a}{2}f^{-\frac{b^2}{4c}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}x + \frac{b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}} + \frac{ef^{cx^2}f^{bx}f^a}{2c\ln(f)} + \frac{be\sqrt{\pi}f^a}{4c}f^{-\frac{b^2}{4c}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}x + \frac{b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*(e\*x+d),x)

[Out] -1/2\*d\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x + 1/2\*b\*ln(f)/(-c\*ln(f))^(1/2)) + 1/2\*e/c/ln(f)\*f^(c\*x^2)\*f^(b\*x)\*f^a + 1/4\*e\*b/c\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x + 1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [B]** time = 1.14236, size = 216, normalized size = 2.4

$$\frac{\left( \frac{\sqrt{\pi}(2cx+b)b \left( \operatorname{erf} \left( \frac{1}{2} \sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} \right) - 1 \right) \log(f)^2}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c}} \log(f)}{(c \log(f))^{\frac{3}{2}}} \right) e f^{a-\frac{b^2}{4c}}}{4\sqrt{c \log(f)}} + \frac{\sqrt{\pi} d f^a \operatorname{erf} \left( \sqrt{-c \log(f)} x - \frac{b \log(f)}{2\sqrt{-c \log(f)}} \right)}{2\sqrt{-c \log(f)} f^{\frac{b^2}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(e\*x+d),x, algorithm="maxima")

[Out]  $-1/4*(\text{sqrt}(\pi)*(2*c*x + b)*b*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)) - 1)*\log(f)^2/(\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)*(c*\log(f))^{3/2}) - 2*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)/(c*\log(f))^{3/2})*e*f^{(a - 1/4*b^2/c)/\text{sqrt}(c*\log(f))} + 1/2*\text{sqrt}(\pi)*d*f^a*\text{erf}(\text{sqrt}(-c*\log(f))*x - 1/2*b*\log(f)/\text{sqrt}(-c*\log(f)))/(\text{sqrt}(-c*\log(f))*f^{1/4*b^2/c})$

---

**Fricas [A]** time = 1.56755, size = 203, normalized size = 2.26

$$\frac{2ce^{cx^2+bx+a} - \frac{\sqrt{\pi}(2cd-be)\sqrt{-c\log(f)}\text{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{4c^2\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)*(e*x+d),x, algorithm="fricas")`

[Out]  $1/4*(2*c*e*f^{(c*x^2 + b*x + a)} - \text{sqrt}(\pi)*(2*c*d - b*e)*\text{sqrt}(-c*\log(f))*\text{erf}(1/2*(2*c*x + b)*\text{sqrt}(-c*\log(f))/c)/f^{1/4*(b^2 - 4*a*c)/c})/(c^2*\log(f))$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x+a)*(e*x+d),x)`

[Out] `Integral(f**(a + b*x + c*x**2)*(d + e*x), x)`

---

**Giac [A]** time = 1.25363, size = 184, normalized size = 2.04

$$\frac{\sqrt{\pi}d\text{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{2\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}b\text{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x + \frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)-4c}{4c}\right)}}{\sqrt{-c\log(f)}} + \frac{2e^{(cx^2\log(f))}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)*(e*x+d),x, algorithm="giac")`

[Out]  $-1/2*\text{sqrt}(\pi)*d*\text{erf}(-1/2*\text{sqrt}(-c*\log(f))*(2*x + b/c))*e^{(-1/4*(b^2*\log(f) - 4*a*c*\log(f))/c)/\text{sqrt}(-c*\log(f))} + 1/4*(\text{sqrt}(\pi)*b*\text{erf}(-1/2*\text{sqrt}(-c*\log(f))*(2*x + b/c))*e^{(-1/4*(b^2*\log(f) - 4*a*c*\log(f) - 4*c)/c)/\text{sqrt}(-c*\log(f))} + 2*e^{(c*x^2*\log(f) + b*x*\log(f) + a*\log(f) + 1)/\log(f)})/c$

$$3.447 \quad \int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)$$

[Out] Unintegrable[f^(a + b\*x + c\*x^2)/(d + e\*x), x]

**Rubi [A]** time = 0.0305396, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

Verification is Not applicable to the result.

[In] Int[f^(a + b\*x + c\*x^2)/(d + e\*x), x]

[Out] Defer[Int][f^(a + b\*x + c\*x^2)/(d + e\*x), x]

Rubi steps

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx = \int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

**Mathematica [A]** time = 0.209346, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x), x]

[Out] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x), x]

**Maple [A]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/(e\*x+d), x)

[Out] int(f^(c\*x^2+b\*x+a)/(e\*x+d), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d),x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(e\*x + d), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{cx^2+bx+a}}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d),x, algorithm="fricas")

[Out] integral(f^(c\*x^2 + b\*x + a)/(e\*x + d), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)/(e\*x+d),x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)/(d + e\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d),x, algorithm="giac")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(e\*x + d), x)

$$3.448 \quad \int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=119

$$\frac{\log(f)(2cd - be)\text{Unintegrable}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)}{e^2} + \frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{e^2} - \frac{f^{a+bx+cx^2}}{e(d+ex)}$$

[Out]  $-(f^{a+bx+cx^2}/(e*(d+e*x))) + (\text{Sqrt}[c]*f^{a-b^2/(4*c)}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(b+2*c*x)*\text{Sqrt}[\text{Log}[f]]]/(2*\text{Sqrt}[c]))*\text{Sqrt}[\text{Log}[f]]/e^2 - ((2*c*d - b*e)*\text{Log}[f]*\text{Unintegrable}[f^{a+bx+cx^2}/(d+e*x), x])/e^2$

**Rubi [A]** time = 0.103133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$$

Verification is Not applicable to the result.

[In] Int[f^(a + b\*x + c\*x^2)/(d + e\*x)^2, x]

[Out]  $-(f^{a+bx+cx^2}/(e*(d+e*x))) + (\text{Sqrt}[c]*f^{a-b^2/(4*c)}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(b+2*c*x)*\text{Sqrt}[\text{Log}[f]]]/(2*\text{Sqrt}[c]))*\text{Sqrt}[\text{Log}[f]]/e^2 - ((2*c*d - b*e)*\text{Log}[f]*\text{Defer}[\text{Int}[f^{a+bx+cx^2}/(d+e*x), x])/e^2$

Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx &= -\frac{f^{a+bx+cx^2}}{e(d+ex)} + \frac{(2c \log(f)) \int f^{a+bx+cx^2} dx}{e^2} - \frac{((2cd - be) \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} \\ &= -\frac{f^{a+bx+cx^2}}{e(d+ex)} - \frac{((2cd - be) \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} + \frac{\left(2cf^{a-\frac{b^2}{4c}} \log(f)\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{e^2} \\ &= -\frac{f^{a+bx+cx^2}}{e(d+ex)} + \frac{\sqrt{c}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}}{e^2} - \frac{((2cd - be) \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} \end{aligned}$$

**Mathematica [A]** time = 0.583072, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x)^2, x]

[Out] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x)^2, x]



**Maple [A]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/(e\*x+d)^2,x)

[Out] int(f^(c\*x^2+b\*x+a)/(e\*x+d)^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d)^2,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(e\*x + d)^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{cx^2+bx+a}}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d)^2,x, algorithm="fricas")

[Out] integral(f^(c\*x^2 + b\*x + a)/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*2,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)/(d + e\*x)\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)/(e*x + d)^2, x)
```

**3.449**  $\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx$

**Optimal.** Leaf size=205

$$\frac{\log^2(f)(2cd - be)^2 \text{Unintegrable}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)}{2e^4} + \frac{c \log(f) \text{Unintegrable}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)}{e^2} - \frac{\sqrt{\pi} \sqrt{c} \log^{\frac{3}{2}}(f) f^{a-\frac{b^2}{4c}} (2cd - be)}{2e^4}$$

```
[Out] -f^(a + b*x + c*x^2)/(2*e*(d + e*x)^2) + ((2*c*d - b*e)*f^(a + b*x + c*x^2)
*Log[f]/(2*e^3*(d + e*x)) - (Sqrt[c]*(2*c*d - b*e)*f^(a - b^2/(4*c))*Sqrt[
Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Log[f]^(3/2))/(2*e^4) + (c
*Log[f]*Unintegrable[f^(a + b*x + c*x^2)/(d + e*x), x])/e^2 + ((2*c*d - b*e)
)^2*Log[f]^2*Unintegrable[f^(a + b*x + c*x^2)/(d + e*x), x]/(2*e^4)
```

**Rubi [A]** time = 0.208053, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[f^(a + b*x + c*x^2)/(d + e*x)^3, x]
```

```
[Out] -f^(a + b*x + c*x^2)/(2*e*(d + e*x)^2) + ((2*c*d - b*e)*f^(a + b*x + c*x^2)
*Log[f]/(2*e^3*(d + e*x)) - (Sqrt[c]*(2*c*d - b*e)*f^(a - b^2/(4*c))*Sqrt[
Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Log[f]^(3/2))/(2*e^4) + (c
*Log[f]*Defer[Int][f^(a + b*x + c*x^2)/(d + e*x), x])/e^2 + ((2*c*d - b*e)
)^2*Log[f]^2*Defer[Int][f^(a + b*x + c*x^2)/(d + e*x), x]/(2*e^4)
```

Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx &= -\frac{f^{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(c \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} - \frac{((2cd - be) \log(f)) \int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx}{2e^2} \\ &= -\frac{f^{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(2cd - be) f^{a+bx+cx^2} \log(f)}{2e^3(d+ex)} + \frac{(c \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} - \frac{(c(2cd - be) \log^2(f)) \int f^{a+bx+cx^2}}{e^4} \\ &= -\frac{f^{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(2cd - be) f^{a+bx+cx^2} \log(f)}{2e^3(d+ex)} + \frac{(c \log(f)) \int \frac{f^{a+bx+cx^2}}{d+ex} dx}{e^2} + \frac{((2cd - be)^2 \log^2(f)) \int \frac{f^{a+bx+cx^2}}{d+ex}}{2e^4} \\ &= -\frac{f^{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(2cd - be) f^{a+bx+cx^2} \log(f)}{2e^3(d+ex)} - \frac{\sqrt{c}(2cd - be) f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) \log^{\frac{3}{2}}(f)}{2e^4} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.83303, size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x)^3,x]

[Out] Integrate[f^(a + b\*x + c\*x^2)/(d + e\*x)^3, x]

**Maple [A]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/(e\*x+d)^3,x)

[Out] int(f^(c\*x^2+b\*x+a)/(e\*x+d)^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d)^3,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(e\*x + d)^3, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^{cx^2+bx+a}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d)^3,x, algorithm="fricas")

[Out] integral(f^(c\*x^2 + b\*x + a)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*3,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)/(d + e\*x)\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(e\*x+d)^3,x, algorithm="giac")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(e\*x + d)^3, x)

$$3.450 \quad \int f^{a+bx+cx^2} (b + 2cx)^3 dx$$

**Optimal.** Leaf size=45

$$\frac{(b + 2cx)^2 f^{a+bx+cx^2}}{\log(f)} - \frac{4c f^{a+bx+cx^2}}{\log^2(f)}$$

[Out]  $(-4*c*f^{(a + b*x + c*x^2)})/Log[f]^2 + (f^{(a + b*x + c*x^2)}*(b + 2*c*x)^2)/Log[f]$

**Rubi [A]** time = 0.0556069, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2237, 2236}

$$\frac{(b + 2cx)^2 f^{a+bx+cx^2}}{\log(f)} - \frac{4c f^{a+bx+cx^2}}{\log^2(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)^3,x]

[Out]  $(-4*c*f^{(a + b*x + c*x^2)})/Log[f]^2 + (f^{(a + b*x + c*x^2)}*(b + 2*c*x)^2)/Log[f]$

#### Rule 2237

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)^m), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2236

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx+cx^2} (b + 2cx)^3 dx &= \frac{f^{a+bx+cx^2} (b + 2cx)^2}{\log(f)} - \frac{(4c) \int f^{a+bx+cx^2} (b + 2cx) dx}{\log(f)} \\ &= -\frac{4c f^{a+bx+cx^2}}{\log^2(f)} + \frac{f^{a+bx+cx^2} (b + 2cx)^2}{\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.124839, size = 31, normalized size = 0.69

$$\frac{f^{a+x(b+cx)} (\log(f)(b + 2cx)^2 - 4c)}{\log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)^3,x]

[Out] (f^(a + x\*(b + c\*x))\*(-4\*c + (b + 2\*c\*x)^2\*Log[f]))/Log[f]^2

**Maple [A]** time = 0.004, size = 45, normalized size = 1.

$$\frac{(4 \ln(f) c^2 x^2 + 4 b c x \ln(f) + \ln(f) b^2 - 4 c) f^{c x^2 + b x + a}}{(\ln(f))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^3,x)

[Out] (4\*ln(f)\*c^2\*x^2+4\*b\*c\*x\*ln(f)+ln(f)\*b^2-4\*c)\*f^(c\*x^2+b\*x+a)/ln(f)^2

**Maxima [C]** time = 1.32112, size = 728, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -3/2*(\text{sqrt}(\pi)*(2*c*x + b)*b*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)) - 1)*\log(f)^2/(\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)*(c*\log(f))^{3/2}) - 2*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)/(c*\log(f))^{3/2})*b^2*c*f^{a - 1/4*b^2/c}/\text{sqrt}(c*\log(f)) \\ & + 3/2*(\text{sqrt}(\pi)*(2*c*x + b)*b^2*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)) - 1)*\log(f)^3/(\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)*(c*\log(f))^{5/2}) - 4*(2*c*x + b)^3*\text{gamma}(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^3/((-2*c*x + b)^2*\log(f)/c)^{3/2}*(c*\log(f))^{5/2}) - 4*b*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)^2/(c*\log(f))^{5/2})*b*c^2*f^{a - 1/4*b^2/c}/\text{sqrt}(c*\log(f)) - 1/2*(\text{sqrt}(\pi)*(2*c*x + b)*b^3*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)) - 1)*\log(f)^4/(\text{sqrt}(-(2*c*x + b)^2*\log(f)/c)*(c*\log(f))^{7/2}) - 12*(2*c*x + b)^3*b*\text{gamma}(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^4/((-2*c*x + b)^2*\log(f)/c)^{3/2}*(c*\log(f))^{7/2}) - 6*b^2*c*f^{1/4*(2*c*x + b)^2/c}*\log(f)^3/(c*\log(f))^{7/2} + 8*c^2*\text{gamma}(2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^2/(c*\log(f))^{7/2})*c^3*f^{a - 1/4*b^2/c}/\text{sqrt}(c*\log(f)) + 1/2*\text{sqrt}(\pi)*b^3*f^a*\text{erf}(\text{sqrt}(-c*\log(f))*x - 1/2*b*\log(f)/\text{sqrt}(-c*\log(f)))/(\text{sqrt}(-c*\log(f))*f^{1/4*b^2/c}) \end{aligned}$$

**Fricas [A]** time = 1.51865, size = 99, normalized size = 2.2

$$\frac{((4c^2x^2 + 4bcx + b^2)\log(f) - 4c)f^{cx^2+bx+a}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^3,x, algorithm="fricas")

[Out] ((4\*c^2\*x^2 + 4\*b\*c\*x + b^2)\*log(f) - 4\*c)\*f^(c\*x^2 + b\*x + a)/log(f)^2

---

**Sympy [A]** time = 0.152633, size = 85, normalized size = 1.89

$$\begin{cases} \frac{f^{a+bx+cx^2}(b^2 \log(f) + 4bcx \log(f) + 4c^2x^2 \log(f) - 4c)}{\log(f)^2} & \text{for } \log(f)^2 \neq 0 \\ b^3x + 3b^2cx^2 + 4bc^2x^3 + 2c^3x^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*\*3,x)

[Out] Piecewise((f\*\*(a + b\*x + c\*x\*\*2)\*(b\*\*2\*log(f) + 4\*b\*c\*x\*log(f) + 4\*c\*\*2\*x\*\*2\*log(f) - 4\*c)/log(f)\*\*2, Ne(log(f)\*\*2, 0)), (b\*\*3\*x + 3\*b\*\*2\*c\*x\*\*2 + 4\*b\*c\*\*2\*x\*\*3 + 2\*c\*\*3\*x\*\*4, True))

---

**Giac [A]** time = 1.29599, size = 59, normalized size = 1.31

$$\frac{\left(c^2\left(2x + \frac{b}{c}\right)^2 \log(f) - 4c\right)e^{(cx^2 \log(f) + bx \log(f) + a \log(f))}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^3,x, algorithm="giac")

[Out] (c^2\*(2\*x + b/c)^2\*log(f) - 4\*c)\*e^(c\*x^2\*log(f) + b\*x\*log(f) + a\*log(f))/log(f)^2



### 3.451 $\int f^{a+bx+cx^2} (b + 2cx)^2 dx$

**Optimal.** Leaf size=78

$$\frac{(b + 2cx)f^{a+bx+cx^2}}{\log(f)} - \frac{\sqrt{\pi}\sqrt{c}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

[Out] -((Sqrt[c]\*f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])))/Log[f]^(3/2)) + (f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/Log[f]

**Rubi [A]** time = 0.0555783, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2237, 2234, 2204}

$$\frac{(b + 2cx)f^{a+bx+cx^2}}{\log(f)} - \frac{\sqrt{\pi}\sqrt{c}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)^2,x]

[Out] -((Sqrt[c]\*f^(a - b^2/(4\*c))\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])))/Log[f]^(3/2)) + (f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/Log[f]

#### Rule 2237

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}(b+2cx)^2 dx &= \frac{f^{a+bx+cx^2}(b+2cx)}{\log(f)} - \frac{(2c) \int f^{a+bx+cx^2} dx}{\log(f)} \\
&= \frac{f^{a+bx+cx^2}(b+2cx)}{\log(f)} - \frac{\left(2cf^{a-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{\log(f)} \\
&= -\frac{\sqrt{c}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)} + \frac{f^{a+bx+cx^2}(b+2cx)}{\log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.104872, size = 86, normalized size = 1.1

$$\frac{f^{a-\frac{b^2}{4c}}\left(\sqrt{\log(f)}(b+2cx)f^{\frac{(b+2cx)^2}{4c}} - \sqrt{\pi}\sqrt{c}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)\right)}{\log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)^2,x]

[Out] (f^(a - b^2/(4\*c))\*(-(Sqrt[c]\*Sqrt[Pi]\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])))) + f^((b + 2\*c\*x)^2/(4\*c))\*(b + 2\*c\*x)\*Sqrt[Log[f]])/Log[f]^(3/2)

**Maple [A]** time = 0.039, size = 99, normalized size = 1.3

$$2 \frac{cx f^{cx^2} f^{bx} f^a}{\ln(f)} + \frac{b f^{cx^2} f^{bx} f^a}{\ln(f)} + \frac{c \sqrt{\pi} f^a}{\ln(f)} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^2,x)

[Out] 2\*c/ln(f)\*x\*f^(c\*x^2)\*f^(b\*x)\*f^a+b/ln(f)\*f^(c\*x^2)\*f^(b\*x)\*f^a+c/ln(f)\*Pi^(1/2)\*f^a\*f^(-1/4\*b^2/c)/(-c\*ln(f))^(1/2)\*erf(-(-c\*ln(f))^(1/2)\*x+1/2\*b\*ln(f)/(-c\*ln(f))^(1/2))

**Maxima [B]** time = 1.25482, size = 448, normalized size = 5.74

$$\frac{\left(\frac{\sqrt{\pi}(2cx+b)b\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{3}{2}}}-\frac{2cf^{\frac{(2cx+b)^2}{4c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)bcf^{a-\frac{b^2}{4c}}}{\sqrt{c\log(f)}} + \frac{\left(\frac{\sqrt{\pi}(2cx+b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^3}{\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{5}{2}}}-\frac{4(2cx+b)f^{\frac{(2cx+b)^2}{4c}}\log(f)}{(c\log(f))^{\frac{5}{2}}}\right)}{2\sqrt{c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)\*(2\*c\*x+b)^2,x, algorithm="maxima")

```
[Out] -(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)
)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*c*x +
b)^2/c)*log(f)/(c*log(f))^(3/2))*b*c*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2
*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log
(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*x + b)^3*ga
mma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-2*c*x + b)^2*log(f)/c)^(
3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^2/(c*log(f))^(
5/2))*c^2*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2*sqrt(pi)*b^2*f^a*erf(sqrt
(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c
))
```

**Fricas [A]** time = 1.53933, size = 190, normalized size = 2.44

$$\frac{(2cx + b)f^{cx^2+bx+a} \log(f) + \frac{\sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c \log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x, algorithm="fricas")
```

```
[Out] ((2*c*x + b)*f^(c*x^2 + b*x + a)*log(f) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*
(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/log(f)^2
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} (b + 2cx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*(2*c*x+b)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*(b + 2*c*x)**2, x)
```

**Giac [A]** time = 1.33836, size = 119, normalized size = 1.53

$$\frac{c\left(2x + \frac{b}{c}\right)e^{(cx^2 \log(f) + bx \log(f) + a \log(f))}}{\log(f)} + \frac{\sqrt{\pi}c \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c \log(f)}\left(2x + \frac{b}{c}\right)\right)e^{-\frac{b^2 \log(f) - 4ac \log(f)}{4c}}}{\sqrt{-c \log(f)} \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x, algorithm="giac")
```

```
[Out] c*(2*x + b/c)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log(f) + sqrt(pi)*c*
erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c
)/(sqrt(-c*log(f))*log(f))
```

$$3.452 \quad \int f^{a+bx+cx^2} (b + 2cx) dx$$

**Optimal.** Leaf size=17

$$\frac{f^{a+bx+cx^2}}{\log(f)}$$

[Out] f^(a + b\*x + c\*x^2)/Log[f]

**Rubi [A]** time = 0.0174103, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2236}

$$\frac{f^{a+bx+cx^2}}{\log(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x),x]

[Out] f^(a + b\*x + c\*x^2)/Log[f]

**Rule 2236**

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] := Simp[(e*F^(a + b*x + c*x^2))/(2*c*Log[F]), x] /; FreeQ[{F, a, b, c, d,
e}, x] && EqQ[b*e - 2*c*d, 0]
```

**Rubi steps**

$$\int f^{a+bx+cx^2} (b + 2cx) dx = \frac{f^{a+bx+cx^2}}{\log(f)}$$

**Mathematica [A]** time = 0.0384347, size = 17, normalized size = 1.

$$\frac{f^{a+bx+cx^2}}{\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)\*(b + 2\*c\*x),x]

[Out] f^(a + b\*x + c\*x^2)/Log[f]

**Maple [A]** time = 0.001, size = 18, normalized size = 1.1

$$\frac{f^{cx^2+bx+a}}{\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x+a)*(2*c*x+b),x)`

[Out]  $f^{(c*x^2+b*x+a)}/\ln(f)$

**Maxima [A]** time = 0.96956, size = 23, normalized size = 1.35

$$\frac{f^{cx^2+bx+a}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="maxima")`

[Out]  $f^{(c*x^2 + b*x + a)}/\log(f)$

**Fricas [A]** time = 1.55734, size = 38, normalized size = 2.24

$$\frac{f^{cx^2+bx+a}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="fricas")`

[Out]  $f^{(c*x^2 + b*x + a)}/\log(f)$

**Sympy [A]** time = 0.116197, size = 24, normalized size = 1.41

$$\begin{cases} \frac{f^{a+bx+cx^2}}{\log(f)} & \text{for } \log(f) \neq 0 \\ bx + cx^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x+a)*(2*c*x+b),x)`

[Out] `Piecewise((f**(a + b*x + c*x**2)/log(f), Ne(log(f), 0)), (b*x + c*x**2, True))`

**Giac [A]** time = 1.2334, size = 23, normalized size = 1.35

$$\frac{f^{cx^2+bx+a}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="giac")`

[Out]  $f^{(c*x^2 + b*x + a)}/\log(f)$

$$3.453 \quad \int \frac{f^{a+bx+cx^2}}{b+2cx} dx$$

**Optimal.** Leaf size=39

$$\frac{f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

[Out] (f^(a - b^2/(4\*c))\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c))]/(4\*c)

**Rubi [A]** time = 0.0371917, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2238}

$$\frac{f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x + c\*x^2)/(b + 2\*c\*x), x]

[Out] (f^(a - b^2/(4\*c))\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c))]/(4\*c)

#### Rule 2238

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(1\*F^(a - b^2/(4\*c))\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[F])/(4\*c))]/(2\*e), x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0]

#### Rubi steps

$$\int \frac{f^{a+bx+cx^2}}{b+2cx} dx = \frac{f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

**Mathematica [A]** time = 0.0545218, size = 39, normalized size = 1.

$$\frac{f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)/(b + 2\*c\*x), x]

[Out] (f^(a - b^2/(4\*c))\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c))]/(4\*c)

**Maple [A]** time = 0.023, size = 40, normalized size = 1.

$$-\frac{1}{4c} f^{\frac{4ac-b^2}{4c}} \operatorname{Ei}\left(1, -\frac{(2cx+b)^2 \ln(f)}{4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x+a)/(2*c*x+b),x)`

[Out]  $-1/4/c*f^{(1/4*(4*a*c-b^2)/c)}*Ei(1,-1/4*(2*c*x+b)^2*\ln(f)/c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{2cx+b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="maxima")`

[Out] `integrate(f^(c*x^2 + b*x + a)/(2*c*x + b), x)`

**Fricas [A]** time = 1.50717, size = 105, normalized size = 2.69

$$\frac{Ei\left(\frac{(4c^2x^2+4bcx+b^2)\log(f)}{4c}\right)}{4cf^{\frac{b^2-4ac}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="fricas")`

[Out]  $1/4*Ei(1/4*(4*c^2*x^2 + 4*b*c*x + b^2)*\log(f)/c)/(c*f^{(1/4*(b^2 - 4*a*c)/c)})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{b+2cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x+a)/(2*c*x+b),x)`

[Out] `Integral(f**(a + b*x + c*x**2)/(b + 2*c*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{2cx+b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="giac")`

[Out] `integrate(f^(c*x^2 + b*x + a)/(2*c*x + b), x)`

$$3.454 \quad \int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} dx$$

**Optimal.** Leaf size=84

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{f^{a+bx+cx^2}}{2c(b+2cx)}$$

[Out]  $-f^{(a + b*x + c*x^2)/(2*c*(b + 2*c*x))} + (f^{(a - b^2/(4*c))}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(b + 2*c*x)*\operatorname{Sqrt}[\operatorname{Log}[f]])/(2*\operatorname{Sqrt}[c])]*\operatorname{Sqrt}[\operatorname{Log}[f]])/(4*c^{(3/2)})$

**Rubi [A]** time = 0.0551733, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2239, 2234, 2204}

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{f^{a+bx+cx^2}}{2c(b+2cx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(a + b*x + c*x^2)/(b + 2*c*x)^2}, x]$

[Out]  $-f^{(a + b*x + c*x^2)/(2*c*(b + 2*c*x))} + (f^{(a - b^2/(4*c))}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(b + 2*c*x)*\operatorname{Sqrt}[\operatorname{Log}[f]])/(2*\operatorname{Sqrt}[c])]*\operatorname{Sqrt}[\operatorname{Log}[f]])/(4*c^{(3/2)})$

#### Rule 2239

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*((d_.) + (e_.)*(x_.))^m}, x\_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{m+1}*F^{(a + b*x + c*x^2)/(e*(m+1))}, x] - \operatorname{Dist}[(2*c*\operatorname{Log}[F])/(e^2*(m+1)), \operatorname{Int}[(d + e*x)^{m+2}*F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x$  &&  $\operatorname{EqQ}[b*e - 2*c*d, 0]$  &&  $\operatorname{LtQ}[m, -1]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c\}, x$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.)^2))}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} dx &= -\frac{f^{a+bx+cx^2}}{2c(b+2cx)} + \frac{\log(f) \int f^{a+bx+cx^2} dx}{2c} \\ &= -\frac{f^{a+bx+cx^2}}{2c(b+2cx)} + \frac{\left(f^{a-\frac{b^2}{4c}} \log(f)\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c} \\ &= -\frac{f^{a+bx+cx^2}}{2c(b+2cx)} + \frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) \sqrt{\log(f)}}{4c^{3/2}} \end{aligned}$$



**Mathematica [A]** time = 0.0983567, size = 96, normalized size = 1.14

$$\frac{f^{a-\frac{b^2}{4c}} \left( \sqrt{\pi} \sqrt{\log(f)} (b+2cx) \operatorname{Erfi} \left( \frac{\sqrt{\log(f)} (b+2cx)}{2\sqrt{c}} \right) - 2\sqrt{c} f^{\frac{(b+2cx)^2}{4c}} \right)}{4c^{3/2} (b+2cx)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)/(b + 2\*c\*x)^2,x]

[Out] (f^(a - b^2/(4\*c)))\*(-2\*Sqrt[c]\*f^((b + 2\*c\*x)^2/(4\*c)) + Sqrt[Pi]\*(b + 2\*c\*x)\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])]\*Sqrt[Log[f]])/(4\*c^(3/2)\*(b + 2\*c\*x))

**Maple [A]** time = 0.062, size = 101, normalized size = 1.2

$$-\frac{1}{2c(2cx+b)} f^{\frac{(2cx+b)^2}{4c}} f^{\frac{4ac-b^2}{4c}} + \frac{\ln(f) \sqrt{\pi}}{4c^2} f^{\frac{4ac-b^2}{4c}} \operatorname{Erf} \left( \frac{2cx+b}{2} \sqrt{-\frac{\ln(f)}{c}} \right) \frac{1}{\sqrt{-\frac{\ln(f)}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^2,x)

[Out] -1/2/c/(2\*c\*x+b)\*f^(1/4\*(2\*c\*x+b)^2/c)\*f^(1/4\*(4\*a\*c-b^2)/c)+1/4/c^2\*ln(f)\*Pi^(1/2)\*f^(1/4\*(4\*a\*c-b^2)/c)/(-ln(f)/c)^(1/2)\*erf(1/2\*(-ln(f)/c)^(1/2)\*(2\*c\*x+b))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(2cx+b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^2,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(2\*c\*x + b)^2, x)

**Fricas [A]** time = 1.54331, size = 205, normalized size = 2.44

$$\frac{2c f^{cx^2+bx+a} + \frac{\sqrt{\pi}(2cx+b)\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{4(2c^3x+bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^2,x, algorithm="fricas")

[Out]  $-1/4*(2*c*f^(c*x^2 + b*x + a) + \sqrt{\pi}*(2*c*x + b)*\sqrt{-c*\log(f)}*\operatorname{erf}(1/2*(2*c*x + b)*\sqrt{-c*\log(f)})/c)/f^(1/4*(b^2 - 4*a*c)/c))/(2*c^3*x + b*c^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x+a)/(2*c*x+b)**2,x)`

[Out] `Integral(f**(a + b*x + c*x**2)/(b + 2*c*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(2cx+b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x, algorithm="giac")`

[Out] `integrate(f^(c*x^2 + b*x + a)/(2*c*x + b)^2, x)`

$$3.455 \quad \int \frac{f^{a+bx+cx^2}}{(b+2cx)^3} dx$$

**Optimal.** Leaf size=69

$$\frac{\log(f)f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{16c^2} - \frac{f^{a+bx+cx^2}}{4c(b+2cx)^2}$$

[Out]  $-f^{a+b*x+c*x^2}/(4*c*(b+2*c*x)^2) + (f^{a-b^2/(4*c)})*\operatorname{ExpIntegralEi}[\frac{((b+2*c*x)^2*\operatorname{Log}[f])}{(4*c)}]*\operatorname{Log}[f]/(16*c^2)$

**Rubi [A]** time = 0.0706892, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2239, 2238}

$$\frac{\log(f)f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{16c^2} - \frac{f^{a+bx+cx^2}}{4c(b+2cx)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{a+b*x+c*x^2}/(b+2*c*x)^3, x]$

[Out]  $-f^{a+b*x+c*x^2}/(4*c*(b+2*c*x)^2) + (f^{a-b^2/(4*c)})*\operatorname{ExpIntegralEi}[\frac{((b+2*c*x)^2*\operatorname{Log}[f])}{(4*c)}]*\operatorname{Log}[f]/(16*c^2)$

#### Rule 2239

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^{(m_)}, x\_Symbol] :> \operatorname{Simp}[\frac{(d + e*x)^{(m+1)}*F^{(a + b*x + c*x^2)}}{(e*(m+1))}, x] - \operatorname{Dist}[\frac{(2*c*\operatorname{Log}[F])}{(e^2*(m+1))}, \operatorname{Int}[(d + e*x)^{(m+2)}*F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \ \operatorname{EqQ}[b*e - 2*c*d, 0] \ \&\& \ \operatorname{LtQ}[m, -1]$

#### Rule 2238

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_))}, x\_Symbol] :> \operatorname{Simp}[\frac{(1*F^{(a - b^2/(4*c)})*\operatorname{ExpIntegralEi}[\frac{((b+2*c*x)^2*\operatorname{Log}[F])}{(4*c)})]}{(2*e)}, x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \ \operatorname{EqQ}[b*e - 2*c*d, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{a+bx+cx^2}}{(b+2cx)^3} dx &= -\frac{f^{a+bx+cx^2}}{4c(b+2cx)^2} + \frac{\log(f) \int \frac{f^{a+bx+cx^2}}{b+2cx} dx}{4c} \\ &= -\frac{f^{a+bx+cx^2}}{4c(b+2cx)^2} + \frac{f^{a-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) \log(f)}{16c^2} \end{aligned}$$

**Mathematica [A]** time = 0.0877809, size = 79, normalized size = 1.14

$$\frac{f^{a-\frac{b^2}{4c}} \left( \log(f)(b+2cx)^2 \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) - 4cf^{\frac{(b+2cx)^2}{4c}} \right)}{16c^2(b+2cx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x + c\*x^2)/(b + 2\*c\*x)^3,x]

[Out] (f^(a - b^2/(4\*c))\*(-4\*c\*f^((b + 2\*c\*x)^2/(4\*c)) + (b + 2\*c\*x)^2\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c)]\*Log[f]))/(16\*c^2\*(b + 2\*c\*x)^2)

**Maple [A]** time = 0.031, size = 88, normalized size = 1.3

$$-\frac{1}{4c(2cx+b)^2}f^{\frac{(2cx+b)^2}{4c}}f^{\frac{4ac-b^2}{4c}}-\frac{\ln(f)}{16c^2}f^{\frac{4ac-b^2}{4c}}\text{Ei}\left(1,-\frac{(2cx+b)^2\ln(f)}{4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^3,x)

[Out] -1/4/c/(2\*c\*x+b)^2\*f^(1/4\*(2\*c\*x+b)^2/c)\*f^(1/4\*(4\*a\*c-b^2)/c)-1/16/c^2\*ln(f)\*f^(1/4\*(4\*a\*c-b^2)/c)\*Ei(1,-1/4\*(2\*c\*x+b)^2\*ln(f)/c)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(2cx+b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^3,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(2\*c\*x + b)^3, x)

**Fricas [A]** time = 1.56674, size = 234, normalized size = 3.39

$$\frac{4cf^{cx^2+bx+a}-\frac{(4c^2x^2+4bcx+b^2)\text{Ei}\left(\frac{(4c^2x^2+4bcx+b^2)\log(f)}{4c}\right)\log(f)}{f^{\frac{b^2-4ac}{4c}}}}{16(4c^4x^2+4bc^3x+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^3,x, algorithm="fricas")

[Out] -1/16\*(4\*c\*f^(c\*x^2 + b\*x + a) - (4\*c^2\*x^2 + 4\*b\*c\*x + b^2)\*Ei(1/4\*(4\*c^2\*x^2 + 4\*b\*c\*x + b^2)\*log(f)/c)\*log(f)/f^(1/4\*(b^2 - 4\*a\*c)/c))/(4\*c^4\*x^2 + 4\*b\*c^3\*x + b^2\*c^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x+a)/(2\*c\*x+b)\*\*3,x)

[Out] Integral(f\*\*(a + b\*x + c\*x\*\*2)/(b + 2\*c\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx+a}}{(2cx+b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x+a)/(2\*c\*x+b)^3,x, algorithm="giac")

[Out] integrate(f^(c\*x^2 + b\*x + a)/(2\*c\*x + b)^3, x)

$$3.456 \quad \int f^{bx+cx^2} (b + 2cx)^3 dx$$

**Optimal.** Leaf size=43

$$\frac{(b + 2cx)^2 f^{bx+cx^2}}{\log(f)} - \frac{4c f^{bx+cx^2}}{\log^2(f)}$$

[Out]  $(-4*c*f^{(b*x + c*x^2)})/Log[f]^2 + (f^{(b*x + c*x^2)}*(b + 2*c*x)^2)/Log[f]$

**Rubi [A]** time = 0.0433466, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2237, 2236}

$$\frac{(b + 2cx)^2 f^{bx+cx^2}}{\log(f)} - \frac{4c f^{bx+cx^2}}{\log^2(f)}$$

Antiderivative was successfully verified.

[In] Int[f^(b\*x + c\*x^2)\*(b + 2\*c\*x)^3,x]

[Out]  $(-4*c*f^{(b*x + c*x^2)})/Log[f]^2 + (f^{(b*x + c*x^2)}*(b + 2*c*x)^2)/Log[f]$

#### Rule 2237

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^(m - 1)\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] - Dist[((m - 1)\*e^2)/(2\*c\*Log[F]), Int[(d + e\*x)^(m - 2)\*F^(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0] && GtQ[m, 1]

#### Rule 2236

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(e\*F^(a + b\*x + c\*x^2))/(2\*c\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0]

#### Rubi steps

$$\begin{aligned} \int f^{bx+cx^2} (b + 2cx)^3 dx &= \frac{f^{bx+cx^2} (b + 2cx)^2}{\log(f)} - \frac{(4c) \int f^{bx+cx^2} (b + 2cx) dx}{\log(f)} \\ &= -\frac{4c f^{bx+cx^2}}{\log^2(f)} + \frac{f^{bx+cx^2} (b + 2cx)^2}{\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0946798, size = 29, normalized size = 0.67

$$\frac{f^{x(b+cx)} (\log(f)(b + 2cx)^2 - 4c)}{\log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(b\*x + c\*x^2)\*(b + 2\*c\*x)^3,x]

[Out]  $(f^{(x*(b + c*x))*(-4*c + (b + 2*c*x)^2*\text{Log}[f])})/\text{Log}[f]^2$

**Maple [A]** time = 0.004, size = 44, normalized size = 1.

$$\frac{(4 \ln(f) c^2 x^2 + 4 b c x \ln(f) + \ln(f) b^2 - 4 c) f^{c x^2 + b x}}{(\ln(f))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x)*(2*c*x+b)^3,x)`

[Out]  $(4*\ln(f)*c^2*x^2+4*b*c*x*\ln(f)+\ln(f)*b^2-4*c)*f^{(c*x^2+b*x)}/\ln(f)^2$

**Maxima [C]** time = 1.34087, size = 724, normalized size = 16.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)*(2*c*x+b)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2}*\sqrt{\pi}*b^3*\text{erf}(\sqrt{-c*\log(f)})*x - \frac{1}{2}*b*\log(f)/\sqrt{-c*\log(f)}}/(\sqrt{-c*\log(f)}*f^{(1/4*b^2/c)} - \frac{3}{2}*(\sqrt{\pi}*(2*c*x + b)*b*(\text{erf}(1/2*\sqrt{-c*\log(f)}) - 1)*\log(f)^2/(\sqrt{-c*\log(f)})*(c*\log(f))^{(3/2)} - 2*c*f^{(1/4*(2*c*x + b)^2/c})*\log(f)/(c*\log(f))^{(3/2)})*b^2*c/\sqrt{c*\log(f)}*f^{(1/4*b^2/c)} + \frac{3}{2}*(\sqrt{\pi}*(2*c*x + b)*b^2*(\text{erf}(1/2*\sqrt{-c*\log(f)}) - 1)*\log(f)^3/(\sqrt{-c*\log(f)})*(c*\log(f))^{(5/2)} - 4*(2*c*x + b)^3*\gamma(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^3/((-c*\log(f))^{(3/2)}*(c*\log(f))^{(5/2)}) - 4*b*c*f^{(1/4*(2*c*x + b)^2/c})*\log(f)^2/(c*\log(f))^{(5/2)})*b*c^2/\sqrt{c*\log(f)}*f^{(1/4*b^2/c)} - \frac{1}{2}*(\sqrt{\pi}*(2*c*x + b)*b^3*(\text{erf}(1/2*\sqrt{-c*\log(f)}) - 1)*\log(f)^4/(\sqrt{-c*\log(f)})*(c*\log(f))^{(7/2)} - 12*(2*c*x + b)^3*b*\gamma(3/2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^4/((-c*\log(f))^{(3/2)}*(c*\log(f))^{(7/2)}) - 6*b^2*c*f^{(1/4*(2*c*x + b)^2/c})*\log(f)^3/(c*\log(f))^{(7/2)} + 8*c^2*\gamma(2, -1/4*(2*c*x + b)^2*\log(f)/c)*\log(f)^2/(c*\log(f))^{(7/2)})*c^3/\sqrt{c*\log(f)}*f^{(1/4*b^2/c)})$

**Fricas [A]** time = 1.52657, size = 93, normalized size = 2.16

$$\frac{((4c^2x^2 + 4bcx + b^2)\log(f) - 4c)f^{cx^2+bx}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)*(2*c*x+b)^3,x, algorithm="fricas")`

[Out]  $((4*c^2*x^2 + 4*b*c*x + b^2)*\log(f) - 4*c)*f^{(c*x^2 + b*x)}/\log(f)^2$

**Sympy [A]** time = 0.149724, size = 83, normalized size = 1.93

$$\begin{cases} \frac{f^{bx+cx^2}(b^2 \log(f) + 4bcx \log(f) + 4c^2x^2 \log(f) - 4c)}{\log(f)^2} & \text{for } \log(f)^2 \neq 0 \\ b^3x + 3b^2cx^2 + 4bc^2x^3 + 2c^3x^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x)\*(2\*c\*x+b)\*\*3,x)

[Out] Piecewise((f\*\*(b\*x + c\*x\*\*2)\*(b\*\*2\*log(f) + 4\*b\*c\*x\*log(f) + 4\*c\*\*2\*x\*\*2\*log(f) - 4\*c)/log(f)\*\*2, Ne(log(f)\*\*2, 0)), (b\*\*3\*x + 3\*b\*\*2\*c\*x\*\*2 + 4\*b\*c\*\*2\*x\*\*3 + 2\*c\*\*3\*x\*\*4, True))

**Giac [A]** time = 1.31869, size = 54, normalized size = 1.26

$$\frac{\left(c^2\left(2x + \frac{b}{c}\right)^2 \log(f) - 4c\right) e^{(cx^2 \log(f) + bx \log(f))}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)\*(2\*c\*x+b)^3,x, algorithm="giac")

[Out] (c^2\*(2\*x + b/c)^2\*log(f) - 4\*c)\*e^(c\*x^2\*log(f) + b\*x\*log(f))/log(f)^2



### 3.457 $\int f^{bx+cx^2} (b + 2cx)^2 dx$

**Optimal.** Leaf size=75

$$\frac{(b + 2cx)f^{bx+cx^2}}{\log(f)} - \frac{\sqrt{\pi}\sqrt{c}f^{-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

[Out]  $-\left(\frac{\sqrt{c}\sqrt{\pi}\operatorname{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{2\sqrt{c}}\right)/\left(f^{b^2/(4c)}\log(f)^{3/2}\right) + (f^{bx+cx^2}(b+2cx))/\log(f)$

**Rubi [A]** time = 0.0575022, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2237, 2234, 2204}

$$\frac{(b + 2cx)f^{bx+cx^2}}{\log(f)} - \frac{\sqrt{\pi}\sqrt{c}f^{-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In]  $\int f^{bx+cx^2}(b+2cx)^2 dx$

[Out]  $-\left(\frac{\sqrt{c}\sqrt{\pi}\operatorname{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{2\sqrt{c}}\right)/\left(f^{b^2/(4c)}\log(f)^{3/2}\right) + (f^{bx+cx^2}(b+2cx))/\log(f)$

#### Rule 2237

$\operatorname{Int}[(F_)^{\left((a_.) + (b_.)(x_) + (c_.)(x_)^2\right)} \left((d_.) + (e_.)(x_)\right)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e*(d + e*x)^{(m-1)}*F^{(a + b*x + c*x^2)})/(2*c*\operatorname{Log}[F]), x] - \operatorname{Dist}[\left((m-1)*e^2\right)/(2*c*\operatorname{Log}[F]), \operatorname{Int}[(d + e*x)^{(m-2)}*F^{(a + b*x + c*x^2)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[b*e - 2*c*d, 0] \&\& \operatorname{GtQ}[m, 1]$

#### Rule 2234

$\operatorname{Int}[(F_)^{\left((a_.) + (b_.)(x_) + (c_.)(x_)^2\right)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{(b + 2*c*x)^2/(4*c)}, x], x] /; \operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2204

$\operatorname{Int}[(F_)^{\left((a_.) + (b_.)\left((c_.) + (d_.)(x_)\right)^2\right)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int f^{bx+cx^2} (b+2cx)^2 dx &= \frac{f^{bx+cx^2} (b+2cx)}{\log(f)} - \frac{(2c) \int f^{bx+cx^2} dx}{\log(f)} \\
&= \frac{f^{bx+cx^2} (b+2cx)}{\log(f)} - \frac{\left(2cf^{-\frac{b^2}{4c}}\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{\log(f)} \\
&= -\frac{\sqrt{c} f^{-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)} + \frac{f^{bx+cx^2} (b+2cx)}{\log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0619255, size = 84, normalized size = 1.12

$$\frac{f^{-\frac{b^2}{4c}} \left( \sqrt{\log(f)} (b+2cx) f^{\frac{(b+2cx)^2}{4c}} - \sqrt{\pi} \sqrt{c} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)} (b+2cx)}{2\sqrt{c}}\right) \right)}{\log^{\frac{3}{2}}(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(b\*x + c\*x^2)\*(b + 2\*c\*x)^2,x]

[Out]  $(-\sqrt{c} \sqrt{\pi} \operatorname{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) + f^{bx+cx^2} (b+2cx)) / \log^{\frac{3}{2}}(f)$

**Maple [A]** time = 0.036, size = 90, normalized size = 1.2

$$2 \frac{cx f^{cx^2} f^{bx}}{\ln(f)} + \frac{b f^{cx^2} f^{bx}}{\ln(f)} + \frac{c \sqrt{\pi}}{\ln(f)} f^{-\frac{b^2}{4c}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)} x + \frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x)\*(2\*c\*x+b)^2,x)

[Out]  $2cx/\ln(f) * x * f^{cx^2} * f^{bx} + b/\ln(f) * f^{cx^2} * f^{bx} + c/\ln(f) * \pi^{1/2} * f^{bx} * (-1/4 * b^2/c) / (-c * \ln(f))^{1/2} * \operatorname{erf}(-(-c * \ln(f))^{1/2} * x + 1/2 * b * \ln(f) / (-c * \ln(f))^{1/2})$

**Maxima [B]** time = 1.24178, size = 444, normalized size = 5.92

$$\frac{\sqrt{\pi} b^2 \operatorname{erf}\left(\sqrt{-c \log(f)} x - \frac{b \log(f)}{2\sqrt{-c \log(f)}}\right)}{2\sqrt{-c \log(f)} f^{\frac{b^2}{4c}}} - \frac{\left(\frac{\sqrt{\pi}(2cx+b)b \left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}}\right) - 1\right) \log(f)^2}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c} \log(f)}}{(c \log(f))^{\frac{3}{2}}}\right) bc}{\sqrt{c \log(f)} f^{\frac{b^2}{4c}}} + \frac{\left(\frac{\sqrt{\pi}(2cx+b)b^2 \left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}}\right) - 1\right) \log(f)^2}{\sqrt{-\frac{(2cx+b)^2 \log(f)}{c}} (c \log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c} \log(f)}}{(c \log(f))^{\frac{3}{2}}}\right) bc}{\sqrt{c \log(f)} f^{\frac{b^2}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)\*(2\*c\*x+b)^2,x, algorithm="maxima")

```
[Out] 1/2*sqrt(pi)*b^2*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c)) - (sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*b*c/(sqrt(c*log(f))*f^(1/4*b^2/c)) + 1/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-2*c*x + b)^2*log(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^2/(c*log(f))^(5/2))*c^2/(sqrt(c*log(f))*f^(1/4*b^2/c))
```

**Fricas [A]** time = 1.54022, size = 171, normalized size = 2.28

$$\frac{(2cx + b)f^{cx^2+bx} \log(f) + \frac{\sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c \log(f)}}{2c}\right)}{f^{\frac{b^2}{4c}}}}{\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x)*(2*c*x+b)^2,x, algorithm="fricas")
```

```
[Out] ((2*c*x + b)*f^(c*x^2 + b*x)*log(f) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*b^2/c))/log(f)^2
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx+cx^2} (b + 2cx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x)*(2*c*x+b)**2,x)
```

```
[Out] Integral(f**(b*x + c*x**2)*(b + 2*c*x)**2, x)
```

**Giac [A]** time = 1.23602, size = 104, normalized size = 1.39

$$\frac{c\left(2x + \frac{b}{c}\right)e^{(cx^2 \log(f) + bx \log(f))}}{\log(f)} + \frac{\sqrt{\pi}c \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c \log(f)}\left(2x + \frac{b}{c}\right)\right)}{\sqrt{-c \log(f)}f^{\frac{b^2}{4c}} \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x)*(2*c*x+b)^2,x, algorithm="giac")
```

```
[Out] c*(2*x + b/c)*e^(c*x^2*log(f) + b*x*log(f))/log(f) + sqrt(pi)*c*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))/(sqrt(-c*log(f))*f^(1/4*b^2/c)*log(f))
```

$$3.458 \quad \int f^{bx+cx^2} (b + 2cx) dx$$

**Optimal.** Leaf size=16

$$\frac{f^{bx+cx^2}}{\log(f)}$$

[Out]  $f^{(b*x + c*x^2)}/\text{Log}[f]$

**Rubi [A]** time = 0.0141611, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2236}

$$\frac{f^{bx+cx^2}}{\log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[f^{(b*x + c*x^2)}*(b + 2*c*x), x]$

[Out]  $f^{(b*x + c*x^2)}/\text{Log}[f]$

**Rule 2236**

$\text{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))}, x\_Symbol]$   
 $]:> \text{Simp}[(e*F^{(a + b*x + c*x^2)})/(2*c*\text{Log}[F]), x] /;$   $\text{FreeQ}[\{F, a, b, c, d, e\}, x]$  &&  $\text{EqQ}[b*e - 2*c*d, 0]$

**Rubi steps**

$$\int f^{bx+cx^2} (b + 2cx) dx = \frac{f^{bx+cx^2}}{\log(f)}$$

**Mathematica [A]** time = 0.0272758, size = 16, normalized size = 1.

$$\frac{f^{bx+cx^2}}{\log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[f^{(b*x + c*x^2)}*(b + 2*c*x), x]$

[Out]  $f^{(b*x + c*x^2)}/\text{Log}[f]$

**Maple [A]** time = 0.002, size = 17, normalized size = 1.1

$$\frac{f^{cx^2+bx}}{\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x)*(2*c*x+b),x)`

[Out]  $f^{(c*x^2+b*x)}/\ln(f)$

**Maxima [A]** time = 0.994273, size = 22, normalized size = 1.38

$$\frac{f^{cx^2+bx}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="maxima")`

[Out]  $f^{(c*x^2 + b*x)}/\log(f)$

**Fricas [A]** time = 1.52006, size = 32, normalized size = 2.

$$\frac{f^{bx+cx^2}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="fricas")`

[Out]  $f^{(c*x^2 + b*x)}/\log(f)$

**Sympy [A]** time = 0.113387, size = 22, normalized size = 1.38

$$\begin{cases} \frac{f^{bx+cx^2}}{\log(f)} & \text{for } \log(f) \neq 0 \\ bx + cx^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x)*(2*c*x+b),x)`

[Out] `Piecewise((f**(b*x + c*x**2)/log(f), Ne(log(f), 0)), (b*x + c*x**2, True))`

**Giac [A]** time = 1.26103, size = 22, normalized size = 1.38

$$\frac{f^{cx^2+bx}}{\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="giac")`

[Out]  $f^{(c*x^2 + b*x)}/\log(f)$

$$3.459 \quad \int \frac{f^{bx+cx^2}}{b+2cx} dx$$

**Optimal.** Leaf size=37

$$\frac{f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

[Out] ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c)]/(4\*c\*f^(b^2/(4\*c)))

**Rubi [A]** time = 0.0294125, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {2238}

$$\frac{f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Int[f^(b\*x + c\*x^2)/(b + 2\*c\*x), x]

[Out] ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c)]/(4\*c\*f^(b^2/(4\*c)))

#### Rule 2238

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(1\*F^(a - b^2/(4\*c))\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[F])/(4\*c)]]/(2\*e), x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b\*e - 2\*c\*d, 0]

#### Rubi steps

$$\int \frac{f^{bx+cx^2}}{b+2cx} dx = \frac{f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

**Mathematica [A]** time = 0.0332878, size = 37, normalized size = 1.

$$\frac{f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[f^(b\*x + c\*x^2)/(b + 2\*c\*x), x]

[Out] ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c)]/(4\*c\*f^(b^2/(4\*c)))

**Maple [A]** time = 0.017, size = 33, normalized size = 0.9

$$-\frac{1}{4c} f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(1, -\frac{(2cx+b)^2 \ln(f)}{4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(c*x^2+b*x)/(2*c*x+b),x)`

[Out]  $-1/4/c*f^{(-1/4*b^2/c)}*Ei(1,-1/4*(2*c*x+b)^2*\ln(f)/c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{2cx+b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="maxima")`

[Out] `integrate(f^(c*x^2 + b*x)/(2*c*x + b), x)`

**Fricas [A]** time = 1.56193, size = 92, normalized size = 2.49

$$\frac{Ei\left(\frac{(4c^2x^2+4bcx+b^2)\log(f)}{4c}\right)}{4cf^{\frac{b^2}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="fricas")`

[Out]  $1/4*Ei(1/4*(4*c^2*x^2 + 4*b*c*x + b^2)*\log(f)/c)/(c*f^{(1/4*b^2/c)})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx+cx^2}}{b+2cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x)/(2*c*x+b),x)`

[Out] `Integral(f**(b*x + c*x**2)/(b + 2*c*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{2cx+b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="giac")`

[Out] `integrate(f^(c*x^2 + b*x)/(2*c*x + b), x)`

$$3.460 \quad \int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx$$

**Optimal.** Leaf size=81

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{f^{bx+cx^2}}{2c(b+2cx)}$$

[Out]  $-f^{(b*x + c*x^2)/(2*c*(b + 2*c*x))} + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\frac{(b + 2*c*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]}{2*\operatorname{Sqrt}[c]}])* \operatorname{Sqrt}[\operatorname{Log}[f]]/(4*c^{(3/2)}*f^{(b^2/(4*c))})$

**Rubi [A]** time = 0.0485183, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2239, 2234, 2204}

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}} - \frac{f^{bx+cx^2}}{2c(b+2cx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(b*x + c*x^2)/(b + 2*c*x)^2}, x]$

[Out]  $-f^{(b*x + c*x^2)/(2*c*(b + 2*c*x))} + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\frac{(b + 2*c*x)*\operatorname{Sqrt}[\operatorname{Log}[f]]}{2*\operatorname{Sqrt}[c]}])* \operatorname{Sqrt}[\operatorname{Log}[f]]/(4*c^{(3/2)}*f^{(b^2/(4*c))})$

#### Rule 2239

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*((d_.) + (e_.)*(x_.))^m}, x\_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{m+1}*F^{(a + b*x + c*x^2)/(e*(m+1))}, x] - \operatorname{Dist}[(2*c*\operatorname{Log}[F])/(e^2*(m+1)), \operatorname{Int}[(d + e*x)^{m+2}*F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x$  &&  $\operatorname{EqQ}[b*e - 2*c*d, 0]$  &&  $\operatorname{LtQ}[m, -1]$

#### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c\}, x$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.)^2))}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx &= -\frac{f^{bx+cx^2}}{2c(b+2cx)} + \frac{\log(f) \int f^{bx+cx^2} dx}{2c} \\ &= -\frac{f^{bx+cx^2}}{2c(b+2cx)} + \frac{\left(f^{-\frac{b^2}{4c}} \log(f)\right) \int f^{\frac{(b+2cx)^2}{4c}} dx}{2c} \\ &= -\frac{f^{bx+cx^2}}{2c(b+2cx)} + \frac{f^{-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) \sqrt{\log(f)}}{4c^{3/2}} \end{aligned}$$



**Mathematica [A]** time = 0.0589249, size = 94, normalized size = 1.16

$$\frac{f^{-\frac{b^2}{4c}} \left( \sqrt{\pi} \sqrt{\log(f)} (b + 2cx) \operatorname{Erfi} \left( \frac{\sqrt{\log(f)} (b + 2cx)}{2\sqrt{c}} \right) - 2\sqrt{c} f^{\frac{(b+2cx)^2}{4c}} \right)}{4c^{3/2} (b + 2cx)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(b\*x + c\*x^2)/(b + 2\*c\*x)^2,x]

[Out] (-2\*Sqrt[c]\*f^((b + 2\*c\*x)^2/(4\*c)) + Sqrt[Pi]\*(b + 2\*c\*x)\*Erfi[((b + 2\*c\*x)\*Sqrt[Log[f]])/(2\*Sqrt[c])]\*Sqrt[Log[f]])/(4\*c^(3/2)\*f^(b^2/(4\*c))\*(b + 2\*c\*x))

**Maple [A]** time = 0.034, size = 87, normalized size = 1.1

$$-\frac{1}{2c(2cx+b)} f^{\frac{(2cx+b)^2}{4c}} f^{-\frac{b^2}{4c}} + \frac{\ln(f) \sqrt{\pi}}{4c^2} f^{-\frac{b^2}{4c}} \operatorname{Erf} \left( \frac{2cx+b}{2} \sqrt{-\frac{\ln(f)}{c}} \right) \frac{1}{\sqrt{-\frac{\ln(f)}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x)/(2\*c\*x+b)^2,x)

[Out] -1/2/c/(2\*c\*x+b)\*f^(1/4\*(2\*c\*x+b)^2/c)\*f^(-1/4\*b^2/c)+1/4/c^2\*ln(f)\*Pi^(1/2)\*f^(-1/4\*b^2/c)/(-ln(f)/c)^(1/2)\*erf(1/2\*(-ln(f)/c)^(1/2)\*(2\*c\*x+b))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{(2cx+b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)/(2\*c\*x+b)^2,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x)/(2\*c\*x + b)^2, x)

**Fricas [A]** time = 1.62962, size = 186, normalized size = 2.3

$$\frac{2cf^{cx^2+bx} + \frac{\sqrt{\pi}(2cx+b)\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{4c}}}{4(2c^3x + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)/(2\*c\*x+b)^2,x, algorithm="fricas")

[Out]  $-1/4*(2*c*f^(c*x^2 + b*x) + \sqrt{\pi}*(2*c*x + b)*\sqrt{-c*\log(f)}*\operatorname{erf}(1/2*(2*c*x + b)*\sqrt{-c*\log(f)}/c)/f^(1/4*b^2/c))/(2*c^3*x + b*c^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(c*x**2+b*x)/(2*c*x+b)**2,x)`

[Out] `Integral(f**(b*x + c*x**2)/(b + 2*c*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{(2cx+b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(c*x^2+b*x)/(2*c*x+b)^2,x, algorithm="giac")`

[Out] `integrate(f^(c*x^2 + b*x)/(2*c*x + b)^2, x)`

### 3.461 $\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx$

**Optimal.** Leaf size=66

$$\frac{\log(f)f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{16c^2} - \frac{f^{bx+cx^2}}{4c(b+2cx)^2}$$

[Out]  $-f^{(b*x + c*x^2)}/(4*c*(b + 2*c*x)^2) + (\operatorname{ExpIntegralEi}[(b + 2*c*x)^2*\operatorname{Log}[f]]/(4*c))*\operatorname{Log}[f]/(16*c^2*f^{(b^2/(4*c))})$

**Rubi [A]** time = 0.0632942, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2239, 2238}

$$\frac{\log(f)f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{16c^2} - \frac{f^{bx+cx^2}}{4c(b+2cx)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[f^{(b*x + c*x^2)}/(b + 2*c*x)^3, x]$

[Out]  $-f^{(b*x + c*x^2)}/(4*c*(b + 2*c*x)^2) + (\operatorname{ExpIntegralEi}[(b + 2*c*x)^2*\operatorname{Log}[f]]/(4*c))*\operatorname{Log}[f]/(16*c^2*f^{(b^2/(4*c))})$

#### Rule 2239

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{(m+1)}*F^{(a + b*x + c*x^2)}]/(e*(m+1)), x] - \operatorname{Dist}[(2*c*\operatorname{Log}[F])/(e^2*(m+1)), \operatorname{Int}[(d + e*x)^{(m+2)}*F^{(a + b*x + c*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \operatorname{EqQ}[b*e - 2*c*d, 0] \ \&\& \operatorname{LtQ}[m, -1]$

#### Rule 2238

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*F^{(a - b^2/(4*c))})*\operatorname{ExpIntegralEi}[(b + 2*c*x)^2*\operatorname{Log}[F]]/(4*c)]/(2*e), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \operatorname{EqQ}[b*e - 2*c*d, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx &= -\frac{f^{bx+cx^2}}{4c(b+2cx)^2} + \frac{\log(f) \int \frac{f^{bx+cx^2}}{b+2cx} dx}{4c} \\ &= -\frac{f^{bx+cx^2}}{4c(b+2cx)^2} + \frac{f^{-\frac{b^2}{4c}} \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) \log(f)}{16c^2} \end{aligned}$$

**Mathematica [A]** time = 0.0494255, size = 77, normalized size = 1.17

$$\frac{f^{-\frac{b^2}{4c}} \left( \log(f)(b+2cx)^2 \operatorname{Ei}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) - 4c f^{\frac{(b+2cx)^2}{4c}} \right)}{16c^2(b+2cx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[f^(b\*x + c\*x^2)/(b + 2\*c\*x)^3,x]

[Out] (-4\*c\*f^((b + 2\*c\*x)^2/(4\*c)) + (b + 2\*c\*x)^2\*ExpIntegralEi[((b + 2\*c\*x)^2\*Log[f])/(4\*c)]\*Log[f])/(16\*c^2\*f^(b^2/(4\*c))\*(b + 2\*c\*x)^2)

**Maple [A]** time = 0.027, size = 74, normalized size = 1.1

$$-\frac{1}{4c(2cx+b)^2}f^{\frac{(2cx+b)^2}{4c}}f^{-\frac{b^2}{4c}}-\frac{\ln(f)}{16c^2}f^{-\frac{b^2}{4c}}\text{Ei}\left(1,-\frac{(2cx+b)^2\ln(f)}{4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c\*x^2+b\*x)/(2\*c\*x+b)^3,x)

[Out] -1/4/c/(2\*c\*x+b)^2\*f^(1/4\*(2\*c\*x+b)^2/c)\*f^(-1/4\*b^2/c)-1/16/c^2\*ln(f)\*f^(-1/4\*b^2/c)\*Ei(1,-1/4\*(2\*c\*x+b)^2\*ln(f)/c)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{(2cx+b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)/(2\*c\*x+b)^3,x, algorithm="maxima")

[Out] integrate(f^(c\*x^2 + b\*x)/(2\*c\*x + b)^3, x)

**Fricas [A]** time = 1.52955, size = 215, normalized size = 3.26

$$\frac{4cf^{cx^2+bx}-\frac{(4c^2x^2+4bcx+b^2)\text{Ei}\left(\frac{(4c^2x^2+4bcx+b^2)\log(f)}{4c}\right)\log(f)}{f^{\frac{b^2}{4c}}}}{16(4c^4x^2+4bc^3x+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)/(2\*c\*x+b)^3,x, algorithm="fricas")

[Out] -1/16\*(4\*c\*f^(c\*x^2 + b\*x) - (4\*c^2\*x^2 + 4\*b\*c\*x + b^2)\*Ei(1/4\*(4\*c^2\*x^2 + 4\*b\*c\*x + b^2)\*log(f)/c)\*log(f)/f^(1/4\*b^2/c))/(4\*c^4\*x^2 + 4\*b\*c^3\*x + b^2\*c^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(c\*x\*\*2+b\*x)/(2\*c\*x+b)\*\*3,x)

[Out] Integral(f\*\*(b\*x + c\*x\*\*2)/(b + 2\*c\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f^{cx^2+bx}}{(2cx+b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c\*x^2+b\*x)/(2\*c\*x+b)^3,x, algorithm="giac")

[Out] integrate(f^(c\*x^2 + b\*x)/(2\*c\*x + b)^3, x)

$$3.462 \quad \int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$$

**Optimal.** Leaf size=145

$$\frac{\sqrt{d} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{e^a b \operatorname{Ei}(bx)}{c} - \frac{e^{a+bx}}{cx}$$

[Out]  $-(E^{(a + b*x)/(c*x)}) + (b*E^a*ExpIntegralEi[b*x])/c + (Sqrt[d]*E^{(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d])])/(2*(-c)^{(3/2)}) - (Sqrt[d]*E^{(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d])])/(2*(-c)^{(3/2)})$

**Rubi [A]** time = 0.35187, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2271, 2177, 2178, 2269}

$$\frac{\sqrt{d} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{e^a b \operatorname{Ei}(bx)}{c} - \frac{e^{a+bx}}{cx}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x)/(x^2\*(c + d\*x^2)),x]

[Out]  $-(E^{(a + b*x)/(c*x)}) + (b*E^a*ExpIntegralEi[b*x])/c + (Sqrt[d]*E^{(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d])])/(2*(-c)^{(3/2)}) - (Sqrt[d]*E^{(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d])])/(2*(-c)^{(3/2)})$

#### Rule 2271

Int[((F\_)^((g\_)\*((d\_)+(e\_)\*(x\_)))^(n\_))\*(u\_)^(m\_)]/((a\_)+(c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), u^m/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*((e\_)+(f\_)\*(x\_))))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_)+(f\_)\*(x\_)))/((c\_)+(d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2269

Int[(F\_)^((g\_)\*((d\_)+(e\_)\*(x\_)))^(n\_)]/((a\_)+(c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), 1/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx &= \int \left( \frac{e^{a+bx}}{cx^2} - \frac{de^{a+bx}}{c(c+dx^2)} \right) dx \\
&= \frac{\int \frac{e^{a+bx}}{x^2} dx}{c} - \frac{d \int \frac{e^{a+bx}}{c+dx^2} dx}{c} \\
&= -\frac{e^{a+bx}}{cx} + \frac{b \int \frac{e^{a+bx}}{x} dx}{c} - \frac{d \int \left( \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}-\sqrt{dx})} + \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}+\sqrt{dx})} \right) dx}{c} \\
&= -\frac{e^{a+bx}}{cx} + \frac{be^a \operatorname{Ei}(bx)}{c} - \frac{d \int \frac{e^{a+bx}}{\sqrt{-c}-\sqrt{dx}} dx}{2(-c)^{3/2}} - \frac{d \int \frac{e^{a+bx}}{\sqrt{-c}+\sqrt{dx}} dx}{2(-c)^{3/2}} \\
&= -\frac{e^{a+bx}}{cx} + \frac{be^a \operatorname{Ei}(bx)}{c} + \frac{\sqrt{d} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.243512, size = 133, normalized size = 0.92

$$\frac{e^a \left( i\sqrt{d} x e^{\frac{ib\sqrt{c}}{\sqrt{d}}} \operatorname{Ei}\left(b\left(x - \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) - i\sqrt{d} x e^{-\frac{ib\sqrt{c}}{\sqrt{d}}} \operatorname{Ei}\left(b\left(x + \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + 2b\sqrt{cx} \operatorname{Ei}(bx) - 2\sqrt{c} e^{bx} \right)}{2c^{3/2}x}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x)/(x^2\*(c + d\*x^2)), x]

[Out] (E^a\*(-2\*Sqrt[c]\*E^(b\*x) + 2\*b\*Sqrt[c]\*x\*ExpIntegralEi[b\*x] + I\*Sqrt[d]\*E^((I\*b\*Sqrt[c])/Sqrt[d])\*x\*ExpIntegralEi[b\*((-I)\*Sqrt[c])/Sqrt[d] + x]) - (I\*Sqrt[d]\*x\*ExpIntegralEi[b\*((I\*Sqrt[c])/Sqrt[d] + x)])/E^((I\*b\*Sqrt[c])/Sqrt[d]))/(2\*c^(3/2)\*x)

**Maple [A]** time = 0.023, size = 142, normalized size = 1.

$$b \left( -\frac{e^{bx+a}}{bcx} - \frac{e^a \operatorname{Ei}(1, -bx)}{c} + \frac{d}{2bc} \left( e^{\frac{1}{d}(b\sqrt{-cd}+ad)} \operatorname{Ei}\left(1, \frac{1}{d}(b\sqrt{-cd} - d(bx+a) + ad)\right) - e^{-\frac{1}{d}(b\sqrt{-cd}-ad)} \operatorname{Ei}\left(1, -\frac{1}{d}(b\sqrt{-cd} + d(bx+a) - ad)\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)/x^2/(d\*x^2+c), x)

[Out] b\*(-exp(b\*x+a)/c/b/x-1/c\*exp(a)\*Ei(1, -b\*x)+1/2\*d\*(exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)-exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d))/c/b/(-c\*d)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{(dx^2+c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x^2/(d\*x^2+c),x, algorithm="maxima")

[Out] integrate(e^(b\*x + a)/((d\*x^2 + c)\*x^2), x)

**Fricas [A]** time = 1.52924, size = 266, normalized size = 1.83

$$\frac{2b^2cx\operatorname{Ei}(bx)e^a + \sqrt{-\frac{b^2c}{d}}dx\operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} - \sqrt{-\frac{b^2c}{d}}dx\operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} - 2bce^{(bx+a)}}{2bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x^2/(d\*x^2+c),x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*c\*x\*Ei(b\*x)\*e^a + sqrt(-b^2\*c/d)\*d\*x\*Ei(b\*x - sqrt(-b^2\*c/d))\*e^(a + sqrt(-b^2\*c/d)) - sqrt(-b^2\*c/d)\*d\*x\*Ei(b\*x + sqrt(-b^2\*c/d))\*e^(a - sqrt(-b^2\*c/d)) - 2\*b\*c\*e^(b\*x + a))/(b\*c^2\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{e^{bx}}{cx^2 + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x\*\*2/(d\*x\*\*2+c),x)

[Out] exp(a)\*Integral(exp(b\*x)/(c\*x\*\*2 + d\*x\*\*4), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{(dx^2 + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x^2/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(e^(b\*x + a)/((d\*x^2 + c)\*x^2), x)



$$3.463 \quad \int \frac{e^{a+bx}}{x(c+dx^2)} dx$$

**Optimal.** Leaf size=111

$$-\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}}\operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2c}-\frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}}\operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2c}+\frac{e^a\operatorname{Ei}(bx)}{c}$$

[Out] (E^a\*ExpIntegralEi[b\*x])/c - (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-(b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d]])/(2\*c) - (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*c)

**Rubi [A]** time = 0.246697, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2271, 2178}

$$-\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}}\operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2c}-\frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}}\operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2c}+\frac{e^a\operatorname{Ei}(bx)}{c}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x)/(x\*(c + d\*x^2)), x]

[Out] (E^a\*ExpIntegralEi[b\*x])/c - (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-(b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d]])/(2\*c) - (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*c)

#### Rule 2271

Int[((F\_)^((g\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.))\*(u\_)^(m\_.))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), u^m/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx}}{x(c+dx^2)} dx &= \int \left( \frac{e^{a+bx}}{cx} - \frac{de^{a+bx}x}{c(c+dx^2)} \right) dx \\
&= \frac{\int \frac{e^{a+bx}}{x} dx}{c} - \frac{d \int \frac{e^{a+bx}x}{c+dx^2} dx}{c} \\
&= \frac{e^a \operatorname{Ei}(bx)}{c} - \frac{d \int \left( -\frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c}-\sqrt{dx})} + \frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c}+\sqrt{dx})} \right) dx}{c} \\
&= \frac{e^a \operatorname{Ei}(bx)}{c} + \frac{\sqrt{d} \int \frac{e^{a+bx}}{\sqrt{-c}-\sqrt{dx}} dx}{2c} - \frac{\sqrt{d} \int \frac{e^{a+bx}}{\sqrt{-c}+\sqrt{dx}} dx}{2c} \\
&= \frac{e^a \operatorname{Ei}(bx)}{c} - \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2c} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2c}
\end{aligned}$$

**Mathematica [C]** time = 0.120942, size = 93, normalized size = 0.84

$$\frac{e^a \left( 2\operatorname{Ei}(bx) - e^{-\frac{ib\sqrt{c}}{\sqrt{d}}} \left( e^{\frac{2ib\sqrt{c}}{\sqrt{d}}} \operatorname{Ei}\left(b\left(x - \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + \operatorname{Ei}\left(b\left(x + \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) \right) \right)}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x)/(x\*(c + d\*x^2)),x]

[Out] (E^a\*(2\*ExpIntegralEi[b\*x] - (E^(((2\*I)\*b\*Sqrt[c])/Sqrt[d])\*ExpIntegralEi[b\*((-I)\*Sqrt[c])/Sqrt[d] + x]) + ExpIntegralEi[b\*((I\*Sqrt[c])/Sqrt[d] + x] )/E^(((I\*b\*Sqrt[c])/Sqrt[d])))/(2\*c)

**Maple [A]** time = 0.018, size = 112, normalized size = 1.

$$-\frac{e^a \operatorname{Ei}(1, -bx)}{c} + \frac{1}{2c} \left( e^{\frac{1}{d}(b\sqrt{-cd}+ad)} \operatorname{Ei}\left(1, \frac{1}{d} \left( b\sqrt{-cd} - d(bx+a) + ad \right) \right) + e^{-\frac{1}{d}(b\sqrt{-cd}-ad)} \operatorname{Ei}\left(1, -\frac{1}{d} \left( b\sqrt{-cd} + d(bx+a) - ad \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)/x/(d\*x^2+c),x)

[Out] -1/c\*exp(a)\*Ei(1,-b\*x)+1/2\*(exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1,(b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)+exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1,-(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d))/c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{(dx^2+c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x/(d\*x^2+c),x, algorithm="maxima")

[Out] integrate(e^(b\*x + a)/((d\*x^2 + c)\*x), x)

**Fricas [A]** time = 1.55933, size = 167, normalized size = 1.5

$$\frac{\operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} + \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} - 2\operatorname{Ei}(bx)e^a}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x/(d\*x^2+c), x, algorithm="fricas")

[Out] -1/2\*(Ei(b\*x - sqrt(-b^2\*c/d))\*e^(a + sqrt(-b^2\*c/d)) + Ei(b\*x + sqrt(-b^2\*c/d))\*e^(a - sqrt(-b^2\*c/d)) - 2\*Ei(b\*x)\*e^a)/c

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{e^{bx}}{cx + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x/(d\*x\*\*2+c), x)

[Out] exp(a)\*Integral(exp(b\*x)/(c\*x + d\*x\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{(dx^2 + c)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/x/(d\*x^2+c), x, algorithm="giac")

[Out] integrate(e^(b\*x + a)/((d\*x^2 + c)\*x), x)

$$3.464 \quad \int \frac{e^{a+bx}}{c+dx^2} dx$$

**Optimal.** Leaf size=118

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

[Out] (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-((b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d])))/(2\*Sqrt[-c]\*Sqrt[d]) - (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*Sqrt[-c]\*Sqrt[d])

**Rubi [A]** time = 0.119093, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2269, 2178}

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x)/(c + d\*x^2), x]

[Out] (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-((b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d])))/(2\*Sqrt[-c]\*Sqrt[d]) - (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*Sqrt[-c]\*Sqrt[d])

#### Rule 2269

Int[(F\_)^((g\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.))/((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), 1/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx}}{c+dx^2} dx &= \int \left( \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}-\sqrt{dx})} + \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}+\sqrt{dx})} \right) dx \\ &= -\frac{\int \frac{e^{a+bx}}{\sqrt{-c}-\sqrt{dx}} dx}{2\sqrt{-c}} - \frac{\int \frac{e^{a+bx}}{\sqrt{-c}+\sqrt{dx}} dx}{2\sqrt{-c}} \\ &= \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} \end{aligned}$$

**Mathematica [C]** time = 0.0537059, size = 94, normalized size = 0.8

$$\frac{ie^{a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\operatorname{Ei}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)-\operatorname{Ei}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)\right)}{2\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x)/(c + d\*x^2), x]

[Out]  $((-I/2)*E^{(a - (I*b*Sqrt[c])/Sqrt[d])}*(E^{((2*I)*b*Sqrt[c])/Sqrt[d]}*ExpIntegralEi[b*((-I)*Sqrt[c])/Sqrt[d] + x]) - ExpIntegralEi[b*((I*Sqrt[c])/Sqrt[d] + x)])/(Sqrt[c]*Sqrt[d])$

**Maple [A]** time = 0.01, size = 102, normalized size = 0.9

$$-\frac{1}{2}\left(e^{\frac{1}{d}(b\sqrt{-cd}+ad)}\operatorname{Ei}\left(1, \frac{1}{d}\left(b\sqrt{-cd}-d(bx+a)+ad\right)\right)-e^{-\frac{1}{d}(b\sqrt{-cd}-ad)}\operatorname{Ei}\left(1, -\frac{1}{d}\left(b\sqrt{-cd}+d(bx+a)-ad\right)\right)\right)\frac{1}{\sqrt{-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)/(d\*x^2+c), x)

[Out]  $-1/2*(\exp((b*(-c*d)^{(1/2)}+a*d)/d)*\operatorname{Ei}(1, (b*(-c*d)^{(1/2)}-d*(b*x+a)+a*d)/d)-\exp(-(b*(-c*d)^{(1/2)}-a*d)/d)*\operatorname{Ei}(1, -(b*(-c*d)^{(1/2)}+d*(b*x+a)-a*d)/d))/(-c*d)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/(d\*x^2+c), x, algorithm="maxima")

[Out] integrate(e^(b\*x + a)/(d\*x^2 + c), x)

**Fricas [A]** time = 1.51285, size = 192, normalized size = 1.63

$$\frac{\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)e^{\left(a+\sqrt{-\frac{b^2c}{d}}\right)}-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)e^{\left(a-\sqrt{-\frac{b^2c}{d}}\right)}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/(d\*x^2+c), x, algorithm="fricas")

[Out]  $-1/2*(\sqrt{-b^2*c/d}*\operatorname{Ei}(b*x - \sqrt{-b^2*c/d})*e^{(a + \sqrt{-b^2*c/d})} - \sqrt{-b^2*c/d}*\operatorname{Ei}(b*x + \sqrt{-b^2*c/d})*e^{(a - \sqrt{-b^2*c/d})})/(b*c)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{e^{bx}}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/(d\*x\*\*2+c),x)

[Out] exp(a)\*Integral(exp(b\*x)/(c + d\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(bx+a)}}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(e^(b\*x + a)/(d\*x^2 + c), x)

$$3.465 \quad \int \frac{e^{a+bx} x}{c+dx^2} dx$$

**Optimal.** Leaf size=100

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d} + \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d}$$

[Out] (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-((b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d])))/(2\*d) + (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*d)

**Rubi [A]** time = 0.128842, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2271, 2178}

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d} + \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(E^(a + b\*x)\*x)/(c + d\*x^2), x]

[Out] (E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-((b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d])))/(2\*d) + (E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*d)

#### Rule 2271

Int[((F\_)^((g\_)\*((d\_) + (e\_)\*(x\_))^(n\_)))\*(u\_)^(m\_)]/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), u^m/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d]/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx} x}{c+dx^2} dx &= \int \left( \frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c}-\sqrt{dx})} + \frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c}+\sqrt{dx})} \right) dx \\ &= \frac{\int \frac{e^{a+bx}}{\sqrt{-c}-\sqrt{dx}} dx}{2\sqrt{d}} + \frac{\int \frac{e^{a+bx}}{\sqrt{-c}+\sqrt{dx}} dx}{2\sqrt{d}} \\ &= \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d} + \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2d} \end{aligned}$$

**Mathematica [C]** time = 0.0542735, size = 83, normalized size = 0.83

$$\frac{e^{a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\operatorname{Ei}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\operatorname{Ei}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x)\*x)/(c + d\*x^2), x]

[Out] (E^(a - (I\*b\*Sqrt[c])/Sqrt[d])\*(E^(((2\*I)\*b\*Sqrt[c])/Sqrt[d])\*ExpIntegralEi[b\*((( -I)\*Sqrt[c])/Sqrt[d] + x)] + ExpIntegralEi[b\*((I\*Sqrt[c])/Sqrt[d] + x)]))/(2\*d)

**Maple [B]** time = 0.012, size = 323, normalized size = 3.2

$$\frac{1}{b^2}\left(-\frac{b}{2d}\left(\sqrt{-cd}e^{\frac{1}{d}(b\sqrt{-cd}+ad)}\operatorname{Ei}\left(1,\frac{1}{d}\left(b\sqrt{-cd}-d(bx+a)+ad\right)\right)b+\sqrt{-cd}e^{-\frac{1}{d}(b\sqrt{-cd}-ad)}\operatorname{Ei}\left(1,-\frac{1}{d}\left(b\sqrt{-cd}+d(bx+a)-ad\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)\*x/(d\*x^2+c), x)

[Out] 1/b^2\*(-1/2\*b/d\*((-c\*d)^(1/2)\*exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1,(b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*b+(-c\*d)^(1/2)\*exp(-b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1,-(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*b+exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1,(b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*a\*d-exp(-b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1,-(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*a\*d)/(-c\*d)^(1/2)+1/2\*a\*b\*(exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1,(b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)-exp(-b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1,-(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d))/(-c\*d)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(bx+a)}}{bdx^2+bc}+\int\frac{(dx^2e^a-ce^a)e^{(bx)}}{bd^2x^4+2bcdx^2+bc^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x/(d\*x^2+c), x, algorithm="maxima")

[Out] x\*e^(b\*x + a)/(b\*d\*x^2 + b\*c) + integrate((d\*x^2\*e^a - c\*e^a)\*e^(b\*x)/(b\*d^2\*x^4 + 2\*b\*c\*d\*x^2 + b\*c^2), x)

**Fricas [A]** time = 1.57449, size = 144, normalized size = 1.44

$$\frac{\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)e^{\left(a+\sqrt{-\frac{b^2c}{d}}\right)}+\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)e^{\left(a-\sqrt{-\frac{b^2c}{d}}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(exp(b\*x+a)\*x/(d\*x^2+c),x, algorithm="fricas")

[Out] 1/2\*(Ei(b\*x - sqrt(-b^2\*c/d))\*e^(a + sqrt(-b^2\*c/d)) + Ei(b\*x + sqrt(-b^2\*c/d))\*e^(a - sqrt(-b^2\*c/d)))/d

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{x e^{bx}}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x/(d\*x\*\*2+c),x)

[Out] exp(a)\*Integral(x\*exp(b\*x)/(c + d\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x e^{(bx+a)}}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(x\*e^(b\*x + a)/(d\*x^2 + c), x)

$$3.466 \quad \int \frac{e^{a+bx}x^2}{c+dx^2} dx$$

**Optimal.** Leaf size=132

$$\frac{\sqrt{-c}e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{\sqrt{-c}e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{e^{a+bx}}{bd}$$

[Out] E^(a + b\*x)/(b\*d) + (Sqrt[-c]\*E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-(b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d]])/(2\*d^(3/2)) - (Sqrt[-c]\*E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*d^(3/2))

**Rubi [A]** time = 0.23545, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2271, 2194, 2269, 2178}

$$\frac{\sqrt{-c}e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{\sqrt{-c}e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{e^{a+bx}}{bd}$$

Antiderivative was successfully verified.

[In] Int[(E^(a + b\*x)\*x^2)/(c + d\*x^2), x]

[Out] E^(a + b\*x)/(b\*d) + (Sqrt[-c]\*E^(a + (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[-(b\*(Sqrt[-c] - Sqrt[d]\*x))/Sqrt[d]])/(2\*d^(3/2)) - (Sqrt[-c]\*E^(a - (b\*Sqrt[-c])/Sqrt[d])\*ExpIntegralEi[(b\*(Sqrt[-c] + Sqrt[d]\*x))/Sqrt[d]])/(2\*d^(3/2))

#### Rule 2271

Int[((F\_)^((g\_)\*((d\_) + (e\_)\*(x\_))^(n\_)))\*(u\_)^(m\_)]/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), u^m/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2269

Int[(F\_)^((g\_)\*((d\_) + (e\_)\*(x\_))^(n\_))]/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), 1/(a + c\*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x]

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx} x^2}{c+dx^2} dx &= \int \left( \frac{e^{a+bx}}{d} - \frac{ce^{a+bx}}{d(c+dx^2)} \right) dx \\
&= \frac{\int e^{a+bx} dx}{d} - \frac{c \int \frac{e^{a+bx}}{c+dx^2} dx}{d} \\
&= \frac{e^{a+bx}}{bd} - \frac{c \int \left( \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}-\sqrt{dx})} + \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}+\sqrt{dx})} \right) dx}{d} \\
&= \frac{e^{a+bx}}{bd} - \frac{\sqrt{-c} \int \frac{e^{a+bx}}{\sqrt{-c}-\sqrt{dx}} dx}{2d} - \frac{\sqrt{-c} \int \frac{e^{a+bx}}{\sqrt{-c}+\sqrt{dx}} dx}{2d} \\
&= \frac{e^{a+bx}}{bd} + \frac{\sqrt{-c} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei} \left( -\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}} \right)}{2d^{3/2}} - \frac{\sqrt{-c} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{Ei} \left( \frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}} \right)}{2d^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.122203, size = 120, normalized size = 0.91

$$\frac{e^a \left( ib\sqrt{ce} \frac{ib\sqrt{c}}{\sqrt{d}} \operatorname{Ei} \left( b \left( x - \frac{i\sqrt{c}}{\sqrt{d}} \right) \right) - ib\sqrt{ce} \frac{-ib\sqrt{c}}{\sqrt{d}} \operatorname{Ei} \left( b \left( x + \frac{i\sqrt{c}}{\sqrt{d}} \right) \right) + 2\sqrt{d}e^{bx} \right)}{2bd^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x)\*x^2)/(c + d\*x^2), x]

[Out] (E^a\*(2\*Sqrt[d]\*E^(b\*x) + I\*b\*Sqrt[c]\*E^((I\*b\*Sqrt[c])/Sqrt[d])\*ExpIntegralEi[b\*((-I)\*Sqrt[c])/Sqrt[d] + x]) - (I\*b\*Sqrt[c]\*ExpIntegralEi[b\*(I\*Sqrt[c])/Sqrt[d] + x])/E^((I\*b\*Sqrt[c])/Sqrt[d]))/(2\*b\*d^(3/2))

**Maple [B]** time = 0.017, size = 660, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)\*x^2/(d\*x^2+c), x)

[Out] 1/b^3\*(b^2/d\*exp(b\*x+a)-1/2/d\*b\*(2\*exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*(-c\*d)^(1/2)\*a\*b+exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*a^2\*d-exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*b^2\*c+2\*exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*(-c\*d)^(1/2)\*a\*b-exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*a^2\*d+exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*b^2\*c)/(-c\*d)^(1/2)-1/2\*a^2\*b\*(exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)-exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d))/(-c\*d)^(1/2)+a\*b/d\*((-c\*d)^(1/2)\*exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*b+(-c\*d)^(1/2)\*exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*b+exp((b\*(-c\*d)^(1/2)+a\*d)/d)\*Ei(1, (b\*(-c\*d)^(1/2)-d\*(b\*x+a)+a\*d)/d)\*a\*d-exp(-(b\*(-c\*d)^(1/2)-a\*d)/d)\*Ei(1, -(b\*(-c\*d)^(1/2)+d\*(b\*x+a)-a\*d)/d)\*a\*d)/(-c\*d)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^2 e^{(bx+a)}}{bdx^2 + bc} - 2c \int \frac{x e^{(bx+a)}}{bd^2 x^4 + 2bcdx^2 + bc^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x^2/(d\*x^2+c),x, algorithm="maxima")

[Out] x^2\*e^(b\*x + a)/(b\*d\*x^2 + b\*c) - 2\*c\*integrate(x\*e^(b\*x + a)/(b\*d^2\*x^4 + 2\*b\*c\*d\*x^2 + b\*c^2), x)

**Fricas [A]** time = 1.56955, size = 212, normalized size = 1.61

$$\frac{\sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} - \sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} + 2e^{(bx+a)}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x^2/(d\*x^2+c),x, algorithm="fricas")

[Out] 1/2\*(sqrt(-b^2\*c/d)\*Ei(b\*x - sqrt(-b^2\*c/d))\*e^(a + sqrt(-b^2\*c/d)) - sqrt(-b^2\*c/d)\*Ei(b\*x + sqrt(-b^2\*c/d))\*e^(a - sqrt(-b^2\*c/d)) + 2\*e^(b\*x + a))/(b\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a \int \frac{x^2 e^{bx}}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x\*\*2/(d\*x\*\*2+c),x)

[Out] exp(a)\*Integral(x\*\*2\*exp(b\*x)/(c + d\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{(bx+a)}}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*x^2/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(x^2\*e^(b\*x + a)/(d\*x^2 + c), x)

**3.467**  $\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$

**Optimal.** Leaf size=212

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \text{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2a^2} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \text{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2a^2} - \frac{be^d \text{Ei}(ex)}{a^2} + \frac{e^d e \text{Ei}(ex)}{a}$$

```
[Out] -(E^(d + e*x)/(a*x)) - (b*E^d*ExpIntegralEi[e*x])/a^2 + (e*E^d*ExpIntegralEi[e*x])/a + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)])/((2*a^2) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)])/((2*a^2)
```

**Rubi [A]** time = 0.620086, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2270, 2177, 2178}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \text{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2a^2} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \text{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2a^2} - \frac{be^d \text{Ei}(ex)}{a^2} + \frac{e^d e \text{Ei}(ex)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(d + e*x)/(x^2*(a + b*x + c*x^2)), x]
```

```
[Out] -(E^(d + e*x)/(a*x)) - (b*E^d*ExpIntegralEi[e*x])/a^2 + (e*E^d*ExpIntegralEi[e*x])/a + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)])/((2*a^2) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)])/((2*a^2)
```

**Rule 2270**

```
Int[((F_)^((g_)*((d_) + (e_)*(x_)))^(n_))*(u_)^(m_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]
```

**Rule 2177**

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1)), x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma == True
```

**Rule 2178**

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

## Rubi steps

$$\begin{aligned}
\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx &= \int \left( \frac{e^{d+ex}}{ax^2} - \frac{be^{d+ex}}{a^2x} + \frac{e^{d+ex}(b^2-ac+bcx)}{a^2(a+bx+cx^2)} \right) dx \\
&= \frac{\int \frac{e^{d+ex}(b^2-ac+bcx)}{a+bx+cx^2} dx}{a^2} + \frac{\int \frac{e^{d+ex}}{x^2} dx}{a} - \frac{b \int \frac{e^{d+ex}}{x} dx}{a^2} \\
&= -\frac{e^{d+ex}}{ax} - \frac{be^d \text{Ei}(ex)}{a^2} + \frac{\int \left( \frac{\left( bc + \frac{c(b^2-2ac)}{\sqrt{b^2-4ac}} \right) e^{d+ex}}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left( bc - \frac{c(b^2-2ac)}{\sqrt{b^2-4ac}} \right) e^{d+ex}}{b+\sqrt{b^2-4ac}+2cx} \right) dx}{a^2} + \frac{e \int \frac{e^{d+ex}}{x} dx}{a} \\
&= -\frac{e^{d+ex}}{ax} - \frac{be^d \text{Ei}(ex)}{a^2} + \frac{ee^d \text{Ei}(ex)}{a} + \frac{\left( c \left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \right) \int \frac{e^{d+ex}}{b+\sqrt{b^2-4ac}+2cx} dx}{a^2} + \frac{\left( c \left( b + \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \right) \int \frac{e^{d+ex}}{b-\sqrt{b^2-4ac}+2cx} dx}{a^2} \\
&= -\frac{e^{d+ex}}{ax} - \frac{be^d \text{Ei}(ex)}{a^2} + \frac{ee^d \text{Ei}(ex)}{a} + \frac{\left( b + \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{Ei}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a^2} + \frac{\left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{Ei}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a^2}
\end{aligned}$$

**Mathematica [A]** time = 1.13093, size = 232, normalized size = 1.09

$$e^d \left( \frac{e^{\frac{\sqrt{b^2-4ac}+b}{2c}} \left( x \left( b\sqrt{b^2-4ac}-2ac+b^2 \right) e^{\frac{\sqrt{b^2-4ac}}{c}} \text{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right) + x \left( b\sqrt{b^2-4ac}+2ac-b^2 \right) \text{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right) - 2a\sqrt{b^2-4ac} e^{\frac{\sqrt{b^2-4ac}+b+2cx}{2c}} \right)}{x\sqrt{b^2-4ac}} \right) - 2(b)$$


---


$$2a^2$$

Antiderivative was successfully verified.

[In] Integrate[E^(d + e\*x)/(x^2\*(a + b\*x + c\*x^2)), x]

[Out] (E^d\*(-2\*(b - a\*e)\*ExpIntegralEi[e\*x] + (-2\*a\*Sqrt[b^2 - 4\*a\*c]\*E^((e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)) + (b^2 - 2\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*E^((Sqrt[b^2 - 4\*a\*c]\*e)/c)\*x\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]) + (-b^2 + 2\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*x\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c]))/(Sqrt[b^2 - 4\*a\*c]\*E^((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*x)/(2\*a^2)

**Maple [B]** time = 0.023, size = 561, normalized size = 2.7

$$e \left( -\frac{e^{ex+d}}{aex} - \frac{(ae-b)e^d \text{Ei}(1, -ex)}{a^2 e} - \frac{1}{2a^2 e} \left( -2e^{1/2} e^{\frac{-be+2cd+\sqrt{-4ace^2+b^2e^2}}{c}} \text{Ei}\left(1, 1/2 \frac{-2c(ex+d)-be+2cd+\sqrt{-4ace^2+b^2e^2}}{c}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*x+d)/x^2/(c\*x^2+b\*x+a), x)

[Out] e\*(-exp(e\*x+d)/a/x/e-1/a^2/e\*(a\*e-b)\*exp(d)\*Ei(1, -e\*x)-1/2\*(-2\*exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*a\*c\*e+exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*b^2\*e+2\*exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e

$$\begin{aligned} & *x+d)+b*e^{-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}}/c)*a*c*e^{-\exp(-1/2*(b*e^{-2*c*d}+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c)*\text{Ei}(1,-1/2*(2*c*(e*x+d)+b*e^{-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}})/c)*b^2*e+\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2})))* \\ & \text{Ei}(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c)*(-4*a*c*e^2+b^2*e^2)^{1/2}*b+\exp(-1/2*(b*e^{-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}})/c)*\text{Ei}(1,- \\ & 1/2*(2*c*(e*x+d)+b*e^{-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}})/c)*(-4*a*c*e^2+b^2*e^2)^{1/2}*b)/a^2/e/(-4*a*c*e^2+b^2*e^2)^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{(cx^2 + bx + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x^2/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] integrate(e^(e\*x + d)/((c\*x^2 + b\*x + a)\*x^2), x)

**Fricas [A]** time = 1.58163, size = 682, normalized size = 3.22

$$2\left(\left(ab^2 - 4a^2c\right)e^2 - \left(b^3 - 4abc\right)e\right)x\text{Ei}(ex)e^d - 2\left(ab^2 - 4a^2c\right)ee^{(ex+d)} + \left(\left(b^3 - 4abc\right)ex + \left(b^2c - 2ac^2\right)\sqrt{\frac{(b^2-4ac)e^2}{c^2}}x\right)\text{Ei}\left(\frac{1}{2}\left(\frac{b^2-4ac}{c^2}\right)^{1/2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x^2/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(2*((a*b^2 - 4*a^2*c)*e^2 - (b^3 - 4*a*b*c)*e)*x*\text{Ei}(e*x)*e^d - 2*(a*b^2 - 4*a^2*c)*e*e^{(e*x + d)} + ((b^3 - 4*a*b*c)*e*x + (b^2*c - 2*a*c^2)*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)*x)*\text{Ei}(1/2*(2*c*e*x + b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)))/c)*e^{(1/2*(2*c*d - b*e + c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)))/c} + ((b^3 - 4*a*b*c)*e*x - (b^2*c - 2*a*c^2)*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)*x)*\text{Ei}(1/2*(2*c*e*x + b*e + c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)))/c)*e^{(1/2*(2*c*d - b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)))/c)}/((a^2*b^2 - 4*a^3*c)*e*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x\*\*2/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{(cx^2 + bx + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```



$$3.468 \quad \int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=169

$$\frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(b - \sqrt{b^2-4ac})}{2c}} \text{Ei}\left(\frac{e(b+2cx - \sqrt{b^2-4ac})}{2c}\right)}{2a} - \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(\sqrt{b^2-4ac} + b)}{2c}} \text{Ei}\left(\frac{e(b+2cx + \sqrt{b^2-4ac})}{2c}\right)}{2a} + \frac{e^d \text{Ei}(ex)}{a}$$

[Out] (E^d\*ExpIntegralEi[e\*x])/a - ((1 + b/Sqrt[b^2 - 4\*a\*c])\*E^(d - ((b - Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c]])/ (2\*a) - ((1 - b/Sqrt[b^2 - 4\*a\*c])\*E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c]])/ (2\*a)

**Rubi [A]** time = 0.411215, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2270, 2178}

$$\frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(b - \sqrt{b^2-4ac})}{2c}} \text{Ei}\left(\frac{e(b+2cx - \sqrt{b^2-4ac})}{2c}\right)}{2a} - \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(\sqrt{b^2-4ac} + b)}{2c}} \text{Ei}\left(\frac{e(b+2cx + \sqrt{b^2-4ac})}{2c}\right)}{2a} + \frac{e^d \text{Ei}(ex)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(d + e\*x)/(x\*(a + b\*x + c\*x^2)), x]

[Out] (E^d\*ExpIntegralEi[e\*x])/a - ((1 + b/Sqrt[b^2 - 4\*a\*c])\*E^(d - ((b - Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c]])/ (2\*a) - ((1 - b/Sqrt[b^2 - 4\*a\*c])\*E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c]])/ (2\*a)

#### Rule 2270

Int[((F\_)^((g\_)\*((d\_) + (e\_)\*(x\_))^(n\_))\*(u\_)^(m\_))]/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), u^m/(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx &= \int \left( \frac{e^{d+ex}}{ax} + \frac{e^{d+ex}(-b-cx)}{a(a+bx+cx^2)} \right) dx \\
&= \frac{\int \frac{e^{d+ex}}{x} dx}{a} + \frac{\int \frac{e^{d+ex}(-b-cx)}{a+bx+cx^2} dx}{a} \\
&= \frac{e^d \operatorname{Ei}(ex)}{a} + \frac{\int \left( \frac{\left(-c - \frac{bc}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} + \frac{\left(-c + \frac{bc}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} \right) dx}{a} \\
&= \frac{e^d \operatorname{Ei}(ex)}{a} - \frac{\left(c \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} dx}{a} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} dx}{a} \\
&= \frac{e^d \operatorname{Ei}(ex)}{a} - \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b - \sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b - \sqrt{b^2-4ac} + 2cx)}{2c}\right)}{2a} - \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b + \sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b + \sqrt{b^2-4ac} + 2cx)}{2c}\right)}{2a}
\end{aligned}$$

**Mathematica [A]** time = 0.554525, size = 163, normalized size = 0.96

$$\frac{e^d \left( \frac{e^{\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \left( (b - \sqrt{b^2-4ac}) \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right) - (\sqrt{b^2-4ac}+b) e^{\frac{e\sqrt{b^2-4ac}}{c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right) \right)}{\sqrt{b^2-4ac}} + 2 \operatorname{Ei}(ex) \right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(d + e\*x)/(x\*(a + b\*x + c\*x^2)),x]

[Out] (E^d\*(2\*ExpIntegralEi[e\*x] + (-((b + Sqrt[b^2 - 4\*a\*c])\*E^((Sqrt[b^2 - 4\*a\*c]\*e)/c)\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]) + (b - Sqrt[b^2 - 4\*a\*c])\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)])/(Sqrt[b^2 - 4\*a\*c]\*E^(((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c)))))/(2\*a)

**Maple [B]** time = 0.018, size = 369, normalized size = 2.2

$$-\frac{e^d \operatorname{Ei}(1, -ex)}{a} + \frac{1}{2a} \left( e^{\frac{1}{2c}(-be+2cd+\sqrt{-4ace^2+b^2e^2})} \operatorname{Ei}\left(1, \frac{1}{2c}(-2c(ex+d) - be + 2cd + \sqrt{-4ace^2+b^2e^2})\right) be - e^{-\frac{1}{2c}(be-2cd+\sqrt{-4ace^2+b^2e^2})} \operatorname{Ei}\left(1, \frac{1}{2c}(-2c(ex+d) + be - 2cd + \sqrt{-4ace^2+b^2e^2})\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*x+d)/x/(c\*x^2+b\*x+a), x)

[Out] -1/a\*exp(d)\*Ei(1, -e\*x)+1/2\*(exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*b\*e-exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*b\*e+exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)+exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/a/(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{(cx^2 + bx + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] integrate(e^(e\*x + d)/((c\*x^2 + b\*x + a)\*x), x)

**Fricas [A]** time = 1.61726, size = 533, normalized size = 3.15

$$\frac{2(b^2 - 4ac)e\text{Ei}(ex)e^d - \left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} + (b^2 - 4ac)e \right) \text{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}} + \left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} - (b^2 - 4ac)e \right) \text{Ei}\left(\frac{2cex+be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\frac{2cd+be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}}}{2(ab^2 - 4a^2c)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] 1/2\*(2\*(b^2 - 4\*a\*c)\*e\*Ei(e\*x)\*e^d - (b\*c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2) + (b^2 - 4\*a\*c)\*e)\*Ei(1/2\*(2\*c\*e\*x + b\*e - c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c)\*e^(1/2\*(2\*c\*d - b\*e + c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c) + (b\*c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2) - (b^2 - 4\*a\*c)\*e)\*Ei(1/2\*(2\*c\*e\*x + b\*e + c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c)\*e^(1/2\*(2\*c\*d - b\*e - c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c))/((a\*b^2 - 4\*a^2\*c)\*e)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^d \int \frac{e^{ex}}{ax + bx^2 + cx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x/(c\*x\*\*2+b\*x+a),x)

[Out] exp(d)\*Integral(exp(e\*x)/(a\*x + b\*x\*\*2 + c\*x\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{(cx^2 + bx + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/x/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] integrate(e^(e\*x + d)/((c\*x^2 + b\*x + a)\*x), x)

$$3.469 \quad \int \frac{e^{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=138

$$\frac{e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}} - \frac{e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}}$$

[Out] (E^(d - ((b - Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]/Sqrt[b^2 - 4\*a\*c] - (E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]/Sqrt[b^2 - 4\*a\*c])

**Rubi [A]** time = 0.194263, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2268, 2178}

$$\frac{e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}} - \frac{e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[E^(d + e\*x)/(a + b\*x + c\*x^2), x]

[Out] (E^(d - ((b - Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]/Sqrt[b^2 - 4\*a\*c] - (E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]/Sqrt[b^2 - 4\*a\*c])

#### Rule 2268

Int[(F\_)^((g\_)\*((d\_) + (e\_)\*(x\_))^(n\_)))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[F^(g\*(d + e\*x)^n), 1/(a + b\*x + c\*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \int \left( \frac{2ce^{d+ex}}{\sqrt{b^2-4ac}(b-\sqrt{b^2-4ac}+2cx)} - \frac{2ce^{d+ex}}{\sqrt{b^2-4ac}(b+\sqrt{b^2-4ac}+2cx)} \right) dx$$

$$= \frac{(2c) \int \frac{e^{d+ex}}{b-\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{e^{d+ex}}{b+\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{\sqrt{b^2-4ac}} - \frac{e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{\sqrt{b^2-4ac}}$$

**Mathematica [A]** time = 0.15274, size = 127, normalized size = 0.92

$$\frac{e^{\frac{e(\sqrt{b^2-4ac}-b)}{2c}+d} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right) - e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(d + e\*x)/(a + b\*x + c\*x^2), x]

[Out] (E^(d + ((-b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)] - E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)])/Sqrt[b^2 - 4\*a\*c]

**Maple [A]** time = 0.011, size = 169, normalized size = 1.2

$$-e \left( e^{\frac{1}{2c}(-be+2cd+\sqrt{-4ace^2+b^2e^2})} \operatorname{Ei}\left(1, \frac{1}{2c}(-2c(ex+d) - be + 2cd + \sqrt{-4ace^2 + b^2e^2})\right) - e^{-\frac{1}{2c}(be-2cd+\sqrt{-4ace^2+b^2e^2})} \operatorname{Ei}\left(1, \frac{1}{2c}(-2c(ex+d) + be - 2cd + \sqrt{-4ace^2 + b^2e^2})\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*x+d)/(c\*x^2+b\*x+a), x)

[Out] -e\*(exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c) - exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c))/(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/(c\*x^2+b\*x+a), x, algorithm="maxima")

[Out] integrate(e^(e\*x + d)/(c\*x^2 + b\*x + a), x)

---

**Fricas [A]** time = 1.57157, size = 420, normalized size = 3.04

$$\frac{c\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \operatorname{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)} - c\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \operatorname{Ei}\left(\frac{2cex+be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)}}{(b^2-4ac)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] (c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2)\*Ei(1/2\*(2\*c\*e\*x + b\*e - c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c)\*e^(1/2\*(2\*c\*d - b\*e + c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c) - c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2)\*Ei(1/2\*(2\*c\*e\*x + b\*e + c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c)\*e^(1/2\*(2\*c\*d - b\*e - c\*sqrt((b^2 - 4\*a\*c)\*e^2/c^2))/c))/((b^2 - 4\*a\*c)\*e)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^d \int \frac{e^{ex}}{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/(c\*x\*\*2+b\*x+a),x)

[Out] exp(d)\*Integral(exp(e\*x)/(a + b\*x + c\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ex+d)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] integrate(e^(e\*x + d)/(c\*x^2 + b\*x + a), x)

$$3.470 \quad \int \frac{e^{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=158

$$\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c} + \frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c}$$

[Out]  $((1 - b/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\operatorname{ExpIntegralEi}[(e*(b - \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c) + ((1 + b/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\operatorname{ExpIntegralEi}[(e*(b + \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c)$

**Rubi [A]** time = 0.214408, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2270, 2178}

$$\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c} + \frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{(d + e*x)*x})/(a + b*x + c*x^2), x]$

[Out]  $((1 - b/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\operatorname{ExpIntegralEi}[(e*(b - \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c) + ((1 + b/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\operatorname{ExpIntegralEi}[(e*(b + \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c)$

#### Rule 2270

$\operatorname{Int}[(F^((g_*)*((d_*) + (e_*)(x_)))^{(n_*)}*(u_*)^{(m_*)})/((a_*) + (b_*)(x_*) + (c_*)(x_*)^2), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[F^{(g*(d + e*x)^n}], u^m/(a + b*x + c*x^2), x], x] /;$  FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2178

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)(x_)))}/((c_*) + (d_*)(x_*)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d)}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{e^{d+ex}}{a+bx+cx^2} dx &= \int \left( \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} + \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} \right) dx \\ &= \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} dx + \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} dx \\ &= \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b-\sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c} + \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b+\sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c} \end{aligned}$$

**Mathematica [A]** time = 0.178503, size = 153, normalized size = 0.97

$$\frac{e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \left( \left( \sqrt{b^2-4ac}-b \right) e^{\frac{e\sqrt{b^2-4ac}}{c}} \operatorname{Ei} \left( \frac{e(b+2cx-\sqrt{b^2-4ac})}{2c} \right) + \left( \sqrt{b^2-4ac}+b \right) \operatorname{Ei} \left( \frac{e(b+2cx+\sqrt{b^2-4ac})}{2c} \right) \right)}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(d + e\*x)\*x)/(a + b\*x + c\*x^2), x]

[Out] (E^(d - ((b + Sqrt[b^2 - 4\*a\*c])\*e)/(2\*c))\*((-b + Sqrt[b^2 - 4\*a\*c])\*E^((Sqrt[b^2 - 4\*a\*c]\*e)/c)\*ExpIntegralEi[(e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)] + (b + Sqrt[b^2 - 4\*a\*c])\*ExpIntegralEi[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(2\*c)]))/(2\*c\*Sqrt[b^2 - 4\*a\*c])

**Maple [B]** time = 0.011, size = 685, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*x+d)\*x/(c\*x^2+b\*x+a), x)

[Out] 1/e^2\*(-1/2\*e^2\*(-exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*b\*e+2\*exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*c\*d+exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*b\*e-2\*exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*c\*d+exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)+exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c/(-4\*a\*c\*e^2+b^2\*e^2)^(1/2)+d\*e^2\*(exp(1/2/c\*(-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))\*Ei(1, 1/2\*(-2\*c\*(e\*x+d)-b\*e+2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)-exp(-1/2\*(b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c)\*Ei(1, -1/2\*(2\*c\*(e\*x+d)+b\*e-2\*c\*d+(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))/c))/(-4\*a\*c\*e^2+b^2\*e^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x e^{(ex+d)}}{cex^2 + bex + ae} + \int \frac{(cx^2 e^d - ae^d) e^{(ex)}}{c^2 ex^4 + 2bcex^3 + 2abex + a^2 e + (b^2 e + 2ace)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x/(c\*x^2+b\*x+a), x, algorithm="maxima")

[Out] x\*e^(e\*x + d)/(c\*e\*x^2 + b\*e\*x + a\*e) + integrate((c\*x^2\*e^d - a\*e^d)\*e^(e\*x)/(c^2\*e\*x^4 + 2\*b\*c\*e\*x^3 + 2\*a\*b\*e\*x + a^2\*e + (b^2\*e + 2\*a\*c\*e)\*x^2), x)



---

**Fricas [A]** time = 1.53259, size = 491, normalized size = 3.11

$$\frac{\left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} - (b^2 - 4ac)e \right) \operatorname{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)} - \left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} + (b^2 - 4ac)e \right) \operatorname{Ei}\left(\frac{2cex+be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)}{2(b^2c - 4ac^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] 
$$-1/2*((b*c*\sqrt{(b^2 - 4*a*c)*e^2/c^2} - (b^2 - 4*a*c)*e)*\operatorname{Ei}(1/2*(2*c*e*x + b*e - c*\sqrt{(b^2 - 4*a*c)*e^2/c^2}))/c)*e^{(1/2*(2*c*d - b*e + c*\sqrt{(b^2 - 4*a*c)*e^2/c^2}))/c} - (b*c*\sqrt{(b^2 - 4*a*c)*e^2/c^2} + (b^2 - 4*a*c)*e)*\operatorname{Ei}(1/2*(2*c*e*x + b*e + c*\sqrt{(b^2 - 4*a*c)*e^2/c^2}))/c)*e^{(1/2*(2*c*d - b*e - c*\sqrt{(b^2 - 4*a*c)*e^2/c^2}))/c}))/((b^2*c - 4*a*c^2)*e)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^d \int \frac{x e^{ex}}{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x/(c\*x\*\*2+b\*x+a),x)

[Out] exp(d)\*Integral(x\*exp(e\*x)/(a + b\*x + c\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x e^{(ex+d)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] integrate(x\*e^(e\*x + d)/(c\*x^2 + b\*x + a), x)

$$3.471 \quad \int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=186

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c^2} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) e^{d - \frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c^2} + \frac{e^{d+ex}}{ce}$$

[Out]  $E^{(d + e*x)/(c*e)} - ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \text{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\text{ExpIntegralEi}[(e*(b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^2) - ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \text{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\text{ExpIntegralEi}[(e*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^2)$

**Rubi [A]** time = 0.407889, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2270, 2194, 2178}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c^2} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) e^{d - \frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c^2} + \frac{e^{d+ex}}{ce}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{(d + e*x)*x^2})/(a + b*x + c*x^2), x]$

[Out]  $E^{(d + e*x)/(c*e)} - ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \text{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\text{ExpIntegralEi}[(e*(b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^2) - ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \text{Sqrt}[b^2 - 4*a*c])*e)/(2*c))}*\text{ExpIntegralEi}[(e*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^2)$

#### Rule 2270

$\text{Int}[(F_{-})^{((g_{-})*((d_{-}) + (e_{-})*(x_{-}))^{(n_{-}))}*(u_{-})^{(m_{-})})/((a_{-}) + (b_{-})*(x_{-}) + (c_{-})*(x_{-})^2), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(g*(d + e*x)^n)}, u^m/(a + b*x + c*x^2), x], x] /;$  FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]

#### Rule 2194

$\text{Int}[(F_{-})^{((c_{-})*((a_{-}) + (b_{-})*(x_{-})))^{(n_{-})}), x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)})^n/(b*c*n*\text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, n}, x]

#### Rule 2178

$\text{Int}[(F_{-})^{((g_{-})*((e_{-}) + (f_{-})*(x_{-}))) / ((c_{-}) + (d_{-})*(x_{-})), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx &= \int \left( \frac{e^{d+ex}}{c} - \frac{e^{d+ex}(a+bx)}{c(a+bx+cx^2)} \right) dx \\
&= \frac{\int e^{d+ex} dx}{c} - \frac{\int \frac{e^{d+ex}(a+bx)}{a+bx+cx^2} dx}{c} \\
&= \frac{e^{d+ex}}{ce} - \frac{\int \left( \frac{\left(b + \frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} + \frac{\left(b - \frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} \right) dx}{c} \\
&= \frac{e^{d+ex}}{ce} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{e^{d+ex}}{b - \sqrt{b^2-4ac} + 2cx} dx}{c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{e^{d+ex}}{b + \sqrt{b^2-4ac} + 2cx} dx}{c} \\
&= \frac{e^{d+ex}}{ce} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b - \sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b - \sqrt{b^2-4ac} + 2cx)}{2c}\right)}{2c^2} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b + \sqrt{b^2-4ac})e}{2c}} \operatorname{Ei}\left(\frac{e(b + \sqrt{b^2-4ac} + 2cx)}{2c}\right)}{2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.519179, size = 217, normalized size = 1.17

$$\frac{e^{d - \frac{e(\sqrt{b^2-4ac}+b)}{2c}} \left( e \left( b\sqrt{b^2-4ac} + 2ac - b^2 \right) e^{\frac{e\sqrt{b^2-4ac}}{c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right) + e \left( b\sqrt{b^2-4ac} - 2ac + b^2 \right) \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right) \right)}{2c^2 e \sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(d + e\*x)\*x^2)/(a + b\*x + c\*x^2), x]

[Out]  $-(E^{(d - ((b + \text{Sqrt}[b^2 - 4*a*c]))*e)/(2*c))}*(-2*c*\text{Sqrt}[b^2 - 4*a*c]*E^{((e*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c))} + (-b^2 + 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*e*E^{((\text{Sqrt}[b^2 - 4*a*c]*e)/c)*\text{ExpIntegralEi}[(e*(b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)] + (b^2 - 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*e*\text{ExpIntegralEi}[(e*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c])])/(2*c^2*\text{Sqrt}[b^2 - 4*a*c]*e)}$

**Maple [B]** time = 0.017, size = 1730, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(e\*x+d)\*x^2/(c\*x^2+b\*x+a), x)

[Out]  $1/e^3*(e^2/c*\exp(e*x+d)+1/2/c^2*e^2*(2*\operatorname{Ei}(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c)*\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))) * a*c*e^2 - \operatorname{Ei}(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2})) * b^2*e^2 + 2*\operatorname{Ei}(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2})) * b*c*d*e - 2*\operatorname{Ei}(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2})) * c^2*d^2 - 2*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \operatorname{Ei}(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * a*c*e^2 + \exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \operatorname{Ei}(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * b^2*e^2 - 2*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) * \operatorname{Ei}(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{1/2}))/c) *$

$b*c*d*e+2*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*c^2*d^2+Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2)*\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*b*e-2*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2)*\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*c*d+\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2)*b*e-2*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2)*c*d/(-4*a*c*e^2+b^2*e^2)^(1/2)-d^2*e^2*(\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)-\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c))*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c))/(-4*a*c*e^2+b^2*e^2)^(1/2)+d*e^2*(-\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b*e+2*\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*c*d+\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b*e-2*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*c*d+\exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2)+\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*(-4*a*c*e^2+b^2*e^2)^(1/2))/(-4*a*c*e^2+b^2*e^2)^(1/2))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^2 e^{(ex+d)}}{cex^2 + bex + ae} - \int \frac{(bx^2 e^d + 2axe^d) e^{(ex)}}{c^2 ex^4 + 2bcex^3 + 2abex + a^2 e + (b^2 e + 2ace)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x^2/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out]  $x^2 * e^{(e*x + d)} / (c * e * x^2 + b * e * x + a * e) - \text{integrate}((b * x^2 * e^d + 2 * a * x * e^d) * e^{(e * x)} / (c^2 * e * x^4 + 2 * b * c * e * x^3 + 2 * a * b * e * x + a^2 * e + (b^2 * e + 2 * a * c * e) * x^2), x)$

**Fricas [A]** time = 1.64275, size = 583, normalized size = 3.13

$$\frac{\left( (b^3 - 4abc)e - (b^2c - 2ac^2) \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}} \right) Ei \left( \frac{2cex + be - c \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}}}{2c} \right) e^{\left( \frac{2cd - be + c \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}}}{2c} \right)} + \left( (b^3 - 4abc)e + (b^2c - 2ac^2) \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}} \right) Ei \left( \frac{2cex + be + c \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}}}{2c} \right) e^{\left( \frac{2cd - be - c \sqrt{\frac{(b^2 - 4ac)e^2}{c^2}}}{2c} \right)}}{2(b^2c^2 - 4ac^3)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x^2/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out]  $-1/2 * (((b^3 - 4*a*b*c)*e - (b^2*c - 2*a*c^2)*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2)) * Ei(1/2*(2*c*e*x + b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c) * e^{(1/2*(2*c*d - b*$

$$e + c\sqrt{(b^2 - 4ac)e^{2/c^2}}/c + ((b^3 - 4abc)e + (b^2c - 2ac^2)\sqrt{(b^2 - 4ac)e^{2/c^2}})Ei(1/2(2cex + be + c\sqrt{(b^2 - 4ac)e^{2/c^2}})/c) * e^{(1/2(2cd - be - c\sqrt{(b^2 - 4ac)e^{2/c^2}})/c)} - 2 * (b^2c - 4ac^2)e^{(ex + d)} / ((b^2c^2 - 4ac^3)e)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^d \int \frac{x^2 e^{ex}}{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x\*\*2/(c\*x\*\*2+b\*x+a), x)

[Out] exp(d)\*Integral(x\*\*2\*exp(e\*x)/(a + b\*x + c\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^{(ex+d)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x^2/(c\*x^2+b\*x+a), x, algorithm="giac")

[Out] integrate(x^2\*e^(e\*x + d)/(c\*x^2 + b\*x + a), x)

$$3.472 \quad \int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=232

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c^3} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c^3} - \frac{be^{d+ex}}{c^2e}$$

[Out]  $-(E^{(d+e*x)/(c*e^2)}) - (b*E^{(d+e*x)})/(c^2*e) + (E^{(d+e*x)*x})/(c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c)))/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))*\operatorname{ExpIntegralEi}[(e*(b - \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c)))/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))*\operatorname{ExpIntegralEi}[(e*(b + \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^3)$

**Rubi [A]** time = 0.509539, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {2270, 2194, 2176, 2178}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c^3} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{Ei}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c^3} - \frac{be^{d+ex}}{c^2e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{(d+e*x)*x^3})/(a+b*x+c*x^2), x]$

[Out]  $-(E^{(d+e*x)/(c*e^2)}) - (b*E^{(d+e*x)})/(c^2*e) + (E^{(d+e*x)*x})/(c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c)))/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b - \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))*\operatorname{ExpIntegralEi}[(e*(b - \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c)))/\operatorname{Sqrt}[b^2 - 4*a*c])*E^{(d - ((b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)/(2*c))*\operatorname{ExpIntegralEi}[(e*(b + \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/(2*c)])/(2*c^3)$

#### Rule 2270

$\operatorname{Int}[(F^{((g_.)*((d_.) + (e_.)*(x_)))^{(n_.)})}*(u_)^{(m_.)})/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[F^{(g*(d+e*x)^n)}, u^m/(a+b*x+c*x^2), x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, g, n\}, x\} \ \&\& \ \operatorname{PolynomialQ}[u, x] \ \&\& \ \operatorname{IntegerQ}[m]$

#### Rule 2194

$\operatorname{Int}[(F^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)})})/((b*c*n*\operatorname{Log}[F])^n), x\_Symbol] := \operatorname{Simp}[(F^{(c*(a+b*x))})^n/(b*c*n*\operatorname{Log}[F]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, n\}, x\}$

#### Rule 2176

$\operatorname{Int}[(b_.)*(F^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})}*((c_.) + (d_.)*(x_.)^m)]/((c_.) + (d_.)*(x_.)^m), x\_Symbol] := \operatorname{Simp}[(c+d*x)^m*(b*F^{(g*(e+f*x))})^n/(f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*(b*F^{(g*(e+f*x))})^n, x], x] /;$   $\operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x\} \ \&\& \ \operatorname{GtQ}[m, 0] \ \&\& \ \operatorname{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == True$

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = \int \left( -\frac{be^{d+ex}}{c^2} + \frac{e^{d+ex} x}{c} + \frac{e^{d+ex} (ab + (b^2 - ac)x)}{c^2 (a + bx + cx^2)} \right) dx$$

$$= \frac{\int \frac{e^{d+ex} (ab + (b^2 - ac)x)}{a + bx + cx^2} dx}{c^2} - \frac{b \int e^{d+ex} dx}{c^2} + \frac{\int e^{d+ex} x dx}{c}$$

$$= -\frac{be^{d+ex}}{c^2 e} + \frac{e^{d+ex} x}{ce} + \frac{\int \left( \frac{(b^2 - ac + \frac{b(-b^2 + 3ac)}{\sqrt{b^2 - 4ac}}) e^{d+ex}}{b - \sqrt{b^2 - 4ac} + 2cx} + \frac{(b^2 - ac - \frac{b(-b^2 + 3ac)}{\sqrt{b^2 - 4ac}}) e^{d+ex}}{b + \sqrt{b^2 - 4ac} + 2cx} \right) dx}{c^2} - \frac{\int e^{d+ex} dx}{ce}$$

$$= -\frac{e^{d+ex}}{ce^2} - \frac{be^{d+ex}}{c^2 e} + \frac{e^{d+ex} x}{ce} + \frac{\left( b^2 - ac - \frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} \right) \int \frac{e^{d+ex}}{b - \sqrt{b^2 - 4ac} + 2cx} dx}{c^2} + \frac{\left( b^2 - ac + \frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} \right) \int \frac{e^{d+ex}}{b + \sqrt{b^2 - 4ac} + 2cx} dx}{c^2}$$

$$= -\frac{e^{d+ex}}{ce^2} - \frac{be^{d+ex}}{c^2 e} + \frac{e^{d+ex} x}{ce} + \frac{\left( b^2 - ac - \frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} \right) e^{d - \frac{(b - \sqrt{b^2 - 4ac})e}{2c}} \text{Ei} \left( \frac{e(b - \sqrt{b^2 - 4ac} + 2cx)}{2c} \right)}{2c^3} + \frac{\left( b^2 - ac + \frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} \right) e^{d - \frac{(b + \sqrt{b^2 - 4ac})e}{2c}} \text{Ei} \left( \frac{e(b + \sqrt{b^2 - 4ac} + 2cx)}{2c} \right)}{2c^3}$$

**Mathematica [A]** time = 0.639057, size = 268, normalized size = 1.16

$$\frac{e^{d - \frac{be}{c}} \left( e^2 \left( b^2 \sqrt{b^2 - 4ac} - ac \sqrt{b^2 - 4ac} + 3abc - b^3 \right) e^{\frac{e(\sqrt{b^2 - 4ac} + b)}{2c}} \text{Ei} \left( \frac{e(b + 2cx - \sqrt{b^2 - 4ac})}{2c} \right) + e^2 \left( b^2 \sqrt{b^2 - 4ac} - ac \sqrt{b^2 - 4ac} - 3abc + b^3 \right) e^{\frac{e(\sqrt{b^2 - 4ac} - b)}{2c}} \text{Ei} \left( \frac{e(b - \sqrt{b^2 - 4ac} + 2cx)}{2c} \right) \right)}{2c^3 e^2 \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(d + e*x)*x^3)/(a + b*x + c*x^2), x]
```

```
[Out] (E^(d - (b*e)/c)*(-2*c*Sqrt[b^2 - 4*a*c]*E^(e*(b/c + x))*(c + b*e - c*e*x)
+ (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^2*E^((
(b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] +
2*c*x))/(2*c)] + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4
*a*c])*e^2*E^(((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x))/(2*c)]))/(2*c^3*Sqrt[b^2 - 4*a*c]*e^2)
```

**Maple [B]** time = 0.022, size = 3532, normalized size = 15.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(e*x+d)*x^3/(c*x^2+b*x+a), x)
```

```
[Out] 1/e^4*(-e^2*exp(e*x+d)*(-c*(e*x+d)+b*e-2*c*d+c)/c^2+1/2/c^3*e^2*(-3*exp(1/2
/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*
d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a*b*c*e^3+6*exp(1/2/c*(-b*e+2*c*d+(-4*a*c*
e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))
```





$(1/2)/c) * Ei(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * b * c*d*e+2*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * Ei(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * c^2*d^2+Ei(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * (-4*a*c*e^2+b^2*e^2)^(1/2) * exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))) * b*e-2*Ei(1, 1/2*(-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * (-4*a*c*e^2+b^2*e^2)^(1/2) * exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))) * c*d+exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * Ei(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * (-4*a*c*e^2+b^2*e^2)^(1/2) * b*e-2*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * Ei(1, -1/2*(2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c) * (-4*a*c*e^2+b^2*e^2)^(1/2) * c*d / (-4*a*c*e^2+b^2*e^2)^(1/2))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(cex^3e^d - cx^2e^d - bxe^d)e^{(ex)}}{c^2e^2x^2 + bce^2x + ace^2} - \int \frac{((bee^d + 2ce^d)ax + (b^2ee^d - 2acee^d)x^2 + abe^d)e^{(ex)}}{c^3e^2x^4 + 2bc^2e^2x^3 + 2abce^2x + a^2ce^2 + (b^2ce^2 + 2ac^2e^2)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x^3/(c\*x^2+b\*x+a), x, algorithm="maxima")

[Out]  $(c*e*x^3*e^d - c*x^2*e^d - b*x*e^d)*e^{(e*x)}/(c^2*e^2*x^2 + b*c*e^2*x + a*c*e^2) - \text{integrate}(-((b*e*e^d + 2*c*e^d)*a*x + (b^2*e*e^d - 2*a*c*e*e^d)*x^2 + a*b*e^d)*e^{(e*x)}/(c^3*e^2*x^4 + 2*b*c^2*e^2*x^3 + 2*a*b*c*e^2*x + a^2*c*e^2 + (b^2*c*e^2 + 2*a*c^2*e^2)*x^2), x)$

**Fricas [A]** time = 1.65116, size = 709, normalized size = 3.06

$$\left( (b^4 - 5ab^2c + 4a^2c^2)e^2 - (b^3c - 3abc^2)e\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \right) Ei\left( \frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right) e^{\left( \frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right)} + \left( (b^4 - 5ab^2c + 4a^2c^2)e^2 - (b^3c - 3abc^2)e\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(e\*x+d)\*x^3/(c\*x^2+b\*x+a), x, algorithm="fricas")

[Out]  $1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2 - (b^3*c - 3*a*b*c^2)*e*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))*Ei(1/2*(2*c*e*x + b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c) * e^{(1/2*(2*c*d - b*e + c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c)} + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2 + (b^3*c - 3*a*b*c^2)*e*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))*Ei(1/2*(2*c*e*x + b*e + c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c) * e^{(1/2*(2*c*d - b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c)} - 2*(b^2*c^2 - 4*a*c^3 - (b^2*c^2 - 4*a*c^3)*e*x + (b^3*c - 4*a*b*c^2)*e)*e^{(e*x + d)}/((b^2*c^3 - 4*a*c^4)*e^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*x+d)*x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 e^{(ex+d)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(e*x+d)*x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(e*x + d)/(c*x^2 + b*x + a), x)
```

$$3.473 \quad \int \frac{4^x}{a+2^x b} dx$$

**Optimal.** Leaf size=30

$$\frac{2^x}{b \log(2)} - \frac{a \log(a + b2^x)}{b^2 \log(2)}$$

[Out]  $2^x/(b*\text{Log}[2]) - (a*\text{Log}[a + 2^x*b])/(b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0372135, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2248, 43}

$$\frac{2^x}{b \log(2)} - \frac{a \log(a + b2^x)}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In] Int[ $4^x/(a + 2^x*b)$ , x]

[Out]  $2^x/(b*\text{Log}[2]) - (a*\text{Log}[a + 2^x*b])/(b^2*\text{Log}[2])$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{a+2^x b} dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, 2^x\right)}{\log(2)} \\ &= \frac{2^x}{b \log(2)} - \frac{a \log(a + 2^x b)}{b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0186802, size = 27, normalized size = 0.9

$$\frac{\frac{2^x}{b} - \frac{a \log(a+b2^x)}{b^2}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/(a + 2^x\*b),x]

[Out] (2^x/b - (a\*Log[a + 2^x\*b])/b^2)/Log[2]

**Maple [A]** time = 0.013, size = 35, normalized size = 1.2

$$\frac{e^{x \ln(2)}}{\ln(2)b} - \frac{a \ln(a + e^{x \ln(2)}b)}{\ln(2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a+2^x\*b),x)

[Out] 1/ln(2)/b\*exp(x\*ln(2))-1/ln(2)/b^2\*a\*ln(a+exp(x\*ln(2))\*b)

**Maxima [A]** time = 1.43986, size = 41, normalized size = 1.37

$$\frac{2^x}{b \log(2)} - \frac{a \log(2^x b + a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+2^x\*b),x, algorithm="maxima")

[Out] 2^x/(b\*log(2)) - a\*log(2^x\*b + a)/(b^2\*log(2))

**Fricas [A]** time = 1.56517, size = 55, normalized size = 1.83

$$\frac{2^x b - a \log(2^x b + a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+2^x\*b),x, algorithm="fricas")

[Out] (2^x\*b - a\*log(2^x\*b + a))/(b^2\*log(2))

**Sympy [A]** time = 0.310819, size = 41, normalized size = 1.37

$$-\frac{a \log\left(\frac{a}{b} + e^{\frac{x \log(4)}{2}}\right)}{b^2 \log(2)} + \begin{cases} \frac{e^{\frac{x \log(4)}{2}}}{b \log(2)} & \text{for } b \log(2) \neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a+2\*\*x\*b),x)

```
[Out] -a*log(a/b + exp(x*log(4)/2))/(b**2*log(2)) + Piecewise((exp(x*log(4)/2)/(b
*log(2)), Ne(b*log(2), 0)), (x/b, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{2^{xb+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a+2^x*b),x, algorithm="giac")
```

```
[Out] integrate(4^x/(2^x*b + a), x)
```

$$3.474 \quad \int \frac{2^{2x}}{a+2^x b} dx$$

**Optimal.** Leaf size=30

$$\frac{2^x}{b \log(2)} - \frac{a \log(a + b2^x)}{b^2 \log(2)}$$

[Out]  $2^x/(b*\text{Log}[2]) - (a*\text{Log}[a + 2^x*b])/(b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0337079, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{2^x}{b \log(2)} - \frac{a \log(a + b2^x)}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{(2*x)}/(a + 2^x*b), x]$

[Out]  $2^x/(b*\text{Log}[2]) - (a*\text{Log}[a + 2^x*b])/(b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))^{(p_.)*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}}$ , x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})$ , x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{a + 2^x b} dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, 2^x\right)}{\log(2)} \\ &= \frac{2^x}{b \log(2)} - \frac{a \log(a + 2^x b)}{b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0117894, size = 27, normalized size = 0.9

$$\frac{\frac{2^x}{b} - \frac{a \log(a+b2^x)}{b^2}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/(a + 2^x\*b), x]

[Out] (2^x/b - (a\*Log[a + 2^x\*b])/b^2)/Log[2]

**Maple [A]** time = 0.009, size = 35, normalized size = 1.2

$$\frac{e^{x \ln(2)}}{\ln(2)b} - \frac{a \ln(a + e^{x \ln(2)}b)}{\ln(2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a+2^x\*b), x)

[Out] 1/ln(2)/b\*exp(x\*ln(2))-1/ln(2)/b^2\*a\*ln(a+exp(x\*ln(2))\*b)

**Maxima [A]** time = 0.965378, size = 41, normalized size = 1.37

$$\frac{2^x}{b \log(2)} - \frac{a \log(2^x b + a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+2^x\*b), x, algorithm="maxima")

[Out] 2^x/(b\*log(2)) - a\*log(2^x\*b + a)/(b^2\*log(2))

**Fricas [A]** time = 1.54493, size = 55, normalized size = 1.83

$$\frac{2^x b - a \log(2^x b + a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+2^x\*b), x, algorithm="fricas")

[Out] (2^x\*b - a\*log(2^x\*b + a))/(b^2\*log(2))

**Sympy [A]** time = 0.15167, size = 31, normalized size = 1.03

$$-\frac{a \log\left(2^x + \frac{a}{b}\right)}{b^2 \log(2)} + \begin{cases} \frac{2^x}{b \log(2)} & \text{for } b \log(2) \neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a+2\*\*x\*b), x)

[Out]  $-a \cdot \log(2^{2x} + a/b) / (b \cdot 2 \cdot \log(2)) + \text{Piecewise}((2^{2x} / (b \cdot \log(2))), \text{Ne}(b \cdot \log(2), 0)), (x/b, \text{True}))$

---

**Giac [A]** time = 1.21052, size = 42, normalized size = 1.4

$$\frac{2^x}{b \log(2)} - \frac{a \log(|2^x b + a|)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(2*x)/(a+2^x*b),x, algorithm="giac")`

[Out]  $2^x / (b \cdot \log(2)) - a \cdot \log(\text{abs}(2^x \cdot b + a)) / (b^2 \cdot \log(2))$



$$3.475 \quad \int \frac{4^x}{a-2^x b} dx$$

**Optimal.** Leaf size=32

$$-\frac{a \log(a - b2^x)}{b^2 \log(2)} - \frac{2^x}{b \log(2)}$$

[Out]  $-(2^x/(b*\text{Log}[2])) - (a*\text{Log}[a - 2^x*b])/(b^2*\text{Log}[2])$

**Rubi [A]** time = 0.038245, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2248, 43}

$$-\frac{a \log(a - b2^x)}{b^2 \log(2)} - \frac{2^x}{b \log(2)}$$

Antiderivative was successfully verified.

[In] Int[4^x/(a - 2^x\*b), x]

[Out]  $-(2^x/(b*\text{Log}[2])) - (a*\text{Log}[a - 2^x*b])/(b^2*\text{Log}[2])$

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{a-2^x b} dx &= \frac{\text{Subst}\left(\int \frac{x}{a-bx} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{b} - \frac{a}{b(-a+bx)}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2^x}{b \log(2)} - \frac{a \log(a - 2^x b)}{b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0219295, size = 26, normalized size = 0.81

$$-\frac{a \log(a - b2^x) + b2^x}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/(a - 2^x\*b), x]

[Out] -((2^x\*b + a\*Log[a - 2^x\*b])/(b^2\*Log[2]))

**Maple [A]** time = 0.017, size = 37, normalized size = 1.2

$$-\frac{e^{x \ln(2)}}{\ln(2)b} - \frac{a \ln(a - e^{x \ln(2)}b)}{\ln(2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a-2^x\*b), x)

[Out] -1/ln(2)/b\*exp(x\*ln(2))-1/ln(2)/b^2\*a\*ln(a-exp(x\*ln(2))\*b)

**Maxima [A]** time = 1.46579, size = 45, normalized size = 1.41

$$-\frac{2^x}{b \log(2)} - \frac{a \log(2^x b - a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-2^x\*b), x, algorithm="maxima")

[Out] -2^x/(b\*log(2)) - a\*log(2^x\*b - a)/(b^2\*log(2))

**Fricas [A]** time = 1.57427, size = 57, normalized size = 1.78

$$\frac{2^x b + a \log(2^x b - a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-2^x\*b), x, algorithm="fricas")

[Out] -(2^x\*b + a\*log(2^x\*b - a))/(b^2\*log(2))

**Sympy [A]** time = 0.321217, size = 44, normalized size = 1.38

$$-\frac{a \log\left(-\frac{a}{b} + e^{\frac{x \log(4)}{2}}\right)}{b^2 \log(2)} + \begin{cases} -\frac{e^{\frac{x \log(4)}{2}}}{b \log(2)} & \text{for } b \log(2) \neq 0 \\ -\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a-2\*\*x\*b), x)

```
[Out] -a*log(-a/b + exp(x*log(4)/2))/(b**2*log(2)) + Piecewise((-exp(x*log(4)/2)/
(b*log(2)), Ne(b*log(2), 0)), (-x/b, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{4^x}{2^{xb} - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a-2^x*b),x, algorithm="giac")
```

```
[Out] integrate(-4^x/(2^x*b - a), x)
```

$$3.476 \quad \int \frac{2^{2x}}{a-2^x b} dx$$

**Optimal.** Leaf size=32

$$-\frac{a \log(a - b2^x)}{b^2 \log(2)} - \frac{2^x}{b \log(2)}$$

[Out]  $-(2^x/(b*\text{Log}[2])) - (a*\text{Log}[a - 2^x*b])/(b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0354631, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2248, 43}

$$-\frac{a \log(a - b2^x)}{b^2 \log(2)} - \frac{2^x}{b \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{(2*x)/(a - 2^x*b)}, x]$

[Out]  $-(2^x/(b*\text{Log}[2])) - (a*\text{Log}[a - 2^x*b])/(b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x*\text{Denominator}[m])^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{a-2^x b} dx &= \frac{\text{Subst}\left(\int \frac{x}{a-bx} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{b} - \frac{a}{b(-a+bx)}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2^x}{b \log(2)} - \frac{a \log(a - 2^x b)}{b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0130584, size = 26, normalized size = 0.81

$$-\frac{a \log(a - b2^x) + b2^x}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/(a - 2^x\*b), x]

[Out] -((2^x\*b + a\*Log[a - 2^x\*b])/(b^2\*Log[2]))

**Maple [A]** time = 0.009, size = 37, normalized size = 1.2

$$-\frac{e^{x \ln(2)}}{\ln(2)b} - \frac{a \ln(a - e^{x \ln(2)}b)}{\ln(2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a-2^x\*b), x)

[Out] -1/ln(2)/b\*exp(x\*ln(2))-1/ln(2)/b^2\*a\*ln(a-exp(x\*ln(2))\*b)

**Maxima [A]** time = 0.967473, size = 45, normalized size = 1.41

$$-\frac{2^x}{b \log(2)} - \frac{a \log(2^x b - a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-2^x\*b), x, algorithm="maxima")

[Out] -2^x/(b\*log(2)) - a\*log(2^x\*b - a)/(b^2\*log(2))

**Fricas [A]** time = 1.59681, size = 57, normalized size = 1.78

$$-\frac{2^x b + a \log(2^x b - a)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-2^x\*b), x, algorithm="fricas")

[Out] -(2^x\*b + a\*log(2^x\*b - a))/(b^2\*log(2))

**Sympy [A]** time = 0.155308, size = 34, normalized size = 1.06

$$-\frac{a \log\left(2^x - \frac{a}{b}\right)}{b^2 \log(2)} + \begin{cases} -\frac{2^x}{b \log(2)} & \text{for } b \log(2) \neq 0 \\ -\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a-2\*\*x\*b), x)

[Out] -a\*log(2\*\*x - a/b)/(b\*\*2\*log(2)) + Piecewise((-2\*\*x/(b\*log(2)), Ne(b\*log(2), 0)), (-x/b, True))

---

**Giac [A]** time = 1.1736, size = 46, normalized size = 1.44

$$-\frac{2^x}{b \log(2)} - \frac{a \log(|2^x b - a|)}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-2^x\*b),x, algorithm="giac")

[Out] -2^x/(b\*log(2)) - a\*log(abs(2^x\*b - a))/(b^2\*log(2))

$$3.477 \quad \int \frac{4^x}{a+2^{-x}b} dx$$

**Optimal.** Leaf size=58

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a + b2^{-x})}{a^3 \log(2)} - \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) - (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a + b/2^x])/(a^3\*Log[2])

**Rubi [A]** time = 0.0578888, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 44}

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a + b2^{-x})}{a^3 \log(2)} - \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[4^x/(a + b/2^x),x]

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) - (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a + b/2^x])/(a^3\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])]

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{a+2^{-x}b} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)} dx, x, 2^{-x}\right)}{\log(2)} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{b^2x}{a^3} + \frac{2^{-1+2x}}{a \log(2)} - \frac{2^xb}{a^2 \log(2)} + \frac{b^2 \log(a + 2^{-x}b)}{a^3 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0333008, size = 36, normalized size = 0.62

$$\frac{2b^2 \log(a2^x + b) + a2^x(a2^x - 2b)}{a^3 \log(4)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/(a + b/2^x),x]

[Out] (2^x\*a\*(2^x\*a - 2\*b) + 2\*b^2\*Log[2^x\*a + b])/(a^3\*Log[4])

**Maple [A]** time = 0.015, size = 54, normalized size = 0.9

$$\frac{(e^{x \ln(2)})^2}{2 a \ln(2)} - \frac{e^{x \ln(2)} b}{\ln(2) a^2} + \frac{b^2 \ln(a e^{x \ln(2)} + b)}{a^3 \ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a+b/(2^x)),x)

[Out] 1/2/a/ln(2)\*exp(x\*ln(2))^2-1/a^2/ln(2)\*b\*exp(x\*ln(2))+1/a^3/ln(2)\*b^2\*ln(a\*exp(x\*ln(2))+b)

**Maxima [A]** time = 1.45123, size = 80, normalized size = 1.38

$$\frac{b^2 x}{a^3} - \frac{(2^{-x+1} b - a) 2^{2x-1}}{a^2 \log(2)} + \frac{b^2 \log\left(a + \frac{b}{2^x}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+b/(2^x)),x, algorithm="maxima")

[Out] b^2\*x/a^3 - (2^(-x + 1)\*b - a)\*2^(2\*x - 1)/(a^2\*log(2)) + b^2\*log(a + b/2^x)/(a^3\*log(2))

**Fricas [A]** time = 1.58735, size = 90, normalized size = 1.55

$$\frac{2^{2x} a^2 - 2 \cdot 2^x a b + 2 b^2 \log(2^x a + b)}{2 a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+b/(2^x)),x, algorithm="fricas")

[Out] 1/2\*(2^(2\*x)\*a^2 - 2\*2^x\*a\*b + 2\*b^2\*log(2^x\*a + b))/(a^3\*log(2))

**Sympy [A]** time = 0.362389, size = 76, normalized size = 1.31

$$\begin{cases} \frac{4^x a^2 \log(2) - 2 a b e^{\frac{x \log(4)}{2}} \log(2)}{2 a^3 \log(2)^2} & \text{for } 2 a^3 \log(2)^2 \neq 0 \\ \frac{x(a-b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2 \log\left(e^{\frac{x \log(4)}{2}} + \frac{b}{a}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(4**x/(a+b/(2**x)),x)
```

```
[Out] Piecewise(((4**x*a**2*log(2) - 2*a*b*exp(x*log(4)/2)*log(2))/(2*a**3*log(2)
**2), Ne(2*a**3*log(2)**2, 0)), (x*(a - b)/a**2, True)) + b**2*log(exp(x*lo
g(4)/2) + b/a)/(a**3*log(2))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{a + \frac{b}{2^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a+b/(2^x)),x, algorithm="giac")
```

```
[Out] integrate(4^x/(a + b/2^x), x)
```

$$3.478 \quad \int \frac{2^{2x}}{a+2^{-x}b} dx$$

**Optimal.** Leaf size=58

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a + b2^{-x})}{a^3 \log(2)} - \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) - (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a + b/2^x])/(a^3\*Log[2])

**Rubi [A]** time = 0.0508679, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 44}

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a + b2^{-x})}{a^3 \log(2)} - \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^(2\*x)/(a + b/2^x), x]

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) - (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a + b/2^x])/(a^3\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{a + 2^{-x}b} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)} dx, x, 2^{-x}\right)}{\log(2)} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{b^2x}{a^3} + \frac{2^{-1+2x}}{a \log(2)} - \frac{2^xb}{a^2 \log(2)} + \frac{b^2 \log(a + 2^{-x}b)}{a^3 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0176555, size = 36, normalized size = 0.62

$$\frac{2b^2 \log(a2^x + b) + a2^x(a2^x - 2b)}{a^3 \log(4)}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/(a + b/2^x),x]

[Out] (2^x\*a\*(2^x\*a - 2\*b) + 2\*b^2\*Log[2^x\*a + b])/(a^3\*Log[4])

**Maple [A]** time = 0.01, size = 54, normalized size = 0.9

$$\frac{(e^{x \ln(2)})^2}{2 a \ln(2)} - \frac{e^{x \ln(2)} b}{\ln(2) a^2} + \frac{b^2 \ln(a e^{x \ln(2)} + b)}{a^3 \ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a+b/(2^x)),x)

[Out] 1/2/a/ln(2)\*exp(x\*ln(2))^2-1/a^2/ln(2)\*b\*exp(x\*ln(2))+1/a^3/ln(2)\*b^2\*ln(a\*exp(x\*ln(2))+b)

**Maxima [A]** time = 0.992382, size = 80, normalized size = 1.38

$$\frac{b^2 x}{a^3} - \frac{(2^{-x+1} b - a) 2^{2x-1}}{a^2 \log(2)} + \frac{b^2 \log\left(a + \frac{b}{2^x}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+b/(2^x)),x, algorithm="maxima")

[Out] b^2\*x/a^3 - (2^(-x + 1)\*b - a)\*2^(2\*x - 1)/(a^2\*log(2)) + b^2\*log(a + b/2^x)/(a^3\*log(2))

**Fricas [A]** time = 1.57711, size = 90, normalized size = 1.55

$$\frac{2^{2x} a^2 - 2 \cdot 2^x a b + 2 b^2 \log(2^x a + b)}{2 a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+b/(2^x)),x, algorithm="fricas")

[Out] 1/2\*(2^(2\*x)\*a^2 - 2\*2^x\*a\*b + 2\*b^2\*log(2^x\*a + b))/(a^3\*log(2))

**Sympy [A]** time = 0.192232, size = 68, normalized size = 1.17

$$\begin{cases} \frac{2^{2x} a^2 \log(2) - 2 \cdot 2^x a b \log(2)}{2 a^3 \log(2)^2} & \text{for } 2 a^3 \log(2)^2 \neq 0 \\ \frac{x(a-b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2 \log\left(2^x + \frac{b}{a}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(2*x)/(a+b/(2**x)),x)
```

```
[Out] Piecewise(((2**(2*x)*a**2*log(2) - 2*2**x*a*b*log(2))/(2*a**3*log(2)**2), N
e(2*a**3*log(2)**2, 0)), (x*(a - b)/a**2, True)) + b**2*log(2**x + b/a)/(a*
*3*log(2))
```

---

**Giac [A]** time = 1.21713, size = 65, normalized size = 1.12

$$\frac{b^2 \log(|2^x a + b|)}{a^3 \log(2)} + \frac{2^{2x} a \log(2) - 2 \cdot 2^x b \log(2)}{2 a^2 \log(2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(2*x)/(a+b/(2^x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2^x*a + b))/(a^3*log(2)) + 1/2*(2^(2*x)*a*log(2) - 2*2^x*b*log(
2))/(a^2*log(2)^2)
```

$$3.479 \quad \int \frac{4^x}{a-2^{-x}b} dx$$

**Optimal.** Leaf size=58

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a - b2^{-x})}{a^3 \log(2)} + \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) + (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a - b/2^x])/(a^3\*Log[2])

**Rubi [A]** time = 0.0540729, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2248, 44}

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a - b2^{-x})}{a^3 \log(2)} + \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[4^x/(a - b/2^x),x]

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) + (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a - b/2^x])/(a^3\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{a-2^{-x}b} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3(a-bx)} dx, x, 2^{-x}\right)}{\log(2)} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{b}{a^2x^2} + \frac{b^2}{a^3x} + \frac{b^3}{a^3(a-bx)}\right) dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{b^2x}{a^3} + \frac{2^{-1+2x}}{a \log(2)} + \frac{2^xb}{a^2 \log(2)} + \frac{b^2 \log(a - 2^{-x}b)}{a^3 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0291611, size = 38, normalized size = 0.66

$$\frac{2b^2 \log(a2^x - b) + a2^x (a2^x + 2b)}{a^3 \log(4)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/(a - b/2^x),x]

[Out] (2^x\*a\*(2^x\*a + 2\*b) + 2\*b^2\*Log[2^x\*a - b])/(a^3\*Log[4])

**Maple [A]** time = 0.012, size = 55, normalized size = 1.

$$\frac{e^{x \ln(2)} b}{\ln(2) a^2} + \frac{(e^{x \ln(2)})^2}{2 a \ln(2)} + \frac{b^2 \ln(a e^{x \ln(2)} - b)}{a^3 \ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a-b/(2^x)),x)

[Out] 1/a^2/ln(2)\*b\*exp(x\*ln(2))+1/2/a/ln(2)\*exp(x\*ln(2))^2+1/a^3/ln(2)\*b^2\*ln(a\*exp(x\*ln(2))-b)

**Maxima [A]** time = 1.45343, size = 78, normalized size = 1.34

$$\frac{b^2 x}{a^3} + \frac{(2^{-x+1} b + a) 2^{2x-1}}{a^2 \log(2)} + \frac{b^2 \log\left(-a + \frac{b}{2^x}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-b/(2^x)),x, algorithm="maxima")

[Out] b^2\*x/a^3 + (2^(-x + 1)\*b + a)\*2^(2\*x - 1)/(a^2\*log(2)) + b^2\*log(-a + b/2^x)/(a^3\*log(2))

**Fricas [A]** time = 1.54281, size = 90, normalized size = 1.55

$$\frac{2^{2x} a^2 + 2 \cdot 2^x a b + 2 b^2 \log(2^x a - b)}{2 a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-b/(2^x)),x, algorithm="fricas")

[Out] 1/2\*(2^(2\*x)\*a^2 + 2\*2^x\*a\*b + 2\*b^2\*log(2^x\*a - b))/(a^3\*log(2))

**Sympy [A]** time = 0.359911, size = 76, normalized size = 1.31

$$\begin{cases} \frac{4^x a^2 \log(2) + 2 a b e^{\frac{x \log(4)}{2}} \log(2)}{2 a^3 \log(2)^2} & \text{for } 2 a^3 \log(2)^2 \neq 0 \\ \frac{x(a+b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2 \log\left(e^{\frac{x \log(4)}{2}} - \frac{b}{a}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4**x/(a-b/(2**x)),x)
```

```
[Out] Piecewise(((4**x*a**2*log(2) + 2*a*b*exp(x*log(4)/2)*log(2))/(2*a**3*log(2)
**2), Ne(2*a**3*log(2)**2, 0)), (x*(a + b)/a**2, True)) + b**2*log(exp(x*lo
g(4)/2) - b/a)/(a**3*log(2))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{a - \frac{b}{2^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a-b/(2^x)),x, algorithm="giac")
```

```
[Out] integrate(4^x/(a - b/2^x), x)
```

$$3.480 \quad \int \frac{2^{2x}}{a-2^{-x}b} dx$$

**Optimal.** Leaf size=58

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a - b2^{-x})}{a^3 \log(2)} + \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) + (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a - b/2^x])/(a^3\*Log[2])

**Rubi [A]** time = 0.0520763, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 44}

$$\frac{b^2x}{a^3} + \frac{b^2 \log(a - b2^{-x})}{a^3 \log(2)} + \frac{b2^x}{a^2 \log(2)} + \frac{2^{2x-1}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^(2\*x)/(a - b/2^x), x]

[Out] (b^2\*x)/a^3 + 2^(-1 + 2\*x)/(a\*Log[2]) + (2^x\*b)/(a^2\*Log[2]) + (b^2\*Log[a - b/2^x])/(a^3\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{a-2^{-x}b} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3(a-bx)} dx, x, 2^{-x}\right)}{\log(2)} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{b}{a^2x^2} + \frac{b^2}{a^3x} + \frac{b^3}{a^3(a-bx)}\right) dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{b^2x}{a^3} + \frac{2^{-1+2x}}{a \log(2)} + \frac{2^x b}{a^2 \log(2)} + \frac{b^2 \log(a - 2^{-x}b)}{a^3 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0170307, size = 38, normalized size = 0.66

$$\frac{2b^2 \log(a2^x - b) + a2^x (a2^x + 2b)}{a^3 \log(4)}$$



Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/(a - b/2^x),x]

[Out] (2^x\*a\*(2^x\*a + 2\*b) + 2\*b^2\*Log[2^x\*a - b])/(a^3\*Log[4])

**Maple [A]** time = 0.011, size = 55, normalized size = 1.

$$\frac{e^{x \ln(2)} b}{\ln(2) a^2} + \frac{(e^{x \ln(2)})^2}{2 a \ln(2)} + \frac{b^2 \ln(a e^{x \ln(2)} - b)}{a^3 \ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a-b/(2^x)),x)

[Out] 1/a^2/ln(2)\*b\*exp(x\*ln(2))+1/2/a/ln(2)\*exp(x\*ln(2))^2+1/a^3/ln(2)\*b^2\*ln(a\*exp(x\*ln(2))-b)

**Maxima [A]** time = 0.98152, size = 78, normalized size = 1.34

$$\frac{b^2 x}{a^3} + \frac{(2^{-x+1} b + a) 2^{2x-1}}{a^2 \log(2)} + \frac{b^2 \log\left(-a + \frac{b}{2^x}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-b/(2^x)),x, algorithm="maxima")

[Out] b^2\*x/a^3 + (2^(-x + 1)\*b + a)\*2^(2\*x - 1)/(a^2\*log(2)) + b^2\*log(-a + b/2^x)/(a^3\*log(2))

**Fricas [A]** time = 1.52623, size = 90, normalized size = 1.55

$$\frac{2^{2x} a^2 + 2 \cdot 2^x a b + 2 b^2 \log(2^x a - b)}{2 a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-b/(2^x)),x, algorithm="fricas")

[Out] 1/2\*(2^(2\*x)\*a^2 + 2\*2^x\*a\*b + 2\*b^2\*log(2^x\*a - b))/(a^3\*log(2))

**Sympy [A]** time = 0.193623, size = 68, normalized size = 1.17

$$\begin{cases} \frac{2^{2x} a^2 \log(2) + 2 \cdot 2^x a b \log(2)}{2 a^3 \log(2)^2} & \text{for } 2 a^3 \log(2)^2 \neq 0 \\ \frac{x(a+b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2 \log\left(2^x - \frac{b}{a}\right)}{a^3 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(2*x)/(a-b/(2**x)),x)
```

```
[Out] Piecewise(((2**(2*x)*a**2*log(2) + 2*2**x*a*b*log(2))/(2*a**3*log(2)**2), N
e(2*a**3*log(2)**2, 0)), (x*(a + b)/a**2, True)) + b**2*log(2**x - b/a)/(a*
*3*log(2))
```

---

**Giac [A]** time = 1.24112, size = 68, normalized size = 1.17

$$\frac{b^2 \log(|2^x a - b|)}{a^3 \log(2)} + \frac{2^{2x} a \log(2) + 2 \cdot 2^x b \log(2)}{2 a^2 \log(2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(2*x)/(a-b/(2^x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2^x*a - b))/(a^3*log(2)) + 1/2*(2^(2*x)*a*log(2) + 2*2^x*b*log(
2))/(a^2*log(2)^2)
```

$$3.481 \quad \int \frac{2^x}{a+4^x b} dx$$

**Optimal.** Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0295758, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/(a + 4^x\*b), x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{a+4^x b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\tan^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0074805, size = 30, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a + 4^x\*b), x]

[Out] ArcTan[(2^x\*sqrt[b])/sqrt[a]]/(sqrt[a]\*sqrt[b]\*Log[2])

**Maple [B]** time = 0.029, size = 53, normalized size = 1.8

$$-\frac{1}{2 \ln(2)} \ln\left(2^x - a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{2 \ln(2)} \ln\left(2^x + a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+4^x\*b), x)

[Out] -1/2/(-a\*b)^(1/2)/ln(2)\*ln(2^x-1/(-a\*b)^(1/2)\*a)+1/2/(-a\*b)^(1/2)/ln(2)\*ln(2^x+1/(-a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+4^x\*b), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.56519, size = 190, normalized size = 6.33

$$\left[ \frac{\sqrt{-ab} \log\left(\frac{2^{2xb} - 2\sqrt{-ab}2^{x-a}}{2^{2xb+a}}\right)}{2ab \log(2)}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}}{2^xb}\right)}{ab \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+4^x\*b), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-a\*b)\*log((2^(2\*x)\*b - 2\*sqrt(-a\*b)\*2^x - a)/(2^(2\*x)\*b + a))/(a\*b\*log(2)), -sqrt(a\*b)\*arctan(sqrt(a\*b)/(2^x\*b))/(a\*b\*log(2))]

**Sympy [A]** time = 0.326304, size = 29, normalized size = 0.97

$$\frac{\text{RootSum}\left(4z^2ab + 1, \left(i \mapsto i \log\left(2ia + e^{\frac{x \log(4)}{2}}\right)\right)\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+4\*\*x\*b), x)

```
[Out] RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*a + exp(x*log(4)/2))))/log(2)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{4^{xb+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^x/(a+4^x*b),x, algorithm="giac")
```

```
[Out] integrate(2^x/(4^x*b + a), x)
```

$$3.482 \quad \int \frac{2^x}{a+2^{2x}b} dx$$

**Optimal.** Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0288414, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/(a + 2^(2\*x)\*b), x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{a+2^{2x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\tan^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0040952, size = 30, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a + 2^(2\*x)\*b), x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Maple [B]** time = 0.026, size = 53, normalized size = 1.8

$$-\frac{1}{2 \ln(2)} \ln\left(2^x - a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}} + \frac{1}{2 \ln(2)} \ln\left(2^x + a \frac{1}{\sqrt{-ab}}\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+2^(2\*x)\*b), x)

[Out] -1/2/(-a\*b)^(1/2)/ln(2)\*ln(2^x-1/(-a\*b)^(1/2)\*a)+1/2/(-a\*b)^(1/2)/ln(2)\*ln(2^x+1/(-a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+2^(2\*x)\*b), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.51252, size = 190, normalized size = 6.33

$$\left[ -\frac{\sqrt{-ab} \log\left(\frac{2^{2x}b - 2\sqrt{-ab}2^x - a}{2^{2x}b + a}\right)}{2ab \log(2)}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}}{2^x b}\right)}{ab \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+2^(2\*x)\*b), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-a\*b)\*log((2^(2\*x)\*b - 2\*sqrt(-a\*b)\*2^x - a)/(2^(2\*x)\*b + a))/(a\*b\*log(2)), -sqrt(a\*b)\*arctan(sqrt(a\*b)/(2^x\*b))/(a\*b\*log(2))]

**Sympy [A]** time = 0.169882, size = 24, normalized size = 0.8

$$\frac{\text{RootSum}\left(4z^2ab + 1, (i \mapsto i \log(2^x + 2ia))\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+2\*\*(2\*x)\*b), x)

[Out]  $\text{RootSum}(4*_z^{**2}*a*b + 1, \text{Lambda}(_i, _i*\log(2**x + 2*_i*a)))/\log(2)$

---

**Giac [A]** time = 1.1458, size = 28, normalized size = 0.93

$$\frac{\arctan\left(\frac{2^x b}{\sqrt{ab}}\right)}{\sqrt{ab} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(2^x/(a+2^{(2*x)*b}),x, \text{algorithm}="giac")$

[Out]  $\arctan(2^x*b/\text{sqrt}(a*b))/(\text{sqrt}(a*b)*\log(2))$



$$3.483 \quad \int \frac{2^x}{a-4^x b} dx$$

**Optimal.** Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.028477, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2249, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/(a - 4^x\*b), x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{a-4^x b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a-bx^2} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\tanh^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0070214, size = 30, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a - 4^x\*b), x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Maple [B]** time = 0.026, size = 49, normalized size = 1.6

$$\frac{1}{2 \ln(2)} \ln\left(2^x + a \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} - \frac{1}{2 \ln(2)} \ln\left(2^x - a \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-4^x\*b), x)

[Out] 1/2/(a\*b)^(1/2)/ln(2)\*ln(2^x+1/(a\*b)^(1/2)\*a)-1/2/(a\*b)^(1/2)/ln(2)\*ln(2^x-1/(a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.5538, size = 189, normalized size = 6.3

$$\left[ \frac{\sqrt{ab} \log\left(\frac{2^{2x}b+2\sqrt{ab}2^x+a}{2^{2x}b-a}\right)}{2ab \log(2)}, -\frac{\sqrt{-ab} \arctan\left(\frac{\sqrt{-ab}}{2^x b}\right)}{ab \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b), x, algorithm="fricas")

[Out] [1/2\*sqrt(a\*b)\*log((2^(2\*x)\*b + 2\*sqrt(a\*b)\*2^x + a)/(2^(2\*x)\*b - a))/(a\*b\*log(2)), -sqrt(-a\*b)\*arctan(sqrt(-a\*b)/(2^x\*b))/(a\*b\*log(2))]

**Sympy [A]** time = 0.340263, size = 29, normalized size = 0.97

$$\frac{\text{RootSum}\left(4z^2ab - 1, \left(i \mapsto i \log\left(2ia + e^{\frac{x \log(4)}{2}}\right)\right)\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-4\*\*x\*b), x)

[Out] RootSum(4\*\_z\*\*2\*a\*b - 1, Lambda(\_i, \_i\*log(2\*\_i\*a + exp(x\*log(4)/2))))/log(2)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{2^x}{4^{xb} - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b),x, algorithm="giac")

[Out] integrate(-2^x/(4^x\*b - a), x)

$$3.484 \quad \int \frac{2^x}{a-2^{2x}b} dx$$

**Optimal.** Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0301893, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2249, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/(a - 2^(2\*x)\*b), x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{a-2^{2x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a-bx^2} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\tanh^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0052984, size = 30, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a - 2^(2\*x)\*b), x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a]]/(Sqrt[a]\*Sqrt[b]\*Log[2])

**Maple [B]** time = 0.024, size = 49, normalized size = 1.6

$$\frac{1}{2 \ln(2)} \ln\left(2^x + a \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} - \frac{1}{2 \ln(2)} \ln\left(2^x - a \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-2^(2\*x)\*b), x)

[Out] 1/2/(a\*b)^(1/2)/ln(2)\*ln(2^x+1/(a\*b)^(1/2)\*a)-1/2/(a\*b)^(1/2)/ln(2)\*ln(2^x-1/(a\*b)^(1/2)\*a)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-2^(2\*x)\*b), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.55946, size = 189, normalized size = 6.3

$$\left[ \frac{\sqrt{ab} \log\left(\frac{2^{2x}b+2\sqrt{ab}2^x+a}{2^{2x}b-a}\right)}{2ab \log(2)}, -\frac{\sqrt{-ab} \arctan\left(\frac{\sqrt{-ab}}{2^x b}\right)}{ab \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-2^(2\*x)\*b), x, algorithm="fricas")

[Out] [1/2\*sqrt(a\*b)\*log((2^(2\*x)\*b + 2\*sqrt(a\*b)\*2^x + a)/(2^(2\*x)\*b - a))/(a\*b\*log(2)), -sqrt(-a\*b)\*arctan(sqrt(-a\*b)/(2^x\*b))/(a\*b\*log(2))]

**Sympy [A]** time = 0.182175, size = 24, normalized size = 0.8

$$\frac{\text{RootSum}\left(4z^2ab - 1, (i \mapsto i \log(2^x + 2ia))\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-2\*\*(2\*x)\*b), x)

[Out]  $\text{RootSum}(4*_z^{**2}*a*b - 1, \text{Lambda}(_i, _i*\log(2**x + 2*_i*a)))/\log(2)$

---

**Giac [A]** time = 1.26682, size = 32, normalized size = 1.07

$$-\frac{\arctan\left(\frac{2^x b}{\sqrt{-ab}}\right)}{\sqrt{-ab} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(2^x/(a-2^{(2*x)*b}), x, \text{algorithm}="giac")$

[Out]  $-\arctan(2^x*b/\text{sqrt}(-a*b))/(\text{sqrt}(-a*b)*\log(2))$

$$3.485 \quad \int \frac{2^x}{a+4^{-x}b} dx$$

**Optimal.** Leaf size=43

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTan}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

**Rubi [A]** time = 0.0431895, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2249, 193, 321, 205}

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In] Int[ $2^x/(a + b/4^x)$ , x]

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTan}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 193

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int \frac{2^x}{a + 4^{-x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a + \frac{b}{x^2}} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{b + ax^2} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{b \text{Subst}\left(\int \frac{1}{b + ax^2} dx, x, 2^x\right)}{a \log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{2^x \sqrt{a}}{\sqrt{b}}\right)}{a^{3/2} \log(2)}
\end{aligned}$$

**Mathematica [A]** time = 0.0200666, size = 40, normalized size = 0.93

$$\frac{\frac{2^x}{a} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} 2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a + b/4^x), x]

[Out] (2^x/a - (Sqrt[b]\*ArcTan[(2^x\*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]

**Maple [B]** time = 0.029, size = 74, normalized size = 1.7

$$\frac{2^x}{a \ln(2)} + \frac{1}{2 a^2 \ln(2)} \sqrt{-ab} \ln\left(2^x - \frac{1}{a} \sqrt{-ab}\right) - \frac{1}{2 a^2 \ln(2)} \sqrt{-ab} \ln\left(2^x + \frac{1}{a} \sqrt{-ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+b/(4^x)), x)

[Out] 2^x/a/ln(2)+1/2/a^2\*(-a\*b)^(1/2)/ln(2)\*ln(2^x-1/a\*(-a\*b)^(1/2))-1/2/a^2\*(-a\*b)^(1/2)/ln(2)\*ln(2^x+1/a\*(-a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x)), x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [A]** time = 1.55328, size = 212, normalized size = 4.93

$$\left[ \frac{\sqrt{-\frac{b}{a}} \log\left(-\frac{2 \cdot 2^x a \sqrt{-\frac{b}{a}} - 2^{2x} a + b}{2^{2x} a + b}\right) + 2 \cdot 2^x}{2 a \log(2)}, -\frac{\sqrt{\frac{b}{a}} \arctan\left(\frac{2^x a \sqrt{\frac{b}{a}}}{b}\right) - 2^x}{a \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x)),x, algorithm="fricas")

[Out] [1/2\*(sqrt(-b/a)\*log(-(2\*2^x\*a\*sqrt(-b/a) - 2^(2\*x)\*a + b)/(2^(2\*x)\*a + b)) + 2\*2^x)/(a\*log(2)), -(sqrt(b/a)\*arctan(2^x\*a\*sqrt(b/a)/b) - 2^x)/(a\*log(2)))]

**Sympy [A]** time = 0.201869, size = 39, normalized size = 0.91

$$\begin{cases} \frac{2^x}{a \log(2)} & \text{for } a \log(2) \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\text{RootSum}(4z^2a^3 + b, (i \mapsto i \log(2^x - 2ia)))}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+b/(4\*\*x)),x)

[Out] Piecewise((2\*\*x/(a\*log(2)), Ne(a\*log(2), 0)), (x/a, True)) + RootSum(4\*\_z\*\*2\*a\*\*3 + b, Lambda(\_i, \_i\*log(2\*\*x - 2\*\_i\*a)))/log(2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{a + \frac{b}{4^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x)),x, algorithm="giac")

[Out] integrate(2^x/(a + b/4^x), x)

$$3.486 \quad \int \frac{2^x}{a+2^{-2x}b} dx$$

**Optimal.** Leaf size=43

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTan}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

**Rubi [A]** time = 0.0400896, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2249, 193, 321, 205}

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^x/(a + b/2^{(2*x)}), x]$

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTan}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

#### Rule 2249

$\text{Int}[(a + (b_*)(F_*)^{((e_*)((c_*) + (d_*)(x_))))})^{(p_*)}(G_*)^{((h_*)((f_*) + (g_*)(x_)))}, x\_Symbol] \rightarrow \text{With}\{m = \text{FullSimplify}[(d*e*\text{Log}[F])/(g*h*\text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*\text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)}*(a + b*F^{(c*e - (d*e*f)/g})*x^{\text{Numerator}[m]})^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \|\ \text{GtQ}[m, 1] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 193

$\text{Int}[(a + (b_*)(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[x^{(n*p)}*(b + a/x^n)^p, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{LtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$

#### Rule 321

$\text{Int}[(c_*)(x_*)^{(m_*)}(a + (b_*)(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^{(n*(m - n + 1))})/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 205

$\text{Int}[(a + (b_*)(x_*)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{2^x}{a + 2^{-2x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a + \frac{b}{x^2}} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{b + ax^2} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{b \text{Subst}\left(\int \frac{1}{b + ax^2} dx, x, 2^x\right)}{a \log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{2^x \sqrt{a}}{\sqrt{b}}\right)}{a^{3/2} \log(2)}
\end{aligned}$$

**Mathematica [A]** time = 0.0081684, size = 40, normalized size = 0.93

$$\frac{2^x}{a} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} 2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a + b/2^(2\*x)), x]

[Out] (2^x/a - (Sqrt[b]\*ArcTan[(2^x\*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]

**Maple [B]** time = 0.029, size = 74, normalized size = 1.7

$$\frac{2^x}{a \ln(2)} + \frac{1}{2 a^2 \ln(2)} \sqrt{-ab} \ln\left(2^x - \frac{1}{a} \sqrt{-ab}\right) - \frac{1}{2 a^2 \ln(2)} \sqrt{-ab} \ln\left(2^x + \frac{1}{a} \sqrt{-ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+b/(2^(2\*x))), x)

[Out] 2^x/a/ln(2)+1/2/a^2\*(-a\*b)^(1/2)/ln(2)\*ln(2^x-1/a\*(-a\*b)^(1/2))-1/2/a^2\*(-a\*b)^(1/2)/ln(2)\*ln(2^x+1/a\*(-a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.55053, size = 212, normalized size = 4.93

$$\left[ \frac{\sqrt{-\frac{b}{a}} \log\left(-\frac{2 \cdot 2^x a \sqrt{-\frac{b}{a}} - 2^{2x} a + b}{2^{2x} a + b}\right) + 2 \cdot 2^x}{2 a \log(2)}, -\frac{\sqrt{\frac{b}{a}} \arctan\left(\frac{2^x a \sqrt{\frac{b}{a}}}{b}\right) - 2^x}{a \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))),x, algorithm="fricas")

[Out] [1/2\*(sqrt(-b/a)\*log(-(2\*2^x\*a\*sqrt(-b/a) - 2^(2\*x)\*a + b)/(2^(2\*x)\*a + b)) + 2\*2^x)/(a\*log(2)), -(sqrt(b/a)\*arctan(2^x\*a\*sqrt(b/a)/b) - 2^x)/(a\*log(2))]

**Sympy [A]** time = 0.201809, size = 39, normalized size = 0.91

$$\begin{cases} \frac{2^x}{a \log(2)} & \text{for } a \log(2) \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\text{RootSum}\left(4z^2a^3 + b, (i \mapsto i \log(2^x - 2ia))\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+b/(2\*\*(2\*x))),x)

[Out] Piecewise((2\*\*x/(a\*log(2)), Ne(a\*log(2), 0)), (x/a, True)) + RootSum(4\*\_z\*\*2\*a\*\*3 + b, Lambda(\_i, \_i\*log(2\*\*x - 2\*\_i\*a)))/log(2)

**Giac [A]** time = 1.22902, size = 51, normalized size = 1.19

$$-\frac{b \arctan\left(\frac{2^x a}{\sqrt{ab}}\right)}{\sqrt{ab} a \log(2)} + \frac{2^x}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))),x, algorithm="giac")

[Out] -b\*arctan(2^x\*a/sqrt(a\*b))/(sqrt(a\*b)\*a\*log(2)) + 2^x/(a\*log(2))

$$3.487 \quad \int \frac{2^x}{a-4^{-x}b} dx$$

**Optimal.** Leaf size=43

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTanh}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

**Rubi [A]** time = 0.0432581, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2249, 193, 321, 208}

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In] Int[ $2^x/(a - b/4^x)$ , x]

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTanh}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 193

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int \frac{2^x}{a - 4^{-x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a - \frac{b}{x^2}} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{-b + ax^2} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{2^x}{a \log(2)} + \frac{b \text{Subst}\left(\int \frac{1}{-b + ax^2} dx, x, 2^x\right)}{a \log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{2^x \sqrt{a}}{\sqrt{b}}\right)}{a^{3/2} \log(2)}
\end{aligned}$$

**Mathematica [A]** time = 0.0178357, size = 40, normalized size = 0.93

$$\frac{2^x}{a} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a} 2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a - b/4^x), x]

[Out] (2^x/a - (Sqrt[b]\*ArcTanh[(2^x\*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]

**Maple [A]** time = 0.032, size = 70, normalized size = 1.6

$$\frac{2^x}{a \ln(2)} + \frac{1}{2 a^2 \ln(2)} \sqrt{ab} \ln\left(2^x - \frac{1}{a} \sqrt{ab}\right) - \frac{1}{2 a^2 \ln(2)} \sqrt{ab} \ln\left(2^x + \frac{1}{a} \sqrt{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-b/(4^x)), x)

[Out] 2^x/a/ln(2)+1/2/a^2\*(a\*b)^(1/2)/ln(2)\*ln(2^x-1/a\*(a\*b)^(1/2))-1/2/a^2\*(a\*b)^(1/2)/ln(2)\*ln(2^x+1/a\*(a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.62698, size = 211, normalized size = 4.91

$$\left[ \frac{\sqrt{\frac{b}{a}} \log\left(-\frac{2 \cdot 2^x a \sqrt{\frac{b}{a}} - 2^{2x} a - b}{2^{2x} a - b}\right) + 2 \cdot 2^x}{2 a \log(2)}, \frac{\sqrt{-\frac{b}{a}} \arctan\left(\frac{2^x a \sqrt{-\frac{b}{a}}}{b}\right) + 2^x}{a \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x)),x, algorithm="fricas")

[Out] [1/2\*(sqrt(b/a)\*log(-(2\*2^x\*a\*sqrt(b/a) - 2^(2\*x)\*a - b)/(2^(2\*x)\*a - b)) + 2\*2^x)/(a\*log(2)), (sqrt(-b/a)\*arctan(2^x\*a\*sqrt(-b/a)/b) + 2^x)/(a\*log(2))]

**Sympy [A]** time = 0.205229, size = 39, normalized size = 0.91

$$\begin{cases} \frac{2^x}{a \log(2)} & \text{for } a \log(2) \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\text{RootSum}(4z^2a^3 - b, (i \mapsto i \log(2^x - 2ia)))}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-b/(4\*\*x)),x)

[Out] Piecewise((2\*\*x/(a\*log(2)), Ne(a\*log(2), 0)), (x/a, True)) + RootSum(4\*\_z\*\*2\*a\*\*3 - b, Lambda(\_i, \_i\*log(2\*\*x - 2\*\_i\*a)))/log(2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{a - \frac{b}{4^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x)),x, algorithm="giac")

[Out] integrate(2^x/(a - b/4^x), x)

$$3.488 \quad \int \frac{2^x}{a-2^{-2x}b} dx$$

**Optimal.** Leaf size=43

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTanh}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

**Rubi [A]** time = 0.0432004, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2249, 193, 321, 208}

$$\frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2} \log(2)}$$

Antiderivative was successfully verified.

[In] Int[ $2^x/(a - b/2^{(2*x)})$ ], x]

[Out]  $2^x/(a*\text{Log}[2]) - (\text{Sqrt}[b]*\text{ArcTanh}[(2^x*\text{Sqrt}[a])/ \text{Sqrt}[b]])/(a^{(3/2)}*\text{Log}[2])$

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 193

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps



$$\begin{aligned}
\int \frac{2^x}{a - 2^{-2x}b} dx &= \frac{\text{Subst}\left(\int \frac{1}{a - \frac{b}{x^2}} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{-b + ax^2} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{2^x}{a \log(2)} + \frac{b \text{Subst}\left(\int \frac{1}{-b + ax^2} dx, x, 2^x\right)}{a \log(2)} \\
&= \frac{2^x}{a \log(2)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{2^x \sqrt{a}}{\sqrt{b}}\right)}{a^{3/2} \log(2)}
\end{aligned}$$

**Mathematica [A]** time = 0.0089197, size = 40, normalized size = 0.93

$$\frac{\frac{2^x}{a} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a} 2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/(a - b/2^(2\*x)), x]

[Out] (2^x/a - (Sqrt[b]\*ArcTanh[(2^x\*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]

**Maple [A]** time = 0.029, size = 70, normalized size = 1.6

$$\frac{2^x}{a \ln(2)} + \frac{1}{2 a^2 \ln(2)} \sqrt{ab} \ln\left(2^x - \frac{1}{a} \sqrt{ab}\right) - \frac{1}{2 a^2 \ln(2)} \sqrt{ab} \ln\left(2^x + \frac{1}{a} \sqrt{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-b/(2^(2\*x))), x)

[Out] 2^x/a/ln(2)+1/2/a^2\*(a\*b)^(1/2)/ln(2)\*ln(2^x-1/a\*(a\*b)^(1/2))-1/2/a^2\*(a\*b)^(1/2)/ln(2)\*ln(2^x+1/a\*(a\*b)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x))), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.6537, size = 211, normalized size = 4.91

$$\left[ \frac{\sqrt{\frac{b}{a}} \log\left(-\frac{2 \cdot 2^x a \sqrt{\frac{b}{a}} - 2^{2x} a - b}{2^{2x} a - b}\right) + 2 \cdot 2^x \sqrt{\frac{-b}{a}} \arctan\left(\frac{2^x a \sqrt{\frac{-b}{a}}}{b}\right) + 2^x}{2 a \log(2)}, \frac{\sqrt{\frac{-b}{a}} \arctan\left(\frac{2^x a \sqrt{\frac{-b}{a}}}{b}\right) + 2^x}{a \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x))),x, algorithm="fricas")

[Out] [1/2\*(sqrt(b/a)\*log(-(2\*2^x\*a\*sqrt(b/a) - 2^(2\*x)\*a - b)/(2^(2\*x)\*a - b)) + 2\*2^x)/(a\*log(2)), (sqrt(-b/a)\*arctan(2^x\*a\*sqrt(-b/a)/b) + 2^x)/(a\*log(2))]

**Sympy [A]** time = 0.205547, size = 39, normalized size = 0.91

$$\begin{cases} \frac{2^x}{a \log(2)} & \text{for } a \log(2) \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\text{RootSum}(4z^2a^3 - b, (i \mapsto i \log(2^x - 2ia))}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-b/(2\*\*(2\*x))),x)

[Out] Piecewise((2\*\*x/(a\*log(2)), Ne(a\*log(2), 0)), (x/a, True)) + RootSum(4\*\_z\*\*2\*a\*\*3 - b, Lambda(\_i, \_i\*log(2\*\*x - 2\*\_i\*a)))/log(2)

**Giac [A]** time = 1.36991, size = 53, normalized size = 1.23

$$\frac{b \arctan\left(\frac{2^x a}{\sqrt{-ab}}\right)}{\sqrt{-ab} a \log(2)} + \frac{2^x}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x))),x, algorithm="giac")

[Out] b\*arctan(2^x\*a/sqrt(-a\*b))/(sqrt(-a\*b)\*a\*log(2)) + 2^x/(a\*log(2))

$$3.489 \quad \int \frac{2^x}{\sqrt{a+4^x b}} dx$$

**Optimal.** Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+4^x b}}\right)}{\sqrt{b}\log(2)}$$

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 4^x\*b]]/(Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0391978, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+4^x b}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a + 4^x\*b],x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 4^x\*b]]/(Sqrt[b]\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{\sqrt{a+4^x b}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{2^x}{\sqrt{a+4^x b}}\right)}{\log(2)} \\ &= \frac{\tanh^{-1}\left(\frac{2^x \sqrt{b}}{\sqrt{a+4^x b}}\right)}{\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0127325, size = 33, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a + 4^x\*b], x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 2^(2\*x)\*b]]/(Sqrt[b]\*Log[2])

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int 2^x \frac{1}{\sqrt{a + 4^x b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+4^x\*b)^(1/2), x)

[Out] int(2^x/(a+4^x\*b)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{4^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+4^x\*b)^(1/2), x, algorithm="maxima")

[Out] integrate(2^x/sqrt(4^x\*b + a), x)

**Fricas [A]** time = 1.62632, size = 198, normalized size = 6.39

$$\left[ \frac{\log\left(-2\sqrt{2^{2x}b+a}2^x\sqrt{b}-2\cdot 2^{2x}b-a\right)}{2\sqrt{b}\log(2)}, -\frac{\sqrt{-b}\arctan\left(\frac{2^x\sqrt{-b}}{\sqrt{2^{2x}b+a}}\right)}{b\log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+4^x\*b)^(1/2), x, algorithm="fricas")

[Out] [1/2\*log(-2\*sqrt(2^(2\*x)\*b + a)\*2^x\*sqrt(b) - 2\*2^(2\*x)\*b - a)/(sqrt(b)\*log(2)), -sqrt(-b)\*arctan(2^x\*sqrt(-b)/sqrt(2^(2\*x)\*b + a))/(b\*log(2))]

**Sympy [A]** time = 0.755857, size = 85, normalized size = 2.74

$$\frac{\begin{cases} \frac{\sqrt{-\frac{a}{b}} \operatorname{asin}\left(2^x \sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b < 0 \\ \frac{\sqrt{\frac{a}{b}} \operatorname{asinh}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b > 0 \\ \frac{\sqrt{-\frac{a}{b}} \operatorname{acosh}\left(2^x \sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b > 0 \wedge a < 0 \end{cases}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+4\*\*x\*b)\*\*(1/2),x)

[Out] Piecewise((sqrt(-a/b)\*asin(2\*\*x\*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)\*asinh(2\*\*x\*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)\*acosh(2\*\*x\*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))/log(2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{4^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+4^x\*b)^(1/2),x, algorithm="giac")

[Out] integrate(2^x/sqrt(4^x\*b + a), x)

$$3.490 \quad \int \frac{2^x}{\sqrt{a+2^{2x}b}} dx$$

**Optimal.** Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b4^x}}\right)}{\sqrt{b}\log(2)}$$

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 4^x\*b]]/(Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0375751, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2249, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b4^x}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a + 2^(2\*x)\*b], x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 4^x\*b]]/(Sqrt[b]\*Log[2])

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{\sqrt{a+2^{2x}b}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{2^x}{\sqrt{a+4^x b}}\right)}{\log(2)} \\ &= \frac{\tanh^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a+4^x b}}\right)}{\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0043184, size = 33, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a + 2^(2\*x)\*b], x]

[Out] ArcTanh[(2^x\*Sqrt[b])/Sqrt[a + 2^(2\*x)\*b]]/(Sqrt[b]\*Log[2])

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int 2^x \frac{1}{\sqrt{a + 2^{2x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+2^(2\*x)\*b)^(1/2), x)

[Out] int(2^x/(a+2^(2\*x)\*b)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+2^(2\*x)\*b)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.62163, size = 198, normalized size = 6.39

$$\left[ \frac{\log\left(-2\sqrt{2^{2x}b+a}2^x\sqrt{b}-2\cdot 2^{2x}b-a\right)}{2\sqrt{b}\log(2)}, -\frac{\sqrt{-b}\arctan\left(\frac{2^x\sqrt{-b}}{\sqrt{2^{2x}b+a}}\right)}{b\log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+2^(2\*x)\*b)^(1/2), x, algorithm="fricas")

[Out] [1/2\*log(-2\*sqrt(2^(2\*x)\*b + a)\*2^x\*sqrt(b) - 2\*2^(2\*x)\*b - a)/(sqrt(b)\*log(2)), -sqrt(-b)\*arctan(2^x\*sqrt(-b)/sqrt(2^(2\*x)\*b + a))/(b\*log(2))]

**Sympy [A]** time = 0.823697, size = 85, normalized size = 2.74

$$\frac{\begin{cases} \frac{\sqrt{-\frac{a}{b}} \operatorname{asin}\left(2^x \sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b < 0 \\ \frac{\sqrt{\frac{a}{b}} \operatorname{asinh}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b > 0 \\ \frac{\sqrt{-\frac{a}{b}} \operatorname{acosh}\left(2^x \sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b > 0 \wedge a < 0 \end{cases}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+2\*\*(2\*x)\*b)\*\*(1/2),x)

[Out] Piecewise((sqrt(-a/b)\*asin(2\*\*x\*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)\*asinh(2\*\*x\*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)\*acosh(2\*\*x\*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))/log(2)

**Giac [A]** time = 1.24596, size = 42, normalized size = 1.35

$$-\frac{\log\left(\left|-2^x\sqrt{b} + \sqrt{2^{2x}b + a}\right|\right)}{\sqrt{b}\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+2^(2\*x)\*b)^(1/2),x, algorithm="giac")

[Out] -log(abs(-2^x\*sqrt(b) + sqrt(2^(2\*x)\*b + a)))/(sqrt(b)\*log(2))



$$3.491 \quad \int \frac{2^x}{\sqrt{a-4^x b}} dx$$

**Optimal.** Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b4^x}}\right)}{\sqrt{b}\log(2)}$$

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 4^x\*b]]/(Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0398257, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2249, 217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b4^x}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a - 4^x\*b],x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 4^x\*b]]/(Sqrt[b]\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m])^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{\sqrt{a-4^x b}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{2^x}{\sqrt{a-4^x b}}\right)}{\log(2)} \\ &= \frac{\tan^{-1}\left(\frac{2^x \sqrt{b}}{\sqrt{a-4^x b}}\right)}{\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0134453, size = 34, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a - 4^x\*b], x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 2^(2\*x)\*b]]/(Sqrt[b]\*Log[2])

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int 2^x \frac{1}{\sqrt{a - 4^x b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-4^x\*b)^(1/2), x)

[Out] int(2^x/(a-4^x\*b)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{-4^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b)^(1/2), x, algorithm="maxima")

[Out] integrate(2^x/sqrt(-4^x\*b + a), x)

**Fricas [A]** time = 1.58186, size = 224, normalized size = 7.

$$\left[ \frac{\sqrt{-b} \log\left(-2 \sqrt{-2^{2x} b + a} 2^x \sqrt{-b} + 2 \cdot 2^{2x} b - a\right)}{2 b \log(2)}, \frac{\arctan\left(\frac{\sqrt{-2^{2x} b + a} 2^x \sqrt{b}}{2^{2x} b - a}\right)}{\sqrt{b} \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b)^(1/2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(-2\*sqrt(-2^(2\*x)\*b + a)\*2^x\*sqrt(-b) + 2\*2^(2\*x)\*b - a)/(b\*log(2)), -arctan(sqrt(-2^(2\*x)\*b + a)\*2^x\*sqrt(b)/(2^(2\*x)\*b - a))/(sqrt(b)\*log(2))]

**Sympy [A]** time = 0.771352, size = 82, normalized size = 2.56

$$\frac{\begin{cases} \frac{\sqrt{\frac{a}{b}} \operatorname{asin}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b > 0 \\ \frac{\sqrt{\frac{-a}{b}} \operatorname{asinh}\left(2^x \sqrt{\frac{-b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b < 0 \\ \frac{\sqrt{\frac{a}{b}} \operatorname{acosh}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } a < 0 \wedge b < 0 \end{cases}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-4\*\*x\*b)\*\*(1/2), x)

[Out] Piecewise((sqrt(a/b)\*asin(2\*\*x\*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)\*asinh(2\*\*x\*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)\*acosh(2\*\*x\*sqrt(b/a))/sqrt(-a), (a < 0) & (b < 0)))/log(2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{-4^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-4^x\*b)^(1/2), x, algorithm="giac")

[Out] integrate(2^x/sqrt(-4^x\*b + a), x)

$$3.492 \quad \int \frac{2^x}{\sqrt{a-2^{2x}b}} dx$$

**Optimal.** Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b4^x}}\right)}{\sqrt{b}\log(2)}$$

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 4^x\*b]]/(Sqrt[b]\*Log[2])

**Rubi [A]** time = 0.0396528, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2249, 217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b4^x}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a - 2^(2\*x)\*b], x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 4^x\*b]]/(Sqrt[b]\*Log[2])

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{2^x}{\sqrt{a-2^{2x}b}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{2^x}{\sqrt{a-4^x b}}\right)}{\log(2)} \\ &= \frac{\tan^{-1}\left(\frac{2^x\sqrt{b}}{\sqrt{a-4^x b}}\right)}{\sqrt{b}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0044745, size = 34, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}2^x}{\sqrt{a-b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a - 2^(2\*x)\*b], x]

[Out] ArcTan[(2^x\*Sqrt[b])/Sqrt[a - 2^(2\*x)\*b]]/(Sqrt[b]\*Log[2])

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int 2^x \frac{1}{\sqrt{a - 2^{2x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-2^(2\*x)\*b)^(1/2), x)

[Out] int(2^x/(a-2^(2\*x)\*b)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-2^(2\*x)\*b)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.6548, size = 224, normalized size = 7.

$$\left[ \frac{\sqrt{-b}\log\left(-2\sqrt{-2^{2x}b+a}2^x\sqrt{-b}+2\cdot 2^{2x}b-a\right)}{2b\log(2)}, -\frac{\arctan\left(\frac{\sqrt{-2^{2x}b+a}2^x\sqrt{b}}{2^{2x}b-a}\right)}{\sqrt{b}\log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-2^(2\*x)\*b)^(1/2), x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(-2\*sqrt(-2^(2\*x)\*b + a)\*2^x\*sqrt(-b) + 2\*2^(2\*x)\*b - a)/(b\*log(2)), -arctan(sqrt(-2^(2\*x)\*b + a)\*2^x\*sqrt(b)/(2^(2\*x)\*b - a))/(sqrt(b)\*log(2))]

**Sympy [A]** time = 0.855239, size = 82, normalized size = 2.56

$$\frac{\begin{cases} \frac{\sqrt{\frac{a}{b}} \operatorname{asin}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b > 0 \\ \frac{\sqrt{-\frac{a}{b}} \operatorname{asinh}\left(2^x \sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a > 0 \wedge b < 0 \\ \frac{\sqrt{\frac{a}{b}} \operatorname{acosh}\left(2^x \sqrt{\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } a < 0 \wedge b < 0 \end{cases}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-2\*\*(2\*x)\*b)\*\*(1/2),x)

[Out] Piecewise((sqrt(a/b)\*asin(2\*\*x\*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)\*asinh(2\*\*x\*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)\*acosh(2\*\*x\*sqrt(b/a))/sqrt(-a), (a < 0) & (b < 0)))/log(2)

**Giac [A]** time = 1.37195, size = 49, normalized size = 1.53

$$-\frac{\log\left(\left|-2^x\sqrt{-b} + \sqrt{-2^{2x}b + a}\right|\right)}{\sqrt{-b}\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-2^(2\*x)\*b)^(1/2),x, algorithm="giac")

[Out] -log(abs(-2^x\*sqrt(-b) + sqrt(-2^(2\*x)\*b + a)))/(sqrt(-b)\*log(2))

$$3.493 \quad \int \frac{2^x}{\sqrt{a+4^{-x}b}} dx$$

**Optimal.** Leaf size=24

$$\frac{2^x \sqrt{a + b2^{-2x}}}{a \log(2)}$$

[Out] (2^x\*Sqrt[a + b/2^(2\*x)])/(a\*Log[2])

**Rubi [A]** time = 0.0479708, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2249, 191}

$$\frac{2^x \sqrt{a + b2^{-2x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a + b/4^x], x]

[Out] (2^x\*Sqrt[a + b/2^(2\*x)])/(a\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 191

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\int \frac{2^x}{\sqrt{a + 4^{-x}b}} dx = \frac{\text{Subst} \left( \int \frac{1}{\sqrt{a + \frac{b}{x^2}}} dx, x, 2^x \right)}{\log(2)} = \frac{2^x \sqrt{a + 2^{-2x}b}}{a \log(2)}$$

**Mathematica [A]** time = 0.0312534, size = 35, normalized size = 1.46

$$\frac{2^{-x} (a2^{2x} + b)}{a \log(2) \sqrt{a + b2^{-2x}}}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a + b/4^x],x]

[Out] (2^(2\*x)\*a + b)/(2^x\*a\*Sqrt[a + b/2^(2\*x)]\*Log[2])

**Maple [A]** time = 0.029, size = 40, normalized size = 1.7

$$\frac{a(2^x)^2 + b}{a2^x \ln(2)} \frac{1}{\sqrt{\frac{a(2^x)^2 + b}{(2^x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+b/(4^x))^(1/2),x)

[Out] 1/((a\*(2^x)^2+b)/(2^x)^2)^(1/2)\*(a\*(2^x)^2+b)/(2^x)/a/ln(2)

**Maxima [A]** time = 1.65125, size = 26, normalized size = 1.08

$$\frac{\sqrt{2^{2x}a + b}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="maxima")

[Out] sqrt(2^(2\*x)\*a + b)/(a\*log(2))

**Fricas [A]** time = 1.52985, size = 62, normalized size = 2.58

$$\frac{2^x \sqrt{\frac{2^{2x}a+b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="fricas")

[Out] 2^x\*sqrt((2^(2\*x)\*a + b)/2^(2\*x))/(a\*log(2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a + 4^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+b/(4\*\*x))\*\*(1/2),x)



[Out] Integral(2\*\*x/sqrt(a + 4\*\*(-x)\*b), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a + \frac{b}{4^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="giac")

[Out] integrate(2^x/sqrt(a + b/4^x), x)

$$3.494 \quad \int \frac{2^x}{\sqrt{a+2^{-2x}b}} dx$$

**Optimal.** Leaf size=24

$$\frac{2^x \sqrt{a + b2^{-2x}}}{a \log(2)}$$

[Out]  $(2^x \text{Sqrt}[a + b/2^{(2*x)}]) / (a \text{Log}[2])$

**Rubi [A]** time = 0.0471247, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2249, 191}

$$\frac{2^x \sqrt{a + b2^{-2x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^x/\text{Sqrt}[a + b/2^{(2*x)}], x]$

[Out]  $(2^x \text{Sqrt}[a + b/2^{(2*x)}]) / (a \text{Log}[2])$

#### Rule 2249

$\text{Int}[(a_ + (b_ \cdot) \cdot (F_ )^{((e_ \cdot) \cdot ((c_ \cdot) + (d_ \cdot) \cdot (x_ ))))^{(p_ \cdot) \cdot (G_ )^{((h_ \cdot) \cdot ((f_ \cdot) + (g_ \cdot) \cdot (x_ ))))}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(d \cdot e \cdot \text{Log}[F]) / (g \cdot h \cdot \text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m] / (g \cdot h \cdot \text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1) \cdot (a + b \cdot F^{(c \cdot e - (d \cdot e \cdot f) / g)} \cdot x^{\text{Numerator}[m]})^p, x], x, G^{((h \cdot (f + g \cdot x)) / \text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \ || \ \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 191

$\text{Int}[(a_ + (b_ \cdot) \cdot (x_ )^{(n_ )})^{(p_ )}, x\_Symbol] := \text{Simp}[(x \cdot (a + b \cdot x^n)^{(p + 1)}) / a, x] /; \text{FreeQ}[\{a, b, n, p\}, x] \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

#### Rubi steps

$$\int \frac{2^x}{\sqrt{a + 2^{-2x}b}} dx = \frac{\text{Subst} \left( \int \frac{1}{\sqrt{a + \frac{b}{x^2}}} dx, x, 2^x \right)}{\log(2)} = \frac{2^x \sqrt{a + 2^{-2x}b}}{a \log(2)}$$

**Mathematica [A]** time = 0.0063918, size = 35, normalized size = 1.46

$$\frac{2^{-x} (a2^{2x} + b)}{a \log(2) \sqrt{a + b2^{-2x}}}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a + b/2^(2\*x)], x]

[Out] (2^(2\*x)\*a + b)/(2^x\*a\*Sqrt[a + b/2^(2\*x)]\*Log[2])

**Maple [A]** time = 0.016, size = 40, normalized size = 1.7

$$\frac{a(2^x)^2 + b}{a2^x \ln(2)} \frac{1}{\sqrt{\frac{a(2^x)^2 + b}{(2^x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a+b/(2^(2\*x))))^(1/2), x)

[Out] 1/((a\*(2^x)^2+b)/(2^x)^2)^(1/2)\*(a\*(2^x)^2+b)/(2^x)/a/ln(2)

**Maxima [A]** time = 0.967163, size = 32, normalized size = 1.33

$$\frac{2^x \sqrt{a + \frac{b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))))^(1/2), x, algorithm="maxima")

[Out] 2^x\*sqrt(a + b/2^(2\*x))/(a\*log(2))

**Fricas [A]** time = 1.57491, size = 62, normalized size = 2.58

$$\frac{2^x \sqrt{\frac{2^{2x}a+b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))))^(1/2), x, algorithm="fricas")

[Out] 2^x\*sqrt((2^(2\*x)\*a + b)/2^(2\*x))/(a\*log(2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a + 2^{-2x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a+b/(2\*\*(2\*x))))\*\*(1/2), x)

[Out] Integral(2\*\*x/sqrt(a + 2\*\*(-2\*x)\*b), x)

---

**Giac [A]** time = 1.29711, size = 39, normalized size = 1.62

$$\frac{\frac{\sqrt{2^x a + b}}{a} - \frac{\sqrt{b}}{a}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a+b/(2^(2\*x))))^(1/2),x, algorithm="giac")

[Out] (sqrt(2^(2\*x)\*a + b)/a - sqrt(b)/a)/log(2)

$$3.495 \quad \int \frac{2^x}{\sqrt{a-4^{-x}b}} dx$$

**Optimal.** Leaf size=25

$$\frac{2^x \sqrt{a - b2^{-2x}}}{a \log(2)}$$

[Out] (2^x\*Sqrt[a - b/2^(2\*x)])/(a\*Log[2])

**Rubi [A]** time = 0.0504276, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2249, 191}

$$\frac{2^x \sqrt{a - b2^{-2x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^x/Sqrt[a - b/4^x], x]

[Out] (2^x\*Sqrt[a - b/2^(2\*x)])/(a\*Log[2])

#### Rule 2249

Int[((a\_) + (b\_)\*(F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(p\_)\*(G\_)^(h\_)\*((f\_) + (g\_)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 191

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}} dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{b}{x^2}}} dx, x, 2^x\right)}{\log(2)} = \frac{2^x \sqrt{a - 2^{-2x}b}}{a \log(2)}$$

**Mathematica [A]** time = 0.0305713, size = 38, normalized size = 1.52

$$\frac{2^{-x} (a2^{2x} - b)}{a \log(2) \sqrt{a - b2^{-2x}}}$$

Antiderivative was successfully verified.

[In] Integrate[2^x/Sqrt[a - b/4^x],x]

[Out] (2^(2\*x)\*a - b)/(2^x\*a\*Sqrt[a - b/2^(2\*x)]\*Log[2])

**Maple [A]** time = 0.024, size = 44, normalized size = 1.8

$$\frac{a(2^x)^2 - b}{a2^x \ln(2)} \frac{1}{\sqrt{\frac{a(2^x)^2 - b}{(2^x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-b/(4^x))^(1/2),x)

[Out] 1/((a\*(2^x)^2-b)/(2^x)^2)^(1/2)\*(a\*(2^x)^2-b)/(2^x)/a/ln(2)

**Maxima [A]** time = 1.64971, size = 28, normalized size = 1.12

$$\frac{\sqrt{2^{2x}a - b}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="maxima")

[Out] sqrt(2^(2\*x)\*a - b)/(a\*log(2))

**Fricas [A]** time = 1.61368, size = 62, normalized size = 2.48

$$\frac{2^x \sqrt{\frac{2^{2x}a - b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="fricas")

[Out] 2^x\*sqrt((2^(2\*x)\*a - b)/2^(2\*x))/(a\*log(2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a - 4^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-b/(4\*\*x))\*\*(1/2),x)

[Out] Integral(2\*\*x/sqrt(a - 4\*\*(-x)\*b), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a - \frac{b}{4^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="giac")

[Out] integrate(2^x/sqrt(a - b/4^x), x)

$$3.496 \quad \int \frac{2^x}{\sqrt{a-2^{-2x}b}} dx$$

**Optimal.** Leaf size=25

$$\frac{2^x \sqrt{a - b2^{-2x}}}{a \log(2)}$$

[Out]  $(2^x \text{Sqrt}[a - b/2^{(2*x)}]) / (a \text{Log}[2])$

**Rubi [A]** time = 0.0500951, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2249, 191}

$$\frac{2^x \sqrt{a - b2^{-2x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^x/\text{Sqrt}[a - b/2^{(2*x)}], x]$

[Out]  $(2^x \text{Sqrt}[a - b/2^{(2*x)}]) / (a \text{Log}[2])$

#### Rule 2249

$\text{Int}[(a_ + (b_ \cdot)(F_ )^{((e_ ) \cdot ((c_ ) + (d_ ) \cdot (x_ ))))^{(p_ )} \cdot (G_ )^{((h_ ) \cdot ((f_ ) + (g_ ) \cdot (x_ ))))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(d \cdot e \cdot \text{Log}[F]) / (g \cdot h \cdot \text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m] / (g \cdot h \cdot \text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1) \cdot (a + b \cdot F^{(c \cdot e - (d \cdot e \cdot f) / g)} \cdot x^{\text{Numerator}[m]})^p, x], x, G^{((h \cdot (f + g \cdot x)) / \text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \parallel \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 191

$\text{Int}[(a_ + (b_ \cdot)(x_ )^{(n_ )})^{(p_ )}, x\_Symbol] \rightarrow \text{Simp}[(x \cdot (a + b \cdot x^n)^{(p + 1)}) / a, x] /; \text{FreeQ}[\{a, b, n, p\}, x] \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

#### Rubi steps

$$\int \frac{2^x}{\sqrt{a - 2^{-2x}b}} dx = \frac{\text{Subst} \left( \int \frac{1}{\sqrt{a - \frac{b}{x^2}}} dx, x, 2^x \right)}{\log(2)} = \frac{2^x \sqrt{a - 2^{-2x}b}}{a \log(2)}$$

**Mathematica [A]** time = 0.0068336, size = 38, normalized size = 1.52

$$\frac{2^{-x} (a2^{2x} - b)}{a \log(2) \sqrt{a - b2^{-2x}}}$$

Antiderivative was successfully verified.



[In] Integrate[2^x/Sqrt[a - b/2^(2\*x)], x]

[Out] (2^(2\*x)\*a - b)/(2^x\*a\*Sqrt[a - b/2^(2\*x)]\*Log[2])

**Maple [A]** time = 0.016, size = 44, normalized size = 1.8

$$\frac{a(2^x)^2 - b}{a2^x \ln(2)} \frac{1}{\sqrt{\frac{a(2^x)^2 - b}{(2^x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^x/(a-b/(2^(2\*x))))^(1/2), x)

[Out] 1/((a\*(2^x)^2-b)/(2^x)^2)^(1/2)\*(a\*(2^x)^2-b)/(2^x)/a/ln(2)

**Maxima [A]** time = 0.979881, size = 34, normalized size = 1.36

$$\frac{2^x \sqrt{a - \frac{b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x))))^(1/2), x, algorithm="maxima")

[Out] 2^x\*sqrt(a - b/2^(2\*x))/(a\*log(2))

**Fricas [A]** time = 1.49888, size = 62, normalized size = 2.48

$$\frac{2^x \sqrt{\frac{2^{2x}a-b}{2^{2x}}}}{a \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x))))^(1/2), x, algorithm="fricas")

[Out] 2^x\*sqrt((2^(2\*x)\*a - b)/2^(2\*x))/(a\*log(2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^x}{\sqrt{a - 2^{-2x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*x/(a-b/(2\*\*(2\*x))))\*\*(1/2), x)

[Out] Integral(2\*\*x/sqrt(a - 2\*\*(-2\*x)\*b), x)

---

**Giac [A]** time = 1.31378, size = 45, normalized size = 1.8

$$\frac{\frac{\sqrt{2^{2x}a-b}}{a} - \frac{\sqrt{-b}}{a}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^x/(a-b/(2^(2\*x)))^(1/2),x, algorithm="giac")

[Out] (sqrt(2^(2\*x)\*a - b)/a - sqrt(-b)/a)/log(2)

$$3.497 \quad \int \frac{4^x}{\sqrt{a+2^x b}} dx$$

**Optimal.** Leaf size=44

$$\frac{2(a+b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a+b2^x}}{b^2 \log(2)}$$

[Out]  $(-2*a*\text{Sqrt}[a + 2^x*b])/(b^2*\text{Log}[2]) + (2*(a + 2^x*b)^(3/2))/(3*b^2*\text{Log}[2])$

**Rubi [A]** time = 0.040342, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{2(a+b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a+b2^x}}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[4^x/\text{Sqrt}[a + 2^x*b], x]$

[Out]  $(-2*a*\text{Sqrt}[a + 2^x*b])/(b^2*\text{Log}[2]) + (2*(a + 2^x*b)^(3/2))/(3*b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1]] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{\sqrt{a+2^x b}} dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2a\sqrt{a+2^x b}}{b^2 \log(2)} + \frac{2(a+2^x b)^{3/2}}{3b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0215607, size = 29, normalized size = 0.66

$$\frac{2(b2^x - 2a)\sqrt{a+b2^x}}{b^2 \log(8)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/Sqrt[a + 2^x\*b],x]

[Out] (2\*(-2\*a + 2^x\*b)\*Sqrt[a + 2^x\*b])/(b^2\*Log[8])

**Maple [A]** time = 0.015, size = 29, normalized size = 0.7

$$-\frac{-2 \cdot 2^{x b} + 4 a}{3 b^2 \ln(2)} \sqrt{a + 2^{x b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a+2^x\*b)^(1/2),x)

[Out] -2/3\*(-2^x\*b+2\*a)\*(a+2^x\*b)^(1/2)/b^2/ln(2)

**Maxima [A]** time = 1.46217, size = 92, normalized size = 2.09

$$\frac{2^{2x+1}}{3\sqrt{2^x b + a} \log(2)} - \frac{2^{x+1} a}{3\sqrt{2^x b + ab} \log(2)} - \frac{4 a^2}{3\sqrt{2^x b + ab^2} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+2^x\*b)^(1/2),x, algorithm="maxima")

[Out] 1/3\*2^(2\*x + 1)/(sqrt(2^x\*b + a)\*log(2)) - 1/3\*2^(x + 1)\*a/(sqrt(2^x\*b + a)\*b\*log(2)) - 4/3\*a^2/(sqrt(2^x\*b + a)\*b^2\*log(2))

**Fricas [A]** time = 1.55118, size = 65, normalized size = 1.48

$$\frac{2\sqrt{2^x b + a}(2^x b - 2a)}{3b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+2^x\*b)^(1/2),x, algorithm="fricas")

[Out] 2/3\*sqrt(2^x\*b + a)\*(2^x\*b - 2\*a)/(b^2\*log(2))

**Sympy [A]** time = 0.969486, size = 56, normalized size = 1.27

$$\begin{cases} \frac{2 \cdot 2^x \sqrt{2^x b + a}}{3 b \log(2)} - \frac{4 a \sqrt{2^x b + a}}{3 b^2 \log(2)} & \text{for } b \neq 0 \\ \frac{4^x}{2 \sqrt{a} \log(2)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a+2\*\*x\*b)\*\*(1/2),x)

```
[Out] Piecewise((2*2**x*sqrt(2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(2**x*b + a)/(3*b
**2*log(2)), Ne(b, 0)), (4**x/(2*sqrt(a)*log(2)), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{2^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a+2^x*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(4^x/sqrt(2^x*b + a), x)
```

$$3.498 \quad \int \frac{2^{2x}}{\sqrt{a+2^x b}} dx$$

Optimal. Leaf size=44

$$\frac{2(a+b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a+b2^x}}{b^2 \log(2)}$$

[Out]  $(-2*a*\text{Sqrt}[a + 2^x*b])/(b^2*\text{Log}[2]) + (2*(a + 2^x*b)^{(3/2)})/(3*b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0410894, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 43}

$$\frac{2(a+b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a+b2^x}}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{(2*x)}/\text{Sqrt}[a + 2^x*b], x]$

[Out]  $(-2*a*\text{Sqrt}[a + 2^x*b])/(b^2*\text{Log}[2]) + (2*(a + 2^x*b)^{(3/2)})/(3*b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))}^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{\sqrt{a+2^x b}} dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2a\sqrt{a+2^x b}}{b^2 \log(2)} + \frac{2(a+2^x b)^{3/2}}{3b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0115383, size = 29, normalized size = 0.66

$$\frac{2(b2^x - 2a)\sqrt{a+b2^x}}{b^2 \log(8)}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/Sqrt[a + 2^x\*b], x]

[Out] (2\*(-2\*a + 2^x\*b)\*Sqrt[a + 2^x\*b])/(b^2\*Log[8])

**Maple [A]** time = 0.01, size = 29, normalized size = 0.7

$$\frac{-2 \cdot 2^x b + 4 a}{3 b^2 \ln(2)} \sqrt{a + 2^x b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a+2^x\*b)^(1/2), x)

[Out] -2/3\*(-2^x\*b+2\*a)\*(a+2^x\*b)^(1/2)/b^2/ln(2)

**Maxima [A]** time = 0.973952, size = 51, normalized size = 1.16

$$\frac{2(2^x b + a)^{\frac{3}{2}}}{3 b^2 \log(2)} - \frac{2 \sqrt{2^x b + a} a}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+2^x\*b)^(1/2), x, algorithm="maxima")

[Out] 2/3\*(2^x\*b + a)^(3/2)/(b^2\*log(2)) - 2\*sqrt(2^x\*b + a)\*a/(b^2\*log(2))

**Fricas [A]** time = 1.56375, size = 65, normalized size = 1.48

$$\frac{2 \sqrt{2^x b + a} (2^x b - 2 a)}{3 b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+2^x\*b)^(1/2), x, algorithm="fricas")

[Out] 2/3\*sqrt(2^x\*b + a)\*(2^x\*b - 2\*a)/(b^2\*log(2))

**Sympy [A]** time = 0.955978, size = 58, normalized size = 1.32

$$\begin{cases} \frac{2 \cdot 2^x \sqrt{2^x b + a}}{3 b \log(2)} - \frac{4 a \sqrt{2^x b + a}}{3 b^2 \log(2)} & \text{for } b \neq 0 \\ \frac{2^{2x}}{2 \sqrt{a} \log(2)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a+2\*\*x\*b)\*\*(1/2), x)

[Out] Piecewise((2\*2\*\*x\*sqrt(2\*\*x\*b + a)/(3\*b\*log(2)) - 4\*a\*sqrt(2\*\*x\*b + a)/(3\*b\*\*2\*log(2)), Ne(b, 0)), (2\*\*(2\*x)/(2\*sqrt(a)\*log(2)), True))

**Giac [A]** time = 1.28933, size = 42, normalized size = 0.95

$$\frac{2 \left( (2^x b + a)^{\frac{3}{2}} - 3 \sqrt{2^x b + a} a \right)}{3 b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+2^x\*b)^(1/2),x, algorithm="giac")

[Out] 2/3\*((2^x\*b + a)^(3/2) - 3\*sqrt(2^x\*b + a)\*a)/(b^2\*log(2))



$$3.499 \quad \int \frac{4^x}{\sqrt{a-2^x b}} dx$$

**Optimal.** Leaf size=46

$$\frac{2(a-b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a-b2^x}}{b^2 \log(2)}$$

[Out]  $(-2*a*\text{Sqrt}[a - 2^x*b])/(b^2*\text{Log}[2]) + (2*(a - 2^x*b)^(3/2))/(3*b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0433091, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2248, 43}

$$\frac{2(a-b2^x)^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a-b2^x}}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[4^x/\text{Sqrt}[a - 2^x*b], x]$

[Out]  $(-2*a*\text{Sqrt}[a - 2^x*b])/(b^2*\text{Log}[2]) + (2*(a - 2^x*b)^(3/2))/(3*b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d)})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{(e*(c + d*x))/\text{Denominator}[m]}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{4^x}{\sqrt{a-2^x b}} dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a-bx}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{b\sqrt{a-bx}} - \frac{\sqrt{a-bx}}{b}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2a\sqrt{a-2^x b}}{b^2 \log(2)} + \frac{2(a-2^x b)^{3/2}}{3b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0223727, size = 30, normalized size = 0.65

$$-\frac{2\sqrt{a-b2^x}(2a+b2^x)}{b^2 \log(8)}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/Sqrt[a - 2^x\*b],x]

[Out] (-2\*Sqrt[a - 2^x\*b]\*(2\*a + 2^x\*b))/(b^2\*Log[8])

**Maple [A]** time = 0.015, size = 29, normalized size = 0.6

$$-\frac{2 \cdot 2^{x b} + 4 a}{3 b^2 \ln(2)} \sqrt{a - 2^{x b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a-2^x\*b)^(1/2),x)

[Out] -2/3\*(2^x\*b+2\*a)/b^2\*(a-2^x\*b)^(1/2)/ln(2)

**Maxima [A]** time = 1.48326, size = 96, normalized size = 2.09

$$\frac{2^{2x+1}}{3 \sqrt{-2^x b + a} \log(2)} + \frac{2^{x+1} a}{3 \sqrt{-2^x b + a b} \log(2)} - \frac{4 a^2}{3 \sqrt{-2^x b + a b^2} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-2^x\*b)^(1/2),x, algorithm="maxima")

[Out] 1/3\*2^(2\*x + 1)/(sqrt(-2^x\*b + a)\*log(2)) + 1/3\*2^(x + 1)\*a/(sqrt(-2^x\*b + a)\*b\*log(2)) - 4/3\*a^2/(sqrt(-2^x\*b + a)\*b^2\*log(2))

**Fricas [A]** time = 1.51855, size = 68, normalized size = 1.48

$$-\frac{2(2^{x b} + 2 a) \sqrt{-2^{x b} + a}}{3 b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-2^x\*b)^(1/2),x, algorithm="fricas")

[Out] -2/3\*(2^x\*b + 2\*a)\*sqrt(-2^x\*b + a)/(b^2\*log(2))

**Sympy [A]** time = 0.986447, size = 58, normalized size = 1.26

$$\begin{cases} -\frac{2 \cdot 2^x \sqrt{-2^{x b} + a}}{3 b \log(2)} - \frac{4 a \sqrt{-2^{x b} + a}}{3 b^2 \log(2)} & \text{for } b \neq 0 \\ \frac{4^x}{2 \sqrt{a} \log(2)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a-2\*\*x\*b)\*\*(1/2),x)

```
[Out] Piecewise((-2*2**x*sqrt(-2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(-2**x*b + a)/(3*b**2*log(2)), Ne(b, 0)), (4**x/(2*sqrt(a)*log(2)), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{-2^x b + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x/(a-2^x*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(4^x/sqrt(-2^x*b + a), x)
```

$$3.500 \quad \int \frac{2^{2x}}{\sqrt{a-2^x b}} dx$$

**Optimal.** Leaf size=46

$$\frac{2(a-2^{2x})^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a-2^x b}}{b^2 \log(2)}$$

[Out]  $(-2*a*\text{Sqrt}[a - 2^x*b])/(b^2*\text{Log}[2]) + (2*(a - 2^x*b)^{(3/2)})/(3*b^2*\text{Log}[2])$

**Rubi [A]** time = 0.0436841, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2248, 43}

$$\frac{2(a-2^{2x})^{3/2}}{3b^2 \log(2)} - \frac{2a\sqrt{a-2^x b}}{b^2 \log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{(2*x)}/\text{Sqrt}[a - 2^x*b], x]$

[Out]  $(-2*a*\text{Sqrt}[a - 2^x*b])/(b^2*\text{Log}[2]) + (2*(a - 2^x*b)^{(3/2)})/(3*b^2*\text{Log}[2])$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))}^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \|\ \text{GeQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{2^{2x}}{\sqrt{a-2^x b}} dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a-bx}} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{b\sqrt{a-bx}} - \frac{\sqrt{a-bx}}{b}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2a\sqrt{a-2^x b}}{b^2 \log(2)} + \frac{2(a-2^x b)^{3/2}}{3b^2 \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0117797, size = 30, normalized size = 0.65

$$-\frac{2\sqrt{a-2^x b}(2a+b2^x)}{b^2 \log(8)}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/Sqrt[a - 2^x\*b], x]

[Out] (-2\*Sqrt[a - 2^x\*b]\*(2\*a + 2^x\*b))/(b^2\*Log[8])

**Maple [A]** time = 0.01, size = 29, normalized size = 0.6

$$-\frac{2 \cdot 2^x b + 4 a}{3 b^2 \ln(2)} \sqrt{a - 2^x b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a-2^x\*b)^(1/2), x)

[Out] -2/3\*(2^x\*b+2\*a)/b^2\*(a-2^x\*b)^(1/2)/ln(2)

**Maxima [A]** time = 0.978245, size = 54, normalized size = 1.17

$$\frac{2(-2^x b + a)^{\frac{3}{2}}}{3 b^2 \log(2)} - \frac{2 \sqrt{-2^x b + a}}{b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-2^x\*b)^(1/2), x, algorithm="maxima")

[Out] 2/3\*(-2^x\*b + a)^(3/2)/(b^2\*log(2)) - 2\*sqrt(-2^x\*b + a)\*a/(b^2\*log(2))

**Fricas [A]** time = 1.49874, size = 68, normalized size = 1.48

$$-\frac{2(2^x b + 2 a) \sqrt{-2^x b + a}}{3 b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-2^x\*b)^(1/2), x, algorithm="fricas")

[Out] -2/3\*(2^x\*b + 2\*a)\*sqrt(-2^x\*b + a)/(b^2\*log(2))

**Sympy [A]** time = 0.952638, size = 60, normalized size = 1.3

$$\begin{cases} -\frac{2 \cdot 2^x \sqrt{-2^x b + a}}{3 b \log(2)} - \frac{4 a \sqrt{-2^x b + a}}{3 b^2 \log(2)} & \text{for } b \neq 0 \\ \frac{2^{2x}}{2 \sqrt{a} \log(2)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a-2\*\*x\*b)\*\*(1/2), x)

```
[Out] Piecewise((-2*2**x*sqrt(-2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(-2**x*b + a)/(3*b**2*log(2)), Ne(b, 0)), (2**(2*x)/(2*sqrt(a)*log(2)), True))
```

**Giac [A]** time = 1.15602, size = 45, normalized size = 0.98

$$\frac{2\left((-2^x b + a)^{\frac{3}{2}} - 3\sqrt{-2^x b + a}a\right)}{3b^2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(2*x)/(a-2^x*b)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((-2^x*b + a)^(3/2) - 3*sqrt(-2^x*b + a)*a)/(b^2*log(2))
```

$$3.501 \quad \int \frac{4^x}{\sqrt{a+2^{-x}b}} dx$$

**Optimal.** Leaf size=93

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} - \frac{3b2^{x-2}\sqrt{a+b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a \log(2)}$$

[Out] (2^(-1 + 2\*x)\*Sqrt[a + b/2^x])/(a\*Log[2]) - (3\*2^(-2 + x)\*b\*Sqrt[a + b/2^x])/(a^2\*Log[2]) + (3\*b^2\*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/(4\*a^(5/2)\*Log[2])

**Rubi [A]** time = 0.0743125, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2248, 51, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} - \frac{3b2^{x-2}\sqrt{a+b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[4^x/Sqrt[a + b/2^x], x]

[Out] (2^(-1 + 2\*x)\*Sqrt[a + b/2^x])/(a\*Log[2]) - (3\*2^(-2 + x)\*b\*Sqrt[a + b/2^x])/(a^2\*Log[2]) + (3\*b^2\*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/(4\*a^(5/2)\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 51

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned} \int \frac{4^x}{\sqrt{a+2^{-x}b}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}} dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, 2^{-x}\right)}{4a\log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)} - \frac{3\ 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2\log(2)} - \frac{(3b^2)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, 2^{-x}\right)}{8a^2\log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)} - \frac{3\ 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2\log(2)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{\frac{-a}{-b}+\frac{x^2}{b}} dx, x, \sqrt{a+2^{-x}b}\right)}{4a^2\log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)} - \frac{3\ 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2\log(2)} + \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0861008, size = 111, normalized size = 1.19

$$\frac{2^{-\frac{x}{2}-2}\left(\sqrt{a}2^{x/2}\left(a^22^{2x+1}-ab2^x-3b^2\right)+3b^2\sqrt{a2^x+b}\tanh^{-1}\left(\frac{\sqrt{a}2^{x/2}}{\sqrt{a2^x+b}}\right)\right)}{a^{5/2}\log(2)\sqrt{a+b2^{-x}}}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/Sqrt[a + b/2^x], x]

[Out] (2^(-2 - x/2)\*(2^(x/2)\*Sqrt[a]\*(2^(1 + 2\*x)\*a^2 - 2^x\*a\*b - 3\*b^2) + 3\*b^2\*Sqrt[2^x\*a + b]\*ArcTanh[(2^(x/2)\*Sqrt[a])/Sqrt[2^x\*a + b]])/(a^(5/2)\*Sqrt[a + b/2^x]\*Log[2])

**Maple [F]** time = 0.024, size = 0, normalized size = 0.

$$\int 4^x \frac{1}{\sqrt{a + \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a+b/(2^x))^(1/2), x)

[Out] int(4^x/(a+b/(2^x))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a + \frac{b}{2^x}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="maxima")

[Out] integrate(4^x/sqrt(a + b/2^x), x)

**Fricas [A]** time = 1.54, size = 373, normalized size = 4.01

$$\left[ \frac{3\sqrt{ab^2} \log\left(2 \cdot 2^x a + 2 \cdot 2^x \sqrt{a} \sqrt{\frac{2^x a + b}{2^x}} + b\right) + 2\left(2 \cdot 2^{2x} a^2 - 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a + b}{2^x}}}{8a^3 \log(2)}, -\frac{3\sqrt{-ab^2} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{2^x a + b}{2^x}}}{a}\right) - \left(2 \cdot 2^{2x} a^2 - 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a + b}{2^x}}}{4a^3 \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="fricas")

[Out] [1/8\*(3\*sqrt(a)\*b^2\*log(2\*2^x\*a + 2\*2^x\*sqrt(a)\*sqrt((2^x\*a + b)/2^x) + b) + 2\*(2\*2^(2\*x)\*a^2 - 3\*2^x\*a\*b)\*sqrt((2^x\*a + b)/2^x))/(a^3\*log(2)), -1/4\*(3\*sqrt(-a)\*b^2\*arctan(sqrt(-a)\*sqrt((2^x\*a + b)/2^x)/a) - (2\*2^(2\*x)\*a^2 - 3\*2^x\*a\*b)\*sqrt((2^x\*a + b)/2^x))/(a^3\*log(2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a + 2^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a+b/(2\*\*x))\*\*(1/2),x)

[Out] Integral(4\*\*x/sqrt(a + 2\*\*(-x)\*b), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a + \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="giac")

[Out] integrate(4^x/sqrt(a + b/2^x), x)

$$3.502 \quad \int \frac{2^{2x}}{\sqrt{a+2^{-x}b}} dx$$

**Optimal.** Leaf size=93

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} - \frac{3b2^{x-2}\sqrt{a+b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a \log(2)}$$

```
[Out] (2^(-1 + 2*x)*Sqrt[a + b/2^x])/(a*Log[2]) - (3*2^(-2 + x)*b*Sqrt[a + b/2^x])/(a^2*Log[2]) + (3*b^2*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/(4*a^(5/2)*Log[2])
```

**Rubi [A]** time = 0.0729013, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2248, 51, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} - \frac{3b2^{x-2}\sqrt{a+b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a \log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^(2*x)/Sqrt[a + b/2^x], x]
```

```
[Out] (2^(-1 + 2*x)*Sqrt[a + b/2^x])/(a*Log[2]) - (3*2^(-2 + x)*b*Sqrt[a + b/2^x])/(a^2*Log[2]) + (3*b^2*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/(4*a^(5/2)*Log[2])
```

### Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(g*h*Log[G])/(d*e*Log[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{2^{2x}}{\sqrt{a+2^{-x}b}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}} dx, x, 2^{-x}\right)}{\log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a \log(2)} + \frac{(3b) \text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, 2^{-x}\right)}{4a \log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a \log(2)} - \frac{3 \cdot 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2 \log(2)} - \frac{(3b^2) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, 2^{-x}\right)}{8a^2 \log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a \log(2)} - \frac{3 \cdot 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2 \log(2)} - \frac{(3b) \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+2^{-x}b}\right)}{4a^2 \log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a \log(2)} - \frac{3 \cdot 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2 \log(2)} + \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0348687, size = 111, normalized size = 1.19

$$\frac{2^{-\frac{x}{2}-2} \left( \sqrt{a} 2^{x/2} (a^2 2^{2x+1} - ab 2^x - 3b^2) + 3b^2 \sqrt{a} 2^x + b \tanh^{-1} \left( \frac{\sqrt{a} 2^{x/2}}{\sqrt{a} 2^x + b} \right) \right)}{a^{5/2} \log(2) \sqrt{a + b 2^{-x}}}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/Sqrt[a + b/2^x], x]

[Out] (2^(-2 - x/2)\*(2^(x/2)\*Sqrt[a]\*(2^(1 + 2\*x)\*a^2 - 2^x\*a\*b - 3\*b^2) + 3\*b^2\*Sqrt[2^x\*a + b]\*ArcTanh[(2^(x/2)\*Sqrt[a])/Sqrt[2^x\*a + b]])/(a^(5/2)\*Sqrt[a + b/2^x]\*Log[2])

**Maple [F]** time = 0.01, size = 0, normalized size = 0.

$$\int 2^{2x} \frac{1}{\sqrt{a + \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a+b/(2^x))^(1/2), x)

[Out] int(2^(2\*x)/(a+b/(2^x))^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+b/(2^x))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.58759, size = 373, normalized size = 4.01

$$\left[ \frac{3\sqrt{ab^2} \log\left(2 \cdot 2^x a + 2 \cdot 2^x \sqrt{a} \sqrt{\frac{2^x a + b}{2^x}} + b\right) + 2\left(2 \cdot 2^{2x} a^2 - 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a + b}{2^x}}}{8a^3 \log(2)}, \frac{3\sqrt{-ab^2} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{2^x a + b}{2^x}}}{a}\right) - \left(2 \cdot 2^{2x} a^2 - 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a + b}{2^x}}}{4a^3 \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+b/(2^x))^(1/2),x, algorithm="fricas")

[Out] [1/8\*(3\*sqrt(a)\*b^2\*log(2\*2^x\*a + 2\*2^x\*sqrt(a)\*sqrt((2^x\*a + b)/2^x) + b) + 2\*(2\*2^(2\*x)\*a^2 - 3\*2^x\*a\*b)\*sqrt((2^x\*a + b)/2^x))/(a^3\*log(2)), -1/4\*(3\*sqrt(-a)\*b^2\*arctan(sqrt(-a)\*sqrt((2^x\*a + b)/2^x)/a) - (2\*2^(2\*x)\*a^2 - 3\*2^x\*a\*b)\*sqrt((2^x\*a + b)/2^x))/(a^3\*log(2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^{2x}}{\sqrt{a + 2^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a+b/(2\*\*x))\*\*(1/2),x)

[Out] Integral(2\*\*(2\*x)/sqrt(a + 2\*\*(-x)\*b), x)

**Giac [A]** time = 1.3435, size = 127, normalized size = 1.37

$$\frac{2\sqrt{2^{2x}a + 2^x b} \left( \frac{2 \cdot 2^x}{a} - \frac{3b}{a^2} \right) - \frac{3b^2 \log\left(\left| -2 \left( 2^x \sqrt{a} - \sqrt{2^{2x}a + 2^x b} \right) \sqrt{a-b} \right| \right)}{a^{\frac{5}{2}}} + \frac{3b^2 \log(|b|)}{a^{\frac{5}{2}}}}{8 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a+b/(2^x))^(1/2),x, algorithm="giac")

[Out] 1/8\*(2\*sqrt(2^(2\*x)\*a + 2^x\*b)\*(2\*2^x/a - 3\*b/a^2) - 3\*b^2\*log(abs(-2\*(2^x\*sqrt(a) - sqrt(2^(2\*x)\*a + 2^x\*b))\*sqrt(a) - b))/a^(5/2) + 3\*b^2\*log(abs(b))/a^(5/2))/log(2)

$$3.503 \quad \int \frac{4^x}{\sqrt{a-2^{-x}b}} dx$$

**Optimal.** Leaf size=96

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} + \frac{3b2^{x-2}\sqrt{a-b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a \log(2)}$$

[Out] (2^(-1 + 2\*x)\*Sqrt[a - b/2^x])/(a\*Log[2]) + (3\*2^(-2 + x)\*b\*Sqrt[a - b/2^x])/(a^2\*Log[2]) + (3\*b^2\*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/(4\*a^(5/2)\*Log[2])

**Rubi [A]** time = 0.0757727, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2248, 51, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} + \frac{3b2^{x-2}\sqrt{a-b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a \log(2)}$$

Antiderivative was successfully verified.

[In] Int[4^x/Sqrt[a - b/2^x], x]

[Out] (2^(-1 + 2\*x)\*Sqrt[a - b/2^x])/(a\*Log[2]) + (3\*2^(-2 + x)\*b\*Sqrt[a - b/2^x])/(a^2\*Log[2]) + (3\*b^2\*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/(4\*a^(5/2)\*Log[2])

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 51

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned} \int \frac{4^x}{\sqrt{a-2^{-x}b}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a-bx}} dx, x, 2^{-x}\right)}{\log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a \log(2)} - \frac{(3b) \text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx}} dx, x, 2^{-x}\right)}{4a \log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a \log(2)} + \frac{3 \cdot 2^{-2+2x}b\sqrt{a-2^{-x}b}}{a^2 \log(2)} - \frac{(3b^2) \text{Subst}\left(\int \frac{1}{x\sqrt{a-bx}} dx, x, 2^{-x}\right)}{8a^2 \log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a \log(2)} + \frac{3 \cdot 2^{-2+2x}b\sqrt{a-2^{-x}b}}{a^2 \log(2)} + \frac{(3b) \text{Subst}\left(\int \frac{1}{\frac{a-x^2}{b}} dx, x, \sqrt{a-2^{-x}b}\right)}{4a^2 \log(2)} \\ &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a \log(2)} + \frac{3 \cdot 2^{-2+2x}b\sqrt{a-2^{-x}b}}{a^2 \log(2)} + \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0811319, size = 115, normalized size = 1.2

$$\frac{2^{-\frac{x}{2}-2} \left( \sqrt{a} 2^{x/2} (a^2 2^{2x+1} + ab 2^x - 3b^2) + 3b^2 \sqrt{a 2^x - b} \tanh^{-1} \left( \frac{\sqrt{a} 2^{x/2}}{\sqrt{a 2^x - b}} \right) \right)}{a^{5/2} \log(2) \sqrt{a - b 2^{-x}}}$$

Antiderivative was successfully verified.

[In] Integrate[4^x/Sqrt[a - b/2^x], x]

[Out] (2^(-2 - x/2)\*(2^(x/2)\*Sqrt[a]\*(2^(1 + 2\*x))\*a^2 + 2^x\*a\*b - 3\*b^2) + 3\*Sqrt[2^x\*a - b]\*b^2\*ArcTanh[(2^(x/2)\*Sqrt[a])/Sqrt[2^x\*a - b]])/(a^(5/2)\*Sqrt[a - b/2^x]\*Log[2])

**Maple [F]** time = 0.022, size = 0, normalized size = 0.

$$\int 4^x \frac{1}{\sqrt{a - \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4^x/(a-b/(2^x))^(1/2), x)

[Out] int(4^x/(a-b/(2^x))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a - \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="maxima")

[Out] integrate(4^x/sqrt(a - b/2^x), x)

**Fricas [A]** time = 1.53072, size = 374, normalized size = 3.9

$$\left[ \frac{3 \sqrt{ab^2} \log\left(-2 \cdot 2^x a - 2 \cdot 2^x \sqrt{a} \sqrt{\frac{2^x a - b}{2^x}} + b\right) + 2\left(2 \cdot 2^{2x} a^2 + 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a - b}{2^x}}}{8 a^3 \log(2)}, -\frac{3 \sqrt{-ab^2} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{2^x a - b}{2^x}}}{a}\right) - (2 \cdot 2^{2x} a^2 + 3 \cdot 2^x ab) \sqrt{\frac{2^x a - b}{2^x}}}{4 a^3 \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="fricas")

[Out] [1/8\*(3\*sqrt(a)\*b^2\*log(-2\*2^x\*a - 2\*2^x\*sqrt(a)\*sqrt((2^x\*a - b)/2^x) + b) + 2\*(2\*2^(2\*x)\*a^2 + 3\*2^x\*a\*b)\*sqrt((2^x\*a - b)/2^x))/(a^3\*log(2)), -1/4\*(3\*sqrt(-a)\*b^2\*arctan(sqrt(-a)\*sqrt((2^x\*a - b)/2^x)/a) - (2\*2^(2\*x)\*a^2 + 3\*2^x\*a\*b)\*sqrt((2^x\*a - b)/2^x))/(a^3\*log(2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a - 2^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4\*\*x/(a-b/(2\*\*x))\*\*(1/2),x)

[Out] Integral(4\*\*x/sqrt(a - 2\*\*(-x)\*b), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x}{\sqrt{a - \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="giac")

[Out] integrate(4^x/sqrt(a - b/2^x), x)

$$3.504 \quad \int \frac{2^{2x}}{\sqrt{a-2^{-x}b}} dx$$

**Optimal.** Leaf size=96

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} + \frac{3b2^{x-2}\sqrt{a-b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a \log(2)}$$

```
[Out] (2^(-1 + 2*x)*Sqrt[a - b/2^x])/(a*Log[2]) + (3*2^(-2 + x)*b*Sqrt[a - b/2^x])/(a^2*Log[2]) + (3*b^2*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/(4*a^(5/2)*Log[2])
```

**Rubi [A]** time = 0.0774554, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2248, 51, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{4a^{5/2} \log(2)} + \frac{3b2^{x-2}\sqrt{a-b2^{-x}}}{a^2 \log(2)} + \frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a \log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^(2*x)/Sqrt[a - b/2^x], x]
```

```
[Out] (2^(-1 + 2*x)*Sqrt[a - b/2^x])/(a*Log[2]) + (3*2^(-2 + x)*b*Sqrt[a - b/2^x])/(a^2*Log[2]) + (3*b^2*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/(4*a^(5/2)*Log[2])
```

#### Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(g*h*Log[G])/(d*e*Log[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208



Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{2^{2x}}{\sqrt{a-2^{-x}b}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a-bx}} dx, x, 2^{-x}\right)}{\log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx}} dx, x, 2^{-x}\right)}{4a\log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)} + \frac{3\ 2^{-2+x}b\sqrt{a-2^{-x}b}}{a^2\log(2)} - \frac{(3b^2)\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx}} dx, x, 2^{-x}\right)}{8a^2\log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)} + \frac{3\ 2^{-2+x}b\sqrt{a-2^{-x}b}}{a^2\log(2)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a-2^{-x}b}\right)}{4a^2\log(2)} \\
 &= \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)} + \frac{3\ 2^{-2+x}b\sqrt{a-2^{-x}b}}{a^2\log(2)} + \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0127632, size = 115, normalized size = 1.2

$$\frac{2^{-\frac{x}{2}-2} \left( \sqrt{a} 2^{x/2} \left( a^2 2^{2x+1} + ab 2^x - 3b^2 \right) + 3b^2 \sqrt{a} 2^x - b \tanh^{-1} \left( \frac{\sqrt{a} 2^{x/2}}{\sqrt{a} 2^x - b} \right) \right)}{a^{5/2} \log(2) \sqrt{a - b 2^{-x}}}$$

Antiderivative was successfully verified.

[In] Integrate[2^(2\*x)/Sqrt[a - b/2^x], x]

[Out] (2^(-2 - x/2)\*(2^(x/2)\*Sqrt[a]\*(2^(1 + 2\*x)\*a^2 + 2^x\*a\*b - 3\*b^2) + 3\*Sqrt[2^x\*a - b]\*b^2\*ArcTanh[(2^(x/2)\*Sqrt[a])/Sqrt[2^x\*a - b]])/(a^(5/2)\*Sqrt[a - b/2^x]\*Log[2])

**Maple [F]** time = 0.013, size = 0, normalized size = 0.

$$\int 2^{2x} \frac{1}{\sqrt{a - \frac{b}{2^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^(2\*x)/(a-b/(2^x))^(1/2), x)

[Out] int(2^(2\*x)/(a-b/(2^x))^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-b/(2^x))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.59552, size = 374, normalized size = 3.9

$$\left[ \frac{3\sqrt{ab^2} \log\left(-2 \cdot 2^x a - 2 \cdot 2^x \sqrt{a} \sqrt{\frac{2^x a - b}{2^x}} + b\right) + 2\left(2 \cdot 2^{2x} a^2 + 3 \cdot 2^x ab\right) \sqrt{\frac{2^x a - b}{2^x}}}{8 a^3 \log(2)}, \frac{3\sqrt{-ab^2} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{2^x a - b}{2^x}}}{a}\right) - \left(2 \cdot 2^{2x} a^2\right)}{4 a^3 \log(2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-b/(2^x))^(1/2),x, algorithm="fricas")

[Out] [1/8\*(3\*sqrt(a)\*b^2\*log(-2\*2^x\*a - 2\*2^x\*sqrt(a)\*sqrt((2^x\*a - b)/2^x) + b) + 2\*(2\*2^(2\*x)\*a^2 + 3\*2^x\*a\*b)\*sqrt((2^x\*a - b)/2^x))/(a^3\*log(2)), -1/4\*(3\*sqrt(-a)\*b^2\*arctan(sqrt(-a)\*sqrt((2^x\*a - b)/2^x)/a) - (2\*2^(2\*x)\*a^2 + 3\*2^x\*a\*b)\*sqrt((2^x\*a - b)/2^x))/(a^3\*log(2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2^{2x}}{\sqrt{a - 2^{-x}b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*\*(2\*x)/(a-b/(2\*\*x))\*\*(1/2),x)

[Out] Integral(2\*\*(2\*x)/sqrt(a - 2\*\*(-x)\*b), x)

**Giac [A]** time = 1.37684, size = 136, normalized size = 1.42

$$\frac{2\sqrt{2^{2x}a - 2^x b} \left( \frac{2 \cdot 2^x}{a} + \frac{3b}{a^2} \right) + \frac{3b^2 \log(\sqrt{|a|}|b|)}{a^{\frac{5}{2}}} - \frac{3b^2 \log\left(\left| -2\left(2^x \sqrt{a} - \sqrt{2^{2x}a - 2^x b}\right) a + \sqrt{ab} \right|\right)}{a^{\frac{5}{2}}}}{8 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^(2\*x)/(a-b/(2^x))^(1/2),x, algorithm="giac")

[Out] 1/8\*(2\*sqrt(2^(2\*x)\*a - 2^x\*b)\*(2\*2^x/a + 3\*b/a^2) + 3\*b^2\*log(sqrt(abs(a))\*abs(b))/a^(5/2) - 3\*b^2\*log(abs(-2\*(2^x\*sqrt(a) - sqrt(2^(2\*x)\*a - 2^x\*b))\*a + sqrt(a)\*b))/a^(5/2))/log(2)

$$3.505 \quad \int \frac{1}{1+2e^x+e^{2x}} dx$$

**Optimal.** Leaf size=17

$$x + \frac{1}{e^x + 1} - \log(e^x + 1)$$

[Out] (1 + E^x)^(-1) + x - Log[1 + E^x]

**Rubi [A]** time = 0.0147249, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2282, 44}

$$x + \frac{1}{e^x + 1} - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*E^x + E^(2\*x))^(-1), x]

[Out] (1 + E^x)^(-1) + x - Log[1 + E^x]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{1+2e^x+e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{x(1+x)^2} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2} \right) dx, x, e^x \right) \\ &= \frac{1}{1+e^x} + x - \log(1+e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0141032, size = 17, normalized size = 1.

$$x + \frac{1}{e^x + 1} - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*E^x + E^(2\*x))^(-1), x]

[Out]  $(1 + E^x)^{-1} + x - \text{Log}[1 + E^x]$

---

**Maple [A]** time = 0.008, size = 18, normalized size = 1.1

$$\ln(e^x) + (1 + e^x)^{-1} - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+2*exp(x)+exp(2*x)),x)`

[Out]  $\ln(\exp(x))+1/(1+\exp(x))-\ln(1+\exp(x))$

---

**Maxima [A]** time = 0.977773, size = 20, normalized size = 1.18

$$x + \frac{1}{e^x + 1} - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")`

[Out]  $x + 1/(e^x + 1) - \log(e^x + 1)$

---

**Fricas [A]** time = 1.55972, size = 70, normalized size = 4.12

$$\frac{x e^x - (e^x + 1) \log(e^x + 1) + x + 1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")`

[Out]  $(x e^x - (e^x + 1) \log(e^x + 1) + x + 1) / (e^x + 1)$

---

**Sympy [A]** time = 0.079514, size = 14, normalized size = 0.82

$$x - \log(e^x + 1) + \frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+2*exp(x)+exp(2*x)),x)`

[Out]  $x - \log(\exp(x) + 1) + 1/(\exp(x) + 1)$

---

**Giac [A]** time = 1.32998, size = 20, normalized size = 1.18

$$x + \frac{1}{e^x + 1} - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] x + 1/(e^x + 1) - log(e^x + 1)
```

$$3.506 \quad \int \frac{1}{2+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=24

$$\frac{x}{2} - \log(e^x + 1) + \frac{1}{2} \log(e^x + 2)$$

[Out] x/2 - Log[1 + E^x] + Log[2 + E^x]/2

**Rubi [A]** time = 0.0198411, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {2282, 705, 29, 632, 31}

$$\frac{x}{2} - \log(e^x + 1) + \frac{1}{2} \log(e^x + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*E^x + E^(2\*x))^(-1), x]

[Out] x/2 - Log[1 + E^x] + Log[2 + E^x]/2

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 31

```
Int[((a_.) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{2 + 3e^x + e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{x(2 + 3x + x^2)} dx, x, e^x \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{1}{x} dx, x, e^x \right) + \frac{1}{2} \text{Subst} \left( \int \frac{-3 - x}{2 + 3x + x^2} dx, x, e^x \right) \\
&= \frac{x}{2} + \frac{1}{2} \text{Subst} \left( \int \frac{1}{2 + x} dx, x, e^x \right) - \text{Subst} \left( \int \frac{1}{1 + x} dx, x, e^x \right) \\
&= \frac{x}{2} - \log(1 + e^x) + \frac{1}{2} \log(2 + e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.010407, size = 24, normalized size = 1.

$$\frac{x}{2} - \log(e^x + 1) + \frac{1}{2} \log(e^x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*E^x + E^(2\*x))^(-1), x]

[Out] x/2 - Log[1 + E^x] + Log[2 + E^x]/2

**Maple [A]** time = 0.007, size = 21, normalized size = 0.9

$$\frac{\ln(e^x)}{2} + \frac{\ln(2 + e^x)}{2} - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+3\*exp(x)+exp(2\*x)), x)

[Out] 1/2\*ln(exp(x))+1/2\*ln(2+exp(x))-ln(1+exp(x))

**Maxima [A]** time = 0.962841, size = 24, normalized size = 1.

$$\frac{1}{2} x + \frac{1}{2} \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3\*exp(x)+exp(2\*x)), x, algorithm="maxima")

[Out] 1/2\*x + 1/2\*log(e^x + 2) - log(e^x + 1)

**Fricas [A]** time = 1.4946, size = 55, normalized size = 2.29

$$\frac{1}{2} x + \frac{1}{2} \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*x + 1/2\*log(e^x + 2) - log(e^x + 1)

**Sympy [A]** time = 0.105901, size = 17, normalized size = 0.71

$$\frac{x}{2} - \log(e^x + 1) + \frac{\log(e^x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3\*exp(x)+exp(2\*x)),x)

[Out] x/2 - log(exp(x) + 1) + log(exp(x) + 2)/2

**Giac [A]** time = 1.27243, size = 24, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{2}\log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] 1/2\*x + 1/2\*log(e^x + 2) - log(e^x + 1)



$$3.507 \quad \int \frac{1}{-1+e^x+e^{2x}} dx$$

**Optimal.** Leaf size=56

$$-x + \frac{1}{10} (5 + \sqrt{5}) \log(2e^x + 1 - \sqrt{5}) + \frac{1}{10} (5 - \sqrt{5}) \log(2e^x + 1 + \sqrt{5})$$

[Out]  $-x + ((5 + \text{Sqrt}[5]) * \text{Log}[1 - \text{Sqrt}[5] + 2 * E^x]) / 10 + ((5 - \text{Sqrt}[5]) * \text{Log}[1 + \text{Sqrt}[5] + 2 * E^x]) / 10$

**Rubi [A]** time = 0.0316725, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {2282, 705, 29, 632, 31}

$$-x + \frac{1}{10} (5 + \sqrt{5}) \log(2e^x + 1 - \sqrt{5}) + \frac{1}{10} (5 - \sqrt{5}) \log(2e^x + 1 + \sqrt{5})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(-1 + E^x + E^{(2*x)})^{(-1)}, x]$

[Out]  $-x + ((5 + \text{Sqrt}[5]) * \text{Log}[1 - \text{Sqrt}[5] + 2 * E^x]) / 10 + ((5 - \text{Sqrt}[5]) * \text{Log}[1 + \text{Sqrt}[5] + 2 * E^x]) / 10$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_))^{(m\_)} /; \text{FreeQ}[\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n]] \&\& \text{!MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))} (F\_)[v_] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

#### Rule 705

$\text{Int}[1/(((d\_)+(e\_)*(x\_))*((a\_)+(b\_)*(x\_)+(c\_)*(x\_)^2)), x\_Symbol] \rightarrow \text{Dist}[e^2/(c*d^2 - b*d*e + a*e^2), \text{Int}[1/(d + e*x), x], x] + \text{Dist}[1/(c*d^2 - b*d*e + a*e^2), \text{Int}[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

#### Rule 29

$\text{Int}[(x\_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

#### Rule 632

$\text{Int}[(d\_)+(e\_)*(x\_)]/((a\_)+(b\_)*(x\_)+(c\_)*(x\_)^2), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(c*d - e*(b/2 - q/2))/q, \text{Int}[1/(b/2 - q/2 + c*x), x], x] - \text{Dist}[(c*d - e*(b/2 + q/2))/q, \text{Int}[1/(b/2 + q/2 + c*x), x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NiceSqrtQ}[b^2 - 4*a*c]$

#### Rule 31

$\text{Int}[(a\_)+(b\_)*(x\_)]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1 + e^x + e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{x(-1 + x + x^2)} dx, x, e^x \right) \\
&= -\text{Subst} \left( \int \frac{1}{x} dx, x, e^x \right) - \text{Subst} \left( \int \frac{-1 - x}{-1 + x + x^2} dx, x, e^x \right) \\
&= -x + \frac{1}{10} (5 - \sqrt{5}) \text{Subst} \left( \int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx, x, e^x \right) + \frac{1}{10} (5 + \sqrt{5}) \text{Subst} \left( \int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx, x, e^x \right) \\
&= -x + \frac{1}{10} (5 + \sqrt{5}) \log(1 - \sqrt{5} + 2e^x) + \frac{1}{10} (5 - \sqrt{5}) \log(1 + \sqrt{5} + 2e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.0249438, size = 44, normalized size = 0.79

$$-x + \frac{1}{2} \log(-e^x - e^{2x} + 1) - \frac{\tanh^{-1}\left(\frac{2e^x+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + E^x + E^(2\*x))^(-1), x]

[Out] -x - ArcTanh[(1 + 2\*E^x)/Sqrt[5]]/Sqrt[5] + Log[1 - E^x - E^(2\*x)]/2

**Maple [A]** time = 0.006, size = 35, normalized size = 0.6

$$-\ln(e^x) + \frac{\ln(-1 + e^x + (e^x)^2)}{2} - \frac{\sqrt{5}}{5} \text{Artanh}\left(\frac{(1 + 2e^x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+exp(x)+exp(2\*x)), x)

[Out] -ln(exp(x))+1/2\*ln(-1+exp(x)+exp(x)^2)-1/5\*5^(1/2)\*arctanh(1/5\*(1+2\*exp(x))\*5^(1/2))

**Maxima [A]** time = 1.47232, size = 58, normalized size = 1.04

$$\frac{1}{10} \sqrt{5} \log\left(-\frac{\sqrt{5} - 2e^x - 1}{\sqrt{5} + 2e^x + 1}\right) - x + \frac{1}{2} \log(e^{(2x)} + e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+exp(x)+exp(2\*x)), x, algorithm="maxima")

[Out] 1/10\*sqrt(5)\*log(-(sqrt(5) - 2\*e^x - 1)/(sqrt(5) + 2\*e^x + 1)) - x + 1/2\*log(e^(2\*x) + e^x - 1)

**Fricas [A]** time = 1.52817, size = 163, normalized size = 2.91

$$\frac{1}{10} \sqrt{5} \log \left( -\frac{2(\sqrt{5}-1)e^x + \sqrt{5} - 2e^{(2x)} - 3}{e^{(2x)} + e^x - 1} \right) - x + \frac{1}{2} \log(e^{(2x)} + e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] 1/10\*sqrt(5)\*log(-(2\*(sqrt(5) - 1)\*e^x + sqrt(5) - 2\*e^(2\*x) - 3)/(e^(2\*x) + e^x - 1)) - x + 1/2\*log(e^(2\*x) + e^x - 1)

**Sympy [A]** time = 0.122494, size = 22, normalized size = 0.39

$$-x + \text{RootSum}(5z^2 - 5z + 1, (i \mapsto i \log(-5i + e^x + 3)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+exp(x)+exp(2\*x)),x)

[Out] -x + RootSum(5\*\_z\*\*2 - 5\*\_z + 1, Lambda(\_i, \_i\*log(-5\*\_i + exp(x) + 3)))

**Giac [A]** time = 1.29618, size = 62, normalized size = 1.11

$$\frac{1}{10} \sqrt{5} \log \left( \frac{|-\sqrt{5} + 2e^x + 1|}{\sqrt{5} + 2e^x + 1} \right) - x + \frac{1}{2} \log(|e^{(2x)} + e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] 1/10\*sqrt(5)\*log(abs(-sqrt(5) + 2\*e^x + 1)/(sqrt(5) + 2\*e^x + 1)) - x + 1/2\*log(abs(e^(2\*x) + e^x - 1))

$$3.508 \quad \int \frac{1}{3+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=44

$$\frac{x}{3} - \frac{1}{6} \log(3e^x + e^{2x} + 3) - \frac{\tan^{-1}\left(\frac{2e^x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] x/3 - ArcTan[(3 + 2\*E^x)/Sqrt[3]]/Sqrt[3] - Log[3 + 3\*E^x + E^(2\*x)]/6

**Rubi [A]** time = 0.036331, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {2282, 705, 29, 634, 618, 204, 628}

$$\frac{x}{3} - \frac{1}{6} \log(3e^x + e^{2x} + 3) - \frac{\tan^{-1}\left(\frac{2e^x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 3\*E^x + E^(2\*x))^(-1), x]

[Out] x/3 - ArcTan[(3 + 2\*E^x)/Sqrt[3]]/Sqrt[3] - Log[3 + 3\*E^x + E^(2\*x)]/6

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{3 + 3e^x + e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{x(3 + 3x + x^2)} dx, x, e^x \right) \\
 &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{x} dx, x, e^x \right) + \frac{1}{3} \text{Subst} \left( \int \frac{-3 - x}{3 + 3x + x^2} dx, x, e^x \right) \\
 &= \frac{x}{3} - \frac{1}{6} \text{Subst} \left( \int \frac{3 + 2x}{3 + 3x + x^2} dx, x, e^x \right) - \frac{1}{2} \text{Subst} \left( \int \frac{1}{3 + 3x + x^2} dx, x, e^x \right) \\
 &= \frac{x}{3} - \frac{1}{6} \log(3 + 3e^x + e^{2x}) + \text{Subst} \left( \int \frac{1}{-3 - x^2} dx, x, 3 + 2e^x \right) \\
 &= \frac{x}{3} - \frac{\tan^{-1} \left( \frac{3 + 2e^x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{1}{6} \log(3 + 3e^x + e^{2x})
 \end{aligned}$$

**Mathematica [A]** time = 0.0179097, size = 44, normalized size = 1.

$$\frac{x}{3} - \frac{1}{6} \log(3e^x + e^{2x} + 3) - \frac{\tan^{-1} \left( \frac{2e^x + 3}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 3\*E^x + E^(2\*x))^(-1), x]

[Out] x/3 - ArcTan[(3 + 2\*E^x)/Sqrt[3]]/Sqrt[3] - Log[3 + 3\*E^x + E^(2\*x)]/6

**Maple [A]** time = 0.006, size = 37, normalized size = 0.8

$$-\frac{\ln(3 + 3e^x + (e^x)^2)}{6} - \frac{\sqrt{3}}{3} \arctan\left(\frac{(3 + 2e^x)\sqrt{3}}{3}\right) + \frac{\ln(e^x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3+3\*exp(x)+exp(2\*x)), x)

[Out] -1/6\*ln(3+3\*exp(x)+exp(x)^2)-1/3\*arctan(1/3\*(3+2\*exp(x))\*3^(1/2))\*3^(1/2)+1/3\*ln(exp(x))

**Maxima [A]** time = 1.46327, size = 46, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+3)\right)+\frac{1}{3}x-\frac{1}{6}\log(e^{2x}+3e^x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+3\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] -1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*e^x + 3)) + 1/3\*x - 1/6\*log(e^(2\*x) + 3\*e^x + 3)

**Fricas [A]** time = 1.5249, size = 117, normalized size = 2.66

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}e^x+\sqrt{3}\right)+\frac{1}{3}x-\frac{1}{6}\log(e^{2x}+3e^x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] -1/3\*sqrt(3)\*arctan(2/3\*sqrt(3)\*e^x + sqrt(3)) + 1/3\*x - 1/6\*log(e^(2\*x) + 3\*e^x + 3)

**Sympy [A]** time = 0.120918, size = 24, normalized size = 0.55

$$\frac{x}{3} + \text{RootSum}\left(9z^2 + 3z + 1, (i \mapsto i \log(-3i + e^x + 1))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+3\*exp(x)+exp(2\*x)),x)

[Out] x/3 + RootSum(9\*\_z\*\*2 + 3\*\_z + 1, Lambda(\_i, \_i\*log(-3\*\_i + exp(x) + 1)))

**Giac [A]** time = 1.26904, size = 46, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+3)\right)+\frac{1}{3}x-\frac{1}{6}\log(e^{2x}+3e^x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+3\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*e^x + 3)) + 1/3\*x - 1/6\*log(e^(2\*x) + 3\*e^x + 3)

### 3.509 $\int \frac{1}{a+be^x+ce^{2x}} dx$

**Optimal.** Leaf size=67

$$\frac{b \tanh^{-1}\left(\frac{b+2ce^x}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+be^x+ce^{2x})}{2a} + \frac{x}{a}$$

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*E^x)/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]) - Log[a + b\*E^x + c\*E^(2\*x)]/(2\*a)

**Rubi [A]** time = 0.0646843, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {2282, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2ce^x}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+be^x+ce^{2x})}{2a} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x + c\*E^(2\*x))^(-1), x]

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*E^x)/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]) - Log[a + b\*E^x + c\*E^(2\*x)]/(2\*a)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_ )^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 628

$\text{Int}[(d_ + (e_ \cdot)(x_ )) / ((a_ + (b_ \cdot)(x_ ) + (c_ \cdot)(x_ )^2), x\_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{a + be^x + ce^{2x}} dx &= \text{Subst} \left( \int \frac{1}{x(a + bx + cx^2)} dx, x, e^x \right) \\ &= \frac{\text{Subst} \left( \int \frac{1}{x} dx, x, e^x \right)}{a} + \frac{\text{Subst} \left( \int \frac{-b-cx}{a+bx+cx^2} dx, x, e^x \right)}{a} \\ &= \frac{x}{a} - \frac{\text{Subst} \left( \int \frac{b+2cx}{a+bx+cx^2} dx, x, e^x \right)}{2a} - \frac{b \text{Subst} \left( \int \frac{1}{a+bx+cx^2} dx, x, e^x \right)}{2a} \\ &= \frac{x}{a} - \frac{\log(a + be^x + ce^{2x})}{2a} + \frac{b \text{Subst} \left( \int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2ce^x \right)}{a} \\ &= \frac{x}{a} + \frac{b \tanh^{-1} \left( \frac{b+2ce^x}{\sqrt{b^2-4ac}} \right)}{a\sqrt{b^2-4ac}} - \frac{\log(a + be^x + ce^{2x})}{2a} \end{aligned}$$

**Mathematica [A]** time = 0.107642, size = 66, normalized size = 0.99

$$\frac{\frac{2b \tan^{-1} \left( \frac{b+2ce^x}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} + \log(a + e^x(b + ce^x)) - 2x}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x + c\*E^(2\*x))^(-1), x]

[Out]  $-( -2*x + (2*b*ArcTan[(b + 2*c*E^x)/Sqrt[-b^2 + 4*a*c]]) / Sqrt[-b^2 + 4*a*c] + \text{Log}[a + E^x*(b + c*E^x)]) / (2*a)$

**Maple [A]** time = 0.008, size = 66, normalized size = 1.

$$\frac{\ln(e^x)}{a} - \frac{\ln(a + be^x + c(e^x)^2)}{2a} - \frac{b}{a} \arctan \left( (b + 2ce^x) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*exp(x)+c\*exp(2\*x)), x)



[Out]  $\frac{1}{a} \ln(\exp(x)) - \frac{1}{2} \frac{1}{a} \ln(a + b \exp(x) + c \exp(x)^2) - \frac{1}{a} \frac{b}{(4ac - b^2)^{1/2}} \arctan\left(\frac{b + 2c \exp(x)}{(4ac - b^2)^{1/2}}\right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.53573, size = 518, normalized size = 7.73

$$\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2 e^{2x} + 2bce^x + b^2 - 2ac + \sqrt{b^2 - 4ac}(2ce^x + b)}{ce^{2x} + be^x + a}\right) + 2(b^2 - 4ac)x - (b^2 - 4ac) \log(ce^{2x} + be^x + a) + 2\sqrt{-b^2 + 4ac}}{2(ab^2 - 4a^2c)},$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{2} \sqrt{b^2 - 4ac} b \log\left(\frac{2c^2 e^{2x} + 2bce^x + b^2 - 2ac + \sqrt{b^2 - 4ac}(2ce^x + b)}{ce^{2x} + be^x + a}\right) + 2(b^2 - 4ac)x - (b^2 - 4ac) \log(ce^{2x} + be^x + a) + 2\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2ce^x + b)}{b^2 - 4ac}\right) + 2(b^2 - 4ac)x - (b^2 - 4ac) \log(ce^{2x} + be^x + a)}{2(ab^2 - 4a^2c)} \right]$

**Sympy [A]** time = 0.285306, size = 63, normalized size = 0.94

$$\text{RootSum}\left(z^2(4a^2c - ab^2) + z(4ac - b^2) + c, \left(i \mapsto i \log\left(e^x + \frac{-4ia^2c + iab^2 - 2ac + b^2}{bc}\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(x)+c*exp(2*x)),x)`

[Out] `RootSum(_z**2*(4*a**2*c - a*b**2) + _z*(4*a*c - b**2) + c, Lambda(_i, _i*log(exp(x) + (-4*_i*a**2*c + _i*a*b**2 - 2*a*c + b**2)/(b*c)))) + x/a`

**Giac [A]** time = 1.21214, size = 85, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{2ce^x + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac}} + \frac{x}{a} - \frac{\log(ce^{2x} + be^x + a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*e^x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) + x/a -  
1/2*log(c*e^(2*x) + b*e^x + a)/a
```

$$3.510 \quad \int \frac{x}{1+2e^x+e^{2x}} dx$$

**Optimal.** Leaf size=44

$$-\text{PolyLog}(2, -e^x) + \frac{x^2}{2} + \frac{x}{e^x+1} - x - x \log(e^x + 1) + \log(e^x + 1)$$

[Out]  $-x + x/(1 + E^x) + x^2/2 + \text{Log}[1 + E^x] - x*\text{Log}[1 + E^x] - \text{PolyLog}[2, -E^x]$

**Rubi [A]** time = 0.12767, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {6688, 2185, 2184, 2190, 2279, 2391, 2191, 2282, 36, 29, 31}

$$-\text{PolyLog}(2, -e^x) + \frac{x^2}{2} + \frac{x}{e^x+1} - x - x \log(e^x + 1) + \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] `Int[x/(1 + 2*E^x + E^(2*x)),x]`

[Out]  $-x + x/(1 + E^x) + x^2/2 + \text{Log}[1 + E^x] - x*\text{Log}[1 + E^x] - \text{PolyLog}[2, -E^x]$

#### Rule 6688

`Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplerIntegrandQ[v, u, x]]`

#### Rule 2185

`Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e + f*x))))^n]^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x))))^n*(a + b*(F^(g*(e + f*x))))^n]^(p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]`

#### Rule 2184

`Int[((c_) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x))))^n/(a + b*(F^(g*(e + f*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(e_.) + (f_.)*(x_))))^(p_.)*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x))))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{1+2e^x+e^{2x}} dx &= \int \frac{x}{(1+e^x)^2} dx \\
 &= -\int \frac{e^x x}{(1+e^x)^2} dx + \int \frac{x}{1+e^x} dx \\
 &= \frac{x}{1+e^x} + \frac{x^2}{2} - \int \frac{1}{1+e^x} dx - \int \frac{e^x x}{1+e^x} dx \\
 &= \frac{x}{1+e^x} + \frac{x^2}{2} - x \log(1+e^x) + \int \log(1+e^x) dx - \text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, e^x\right) \\
 &= \frac{x}{1+e^x} + \frac{x^2}{2} - x \log(1+e^x) - \text{Subst}\left(\int \frac{1}{x} dx, x, e^x\right) + \text{Subst}\left(\int \frac{1}{1+x} dx, x, e^x\right) + \text{Subst}\left(\int \frac{\log(x)}{x} dx, x, e^x\right) \\
 &= -x + \frac{x}{1+e^x} + \frac{x^2}{2} + \log(1+e^x) - x \log(1+e^x) - \text{Li}_2(-e^x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0546394, size = 38, normalized size = 0.86

$$-\text{PolyLog}(2, -e^x) + \frac{1}{2}x\left(x + \frac{2}{e^x + 1} - 2\right) - (x - 1)\log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + 2\*E^x + E^(2\*x)),x]

[Out] (x\*(-2 + 2/(1 + E^x) + x))/2 - (-1 + x)\*Log[1 + E^x] - PolyLog[2, -E^x]

**Maple [A]** time = 0.012, size = 38, normalized size = 0.9

$$\ln(1 + e^x) - \frac{e^x x}{1 + e^x} + \frac{x^2}{2} - \operatorname{dilog}(1 + e^x) - x \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+2\*exp(x)+exp(2\*x)),x)

[Out] ln(1+exp(x))-x\*exp(x)/(1+exp(x))+1/2\*x^2-dilog(1+exp(x))-x\*ln(1+exp(x))

**Maxima [A]** time = 0.985748, size = 50, normalized size = 1.14

$$\frac{1}{2}x^2 - x \log(e^x + 1) - x + \frac{x}{e^x + 1} - \operatorname{Li}_2(-e^x) + \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] 1/2\*x^2 - x\*log(e^x + 1) - x + x/(e^x + 1) - dilog(-e^x) + log(e^x + 1)

**Fricas [A]** time = 1.51689, size = 140, normalized size = 3.18

$$\frac{x^2 - 2(e^x + 1)\operatorname{Li}_2(-e^x) + (x^2 - 2x)e^x - 2((x - 1)e^x + x - 1)\log(e^x + 1)}{2(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*(x^2 - 2\*(e^x + 1)\*dilog(-e^x) + (x^2 - 2\*x)\*e^x - 2\*((x - 1)\*e^x + x - 1)\*log(e^x + 1))/(e^x + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x}{e^x + 1} + \int \frac{x - 1}{e^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*exp(x)+exp(2\*x)),x)

[Out] x/(exp(x) + 1) + Integral((x - 1)/(exp(x) + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{(2x)} + 2e^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] integrate(x/(e^(2\*x) + 2\*e^x + 1), x)

### 3.511 $\int \frac{x}{2+3e^x+e^{2x}} dx$

**Optimal.** Leaf size=54

$$-\text{PolyLog}(2, -e^x) + \frac{1}{2}\text{PolyLog}\left(2, -\frac{e^x}{2}\right) + \frac{x^2}{4} + \frac{1}{2}x \log\left(\frac{e^x}{2} + 1\right) - x \log(e^x + 1)$$

[Out]  $x^2/4 + (x*\text{Log}[1 + E^x/2])/2 - x*\text{Log}[1 + E^x] - \text{PolyLog}[2, -E^x] + \text{PolyLog}[2, -E^x/2]/2$

**Rubi [A]** time = 0.124028, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2263, 2184, 2190, 2279, 2391}

$$-\text{PolyLog}(2, -e^x) + \frac{1}{2}\text{PolyLog}\left(2, -\frac{e^x}{2}\right) + \frac{x^2}{4} + \frac{1}{2}x \log\left(\frac{e^x}{2} + 1\right) - x \log(e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/(2 + 3E^x + E^{(2*x)}), x]$

[Out]  $x^2/4 + (x*\text{Log}[1 + E^x/2])/2 - x*\text{Log}[1 + E^x] - \text{PolyLog}[2, -E^x] + \text{PolyLog}[2, -E^x/2]/2$

#### Rule 2263

$\text{Int}[\frac{(f_.) + (g_.)*(x_.)^{(m_.)}}{(a_.) + (b_.)*(F_.)^{(u_.)} + (c_.)*(F_.)^{(v_.)}}, x\_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m/(b - q + 2*c*F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m/(b + q + 2*c*F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

#### Rule 2184

$\text{Int}[\frac{(c_.) + (d_.)*(x_.)^{(m_.)}}{(a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)^{(n_.)}))})^{(n_.)})}, x\_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[\frac{(c + d*x)^m*(F^{(g*(e + f*x)))^n}}{(a + b*(F^{(g*(e + f*x)))^n}), x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2190

$\text{Int}[\frac{((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)^{(n_.)}))})^{(n_.)})*((c_.) + (d_.)*(x_.)^{(m_.)})}{((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)^{(n_.)}))})^{(n_.)})}, x\_Symbol] := \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{(b*f*g*n*\text{Log}[F])}, x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_.)^{(n_.)}))})^{(n_.)})], x\_Symbol] := \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{2+3e^x+e^{2x}} dx &= 2 \int \frac{x}{2+2e^x} dx - 2 \int \frac{x}{4+2e^x} dx \\
 &= \frac{x^2}{4} - 2 \int \frac{e^x x}{2+2e^x} dx + \int \frac{e^x x}{4+2e^x} dx \\
 &= \frac{x^2}{4} + \frac{1}{2} x \log\left(1 + \frac{e^x}{2}\right) - x \log(1 + e^x) - \frac{1}{2} \int \log\left(1 + \frac{e^x}{2}\right) dx + \int \log(1 + e^x) dx \\
 &= \frac{x^2}{4} + \frac{1}{2} x \log\left(1 + \frac{e^x}{2}\right) - x \log(1 + e^x) - \frac{1}{2} \text{Subst}\left(\int \frac{\log\left(1 + \frac{x}{2}\right)}{x} dx, x, e^x\right) + \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^x\right) \\
 &= \frac{x^2}{4} + \frac{1}{2} x \log\left(1 + \frac{e^x}{2}\right) - x \log(1 + e^x) - \text{Li}_2(-e^x) + \frac{1}{2} \text{Li}_2\left(-\frac{e^x}{2}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0034946, size = 49, normalized size = 0.91

$$-\frac{1}{2} \text{PolyLog}(2, -2e^{-x}) + \text{PolyLog}(2, -e^{-x}) - x \log(e^{-x} + 1) + \frac{1}{2} x \log(2e^{-x} + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 3*E^x + E^(2*x)), x]
```

```
[Out] -(x*Log[1 + E^(-x)]) + (x*Log[1 + 2/E^x])/2 - PolyLog[2, -2/E^x]/2 + PolyLog[2, -E^(-x)]
```

**Maple [A]** time = 0.006, size = 41, normalized size = 0.8

$$\frac{x^2}{4} + \frac{x}{2} \ln\left(1 + \frac{e^x}{2}\right) - x \ln(1 + e^x) - \text{polylog}(2, -e^x) + \frac{1}{2} \text{polylog}\left(2, -\frac{e^x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+3*exp(x)+exp(2*x)), x)
```

```
[Out] 1/4*x^2+1/2*x*ln(1+1/2*exp(x))-x*ln(1+exp(x))-polylog(2,-exp(x))+1/2*polylog(2,-1/2*exp(x))
```

**Maxima [A]** time = 0.981413, size = 51, normalized size = 0.94

$$\frac{1}{4} x^2 - x \log(e^x + 1) + \frac{1}{2} x \log\left(\frac{1}{2} e^x + 1\right) + \frac{1}{2} \text{Li}_2\left(-\frac{1}{2} e^x\right) - \text{Li}_2(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+3*exp(x)+exp(2*x)), x, algorithm="maxima")
```



[Out]  $\frac{1}{4}x^2 - x \log(e^x + 1) + \frac{1}{2}x \log\left(\frac{1}{2}e^x + 1\right) + \frac{1}{2} \operatorname{dilog}\left(-\frac{1}{2}e^x\right) - \operatorname{dilog}(-e^x)$

**Fricas [A]** time = 1.56201, size = 117, normalized size = 2.17

$$\frac{1}{4}x^2 - x \log(e^x + 1) + \frac{1}{2}x \log\left(\frac{1}{2}e^x + 1\right) + \frac{1}{2} \operatorname{Li}_2\left(-\frac{1}{2}e^x\right) - \operatorname{Li}_2(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}x^2 - x \log(e^x + 1) + \frac{1}{2}x \log\left(\frac{1}{2}e^x + 1\right) + \frac{1}{2} \operatorname{dilog}\left(-\frac{1}{2}e^x\right) - \operatorname{dilog}(-e^x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(e^x + 1)(e^x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+3*exp(x)+exp(2*x)),x)`

[Out] `Integral(x/((exp(x) + 1)*(exp(x) + 2)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{(2x)} + 3e^x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")`

[Out] `integrate(x/(e^(2*x) + 3*e^x + 2), x)`

$$3.512 \quad \int \frac{x}{-1+e^x+e^{2x}} dx$$

**Optimal.** Leaf size=180

$$-\frac{2\text{PolyLog}\left(2, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2\text{PolyLog}\left(2, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} + \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{2x \log\left(\frac{2e^x}{1-\sqrt{5}} + 1\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2x \log\left(\frac{2e^x}{1+\sqrt{5}} + 1\right)}{\sqrt{5}(1+\sqrt{5})}$$

[Out] x^2/(Sqrt[5]\*(1 - Sqrt[5])) - x^2/(Sqrt[5]\*(1 + Sqrt[5])) - (2\*x\*Log[1 + (2\*E^x)/(1 - Sqrt[5])])/(Sqrt[5]\*(1 - Sqrt[5])) + (2\*x\*Log[1 + (2\*E^x)/(1 + Sqrt[5])])/(Sqrt[5]\*(1 + Sqrt[5])) - (2\*PolyLog[2, (-2\*E^x)/(1 - Sqrt[5])])/(Sqrt[5]\*(1 - Sqrt[5])) + (2\*PolyLog[2, (-2\*E^x)/(1 + Sqrt[5])])/(Sqrt[5]\*(1 + Sqrt[5]))

**Rubi [A]** time = 0.188071, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {2263, 2184, 2190, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2\text{PolyLog}\left(2, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} + \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{2x \log\left(\frac{2e^x}{1-\sqrt{5}} + 1\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2x \log\left(\frac{2e^x}{1+\sqrt{5}} + 1\right)}{\sqrt{5}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Int[x/(-1 + E^x + E^(2\*x)), x]

[Out] x^2/(Sqrt[5]\*(1 - Sqrt[5])) - x^2/(Sqrt[5]\*(1 + Sqrt[5])) - (2\*x\*Log[1 + (2\*E^x)/(1 - Sqrt[5])])/(Sqrt[5]\*(1 - Sqrt[5])) + (2\*x\*Log[1 + (2\*E^x)/(1 + Sqrt[5])])/(Sqrt[5]\*(1 + Sqrt[5])) - (2\*PolyLog[2, (-2\*E^x)/(1 - Sqrt[5])])/(Sqrt[5]\*(1 - Sqrt[5])) + (2\*PolyLog[2, (-2\*E^x)/(1 + Sqrt[5])])/(Sqrt[5]\*(1 + Sqrt[5]))

#### Rule 2263

Int[((f\_.) + (g\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m/(b + q + 2\*c\*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{-1 + e^x + e^{2x}} dx &= \frac{2 \int \frac{x}{1-\sqrt{5}+2e^x} dx}{\sqrt{5}} - \frac{2 \int \frac{x}{1+\sqrt{5}+2e^x} dx}{\sqrt{5}} \\ &= \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} - \frac{4 \int \frac{e^x x}{1-\sqrt{5}+2e^x} dx}{\sqrt{5}(1-\sqrt{5})} + \frac{4 \int \frac{e^x x}{1+\sqrt{5}+2e^x} dx}{\sqrt{5}(1+\sqrt{5})} \\ &= \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} - \frac{2x \log\left(1 + \frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2x \log\left(1 + \frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} + \frac{2 \int \log\left(1 + \frac{2e^x}{1-\sqrt{5}}\right) dx}{\sqrt{5}(1-\sqrt{5})} \\ &= \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} - \frac{2x \log\left(1 + \frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2x \log\left(1 + \frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} + \frac{2 \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{2x}{1-\sqrt{5}}\right)}{x} dx\right)}{\sqrt{5}(1-\sqrt{5})} \\ &= \frac{x^2}{\sqrt{5}(1-\sqrt{5})} - \frac{x^2}{\sqrt{5}(1+\sqrt{5})} - \frac{2x \log\left(1 + \frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2x \log\left(1 + \frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} - \frac{2 \operatorname{Li}_2\left(-\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{2 \operatorname{Li}_2\left(-\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} \end{aligned}$$

**Mathematica [A]** time = 0.0878204, size = 120, normalized size = 0.67

$$\frac{-(1 + \sqrt{5}) \operatorname{PolyLog}\left(2, \frac{1}{2}(\sqrt{5} - 1)e^{-x}\right) - (\sqrt{5} - 1) \operatorname{PolyLog}\left(2, -\frac{1}{2}(1 + \sqrt{5})e^{-x}\right) + (1 + \sqrt{5})x \log\left(1 - \frac{1}{2}(\sqrt{5} - 1)e^{-x}\right) - (\sqrt{5} - 1)x \log\left(1 + \frac{1}{2}(1 + \sqrt{5})e^{-x}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + E^x + E^(2*x)), x]
```

```
[Out] ((1 + Sqrt[5])*x*Log[1 - (-1 + Sqrt[5])/(2*E^x)] + (-1 + Sqrt[5])*x*Log[1 +
(1 + Sqrt[5])/(2*E^x)] - (1 + Sqrt[5])*PolyLog[2, (-1 + Sqrt[5])/(2*E^x)]
- (-1 + Sqrt[5])*PolyLog[2, -(1 + Sqrt[5])/(2*E^x)])/(2*Sqrt[5])
```

**Maple [A]** time = 0.01, size = 183, normalized size = 1.

$$-\frac{x^2}{2} + \frac{x}{2} \ln\left(\frac{\sqrt{5}-1-2e^x}{\sqrt{5}-1}\right) + \frac{\sqrt{5}x}{10} \ln\left(\frac{\sqrt{5}-1-2e^x}{\sqrt{5}-1}\right) - \frac{\sqrt{5}x}{10} \ln\left(\frac{1+2e^x+\sqrt{5}}{\sqrt{5}+1}\right) + \frac{x}{2} \ln\left(\frac{1+2e^x+\sqrt{5}}{\sqrt{5}+1}\right) + \frac{1}{2} \operatorname{dilog}\left(\frac{1+2e^x+\sqrt{5}}{\sqrt{5}+1}\right) - \frac{1}{2} \operatorname{dilog}\left(\frac{1+2e^x+\sqrt{5}}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+exp(x)+exp(2*x)), x)
```

```
[Out] -1/2*x^2+1/2*x*ln((5^(1/2)-1-2*exp(x))/(5^(1/2)-1))+1/10*5^(1/2)*x*ln((5^(1/2)-1-2*exp(x))/(5^(1/2)-1))-1/10*5^(1/2)*x*ln((1+2*exp(x)+5^(1/2))/(5^(1/2)+1))+x/2*ln((1+2*exp(x)+5^(1/2))/(5^(1/2)+1))+1/2*dilog((1+2*exp(x)+5^(1/2))/(5^(1/2)+1))-1/2*dilog((1+2*exp(x)+5^(1/2))/(5^(1/2)+1))
```

)+1)))+1/2\*x\*ln((1+2\*exp(x)+5^(1/2))/(5^(1/2)+1))+1/2\*dilog((5^(1/2)-1-2\*exp(x))/(5^(1/2)-1))+1/10\*5^(1/2)\*dilog((5^(1/2)-1-2\*exp(x))/(5^(1/2)-1))-1/10\*5^(1/2)\*dilog((1+2\*exp(x)+5^(1/2))/(5^(1/2)+1))+1/2\*dilog((1+2\*exp(x)+5^(1/2))/(5^(1/2)+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] integrate(x/(e^(2\*x) + e^x - 1), x)

**Fricas [A]** time = 1.57597, size = 302, normalized size = 1.68

$$-\frac{1}{2}x^2 + \frac{1}{10}(\sqrt{5}+5)\text{Li}_2\left(\frac{1}{2}(\sqrt{5}+1)e^x\right) - \frac{1}{10}(\sqrt{5}-5)\text{Li}_2\left(-\frac{1}{2}(\sqrt{5}-1)e^x\right) + \frac{1}{10}(\sqrt{5}x+5x)\log\left(-\frac{1}{2}(\sqrt{5}+1)e^x+1\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] -1/2\*x^2 + 1/10\*(sqrt(5) + 5)\*dilog(1/2\*(sqrt(5) + 1)\*e^x) - 1/10\*(sqrt(5) - 5)\*dilog(-1/2\*(sqrt(5) - 1)\*e^x) + 1/10\*(sqrt(5)\*x + 5\*x)\*log(-1/2\*(sqrt(5) + 1)\*e^x + 1) - 1/10\*(sqrt(5)\*x - 5\*x)\*log(1/2\*(sqrt(5) - 1)\*e^x + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+exp(x)+exp(2\*x)),x)

[Out] Integral(x/(exp(2\*x) + exp(x) - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] integrate(x/(e^(2\*x) + e^x - 1), x)

### 3.513 $\int \frac{x}{3+3e^x+e^{2x}} dx$

**Optimal.** Leaf size=204

$$-\frac{2\text{PolyLog}\left(2, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{2\text{PolyLog}\left(2, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{x^2}{\sqrt{3}(\sqrt{3}+3i)} - \frac{x^2}{\sqrt{3}(-\sqrt{3}+3i)} - \frac{2x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{2x \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)}$$

```
[Out] -(x^2/(Sqrt[3]*(3*I - Sqrt[3]))) + x^2/(Sqrt[3]*(3*I + Sqrt[3])) - (2*x*Log
[1 + (2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*x*Log[1 + (2*
E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) - (2*PolyLog[2, (-2*E^x)/(
3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*PolyLog[2, (-2*E^x)/(3 + I*
Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3]))
```

**Rubi [A]** time = 0.195367, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2263, 2184, 2190, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{2\text{PolyLog}\left(2, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{x^2}{\sqrt{3}(\sqrt{3}+3i)} - \frac{x^2}{\sqrt{3}(-\sqrt{3}+3i)} - \frac{2x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{2x \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 3*E^x + E^(2*x)),x]
```

```
[Out] -(x^2/(Sqrt[3]*(3*I - Sqrt[3]))) + x^2/(Sqrt[3]*(3*I + Sqrt[3])) - (2*x*Log
[1 + (2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*x*Log[1 + (2*
E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) - (2*PolyLog[2, (-2*E^x)/(
3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*PolyLog[2, (-2*E^x)/(3 + I*
Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3]))
```

#### Rule 2263

```
Int[((f_.) + (g_.)*(x_)^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_.)),
x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m/
(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m/(b + q + 2*c*F^u)
, x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] &&
& NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_
_)))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int((((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_)^(m_.))/
((a_.) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{3 + 3e^x + e^{2x}} dx &= -\frac{(2i) \int \frac{x}{3-i\sqrt{3}+2e^x} dx}{\sqrt{3}} + \frac{(2i) \int \frac{x}{3+i\sqrt{3}+2e^x} dx}{\sqrt{3}} \\ &= -\frac{x^2}{\sqrt{3}(3-i\sqrt{3})} + \frac{x^2}{\sqrt{3}(3+i\sqrt{3})} + \frac{(4i) \int \frac{e^x x}{3-i\sqrt{3}+2e^x} dx}{\sqrt{3}(3-i\sqrt{3})} - \frac{(4i) \int \frac{e^x x}{3+i\sqrt{3}+2e^x} dx}{\sqrt{3}(3+i\sqrt{3})} \\ &= -\frac{x^2}{\sqrt{3}(3-i\sqrt{3})} + \frac{x^2}{\sqrt{3}(3+i\sqrt{3})} - \frac{2x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3+i\sqrt{3})} + \frac{2x \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3-i\sqrt{3})} - \frac{(2i) \int \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3-i\sqrt{3})} \\ &= -\frac{x^2}{\sqrt{3}(3-i\sqrt{3})} + \frac{x^2}{\sqrt{3}(3+i\sqrt{3})} - \frac{2x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3+i\sqrt{3})} + \frac{2x \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3-i\sqrt{3})} - \frac{(2i) \operatorname{Subst}\left[\int \frac{\log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{x}\right]}{\sqrt{3}(3-i\sqrt{3})} \\ &= -\frac{x^2}{\sqrt{3}(3-i\sqrt{3})} + \frac{x^2}{\sqrt{3}(3+i\sqrt{3})} - \frac{2x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3+i\sqrt{3})} + \frac{2x \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3-i\sqrt{3})} - \frac{2\operatorname{Li}_2\left(-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3+i\sqrt{3})} + \frac{2\operatorname{Li}_2\left(-\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3-i\sqrt{3})} \end{aligned}$$

**Mathematica [A]** time = 0.0902245, size = 144, normalized size = 0.71

$$\frac{(\sqrt{3} + 3i) \operatorname{PolyLog}\left(2, -\frac{1}{2}(3 + i\sqrt{3})e^{-x}\right) + (\sqrt{3} - 3i) \operatorname{PolyLog}\left(2, \frac{1}{2}i(\sqrt{3} + 3i)e^{-x}\right) - x\left((\sqrt{3} - 3i) \log\left(1 + \frac{1}{2}(3 - i\sqrt{3})e^{-x}\right) + (\sqrt{3} + 3i) \log\left(1 + \frac{1}{2}(3 + i\sqrt{3})e^{-x}\right)\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 3*E^x + E^(2*x)), x]
```

```
[Out] (-x*((-3*I + Sqrt[3])*Log[1 + (3 - I*Sqrt[3])/(2*E^x)] + (3*I + Sqrt[3])*Log[1 + (3 + I*Sqrt[3])/(2*E^x)])) + (3*I + Sqrt[3])*PolyLog[2, -(3 + I*Sqrt[3])/(2*E^x)] + (-3*I + Sqrt[3])*PolyLog[2, ((I/2)*(3*I + Sqrt[3]))/E^x]/(6*Sqrt[3])
```

**Maple [A]** time = 0.01, size = 235, normalized size = 1.2

$$\frac{i}{6}\sqrt{3}x \ln\left(\frac{i\sqrt{3}-2e^x-3}{i\sqrt{3}-3}\right) - \frac{x}{6} \ln\left(\frac{i\sqrt{3}-2e^x-3}{i\sqrt{3}-3}\right) - \frac{i}{6}\sqrt{3}x \ln\left(\frac{i\sqrt{3}+2e^x+3}{3+i\sqrt{3}}\right) - \frac{x}{6} \ln\left(\frac{i\sqrt{3}+2e^x+3}{3+i\sqrt{3}}\right) + \frac{i}{6}\sqrt{3} \operatorname{dilog}\left(\frac{i\sqrt{3}-2e^x-3}{i\sqrt{3}-3}\right) - \frac{i}{6}\sqrt{3} \operatorname{dilog}\left(\frac{i\sqrt{3}+2e^x+3}{3+i\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+3*exp(x)+exp(2*x)), x)
```

```
[Out] 1/6*I*3^(1/2)*x*ln((I*3^(1/2)-2*exp(x)-3)/(I*3^(1/2)-3))-1/6*x*ln((I*3^(1/2)+2*exp(x)+3)/(3+I*3^(1/2)))-1/6*I*3^(1/2)*x*ln((I*3^(1/2)+2*exp(x)+3)/(3+I*3^(1/2)))+1/6*I*3^(1/2)*ln((I*3^(1/2)-2*exp(x)-3)/(I*3^(1/2)-3))-1/6*I*3^(1/2)*ln((I*3^(1/2)+2*exp(x)+3)/(3+I*3^(1/2)))
```

$*3^{(1/2)}) - 1/6*x*\ln((I*3^{(1/2)}+2*\exp(x)+3)/(3+I*3^{(1/2)}))+1/6*I*3^{(1/2)}*\operatorname{dilog}((I*3^{(1/2)}-2*\exp(x)-3)/(I*3^{(1/2)}-3))-1/6*\operatorname{dilog}((I*3^{(1/2)}-2*\exp(x)-3)/(I*3^{(1/2)}-3))-1/6*I*3^{(1/2)}*\operatorname{dilog}((I*3^{(1/2)}+2*\exp(x)+3)/(3+I*3^{(1/2)}))-1/6*\operatorname{dilog}((I*3^{(1/2)}+2*\exp(x)+3)/(3+I*3^{(1/2)}))+1/6*x^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3+3\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] integrate(x/(e^(2\*x) + 3\*e^x + 3), x)

**Fricas [A]** time = 1.60891, size = 317, normalized size = 1.55

$$\frac{1}{6}x^2 + \frac{1}{6}(i\sqrt{3}-1)\operatorname{Li}_2\left(-\frac{1}{6}(i\sqrt{3}+3)e^x\right) + \frac{1}{6}(-i\sqrt{3}-1)\operatorname{Li}_2\left(-\frac{1}{6}(-i\sqrt{3}+3)e^x\right) + \frac{1}{6}(i\sqrt{3}x-x)\log\left(\frac{1}{6}(i\sqrt{3}+3)e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] 1/6\*x^2 + 1/6\*(I\*sqrt(3) - 1)\*dilog(-1/6\*(I\*sqrt(3) + 3)\*e^x) + 1/6\*(-I\*sqrt(3) - 1)\*dilog(-1/6\*(-I\*sqrt(3) + 3)\*e^x) + 1/6\*(I\*sqrt(3)\*x - x)\*log(1/6\*(I\*sqrt(3) + 3)\*e^x + 1) + 1/6\*(-I\*sqrt(3)\*x - x)\*log(1/6\*(-I\*sqrt(3) + 3)\*e^x + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3+3\*exp(x)+exp(2\*x)),x)

[Out] Integral(x/(exp(2\*x) + 3\*exp(x) + 3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{e^{2x} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3+3\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] integrate(x/(e^(2\*x) + 3\*e^x + 3), x)

### 3.514 $\int \frac{x}{a+be^x+ce^{2x}} dx$

**Optimal.** Leaf size=276

$$\frac{2c \operatorname{PolyLog}\left(2, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{2c \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{cx^2}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{cx^2}{b\sqrt{b^2-4ac}-4ac+b^2} + \frac{2cx \log}{-b\sqrt{b^2-4ac}}$$

[Out]  $-\left(\frac{c*x^2}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})}\right) - \frac{c*x^2}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*x*\operatorname{Log}[1 + (2*c*E^x)/(b - \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*x*\operatorname{Log}[1 + (2*c*E^x)/(b + \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*\operatorname{PolyLog}[2, (-2*c*E^x)/(b - \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*\operatorname{PolyLog}[2, (-2*c*E^x)/(b + \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})}$

**Rubi [A]** time = 0.429495, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {2263, 2184, 2190, 2279, 2391}

$$\frac{2c \operatorname{PolyLog}\left(2, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{2c \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{cx^2}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{cx^2}{b\sqrt{b^2-4ac}-4ac+b^2} + \frac{2cx \log}{-b\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x/(a + b*E^x + c*E^{(2*x)}), x]$

[Out]  $-\left(\frac{c*x^2}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})}\right) - \frac{c*x^2}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*x*\operatorname{Log}[1 + (2*c*E^x)/(b - \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*x*\operatorname{Log}[1 + (2*c*E^x)/(b + \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*\operatorname{PolyLog}[2, (-2*c*E^x)/(b - \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c - b*\sqrt{b^2 - 4*a*c})} + \frac{(2*c*\operatorname{PolyLog}[2, (-2*c*E^x)/(b + \sqrt{b^2 - 4*a*c})])}{(b^2 - 4*a*c + b*\sqrt{b^2 - 4*a*c})}$

#### Rule 2263

$\operatorname{Int}[\left(\frac{(f_.) + (g_.)*(x_.)^{(m_.)}}{(a_.) + (b_.)*(F_.)^{(u_.)} + (c_.)*(F_.)^{(v_.)}}\right), x\_Symbol] :> \operatorname{With}[\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f + g*x)^m/(b - q + 2*c*F^u), x], x] - \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f + g*x)^m/(b + q + 2*c*F^u), x], x]] /; \operatorname{FreeQ}[\{F, a, b, c, f, g\}, x] \&\& \operatorname{EqQ}[v, 2*u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2184

$\operatorname{Int}[\left(\frac{(c_.) + (d_.)*(x_.)^{(m_.)}}{(a_.) + (b_.)*((F_.)^{(g_.)})*((e_.) + (f_.)*(x_.)^{(n_.)})}\right), x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m+1)}/(a*d*(m+1)), x] - \operatorname{Dist}[b/a, \operatorname{Int}[\left(\frac{(c + d*x)^m*(F^{(g*(e + f*x)))^n}}{(a + b*(F^{(g*(e + f*x)))^n})}\right), x], x] /; \operatorname{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2190

$\operatorname{Int}[\left(\frac{(F_.)^{(g_.)*((e_.) + (f_.)*(x_.)^{(n_.)})}}{(a_.) + (b_.)*((F_.)^{(g_.)})*((e_.) + (f_.)*(x_.)^{(n_.)})}\right), x\_Symbol] :> \operatorname{Simp}[\left(\frac{(c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{(b*f*g*n*\operatorname{Log}[F])}\right), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x]$



)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2279**

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

**Rule 2391**

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

**Rubi steps**

$$\int \frac{x}{a + be^x + ce^{2x}} dx = \frac{(2c) \int \frac{x}{b - \sqrt{b^2 - 4ac} + 2ce^x} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x}{b + \sqrt{b^2 - 4ac} + 2ce^x} dx}{\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{(4c^2) \int \frac{e^{2x}}{b - \sqrt{b^2 - 4ac} + 2ce^x} dx}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{(4c^2) \int \frac{e^{2x}}{b + \sqrt{b^2 - 4ac} + 2ce^x} dx}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

**Mathematica [A]** time = 0.209777, size = 205, normalized size = 0.74

$$\frac{-\left(\sqrt{b^2 - 4ac} + b\right) \text{PolyLog}\left(2, \frac{2ce^x}{\sqrt{b^2 - 4ac} - b}\right) + \left(b - \sqrt{b^2 - 4ac}\right) \text{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{b^2 - 4ac} + b}\right) + x\left(x\sqrt{b^2 - 4ac} - \left(\sqrt{b^2 - 4ac} - b\right)\right)}{2a\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*E^x + c\*E^(2\*x)), x]

[Out] (x\*(Sqrt[b^2 - 4\*a\*c]\*x - (b + Sqrt[b^2 - 4\*a\*c])\*Log[1 + (2\*c\*E^x)/(b - Sqrt[b^2 - 4\*a\*c]]) + (b - Sqrt[b^2 - 4\*a\*c])\*Log[1 + (2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c]]) - (b + Sqrt[b^2 - 4\*a\*c])\*PolyLog[2, (2\*c\*E^x)/(-b + Sqrt[b^2 - 4\*a\*c]]) + (b - Sqrt[b^2 - 4\*a\*c])\*PolyLog[2, (-2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c]])/(2\*a\*Sqrt[b^2 - 4\*a\*c])

**Maple [A]** time = 0.013, size = 378, normalized size = 1.4

$$\frac{x^2}{2a} - \frac{x}{2a} \ln\left(\left(-2ce^x + \sqrt{-4ac + b^2} - b\right)\left(-b + \sqrt{-4ac + b^2}\right)^{-1}\right) - \frac{bx}{2a} \ln\left(\left(-2ce^x + \sqrt{-4ac + b^2} - b\right)\left(-b + \sqrt{-4ac + b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*exp(x)+c*exp(2*x)),x)`

[Out]  $\frac{1}{2}x^2/a - \frac{1}{2}ax \ln\left(\frac{-2c\exp(x) + (-4ac + b^2)^{1/2} - b}{-b + (-4ac + b^2)^{1/2}}\right) - \frac{1}{2}ax \ln\left(\frac{-2c\exp(x) + (-4ac + b^2)^{1/2} - b}{-b + (-4ac + b^2)^{1/2}}\right) * b - \frac{1}{2}ax \ln\left(\frac{2c\exp(x) + (-4ac + b^2)^{1/2} + b}{b + (-4ac + b^2)^{1/2}}\right) + \frac{1}{2}ax \ln\left(\frac{2c\exp(x) + (-4ac + b^2)^{1/2} + b}{b + (-4ac + b^2)^{1/2}}\right) * b - \frac{1}{2}a \operatorname{dilog}\left(\frac{2c\exp(x) + (-4ac + b^2)^{1/2} + b}{b + (-4ac + b^2)^{1/2}}\right) + \frac{1}{2}a \operatorname{dilog}\left(\frac{2c\exp(x) + (-4ac + b^2)^{1/2} + b}{b + (-4ac + b^2)^{1/2}}\right) * b - \frac{1}{2}a \operatorname{dilog}\left(\frac{-2c\exp(x) + (-4ac + b^2)^{1/2} - b}{-b + (-4ac + b^2)^{1/2}}\right) - \frac{1}{2}a \operatorname{dilog}\left(\frac{-2c\exp(x) + (-4ac + b^2)^{1/2} - b}{-b + (-4ac + b^2)^{1/2}}\right) * b$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.52882, size = 649, normalized size = 2.35

$$\frac{(b^2 - 4ac)x^2 - \left(ab\sqrt{\frac{b^2 - 4ac}{a^2}} + b^2 - 4ac\right) \operatorname{Li}_2\left(-\frac{a\sqrt{\frac{b^2 - 4ac}{a^2}}e^{x+be^x+2a}}{2a} + 1\right) + \left(ab\sqrt{\frac{b^2 - 4ac}{a^2}} - b^2 + 4ac\right) \operatorname{Li}_2\left(\frac{a\sqrt{\frac{b^2 - 4ac}{a^2}}e^{x-be^x-2a}}{2a} + 1\right)}{2(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{2}((b^2 - 4ac)x^2 - (ab\sqrt{(b^2 - 4ac)/a^2} + b^2 - 4ac) \operatorname{dilog}\left(-\frac{1}{2}(a\sqrt{(b^2 - 4ac)/a^2})e^x + b e^x + 2a\right)/a + 1) + (ab\sqrt{(b^2 - 4ac)/a^2} - b^2 + 4ac) \operatorname{dilog}\left(\frac{1}{2}(a\sqrt{(b^2 - 4ac)/a^2})e^x - b e^x - 2a\right)/a + 1) - (abx\sqrt{(b^2 - 4ac)/a^2} + (b^2 - 4ac)x) \log\left(\frac{1}{2}(a\sqrt{(b^2 - 4ac)/a^2})e^x + b e^x + 2a\right)/a + (abx\sqrt{(b^2 - 4ac)/a^2} - (b^2 - 4ac)x) \log\left(-\frac{1}{2}(a\sqrt{(b^2 - 4ac)/a^2})e^x - b e^x - 2a\right)/a) / (ab^2 - 4a^2c)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + be^x + ce^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*exp(x)+c*exp(2*x)),x)`

[Out] `Integral(x/(a + b*exp(x) + c*exp(2*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{ce^{(2x)} + be^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(x/(c*e^(2*x) + b*e^x + a), x)
```

$$3.515 \quad \int \frac{x^2}{1+2e^x+e^{2x}} dx$$

**Optimal.** Leaf size=72

$$-2x\text{PolyLog}(2, -e^x) + 2\text{PolyLog}(2, -e^x) + 2\text{PolyLog}(3, -e^x) + \frac{x^3}{3} + \frac{x^2}{e^x+1} - x^2 - x^2 \log(e^x + 1) + 2x \log(e^x + 1)$$

[Out]  $-x^2 + x^2/(1 + E^x) + x^3/3 + 2*x*\text{Log}[1 + E^x] - x^2*\text{Log}[1 + E^x] + 2*\text{PolyLog}[2, -E^x] - 2*x*\text{PolyLog}[2, -E^x] + 2*\text{PolyLog}[3, -E^x]$

**Rubi [A]** time = 0.229501, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6688, 2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$-2x\text{PolyLog}(2, -e^x) + 2\text{PolyLog}(2, -e^x) + 2\text{PolyLog}(3, -e^x) + \frac{x^3}{3} + \frac{x^2}{e^x+1} - x^2 - x^2 \log(e^x + 1) + 2x \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + 2\*E^x + E^(2\*x)),x]

[Out]  $-x^2 + x^2/(1 + E^x) + x^3/3 + 2*x*\text{Log}[1 + E^x] - x^2*\text{Log}[1 + E^x] + 2*\text{PolyLog}[2, -E^x] - 2*x*\text{PolyLog}[2, -E^x] + 2*\text{PolyLog}[3, -E^x]$

#### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 2185

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Dist[1/a, Int[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n]^(p + 1), x], x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x))))^n]^(a + b\*(F^(g\*(e + f\*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]

#### Rule 2184

Int[((c\_) + (d\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n]/(a + b\*(F^(g\*(e + f\*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -

1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :=> Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1)/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :=> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :=> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{1 + 2e^x + e^{2x}} dx &= \int \frac{x^2}{(1 + e^x)^2} dx \\
 &= - \int \frac{e^x x^2}{(1 + e^x)^2} dx + \int \frac{x^2}{1 + e^x} dx \\
 &= \frac{x^2}{1 + e^x} + \frac{x^3}{3} - 2 \int \frac{x}{1 + e^x} dx - \int \frac{e^x x^2}{1 + e^x} dx \\
 &= -x^2 + \frac{x^2}{1 + e^x} + \frac{x^3}{3} - x^2 \log(1 + e^x) + 2 \int \frac{e^x x}{1 + e^x} dx + 2 \int x \log(1 + e^x) dx \\
 &= -x^2 + \frac{x^2}{1 + e^x} + \frac{x^3}{3} + 2x \log(1 + e^x) - x^2 \log(1 + e^x) - 2x \text{Li}_2(-e^x) - 2 \int \log(1 + e^x) dx + 2 \int \log(1 + e^x) dx \\
 &= -x^2 + \frac{x^2}{1 + e^x} + \frac{x^3}{3} + 2x \log(1 + e^x) - x^2 \log(1 + e^x) - 2x \text{Li}_2(-e^x) - 2 \text{Subst} \left( \int \frac{\log(1 + x)}{x} dx, x, e^x \right) \\
 &= -x^2 + \frac{x^2}{1 + e^x} + \frac{x^3}{3} + 2x \log(1 + e^x) - x^2 \log(1 + e^x) + 2 \text{Li}_2(-e^x) - 2x \text{Li}_2(-e^x) + 2 \text{Li}_3(-e^x)
 \end{aligned}$$

**Mathematica [A]** time = 0.091594, size = 57, normalized size = 0.79

$$-2(x-1)\text{PolyLog}(2, -e^x) + 2\text{PolyLog}(3, -e^x) + \frac{(e^x(x-3) + x)x^2}{3(e^x + 1)} - (x-2)x \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + 2\*E^x + E^(2\*x)), x]

[Out] (x^2\*(E^x\*(-3 + x) + x))/(3\*(1 + E^x)) - (-2 + x)\*x\*Log[1 + E^x] - 2\*(-1 + x)\*PolyLog[2, -E^x] + 2\*PolyLog[3, -E^x]

**Maple [A]** time = 0.033, size = 65, normalized size = 0.9

$$-x^2 + \frac{x^2}{1 + e^x} + \frac{x^3}{3} + 2x \ln(1 + e^x) - x^2 \ln(1 + e^x) + 2 \text{polylog}(2, -e^x) - 2x \text{polylog}(2, -e^x) + 2 \text{polylog}(3, -e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+2\*exp(x)+exp(2\*x)), x)

[Out] -x^2+x^2/(1+exp(x))+1/3\*x^3+2\*x\*ln(1+exp(x))-x^2\*ln(1+exp(x))+2\*polylog(2, -exp(x))-2\*x\*polylog(2, -exp(x))+2\*polylog(3, -exp(x))

**Maxima [A]** time = 0.985644, size = 84, normalized size = 1.17

$$\frac{1}{3}x^3 - x^2 \log(e^x + 1) - x^2 - 2x \text{Li}_2(-e^x) + 2x \log(e^x + 1) + \frac{x^2}{e^x + 1} + 2 \text{Li}_2(-e^x) + 2 \text{Li}_3(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2\*exp(x)+exp(2\*x)), x, algorithm="maxima")

[Out] 1/3\*x^3 - x^2\*log(e^x + 1) - x^2 - 2\*x\*dilog(-e^x) + 2\*x\*log(e^x + 1) + x^2/(e^x + 1) + 2\*dilog(-e^x) + 2\*polylog(3, -e^x)

**Fricas [C]** time = 1.49997, size = 212, normalized size = 2.94

$$\frac{x^3 - 6((x-1)e^x + x-1)\text{Li}_2(-e^x) + (x^3 - 3x^2)e^x - 3(x^2 + (x^2 - 2x)e^x - 2x)\log(e^x + 1) + 6(e^x + 1)\text{polylog}(3, -e^x)}{3(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2\*exp(x)+exp(2\*x)), x, algorithm="fricas")

[Out] 1/3\*(x^3 - 6\*((x - 1)\*e^x + x - 1)\*dilog(-e^x) + (x^3 - 3\*x^2)\*e^x - 3\*(x^2 + (x^2 - 2\*x)\*e^x - 2\*x)\*log(e^x + 1) + 6\*(e^x + 1)\*polylog(3, -e^x))/(e^x + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{e^x + 1} + \int \frac{x(x-2)}{e^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(1+2\*exp(x)+exp(2\*x)),x)

[Out] x\*\*2/(exp(x) + 1) + Integral(x\*(x - 2)/(exp(x) + 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + 2e^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] integrate(x^2/(e^(2\*x) + 2\*e^x + 1), x)

$$3.516 \quad \int \frac{x^2}{2+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=77

$$-2x \operatorname{PolyLog}(2, -e^x) + x \operatorname{PolyLog}\left(2, -\frac{e^x}{2}\right) + 2 \operatorname{PolyLog}(3, -e^x) - \operatorname{PolyLog}\left(3, -\frac{e^x}{2}\right) + \frac{x^3}{6} + \frac{1}{2}x^2 \log\left(\frac{e^x}{2} + 1\right) - x^2 \log(2)$$

[Out]  $x^3/6 + (x^2 \cdot \operatorname{Log}[1 + E^x/2])/2 - x^2 \cdot \operatorname{Log}[1 + E^x] - 2 \cdot x \cdot \operatorname{PolyLog}[2, -E^x] + x \cdot \operatorname{PolyLog}[2, -E^x/2] + 2 \cdot \operatorname{PolyLog}[3, -E^x] - \operatorname{PolyLog}[3, -E^x/2]$

**Rubi [A]** time = 0.222342, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2263, 2184, 2190, 2531, 2282, 6589}

$$-2x \operatorname{PolyLog}(2, -e^x) + x \operatorname{PolyLog}\left(2, -\frac{e^x}{2}\right) + 2 \operatorname{PolyLog}(3, -e^x) - \operatorname{PolyLog}\left(3, -\frac{e^x}{2}\right) + \frac{x^3}{6} + \frac{1}{2}x^2 \log\left(\frac{e^x}{2} + 1\right) - x^2 \log(2)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(2 + 3 \cdot E^x + E^{(2 \cdot x)}), x]$

[Out]  $x^3/6 + (x^2 \cdot \operatorname{Log}[1 + E^x/2])/2 - x^2 \cdot \operatorname{Log}[1 + E^x] - 2 \cdot x \cdot \operatorname{PolyLog}[2, -E^x] + x \cdot \operatorname{PolyLog}[2, -E^x/2] + 2 \cdot \operatorname{PolyLog}[3, -E^x] - \operatorname{PolyLog}[3, -E^x/2]$

#### Rule 2263

$\operatorname{Int}[(f + (g \cdot x)^m)/(a + (b \cdot F)^u + (c \cdot F)^v), x\_Symbol] \rightarrow \operatorname{With}[q = \operatorname{Rt}[b^2 - 4ac, 2], \operatorname{Dist}[(2c)/q, \operatorname{Int}[(f + gx)^m/(b - q + 2cF^u), x], x] - \operatorname{Dist}[(2c)/q, \operatorname{Int}[(f + gx)^m/(b + q + 2cF^u), x], x]] /; \operatorname{FreeQ}\{F, a, b, c, f, g, x\} \&\& \operatorname{EqQ}[v, 2u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2184

$\operatorname{Int}[(c + (d \cdot x)^m)/(a + (b \cdot F)^{(g \cdot (e + f \cdot x))^n}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^{m+1}/(a \cdot d \cdot (m+1)), x] - \operatorname{Dist}[b/a, \operatorname{Int}[(c + dx)^m \cdot (F^{g(e+fx)})^n]/(a + b \cdot (F^{g(e+fx)})^n), x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2190

$\operatorname{Int}[(F^{(g \cdot (e + f \cdot x))^n} \cdot (c + (d \cdot x)^m))/(a + (b \cdot F)^{(g \cdot (e + f \cdot x))^n}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^m \cdot \operatorname{Log}[1 + (b \cdot F^{g(e+fx)})^n/a]/(b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), x] - \operatorname{Dist}[(d \cdot m)/(b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(c + dx)^{m-1} \cdot \operatorname{Log}[1 + (b \cdot F^{g(e+fx)})^n]/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e \cdot F^{(c \cdot (a + b \cdot x))^n})] \cdot (f + (g \cdot x)^m), x\_Symbol] \rightarrow -\operatorname{Simp}[(f + gx)^m \cdot \operatorname{PolyLog}[2, -(e \cdot F^{c(a+bx)})^n]/(b \cdot c \cdot n \cdot \operatorname{Log}[F]), x] + \operatorname{Dist}[(g \cdot m)/(b \cdot c \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(f + gx)^{m-1} \cdot \operatorname{PolyLog}[2, -(e \cdot F^{c(a+bx)})^n]], x], x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n, x\} \&\& \operatorname{GtQ}[m, 0]$

#### Rule 2282



```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_)+(b_)*(x_))^(p_)]/((d_)+(e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n+1, c*(a+b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{2+3e^x+e^{2x}} dx &= 2 \int \frac{x^2}{2+2e^x} dx - 2 \int \frac{x^2}{4+2e^x} dx \\ &= \frac{x^3}{6} - 2 \int \frac{e^x x^2}{2+2e^x} dx + \int \frac{e^x x^2}{4+2e^x} dx \\ &= \frac{x^3}{6} + \frac{1}{2} x^2 \log\left(1 + \frac{e^x}{2}\right) - x^2 \log(1+e^x) + 2 \int x \log(1+e^x) dx - \int x \log\left(1 + \frac{e^x}{2}\right) dx \\ &= \frac{x^3}{6} + \frac{1}{2} x^2 \log\left(1 + \frac{e^x}{2}\right) - x^2 \log(1+e^x) - 2x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2\left(-\frac{e^x}{2}\right) + 2 \int \operatorname{Li}_2(-e^x) dx - \int \operatorname{Li}_2\left(-\frac{e^x}{2}\right) dx \\ &= \frac{x^3}{6} + \frac{1}{2} x^2 \log\left(1 + \frac{e^x}{2}\right) - x^2 \log(1+e^x) - 2x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2\left(-\frac{e^x}{2}\right) + 2 \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, x, e^x\right) \\ &= \frac{x^3}{6} + \frac{1}{2} x^2 \log\left(1 + \frac{e^x}{2}\right) - x^2 \log(1+e^x) - 2x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2\left(-\frac{e^x}{2}\right) + 2 \operatorname{Li}_3(-e^x) - \operatorname{Li}_3\left(-\frac{e^x}{2}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0098889, size = 77, normalized size = 1.

$$-x \operatorname{PolyLog}[2, -2e^{-x}] + 2x \operatorname{PolyLog}[2, -e^{-x}] - \operatorname{PolyLog}[3, -2e^{-x}] + 2 \operatorname{PolyLog}[3, -e^{-x}] + x^2 (-\log(e^{-x} + 1)) + \frac{1}{2} x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*E^x + E^(2*x)), x]
```

```
[Out] -(x^2*Log[1 + E^(-x)]) + (x^2*Log[1 + 2/E^x])/2 - x*PolyLog[2, -2/E^x] + 2*
x*PolyLog[2, -E^(-x)] - PolyLog[3, -2/E^x] + 2*PolyLog[3, -E^(-x)]
```

**Maple [A]** time = 0.006, size = 62, normalized size = 0.8

$$\frac{x^3}{6} + \frac{x^2}{2} \ln\left(1 + \frac{e^x}{2}\right) - x^2 \ln(1+e^x) - 2x \operatorname{polylog}(2, -e^x) + x \operatorname{polylog}\left(2, -\frac{e^x}{2}\right) + 2 \operatorname{polylog}(3, -e^x) - \operatorname{polylog}\left(3, -\frac{e^x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+3*exp(x)+exp(2*x)), x)
```

```
[Out] 1/6*x^3+1/2*x^2*ln(1+1/2*exp(x))-x^2*ln(1+exp(x))-2*x*polylog(2,-exp(x))+x*
polylog(2,-1/2*exp(x))+2*polylog(3,-exp(x))-polylog(3,-1/2*exp(x))
```

**Maxima [A]** time = 0.989142, size = 80, normalized size = 1.04

$$\frac{1}{6}x^3 - x^2 \log(e^x + 1) + \frac{1}{2}x^2 \log\left(\frac{1}{2}e^x + 1\right) + x\text{Li}_2\left(-\frac{1}{2}e^x\right) - 2x\text{Li}_2(-e^x) - \text{Li}_3\left(-\frac{1}{2}e^x\right) + 2\text{Li}_3(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+3\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] 1/6\*x^3 - x^2\*log(e^x + 1) + 1/2\*x^2\*log(1/2\*e^x + 1) + x\*dilog(-1/2\*e^x) - 2\*x\*dilog(-e^x) - polylog(3, -1/2\*e^x) + 2\*polylog(3, -e^x)

**Fricas [C]** time = 1.48589, size = 185, normalized size = 2.4

$$\frac{1}{6}x^3 - x^2 \log(e^x + 1) + \frac{1}{2}x^2 \log\left(\frac{1}{2}e^x + 1\right) + x\text{Li}_2\left(-\frac{1}{2}e^x\right) - 2x\text{Li}_2(-e^x) - \text{polylog}\left(3, -\frac{1}{2}e^x\right) + 2\text{polylog}(3, -e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] 1/6\*x^3 - x^2\*log(e^x + 1) + 1/2\*x^2\*log(1/2\*e^x + 1) + x\*dilog(-1/2\*e^x) - 2\*x\*dilog(-e^x) - polylog(3, -1/2\*e^x) + 2\*polylog(3, -e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(e^x + 1)(e^x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(2+3\*exp(x)+exp(2\*x)),x)

[Out] Integral(x\*\*2/((exp(x) + 1)\*(exp(x) + 2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + 3e^x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+3\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] integrate(x^2/(e^(2\*x) + 3\*e^x + 2), x)

$$3.517 \quad \int \frac{x^2}{-1+e^x+e^{2x}} dx$$

**Optimal.** Leaf size=259

$$\frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} + \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} - \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1+\sqrt{5})}$$

```
[Out] (2*x^3)/(3*Sqrt[5]*(1 - Sqrt[5])) - (2*x^3)/(3*Sqrt[5]*(1 + Sqrt[5])) - (2*
x^2*Log[1 + (2*E^x)/(1 - Sqrt[5])])/(Sqrt[5]*(1 - Sqrt[5])) + (2*x^2*Log[1
+ (2*E^x)/(1 + Sqrt[5])])/(Sqrt[5]*(1 + Sqrt[5])) - (4*x*PolyLog[2, (-2*E^x
)/(1 - Sqrt[5])])/(Sqrt[5]*(1 - Sqrt[5])) + (4*x*PolyLog[2, (-2*E^x)/(1 + S
qrt[5])])/(Sqrt[5]*(1 + Sqrt[5])) + (4*PolyLog[3, (-2*E^x)/(1 - Sqrt[5])])/(
Sqrt[5]*(1 - Sqrt[5])) - (4*PolyLog[3, (-2*E^x)/(1 + Sqrt[5])])/(Sqrt[5]*(
1 + Sqrt[5]))
```

**Rubi [A]** time = 0.296272, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2263, 2184, 2190, 2531, 2282, 6589}

$$\frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} + \frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} + \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}(1-\sqrt{5})} - \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}(1+\sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + E^x + E^(2*x)), x]
```

```
[Out] (2*x^3)/(3*Sqrt[5]*(1 - Sqrt[5])) - (2*x^3)/(3*Sqrt[5]*(1 + Sqrt[5])) - (2*
x^2*Log[1 + (2*E^x)/(1 - Sqrt[5])])/(Sqrt[5]*(1 - Sqrt[5])) + (2*x^2*Log[1
+ (2*E^x)/(1 + Sqrt[5])])/(Sqrt[5]*(1 + Sqrt[5])) - (4*x*PolyLog[2, (-2*E^x
)/(1 - Sqrt[5])])/(Sqrt[5]*(1 - Sqrt[5])) + (4*x*PolyLog[2, (-2*E^x)/(1 + S
qrt[5])])/(Sqrt[5]*(1 + Sqrt[5])) + (4*PolyLog[3, (-2*E^x)/(1 - Sqrt[5])])/(
Sqrt[5]*(1 - Sqrt[5])) - (4*PolyLog[3, (-2*E^x)/(1 + Sqrt[5])])/(Sqrt[5]*(
1 + Sqrt[5]))
```

#### Rule 2263

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m/
(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m/(b + q + 2*c*F^u)
, x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x
_))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x))))^n/(a + b*(F^(g*(e + f*x))))^n, x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{-1 + e^x + e^{2x}} dx = \frac{2 \int \frac{x^2}{1 - \sqrt{5} + 2e^x} dx}{\sqrt{5}} - \frac{2 \int \frac{x^2}{1 + \sqrt{5} + 2e^x} dx}{\sqrt{5}}$$

$$= \frac{2x^3}{3\sqrt{5}(1 - \sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1 + \sqrt{5})} - \frac{4 \int \frac{e^x x^2}{1 - \sqrt{5} + 2e^x} dx}{\sqrt{5}(1 - \sqrt{5})} + \frac{4 \int \frac{e^x x^2}{1 + \sqrt{5} + 2e^x} dx}{\sqrt{5}(1 + \sqrt{5})}$$

$$= \frac{2x^3}{3\sqrt{5}(1 - \sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1 + \sqrt{5})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{1 + \sqrt{5}}\right)}{\sqrt{5}(1 + \sqrt{5})} + \frac{4 \int x \log\left(1 + \frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})}$$

$$= \frac{2x^3}{3\sqrt{5}(1 - \sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1 + \sqrt{5})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{1 + \sqrt{5}}\right)}{\sqrt{5}(1 + \sqrt{5})} - \frac{4x \operatorname{Li}_2\left(-\frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})} + \frac{4x \operatorname{Li}_2\left(-\frac{2e^x}{1 + \sqrt{5}}\right)}{\sqrt{5}(1 + \sqrt{5})}$$

$$= \frac{2x^3}{3\sqrt{5}(1 - \sqrt{5})} - \frac{2x^3}{3\sqrt{5}(1 + \sqrt{5})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{1 + \sqrt{5}}\right)}{\sqrt{5}(1 + \sqrt{5})} - \frac{4x \operatorname{Li}_2\left(-\frac{2e^x}{1 - \sqrt{5}}\right)}{\sqrt{5}(1 - \sqrt{5})} + \frac{4x \operatorname{Li}_2\left(-\frac{2e^x}{1 + \sqrt{5}}\right)}{\sqrt{5}(1 + \sqrt{5})}$$

**Mathematica [A]** time = 0.150859, size = 172, normalized size = 0.66

$$2 \left( \frac{2 \left( x \operatorname{PolyLog}\left(2, \frac{1}{2}(\sqrt{5}-1)e^{-x}\right) + \operatorname{PolyLog}\left(3, \frac{1}{2}(\sqrt{5}-1)e^{-x}\right) \right)}{\sqrt{5}-1} - \frac{2 \left( x \operatorname{PolyLog}\left(2, -\frac{1}{2}(1+\sqrt{5})e^{-x}\right) + \operatorname{PolyLog}\left(3, -\frac{1}{2}(1+\sqrt{5})e^{-x}\right) \right)}{1+\sqrt{5}} + \frac{x^2 \log\left(1 - \frac{1}{2}(\sqrt{5}-1)e^{-x}\right)}{\sqrt{5}-1} \right) \frac{1}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-1 + E^x + E^(2\*x)), x]

[Out] (2\*((x^2\*Log[1 - (-1 + Sqrt[5])/(2\*E^x)])/(-1 + Sqrt[5]) + (x^2\*Log[1 + (1 + Sqrt[5])/(2\*E^x)])/(1 + Sqrt[5]) - (2\*(x\*PolyLog[2, (-1 + Sqrt[5])/(2\*E^x)]) + PolyLog[3, (-1 + Sqrt[5])/(2\*E^x)])))/(-1 + Sqrt[5]) - (2\*(x\*PolyLog[2, -(1 + Sqrt[5])/(2\*E^x)] + PolyLog[3, -(1 + Sqrt[5])/(2\*E^x)])))/(1 + Sqrt[5]))/Sqrt[5]

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int \frac{x^2}{-1 + e^x + e^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-1+exp(x)+exp(2\*x)), x)

[Out] int(x^2/(-1+exp(x)+exp(2\*x)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+exp(x)+exp(2\*x)), x, algorithm="maxima")

[Out] integrate(x^2/(e^(2\*x) + e^x - 1), x)

**Fricas [C]** time = 1.49855, size = 468, normalized size = 1.81

$$-\frac{1}{3}x^3 + \frac{1}{5}(\sqrt{5}x + 5x)\text{Li}_2\left(\frac{1}{2}(\sqrt{5} + 1)e^x\right) - \frac{1}{5}(\sqrt{5}x - 5x)\text{Li}_2\left(-\frac{1}{2}(\sqrt{5} - 1)e^x\right) + \frac{1}{10}(\sqrt{5}x^2 + 5x^2)\log\left(-\frac{1}{2}(\sqrt{5} + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+exp(x)+exp(2\*x)), x, algorithm="fricas")

[Out] -1/3\*x^3 + 1/5\*(sqrt(5)\*x + 5\*x)\*dilog(1/2\*(sqrt(5) + 1)\*e^x) - 1/5\*(sqrt(5)\*x - 5\*x)\*dilog(-1/2\*(sqrt(5) - 1)\*e^x) + 1/10\*(sqrt(5)\*x^2 + 5\*x^2)\*log(-1/2\*(sqrt(5) + 1)\*e^x + 1) - 1/10\*(sqrt(5)\*x^2 - 5\*x^2)\*log(1/2\*(sqrt(5) - 1)\*e^x + 1) - 1/5\*(sqrt(5) + 5)\*polylog(3, 1/2\*(sqrt(5) + 1)\*e^x) + 1/5\*(sqrt(5) - 5)\*polylog(3, -1/2\*(sqrt(5) - 1)\*e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{2x} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+exp(x)+exp(2*x)),x)
```

```
[Out] Integral(x**2/(exp(2*x) + exp(x) - 1), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + e^x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(e^(2*x) + e^x - 1), x)
```

$$3.518 \quad \int \frac{x^2}{3+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=293

$$-\frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} - \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{2x^3}{3\sqrt{3}(\sqrt{3}+3i)}$$

```
[Out] (-2*x^3)/(3*Sqrt[3]*(3*I - Sqrt[3])) + (2*x^3)/(3*Sqrt[3]*(3*I + Sqrt[3]))
- (2*x^2*Log[1 + (2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*x
^2*Log[1 + (2*E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) - (4*x*PolyL
og[2, (-2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (4*x*PolyLog[2
, (-2*E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) + (4*PolyLog[3, (-2*
E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) - (4*PolyLog[3, (-2*E^x)/(
3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3]))
```

**Rubi [A]** time = 0.308999, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2263, 2184, 2190, 2531, 2282, 6589}

$$-\frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} + \frac{4x \operatorname{PolyLog}\left(2, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(\sqrt{3}+3i)} - \frac{4 \operatorname{PolyLog}\left(3, -\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(-\sqrt{3}+3i)} + \frac{2x^3}{3\sqrt{3}(\sqrt{3}+3i)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + 3*E^x + E^(2*x)), x]
```

```
[Out] (-2*x^3)/(3*Sqrt[3]*(3*I - Sqrt[3])) + (2*x^3)/(3*Sqrt[3]*(3*I + Sqrt[3]))
- (2*x^2*Log[1 + (2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (2*x
^2*Log[1 + (2*E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) - (4*x*PolyL
og[2, (-2*E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) + (4*x*PolyLog[2
, (-2*E^x)/(3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3])) + (4*PolyLog[3, (-2*
E^x)/(3 - I*Sqrt[3])])/(Sqrt[3]*(3*I + Sqrt[3])) - (4*PolyLog[3, (-2*E^x)/(
3 + I*Sqrt[3])])/(Sqrt[3]*(3*I - Sqrt[3]))
```

#### Rule 2263

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m/
(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m/(b + q + 2*c*F^u)
, x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x
_))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/
((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{3 + 3e^x + e^{2x}} dx = -\frac{(2i) \int \frac{x^2}{3-i\sqrt{3}+2e^x} dx}{\sqrt{3}} + \frac{(2i) \int \frac{x^2}{3+i\sqrt{3}+2e^x} dx}{\sqrt{3}}$$

$$= -\frac{2x^3}{3\sqrt{3}(3i - \sqrt{3})} + \frac{2x^3}{3\sqrt{3}(3i + \sqrt{3})} + \frac{(4i) \int \frac{e^x x^2}{3-i\sqrt{3}+2e^x} dx}{\sqrt{3}(3 - i\sqrt{3})} - \frac{(4i) \int \frac{e^x x^2}{3+i\sqrt{3}+2e^x} dx}{\sqrt{3}(3 + i\sqrt{3})}$$

$$= -\frac{2x^3}{3\sqrt{3}(3i - \sqrt{3})} + \frac{2x^3}{3\sqrt{3}(3i + \sqrt{3})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3i - \sqrt{3})} - \frac{(4i) \int x \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3 - i\sqrt{3})}$$

$$= -\frac{2x^3}{3\sqrt{3}(3i - \sqrt{3})} + \frac{2x^3}{3\sqrt{3}(3i + \sqrt{3})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3i - \sqrt{3})} - \frac{4x \text{Li}_2\left(-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})}$$

$$= -\frac{2x^3}{3\sqrt{3}(3i - \sqrt{3})} + \frac{2x^3}{3\sqrt{3}(3i + \sqrt{3})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3i - \sqrt{3})} - \frac{4x \text{Li}_2\left(-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})}$$

$$= -\frac{2x^3}{3\sqrt{3}(3i - \sqrt{3})} + \frac{2x^3}{3\sqrt{3}(3i + \sqrt{3})} - \frac{2x^2 \log\left(1 + \frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})} + \frac{2x^2 \log\left(1 + \frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}(3i - \sqrt{3})} - \frac{4x \text{Li}_2\left(-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}(3i + \sqrt{3})}$$

**Mathematica [A]** time = 0.165446, size = 216, normalized size = 0.74

$$2i \left( \frac{2 \left( x \text{PolyLog}\left(2, -\frac{1}{2}(3+i\sqrt{3})e^{-x}\right) + \text{PolyLog}\left(3, -\frac{1}{2}i(\sqrt{3}-3i)e^{-x}\right) \right)}{3+i\sqrt{3}} - \frac{2i \left( x \text{PolyLog}\left(2, \frac{1}{2}i(\sqrt{3}+3i)e^{-x}\right) + \text{PolyLog}\left(3, \frac{1}{2}i(\sqrt{3}+3i)e^{-x}\right) \right)}{\sqrt{3}+3i} + \frac{ix^2 \log\left(1 + \frac{1}{2}(3-i\sqrt{3})e^{-x}\right)}{\sqrt{3}+3i} \right) \sqrt{3}$$

Antiderivative was successfully verified.



[In] Integrate[x^2/(3 + 3\*E^x + E^(2\*x)),x]

[Out]  $((2I)*((I*x^2*\text{Log}[1 + (3 - I*\text{Sqrt}[3])/(2*E^x)])/(3I + \text{Sqrt}[3]) + (I*x^2*\text{Log}[1 + (3 + I*\text{Sqrt}[3])/(2*E^x)])/(-3I + \text{Sqrt}[3]) + (2*(x*\text{PolyLog}[2, -(3 + I*\text{Sqrt}[3])/(2*E^x)] + \text{PolyLog}[3, ((-I/2)*(-3I + \text{Sqrt}[3]))/E^x]))/(3 + I*\text{Sqrt}[3]) - ((2I)*(x*\text{PolyLog}[2, ((I/2)*(3I + \text{Sqrt}[3]))/E^x] + \text{PolyLog}[3, ((I/2)*(3I + \text{Sqrt}[3]))/E^x]))/(3I + \text{Sqrt}[3])))/\text{Sqrt}[3]$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{x^2}{3 + 3e^x + e^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(3+3\*exp(x)+exp(2\*x)),x)

[Out] int(x^2/(3+3\*exp(x)+exp(2\*x)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3+3\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] integrate(x^2/(e^(2\*x) + 3\*e^x + 3), x)

**Fricas [C]** time = 1.65442, size = 520, normalized size = 1.77

$$\frac{1}{9}x^3 + \frac{1}{18}(6i\sqrt{3}x - 6x)\text{Li}_2\left(-\frac{1}{6}(i\sqrt{3} + 3)e^x\right) + \frac{1}{18}(-6i\sqrt{3}x - 6x)\text{Li}_2\left(-\frac{1}{6}(-i\sqrt{3} + 3)e^x\right) + \frac{1}{18}(3i\sqrt{3}x^2 - 3x^2)\log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out]  $1/9*x^3 + 1/18*(6*I*\text{sqrt}(3)*x - 6*x)*\text{dilog}(-1/6*(I*\text{sqrt}(3) + 3)*e^x) + 1/18*(-6*I*\text{sqrt}(3)*x - 6*x)*\text{dilog}(-1/6*(-I*\text{sqrt}(3) + 3)*e^x) + 1/18*(3*I*\text{sqrt}(3)*x^2 - 3*x^2)*\log(1/6*(I*\text{sqrt}(3) + 3)*e^x + 1) + 1/18*(-3*I*\text{sqrt}(3)*x^2 - 3*x^2)*\log(1/6*(-I*\text{sqrt}(3) + 3)*e^x + 1) - 1/3*(-I*\text{sqrt}(3) - 1)*\text{polylog}(3, 1/6*(I*\text{sqrt}(3) - 3)*e^x) - 1/3*(I*\text{sqrt}(3) - 1)*\text{polylog}(3, 1/6*(-I*\text{sqrt}(3) - 3)*e^x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{2x} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3+3*exp(x)+exp(2*x)),x)
```

```
[Out] Integral(x**2/(exp(2*x) + 3*exp(x) + 3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(2x)} + 3e^x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(e^(2*x) + 3*e^x + 3), x)
```

$$3.519 \quad \int \frac{x^2}{a+be^x+ce^{2x}} dx$$

**Optimal.** Leaf size=391

$$\frac{4cx \operatorname{PolyLog}\left(2, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{4cx \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{4c \operatorname{PolyLog}\left(3, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{4c \operatorname{PolyLog}\left(3, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x^3)/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3)/(3*(b^2 - 4*
a*c + b*Sqrt[b^2 - 4*a*c])) + (2*c*x^2*Log[1 + (2*c*E^x)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*x^2*Log[1 + (2*c*E^x)/(b
+ Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (4*c*x*PolyLo
g[2, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c
]) + (4*c*x*PolyLog[2, (-2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c +
b*Sqrt[b^2 - 4*a*c]) - (4*c*PolyLog[3, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])])
/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (4*c*PolyLog[3, (-2*c*E^x)/(b + Sqrt
[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

**Rubi [A]** time = 0.665303, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2263, 2184, 2190, 2531, 2282, 6589}

$$\frac{4cx \operatorname{PolyLog}\left(2, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{4cx \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} - \frac{4c \operatorname{PolyLog}\left(3, -\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{4c \operatorname{PolyLog}\left(3, -\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*E^x + c*E^(2*x)), x]
```

```
[Out] (-2*c*x^3)/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3)/(3*(b^2 - 4*
a*c + b*Sqrt[b^2 - 4*a*c])) + (2*c*x^2*Log[1 + (2*c*E^x)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*x^2*Log[1 + (2*c*E^x)/(b
+ Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (4*c*x*PolyLo
g[2, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c
]) + (4*c*x*PolyLog[2, (-2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c +
b*Sqrt[b^2 - 4*a*c]) - (4*c*PolyLog[3, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])])
/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (4*c*PolyLog[3, (-2*c*E^x)/(b + Sqrt
[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])
```

#### Rule 2263

```
Int[((f_.) + (g_.)*(x_.))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_.)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m/
(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m/(b + q + 2*c*F^u)
, x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] &&
& NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x
_))))^(n_.), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a + be^x + ce^{2x}} dx = \frac{(2c) \int \frac{x^2}{b - \sqrt{b^2 - 4ac} + 2ce^x} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x^2}{b + \sqrt{b^2 - 4ac} + 2ce^x} dx}{\sqrt{b^2 - 4ac}}$$

$$= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{(4c^2) \int \frac{e^x x^2}{b - \sqrt{b^2 - 4ac} + 2ce^x} dx}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{(4c^2) \int \frac{e^x x^2}{b + \sqrt{b^2 - 4ac} + 2ce^x} dx}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{2cx^2 \log\left(1 + \frac{2ce^x}{b + \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

**Mathematica [A]** time = 0.184272, size = 407, normalized size = 1.04

$$-6x\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{2ce^x}{\sqrt{b^2-4ac}-b}\right)+6x\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)+6b\text{PolyLog}\left(3,\frac{2ce^x}{\sqrt{b^2-4ac}-b}\right)+6b\text{PolyLog}\left(3,-\frac{2ce^x}{\sqrt{b^2-4ac}+b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*E^x + c\*E^(2\*x)),x]

[Out] (2\*Sqrt[b^2 - 4\*a\*c]\*x^3 - 3\*b\*x^2\*Log[1 + (2\*c\*E^x)/(b - Sqrt[b^2 - 4\*a\*c])] - 3\*Sqrt[b^2 - 4\*a\*c]\*x^2\*Log[1 + (2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c])] + 3\*b\*x^2\*Log[1 + (2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c])] - 3\*Sqrt[b^2 - 4\*a\*c]\*x^2\*Log[1 + (2\*c\*E^x)/(b - Sqrt[b^2 - 4\*a\*c])] - 6\*(b + Sqrt[b^2 - 4\*a\*c])\*x\*PolyLog[2, (2\*c\*E^x)/(-b + Sqrt[b^2 - 4\*a\*c])] + 6\*(b - Sqrt[b^2 - 4\*a\*c])\*x\*PolyLog[2, (-2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c])] + 6\*b\*PolyLog[3, (2\*c\*E^x)/(-b + Sqrt[b^2 - 4\*a\*c])] + 6\*Sqrt[b^2 - 4\*a\*c]\*PolyLog[3, (2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c])] - 6\*b\*PolyLog[3, (-2\*c\*E^x)/(b + Sqrt[b^2 - 4\*a\*c])] + 6\*Sqrt[b^2 - 4\*a\*c]\*PolyLog[3, (-2\*c\*E^x)/(-b + Sqrt[b^2 - 4\*a\*c])])/(6\*a\*Sqrt[b^2 - 4\*a\*c])

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + be^x + ce^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*exp(x)+c\*exp(2\*x)),x)

[Out] int(x^2/(a+b\*exp(x)+c\*exp(2\*x)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(x)+c\*exp(2\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.55385, size = 977, normalized size = 2.5

$$2(b^2-4ac)x^3-6\left(abx\sqrt{\frac{b^2-4ac}{a^2}}+(b^2-4ac)x\right)\text{Li}_2\left(-\frac{a\sqrt{\frac{b^2-4ac}{a^2}}e^{x+be^x+2a}}{2a}+1\right)+6\left(abx\sqrt{\frac{b^2-4ac}{a^2}}-(b^2-4ac)x\right)\text{Li}_2\left(\frac{a\sqrt{\frac{b^2-4ac}{a^2}}e^{x+be^x+2a}}{2a}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(x)+c\*exp(2\*x)),x, algorithm="fricas")

```
[Out] 1/6*(2*(b^2 - 4*a*c)*x^3 - 6*(a*b*x*sqrt((b^2 - 4*a*c)/a^2) + (b^2 - 4*a*c)*x)*dilog(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a)/a + 1) + 6*(a*b*x*sqrt((b^2 - 4*a*c)/a^2) - (b^2 - 4*a*c)*x)*dilog(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x - b*e^x - 2*a)/a + 1) - 3*(a*b*x^2*sqrt((b^2 - 4*a*c)/a^2) + (b^2 - 4*a*c)*x^2)*log(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a)/a) + 3*(a*b*x^2*sqrt((b^2 - 4*a*c)/a^2) - (b^2 - 4*a*c)*x^2)*log(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x - b*e^x - 2*a)/a) + 6*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 4*a*c)*polylog(3, -1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x)/a) - 6*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 4*a*c)*polylog(3, 1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x - b*e^x)/a))/(a*b^2 - 4*a^2*c)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + be^x + ce^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*exp(x)+c*exp(2*x)),x)
```

```
[Out] Integral(x**2/(a + b*exp(x) + c*exp(2*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ce^{(2x)} + be^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*e^(2*x) + b*e^x + a), x)
```

$$3.520 \quad \int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}} dx$$

**Optimal.** Leaf size=40

$$-\frac{\log(f^{c+dx}+1)}{d \log(f)} + \frac{1}{d \log(f)(f^{c+dx}+1)} + x$$

[Out] x + 1/(d\*(1 + f^(c + d\*x))\*Log[f]) - Log[1 + f^(c + d\*x)]/(d\*Log[f])

**Rubi [A]** time = 0.0265826, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2282, 44}

$$-\frac{\log(f^{c+dx}+1)}{d \log(f)} + \frac{1}{d \log(f)(f^{c+dx}+1)} + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*f^(c + d\*x) + f^(2\*c + 2\*d\*x))^(-1), x]

[Out] x + 1/(d\*(1 + f^(c + d\*x))\*Log[f]) - Log[1 + f^(c + d\*x)]/(d\*Log[f])

#### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_)+(b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :=> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)^2} dx, x, f^{c+dx}\right)}{d \log(f)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2}\right) dx, x, f^{c+dx}\right)}{d \log(f)} \\ &= x + \frac{1}{d(1+f^{c+dx}) \log(f)} - \frac{\log(1+f^{c+dx})}{d \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0338202, size = 37, normalized size = 0.92

$$\frac{1}{f^{c+dx}+1} - \log(f^{c+dx}+1) + dx \log(f)$$

$$\frac{\quad}{d \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*f^(c + d\*x) + f^(2\*c + 2\*d\*x))^(-1), x]

[Out] ((1 + f^(c + d\*x))^(-1) + d\*x\*Log[f] - Log[1 + f^(c + d\*x)])/(d\*Log[f])

**Maple [A]** time = 0.013, size = 68, normalized size = 1.7

$$\frac{1}{e^{(dx+c)\ln(f)} + 1} \left( x + x e^{(dx+c)\ln(f)} - \frac{e^{(dx+c)\ln(f)}}{d \ln(f)} \right) - \frac{\ln(e^{(dx+c)\ln(f)} + 1)}{d \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)), x)

[Out] (x+x\*exp((d\*x+c)\*ln(f))-1/d/ln(f)\*exp((d\*x+c)\*ln(f)))/(exp((d\*x+c)\*ln(f))+1)-1/d/ln(f)\*ln(exp((d\*x+c)\*ln(f))+1)

**Maxima [A]** time = 0.989898, size = 74, normalized size = 1.85

$$-\frac{\log(f^{dx+c} + 1)}{d \log(f)} + \frac{\log(f^{dx+c})}{d \log(f)} + \frac{1}{d(f^{dx+c} + 1) \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)), x, algorithm="maxima")

[Out] -log(f^(d\*x + c) + 1)/(d\*log(f)) + log(f^(d\*x + c))/(d\*log(f)) + 1/(d\*(f^(d\*x + c) + 1)\*log(f))

**Fricas [A]** time = 1.60264, size = 159, normalized size = 3.98

$$\frac{d f^{dx+c} x \log(f) + dx \log(f) - (f^{dx+c} + 1) \log(f^{dx+c} + 1) + 1}{d f^{dx+c} \log(f) + d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)), x, algorithm="fricas")

[Out] (d\*f^(d\*x + c)\*x\*log(f) + d\*x\*log(f) - (f^(d\*x + c) + 1)\*log(f^(d\*x + c) + 1) + 1)/(d\*f^(d\*x + c)\*log(f) + d\*log(f))

**Sympy [A]** time = 0.116324, size = 34, normalized size = 0.85

$$x + \frac{1}{d f^{c+dx} \log(f) + d \log(f)} - \frac{\log(f^{c+dx} + 1)}{d \log(f)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2\*f\*\*(d\*x+c)+f\*\*(2\*d\*x+2\*c)),x)

[Out] x + 1/(d\*f\*\*(c + d\*x)\*log(f) + d\*log(f)) - log(f\*\*(c + d\*x) + 1)/(d\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{f^{2dx+2c} + 2f^{dx+c} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="giac")

[Out] integrate(1/(f^(2\*d\*x + 2\*c) + 2\*f^(d\*x + c) + 1), x)

$$3.521 \quad \int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}} dx$$

**Optimal.** Leaf size=94

$$\frac{b \tanh^{-1}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{ad \log(f)\sqrt{b^2-4ac}} - \frac{\log(a+bf^{c+dx}+cf^{2c+2dx})}{2ad \log(f)} + \frac{x}{a}$$

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*f^(c + d\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*d\*Log[f]) - Log[a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x)]/(2\*a\*d\*Log[f])

**Rubi [A]** time = 0.108036, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$ , Rules used = {2282, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{ad \log(f)\sqrt{b^2-4ac}} - \frac{\log(a+bf^{c+dx}+cf^{2c+2dx})}{2ad \log(f)} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x))^(-1), x]

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*f^(c + d\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*d\*Log[f]) - Log[a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x)]/(2\*a\*d\*Log[f])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

$x]$  && NeQ[ $b^2 - 4*a*c, 0]$

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned} \int \frac{1}{a + bf^{c+dx} + cf^{2c+2dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)} dx, x, f^{c+dx}\right)}{d \log(f)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, f^{c+dx}\right)}{ad \log(f)} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, f^{c+dx}\right)}{ad \log(f)} \\ &= \frac{x}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, f^{c+dx}\right)}{2ad \log(f)} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, f^{c+dx}\right)}{2ad \log(f)} \\ &= \frac{x}{a} - \frac{\log(a + bf^{c+dx} + cf^{2c+2dx})}{2ad \log(f)} + \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cf^{c+dx}\right)}{ad \log(f)} \\ &= \frac{x}{a} + \frac{b \tanh^{-1}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}d \log(f)} - \frac{\log(a + bf^{c+dx} + cf^{2c+2dx})}{2ad \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.137525, size = 93, normalized size = 0.99

$$\frac{\frac{2b \tan^{-1}\left(\frac{b+2cf^{c+dx}}{\sqrt{4ac-b^2}}\right)}{d \log(f)\sqrt{4ac-b^2}} + \frac{\log(a + f^{c+dx}(b + cf^{c+dx}))}{d \log(f)} - 2x}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x))^(-1), x]

[Out] -(-2\*x + (2\*b\*ArcTan[(b + 2\*c\*f^(c + d\*x))/Sqrt[-b^2 + 4\*a\*c]])/(Sqrt[-b^2 + 4\*a\*c]\*d\*Log[f]) + Log[a + f^(c + d\*x)\*(b + c\*f^(c + d\*x))]/(d\*Log[f]))/(2\*a)

**Maple [B]** time = 0.085, size = 547, normalized size = 5.8

$$4 \frac{(\ln(f))^2 acd^2 x}{4 (\ln(f))^2 a^2 cd^2 - (\ln(f))^2 ab^2 d^2} - \frac{(\ln(f))^2 b^2 d^2 x}{4 (\ln(f))^2 a^2 cd^2 - (\ln(f))^2 ab^2 d^2} + 4 \frac{(\ln(f))^2 ac^2 d}{4 (\ln(f))^2 a^2 cd^2 - (\ln(f))^2 ab^2 d^2} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)`

[Out] 
$$\frac{4/(4*\ln(f)^2*a^2*c*d^2-\ln(f)^2*a*b^2*d^2)*\ln(f)^2*a*c*d^2*x-1/(4*\ln(f)^2*a^2*c*d^2-\ln(f)^2*a*b^2*d^2)*\ln(f)^2*b^2*d^2*x+4/(4*\ln(f)^2*a^2*c*d^2-\ln(f)^2*a*b^2*d^2)*\ln(f)^2*b^2*c*d-2/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})-1/2*(-b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*c+1/2/a/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})-1/2*(-b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*b^2+1/2/a/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})-1/2*(-b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*(-4*a*b^2*c+b^4)^{(1/2)}-2/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})+1/2*(b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*c+1/2/a/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})+1/2*(b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*b^2-1/2/a/(4*a*c-b^2)/d/\ln(f)*\ln(f^{d*x+c})+1/2*(b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*(-4*a*b^2*c+b^4)^{(1/2)}}{2*(ab^2-4a^2c)d\log(f)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.68393, size = 711, normalized size = 7.56

$$\frac{2(b^2-4ac)dx \log(f) + \sqrt{b^2-4ac}b \log\left(\frac{2c^2 f^{2dx+2c} + b^2 - 2ac + 2(bc + \sqrt{b^2-4ac})f^{dx+c} + \sqrt{b^2-4ac}b}{cf^{2dx+2c} + bf^{dx+c} + a}\right) - (b^2-4ac) \log(cf^{2dx+2c} + bf^{dx+c} + a)}{2(ab^2-4a^2c)d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * x * \log(f) + \sqrt{b^2 - 4 * a * c} * b * \log((2 * c^2 * f^{2 * d * x + 2 * c} + b^2 - 2 * a * c + 2 * (b * c + \sqrt{b^2 - 4 * a * c}) * c) * f^{d * x + c} + \sqrt{b^2 - 4 * a * c} * b) / (c * f^{2 * d * x + 2 * c} + b * f^{d * x + c} + a)) - (b^2 - 4 * a * c) * \log(c * f^{2 * d * x + 2 * c} + b * f^{d * x + c} + a)) / ((a * b^2 - 4 * a^2 * c) * d * \log(f)), \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * x * \log(f) + 2 * \sqrt{-b^2 + 4 * a * c} * b * \arctan(-2 * \sqrt{-b^2 + 4 * a * c} * c * f^{d * x + c} + \sqrt{-b^2 + 4 * a * c} * b) / (b^2 - 4 * a * c)) - (b^2 - 4 * a * c) * \log(c * f^{2 * d * x + 2 * c} + b * f^{d * x + c} + a)) / ((a * b^2 - 4 * a^2 * c) * d * \log(f)) \right]$$

**Sympy [A]** time = 0.420756, size = 104, normalized size = 1.11

$$\text{RootSum}\left(z^2\left(4a^2cd^2\log(f)^2 - ab^2d^2\log(f)^2\right) + z\left(4acd\log(f) - b^2d\log(f)\right) + c, \left(i \mapsto i\log\left(f^{c+dx} + \frac{-4ia^2cd\log(f)}{2z}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f**(d*x+c)+c*f**(2*d*x+2*c)),x)`

```
[Out] RootSum(_z**2*(4*a**2*c*d**2*log(f)**2 - a*b**2*d**2*log(f)**2) + _z*(4*a*c
*d*log(f) - b**2*d*log(f)) + c, Lambda(_i, _i*log(f**(c + d*x) + (-4*_i*a**
2*c*d*log(f) + _i*a*b**2*d*log(f) - 2*a*c + b**2)/(b*c)))) + x/a
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cf^{2dx+2c} + bf^{dx+c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="giac")
```

```
[Out] integrate(1/(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a), x)
```

$$3.522 \quad \int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}} dx$$

**Optimal.** Leaf size=94

$$\frac{b \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{\log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{x}{a}$$

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)]/(2\*a\*h\*Log[f])

**Rubi [A]** time = 0.0926453, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {2282, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{\log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*f^(g + h\*x) + c\*f^(2\*(g + h\*x)))^(-1), x]

[Out] x/a + (b\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)]/(2\*a\*h\*Log[f])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || LtQ}[b, 0])$

### Rule 628

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \text{ :> Simp}[d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b, x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{a + bfg^{hx} + cf^{2(g+hx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)} dx, x, fg^{hx}\right)}{h \log(f)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, fg^{hx}\right)}{ah \log(f)} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, fg^{hx}\right)}{ah \log(f)} \\ &= \frac{x}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, fg^{hx}\right)}{2ah \log(f)} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, fg^{hx}\right)}{2ah \log(f)} \\ &= \frac{x}{a} - \frac{\log(a + bfg^{hx} + cf^{2g+2hx})}{2ah \log(f)} + \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2c fg^{hx}\right)}{ah \log(f)} \\ &= \frac{x}{a} + \frac{b \tanh^{-1}\left(\frac{b+2c fg^{hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}h \log(f)} - \frac{\log(a + bfg^{hx} + cf^{2g+2hx})}{2ah \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.144175, size = 93, normalized size = 0.99

$$\frac{\frac{2b \tan^{-1}\left(\frac{b+2c fg^{hx}}{\sqrt{4ac-b^2}}\right)}{h \log(f)\sqrt{4ac-b^2}} + \frac{\log(a + fg^{hx}(b + c fg^{hx}))}{h \log(f)} - 2x}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*f^(g + h\*x) + c\*f^(2\*(g + h\*x)))^(-1), x]

[Out]  $-\frac{(-2*x + (2*b*\text{ArcTan}[(b + 2*c*f^(g + h*x))/\text{Sqrt}[-b^2 + 4*a*c]])/\text{Sqrt}[-b^2 + 4*a*c]*h*\text{Log}[f] + \text{Log}[a + f^(g + h*x)*(b + c*f^(g + h*x))]/(h*\text{Log}[f])}{2*a}$

**Maple [B]** time = 0.076, size = 546, normalized size = 5.8

$$4 \frac{(\ln(f))^2 ach^2 x}{4 (\ln(f))^2 a^2 ch^2 - (\ln(f))^2 ab^2 h^2} - \frac{(\ln(f))^2 b^2 h^2 x}{4 (\ln(f))^2 a^2 ch^2 - (\ln(f))^2 ab^2 h^2} + 4 \frac{(\ln(f))^2 acgh}{4 (\ln(f))^2 a^2 ch^2 - (\ln(f))^2 ab^2 h^2} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x)`

[Out] 
$$\frac{4/(4*\ln(f)^2*a^2*c*h^2-\ln(f)^2*a*b^2*h^2)*\ln(f)^2*a*c*h^2*x-1/(4*\ln(f)^2*a^2*c*h^2-\ln(f)^2*a*b^2*h^2)*\ln(f)^2*b^2*h^2*x+4/(4*\ln(f)^2*a^2*c*h^2-\ln(f)^2*a*b^2*h^2)*\ln(f)^2*a*c*g*h-1/(4*\ln(f)^2*a^2*c*h^2-\ln(f)^2*a*b^2*h^2)*\ln(f)^2*b^2*g*h-2/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)-1/2*(-b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*c+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)-1/2*(-b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*b^2+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)-1/2*(-b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*(-4*a*b^2*c+b^4)^{1/2}-2/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*c+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*b^2-1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^{1/2})})/b/c)*(-4*a*b^2*c+b^4)^{1/2}}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.80076, size = 711, normalized size = 7.56

$$\frac{2(b^2 - 4ac)hx \log(f) + \sqrt{b^2 - 4ac}b \log\left(\frac{2c^2 f^{2hx+2g} + b^2 - 2ac + 2(bc + \sqrt{b^2 - 4ac})f^{hx+g} + \sqrt{b^2 - 4ac}b}{c f^{2hx+2g} + b f^{hx+g} + a}\right) - (b^2 - 4ac) \log(c f^{2hx+2g} + b f^{hx+g} + a)}{2(ab^2 - 4a^2c)h \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{2} * (2 * (b^2 - 4 * a * c) * h * x * \log(f) + \sqrt{b^2 - 4 * a * c} * b * \log((2 * c^2 * f^{2 * h * x + 2 * g} + b^2 - 2 * a * c + 2 * (b * c + \sqrt{b^2 - 4 * a * c}) * c) * f^{h * x + g} + \sqrt{b^2 - 4 * a * c} * b) / (c * f^{2 * h * x + 2 * g} + b * f^{h * x + g} + a)) - (b^2 - 4 * a * c) * \log(c * f^{2 * h * x + 2 * g} + b * f^{h * x + g} + a)) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)), \frac{1}{2} * (2 * (b^2 - 4 * a * c) * h * x * \log(f) + 2 * \sqrt{-b^2 + 4 * a * c} * b * \arctan(-2 * \sqrt{-b^2 + 4 * a * c} * c * f^{h * x + g} + \sqrt{-b^2 + 4 * a * c} * b) / (b^2 - 4 * a * c)) - (b^2 - 4 * a * c) * \log(c * f^{2 * h * x + 2 * g} + b * f^{h * x + g} + a)) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)) \right]$$

**Sympy [A]** time = 0.424013, size = 104, normalized size = 1.11

$$\text{RootSum}\left(z^2\left(4a^2ch^2\log(f)^2 - ab^2h^2\log(f)^2\right) + z\left(4ach\log(f) - b^2h\log(f)\right) + c, \left(i \mapsto i \log\left(f^{g+hx} + \frac{-4ia^2ch\log(f)}{2z}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*f**(h*x+g)+c*f**(2*h*x+2*g)),x)`



```
[Out] RootSum(_z**2*(4*a**2*c*h**2*log(f)**2 - a*b**2*h**2*log(f)**2) + _z*(4*a*c
*h*log(f) - b**2*h*log(f)) + c, Lambda(_i, _i*log(f**(g + h*x) + (-4*_i*a**
2*c*h*log(f) + _i*a*b**2*h*log(f) - 2*a*c + b**2)/(b*c)))) + x/a
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cf^{2hx+2g} + bf^{hx+g} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="giac")
```

```
[Out] integrate(1/(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a), x)
```

$$3.523 \quad \int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}} dx$$

**Optimal.** Leaf size=96

$$-\frac{\text{PolyLog}(2, -f^{c+dx})}{d^2 \log^2(f)} + \frac{\log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x \log(f^{c+dx} + 1)}{d \log(f)} + \frac{x}{d \log(f)(f^{c+dx} + 1)} - \frac{x}{d \log(f)} + \frac{x^2}{2}$$

[Out]  $x^2/2 - x/(d*\text{Log}[f]) + x/(d*(1 + f^{(c + d*x)})*\text{Log}[f]) + \text{Log}[1 + f^{(c + d*x)}] / (d^2*\text{Log}[f]^2) - (x*\text{Log}[1 + f^{(c + d*x)}]) / (d*\text{Log}[f]) - \text{PolyLog}[2, -f^{(c + d*x)}] / (d^2*\text{Log}[f]^2)$

**Rubi [A]** time = 0.267543, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$ , Rules used = {6688, 2185, 2184, 2190, 2279, 2391, 2191, 2282, 36, 29, 31}

$$-\frac{\text{PolyLog}(2, -f^{c+dx})}{d^2 \log^2(f)} + \frac{\log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x \log(f^{c+dx} + 1)}{d \log(f)} + \frac{x}{d \log(f)(f^{c+dx} + 1)} - \frac{x}{d \log(f)} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] `Int[x/(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x)),x]`

[Out]  $x^2/2 - x/(d*\text{Log}[f]) + x/(d*(1 + f^{(c + d*x)})*\text{Log}[f]) + \text{Log}[1 + f^{(c + d*x)}] / (d^2*\text{Log}[f]^2) - (x*\text{Log}[1 + f^{(c + d*x)}]) / (d*\text{Log}[f]) - \text{PolyLog}[2, -f^{(c + d*x)}] / (d^2*\text{Log}[f]^2)$

#### Rule 6688

`Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]`

#### Rule 2185

`Int[((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e + f*x))))^n^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x))))^n*(a + b*(F^(g*(e + f*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]`

#### Rule 2184

`Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x))))^n/(a + b*(F^(g*(e + f*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^n_)]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2191

```
Int[((F_)^((g_)*((e_) + (f_)*(x_)))^n_)*((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^n_))^p_)*((c_) + (d_)*(x_)^m_), x_Symbol] :>
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^n_)^m_] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}} dx &= \int \frac{x}{(1+f^{c+dx})^2} dx \\
&= -\int \frac{f^{c+dx}x}{(1+f^{c+dx})^2} dx + \int \frac{x}{1+f^{c+dx}} dx \\
&= \frac{x^2}{2} + \frac{x}{d(1+f^{c+dx})\log(f)} - \frac{\int \frac{1}{1+f^{c+dx}} dx}{d\log(f)} - \int \frac{f^{c+dx}x}{1+f^{c+dx}} dx \\
&= \frac{x^2}{2} + \frac{x}{d(1+f^{c+dx})\log(f)} - \frac{x\log(1+f^{c+dx})}{d\log(f)} - \frac{\text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} + \frac{\int \log(1+f^{c+dx})}{d\log(f)} \\
&= \frac{x^2}{2} + \frac{x}{d(1+f^{c+dx})\log(f)} - \frac{x\log(1+f^{c+dx})}{d\log(f)} - \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} + \frac{\text{Subst}\left(\int \frac{1}{1+x} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} \\
&= \frac{x^2}{2} - \frac{x}{d\log(f)} + \frac{x}{d(1+f^{c+dx})\log(f)} + \frac{\log(1+f^{c+dx})}{d^2\log^2(f)} - \frac{x\log(1+f^{c+dx})}{d\log(f)} - \frac{\text{Li}_2(-f^{c+dx})}{d^2\log^2(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.176088, size = 88, normalized size = 0.92

$$-\frac{\text{PolyLog}\left(2, -f^{c+dx}\right)}{d^2\log^2(f)} + \frac{\log\left(f^{c+dx}+1\right)}{d^2\log^2(f)} + \frac{1}{2}x\left(\frac{2}{d\log(f)f^{c+dx}+d\log(f)}+x\right) - \frac{x\left(\log\left(f^{c+dx}+1\right)+1\right)}{d\log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1+2\*f^(c+d\*x)+f^(2\*c+2\*d\*x)),x]

[Out] (x\*(x+2/(d\*Log[f]+d\*f^(c+d\*x)\*Log[f])))/2+Log[1+f^(c+d\*x)]/(d^2\*Log[f]^2)-(x\*(1+Log[1+f^(c+d\*x)]))/(d\*Log[f])-PolyLog[2,-f^(c+d\*x)]/(d^2\*Log[f]^2)

**Maple [A]** time = 0.059, size = 143, normalized size = 1.5

$$\frac{x}{d(1+f^{dx+c})\ln(f)} + \frac{x^2}{2} + \frac{cx}{d} + \frac{c^2}{2d^2} - \frac{\ln(f^{dx}fc+1)x}{d\ln(f)} - \frac{\text{polylog}\left(2, -f^{dx}fc\right)}{(\ln(f))^2d^2} - \frac{\ln(f^{dx}fc)}{(\ln(f))^2d^2} + \frac{\ln(f^{dx}fc+1)}{(\ln(f))^2d^2} - \frac{c\ln(f)}{\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x)

[Out] x/d/(1+f^(d\*x+c))/ln(f)+1/2\*x^2+c/d\*x+1/2\*c^2/d^2-1/ln(f)/d\*ln(f^(d\*x)\*f^c+1)\*x-1/ln(f)^2/d^2\*polylog(2,-f^(d\*x)\*f^c)-1/ln(f)^2/d^2\*ln(f^(d\*x)\*f^c)+1/ln(f)^2/d^2\*ln(f^(d\*x)\*f^c+1)-1/ln(f)/d^2\*c\*ln(f^(d\*x)\*f^c)

**Maxima [A]** time = 1.00549, size = 154, normalized size = 1.6

$$\frac{x}{df^{dx}fc\log(f)+d\log(f)} + \frac{\log(f^{dx})^2}{2d^2\log(f)^2} - \frac{\log(f^{dx}fc+1)\log(f^{dx})+\text{Li}_2(-f^{dx}fc)}{d^2\log(f)^2} + \frac{\log(f^{dx}fc+1)}{d^2\log(f)^2} - \frac{\log(f^{dx})}{d^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="maxima")

[Out] x/(d\*f^(d\*x)\*f^c\*log(f) + d\*log(f)) + 1/2\*log(f^(d\*x))^2/(d^2\*log(f)^2) - (log(f^(d\*x)\*f^c + 1)\*log(f^(d\*x)) + dilog(-f^(d\*x)\*f^c))/(d^2\*log(f)^2) + 1\*log(f^(d\*x)\*f^c + 1)/(d^2\*log(f)^2) - log(f^(d\*x))/(d^2\*log(f)^2)

**Fricas [A]** time = 1.54127, size = 356, normalized size = 3.71

$$\frac{(d^2x^2 - c^2)\log(f)^2 + \left((d^2x^2 - c^2)\log(f)^2 - 2(dx + c)\log(f)\right)f^{dx+c} - 2(f^{dx+c} + 1)\text{Li}_2(-f^{dx+c}) - 2(dx\log(f) + (d^2x^2 - c^2)\log(f)^2)}{2\left(d^2f^{dx+c}\log(f)^2 + d^2\log(f)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="fricas")

[Out] 1/2\*((d^2\*x^2 - c^2)\*log(f)^2 + ((d^2\*x^2 - c^2)\*log(f)^2 - 2\*(d\*x + c)\*log(f))\*f^(d\*x + c) - 2\*(f^(d\*x + c) + 1)\*dilog(-f^(d\*x + c)) - 2\*(d\*x\*log(f) + (d^2\*x^2 - c^2)\*log(f)^2) + (d\*x\*log(f) - 1)\*f^(d\*x + c) - 1\*log(f^(d\*x + c) + 1) - 2\*c\*log(f))/(d^2\*f^(d\*x + c)\*log(f)^2 + d^2\*log(f)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x}{df^{c+dx}\log(f) + d\log(f)} + \frac{\int \frac{dx\log(f)}{e^{c\log(f)}e^{dx\log(f)}+1} dx + \int -\frac{1}{e^{c\log(f)}e^{dx\log(f)}+1} dx}{d\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*f\*\*(d\*x+c)+f\*\*(2\*d\*x+2\*c)),x)

[Out] x/(d\*f\*\*(c + d\*x)\*log(f) + d\*log(f)) + (Integral(d\*x\*log(f)/(exp(c\*log(f))\*exp(d\*x\*log(f)) + 1), x) + Integral(-1/(exp(c\*log(f))\*exp(d\*x\*log(f)) + 1), x))/(d\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{f^{2dx+2c} + 2f^{dx+c} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="giac")

[Out] integrate(x/(f^(2\*d\*x + 2\*c) + 2\*f^(d\*x + c) + 1), x)

$$3.524 \quad \int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}} dx$$

**Optimal.** Leaf size=338

$$-\frac{2c \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^2 \log^2(f) \sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})} + \frac{2c \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}\right)}{d^2 \log^2(f) \sqrt{b^2-4ac} (\sqrt{b^2-4ac}+b)} - \frac{2cx \log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}} + 1\right)}{d \log(f) \sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})} + \dots$$

```
[Out] -((c*x^2)/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2)/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Log[1 + (2*c*f^(c + d*x))/(b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*Log[f]) + (2*c*x*Log[1 + (2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*Log[f]) - (2*c*PolyLog[2, (-2*c*f^(c + d*x))/(b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*Log[f]^2) + (2*c*PolyLog[2, (-2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*Log[f]^2)
```

**Rubi [A]** time = 0.685755, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {2263, 2184, 2190, 2279, 2391}

$$-\frac{2c \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^2 \log^2(f) \sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})} + \frac{2c \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}\right)}{d^2 \log^2(f) \sqrt{b^2-4ac} (\sqrt{b^2-4ac}+b)} - \frac{2cx \log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}} + 1\right)}{d \log(f) \sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)), x]
```

```
[Out] -((c*x^2)/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2)/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Log[1 + (2*c*f^(c + d*x))/(b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*Log[f]) + (2*c*x*Log[1 + (2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*Log[f]) - (2*c*PolyLog[2, (-2*c*f^(c + d*x))/(b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*Log[f]^2) + (2*c*PolyLog[2, (-2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*Log[f]^2)
```

#### Rule 2263

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))]^(n_), x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a + b f^{c+dx} + c f^{2c+2dx}} dx = \frac{(2c) \int \frac{x}{b - \sqrt{b^2 - 4ac + 2c f^{c+dx}}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x}{b + \sqrt{b^2 - 4ac + 2c f^{c+dx}}} dx}{\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{(4c^2) \int \frac{f^{c+dx} x}{b - \sqrt{b^2 - 4ac + 2c f^{c+dx}}} dx}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \frac{(4c^2) \int \frac{f^{c+dx} x}{b + \sqrt{b^2 - 4ac + 2c f^{c+dx}}} dx}{b^2 - 4ac + b\sqrt{b^2 - 4ac}}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log(f)}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log(f)}$$

$$= -\frac{cx^2}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{cx^2}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} + \frac{2cx \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log(f)}$$

**Mathematica [F]** time = 5.06031, size = 0, normalized size = 0.

$$\int \frac{x}{a + b f^{c+dx} + c f^{2c+2dx}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)), x]
```

```
[Out] Integrate[x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)), x]
```

**Maple [B]** time = 0.078, size = 855, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x)

[Out] 
$$\begin{aligned} & -1/2/\ln(f)/d/a*\ln((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & *x-1/2/\ln(f)/d^2/a*\ln((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & *c-1/2/\ln(f)/d/a/(-4*a*c+b^2)^(1/2)*\ln((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & *b*x-1/2/\ln(f)/d^2/a/(-4*a*c+b^2)^(1/2)*\ln((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & *b*c-1/2/\ln(f)/d/a*\ln((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & *x-1/2/\ln(f)/d^2/a*\ln((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & *c+1/2/\ln(f)/d/a/(-4*a*c+b^2)^(1/2)*\ln((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & *b*x+1/2/\ln(f)/d^2/a/(-4*a*c+b^2)^(1/2)*\ln((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & *b*c-1/2/\ln(f)^2/d^2/a*\operatorname{dilog}((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & -1/2/\ln(f)^2/d^2/a/(-4*a*c+b^2)^(1/2)*\operatorname{dilog}((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2))) \\ & *b-1/2/\ln(f)^2/d^2/a*\operatorname{dilog}((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & +1/2/\ln(f)^2/d^2/a/(-4*a*c+b^2)^(1/2)*\operatorname{dilog}((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))) \\ & *b+1/2*x^2/a+1/d/a*x*c+1/2/d^2/a*c^2+1/2/\ln(f)/d^2*c/a*\ln(a+b*f^(d*x)*f^c+c*(f^(d*x))^2*(f^c)^2)+1/\ln(f)/d^2*c/a*b/(4*a*c-b^2)^(1/2) \\ & *arctan((2*c*f^(d*x)*f^c+b)/(4*a*c-b^2)^(1/2))-1/\ln(f)/d^2*c/a*\ln(f^(d*x)*f^c) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.51207, size = 1176, normalized size = 3.48

$$(b^2 - 4ac)d^2x^2 \log(f)^2 - \left( ab\sqrt{\frac{b^2-4ac}{a^2}} + b^2 - 4ac \right) \operatorname{Li}_2 \left( -\frac{\left( a\sqrt{\frac{b^2-4ac}{a^2}} + b \right) f^{dx+c+2a}}{2a} + 1 \right) + \left( ab\sqrt{\frac{b^2-4ac}{a^2}} - b^2 + 4ac \right) \operatorname{Li}_2 \left( \frac{a\sqrt{\frac{b^2-4ac}{a^2}}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/2*((b^2 - 4*a*c)*d^2*x^2*\log(f)^2 - (a*b*\sqrt{(b^2 - 4*a*c)/a^2} + b^2 - 4*a*c) \\ & *dilog(-1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} + b)*f^(d*x + c) + 2*a)/a + 1) + (a*b*\sqrt{(b^2 - 4*a*c)/a^2} - b^2 + 4*a*c) \\ & *dilog(1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} - b)*f^(d*x + c) - 2*a)/a + 1) - (a*b*c*\sqrt{(b^2 - 4*a*c)/a^2} * \\ & \log(f) - (b^2*c - 4*a*c^2)*\log(f))*\log(2*c*f^(d*x + c) + a*\sqrt{(b^2 - 4*a*c)/a^2} + b) + (a*b*c*\sqrt{(b^2 - 4*a*c)/a^2} * \\ & \log(f) + (b^2*c - 4*a*c^2)*\log(f))*\log(2*c*f^(d*x + c) - a*\sqrt{(b^2 - 4*a*c)/a^2} + b) - ((a*b*d*x + a*b*c) \\ & *\sqrt{(b^2 - 4*a*c)/a^2}*\log(f) + (b^2*c - 4*a*c^2 + (b^2 - 4*a*c)*d*x)*\log(f))*\log(1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} + b)*f^(d*x + c) + 2*a)/a) + \\ & ((a*b*d*x + a*b*c)*\sqrt{(b^2 - 4*a*c)/a^2}*\log(f) - (b^2*c - 4*a*c^2 + (b^2 - 4*a*c)*d*x) \end{aligned}$$



$$- 4*a*c)*d*x)*\log(f))*\log(-1/2*((a*\sqrt{(b^2 - 4*a*c)}/a^2) - b)*f^(d*x + c) - 2*a)/a))/((a*b^2 - 4*a^2*c)*d^2*\log(f)^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b f^c f^{dx} + c f^{2c} f^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f\*\*(d\*x+c)+c\*f\*\*(2\*d\*x+2\*c)),x)

[Out] Integral(x/(a + b\*f\*\*c\*f\*\*(d\*x) + c\*f\*\*(2\*c)\*f\*\*(2\*d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{c f^{2 dx+2c} + b f^{dx+c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="giac")

[Out] integrate(x/(c\*f^(2\*d\*x + 2\*c) + b\*f^(d\*x + c) + a), x)

$$3.525 \quad \int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}} dx$$

**Optimal.** Leaf size=145

$$-\frac{2x \operatorname{PolyLog}(2, -f^{c+dx})}{d^2 \log^2(f)} + \frac{2 \operatorname{PolyLog}(2, -f^{c+dx})}{d^3 \log^3(f)} + \frac{2 \operatorname{PolyLog}(3, -f^{c+dx})}{d^3 \log^3(f)} + \frac{2x \log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x^2 \log(f^{c+dx} + 1)}{d \log(f)} +$$

[Out]  $x^3/3 - x^2/(d \operatorname{Log}[f]) + x^2/(d(1 + f^{(c + d*x)}) \operatorname{Log}[f]) + (2*x \operatorname{Log}[1 + f^{(c + d*x)}])/(d^2 \operatorname{Log}[f]^2) - (x^2 \operatorname{Log}[1 + f^{(c + d*x)}])/(d \operatorname{Log}[f]) + (2 \operatorname{PolyLog}[2, -f^{(c + d*x)}])/(d^3 \operatorname{Log}[f]^3) - (2*x \operatorname{PolyLog}[2, -f^{(c + d*x)}])/(d^2 \operatorname{Log}[f]^2) + (2 \operatorname{PolyLog}[3, -f^{(c + d*x)}])/(d^3 \operatorname{Log}[f]^3)$

**Rubi [A]** time = 0.420712, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$ , Rules used = {6688, 2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$-\frac{2x \operatorname{PolyLog}(2, -f^{c+dx})}{d^2 \log^2(f)} + \frac{2 \operatorname{PolyLog}(2, -f^{c+dx})}{d^3 \log^3(f)} + \frac{2 \operatorname{PolyLog}(3, -f^{c+dx})}{d^3 \log^3(f)} + \frac{2x \log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x^2 \log(f^{c+dx} + 1)}{d \log(f)} +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(1 + 2*f^{(c + d*x)} + f^{(2*c + 2*d*x)}), x]$

[Out]  $x^3/3 - x^2/(d \operatorname{Log}[f]) + x^2/(d(1 + f^{(c + d*x)}) \operatorname{Log}[f]) + (2*x \operatorname{Log}[1 + f^{(c + d*x)}])/(d^2 \operatorname{Log}[f]^2) - (x^2 \operatorname{Log}[1 + f^{(c + d*x)}])/(d \operatorname{Log}[f]) + (2 \operatorname{PolyLog}[2, -f^{(c + d*x)}])/(d^3 \operatorname{Log}[f]^3) - (2*x \operatorname{PolyLog}[2, -f^{(c + d*x)}])/(d^2 \operatorname{Log}[f]^2) + (2 \operatorname{PolyLog}[3, -f^{(c + d*x)}])/(d^3 \operatorname{Log}[f]^3)$

### Rule 6688

$\operatorname{Int}[u, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{SimplifyIntegrand}[u, x]\}, \operatorname{Int}[v, x] /; \operatorname{SimplerIntegrandQ}[v, u, x]$

### Rule 2185

$\operatorname{Int}[(a + (b \cdot (F)^{(g \cdot (e + (f \cdot x))))^n)^p \cdot ((c + (d \cdot x))^m), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(c + d*x)^m \cdot (a + b \cdot (F^{(g \cdot (e + f*x))})^n)^{p+1}, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(c + d*x)^m \cdot (F^{(g \cdot (e + f*x))})^n \cdot (a + b \cdot (F^{(g \cdot (e + f*x))})^n)^p, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \&\amp; \operatorname{ILtQ}[p, 0] \&\amp; \operatorname{IGtQ}[m, 0]$

### Rule 2184

$\operatorname{Int}[(c + (d \cdot x))^m / ((a + (b \cdot (F)^{(g \cdot (e + (f \cdot x))))^n)^p), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1} / (a \cdot d \cdot (m+1)), x] - \operatorname{Dist}[b/a, \operatorname{Int}[(c + d*x)^m \cdot (F^{(g \cdot (e + f*x))})^n / (a + b \cdot (F^{(g \cdot (e + f*x))})^n), x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \&\amp; \operatorname{IGtQ}[m, 0]$

### Rule 2190

$\operatorname{Int}[(F)^{(g \cdot (e + (f \cdot x)))^n \cdot ((c + (d \cdot x))^m) / ((a + (b \cdot (F)^{(g \cdot (e + (f \cdot x))))^n)^p), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f*x))})^n) / a] / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), x] - \operatorname{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{m-1} \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f*x))})^n) / a], x]$

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*(a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2191

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((a\_.) + (b\_.)\*(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.))], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}} dx &= \int \frac{x^2}{(1+f^{c+dx})^2} dx \\
&= -\int \frac{f^{c+dx}x^2}{(1+f^{c+dx})^2} dx + \int \frac{x^2}{1+f^{c+dx}} dx \\
&= \frac{x^3}{3} + \frac{x^2}{d(1+f^{c+dx})\log(f)} - \frac{2\int \frac{x}{1+f^{c+dx}} dx}{d\log(f)} - \int \frac{f^{c+dx}x^2}{1+f^{c+dx}} dx \\
&= \frac{x^3}{3} - \frac{x^2}{d\log(f)} + \frac{x^2}{d(1+f^{c+dx})\log(f)} - \frac{x^2\log(1+f^{c+dx})}{d\log(f)} + \frac{2\int \frac{f^{c+dx}x}{1+f^{c+dx}} dx}{d\log(f)} + \frac{2\int x\log(1+f^{c+dx})}{d\log(f)} \\
&= \frac{x^3}{3} - \frac{x^2}{d\log(f)} + \frac{x^2}{d(1+f^{c+dx})\log(f)} + \frac{2x\log(1+f^{c+dx})}{d^2\log^2(f)} - \frac{x^2\log(1+f^{c+dx})}{d\log(f)} - \frac{2x\text{Li}_2(-f^{c+dx})}{d^2\log^2(f)} \\
&= \frac{x^3}{3} - \frac{x^2}{d\log(f)} + \frac{x^2}{d(1+f^{c+dx})\log(f)} + \frac{2x\log(1+f^{c+dx})}{d^2\log^2(f)} - \frac{x^2\log(1+f^{c+dx})}{d\log(f)} - \frac{2x\text{Li}_2(-f^{c+dx})}{d^2\log^2(f)} \\
&= \frac{x^3}{3} - \frac{x^2}{d\log(f)} + \frac{x^2}{d(1+f^{c+dx})\log(f)} + \frac{2x\log(1+f^{c+dx})}{d^2\log^2(f)} - \frac{x^2\log(1+f^{c+dx})}{d\log(f)} + \frac{2\text{Li}_2(-f^{c+dx})}{d^3\log^3(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.230603, size = 123, normalized size = 0.85

$$\frac{6\text{PolyLog}\left(3, -f^{c+dx}\right) + (6 - 6dx \log(f))\text{PolyLog}\left(2, -f^{c+dx}\right) - \frac{3d^2x^2 \log^2(f)(f^{c+dx} + (f^{c+dx} + 1)\log(f^{c+dx} + 1))}{f^{c+dx} + 1} + 6dx \log(f) \log(f)}{3d^3 \log^3(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + 2\*f^(c + d\*x) + f^(2\*c + 2\*d\*x)), x]

[Out] (d^3\*x^3\*Log[f]^3 + 6\*d\*x\*Log[f]\*Log[1 + f^(c + d\*x)] - (3\*d^2\*x^2\*Log[f]^2\*(f^(c + d\*x) + (1 + f^(c + d\*x))\*Log[1 + f^(c + d\*x)]))/(1 + f^(c + d\*x)) + (6 - 6\*d\*x\*Log[f])\*PolyLog[2, -f^(c + d\*x)] + 6\*PolyLog[3, -f^(c + d\*x)]/(3\*d^3\*Log[f]^3)

**Maple [A]** time = 0.061, size = 232, normalized size = 1.6

$$\frac{x^2}{d(1+f^{dx+c})\ln(f)} + \frac{x^3}{3} - \frac{c^2x}{d^2} - \frac{2c^3}{3d^3} - \frac{\ln(f^{dx}f^c + 1)x^2}{d\ln(f)} - 2\frac{\text{polylog}\left(2, -f^{dx}f^c\right)x}{(\ln(f))^2d^2} + 2\frac{\text{polylog}\left(3, -f^{dx}f^c\right)}{(\ln(f))^3d^3} + \frac{c^2\ln(f)}{\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)), x)

[Out] x^2/d/(1+f^(d\*x+c))/ln(f)+1/3\*x^3-c^2/d^2\*x-2/3\*c^3/d^3-1/ln(f)/d\*ln(f^(d\*x)\*f^c+1)\*x^2-2/ln(f)^2/d^2\*polylog(2, -f^(d\*x)\*f^c)\*x+2/ln(f)^3/d^3\*polylog(3, -f^(d\*x)\*f^c)+1/ln(f)/d^3\*c^2\*ln(f^(d\*x)\*f^c)-x^2/d/ln(f)-2/ln(f)/d^2\*c\*x-1/ln(f)/d^3\*c^2+2/ln(f)^2/d^2\*ln(f^(d\*x)\*f^c+1)\*x+2/ln(f)^3/d^3\*polylog(2, -f^(d\*x)\*f^c)+2/ln(f)^2/d^3\*c\*ln(f^(d\*x)\*f^c)

**Maxima [A]** time = 1.01225, size = 211, normalized size = 1.46

$$\frac{x^2}{d f^{dx} f^c \log(f) + d \log(f)} - \frac{\log(f^{dx} f^c + 1) \log(f^{dx})^2 + 2 \operatorname{Li}_2(-f^{dx} f^c) \log(f^{dx}) - 2 \operatorname{Li}_3(-f^{dx} f^c)}{d^3 \log(f)^3} + \frac{\log(f^{dx})^3 - 3 \log(f^{dx}) \log(f)}{3 d^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="maxima")

[Out] x^2/(d\*f^(d\*x)\*f^c\*log(f) + d\*log(f)) - (log(f^(d\*x)\*f^c + 1)\*log(f^(d\*x)))^2 + 2\*dilog(-f^(d\*x)\*f^c)\*log(f^(d\*x)) - 2\*polylog(3, -f^(d\*x)\*f^c))/(d^3\*log(f)^3) + 1/3\*(log(f^(d\*x)))^3 - 3\*log(f^(d\*x))^2)/(d^3\*log(f)^3) + 2\*(log(f^(d\*x)\*f^c + 1)\*log(f^(d\*x)) + dilog(-f^(d\*x)\*f^c))/(d^3\*log(f)^3)

**Fricas [C]** time = 1.36791, size = 522, normalized size = 3.6

$$\frac{3 c^2 \log(f)^2 + (d^3 x^3 + c^3) \log(f)^3 + ((d^3 x^3 + c^3) \log(f)^3 - 3(d^2 x^2 - c^2) \log(f)^2) f^{dx+c} - 6(dx \log(f) + (dx \log(f))^2)}{d^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2\*f^(d\*x+c)+f^(2\*d\*x+2\*c)),x, algorithm="fricas")

[Out] 1/3\*(3\*c^2\*log(f)^2 + (d^3\*x^3 + c^3)\*log(f)^3 + ((d^3\*x^3 + c^3)\*log(f)^3 - 3\*(d^2\*x^2 - c^2)\*log(f)^2)\*f^(d\*x + c) - 6\*(d\*x\*log(f) + (d\*x\*log(f) - 1)\*f^(d\*x + c) - 1)\*dilog(-f^(d\*x + c)) - 3\*(d^2\*x^2\*log(f)^2 - 2\*d\*x\*log(f) + (d^2\*x^2\*log(f)^2 - 2\*d\*x\*log(f))\*f^(d\*x + c))\*log(f^(d\*x + c) + 1) + 6\*(f^(d\*x + c) + 1)\*polylog(3, -f^(d\*x + c)))/(d^3\*f^(d\*x + c)\*log(f)^3 + d^3\*log(f)^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{d f^{c+dx} \log(f) + d \log(f)} + \frac{\int -\frac{2x}{e^{c \log(f)} e^{dx \log(f)} + 1} dx + \int \frac{dx^2 \log(f)}{e^{c \log(f)} e^{dx \log(f)} + 1} dx}{d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(1+2\*f\*\*(d\*x+c)+f\*\*(2\*d\*x+2\*c)),x)

[Out] x\*\*2/(d\*f\*\*(c + d\*x)\*log(f) + d\*log(f)) + (Integral(-2\*x/(exp(c\*log(f))\*exp(d\*x\*log(f)) + 1), x) + Integral(d\*x\*\*2\*log(f)/(exp(c\*log(f))\*exp(d\*x\*log(f)) + 1), x))/(d\*log(f))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{f^{2dx+2c} + 2 f^{dx+c} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(f^(2*d*x + 2*c) + 2*f^(d*x + c) + 1), x)
```

$$3.526 \quad \int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}} dx$$

**Optimal.** Leaf size=484

$$\frac{4cx \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^2 \log^2(f)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} + \frac{4cx \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}\right)}{d^2 \log^2(f)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)} + \frac{4c \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^3 \log^3(f)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

[Out]  $(-2cx^3)/(3(b^2 - 4ac - b\sqrt{b^2 - 4ac})) - (2cx^3)/(3(b^2 - 4ac + b\sqrt{b^2 - 4ac})) - (2cx^2 \operatorname{Log}[1 + (2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d \operatorname{Log}[f]) + (2cx^2 \operatorname{Log}[1 + (2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d \operatorname{Log}[f]) - (4cx \operatorname{PolyLog}[2, (-2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d^2 \operatorname{Log}[f]^2) + (4cx \operatorname{PolyLog}[2, (-2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d^2 \operatorname{Log}[f]^2) + (4c \operatorname{PolyLog}[3, (-2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d^3 \operatorname{Log}[f]^3) - (4c \operatorname{PolyLog}[3, (-2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d^3 \operatorname{Log}[f]^3)$

**Rubi [A]** time = 0.87472, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {2263, 2184, 2190, 2531, 2282, 6589}

$$\frac{4cx \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^2 \log^2(f)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} + \frac{4cx \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}\right)}{d^2 \log^2(f)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)} + \frac{4c \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^3 \log^3(f)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(a + bf^{c+dx} + cf^{2c+2dx}), x]$

[Out]  $(-2cx^3)/(3(b^2 - 4ac - b\sqrt{b^2 - 4ac})) - (2cx^3)/(3(b^2 - 4ac + b\sqrt{b^2 - 4ac})) - (2cx^2 \operatorname{Log}[1 + (2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d \operatorname{Log}[f]) + (2cx^2 \operatorname{Log}[1 + (2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d \operatorname{Log}[f]) - (4cx \operatorname{PolyLog}[2, (-2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d^2 \operatorname{Log}[f]^2) + (4cx \operatorname{PolyLog}[2, (-2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d^2 \operatorname{Log}[f]^2) + (4c \operatorname{PolyLog}[3, (-2cf^{c+dx})/(b - \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})d^3 \operatorname{Log}[f]^3) - (4c \operatorname{PolyLog}[3, (-2cf^{c+dx})/(b + \sqrt{b^2 - 4ac})]) / (\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})d^3 \operatorname{Log}[f]^3)$

#### Rule 2263

$\operatorname{Int}(((f_{.}) + (g_{.})*(x_{.}))^{(m_{.})}/((a_{.}) + (b_{.})*(F_{.})^{(u_{.})} + (c_{.})*(F_{.})^{(v_{.})}), x\_Symbol) \rightarrow \operatorname{With}[\{q = \operatorname{Rt}[b^2 - 4ac, 2]\}, \operatorname{Dist}[(2c)/q, \operatorname{Int}[(f + gx)^m/(b - q + 2cF^u), x], x] - \operatorname{Dist}[(2c)/q, \operatorname{Int}[(f + gx)^m/(b + q + 2cF^u), x], x] /; \operatorname{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \operatorname{EqQ}[v, 2u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2184

$\operatorname{Int}(((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}/((a_{.}) + (b_{.})*((F_{.})^{(g_{.})}*((e_{.}) + (f_{.})*(x_{.}))))^{(n_{.})}, x\_Symbol) \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)}/(a*d*(m+1)), x] - \operatorname{Dist}[\dots]$

b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^((n\_)\*((c\_) + (d\_)\*(x\_))^(m\_)))/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^((n\_))), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^((n\_)))\*((f\_) + (g\_)\*(x\_))^(m\_)], x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{a + b f^{c+dx} + c f^{2c+2dx}} dx &= \frac{(2c) \int \frac{x^2}{b - \sqrt{b^2 - 4ac} + 2c f^{c+dx}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x^2}{b + \sqrt{b^2 - 4ac} + 2c f^{c+dx}} dx}{\sqrt{b^2 - 4ac}} \\ &= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{(4c^2) \int \frac{f^{c+dx} x^2}{b - \sqrt{b^2 - 4ac} + 2c f^{c+dx}} dx}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} + \\ &= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log} \\ &= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log} \\ &= -\frac{2cx^3}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} + \frac{2cx^2 \log\left(1 + \frac{2c f^{c+dx}}{b - \sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac - b\sqrt{b^2 - 4ac}) d \log} \end{aligned}$$



**Mathematica [F]** time = 2.77562, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b f^{c+dx} + c f^{2c+2dx}} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/(a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x)),x]

[Out] Integrate[x^2/(a + b\*f^(c + d\*x) + c\*f^(2\*c + 2\*d\*x)), x]

**Maple [F]** time = 0.126, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b f^{dx+c} + c f^{2dx+2c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x)

[Out] int(x^2/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.4928, size = 1623, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/6*(2*(b^2 - 4*a*c)*d^3*x^3*\log(f)^3 - 6*(a*b*d*x*\sqrt{(b^2 - 4*a*c)/a^2})* \\ & \log(f) + (b^2 - 4*a*c)*d*x*\log(f))*\operatorname{dilog}(-1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} + \\ & b)*f^{(d*x + c) + 2*a}/a + 1) + 6*(a*b*d*x*\sqrt{(b^2 - 4*a*c)/a^2})*\log(f) - \\ & (b^2 - 4*a*c)*d*x*\log(f))*\operatorname{dilog}(1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} - b)*f^{(d* \\ & x + c) - 2*a}/a + 1) + 3*(a*b*c^2*\sqrt{(b^2 - 4*a*c)/a^2})*\log(f)^2 - (b^2*c \\ & ^2 - 4*a*c^3)*\log(f)^2)*\log(2*c*f^{(d*x + c) + a*\sqrt{(b^2 - 4*a*c)/a^2} + b \\ & ) - 3*(a*b*c^2*\sqrt{(b^2 - 4*a*c)/a^2})*\log(f)^2 + (b^2*c^2 - 4*a*c^3)*\log(f \\ & )^2)*\log(2*c*f^{(d*x + c) - a*\sqrt{(b^2 - 4*a*c)/a^2} + b) - 3*((a*b*d^2*x^2 \\ & - a*b*c^2)*\sqrt{(b^2 - 4*a*c)/a^2})*\log(f)^2 + ((b^2 - 4*a*c)*d^2*x^2 - b^2 \\ & *c^2 + 4*a*c^3)*\log(f)^2)*\log(1/2*((a*\sqrt{(b^2 - 4*a*c)/a^2} + b)*f^{(d*x + \\ & c) + 2*a}/a) + 3*((a*b*d^2*x^2 - a*b*c^2)*\sqrt{(b^2 - 4*a*c)/a^2})*\log(f)^2 \end{aligned}$$

-  $((b^2 - 4ac)d^2x^2 - b^2c^2 + 4ac^3)\log(f)^2 \cdot \log(-1/2((a\sqrt{(b^2 - 4ac)/a^2} - b)f^{(dx+c) - 2a/a} + 6(ab\sqrt{(b^2 - 4ac)/a^2} + b^2 - 4ac)\text{polylog}(3, -1/2(a\sqrt{(b^2 - 4ac)/a^2} + b)f^{(dx+c)/a} - 6(ab\sqrt{(b^2 - 4ac)/a^2} - b^2 + 4ac)\text{polylog}(3, 1/2(a\sqrt{(b^2 - 4ac)/a^2} - b)f^{(dx+c)/a}))/((ab^2 - 4a^2c)d^3\log(f)^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + bf^c f^{dx} + cf^{2c} f^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*f\*\*(d\*x+c)+c\*f\*\*(2\*d\*x+2\*c)),x)

[Out] Integral(x\*\*2/(a + b\*f\*\*c\*f\*\*(d\*x) + c\*f\*\*(2\*c)\*f\*\*(2\*d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cf^{2dx+2c} + bf^{dx+c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*f^(d\*x+c)+c\*f^(2\*d\*x+2\*c)),x, algorithm="giac")

[Out] integrate(x^2/(c\*f^(2\*d\*x + 2\*c) + b\*f^(d\*x + c) + a), x)

$$3.527 \quad \int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}} dx$$

**Optimal.** Leaf size=103

$$\frac{(bd-2ae) \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{d \log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{dx}{a}$$

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - (d\*Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)])/(2\*a\*h\*Log[f])

**Rubi [A]** time = 0.156565, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$ , Rules used = {2282, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae) \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{d \log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{dx}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*f^(g + h\*x))/(a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)),x]

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - (d\*Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)])/(2\*a\*h\*Log[f])

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 800

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + e f^{g+hx}}{a + b f^{g+hx} + c f^{2g+2hx}} dx &= \frac{\text{Subst}\left(\int \frac{d+ex}{x(a+bx+cx^2)} dx, x, f^{g+hx}\right)}{h \log(f)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{d}{ax} + \frac{-bd+ae-cdx}{a(a+bx+cx^2)}\right) dx, x, f^{g+hx}\right)}{h \log(f)} \\ &= \frac{dx}{a} + \frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{ah \log(f)} \\ &= \frac{dx}{a} - \frac{d \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{2ah \log(f)} - \frac{(bd - 2ae) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{2ah \log(f)} \\ &= \frac{dx}{a} - \frac{d \log(a + b f^{g+hx} + c f^{2g+2hx})}{2ah \log(f)} + \frac{(bd - 2ae) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2c f^{g+hx}\right)}{ah \log(f)} \\ &= \frac{dx}{a} + \frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2c f^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2 - 4ac} h \log(f)} - \frac{d \log(a + b f^{g+hx} + c f^{2g+2hx})}{2ah \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.157817, size = 102, normalized size = 0.99

$$-\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{4ac-b^2}}\right)}{h\log(f)\sqrt{4ac-b^2}} + \frac{d\log(a+f^{g+hx}(b+cf^{g+hx}))}{h\log(f)} - 2dx$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)), x]
```

```
[Out] -(-2*d*x + (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*f^(g + h*x))/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*h*Log[f]) + (d*Log[a + f^(g + h*x)*(b + c*f^(g + h*x))])/(h*Log[f])/(2*a)
```

**Maple [B]** time = 0.134, size = 993, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x)

[Out] 
$$\frac{4/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*a*c*d*h^2*x-1}/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*b^2*d*h^2*x+4}/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*b^2*d*g*h-2}/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*b^2*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}-2/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*b^2*d-1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.33769, size = 755, normalized size = 7.33

$$\frac{2(b^2 - 4ac)dhx \log(f) - (b^2 - 4ac)d \log(cf^{2hx+2g} + bf^{hx+g} + a) - \sqrt{b^2 - 4ac}(bd - 2ae) \log\left(\frac{2c^2f^{2hx+2g} + b^2 - 2ac + 2(bc + af^{hx+g})}{cf^{2hx+2g} + bf^{hx+g} + a}\right)}{2(ab^2 - 4a^2c)h \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="fricas")

[Out] 
$$\left[ \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * h * x * \log(f) - (b^2 - 4 * a * c) * d * \log(c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a) - \sqrt{b^2 - 4 * a * c} * (b * d - 2 * a * e) * \log((2 * c^2 * f^{(2 * h * x + 2 * g)} + b^2 - 2 * a * c + 2 * (b * c - \sqrt{b^2 - 4 * a * c}) * c) * f^{(h * x + g)} - \sqrt{b^2 - 4 * a * c} * b) / (c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a))) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)), \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * h * x * \log(f) - (b^2 - 4 * a * c) * d * \log(c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a) + 2 * \sqrt{-b^2 + 4 * a * c} * (b * d - 2 * a * e) * \arctan(- (2 * \sqrt{-b^2 + 4 * a * c}) * c * f^{(h * x + g)} + \sqrt{-b^2 + 4 * a * c} * b) / (b^2 - 4 * a * c))) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)) \right]$$

---

**Sympy [A]** time = 0.878698, size = 139, normalized size = 1.35

$$\text{RootSum}\left(z^2\left(4a^2ch^2\log(f)^2 - ab^2h^2\log(f)^2\right) + z\left(4acdh\log(f) - b^2dh\log(f)\right) + ae^2 - bde + cd^2, \left(i \mapsto i\log\left(f^{g+hx} - \dots\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f\*\*(h\*x+g))/(a+b\*f\*\*(h\*x+g)+c\*f\*\*(2\*h\*x+2\*g)),x)

[Out] RootSum(\_z\*\*2\*(4\*a\*\*2\*c\*h\*\*2\*log(f)\*\*2 - a\*b\*\*2\*h\*\*2\*log(f)\*\*2) + \_z\*(4\*a\*c\*d\*h\*log(f) - b\*\*2\*d\*h\*log(f)) + a\*e\*\*2 - b\*d\*e + c\*d\*\*2, Lambda(\_i, \_i\*log(f\*\*(g + h\*x) + (4\*\_i\*a\*\*2\*c\*h\*log(f) - \_i\*a\*b\*\*2\*h\*log(f) + a\*b\*e + 2\*a\*c\*d - b\*\*2\*d)/(2\*a\*c\*e - b\*c\*d)))) + d\*x/a

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{fx+g} + d}{cf^{2hx+2g} + bf^{hx+g} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="giac")

[Out] integrate((e\*f^(h\*x + g) + d)/(c\*f^(2\*h\*x + 2\*g) + b\*f^(h\*x + g) + a), x)

$$3.528 \quad \int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}} dx$$

**Optimal.** Leaf size=103

$$\frac{(bd-2ae) \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{d \log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{dx}{a}$$

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - (d\*Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)])/(2\*a\*h\*Log[f])

**Rubi [A]** time = 0.141365, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae) \tanh^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{ah \log(f) \sqrt{b^2-4ac}} - \frac{d \log(a+bf^{g+hx}+cf^{2g+2hx})}{2ah \log(f)} + \frac{dx}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*f^(g + h\*x))/(a + b\*f^(g + h\*x) + c\*f^(2\*(g + h\*x))),x]

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*f^(g + h\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*h\*Log[f]) - (d\*Log[a + b\*f^(g + h\*x) + c\*f^(2\*g + 2\*h\*x)])/(2\*a\*h\*Log[f])

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 800

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + e f^{g+hx}}{a + b f^{g+hx} + c f^{2(g+hx)}} dx &= \frac{\text{Subst}\left(\int \frac{d+ex}{x(a+bx+cx^2)} dx, x, f^{g+hx}\right)}{h \log(f)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{d}{ax} + \frac{-bd+ae-cdx}{a(a+bx+cx^2)}\right) dx, x, f^{g+hx}\right)}{h \log(f)} \\ &= \frac{dx}{a} + \frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{ah \log(f)} \\ &= \frac{dx}{a} - \frac{d \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{2ah \log(f)} - \frac{(bd - 2ae) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, f^{g+hx}\right)}{2ah \log(f)} \\ &= \frac{dx}{a} - \frac{d \log(a + b f^{g+hx} + c f^{2g+2hx})}{2ah \log(f)} + \frac{(bd - 2ae) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2c f^{g+hx}\right)}{ah \log(f)} \\ &= \frac{dx}{a} + \frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2c f^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac} h \log(f)} - \frac{d \log(a + b f^{g+hx} + c f^{2g+2hx})}{2ah \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0330984, size = 102, normalized size = 0.99

$$-\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cf^{g+hx}}{\sqrt{4ac-b^2}}\right)}{h\log(f)\sqrt{4ac-b^2}} + \frac{d\log(a+f^{g+hx}(b+cf^{g+hx}))}{h\log(f)} - 2dx$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*(g + h*x))),x]
```

```
[Out] -(-2*d*x + (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*f^(g + h*x))/Sqrt[-b^2 + 4*a*c]
])/ (Sqrt[-b^2 + 4*a*c]*h*Log[f]) + (d*Log[a + f^(g + h*x)*(b + c*f^(g + h*x
))])/ (h*Log[f]))/(2*a)
```

**Maple [B]** time = 0.002, size = 993, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] int((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x)

[Out] 
$$\frac{4/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*a*c*d*h^2*x-1}/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*b^2*d*h^2*x+4}/(4*\ln(f)^{2*a^2*c*h^2-\ln(f)^{2*a*b^2*h^2}}*\ln(f)^{2*b^2*d*g*h-2}/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*b^2*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}-2/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*b^2*d-1/2/a/(4*a*c-b^2)/h/\ln(f)*\ln(f^{(h*x+g)}-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^{1/2}}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.42942, size = 755, normalized size = 7.33

$$\frac{2(b^2 - 4ac)dhx \log(f) - (b^2 - 4ac)d \log(cf^{2hx+2g} + bf^{hx+g} + a) - \sqrt{b^2 - 4ac}(bd - 2ae) \log\left(\frac{2c^2f^{2hx+2g} + b^2 - 2ac + 2(bc + af^{hx+g})}{cf^{2hx+2g} + bf^{hx+g} + a}\right)}{2(ab^2 - 4a^2c)h \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="fricas")

[Out] 
$$\left[ \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * h * x * \log(f) - (b^2 - 4 * a * c) * d * \log(c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a) - \sqrt{b^2 - 4 * a * c} * (b * d - 2 * a * e) * \log((2 * c^2 * f^{(2 * h * x + 2 * g)} + b^2 - 2 * a * c + 2 * (b * c - \sqrt{b^2 - 4 * a * c}) * c) * f^{(h * x + g)} - \sqrt{b^2 - 4 * a * c} * b) / (c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a))) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)), \frac{1}{2} * (2 * (b^2 - 4 * a * c) * d * h * x * \log(f) - (b^2 - 4 * a * c) * d * \log(c * f^{(2 * h * x + 2 * g)} + b * f^{(h * x + g)} + a) + 2 * \sqrt{-b^2 + 4 * a * c} * (b * d - 2 * a * e) * \arctan(- (2 * \sqrt{-b^2 + 4 * a * c}) * c * f^{(h * x + g)} + \sqrt{-b^2 + 4 * a * c} * b) / (b^2 - 4 * a * c))) / ((a * b^2 - 4 * a^2 * c) * h * \log(f)) \right]$$

---

**Sympy [A]** time = 0.893791, size = 139, normalized size = 1.35

$$\text{RootSum}\left(z^2\left(4a^2ch^2\log(f)^2 - ab^2h^2\log(f)^2\right) + z\left(4acdh\log(f) - b^2dh\log(f)\right) + ae^2 - bde + cd^2, \left(i \mapsto i\log\left(f^{g+hx}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f\*\*(h\*x+g))/(a+b\*f\*\*(h\*x+g)+c\*f\*\*(2\*h\*x+2\*g)),x)

[Out] RootSum(\_z\*\*2\*(4\*a\*\*2\*c\*h\*\*2\*log(f)\*\*2 - a\*b\*\*2\*h\*\*2\*log(f)\*\*2) + \_z\*(4\*a\*c\*d\*h\*log(f) - b\*\*2\*d\*h\*log(f)) + a\*e\*\*2 - b\*d\*e + c\*d\*\*2, Lambda(\_i, \_i\*log(f\*\*(g + h\*x) + (4\*\_i\*a\*\*2\*c\*h\*log(f) - \_i\*a\*b\*\*2\*h\*log(f) + a\*b\*e + 2\*a\*c\*d - b\*\*2\*d)/(2\*a\*c\*e - b\*c\*d)))) + d\*x/a

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{fx+g} + d}{cf^{2hx+2g} + bf^{hx+g} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*f^(h\*x+g))/(a+b\*f^(h\*x+g)+c\*f^(2\*h\*x+2\*g)),x, algorithm="giac")

[Out] integrate((e\*f^(h\*x + g) + d)/(c\*f^(2\*h\*x + 2\*g) + b\*f^(h\*x + g) + a), x)

$$3.529 \quad \int \frac{1}{2+e^{-x}+e^x} dx$$

**Optimal.** Leaf size=9

$$-\frac{1}{e^x+1}$$

[Out]  $-(1 + E^x)^{-1}$

**Rubi [A]** time = 0.0108058, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 32}

$$-\frac{1}{e^x+1}$$

Antiderivative was successfully verified.

[In] Int[(2 + E^(-x) + E^x)^(-1), x]

[Out]  $-(1 + E^x)^{-1}$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{2+e^{-x}+e^x} dx &= \text{Subst} \left( \int \frac{1}{(1+x)^2} dx, x, e^x \right) \\ &= -\frac{1}{1+e^x} \end{aligned}$$

**Mathematica [A]** time = 0.0077872, size = 9, normalized size = 1.

$$-\frac{1}{e^x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + E^(-x) + E^x)^(-1), x]

[Out]  $-(1 + E^x)^{-1}$

**Maple [A]** time = 0.004, size = 9, normalized size = 1.

$$-(1 + e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2+exp(-x)+exp(x)),x)`

[Out] `-1/(1+exp(x))`

---

**Maxima [A]** time = 0.960909, size = 11, normalized size = 1.22

$$\frac{1}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+exp(-x)+exp(x)),x, algorithm="maxima")`

[Out] `1/(e^(-x) + 1)`

---

**Fricas [A]** time = 1.24593, size = 19, normalized size = 2.11

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+exp(-x)+exp(x)),x, algorithm="fricas")`

[Out] `-1/(e^x + 1)`

---

**Sympy [A]** time = 0.071179, size = 7, normalized size = 0.78

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+exp(-x)+exp(x)),x)`

[Out] `-1/(exp(x) + 1)`

---

**Giac [A]** time = 1.22883, size = 11, normalized size = 1.22

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+exp(-x)+exp(x)),x, algorithm="giac")`

[Out] `-1/(e^x + 1)`

$$3.530 \quad \int \frac{x}{2+e^{-x}+e^x} dx$$

**Optimal.** Leaf size=20

$$-\frac{x}{e^x+1} + x - \log(e^x+1)$$

[Out] x - x/(1 + E^x) - Log[1 + E^x]

**Rubi [A]** time = 0.13448, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {2267, 6688, 2191, 2282, 36, 29, 31}

$$-\frac{x}{e^x+1} + x - \log(e^x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(2 + E^(-x) + E^x),x]

[Out] x - x/(1 + E^x) - Log[1 + E^x]

#### Rule 2267

Int[(u\_)/((a\_) + (b\_)\*(F\_)^(v\_) + (c\_)\*(F\_)^(w\_)), x\_Symbol] := Int[(u\*F^v)/(c + a\*F^v + b\*F^(2\*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && LinearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1], 0], LtQ[LeafCount[v], LeafCount[w]]]

#### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

**Rule 29**

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

**Rule 31**

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

**Rubi steps**

$$\begin{aligned}
 \int \frac{x}{2 + e^{-x} + e^x} dx &= \int \frac{e^x x}{1 + 2e^x + e^{2x}} dx \\
 &= \int \frac{e^x x}{(1 + e^x)^2} dx \\
 &= -\frac{x}{1 + e^x} + \int \frac{1}{1 + e^x} dx \\
 &= -\frac{x}{1 + e^x} + \text{Subst}\left(\int \frac{1}{x(1 + x)} dx, x, e^x\right) \\
 &= -\frac{x}{1 + e^x} + \text{Subst}\left(\int \frac{1}{x} dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1 + x} dx, x, e^x\right) \\
 &= x - \frac{x}{1 + e^x} - \log(1 + e^x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0251414, size = 20, normalized size = 1.

$$-\frac{x}{e^x + 1} + x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + E^(-x) + E^x), x]
```

```
[Out] x - x/(1 + E^x) - Log[1 + E^x]
```

**Maple [A]** time = 0.009, size = 19, normalized size = 1.

$$-\ln(1 + e^x) + \frac{e^x x}{1 + e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+exp(-x)+exp(x)), x)
```

```
[Out] -ln(1+exp(x))+x*exp(x)/(1+exp(x))
```

**Maxima [A]** time = 0.987925, size = 24, normalized size = 1.2

$$\frac{x e^x}{e^x + 1} - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+exp(-x)+exp(x)),x, algorithm="maxima")

[Out]  $x e^x / (e^x + 1) - \log(e^x + 1)$

**Fricas [A]** time = 1.383, size = 59, normalized size = 2.95

$$\frac{x e^x - (e^x + 1) \log(e^x + 1)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+exp(-x)+exp(x)),x, algorithm="fricas")

[Out]  $(x e^x - (e^x + 1) \log(e^x + 1)) / (e^x + 1)$

**Sympy [A]** time = 0.088783, size = 14, normalized size = 0.7

$$x - \frac{x}{e^x + 1} - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+exp(-x)+exp(x)),x)

[Out]  $x - x / (\exp(x) + 1) - \log(\exp(x) + 1)$

**Giac [A]** time = 1.42235, size = 38, normalized size = 1.9

$$\frac{x e^x - e^x \log(e^x + 1) - \log(e^x + 1)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+exp(-x)+exp(x)),x, algorithm="giac")

[Out]  $(x e^x - e^x \log(e^x + 1) - \log(e^x + 1)) / (e^x + 1)$

$$3.531 \quad \int \frac{x^2}{2+e^{-x}+e^x} dx$$

**Optimal.** Leaf size=34

$$-2\text{PolyLog}(2, -e^x) - \frac{x^2}{e^x + 1} + x^2 - 2x \log(e^x + 1)$$

[Out]  $x^2 - x^2/(1 + E^x) - 2*x*\text{Log}[1 + E^x] - 2*\text{PolyLog}[2, -E^x]$

**Rubi [A]** time = 0.249505, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {2267, 6688, 2191, 2184, 2190, 2279, 2391}

$$-2\text{PolyLog}(2, -e^x) - \frac{x^2}{e^x + 1} + x^2 - 2x \log(e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(2 + E^{-x} + E^x), x]$

[Out]  $x^2 - x^2/(1 + E^x) - 2*x*\text{Log}[1 + E^x] - 2*\text{PolyLog}[2, -E^x]$

#### Rule 2267

$\text{Int}[(u_)/((a_.) + (b_.)*(F_)^{(v_)} + (c_.)*(F_)^{(w_)}), x\_Symbol] \rightarrow \text{Int}[(u*F^v)/(c + a*F^v + b*F^{(2*v)}), x] /;$  FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && LinearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1], 0], LtQ[LeafCount[v], LeafCount[w]]]

#### Rule 6688

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}\{v = \text{SimplifyIntegrand}[u, x]\}, \text{Int}[v, x] /;$  SimplifierIntegrandQ[v, u, x]

#### Rule 2191

$\text{Int}[(F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}*((a_.) + (b_.)*(F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^{(p + 1)}/(b*f*g*n*(p + 1)*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*(p + 1)*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*(a + b*(F^(g*(e + f*x)))^n)^{(p + 1)}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2184

$\text{Int}[(c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*(F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n), x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[(F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*(F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)], x], x]$



))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
 :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
 )^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2,  
 -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{2 + e^{-x} + e^x} dx &= \int \frac{e^x x^2}{1 + 2e^x + e^{2x}} dx \\
 &= \int \frac{e^x x^2}{(1 + e^x)^2} dx \\
 &= -\frac{x^2}{1 + e^x} + 2 \int \frac{x}{1 + e^x} dx \\
 &= x^2 - \frac{x^2}{1 + e^x} - 2 \int \frac{e^x x}{1 + e^x} dx \\
 &= x^2 - \frac{x^2}{1 + e^x} - 2x \log(1 + e^x) + 2 \int \log(1 + e^x) dx \\
 &= x^2 - \frac{x^2}{1 + e^x} - 2x \log(1 + e^x) + 2 \operatorname{Subst}\left(\int \frac{\log(1 + x)}{x} dx, x, e^x\right) \\
 &= x^2 - \frac{x^2}{1 + e^x} - 2x \log(1 + e^x) - 2\operatorname{Li}_2(-e^x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0469111, size = 33, normalized size = 0.97

$$x \left( \frac{e^x x}{e^x + 1} - 2 \log(e^x + 1) \right) - 2 \operatorname{PolyLog}(2, -e^x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 + E^(-x) + E^x), x]

[Out] x\*(E^x\*x)/(1 + E^x) - 2\*Log[1 + E^x] - 2\*PolyLog[2, -E^x]

**Maple [A]** time = 0.02, size = 32, normalized size = 0.9

$$x^2 - \frac{x^2}{1 + e^x} - 2x \ln(1 + e^x) - 2 \operatorname{polylog}(2, -e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+exp(-x)+exp(x)), x)

[Out] x^2-x^2/(1+exp(x))-2\*x\*ln(1+exp(x))-2\*polylog(2,-exp(x))

**Maxima [A]** time = 0.980707, size = 41, normalized size = 1.21

$$x^2 - 2x \log(e^x + 1) - \frac{x^2}{e^x + 1} - 2 \operatorname{Li}_2(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="maxima")

[Out] x^2 - 2\*x\*log(e^x + 1) - x^2/(e^x + 1) - 2\*dilog(-e^x)

---

**Fricas [A]** time = 1.31795, size = 103, normalized size = 3.03

$$\frac{x^2 e^x - 2(e^x + 1) \operatorname{Li}_2(-e^x) - 2(xe^x + x) \log(e^x + 1)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="fricas")

[Out] (x^2\*e^x - 2\*(e^x + 1)\*dilog(-e^x) - 2\*(x\*e^x + x)\*log(e^x + 1))/(e^x + 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^2}{e^x + 1} + 2 \int \frac{x}{e^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(2+exp(-x)+exp(x)),x)

[Out] -x\*\*2/(exp(x) + 1) + 2\*Integral(x/(exp(x) + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{e^{(-x)} + e^x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="giac")

[Out] integrate(x^2/(e^(-x) + e^x + 2), x)

$$3.532 \quad \int \frac{1}{2+f^{-c-dx}+f^{c+dx}} dx$$

**Optimal.** Leaf size=20

$$-\frac{1}{d \log(f) (f^{c+dx} + 1)}$$

[Out]  $-(1/(d*(1 + f^{(c + d*x)})*Log[f]))$

**Rubi [A]** time = 0.0203003, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2282, 32}

$$-\frac{1}{d \log(f) (f^{c+dx} + 1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(2 + f^{(-c - d*x)} + f^{(c + d*x)})^{-1}, x]$

[Out]  $-(1/(d*(1 + f^{(c + d*x)})*Log[f]))$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \text{FreeQ}[\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n]] \&\& \text{!MatchQ}[u, E^{((c_)*((a_)+(b_)*x))} (F_)[v_] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

#### Rule 32

$\text{Int}[(a_)+(b_)*(x_)^{(m_)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \text{FreeQ}[\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{2+f^{-c-dx}+f^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)^2} dx, x, f^{c+dx}\right)}{d \log(f)} \\ &= -\frac{1}{d(1+f^{c+dx}) \log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.0172876, size = 20, normalized size = 1.

$$-\frac{1}{d \log(f) (f^{c+dx} + 1)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(2 + f^{(-c - d*x)} + f^{(c + d*x)})^{-1}, x]$

[Out]  $-(1/(d*(1 + f^{(c + d*x)})*\text{Log}[f]))$

**Maple [A]** time = 0.011, size = 25, normalized size = 1.3

$$\frac{1}{d \ln(f) \left( e^{(-dx-c) \ln(f)} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2+f^(-d*x-c)+f^(d*x+c)),x)`

[Out]  $1/d/\ln(f)/(\exp((-d*x-c)*\ln(f))+1)$

**Maxima [A]** time = 0.953765, size = 30, normalized size = 1.5

$$\frac{1}{d(f^{-dx-c} + 1) \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="maxima")`

[Out]  $1/(d*(f^{-d*x - c} + 1)*\log(f))$

**Fricas [A]** time = 1.25229, size = 51, normalized size = 2.55

$$-\frac{1}{df^{dx+c} \log(f) + d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/(d*f^{(d*x + c)}*\log(f) + d*\log(f))$

**Sympy [A]** time = 0.102194, size = 19, normalized size = 0.95

$$-\frac{1}{df^{c+dx} \log(f) + d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+f**(-d*x-c)+f**(d*x+c)),x)`

[Out]  $-1/(d*f^{(c + d*x)}*\log(f) + d*\log(f))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{f^{dx+c} + f^{-dx-c} + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(f^(d*x + c) + f^(-d*x - c) + 2), x)
```

$$3.533 \quad \int \frac{x}{2+f^{-c-dx}+f^{c+dx}} dx$$

**Optimal.** Leaf size=50

$$-\frac{\log(f^{c+dx}+1)}{d^2 \log^2(f)} - \frac{x}{d \log(f)(f^{c+dx}+1)} + \frac{x}{d \log(f)}$$

[Out] x/(d\*Log[f]) - x/(d\*(1 + f^(c + d\*x))\*Log[f]) - Log[1 + f^(c + d\*x)]/(d^2\*Log[f]^2)

**Rubi [A]** time = 0.288148, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {2267, 6688, 2191, 2282, 36, 29, 31}

$$-\frac{\log(f^{c+dx}+1)}{d^2 \log^2(f)} - \frac{x}{d \log(f)(f^{c+dx}+1)} + \frac{x}{d \log(f)}$$

Antiderivative was successfully verified.

[In] Int[x/(2 + f^(-c - d\*x) + f^(c + d\*x)),x]

[Out] x/(d\*Log[f]) - x/(d\*(1 + f^(c + d\*x))\*Log[f]) - Log[1 + f^(c + d\*x)]/(d^2\*Log[f]^2)

#### Rule 2267

Int[(u\_)/((a\_) + (b\_)\*(F\_)^(v\_) + (c\_)\*(F\_)^(w\_)), x\_Symbol] := Int[(u\*F^v)/(c + a\*F^v + b\*F^(2\*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && LinearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1], 0], LtQ[LeafCount[v], LeafCount[w]]]

#### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{2 + f^{-c-dx} + f^{c+dx}} dx &= \int \frac{f^{c+dx}x}{1 + 2f^{c+dx} + f^{2(c+dx)}} dx \\
&= \int \frac{f^{c+dx}x}{(1 + f^{c+dx})^2} dx \\
&= -\frac{x}{d(1 + f^{c+dx})\log(f)} + \frac{\int \frac{1}{1+f^{c+dx}} dx}{d\log(f)} \\
&= -\frac{x}{d(1 + f^{c+dx})\log(f)} + \frac{\text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} \\
&= -\frac{x}{d(1 + f^{c+dx})\log(f)} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} - \frac{\text{Subst}\left(\int \frac{1}{1+x} dx, x, f^{c+dx}\right)}{d^2\log^2(f)} \\
&= \frac{x}{d\log(f)} - \frac{x}{d(1 + f^{c+dx})\log(f)} - \frac{\log(1 + f^{c+dx})}{d^2\log^2(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0652724, size = 44, normalized size = 0.88

$$\frac{\frac{dx \log(f)^{f^{c+dx}}}{f^{c+dx}+1} - \log(f^{c+dx} + 1)}{d^2 \log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

```
[Out] ((d*f^(c + d*x)*x*Log[f])/(1 + f^(c + d*x)) - Log[1 + f^(c + d*x)])/(d^2*Lo
g[f]^2)
```

**Maple [A]** time = 0.014, size = 64, normalized size = 1.3

$$-\frac{x e^{(-dx-c)\ln(f)}}{d \ln(f) \left(e^{(-dx-c)\ln(f)} + 1\right)} - \frac{\ln\left(e^{(-dx-c)\ln(f)} + 1\right)}{(\ln(f))^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(2+f^(-d*x-c)+f^(d*x+c)),x)`

[Out] 
$$-1/d/\ln(f)*x*\exp((-d*x-c)*\ln(f))/(\exp((-d*x-c)*\ln(f))+1)-1/d^2/\ln(f)^2*\ln(\exp((-d*x-c)*\ln(f))+1)$$

**Maxima [A]** time = 0.98975, size = 77, normalized size = 1.54

$$\frac{f^{dx} f^c x}{d f^{dx} f^c \log(f) + d \log(f)} - \frac{\log\left(\frac{f^{dx} f^c + 1}{f^c}\right)}{d^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="maxima")`

[Out] 
$$f^{(d*x)}*f^c*x/(d*f^{(d*x)}*f^c*\log(f) + d*\log(f)) - \log((f^{(d*x)}*f^c + 1)/f^c)/(d^2*\log(f)^2)$$

**Fricas [A]** time = 1.29051, size = 147, normalized size = 2.94

$$\frac{d f^{dx+c} x \log(f) - (f^{dx+c} + 1) \log(f^{dx+c} + 1)}{d^2 f^{dx+c} \log(f)^2 + d^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="fricas")`

[Out] 
$$(d*f^{(d*x + c)}*x*\log(f) - (f^{(d*x + c)} + 1)*\log(f^{(d*x + c)} + 1))/(d^2*f^{(d*x + c)}*\log(f)^2 + d^2*\log(f)^2)$$

**Sympy [A]** time = 0.137308, size = 42, normalized size = 0.84

$$-\frac{x}{d f^{c+dx} \log(f) + d \log(f)} + \frac{x}{d \log(f)} - \frac{\log(f^{c+dx} + 1)}{d^2 \log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(2+f**(-d*x-c)+f**(d*x+c)),x)`

[Out] 
$$-x/(d*f^{(c + d*x)}*\log(f) + d*\log(f)) + x/(d*\log(f)) - \log(f^{(c + d*x)} + 1)/(d^2*\log(f)^2)$$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{f^{dx+c} + f^{-dx-c} + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x/(f^(d*x + c) + f^(-d*x - c) + 2), x)
```

$$3.534 \quad \int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}} dx$$

**Optimal.** Leaf size=75

$$-\frac{2\text{PolyLog}(2, -f^{c+dx})}{d^3 \log^3(f)} - \frac{2x \log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x^2}{d \log(f)(f^{c+dx} + 1)} + \frac{x^2}{d \log(f)}$$

[Out]  $x^2/(d*\text{Log}[f]) - x^2/(d*(1 + f^{(c + d*x))*\text{Log}[f])} - (2*x*\text{Log}[1 + f^{(c + d*x)}])/(d^2*\text{Log}[f]^2) - (2*\text{PolyLog}[2, -f^{(c + d*x)}])/(d^3*\text{Log}[f]^3)$

**Rubi [A]** time = 0.490936, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$ , Rules used = {2267, 6688, 2191, 2184, 2190, 2279, 2391}

$$-\frac{2\text{PolyLog}(2, -f^{c+dx})}{d^3 \log^3(f)} - \frac{2x \log(f^{c+dx} + 1)}{d^2 \log^2(f)} - \frac{x^2}{d \log(f)(f^{c+dx} + 1)} + \frac{x^2}{d \log(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(2 + f^{(-c - d*x)} + f^{(c + d*x)}), x]$

[Out]  $x^2/(d*\text{Log}[f]) - x^2/(d*(1 + f^{(c + d*x))*\text{Log}[f])} - (2*x*\text{Log}[1 + f^{(c + d*x)}])/(d^2*\text{Log}[f]^2) - (2*\text{PolyLog}[2, -f^{(c + d*x)}])/(d^3*\text{Log}[f]^3)$

#### Rule 2267

$\text{Int}[(u_)/((a_.) + (b_.)*(F_)^{(v_)} + (c_.)*(F_)^{(w_)}), x\_Symbol] \rightarrow \text{Int}[(u*F^v)/(c + a*F^v + b*F^{(2*v)}), x] /; \text{FreeQ}\{F, a, b, c, x\} \ \&\& \ \text{EqQ}[w, -v] \ \&\& \ \text{LinearQ}[v, x] \ \&\& \ \text{If}[\text{RationalQ}[\text{Coefficient}[v, x, 1]], \text{GtQ}[\text{Coefficient}[v, x, 1], 0], \text{LtQ}[\text{LeafCount}[v], \text{LeafCount}[w]]]$

#### Rule 6688

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{SimplifyIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SimplerIntegrandQ}[v, u, x]$

#### Rule 2191

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}*((a_.) + (b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_))^{(m_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(a + b*(F^{(g*(e + f*x)))^n)^{(p + 1)})/(b*f*g*n*(p + 1)*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*(p + 1)*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*(a + b*(F^{(g*(e + f*x)))^n)^{(p + 1)}, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n, p\}, x \ \&\& \ \text{NeQ}[p, -1]$

#### Rule 2184

$\text{Int}[(c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n}/(a + b*(F^{(g*(e + f*x)))^n}), x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2 + f^{-c-dx} + f^{c+dx}} dx &= \int \frac{f^{c+dx} x^2}{1 + 2f^{c+dx} + f^{2(c+dx)}} dx \\ &= \int \frac{f^{c+dx} x^2}{(1 + f^{c+dx})^2} dx \\ &= -\frac{x^2}{d(1 + f^{c+dx}) \log(f)} + \frac{2 \int \frac{x}{1 + f^{c+dx}} dx}{d \log(f)} \\ &= \frac{x^2}{d \log(f)} - \frac{x^2}{d(1 + f^{c+dx}) \log(f)} - \frac{2 \int \frac{f^{c+dx} x}{1 + f^{c+dx}} dx}{d \log(f)} \\ &= \frac{x^2}{d \log(f)} - \frac{x^2}{d(1 + f^{c+dx}) \log(f)} - \frac{2x \log(1 + f^{c+dx})}{d^2 \log^2(f)} + \frac{2 \int \log(1 + f^{c+dx}) dx}{d^2 \log^2(f)} \\ &= \frac{x^2}{d \log(f)} - \frac{x^2}{d(1 + f^{c+dx}) \log(f)} - \frac{2x \log(1 + f^{c+dx})}{d^2 \log^2(f)} + \frac{2 \text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, f^{c+dx}\right)}{d^3 \log^3(f)} \\ &= \frac{x^2}{d \log(f)} - \frac{x^2}{d(1 + f^{c+dx}) \log(f)} - \frac{2x \log(1 + f^{c+dx})}{d^2 \log^2(f)} - \frac{2 \text{Li}_2(-f^{c+dx})}{d^3 \log^3(f)} \end{aligned}$$

**Mathematica [A]** time = 0.112142, size = 63, normalized size = 0.84

$$\frac{dx \log(f) \left( \frac{f^{c+dx}}{f^{c+dx} + 1} - 2 \log(f^{c+dx} + 1) \right) - 2 \text{PolyLog}(2, -f^{c+dx})}{d^3 \log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

```
[Out] (d*x*Log[f]*((d*f^(c + d*x))*x*Log[f])/(1 + f^(c + d*x)) - 2*Log[1 + f^(c +
d*x)]) - 2*PolyLog[2, -f^(c + d*x)]/(d^3*Log[f]^3)
```

**Maple [A]** time = 0.06, size = 134, normalized size = 1.8

$$\frac{x^2}{d \ln(f)(f^{-dx-c} + 1)} - \frac{x^2}{d \ln(f)} - 2 \frac{cx}{\ln(f)d^2} - \frac{c^2}{\ln(f)d^3} - 2 \frac{\ln(f^{-dx}f^{-c} + 1)x}{(\ln(f))^2 d^2} + 2 \frac{\text{polylog}(2, -f^{-dx}f^{-c})}{d^3 (\ln(f))^3} - 2 \frac{c \ln(f^{-dx})}{d^3 (\ln(f))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+f^(-d\*x-c)+f^(d\*x+c)),x)

[Out] 1/d/ln(f)\*x^2/(f^(-d\*x-c)+1)-x^2/d/ln(f)-2/ln(f)/d^2\*c\*x-1/ln(f)/d^3\*c^2-2/d^2/ln(f)^2\*ln(f^(-d\*x)\*f^(-c)+1)\*x+2/d^3/ln(f)^3\*polylog(2,-f^(-d\*x)\*f^(-c))-2/d^3/ln(f)^2\*c\*ln(f^(-d\*x)\*f^(-c))

**Maxima [A]** time = 1.03322, size = 109, normalized size = 1.45

$$-\frac{x^2}{d f^{dx} f^c \log(f) + d \log(f)} + \frac{\log(f^{dx})^2}{d^3 \log(f)^3} - \frac{2(\log(f^{dx} f^c + 1) \log(f^{dx}) + \text{Li}_2(-f^{dx} f^c))}{d^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+f^(-d\*x-c)+f^(d\*x+c)),x, algorithm="maxima")

[Out] -x^2/(d\*f^(d\*x)\*f^c\*log(f) + d\*log(f)) + log(f^(d\*x))^2/(d^3\*log(f)^3) - 2\*(log(f^(d\*x)\*f^c + 1)\*log(f^(d\*x)) + dilog(-f^(d\*x)\*f^c))/(d^3\*log(f)^3)

**Fricas [A]** time = 1.34936, size = 274, normalized size = 3.65

$$\frac{c^2 \log(f)^2 - (d^2 x^2 - c^2) f^{dx+c} \log(f)^2 + 2(f^{dx+c} + 1) \text{Li}_2(-f^{dx+c}) + 2(d f^{dx+c} x \log(f) + dx \log(f)) \log(f^{dx+c} + 1)}{d^3 f^{dx+c} \log(f)^3 + d^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+f^(-d\*x-c)+f^(d\*x+c)),x, algorithm="fricas")

[Out] -(c^2\*log(f)^2 - (d^2\*x^2 - c^2)\*f^(d\*x + c)\*log(f)^2 + 2\*(f^(d\*x + c) + 1)\*dilog(-f^(d\*x + c)) + 2\*(d\*f^(d\*x + c)\*x\*log(f) + d\*x\*log(f))\*log(f^(d\*x + c) + 1))/(d^3\*f^(d\*x + c)\*log(f)^3 + d^3\*log(f)^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^2}{d f^{c+dx} \log(f) + d \log(f)} + \frac{2 \int \frac{x}{e^{c \log(f)} e^{dx \log(f)} + 1} dx}{d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(2+f\*\*(-d\*x-c)+f\*\*(d\*x+c)),x)

```
[Out] -x**2/(d*f**(c + d*x)*log(f) + d*log(f)) + 2*Integral(x/(exp(c*log(f))*exp(
d*x*log(f)) + 1), x)/(d*log(f))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{f^{dx+c} + f^{-dx-c} + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(f^(d*x + c) + f^(-d*x - c) + 2), x)
```

$$3.535 \quad \int \frac{1}{2+3^{-x}+3^x} dx$$

**Optimal.** Leaf size=13

$$-\frac{1}{(3^x + 1)\log(3)}$$

[Out] -(1/((1 + 3^x)\*Log[3]))

**Rubi [A]** time = 0.0120727, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 32}

$$-\frac{1}{(3^x + 1)\log(3)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3^(-x) + 3^x)^(-1), x]

[Out] -(1/((1 + 3^x)\*Log[3]))

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{2+3^{-x}+3^x} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)^2} dx, x, 3^x\right)}{\log(3)} \\ &= -\frac{1}{(1+3^x)\log(3)} \end{aligned}$$

**Mathematica [A]** time = 0.010441, size = 13, normalized size = 1.

$$-\frac{1}{(3^x + 1)\log(3)}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3^(-x) + 3^x)^(-1), x]

[Out] -(1/((1 + 3^x)\*Log[3]))

---

**Maple [A]** time = 0.003, size = 14, normalized size = 1.1

$$-\frac{1}{(1+3^x)\ln(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+1/(3^x)+3^x),x)

[Out] -1/(1+3^x)/ln(3)

---

**Maxima [A]** time = 0.987349, size = 19, normalized size = 1.46

$$\frac{1}{\left(\frac{1}{3^x} + 1\right)\log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+1/(3^x)+3^x),x, algorithm="maxima")

[Out] 1/((1/3^x + 1)\*log(3))

---

**Fricas [A]** time = 1.22765, size = 35, normalized size = 2.69

$$-\frac{1}{3^x \log(3) + \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+1/(3^x)+3^x),x, algorithm="fricas")

[Out] -1/(3^x\*log(3) + log(3))

---

**Sympy [A]** time = 0.088442, size = 12, normalized size = 0.92

$$-\frac{1}{3^x \log(3) + \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+1/(3\*\*x)+3\*\*x),x)

[Out] -1/(3\*\*x\*log(3) + log(3))

---

**Giac [A]** time = 1.19945, size = 18, normalized size = 1.38

$$-\frac{1}{(3^x + 1)\log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+1/(3^x)+3^x),x, algorithm="giac")
```

```
[Out] -1/((3^x + 1)*log(3))
```



$$3.536 \quad \int \frac{1}{1-e^{-x}+2e^x} dx$$

**Optimal.** Leaf size=23

$$\frac{1}{3} \log(1-2e^x) - \frac{1}{3} \log(e^x+1)$$

[Out] Log[1 - 2\*E^x]/3 - Log[1 + E^x]/3

**Rubi [A]** time = 0.0192433, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2282, 616, 31}

$$\frac{1}{3} \log(1-2e^x) - \frac{1}{3} \log(e^x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - E^(-x) + 2\*E^x)^(-1), x]

[Out] Log[1 - 2\*E^x]/3 - Log[1 + E^x]/3

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{1-e^{-x}+2e^x} dx &= \text{Subst} \left( \int \frac{1}{-1+x+2x^2} dx, x, e^x \right) \\ &= \frac{2}{3} \text{Subst} \left( \int \frac{1}{-1+2x} dx, x, e^x \right) - \frac{2}{3} \text{Subst} \left( \int \frac{1}{2+2x} dx, x, e^x \right) \\ &= \frac{1}{3} \log(1-2e^x) - \frac{1}{3} \log(1+e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0107146, size = 16, normalized size = 0.7

$$-\frac{2}{3} \tanh^{-1} \left( \frac{1}{3} (4e^x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - E^(-x) + 2\*E^x)^(-1),x]

[Out] (-2\*ArcTanh[(1 + 4\*E^x)/3])/3

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$\frac{\ln(2e^x - 1)}{3} - \frac{\ln(1 + e^x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-1/exp(x)+2\*exp(x)),x)

[Out] 1/3\*ln(2\*exp(x)-1)-1/3\*ln(1+exp(x))

**Maxima [A]** time = 0.988433, size = 26, normalized size = 1.13

$$-\frac{1}{3} \log(e^{-x} + 1) + \frac{1}{3} \log(e^{-x} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-1/exp(x)+2\*exp(x)),x, algorithm="maxima")

[Out] -1/3\*log(e^(-x) + 1) + 1/3\*log(e^(-x) - 2)

**Fricas [A]** time = 1.17712, size = 53, normalized size = 2.3

$$\frac{1}{3} \log(2e^x - 1) - \frac{1}{3} \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-1/exp(x)+2\*exp(x)),x, algorithm="fricas")

[Out] 1/3\*log(2\*e^x - 1) - 1/3\*log(e^x + 1)

**Sympy [A]** time = 0.112045, size = 17, normalized size = 0.74

$$\frac{\log\left(e^x - \frac{1}{2}\right)}{3} - \frac{\log(e^x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-1/exp(x)+2\*exp(x)),x)

[Out] log(exp(x) - 1/2)/3 - log(exp(x) + 1)/3

---

**Giac [A]** time = 1.22618, size = 24, normalized size = 1.04

$$-\frac{1}{3} \log(e^x + 1) + \frac{1}{3} \log(|2e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-1/exp(x)+2\*exp(x)),x, algorithm="giac")

[Out] -1/3\*log(e^x + 1) + 1/3\*log(abs(2\*e^x - 1))

$$3.537 \quad \int \frac{1}{a+be^{-x}+ce^x} dx$$

**Optimal.** Leaf size=36

$$-\frac{2 \tanh^{-1}\left(\frac{a+2ce^x}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

[Out] (-2\*ArcTanh[(a + 2\*c\*E^x)/Sqrt[a^2 - 4\*b\*c]])/Sqrt[a^2 - 4\*b\*c]

**Rubi [A]** time = 0.0568149, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2282, 1386, 618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{a+2ce^x}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/E^x + c\*E^x)^(-1), x]

[Out] (-2\*ArcTanh[(a + 2\*c\*E^x)/Sqrt[a^2 - 4\*b\*c]])/Sqrt[a^2 - 4\*b\*c]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{a + be^{-x} + ce^x} dx &= \text{Subst} \left( \int \frac{1}{x \left( a + \frac{b}{x} + cx \right)} dx, x, e^x \right) \\
&= \text{Subst} \left( \int \frac{1}{b + ax + cx^2} dx, x, e^x \right) \\
&= - \left( 2 \text{Subst} \left( \int \frac{1}{a^2 - 4bc - x^2} dx, x, a + 2ce^x \right) \right) \\
&= - \frac{2 \tanh^{-1} \left( \frac{a+2ce^x}{\sqrt{a^2-4bc}} \right)}{\sqrt{a^2-4bc}}
\end{aligned}$$

**Mathematica [A]** time = 0.0235092, size = 36, normalized size = 1.

$$-\frac{2 \tanh^{-1} \left( \frac{a+2ce^x}{\sqrt{a^2-4bc}} \right)}{\sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/E^x + c\*E^x)^(-1), x]

[Out] (-2\*ArcTanh[(a + 2\*c\*E^x)/Sqrt[a^2 - 4\*b\*c]])/Sqrt[a^2 - 4\*b\*c]

**Maple [A]** time = 0.006, size = 36, normalized size = 1.

$$2 \frac{1}{\sqrt{-a^2 + 4bc}} \arctan \left( \frac{a + 2ce^x}{\sqrt{-a^2 + 4bc}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/exp(x)+c\*exp(x)), x)

[Out] 2/(-a^2+4\*b\*c)^(1/2)\*arctan((a+2\*c\*exp(x))/(-a^2+4\*b\*c)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/exp(x)+c\*exp(x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.26063, size = 298, normalized size = 8.28

$$\left[ \frac{\log \left( \frac{2c^2e^{2x} + 2ace^x + a^2 - 2bc - \sqrt{a^2 - 4bc}(2ce^x + a)}{ce^{2x} + ae^x + b} \right)}{\sqrt{a^2 - 4bc}}, - \frac{2\sqrt{-a^2 + 4bc} \arctan \left( - \frac{\sqrt{-a^2 + 4bc}(2ce^x + a)}{a^2 - 4bc} \right)}{a^2 - 4bc} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/exp(x)+c\*exp(x)),x, algorithm="fricas")

[Out] [log((2\*c^2\*e^(2\*x) + 2\*a\*c\*e^x + a^2 - 2\*b\*c - sqrt(a^2 - 4\*b\*c))\*(2\*c\*e^x + a))/(c\*e^(2\*x) + a\*e^x + b))/sqrt(a^2 - 4\*b\*c), -2\*sqrt(-a^2 + 4\*b\*c)\*arc tan(-sqrt(-a^2 + 4\*b\*c)\*(2\*c\*e^x + a)/(a^2 - 4\*b\*c))/(a^2 - 4\*b\*c)]

**Sympy [A]** time = 0.229591, size = 36, normalized size = 1.

$$\text{RootSum}\left(z^2(a^2 - 4bc) - 1, \left(i \mapsto i \log\left(e^x + \frac{-ia^2 + 4ibc + a}{2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/exp(x)+c\*exp(x)),x)

[Out] RootSum(\_z\*\*2\*(a\*\*2 - 4\*b\*c) - 1, Lambda(\_i, \_i\*log(exp(x) + (-\_i\*a\*\*2 + 4\*\_i\*b\*c + a)/(2\*c))))

**Giac [A]** time = 1.14394, size = 47, normalized size = 1.31

$$\frac{2 \arctan\left(\frac{2ce^x+a}{\sqrt{-a^2+4bc}}\right)}{\sqrt{-a^2+4bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/exp(x)+c\*exp(x)),x, algorithm="giac")

[Out] 2\*arctan((2\*c\*e^x + a)/sqrt(-a^2 + 4\*b\*c))/sqrt(-a^2 + 4\*b\*c)

### 3.538 $\int \frac{x}{a+be^{-x}+ce^x} dx$

**Optimal.** Leaf size=159

$$\frac{\text{PolyLog}\left(2, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{\text{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2-4bc}+a}\right)}{\sqrt{a^2-4bc}} + \frac{x \log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}} + 1\right)}{\sqrt{a^2-4bc}} - \frac{x \log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a} + 1\right)}{\sqrt{a^2-4bc}}$$

[Out] (x\*Log[1 + (2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (x\*Log[1 + (2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + PolyLog[2, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])]/Sqrt[a^2 - 4\*b\*c] - PolyLog[2, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])]/Sqrt[a^2 - 4\*b\*c]

**Rubi [A]** time = 0.303915, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {2267, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{\text{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2-4bc}+a}\right)}{\sqrt{a^2-4bc}} + \frac{x \log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}} + 1\right)}{\sqrt{a^2-4bc}} - \frac{x \log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a} + 1\right)}{\sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b/E^x + c\*E^x), x]

[Out] (x\*Log[1 + (2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (x\*Log[1 + (2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + PolyLog[2, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])]/Sqrt[a^2 - 4\*b\*c] - PolyLog[2, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])]/Sqrt[a^2 - 4\*b\*c]

#### Rule 2267

Int[(u\_)/((a\_) + (b\_)\*(F\_)^(v\_) + (c\_)\*(F\_)^(w\_)), x\_Symbol] :> Int[(u\*F^v)/(c + a\*F^v + b\*F^(2\*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && LinearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1], 0], LtQ[LeafCount[v], LeafCount[w]]]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))]

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{x}{a + be^{-x} + ce^x} dx &= \int \frac{e^x x}{b + ae^x + ce^{2x}} dx \\ &= \frac{(2c) \int \frac{e^x x}{a - \sqrt{a^2 - 4bc} + 2ce^x} dx}{\sqrt{a^2 - 4bc}} - \frac{(2c) \int \frac{e^x x}{a + \sqrt{a^2 - 4bc} + 2ce^x} dx}{\sqrt{a^2 - 4bc}} \\ &= \frac{x \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{\int \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bc}} + \frac{\int \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bc}} \\ &= \frac{x \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{2cx}{a - \sqrt{a^2 - 4bc}}\right)}{x} dx, x, e^x\right)}{\sqrt{a^2 - 4bc}} + \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{2cx}{a + \sqrt{a^2 - 4bc}}\right)}{x} dx, x, e^x\right)}{\sqrt{a^2 - 4bc}} \\ &= \frac{x \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} + \frac{\text{Li}_2\left(-\frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{\text{Li}_2\left(-\frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} \end{aligned}$$

**Mathematica [A]** time = 0.0670996, size = 123, normalized size = 0.77

$$\frac{\text{PolyLog}\left(2, \frac{2ce^x}{\sqrt{a^2 - 4bc} - a}\right) - \text{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2 - 4bc} + a}\right) + x\left(\log\left(\frac{2ce^x}{a - \sqrt{a^2 - 4bc}} + 1\right) - \log\left(\frac{2ce^x}{\sqrt{a^2 - 4bc} + a} + 1\right)\right)}{\sqrt{a^2 - 4bc}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b/E^x + c\*E^x), x]

[Out] (x\*(Log[1 + (2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c]]) - Log[1 + (2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c]]) + PolyLog[2, (2\*c\*E^x)/(-a + Sqrt[a^2 - 4\*b\*c]]) - PolyLog[2, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c]])/Sqrt[a^2 - 4\*b\*c])

**Maple [A]** time = 0.008, size = 181, normalized size = 1.1

$$-x \left( \ln \left( \left( 2ce^x + \sqrt{a^2 - 4bc} + a \right) \left( a + \sqrt{a^2 - 4bc} \right)^{-1} \right) - \ln \left( \left( -2ce^x + \sqrt{a^2 - 4bc} - a \right) \left( -a + \sqrt{a^2 - 4bc} \right)^{-1} \right) \right) \frac{1}{\sqrt{a^2 - 4bc}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b/exp(x)+c\*exp(x)), x)

[Out] -x\*(ln((2\*c\*exp(x)+(a^2-4\*b\*c)^(1/2)+a)/(a+(a^2-4\*b\*c)^(1/2))))-ln((-2\*c\*exp(x)+(a^2-4\*b\*c)^(1/2)-a)/(-a+(a^2-4\*b\*c)^(1/2))))/(a^2-4\*b\*c)^(1/2)+1/(a^2-4\*b\*c)^(1/2)\*dilog((-2\*c\*exp(x)+(a^2-4\*b\*c)^(1/2)-a)/(-a+(a^2-4\*b\*c)^(1/2)))-1/(a^2-4\*b\*c)^(1/2)\*dilog((2\*c\*exp(x)+(a^2-4\*b\*c)^(1/2)+a)/(a+(a^2-4\*b\*c)^(1/2)))



---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/exp(x)+c\*exp(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.36157, size = 504, normalized size = 3.17

$$\frac{bx\sqrt{\frac{a^2-4bc}{b^2}} \log\left(\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x+ae^x+2b}}{2b}\right) - bx\sqrt{\frac{a^2-4bc}{b^2}} \log\left(-\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x-ae^x-2b}}{2b}\right) + b\sqrt{\frac{a^2-4bc}{b^2}} \operatorname{Li}_2\left(-\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x+ae^x+2b}}{2b} + 1\right) - b\sqrt{\frac{a^2-4bc}{b^2}} \operatorname{Li}_2\left(-\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x-ae^x-2b}}{2b} + 1\right)}{a^2 - 4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/exp(x)+c\*exp(x)),x, algorithm="fricas")

[Out] (b\*x\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x + a\*e^x + 2\*b)/b) - b\*x\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(-1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x - a\*e^x - 2\*b)/b) + b\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(-1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x + a\*e^x + 2\*b)/b + 1) - b\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x - a\*e^x - 2\*b)/b + 1)/(a^2 - 4\*b\*c)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{ae^x + b + ce^{2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/exp(x)+c\*exp(x)),x)

[Out] Integral(x\*exp(x)/(a\*exp(x) + b + c\*exp(2\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{be^{(-x)} + ce^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/exp(x)+c\*exp(x)),x, algorithm="giac")

[Out] integrate(x/(b\*e^(-x) + c\*e^x + a), x)

$$3.539 \quad \int \frac{x^2}{a+be^{-x}+ce^x} dx$$

**Optimal.** Leaf size=244

$$\frac{2x \operatorname{PolyLog}\left(2, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{2x \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2-4bc+a}}\right)}{\sqrt{a^2-4bc}} - \frac{2 \operatorname{PolyLog}\left(3, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} + \frac{2 \operatorname{PolyLog}\left(3, -\frac{2ce^x}{\sqrt{a^2-4bc+a}}\right)}{\sqrt{a^2-4bc}} +$$

[Out] (x^2\*Log[1 + (2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (x^2\*Log[1 + (2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + (2\*x\*PolyLog[2, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (2\*x\*PolyLog[2, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (2\*PolyLog[3, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + (2\*PolyLog[3, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c]

**Rubi [A]** time = 0.498767, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.3, Rules used = {2267, 2264, 2190, 2531, 2282, 6589}

$$\frac{2x \operatorname{PolyLog}\left(2, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{2x \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2-4bc+a}}\right)}{\sqrt{a^2-4bc}} - \frac{2 \operatorname{PolyLog}\left(3, -\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} + \frac{2 \operatorname{PolyLog}\left(3, -\frac{2ce^x}{\sqrt{a^2-4bc+a}}\right)}{\sqrt{a^2-4bc}} +$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b/E^x + c\*E^x), x]

[Out] (x^2\*Log[1 + (2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (x^2\*Log[1 + (2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + (2\*x\*PolyLog[2, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (2\*x\*PolyLog[2, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] - (2\*PolyLog[3, (-2\*c\*E^x)/(a - Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c] + (2\*PolyLog[3, (-2\*c\*E^x)/(a + Sqrt[a^2 - 4\*b\*c])])/Sqrt[a^2 - 4\*b\*c]

#### Rule 2267

Int[(u\_)/((a\_) + (b\_)\*(F\_)^(v\_) + (c\_)\*(F\_)^(w\_)), x\_Symbol] :> Int[(u\*F^v)/(c + a\*F^v + b\*F^(2\*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && LinearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1], 0], LtQ[LeafCount[v], LeafCount[w]]]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a + be^{-x} + ce^x} dx &= \int \frac{e^x x^2}{b + ae^x + ce^{2x}} dx \\ &= \frac{(2c) \int \frac{e^x x^2}{a - \sqrt{a^2 - 4bc} + 2ce^x} dx}{\sqrt{a^2 - 4bc}} - \frac{(2c) \int \frac{e^x x^2}{a + \sqrt{a^2 - 4bc} + 2ce^x} dx}{\sqrt{a^2 - 4bc}} \\ &= \frac{x^2 \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x^2 \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{2 \int x \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bc}} + \frac{2 \int x \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bc}} \\ &= \frac{x^2 \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x^2 \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} + \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} \\ &= \frac{x^2 \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x^2 \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} + \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} \\ &= \frac{x^2 \log\left(1 + \frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{x^2 \log\left(1 + \frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} + \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} - \frac{2x \operatorname{Li}_2\left(-\frac{2ce^x}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bc}} \end{aligned}$$

**Mathematica [A]** time = 0.0374542, size = 185, normalized size = 0.76

$$\frac{2x \operatorname{PolyLog}\left(2, \frac{2ce^x}{\sqrt{a^2 - 4bc} - a}\right) - 2x \operatorname{PolyLog}\left(2, -\frac{2ce^x}{\sqrt{a^2 - 4bc} + a}\right) - 2 \operatorname{PolyLog}\left(3, \frac{2ce^x}{\sqrt{a^2 - 4bc} - a}\right) + 2 \operatorname{PolyLog}\left(3, -\frac{2ce^x}{\sqrt{a^2 - 4bc} + a}\right) + x^2}{\sqrt{a^2 - 4bc}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/E^x + c*E^x), x]
```

```
[Out] (x^2*Log[1 + (2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])] - x^2*Log[1 + (2*c*E^x)/(a
+ Sqrt[a^2 - 4*b*c])] + 2*x*PolyLog[2, (2*c*E^x)/(-a + Sqrt[a^2 - 4*b*c])]
- 2*x*PolyLog[2, (-2*c*E^x)/(a + Sqrt[a^2 - 4*b*c])] - 2*PolyLog[3, (2*c*E^
x)/(-a + Sqrt[a^2 - 4*b*c])] + 2*PolyLog[3, (-2*c*E^x)/(a + Sqrt[a^2 - 4*b*
```

c]))/Sqrt[a^2 - 4\*b\*c]

**Maple [F]** time = 0.017, size = 0, normalized size = 0.

$$\int x^2 \left( a + \frac{b}{e^x} + ce^x \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b/exp(x)+c\*exp(x)),x)

[Out] int(x^2/(a+b/exp(x)+c\*exp(x)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b/exp(x)+c\*exp(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.38465, size = 759, normalized size = 3.11

$$bx^2 \sqrt{\frac{a^2-4bc}{b^2}} \log\left(\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x+ae^x+2b}}{2b}\right) - bx^2 \sqrt{\frac{a^2-4bc}{b^2}} \log\left(-\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x-ae^x-2b}}{2b}\right) + 2bx \sqrt{\frac{a^2-4bc}{b^2}} \operatorname{Li}_2\left(-\frac{b\sqrt{\frac{a^2-4bc}{b^2}}e^{x+ae^x+2b}}{2b} + 1\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b/exp(x)+c\*exp(x)),x, algorithm="fricas")

[Out] (b\*x^2\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x + a\*e^x + 2\*b)/b) - b\*x^2\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(-1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x - a\*e^x - 2\*b)/b) + 2\*b\*x\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(-1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x + a\*e^x + 2\*b)/b + 1) - 2\*b\*x\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x - a\*e^x - 2\*b)/b + 1) - 2\*b\*sqrt((a^2 - 4\*b\*c)/b^2)\*polylog(3, -1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x + a\*e^x)/b) + 2\*b\*sqrt((a^2 - 4\*b\*c)/b^2)\*polylog(3, 1/2\*(b\*sqrt((a^2 - 4\*b\*c)/b^2)\*e^x - a\*e^x)/b))/(a^2 - 4\*b\*c)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/exp(x)+c*exp(x)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{be^{-x} + ce^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/exp(x)+c*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*e^(-x) + c*e^x + a), x)
```

$$3.540 \quad \int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}} dx$$

**Optimal.** Leaf size=47

$$\frac{2 \tanh^{-1} \left( \frac{a+2cf^{c+dx}}{\sqrt{a^2-4bc}} \right)}{d \log(f) \sqrt{a^2-4bc}}$$

[Out] (-2\*ArcTanh[(a + 2\*c\*f^(c + d\*x))/Sqrt[a^2 - 4\*b\*c]])/(Sqrt[a^2 - 4\*b\*c]\*d\*Log[f])

**Rubi [A]** time = 0.0641623, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2282, 1386, 618, 206}

$$\frac{2 \tanh^{-1} \left( \frac{a+2cf^{c+dx}}{\sqrt{a^2-4bc}} \right)}{d \log(f) \sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*f^(-c - d\*x) + c\*f^(c + d\*x))^(-1),x]

[Out] (-2\*ArcTanh[(a + 2\*c\*f^(c + d\*x))/Sqrt[a^2 - 4\*b\*c]])/(Sqrt[a^2 - 4\*b\*c]\*d\*Log[f])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b f^{-c-dx} + c f^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a + \frac{b}{x} + cx\right)} dx, x, f^{c+dx}\right)}{d \log(f)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{b+ax+cx^2} dx, x, f^{c+dx}\right)}{d \log(f)} \\
&= -\frac{2 \text{Subst}\left(\int \frac{1}{a^2-4bc-x^2} dx, x, a + 2c f^{c+dx}\right)}{d \log(f)} \\
&= -\frac{2 \tanh^{-1}\left(\frac{a+2c f^{c+dx}}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc} d \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0544174, size = 47, normalized size = 1.

$$-\frac{2 \tanh^{-1}\left(\frac{a+2c f^{c+dx}}{\sqrt{a^2-4bc}}\right)}{d \log(f) \sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*f^(-c - d\*x) + c\*f^(c + d\*x))^(-1), x]

[Out] (-2\*ArcTanh[(a + 2\*c\*f^(c + d\*x))/Sqrt[a^2 - 4\*b\*c]])/(Sqrt[a^2 - 4\*b\*c]\*d\*Log[f])

**Maple [B]** time = 0.029, size = 135, normalized size = 2.9

$$\frac{1}{d \ln(f)} \ln\left(f^{-dx-c} + \frac{1}{2b} \left(a\sqrt{a^2-4bc} + a^2 - 4bc\right) \frac{1}{\sqrt{a^2-4bc}}\right) \frac{1}{\sqrt{a^2-4bc}} - \frac{1}{d \ln(f)} \ln\left(f^{-dx-c} + \frac{1}{2b} \left(a\sqrt{a^2-4bc} - a^2 + 4bc\right) \frac{1}{\sqrt{a^2-4bc}}\right) \frac{1}{\sqrt{a^2-4bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)), x)

[Out] 1/(a^2-4\*b\*c)^(1/2)/d/ln(f)\*ln(f^(-d\*x-c)+1/2\*(a\*(a^2-4\*b\*c)^(1/2)+a^2-4\*b\*c)/b/(a^2-4\*b\*c)^(1/2))-1/(a^2-4\*b\*c)^(1/2)/d/ln(f)\*ln(f^(-d\*x-c)+1/2\*(a\*(a^2-4\*b\*c)^(1/2)-a^2+4\*b\*c)/b/(a^2-4\*b\*c)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.37557, size = 424, normalized size = 9.02

$$\left[ \frac{\log\left(\frac{2c^2f^{2dx+2c}+a^2-2bc+2(ac-\sqrt{a^2-4bcc})f^{dx+c}-\sqrt{a^2-4bca}}{cf^{2dx+2c}+af^{dx+c}+b}\right)}{\sqrt{a^2-4bcd}\log(f)}, -\frac{2\sqrt{-a^2+4bc}\arctan\left(-\frac{2\sqrt{-a^2+4bcc}f^{dx+c}+\sqrt{-a^2+4bca}}{a^2-4bc}\right)}{(a^2-4bc)d\log(f)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="fricas")

[Out] [log((2\*c^2\*f^(2\*d\*x + 2\*c) + a^2 - 2\*b\*c + 2\*(a\*c - sqrt(a^2 - 4\*b\*c))\*c)\*f^(d\*x + c) - sqrt(a^2 - 4\*b\*c)\*a)/(c\*f^(2\*d\*x + 2\*c) + a\*f^(d\*x + c) + b))/ (sqrt(a^2 - 4\*b\*c)\*d\*log(f)), -2\*sqrt(-a^2 + 4\*b\*c)\*arctan(-(2\*sqrt(-a^2 + 4\*b\*c))\*c\*f^(d\*x + c) + sqrt(-a^2 + 4\*b\*c)\*a)/(a^2 - 4\*b\*c))/((a^2 - 4\*b\*c)\*d\*log(f))]

**Sympy [A]** time = 0.328432, size = 66, normalized size = 1.4

$$\text{RootSum}\left(z^2\left(a^2d^2\log(f)^2 - 4bcd^2\log(f)^2\right) - 1, \left(i \mapsto i\log\left(f^{c+dx} + \frac{-ia^2d\log(f) + 4ibcd\log(f) + a}{2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*f\*\*(-d\*x-c)+c\*f\*\*(d\*x+c)),x)

[Out] RootSum(\_z\*\*2\*(a\*\*2\*d\*\*2\*log(f)\*\*2 - 4\*b\*c\*d\*\*2\*log(f)\*\*2) - 1, Lambda(\_i, \_i\*log(f\*\*(c + d\*x) + (-\_i\*a\*\*2\*d\*log(f) + 4\*\_i\*b\*c\*d\*log(f) + a)/(2\*c))))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cf^{dx+c} + bf^{-dx-c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/(c\*f^(d\*x + c) + b\*f^(-d\*x - c) + a), x)



$$3.541 \quad \int \frac{x}{a+bf^{-c-dx}+cfc+dx} dx$$

**Optimal.** Leaf size=203

$$\frac{\text{PolyLog}\left(2, -\frac{2cfc+dx}{a-\sqrt{a^2-4bc}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{\text{PolyLog}\left(2, -\frac{2cfc+dx}{\sqrt{a^2-4bc+a}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} + \frac{x \log\left(\frac{2cfc+dx}{a-\sqrt{a^2-4bc}} + 1\right)}{d \log(f)\sqrt{a^2-4bc}} - \frac{x \log\left(\frac{2cfc+dx}{\sqrt{a^2-4bc+a}} + 1\right)}{d \log(f)\sqrt{a^2-4bc}}$$

```
[Out] (x*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d
*Log[f]) - (x*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2
- 4*b*c]*d*Log[f]) + PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])
]/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - PolyLog[2, (-2*c*f^(c + d*x))/(a + Sqr
t[a^2 - 4*b*c])]/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2)
```

**Rubi [A]** time = 0.406733, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {2267, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -\frac{2cfc+dx}{a-\sqrt{a^2-4bc}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{\text{PolyLog}\left(2, -\frac{2cfc+dx}{\sqrt{a^2-4bc+a}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} + \frac{x \log\left(\frac{2cfc+dx}{a-\sqrt{a^2-4bc}} + 1\right)}{d \log(f)\sqrt{a^2-4bc}} - \frac{x \log\left(\frac{2cfc+dx}{\sqrt{a^2-4bc+a}} + 1\right)}{d \log(f)\sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

```
[Out] (x*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d
*Log[f]) - (x*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2
- 4*b*c]*d*Log[f]) + PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])
]/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - PolyLog[2, (-2*c*f^(c + d*x))/(a + Sqr
t[a^2 - 4*b*c])]/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2)
```

#### Rule 2267

```
Int[(u_)/((a_) + (b_)*(F_)^(v_) + (c_)*(F_)^(w_)), x_Symbol] := Int[(u*F^
v)/(c + a*F^v + b*F^(2*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1
], 0], LtQ[LeafCount[v], LeafCount[w]]]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*(F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

**Rule 2391**

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}} dx = \int \frac{f^{c+dx}x}{b + af^{c+dx} + cf^{2(c+dx)}} dx$$

$$= \frac{(2c) \int \frac{f^{c+dx}x}{a-\sqrt{a^2-4bc}+2cf^{c+dx}} dx}{\sqrt{a^2-4bc}} - \frac{(2c) \int \frac{f^{c+dx}x}{a+\sqrt{a^2-4bc}+2cf^{c+dx}} dx}{\sqrt{a^2-4bc}}$$

$$= \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} - \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} - \frac{\int \log\left(1 + \frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right) dx}{\sqrt{a^2-4bcd} \log(f)} + \frac{\int \log\left(1 + \frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right) dx}{\sqrt{a^2-4bcd} \log(f)}$$

$$= \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} - \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} - \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{2cx}{a-\sqrt{a^2-4bc}}\right)}{x} dx, x, f^{c+dx}\right)}{\sqrt{a^2-4bcd^2} \log^2(f)} + \dots$$

$$= \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} - \frac{x \log\left(1 + \frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd} \log(f)} + \frac{\text{Li}_2\left(-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd^2} \log^2(f)} - \frac{\text{Li}_2\left(-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bcd^2} \log^2(f)}$$

**Mathematica [F]** time = 0.450121, size = 0, normalized size = 0.

$$\int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*f^(-c - d*x) + c*f^(c + d*x)), x]
```

```
[Out] Integrate[x/(a + b*f^(-c - d*x) + c*f^(c + d*x)), x]
```

**Maple [B]** time = 0.067, size = 433, normalized size = 2.1

$$\frac{x}{d \ln(f)} \ln\left(\left(2bf^{-dx}f^{-c} + \sqrt{a^2-4bc} + a\right)\left(a + \sqrt{a^2-4bc}\right)^{-1}\right) \frac{1}{\sqrt{a^2-4bc}} - \frac{x}{d \ln(f)} \ln\left(\left(-2bf^{-dx}f^{-c} + \sqrt{a^2-4bc} - a\right)\left(a - \sqrt{a^2-4bc}\right)^{-1}\right) \frac{1}{\sqrt{a^2-4bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)), x)
```

```
[Out] 1/ln(f)/d/(a^2-4*b*c)^(1/2)*ln((2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)+a)/(a
+(a^2-4*b*c)^(1/2)))*x-1/ln(f)/d/(a^2-4*b*c)^(1/2)*ln((-2*b*f^(-d*x)*f^(-c)
+(a^2-4*b*c)^(1/2)-a)/(-a+(a^2-4*b*c)^(1/2)))*x+1/ln(f)/d^2/(a^2-4*b*c)^(1/
2)*ln((2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)+a)/(a+(a^2-4*b*c)^(1/2)))*c-1/
ln(f)/d^2/(a^2-4*b*c)^(1/2)*ln((-2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)-a)/(-
```

$$-a+(a^2-4*bc)^{(1/2)}) * c + 1/\ln(f)^2/d^2/(a^2-4*bc)^{(1/2)} * \operatorname{dilog}((-2*bf^{(-d*x)} * f^{(-c)} + (a^2-4*bc)^{(1/2)} - a)/(-a+(a^2-4*bc)^{(1/2)})) - 1/\ln(f)^2/d^2/(a^2-4*bc)^{(1/2)} * \operatorname{dilog}((2*bf^{(-d*x)} * f^{(-c)} + (a^2-4*bc)^{(1/2)} + a)/(a+(a^2-4*bc)^{(1/2)})) + 2/\ln(f)/d^2 * c/(-a^2+4*bc)^{(1/2)} * \arctan((2*bf^{(-d*x)} * f^{(-c)} + a)/(-a^2+4*bc)^{(1/2)})$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.3956, size = 845, normalized size = 4.16

$$bc\sqrt{\frac{a^2-4bc}{b^2}} \log\left(2cf^{dx+c} + b\sqrt{\frac{a^2-4bc}{b^2}} + a\right) \log(f) - bc\sqrt{\frac{a^2-4bc}{b^2}} \log\left(2cf^{dx+c} - b\sqrt{\frac{a^2-4bc}{b^2}} + a\right) \log(f) + (bdx + bc)\sqrt{\frac{a^2-4bc}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="fricas")

[Out] (b\*c\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(2\*c\*f^(d\*x + c) + b\*sqrt((a^2 - 4\*b\*c)/b^2) + a)\*log(f) - b\*c\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(2\*c\*f^(d\*x + c) - b\*sqrt((a^2 - 4\*b\*c)/b^2) + a)\*log(f) + (b\*d\*x + b\*c)\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(f) \*log(1/2\*((b\*sqrt((a^2 - 4\*b\*c)/b^2) + a)\*f^(d\*x + c) + 2\*b)/b) - (b\*d\*x + b\*c)\*sqrt((a^2 - 4\*b\*c)/b^2)\*log(f)\*log(-1/2\*((b\*sqrt((a^2 - 4\*b\*c)/b^2) - a)\*f^(d\*x + c) - 2\*b)/b) + b\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(-1/2\*((b\*sqrt((a^2 - 4\*b\*c)/b^2) + a)\*f^(d\*x + c) + 2\*b)/b + 1) - b\*sqrt((a^2 - 4\*b\*c)/b^2)\*dilog(1/2\*((b\*sqrt((a^2 - 4\*b\*c)/b^2) - a)\*f^(d\*x + c) - 2\*b)/b + 1))/((a^2 - 4\*b\*c)\*d^2\*log(f)^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*f\*\*(-d\*x-c)+c\*f\*\*(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cf^{dx+c} + bf^{-dx-c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x/(c*f^(d*x + c) + b*f^(-d*x - c) + a), x)
```

$$3.542 \quad \int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}} dx$$

**Optimal.** Leaf size=310

$$\frac{2x \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{2x \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{a^2-4bc+a}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{2 \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d^3 \log^3(f)\sqrt{a^2-4bc}} + \frac{2 \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{\sqrt{a^2-4bc+a}}\right)}{d^3 \log^3(f)\sqrt{a^2-4bc}}$$

```
[Out] (x^2*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]
*d*Log[f]) - (x^2*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt
[a^2 - 4*b*c]*d*Log[f]) + (2*x*PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2
- 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - (2*x*PolyLog[2, (-2*c*f^(c +
d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - (2*Poly
Log[3, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^3*
Log[f]^3) + (2*PolyLog[3, (-2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqr
t[a^2 - 4*b*c]*d^3*Log[f]^3)
```

**Rubi [A]** time = 0.659629, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {2267, 2264, 2190, 2531, 2282, 6589}

$$\frac{2x \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{2x \operatorname{PolyLog}\left(2, -\frac{2cf^{c+dx}}{\sqrt{a^2-4bc+a}}\right)}{d^2 \log^2(f)\sqrt{a^2-4bc}} - \frac{2 \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d^3 \log^3(f)\sqrt{a^2-4bc}} + \frac{2 \operatorname{PolyLog}\left(3, -\frac{2cf^{c+dx}}{\sqrt{a^2-4bc+a}}\right)}{d^3 \log^3(f)\sqrt{a^2-4bc}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*f^(-c - d*x) + c*f^(c + d*x)), x]
```

```
[Out] (x^2*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]
*d*Log[f]) - (x^2*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt
[a^2 - 4*b*c]*d*Log[f]) + (2*x*PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2
- 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - (2*x*PolyLog[2, (-2*c*f^(c +
d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2) - (2*Poly
Log[3, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^3*
Log[f]^3) + (2*PolyLog[3, (-2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])])/(Sqr
t[a^2 - 4*b*c]*d^3*Log[f]^3)
```

#### Rule 2267

```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] := Int[(u*F^
v)/(c + a*F^v + b*F^(2*v)), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[Coefficient[v, x, 1]], GtQ[Coefficient[v, x, 1
], 0], LtQ[LeafCount[v], LeafCount[w]]]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_)^(m_.)))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/
((a_) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp
```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}} dx &= \int \frac{f^{c+dx} x^2}{b + af^{c+dx} + cf^{2(c+dx)}} dx \\
&= \frac{(2c) \int \frac{f^{c+dx} x^2}{a - \sqrt{a^2 - 4bc} + 2cf^{c+dx}} dx}{\sqrt{a^2 - 4bc}} - \frac{(2c) \int \frac{f^{c+dx} x^2}{a + \sqrt{a^2 - 4bc} + 2cf^{c+dx}} dx}{\sqrt{a^2 - 4bc}} \\
&= \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} - \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} - \frac{2 \int x \log\left(1 + \frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bcd} \log(f)} + \frac{2 \int x \log\left(1 + \frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right) dx}{\sqrt{a^2 - 4bcd} \log(f)} \\
&= \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} - \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} + \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)} - \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)} \\
&= \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} - \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} + \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)} - \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)} \\
&= \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} - \frac{x^2 \log\left(1 + \frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd} \log(f)} + \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a - \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)} - \frac{2x \operatorname{Li}_2\left(-\frac{2cf^{c+dx}}{a + \sqrt{a^2 - 4bc}}\right)}{\sqrt{a^2 - 4bcd^2} \log^2(f)}
\end{aligned}$$

**Mathematica [F]** time = 0.191929, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

[Out] Integrate[x^2/(a + b\*f^(-c - d\*x) + c\*f^(c + d\*x)), x]

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b f^{-dx-c} + c f^{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x)

[Out] int(x^2/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 1.41278, size = 1164, normalized size = 3.75

$$bc^2 \sqrt{\frac{a^2-4bc}{b^2}} \log\left(2cf^{dx+c} + b\sqrt{\frac{a^2-4bc}{b^2}} + a\right) \log(f)^2 - bc^2 \sqrt{\frac{a^2-4bc}{b^2}} \log\left(2cf^{dx+c} - b\sqrt{\frac{a^2-4bc}{b^2}} + a\right) \log(f)^2 - 2bdx \sqrt{\frac{a^2-4bc}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*f^(-d\*x-c)+c\*f^(d\*x+c)),x, algorithm="fricas")

[Out]  $-(b*c^2*\sqrt{(a^2 - 4*b*c)/b^2})*\log(2*c*f^{(d*x + c) + b*\sqrt{(a^2 - 4*b*c)/b^2}} + a)*\log(f)^2 - b*c^2*\sqrt{(a^2 - 4*b*c)/b^2}*\log(2*c*f^{(d*x + c) - b*\sqrt{(a^2 - 4*b*c)/b^2}} + a)*\log(f)^2 - 2*b*d*x*\sqrt{(a^2 - 4*b*c)/b^2}*dilog(-1/2*((b*\sqrt{(a^2 - 4*b*c)/b^2} + a)*f^{(d*x + c) + 2*b}/b + 1)*\log(f) + 2*b*d*x*\sqrt{(a^2 - 4*b*c)/b^2})*dilog(1/2*((b*\sqrt{(a^2 - 4*b*c)/b^2} - a)*f^{(d*x + c) - 2*b}/b + 1)*\log(f) - (b*d^2*x^2 - b*c^2)*\sqrt{(a^2 - 4*b*c)/b^2})*\log(f)^2*\log(1/2*((b*\sqrt{(a^2 - 4*b*c)/b^2} + a)*f^{(d*x + c) + 2*b}/b) + (b*d^2*x^2 - b*c^2)*\sqrt{(a^2 - 4*b*c)/b^2})*\log(f)^2*\log(-1/2*((b*\sqrt{(a^2 - 4*b*c)/b^2} - a)*f^{(d*x + c) - 2*b}/b) + 2*b*\sqrt{(a^2 - 4*b*c)/b^2})*polylog(3, -1/2*(b*\sqrt{(a^2 - 4*b*c)/b^2} + a)*f^{(d*x + c)}/b) - 2*b*\sqrt{(a^2 - 4*b*c)/b^2})*polylog(3, 1/2*(b*\sqrt{(a^2 - 4*b*c)/b^2} - a)*f^{(d*x + c)}/b))/((a^2 - 4*b*c)*d^3*\log(f)^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*f**(-d*x-c)+c*f**(d*x+c)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cf^{dx+c} + bf^{-dx-c} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="giac")`

[Out] `integrate(x^2/(c*f^(d*x + c) + b*f^(-d*x - c) + a), x)`



$$3.543 \quad \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{df + (ef+dg)x + egx^2} dx$$

**Optimal.** Leaf size=52

$$\text{Unintegrable} \left( \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{x(dg + ef) + df + egx^2}, x \right)$$

[Out] Unintegrable[(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^n/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

**Rubi [A]** time = 0.149229, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{df + (ef + dg)x + egx^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^n/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] Defer[Int][(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^n/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

Rubi steps

$$\int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{df + (ef + dg)x + egx^2} dx = \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{df + (ef + dg)x + egx^2} dx$$

**Mathematica [A]** time = 0.663491, size = 0, normalized size = 0.

$$\int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^n}{df + (ef + dg)x + egx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^n/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^n/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

**Maple [A]** time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left( a + bF^{c\sqrt{ex+d}\frac{1}{\sqrt{gx+f}}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} b + a \right)^n}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="maxima")

[Out] integrate((F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*b + a)^n/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)))\*\*n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x\*\*2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} b + a \right)^n}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^n/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="giac")

[Out] integrate((F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*b + a)^n/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x), x)

$$3.544 \quad \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^3}{df + (ef+dg)x + egx^2} dx$$

**Optimal.** Leaf size=154

$$\frac{6a^2b\text{Ei}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{6ab^2\text{Ei}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b^3\text{Ei}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

[Out] (6\*a^2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (6\*a\*b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*b^3\*ExpIntegralEi[(3\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*a^3\*Log[Sqrt[d + e\*x]/Sqrt[f + g\*x]])/(e\*f - d\*g)

**Rubi [A]** time = 0.264809, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.06$ , Rules used = {2290, 2183, 2178}

$$\frac{6a^2b\text{Ei}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{6ab^2\text{Ei}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b^3\text{Ei}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

Antiderivative was successfully verified.

[In] Int[(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^3/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] (6\*a^2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (6\*a\*b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*b^3\*ExpIntegralEi[(3\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*a^3\*Log[Sqrt[d + e\*x]/Sqrt[f + g\*x]])/(e\*f - d\*g)

#### Rule 2290

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + bF^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[B\*e\*g - C\*(e\*f + d\*g), 0] && IGtQ[n, 0]

#### Rule 2183

Int[((a\_) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^((n\_.)^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

#### Rule 2178

Int[(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df + (ef + dg)x + egx^2} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{(a+bF^{cx})^3}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(\frac{a^3}{x} + \frac{3a^2bF^{cx}}{x} + \frac{3ab^2F^{2cx}}{x} + \frac{b^3F^{3cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2a^3 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{(6a^2b) \operatorname{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{(6ab^2) \operatorname{Subst}\left(\int \frac{F^{2cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{6a^2b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{6ab^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{2b^3 \operatorname{Ei}\left(\frac{3c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{2a^3 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg}
\end{aligned}$$

**Mathematica [F]** time = 1.51019, size = 0, normalized size = 0.

$$\int \frac{\left(a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df + (ef + dg)x + egx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^3/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^3/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left(a + bF^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^3/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^3/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3 \left( \frac{\log(ex + d)}{ef - dg} - \frac{\log(gx + f)}{ef - dg} \right) + b^3 \int \frac{F^{\frac{3\sqrt{ex+dc}}{\sqrt{gx+f}}}}{egx^2 + df + (ef + dg)x} dx + 3ab^2 \int \frac{F^{\frac{2\sqrt{ex+dc}}{\sqrt{gx+f}}}}{egx^2 + df + (ef + dg)x} dx + 3a^2b \int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}}}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^3/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x, algorithm="maxima")

```
[Out] a^3*(log(e*x + d)/(e*f - d*g) - log(g*x + f)/(e*f - d*g)) + b^3*integrate(F
^(3*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g)*x), x) + 3*
a*b^2*integrate(F^(2*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f +
d*g)*x), x) + 3*a^2*b*integrate(F^(sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2
+ d*f + (e*f + d*g)*x), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x^
2),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**3/(d*f+(d*g+e*f)*x+e*
g*x**2),x)
```

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( F \frac{\sqrt{ex+dc}}{\sqrt{gx+f}} b + a \right)^3}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x^
2),x, algorithm="giac")
```

```
[Out] integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^3/(e*g*x^2 + d*f + (e*f
+ d*g)*x), x)
```

$$3.545 \quad \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}} \right)^2}{df + (ef+dg)x + egx^2} dx$$

**Optimal.** Leaf size=112

$$\frac{2a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{4ab \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

[Out]  $(4*a*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]])/(e*f - d*g) + (2*b^2*ExpIntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]])/(e*f - d*g) + (2*a^2*Log[Sqrt[d + e*x]/Sqrt[f + g*x]])/(e*f - d*g)$

**Rubi [A]** time = 0.22729, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.06$ , Rules used = {2290, 2183, 2178}

$$\frac{2a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{4ab \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + bF^{((c\sqrt{d + ex})/\sqrt{f + gx}))^2}/(df + (ef + dg)x + egx^2), x]$

[Out]  $(4*a*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]])/(e*f - d*g) + (2*b^2*ExpIntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]])/(e*f - d*g) + (2*a^2*Log[Sqrt[d + e*x]/Sqrt[f + g*x]])/(e*f - d*g)$

#### Rule 2290

$\text{Int}[(a + b(F)^{((c)\sqrt{(d) + (e)(x)})}/\sqrt{(f) + (g)(x)})^n]/((A) + (B)(x) + (C)(x)^2), x\_Symbol] \rightarrow \text{Dist}[(2*eg)/(C*(ef - dg)), \text{Subst}[\text{Int}[(a + bF^{(c*x)})^n/x, x], x, \text{Sqrt}[d + e*x]/\text{Sqrt}[f + g*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, A, B, C, F\}, x] \&\& \text{EqQ}[C*d*f - A*e*g, 0] \&\& \text{EqQ}[B*e*g - C*(e*f + d*g), 0] \&\& \text{IGtQ}[n, 0]$

#### Rule 2183

$\text{Int}[(a + b(F)^{((g)(e) + (f)(x))})^n]^{(p)}((c) + (d)(x))^m, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b(F)^{(g*(e + f*x))})^n]^p, x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

#### Rule 2178

$\text{Int}[(F)^{((g)(e) + (f)(x))}/((c) + (d)(x)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}) * \text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2}{df + (ef + dg)x + egx^2} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{(a+bF^{cx})^2}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(\frac{a^2}{x} + \frac{2abF^{cx}}{x} + \frac{b^2F^{2cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{(4ab) \operatorname{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{(2b^2) \operatorname{Subst}\left(\int \frac{F^{2cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{4ab \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{2b^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{2a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg}
\end{aligned}$$

**Mathematica [F]** time = 1.3883, size = 0, normalized size = 0.

$$\int \frac{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2}{df + (ef + dg)x + egx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

**Maple [F]** time = 0.071, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left(a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{gx+f}}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \left( \frac{\log(ex + d)}{ef - dg} - \frac{\log(gx + f)}{ef - dg} \right) + b^2 \int \frac{F \frac{2\sqrt{ex+dc}}{\sqrt{gx+f}}}{egx^2 + df + (ef + dg)x} dx + 2ab \int \frac{F \frac{\sqrt{ex+dc}}{\sqrt{gx+f}}}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x, algorithm="maxima")



```
[Out] a^2*(log(e*x + d)/(e*f - d*g) - log(g*x + f)/(e*f - d*g)) + b^2*integrate(F
^(2*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g)*x), x) + 2*
a*b*integrate(F^(sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g
)*x), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^
2),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**2/(d*f+(d*g+e*f)*x+e*
g*x**2),x)
```

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( F \frac{\sqrt{ex+dc}}{\sqrt{gx+f}} b + a \right)^2}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^
2),x, algorithm="giac")
```

```
[Out] integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^2/(e*g*x^2 + d*f + (e*f
+ d*g)*x), x)
```

$$3.546 \quad \int \frac{a+bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}{df+(ef+dg)x+egx^2} dx$$

**Optimal.** Leaf size=70

$$\frac{2a \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

[Out] (2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*a\*Log[Sqrt[d + e\*x]/Sqrt[f + g\*x]])/(e\*f - d\*g)

**Rubi [A]** time = 0.122379, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2290, 14, 2178}

$$\frac{2a \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg} + \frac{2b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef-dg}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] (2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[f + g\*x]])/(e\*f - d\*g) + (2\*a\*Log[Sqrt[d + e\*x]/Sqrt[f + g\*x]])/(e\*f - d\*g)

#### Rule 2290

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[B\*e\*g - C\*(e\*f + d\*g), 0] && IGtQ[n, 0]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_)^m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df + (ef + dg)x + egx^2} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{a+bF^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(\frac{a}{x} + \frac{bF^{cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2a \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{(2b) \operatorname{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg} \\
&= \frac{2b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{f+gx}}\right)}{ef - dg} + \frac{2a \log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef - dg}
\end{aligned}$$

**Mathematica [F]** time = 0.508173, size = 0, normalized size = 0.

$$\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df + (ef + dg)x + egx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))/(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2), x]

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left( a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{gx+f}}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a \left( \frac{\log(ex + d)}{ef - dg} - \frac{\log(gx + f)}{ef - dg} \right) + b \int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}}}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x, algorithm="maxima")

[Out] a\*(log(e\*x + d)/(e\*f - d\*g) - log(g\*x + f)/(e\*f - d\*g)) + b\*integrate(F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} b + a}{(d+ex)(f+gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x\*\*2),x)

[Out] Integral((F\*\*(c\*sqrt(d + e\*x)/sqrt(f + g\*x))\*b + a)/((d + e\*x)\*(f + g\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} b + a}{egx^2 + df + (ef + dg)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="giac")

[Out] integrate((F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*b + a)/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x), x)

$$3.547 \quad \int \frac{1}{df+(ef+dg)x+egx^2} dx$$

**Optimal.** Leaf size=36

$$\frac{\log(d+ex)}{ef-dg} - \frac{\log(f+gx)}{ef-dg}$$

[Out] Log[d + e\*x]/(e\*f - d\*g) - Log[f + g\*x]/(e\*f - d\*g)

**Rubi [A]** time = 0.0145575, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {616, 31}

$$\frac{\log(d+ex)}{ef-dg} - \frac{\log(f+gx)}{ef-dg}$$

Antiderivative was successfully verified.

[In] Int[(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)^(-1), x]

[Out] Log[d + e\*x]/(e\*f - d\*g) - Log[f + g\*x]/(e\*f - d\*g)

#### Rule 616

Int[((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(-1), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c\*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c\*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c] && PerfectSquareQ[b^2 - 4\*a\*c]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{df+(ef+dg)x+egx^2} dx &= -\frac{(eg) \int \frac{1}{ef+egx} dx}{ef-dg} + \frac{(eg) \int \frac{1}{dg+egx} dx}{ef-dg} \\ &= \frac{\log(d+ex)}{ef-dg} - \frac{\log(f+gx)}{ef-dg} \end{aligned}$$

**Mathematica [A]** time = 0.0103459, size = 26, normalized size = 0.72

$$\frac{\log(d+ex) - \log(f+gx)}{ef-dg}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)^(-1), x]

[Out] (Log[d + e\*x] - Log[f + g\*x])/(e\*f - d\*g)

---

**Maple [A]** time = 0.007, size = 37, normalized size = 1.

$$-\frac{\ln(ex + d)}{dg - fe} + \frac{\ln(gx + f)}{dg - fe}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x)

[Out] -1/(d\*g-e\*f)\*ln(e\*x+d)+1/(d\*g-e\*f)\*ln(g\*x+f)

---

**Maxima [A]** time = 0.974656, size = 49, normalized size = 1.36

$$\frac{\log(ex + d)}{ef - dg} - \frac{\log(gx + f)}{ef - dg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="maxima")

[Out] log(e\*x + d)/(e\*f - d\*g) - log(g\*x + f)/(e\*f - d\*g)

---

**Fricas [A]** time = 1.61521, size = 58, normalized size = 1.61

$$\frac{\log(ex + d) - \log(gx + f)}{ef - dg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="fricas")

[Out] (log(e\*x + d) - log(g\*x + f))/(e\*f - d\*g)

---

**Sympy [B]** time = 0.307197, size = 128, normalized size = 3.56

$$\frac{\log\left(x + \frac{-\frac{d^2g^2}{dg-ef} + \frac{2defg}{dg-ef} + dg - \frac{e^2f^2}{dg-ef} + ef}{2eg}\right)}{dg - ef} - \frac{\log\left(x + \frac{\frac{d^2g^2}{dg-ef} - \frac{2defg}{dg-ef} + dg + \frac{e^2f^2}{dg-ef} + ef}{2eg}\right)}{dg - ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*f+(d\*g+e\*f)\*x+e\*g\*x\*\*2),x)

[Out] log(x + (-d\*\*2\*g\*\*2/(d\*g - e\*f) + 2\*d\*e\*f\*g/(d\*g - e\*f) + d\*g - e\*\*2\*f\*\*2/(d\*g - e\*f) + e\*f)/(2\*e\*g))/(d\*g - e\*f) - log(x + (d\*\*2\*g\*\*2/(d\*g - e\*f) - 2\*d\*e\*f\*g/(d\*g - e\*f) + d\*g + e\*\*2\*f\*\*2/(d\*g - e\*f) + e\*f)/(2\*e\*g))/(d\*g - e\*f)

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*f+(d*g+e*f)*x+e*g*x^2),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.548 \quad \int \frac{1}{\left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right) (df + (ef + dg)x + egx^2)} dx$$

**Optimal.** Leaf size=52

$$\text{Unintegrable} \left[ \frac{1}{\left( x(dg + ef) + df + egx^2 \right) \left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right)}, x \right]$$

[Out] Unintegrable[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

**Rubi [A]** time = 0.152223, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right) (df + (ef + dg)x + egx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

[Out] Defer[Int][1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

Rubi steps

$$\int \frac{1}{\left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right) (df + (ef + dg)x + egx^2)} dx = \int \frac{1}{\left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right) (df + (ef + dg)x + egx^2)} dx$$

**Mathematica [A]** time = 0.378046, size = 0, normalized size = 0.

$$\int \frac{1}{\left( a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}} \right) (df + (ef + dg)x + egx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

[Out] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]



---

**Maple [A]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left( a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{gx+f}}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x)

[Out] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(egx^2 + df + (ef + dg)x) \left( F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} b + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="maxima")

[Out] integrate(1/((e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)\*(F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*b + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{aegx^2 + adf + (begx^2 + bdf + (bef + bdg)x) F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} + (aef + adg)x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="fricas")

[Out] integral(1/(a\*e\*g\*x^2 + a\*d\*f + (b\*e\*g\*x^2 + b\*d\*f + (b\*e\*f + b\*d\*g)\*x)\*F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f)) + (a\*e\*f + a\*d\*g)\*x), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x\*\*2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(egx^2 + df + (ef + dg)x) \left( F^{\frac{\sqrt{ex+d}}{\sqrt{gx+f}}} b + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2),x, algorithm="giac")

[Out] integrate(1/((e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)\*(F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*b + a)), x)

$$3.549 \quad \int \frac{1}{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2 (df + (ef+dg)x + egx^2)} dx$$

**Optimal.** Leaf size=52

$$\text{Unintegrable} \left[ \frac{1}{(x(dg + ef) + df + egx^2) \left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2}, x \right]$$

[Out] Unintegrable[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

**Rubi [A]** time = 0.148908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2 (df + (ef + dg)x + egx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

[Out] Defer[Int][1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

Rubi steps

$$\int \frac{1}{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2 (df + (ef + dg)x + egx^2)} dx = \int \frac{1}{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2 (df + (ef + dg)x + egx^2)} dx$$

**Mathematica [A]** time = 1.2715, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{f+gx}}\right)^2 (df + (ef + dg)x + egx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

[Out] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[f + g\*x]))^2\*(d\*f + (e\*f + d\*g)\*x + e\*g\*x^2)), x]

**Maple [A]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{df + (dg + fe)x + egx^2} \left( a + bF^{c\sqrt{ex+d}\frac{1}{\sqrt{gx+f}}} \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

[Out] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{gx+f}}{(ef-dg)\sqrt{ex+d}F^{\frac{\sqrt{ex+d}}{\sqrt{gx+f}}}} + \int \frac{abc \log(F) + (ef-dg)\sqrt{ex+d}a^2c \log(F)}{(abceg x^2 \log(F) + abcd f \log(F) + (ef+dg)abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x, algorithm="maxima")

[Out] 2\*sqrt(g\*x + f)/((e\*f - d\*g)\*sqrt(e\*x + d)\*F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f))\*a\*b\*c\*log(F) + (e\*f - d\*g)\*sqrt(e\*x + d)\*a^2\*c\*log(F)) + integrate((sqrt(e\*x + d)\*c\*log(F) + sqrt(g\*x + f))/((a\*b\*c\*e\*g\*x^2\*log(F) + a\*b\*c\*d\*f\*log(F) + (e\*f + d\*g)\*a\*b\*c\*x\*log(F))\*sqrt(e\*x + d)\*F^(sqrt(e\*x + d)\*c/sqrt(g\*x + f)) + (a^2\*c\*e\*g\*x^2\*log(F) + a^2\*c\*d\*f\*log(F) + (e\*f + d\*g)\*a^2\*c\*x\*log(F))\*sqrt(e\*x + d)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(g\*x+f)^(1/2)))^2/(d\*f+(d\*g+e\*f)\*x+e\*g\*x^2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))*2/(d*f+(d*g+e*f)*x+
e*g*x**2),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(egx^2 + df + (ef + dg)x) \left( F^{\frac{\sqrt{ex+dc}}{\sqrt{gx+f}}} b + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*
x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*g*x^2 + d*f + (e*f + d*g)*x)*(F^(sqrt(e*x + d)*c/sqrt(g*x +
f))*b + a)^2), x)
```

$$3.550 \quad \int \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=49

$$\text{Unintegrable} \left( \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2}, x \right)$$

[Out] Unintegrable[(a + bF^((c\*sqrt[d + e\*x])/sqrt[d\*f - e\*f\*x]))^n/(d^2 - e^2\*x^2), x]

**Rubi [A]** time = 0.229538, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + bF^((c\*sqrt[d + e\*x])/sqrt[d\*f - e\*f\*x]))^n/(d^2 - e^2\*x^2), x]

[Out] Defer[Int] [(a + bF^((c\*sqrt[d + e\*x])/sqrt[d\*f - e\*f\*x]))^n/(d^2 - e^2\*x^2), x]

Rubi steps

$$\int \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2} dx = \int \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2} dx$$

**Mathematica [A]** time = 0.71925, size = 0, normalized size = 0.

$$\int \frac{\left( \frac{c\sqrt{d+ex}}{a+bF\sqrt{df-efx}} \right)^n}{d^2-e^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + bF^((c\*sqrt[d + e\*x])/sqrt[d\*f - e\*f\*x]))^n/(d^2 - e^2\*x^2), x]

[Out] Integrate[(a + bF^((c\*sqrt[d + e\*x])/sqrt[d\*f - e\*f\*x]))^n/(d^2 - e^2\*x^2), x]

---

**Maple [A]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2 x^2 + d^2} \left( a + b F^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^n/(-e^2\*x^2+d^2),x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^n/(-e^2\*x^2+d^2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\left( F^{\frac{\sqrt{ex+dc}}{\sqrt{-efx+df}}} b + a \right)^n}{e^2 x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^n/(-e^2\*x^2+d^2),x,  
algorithm="maxima")

[Out] -integrate((F^(sqrt(e\*x + d)\*c/sqrt(-e\*f\*x + d\*f))\*b + a)^n/(e^2\*x^2 - d^2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^n/(-e^2\*x^2+d^2),x,  
algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(-e\*f\*x+d\*f)\*\*(1/2)))\*\*n/(-e\*\*2\*x\*\*2+d\*  
\*2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(F^{\frac{\sqrt{ex+dc}}{\sqrt{-efx+df}}b+a}\right)^n}{e^2x^2-d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f))^(1/2)))^n/(-e^2\*x^2+d^2),x,  
algorithm="giac")

[Out] integrate(-(F^(sqrt(e\*x + d)\*c/sqrt(-e\*f\*x + d\*f))\*b + a)^n/(e^2\*x^2 - d^2)  
, x)



$$3.551 \quad \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^3}{d^2 - e^2x^2} dx$$

**Optimal.** Leaf size=152

$$\frac{3a^2b\text{Ei}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{3ab^2\text{Ei}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^3\text{Ei}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}$$

[Out] (3\*a^2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (3\*a\*b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (b^3\*ExpIntegralEi[(3\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a^3\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

**Rubi [A]** time = 0.326986, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$ , Rules used = {2291, 2183, 2178}

$$\frac{3a^2b\text{Ei}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{3ab^2\text{Ei}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^3\text{Ei}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + bF^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^3/(d^2 - e^2\*x^2), x]

[Out] (3\*a^2\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (3\*a\*b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (b^3\*ExpIntegralEi[(3\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a^3\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/(A\_) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + bF^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2183

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^((n\_.))^((p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}}\right)^3}{d^2 - e^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+bF^{cx})^3}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x} + \frac{3a^2bF^{cx}}{x} + \frac{3ab^2F^{2cx}}{x} + \frac{b^3F^{3cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{a^3 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{(3a^2b) \text{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{(3ab^2) \text{Subst}\left(\int \frac{F^{2cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{3a^2b \text{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{3ab^2 \text{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^3 \text{Ei}\left(\frac{3c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a^3 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}
\end{aligned}$$

**Mathematica [F]** time = 1.31904, size = 0, normalized size = 0.

$$\int \frac{\left(a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}}\right)^3}{d^2 - e^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^3/(d^2 - e^2\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^3/(d^2 - e^2\*x^2), x]

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2x^2 + d^2} \left(a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^3/(-e^2\*x^2+d^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^3/(-e^2\*x^2+d^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^3 \left( \frac{\log(ex + d)}{de} - \frac{\log(ex - d)}{de} \right) - b^3 \int \frac{F^{\frac{3\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx - 3ab^2 \int \frac{F^{\frac{2\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx - 3a^2b \int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^3/(-e^2\*x^2+d^2), x, algorithm="maxima")

```
[Out] 1/2*a^3*(log(e*x + d)/(d*e) - log(e*x - d)/(d*e)) - b^3*integrate(F^(3*sqrt
(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x) - 3*a*b^2*integrate
(F^(2*sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x) - 3*a
^2*b*integrate(F^(sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2)
, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{a^3 + \frac{3a^2b}{F \frac{\sqrt{-efx+df}\sqrt{ex+dc}}{efx-df}} + \frac{3ab^2}{F \frac{2\sqrt{-efx+df}\sqrt{ex+dc}}{efx-df}} + \frac{b^3}{F \frac{3\sqrt{-efx+df}\sqrt{ex+dc}}{efx-df}}}{e^2x^2 - d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x,
algorithm="fricas")
```

```
[Out] integral(-(a^3 + 3*a^2*b/F^(sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f
)) + 3*a*b^2/F^(2*sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)) + b^3/F
^(3*sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)))/(e^2*x^2 - d^2), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^3}{-d^2 + e^2x^2} dx - \int \frac{F \frac{3c\sqrt{d+ex}}{\sqrt{df-efx}} b^3}{-d^2 + e^2x^2} dx - \int \frac{3F \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} a^2b}{-d^2 + e^2x^2} dx - \int \frac{3F \frac{2c\sqrt{d+ex}}{\sqrt{df-efx}} ab^2}{-d^2 + e^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))**3/(-e**2*x**2+d*
*2),x)
```

```
[Out] -Integral(a**3/(-d**2 + e**2*x**2), x) - Integral(F**(3*c*sqrt(d + e*x)/sqr
t(d*f - e*f*x))*b**3/(-d**2 + e**2*x**2), x) - Integral(3*F**(c*sqrt(d + e*
x)/sqrt(d*f - e*f*x))*a**2*b/(-d**2 + e**2*x**2), x) - Integral(3*F**(2*c*s
qrt(d + e*x)/sqrt(d*f - e*f*x))*a*b**2/(-d**2 + e**2*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left( F \frac{\sqrt{ex+dc}}{\sqrt{-efx+df}} b + a \right)^3}{e^2x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x,
algorithm="giac")
```

```
[Out] integrate(-(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)^3/(e^2*x^2 - d^2)
, x)
```

$$3.552 \quad \int \frac{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2}{d^2 - e^2x^2} dx$$

**Optimal.** Leaf size=110

$$\frac{a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{2ab \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de}$$

[Out] (2\*a\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a^2\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

**Rubi [A]** time = 0.303162, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$ , Rules used = {2291, 2183, 2178}

$$\frac{a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{2ab \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^2 \operatorname{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2/(d^2 - e^2\*x^2), x]

[Out] (2\*a\*b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (b^2\*ExpIntegralEi[(2\*c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a^2\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2)), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2183

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^((n\_.))^((p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2 - e^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+bF^{cx})^2}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x} + \frac{2abF^{cx}}{x} + \frac{b^2F^{2cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{(2ab) \text{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{b^2 \text{Subst}\left(\int \frac{F^{2cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{2ab \text{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^2 \text{Ei}\left(\frac{2c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a^2 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}
\end{aligned}$$

**Mathematica [F]** time = 1.2677, size = 0, normalized size = 0.

$$\int \frac{\left(a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2 - e^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2/(d^2 - e^2\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2/(d^2 - e^2\*x^2), x]

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2x^2 + d^2} \left(a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 \left( \frac{\log(ex+d)}{de} - \frac{\log(ex-d)}{de} \right) - b^2 \int \frac{F^{\frac{2\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx - 2ab \int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x, algorithm="maxima")

[Out]  $\frac{1}{2}a^2(\log(ex + d)/(d*e) - \log(ex - d)/(d*e)) - b^2 \int \frac{F^2 \sqrt{c} / (\sqrt{-ex + d} \sqrt{f})}{e^2 x^2 - d^2} dx - 2ab \int \frac{F \sqrt{c} / (\sqrt{-ex + d} \sqrt{f})}{e^2 x^2 - d^2} dx$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( a^2 + \frac{2ab}{\frac{\sqrt{-efx+df}\sqrt{ex+dc}}{F} \frac{efx-df}{F}} + \frac{b^2}{\frac{2\sqrt{-efx+df}\sqrt{ex+dc}}{F} \frac{efx-df}{F}} \right)}{e^2 x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*F^(c*(ex+d)^(1/2)/(-ef*x+df)^(1/2)))^2/(-e^2*x^2+d^2), x, algorithm="fricas")`

[Out]  $\int \frac{-(a^2 + 2ab/F(\sqrt{-ef*x + df})\sqrt{ex + d}*c/(ef*x - df)) + b^2/F^2(\sqrt{-ef*x + df})\sqrt{ex + d}*c/(ef*x - df))}{e^2 x^2 - d^2} dx$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2}{-d^2 + e^2 x^2} dx - \int \frac{\frac{2c\sqrt{d+ex}}{F\sqrt{df-efx}} b^2}{-d^2 + e^2 x^2} dx - \int \frac{\frac{c\sqrt{d+ex}}{2F\sqrt{df-efx}} ab}{-d^2 + e^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*F**(c*(ex+d)**(1/2)/(-ef*x+df)**(1/2)))**2/(-e**2*x**2+d**2), x)`

[Out]  $-\text{Integral}(a^2/(-d^2 + e^2 x^2), x) - \text{Integral}(F^2 c \sqrt{d + ex} / \sqrt{df - ef x}) b^2 / (-d^2 + e^2 x^2), x) - \text{Integral}(2 F c \sqrt{d + ex} / \sqrt{df - ef x}) a b / (-d^2 + e^2 x^2), x)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( \frac{\sqrt{ex+dc}}{F\sqrt{-efx+df}} b + a \right)^2}{e^2 x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*F^(c*(ex+d)^(1/2)/(-ef*x+df)^(1/2)))^2/(-e^2*x^2+d^2), x, algorithm="giac")`

[Out]  $\int \frac{-(F(\sqrt{ex + d})c/\sqrt{-ef*x + df})b + a)^2}{e^2 x^2 - d^2} dx$

$$3.553 \quad \int \frac{a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}{d^2 - e^2x^2} dx$$

**Optimal.** Leaf size=68

$$\frac{a \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de}$$

[Out] (b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

**Rubi [A]** time = 0.179581, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2291, 14, 2178}

$$\frac{a \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{b \operatorname{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))/(d^2 - e^2\*x^2), x]

[Out] (b\*ExpIntegralEi[(c\*Sqrt[d + e\*x]\*Log[F])/Sqrt[d\*f - e\*f\*x]])/(d\*e) + (a\*Log[Sqrt[d + e\*x]/Sqrt[d\*f - e\*f\*x]])/(d\*e)

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2)), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2 - e^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{a+bF^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x} + \frac{bF^{cx}}{x}\right) dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{a \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} + \frac{b \text{Subst}\left(\int \frac{F^{cx}}{x} dx, x, \frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de} \\
&= \frac{b \text{Ei}\left(\frac{c\sqrt{d+ex} \log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}
\end{aligned}$$

**Mathematica [F]** time = 0.454371, size = 0, normalized size = 0.

$$\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2 - e^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))/(d^2 - e^2\*x^2), x]

[Out] Integrate[(a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))/(d^2 - e^2\*x^2), x]

**Maple [F]** time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2x^2 + d^2} \left( a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x)

[Out] int((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a \left( \frac{\log(ex + d)}{de} - \frac{\log(ex - d)}{de} \right) - b \int \frac{F^{\frac{\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}}{e^2x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x, algorithm="maxima")

[Out] 1/2\*a\*(log(e\*x + d)/(d\*e) - log(e\*x - d)/(d\*e)) - b\*integrate(F^(sqrt(e\*x + d)\*c/(sqrt(-e\*x + d)\*sqrt(f)))/(e^2\*x^2 - d^2), x)



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{a + \frac{b}{\sqrt{-efx+df}\sqrt{ex+dc}}}{\frac{F}{efx-df}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x, algorithm="fricas")

[Out] integral(-(a + b/F^(sqrt(-e\*f\*x + d\*f))\*sqrt(e\*x + d)\*c/(e\*f\*x - d\*f)))/(-e^2\*x^2 - d^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{-d^2 + e^2x^2} dx - \int \frac{F \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} b}{-d^2 + e^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(-e\*f\*x+d\*f)\*\*(1/2)))/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out] -Integral(a/(-d\*\*2 + e\*\*2\*x\*\*2), x) - Integral(F\*\*(c\*sqrt(d + e\*x)/sqrt(d\*f - e\*f\*x))\*b/(-d\*\*2 + e\*\*2\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{F \frac{\sqrt{ex+dc}}{\sqrt{-efx+df}} b + a}{e^2x^2 - d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x, algorithm="giac")

[Out] integrate(-(F^(sqrt(e\*x + d))\*c/sqrt(-e\*f\*x + d\*f))\*b + a)/(-e^2\*x^2 - d^2), x)

$$3.554 \quad \int \frac{1}{d^2 - e^2 x^2} dx$$

**Optimal.** Leaf size=14

$$\frac{\tanh^{-1}\left(\frac{ex}{d}\right)}{de}$$

[Out] ArcTanh[(e\*x)/d]/(d\*e)

**Rubi [A]** time = 0.0078265, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{ex}{d}\right)}{de}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(-1), x]

[Out] ArcTanh[(e\*x)/d]/(d\*e)

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\int \frac{1}{d^2 - e^2 x^2} dx = \frac{\tanh^{-1}\left(\frac{ex}{d}\right)}{de}$$

**Mathematica [A]** time = 0.0029067, size = 14, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{ex}{d}\right)}{de}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(-1), x]

[Out] ArcTanh[(e\*x)/d]/(d\*e)

**Maple [B]** time = 0.006, size = 32, normalized size = 2.3

$$\frac{\ln(ex + d)}{2de} - \frac{\ln(ex - d)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-e^2*x^2+d^2),x)`

[Out]  $1/2/e/d*\ln(e*x+d)-1/2/e/d*\ln(e*x-d)$

**Maxima [B]** time = 0.974911, size = 42, normalized size = 3.

$$\frac{\log(ex+d)}{2de} - \frac{\log(ex-d)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-e^2*x^2+d^2),x, algorithm="maxima")`

[Out]  $1/2*\log(e*x + d)/(d*e) - 1/2*\log(e*x - d)/(d*e)$

**Fricas [A]** time = 1.20058, size = 55, normalized size = 3.93

$$\frac{\log(ex+d) - \log(ex-d)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-e^2*x^2+d^2),x, algorithm="fricas")`

[Out]  $1/2*(\log(e*x + d) - \log(e*x - d))/(d*e)$

**Sympy [B]** time = 0.133459, size = 20, normalized size = 1.43

$$-\frac{\frac{\log\left(-\frac{d}{e}+x\right)}{2} - \frac{\log\left(\frac{d}{e}+x\right)}{2}}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-e**2*x**2+d**2),x)`

[Out]  $-(\log(-d/e + x)/2 - \log(d/e + x)/2)/(d*e)$

**Giac [B]** time = 1.3071, size = 51, normalized size = 3.64

$$-\frac{e^{(-1)} \log\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{2|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-e^2*x^2+d^2),x, algorithm="giac")`

[Out]  $-1/2*e^{(-1)}*\log(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/\text{abs}(d)$

$$3.555 \quad \int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right) (d^2 - e^2x^2)} dx$$

**Optimal.** Leaf size=49

$$\text{Unintegrable} \left( \frac{1}{(d^2 - e^2x^2) \left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)}, x \right)$$

[Out] Unintegrable[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))\*(d^2 - e^2\*x^2)), x]

**Rubi [A]** time = 0.23251, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right) (d^2 - e^2x^2)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))\*(d^2 - e^2\*x^2)),x]

[Out] Defer[Int][1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))\*(d^2 - e^2\*x^2)), x]

Rubi steps

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right) (d^2 - e^2x^2)} dx = \int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right) (d^2 - e^2x^2)} dx$$

**Mathematica [A]** time = 0.4697, size = 0, normalized size = 0.

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right) (d^2 - e^2x^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))\*(d^2 - e^2\*x^2)),x]

[Out] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))\*(d^2 - e^2\*x^2)), x]

---

**Maple [A]** time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2x^2 + d^2} \left( a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x)

[Out] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(e^2x^2 - d^2) \left( F^{\frac{\sqrt{ex+dc}}{\sqrt{-efx+df}}} b + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x,  
algorithm="maxima")

[Out] -integrate(1/((e^2\*x^2 - d^2)\*(F^(sqrt(e\*x + d)\*c/sqrt(-e\*f\*x + d\*f))\*b + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{ae^2x^2 - ad^2 + \frac{be^2x^2 - bd^2}{F^{\frac{\sqrt{-efx+df}\sqrt{ex+dc}}{efx-df}}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))/(-e^2\*x^2+d^2), x,  
algorithm="fricas")

[Out] integral(-1/(a\*e^2\*x^2 - a\*d^2 + (b\*e^2\*x^2 - b\*d^2)/F^(sqrt(-e\*f\*x + d\*f)\*  
sqrt(e\*x + d)\*c/(e\*f\*x - d\*f))), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}} bd^2 + F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}} be^2x^2 - ad^2 + ae^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(-e\*f\*x+d\*f)\*\*(1/2)))/(-e\*\*2\*x\*\*2+d\*\*  
2), x)

```
[Out] -Integral(1/(-F**(c*sqrt(d + e*x)/sqrt(d*f - e*f*x))*b*d**2 + F**(c*sqrt(d
+ e*x)/sqrt(d*f - e*f*x))*b*e**2*x**2 - a*d**2 + a*e**2*x**2), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(e^2x^2 - d^2) \left( F^{\frac{\sqrt{ex+dc}}{\sqrt{-efx+df}}} b + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2), x,
algorithm="giac")
```

```
[Out] integrate(-1/((e^2*x^2 - d^2)*(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a
)), x)
```

$$3.556 \quad \int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2 (d^2 - e^2x^2)} dx$$

**Optimal.** Leaf size=49

$$\text{Unintegrable} \left( \frac{1}{(d^2 - e^2x^2) \left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2}, x \right)$$

[Out] Unintegrable[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2\*(d^2 - e^2\*x^2)), x]

**Rubi [A]** time = 0.224854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2 (d^2 - e^2x^2)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2\*(d^2 - e^2\*x^2)), x]

[Out] Defer[Int][1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2\*(d^2 - e^2\*x^2)), x]

Rubi steps

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2 (d^2 - e^2x^2)} dx = \int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2 (d^2 - e^2x^2)} dx$$

**Mathematica [A]** time = 1.40698, size = 0, normalized size = 0.

$$\int \frac{1}{\left( a + bF \frac{c\sqrt{d+ex}}{\sqrt{df-efx}} \right)^2 (d^2 - e^2x^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + bF^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2\*(d^2 - e^2\*x^2)), x]

[Out] Integrate[1/((a + b\*F^((c\*Sqrt[d + e\*x])/Sqrt[d\*f - e\*f\*x]))^2\*(d^2 - e^2\*x^2)), x]

**Maple [A]** time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{-e^2x^2 + d^2} \left( a + bF^{c\sqrt{ex+d} \frac{1}{\sqrt{-efx+df}}} \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x)

[Out] int(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{-ex+d}\sqrt{f}}{\sqrt{ex+d}F^{\frac{\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}} abcde \log(F) + \sqrt{ex+d}a^2cde \log(F) - \int \frac{\sqrt{ex+dc} \log(F) + \sqrt{-ex+d}\sqrt{f}}{(abce^2x^2 \log(F) - abcd^2 \log(F))\sqrt{ex+d}F^{\frac{\sqrt{ex+dc}}{\sqrt{-ex+d}\sqrt{f}}}} + (a^2ce^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x, algorithm="maxima")

[Out] sqrt(-e\*x + d)\*sqrt(f)/(sqrt(e\*x + d)\*F^(sqrt(e\*x + d)\*c/(sqrt(-e\*x + d)\*sqrt(f))))\*a\*b\*c\*d\*e\*log(F) + sqrt(e\*x + d)\*a^2\*c\*d\*e\*log(F) - integrate((sqrt(e\*x + d)\*c\*log(F) + sqrt(-e\*x + d)\*sqrt(f))/((a\*b\*c\*e^2\*x^2\*log(F) - a\*b\*c\*d^2\*log(F))\*sqrt(e\*x + d)\*F^(sqrt(e\*x + d)\*c/(sqrt(-e\*x + d)\*sqrt(f)))) + (a^2\*c\*e^2\*x^2\*log(F) - a^2\*c\*d^2\*log(F))\*sqrt(e\*x + d)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(a+b\*F\*\*(c\*(e\*x+d)\*\*(1/2)/(-e\*f\*x+d\*f)\*\*(1/2))\*\*2/(-e\*\*2\*x\*\*2+d\*\*2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(e^2x^2 - d^2) \left( F^{\frac{\sqrt{ex+dc}}{\sqrt{-efx+df}}} b + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*F^(c\*(e\*x+d)^(1/2)/(-e\*f\*x+d\*f)^(1/2)))^2/(-e^2\*x^2+d^2),x, algorithm="giac")

[Out] integrate(-1/((e^2\*x^2 - d^2)\*(F^(sqrt(e\*x + d)\*c/sqrt(-e\*f\*x + d\*f))\*b + a)^2), x)

$$3.557 \quad \int \frac{\left( \frac{\sqrt{1-ax}}{F\sqrt{1+ax}} \right)^n}{1-a^2x^2} dx$$

**Optimal.** Leaf size=77

$$\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}} \left( \frac{\sqrt{1-ax}}{F\sqrt{ax+1}} \right)^n \operatorname{Ei} \left( \frac{n\sqrt{1-ax} \log(F)}{\sqrt{ax+1}} \right)}{a}$$

[Out] -(((F^(Sqrt[1 - a\*x])/Sqrt[1 + a\*x]))^n\*ExpIntegralEi[(n\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/(a\*F^((n\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])))

**Rubi [A]** time = 0.240574, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$ , Rules used = {2281, 2291, 2178}

$$\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}} \left( \frac{\sqrt{1-ax}}{F\sqrt{ax+1}} \right)^n \operatorname{Ei} \left( \frac{n\sqrt{1-ax} \log(F)}{\sqrt{ax+1}} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(F^(Sqrt[1 - a\*x])/Sqrt[1 + a\*x]))^n/(1 - a^2\*x^2), x]

[Out] -(((F^(Sqrt[1 - a\*x])/Sqrt[1 + a\*x]))^n\*ExpIntegralEi[(n\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/(a\*F^((n\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])))

#### Rule 2281

Int[(u\_)\*((a\_)\*(F\_)^(v\_))^(n\_), x\_Symbol] :> Dist[(a\*F^v)^n/F^(n\*v), Int[u\*F^(n\*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(c\_\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^(n\_)/((A\_) + (C\_)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^(g\_)\*((e\_.) + (f\_.)\*(x\_))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(F \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)^n}{1-a^2x^2} dx &= \left(F^{-\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}} \left(F \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)^n\right) \int \frac{F^{\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx \\
&= \frac{\left(F^{-\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}} \left(F \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)^n\right) \text{Subst}\left(\int \frac{F^{nx}}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}} \left(F \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)^n \text{Ei}\left(\frac{n\sqrt{1-ax} \log(F)}{\sqrt{1+ax}}\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.317459, size = 77, normalized size = 1.

$$-\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}} \left(F \frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^n \text{Ei}\left(\frac{n\sqrt{1-ax} \log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x]))^n/(1 - a^2\*x^2),x]

[Out] -(((F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x]))^n\*ExpIntegralEi[(n\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/(a\*F^((n\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])))

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} \left(F^{\sqrt{-ax+1} \frac{1}{\sqrt{ax+1}}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))^n/(-a^2\*x^2+1),x)

[Out] int((F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))^n/(-a^2\*x^2+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(F \frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^n}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))^n/(-a^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate((F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1)))^n/(a^2\*x^2 - 1), x)

**Fricas [A]** time = 2.87811, size = 62, normalized size = 0.81

$$\frac{\operatorname{Ei}\left(\frac{\sqrt{-ax+1}n\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))^n/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] -Ei(sqrt(-a\*x + 1)\*n\*log(F)/sqrt(a\*x + 1))/a

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F\*\*((-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2)))\*\*n/(-a\*\*2\*x\*\*2+1),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(F \frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^n}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))^n/(-a^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-(F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1)))^n/(a^2\*x^2 - 1), x)

$$3.558 \quad \int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$$

**Optimal.** Leaf size=29

$$-\frac{\text{Ei}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

[Out] -(ExpIntegralEi[(3\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Rubi [A]** time = 0.106451, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2291, 2178}

$$-\frac{\text{Ei}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[F^((3\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])/(1 - a^2\*x^2),x]

[Out] -(ExpIntegralEi[(3\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{F^{3x}}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\text{Ei}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.25304, size = 29, normalized size = 1.

$$-\frac{\text{Ei}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[F^((3\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])/(1 - a^2\*x^2),x]

[Out] -(ExpIntegralEi[(3\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} F^{3 \frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(3\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

[Out] int(F^(3\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(3\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate(F^(3\*sqrt(-a\*x + 1)/sqrt(a\*x + 1)))/(a^2\*x^2 - 1), x)

**Fricas [A]** time = 2.49736, size = 62, normalized size = 2.14

$$\frac{\text{Ei}\left(\frac{3\sqrt{-ax+1}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(3\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] -Ei(3\*sqrt(-a\*x + 1)\*log(F)/sqrt(a\*x + 1))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(3\*(-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2)))/(-a\*\*2\*x\*\*2+1),x)

[Out] -Integral(F\*\*(3\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a\*\*2\*x\*\*2 - 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{F \frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(3\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-F^(3\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a^2\*x^2 - 1), x)

$$3.559 \quad \int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$$

**Optimal.** Leaf size=29

$$-\frac{\operatorname{Ei}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

[Out] -(ExpIntegralEi[(2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Rubi [A]** time = 0.107793, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2291, 2178}

$$-\frac{\operatorname{Ei}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[F^((2\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])/(1 - a^2\*x^2), x]

[Out] -(ExpIntegralEi[(2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2)), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx = -\frac{\operatorname{Subst}\left(\int \frac{F^{2x}}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} = -\frac{\operatorname{Ei}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

**Mathematica [A]** time = 0.246192, size = 29, normalized size = 1.

$$-\frac{\operatorname{Ei}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.



[In] Integrate[F^((2\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])/(1 - a^2\*x^2),x]

[Out] -(ExpIntegralEi[(2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} F^{2\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x)

[Out] int(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate(F^(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(-a^2\*x^2 - 1), x)

**Fricas [A]** time = 2.42966, size = 62, normalized size = 2.14

$$-\frac{\text{Ei}\left(\frac{2\sqrt{-ax+1}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] -Ei(2\*sqrt(-a\*x + 1)\*log(F)/sqrt(a\*x + 1))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(2\*(-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2))/(-a\*\*2\*x\*\*2+1),x)

[Out] -Integral(F\*\*(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a\*\*2\*x\*\*2 - 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="gias")

[Out] integrate(-F^(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a^2\*x^2 - 1), x)

$$3.560 \quad \int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$$

**Optimal.** Leaf size=28

$$-\frac{\operatorname{Ei}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

[Out] -(ExpIntegralEi[(Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a)

**Rubi [A]** time = 0.0958213, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$ , Rules used = {2291, 2178}

$$-\frac{\operatorname{Ei}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x])/(1 - a^2\*x^2), x]

[Out] -(ExpIntegralEi[(Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a)

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{F^x}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\operatorname{Ei}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.229814, size = 28, normalized size = 1.

$$-\frac{\operatorname{Ei}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x])/(1 - a^2\*x^2), x]

[Out] -(ExpIntegralEi[(Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x)

[Out] int(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x, algorithm="maxima")

[Out] -integrate(F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(-a^2\*x^2 - 1), x)

**Fricas [A]** time = 2.45206, size = 59, normalized size = 2.11

$$-\frac{\text{Ei}\left(\frac{\sqrt{-ax+1}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1), x, algorithm="fricas")

[Out] -Ei(sqrt(-a\*x + 1)\*log(F)/sqrt(a\*x + 1))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*((-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2))/(-a\*\*2\*x\*\*2+1), x)

[Out] -Integral(F\*\*(sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a\*\*2\*x\*\*2 - 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{F \frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2))/(-a^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1))/(a^2\*x^2 - 1), x)

$$3.561 \quad \int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$$

**Optimal.** Leaf size=29

$$-\frac{\text{Ei}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

[Out] -(ExpIntegralEi[-((Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x])]/a)

**Rubi [A]** time = 0.104185, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2291, 2178}

$$-\frac{\text{Ei}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x])\*(1 - a^2\*x^2)),x]

[Out] -(ExpIntegralEi[-((Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x])]/a)

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2)), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{F^{-x}}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\text{Ei}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.242674, size = 29, normalized size = 1.

$$-\frac{\text{Ei}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(F^(Sqrt[1 - a\*x]/Sqrt[1 + a\*x])\*(1 - a^2\*x^2)),x]

[Out] -(ExpIntegralEi[-((Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x])]/a)

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} \left( F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} \frac{1}{\sqrt{ax+1}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

[Out] int(1/(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{1}{(a^2x^2 - 1) F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate(1/((a^2\*x^2 - 1)\*F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1))), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(a^2x^2 - 1) F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F^((-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-1/((a^2\*x^2 - 1)\*F^(sqrt(-a\*x + 1)/sqrt(a\*x + 1))), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{1}{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} a^2x^2 - F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(F**((-a*x+1)**(1/2)/(a*x+1)**(1/2)))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(1/(F**(sqrt(-a*x + 1)/sqrt(a*x + 1))*a**2*x**2 - F**(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2 - 1)F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*F^(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```



$$3.562 \quad \int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$$

**Optimal.** Leaf size=29

$$-\frac{\text{Ei}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

[Out] -(ExpIntegralEi[(-2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

**Rubi [A]** time = 0.104807, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2291, 2178}

$$-\frac{\text{Ei}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(F^((2\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])\*(1 - a^2\*x^2)),x]

[Out] -(ExpIntegralEi[(-2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]])/a

#### Rule 2291

Int[((a\_.) + (b\_.)\*(F\_)^(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^((n\_.)/((A\_) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F^(c\*x))^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{F^{-2x}}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\text{Ei}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.245116, size = 29, normalized size = 1.

$$-\frac{\text{Ei}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(F^((2\*Sqrt[1 - a\*x])/Sqrt[1 + a\*x])\*(1 - a^2\*x^2)),x]

[Out] -(ExpIntegralEi[(-2\*Sqrt[1 - a\*x]\*Log[F])/Sqrt[1 + a\*x]]/a)

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2 + 1} \left( F^{2 \frac{\sqrt{-ax+1}}{\sqrt{ax+1}}} \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

[Out] int(1/(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{1}{(a^2x^2 - 1) F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate(1/((a^2\*x^2 - 1)\*F^(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(a^2x^2 - 1) F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-1/((a^2\*x^2 - 1)\*F^(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F\*\*(2\*(-a\*x+1)\*\*(1/2)/(a\*x+1)\*\*(1/2)))/(-a\*\*2\*x\*\*2+1),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2 - 1)F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(F^(2\*(-a\*x+1)^(1/2)/(a\*x+1)^(1/2)))/(-a^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-1/((a^2\*x^2 - 1)\*F^(2\*sqrt(-a\*x + 1)/sqrt(a\*x + 1))), x)

### 3.563 $\int a^x b^x x^2 dx$

**Optimal.** Leaf size=49

$$\frac{x^2 a^x b^x}{\log(a) + \log(b)} - \frac{2xa^x b^x}{(\log(a) + \log(b))^2} + \frac{2a^x b^x}{(\log(a) + \log(b))^3}$$

[Out]  $(2*a^x*b^x)/(Log[a] + Log[b])^3 - (2*a^x*b^x*x)/(Log[a] + Log[b])^2 + (a^x*b^x*x^2)/(Log[a] + Log[b])$

**Rubi [A]** time = 0.0638647, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2287, 2176, 2194}

$$\frac{x^2 a^x b^x}{\log(a) + \log(b)} - \frac{2xa^x b^x}{(\log(a) + \log(b))^2} + \frac{2a^x b^x}{(\log(a) + \log(b))^3}$$

Antiderivative was successfully verified.

[In] Int[a^x\*b^x\*x^2,x]

[Out]  $(2*a^x*b^x)/(Log[a] + Log[b])^3 - (2*a^x*b^x*x)/(Log[a] + Log[b])^2 + (a^x*b^x*x^2)/(Log[a] + Log[b])$

#### Rule 2287

Int[(u\_)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2176

Int[((b\_)\*(F\_)^(g\_)\*((e\_)+(f\_)\*(x\_)))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rule 2194

Int[((F\_)^(c\_)\*((a\_)+(b\_)\*(x\_)))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int a^x b^x x^2 dx &= \int e^{x(\log(a)+\log(b))} x^2 dx \\ &= \frac{a^x b^x x^2}{\log(a) + \log(b)} - \frac{2 \int e^{x(\log(a)+\log(b))} x dx}{\log(a) + \log(b)} \\ &= -\frac{2a^x b^x x}{(\log(a) + \log(b))^2} + \frac{a^x b^x x^2}{\log(a) + \log(b)} + \frac{2 \int e^{x(\log(a)+\log(b))} dx}{(\log(a) + \log(b))^2} \\ &= \frac{2a^x b^x}{(\log(a) + \log(b))^3} - \frac{2a^x b^x x}{(\log(a) + \log(b))^2} + \frac{a^x b^x x^2}{\log(a) + \log(b)} \end{aligned}$$

**Mathematica [A]** time = 0.0291436, size = 35, normalized size = 0.71

$$\frac{a^x b^x (x^2 (\log(a) + \log(b))^2 - 2x (\log(a) + \log(b)) + 2)}{(\log(a) + \log(b))^3}$$

Antiderivative was successfully verified.

[In] Integrate[a^x\*b^x\*x^2,x]

[Out] (a^x\*b^x\*(2 - 2\*x\*(Log[a] + Log[b]) + x^2\*(Log[a] + Log[b])^2))/(Log[a] + Log[b])^3

**Maple [A]** time = 0.007, size = 69, normalized size = 1.4

$$\frac{((\ln(a))^2 x^2 + 2 \ln(a) \ln(b) x^2 + (\ln(b))^2 x^2 - 2 \ln(a) x - 2 \ln(b) x + 2) a^x b^x}{(\ln(a) + \ln(b)) ((\ln(a))^2 + 2 \ln(a) \ln(b) + (\ln(b))^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*b^x\*x^2,x)

[Out] (ln(a)^2\*x^2+2\*ln(a)\*ln(b)\*x^2+ln(b)^2\*x^2-2\*ln(a)\*x-2\*ln(b)\*x+2)\*a^x\*b^x/(ln(a)+ln(b))/(ln(a)^2+2\*ln(a)\*ln(b)+ln(b)^2)

**Maxima [A]** time = 0.976534, size = 90, normalized size = 1.84

$$\frac{((\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2) x^2 - 2x (\log(a) + \log(b)) + 2) e^{(x \log(a) + x \log(b))}}{\log(a)^3 + 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 + \log(b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x\*x^2,x, algorithm="maxima")

[Out] ((log(a)^2 + 2\*log(a)\*log(b) + log(b)^2)\*x^2 - 2\*x\*(log(a) + log(b)) + 2)\*e^(x\*log(a) + x\*log(b))/(log(a)^3 + 3\*log(a)^2\*log(b) + 3\*log(a)\*log(b)^2 + log(b)^3)

**Fricas [A]** time = 1.28376, size = 197, normalized size = 4.02

$$\frac{(x^2 \log(a)^2 + x^2 \log(b)^2 - 2x \log(a) + 2(x^2 \log(a) - x) \log(b) + 2) a^x b^x}{\log(a)^3 + 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 + \log(b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x\*x^2,x, algorithm="fricas")

[Out] (x^2\*log(a)^2 + x^2\*log(b)^2 - 2\*x\*log(a) + 2\*(x^2\*log(a) - x)\*log(b) + 2)\*a^x\*b^x/(log(a)^3 + 3\*log(a)^2\*log(b) + 3\*log(a)\*log(b)^2 + log(b)^3)

**Sympy [A]** time = 2.35986, size = 279, normalized size = 5.69

$$\left\{ \frac{a^x b^x x^2 \log(a)^2}{\log(a)^3 + 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 + \log(b)^3} + \frac{2 a^x b^x x^2 \log(a) \log(b)}{\log(a)^3 + 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 + \log(b)^3} + \frac{a^x b^x x^2 \log(b)^2}{\log(a)^3 + 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 + \log(b)^3} \right\} \approx b^x \left( \frac{1}{b} \right)^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x\*x\*\*2,x)

[Out] Piecewise((a\*\*x\*b\*\*x\*x\*\*2\*log(a)\*\*2/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3) + 2\*a\*\*x\*b\*\*x\*x\*\*2\*log(a)\*log(b)/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3) + a\*\*x\*b\*\*x\*x\*\*2\*log(b)\*\*2/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3) - 2\*a\*\*x\*b\*\*x\*x\*log(a)/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3) - 2\*a\*\*x\*b\*\*x\*x\*log(b)/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3) + 2\*a\*\*x\*b\*\*x/(log(a)\*\*3 + 3\*log(a)\*\*2\*log(b) + 3\*log(a)\*log(b)\*\*2 + log(b)\*\*3), Ne(a, 1/b)), (zoo\*b\*\*x\*(1/b)\*\*x, True))

**Giac [B]** time = 1.4313, size = 3617, normalized size = 73.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x\*x^2,x, algorithm="giac")

[Out] ((2\*(pi\*x^2\*log(abs(a))\*sgn(a) + pi\*x^2\*log(abs(b))\*sgn(a) + pi\*x^2\*log(abs(a))\*sgn(b) + pi\*x^2\*log(abs(b))\*sgn(b) - 2\*pi\*x^2\*log(abs(a)) - 2\*pi\*x^2\*log(abs(b)) - pi\*x\*sgn(a) - pi\*x\*sgn(b) + 2\*pi\*x)\*(3\*pi^3\*sgn(a)\*sgn(b) - 4\*pi^3\*sgn(a) + 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) + 3\*pi\*log(abs(b))^2\*sgn(a) - 4\*pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b) + 5\*pi^3 - 6\*pi\*log(abs(a))^2 - 12\*pi\*log(abs(a))\*log(abs(b)) - 6\*pi\*log(abs(b))^2)/(3\*pi^3\*sgn(a)\*sgn(b) - 4\*pi^3\*sgn(a) + 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) + 3\*pi\*log(abs(b))^2\*sgn(a) - 4\*pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b) + 5\*pi^3 - 6\*pi\*log(abs(a))^2 - 12\*pi\*log(abs(a))\*log(abs(b)) - 6\*pi\*log(abs(b))^2)^2 + (3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) + 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 6\*pi^2\*log(abs(a))\*sgn(a) - 6\*pi^2\*log(abs(b))\*sgn(a) - 6\*pi^2\*log(abs(a))\*sgn(b) - 6\*pi^2\*log(abs(b))\*sgn(b) + 9\*pi^2\*log(abs(a)) - 2\*log(abs(a))^3 + 9\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) - 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)^2) + (pi^2\*x^2\*sgn(a)\*sgn(b) - 2\*pi^2\*x^2\*sgn(a) - 2\*pi^2\*x^2\*sgn(b) + 3\*pi^2\*x^2 - 2\*x^2\*log(abs(a)))^2 - 4\*x^2\*log(abs(a))\*log(abs(b)) - 2\*x^2\*log(abs(b))^2 + 4\*x\*log(abs(a)) + 4\*x\*log(abs(b)) - 4)\*(3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) + 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 6\*pi^2\*log(abs(a))\*sgn(a) - 6\*pi^2\*log(abs(b))\*sgn(a) - 6\*pi^2\*log(abs(a))\*sgn(b) - 6\*pi^2\*log(abs(b))\*sgn(b) + 9\*pi^2\*log(abs(a)) - 2\*log(abs(a))^3 + 9\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) - 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)/(3\*pi^3\*sgn(a)\*sgn(b) - 4\*pi^3\*sgn(a) + 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) + 3\*pi\*log(abs(b))^2\*sgn(a) - 4\*pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b) + 5\*pi^3 - 6\*pi\*log(abs(a))^2 - 12\*pi\*log(abs(a))\*log(abs(b)) - 6\*pi\*log(abs(b))^2)^2 + (3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) + 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 6\*pi^2\*log(abs(a))\*sgn(a) - 6\*pi^2\*log(abs(b))\*sgn(a) - 6\*pi^2\*log(abs(a))\*sgn(b) - 6\*pi^2\*log(abs(b))\*sgn(b) + 9\*pi^2\*log(abs(a)) - 2\*log(abs(a))^3 + 9\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) - 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)



$$\begin{aligned}
& *i*\operatorname{sgn}(b) + 3\pi*i*\log(\operatorname{abs}(a))^2*\operatorname{sgn}(b) + 6\pi*i*\log(\operatorname{abs}(a))*\log(\operatorname{abs}(b))*\operatorname{sgn}(b) \\
& + 3\pi*i*\log(\operatorname{abs}(b))^2*\operatorname{sgn}(b) + 3\pi^2*\log(\operatorname{abs}(a))*\operatorname{sgn}(a)*\operatorname{sgn}(b) + 3\pi^2*\log(\operatorname{abs}(b))*\operatorname{sgn}(a)*\operatorname{sgn}(b) \\
& + 5\pi^3*i - 6\pi*i*\log(\operatorname{abs}(a))^2 - 12\pi*i*\log(\operatorname{abs}(a))*\log(\operatorname{abs}(b)) - 6\pi*i*\log(\operatorname{abs}(b))^2 - 6\pi^2*\log(\operatorname{abs}(a))*\operatorname{sgn}(a) - \\
& 6\pi^2*\log(\operatorname{abs}(b))*\operatorname{sgn}(a) - 6\pi^2*\log(\operatorname{abs}(a))*\operatorname{sgn}(b) - 6\pi^2*\log(\operatorname{abs}(b))*\operatorname{sgn}(b) \\
& + 9\pi^2*\log(\operatorname{abs}(a)) - 2*\log(\operatorname{abs}(a))^3 + 9\pi^2*\log(\operatorname{abs}(b)) - 6*\log(\operatorname{abs}(a))^2*\log(\operatorname{abs}(b)) \\
& - 6*\log(\operatorname{abs}(a))*\log(\operatorname{abs}(b))^2 - 2*\log(\operatorname{abs}(b))^3)) * e^{(x*(\log(\operatorname{abs}(a)) + \log(\operatorname{abs}(b))))} / i
\end{aligned}$$



### 3.564 $\int a^x b^x x dx$

**Optimal.** Leaf size=31

$$\frac{xa^x b^x}{\log(a) + \log(b)} - \frac{a^x b^x}{(\log(a) + \log(b))^2}$$

[Out]  $-(a^x b^x)/(\log[a] + \log[b])^2 + (a^x b^x x)/(\log[a] + \log[b])$

**Rubi [A]** time = 0.0286941, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2287, 2176, 2194}

$$\frac{xa^x b^x}{\log(a) + \log(b)} - \frac{a^x b^x}{(\log(a) + \log(b))^2}$$

Antiderivative was successfully verified.

[In] Int[a^x\*b^x\*x,x]

[Out]  $-(a^x b^x)/(\log[a] + \log[b])^2 + (a^x b^x x)/(\log[a] + \log[b])$

#### Rule 2287

Int[(u\_)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int a^x b^x x dx &= \int e^{x(\log(a)+\log(b))} x dx \\ &= \frac{a^x b^x x}{\log(a) + \log(b)} - \frac{\int e^{x(\log(a)+\log(b))} dx}{\log(a) + \log(b)} \\ &= -\frac{a^x b^x}{(\log(a) + \log(b))^2} + \frac{a^x b^x x}{\log(a) + \log(b)} \end{aligned}$$

**Mathematica [A]** time = 0.0123487, size = 26, normalized size = 0.84

$$a^x b^x \left( \frac{x}{\log(a) + \log(b)} - \frac{1}{(\log(a) + \log(b))^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[a<sup>x</sup>\*b<sup>x</sup>\*x,x]

[Out] a<sup>x</sup>\*b<sup>x</sup>\*(-(Log[a] + Log[b])<sup>-2</sup>) + x/(Log[a] + Log[b])

**Maple [A]** time = 0.005, size = 25, normalized size = 0.8

$$\frac{(\ln(b)x + \ln(a)x - 1)a^x b^x}{(\ln(a) + \ln(b))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a<sup>x</sup>\*b<sup>x</sup>\*x,x)

[Out] (ln(b)\*x+ln(a)\*x-1)\*a<sup>x</sup>\*b<sup>x</sup>/(ln(a)+ln(b))<sup>2</sup>

**Maxima [A]** time = 0.99224, size = 50, normalized size = 1.61

$$\frac{(x(\log(a) + \log(b)) - 1)e^{(x \log(a) + x \log(b))}}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a<sup>x</sup>\*b<sup>x</sup>\*x,x, algorithm="maxima")

[Out] (x\*(log(a) + log(b)) - 1)\*e<sup>(x\*log(a) + x\*log(b))</sup>/(log(a)<sup>2</sup> + 2\*log(a)\*log(b) + log(b)<sup>2</sup>)

**Fricas [A]** time = 1.357, size = 101, normalized size = 3.26

$$\frac{(x \log(a) + x \log(b) - 1)a^x b^x}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a<sup>x</sup>\*b<sup>x</sup>\*x,x, algorithm="fricas")

[Out] (x\*log(a) + x\*log(b) - 1)\*a<sup>x</sup>\*b<sup>x</sup>/(log(a)<sup>2</sup> + 2\*log(a)\*log(b) + log(b)<sup>2</sup>)

**Sympy [A]** time = 1.17821, size = 97, normalized size = 3.13

$$\begin{cases} \frac{a^x b^x x \log(a)}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2} + \frac{a^x b^x x \log(b)}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2} - \frac{a^x b^x}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2} & \text{for } a \neq \frac{1}{b} \\ \infty b^x \left(\frac{1}{b}\right)^x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x\*x,x)

```
[Out] Piecewise((a**x*b**x*log(a)/(log(a)**2 + 2*log(a)*log(b) + log(b)**2) + a
**x*b**x*log(b)/(log(a)**2 + 2*log(a)*log(b) + log(b)**2) - a**x*b**x/(lo
g(a)**2 + 2*log(a)*log(b) + log(b)**2), Ne(a, 1/b)), (zoo*b**x*(1/b)**x, Tr
ue))
```

**Giac [B]** time = 1.26627, size = 1377, normalized size = 44.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x*x,x, algorithm="giac")
```

```
[Out] (2*((pi*x*sgn(a) + pi*x*sgn(b) - 2*pi*x)*(pi*log(abs(a))*sgn(a) + pi*log(ab
s(b))*sgn(a) + pi*log(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs
(a)) - 2*pi*log(abs(b)))/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(
b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b))^2
)^2 + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log(abs(a))*sgn
(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b)))^2) - (pi
^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2
- 4*log(abs(a))*log(abs(b)) - 2*log(abs(b))^2)*(x*log(abs(a)) + x*log(abs(
b)) - 1)/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*
log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b))^2)^2 + 4*(pi*log(
abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log(abs(a))*sgn(b) + pi*log(abs
(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b)))^2))*cos(-1/2*pi*x*sgn(a)
- 1/2*pi*x*sgn(b) + pi*x) - ((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*
sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b
))^2)*(pi*x*sgn(a) + pi*x*sgn(b) - 2*pi*x)/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sg
n(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b))
- 2*log(abs(b))^2)^2 + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) +
pi*log(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log
(abs(b)))^2) + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log(ab
s(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b)))
*(x*log(abs(a)) + x*log(abs(b)) - 1)/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) -
2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*lo
g(abs(b))^2)^2 + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log
(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b
)))^2))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi*x))*e^(x*(log(abs(a)) +
log(abs(b)))) - 1/2*((2*i*x*log(abs(a)) + 2*i*x*log(abs(b)) - pi*x*sgn(a)
- pi*x*sgn(b) + 2*pi*x - 2*i)*e^(1/2*(pi*(sgn(a) - 1) + pi*(sgn(b) - 1))*i*
x)/(2*pi*i*log(abs(a))*sgn(a) + 2*pi*i*log(abs(b))*sgn(a) + 2*pi*i*log(abs(
a))*sgn(b) + 2*pi*i*log(abs(b))*sgn(b) - pi^2*sgn(a)*sgn(b) - 4*pi*i*log(ab
s(a)) - 4*pi*i*log(abs(b)) + 2*pi^2*sgn(a) + 2*pi^2*sgn(b) - 3*pi^2 + 2*log
(abs(a))^2 + 4*log(abs(a))*log(abs(b)) + 2*log(abs(b))^2) + (2*i*x*log(abs(
a)) + 2*i*x*log(abs(b)) + pi*x*sgn(a) + pi*x*sgn(b) - 2*pi*x - 2*i)*e^(-1/2
*(pi*(sgn(a) - 1) + pi*(sgn(b) - 1))*i*x)/(2*pi*i*log(abs(a))*sgn(a) + 2*pi
*i*log(abs(b))*sgn(a) + 2*pi*i*log(abs(a))*sgn(b) + 2*pi*i*log(abs(b))*sgn(
b) + pi^2*sgn(a)*sgn(b) - 4*pi*i*log(abs(a)) - 4*pi*i*log(abs(b)) - 2*pi^2*
sgn(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b
))) - 2*log(abs(b))^2))*e^(x*(log(abs(a)) + log(abs(b))))/i
```

### 3.565 $\int a^x b^x dx$

**Optimal.** Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

[Out]  $(a^x b^x)/(\text{Log}[a] + \text{Log}[b])$

**Rubi [A]** time = 0.0123337, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2287, 2194}

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[a^x b^x, x]$

[Out]  $(a^x b^x)/(\text{Log}[a] + \text{Log}[b])$

#### Rule 2287

$\text{Int}[(u_.)*(F_.)^{(v_)}*(G_.)^{(w_)}, x\_Symbol] := \text{With}[\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /; \text{BinomialQ}[z, x] \|\| (\text{PolynomialQ}[z, x] \&\& \text{LeQ}[\text{Exponent}[z, x], 2])] /; \text{FreeQ}[\{F, G\}, x]$

#### Rule 2194

$\text{Int}[(F_.)^{((c_.)*((a_.) + (b_.)*(x_)))})^{(n_.)}, x\_Symbol] := \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int a^x b^x dx &= \int e^{x(\log(a)+\log(b))} dx \\ &= \frac{a^x b^x}{\log(a) + \log(b)} \end{aligned}$$

**Mathematica [A]** time = 0.0055033, size = 14, normalized size = 1.

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[a^x b^x, x]$

[Out]  $(a^x b^x)/(\text{Log}[a] + \text{Log}[b])$

**Maple [A]** time = 0.003, size = 15, normalized size = 1.1

$$\frac{a^x b^x}{\ln(a) + \ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*b^x,x)

[Out] a^x\*b^x/(ln(a)+ln(b))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.24828, size = 36, normalized size = 2.57

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x,x, algorithm="fricas")

[Out] a^x\*b^x/(log(a) + log(b))

**Sympy [A]** time = 0.564912, size = 24, normalized size = 1.71

$$\begin{cases} \frac{a^x b^x}{\log(a) + \log(b)} & \text{for } a \neq \frac{1}{b} \\ \infty b^x \left(\frac{1}{b}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x,x)

[Out] Piecewise((a\*\*x\*b\*\*x/(log(a) + log(b)), Ne(a, 1/b)), (zoo\*b\*\*x\*(1/b)\*\*x, True))

**Giac [B]** time = 1.25521, size = 327, normalized size = 23.36

$$2 \left( \frac{2 (\log(|a|) + \log(|b|)) \cos\left(-\frac{1}{2} \pi x \operatorname{sgn}(a) - \frac{1}{2} \pi x \operatorname{sgn}(b) + \pi x\right)}{(2 \pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4 (\log(|a|) + \log(|b|))^2} + \frac{(2 \pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b)) \sin\left(-\frac{1}{2} \pi x \operatorname{sgn}(a) - \frac{1}{2} \pi x \operatorname{sgn}(b) + \pi x\right)}{(2 \pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4 (\log(|a|) + \log(|b|))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x,x, algorithm="giac")

[Out] 
$$\frac{2(2(\log(\text{abs}(a)) + \log(\text{abs}(b)))\cos(-\frac{1}{2}\pi x \text{sgn}(a) - \frac{1}{2}\pi x \text{sgn}(b) + \pi i x) + (2\pi - \pi \text{sgn}(a) - \pi \text{sgn}(b))\sin(-\frac{1}{2}\pi x \text{sgn}(a) - \frac{1}{2}\pi x \text{sgn}(b) + \pi x))}{((2\pi - \pi \text{sgn}(a) - \pi \text{sgn}(b))^2 + 4(\log(\text{abs}(a)) + \log(\text{abs}(b)))^2)} e^{x(\log(\text{abs}(a)) + \log(\text{abs}(b)))} - \frac{(i e^{\frac{1}{2}(\pi(\text{sgn}(a) - 1) + \pi(\text{sgn}(b) - 1))i x}}{(\pi i \text{sgn}(a) + \pi i \text{sgn}(b) - 2\pi i + 2\log(\text{abs}(a)) + 2\log(\text{abs}(b)))} + i e^{-\frac{1}{2}(\pi(\text{sgn}(a) - 1) + \pi(\text{sgn}(b) - 1))i x}}{(\pi i \text{sgn}(a) + \pi i \text{sgn}(b) - 2\pi i - 2\log(\text{abs}(a)) - 2\log(\text{abs}(b)))}) e^{x(\log(\text{abs}(a)) + \log(\text{abs}(b)))}}{i}$$

$$3.566 \quad \int \frac{a^x b^x}{x} dx$$

**Optimal.** Leaf size=8

$$\text{Ei}(x(\log(a) + \log(b)))$$

[Out] ExpIntegralEi[x\*(Log[a] + Log[b])]

**Rubi [A]** time = 0.0416269, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2287, 2178}

$$\text{Ei}(x(\log(a) + \log(b)))$$

Antiderivative was successfully verified.

[In] Int[(a^x\*b^x)/x,x]

[Out] ExpIntegralEi[x\*(Log[a] + Log[b])]

**Rule 2287**

Int[(u\_)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

**Rule 2178**

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

**Rubi steps**

$$\begin{aligned} \int \frac{a^x b^x}{x} dx &= \int \frac{e^{x(\log(a) + \log(b))}}{x} dx \\ &= \text{Ei}(x(\log(a) + \log(b))) \end{aligned}$$

**Mathematica [A]** time = 0.0109685, size = 10, normalized size = 1.25

$$\text{Ei}(x \log(a) + x \log(b))$$

Antiderivative was successfully verified.

[In] Integrate[(a^x\*b^x)/x,x]

[Out] ExpIntegralEi[x\*Log[a] + x\*Log[b]]

**Maple [C]** time = 0.034, size = 56, normalized size = 7.

$$\ln(x) + i\pi + \ln(\ln(b)) + \ln\left(1 + \frac{\ln(a)}{\ln(b)}\right) - \ln\left(-x \ln(b) \left(1 + \frac{\ln(a)}{\ln(b)}\right)\right) - \text{Ei}\left(1, -x \ln(b) \left(1 + \frac{\ln(a)}{\ln(b)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^x*b^x/x,x)`

[Out]  $\ln(x)+I\pi+\ln(\ln(b))+\ln(1+\ln(a)/\ln(b))-\ln(-x*\ln(b)*(1+\ln(a)/\ln(b)))-Ei(1,-x*\ln(b)*(1+\ln(a)/\ln(b)))$

**Maxima [A]** time = 1.09311, size = 11, normalized size = 1.38

$Ei(x(\log(a) + \log(b)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*b^x/x,x, algorithm="maxima")`

[Out]  $Ei(x*(\log(a) + \log(b)))$

**Fricas [A]** time = 1.22634, size = 34, normalized size = 4.25

$Ei(x \log(a) + x \log(b))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*b^x/x,x, algorithm="fricas")`

[Out]  $Ei(x*\log(a) + x*\log(b))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a**x*b**x/x,x)`

[Out] `Integral(a**x*b**x/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*b^x/x,x, algorithm="giac")`

[Out] `integrate(a^x*b^x/x, x)`



$$3.567 \quad \int \frac{a^x b^x}{x^2} dx$$

**Optimal.** Leaf size=26

$$(\log(a) + \log(b))\text{Ei}(x(\log(a) + \log(b))) - \frac{a^x b^x}{x}$$

[Out]  $-(a^x b^x)/x + \text{ExpIntegralEi}[x*(\text{Log}[a] + \text{Log}[b])]*(\text{Log}[a] + \text{Log}[b])$

**Rubi [A]** time = 0.0619315, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2287, 2177, 2178}

$$(\log(a) + \log(b))\text{Ei}(x(\log(a) + \log(b))) - \frac{a^x b^x}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a^x b^x)/x^2, x]$

[Out]  $-(a^x b^x)/x + \text{ExpIntegralEi}[x*(\text{Log}[a] + \text{Log}[b])]*(\text{Log}[a] + \text{Log}[b])$

#### Rule 2287

$\text{Int}[(u_.)*(F_)^(v_)*(G_)^(w_), x\_Symbol] \rightarrow \text{With}[\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /; \text{BinomialQ}[z, x] \mid\mid (\text{PolynomialQ}[z, x] \ \&\& \ \text{LeQ}[\text{Exponent}[z, x], 2])] /; \text{FreeQ}[\{F, G\}, x]$

#### Rule 2177

$\text{Int}[(b_.)*(F_)^(g_.*((e_.) + (f_.)*(x_)))^(n_.*((c_.) + (d_.)*(x_)))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/(d*(m + 1)), \text{Int}[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == True$

#### Rule 2178

$\text{Int}[(F_)^(g_.*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^(g*(e - (c*f)/d))*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \ !\$UseGamma == True$

#### Rubi steps

$$\begin{aligned} \int \frac{a^x b^x}{x^2} dx &= \int \frac{e^{x(\log(a)+\log(b))}}{x^2} dx \\ &= -\frac{a^x b^x}{x} - (-\log(a) - \log(b)) \int \frac{e^{x(\log(a)+\log(b))}}{x} dx \\ &= -\frac{a^x b^x}{x} + \text{Ei}(x(\log(a) + \log(b)))(\log(a) + \log(b)) \end{aligned}$$

**Mathematica [F]** time = 0.0489283, size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a^x\*b^x)/x^2,x]

[Out] Integrate[(a^x\*b^x)/x^2, x]

**Maple [C]** time = 0.036, size = 160, normalized size = 6.2

$$-\ln(b) \left(1 + \frac{\ln(a)}{\ln(b)}\right) \left(\frac{1}{\ln(b)x} \left(1 + \frac{\ln(a)}{\ln(b)}\right)^{-1} + 1 - \ln(x) - i\pi - \ln(\ln(b)) - \ln\left(1 + \frac{\ln(a)}{\ln(b)}\right) - \frac{1}{2 \ln(b)x} \left(2 + 2x \ln(b) \left(1 + \frac{\ln(a)}{\ln(b)}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*b^x/x^2,x)

[Out]  $-\ln(b) * (1 + \ln(a) / \ln(b)) * (1/x / \ln(b) / (1 + \ln(a) / \ln(b)) + 1 - \ln(x) - I * \text{Pi} - \ln(\ln(b)) - \ln(1 + \ln(a) / \ln(b)) - 1/2/x / \ln(b) / (1 + \ln(a) / \ln(b)) * (2 + 2*x*\ln(b) * (1 + \ln(a) / \ln(b))) + 1/x / \ln(b) / (1 + \ln(a) / \ln(b)) * \exp(x*\ln(b) * (1 + \ln(a) / \ln(b))) + \ln(-x*\ln(b) * (1 + \ln(a) / \ln(b))) + \text{Ei}(1, -x*\ln(b) * (1 + \ln(a) / \ln(b))))$

**Maxima [A]** time = 1.11111, size = 22, normalized size = 0.85

$$(\log(a) + \log(b))\Gamma(-1, -x(\log(a) + \log(b)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^2,x, algorithm="maxima")

[Out]  $(\log(a) + \log(b)) * \text{gamma}(-1, -x * (\log(a) + \log(b)))$

**Fricas [A]** time = 1.27189, size = 84, normalized size = 3.23

$$-\frac{a^x b^x - (x \log(a) + x \log(b)) \text{Ei}(x \log(a) + x \log(b))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^2,x, algorithm="fricas")

[Out]  $-(a^x*b^x - (x*\log(a) + x*\log(b))*\text{Ei}(x*\log(a) + x*\log(b)))/x$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x/x\*\*2,x)

[Out] Integral(a\*\*x\*b\*\*x/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^2,x, algorithm="giac")

[Out] integrate(a^x\*b^x/x^2, x)

### 3.568 $\int \frac{a^x b^x}{x^3} dx$

**Optimal.** Leaf size=51

$$-\frac{a^x b^x}{2x^2} - \frac{a^x b^x (\log(a) + \log(b))}{2x} + \frac{1}{2} (\log(a) + \log(b))^2 \text{Ei}(x(\log(a) + \log(b)))$$

[Out]  $-(a^x b^x)/(2x^2) - (a^x b^x (\text{Log}[a] + \text{Log}[b]))/(2x) + (\text{ExpIntegralEi}[x(\text{Log}[a] + \text{Log}[b])]) (\text{Log}[a] + \text{Log}[b])^2/2$

**Rubi [A]** time = 0.0853705, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2287, 2177, 2178}

$$-\frac{a^x b^x}{2x^2} - \frac{a^x b^x (\log(a) + \log(b))}{2x} + \frac{1}{2} (\log(a) + \log(b))^2 \text{Ei}(x(\log(a) + \log(b)))$$

Antiderivative was successfully verified.

[In] Int[(a^x\*b^x)/x^3,x]

[Out]  $-(a^x b^x)/(2x^2) - (a^x b^x (\text{Log}[a] + \text{Log}[b]))/(2x) + (\text{ExpIntegralEi}[x(\text{Log}[a] + \text{Log}[b])]) (\text{Log}[a] + \text{Log}[b])^2/2$

#### Rule 2287

Int[(u\_)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2177

Int[((b\_)\*(F\_)^(g\_)\*((e\_)+(f\_)\*(x\_)))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c+d\*x)^(m+1)\*(b\*F^(g\*(e+f\*x)))^n)/(d\*(m+1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m+1)), Int[(c+d\*x)^(m+1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2178

Int[(F\_)^(g\_)\*((e\_)+(f\_)\*(x\_)))/((c\_)+(d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e-(c\*f)/d))\*ExpIntegralEi[(f\*g\*(c+d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{a^x b^x}{x^3} dx &= \int \frac{e^{x(\log(a)+\log(b))}}{x^3} dx \\ &= -\frac{a^x b^x}{2x^2} - \frac{1}{2} (-\log(a) - \log(b)) \int \frac{e^{x(\log(a)+\log(b))}}{x^2} dx \\ &= -\frac{a^x b^x}{2x^2} - \frac{a^x b^x (\log(a) + \log(b))}{2x} + \frac{1}{2} (\log(a) + \log(b))^2 \int \frac{e^{x(\log(a)+\log(b))}}{x} dx \\ &= -\frac{a^x b^x}{2x^2} - \frac{a^x b^x (\log(a) + \log(b))}{2x} + \frac{1}{2} \text{Ei}(x(\log(a) + \log(b))) (\log(a) + \log(b))^2 \end{aligned}$$

**Mathematica [F]** time = 0.0573304, size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a^x\*b^x)/x^3,x]

[Out] Integrate[(a^x\*b^x)/x^3, x]

**Maple [C]** time = 0.044, size = 225, normalized size = 4.4

$$(\ln(b))^2 \left(1 + \frac{\ln(a)}{\ln(b)}\right)^2 \left(-\frac{1}{2(\ln(b))^2 x^2} \left(1 + \frac{\ln(a)}{\ln(b)}\right)^{-2} - \frac{1}{\ln(b)x} \left(1 + \frac{\ln(a)}{\ln(b)}\right)^{-1} - \frac{3}{4} + \frac{\ln(x)}{2} + \frac{i}{2}\pi + \frac{\ln(\ln(b))}{2} + \frac{1}{2} \ln\left(\frac{\ln(b)}{\ln(a)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*b^x/x^3,x)

[Out]  $\ln(b)^2 (1 + \ln(a)/\ln(b))^2 (-1/2/x^2/\ln(b)^2/(1 + \ln(a)/\ln(b))^2 - 1/x/\ln(b)/(1 + \ln(a)/\ln(b)) - 3/4 + 1/2*\ln(x) + 1/2*I*\pi + 1/2*\ln(\ln(b)) + 1/2*\ln(1 + \ln(a)/\ln(b)) + 1/12/x^2/\ln(b)^2/(1 + \ln(a)/\ln(b))^2 (9*x^2*\ln(b)^2*(1 + \ln(a)/\ln(b))^2 + 12*x*\ln(b)*(1 + \ln(a)/\ln(b)) + 6) - 1/6/x^2/\ln(b)^2/(1 + \ln(a)/\ln(b))^2 (3 + 3*x*\ln(b)*(1 + \ln(a)/\ln(b))) * \exp(x*\ln(b)*(1 + \ln(a)/\ln(b))) - 1/2*\ln(-x*\ln(b)*(1 + \ln(a)/\ln(b))) - 1/2*Ei(1, -x*\ln(b)*(1 + \ln(a)/\ln(b)))$

**Maxima [A]** time = 1.0967, size = 26, normalized size = 0.51

$$-(\log(a) + \log(b))^2 \Gamma(-2, -x(\log(a) + \log(b)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^3,x, algorithm="maxima")

[Out]  $-(\log(a) + \log(b))^2 \text{gamma}(-2, -x*(\log(a) + \log(b)))$

**Fricas [A]** time = 1.33229, size = 167, normalized size = 3.27

$$\frac{(x \log(a) + x \log(b) + 1) a^x b^x - (x^2 \log(a)^2 + 2x^2 \log(a) \log(b) + x^2 \log(b)^2) \text{Ei}(x \log(a) + x \log(b))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^3,x, algorithm="fricas")

[Out]  $-1/2*((x*\log(a) + x*\log(b) + 1)*a^x*b^x - (x^2*\log(a)^2 + 2*x^2*\log(a)*\log(b) + x^2*\log(b)^2)*Ei(x*\log(a) + x*\log(b)))/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x/x\*\*3,x)

[Out] Integral(a\*\*x\*b\*\*x/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x b^x}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x/x^3,x, algorithm="giac")

[Out] integrate(a^x\*b^x/x^3, x)

### 3.569 $\int a^x b^x c^x dx$

**Optimal.** Leaf size=19

$$\frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)}$$

[Out] (a<sup>x</sup>\*b<sup>x</sup>\*c<sup>x</sup>)/(Log[a] + Log[b] + Log[c])

**Rubi [A]** time = 0.0410007, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2287, 2194}

$$\frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)}$$

Antiderivative was successfully verified.

[In] Int[a<sup>x</sup>\*b<sup>x</sup>\*c<sup>x</sup>,x]

[Out] (a<sup>x</sup>\*b<sup>x</sup>\*c<sup>x</sup>)/(Log[a] + Log[b] + Log[c])

#### Rule 2287

Int[(u\_.)\*(F\_)^(v\_.)\*(G\_)^(w\_.), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E<sup>z</sup>, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2194

Int[((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] := Simp[(F<sup>c</sup>\*(a + b\*x))<sup>n</sup>/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int a^x b^x c^x dx &= \int c^x e^{x(\log(a)+\log(b))} dx \\ &= \int e^{x(\log(a)+\log(b)+\log(c))} dx \\ &= \frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)} \end{aligned}$$

**Mathematica [A]** time = 0.0162939, size = 21, normalized size = 1.11

$$\frac{e^{x(\log(a)+\log(b)+\log(c))}}{\log(a) + \log(b) + \log(c)}$$

Antiderivative was successfully verified.

[In] Integrate[a<sup>x</sup>\*b<sup>x</sup>\*c<sup>x</sup>,x]

[Out] E<sup>(x\*(Log[a] + Log[b] + Log[c]))</sup>/(Log[a] + Log[b] + Log[c])

**Maple [A]** time = 0.005, size = 20, normalized size = 1.1

$$\frac{a^x b^x c^x}{\ln(a) + \ln(b) + \ln(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*b^x\*c^x,x)

[Out] a^x\*b^x\*c^x/(ln(a)+ln(b)+ln(c))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x\*c^x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.2876, size = 54, normalized size = 2.84

$$\frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*b^x\*c^x,x, algorithm="fricas")

[Out] a^x\*b^x\*c^x/(log(a) + log(b) + log(c))

**Sympy [A]** time = 2.65143, size = 41, normalized size = 2.16

$$\begin{cases} \frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)} & \text{for } a \neq \frac{1}{bc} \\ \infty b^x c^x \left(\frac{1}{b}\right)^x \left(\frac{1}{c}\right)^x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*b\*\*x\*c\*\*x,x)

[Out] Piecewise((a\*\*x\*b\*\*x\*c\*\*x/(log(a) + log(b) + log(c)), Ne(a, 1/(b\*c))), (zoo\*b\*\*x\*c\*\*x\*(1/b)\*\*x\*(1/c)\*\*x, True))

**Giac [B]** time = 1.47958, size = 429, normalized size = 22.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(a^x\*b^x\*c^x,x, algorithm="giac")

[Out] 
$$2*(2*(\log(\text{abs}(a)) + \log(\text{abs}(b)) + \log(\text{abs}(c)))\cos(-1/2*\pi*x*\text{sgn}(a) - 1/2*\pi*x*\text{sgn}(b) - 1/2*\pi*x*\text{sgn}(c) + 3/2*\pi*x)/((3*\pi - \pi*\text{sgn}(a) - \pi*\text{sgn}(b) - \pi*\text{sgn}(c))^2 + 4*(\log(\text{abs}(a)) + \log(\text{abs}(b)) + \log(\text{abs}(c)))^2) + (3*\pi - \pi*\text{sgn}(a) - \pi*\text{sgn}(b) - \pi*\text{sgn}(c))*\sin(-1/2*\pi*x*\text{sgn}(a) - 1/2*\pi*x*\text{sgn}(b) - 1/2*\pi*x*\text{sgn}(c) + 3/2*\pi*x)/((3*\pi - \pi*\text{sgn}(a) - \pi*\text{sgn}(b) - \pi*\text{sgn}(c))^2 + 4*(\log(\text{abs}(a)) + \log(\text{abs}(b)) + \log(\text{abs}(c)))^2))*e^{x*(\log(\text{abs}(a)) + \log(\text{abs}(b)) + \log(\text{abs}(c)))} - (i*e^{1/2*(\pi*(\text{sgn}(a) - 1) + \pi*(\text{sgn}(b) - 1) + \pi*(\text{sgn}(c) - 1))*x}/(\pi*i*\text{sgn}(a) + \pi*i*\text{sgn}(b) + \pi*i*\text{sgn}(c) - 3*\pi*i + 2*\log(\text{abs}(a)) + 2*\log(\text{abs}(b)) + 2*\log(\text{abs}(c))) + i*e^{-1/2*(\pi*(\text{sgn}(a) - 1) + \pi*(\text{sgn}(b) - 1) + \pi*(\text{sgn}(c) - 1))*x}/(\pi*i*\text{sgn}(a) + \pi*i*\text{sgn}(b) + \pi*i*\text{sgn}(c) - 3*\pi*i - 2*\log(\text{abs}(a)) - 2*\log(\text{abs}(b)) - 2*\log(\text{abs}(c))))*e^{x*(\log(\text{abs}(a)) + \log(\text{abs}(b)) + \log(\text{abs}(c)))})/i$$

### 3.570 $\int a^x b^{-x} dx$

**Optimal.** Leaf size=18

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

[Out]  $a^x/(b^x*(\text{Log}[a] - \text{Log}[b]))$

**Rubi [A]** time = 0.0211684, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2287, 2194}

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

[In] Int[a<sup>x</sup>/b<sup>x</sup>, x]

[Out]  $a^x/(b^x*(\text{Log}[a] - \text{Log}[b]))$

#### Rule 2287

Int[(u\_.)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E<sup>z</sup>, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int a^x b^{-x} dx &= \int e^{x(\log(a) - \log(b))} dx \\ &= \frac{a^x b^{-x}}{\log(a) - \log(b)} \end{aligned}$$

**Mathematica [A]** time = 0.0096114, size = 18, normalized size = 1.

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

[In] Integrate[a<sup>x</sup>/b<sup>x</sup>, x]

[Out]  $a^x/(b^x*(\text{Log}[a] - \text{Log}[b]))$

**Maple [A]** time = 0.003, size = 19, normalized size = 1.1

$$\frac{a^x}{b^x (\ln(a) - \ln(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x/(b^x),x)

[Out] a^x/(b^x)/(ln(a)-ln(b))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x/(b^x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.28415, size = 39, normalized size = 2.17

$$\frac{a^x}{b^x(\log(a) - \log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x/(b^x),x, algorithm="fricas")

[Out] a^x/(b^x\*(log(a) - log(b)))

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x/(b\*\*x),x)

[Out] Exception raised: TypeError

**Giac [B]** time = 1.49597, size = 312, normalized size = 17.33

$$2 \left( \frac{2 (\log(|a|) - \log(|b|)) \cos\left(-\frac{1}{2} \pi x \operatorname{sgn}(a) + \frac{1}{2} \pi x \operatorname{sgn}(b)\right)}{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4 (\log(|a|) - \log(|b|))^2} - \frac{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b)) \sin\left(-\frac{1}{2} \pi x \operatorname{sgn}(a) + \frac{1}{2} \pi x \operatorname{sgn}(b)\right)}{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4 (\log(|a|) - \log(|b|))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) -
(i*e^(1/2*(pi*(sgn(a) - 1) - pi*(sgn(b) - 1))*x)/(pi*i*sgn(a) - pi*i*sgn(
b) + 2*log(abs(a)) - 2*log(abs(b))) + i*e^(-1/2*(pi*(sgn(a) - 1) - pi*(sgn(
b) - 1))*x)/(pi*i*sgn(a) - pi*i*sgn(b) - 2*log(abs(a)) + 2*log(abs(b)))))*
e^(x*(log(abs(a)) - log(abs(b))))/i
```

### 3.571 $\int a^x b^{-x} x^2 dx$

**Optimal.** Leaf size=61

$$\frac{x^2 a^x b^{-x}}{\log(a) - \log(b)} - \frac{2x a^x b^{-x}}{(\log(a) - \log(b))^2} + \frac{2a^x b^{-x}}{(\log(a) - \log(b))^3}$$

[Out]  $(2*a^x)/(b^x*(\text{Log}[a] - \text{Log}[b])^3) - (2*a^x*x)/(b^x*(\text{Log}[a] - \text{Log}[b])^2) + (a^x*x^2)/(b^x*(\text{Log}[a] - \text{Log}[b]))$

**Rubi [A]** time = 0.0697103, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2287, 2176, 2194}

$$\frac{x^2 a^x b^{-x}}{\log(a) - \log(b)} - \frac{2x a^x b^{-x}}{(\log(a) - \log(b))^2} + \frac{2a^x b^{-x}}{(\log(a) - \log(b))^3}$$

Antiderivative was successfully verified.

[In] Int[(a^x\*x^2)/b^x,x]

[Out]  $(2*a^x)/(b^x*(\text{Log}[a] - \text{Log}[b])^3) - (2*a^x*x)/(b^x*(\text{Log}[a] - \text{Log}[b])^2) + (a^x*x^2)/(b^x*(\text{Log}[a] - \text{Log}[b]))$

#### Rule 2287

Int[(u\_)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_)+(f\_)\*(x\_))))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c+d\*x)^m\*(b\*F^(g\*(e+f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_)+(b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a+b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int a^x b^{-x} x^2 dx &= \int e^{x(\log(a)-\log(b))} x^2 dx \\ &= \frac{a^x b^{-x} x^2}{\log(a) - \log(b)} - \frac{2 \int e^{x(\log(a)-\log(b))} x dx}{\log(a) - \log(b)} \\ &= -\frac{2a^x b^{-x} x}{(\log(a) - \log(b))^2} + \frac{a^x b^{-x} x^2}{\log(a) - \log(b)} + \frac{2 \int e^{x(\log(a)-\log(b))} dx}{(\log(a) - \log(b))^2} \\ &= \frac{2a^x b^{-x}}{(\log(a) - \log(b))^3} - \frac{2a^x b^{-x} x}{(\log(a) - \log(b))^2} + \frac{a^x b^{-x} x^2}{\log(a) - \log(b)} \end{aligned}$$

**Mathematica [A]** time = 0.0281628, size = 43, normalized size = 0.7

$$\frac{a^x b^{-x} (x^2 (\log(a) - \log(b))^2 - 2x (\log(a) - \log(b)) + 2)}{(\log(a) - \log(b))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a^x\*x^2)/b^x,x]

[Out] (a^x\*(2 - 2\*x\*(Log[a] - Log[b]) + x^2\*(Log[a] - Log[b])^2))/(b^x\*(Log[a] - Log[b])^3)

**Maple [A]** time = 0.007, size = 73, normalized size = 1.2

$$\frac{((\ln(a))^2 x^2 - 2 \ln(a) \ln(b) x^2 + (\ln(b))^2 x^2 - 2 \ln(a) x + 2 \ln(b) x + 2) a^x}{(\ln(a) - \ln(b)) ((\ln(a))^2 - 2 \ln(a) \ln(b) + (\ln(b))^2) b^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x\*x^2/(b^x),x)

[Out] (ln(a)^2\*x^2-2\*ln(a)\*ln(b)\*x^2+ln(b)^2\*x^2-2\*ln(a)\*x+2\*ln(b)\*x+2)\*a^x/(ln(a)-ln(b))/(ln(a)^2-2\*ln(a)\*ln(b)+ln(b)^2)/(b^x)

**Maxima [A]** time = 1.03328, size = 97, normalized size = 1.59

$$\frac{((\log(a))^2 - 2 \log(a) \log(b) + \log(b)^2) x^2 - 2x (\log(a) - \log(b)) + 2) e^{x \log(a) - x \log(b)}}{\log(a)^3 - 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 - \log(b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*x^2/(b^x),x, algorithm="maxima")

[Out] ((log(a)^2 - 2\*log(a)\*log(b) + log(b)^2)\*x^2 - 2\*x\*(log(a) - log(b)) + 2)\*e^(x\*log(a) - x\*log(b))/(log(a)^3 - 3\*log(a)^2\*log(b) + 3\*log(a)\*log(b)^2 - log(b)^3)

**Fricas [A]** time = 1.31193, size = 200, normalized size = 3.28

$$\frac{(x^2 \log(a)^2 + x^2 \log(b)^2 - 2x \log(a) - 2(x^2 \log(a) - x) \log(b) + 2) a^x}{(\log(a)^3 - 3 \log(a)^2 \log(b) + 3 \log(a) \log(b)^2 - \log(b)^3) b^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*x^2/(b^x),x, algorithm="fricas")

[Out] (x^2\*log(a)^2 + x^2\*log(b)^2 - 2\*x\*log(a) - 2\*(x^2\*log(a) - x)\*log(b) + 2)\*a^x/((log(a)^3 - 3\*log(a)^2\*log(b) + 3\*log(a)\*log(b)^2 - log(b)^3)\*b^x)

---

**Sympy [A]** time = 1.66017, size = 333, normalized size = 5.46

$$\left\{ \begin{array}{l} \frac{a^x x^2 \log(a)^2}{b^x \log(a)^3 - 3b^x \log(a)^2 \log(b) + 3b^x \log(a) \log(b)^2 - b^x \log(b)^3} - \frac{2a^x x^2 \log(a) \log(b)}{b^x \log(a)^3 - 3b^x \log(a)^2 \log(b) + 3b^x \log(a) \log(b)^2 - b^x \log(b)^3} + \frac{x^3}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*\*x\*x\*\*2/(b\*\*x),x)

[Out] Piecewise((a\*\*x\*x\*\*2\*log(a)\*\*2/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3) - 2\*a\*\*x\*x\*\*2\*log(a)\*log(b)/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3) + a\*\*x\*x\*\*2\*log(b)\*\*2/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3) - 2\*a\*\*x\*x\*log(a)/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3) + 2\*a\*\*x\*x\*log(b)/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3) + 2\*a\*\*x/(b\*\*x\*log(a)\*\*3 - 3\*b\*\*x\*log(a)\*\*2\*log(b) + 3\*b\*\*x\*log(a)\*log(b)\*\*2 - b\*\*x\*log(b)\*\*3), Ne(a, b)), (x\*\*3/3, True))

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**Giac [B]** time = 1.31603, size = 2510, normalized size = 41.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x\*x^2/(b^x),x, algorithm="giac")

[Out] (((pi^2\*x^2\*sgn(a)\*sgn(b) - pi^2\*x^2 + 2\*x^2\*log(abs(a))^2 - 4\*x^2\*log(abs(a))\*log(abs(b)) + 2\*x^2\*log(abs(b))^2 - 4\*x\*log(abs(a)) + 4\*x\*log(abs(b)) + 4)\*(3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(a)) + 2\*log(abs(a))^3 + 3\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) + 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)/((3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(a)) + 2\*log(abs(a))^3 + 3\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) + 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)^2 + (pi^3\*sgn(a) - 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) - 3\*pi\*log(abs(b))^2\*sgn(a) - pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) - 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b))^2) - 2\*(pi\*x^2\*log(abs(a))\*sgn(a) - pi\*x^2\*log(abs(b))\*sgn(a) - pi\*x^2\*log(abs(a))\*sgn(b) + pi\*x^2\*log(abs(b))\*sgn(b) - pi\*x\*sgn(a) + pi\*x\*sgn(b))\*(pi^3\*sgn(a) - 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) - 3\*pi\*log(abs(b))^2\*sgn(a) - pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) - 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b))/((3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(a)) + 2\*log(abs(a))^3 + 3\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) + 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)^2 + (pi^3\*sgn(a) - 3\*pi\*log(abs(a))^2\*sgn(a) + 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(a) - 3\*pi\*log(abs(b))^2\*sgn(a) - pi^3\*sgn(b) + 3\*pi\*log(abs(a))^2\*sgn(b) - 6\*pi\*log(abs(a))\*log(abs(b))\*sgn(b) + 3\*pi\*log(abs(b))^2\*sgn(b))^2)\*cos(-1/2\*pi\*x\*sgn(a) + 1/2\*pi\*x\*sgn(b)) + (2\*(pi\*x^2\*log(abs(a))\*sgn(a) - pi\*x^2\*log(abs(b))\*sgn(a) - pi\*x^2\*log(abs(a))\*sgn(b) + pi\*x^2\*log(abs(b))\*sgn(b) - pi\*x\*sgn(a) + pi\*x\*sgn(b))\*(3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(a)) + 2\*log(abs(a))^3 + 3\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) + 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)/((3\*pi^2\*log(abs(a))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(b))\*sgn(a)\*sgn(b) - 3\*pi^2\*log(abs(a)) + 2\*log(abs(a))^3 + 3\*pi^2\*log(abs(b)) - 6\*log(abs(a))^2\*log(abs(b)) + 6\*log(abs(a))\*log(abs(b))^2 - 2\*log(abs(b))^3)





$$3.572 \quad \int \frac{(d+ee^{h+ix})(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}} dx$$

**Optimal.** Leaf size=770

$$\frac{6g^2(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^3\left(b-\sqrt{b^2-4ac}\right)} + \frac{6g^2(f+gx)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(3,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^3\left(\sqrt{b^2-4ac}+b\right)} - \frac{3g(f+gx)}{i}$$

[Out]  $((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^4)/(4*(b + \text{Sqrt}[b^2 - 4*a*c])*g) + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^4)/(4*(b - \text{Sqrt}[b^2 - 4*a*c])*g) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3*\text{Log}[1 + (2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3*\text{Log}[1 + (2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i) - (3*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)^2*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^2) - (3*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)^2*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^2) + (6*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*(f + g*x)*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^3) + (6*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*(f + g*x)*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^3) - (6*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^3*\text{PolyLog}[4, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^4) - (6*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^3*\text{PolyLog}[4, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^4)$

**Rubi [A]** time = 1.37399, antiderivative size = 770, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$ , Rules used = {2265, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{6g^2(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^3\left(b-\sqrt{b^2-4ac}\right)} + \frac{6g^2(f+gx)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(3,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^3\left(\sqrt{b^2-4ac}+b\right)} - \frac{3g(f+gx)}{i}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*E^{(h + i*x)})*(f + g*x)^3)/(a + b*E^{(h + i*x)} + c*E^{(2*h + 2*i*x)}), x)$

[Out]  $((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^4)/(4*(b + \text{Sqrt}[b^2 - 4*a*c])*g) + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^4)/(4*(b - \text{Sqrt}[b^2 - 4*a*c])*g) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3*\text{Log}[1 + (2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3*\text{Log}[1 + (2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i) - (3*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)^2*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^2) - (3*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)^2*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^2) + (6*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*(f + g*x)*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^3) + (6*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*(f + g*x)*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^3) - (6*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^3*\text{PolyLog}[4, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^4) - (6*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^3*\text{PolyLog}[4, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^4)$

$*E^{(h + i*x)} / (b + \text{Sqrt}[b^2 - 4*a*c]) / ((b + \text{Sqrt}[b^2 - 4*a*c])*i^4)$

### Rule 2265

$\text{Int}[(((i_.)*(F_)^{(u_)} + (h_))*((f_.) + (g_.)*(x_))^{(m_.)})/((a_.) + (b_.)*(F_)^{(u_)} + (c_.)*(F_)^{(v_)}), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[\text{Simplify}[(2*c*h - b*i)/q] + i, \text{Int}[(f + g*x)^m/(b - q + 2*c*F^u), x], x] - \text{Dist}[\text{Simplify}[(2*c*h - b*i)/q] - i, \text{Int}[(f + g*x)^m/(b + q + 2*c*F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g, h, i\}, x\} \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2184

$\text{Int}[((c_.) + (d_.)*(x_))^{(m_.)}/((a_.) + (b_.)*((F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n})/(a + b*(F^{(g*(e + f*x)))^n}), x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(((F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)})/((a_.) + (b_.)*((F_)^{(g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{(c_.)*((a_.) + (b_.)*(x_)))^{(n_.)})*((f_.) + (g_.)*(x_))^{(m_.)}), x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$

### Rule 6609

$\text{Int}[((e_.) + (f_.)*(x_))^{(m_.)}* \text{PolyLog}[n_, (d_.)*((F_)^{(c_.)*((a_.) + (b_.)*(x_)))^{(p_.)}), x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^m*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}]/(b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m)/(b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m - 1)}*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n, p\}, x\} \&\& \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_)*((a_.)*(v_))^{(n_)}] /; \text{FreeQ}\{a, m, n\}, x\} \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{(c_.)*((a_.) + (b_.)*x)}*(F_)[v_] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_))^{(p_.)}]/((d_.) + (e_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\} \&\& \text{EqQ}[b*d, a*e]$

### Rubi steps

$$\int \frac{(d + ee^{h+572x})(f + gx)^3}{a + be^{h+572x} + ce^{2h+1144x}} dx = -\left(\left(-e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{(f + gx)^3}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}} dx\right) + \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{b - \sqrt{b^2 - 4ac}}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(2c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{e^{h+572x}(f + gx)^3}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}} dx}{b + \sqrt{b^2 - 4ac}}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3 \log\left(1 + \frac{2ce^{h+572x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}}\right)}{572(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3 \log\left(1 + \frac{2ce^{h+572x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}}\right)}{572(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3 \log\left(1 + \frac{2ce^{h+572x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}}\right)}{572(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3 \log\left(1 + \frac{2ce^{h+572x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}}\right)}{572(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^4}{4(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3 \log\left(1 + \frac{2ce^{h+572x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+572x}}\right)}{572(b - \sqrt{b^2 - 4ac})}$$

**Mathematica [B]** time = 4.47916, size = 2441, normalized size = 3.17

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((d + eE^(h + i\*x))\*(f + g\*x)^3)/(a + bE^(h + i\*x) + cE^(2\*h + 2\*i\*x)), x]

[Out] 
$$-(-4\sqrt{-(b^2 - 4ac)}df^3i^4x - 6\sqrt{-(b^2 - 4ac)}df^2g^2i^4x^2 - 4\sqrt{-(b^2 - 4ac)}dfg^2i^4x^3 - \sqrt{-(b^2 - 4ac)}d^2g^3i^4x^4 + 4b\sqrt{b^2 - 4ac}df^3i^3\text{ArcTan}\left[\frac{b + 2cE^{h + ix}}{\sqrt{b^2 - 4ac}}\right] + 8a\sqrt{-b^2 + 4ac}ef^3i^3\text{ArcTanh}\left[\frac{b + 2cE^{h + ix}}{\sqrt{b^2 - 4ac}}\right] + 6\sqrt{-(b^2 - 4ac)}df^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 6b\sqrt{-b^2 + 4ac}df^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] - 12a\sqrt{-b^2 + 4ac}ef^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 6\sqrt{-(b^2 - 4ac)}dfg^2i^3x^2\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 6b\sqrt{-b^2 + 4ac}dfg^2i^3x^2\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] - 12a\sqrt{-b^2 + 4ac}efg^2i^3x^2\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 2\sqrt{-(b^2 - 4ac)}d^2g^3i^3x^3\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 2b\sqrt{-b^2 + 4ac}d^2g^3i^3x^3\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] - 4a\sqrt{-b^2 + 4ac}eg^3i^3x^3\text{Log}\left[1 + \frac{2cE^{h + ix}}{b - \sqrt{b^2 - 4ac}}\right] + 6\sqrt{-(b^2 - 4ac)}df^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b + \sqrt{b^2 - 4ac}}\right] - 6b\sqrt{-b^2 + 4ac}df^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b + \sqrt{b^2 - 4ac}}\right] + 12a\sqrt{-b^2 + 4ac}ef^2gi^3x\text{Log}\left[1 + \frac{2cE^{h + ix}}{b + \sqrt{b^2 - 4ac}}\right] + 6\sqrt{-(b^2 - 4ac)}dfg^2i^3x^2\text{Log}\left[1 + \frac{2cE^{h + ix}}{b + \sqrt{b^2 - 4ac}}\right] - 6b\sqrt{-b^2 + 4ac}dfg^2i^3x^2\text{Log}\left[1 + \frac{2cE^{h + ix}}{b + \sqrt{b^2 - 4ac}}\right]$$

```

c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c]) + 12*a*Sqrt[-b^2 + 4*a*c]*e*f*g^2*i
^3*x^2*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 2*Sqrt[-(b^2 -
4*a*c)^2]*d*g^3*i^3*x^3*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])]
- 2*b*Sqrt[-b^2 + 4*a*c]*d*g^3*i^3*x^3*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[
b^2 - 4*a*c])] + 4*a*Sqrt[-b^2 + 4*a*c]*e*g^3*i^3*x^3*Log[1 + (2*c*E^(h + i
*x))/(b + Sqrt[b^2 - 4*a*c])] + 2*Sqrt[-(b^2 - 4*a*c)^2]*d*f^3*i^3*Log[a +
E^(h + i*x)*(b + c*E^(h + i*x))] + 6*(Sqrt[-(b^2 - 4*a*c)^2]*d + b*Sqrt[-b^
2 + 4*a*c]*d - 2*a*Sqrt[-b^2 + 4*a*c]*e)*g*i^2*(f + g*x)^2*PolyLog[2, (2*c*
E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 6*(Sqrt[-(b^2 - 4*a*c)^2]*d - b*Sq
rt[-b^2 + 4*a*c]*d + 2*a*Sqrt[-b^2 + 4*a*c]*e)*g*i^2*(f + g*x)^2*PolyLog[2,
(-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 12*Sqrt[-(b^2 - 4*a*c)^2]*d*
f*g^2*i*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 12*b*Sqrt[
-b^2 + 4*a*c]*d*f*g^2*i*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c
])] + 24*a*Sqrt[-b^2 + 4*a*c]*e*f*g^2*i*PolyLog[3, (2*c*E^(h + i*x))/(-b +
Sqrt[b^2 - 4*a*c])] - 12*Sqrt[-(b^2 - 4*a*c)^2]*d*g^3*i*x*PolyLog[3, (2*c*E
^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 12*b*Sqrt[-b^2 + 4*a*c]*d*g^3*i*x*P
olyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 24*a*Sqrt[-b^2 + 4*
a*c]*e*g^3*i*x*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 12*
Sqrt[-(b^2 - 4*a*c)^2]*d*f*g^2*i*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^
2 - 4*a*c])] + 12*b*Sqrt[-b^2 + 4*a*c]*d*f*g^2*i*PolyLog[3, (-2*c*E^(h + i*
x))/(b + Sqrt[b^2 - 4*a*c])] - 24*a*Sqrt[-b^2 + 4*a*c]*e*f*g^2*i*PolyLog[3,
(-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 12*Sqrt[-(b^2 - 4*a*c)^2]*d*
g^3*i*x*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 12*b*Sqrt[
-b^2 + 4*a*c]*d*g^3*i*x*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c
])] - 24*a*Sqrt[-b^2 + 4*a*c]*e*g^3*i*x*PolyLog[3, (-2*c*E^(h + i*x))/(b +
Sqrt[b^2 - 4*a*c])] + 12*Sqrt[-(b^2 - 4*a*c)^2]*d*g^3*PolyLog[4, (2*c*E^(h
+ i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 12*b*Sqrt[-b^2 + 4*a*c]*d*g^3*PolyLog[4
, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 24*a*Sqrt[-b^2 + 4*a*c]*e*g
^3*PolyLog[4, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 12*Sqrt[-(b^2 -
4*a*c)^2]*d*g^3*PolyLog[4, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 1
2*b*Sqrt[-b^2 + 4*a*c]*d*g^3*PolyLog[4, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 -
4*a*c])] + 24*a*Sqrt[-b^2 + 4*a*c]*e*g^3*PolyLog[4, (-2*c*E^(h + i*x))/(b +
Sqrt[b^2 - 4*a*c])]/(4*a*Sqrt[-(b^2 - 4*a*c)^2]*i^4)

```

---

**Maple [F]** time = 0.331, size = 0, normalized size = 0.

$$\int \frac{(d + e^{ix+h})(gx + f)^3}{a + be^{ix+h} + ce^{2ix+2h}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*exp(i\*x+h))\*(g\*x+f)^3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x)

[Out] int((d+e\*exp(i\*x+h))\*(g\*x+f)^3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)^3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, a  
lgorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 1.7657, size = 4178, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)^3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, a  
lgorithm="fricas")

[Out] 
$$\frac{1}{4}((b^2 - 4ac)dg^3i^4x^4 + 4(b^2 - 4ac)d^2fg^2i^4x^3 + 6(b^2 - 4ac)d^3f^2gi^4x^2 + 4(b^2 - 4ac)d^4f^3i^4x - 6((b^2 - 4ac)d^2g^3i^2x^2 + 2(b^2 - 4ac)d^3fg^2i^2x + (b^2 - 4ac)d^4f^2gi^2 + ((abd - 2a^2e)g^3i^2x^2 + 2(abd - 2a^2e)fg^2i^2x + (abd - 2a^2e)f^2gi^2)\sqrt{(b^2 - 4ac)/a^2})\operatorname{dilog}(-1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} + be^{ix+h} + 2a)/a + 1) - 6((b^2 - 4ac)d^2g^3i^2x^2 + 2(b^2 - 4ac)d^3fg^2i^2x + (b^2 - 4ac)d^4f^2gi^2 - ((abd - 2a^2e)g^3i^2x^2 + 2(abd - 2a^2e)fg^2i^2x + (abd - 2a^2e)f^2gi^2)\sqrt{(b^2 - 4ac)/a^2})\operatorname{dilog}(1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} - be^{ix+h} - 2a)/a + 1) + 2((b^2 - 4ac)d^2g^3h^3 - 3(b^2 - 4ac)d^3fg^2h^2i + 3(b^2 - 4ac)d^4f^2g^2hi^2 - (b^2 - 4ac)d^5f^3i^3 - ((abd - 2a^2e)g^3h^3 - 3(abd - 2a^2e)fg^2h^2i + 3(abd - 2a^2e)f^2g^2hi^2 - (abd - 2a^2e)f^3i^3)\sqrt{(b^2 - 4ac)/a^2})\log(2ce^{ix+h} + a\sqrt{(b^2 - 4ac)/a^2} + b) + 2((b^2 - 4ac)d^2g^3h^3 - 3(b^2 - 4ac)d^3fg^2h^2i + 3(b^2 - 4ac)d^4f^2g^2hi^2 - (b^2 - 4ac)d^5f^3i^3 + ((abd - 2a^2e)g^3h^3 - 3(abd - 2a^2e)fg^2h^2i + 3(abd - 2a^2e)f^2g^2hi^2 - (abd - 2a^2e)f^3i^3)\sqrt{(b^2 - 4ac)/a^2})\log(2ce^{ix+h} - a\sqrt{(b^2 - 4ac)/a^2} + b) - 2((b^2 - 4ac)d^2g^3i^3x^3 + 3(b^2 - 4ac)d^3fg^2i^3x^2 + 3(b^2 - 4ac)d^4f^2gi^3x + (b^2 - 4ac)d^5f^3h^3 - 3(b^2 - 4ac)d^6fg^2h^2i + 3(b^2 - 4ac)d^7f^2g^2hi^2 + ((abd - 2a^2e)g^3i^3x^3 + 3(abd - 2a^2e)fg^2i^3x^2 + 3(abd - 2a^2e)f^2g^2hi^2 + 3(abd - 2a^2e)g^3h^3 - 3(abd - 2a^2e)fg^2h^2i + 3(abd - 2a^2e)f^2g^2hi^2)\sqrt{(b^2 - 4ac)/a^2})\log(1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} + be^{ix+h} + 2a)/a - 2((b^2 - 4ac)d^2g^3i^3x^3 + 3(b^2 - 4ac)d^3fg^2i^3x^2 + 3(b^2 - 4ac)d^4f^2gi^3x + (b^2 - 4ac)d^5f^3h^3 - 3(b^2 - 4ac)d^6fg^2h^2i + 3(b^2 - 4ac)d^7f^2g^2hi^2 - ((abd - 2a^2e)g^3i^3x^3 + 3(abd - 2a^2e)fg^2i^3x^2 + 3(abd - 2a^2e)f^2g^2hi^2 - (abd - 2a^2e)g^3h^3 - 3(abd - 2a^2e)fg^2h^2i + 3(abd - 2a^2e)f^2g^2hi^2)\sqrt{(b^2 - 4ac)/a^2})\log(-1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} - be^{ix+h} - 2a)/a - 12((b^2 - 4ac)d^2g^3 + (abd - 2a^2e)g^3\sqrt{(b^2 - 4ac)/a^2})\operatorname{polylog}(4, -1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} + be^{ix+h})/a - 12((b^2 - 4ac)d^2g^3 - (abd - 2a^2e)g^3\sqrt{(b^2 - 4ac)/a^2})\operatorname{polylog}(4, 1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} - be^{ix+h})/a + 12((b^2 - 4ac)d^2g^3ix + (b^2 - 4ac)d^3fg^2i + ((abd - 2a^2e)g^3ix + (abd - 2a^2e)fg^2i)\sqrt{(b^2 - 4ac)/a^2})\operatorname{polylog}(3, -1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} + be^{ix+h})/a + 12((b^2 - 4ac)d^2g^3ix + (b^2 - 4ac)d^3fg^2i - ((abd - 2a^2e)g^3ix + (abd - 2a^2e)fg^2i)\sqrt{(b^2 - 4ac)/a^2})\operatorname{polylog}(3, 1/2(a\sqrt{(b^2 - 4ac)/a^2})e^{ix+h} - be^{ix+h})/a)/((ab^2 - 4a^2c)i^4)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)\*\*3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^3 (e^{ix+h} + d)}{ce^{2ix+2h} + be^{ix+h} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)^3/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, algorithm="giac")

[Out] integrate((g\*x + f)^3\*(e\*e^(i\*x + h) + d)/(c\*e^(2\*i\*x + 2\*h) + b\*e^(i\*x + h) + a), x)

$$3.573 \quad \int \frac{(d+ee^{h+ix})(f+gx)^2}{a+be^{h+ix}+ce^{2h+2ix}} dx$$

**Optimal.** Leaf size=599

$$\frac{2g(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^2\left(b-\sqrt{b^2-4ac}\right)} - \frac{2g(f+gx)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^2\left(\sqrt{b^2-4ac}+b\right)} + \frac{2g^2\left(\frac{2cd-b}{\sqrt{b^2-4ac}}\right)}{i^2}$$

[Out]  $((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3)/(3*(b + \text{Sqrt}[b^2 - 4*a*c])*g) + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3)/(3*(b - \text{Sqrt}[b^2 - 4*a*c])*g) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^2*\text{Log}[1 + (2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^2*\text{Log}[1 + (2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i) - (2*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^2) - (2*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^2) + (2*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^3) + (2*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^3)$

**Rubi [A]** time = 1.00222, antiderivative size = 599, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2265, 2184, 2190, 2531, 2282, 6589}

$$\frac{2g(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^2\left(b-\sqrt{b^2-4ac}\right)} - \frac{2g(f+gx)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^2\left(\sqrt{b^2-4ac}+b\right)} + \frac{2g^2\left(\frac{2cd-b}{\sqrt{b^2-4ac}}\right)}{i^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d + e*E^(h + i*x))*(f + g*x)^2}{(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))}, x]$

[Out]  $((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3)/(3*(b + \text{Sqrt}[b^2 - 4*a*c])*g) + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^3)/(3*(b - \text{Sqrt}[b^2 - 4*a*c])*g) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^2*\text{Log}[1 + (2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*(f + g*x)^2*\text{Log}[1 + (2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i) - (2*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^2) - (2*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g*(f + g*x)*\text{PolyLog}[2, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^2) + (2*(e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b - \text{Sqrt}[b^2 - 4*a*c])])/(b - \text{Sqrt}[b^2 - 4*a*c])*i^3) + (2*(e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*g^2*\text{PolyLog}[3, (-2*c*E^(h + i*x))/(b + \text{Sqrt}[b^2 - 4*a*c])])/(b + \text{Sqrt}[b^2 - 4*a*c])*i^3)$

**Rule 2265**

$\text{Int}[\frac{(i_*)*(F_*)^{(u_*)} + (h_*)*((f_*) + (g_*)*(x_*)^{(m_*)})}{(a_*) + (b_*)*(F_*)^{(u_*)} + (c_*)*(F_*)^{(v_*)}}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dis}$

```
t[Simplify[(2*c*h - b*i)/q] + i, Int[(f + g*x)^m/(b - q + 2*c*F^u), x], x]
- Dist[Simplify[(2*c*h - b*i)/q] - i, Int[(f + g*x)^m/(b + q + 2*c*F^u), x],
x] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2*u] && LinearQ[u, x]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2184

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps



$$\int \frac{(d + ee^{h+573x})(f + gx)^2}{a + be^{h+573x} + ce^{2h+1146x}} dx = -\left(\left(-e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{(f + gx)^2}{b + \sqrt{b^2 - 4ac} + 2ce^{h+573x}} dx\right) + \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{(f + gx)^2}{b - \sqrt{b^2 - 4ac} + 2ce^{h+573x}} dx$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b - \sqrt{b^2 - 4ac})g} - \frac{\left(2c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{e^{h+573x}(f + gx)^2}{b + \sqrt{b^2 - 4ac} + 2ce^{h+573x}} dx}{b + \sqrt{b^2 - 4ac}}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2 \log\left(1 + \frac{2ce^{h+573x}}{b - \sqrt{b^2 - 4ac}}\right)}{573(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2 \log\left(1 + \frac{2ce^{h+573x}}{b - \sqrt{b^2 - 4ac}}\right)}{573(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2 \log\left(1 + \frac{2ce^{h+573x}}{b - \sqrt{b^2 - 4ac}}\right)}{573(b - \sqrt{b^2 - 4ac})}$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^3}{3(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2 \log\left(1 + \frac{2ce^{h+573x}}{b - \sqrt{b^2 - 4ac}}\right)}{573(b - \sqrt{b^2 - 4ac})}$$

**Mathematica [B]** time = 2.52292, size = 1412, normalized size = 2.36

$$-2\sqrt{-(b^2 - 4ac)^2} dg^2 x^3 i^3 - 6\sqrt{-(b^2 - 4ac)^2} df gx^2 i^3 - 6\sqrt{-(b^2 - 4ac)^2} df^2 xi^3 + 6b\sqrt{b^2 - 4ac} d f^2 \tan^{-1}\left(\frac{b+2ce^{h+ix}}{\sqrt{4ac-b^2}}\right) i^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*E^(h + i*x))*(f + g*x)^2)/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x)), x]
```

```
[Out] -(-6*Sqrt[-(b^2 - 4*a*c)^2]*d*f^2*i^3*x - 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^3*x^2 - 2*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*i^3*x^3 + 6*b*Sqrt[b^2 - 4*a*c]*d*f^2*i^2*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]] + 12*a*Sqrt[-b^2 + 4*a*c]*e*f^2*i^2*ArcTanh[(b + 2*c*E^(h + i*x))/Sqrt[b^2 - 4*a*c]] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*b*Sqrt[-b^2 + 4*a*c]*d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 12*a*Sqrt[-b^2 + 4*a*c]*e*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 3*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 3*b*Sqrt[-b^2 + 4*a*c]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 6*a*Sqrt[-b^2 + 4*a*c]*e*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 6*b*Sqrt[-b^2 + 4*a*c]*d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 12*a*Sqrt[-b^2 + 4*a*c]*e*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 3*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 3*b*Sqrt[-b^2 + 4*a*c]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 6*a*Sqrt[-b^2 + 4*a*c]*e*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 3*Sqrt[-(b^2 - 4*a*c)^2]*d*f^2*i^2*Log[a + E^(h + i*x)*(b + c*E^(h + i*x))] + 6*(Sqrt[-(b^2 - 4*a*c)^2]*d + b*Sqrt[-b^2 + 4*a*c]*d - 2*a*Sqrt[-b^2 + 4*a*c]*e)*g*i*(f + g*x)*PolyLog[2, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 6*(Sqrt[-(b^2 - 4*a*c)^2]*d - b*Sqrt[-b^2 + 4*a*c]
```

```
*d + 2*a*Sqrt[-b^2 + 4*a*c]*e)*g*i*(f + g*x)*PolyLog[2, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 6*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 6*b*Sqrt[-b^2 + 4*a*c]*d*g^2*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + 12*a*Sqrt[-b^2 + 4*a*c]*e*g^2*PolyLog[3, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] - 6*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 6*b*Sqrt[-b^2 + 4*a*c]*d*g^2*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 12*a*Sqrt[-b^2 + 4*a*c]*e*g^2*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])]/(6*a*Sqrt[-(b^2 - 4*a*c)^2]*i^3)
```

**Maple [F]** time = 0.208, size = 0, normalized size = 0.

$$\int \frac{(d + ee^{ix+h})(gx + f)^2}{a + be^{ix+h} + ce^{2ix+2h}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

```
[Out] int((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 1.5907, size = 2732, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b^2 - 4*a*c)*d*g^2*i^3*x^3 + 6*(b^2 - 4*a*c)*d*f*g*i^3*x^2 + 6*(b^2 - 4*a*c)*d*f^2*i^3*x - 6*((b^2 - 4*a*c)*d*g^2*i*x + (b^2 - 4*a*c)*d*f*g*i + ((a*b*d - 2*a^2*e)*g^2*i*x + (a*b*d - 2*a^2*e)*f*g*i)*sqrt((b^2 - 4*a*c)/a^2))*dilog(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x + h) + b*e^(i*x + h) + 2*a)/a + 1) - 6*((b^2 - 4*a*c)*d*g^2*i*x + (b^2 - 4*a*c)*d*f*g*i - ((a*b*d - 2*a^2*e)*g^2*i*x + (a*b*d - 2*a^2*e)*f*g*i)*sqrt((b^2 - 4*a*c)/a^2))*dilog(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x + h) - b*e^(i*x + h) - 2*a)/a + 1) - 3*((b^2 - 4*a*c)*d*g^2*h^2 - 2*(b^2 - 4*a*c)*d*f*g*h*i + (b^2 - 4*a*c)*d*f^2*i^2 - ((a*b*d - 2*a^2*e)*g^2*h^2 - 2*(a*b*d - 2*a^2*e)*f*g*h*i + (a*b*d - 2*a^2*e)*f^2*i^2)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x + h) + a*sqrt((b^2 - 4*a*c)/a^2) + b) - 3*((b^2 - 4*a*c)*d*g^2*h^2 - 2*(b^2 - 4*a*c)*d*f*g
```

$$\begin{aligned}
 & *h*i + (b^2 - 4*a*c)*d*f^2*i^2 + ((a*b*d - 2*a^2*e)*g^2*h^2 - 2*(a*b*d - 2* \\
 & a^2*e)*f*g*h*i + (a*b*d - 2*a^2*e)*f^2*i^2)*\sqrt{(b^2 - 4*a*c)/a^2})*\log(2* \\
 & c*e^{(i*x + h)} - a*\sqrt{(b^2 - 4*a*c)/a^2} + b) - 3*((b^2 - 4*a*c)*d*g^2*i^2 \\
 & *x^2 + 2*(b^2 - 4*a*c)*d*f*g*i^2*x - (b^2 - 4*a*c)*d*g^2*h^2 + 2*(b^2 - 4*a \\
 & *c)*d*f*g*h*i + ((a*b*d - 2*a^2*e)*g^2*i^2*x^2 + 2*(a*b*d - 2*a^2*e)*f*g*i^ \\
 & 2*x - (a*b*d - 2*a^2*e)*g^2*h^2 + 2*(a*b*d - 2*a^2*e)*f*g*h*i)*\sqrt{(b^2 - \\
 & 4*a*c)/a^2})*\log(1/2*(a*\sqrt{(b^2 - 4*a*c)/a^2}*e^{(i*x + h)} + b*e^{(i*x + h)} \\
 & + 2*a)/a) - 3*((b^2 - 4*a*c)*d*g^2*i^2*x^2 + 2*(b^2 - 4*a*c)*d*f*g*i^2*x - \\
 & (b^2 - 4*a*c)*d*g^2*h^2 + 2*(b^2 - 4*a*c)*d*f*g*h*i - ((a*b*d - 2*a^2*e)*g \\
 & ^2*i^2*x^2 + 2*(a*b*d - 2*a^2*e)*f*g*i^2*x - (a*b*d - 2*a^2*e)*g^2*h^2 + 2* \\
 & (a*b*d - 2*a^2*e)*f*g*h*i)*\sqrt{(b^2 - 4*a*c)/a^2})*\log(-1/2*(a*\sqrt{(b^2 - \\
 & 4*a*c)/a^2}*e^{(i*x + h)} - b*e^{(i*x + h)} - 2*a)/a) + 6*((b^2 - 4*a*c)*d*g^2 \\
 & + (a*b*d - 2*a^2*e)*g^2*\sqrt{(b^2 - 4*a*c)/a^2})*\text{polylog}(3, -1/2*(a*\sqrt{(b^2 - \\
 & b^2 - 4*a*c)/a^2}*e^{(i*x + h)} + b*e^{(i*x + h)})/a) + 6*((b^2 - 4*a*c)*d*g^2 \\
 & - (a*b*d - 2*a^2*e)*g^2*\sqrt{(b^2 - 4*a*c)/a^2})*\text{polylog}(3, 1/2*(a*\sqrt{(b^2 - \\
 & 2 - 4*a*c)/a^2}*e^{(i*x + h)} - b*e^{(i*x + h)})/a))/((a*b^2 - 4*a^2*c)*i^3)
 \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + e^h e^{ix})(f + gx)^2}{a + b e^h e^{ix} + c e^{2h} e^{2ix}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)\*\*2/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x)

[Out] Integral((d + e\*exp(h)\*exp(i\*x))\*(f + g\*x)\*\*2/(a + b\*exp(h)\*exp(i\*x) + c\*exp(2\*h)\*exp(2\*i\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2 (e^{(ix+h)} + d)}{c e^{(2ix+2h)} + b e^{(ix+h)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)^2/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, algorithm="giac")

[Out] integrate((g\*x + f)^2\*(e\*e^{(i\*x + h)} + d)/(c\*e^{(2\*i\*x + 2\*h)} + b\*e^{(i\*x + h)} + a), x)

$$3.574 \quad \int \frac{(d+e^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}} dx$$

**Optimal.** Leaf size=428

$$\frac{g\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^2\left(b-\sqrt{b^2-4ac}\right)} - \frac{g\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^2\left(\sqrt{b^2-4ac}+b\right)} - \frac{(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i\left(b-\sqrt{b^2-4ac}\right)}$$

[Out] ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)^2)/(2\*(b + Sqrt[b^2 - 4\*a\*c])\*g) + ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)^2)/(2\*(b - Sqrt[b^2 - 4\*a\*c])\*g) - ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)\*Log[1 + (2\*c\*E^(h + i\*x))/(b - Sqrt[b^2 - 4\*a\*c])])/((b - Sqrt[b^2 - 4\*a\*c])\*i) - ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)\*Log[1 + (2\*c\*E^(h + i\*x))/(b + Sqrt[b^2 - 4\*a\*c])])/((b + Sqrt[b^2 - 4\*a\*c])\*i) - ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*g\*PolyLog[2, (-2\*c\*E^(h + i\*x))/(b - Sqrt[b^2 - 4\*a\*c])])/((b - Sqrt[b^2 - 4\*a\*c])\*i^2) - ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*g\*PolyLog[2, (-2\*c\*E^(h + i\*x))/(b + Sqrt[b^2 - 4\*a\*c])])/((b + Sqrt[b^2 - 4\*a\*c])\*i^2)

**Rubi [A]** time = 0.582517, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 42,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$ , Rules used = {2265, 2184, 2190, 2279, 2391}

$$\frac{g\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^2\left(b-\sqrt{b^2-4ac}\right)} - \frac{g\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}\right)}{i^2\left(\sqrt{b^2-4ac}+b\right)} - \frac{(f+gx)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i\left(b-\sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*E^(h + i\*x))\*(f + g\*x))/(a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x)), x]

[Out] ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)^2)/(2\*(b + Sqrt[b^2 - 4\*a\*c])\*g) + ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)^2)/(2\*(b - Sqrt[b^2 - 4\*a\*c])\*g) - ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)\*Log[1 + (2\*c\*E^(h + i\*x))/(b - Sqrt[b^2 - 4\*a\*c])])/((b - Sqrt[b^2 - 4\*a\*c])\*i) - ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*(f + g\*x)\*Log[1 + (2\*c\*E^(h + i\*x))/(b + Sqrt[b^2 - 4\*a\*c])])/((b + Sqrt[b^2 - 4\*a\*c])\*i) - ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*g\*PolyLog[2, (-2\*c\*E^(h + i\*x))/(b - Sqrt[b^2 - 4\*a\*c])])/((b - Sqrt[b^2 - 4\*a\*c])\*i^2) - ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*g\*PolyLog[2, (-2\*c\*E^(h + i\*x))/(b + Sqrt[b^2 - 4\*a\*c])])/((b + Sqrt[b^2 - 4\*a\*c])\*i^2)

#### Rule 2265

Int[(((i\_.)\*(F\_)^(u\_) + (h\_.))\*((f\_.) + (g\_.)\*(x\_)^(m\_.)))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[Simplify[(2\*c\*h - b\*i)/q] + i, Int[(f + g\*x)^m/(b - q + 2\*c\*F^u), x], x] - Dist[Simplify[(2\*c\*h - b\*i)/q] - i, Int[(f + g\*x)^m/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_)^(m\_.)))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)^(n\_.)))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[

b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{(d + ee^{h+574x})(f + gx)}{a + be^{h+574x} + ce^{2h+1148x}} dx &= -\left(\left(-e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{f + gx}{b + \sqrt{b^2 - 4ac} + 2ce^{h+574x}} dx\right) + \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{f + gx}{b - \sqrt{b^2 - 4ac} + 2ce^{h+574x}} dx \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b - \sqrt{b^2 - 4ac})g} - \frac{2c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \int \frac{e^{h+574x}(f + gx)}{b + \sqrt{b^2 - 4ac} + 2ce^{h+574x}} dx}{b + \sqrt{b^2 - 4ac}} \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx) \log\left(1 + \frac{2ce^{h+574x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+574x}}\right)}{574(b - \sqrt{b^2 - 4ac})} \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx) \log\left(1 + \frac{2ce^{h+574x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+574x}}\right)}{574(b - \sqrt{b^2 - 4ac})} \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b + \sqrt{b^2 - 4ac})g} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx)^2}{2(b - \sqrt{b^2 - 4ac})g} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)(f + gx) \log\left(1 + \frac{2ce^{h+574x}}{b + \sqrt{b^2 - 4ac} + 2ce^{h+574x}}\right)}{574(b - \sqrt{b^2 - 4ac})} \end{aligned}$$

**Mathematica [A]** time = 1.76181, size = 677, normalized size = 1.58

$$g \left( d \sqrt{-(b^2 - 4ac)^2} + bd \sqrt{4ac - b^2} - 2ae \sqrt{4ac - b^2} \right) \text{PolyLog} \left( 2, \frac{2ce^{h+ix}}{\sqrt{b^2 - 4ac} - b} \right) + g \left( d \sqrt{-(b^2 - 4ac)^2} - bd \sqrt{4ac - b^2} + 2ae \sqrt{4ac - b^2} \right) \text{PolyLog} \left( 2, \frac{2ce^{h+ix}}{\sqrt{b^2 - 4ac} + b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*E^(h + i\*x))\*(f + g\*x))/(a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x)), x]

[Out] -(i\*(-2\*Sqrt[-(b^2 - 4\*a\*c)^2]\*d\*f\*i\*x - Sqrt[-(b^2 - 4\*a\*c)^2]\*d\*g\*i\*x^2 + 2\*b\*Sqrt[b^2 - 4\*a\*c]\*d\*f\*ArcTan[(b + 2\*c\*E^(h + i\*x))/Sqrt[-b^2 + 4\*a\*c]])

$$\begin{aligned}
& + 4*a*\text{Sqrt}[-b^2 + 4*a*c]*e*f*\text{ArcTanh}[(b + 2*c*E^{(h + i*x)})/\text{Sqrt}[b^2 - 4*a*c]] \\
& + \text{Sqrt}[-(b^2 - 4*a*c)^2]*d*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b - \text{Sqrt}[b^2 - 4*a*c])] \\
& + b*\text{Sqrt}[-b^2 + 4*a*c]*d*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b - \text{Sqrt}[b^2 - 4*a*c])] \\
& - 2*a*\text{Sqrt}[-b^2 + 4*a*c]*e*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b - \text{Sqrt}[b^2 - 4*a*c])] \\
& + \text{Sqrt}[-(b^2 - 4*a*c)^2]*d*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b + \text{Sqrt}[b^2 - 4*a*c])] \\
& - b*\text{Sqrt}[-b^2 + 4*a*c]*d*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b + \text{Sqrt}[b^2 - 4*a*c])] \\
& + 2*a*\text{Sqrt}[-b^2 + 4*a*c]*e*g*x*\text{Log}[1 + (2*c*E^{(h + i*x)})/(b + \text{Sqrt}[b^2 - 4*a*c])] \\
& + \text{Sqrt}[-(b^2 - 4*a*c)^2]*d*f*\text{Log}[a + E^{(h + i*x)}*(b + c*E^{(h + i*x)})] \\
& + (\text{Sqrt}[-(b^2 - 4*a*c)^2]*d + b*\text{Sqrt}[-b^2 + 4*a*c]*d - 2*a*\text{Sqrt}[-b^2 + 4*a*c]*e)*g*\text{PolyLog}[2, (2*c*E^{(h + i*x)})/(-b + \text{Sqrt}[b^2 - 4*a*c])] \\
& + (\text{Sqrt}[-(b^2 - 4*a*c)^2]*d - b*\text{Sqrt}[-b^2 + 4*a*c]*d + 2*a*\text{Sqrt}[-b^2 + 4*a*c]*e)*g*\text{PolyLog}[2, (-2*c*E^{(h + i*x)})/(b + \text{Sqrt}[b^2 - 4*a*c])] \\
& )/(2*a*\text{Sqrt}[-(b^2 - 4*a*c)^2]*i^2)
\end{aligned}$$

**Maple [B]** time = 0.032, size = 1261, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)`

[Out] 
$$\begin{aligned}
& d*f/i/a*\ln(\exp(i*x))-1/2*d*f/i/a*\ln(a+b*\exp(i*x)*\exp(h)+c*\exp(i*x)^2*\exp(2*h)) \\
& -d*f/i/a*\exp(h)*b/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)}*\arctan((\exp(h)*b+2*\exp(2*h)*\exp(i*x)*c)/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)})+1/2*d*g/a*x^2-1/2*d*g/i/a*x/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\exp(h)*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& *b+1/2*d*g/i/a*x/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\exp(h)*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& *b-1/2*d*g/i/a*x*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& -1/2*d*g/i/a*x*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& +1/2*d*g/i^2/a/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\exp(h)*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& *b-1/2*d*g/i^2/a/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\exp(h)*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& *b-1/2*d*g/i^2/a*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& -1/2*d*g/i^2/a*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& +2*e*\exp(h)*f/i/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)}*\arctan((\exp(h)*b+2*\exp(2*h)*\exp(i*x)*c)/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)})+e*\exp(h)*g/i*x/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& -e*\exp(h)*g/i*x/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\ln((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& +e*\exp(h)*g/i^2/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b-(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})) \\
& -e*\exp(h)*g/i^2/(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}*\text{dilog}((2*\exp(2*h)*\exp(i*x)*c+\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)})/(\exp(h)*b+(\exp(h)^2*b^2-4*a*c*\exp(2*h))^{(1/2)}))
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.34788, size = 1513, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, algorithm="fricas")

[Out] 
$$\frac{1}{2}((b^2 - 4ac)dg^2x^2 + 2(b^2 - 4ac)df^2x - ((b^2 - 4ac)dg + (abd - 2a^2e)g\sqrt{(b^2 - 4ac)/a^2})\operatorname{dilog}(-1/2(a\sqrt{(b^2 - 4ac)/a^2}e^{ix+h} + be^{ix+h} + 2a)/a + 1) - ((b^2 - 4ac)dg - (abd - 2a^2e)g\sqrt{(b^2 - 4ac)/a^2})\operatorname{dilog}(1/2(a\sqrt{(b^2 - 4ac)/a^2}e^{ix+h} - be^{ix+h} - 2a)/a + 1) + ((b^2 - 4ac)dg^2h - (b^2 - 4ac)df^2i - ((abd - 2a^2e)g^2h - (abd - 2a^2e)fi)\sqrt{(b^2 - 4ac)/a^2})\log(2ce^{ix+h} + a\sqrt{(b^2 - 4ac)/a^2} + b) + ((b^2 - 4ac)dg^2h - (b^2 - 4ac)df^2i + ((abd - 2a^2e)g^2h - (abd - 2a^2e)fi)\sqrt{(b^2 - 4ac)/a^2})\log(2ce^{ix+h} - a\sqrt{(b^2 - 4ac)/a^2} + b) - ((b^2 - 4ac)dg^2ix + (b^2 - 4ac)dg^2h + (abd - 2a^2e)g^2ix + (abd - 2a^2e)g^2h)\sqrt{(b^2 - 4ac)/a^2})\log(1/2(a\sqrt{(b^2 - 4ac)/a^2}e^{ix+h} + be^{ix+h} + 2a)/a) - ((b^2 - 4ac)dg^2ix + (b^2 - 4ac)dg^2h - ((abd - 2a^2e)g^2ix + (abd - 2a^2e)g^2h)\sqrt{(b^2 - 4ac)/a^2})\log(-1/2(a\sqrt{(b^2 - 4ac)/a^2}e^{ix+h} - be^{ix+h} - 2a)/a))/((ab^2 - 4a^2c)i^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + e^h e^{ix})(f + gx)}{a + be^h e^{ix} + ce^{2h} e^{2ix}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))\*(g\*x+f)/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x)

[Out] Integral((d + e\*exp(h)\*exp(i\*x))\*(f + g\*x)/(a + b\*exp(h)\*exp(i\*x) + c\*exp(2\*h)\*exp(2\*i\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)(e^{(ix+h)} + d)}{ce^{(2ix+2h)} + be^{(ix+h)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(e*e^(i*x + h) + d)/(c*e^(2*i*x + 2*h) + b*e^(i*x + h) + a), x)
```



$$3.575 \quad \int \frac{d+ce^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}} dx$$

**Optimal.** Leaf size=95

$$\frac{(bd - 2ae) \tanh^{-1} \left( \frac{b+2ce^{h+ix}}{\sqrt{b^2-4ac}} \right)}{ai\sqrt{b^2 - 4ac}} - \frac{d \log(a + be^{h+ix} + ce^{2h+2ix})}{2ai} + \frac{dx}{a}$$

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*E^(h + i\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*i) - (d\*Log[a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x)])/(2\*a\*i)

**Rubi [A]** time = 0.151467, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$ , Rules used = {2282, 800, 634, 618, 206, 628}

$$\frac{(bd - 2ae) \tanh^{-1} \left( \frac{b+2ce^{h+ix}}{\sqrt{b^2-4ac}} \right)}{ai\sqrt{b^2 - 4ac}} - \frac{d \log(a + be^{h+ix} + ce^{2h+2ix})}{2ai} + \frac{dx}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*E^(h + i\*x))/(a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x)),x]

[Out] (d\*x)/a + ((b\*d - 2\*a\*e)\*ArcTanh[(b + 2\*c\*E^(h + i\*x))/Sqrt[b^2 - 4\*a\*c]])/(a\*Sqrt[b^2 - 4\*a\*c]\*i) - (d\*Log[a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x)])/(2\*a\*i)

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 800

Int[(((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{d + e^{h+575x}}{a + be^{h+575x} + ce^{2h+1150x}} dx &= \frac{1}{575} \text{Subst} \left( \int \frac{d + ex}{x(a + bx + cx^2)} dx, x, e^{h+575x} \right) \\ &= \frac{1}{575} \text{Subst} \left( \int \left( \frac{d}{ax} + \frac{-bd + ae - cdx}{a(a + bx + cx^2)} \right) dx, x, e^{h+575x} \right) \\ &= \frac{dx}{a} + \frac{\text{Subst} \left( \int \frac{-bd + ae - cdx}{a + bx + cx^2} dx, x, e^{h+575x} \right)}{575a} \\ &= \frac{dx}{a} - \frac{d \text{Subst} \left( \int \frac{b+2cx}{a+bx+cx^2} dx, x, e^{h+575x} \right)}{1150a} - \frac{(bd - 2ae) \text{Subst} \left( \int \frac{1}{a+bx+cx^2} dx, x, e^{h+575x} \right)}{1150a} \\ &= \frac{dx}{a} - \frac{d \log(a + be^{h+575x} + ce^{2h+1150x})}{1150a} + \frac{(bd - 2ae) \text{Subst} \left( \int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2ce^{h+575x} \right)}{575a} \\ &= \frac{dx}{a} + \frac{(bd - 2ae) \tanh^{-1} \left( \frac{b+2ce^{h+575x}}{\sqrt{b^2 - 4ac}} \right)}{575a\sqrt{b^2 - 4ac}} - \frac{d \log(a + be^{h+575x} + ce^{2h+1150x})}{1150a} \end{aligned}$$

**Mathematica [A]** time = 0.167344, size = 94, normalized size = 0.99

$$\frac{\frac{2(bd-2ae) \tan^{-1} \left( \frac{b+2ce^{h+ix}}{\sqrt{4ac-b^2}} \right)}{i\sqrt{4ac-b^2}} + \frac{d \log(a+e^{h+ix}(b+ce^{h+ix}))}{i} - 2dx}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*E^(h + i*x))/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x)), x]
```

```
[Out] -(-2*d*x + (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]
])/ (Sqrt[-b^2 + 4*a*c]*i) + (d*Log[a + E^(h + i*x)*(b + c*E^(h + i*x))])/i
/(2*a)
```

**Maple [B]** time = 0.003, size = 183, normalized size = 1.9

$$\frac{d \ln(e^{ix})}{ai} - \frac{d \ln(a + be^{ix}e^h + c(e^{ix})^2 e^{2h})}{2ai} - \frac{de^hb}{ai} \arctan \left( (e^hb + 2e^{2h}e^{ix}c) \frac{1}{\sqrt{4ace^{2h} - (e^h)^2 b^2}} \right) \frac{1}{\sqrt{4ace^{2h} - (e^h)^2 b^2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)`

[Out]  $d/i/a*\ln(\exp(i*x))-1/2*d/i/a*\ln(a+b*\exp(i*x)*\exp(h)+c*\exp(i*x)^2*\exp(2*h))-d/i/a*\exp(h)*b/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)}*\arctan((\exp(h)*b+2*\exp(2*h)*\exp(i*x)*c)/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)})+2*e*\exp(h)/i/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)}*\arctan((\exp(h)*b+2*\exp(2*h)*\exp(i*x)*c)/(4*a*c*\exp(2*h)-\exp(h)^2*b^2)^{(1/2)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.4407, size = 686, normalized size = 7.22

$$\frac{2(b^2 - 4ac)dix - (b^2 - 4ac)d \log(ce^{2ix+2h} + be^{ix+h} + a) - \sqrt{b^2 - 4ac}(bd - 2ae) \log\left(\frac{2c^2e^{2ix+2h} + 2bce^{ix+h} + b^2 - 2ac - \sqrt{b^2 - 4ac}}{ce^{2ix+2h} + be^{ix+h} + a}\right)}{2(ab^2 - 4a^2c)i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm="fricas")`

[Out]  $[1/2*(2*(b^2 - 4*a*c)*d*i*x - (b^2 - 4*a*c)*d*\log(c*e^{(2*i*x + 2*h)} + b*e^{(i*x + h)} + a) - \text{sqrt}(b^2 - 4*a*c)*(b*d - 2*a*e)*\log((2*c^2*e^{(2*i*x + 2*h)} + 2*b*c*e^{(i*x + h)} + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c)*(2*c*e^{(i*x + h)} + b))/(c*e^{(2*i*x + 2*h)} + b*e^{(i*x + h)} + a)))/((a*b^2 - 4*a^2*c)*i), 1/2*(2*(b^2 - 4*a*c)*d*i*x - (b^2 - 4*a*c)*d*\log(c*e^{(2*i*x + 2*h)} + b*e^{(i*x + h)} + a) + 2*\text{sqrt}(-b^2 + 4*a*c)*(b*d - 2*a*e)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*e^{(i*x + h)} + b)/(b^2 - 4*a*c)))/((a*b^2 - 4*a^2*c)*i)]$

**Sympy [A]** time = 0.711209, size = 116, normalized size = 1.22

$$\text{RootSum}\left(z^2(4a^2ci^2 - ab^2i^2) + z(4acdi - b^2di) + ae^2 - bde + cd^2, \left(i \mapsto i \log\left(e^{h+ix} + \frac{4ia^2ci - iab^2i + abe + 2acd - bcd}{2ace - bcd}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)`

[Out]  $\text{RootSum}(\_z**2*(4*a**2*c*i**2 - a*b**2*i**2) + \_z*(4*a*c*d*i - b**2*d*i) + a**2 - b*d*e + c*d**2, \text{Lambda}(\_i, \_i*\log(\exp(h + i*x) + (4*\_i*a**2*c*i - \_i*a*b**2*i + a*b*e + 2*a*c*d - b**2*d)/(2*a*c*e - b*c*d)))) + d*x/a$

**Giac [A]** time = 1.2916, size = 161, normalized size = 1.69

$$\frac{1}{2} \left( \frac{2 \left( bde^{(3h)} - 2ae^{(3h+1)} \right) \arctan \left( \frac{(2ce^{(ix+4h)} + be^{(3h)})e^{(-3h)}}{\sqrt{-b^2+4ac}} \right) e^{(-3h)}}{\sqrt{-b^2+4ac}} - \frac{8dh}{a} + \frac{d \log \left( ce^{(2ix+8h)} + be^{(ix+7h)} + ae^{(6h)} \right)}{a} \right) i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h)),x, algorithm="giac")

[Out] 1/2\*(2\*(b\*d\*e^(3\*h) - 2\*a\*e^(3\*h + 1))\*arctan((2\*c\*e^(i\*x + 4\*h) + b\*e^(3\*h)))\*e^(-3\*h)/sqrt(-b^2 + 4\*a\*c))\*e^(-3\*h)/(sqrt(-b^2 + 4\*a\*c)\*a) - 8\*d\*h/a + d\*log(c\*e^(2\*i\*x + 8\*h) + b\*e^(i\*x + 7\*h) + a\*e^(6\*h))/a)\*i

$$3.576 \quad \int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)} dx$$

**Optimal.** Leaf size=82

$$d\text{CannotIntegrate}\left(\frac{1}{(f+gx)(a+be^{h+ix}+ce^{2h+2ix})}, x\right) + e\text{CannotIntegrate}\left(\frac{e^{h+ix}}{(f+gx)(a+be^{h+ix}+ce^{2h+2ix})}, x\right)$$

[Out] d\*CannotIntegrate[1/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x] + e\*CannotIntegrate[E^(h + i\*x)/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x]

**Rubi [A]** time = 1.01787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{d + ee^{h+ix}}{(a + be^{h+ix} + ce^{2h+2ix})(f + gx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x]

[Out] d\*Defer[Int][1/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x] + e\*Defer[Int][E^(h + i\*x)/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x]

Rubi steps

$$\begin{aligned} \int \frac{d + ee^{h+576x}}{(a + be^{h+576x} + ce^{2h+1152x})(f + gx)} dx &= \int \left( \frac{d}{(a + be^{h+576x} + ce^{2h+1152x})(f + gx)} + \frac{ee^{h+576x}}{(a + be^{h+576x} + ce^{2h+1152x})(f + gx)} \right) dx \\ &= d \int \frac{1}{(a + be^{h+576x} + ce^{2h+1152x})(f + gx)} dx + e \int \frac{e^{h+576x}}{(a + be^{h+576x} + ce^{2h+1152x})(f + gx)} dx \end{aligned}$$

**Mathematica [A]** time = 0.412382, size = 0, normalized size = 0.

$$\int \frac{d + ee^{h+ix}}{(a + be^{h+ix} + ce^{2h+2ix})(f + gx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x]

[Out] Integrate[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)), x]

**Maple [A]** time = 0.156, size = 0, normalized size = 0.

$$\int \frac{d + ee^{ix+h}}{(a + be^{ix+h} + ce^{2ix+2h})(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x)

[Out] int((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ee^{(ix+h)} + d}{(gx + f)(ce^{2ix+2h} + be^{(ix+h)} + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x, algorithm="maxima")

[Out] integrate((e\*e^(i\*x + h) + d)/((g\*x + f)\*(c\*e^(2\*i\*x + 2\*h) + b\*e^(i\*x + h) + a)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ee^{(ix+h)} + d}{agx + af + (cgx + cf)e^{2ix+2h} + (bgx + bf)e^{(ix+h)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x, algorithm="fricas")

[Out] integral((e\*e^(i\*x + h) + d)/(a\*g\*x + a\*f + (c\*g\*x + c\*f)\*e^(2\*i\*x + 2\*h) + (b\*g\*x + b\*f)\*e^(i\*x + h)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(ix+h)} + d}{(gx + f)(ce^{2ix+2h} + be^{(ix+h)} + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f),x, algorithm="giac")

[Out] integrate((e\*e^(i\*x + h) + d)/((g\*x + f)\*(c\*e^(2\*i\*x + 2\*h) + b\*e^(i\*x + h) + a)), x)

$$3.577 \quad \int \frac{d + e^{h+ix}}{(a + be^{h+ix} + ce^{2h+2ix})(f+gx)^2} dx$$

**Optimal.** Leaf size=82

$$d\text{CannotIntegrate}\left(\frac{1}{(f+gx)^2(a+be^{h+ix}+ce^{2h+2ix})}, x\right) + e\text{CannotIntegrate}\left(\frac{e^{h+ix}}{(f+gx)^2(a+be^{h+ix}+ce^{2h+2ix})}, x\right)$$

[Out] d\*CannotIntegrate[1/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x] + e\*CannotIntegrate[E^(h + i\*x)/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x]

**Rubi [A]** time = 0.869854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{d + e^{h+ix}}{(a + be^{h+ix} + ce^{2h+2ix})(f+gx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x]

[Out] d\*Defer[Int][1/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x] + e\*Defer[Int][E^(h + i\*x)/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x]

Rubi steps

$$\begin{aligned} \int \frac{d + e^{h+577x}}{(a + be^{h+577x} + ce^{2h+1154x})(f+gx)^2} dx &= \int \left( \frac{d}{(a + be^{h+577x} + ce^{2h+1154x})(f+gx)^2} + \frac{e^{h+577x}}{(a + be^{h+577x} + ce^{2h+1154x})(f+gx)^2} \right) dx \\ &= d \int \frac{1}{(a + be^{h+577x} + ce^{2h+1154x})(f+gx)^2} dx + e \int \frac{e^{h+577x}}{(a + be^{h+577x} + ce^{2h+1154x})(f+gx)^2} dx \end{aligned}$$

**Mathematica [A]** time = 5.76063, size = 0, normalized size = 0.

$$\int \frac{d + e^{h+ix}}{(a + be^{h+ix} + ce^{2h+2ix})(f+gx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x]

[Out] Integrate[(d + e\*E^(h + i\*x))/((a + b\*E^(h + i\*x) + c\*E^(2\*h + 2\*i\*x))\*(f + g\*x)^2), x]



**Maple [A]** time = 0.197, size = 0, normalized size = 0.

$$\int \frac{d + ee^{ix+h}}{(a + be^{ix+h} + ce^{2ix+2h})(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f)^2,x)

[Out] int((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f)^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ee^{(ix+h)} + d}{(gx + f)^2 (ce^{2ix+2h} + be^{(ix+h)} + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f)^2,x, algorithm="maxima")

[Out] integrate((e\*e^(i\*x + h) + d)/((g\*x + f)^2\*(c\*e^(2\*i\*x + 2\*h) + b\*e^(i\*x + h) + a)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ee^{(ix+h)} + d}{ag^2x^2 + 2afgx + af^2 + (cg^2x^2 + 2cfgx + cf^2)e^{2ix+2h} + (bg^2x^2 + 2bfgx + bf^2)e^{(ix+h)}, x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f)^2,x, algorithm="fricas")

[Out] integral((e\*e^(i\*x + h) + d)/(a\*g^2\*x^2 + 2\*a\*f\*g\*x + a\*f^2 + (c\*g^2\*x^2 + 2\*c\*f\*g\*x + c\*f^2)\*e^(2\*i\*x + 2\*h) + (b\*g^2\*x^2 + 2\*b\*f\*g\*x + b\*f^2)\*e^(i\*x + h)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*exp(i\*x+h))/(a+b\*exp(i\*x+h)+c\*exp(2\*i\*x+2\*h))/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ee^{(ix+h)} + d}{(gx + f)^2 (ce^{2ix+2h} + be^{(ix+h)} + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((e*e^(i*x + h) + d)/((g*x + f)^2*(c*e^(2*i*x + 2*h) + b*e^(i*x + h) + a)), x)
```

$$3.578 \quad \int \frac{(be - aee^{c+dx})x}{be - 2aee^{c+dx} - bee^{2(c+dx)}} dx$$

**Optimal.** Leaf size=150

$$-\frac{\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2} - \frac{\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2} - \frac{x \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{2d} - \frac{x \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{2d} + \frac{x^2}{2}$$

[Out]  $x^2/2 - (x*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*d) - (x*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*d) - \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(2*d^2) - \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(2*d^2)$

**Rubi [A]** time = 0.67244, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.106$ , Rules used = {2265, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2} - \frac{\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2} - \frac{x \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{2d} - \frac{x \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{2d} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(b*e - a*e*E^{(c + d*x)})x/(b*e - 2*a*e*E^{(c + d*x)} - b*e*E^{(2*(c + d*x))}), x]$

[Out]  $x^2/2 - (x*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*d) - (x*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*d) - \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(2*d^2) - \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(2*d^2)$

#### Rule 2265

$\text{Int}[(i_*)*(F_*)^{(u_*)} + (h_*)*((f_*) + (g_*)*(x_*)^{(m_*)})]/((a_*) + (b_*)*(F_*)^{(u_*)} + (c_*)*(F_*)^{(v_*)}), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[\text{Simplify}[(2*c*h - b*i)/q] + i, \text{Int}[(f + g*x)^m/(b - q + 2*c*F^u), x], x] - \text{Dist}[\text{Simplify}[(2*c*h - b*i)/q] - i, \text{Int}[(f + g*x)^m/(b + q + 2*c*F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g, h, i\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

#### Rule 2184

$\text{Int}[(c_*) + (d_*)*(x_*)^{(m_*)}]/((a_*) + (b_*)*((F_*)^{(g_*)}*((e_*) + (f_*)*(x_*)^{(n_*)}))^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}/(a*d*(m+1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n}/(a + b*(F^{(g*(e + f*x)))^n}), x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2190

$\text{Int}[(F_*)^{(g_*)}*((e_*) + (f_*)*(x_*)^{(n_*)}))^{(n_*)}]/((a_*) + (b_*)*((F_*)^{(g_*)}*((e_*) + (f_*)*(x_*)^{(n_*)}))^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(be - aee^{c+dx})x}{be - 2aee^{c+dx} - bee^{2(c+dx)}} dx &= - \left( \left( (a - \sqrt{a^2 + b^2})e \right) \int \frac{x}{-2ae + 2\sqrt{a^2 + b^2}e - 2bee^{c+dx}} dx \right) - \left( \left( (a + \sqrt{a^2 + b^2})e \right) \int \frac{x}{-2ae - 2\sqrt{a^2 + b^2}e - 2bee^{c+dx}} dx \right) \\ &= \frac{x^2}{2} + (be) \int \frac{e^{c+dx}x}{-2ae - 2\sqrt{a^2 + b^2}e - 2bee^{c+dx}} dx + (be) \int \frac{e^{c+dx}x}{-2ae + 2\sqrt{a^2 + b^2}e - 2bee^{c+dx}} dx \\ &= \frac{x^2}{2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{2d} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{2d} + \frac{\int \log\left(1 - \frac{2bee^{c+dx}}{-2ae - 2\sqrt{a^2 + b^2}e}\right) dx}{2d} + \frac{\int \log\left(1 - \frac{2bee^{c+dx}}{-2ae + 2\sqrt{a^2 + b^2}e}\right) dx}{2d} \\ &= \frac{x^2}{2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{2d} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{2d} + \frac{\text{Subst}\left(\int \frac{\log\left(1 - \frac{2be^{c+dx}}{-2ae - 2\sqrt{a^2 + b^2}e}\right)}{x} dx, x, e^{c+dx}\right)}{2d^2} + \frac{\text{Subst}\left(\int \frac{\log\left(1 - \frac{2be^{c+dx}}{-2ae + 2\sqrt{a^2 + b^2}e}\right)}{x} dx, x, e^{c+dx}\right)}{2d^2} \\ &= \frac{x^2}{2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{2d} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{2d} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{2d^2} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{2d^2} \end{aligned}$$

**Mathematica [B]** time = 0.413307, size = 398, normalized size = 2.65

$$\left(\sqrt{a^2 + b^2} + a\right) \text{PolyLog}\left(2, \frac{(\sqrt{a^2 + b^2} - a)e^{-c-dx}}{b}\right) + \left(\sqrt{a^2 + b^2} - a\right) \text{PolyLog}\left(2, -\frac{(\sqrt{a^2 + b^2} + a)e^{-c-dx}}{b}\right) + a \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*e - a*e*E^(c + d*x))*x)/(b*e - 2*a*e*E^(c + d*x) - b*e*E^(2*(c + d*x))),x]
```

```
[Out] -(a*d*x*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b]) - Sqrt[a^2 + b^2]*d*x*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] + a*d*x*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - Sqrt[a^2 + b^2]*d*x*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] + a*d*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - a*d*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + (a + Sqrt[a^2 + b^2])*PolyLog[2, (((-a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] + (-a + Sqrt[a^2 + b^2])*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] + a*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - a*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(2*Sqrt[a^2 + b^2]*d^2)
```

**Maple [B]** time = 0.037, size = 285, normalized size = 1.9

$$-\frac{x}{2d} \ln\left(\left(e^{2c}e^{dx}b + e^ca - \sqrt{(e^c)^2 a^2 + e^{2c}b^2}\right)\left(e^ca - \sqrt{(e^c)^2 a^2 + e^{2c}b^2}\right)^{-1}\right) - \frac{x}{2d} \ln\left(\left(e^{2c}e^{dx}b + e^ca + \sqrt{(e^c)^2 a^2 + e^{2c}b^2}\right)\left(e^ca + \sqrt{(e^c)^2 a^2 + e^{2c}b^2}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x)`

[Out] 
$$-1/2/d*x*\ln((\exp(2*c)*\exp(d*x)*b+\exp(c)*a-(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)})/(\exp(c)*a-(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)}))-1/2/d*x*\ln((\exp(2*c)*\exp(d*x)*b+\exp(c)*a+(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)})/(\exp(c)*a+(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)}))-1/2/d^2*dilog((\exp(2*c)*\exp(d*x)*b+\exp(c)*a+(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)})/(\exp(c)*a+(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)}))-1/2/d^2*dilog((\exp(2*c)*\exp(d*x)*b+\exp(c)*a-(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)})/(\exp(c)*a-(\exp(c)^2*a^2+\exp(2*c)*b^2)^{(1/2)}))+1/2*x^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(aee^{(dx+c)} - be)x}{bee^{(2dx+2c)} + 2aee^{(dx+c)} - be} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x, algorithm="maxima")`

[Out] `integrate((a*e*e^(d*x + c) - b*e)*x/(b*e*e^(2*d*x + 2*c) + 2*a*e*e^(d*x + c) - b*e), x)`

**Fricas [A]** time = 1.31385, size = 593, normalized size = 3.95

$$d^2x^2 + c \log\left(2be^{(dx+c)} + 2b\sqrt{\frac{a^2+b^2}{b^2}} + 2a\right) + c \log\left(2be^{(dx+c)} - 2b\sqrt{\frac{a^2+b^2}{b^2}} + 2a\right) - (dx+c) \log\left(-\frac{b\sqrt{\frac{a^2+b^2}{b^2}}e^{(dx+c)} + ae^{(dx+c)} - b}{b}\right)$$

$2d^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x, algorithm="fricas")`

[Out] 
$$1/2*(d^2*x^2 + c*\log(2*b*e^(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + c*\log(2*b*e^(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (d*x + c)*\log(-(b*\sqrt{(a^2 + b^2)/b^2}*e^(d*x + c) + a*e^(d*x + c) - b)/b) - (d*x + c)*\log((b*\sqrt{(a^2 + b^2)/b^2}*e^(d*x + c) - a*e^(d*x + c) + b)/b) - dilog((b*\sqrt{(a^2 + b^2)/b^2}*e^(d*x + c) + a*e^(d*x + c) - b)/b + 1) - dilog(-(b*\sqrt{(a^2 + b^2)/b^2}*e^(d*x + c) - a*e^(d*x + c) + b)/b + 1))/d^2$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(ae^c e^{dx} - b)}{2ae^c e^{dx} + be^{2c} e^{2dx} - b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x)`

[Out] Integral(x\*(a\*exp(c)\*exp(d\*x) - b)/(2\*a\*exp(c)\*exp(d\*x) + b\*exp(2\*c)\*exp(2\*d\*x) - b), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(aee^{dx+c} - be)x}{bee^{2dx+2c} + 2aee^{dx+c} - be} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*e-a\*e\*exp(d\*x+c))\*x/(b\*e-2\*a\*e\*exp(d\*x+c)-b\*e\*exp(2\*d\*x+2\*c)), x, algorithm="giac")

[Out] integrate((a\*e\*e^(d\*x + c) - b\*e)\*x/(b\*e\*e^(2\*d\*x + 2\*c) + 2\*a\*e\*e^(d\*x + c) - b\*e), x)

### 3.579 $\int F^{a+b \log(c+dx^n)} x^2 dx$

**Optimal.** Leaf size=65

$$\frac{1}{3} x^3 F^a (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{3}{n}, -b \log(F); \frac{n+3}{n}; -\frac{dx^n}{c} \right)$$

[Out] (F^a\*x^3\*(c + d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[3/n, -(b\*Log[F]), (3 + n)/n, -((d\*x^n)/c)])/(3\*(1 + (d\*x^n)/c)^(b\*Log[F]))

**Rubi [A]** time = 0.0492867, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2274, 12, 365, 364}

$$\frac{1}{3} x^3 F^a (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{3}{n}, -b \log(F); \frac{n+3}{n}; -\frac{dx^n}{c} \right)$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n])\*x^2,x]

[Out] (F^a\*x^3\*(c + d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[3/n, -(b\*Log[F]), (3 + n)/n, -((d\*x^n)/c)])/(3\*(1 + (d\*x^n)/c)^(b\*Log[F]))

#### Rule 2274

Int[(u\_)\*(F\_)^((a\_)\*(Log[z\_]\*(b\_) + (v\_))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 365

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int F^{a+b\log(c+dx^n)} x^2 dx &= \int F^a x^2 (c+dx^n)^{b\log(F)} dx \\
&= F^a \int x^2 (c+dx^n)^{b\log(F)} dx \\
&= \left( F^a (c+dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} \right) \int x^2 \left( 1 + \frac{dx^n}{c} \right)^{b\log(F)} dx \\
&= \frac{1}{3} F^a x^3 (c+dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} {}_2F_1 \left( \frac{3}{n}, -b\log(F); \frac{3+n}{n}; -\frac{dx^n}{c} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.140509, size = 85, normalized size = 1.31

$$\frac{x^3 \left( -\frac{dx^n}{c} \right)^{-3/n} (c+dx^n)^{F^{a+b\log(c+dx^n)}} {}_2F_1 \left( \frac{n-3}{n}, b\log(F)+1; b\log(F)+2; \frac{dx^n}{c}+1 \right)}{cn(b\log(F)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])\*x^2,x]

[Out] -((F^(a + b\*Log[c + d\*x^n])\*x^3\*(c + d\*x^n)\*Hypergeometric2F1[(-3 + n)/n, 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*(-((d\*x^n)/c))^(3/n)\*(1 + b\*Log[F])))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int F^{a+b\ln(c+dx^n)} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))\*x^2,x)

[Out] int(F^(a+b\*ln(c+d\*x^n))\*x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b\log(dx^n+c)+a} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))\*x^2,x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( F^{b\log(dx^n+c)+a} x^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F^(a+b*log(c+d*x^n))*x^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*log(d*x^n + c) + a)*x^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*ln(c+d*x**n))*x**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b \log(dx^n+c)+a} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n))*x^2,x, algorithm="giac")
```

```
[Out] integrate(F^(b*log(d*x^n + c) + a)*x^2, x)
```

### 3.580 $\int F^{a+b \log(c+dx^n)} x dx$

**Optimal.** Leaf size=65

$$\frac{1}{2} x^{2F^a} (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{2}{n}, -b \log(F); \frac{n+2}{n}; -\frac{dx^n}{c} \right)$$

[Out]  $(F^a x^{2(c + dx^n)^{b \log(F)}} \text{Hypergeometric2F1}[2/n, -(b \log(F)), (2 + n)/n, -((dx^n)/c)]) / (2(1 + (dx^n)/c)^{b \log(F)})$

**Rubi [A]** time = 0.0331369, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2274, 12, 365, 364}

$$\frac{1}{2} x^{2F^a} (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{2}{n}, -b \log(F); \frac{n+2}{n}; -\frac{dx^n}{c} \right)$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n])\*x,x]

[Out]  $(F^a x^{2(c + dx^n)^{b \log(F)}} \text{Hypergeometric2F1}[2/n, -(b \log(F)), (2 + n)/n, -((dx^n)/c)]) / (2(1 + (dx^n)/c)^{b \log(F)})$

#### Rule 2274

Int[(u\_)\*(F\_)^((a\_)\*(Log[z\_]\*(b\_.) + (v\_))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 365

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p]) / (1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)]) / (c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int F^{a+b\log(c+dx^n)} x dx &= \int F^a x (c + dx^n)^{b\log(F)} dx \\
&= F^a \int x (c + dx^n)^{b\log(F)} dx \\
&= \left( F^a (c + dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} \right) \int x \left( 1 + \frac{dx^n}{c} \right)^{b\log(F)} dx \\
&= \frac{1}{2} F^a x^2 (c + dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} {}_2F_1 \left( \frac{2}{n}, -b\log(F); \frac{2+n}{n}; -\frac{dx^n}{c} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.108177, size = 85, normalized size = 1.31

$$\frac{x^2 \left( -\frac{dx^n}{c} \right)^{-2/n} (c + dx^n)^{F^{a+b\log(c+dx^n)}} {}_2F_1 \left( \frac{n-2}{n}, b\log(F) + 1; b\log(F) + 2; \frac{dx^n}{c} + 1 \right)}{cn(b\log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])\*x,x]

[Out] -((F^(a + b\*Log[c + d\*x^n])\*x^2\*(c + d\*x^n)\*Hypergeometric2F1[(-2 + n)/n, 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*(-((d\*x^n)/c))^(2/n)\*(1 + b\*Log[F])))

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int F^{a+b\ln(c+dx^n)} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))\*x,x)

[Out] int(F^(a+b\*ln(c+d\*x^n))\*x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b\log(dx^n+c)+a} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))\*x,x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)\*x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{b\log(dx^n+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n))*x,x, algorithm="fricas")
```

```
[Out] integral(F^(b*log(d*x^n + c) + a)*x, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*ln(c+d*x**n))*x,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b \log(dx^n+c)+a} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n))*x,x, algorithm="giac")
```

```
[Out] integrate(F^(b*log(d*x^n + c) + a)*x, x)
```

### 3.581 $\int F^{a+b \log(c+dx^n)} dx$

**Optimal.** Leaf size=56

$$xF^a (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{1}{n}, -b \log(F); 1 + \frac{1}{n}; -\frac{dx^n}{c} \right)$$

[Out]  $(F^a * x * (c + d * x^n)^{(b * \text{Log}[F])} * \text{Hypergeometric2F1}[n^{-1}, -(b * \text{Log}[F]), 1 + n^{-1}, -((d * x^n)/c)]) / (1 + (d * x^n)/c)^{(b * \text{Log}[F])}$

**Rubi [A]** time = 0.0216632, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2274, 12, 246, 245}

$$xF^a (c + dx^n)^{b \log(F)} \left( \frac{dx^n}{c} + 1 \right)^{-b \log(F)} {}_2F_1 \left( \frac{1}{n}, -b \log(F); 1 + \frac{1}{n}; -\frac{dx^n}{c} \right)$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n]), x]

[Out]  $(F^a * x * (c + d * x^n)^{(b * \text{Log}[F])} * \text{Hypergeometric2F1}[n^{-1}, -(b * \text{Log}[F]), 1 + n^{-1}, -((d * x^n)/c)]) / (1 + (d * x^n)/c)^{(b * \text{Log}[F])}$

#### Rule 2274

Int[(u\_)\*(F\_)^((a\_)\*(Log[z\_]\*(b\_) + (v\_))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 246

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p]) / (1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 245

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int F^{a+b\log(c+dx^n)} dx &= \int F^a (c + dx^n)^{b\log(F)} dx \\
&= F^a \int (c + dx^n)^{b\log(F)} dx \\
&= \left( F^a (c + dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} \right) \int \left( 1 + \frac{dx^n}{c} \right)^{b\log(F)} dx \\
&= F^a x (c + dx^n)^{b\log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b\log(F)} {}_2F_1 \left( \frac{1}{n}, -b\log(F); 1 + \frac{1}{n}; -\frac{dx^n}{c} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0874138, size = 83, normalized size = 1.48

$$-\frac{x \left( -\frac{dx^n}{c} \right)^{-1/n} (c + dx^n)^{F^{a+b\log(c+dx^n)}} {}_2F_1 \left( \frac{n-1}{n}, b\log(F) + 1; b\log(F) + 2; \frac{dx^n}{c} + 1 \right)}{cn(b\log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n]),x]

[Out] -((F^(a + b\*Log[c + d\*x^n])\*x\*(c + d\*x^n)\*Hypergeometric2F1[(-1 + n)/n, 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*(-((d\*x^n)/c))^n^(-1)\*(1 + b\*Log[F])))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int F^{a+b\ln(c+dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n)),x)

[Out] int(F^(a+b\*ln(c+d\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b\log(dx^n+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n)),x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{b\log(dx^n+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*log(d*x^n + c) + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*ln(c+d*x**n)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b \log(dx^n+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*log(d*x^n + c) + a), x)
```

$$3.582 \quad \int \frac{F^{a+b \log(c+dx^n)}}{x} dx$$

**Optimal.** Leaf size=57

$$\frac{F^a (c + dx^n)^{b \log(F)+1} {}_2F_1\left(1, b \log(F) + 1; b \log(F) + 2; \frac{dx^n}{c} + 1\right)}{cn(b \log(F) + 1)}$$

[Out]  $-\left(\left(F^a (c + d x^n)^{(1 + b \operatorname{Log}[F])}\right) \operatorname{Hypergeometric2F1}\left[1, 1 + b \operatorname{Log}[F], 2 + b \operatorname{Log}[F], 1 + (d x^n)/c\right]\right) / (c n (1 + b \operatorname{Log}[F]))$

**Rubi [A]** time = 0.0638044, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2274, 12, 266, 65}

$$\frac{F^a (c + dx^n)^{b \log(F)+1} {}_2F_1\left(1, b \log(F) + 1; b \log(F) + 2; \frac{dx^n}{c} + 1\right)}{cn(b \log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n])/x,x]

[Out]  $-\left(\left(F^a (c + d x^n)^{(1 + b \operatorname{Log}[F])}\right) \operatorname{Hypergeometric2F1}\left[1, 1 + b \operatorname{Log}[F], 2 + b \operatorname{Log}[F], 1 + (d x^n)/c\right]\right) / (c n (1 + b \operatorname{Log}[F]))$

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] :> Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 65

Int[((b\_.)\*(x\_)^(m\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^(n + 1)\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d\*x)/c])/(d\*(n + 1)\*(-(d/(b\*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b\*c)), 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{F^{a+b \log(c+dx^n)}}{x} dx &= \int \frac{F^a (c+dx^n)^{b \log(F)}}{x} dx \\
&= F^a \int \frac{(c+dx^n)^{b \log(F)}}{x} dx \\
&= \frac{F^a \text{Subst} \left( \int \frac{(c+dx)^{b \log(F)}}{x} dx, x, x^n \right)}{n} \\
&= -\frac{F^a (c+dx^n)^{1+b \log(F)} {}_2F_1 \left( 1, 1+b \log(F); 2+b \log(F); 1+\frac{dx^n}{c} \right)}{cn(1+b \log(F))}
\end{aligned}$$

**Mathematica [A]** time = 0.102058, size = 50, normalized size = 0.88

$$-\frac{F^{a+b \log(c+dx^n)} \left( {}_2F_1 \left( 1, b \log(F); b \log(F) + 1; \frac{dx^n}{c} + 1 \right) - 1 \right)}{bn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])/x, x]

[Out] -((F^(a + b\*Log[c + d\*x^n]))\*(-1 + Hypergeometric2F1[1, b\*Log[F], 1 + b\*Log[F], 1 + (d\*x^n)/c]))/(b\*n\*Log[F])

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{F^{a+b \ln(c+dx^n)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))/x, x)

[Out] int(F^(a+b\*ln(c+d\*x^n))/x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b \log(dx^n+c)+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x, x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{b \log(dx^n+c)+a}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x,x, algorithm="fricas")

[Out] integral(F^(b\*log(d\*x^n + c) + a)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*ln(c+d\*x\*\*n))/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b \log(dx^n+c)+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x,x, algorithm="giac")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x, x)

$$3.583 \quad \int \frac{F^{a+b \log(c+dx^n)}}{x^2} dx$$

**Optimal.** Leaf size=66

$$\frac{F^a (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(-\frac{1}{n}, -b \log(F); -\frac{1-n}{n}; -\frac{dx^n}{c}\right)}{x}$$

[Out] -((F^a\*(c + d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[-n^(-1), -(b\*Log[F]), -(1 - n)/n], -((d\*x^n)/c)))/(x\*(1 + (d\*x^n)/c)^(b\*Log[F]))

**Rubi [A]** time = 0.0450553, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2274, 12, 365, 364}

$$\frac{F^a (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(-\frac{1}{n}, -b \log(F); -\frac{1-n}{n}; -\frac{dx^n}{c}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n])/x^2,x]

[Out] -((F^a\*(c + d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[-n^(-1), -(b\*Log[F]), -(1 - n)/n], -((d\*x^n)/c)))/(x\*(1 + (d\*x^n)/c)^(b\*Log[F]))

#### Rule 2274

Int[(u\_)\*(F\_)^((a\_)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] :> Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 365

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b \log(c+dx^n)}}{x^2} dx &= \int \frac{F^a (c + dx^n)^{b \log(F)}}{x^2} dx \\
&= F^a \int \frac{(c + dx^n)^{b \log(F)}}{x^2} dx \\
&= \left( F^a (c + dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} \right) \int \frac{\left( 1 + \frac{dx^n}{c} \right)^{b \log(F)}}{x^2} dx \\
&= - \frac{F^a (c + dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} {}_2F_1 \left( -\frac{1}{n}, -b \log(F); -\frac{1-n}{n}; -\frac{dx^n}{c} \right)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.118012, size = 81, normalized size = 1.23

$$\frac{\left( -\frac{dx^n}{c} \right)^{\frac{1}{n}} (c + dx^n) F^{a+b \log(c+dx^n)} {}_2F_1 \left( 1 + \frac{1}{n}, b \log(F) + 1; b \log(F) + 2; \frac{dx^n}{c} + 1 \right)}{cnx(b \log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])/x^2,x]

[Out] -((F^(a + b\*Log[c + d\*x^n])\*((-(d\*x^n)/c))^n^(-1)\*(c + d\*x^n)\*Hypergeometric2F1[1 + n^(-1), 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*x\*(1 + b\*Log[F])))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{F^{a+b \ln(c+dx^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))/x^2,x)

[Out] int(F^(a+b\*ln(c+d\*x^n))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b \log(dx^n+c)+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^2,x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{b\log(dx^n+c)+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^2,x, algorithm="fricas")

[Out] integral(F^(b\*log(d\*x^n + c) + a)/x^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*ln(c+d\*x\*\*n))/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b\log(dx^n+c)+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^2,x, algorithm="giac")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x^2, x)

$$3.584 \quad \int \frac{F^{a+b \log(c+dx^n)}}{x^3} dx$$

**Optimal.** Leaf size=68

$$\frac{F^a (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(-\frac{2}{n}, -b \log(F); -\frac{2-n}{n}; -\frac{dx^n}{c}\right)}{2x^2}$$

[Out]  $-(F^a(c + d*x^n)^{(b*\text{Log}[F])}*\text{Hypergeometric2F1}[-2/n, -(b*\text{Log}[F]), -((2 - n)/n), -((d*x^n)/c)])/(2*x^2*(1 + (d*x^n)/c)^{(b*\text{Log}[F])})$

**Rubi [A]** time = 0.0449979, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2274, 12, 365, 364}

$$\frac{F^a (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(-\frac{2}{n}, -b \log(F); -\frac{2-n}{n}; -\frac{dx^n}{c}\right)}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(a + b*\text{Log}[c + d*x^n])}/x^3, x]$

[Out]  $-(F^a(c + d*x^n)^{(b*\text{Log}[F])}*\text{Hypergeometric2F1}[-2/n, -(b*\text{Log}[F]), -((2 - n)/n), -((d*x^n)/c)])/(2*x^2*(1 + (d*x^n)/c)^{(b*\text{Log}[F])})$

#### Rule 2274

$\text{Int}[(u_*)*(F_*)^{((a_*)*(\text{Log}[z_*]*(b_*) + (v_*)))}, x\_Symbol] \rightarrow \text{Int}[u*F^{(a*v)*z^{(a*b*\text{Log}[F])}}, x] /; \text{FreeQ}\{F, a, b\}, x]$

#### Rule 12

$\text{Int}[(a_*)(u_*), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b_*)(v_*)] /; \text{FreeQ}[b, x]$

#### Rule 365

$\text{Int}[((c_*)(x_*))^{(m_*)}*((a_*) + (b_*)(x_*))^{(n_*)}^{(p_*)}, x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(c*x)^{m*(1 + (b*x^n)/a)^p}, x], x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\text{IGtQ}[p, 0] \&\& !(ILtQ[p, 0] || GtQ[a, 0])$

#### Rule 364

$\text{Int}[((c_*)(x_*))^{(m_*)}*((a_*) + (b_*)(x_*))^{(n_*)}^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -((b*x^n)/a)])/(c*(m+1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\text{IGtQ}[p, 0] \&\& (ILtQ[p, 0] || GtQ[a, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b \log(c+dx^n)}}{x^3} dx &= \int \frac{F^a (c+dx^n)^{b \log(F)}}{x^3} dx \\
&= F^a \int \frac{(c+dx^n)^{b \log(F)}}{x^3} dx \\
&= \left( F^a (c+dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} \right) \int \frac{\left( 1 + \frac{dx^n}{c} \right)^{b \log(F)}}{x^3} dx \\
&= \frac{F^a (c+dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} {}_2F_1 \left( -\frac{2}{n}, -b \log(F); -\frac{2-n}{n}; -\frac{dx^n}{c} \right)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.117691, size = 85, normalized size = 1.25

$$\frac{\left( -\frac{dx^n}{c} \right)^{2/n} (c+dx^n)^{F^{a+b \log(c+dx^n)}} {}_2F_1 \left( \frac{n+2}{n}, b \log(F) + 1; b \log(F) + 2; \frac{dx^n}{c} + 1 \right)}{cnx^2(b \log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])/x^3,x]

[Out] -((F^(a + b\*Log[c + d\*x^n]))\*((((d\*x^n)/c))^(2/n)\*(c + d\*x^n)\*Hypergeometric2F1[(2 + n)/n, 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*x^2\*(1 + b\*Log[F])))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{F^{a+b \ln(c+dx^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))/x^3,x)

[Out] int(F^(a+b\*ln(c+d\*x^n))/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b \log(dx^n+c)+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^3,x, algorithm="maxima")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{b \log(dx^n+c)+a}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^3,x, algorithm="fricas")

[Out] integral(F^(b\*log(d\*x^n + c) + a)/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*ln(c+d\*x\*\*n))/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{b \log(dx^n+c)+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))/x^3,x, algorithm="giac")

[Out] integrate(F^(b\*log(d\*x^n + c) + a)/x^3, x)



### 3.585 $\int F^{a+b \log(c+dx^n)} (dx)^m dx$

**Optimal.** Leaf size=77

$$\frac{F^a (dx)^{m+1} (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(\frac{m+1}{n}, -b \log(F); \frac{m+n+1}{n}; -\frac{dx^n}{c}\right)}{d(m+1)}$$

[Out] (F^a\*(d\*x)^(1+m)\*(c+d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[(1+m)/n, -(b\*Log[F]), (1+m+n)/n, -((d\*x^n)/c)]/(d\*(1+m)\*(1+(d\*x^n)/c)^(b\*Log[F]))

**Rubi [A]** time = 0.0547465, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2274, 12, 365, 364}

$$\frac{F^a (dx)^{m+1} (c + dx^n)^{b \log(F)} \left(\frac{dx^n}{c} + 1\right)^{-b \log(F)} {}_2F_1\left(\frac{m+1}{n}, -b \log(F); \frac{m+n+1}{n}; -\frac{dx^n}{c}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*Log[c + d\*x^n])\*(d\*x)^m, x]

[Out] (F^a\*(d\*x)^(1+m)\*(c+d\*x^n)^(b\*Log[F])\*Hypergeometric2F1[(1+m)/n, -(b\*Log[F]), (1+m+n)/n, -((d\*x^n)/c)]/(d\*(1+m)\*(1+(d\*x^n)/c)^(b\*Log[F]))

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_\*](b\_.) + (v\_.))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^(m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int F^{a+b \log(c+dx^n)} (dx)^m dx &= \int F^a (dx)^m (c + dx^n)^{b \log(F)} dx \\
&= F^a \int (dx)^m (c + dx^n)^{b \log(F)} dx \\
&= \left( F^a (c + dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} \right) \int (dx)^m \left( 1 + \frac{dx^n}{c} \right)^{b \log(F)} dx \\
&= \frac{F^a (dx)^{1+m} (c + dx^n)^{b \log(F)} \left( 1 + \frac{dx^n}{c} \right)^{-b \log(F)} {}_2F_1 \left( \frac{1+m}{n}, -b \log(F); \frac{1+m+n}{n}; -\frac{dx^n}{c} \right)}{d(1+m)}
\end{aligned}$$

**Mathematica [A]** time = 0.159498, size = 94, normalized size = 1.22

$$\frac{x(dx)^m (c + dx^n) \left( -\frac{dx^n}{c} \right)^{-\frac{m+1}{n}} F^{a+b \log(c+dx^n)} {}_2F_1 \left( 1 - \frac{m+1}{n}, b \log(F) + 1; b \log(F) + 2; \frac{dx^n}{c} + 1 \right)}{cn(b \log(F) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*Log[c + d\*x^n])\*(d\*x)^m,x]

[Out] -((F^(a + b\*Log[c + d\*x^n]))\*x\*(d\*x)^m\*(c + d\*x^n)\*Hypergeometric2F1[1 - (1 + m)/n, 1 + b\*Log[F], 2 + b\*Log[F], 1 + (d\*x^n)/c])/(c\*n\*(-((d\*x^n)/c))^(1 + m)/n)\*(1 + b\*Log[F]))

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int F^{a+b \ln(c+dx^n)} (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*ln(c+d\*x^n))\*(d\*x)^m,x)

[Out] int(F^(a+b\*ln(c+d\*x^n))\*(d\*x)^m,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m F^{b \log(dx^n+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*log(c+d\*x^n))\*(d\*x)^m,x, algorithm="maxima")

[Out] integrate((d\*x)^m \* F^(b\*log(d\*x^n + c) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (dx)^m F^{b \log(dx^n+c)+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n))*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*F^(b*log(d*x^n + c) + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*ln(c+d*x**n))*(d*x)**m,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m F^{b \log(dx^n+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*log(c+d*x^n))*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*F^(b*log(d*x^n + c) + a), x)
```

$$3.586 \quad \int e^{\log^2((d+ex)^n)} (d+ex)^m dx$$

**Optimal.** Leaf size=76

$$\frac{\sqrt{\pi} e^{-\frac{(m+1)^2}{4n^2}} (d+ex)^{m+1} ((d+ex)^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{2n \log((d+ex)^n) + m + 1}{2n}\right)}{2en}$$

[Out] (Sqrt[Pi]\*(d + e\*x)^(1 + m)\*Erfi[(1 + m + 2\*n\*Log[(d + e\*x)^n]/(2\*n))]/(2\*e\*E^((1 + m)^2/(4\*n^2))\*n\*((d + e\*x)^n)^((1 + m)/n))

**Rubi [A]** time = 0.156927, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2276, 2234, 2204}

$$\frac{\sqrt{\pi} e^{-\frac{(m+1)^2}{4n^2}} (d+ex)^{m+1} ((d+ex)^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{2n \log((d+ex)^n) + m + 1}{2n}\right)}{2en}$$

Antiderivative was successfully verified.

[In] Int[E^Log[(d + e\*x)^n]^2\*(d + e\*x)^m,x]

[Out] (Sqrt[Pi]\*(d + e\*x)^(1 + m)\*Erfi[(1 + m + 2\*n\*Log[(d + e\*x)^n]/(2\*n))]/(2\*e\*E^((1 + m)^2/(4\*n^2))\*n\*((d + e\*x)^n)^((1 + m)/n))

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(a\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int e^{\log^2((d+ex)^n)}(d+ex)^m dx &= \frac{\text{Subst}\left(\int e^{\log^2(x^n)}x^m dx, x, d+ex\right)}{e} \\
&= \frac{\left((d+ex)^{1+m}((d+ex)^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}+x^2} dx, x, \log((d+ex)^n)\right)}{en} \\
&= \frac{\left(e^{-\frac{(1+m)^2}{4n^2}}(d+ex)^{1+m}((d+ex)^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{1}{4}\left(\frac{1+m}{n}+2x\right)^2} dx, x, \log((d+ex)^n)\right)}{en} \\
&= \frac{e^{-\frac{(1+m)^2}{4n^2}}\sqrt{\pi}(d+ex)^{1+m}((d+ex)^n)^{-\frac{1+m}{n}}\text{erfi}\left(\frac{1+m+2n\log((d+ex)^n)}{2n}\right)}{2en}
\end{aligned}$$

**Mathematica [F]** time = 0.085411, size = 0, normalized size = 0.

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[E^Log[(d + e\*x)^n]^2\*(d + e\*x)^m, x]

[Out] Integrate[E^Log[(d + e\*x)^n]^2\*(d + e\*x)^m, x]

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int e^{(\ln((ex+d)^n))^2}(ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(ln((e\*x+d)^n)^2)\*(e\*x+d)^m, x)

[Out] int(exp(ln((e\*x+d)^n)^2)\*(e\*x+d)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^m e^{\left(\log((ex+d)^n)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(log((e\*x+d)^n)^2)\*(e\*x+d)^m, x, algorithm="maxima")

[Out] integrate((e\*x + d)^m\*e^(log((e\*x + d)^n)^2), x)

**Fricas [A]** time = 1.02208, size = 151, normalized size = 1.99

$$\frac{\sqrt{\pi}\sqrt{n^2}\text{erfi}\left(\frac{(2n^2\log(ex+d)+m+1)\sqrt{n^2}}{2n^2}\right)e^{\left(-\frac{m^2+2m+1}{4n^2}\right)}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(log((e\*x+d)^n)^2)\*(e\*x+d)^m,x, algorithm="fricas")

[Out] 1/2\*sqrt(pi)\*sqrt(n^2)\*erfi(1/2\*(2\*n^2\*log(e\*x + d) + m + 1)\*sqrt(n^2)/n^2)\*e^(-1/4\*(m^2 + 2\*m + 1)/n^2)/(e\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^m e^{\log((d+ex)^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(ln((e\*x+d)\*\*n)\*\*2)\*(e\*x+d)\*\*m,x)

[Out] Integral((d + e\*x)\*\*m\*exp(log((d + e\*x)\*\*n)\*\*2), x)

**Giac [A]** time = 1.32825, size = 76, normalized size = 1.

$$\frac{\sqrt{\pi}i \operatorname{erf}\left(in \log(xe + d) + \frac{im}{2n} + \frac{i}{2n}\right) e^{\left(-\frac{m^2}{4n^2} - \frac{m}{2n^2} - \frac{1}{4n^2} - 1\right)}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(log((e\*x+d)^n)^2)\*(e\*x+d)^m,x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*i\*erf(i\*n\*log(x\*e + d) + 1/2\*i\*m/n + 1/2\*i/n)\*e^(-1/4\*m^2/n^2 - 1/2\*m/n^2 - 1/4/n^2 - 1)/n

**3.587**  $\int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx)^m dx$

**Optimal.** Leaf size=137

$$\frac{\sqrt{\pi} F^{af} (dg + egx)^{m+1} e^{-\frac{(m+1)^2}{4bf n^2 \log(F)}} (c(d + ex)^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n) + m + 1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fgn}\sqrt{\log(F)}}$$

[Out]  $(F^{(a*f)*\operatorname{Sqrt}[\operatorname{Pi}]*(d*g + e*g*x)^{(1 + m)*\operatorname{Erfi}[(1 + m + 2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])}]/(2*\operatorname{Sqrt}[b]*e*E^{((1 + m)^2/(4*b*f*n^2*\operatorname{Log}[F]))})*\operatorname{Sqrt}[f]*g*n*(c*(d + e*x)^n)^{((1 + m)/n)*\operatorname{Sqrt}[\operatorname{Log}[F]])}$

**Rubi [A]** time = 0.389683, antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {2276, 2234, 2204}

$$\frac{\sqrt{\pi} F^{af} (g(d + ex))^{m+1} e^{-\frac{(m+1)^2}{4bf n^2 \log(F)}} (c(d + ex)^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n) + m + 1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fgn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(f*(a + b*\operatorname{Log}[c*(d + e*x)^n]^2))}*(d*g + e*g*x)^m, x]$

[Out]  $(F^{(a*f)*\operatorname{Sqrt}[\operatorname{Pi}]*(g*(d + e*x))^{(1 + m)*\operatorname{Erfi}[(1 + m + 2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])}]/(2*\operatorname{Sqrt}[b]*e*E^{((1 + m)^2/(4*b*f*n^2*\operatorname{Log}[F]))})*\operatorname{Sqrt}[f]*g*n*(c*(d + e*x)^n)^{((1 + m)/n)*\operatorname{Sqrt}[\operatorname{Log}[F]])}$

**Rule 2276**

$\operatorname{Int}[(F_)^{((a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]^2*(b_.))*(d_.))*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m + 1)}/(e*n*(c*x^n)^{((m + 1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(a*d*\operatorname{Log}[F] + ((m + 1)*x)/n + b*d*\operatorname{Log}[F]*x^2)}, x], x, \operatorname{Log}[c*x^n]], x] /;$  Free Q[{F, a, b, c, d, e, m, n}, x]

**Rule 2234**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$  FreeQ[{F, a, b, c}, x]

**Rule 2204**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

**Rubi steps**

$$\begin{aligned}
\int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx)^m dx &= \frac{\text{Subst} \left( \int F^{f(a+b \log^2(cx^n))} (gx)^m dx, x, d + ex \right)}{e} \\
&= \frac{\left( (g(d + ex))^{1+m} (c(d + ex)^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int e^{\frac{(1+m)x}{n} + af \log(F) + bfx^2 \log(F)} dx, x, \log(c(d + ex)^n) \right)}{egn} \\
&= \frac{\left( e^{-\frac{(1+m)^2}{4bf n^2 \log(F)}} F^{af} (g(d + ex))^{1+m} (c(d + ex)^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int e^{\frac{\left(\frac{1+m}{n} + 2bf x \log(F)\right)^2}{4bf \log(F)}} dx, x, \log(c(d + ex)^n) \right)}{egn} \\
&= \frac{e^{-\frac{(1+m)^2}{4bf n^2 \log(F)}} F^{af} \sqrt{\pi} (g(d + ex))^{1+m} (c(d + ex)^n)^{-\frac{1+m}{n}} \text{erfi} \left( \frac{1+m+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [F]** time = 0.157323, size = 0, normalized size = 0.

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x)^m, x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x)^m, x]

**Maple [F]** time = 80.773, size = 0, normalized size = 0.

$$\int F^{f\left(a+b\left(\ln(c(ex+d)^n)\right)^2\right)} (egx + dg)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^m, x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^m F^{(b \log((ex+d)^n c)^2 + a)f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^m, x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)^m \* F^((b\*log((e\*x + d)^n \* c)^2 + a)\*f), x)



**Fricas [A]** time = 1.02729, size = 382, normalized size = 2.79

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} \operatorname{erf}\left(\frac{(2bf n^2 \log(ex+d) \log(F) + 2bf n \log(F) \log(c) + m + 1)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}\right)}{2en} e^{\left(\frac{4abf^2 n^2 \log(F)^2 + 4bf m n^2 \log(F) \log(g) - 4(bf m + bf)n \log(F)}{4bf n^2 \log(F)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^m,x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-b\*f\*n^2\*log(F))\*erf(1/2\*(2\*b\*f\*n^2\*log(e\*x + d)\*log(F) + 2\*b\*f\*n\*log(F)\*log(c) + m + 1)\*sqrt(-b\*f\*n^2\*log(F))/(b\*f\*n^2\*log(F)))\*e^(1/4\*(4\*a\*b\*f^2\*n^2\*log(F)^2 + 4\*b\*f\*m\*n^2\*log(F)\*log(g) - 4\*(b\*f\*m + b\*f)\*n\*log(F)\*log(c) - m^2 - 2\*m - 1)/(b\*f\*n^2\*log(F)))/(e\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(e\*g\*x+d\*g)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^m F^{(b \log((ex+d)^n c)^2 + a)f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^m,x, algorithm="giac")

[Out] integrate((e\*g\*x + d\*g)^m \* F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

$$3.588 \quad \int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx)^2 dx$$

**Optimal.** Leaf size=123

$$\frac{\sqrt{\pi} g^2 F^{af} (d+ex)^3 e^{-\frac{9}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-3/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+3}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (F^(a\*f)\*g^2\*Sqrt[Pi]\*(d + e\*x)^3\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(9/(4\*b\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(3/n)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.249306, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {12, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} g^2 F^{af} (d+ex)^3 e^{-\frac{9}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-3/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+3}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x)^2,x]

[Out] (F^(a\*f)\*g^2\*Sqrt[Pi]\*(d + e\*x)^3\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(9/(4\*b\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(3/n)\*Sqrt[Log[F]])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^(a\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
\int F^{f(a+b\log^2(c(d+ex)^n))} (dg+egx)^2 dx &= \frac{\text{Subst}\left(\int F^{f(a+b\log^2(cx^n))} g^2 x^2 dx, x, d+ex\right)}{e} \\
&= \frac{g^2 \text{Subst}\left(\int F^{f(a+b\log^2(cx^n))} x^2 dx, x, d+ex\right)}{e} \\
&= \frac{(g^2(d+ex)^3 (c(d+ex)^n)^{-3/n}) \text{Subst}\left(\int e^{\frac{3x}{n}+af\log(F)+bf x^2\log(F)} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{\left(e^{-\frac{9}{4bf n^2\log(F)}} F^{af} g^2 (d+ex)^3 (c(d+ex)^n)^{-3/n}\right) \text{Subst}\left(\int e^{\frac{\left(\frac{3}{n}+2bf x\log(F)\right)^2}{4bf\log(F)}} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{e^{-\frac{9}{4bf n^2\log(F)}} F^{af} g^2 \sqrt{\pi} (d+ex)^3 (c(d+ex)^n)^{-3/n} \text{erfi}\left(\frac{3+2bf n\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.235939, size = 123, normalized size = 1.

$$\frac{\sqrt{\pi} g^2 F^{af} (d+ex)^3 e^{-\frac{9}{4bf n^2\log(F)}} (c(d+ex)^n)^{-3/n} \text{Erfi}\left(\frac{2bf n\log(F)\log(c(d+ex)^n)+3}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x)^2,x]

[Out] (F^(a\*f)\*g^2\*Sqrt[Pi]\*(d + e\*x)^3\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*Sqrt[b]\*e^E^(9/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(3/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.302, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} (egx+dg)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^2,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx+dg)^2 F^{(b\log((ex+d)^n c)^2+a)f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^2,x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)^2\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 1.06633, size = 316, normalized size = 2.57

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)}g^2 \operatorname{erf}\left(\frac{(2bf n^2 \log(ex+d) \log(F)+2bf n \log(F) \log(c)+3)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}\right) e^{\left(\frac{4abf^2 n^2 \log(F)^2-12bf n \log(F) \log(c)-9}{4bf n^2 \log(F)}\right)}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^2,x, algorithm="fricas")

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*g^2*\operatorname{erf}\left(\frac{1}{2}*(2*b*f*n^2*\log(e*x + d)*\log(F) + 2*b*f*n*\log(F)*\log(c) + 3)*\sqrt{-b*f*n^2*\log(F)}\right)/(b*f*n^2*\log(F))$   
 $*e^{(1/4*(4*a*b*f^2*n^2*\log(F)^2 - 12*b*f*n*\log(F)*\log(c) - 9)/(b*f*n^2*\log(F)))/(e*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(e\*g\*x+d\*g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^2 F^{(b \log((ex+d)^n c)^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g)^2,x, algorithm="giac")

[Out] integrate((e\*g\*x + d\*g)^2\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

$$3.589 \quad \int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx) dx$$

**Optimal.** Leaf size=115

$$\frac{\sqrt{\pi} g F^{af} (d+ex)^2 e^{-\frac{1}{bf n^2 \log(F)}} (c(d+ex)^n)^{-2/n} \operatorname{Erfi}\left(\frac{bf n \log(F) \log(c(d+ex)^n)+1}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}}$$

[Out] (F^(a\*f)\*g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.207366, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {12, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} g F^{af} (d+ex)^2 e^{-\frac{1}{bf n^2 \log(F)}} (c(d+ex)^n)^{-2/n} \operatorname{Erfi}\left(\frac{bf n \log(F) \log(c(d+ex)^n)+1}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x), x]

[Out] (F^(a\*f)\*g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(a\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int F^{f(a+b \log^2(c(d+ex)^n))} (dg + egx) dx &= \frac{\text{Subst} \left( \int F^{f(a+b \log^2(cx^n))} gx dx, x, d + ex \right)}{e} \\
&= \frac{g \text{Subst} \left( \int F^{f(a+b \log^2(cx^n))} x dx, x, d + ex \right)}{e} \\
&= \frac{(g(d + ex)^2 (c(d + ex)^n)^{-2/n}) \text{Subst} \left( \int e^{\frac{2x}{n} + af \log(F) + bfx^2 \log(F)} dx, x, \log(c(d + ex)^n) \right)}{en} \\
&= \frac{\left( e^{-\frac{1}{bfn^2 \log(F)}} F^{af} g(d + ex)^2 (c(d + ex)^n)^{-2/n} \right) \text{Subst} \left( \int e^{\frac{\left(\frac{2}{n} + 2bfx \log(F)\right)^2}{4bf \log(F)}} dx, x, \log(c(d + ex)^n) \right)}{en} \\
&= \frac{e^{-\frac{1}{bfn^2 \log(F)}} F^{af} g \sqrt{\pi} (d + ex)^2 (c(d + ex)^n)^{-2/n} \text{erfi} \left( \frac{1 + bfn \log(F) \log(c(d + ex)^n)}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.491363, size = 115, normalized size = 1.

$$\frac{\sqrt{\pi} g F^{af} (d + ex)^2 e^{-\frac{1}{bfn^2 \log(F)}} (c(d + ex)^n)^{-2/n} \text{Erfi} \left( \frac{bfn \log(F) \log(c(d + ex)^n) + 1}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(d\*g + e\*g\*x), x]

[Out] (F^(a\*f)\*g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])]/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*Sqrt[b]\*e\*E^(1/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.307, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} (egx + dg) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg) F^{(b \log((ex+d)^n c)^2 + a) f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g), x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 0.99465, size = 293, normalized size = 2.55

$$\frac{\sqrt{\pi}\sqrt{-bfn^2 \log(F)}g \operatorname{erf}\left(\frac{(bfn^2 \log(ex+d) \log(F)+bfn \log(F) \log(c)+1)\sqrt{-bfn^2 \log(F)}}{bfn^2 \log(F)}\right) e^{\left(\frac{abf^2n^2 \log(F)^2-2bfn \log(F) \log(c)-1}{bfn^2 \log(F)}\right)}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g),x, algorithm="fricas")

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*g*\operatorname{erf}((b*f*n^2*\log(e*x + d)*\log(F) + b*f*n*\log(F)*\log(c) + 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{((a*b*f^2*n^2*\log(F)^2 - 2*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F)))/(e*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(e\*g\*x+d\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)F^{(b \log((ex+d)^n c)^2 + a)} f dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(e\*g\*x+d\*g),x, algorithm="giac")

[Out] integrate((e\*g\*x + d\*g)\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

$$3.590 \quad \int F^{f(a+b \log^2(c(dx+e)^n))} dx$$

**Optimal.** Leaf size=118

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.126591, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2275, 2234, 2204}

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2)), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

#### Rule 2275

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.)), x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(a\*d\*Log[F] + x/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, n}, x]

#### Rule 2234

Int[(F\_)^(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^(((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps



$$\begin{aligned}
\int F^{f(a+b \log^2(c(d+ex)^n))} dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log^2(cx^n))} dx, x, d+ex\right)}{e} \\
&= \frac{\left((d+ex)(c(d+ex)^n)^{-1/n}\right) \text{Subst}\left(\int e^{\frac{x}{n}+af \log(F)+bf x^2 \log(F)} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{\left(e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} (d+ex)(c(d+ex)^n)^{-1/n}\right) \text{Subst}\left(\int e^{\frac{\left(\frac{1}{n}+2bf x \log(F)\right)^2}{4bf \log(F)}} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} \sqrt{\pi} (d+ex)(c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{1+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.095201, size = 118, normalized size = 1.

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2)), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

**Maple [F]** time = 0.125, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2)), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(b \log((ex+d)^n c)^2 + a) f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2)), x, algorithm="maxima")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 1.02131, size = 309, normalized size = 2.62

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} \operatorname{erf}\left(\frac{(2bf n^2 \log(ex+d) \log(F) + 2bf n \log(F) \log(c) + 1)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}\right) e^{\left(\frac{4abf^2 n^2 \log(F)^2 - 4bf n \log(F) \log(c) - 1}{4bf n^2 \log(F)}\right)}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F) + 2*b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/(e*n)
```

**Sympy [A]** time = 107.124, size = 532, normalized size = 4.51

$$\left\{ \begin{array}{l} \frac{2F^{af} F^{bf \log(c)^2} F^{bf n^2 \log(d+ex)^2} F^{2bf n \log(c) \log(d+ex)} bdf n^2 \log(F) \log(d+ex)}{e} - \frac{2F^{af} F^{bf \log(c)^2} F^{bf n^2 \log(d+ex)^2} F^{2bf n \log(c) \log(d+ex)} bdf n^2 \log(F)}{e} - \frac{2F^{af} F^{bf \log(c)^2} F^{bf n^2 \log(d+ex)^2} F^{2bf n \log(c) \log(d+ex)} bdf n^2 \log(F)}{e} \\ F^{f(a+b \log(cd^n)^2)}_x \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2)),x)
```

```
[Out] Piecewise((-2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*b*d*f*n**2*log(F)*log(d + e*x)/e - 2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*b*d*f*n**2*log(F)/e - 2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*b*d*f*n*log(F)*log(c)/e - 2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*b*f*n**2*x*log(F)*log(d + e*x) + 2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*b*f*n**2*x*log(F) - 2*F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F*(2*b*f*n*log(c)*log(d + e*x))*b*f*n*x*log(F)*log(c) + F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*d/e + F**(a*f)*F**(b*f*log(c)**2)*F**(b*f*n**2*log(d + e*x)**2)*F**(2*b*f*n*log(c)*log(d + e*x))*x, Ne(e, 0)), (F**(f*(a + b*log(c*d**n)**2))*x, True))
```

**Giac [A]** time = 1.29082, size = 136, normalized size = 1.15

$$\frac{\sqrt{\pi}F^{af} \operatorname{erf}\left(-\sqrt{-bf \log(F)}n \log(xe + d) - \sqrt{-bf \log(F)} \log(c) - \frac{\sqrt{-bf \log(F)}}{2bf n \log(F)}\right) e^{\left(-\frac{1}{4bf n^2 \log(F)} - 1\right)}}{2\sqrt{-bf \log(F)}c^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(pi)*F^(a*f)*erf(-sqrt(-b*f*log(F))*n*log(x*e + d) - sqrt(-b*f*log(F))*log(c) - 1/2*sqrt(-b*f*log(F))/(b*f*n*log(F)))*e^(-1/4/(b*f*n^2*log(F)) - 1)/(sqrt(-b*f*log(F))*c^(1/n)*n)
```

$$3.591 \quad \int \frac{F^{f(a+b \log^2(c(dx+e)^n))}}{dg+egx} dx$$

**Optimal.** Leaf size=67

$$\frac{\sqrt{\pi} F^{af} \operatorname{Erfi}\left(\sqrt{b}\sqrt{f}\sqrt{\log(F)} \log(c(dx+e)^n)\right)}{2\sqrt{be}\sqrt{fgn}\sqrt{\log(F)}}$$

[Out] (F^(a\*f)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[f]\*Sqrt[Log[F]]\*Log[c\*(d + e\*x)^n]]/(2\*Sqrt[b]\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

**Rubi [A]** time = 0.144066, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {12, 2276, 2204}

$$\frac{\sqrt{\pi} F^{af} \operatorname{Erfi}\left(\sqrt{b}\sqrt{f}\sqrt{\log(F)} \log(c(dx+e)^n)\right)}{2\sqrt{be}\sqrt{fgn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(d\*g + e\*g\*x),x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[f]\*Sqrt[Log[F]]\*Log[c\*(d + e\*x)^n]]/(2\*Sqrt[b]\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(a\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{dg + egx} dx &= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{gx} dx, x, d + ex \right)}{e} \\
&= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{x} dx, x, d + ex \right)}{eg} \\
&= \frac{\text{Subst} \left( \int e^{af \log(F) + bf x^2 \log(F)} dx, x, \log(c(d + ex)^n) \right)}{egn} \\
&= \frac{F^{af} \sqrt{\pi} \text{erfi} \left( \sqrt{b} \sqrt{f} \sqrt{\log(F)} \log(c(d + ex)^n) \right)}{2\sqrt{be} \sqrt{f} gn \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.036934, size = 67, normalized size = 1.

$$\frac{\sqrt{\pi} F^{af} \text{Erfi} \left( \sqrt{b} \sqrt{f} \sqrt{\log(F)} \log(c(d + ex)^n) \right)}{2\sqrt{be} \sqrt{f} gn \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(d\*g + e\*g\*x), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[f]\*Sqrt[Log[F]]\*Log[c\*(d + e\*x)^n]]/(2\*Sqrt[b]\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{egx + dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c^2) + a)f}}{egx + dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g), x, algorithm="maxima")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g), x)

**Fricas [A]** time = 0.981335, size = 146, normalized size = 2.18

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} F^{af} \operatorname{erf}\left(\frac{\sqrt{-bf n^2 \log(F)}(n \log(ex+d)+\log(c))}{n}\right)}{2egn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g),x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-b\*f\*n^2\*log(F))\*F^(a\*f)\*erf(sqrt(-b\*f\*n^2\*log(F))\*(n\*log(e\*x + d) + log(c))/n)/(e\*g\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))/(e\*g\*x+d\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)} f}{egx + dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g),x, algorithm="giac")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g), x)

$$3.592 \quad \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^2} dx$$

**Optimal.** Leaf size=121

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{\frac{1}{n}} \operatorname{Erfi}\left(\frac{1-2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fg^2 n \sqrt{\log(F)}}(d+ex)}$$

[Out]  $-(F^{(a*f)}*\operatorname{Sqrt}[\operatorname{Pi}]*(c*(d+e*x)^n)^n^{(-1)}*\operatorname{Erfi}[(1-2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d+e*x)^n])/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])])/(2*\operatorname{Sqrt}[b]*e*E^{(1/(4*b*f*n^2*\operatorname{Log}[F]))})*\operatorname{Sqrt}[f]*g^2*n*(d+e*x)*\operatorname{Sqrt}[\operatorname{Log}[F]])$

**Rubi [A]** time = 0.233933, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {12, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{\frac{1}{n}} \operatorname{Erfi}\left(\frac{1-2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fg^2 n \sqrt{\log(F)}}(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(f*(a+b*\operatorname{Log}[c*(d+e*x)^n]^2))}/(d*g+e*g*x)^2, x]$

[Out]  $-(F^{(a*f)}*\operatorname{Sqrt}[\operatorname{Pi}]*(c*(d+e*x)^n)^n^{(-1)}*\operatorname{Erfi}[(1-2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d+e*x)^n])/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])])/(2*\operatorname{Sqrt}[b]*e*E^{(1/(4*b*f*n^2*\operatorname{Log}[F]))})*\operatorname{Sqrt}[f]*g^2*n*(d+e*x)*\operatorname{Sqrt}[\operatorname{Log}[F]])$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2276

$\operatorname{Int}[(F_)^{((a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]^2*(b_.))*(d_.))*((e_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{(a*d*\operatorname{Log}[F] + ((m+1)*x)/n + b*d*\operatorname{Log}[F]*x^2)}, x], x, \operatorname{Log}[c*x^n]], x] /;$  FreeQ[{F, a, b, c, d, e, m, n}, x]

### Rule 2234

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a-b^2/(4*c))}, \operatorname{Int}[F^{((b+2*c*x)^2/(4*c))}, x], x] /;$  FreeQ[{F, a, b, c}, x]

### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$  FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^2} dx &= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{g^2 x^2} dx, x, d+ex \right)}{e} \\
&= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{x^2} dx, x, d+ex \right)}{eg^2} \\
&= \frac{(c(d+ex)^n)^{\frac{1}{n}} \text{Subst} \left( \int e^{-\frac{x}{n}+af \log(F)+bf x^2 \log(F)} dx, x, \log(c(d+ex)^n) \right)}{eg^2 n(d+ex)} \\
&= \frac{\left( e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} (c(d+ex)^n)^{\frac{1}{n}} \right) \text{Subst} \left( \int e^{\frac{(-\frac{1}{n}+2bf x \log(F))^2}{4bf \log(F)}} dx, x, \log(c(d+ex)^n) \right)}{eg^2 n(d+ex)} \\
&= \frac{e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} \sqrt{\pi} (c(d+ex)^n)^{\frac{1}{n}} \text{erfi} \left( \frac{1-2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{b} e \sqrt{f} g^2 n(d+ex) \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.201997, size = 121, normalized size = 1.

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{\frac{1}{n}} \text{Erfi} \left( \frac{2bf n \log(F) \log(c(d+ex)^n) - 1}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{b} e \sqrt{f} g^2 n \sqrt{\log(F)} (d+ex)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(d\*g + e\*g\*x)^2,x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(c\*(d + e\*x)^n)^n^(-1)\*Erfi[(-1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*g^2\*n\*(d + e\*x)\*Sqrt[Log[F]])

**Maple [F]** time = 0.533, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{(egx+dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^2,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a) f}}{(egx+dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^2,x, algorithm="maxima")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g)^2, x)

**Fricas [A]** time = 1.03447, size = 315, normalized size = 2.6

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} \operatorname{erf}\left(\frac{(2bf n^2 \log(ex+d) \log(F) + 2bf n \log(F) \log(c) - 1)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}\right) e^{\left(\frac{4abf^2 n^2 \log(F)^2 + 4bf n \log(F) \log(c) - 1}{4bf n^2 \log(F)}\right)}}{2eg^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^2,x, algorithm="fricas")

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b*f*n^2*\log(e*x + d)*\log(F) + 2*b*f*n*\log(F)*\log(c) - 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{1/4*(4*a*b*f^2*n^2*\log(F)^2 + 4*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F))}/(e*g^2*n)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))/(e\*g\*x+d\*g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a) f}}{(egx + dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^2,x, algorithm="giac")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g)^2, x)



$$3.593 \quad \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^3} dx$$

**Optimal.** Leaf size=118

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{bfn^2 \log(F)}} (c(d+ex)^n)^{2/n} \operatorname{Erfi}\left(\frac{1-bfn \log(F) \log(c(d+ex)^n)}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{be} \sqrt{fg^3 n} \sqrt{\log(F)} (d+ex)^2}$$

[Out]  $-(F^{(a*f)} * \operatorname{Sqrt}[\operatorname{Pi}] * (c*(d + e*x)^n)^{(2/n)} * \operatorname{Erfi}[(1 - b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n]) / (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])]) / (2*\operatorname{Sqrt}[b]*e*E^{(1/(b*f*n^2*\operatorname{Log}[F]))} * \operatorname{Sqrt}[f]*g^3*n*(d + e*x)^2*\operatorname{Sqrt}[\operatorname{Log}[F]])$

**Rubi [A]** time = 0.230671, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {12, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{bfn^2 \log(F)}} (c(d+ex)^n)^{2/n} \operatorname{Erfi}\left(\frac{1-bfn \log(F) \log(c(d+ex)^n)}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{be} \sqrt{fg^3 n} \sqrt{\log(F)} (d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(f*(a + b*\operatorname{Log}[c*(d + e*x)^n]^2))} / (d*g + e*g*x)^3, x]$

[Out]  $-(F^{(a*f)} * \operatorname{Sqrt}[\operatorname{Pi}] * (c*(d + e*x)^n)^{(2/n)} * \operatorname{Erfi}[(1 - b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n]) / (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]])]) / (2*\operatorname{Sqrt}[b]*e*E^{(1/(b*f*n^2*\operatorname{Log}[F]))} * \operatorname{Sqrt}[f]*g^3*n*(d + e*x)^2*\operatorname{Sqrt}[\operatorname{Log}[F]])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

#### Rule 2276

$\operatorname{Int}[(F_*)^{((a_*) + \operatorname{Log}[(c_*)*(x_*)^{(n_*)}]^2*(b_*))*(d_*)} * ((e_*)*(x_*)^{(m_*)})$ ,  $x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)} / (e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(a*d*\operatorname{Log}[F] + ((m+1)*x)/n + b*d*\operatorname{Log}[F]*x^2)}, x], x, \operatorname{Log}[c*x^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

#### Rule 2234

$\operatorname{Int}[(F_*)^{((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2)}, x\_Symbol] \rightarrow \operatorname{Dist}[F^{(a - b^2/(4*c))}, \operatorname{Int}[F^{((b + 2*c*x)^2/(4*c))}, x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c\}, x]$

#### Rule 2204

$\operatorname{Int}[(F_*)^{((a_*) + (b_*)*((c_*) + (d_*)*(x_*)^2))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{a*\operatorname{Sqrt}[\operatorname{Pi}]} * \operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]) / (2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(dg+egx)^3} dx &= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{g^3 x^3} dx, x, d+ex \right)}{e} \\
&= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log^2(cx^n))}}{x^3} dx, x, d+ex \right)}{eg^3} \\
&= \frac{(c(d+ex)^n)^{2/n} \text{Subst} \left( \int e^{-\frac{2x}{n}+af \log(F)+bf x^2 \log(F)} dx, x, \log(c(d+ex)^n) \right)}{eg^3 n (d+ex)^2} \\
&= \frac{\left( e^{-\frac{1}{bf n^2 \log(F)}} F^{af} (c(d+ex)^n)^{2/n} \right) \text{Subst} \left( \int e^{\frac{\left(-\frac{2}{n}+2bf x \log(F)\right)^2}{4bf \log(F)}} dx, x, \log(c(d+ex)^n) \right)}{eg^3 n (d+ex)^2} \\
&= \frac{e^{-\frac{1}{bf n^2 \log(F)}} F^{af} \sqrt{\pi} (c(d+ex)^n)^{2/n} \text{erfi} \left( \frac{1-bfn \log(F) \log(c(d+ex)^n)}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{be} \sqrt{f} g^3 n (d+ex)^2 \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.207316, size = 117, normalized size = 0.99

$$\frac{\sqrt{\pi} F^{af} e^{-\frac{1}{bf n^2 \log(F)}} (c(d+ex)^n)^{2/n} \text{Erfi} \left( \frac{bf n \log(F) \log(c(d+ex)^n) - 1}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{be} \sqrt{f} g^3 n \sqrt{\log(F)} (d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(d\*g + e\*g\*x)^3,x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(c\*(d + e\*x)^n)^(2/n)\*Erfi[(-1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e\*E^(1/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*g^3\*n\*(d + e\*x)^2\*Sqrt[Log[F]])

**Maple [F]** time = 0.614, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{(egx+dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^3,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)f}}{(egx+dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^3,x, algorithm="maxima")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g)^3, x)

---

**Fricas [A]** time = 1.05993, size = 296, normalized size = 2.51

$$\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}\operatorname{erf}\left(\frac{(bfn^2\log(ex+d)\log(F)+bfn\log(F)\log(c)-1)\sqrt{-bfn^2\log(F)}}{bfn^2\log(F)}\right)e^{\left(\frac{abf^2n^2\log(F)^2+2bfn\log(F)\log(c)-1}{bfn^2\log(F)}\right)}}{2eg^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^3,x, algorithm="fricas")

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*\operatorname{erf}((b*f*n^2*\log(e*x + d)*\log(F) + b*f*n*\log(F)*\log(c) - 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{((a*b*f^2*n^2*\log(F)^2 + 2*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F)))/(e*g^3*n)}$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))/(e\*g\*x+d\*g)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b\log((ex+d)^n c)^2 + a)} f}{(egx + dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))/(e\*g\*x+d\*g)^3,x, algorithm="giac")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f)/(e\*g\*x + d\*g)^3, x)

$$3.594 \quad \int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx$$

Optimal. Leaf size=30

$$\text{Unintegrable}\left((g+hx)^m F^{f(a+b \log^2(c(d+ex)^n))}, x\right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^m, x]

**Rubi [A]** time = 0.0762954, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^m, x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^m, x]

Rubi steps

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx = \int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx$$

**Mathematica [A]** time = 1.77503, size = 0, normalized size = 0.

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^m, x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^m, x]

**Maple [A]** time = 0.861, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} (hx+g)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^m, x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^m F^{(b \log((ex+d)^n c)^2 + a)^f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^m,x, algorithm="maxima")

[Out] integrate((h\*x + g)^m \* F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(hx + g\right)^m F^{bf \log\left(\left(ex+d\right)^n c\right)^2 + af}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^m,x, algorithm="fricas")

[Out] integral((h\*x + g)^m \* F^(b\*f\*log((e\*x + d)^n\*c)^2 + a\*f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(h\*x+g)\*\*m,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^m F^{(b \log((ex+d)^n c)^2 + a)^f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^m,x, algorithm="giac")

[Out] integrate((h\*x + g)^m \* F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**3.595**  $\int F^{f(a+b \log^2(c(d+ex)^n))} (g + hx)^3 dx$

**Optimal.** Leaf size=502

$$\frac{3\sqrt{\pi}h^2F^{af}(d+ex)^3(eg-dh)e^{-\frac{9}{4bf n^2 \log(F)}}(c(d+ex)^n)^{-3/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+3}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{b}e^4\sqrt{fn}\sqrt{\log(F)}} + \frac{3\sqrt{\pi}hF^{af}(d+ex)^2(eg-dh)^2e^{-\frac{9}{4bf n^2 \log(F)}}}{2}$$

```
[Out] (3*F^(a*f)*h*(e*g - d*h)^2*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e^4*E^(1/(b*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + (F^(a*f)*h^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2 + b*f*n*Log[F]*Log[c*(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e^4*E^(4/(b*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*Sqrt[Log[F]]) + (F^(a*f)*(e*g - d*h)^3*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e^4*E^(1/(4*b*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(-1)*Sqrt[Log[F]]) + (3*F^(a*f)*h^2*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^3*Erfi[(3 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e^4*E^(9/(4*b*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

**Rubi [F]** time = 0.376368, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g + hx)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^3,x]
```

```
[Out] (F^(a*f)*g^3*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + 3*g^2*h*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n]^2))*x, x] + 3*g*h^2*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n]^2))*x^2, x] + h^3*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n]^2))*x^3, x]
```

Rubi steps

$$\begin{aligned}
\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^3 dx &= \int \left( F^{f(a+b \log^2(c(d+ex)^n))} g^3 + 3F^{f(a+b \log^2(c(d+ex)^n))} g^2 hx + 3F^{f(a+b \log^2(c(d+ex)^n))} gh^2 x^2 + \dots \right) dx \\
&= g^3 \int F^{f(a+b \log^2(c(d+ex)^n))} dx + (3g^2 h) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + (3gh^2) \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx + \dots \\
&= \frac{g^3 \operatorname{Subst} \left( \int F^{f(a+b \log^2(cx)^n)} dx, x, d+ex \right)}{e} + (3g^2 h) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + (3gh^2) \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx + \dots \\
&= (3g^2 h) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + (3gh^2) \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx + h^3 \int F^{f(a+b \log^2(c(d+ex)^n))} x^3 dx + \dots \\
&= (3g^2 h) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + (3gh^2) \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx + h^3 \int F^{f(a+b \log^2(c(d+ex)^n))} x^3 dx + \dots \\
&= \frac{e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} g^3 \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \operatorname{erfi} \left( \frac{1+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right)}{2\sqrt{b} e \sqrt{fn} \sqrt{\log(F)}} + (3g^2 h) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + \dots
\end{aligned}$$

**Mathematica [A]** time = 1.55108, size = 396, normalized size = 0.79

$$\sqrt{\pi} F^{af} (d+ex) e^{-\frac{4}{bf n^2 \log(F)}} (c(d+ex)^n)^{-4/n} \left( (eg-dh) e^{\frac{7}{4bf n^2 \log(F)}} (c(d+ex)^n)^{\frac{1}{n}} \left( (eg-dh)^2 e^{\frac{2}{bf n^2 \log(F)}} (c(d+ex)^n)^{2/n} \operatorname{Erfi} \left( \frac{1+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^3, x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*(3\*E^(3/(b\*f\*n^2\*Log[F])))\*h\*(e\*g - d\*h)^2\*(d + e\*x)\*(c\*(d + e\*x)^n)^(2/n)\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + h^3\*(d + e\*x)^3\*Erfi[(2 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + E^(7/(4\*b\*f\*n^2\*Log[F]))\*(e\*g - d\*h)\*(c\*(d + e\*x)^n)^(2/n)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + 3\*h^2\*(d + e\*x)^2\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e^4\*E^(4/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(4/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.552, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} (hx+g)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^3, x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx+g)^3 F^{(b \log((ex+d)^n c)^2 + a)f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^3,x, algorithm="maxima")

[Out] integrate((h\*x + g)^3\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 1.093, size = 1319, normalized size = 2.63

$$\sqrt{\pi}\sqrt{-bfn^2\log(F)}h^3\operatorname{erf}\left(\frac{bfn^2\log(ex+d)\log(F)+bfn\log(F)\log(c)+2\sqrt{-bfn^2\log(F)}}{bfn^2\log(F)}\right)e^{\left(\frac{abf^2n^2\log(F)^2-4bfn\log(F)\log(c)-4}{bfn^2\log(F)}\right)}+\sqrt{\pi}(e^3g^3-3d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^3,x, algorithm="fricas")

[Out] 
$$-1/2*(\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)})*h^3*\operatorname{erf}((b*f*n^2*\log(e*x + d)*\log(F) + b*f*n*\log(F)*\log(c) + 2)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{((a*b*f^2*n^2*\log(F)^2 - 4*b*f*n*\log(F)*\log(c) - 4)/(b*f*n^2*\log(F)))} + \sqrt{\pi}*(e^3*g^3 - 3*d*e^2*g^2*h + 3*d^2*e*g*h^2 - d^3*h^3)*\sqrt{-b*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b*f*n^2*\log(e*x + d)*\log(F) + 2*b*f*n*\log(F)*\log(c) + 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{(1/4*(4*a*b*f^2*n^2*\log(F)^2 - 4*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F)))} + 3*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*(e*g*h^2 - d*h^3)*\operatorname{erf}(1/2*(2*b*f*n^2*\log(e*x + d)*\log(F) + 2*b*f*n*\log(F)*\log(c) + 3)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{(1/4*(4*a*b*f^2*n^2*\log(F)^2 - 12*b*f*n*\log(F)*\log(c) - 9)/(b*f*n^2*\log(F)))} + 3*\sqrt{\pi}*(e^2*g^2*h - 2*d*e*g*h^2 + d^2*h^3)*\sqrt{-b*f*n^2*\log(F)}*\operatorname{erf}((b*f*n^2*\log(e*x + d)*\log(F) + b*f*n*\log(F)*\log(c) + 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{((a*b*f^2*n^2*\log(F)^2 - 2*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F)))}/(e^4*n)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(h\*x+g)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^3 F^{(b\log((ex+d)^n c)^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^3,x, algorithm="giac")

[Out] integrate((h\*x + g)^3\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)



$$3.596 \quad \int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^2 dx$$

**Optimal.** Leaf size=372

$$\frac{\sqrt{\pi} h F^{af} (d+ex)^2 (eg-dh) e^{-\frac{1}{bfn^2 \log(F)}} (c(d+ex)^n)^{-2/n} \operatorname{Erfi}\left(\frac{bfn \log(F) \log(c(d+ex)^n)+1}{\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{\sqrt{be^3} \sqrt{fn} \sqrt{\log(F)}} + \frac{\sqrt{\pi} F^{af} (d+ex) (eg-dh)^2 e^{-\frac{1}{4bfn^2 \log(F)}}}{2\sqrt{be^3} \sqrt{fn} \sqrt{\log(F)}} \quad 2x$$

[Out]  $(F^{(a*f)*h*(e*g - d*h)*\operatorname{Sqrt}[\operatorname{Pi}]}*(d + e*x)^2*\operatorname{Erfi}[(1 + b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]]))/(\operatorname{Sqrt}[b]*e^{3*E^{(1/(b*f*n^2*\operatorname{Log}[F])*)}}*\operatorname{Sqrt}[f]*n*(c*(d + e*x)^n)^{(2/n)*\operatorname{Sqrt}[\operatorname{Log}[F]]}) + (F^{(a*f)*(e*g - d*h)^2*\operatorname{Sqrt}[\operatorname{Pi}]}*(d + e*x)*\operatorname{Erfi}[(1 + 2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])]/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]]))/((2*\operatorname{Sqrt}[b]*e^{3*E^{(1/(4*b*f*n^2*\operatorname{Log}[F])*)}}*\operatorname{Sqrt}[f]*n*(c*(d + e*x)^n)^{(-1)*\operatorname{Sqrt}[\operatorname{Log}[F]]}) + (F^{(a*f)*h^2*\operatorname{Sqrt}[\operatorname{Pi}]}*(d + e*x)^3*\operatorname{Erfi}[(3 + 2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])]/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]]))/((2*\operatorname{Sqrt}[b]*e^{3*E^{(9/(4*b*f*n^2*\operatorname{Log}[F])*)}}*\operatorname{Sqrt}[f]*n*(c*(d + e*x)^n)^{(3/n)*\operatorname{Sqrt}[\operatorname{Log}[F]]})$

**Rubi [F]** time = 0.306164, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^2 dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[F^{(f*(a + b*\operatorname{Log}[c*(d + e*x)^n]^2))}*(g + h*x)^2, x]$

[Out]  $(F^{(a*f)*g^2*\operatorname{Sqrt}[\operatorname{Pi}]}*(d + e*x)*\operatorname{Erfi}[(1 + 2*b*f*n*\operatorname{Log}[F]*\operatorname{Log}[c*(d + e*x)^n])]/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[f]*n*\operatorname{Sqrt}[\operatorname{Log}[F]]))/((2*\operatorname{Sqrt}[b]*e^{E^{(1/(4*b*f*n^2*\operatorname{Log}[F])*)}}*\operatorname{Sqrt}[f]*n*(c*(d + e*x)^n)^{(-1)*\operatorname{Sqrt}[\operatorname{Log}[F]]}) + 2*g*h*\operatorname{Defer}[\operatorname{Int}[F^{(f*(a + b*\operatorname{Log}[c*(d + e*x)^n]^2))}*x, x] + h^2*\operatorname{Defer}[\operatorname{Int}[F^{(f*(a + b*\operatorname{Log}[c*(d + e*x)^n]^2))}*x^2, x]$

Rubi steps

$$\begin{aligned} \int F^{f(a+b \log^2(c(d+ex)^n))} (g+hx)^2 dx &= \int \left( F^{f(a+b \log^2(c(d+ex)^n))} g^2 + 2F^{f(a+b \log^2(c(d+ex)^n))} ghx + F^{f(a+b \log^2(c(d+ex)^n))} h^2 x^2 \right) dx \\ &= g^2 \int F^{f(a+b \log^2(c(d+ex)^n))} dx + (2gh) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + h^2 \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx \\ &= \frac{g^2 \operatorname{Subst}\left(\int F^{f(a+b \log^2(cx^n))} dx, x, d+ex\right)}{e} + (2gh) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + h^2 \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx \\ &= (2gh) \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + h^2 \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx + \frac{(g^2(d+ex)(c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{1+2bfn \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right))}{e^{-\frac{1}{4bfn^2 \log(F)}} F^{af}} \\ &= \frac{e^{-\frac{1}{4bfn^2 \log(F)}} F^{af} g^2 \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{1+2bfn \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{be} \sqrt{fn} \sqrt{\log(F)}} + (2gh) \int F^{f(a+b \log^2(c(d+ex)^n))} x^2 dx \end{aligned}$$

**Mathematica [A]** time = 0.703055, size = 303, normalized size = 0.81

$$\frac{\sqrt{\pi} F^{af} (d + ex) e^{-\frac{9}{4bf n^2 \log(F)}} (c(d + ex)^n)^{-3/n} \left( (eg - dh)^2 e^{\frac{2}{bf n^2 \log(F)}} (c(d + ex)^n)^{2/n} \operatorname{Erfi} \left( \frac{2bf n \log(F) \log(c(d + ex)^n) + 1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}} \right) - 2h(d + ex) \right)}{2\sqrt{b}e^3\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x)^2,x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*(-2\*E^(5/(4\*b\*f\*n^2\*Log[F]))\*h\*(-(e\*g) + d\*h)\*(d + e\*x)\*(c\*(d + e\*x)^n)^n^(-1)\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + E^(2/(b\*f\*n^2\*Log[F]))\*(e\*g - d\*h)^2\*(c\*(d + e\*x)^n)^(2/n)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + h^2\*(d + e\*x)^2\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])]))/(2\*Sqrt[b]\*e^3\*E^(9/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(3/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.458, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(ex+d)^n))^2)} (hx + g)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^2,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^2 F^{(b \log((ex+d)^n c) + a) f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^2,x, algorithm="maxima")

[Out] integrate((h\*x + g)^2\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 1.03795, size = 961, normalized size = 2.58

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} h^2 \operatorname{erf} \left( \frac{(2bf n^2 \log(ex+d) \log(F) + 2bf n \log(F) \log(c) + 3)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)} \right) e^{\left( \frac{4abf^2 n^2 \log(F)^2 - 12bf n \log(F) \log(c) - 9}{4bf n^2 \log(F)} \right)} + \sqrt{\pi}\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^2,x, algorithm="fricas")

[Out] -1/2\*(sqrt(pi)\*sqrt(-b\*f\*n^2\*log(F))\*h^2\*erf(1/2\*(2\*b\*f\*n^2\*log(e\*x + d)\*log(F) + 2\*b\*f\*n\*log(F)\*log(c) + 3)\*sqrt(-b\*f\*n^2\*log(F))/(b\*f\*n^2\*log(F)))\*e

$$\begin{aligned} & \left( \frac{1}{4} (4a^2 b^2 f^{2n} \log(F)^2 - 12 b^2 f^n \log(F) \log(c) - 9) / (b^2 f^n \log(F)^2) \right. \\ & \left. + \sqrt{\pi} \sqrt{-b^2 f^n \log(F)} (e^{2g} - 2d e g h + d^2 h^2) \operatorname{erf} \left( \frac{1}{2} \right. \right. \\ & \left. \left. * (2 b^2 f^n \log(e^x + d) \log(F) + 2 b^2 f^n \log(F) \log(c) + 1) \sqrt{-b^2 f^n \log(F)} / (b^2 f^n \log(F)) \right) \right. \\ & \left. + 2 \sqrt{\pi} \sqrt{-b^2 f^n \log(F)} (e g h - d h^2) \operatorname{erf} \left( (b^2 f^n \log(e^x + d) \log(F) + b^2 f^n \log(F) \log(c) + 1) \sqrt{-b^2 f^n \log(F)} / (b^2 f^n \log(F)) \right) \right. \\ & \left. + 2 \sqrt{\pi} \sqrt{-b^2 f^n \log(F)} (e g h - d h^2) \operatorname{erf} \left( (b^2 f^n \log(e^x + d) \log(F) + b^2 f^n \log(F) \log(c) + 1) \sqrt{-b^2 f^n \log(F)} / (b^2 f^n \log(F)) \right) \right) / (e^{3n}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(h\*x+g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^2 F^{(b \log((ex+d)^n c^2 + a) f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g)^2,x, algorithm="giac")

[Out] integrate((h\*x + g)^2 \* F^((b\*log((e\*x + d)^n \* c)^2 + a) \* f), x)

**3.597**  $\int F^{f(a+b \log^2(c(d+ex)^n))} (g + hx) dx$

**Optimal.** Leaf size=242

$$\frac{\sqrt{\pi} F^{af} (d + ex) (eg - dh) e^{-\frac{1}{4bf n^2 \log(F)} (c(d + ex)^n)^{-1/n}} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be^2}\sqrt{fn}\sqrt{\log(F)}} + \frac{\sqrt{\pi} h F^{af} (d + ex)^2 e^{-\frac{1}{bf n^2 \log(F)} (c(d + ex)^n)^{-1/n}}}{2\sqrt{be^2}\sqrt{fn}}$$

[Out]  $(F^{(a*f)*h*\sqrt{\pi}}*(d + e*x)^2*\operatorname{Erfi}[(1 + b*f*n*\log[F]*\log[c*(d + e*x)^n])]/(\sqrt{b}*\sqrt{f}*n*\sqrt{\log[F]}))/ (2*\sqrt{b}*e^2*E^{(1/(b*f*n^2*\log[F]))}*\sqrt{f}*n*(c*(d + e*x)^n)^{(2/n)*\sqrt{\log[F]}}) + (F^{(a*f)}*(e*g - d*h)*\sqrt{\pi}*(d + e*x)*\operatorname{Erfi}[(1 + 2*b*f*n*\log[F]*\log[c*(d + e*x)^n])]/(2*\sqrt{b}*\sqrt{f}*n*\sqrt{\log[F]}))/ (2*\sqrt{b}*e^2*E^{(1/(4*b*f*n^2*\log[F]))}*\sqrt{f}*n*(c*(d + e*x)^n)^{n*(-1)*\sqrt{\log[F]}})$

**Rubi [F]** time = 0.22953, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log^2(c(d+ex)^n))} (g + hx) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[F^{(f*(a + b*\log[c*(d + e*x)^n]^2))}*(g + h*x), x]$

[Out]  $(F^{(a*f)*g*\sqrt{\pi}}*(d + e*x)*\operatorname{Erfi}[(1 + 2*b*f*n*\log[F]*\log[c*(d + e*x)^n])]/(2*\sqrt{b}*\sqrt{f}*n*\sqrt{\log[F]}))/ (2*\sqrt{b}*e*E^{(1/(4*b*f*n^2*\log[F]))}*\sqrt{f}*n*(c*(d + e*x)^n)^{n*(-1)*\sqrt{\log[F]}}) + h*\operatorname{Defer}[\operatorname{Int}[F^{(f*(a + b*\log[c*(d + e*x)^n]^2))}]*x, x]$

Rubi steps

$$\begin{aligned} \int F^{f(a+b \log^2(c(d+ex)^n))} (g + hx) dx &= \int \left( F^{f(a+b \log^2(c(d+ex)^n))} g + F^{f(a+b \log^2(c(d+ex)^n))} hx \right) dx \\ &= g \int F^{f(a+b \log^2(c(d+ex)^n))} dx + h \int F^{f(a+b \log^2(c(d+ex)^n))} x dx \\ &= \frac{g \operatorname{Subst}\left(\int F^{f(a+b \log^2(cx^n))} dx, x, d + ex\right)}{e} + h \int F^{f(a+b \log^2(c(d+ex)^n))} x dx \\ &= h \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + \frac{(g(d + ex) (c(d + ex)^n)^{-1/n}) \operatorname{Subst}\left(\int e^{\frac{x}{n} + af \log(F) + bf x^2 \log(F)} dx, x, d + ex\right)}{en} \\ &= h \int F^{f(a+b \log^2(c(d+ex)^n))} x dx + \frac{\left(e^{-\frac{1}{4bf n^2 \log(F)} F^{af} g (d + ex) (c(d + ex)^n)^{-1/n}}\right) \operatorname{Subst}\left(\int e^{\frac{x}{n}} dx, x, d + ex\right)}{en} \\ &= \frac{e^{-\frac{1}{4bf n^2 \log(F)} F^{af} g \sqrt{\pi} (d + ex) (c(d + ex)^n)^{-1/n}} \operatorname{erfi}\left(\frac{1+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}} + h \int F^{f(a+b \log^2(c(d+ex)^n))} x dx \end{aligned}$$

**Mathematica [A]** time = 0.374243, size = 204, normalized size = 0.84

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{bf n^2 \log(F)}} (c(d+ex)^n)^{-2/n} \left( (eg-dh) e^{\frac{3}{4bf n^2 \log(F)}} (c(d+ex)^n)^{\frac{1}{n}} \operatorname{Erfi} \left( \frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}} \right) + h(d+ex) \right)}{2\sqrt{b}e^2\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))\*(g + h\*x), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*(h\*(d + e\*x)\*Erfi[(1 + b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])] + E^(3/(4\*b\*f\*n^2\*Log[F]))\*(e\*g - d\*h)\*(c\*(d + e\*x)^n)^(-1)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*Sqrt[b]\*e^2\*E^(1/(b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.372, size = 0, normalized size = 0.

$$\int F^{f(a+b(\ln(c(e^x+d)^n))^2)} (hx + g) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))\*(h\*x+g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g) F^{(b \log((e^x+d)^n c)^2 + a) f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g), x, algorithm="maxima")

[Out] integrate((h\*x + g)\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

**Fricas [A]** time = 1.05963, size = 610, normalized size = 2.52

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)}(eg-dh) \operatorname{erf} \left( \frac{(2bf n^2 \log(ex+d) \log(F)+2bf n \log(F) \log(c)+1)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)} \right) e^{\left( \frac{4abf^2 n^2 \log(F)^2 - 4bf n \log(F) \log(c)-1}{4bf n^2 \log(F)} \right)}}{2e^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g), x, algorithm="fricas")

[Out] -1/2\*(sqrt(pi)\*sqrt(-b\*f\*n^2\*log(F))\*(e\*g - d\*h)\*erf(1/2\*(2\*b\*f\*n^2\*log(e\*x + d)\*log(F) + 2\*b\*f\*n\*log(F)\*log(c) + 1)\*sqrt(-b\*f\*n^2\*log(F))/(b\*f\*n^2\*log(F)))\*e^(1/4\*(4\*a\*b\*f^2\*n^2\*log(F)^2 - 4\*b\*f\*n\*log(F)\*log(c) - 1)/(b\*f\*n^2

$*\log(F)) + \sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*h*\operatorname{erf}((b*f*n^2*\log(e*x + d)*\log(F) + b*f*n*\log(F)*\log(c) + 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F)))*e^{((a*b*f^2*n^2*\log(F)^2 - 2*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F)))}/(e^{2*n})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2))\*(h\*x+g), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)F^{(b \log((ex+d)^n c)^2 + a)f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2))\*(h\*x+g), x, algorithm="giac")

[Out] integrate((h\*x + g)\*F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)

$$3.598 \quad \int F^{f(a+b \log^2(c(d+ex)^n))} dx$$

Optimal. Leaf size=118

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])]/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]]))/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0938932, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2275, 2234, 2204}

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{fn}\sqrt{\log(F)}}\right)}{2\sqrt{be}\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2)), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])]/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]]))/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

#### Rule 2275

Int[(F\_)^((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(a\*d\*Log[F] + x/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int F^{f(a+b \log^2(c(d+ex)^n))} dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log^2(cx^n))} dx, x, d+ex\right)}{e} \\
&= \frac{\left((d+ex)(c(d+ex)^n)^{-1/n}\right) \text{Subst}\left(\int e^{\frac{x}{n}+af \log(F)+bf x^2 \log(F)} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{\left(e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} (d+ex)(c(d+ex)^n)^{-1/n}\right) \text{Subst}\left(\int e^{\frac{\left(\frac{1}{n}+2bf x \log(F)\right)^2}{4bf \log(F)}} dx, x, \log(c(d+ex)^n)\right)}{en} \\
&= \frac{e^{-\frac{1}{4bf n^2 \log(F)}} F^{af} \sqrt{\pi} (d+ex)(c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{1+2bf n \log(F) \log(c(d+ex)^n)}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{b} e \sqrt{fn} \sqrt{\log(F)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0229734, size = 118, normalized size = 1.

$$\frac{\sqrt{\pi} F^{af} (d+ex) e^{-\frac{1}{4bf n^2 \log(F)}} (c(d+ex)^n)^{-1/n} \operatorname{Erfi}\left(\frac{2bf n \log(F) \log(c(d+ex)^n)+1}{2\sqrt{b} \sqrt{fn} \sqrt{\log(F)}}\right)}{2\sqrt{b} e \sqrt{fn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2)), x]

[Out] (F^(a\*f)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*Log[c\*(d + e\*x)^n])]/(2\*Sqrt[b]\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*Sqrt[b]\*e\*E^(1/(4\*b\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

**Maple [F]** time = 0.013, size = 0, normalized size = 0.

$$\int F^{f\left(a+b\left(\ln(c(ex+d)^n)\right)^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2)), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{b \log((ex+d)^n c^2 + a) f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2)), x, algorithm="maxima")

[Out] integrate(F^((b\*log((e\*x + d)^n\*c)^2 + a)\*f), x)



**Fricas [A]** time = 0.986247, size = 309, normalized size = 2.62

$$\frac{\sqrt{\pi}\sqrt{-bf n^2 \log(F)} \operatorname{erf}\left(\frac{(2bf n^2 \log(ex+d) \log(F) + 2bf n \log(F) \log(c) + 1)\sqrt{-bf n^2 \log(F)}}{2bf n^2 \log(F)}\right) e^{\left(\frac{4abf^2 n^2 \log(F)^2 - 4bf n \log(F) \log(c) - 1}{4bf n^2 \log(F)}\right)}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2)),x, algorithm="fricas")

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b*f*n^2*\log(e*x + d)*\log(F) + 2*b*f*n*\log(F)*\log(c) + 1)*\sqrt{-b*f*n^2*\log(F)})/(b*f*n^2*\log(F))*e^{1/4*(4*a*b*f^2*n^2*\log(F)^2 - 4*b*f*n*\log(F)*\log(c) - 1)/(b*f*n^2*\log(F))}/(e*n)$

**Sympy [A]** time = 105.909, size = 532, normalized size = 4.51

$$\left\{ \frac{2F^{af} F^{bf \log(c)^2} F^{bf n^2 \log(d+ex)^2} F^{2bf n \log(c) \log(d+ex)} bdf n^2 \log(F) \log(d+ex)}{e} - \frac{2F^{af} F^{bf \log(c)^2} F^{bf n^2 \log(d+ex)^2} F^{2bf n \log(c) \log(d+ex)} bdf n^2 \log(F)}{e} - 2F^{f(a+b \log(cd^n)^2)} \right\}_x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n)\*\*2)),x)

[Out] Piecewise((-2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*d\*f\*n\*\*2\*log(F)\*log(d + e\*x)/e - 2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*d\*f\*n\*\*2\*log(F)/e - 2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*d\*f\*n\*log(F)\*log(c)/e - 2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*f\*n\*\*2\*x\*log(F)\*log(d + e\*x) + 2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*f\*n\*\*2\*x\*log(F) - 2\*F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*b\*f\*n\*x\*log(F)\*log(c) + F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*d/e + F\*\*(a\*f)\*F\*\*(b\*f\*log(c)\*\*2)\*F\*\*(b\*f\*n\*\*2\*log(d + e\*x)\*\*2)\*F\*\*(2\*b\*f\*n\*log(c)\*log(d + e\*x))\*x, Ne(e, 0)), (F\*\*(f\*(a + b\*log(c\*d\*\*n)\*\*2))\*x, True))

**Giac [A]** time = 1.36132, size = 136, normalized size = 1.15

$$\frac{\sqrt{\pi}F^{af} \operatorname{erf}\left(-\sqrt{-bf \log(F)}n \log(xe + d) - \sqrt{-bf \log(F)} \log(c) - \frac{\sqrt{-bf \log(F)}}{2bf n \log(F)}\right) e^{\left(-\frac{1}{4bf n^2 \log(F)} - 1\right)}}{2\sqrt{-bf \log(F)}c^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n)^2)),x, algorithm="giac")

[Out]  $-1/2*\sqrt{\pi}*F^{(a*f)}*\operatorname{erf}(-\sqrt{-b*f*\log(F)}*n*\log(x*e + d) - \sqrt{-b*f*\log(F)}*\log(c) - 1/2*\sqrt{-b*f*\log(F)})/(b*f*n*\log(F))*e^{(-1/4/(b*f*n^2*\log(F)) - 1)/(sqrt{-b*f*\log(F)})*c^{(1/n)*n}}$

$$3.599 \quad \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x), x]

**Rubi [A]** time = 0.0836023, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x), x]

[Out] Defer[Int] [F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x), x]

Rubi steps

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx = \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx$$

**Mathematica [A]** time = 0.516759, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{g+hx} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x), x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x), x]

**Maple [A]** time = 0.386, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(h\*x+g), x)

[Out]  $\text{int}(F^{(f*(a+b*\ln(c*(e*x+d)^n)^2)})/(h*x+g), x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g), x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{bf \log((ex+d)^n c)^2 + af}}{hx + g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g), x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(F^{(b*f*\log((e*x + d)^n*c)^2 + a*f)/(h*x + g)}, x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{**}(f*(a+b*\ln(c*(e*x+d)**n)**2)})/(h*x+g), x)$

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g), x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)}, x)$

$$3.600 \quad \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^2, x]

**Rubi [A]** time = 0.0852799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^2,x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^2, x]

Rubi steps

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx = \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx$$

**Mathematica [A]** time = 2.48421, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^2,x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^2, x]

**Maple [A]** time = 0.468, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(h\*x+g)^2,x)

[Out]  $\text{int}(F^{(f*(a+b*\ln(c*(e*x+d)^n)^2)})/(h*x+g)^2, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^2, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^2, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{bf \log((ex+d)^n c)^2 + af}}{h^2 x^2 + 2ghx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^2, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(F^{(b*f*\log((e*x + d)^n*c)^2 + a*f)/(h^2*x^2 + 2*g*h*x + g^2), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\ln(c*(e*x+d)**n)**2)})/(h*x+g)**2, x)$

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^2, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^2, x)$

$$3.601 \quad \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^3, x]

**Rubi [A]** time = 0.0834928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^3,x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^3, x]

Rubi steps

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx = \int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx$$

**Mathematica [A]** time = 3.30235, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log^2(c(d+ex)^n))}}{(g+hx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^3,x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n]^2))/(g + h\*x)^3, x]

**Maple [A]** time = 0.657, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b(\ln(c(ex+d)^n))^2)}}{(hx+g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n)^2))/(h\*x+g)^3,x)

[Out]  $\text{int}(F^{(f*(a+b*\ln(c*(e*x+d)^n)^2)})/(h*x+g)^3, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{(hx + g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^3, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^3, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{bf \log((ex+d)^n c)^2 + af}}{h^3 x^3 + 3 g h^2 x^2 + 3 g^2 h x + g^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^3, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(F^{(b*f*\log((e*x + d)^n*c)^2 + a*f)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)}, x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\ln(c*(e*x+d)**n)**2)})/(h*x+g)**3, x)$

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c)^2 + a) f}}{(hx + g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n)^2)})/(h*x+g)^3, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^3, x)$

### 3.602 $\int F^{f(a+b \log(c(d+ex)^n))^2} (dg + egx)^m dx$

**Optimal.** Leaf size=153

$$\frac{\sqrt{\pi} F^{a^2 f} (d+ex)(dg+egx)^m (c(d+ex)^n)^{-\frac{m+1}{n}} \exp\left(-\frac{(2abfn \log(F)+m+1)^2}{4b^2 fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abfn \log(F)+2b^2 fn \log(F) \log(c(d+ex)^n)+m+1}{2b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (F^(a^2\*f)\*Sqrt[Pi]\*(d+e\*x)\*(d\*g+e\*g\*x)^m\*Erfi[(1+m+2\*a\*b\*f\*n\*Log[F]+2\*b^2\*f\*n\*Log[F]\*Log[c\*(d+e\*x)^n])/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1+m+2\*a\*b\*f\*n\*Log[F])^2/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d+e\*x)^n)^((1+m)/n)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.757005, antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {2278, 2274, 15, 20, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} F^{a^2 f} (d+ex)(g(d+ex))^m (c(d+ex)^n)^{-\frac{m+1}{n}} \exp\left(-\frac{(2abfn \log(F)+m+1)^2}{4b^2 fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abfn \log(F)+2b^2 fn \log(F) \log(c(d+ex)^n)+m+1}{2b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a+b\*Log[c\*(d+e\*x)^n])^2)\*(d\*g+e\*g\*x)^m,x]

[Out] (F^(a^2\*f)\*Sqrt[Pi]\*(d+e\*x)\*(g\*(d+e\*x))^m\*Erfi[(1+m+2\*a\*b\*f\*n\*Log[F]+2\*b^2\*f\*n\*Log[F]\*Log[c\*(d+e\*x)^n])/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1+m+2\*a\*b\*f\*n\*Log[F])^2/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d+e\*x)^n)^((1+m)/n)\*Sqrt[Log[F]])

#### Rule 2278

Int[(F\_)^(((a\_.)+Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^2\*(d\_.))\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[(e\*x)^m\*F^(a^2\*d+2\*a\*b\*d\*Log[c\*x^n]+b^2\*d\*Log[c\*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.)+(v\_.))), x\_Symbol] :> Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 15

Int[(u\_.)\*((a\_.)\*(x\_)^(n\_))^(m\_), x\_Symbol] :> Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 2276



```
Int[(F_)^(((a_) + Log[(c_)*(x_)^(n_)]^2*(b_))* (d_))*((e_)*(x_)^(m_))
, x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2234

```
Int[(F_)^((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int F^{f(a+b \log(c(d+ex)^n))^2} (dg + egx)^m dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log(cx^n))^2} (gx)^m dx, x, d + ex\right)}{e} \\ &= \frac{\text{Subst}\left(\int F^{a^2 f + 2abf \log(cx^n) + b^2 f \log^2(cx^n)} (gx)^m dx, x, d + ex\right)}{e} \\ &= \frac{\text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} (gx)^m (cx^n)^{2abf \log(F)} dx, x, d + ex\right)}{e} \\ &= \frac{\left((d + ex)^{-2abfn \log(F)} (c(d + ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x^{2abfn \log(F)} dx, x, d + ex\right)}{e} \\ &= \frac{\left((d + ex)^{-m - 2abfn \log(F)} (g(d + ex))^m (c(d + ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x^{2abfn \log(F)} dx, x, d + ex\right)}{e} \\ &= \frac{\left((d + ex)(g(d + ex))^m (c(d + ex)^n)^{2abf \log(F) - \frac{1+m+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(a^2 f \log^2(cx^n)\right) dx, x, d + ex\right)}{en} \\ &= \frac{\left(\exp\left(-\frac{(1+m+2abfn \log(F))^2}{4b^2fn^2 \log(F)}\right) F^{a^2 f} (d + ex)(g(d + ex))^m (c(d + ex)^n)^{2abf \log(F) - \frac{1+m+2abfn \log(F)}{n}}\right) \text{erfi}\left(\frac{1+m+2abfn \log(F)}{2bn \sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}} \end{aligned}$$

**Mathematica [F]** time = 0.238361, size = 0, normalized size = 0.

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (dg + egx)^m dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^m, x]
```

```
[Out] Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^m, x]
```

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int F^{f(a+b\ln(c(ex+d)^n))^2} (egx + dg)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^m,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^m,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^m F^{(b\log((ex+d)^n c)+a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^m,x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)^m \* F^((b\*log((e\*x + d)^n \* c) + a)^2 \* f), x)

**Fricas [A]** time = 1.02914, size = 443, normalized size = 2.9

$$\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\operatorname{erf}\left(\frac{(2b^2fn^2\log(ex+d)\log(F)+2b^2fn\log(F)\log(c)+2abfn\log(F)+m+1)\sqrt{-b^2fn^2\log(F)}}{2b^2fn^2\log(F)}\right)}{2ben} e^{\left(\frac{4b^2fmn^2\log(F)\log(g)-4(b^2fm+b^2f)}{4b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^m,x, algorithm="fricas")

[Out] 
$$-1/2*\sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + m + 1)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F)))*e^{(1/4*(4*b^2*f*m*n^2*\log(F)*\log(g) - 4*(b^2*f*m + b^2*f)*n*\log(F)*\log(c) - 4*(a*b*f*m + a*b*f)*n*\log(F) - m^2 - 2*m - 1)/(b^2*f*n^2*\log(F)))/(b*e*n)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n))\*\*2)\*(e\*g\*x+d\*g)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^m F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^m,x, algorithm="giac")

[Out] integrate((e\*g\*x + d\*g)^m\*F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

### 3.603 $\int F f(a+b \log(c(d+ex)^n))^2 (dg + egx)^2 dx$

**Optimal.** Leaf size=133

$$\frac{\sqrt{\pi} g^2 (d+ex)^3 (c(d+ex)^n)^{-3/n} \exp\left(-\frac{3(4abfn \log(F)+3)}{4b^2fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

[Out]  $(g^2 \sqrt{\pi} (d+ex)^3 \operatorname{Erfi}[(3/n + 2abf \log(F) + 2b^2f \log(F) \log(c(d+ex)^n))/n]) / (2b \sqrt{f} \sqrt{\log(F)}) / (2b e E^{((3(3 + 4abfn \log(F)))/n)}) / (4b^2fn^2 \log(F)) \sqrt{f} n (c(d+ex)^n)^{3/n} \sqrt{\log(F)}$

**Rubi [A]** time = 0.410737, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {12, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} g^2 (d+ex)^3 (c(d+ex)^n)^{-3/n} \exp\left(-\frac{3(4abfn \log(F)+3)}{4b^2fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(f(a + b \log(c(d + ex)^n))^2) (d * g + e * g * x)^2, x]$

[Out]  $(g^2 \sqrt{\pi} (d+ex)^3 \operatorname{Erfi}[(3/n + 2abf \log(F) + 2b^2f \log(F) \log(c(d+ex)^n))/n]) / (2b \sqrt{f} \sqrt{\log(F)}) / (2b e E^{((3(3 + 4abfn \log(F)))/n)}) / (4b^2fn^2 \log(F)) \sqrt{f} n (c(d+ex)^n)^{3/n} \sqrt{\log(F)}$

#### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

#### Rule 2278

$\text{Int}[(F_*)^{((a_*) + \text{Log}[(c_*)(x_*)^{(n_*)}) (b_*)^2 (d_*) ((e_*)(x_*)^{(m_*)}), x\_Symbol] := \text{Int}[(e*x)^m F^{(a^2*d + 2*a*b*d*\text{Log}[c*x^n] + b^2*d*\text{Log}[c*x^n]^2)}, x] /; \text{FreeQ}[\{F, a, b, c, d, e, m, n\}, x]$

#### Rule 2274

$\text{Int}[(u_*)(F_*)^{((a_*)(\text{Log}[z_*)(b_*) + (v_*)])}, x\_Symbol] := \text{Int}[u * F^{(a*v)} z^{(a*b*\text{Log}[F])}, x] /; \text{FreeQ}[\{F, a, b\}, x]$

#### Rule 15

$\text{Int}[(u_*)(a_*)(x_*)^{(n_*)} (m_*)], x\_Symbol] := \text{Dist}[(a^{\text{IntPart}[m]} (a*x^n)^{\text{racPart}[m]}) / x^{(n*\text{FracPart}[m])}, \text{Int}[u*x^{(m*n)}, x], x] /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 2276

$\text{Int}[(F_*)^{((a_*) + \text{Log}[(c_*)(x_*)^{(n_*)})^2 (b_*) (d_*) ((e_*)(x_*)^{(m_*)}), x\_Symbol] := \text{Dist}[(e*x)^{(m+1)} / (e*n*(c*x^n)^{((m+1)/n)}], \text{Subst}[\text{Int}[E^{(a*d*\text{Log}[F] + ((m+1)*x)/n + b*d*\text{Log}[F]*x^2)}, x], x, \text{Log}[c*x^n]], x] /; \text{FreeQ}[\{F, a, b, c, d, e, m, n\}, x]$

$Q\{F, a, b, c, d, e, m, n\}, x]$

### Rule 2234

$\text{Int}[(F\_)^{\{(a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\}}, x\_Symbol] := \text{Dist}[F^{(a - b^2/(4*c))}, \text{Int}[F^{(b + 2*c*x)^2/(4*c)}, x], x] /; \text{FreeQ}\{F, a, b, c\}, x]$

### Rule 2204

$\text{Int}[(F\_)^{\{(a\_.) + (b\_.)*((c\_.) + (d\_.)*(x\_.)^2\)}, x\_Symbol] := \text{Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2])], x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

### Rubi steps

$$\begin{aligned} \int F^{(a+b \log(c(d+ex)^n))^2} (dg + egx)^2 dx &= \frac{\text{Subst}\left(\int F^{(a+b \log(cx^n))^2} g^2 x^2 dx, x, d+ex\right)}{e} \\ &= \frac{g^2 \text{Subst}\left(\int F^{(a+b \log(cx^n))^2} x^2 dx, x, d+ex\right)}{e} \\ &= \frac{g^2 \text{Subst}\left(\int F^{a^2 f + 2abf \log(cx^n) + b^2 f \log^2(cx^n)} x^2 dx, x, d+ex\right)}{e} \\ &= \frac{g^2 \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x^2 (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} \\ &= \frac{(g^2(d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}) \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x^{2+2abfn} dx, x, d+ex\right)}{e} \\ &= \frac{\left(g^2(d+ex)^3 (c(d+ex)^n)^{2abf \log(F) - \frac{3+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(a^2 f \log(F) + b^2 f \log^2(F)\right) dx, x, d+ex\right)}{en} \\ &= \frac{\left(\exp\left(a^2 f \log(F) - \frac{(3+2abfn \log(F))^2}{4b^2 fn^2 \log(F)}\right) g^2 (d+ex)^3 (c(d+ex)^n)^{2abf \log(F) - \frac{3+2abfn \log(F)}{n}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}} \\ &= \frac{\exp\left(-\frac{3(3+4abfn \log(F))}{4b^2 fn^2 \log(F)}\right) g^2 \sqrt{\pi} (d+ex)^3 (c(d+ex)^n)^{-3/n} \text{erfi}\left(\frac{\frac{3}{n} + 2abf \log(F) + 2b^2 f \log^2(F)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.34262, size = 129, normalized size = 0.97

$$\frac{\sqrt{\pi} g^2 \text{Erfi}\left(\frac{2bfn \log(F)(a+b \log(c(d+ex)^n))+3}{2b\sqrt{fn}\sqrt{\log(F)}}\right) \exp\left(-\frac{3(4bfn \log(F)(a+b \log(c(d+ex)^n)-n \log(d+ex))+3)}{4b^2 fn^2 \log(F)}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(d\*g + e\*g\*x)^2,x]

[Out] (g^2\*Sqrt[Pi]\*Erfi[(3 + 2\*b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*b\*e\*E^((3\*(3 + 4\*b\*f\*n\*Log[F]\*(a + b\*(-(n\*Log[d + e\*x]) + Log[c\*(d + e\*x)^n))))/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*Sqrt[Log[F]]])

---

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int F^{f(a+b\ln(c(ex+d)^n))} (egx + dg)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^2,x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^2 F^{(b\log((ex+d)^n c) + a)^f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^2,x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)^2\*F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

---

**Fricas [A]** time = 1.0033, size = 355, normalized size = 2.67

$$\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}g^2\operatorname{erf}\left(\frac{(2b^2fn^2\log(ex+d)\log(F)+2b^2fn\log(F)\log(c)+2abfn\log(F)+3)\sqrt{-b^2fn^2\log(F)}}{2b^2fn^2\log(F)}\right)}{2ben}e^{\left(-\frac{3(4b^2fn\log(F)\log(c)+4abfn\log(F)+3)}{4b^2fn^2\log(F)}\right)}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g)^2,x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-b^2\*f\*n^2\*log(F))\*g^2\*erf(1/2\*(2\*b^2\*f\*n^2\*log(e\*x + d)\*log(F) + 2\*b^2\*f\*n\*log(F)\*log(c) + 2\*a\*b\*f\*n\*log(F) + 3)\*sqrt(-b^2\*f\*n^2\*log(F))/(b^2\*f\*n^2\*log(F)))\*e^(-3/4\*(4\*b^2\*f\*n\*log(F)\*log(c) + 4\*a\*b\*f\*n\*log(F) + 3)/(b^2\*f\*n^2\*log(F)))/(b\*e\*n)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n))\*\*2)\*(e\*g\*x+d\*g)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)^2 F^{(b \log((ex+d)^n c) + a)^2} f dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x, algorithm="giac")
```

```
[Out] integrate((e*g*x + d*g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

### 3.604 $\int F^{f(a+b \log(c(d+ex)^n))^2} (dg + egx) dx$

**Optimal.** Leaf size=122

$$\frac{\sqrt{\pi} g(d+ex)^2 (c(d+ex)^n)^{-2/n} e^{-\frac{2abfn \log(F)+1}{b^2 f n^2 \log(F)}} \operatorname{Erfi}\left(\frac{abf \log(F)+b^2 f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(n^(-1) + a\*b\*f\*Log[F] + b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F]))) \* Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]

**Rubi [A]** time = 0.369419, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {12, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} g(d+ex)^2 (c(d+ex)^n)^{-2/n} e^{-\frac{2abfn \log(F)+1}{b^2 f n^2 \log(F)}} \operatorname{Erfi}\left(\frac{abf \log(F)+b^2 f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(d\*g + e\*g\*x), x]

[Out] (g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(n^(-1) + a\*b\*f\*Log[F] + b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F]))) \* Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 2278

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)])\*(b\_.))^2\*(d\_.))\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(e\*x)^m\*F^(a^2\*d + 2\*a\*b\*d\*Log[c\*x^n] + b^2\*d\*Log[c\*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 15

Int[(u\_.)\*((a\_.)\*(x\_)^(n\_))^(m\_), x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)])^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^(m+1/n)), Subst[Int[E^(a\*d\*Log[F] + ((m+1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; Free



$Q\{F, a, b, c, d, e, m, n\}, x]$

### Rule 2234

$\text{Int}[(F\_)^{\((a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\)}, x\_Symbol] \text{ :> Dist}[F^{\(a - b^2/(4*c)\)}, \text{Int}[F^{\(b + 2*c*x\)^2/(4*c)}, x], x] \text{ /; FreeQ}\{F, a, b, c\}, x]$

### Rule 2204

$\text{Int}[(F\_)^{\((a\_.) + (b\_.)*((c\_.) + (d\_.)*(x\_.)^2\)}, x\_Symbol] \text{ :> Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2])], x] \text{ /; FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

### Rubi steps

$$\begin{aligned} \int F^{f(a+b \log(c(d+ex)^n))^2} (dg + egx) dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log(cx^n))^2} gx dx, x, d+ex\right)}{e} \\ &= \frac{g \text{Subst}\left(\int F^{f(a+b \log(cx^n))^2} x dx, x, d+ex\right)}{e} \\ &= \frac{g \text{Subst}\left(\int F^{a^2 f + 2abf \log(cx^n) + b^2 f \log^2(cx^n)} x dx, x, d+ex\right)}{e} \\ &= \frac{g \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} \\ &= \frac{(g(d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}) \text{Subst}\left(\int F^{a^2 f + b^2 f \log^2(cx^n)} x^{1+2abfn \log(F)} dx, x, d+ex\right)}{e} \\ &= \frac{\left(g(d+ex)^2 (c(d+ex)^n)^{2abf \log(F) - \frac{2+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(a^2 f \log(F) + b^2 f x^2\right) dx, x, d+ex\right)}{en} \\ &= \frac{\left(\exp\left(a^2 f \log(F) - \frac{(2+2abfn \log(F))^2}{4b^2 fn^2 \log(F)}\right) g(d+ex)^2 (c(d+ex)^n)^{2abf \log(F) - \frac{2+2abfn \log(F)}{n}}\right)}{en} \\ &= \frac{e^{-\frac{1+2abfn \log(F)}{b^2 fn^2 \log(F)}} g \sqrt{\pi} (d+ex)^2 (c(d+ex)^n)^{-2/n} \text{erfi}\left(\frac{\frac{1}{n} + abf \log(F) + b^2 f \log(F) \log(c(d+ex)^n)}{b \sqrt{f} \sqrt{\log(F)}}\right)}{2be \sqrt{fn} \sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]** time = 0.703555, size = 120, normalized size = 0.98

$$\frac{\sqrt{\pi} g (d+ex)^2 (c(d+ex)^n)^{-2/n} e^{-\frac{2abfn \log(F)+1}{b^2 fn^2 \log(F)}} \text{Erfi}\left(\frac{bfn \log(F)(a+b \log(c(d+ex)^n))+1}{b \sqrt{fn} \sqrt{\log(F)}}\right)}{2be \sqrt{fn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(d\*g + e\*g\*x),x]

[Out] (g\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(1 + b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(b\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*b\*e\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.353, size = 0, normalized size = 0.

$$\int F^{f(a+b\ln(c(e^x+d)^n))} (egx + dg) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g),x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)F^{(b\log((e^x+d)^n c)+a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g),x, algorithm="maxima")

[Out] integrate((e\*g\*x + d\*g)\*F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

**Fricas [A]** time = 1.07586, size = 333, normalized size = 2.73

$$\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}g\operatorname{erf}\left(\frac{(b^2fn^2\log(e^x+d)\log(F)+b^2fn\log(F)\log(c)+abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{b^2fn^2\log(F)}\right)e^{\left(-\frac{2b^2fn\log(F)\log(c)+2abfn\log(F)+1}{b^2fn^2\log(F)}\right)}}{2ben}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(e\*g\*x+d\*g),x, algorithm="fricas")

[Out] -1/2\*sqrt(pi)\*sqrt(-b^2\*f\*n^2\*log(F))\*g\*erf((b^2\*f\*n^2\*log(e\*x + d)\*log(F) + b^2\*f\*n\*log(F)\*log(c) + a\*b\*f\*n\*log(F) + 1)\*sqrt(-b^2\*f\*n^2\*log(F))/(b^2\*f\*n^2\*log(F)))\*e^(-(2\*b^2\*f\*n\*log(F)\*log(c) + 2\*a\*b\*f\*n\*log(F) + 1)/(b^2\*f\*n^2\*log(F)))/(b\*e\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n))\*\*2)\*(e\*g\*x+d\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (egx + dg)F^{(b\log((e^x+d)^n c)+a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g),x, algorithm="giac")
```

```
[Out] integrate((e*g*x + d*g)*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

### 3.605 $\int F f(a+b \log(c(d+ex)^n))^2 dx$

**Optimal.** Leaf size=126

$$\frac{\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^(((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]]))

**Rubi [A]** time = 0.232463, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2277, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2), x]

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^(((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]]))

#### Rule 2277

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^2\*(d\_.)), x\_Symbol] := Int[F^(a^2\*d + 2\*a\*b\*d\*Log[c\*x^n] + b^2\*d\*Log[c\*x^n]^2), x] /; FreeQ[{F, a, b, c, d, n}, x]

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 15

Int[(u\_.)\*((a\_.)\*(x\_)^(n\_))^(m\_), x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^(a\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int F^{f(a+b \log(c(d+ex)^n))^2} dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log(cx^n))^2} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} \\
 &= \frac{\left((d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} x^{2abfn \log(F)} dx, x, d+ex\right)}{e} \\
 &= \frac{\left((d+ex) (c(d+ex)^n)^{2abf \log(F)-\frac{1+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(a^2 f \log(F) + b^2 f x^2 \log(F) + \frac{x(1-2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{en} \\
 &= \frac{\left(\exp\left(a^2 f \log(F) - \frac{(1+2abfn \log(F))^2}{4b^2 fn^2 \log(F)}\right) (d+ex) (c(d+ex)^n)^{2abf \log(F)-\frac{1+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{x(1-2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{en} \\
 &= \frac{e^{-\frac{1+4abfn \log(F)}{4b^2 fn^2 \log(F)}} \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \text{erfi}\left(\frac{\frac{1}{n}+2abf \log(F)+2b^2 f \log(F) \log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0918427, size = 123, normalized size = 0.98

$$\frac{\sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2 fn^2 \log(F)}} \text{Erfi}\left(\frac{2bfn \log(F)(a+b \log(c(d+ex)^n))+1}{2b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2), x]

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*b\*e\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(-1)\*Sqrt[Log[F]])

**Maple [F]** time = 0.194, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(ex+d)^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2), x)

[Out]  $\text{int}(F^{(f*(a+b*\ln(c*(e*x+d)^n))^2}), x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(b \log((ex+d)^n c) + a)^2} f dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2}), x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c) + a)^{2*f}), x)$

**Fricas [A]** time = 1.00909, size = 350, normalized size = 2.78

$$\frac{\sqrt{\pi} \sqrt{-b^2 f n^2 \log(F)} \operatorname{erf}\left(\frac{(2 b^2 f n^2 \log(ex+d) \log(F) + 2 b^2 f n \log(F) \log(c) + 2 a b f n \log(F) + 1) \sqrt{-b^2 f n^2 \log(F)}}{2 b^2 f n^2 \log(F)}\right) e^{\left(\frac{-4 b^2 f n \log(F) \log(c) + 4 a b f n \log(F) + 1}{4 b^2 f n^2 \log(F)}\right)}}{2 b e n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2}), x, \text{algorithm}="fricas")$

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F))*e^{(-1/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F)))/(b*e*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\ln(c*(e*x+d)**n))**2}), x)$

[Out] Timed out

**Giac [A]** time = 1.34579, size = 157, normalized size = 1.25

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-f \log(F)} b n \log(xe + d) - \sqrt{-f \log(F)} b \log(c) - \sqrt{-f \log(F)} a - \frac{\sqrt{-f \log(F)}}{2 b f n \log(F)}\right) e^{\left(-\frac{a}{b n} - \frac{1}{4 b^2 f n^2 \log(F)} - 1\right)}}{2 \sqrt{-f \log(F)} b c^{\left(\frac{1}{n}\right) n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2}), x, \text{algorithm}="giac")$

[Out]  $-1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-f*\log(F)}*b*n*\log(x*e + d) - \sqrt{-f*\log(F)}*b*\log(c) - \sqrt{-f*\log(F)}*a - 1/2*\sqrt{-f*\log(F)})/(b*f*n*\log(F))*e^{(-a/(b*n) - 1/4/(b^2*f*n^2*\log(F)) - 1)/(\sqrt{-f*\log(F)}*b*c^{(1/n)*n})}$

$$3.606 \quad \int \frac{F^{f(a+b \log(c(dx)^n))^2}}{dg+egx} dx$$

**Optimal.** Leaf size=70

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(a\sqrt{f}\sqrt{\log(F)} + b\sqrt{f}\sqrt{\log(F)} \log(c(d+ex)^n)\right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

[Out] (Sqrt[Pi]\*Erfi[a\*Sqrt[f]\*Sqrt[Log[F]] + b\*Sqrt[f]\*Sqrt[Log[F]]\*Log[c\*(d + e\*x)^n]])/(2\*b\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

**Rubi [A]** time = 0.267745, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {12, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(a\sqrt{f}\sqrt{\log(F)} + b\sqrt{f}\sqrt{\log(F)} \log(c(d+ex)^n)\right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(d\*g + e\*g\*x),x]

[Out] (Sqrt[Pi]\*Erfi[a\*Sqrt[f]\*Sqrt[Log[F]] + b\*Sqrt[f]\*Sqrt[Log[F]]\*Log[c\*(d + e\*x)^n]])/(2\*b\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2278

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^2\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[(e\*x)^m\*F^(a^2\*d + 2\*a\*b\*d\*Log[c\*x^n] + b^2\*d\*Log[c\*x^n]^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 15

Int[(u\_.)\*((a\_.)\*(x\_)^(n\_.))^m, x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(a\*d\*Log[F] + ((m+1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
 \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{dg + egx} dx &= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log(cx^n))^2}}{gx} dx, x, d + ex \right)}{e} \\
 &= \frac{\text{Subst} \left( \int \frac{F^{f(a+b \log(cx^n))^2}}{x} dx, x, d + ex \right)}{eg} \\
 &= \frac{\text{Subst} \left( \int \frac{F^{a^2 f + 2abf \log(cx^n) + b^2 f \log^2(cx^n)}}{x} dx, x, d + ex \right)}{eg} \\
 &= \frac{\text{Subst} \left( \int \frac{F^{a^2 f + b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)}}{x} dx, x, d + ex \right)}{eg} \\
 &= \frac{\left( (d + ex)^{-2abfn \log(F)} (c(d + ex)^n)^{2abf \log(F)} \right) \text{Subst} \left( \int F^{a^2 f + b^2 f \log^2(cx^n)} x^{-1 + 2abfn \log(F)} dx, x, d + ex \right)}{eg} \\
 &= \frac{\text{Subst} \left( \int \exp \left( a^2 f \log(F) + 2abfx \log(F) + b^2 fx^2 \log(F) \right) dx, x, \log(c(d + ex)^n) \right)}{egn} \\
 &= \frac{\text{Subst} \left( \int \exp \left( \frac{(2abf \log(F) + 2b^2 fx \log(F))^2}{4b^2 f \log(F)} \right) dx, x, \log(c(d + ex)^n) \right)}{egn} \\
 &= \frac{\sqrt{\pi} \text{erfi} \left( a\sqrt{f} \sqrt{\log(F)} + b\sqrt{f} \sqrt{\log(F)} \log(c(d + ex)^n) \right)}{2be\sqrt{f}gn\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0460468, size = 59, normalized size = 0.84

$$\frac{\sqrt{\pi} \text{Erfi} \left( \sqrt{f} \sqrt{\log(F)} (a + b \log(c(d + ex)^n)) \right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(d\*g + e\*g\*x), x]

[Out] (Sqrt[Pi]\*Erfi[Sqrt[f]\*Sqrt[Log[F]]\*(a + b\*Log[c\*(d + e\*x)^n])])/(2\*b\*e\*Sqrt[f]\*g\*n\*Sqrt[Log[F]])

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \ln(c(ex+d)^n))^2}}{egx + dg} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g),x)`

[Out] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2} f}{egx + dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="maxima")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g), x)`

**Fricas [A]** time = 1.00244, size = 159, normalized size = 2.27

$$\frac{\sqrt{\pi} \sqrt{-b^2 f n^2 \log(F)} \operatorname{erf}\left(\frac{\sqrt{-b^2 f n^2 \log(F)} (bn \log(ex+d) + b \log(c) + a)}{bn}\right)}{2 begn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="fricas")`

[Out] `-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(sqrt(-b^2*f*n^2*log(F))*(b*n*log(e*x + d) + b*log(c) + a)/(b*n))/(b*e*g*n)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2} f}{egx + dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="giac")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g), x)`

$$3.607 \quad \int \frac{F^{f(a+b \log(c(dx)^n))^2}}{(dg+egx)^2} dx$$

**Optimal.** Leaf size=128

$$\frac{\sqrt{\pi} (c(dx)^n)^{\frac{1}{n}} e^{\frac{a}{bn} - \frac{1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{-2abf \log(F) - 2b^2f \log(F) \log(c(dx)^n) + \frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n\sqrt{\log(F)}(d+ex)}$$

[Out]  $-(E^{(a/(b*n) - 1/(4*b^2*f*n^2*\log[F]))}*\sqrt{\pi}*(c*(d + e*x)^n)^{(-1)}*\operatorname{Erfi}[(n^{(-1)} - 2*a*b*f*\log[F] - 2*b^2*f*\log[F]*\log[c*(d + e*x)^n])/(2*b*\sqrt{f}*\sqrt{\log[F]})])/(2*b*e*\sqrt{f}*g^2*n*(d + e*x)*\sqrt{\log[F]})$

**Rubi [A]** time = 0.411869, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {12, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} (c(dx)^n)^{\frac{1}{n}} e^{\frac{a}{bn} - \frac{1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{-2abf \log(F) - 2b^2f \log(F) \log(c(dx)^n) + \frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n\sqrt{\log(F)}(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(f*(a + b*\log[c*(d + e*x)^n])^2)/(d*g + e*g*x)^2}, x]$

[Out]  $-(E^{(a/(b*n) - 1/(4*b^2*f*n^2*\log[F]))}*\sqrt{\pi}*(c*(d + e*x)^n)^{(-1)}*\operatorname{Erfi}[(n^{(-1)} - 2*a*b*f*\log[F] - 2*b^2*f*\log[F]*\log[c*(d + e*x)^n])/(2*b*\sqrt{f}*\sqrt{\log[F]})])/(2*b*e*\sqrt{f}*g^2*n*(d + e*x)*\sqrt{\log[F]})$

### Rule 12

$\operatorname{Int}[(a_*)*(u_*) , x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_*)] /; \operatorname{FreeQ}[b, x]$

### Rule 2278

$\operatorname{Int}[(F_*)^{((a_*) + \log[(c_*)*(x_*)^{(n_*)}])*(b_*)^2*(d_*)}*((e_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \operatorname{Int}[(e*x)^m * F^{(a^2*d + 2*a*b*d*\log[c*x^n] + b^2*d*\log[c*x^n])^2}, x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

### Rule 2274

$\operatorname{Int}[(u_*)*(F_*)^{((a_*)*(\log[z_]*(b_*) + (v_*.)))}, x\_Symbol] \rightarrow \operatorname{Int}[u * F^{(a*v)} * z^{(a*b*\log[F])}, x] /; \operatorname{FreeQ}\{F, a, b\}, x]$

### Rule 15

$\operatorname{Int}[(u_*)*((a_*)*(x_*)^{(n_*)})^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(a^{\operatorname{IntPart}[m]}*(a*x^n)^{\operatorname{FracPart}[m]})/x^{(n*\operatorname{FracPart}[m])}, \operatorname{Int}[u*x^{(m*n)}, x], x] /; \operatorname{FreeQ}\{a, m, n\}, x \ \&\& \ !\operatorname{IntegerQ}[m]$

### Rule 2276

$\operatorname{Int}[(F_*)^{((a_*) + \log[(c_*)*(x_*)^{(n_*)}]^2*(b_*)*(d_*)}*((e_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(a$

\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(dg+egx)^2} dx &= \frac{\text{Subst}\left(\int \frac{F^{f(a+b \log(cx^n))^2}}{g^2 x^2} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{f(a+b \log(cx^n))^2}}{x^2} dx, x, d+ex\right)}{eg^2} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)}}{x^2} dx, x, d+ex\right)}{eg^2} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)}}{x^2} dx, x, d+ex\right)}{eg^2} \\
 &= \frac{\left((d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} x^{-2+2abfn \log(F)} dx, x, d+ex\right)}{eg^2} \\
 &= \frac{\left(c(d+ex)^n\right)^{2abf \log(F)-\frac{-1+2abfn \log(F)}{n}} \text{Subst}\left(\int \exp\left(a^2 f \log(F)+b^2 f x^2 \log(F)+\frac{x(-1+2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{eg^2 n(d+ex)} \\
 &= \frac{\left(e^{\frac{a}{bn}-\frac{1}{4b^2fn^2 \log(F)}} (c(d+ex)^n\right)^{2abf \log(F)-\frac{-1+2abfn \log(F)}{n}} \text{Subst}\left(\int \exp\left(\frac{(2b^2fx \log(F)+\frac{-1+2abfn \log(F)}{n})}{4b^2f \log(F)}\right) dx, x, d+ex\right)}{eg^2 n(d+ex)} \\
 &= \frac{e^{\frac{a}{bn}-\frac{1}{4b^2fn^2 \log(F)}} \sqrt{\pi} (c(d+ex)^n)^{\frac{1}{n}} \text{erfi}\left(\frac{\frac{1}{n}-2abf \log(F)-2b^2f \log(F) \log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n(d+ex)\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.256672, size = 126, normalized size = 0.98

$$\frac{\sqrt{\pi} (c(d+ex)^n)^{\frac{1}{n}} e^{\frac{4abfn \log(F)-1}{4b^2fn^2 \log(F)}} \text{Erfi}\left(\frac{2bfn \log(F)(a+b \log(c(d+ex)^n))-1}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n\sqrt{\log(F)}(d+ex)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(d\*g + e\*g\*x)^2,x]

[Out]  $(E^{((-1 + 4*a*b*f*n*\text{Log}[F])/(4*b^2*f*n^2*\text{Log}[F]))}*\text{Sqrt}[\text{Pi}]*(c*(d + e*x)^n)^{n^{(-1)*\text{Erfi}[(-1 + 2*b*f*n*\text{Log}[F]*(a + b*\text{Log}[c*(d + e*x)^n])]/(2*b*\text{Sqrt}[f]*n*\text{Sqrt}[\text{Log}[F]])}]/(2*b*e*\text{Sqrt}[f]*g^{2*n}*(d + e*x)*\text{Sqrt}[\text{Log}[F]])$

**Maple [F]** time = 0.521, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(egx+dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x)`

[Out] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b\log((ex+d)^n)c+a)^2 f}}{(egx+dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="maxima")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^2, x)`

**Fricas [A]** time = 1.01412, size = 354, normalized size = 2.77

$$\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\operatorname{erf}\left(\frac{(2b^2fn^2\log(ex+d)\log(F)+2b^2fn\log(F)\log(c)+2abfn\log(F)-1)\sqrt{-b^2fn^2\log(F)}}{2b^2fn^2\log(F)}\right)e^{\left(\frac{4b^2fn\log(F)\log(c)+4abfn\log(F)-1}{4b^2fn^2\log(F)}\right)}}{2beg^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="fricas")`

[Out]  $-1/2*\text{sqrt}(\text{pi})*\text{sqrt}(-b^2*f*n^2*\log(F))*\text{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) - 1)*\text{sqrt}(-b^2*f*n^2*\log(F)))/(b^2*f*n^2*\log(F))*e^{(1/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) - 1)/(b^2*f*n^2*\log(F)))/(b*e*g^2*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2} f}{(egx + dg)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^2, x)
```

$$3.608 \quad \int \frac{F^{f(a+b \log(c(dx)^n))^2}}{(dg+egx)^3} dx$$

**Optimal.** Leaf size=126

$$\frac{\sqrt{\pi} (c(d+ex)^n)^{2/n} e^{-\frac{1-2abfn \log(F)}{b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{-abf \log(F)+b^2(-f) \log(F) \log(c(dx)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3n\sqrt{\log(F)}(d+ex)^2}$$

[Out]  $-(\operatorname{Sqrt}[\pi]*(c*(d+e*x)^n)^{(2/n)}*\operatorname{Erfi}[(n^{(-1)}-a*b*f*\operatorname{Log}[F]-b^2*f*\operatorname{Log}[F]*\operatorname{Log}[c*(d+e*x)^n])/(b*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[\operatorname{Log}[F]])])/(2*b*e*E^{((1-2*a*b*f*n*\operatorname{Log}[F])/(b^2*f*n^2*\operatorname{Log}[F]))}* \operatorname{Sqrt}[f]*g^3*n*(d+e*x)^2*\operatorname{Sqrt}[\operatorname{Log}[F]])$

**Rubi [A]** time = 0.405278, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {12, 2278, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi} (c(d+ex)^n)^{2/n} e^{-\frac{1-2abfn \log(F)}{b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{-abf \log(F)+b^2(-f) \log(F) \log(c(dx)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3n\sqrt{\log(F)}(d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(f*(a+b*\operatorname{Log}[c*(d+e*x)^n])^2)/(d*g+e*g*x)^3}, x]$

[Out]  $-(\operatorname{Sqrt}[\pi]*(c*(d+e*x)^n)^{(2/n)}*\operatorname{Erfi}[(n^{(-1)}-a*b*f*\operatorname{Log}[F]-b^2*f*\operatorname{Log}[F]*\operatorname{Log}[c*(d+e*x)^n])/(b*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[\operatorname{Log}[F]])])/(2*b*e*E^{((1-2*a*b*f*n*\operatorname{Log}[F])/(b^2*f*n^2*\operatorname{Log}[F]))}* \operatorname{Sqrt}[f]*g^3*n*(d+e*x)^2*\operatorname{Sqrt}[\operatorname{Log}[F]])$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

### Rule 2278

$\operatorname{Int}[(F_)^{((a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}])^2*(b_.)*(d_.)}*(e_.)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Int}[(e*x)^m * F^{(a^2*d + 2*a*b*d*\operatorname{Log}[c*x^n] + b^2*d*\operatorname{Log}[c*x^n]^2)}, x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, m, n\}, x]$

### Rule 2274

$\operatorname{Int}[(u_.)*(F_)^{((a_.)*(\operatorname{Log}[z_]*(b_.) + (v_)))}, x\_Symbol] \rightarrow \operatorname{Int}[u * F^{(a*v)*z^{(a*b*\operatorname{Log}[F])}}, x] /; \operatorname{FreeQ}\{F, a, b\}, x]$

### Rule 15

$\operatorname{Int}[(u_.)*((a_.)*(x_)^{(n_.)})^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(a^{\operatorname{IntPart}[m]}*(a*x^n)^{\operatorname{FracPart}[m]})/x^{(n*\operatorname{FracPart}[m])}, \operatorname{Int}[u*x^{(m*n)}, x], x] /; \operatorname{FreeQ}\{a, m, n\}, x \ \&\& \ !\operatorname{IntegerQ}[m]$

### Rule 2276

$\operatorname{Int}[(F_)^{((a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}])^2*(b_.)*(d_.)}*(e_.)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(a$

\*d\*Log[F] + ((m + 1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
 \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(dg+egx)^3} dx &= \frac{\text{Subst}\left(\int \frac{F^{f(a+b \log(cx^n))^2}}{g^3 x^3} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{f(a+b \log(cx^n))^2}}{x^3} dx, x, d+ex\right)}{eg^3} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)}}{x^3} dx, x, d+ex\right)}{eg^3} \\
 &= \frac{\text{Subst}\left(\int \frac{F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)}}{x^3} dx, x, d+ex\right)}{eg^3} \\
 &= \frac{\left((d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} x^{-3+2abfn \log(F)} dx, x, d+ex\right)}{eg^3} \\
 &= \frac{(c(d+ex)^n)^{2abf \log(F)-\frac{-2+2abfn \log(F)}{n}} \text{Subst}\left(\int \exp\left(a^2 f \log(F) + b^2 f x^2 \log(F) + \frac{x(-2+2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{eg^3 n (d+ex)^2} \\
 &= \frac{\left(\exp\left(a^2 f \log(F) - \frac{(-2+2abfn \log(F))^2}{4b^2 fn^2 \log(F)}\right) (c(d+ex)^n)^{2abf \log(F)-\frac{-2+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{2b^2 f x^2 \log(F)}{n}\right) dx, x, d+ex\right)}{eg^3 n (d+ex)^2} \\
 &= \frac{e^{-\frac{1-2abfn \log(F)}{b^2 fn^2 \log(F)}} \sqrt{\pi} (c(d+ex)^n)^{2/n} \text{erfi}\left(\frac{\frac{1}{n} - abf \log(F) - b^2 f \log(F) \log(c(d+ex)^n)}{b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3 n (d+ex)^2 \sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.244847, size = 121, normalized size = 0.96

$$\frac{\sqrt{\pi} (c(d+ex)^n)^{2/n} e^{\frac{2abfn \log(F)-1}{b^2 fn^2 \log(F)}} \text{Erfi}\left(\frac{bfn \log(F)(a+b \log(c(d+ex)^n))-1}{b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3 n \sqrt{\log(F)}(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(d\*g + e\*g\*x)^3,x]

[Out]  $(E^{((-1 + 2*a*b*f*n*\text{Log}[F])/(b^2*f*n^2*\text{Log}[F]))}*\text{Sqrt}[\text{Pi}]*c*(d + e*x)^n)^{(2/n)}*\text{Erfi}[(-1 + b*f*n*\text{Log}[F]*(a + b*\text{Log}[c*(d + e*x)^n])/(b*\text{Sqrt}[f]*n*\text{Sqrt}[\text{Log}[F]]))]/(2*b*e*\text{Sqrt}[f]*g^3*n*(d + e*x)^2*\text{Sqrt}[\text{Log}[F]])$

**Maple [F]** time = 0.665, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(egx + dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x)`

[Out] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b\log((ex+d)^n)c+a)^2 f}}{(egx + dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="maxima")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^3, x)`

**Fricas [A]** time = 1.0907, size = 335, normalized size = 2.66

$$\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\operatorname{erf}\left(\frac{(b^2fn^2\log(ex+d)\log(F)+b^2fn\log(F)\log(c)+abfn\log(F)-1)\sqrt{-b^2fn^2\log(F)}}{b^2fn^2\log(F)}\right)e^{\left(\frac{2b^2fn\log(F)\log(c)+2abfn\log(F)-1}{b^2fn^2\log(F)}\right)}}{2beg^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="fricas")`

[Out]  $-1/2*\text{sqrt}(\text{pi})*\text{sqrt}(-b^2*f*n^2*\log(F))*\text{erf}((b^2*f*n^2*\log(e*x + d)*\log(F) + b^2*f*n*\log(F)*\log(c) + a*b*f*n*\log(F) - 1)*\text{sqrt}(-b^2*f*n^2*\log(F))/(b^2*f*n^2*\log(F)))*e^{((2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) - 1)/(b^2*f*n^2*\log(F)))/(b*e*g^3*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2} f}{(egx + dg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^3, x)
```

$$3.609 \quad \int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable}\left((g+hx)^m F^{f(a+b \log(c(d+ex)^n))^2}, x\right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^m, x]

**Rubi [A]** time = 0.0886785, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^m, x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^m, x]

Rubi steps

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx = \int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx$$

**Mathematica [A]** time = 1.40233, size = 0, normalized size = 0.

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^m, x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^m, x]

**Maple [A]** time = 0.888, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(e*x+d)^n))^2} (hx+g)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(h\*x+g)^m, x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(h\*x+g)^m, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^m F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g)^m,x, algorithm="maxima")

[Out] integrate((h\*x + g)^m \* F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(hx + g\right)^m F^{b^2 f \log((ex+d)^n c)^2 + 2abf \log((ex+d)^n c) + a^2 f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g)^m,x, algorithm="fricas")

[Out] integral((h\*x + g)^m \* F^(b^2\*f\*log((e\*x + d)^n\*c)^2 + 2\*a\*b\*f\*log((e\*x + d)^n\*c) + a^2\*f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n))\*\*2)\*(h\*x+g)\*\*m,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^m F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g)^m,x, algorithm="giac")

[Out] integrate((h\*x + g)^m \* F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

### 3.610 $\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^3 dx$

**Optimal.** Leaf size=535

$$\frac{3\sqrt{\pi}h^2(d+ex)^3(eg-dh)(c(d+ex)^n)^{-3/n} \exp\left(-\frac{3(4abfn \log(F)+3)}{4b^2fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^4\sqrt{fn}\sqrt{\log(F)}} + \frac{3\sqrt{\pi}h(d+ex)}{2be^4\sqrt{fn}\sqrt{\log(F)}}$$

```
[Out] (3*h*(e*g - d*h)^2*Sqrt[Pi]*(d + e*x)^2*Erfi[(n^(-1) + a*b*f*Log[F] + b^2*f*Log[F]*Log[c*(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])]/(2*b*e^4*E^((1 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + (h^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2/n + a*b*f*Log[F] + b^2*f*Log[F]*Log[c*(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])]/(2*b*e^4*E^((4*(1 + a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*Sqrt[Log[F]]) + ((e*g - d*h)^3*Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])]/(2*b*e^4*E^((1 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]]) + (3*h^2*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^3*Erfi[(3/n + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])]/(2*b*e^4*E^((3*(3 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]]))
```

**Rubi [F]** time = 0.493199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^3,x]
```

```
[Out] (g^3*Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])]/(2*b*e*E^((1 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + 3*g^2*h*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n])^2)*x, x] + 3*g*h^2*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n])^2)*x^2, x] + h^3*Defer[Int][F^(f*(a + b*Log[c*(d + e*x)^n])^2)*x^3, x]
```

Rubi steps

$$\begin{aligned}
 \int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^3 dx &= \int \left( F^{f(a+b \log(c(d+ex)^n))^2} g^3 + 3F^{f(a+b \log(c(d+ex)^n))^2} g^2 hx + 3F^{f(a+b \log(c(d+ex)^n))^2} gh^2 x^2 + \dots \right) dx \\
 &= g^3 \int F^{f(a+b \log(c(d+ex)^n))^2} dx + (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \dots \\
 &= \frac{g^3 \operatorname{Subst}\left(\int F^{f(a+b \log(cx^n))^2} dx, x, d+ex\right)}{e} + (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \dots \\
 &= \frac{g^3 \operatorname{Subst}\left(\int F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)} dx, x, d+ex\right)}{e} + (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \dots \\
 &= \frac{g^3 \operatorname{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} + (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + h^3 \int F^{f(a+b \log(c(d+ex)^n))^2} x^3 dx + \dots \\
 &= (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + h^3 \int F^{f(a+b \log(c(d+ex)^n))^2} x^3 dx + \dots \\
 &= (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + h^3 \int F^{f(a+b \log(c(d+ex)^n))^2} x^3 dx + \dots \\
 &= (3g^2 h) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + (3gh^2) \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + h^3 \int F^{f(a+b \log(c(d+ex)^n))^2} x^3 dx + \dots \\
 &= \frac{e^{-\frac{1+4abfn \log(F)}{4b^2fn^2 \log(F)}} g^3 \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{\frac{1}{n}+2abf \log(F)+2b^2 f \log(F) \log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 2.38039, size = 434, normalized size = 0.81

$$\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-4/n} e^{-\frac{4(abfn \log(F)+1)}{b^2fn^2 \log(F)}} \left( (eg-dh)(c(d+ex)^n)^{\frac{1}{n}} e^{\frac{4abfn \log(F)+7}{4b^2fn^2 \log(F)}} \left( (eg-dh)^2 (c(d+ex)^n)^{2/n} e^{\frac{2abfn \log(F)+2}{b^2fn^2 \log(F)}} E\left(\frac{1}{n}+2abf \log(F)+2b^2 f \log(F) \log(c(d+ex)^n)\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^3,x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*(3*E^((3 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F])))*h*(e*g - d*h)^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n])/(b*Sqrt[f]*n*Sqrt[Log[F]])] + h^3*(d + e*x)^3*Erfi[(2 + b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n])/(b*Sqrt[f]*n*Sqrt[Log[F]])] + E^((7 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*(e*g - d*h)*(c*(d + e*x)^n)^n^(-1)*(E^((2 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*(e*g - d*h)^2*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*n*Sqrt[Log[F]])] + 3*h^2*(d + e*x)^2*Erfi[(3 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*n*Sqrt[Log[F]])]))/(2*b*e^4*E^((4*(1 + a*b*f*n*Log[F])/(b^2*f*n^2*Log[F])))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*Sqrt[Log[F]]])
```

**Maple [F]** time = 0.517, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(ex+d)^n))^2} (hx+g)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^3,x)`

[Out] `int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^3 F^{(b \log((ex+d)^n c) + a)^2} f dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="maxima")`

[Out] `integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c) + a)^2*f), x)`

**Fricas [A]** time = 1.09938, size = 1469, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="fricas")`

[Out] `-1/2*(sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*h^3*erf((b^2*f*n^2*log(e*x + d)*log(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 2)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-4*(b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))) + 3*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e*g*h^2 - d*h^3)*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 3)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-3/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(F) + 3)/(b^2*f*n^2*log(F))) + sqrt(pi)*(e^3*g^3 - 3*d*e^2*g^2*h + 3*d^2*e*g*h^2 - d^3*h^3)*sqrt(-b^2*f*n^2*log(F))*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-1/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))) + 3*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e^2*g^2*h - 2*d*e*g*h^2 + d^2*h^3)*erf((b^2*f*n^2*log(e*x + d)*log(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-(2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))))/(b*e^4*n)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g)**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^3 F^{(b \log((ex+d)^n c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

$$3.611 \quad \int F^{f(a+b \log(c(d+ex)^n))^2} (g + hx)^2 dx$$

**Optimal.** Leaf size=397

$$\frac{\sqrt{\pi} h^2 (d + ex)^3 (c(d + ex)^n)^{-3/n} \exp\left(-\frac{3(4abfn \log(F)+3)}{4b^2fn^2 \log(F)}\right) \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^3\sqrt{fn}\sqrt{\log(F)}} + \frac{\sqrt{\pi} h (d + ex)^2 (eg - dh)}{2b^2\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (h\*(e\*g - d\*h)\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(n^(-1) + a\*b\*f\*Log[F] + b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(b\*Sqrt[f]\*Sqrt[Log[F]])])/(b\*e^3\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]) + ((e\*g - d\*h)^2\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^3\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]) + (h^2\*Sqrt[Pi]\*(d + e\*x)^3\*Erfi[(3/n + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^3\*E^((3\*(3 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(3/n)\*Sqrt[Log[F]])

**Rubi [F]** time = 0.367427, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g + hx)^2 dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x)^2,x]

[Out] (g^2\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^3\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]) + 2\*g\*h\*Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*x, x] + h^2\*Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*x^2, x]

Rubi steps



$$\begin{aligned}
 \int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx)^2 dx &= \int \left( F^{f(a+b \log(c(d+ex)^n))^2} g^2 + 2F^{f(a+b \log(c(d+ex)^n))^2} ghx + F^{f(a+b \log(c(d+ex)^n))^2} h^2 x^2 \right) dx \\
 &= g^2 \int F^{f(a+b \log(c(d+ex)^n))^2} dx + (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx \\
 &= \frac{g^2 \operatorname{Subst}\left(\int F^{f(a+b \log(cx^n))^2} dx, x, d+ex\right)}{e} + (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx \\
 &= \frac{g^2 \operatorname{Subst}\left(\int F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)} dx, x, d+ex\right)}{e} + (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx \\
 &= \frac{g^2 \operatorname{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} + (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx \\
 &= (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \frac{(g^2(d+ex))^{-2abf}}{e} \\
 &= (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \frac{(g^2(d+ex)(c(d+ex)^n))^{-2abf}}{e} \\
 &= (2gh) \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + h^2 \int F^{f(a+b \log(c(d+ex)^n))^2} x^2 dx + \frac{\left(\exp\left(a^2 f \log(F)\right)\right)^{-2abf}}{e} \\
 &= \frac{e^{-\frac{1+4abfn \log(F)}{4b^2fn^2 \log(F)}} g^2 \sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n} \operatorname{erfi}\left(\frac{\frac{1}{n}+2abf \log(F)+2b^2 f \log(F) \log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 1.0468, size = 331, normalized size = 0.83

$$\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-3/n} \exp\left(-\frac{3(4abfn \log(F)+3)}{4b^2fn^2 \log(F)}\right) \left( (eg-dh)^2 (c(d+ex)^n)^{2/n} e^{\frac{2abfn \log(F)+2}{b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2bfn \log(F)(a+b \log(c(d+ex)^n))}{2b\sqrt{fn}\sqrt{\log(F)}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^2,x]
```

```
[Out] (Sqrt[Pi]*(d + e*x)*(-2*E^((5 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F])))*h*(
-(e*g) + d*h)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 + b*f*n*Log[F]*(a +
b*Log[c*(d + e*x)^n]))/(b*Sqrt[f]*n*Sqrt[Log[F]])] + E^((2 + 2*a*b*f*n*Log[
F])/(b^2*f*n^2*Log[F]))*(e*g - d*h)^2*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + 2*b*f
*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f]*n*Sqrt[Log[F]])] + h^2*(
d + e*x)^2*Erfi[(3 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f
]*n*Sqrt[Log[F]])])/(2*b*e^3*E^((3*(3 + 4*a*b*f*n*Log[F]))/(4*b^2*f*n^2*Lo
g[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

**Maple [F]** time = 0.483, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(ex+d)^n))^2} (hx+g)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^2,x)
```

[Out]  $\int (F^{(f*(a+b*\ln(c*(e*x+d)^n))^2})*(h*x+g)^2, x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^2 F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="maxima")`

[Out] `integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)`

**Fricas [A]** time = 1.12098, size = 1076, normalized size = 2.71

$$\sqrt{\pi} \sqrt{-b^2 f n^2 \log(F)} h^2 \operatorname{erf} \left( \frac{(2b^2 f n^2 \log(ex+d) \log(F) + 2b^2 f n \log(F) \log(c) + 2abfn \log(F) + 3) \sqrt{-b^2 f n^2 \log(F)}}{2b^2 f n^2 \log(F)} \right) e^{\left( -\frac{3(4b^2 f n \log(F) \log(c) + 4abfn \log(F) + 3)}{4b^2 f n^2 \log(F)} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="fricas")`

[Out]  $-1/2*(\sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)})*h^2*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 3)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F))*e^{(-3/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) + 3)/(b^2*f*n^2*\log(F)))} + \sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)}*(e^{2*g^2 - 2*d*e*g*h + d^2*h^2}*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F))*e^{(-1/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F)))} + 2*\sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)}*(e*g*h - d*h^2)*\operatorname{erf}((b^2*f*n^2*\log(e*x + d)*\log(F) + b^2*f*n*\log(F)*\log(c) + a*b*f*n*\log(F) + 1)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F))*e^{(-(2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F)))})/(b*e^{3*n})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g)**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)^2 F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

$$3.612 \quad \int F^{f(a+b \log(c(d+ex)^n))^2} (g + hx) dx$$

**Optimal.** Leaf size=257

$$\frac{\sqrt{\pi}(d+ex)(eg-dh)(c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^2\sqrt{fn}\sqrt{\log(F)}} + \frac{\sqrt{\pi}h(d+ex)^2(c(d+ex)^n)^{-2/n}}{2b^2\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (h\*Sqrt[Pi]\*(d + e\*x)^2\*Erfi[(n^(-1) + a\*b\*f\*Log[F] + b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^2\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]) + ((e\*g - d\*h)\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^2\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Rubi [F]** time = 0.305888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int F^{f(a+b \log(c(d+ex)^n))^2} (g + hx) dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x), x]

[Out] (g\*Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e^2\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]]) + h\*D efer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*x, x]

Rubi steps

$$\begin{aligned}
\int F^{f(a+b \log(c(d+ex)^n))^2} (g+hx) dx &= \int \left( F^{f(a+b \log(c(d+ex)^n))^2} g + F^{f(a+b \log(c(d+ex)^n))^2} hx \right) dx \\
&= g \int F^{f(a+b \log(c(d+ex)^n))^2} dx + h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx \\
&= \frac{g \operatorname{Subst} \left( \int F^{f(a+b \log(cx^n))^2} dx, x, d+ex \right)}{e} + h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx \\
&= \frac{g \operatorname{Subst} \left( \int F^{a^2 f + 2abf \log(cx^n) + b^2 f \log^2(cx^n)} dx, x, d+ex \right)}{e} + h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx \\
&= \frac{g \operatorname{Subst} \left( \int F^{a^2 f + b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)} dx, x, d+ex \right)}{e} + h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx \\
&= h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + \frac{(g(d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}) \operatorname{Subst} \left( \int F^{f(a+b \log(cx^n))^2} dx, x, d+ex \right)}{e} \\
&= h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + \frac{\left( g(d+ex) (c(d+ex)^n)^{2abf \log(F) - \frac{1+2abfn \log(F)}{n}} \right) \operatorname{Subst} \left( \int F^{f(a+b \log(cx^n))^2} dx, x, d+ex \right)}{e} \\
&= h \int F^{f(a+b \log(c(d+ex)^n))^2} x dx + \frac{\left( \exp \left( a^2 f \log(F) - \frac{(1+2abfn \log(F))^2}{4b^2 fn^2 \log(F)} \right) g(d+ex) (c(d+ex)^n)^{\frac{1+2abfn \log(F)}{n}} \right) \operatorname{Subst} \left( \int F^{f(a+b \log(cx^n))^2} dx, x, d+ex \right)}{e} \\
&= \frac{e^{-\frac{1+4abfn \log(F)}{4b^2 fn^2 \log(F)}} g \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \operatorname{erfi} \left( \frac{\frac{1}{n} + 2abf \log(F) + 2b^2 f \log(F) \log(c(d+ex)^n)}{2b \sqrt{f} \sqrt{\log(F)}} \right)}{2be \sqrt{fn} \sqrt{\log(F)}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.505997, size = 221, normalized size = 0.86

$$\frac{\sqrt{\pi} (d+ex) (c(d+ex)^n)^{-2/n} e^{-\frac{2abfn \log(F)+1}{b^2 fn^2 \log(F)}} \left( (eg-dh) (c(d+ex)^n)^{\frac{1}{n}} e^{\frac{4abfn \log(F)+3}{4b^2 fn^2 \log(F)}} \operatorname{Erfi} \left( \frac{2bfn \log(F)(a+b \log(c(d+ex)^n))+1}{2b \sqrt{fn} \sqrt{\log(F)}} \right) + h(d+ex) \right)}{2be^2 \sqrt{fn} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)\*(g + h\*x), x]

[Out] (Sqrt[Pi]\*(d + e\*x)\*(h\*(d + e\*x)\*Erfi[(1 + b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(b\*Sqrt[f]\*n\*Sqrt[Log[F]])] + E^((3 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*(e\*g - d\*h)\*(c\*(d + e\*x)^n)^(-1)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])])/(2\*b\*e^2\*E^((1 + 2\*a\*b\*f\*n\*Log[F])/(b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(2/n)\*Sqrt[Log[F]])

**Maple [F]** time = 0.373, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(ex+d)^n))^2} (hx+g) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(h\*x+g), x)

[Out] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)\*(h\*x+g), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g),x, algorithm="maxima")

[Out] integrate((h\*x + g)\*F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

---

**Fricas [A]** time = 1.35583, size = 689, normalized size = 2.68

$$\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}(eg - dh) \operatorname{erf}\left(\frac{(2b^2fn^2\log(ex+d)\log(F)+2b^2fn\log(F)\log(c)+2abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{2b^2fn^2\log(F)}\right) e^{\left(-\frac{4b^2fn\log(F)\log(c)+4abfn}{4b^2fn^2\log(F)}\right)}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g),x, algorithm="fricas")

[Out]  $-1/2*(\operatorname{sqrt}(\pi)*\operatorname{sqrt}(-b^2*f*n^2*\log(F))*(e*g - d*h)*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)*\operatorname{sqrt}(-b^2*f*n^2*\log(F))/(b^2*f*n^2*\log(F)))*e^{(-1/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F))} + \operatorname{sqrt}(\pi)*\operatorname{sqrt}(-b^2*f*n^2*\log(F))*h*\operatorname{erf}((b^2*f*n^2*\log(e*x + d)*\log(F) + b^2*f*n*\log(F)*\log(c) + a*b*f*n*\log(F) + 1)*\operatorname{sqrt}(-b^2*f*n^2*\log(F))/(b^2*f*n^2*\log(F)))*e^{(-(2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F)))})/(b*e^{2*n})$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(f\*(a+b\*ln(c\*(e\*x+d)\*\*n))\*\*2)\*(h\*x+g),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g)F^{(b \log((ex+d)^n c) + a)^2 f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(f\*(a+b\*log(c\*(e\*x+d)^n))^2)\*(h\*x+g),x, algorithm="giac")

[Out] integrate((h\*x + g)\*F^((b\*log((e\*x + d)^n\*c) + a)^2\*f), x)

### 3.613 $\int F f(a+b \log(c(d+ex)^n))^2 dx$

**Optimal.** Leaf size=126

$$\frac{\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

**Rubi [A]** time = 0.142727, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2277, 2274, 15, 2276, 2234, 2204}

$$\frac{\sqrt{\pi}(d+ex)(c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2fn^2 \log(F)}} \operatorname{Erfi}\left(\frac{2abf \log(F)+2b^2f \log(F) \log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2), x]

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(n^(-1) + 2\*a\*b\*f\*Log[F] + 2\*b^2\*f\*Log[F]\*Log[c\*(d + e\*x)^n])/(2\*b\*Sqrt[f]\*Sqrt[Log[F]])])/(2\*b\*e\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F])))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^n^(-1)\*Sqrt[Log[F]])

#### Rule 2277

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^2\*(d\_.)), x\_Symbol] := Int[F^(a^2\*d + 2\*a\*b\*d\*Log[c\*x^n] + b^2\*d\*Log[c\*x^n]^2), x] /; FreeQ[{F, a, b, c, d, n}, x]

#### Rule 2274

Int[(u\_.)\*(F\_)^((a\_.)\*(Log[z\_]\*(b\_.) + (v\_.))), x\_Symbol] := Int[u\*F^(a\*v)\*z^(a\*b\*Log[F]), x] /; FreeQ[{F, a, b}, x]

#### Rule 15

Int[(u\_.)\*((a\_.)\*(x\_)^(n\_.))^m, x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 2276

Int[(F\_)^(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^2\*(b\_.))\*(d\_.))\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(a\*d\*Log[F] + ((m+1)\*x)/n + b\*d\*Log[F]\*x^2), x], x, Log[c\*x^n]], x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]

#### Rule 2234

Int[(F\_)^((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[F^(a - b^2/(4\*c)), Int[F^((b + 2\*c\*x)^2/(4\*c)), x], x] /; FreeQ[{F, a, b, c}, x]

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_.))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int F^{f(a+b \log(c(d+ex)^n))^2} dx &= \frac{\text{Subst}\left(\int F^{f(a+b \log(cx^n))^2} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int F^{a^2 f+2abf \log(cx^n)+b^2 f \log^2(cx^n)} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} (cx^n)^{2abf \log(F)} dx, x, d+ex\right)}{e} \\
 &= \frac{\left((d+ex)^{-2abfn \log(F)} (c(d+ex)^n)^{2abf \log(F)}\right) \text{Subst}\left(\int F^{a^2 f+b^2 f \log^2(cx^n)} x^{2abfn \log(F)} dx, x, d+ex\right)}{e} \\
 &= \frac{\left((d+ex) (c(d+ex)^n)^{2abf \log(F)-\frac{1+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(a^2 f \log(F) + b^2 f x^2 \log(F) + \frac{x(1+2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{en} \\
 &= \frac{\left(\exp\left(a^2 f \log(F) - \frac{(1+2abfn \log(F))^2}{4b^2 fn^2 \log(F)}\right) (d+ex) (c(d+ex)^n)^{2abf \log(F)-\frac{1+2abfn \log(F)}{n}}\right) \text{Subst}\left(\int \exp\left(\frac{x(1+2abfn \log(F))}{n}\right) dx, x, d+ex\right)}{en} \\
 &= \frac{e^{-\frac{1+4abfn \log(F)}{4b^2 fn^2 \log(F)}} \sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} \text{erfi}\left(\frac{\frac{1}{n}+2abf \log(F)+2b^2 f \log(F) \log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0603023, size = 123, normalized size = 0.98

$$\frac{\sqrt{\pi} (d+ex) (c(d+ex)^n)^{-1/n} e^{-\frac{4abfn \log(F)+1}{4b^2 fn^2 \log(F)}} \text{Erfi}\left(\frac{2bfn \log(F)(a+b \log(c(d+ex)^n))+1}{2b\sqrt{fn}\sqrt{\log(F)}}\right)}{2be\sqrt{fn}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2), x]

[Out] (Sqrt[Pi]\*(d + e\*x)\*Erfi[(1 + 2\*b\*f\*n\*Log[F]\*(a + b\*Log[c\*(d + e\*x)^n]))/(2\*b\*Sqrt[f]\*n\*Sqrt[Log[F]])]/(2\*b\*e\*E^((1 + 4\*a\*b\*f\*n\*Log[F])/(4\*b^2\*f\*n^2\*Log[F]))\*Sqrt[f]\*n\*(c\*(d + e\*x)^n)^(-1)\*Sqrt[Log[F]])

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int F^{f(a+b \ln(c(ex+d)^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2), x)



[Out]  $\int F^{(f*(a+b*\ln(c*(e*x+d)^n))^2), x}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{(b \log((ex+d)^n c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2), x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c) + a)^2*f), x)$

**Fricas [A]** time = 1.31762, size = 350, normalized size = 2.78

$$\frac{\sqrt{\pi} \sqrt{-b^2 f n^2 \log(F)} \operatorname{erf}\left(\frac{(2b^2 f n^2 \log(ex+d) \log(F) + 2b^2 f n \log(F) \log(c) + 2abfn \log(F) + 1) \sqrt{-b^2 f n^2 \log(F)}}{2b^2 f n^2 \log(F)}\right) e^{\left(\frac{-4b^2 f n \log(F) \log(c) + 4abfn \log(F) + 1}{4b^2 f n^2 \log(F)}\right)}}{2ben}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2), x, \text{algorithm}="fricas")$

[Out]  $-1/2*\sqrt{\pi}*\sqrt{-b^2*f*n^2*\log(F)}*\operatorname{erf}(1/2*(2*b^2*f*n^2*\log(e*x + d)*\log(F) + 2*b^2*f*n*\log(F)*\log(c) + 2*a*b*f*n*\log(F) + 1)*\sqrt{-b^2*f*n^2*\log(F)})/(b^2*f*n^2*\log(F))*e^{(-1/4*(4*b^2*f*n*\log(F)*\log(c) + 4*a*b*f*n*\log(F) + 1)/(b^2*f*n^2*\log(F)))/(b*e*n)}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\ln(c*(e*x+d)**n))**2), x)$

[Out] Timed out

**Giac [A]** time = 1.30542, size = 157, normalized size = 1.25

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-f \log(F)} b n \log(xe + d) - \sqrt{-f \log(F)} b \log(c) - \sqrt{-f \log(F)} a - \frac{\sqrt{-f \log(F)}}{2 b f n \log(F)}\right) e^{\left(-\frac{a}{b n} - \frac{1}{4 b^2 f n^2 \log(F)} - 1\right)}}{2 \sqrt{-f \log(F)} b c \binom{1}{n} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2), x, \text{algorithm}="giac")$

[Out]  $-1/2*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-f*\log(F)}*b*n*\log(x*e + d) - \sqrt{-f*\log(F)}*b*\log(c) - \sqrt{-f*\log(F)}*a - 1/2*\sqrt{-f*\log(F)}/(b*f*n*\log(F)))*e^{(-a/(b*n) - 1/4/(b^2*f*n^2*\log(F)) - 1)/(sqrt(-f*\log(F))*b*c^(1/n)*n)}$

$$3.614 \quad \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x), x]

**Rubi [A]** time = 0.0942562, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x), x]

[Out] Defer[Int] [F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x), x]

Rubi steps

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx = \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx$$

**Mathematica [A]** time = 0.958549, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{g+hx} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x), x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x), x]

**Maple [A]** time = 0.553, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \ln(c(ex+d)^n))^2}}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)/(h\*x+g), x)

[Out]  $\text{int}(F^{(f*(a+b*\ln(c*(e*x+d)^n))^2)/(h*x+g)}, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2)/(h*x+g)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c) + a)^2*f)/(h*x + g)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{b^2 f \log((ex+d)^n c)^2 + 2 ab f \log((ex+d)^n c) + a^2 f}}{hx + g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2)/(h*x+g)}, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(F^{(b^2*f*\log((e*x + d)^n*c)^2 + 2*a*b*f*\log((e*x + d)^n*c) + a^2*f)/(h*x + g)}, x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\ln(c*(e*x+d)**n))**2)/(h*x+g)}, x)$

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(f*(a+b*\log(c*(e*x+d)^n))^2)/(h*x+g)}, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(F^{((b*\log((e*x + d)^n*c) + a)^2*f)/(h*x + g)}, x)$

$$3.615 \quad \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^2, x]

**Rubi [A]** time = 0.102457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^2,x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^2, x]

Rubi steps

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx = \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx$$

**Mathematica [A]** time = 4.50853, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^2,x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^2, x]

**Maple [A]** time = 0.579, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \ln(c(ex+d)^n))^2}}{(hx+g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)/(h\*x+g)^2,x)

[Out]  $\int (F^{(f*(a+b*\ln(c*(e*x+d)^n))^2})/(h*x+g)^2, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="maxima")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{b^2 f \log((ex+d)^n c)^2 + 2abf \log((ex+d)^n c) + a^2 f}}{h^2 x^2 + 2ghx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="fricas")`

[Out] `integral(F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f)/(h^2*x^2 + 2*g*h*x + g^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(h*x+g)**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="giac")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^2, x)`

$$3.616 \quad \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3}, x \right)$$

[Out] Unintegrable[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^3, x]

**Rubi [A]** time = 0.0993224, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^3,x]

[Out] Defer[Int][F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^3, x]

Rubi steps

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx = \int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

**Mathematica [A]** time = 6.76969, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^3,x]

[Out] Integrate[F^(f\*(a + b\*Log[c\*(d + e\*x)^n])^2)/(g + h\*x)^3, x]

**Maple [A]** time = 0.786, size = 0, normalized size = 0.

$$\int \frac{F^{f(a+b \ln(c(ex+d)^n))^2}}{(hx+g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(f\*(a+b\*ln(c\*(e\*x+d)^n))^2)/(h\*x+g)^3,x)

[Out]  $\int (F^{(f*(a+b*\ln(c*(e*x+d)^n))^2})/(h*x+g)^3, x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{(hx + g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="maxima")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^3, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{F^{b^2 f \log((ex+d)^n c)^2 + 2abf \log((ex+d)^n c) + a^2 f}}{h^3 x^3 + 3gh^2 x^2 + 3g^2 hx + g^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="fricas")`

[Out] `integral(F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*f)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(h*x+g)**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(b \log((ex+d)^n c) + a)^2 f}}{(hx + g)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="giac")`

[Out] `integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^3, x)`

$$3.617 \quad \int F^{a+bx+cx^3} (b + 3cx^2) dx$$

**Optimal.** Leaf size=17

$$\frac{F^{a+bx+cx^3}}{\log(F)}$$

[Out] F^(a + b\*x + c\*x^3)/Log[F]

**Rubi [A]** time = 0.0520096, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {6706}

$$\frac{F^{a+bx+cx^3}}{\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x + c\*x^3)\*(b + 3\*c\*x^2), x]

[Out] F^(a + b\*x + c\*x^3)/Log[F]

**Rule 6706**

Int[(F\_)^(v\_)\*(u\_), x\_Symbol] :> With[{q = DerivativeDivides[v, u, x]}, Simp[(q\*F^v)/Log[F], x] /; !FalseQ[q]] /; FreeQ[F, x]

**Rubi steps**

$$\int F^{a+bx+cx^3} (b + 3cx^2) dx = \frac{F^{a+bx+cx^3}}{\log(F)}$$

**Mathematica [A]** time = 0.0519045, size = 17, normalized size = 1.

$$\frac{F^{a+bx+cx^3}}{\log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x + c\*x^3)\*(b + 3\*c\*x^2), x]

[Out] F^(a + b\*x + c\*x^3)/Log[F]

**Maple [A]** time = 0.043, size = 18, normalized size = 1.1

$$\frac{F^{cx^3+bx+a}}{\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(F^(c*x^3+b*x+a)*(3*c*x^2+b),x)`

[Out]  $F^{(c*x^3+b*x+a)}/\ln(F)$

**Maxima [A]** time = 0.959697, size = 23, normalized size = 1.35

$$\frac{F^{cx^3+bx+a}}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="maxima")`

[Out]  $F^{(c*x^3 + b*x + a)}/\log(F)$

**Fricas [A]** time = 0.924443, size = 38, normalized size = 2.24

$$\frac{F^{cx^3+bx+a}}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="fricas")`

[Out]  $F^{(c*x^3 + b*x + a)}/\log(F)$

**Sympy [A]** time = 0.12406, size = 24, normalized size = 1.41

$$\begin{cases} \frac{F^{a+bx+cx^3}}{\log(F)} & \text{for } \log(F) \neq 0 \\ bx + cx^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*x**3+b*x+a)*(3*c*x**2+b),x)`

[Out] `Piecewise((F**(a + b*x + c*x**3)/log(F), Ne(log(F), 0)), (b*x + c*x**3, True))`

**Giac [A]** time = 1.33893, size = 23, normalized size = 1.35

$$\frac{F^{cx^3+bx+a}}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="giac")`

[Out]  $F^{(c*x^3 + b*x + a)}/\log(F)$

$$3.618 \quad \int \frac{F^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=20

$$-\frac{F^{a+bx+cx^2}}{\log(F)}$$

[Out]  $-(F^{(a + b*x + c*x^2)}^{-1})/\text{Log}[F]$

**Rubi [A]** time = 0.166147, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$ , Rules used = {6706}

$$-\frac{F^{a+bx+cx^2}}{\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{(a + b*x + c*x^2)}^{-1})*(b + 2*c*x)]/(a + b*x + c*x^2)^2, x]$

[Out]  $-(F^{(a + b*x + c*x^2)}^{-1})/\text{Log}[F]$

**Rule 6706**

$\text{Int}[(F_)^{(v_)}*(u_), x\_Symbol] \rightarrow \text{With}[\{q = \text{DerivativeDivides}[v, u, x]\}, \text{Simp}[(q*F^v)/\text{Log}[F], x] /; \text{!FalseQ}[q] /; \text{FreeQ}[F, x]$

**Rubi steps**

$$\int \frac{F^{a+bx+cx^2} (b+2cx)}{(a+bx+cx^2)^2} dx = -\frac{F^{a+bx+cx^2}}{\log(F)}$$

**Mathematica [A]** time = 0.384291, size = 19, normalized size = 0.95

$$-\frac{F^{a+x(b+cx)}}{\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(F^{(a + b*x + c*x^2)}^{-1})*(b + 2*c*x)]/(a + b*x + c*x^2)^2, x]$

[Out]  $-(F^{(a + x*(b + c*x)}^{-1})/\text{Log}[F])$

**Maple [A]** time = 0.042, size = 21, normalized size = 1.1

$$-\frac{F^{(cx^2+bx+a)}^{-1}}{\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x)`

[Out] `-F^(1/(c*x^2+b*x+a))/ln(F)`

**Maxima [A]** time = 0.977901, size = 27, normalized size = 1.35

$$-\frac{F\left(\frac{1}{cx^2+bx+a}\right)}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="maxima")`

[Out] `-F^(1/(c*x^2 + b*x + a))/log(F)`

**Fricas [A]** time = 0.779014, size = 45, normalized size = 2.25

$$-\frac{F\left(\frac{1}{cx^2+bx+a}\right)}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="fricas")`

[Out] `-F^(1/(c*x^2 + b*x + a))/log(F)`

**Sympy [A]** time = 0.679001, size = 32, normalized size = 1.6

$$\begin{cases} -\frac{1}{F^{a+bx+cx^2} \log(F)} & \text{for } \log(F) \neq 0 \\ -\frac{1}{a+bx+cx^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(1/(c*x**2+b*x+a))*(2*c*x+b)/(c*x**2+b*x+a)**2,x)`

[Out] `Piecewise((-F**(1/(a + b*x + c*x**2))/log(F), Ne(log(F), 0)), (-1/(a + b*x + c*x**2), True))`

**Giac [A]** time = 1.30072, size = 27, normalized size = 1.35

$$-\frac{F\left(\frac{1}{cx^2+bx+a}\right)}{\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -F^(1/(c*x^2 + b*x + a))/log(F)
```

$$3.619 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^m dx$$

**Optimal.** Leaf size=49

$$(-a - bx - cx^2)^{-m} (a + bx + cx^2)^m \text{Gamma}(m + 1, -a - bx - cx^2)$$

[Out]  $((a + b*x + c*x^2)^m * \text{Gamma}[1 + m, -a - b*x - c*x^2]) / (-a - b*x - c*x^2)^m$

**Rubi [A]** time = 0.199597, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {6707, 2181}

$$(-a - bx - cx^2)^{-m} (a + bx + cx^2)^m \text{Gamma}(m + 1, -a - bx - cx^2)$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^m,x]

[Out]  $((a + b*x + c*x^2)^m * \text{Gamma}[1 + m, -a - b*x - c*x^2]) / (-a - b*x - c*x^2)^m$

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^m dx &= \text{Subst} \left( \int e^x x^m dx, x, a + bx + cx^2 \right) \\ &= (-a - bx - cx^2)^{-m} (a + bx + cx^2)^m \Gamma(1 + m, -a - bx - cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0523048, size = 44, normalized size = 0.9

$$(-a - x(b + cx))^{-m} (a + x(b + cx))^m \text{Gamma}(m + 1, -a - x(b + cx))$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^m,x]

[Out]  $((a + x*(b + c*x))^m * \text{Gamma}[1 + m, -a - x*(b + c*x)]) / (-a - x*(b + c*x))^m$

**Maple [F]** time = 1.413, size = 0, normalized size = 0.

$$\int e^{cx^2+bx+a} (2cx+b) (cx^2+bx+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^m,x)

[Out] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (2cx+b)(cx^2+bx+a)^m e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^m,x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*(c\*x^2 + b\*x + a)^m\*e^(c\*x^2 + b\*x + a), x)

---

**Fricas [A]** time = 0.732206, size = 57, normalized size = 1.16

$$\cos(\pi m) \Gamma(m+1, -cx^2 - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^m,x, algorithm="fricas")

[Out] cos(pi\*m)\*gamma(m + 1, -c\*x^2 - b\*x - a)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*m,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (2cx+b)(cx^2+bx+a)^m e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^m,x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*(c\*x^2 + b\*x + a)^m\*e^(c\*x^2 + b\*x + a), x)

$$3.620 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^3 dx$$

**Optimal.** Leaf size=90

$$e^{a+bx+cx^2} (a + bx + cx^2)^3 - 3e^{a+bx+cx^2} (a + bx + cx^2)^2 + 6e^{a+bx+cx^2} (a + bx + cx^2) - 6e^{a+bx+cx^2}$$

[Out]  $-6E^{(a + b*x + c*x^2)} + 6E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2) - 3E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^2 + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^3$

**Rubi [A]** time = 0.181703, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6707, 2176, 2194}

$$e^{a+bx+cx^2} (a + bx + cx^2)^3 - 3e^{a+bx+cx^2} (a + bx + cx^2)^2 + 6e^{a+bx+cx^2} (a + bx + cx^2) - 6e^{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^3,x]

[Out]  $-6E^{(a + b*x + c*x^2)} + 6E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2) - 3E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^2 + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^3$

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_.) + (f\_)\*(x\_))))^(n\_.)\*((c\_.) + (d\_)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_.) + (b\_)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^3 dx &= \text{Subst} \left( \int e^x x^3 dx, x, a + bx + cx^2 \right) \\ &= e^{a+bx+cx^2} (a + bx + cx^2)^3 - 3 \text{Subst} \left( \int e^x x^2 dx, x, a + bx + cx^2 \right) \\ &= -3e^{a+bx+cx^2} (a + bx + cx^2)^2 + e^{a+bx+cx^2} (a + bx + cx^2)^3 + 6 \text{Subst} \left( \int e^x x dx, x, a + bx + cx^2 \right) \\ &= 6e^{a+bx+cx^2} (a + bx + cx^2) - 3e^{a+bx+cx^2} (a + bx + cx^2)^2 + e^{a+bx+cx^2} (a + bx + cx^2)^3 - 6e^{a+bx+cx^2} \\ &= -6e^{a+bx+cx^2} + 6e^{a+bx+cx^2} (a + bx + cx^2) - 3e^{a+bx+cx^2} (a + bx + cx^2)^2 + e^{a+bx+cx^2} (a + bx + cx^2)^3 \end{aligned}$$

**Mathematica [A]** time = 0.0374432, size = 49, normalized size = 0.54

$$e^{a+x(b+cx)} \left( (a+x(b+cx))^3 - 3(a+x(b+cx))^2 + 6(a+x(b+cx)) - 6 \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^3,x]

[Out] E^(a + x\*(b + c\*x))\*(-6 + 6\*(a + x\*(b + c\*x)) - 3\*(a + x\*(b + c\*x))^2 + (a + x\*(b + c\*x))^3)

**Maple [A]** time = 0.039, size = 145, normalized size = 1.6

$$(c^3x^6 + 3c^2bx^5 + 3ac^2x^4 + 3b^2cx^4 + 6abcx^3 + b^3x^3 - 3c^2x^4 + 3a^2cx^2 + 3ab^2x^2 - 6bcx^3 + 3a^2bx - 6acx^2 - 3b^2x^2 + a^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^3,x)

[Out] (c^3\*x^6+3\*b\*c^2\*x^5+3\*a\*c^2\*x^4+3\*b^2\*c\*x^4+6\*a\*b\*c\*x^3+b^3\*x^3-3\*c^2\*x^4+3\*a^2\*c\*x^2+3\*a\*b^2\*x^2-6\*b\*c\*x^3+3\*a^2\*b\*x-6\*a\*c\*x^2-3\*b^2\*x^2+a^3-6\*a\*b\*x+6\*c\*x^2-3\*a^2+6\*b\*x+6\*a-6)\*exp(c\*x^2+b\*x+a)

**Maxima [C]** time = 2.13694, size = 3214, normalized size = 35.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^3,x, algorithm="maxima")

[Out] 1/2\*sqrt(pi)\*a^3\*b\*erf(sqrt(-c)\*x - 1/2\*b/sqrt(-c))\*e^(a - 1/4\*b^2/c)/sqrt(-c) - 3/4\*(sqrt(pi)\*(2\*c\*x + b)\*b\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(3/2)) - 2\*e^(1/4\*(2\*c\*x + b)^2/c)/sqrt(c))\*a^2\*b^2\*e^(a - 1/4\*b^2/c)/sqrt(c) + 3/8\*(sqrt(pi)\*(2\*c\*x + b)\*b^2\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(5/2)) - 4\*b\*e^(1/4\*(2\*c\*x + b)^2/c)/c^(3/2) - 4\*(2\*c\*x + b)^3\*gamma(3/2, -1/4\*(2\*c\*x + b)^2/c)/((-2\*c\*x + b)^2/c)^(3/2)\*c^(5/2))\*a\*b^3\*e^(a - 1/4\*b^2/c)/sqrt(c) - 1/16\*(sqrt(pi)\*(2\*c\*x + b)\*b^3\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(7/2)) - 6\*b^2\*e^(1/4\*(2\*c\*x + b)^2/c)/c^(5/2) - 12\*(2\*c\*x + b)^3\*b\*gamma(3/2, -1/4\*(2\*c\*x + b)^2/c)/((-2\*c\*x + b)^2/c)^(3/2)\*c^(7/2)) + 8\*gamma(2, -1/4\*(2\*c\*x + b)^2/c)/c^(3/2))\*b^4\*e^(a - 1/4\*b^2/c)/sqrt(c) - 1/2\*(sqrt(pi)\*(2\*c\*x + b)\*b\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(3/2)) - 2\*e^(1/4\*(2\*c\*x + b)^2/c)/sqrt(c))\*a^3\*sqrt(c)\*e^(a - 1/4\*b^2/c) + 9/8\*(sqrt(pi)\*(2\*c\*x + b)\*b^2\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(5/2)) - 4\*b\*e^(1/4\*(2\*c\*x + b)^2/c)/c^(3/2) - 4\*(2\*c\*x + b)^3\*gamma(3/2, -1/4\*(2\*c\*x + b)^2/c)/((-2\*c\*x + b)^2/c)^(3/2)\*c^(5/2))\*a^2\*b\*sqrt(c)\*e^(a - 1/4\*b^2/c) - 3/4\*(sqrt(pi)\*(2\*c\*x + b)\*b^3\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(7/2)) - 6\*b^2\*e^(1/4\*(2\*c\*x + b)^2/c)/c^(5/2) - 12\*(2\*c\*x + b)^3\*b\*gamma(3/2, -1/4\*(2\*c\*x + b)^2/c)/((-2\*c\*x + b)^2/c)^(3/2)\*c^(7/2)) + 8\*gamma(2, -1/4\*(2\*c\*x + b)^2/c)/c^(3/2))\*a\*b^2\*sqrt(c)\*e^(a - 1/4\*b^2/c) + 5/32\*(sqrt(pi)\*(2\*c\*x + b)\*b^4\*(erf(1/2\*sqrt(-(2\*c\*x + b)^2/c)) - 1)/(sqrt(-(2\*c\*x + b)^2/c)\*c^(9/2)) - 8\*b^3\*e^(1/4\*(2\*c\*x + b)^2/c)/c^(7/2) - 24\*(2\*c\*x + b)^3\*b^2\*gamma



$$\begin{aligned}
& a(3/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)}*c^{(9/2)} + 32*b*\text{gamma} \\
& a(2, -1/4*(2*c*x + b)^2/c)/c^{(5/2)} - 16*(2*c*x + b)^5*\text{gamma}(5/2, -1/4*(2*c*x \\
& x + b)^2/c)/((-2*c*x + b)^2/c)^{(5/2)}*c^{(9/2)})*b^3*\text{sqrt}(c)*e^{(a - 1/4*b^2/c)} \\
& c) - 3/8*(\text{sqrt}(\pi)*(2*c*x + b)*b^3*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2/c)) - 1)/(\text{sqrt} \\
& (-2*c*x + b)^2/c)*c^{(7/2)}) - 6*b^2*e^{(1/4*(2*c*x + b)^2/c)/c^{(5/2)} - 12 \\
& *(2*c*x + b)^3*b*\text{gamma}(3/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)} \\
& *c^{(7/2)} + 8*\text{gamma}(2, -1/4*(2*c*x + b)^2/c)/c^{(3/2)})*a^2*c^{(3/2)}*e^{(a - 1/ \\
& 4*b^2/c)} + 15/32*(\text{sqrt}(\pi)*(2*c*x + b)*b^4*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2/c)) \\
& - 1)/(\text{sqrt}(-(2*c*x + b)^2/c)*c^{(9/2)}) - 8*b^3*e^{(1/4*(2*c*x + b)^2/c)/c^{(7 \\
& /2)} - 24*(2*c*x + b)^3*b^2*\text{gamma}(3/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^ \\
& 2/c)^{(3/2)}*c^{(9/2)} + 32*b*\text{gamma}(2, -1/4*(2*c*x + b)^2/c)/c^{(5/2)} - 16*(2*c \\
& *x + b)^5*\text{gamma}(5/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(5/2)}*c^{(9/2)} \\
& ))*a*b*c^{(3/2)}*e^{(a - 1/4*b^2/c)} - 9/64*(\text{sqrt}(\pi)*(2*c*x + b)*b^5*(\text{erf}(1/2 \\
& *\text{sqrt}(-(2*c*x + b)^2/c)) - 1)/(\text{sqrt}(-(2*c*x + b)^2/c)*c^{(11/2)}) - 10*b^4*e^{ \\
& (1/4*(2*c*x + b)^2/c)/c^{(9/2)} - 40*(2*c*x + b)^3*b^3*\text{gamma}(3/2, -1/4*(2*c*x \\
& + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)}*c^{(11/2)} + 80*b^2*\text{gamma}(2, -1/4*(2*c \\
& *x + b)^2/c)/c^{(7/2)} - 80*(2*c*x + b)^5*b*\text{gamma}(5/2, -1/4*(2*c*x + b)^2/c)/ \\
& (-2*c*x + b)^2/c)^{(5/2)}*c^{(11/2)} - 32*\text{gamma}(3, -1/4*(2*c*x + b)^2/c)/c^{(5 \\
& /2)})*b^2*c^{(3/2)}*e^{(a - 1/4*b^2/c)} - 3/32*(\text{sqrt}(\pi)*(2*c*x + b)*b^5*(\text{erf}(1/ \\
& 2*\text{sqrt}(-(2*c*x + b)^2/c)) - 1)/(\text{sqrt}(-(2*c*x + b)^2/c)*c^{(11/2)}) - 10*b^4*e^{ \\
& (1/4*(2*c*x + b)^2/c)/c^{(9/2)} - 40*(2*c*x + b)^3*b^3*\text{gamma}(3/2, -1/4*(2*c \\
& *x + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)}*c^{(11/2)} + 80*b^2*\text{gamma}(2, -1/4*(2*c \\
& *x + b)^2/c)/c^{(7/2)} - 80*(2*c*x + b)^5*b*\text{gamma}(5/2, -1/4*(2*c*x + b)^2/c)/ \\
& ((-2*c*x + b)^2/c)^{(5/2)}*c^{(11/2)} - 32*\text{gamma}(3, -1/4*(2*c*x + b)^2/c)/c^{(5 \\
& /2)})*a*c^{(5/2)}*e^{(a - 1/4*b^2/c)} + 7/128*(\text{sqrt}(\pi)*(2*c*x + b)*b^6*(\text{erf}(1/ \\
& 2*\text{sqrt}(-(2*c*x + b)^2/c)) - 1)/(\text{sqrt}(-(2*c*x + b)^2/c)*c^{(13/2)}) - 12*b^5*e^{ \\
& (1/4*(2*c*x + b)^2/c)/c^{(11/2)} - 60*(2*c*x + b)^3*b^4*\text{gamma}(3/2, -1/4*(2*c \\
& *x + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)}*c^{(13/2)} + 160*b^3*\text{gamma}(2, -1/4*(2 \\
& *c*x + b)^2/c)/c^{(9/2)} - 240*(2*c*x + b)^5*b^2*\text{gamma}(5/2, -1/4*(2*c*x + b)^ \\
& 2/c)/((-2*c*x + b)^2/c)^{(5/2)}*c^{(13/2)} - 192*b*\text{gamma}(3, -1/4*(2*c*x + b)^ \\
& 2/c)/c^{(7/2)} - 64*(2*c*x + b)^7*\text{gamma}(7/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x \\
& + b)^2/c)^{(7/2)}*c^{(13/2)}))*b*c^{(5/2)}*e^{(a - 1/4*b^2/c)} - 1/128*(\text{sqrt}(\pi)*(2 \\
& *c*x + b)*b^7*(\text{erf}(1/2*\text{sqrt}(-(2*c*x + b)^2/c)) - 1)/(\text{sqrt}(-(2*c*x + b)^2/c) \\
& *c^{(15/2)}) - 14*b^6*e^{(1/4*(2*c*x + b)^2/c)/c^{(13/2)} - 84*(2*c*x + b)^3*b^5 \\
& *\text{gamma}(3/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(3/2)}*c^{(15/2)} + 280 \\
& *b^4*\text{gamma}(2, -1/4*(2*c*x + b)^2/c)/c^{(11/2)} - 560*(2*c*x + b)^5*b^3*\text{gamma}( \\
& 5/2, -1/4*(2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(5/2)}*c^{(15/2)} - 672*b^2*\text{ga} \\
& mma(3, -1/4*(2*c*x + b)^2/c)/c^{(9/2)} - 448*(2*c*x + b)^7*b*\text{gamma}(7/2, -1/4* \\
& (2*c*x + b)^2/c)/((-2*c*x + b)^2/c)^{(7/2)}*c^{(15/2)} + 128*\text{gamma}(4, -1/4*(2 \\
& *c*x + b)^2/c)/c^{(7/2)})*c^{(7/2)}*e^{(a - 1/4*b^2/c)}
\end{aligned}$$


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**Fricas [A]** time = 0.831962, size = 261, normalized size = 2.9

$$(c^3x^6 + 3bc^2x^5 + 3(b^2c + (a-1)c^2)x^4 + (b^3 + 6(a-1)bc)x^3 + a^3 + 3(a^2 - 2a + 2)bx + 3((a-1)b^2 + (a^2 - 2a + 2)c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^3,x, algorithm="fricas")

[Out] (c^3\*x^6 + 3\*b\*c^2\*x^5 + 3\*(b^2\*c + (a - 1)\*c^2)\*x^4 + (b^3 + 6\*(a - 1)\*b\*c) \*x^3 + a^3 + 3\*(a^2 - 2\*a + 2)\*b\*x + 3\*((a - 1)\*b^2 + (a^2 - 2\*a + 2)\*c)\*x ^2 - 3\*a^2 + 6\*a - 6)\*e^(c\*x^2 + b\*x + a)

---

**Sympy [A]** time = 0.235616, size = 160, normalized size = 1.78

$$(a^3 + 3a^2bx + 3a^2cx^2 - 3a^2 + 3ab^2x^2 + 6abcx^3 - 6abx + 3ac^2x^4 - 6acx^2 + 6a + b^3x^3 + 3b^2cx^4 - 3b^2x^2 + 3bc^2x^5 - 6bc$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*3,x)

[Out] (a\*\*3 + 3\*a\*\*2\*b\*x + 3\*a\*\*2\*c\*x\*\*2 - 3\*a\*\*2 + 3\*a\*b\*\*2\*x\*\*2 + 6\*a\*b\*c\*x\*\*3 - 6\*a\*b\*x + 3\*a\*c\*\*2\*x\*\*4 - 6\*a\*c\*x\*\*2 + 6\*a + b\*\*3\*x\*\*3 + 3\*b\*\*2\*c\*x\*\*4 - 3\*b\*\*2\*x\*\*2 + 3\*b\*c\*\*2\*x\*\*5 - 6\*b\*c\*x\*\*3 + 6\*b\*x + c\*\*3\*x\*\*6 - 3\*c\*\*2\*x\*\*4 + 6\*c\*x\*\*2 - 6)\*exp(a + b\*x + c\*x\*\*2)

**Giac [B]** time = 1.26402, size = 360, normalized size = 4.

$$\left(c^6\left(2x + \frac{b}{c}\right)^6 - 3b^2c^4\left(2x + \frac{b}{c}\right)^4 + 12ac^5\left(2x + \frac{b}{c}\right)^4 - 12c^5\left(2x + \frac{b}{c}\right)^4 + 3b^4c^2\left(2x + \frac{b}{c}\right)^2 - 24ab^2c^3\left(2x + \frac{b}{c}\right)^2 + 48a^2c^4\left(2x + \frac{b}{c}\right)^2 - 12c^5\left(2x + \frac{b}{c}\right)^4 + 3b^4c^2\left(2x + \frac{b}{c}\right)^2 - 24ab^2c^3\left(2x + \frac{b}{c}\right)^2 + 48a^2c^4\left(2x + \frac{b}{c}\right)^2 - b^6 + 12ab^4c - 48a^2b^2c^2 + 64a^3c^3 + 96c^4\left(2x + \frac{b}{c}\right)^2 - 12b^4c + 96ab^2c^2 - 192a^2c^3 - 96b^2c^2 + 384ac^3 - 384c^3\right)e^{(cx^2 + bx + a)/c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^3,x, algorithm="giac")

[Out] 1/64\*(c^6\*(2\*x + b/c)^6 - 3\*b^2\*c^4\*(2\*x + b/c)^4 + 12\*a\*c^5\*(2\*x + b/c)^4 - 12\*c^5\*(2\*x + b/c)^4 + 3\*b^4\*c^2\*(2\*x + b/c)^2 - 24\*a\*b^2\*c^3\*(2\*x + b/c)^2 + 48\*a^2\*c^4\*(2\*x + b/c)^2 + 24\*b^2\*c^3\*(2\*x + b/c)^2 - 96\*a\*c^4\*(2\*x + b/c)^2 - b^6 + 12\*a\*b^4\*c - 48\*a^2\*b^2\*c^2 + 64\*a^3\*c^3 + 96\*c^4\*(2\*x + b/c)^2 - 12\*b^4\*c + 96\*a\*b^2\*c^2 - 192\*a^2\*c^3 - 96\*b^2\*c^2 + 384\*a\*c^3 - 384\*c^3)\*e^(c\*x^2 + b\*x + a)/c^3

$$3.621 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^2 dx$$

**Optimal.** Leaf size=64

$$e^{a+bx+cx^2} (a + bx + cx^2)^2 - 2e^{a+bx+cx^2} (a + bx + cx^2) + 2e^{a+bx+cx^2}$$

[Out]  $2E^{(a + b*x + c*x^2)} - 2E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2) + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^2$

**Rubi [A]** time = 0.160683, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6707, 2176, 2194}

$$e^{a+bx+cx^2} (a + bx + cx^2)^2 - 2e^{a+bx+cx^2} (a + bx + cx^2) + 2e^{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^2,x]

[Out]  $2E^{(a + b*x + c*x^2)} - 2E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2) + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)^2$

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^2 dx &= \text{Subst} \left( \int e^x x^2 dx, x, a + bx + cx^2 \right) \\ &= e^{a+bx+cx^2} (a + bx + cx^2)^2 - 2 \text{Subst} \left( \int e^x x dx, x, a + bx + cx^2 \right) \\ &= -2e^{a+bx+cx^2} (a + bx + cx^2) + e^{a+bx+cx^2} (a + bx + cx^2)^2 + 2 \text{Subst} \left( \int e^x dx, x, a + bx + cx^2 \right) \\ &= 2e^{a+bx+cx^2} - 2e^{a+bx+cx^2} (a + bx + cx^2) + e^{a+bx+cx^2} (a + bx + cx^2)^2 \end{aligned}$$

**Mathematica [A]** time = 0.0312684, size = 36, normalized size = 0.56

$$e^{a+x(b+cx)} \left( (a + x(b + cx))^2 - 2(a + x(b + cx)) + 2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^2,x]

[Out] E^(a + x\*(b + c\*x))\*(2 - 2\*(a + x\*(b + c\*x)) + (a + x\*(b + c\*x))^2)

**Maple [A]** time = 0.039, size = 64, normalized size = 1.

$$(c^2x^4 + 2bcx^3 + 2acx^2 + b^2x^2 + 2abx - 2cx^2 + a^2 - 2bx - 2a + 2)e^{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^2,x)

[Out] (c^2\*x^4+2\*b\*c\*x^3+2\*a\*c\*x^2+b^2\*x^2+2\*a\*b\*x-2\*c\*x^2+a^2-2\*b\*x-2\*a+2)\*exp(c\*x^2+b\*x+a)

**Maxima [C]** time = 1.68291, size = 1651, normalized size = 25.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}\sqrt{\pi}a^2b\operatorname{erf}(\sqrt{-c}x - \frac{1}{2}b/\sqrt{-c})e^{(a - \frac{1}{4}b^2/c)/\sqrt{-c}} - \frac{1}{2}(\sqrt{\pi}(2cx + b)b(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{3/2}) - 2e^{(1/4(2cx + b)^2/c)/\sqrt{c}}ab^2e^{(a - \frac{1}{4}b^2/c)/\sqrt{c}} + \frac{1}{8}(\sqrt{\pi}(2cx + b)b^2(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{5/2}) - 4b^2e^{(1/4(2cx + b)^2/c)/c^{3/2}} - 4(2cx + b)^3\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{5/2})b^3e^{(a - \frac{1}{4}b^2/c)/\sqrt{c}} - \frac{1}{2}(\sqrt{\pi}(2cx + b)b(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{3/2}) - 2e^{(1/4(2cx + b)^2/c)/\sqrt{c}}a^2\sqrt{c}e^{(a - \frac{1}{4}b^2/c)} + \frac{3}{4}(\sqrt{\pi}(2cx + b)b^2(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{5/2}) - 4b^2e^{(1/4(2cx + b)^2/c)/c^{3/2}} - 4(2cx + b)^3\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{5/2})a^2b\sqrt{c}e^{(a - \frac{1}{4}b^2/c)} - \frac{1}{4}(\sqrt{\pi}(2cx + b)b^3(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{7/2}) - 6b^2e^{(1/4(2cx + b)^2/c)/c^{5/2}} - 12(2cx + b)^3b\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{7/2}) + 8\gamma(2, -1/4(2cx + b)^2/c)/c^{3/2})b^2\sqrt{c}e^{(a - \frac{1}{4}b^2/c)} - \frac{1}{4}(\sqrt{\pi}(2cx + b)b^3(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{7/2}) - 6b^2e^{(1/4(2cx + b)^2/c)/c^{5/2}} - 12(2cx + b)^3b\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{7/2}) + 8\gamma(2, -1/4(2cx + b)^2/c)/c^{3/2})ac^{3/2}e^{(a - \frac{1}{4}b^2/c)} + \frac{5}{32}(\sqrt{\pi}(2cx + b)b^4(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{9/2}) - 8b^3e^{(1/4(2cx + b)^2/c)/c^{7/2}} - 24(2cx + b)^3b^2\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{9/2}) + 32b\gamma(2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{5/2}c^{9/2})b^2c^{3/2}e^{(a - \frac{1}{4}b^2/c)} - \frac{1}{32}(\sqrt{\pi}(2cx + b)b^5(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{11/2}) - 10b^4e^{(1/4(2cx + b)^2/c)/c^{9/2}} - 40(2cx + b)^3b^3\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{11/2}$

)) + 80\*b^2\*gamma(2, -1/4\*(2\*c\*x + b)^2/c)/c^(7/2) - 80\*(2\*c\*x + b)^5\*b\*gamma(5/2, -1/4\*(2\*c\*x + b)^2/c)/((-2\*c\*x + b)^2/c)^(5/2)\*c^(11/2)) - 32\*gamma(3, -1/4\*(2\*c\*x + b)^2/c)/c^(5/2))\*c^(5/2)\*e^(a - 1/4\*b^2/c)

**Fricas [A]** time = 0.80876, size = 136, normalized size = 2.12

$$(c^2x^4 + 2bcx^3 + 2(a-1)bx + (b^2 + 2(a-1)c)x^2 + a^2 - 2a + 2)e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] (c^2\*x^4 + 2\*b\*c\*x^3 + 2\*(a - 1)\*b\*x + (b^2 + 2\*(a - 1)\*c)\*x^2 + a^2 - 2\*a + 2)\*e^(c\*x^2 + b\*x + a)

**Sympy [A]** time = 0.180534, size = 68, normalized size = 1.06

$$(a^2 + 2abx + 2acx^2 - 2a + b^2x^2 + 2bcx^3 - 2bx + c^2x^4 - 2cx^2 + 2)e^{a+bx+cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] (a\*\*2 + 2\*a\*b\*x + 2\*a\*c\*x\*\*2 - 2\*a + b\*\*2\*x\*\*2 + 2\*b\*c\*x\*\*3 - 2\*b\*x + c\*\*2\*x\*\*4 - 2\*c\*x\*\*2 + 2)\*exp(a + b\*x + c\*x\*\*2)

**Giac [A]** time = 1.31223, size = 161, normalized size = 2.52

$$\frac{\left(c^4\left(2x + \frac{b}{c}\right)^4 - 2b^2c^2\left(2x + \frac{b}{c}\right)^2 + 8ac^3\left(2x + \frac{b}{c}\right)^2 - 8c^3\left(2x + \frac{b}{c}\right)^2 + b^4 - 8ab^2c + 16a^2c^2 + 8b^2c - 32ac^2 + 32c^2\right)e^{(c}}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out] 1/16\*(c^4\*(2\*x + b/c)^4 - 2\*b^2\*c^2\*(2\*x + b/c)^2 + 8\*a\*c^3\*(2\*x + b/c)^2 - 8\*c^3\*(2\*x + b/c)^2 + b^4 - 8\*a\*b^2\*c + 16\*a^2\*c^2 + 8\*b^2\*c - 32\*a\*c^2 + 32\*c^2)\*e^(c\*x^2 + b\*x + a)/c^2

$$3.622 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2) dx$$

**Optimal.** Leaf size=38

$$e^{a+bx+cx^2} (a + bx + cx^2) - e^{a+bx+cx^2}$$

[Out]  $-E^{(a + b*x + c*x^2)} + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)$

**Rubi [A]** time = 0.0956658, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {6707, 2176, 2194}

$$e^{a+bx+cx^2} (a + bx + cx^2) - e^{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(a + b*x + c*x^2)}*(b + 2*c*x)*(a + b*x + c*x^2), x]$

[Out]  $-E^{(a + b*x + c*x^2)} + E^{(a + b*x + c*x^2)}*(a + b*x + c*x^2)$

#### Rule 6707

$\text{Int}[(F_)^{(v_)}*(u_)*(w_)^{(m_)}], x\_Symbol] := \text{With}[\{q = \text{DerivativeDivides}[v, u, x]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^m * F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \text{FreeQ}[\{F, m\}, x] \&\& \text{EqQ}[w, v]$

#### Rule 2176

$\text{Int}[(b_)*(F_)^{((g_)*((e_)+(f_)*(x_)))^{(n_)}*((c_)+(d_)*(x_))^{(m_)}], x\_Symbol] := \text{Simp}[(c + d*x)^m*(b * F^{(g*(e + f*x))})^n / (f * g * n * \text{Log}[F]), x] - \text{Dist}[(d * m) / (f * g * n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2 * m] \&\& \text{!UseGamma} == \text{True}$

#### Rule 2194

$\text{Int}[(F_)^{((c_)*((a_)+(b_)*(x_)))^{(n_)}], x\_Symbol] := \text{Simp}[(F^{(c*(a + b*x))})^n / (b * c * n * \text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2) dx &= \text{Subst} \left( \int e^x x dx, x, a + bx + cx^2 \right) \\ &= e^{a+bx+cx^2} (a + bx + cx^2) - \text{Subst} \left( \int e^x dx, x, a + bx + cx^2 \right) \\ &= -e^{a+bx+cx^2} + e^{a+bx+cx^2} (a + bx + cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0288192, size = 23, normalized size = 0.61

$$e^{a+x(b+cx)} (a + bx + cx^2 - 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2),x]

[Out] E^(a + x\*(b + c\*x))\*(-1 + a + b\*x + c\*x^2)

**Maple [A]** time = 0.037, size = 24, normalized size = 0.6

$$(cx^2 + bx + a - 1)e^{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a),x)

[Out] (c\*x^2+b\*x+a-1)\*exp(c\*x^2+b\*x+a)

**Maxima [C]** time = 1.37013, size = 676, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}\sqrt{\pi}ab\operatorname{erf}(\sqrt{-c}x - \frac{1}{2}b/\sqrt{-c})e^{(a - \frac{1}{4}b^2/c)/\sqrt{-c}} - \frac{1}{4}(\sqrt{\pi}(2cx + b)b(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{3/2}) - 2e^{(1/4(2cx + b)^2/c)/\sqrt{c}}b^2e^{(a - \frac{1}{4}b^2/c)/\sqrt{c}} - \frac{1}{2}(\sqrt{\pi}(2cx + b)b(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{3/2}) - 2e^{(1/4(2cx + b)^2/c)/\sqrt{c}}a\sqrt{c}e^{(a - \frac{1}{4}b^2/c)} + \frac{3}{8}(\sqrt{\pi}(2cx + b)b^2(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{5/2}) - 4b^2e^{(1/4(2cx + b)^2/c)/c^{3/2}} - 4(2cx + b)^3\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{5/2})b\sqrt{c}e^{(a - \frac{1}{4}b^2/c)} - \frac{1}{8}(\sqrt{\pi}(2cx + b)b^3(\operatorname{erf}(\frac{1}{2}\sqrt{-(2cx + b)^2/c}) - 1)/(\sqrt{-(2cx + b)^2/c})c^{7/2}) - 6b^2e^{(1/4(2cx + b)^2/c)/c^{5/2}} - 12(2cx + b)^3b\gamma(3/2, -1/4(2cx + b)^2/c)/((-2cx + b)^2/c)^{3/2}c^{7/2}) + 8\gamma(2, -1/4(2cx + b)^2/c)/c^{3/2}c^{3/2}e^{(a - \frac{1}{4}b^2/c)}$

**Fricas [A]** time = 0.825417, size = 58, normalized size = 1.53

$$(cx^2 + bx + a - 1)e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] (c\*x^2 + b\*x + a - 1)\*e^(c\*x^2 + b\*x + a)

**Sympy [A]** time = 0.135125, size = 22, normalized size = 0.58

$$(a + bx + cx^2 - 1)e^{a+bx+cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a),x)

[Out] (a + b\*x + c\*x\*\*2 - 1)\*exp(a + b\*x + c\*x\*\*2)

**Giac [A]** time = 1.19335, size = 59, normalized size = 1.55

$$\frac{\left(c^2\left(2x + \frac{b}{c}\right)^2 - b^2 + 4ac - 4c\right)e^{(cx^2+bx+a)}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] 1/4\*(c^2\*(2\*x + b/c)^2 - b^2 + 4\*a\*c - 4\*c)\*e^(c\*x^2 + b\*x + a)/c



### 3.623 $\int e^{a+bx+cx^2} (b + 2cx) dx$

**Optimal.** Leaf size=12

$$e^{a+bx+cx^2}$$

[Out]  $E^{(a + b*x + c*x^2)}$

**Rubi [A]** time = 0.0192453, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2236}

$$e^{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(a + b*x + c*x^2)}*(b + 2*c*x), x]$

[Out]  $E^{(a + b*x + c*x^2)}$

#### Rule 2236

$\text{Int}[(F_)^{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))}, x\_Symbol]$   
 $] :> \text{Simp}[(e*F^{(a + b*x + c*x^2)})/(2*c*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[b*e - 2*c*d, 0]$

#### Rubi steps

$$\int e^{a+bx+cx^2} (b + 2cx) dx = e^{a+bx+cx^2}$$

**Mathematica [A]** time = 0.0373032, size = 12, normalized size = 1.

$$e^{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[E^{(a + b*x + c*x^2)}*(b + 2*c*x), x]$

[Out]  $E^{(a + b*x + c*x^2)}$

**Maple [A]** time = 0.036, size = 12, normalized size = 1.

$$e^{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\exp(c*x^2+b*x+a)*(2*c*x+b), x)$

[Out]  $\exp(c*x^2+b*x+a)$

---

**Maxima [A]** time = 0.974464, size = 15, normalized size = 1.25

$$e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] e^(c*x^2 + b*x + a)
```

---

**Fricas [A]** time = 0.706163, size = 28, normalized size = 2.33

$$e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] e^(c*x^2 + b*x + a)
```

---

**Sympy [A]** time = 0.100673, size = 10, normalized size = 0.83

$$e^{a+bx+cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*x**2+b*x+a)*(2*c*x+b),x)
```

```
[Out] exp(a + b*x + c*x**2)
```

---

**Giac [A]** time = 1.25708, size = 15, normalized size = 1.25

$$e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="giac")
```

```
[Out] e^(c*x^2 + b*x + a)
```

$$3.624 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=11

$$\text{Ei}(a + bx + cx^2)$$

[Out] ExpIntegralEi[a + b\*x + c\*x^2]

**Rubi [A]** time = 0.176594, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {6707, 2178}

$$\text{Ei}(a + bx + cx^2)$$

Antiderivative was successfully verified.

[In] Int[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2),x]

[Out] ExpIntegralEi[a + b\*x + c\*x^2]

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2} dx &= \text{Subst} \left( \int \frac{e^x}{x} dx, x, a + bx + cx^2 \right) \\ &= \text{Ei}(a + bx + cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0237218, size = 10, normalized size = 0.91

$$\text{Ei}(a + x(b + cx))$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2),x]

[Out] ExpIntegralEi[a + x\*(b + c\*x)]

**Maple [A]** time = 0.036, size = 19, normalized size = 1.7

$$-\text{Ei}\left(1, -cx^2 - bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a), x)`

[Out] `-Ei(1, -c*x^2-b*x-a)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a), x, algorithm="maxima")`

[Out] `integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a), x)`

---

**Fricas [A]** time = 0.806188, size = 28, normalized size = 2.55

$$\text{Ei}\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a), x, algorithm="fricas")`

[Out] `Ei(c*x^2 + b*x + a)`

---

**Sympy [A]** time = 34.1153, size = 10, normalized size = 0.91

$$\text{Ei}\left(a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a), x)`

[Out] `Ei(a + b*x + c*x**2)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a), x, algorithm="giac")`

[Out] `integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a), x)`

$$3.625 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=38

$$\text{Ei}(cx^2 + bx + a) - \frac{e^{a+bx+cx^2}}{a + bx + cx^2}$$

[Out]  $-(E^{(a + b*x + c*x^2)} / (a + b*x + c*x^2)) + \text{ExpIntegralEi}[a + b*x + c*x^2]$

**Rubi [A]** time = 0.19702, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6707, 2177, 2178}

$$\text{Ei}(cx^2 + bx + a) - \frac{e^{a+bx+cx^2}}{a + bx + cx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{(a + b*x + c*x^2)}*(b + 2*c*x))/(a + b*x + c*x^2)^2, x]$

[Out]  $-(E^{(a + b*x + c*x^2)} / (a + b*x + c*x^2)) + \text{ExpIntegralEi}[a + b*x + c*x^2]$

#### Rule 6707

$\text{Int}[(F_)^{(v_)}*(u_)*(w_)^{(m_)}, x\_Symbol] \rightarrow \text{With}[\{q = \text{DerivativeDivides}[v, u, x]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^m * F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \text{FreeQ}[\{F, m\}, x] \&\& \text{EqQ}[w, v]$

#### Rule 2177

$\text{Int}[(b_)*(F_)^{((g_)*((e_.) + (f_)*(x_)))^{(n_)}*((c_.) + (d_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*(b * F^{(g*(e + f*x))})^n / (d*(m + 1)), x] - \text{Dist}[(f * g * n * \text{Log}[F]) / (d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*(b * F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& \text{!UseGamma} == \text{True}$

#### Rule 2178

$\text{Int}[(F_)^{((g_)*((e_.) + (f_)*(x_))) / ((c_.) + (d_)*(x_))}, x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}) * \text{ExpIntegralEi}[(f * g * (c + d*x) * \text{Log}[F]) / d]) / d, x] /; \text{FreeQ}[\{F, c, d, e, f, g\}, x] \&\& \text{!UseGamma} == \text{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2} dx &= \text{Subst} \left( \int \frac{e^x}{x^2} dx, x, a + bx + cx^2 \right) \\ &= -\frac{e^{a+bx+cx^2}}{a + bx + cx^2} + \text{Subst} \left( \int \frac{e^x}{x} dx, x, a + bx + cx^2 \right) \\ &= -\frac{e^{a+bx+cx^2}}{a + bx + cx^2} + \text{Ei}(a + bx + cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.050047, size = 35, normalized size = 0.92

$$\text{Ei}(a + x(b + cx)) - \frac{e^{a+x(b+cx)}}{a + x(b + cx)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^2,x]

[Out] -(E^(a + x\*(b + c\*x))/(a + x\*(b + c\*x))) + ExpIntegralEi[a + x\*(b + c\*x)]

**Maple [A]** time = 0.037, size = 45, normalized size = 1.2

$$-\frac{e^{cx^2+bx+a}}{cx^2 + bx + a} - \text{Ei}(1, -cx^2 - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^2,x)

[Out] -exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)-Ei(1,-c\*x^2-b\*x-a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^2, x)

**Fricas [A]** time = 0.809427, size = 109, normalized size = 2.87

$$\frac{(cx^2 + bx + a)\text{Ei}(cx^2 + bx + a) - e^{(cx^2+bx+a)}}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] ((c\*x^2 + b\*x + a)\*Ei(c\*x^2 + b\*x + a) - e^(c\*x^2 + b\*x + a))/(c\*x^2 + b\*x + a)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^2, x)

$$3.626 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3} dx$$

**Optimal.** Leaf size=72

$$\frac{1}{2}\text{Ei}(cx^2 + bx + a) - \frac{e^{a+bx+cx^2}}{2(a + bx + cx^2)} - \frac{e^{a+bx+cx^2}}{2(a + bx + cx^2)^2}$$

[Out]  $-E^{(a + b*x + c*x^2)}/(2*(a + b*x + c*x^2)^2) - E^{(a + b*x + c*x^2)}/(2*(a + b*x + c*x^2)) + \text{ExpIntegralEi}[a + b*x + c*x^2]/2$

**Rubi [A]** time = 0.241148, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6707, 2177, 2178}

$$\frac{1}{2}\text{Ei}(cx^2 + bx + a) - \frac{e^{a+bx+cx^2}}{2(a + bx + cx^2)} - \frac{e^{a+bx+cx^2}}{2(a + bx + cx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{(a + b*x + c*x^2)}*(b + 2*c*x))/(a + b*x + c*x^2)^3, x]$

[Out]  $-E^{(a + b*x + c*x^2)}/(2*(a + b*x + c*x^2)^2) - E^{(a + b*x + c*x^2)}/(2*(a + b*x + c*x^2)) + \text{ExpIntegralEi}[a + b*x + c*x^2]/2$

#### Rule 6707

$\text{Int}[(F_)^(v_)*(u_)*(w_)^(m_.), x\_Symbol] := \text{With}[\{q = \text{DerivativeDivides}[v, u, x]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^m * F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \text{FreeQ}[\{F, m\}, x] \ \&\& \ \text{EqQ}[w, v]$

#### Rule 2177

$\text{Int}[(b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x\_Symbol] := \text{Simp}[(c + d*x)^(m + 1)*(b * F^(g*(e + f*x)))^n / (d*(m + 1)), x] - \text{Dist}[(f * g * n * \text{Log}[F]) / (d*(m + 1)), \text{Int}[(c + d*x)^(m + 1)*(b * F^(g*(e + f*x)))^n, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ \text{!UseGamma} == \text{True}$

#### Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] := \text{Simp}[(F^(g*(e - (c*f)/d)) * \text{ExpIntegralEi}[(f * g * (c + d*x) * \text{Log}[F]) / d]) / d, x] /; \text{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \ \text{!UseGamma} == \text{True}$

#### Rubi steps



$$\begin{aligned}
\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3} dx &= \text{Subst} \left( \int \frac{e^x}{x^3} dx, x, a+bx+cx^2 \right) \\
&= -\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2} + \frac{1}{2} \text{Subst} \left( \int \frac{e^x}{x^2} dx, x, a+bx+cx^2 \right) \\
&= -\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2} - \frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)} + \frac{1}{2} \text{Subst} \left( \int \frac{e^x}{x} dx, x, a+bx+cx^2 \right) \\
&= -\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2} - \frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)} + \frac{1}{2} \text{Ei}(a+bx+cx^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0688787, size = 50, normalized size = 0.69

$$\frac{1}{2} \left( \text{Ei}(a+x(b+cx)) - \frac{e^{a+x(b+cx)}(a+bx+cx^2+1)}{(a+x(b+cx))^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^3,x]

[Out] (-((E^(a + x\*(b + c\*x))\*(1 + a + b\*x + c\*x^2))/(a + x\*(b + c\*x))^2) + ExpIntegralEi[a + x\*(b + c\*x)])/2

**Maple [A]** time = 0.041, size = 70, normalized size = 1.

$$-\frac{e^{cx^2+bx+a}}{2(cx^2+bx+a)^2} - \frac{e^{cx^2+bx+a}}{2cx^2+2bx+2a} - \frac{\text{Ei}(1, -cx^2-bx-a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^3,x)

[Out] -1/2\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^2-1/2\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)-1/2\*Ei(1,-c\*x^2-b\*x-a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^3,x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^3, x)

**Fricas [A]** time = 0.906188, size = 252, normalized size = 3.5

$$\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)\text{Ei}(cx^2 + bx + a) - (cx^2 + bx + a + 1)e^{(cx^2+bx+a)}}{2(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^3,x, algorithm="fricas")

[Out] 1/2\*((c^2\*x^4 + 2\*b\*c\*x^3 + 2\*a\*b\*x + (b^2 + 2\*a\*c)\*x^2 + a^2)\*Ei(c\*x^2 + b\*x + a) - (c\*x^2 + b\*x + a + 1)\*e^(c\*x^2 + b\*x + a))/(c^2\*x^4 + 2\*b\*c\*x^3 + 2\*a\*b\*x + (b^2 + 2\*a\*c)\*x^2 + a^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^3,x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^3, x)

$$3.627 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^{7/2} dx$$

**Optimal.** Leaf size=142

$$\frac{105}{16} \sqrt{\pi} \operatorname{Erfi} \left( \sqrt{a + bx + cx^2} \right) + e^{a+bx+cx^2} (a + bx + cx^2)^{7/2} - \frac{7}{2} e^{a+bx+cx^2} (a + bx + cx^2)^{5/2} + \frac{35}{4} e^{a+bx+cx^2} (a + bx + cx^2)^3$$

[Out]  $(-105 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot \operatorname{Sqrt}[a + b \cdot x + c \cdot x^2]) / 8 + (35 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(3/2)}) / 4 - (7 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(5/2)}) / 2 + E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(7/2)} + (105 \cdot \operatorname{Sqrt}[\operatorname{Pi}] \cdot \operatorname{Erfi}[\operatorname{Sqrt}[a + b \cdot x + c \cdot x^2]]) / 16$

**Rubi [A]** time = 0.633317, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2176, 2180, 2204}

$$\frac{105}{16} \sqrt{\pi} \operatorname{Erfi} \left( \sqrt{a + bx + cx^2} \right) + e^{a+bx+cx^2} (a + bx + cx^2)^{7/2} - \frac{7}{2} e^{a+bx+cx^2} (a + bx + cx^2)^{5/2} + \frac{35}{4} e^{a+bx+cx^2} (a + bx + cx^2)^3$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a + b \cdot x + c \cdot x^2)} \cdot (b + 2 \cdot c \cdot x) \cdot (a + b \cdot x + c \cdot x^2)^{(7/2)}, x]$

[Out]  $(-105 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot \operatorname{Sqrt}[a + b \cdot x + c \cdot x^2]) / 8 + (35 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(3/2)}) / 4 - (7 \cdot E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(5/2)}) / 2 + E^{(a + b \cdot x + c \cdot x^2)} \cdot (a + b \cdot x + c \cdot x^2)^{(7/2)} + (105 \cdot \operatorname{Sqrt}[\operatorname{Pi}] \cdot \operatorname{Erfi}[\operatorname{Sqrt}[a + b \cdot x + c \cdot x^2]]) / 16$

#### Rule 6707

$\operatorname{Int}[(F_)^{(v_*)} \cdot (u_*) \cdot (w_)^{(m_*)}, x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m \cdot F^x, x], x, v], x] /; \text{!FalseQ}[q] /; \operatorname{FreeQ}[\{F, m\}, x] \ \&\& \operatorname{EqQ}[w, v]$

#### Rule 2176

$\operatorname{Int}[(b_*) \cdot (F_)^{((g_*) \cdot ((e_*) + (f_*) \cdot (x_)))^{(n_*)} \cdot ((c_*) + (d_*) \cdot (x_))^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d \cdot x)^m \cdot (b \cdot F^{(g \cdot (e + f \cdot x)))^n} / (f \cdot g \cdot n \cdot \operatorname{Log}[F]), x] - \operatorname{Dist}[(d \cdot m) / (f \cdot g \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(c + d \cdot x)^{(m-1)} \cdot (b \cdot F^{(g \cdot (e + f \cdot x)))^n}, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \operatorname{GtQ}[m, 0] \ \&\& \operatorname{IntegerQ}[2 \cdot m] \ \&\& \text{!UseGamma} == \text{True}$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_*) \cdot ((e_*) + (f_*) \cdot (x_))) / \operatorname{Sqrt}[(c_*) + (d_*) \cdot (x_)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g \cdot (e - (c \cdot f)/d) + (f \cdot g \cdot x^2)/d)}, x], x, \operatorname{Sqrt}[c + d \cdot x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \text{!UseGamma} == \text{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_*) + (b_*) \cdot ((c_*) + (d_*) \cdot (x_))^{2})}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a \cdot \operatorname{Sqrt}[\operatorname{Pi}] \cdot \operatorname{Erfi}[(c + d \cdot x) \cdot \operatorname{Rt}[b \cdot \operatorname{Log}[F], 2]]) / (2 \cdot d \cdot \operatorname{Rt}[b \cdot \operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int e^{a+bx+cx^2}(b+2cx)(a+bx+cx^2)^{7/2} dx &= \text{Subst}\left(\int e^x x^{7/2} dx, x, a+bx+cx^2\right) \\
&= e^{a+bx+cx^2}(a+bx+cx^2)^{7/2} - \frac{7}{2} \text{Subst}\left(\int e^x x^{5/2} dx, x, a+bx+cx^2\right) \\
&= -\frac{7}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{7/2} + \frac{35}{4} \text{Subst}\left(\int e^x x^{3/2} dx, x, a+bx+cx^2\right) \\
&= \frac{35}{4} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} - \frac{7}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{7/2} \\
&= -\frac{105}{8} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + \frac{35}{4} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} - \frac{7}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} \\
&= -\frac{105}{8} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + \frac{35}{4} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} - \frac{7}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} \\
&= -\frac{105}{8} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + \frac{35}{4} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} - \frac{7}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.196046, size = 47, normalized size = 0.33

$$\frac{\sqrt{a+x(b+cx)} \text{Gamma}\left(\frac{9}{2}, -a-x(b+cx)\right)}{\sqrt{-a-x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(7/2), x]

[Out] -((Sqrt[a + x\*(b + c\*x)]\*Gamma[9/2, -a - x\*(b + c\*x)])/Sqrt[-a - x\*(b + c\*x)])

**Maple [A]** time = 0.042, size = 119, normalized size = 0.8

$$\frac{35 e^{cx^2+bx+a}}{4} (cx^2 + bx + a)^{\frac{3}{2}} - \frac{7 e^{cx^2+bx+a}}{2} (cx^2 + bx + a)^{\frac{5}{2}} + e^{cx^2+bx+a} (cx^2 + bx + a)^{\frac{7}{2}} + \frac{105 \sqrt{\pi}}{16} \text{erfi}\left(\sqrt{cx^2 + bx + a}\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(7/2), x)

[Out] 35/4\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(3/2)-7/2\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(5/2)+exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(7/2)+105/16\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-105/8\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{7}{2}} (2cx + b) e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(7/2), x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(7/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral((2\*c^4\*x^7 + 7\*b\*c^3\*x^6 + 3\*(3\*b^2\*c^2 + 2\*a\*c^3)\*x^5 + 5\*(b^3\*c + 3\*abc^2)\*x^4 + a^3\*b + (b^4 + 12\*ab^2\*c + 6\*a^2\*c^2)\*x^3 + 3\*(ab^3 + 3\*a^2\*b\*c)\*x^2 + (3\*a^2\*b^2 + 2\*a^3\*c)\*x)\*sqrt(c\*x^2 + b\*x + a)\*e^(c\*x^2 + b\*x + a), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(7/2),x, algorithm="fricas")

[Out] integral((2\*c^4\*x^7 + 7\*b\*c^3\*x^6 + 3\*(3\*b^2\*c^2 + 2\*a\*c^3)\*x^5 + 5\*(b^3\*c + 3\*a\*b\*c^2)\*x^4 + a^3\*b + (b^4 + 12\*a\*b^2\*c + 6\*a^2\*c^2)\*x^3 + 3\*(a\*b^3 + 3\*a^2\*b\*c)\*x^2 + (3\*a^2\*b^2 + 2\*a^3\*c)\*x)\*sqrt(c\*x^2 + b\*x + a)\*e^(c\*x^2 + b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{7}{2}}(2cx + b)e^{(cx^2 + bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(7/2),x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)^(7/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

$$3.628 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^{5/2} dx$$

**Optimal.** Leaf size=112

$$-\frac{15}{8}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} - \frac{5}{2}e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + \frac{15}{4}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}$$

[Out] (15\*E^(a + b\*x + c\*x^2)\*Sqrt[a + b\*x + c\*x^2])/4 - (5\*E^(a + b\*x + c\*x^2)\*(a + b\*x + c\*x^2)^(3/2))/2 + E^(a + b\*x + c\*x^2)\*(a + b\*x + c\*x^2)^(5/2) - (15\*Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]])/8

**Rubi [A]** time = 0.45702, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2176, 2180, 2204}

$$-\frac{15}{8}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} - \frac{5}{2}e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + \frac{15}{4}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(5/2), x]

[Out] (15\*E^(a + b\*x + c\*x^2)\*Sqrt[a + b\*x + c\*x^2])/4 - (5\*E^(a + b\*x + c\*x^2)\*(a + b\*x + c\*x^2)^(3/2))/2 + E^(a + b\*x + c\*x^2)\*(a + b\*x + c\*x^2)^(5/2) - (15\*Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]])/8

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2180

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int e^{a+bx+cx^2}(b+2cx)(a+bx+cx^2)^{5/2} dx &= \text{Subst}\left(\int e^x x^{5/2} dx, x, a+bx+cx^2\right) \\
&= e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} - \frac{5}{2} \text{Subst}\left(\int e^x x^{3/2} dx, x, a+bx+cx^2\right) \\
&= -\frac{5}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} + \frac{15}{4} \text{Subst}\left(\int e^x dx, x, a+bx+cx^2\right) \\
&= \frac{15}{4} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \frac{5}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} \\
&= \frac{15}{4} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \frac{5}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2} \\
&= \frac{15}{4} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \frac{5}{2} e^{a+bx+cx^2}(a+bx+cx^2)^{3/2} + e^{a+bx+cx^2}(a+bx+cx^2)^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.124051, size = 46, normalized size = 0.41

$$\frac{\sqrt{a+x(b+cx)} \Gamma\left(\frac{7}{2}, -a-x(b+cx)\right)}{\sqrt{-a-x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(5/2), x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*Gamma[7/2, -a - x\*(b + c\*x)])/Sqrt[-a - x\*(b + c\*x)]

**Maple [A]** time = 0.041, size = 94, normalized size = 0.8

$$-\frac{5 e^{cx^2+bx+a}}{2} (cx^2 + bx + a)^{\frac{3}{2}} + e^{cx^2+bx+a} (cx^2 + bx + a)^{\frac{5}{2}} - \frac{15 \sqrt{\pi}}{8} \operatorname{erfi}\left(\sqrt{cx^2 + bx + a}\right) + \frac{15 e^{cx^2+bx+a}}{4} \sqrt{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(5/2), x)

[Out] -5/2\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(3/2)+exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(5/2)-15/8\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)+15/4\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{5}{2}} (2cx + b) e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(5/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2c^3x^5 + 5bc^2x^4 + 4(b^2c + ac^2)x^3 + a^2b + (b^3 + 6abc)x^2 + 2(ab^2 + a^2c)x\right)\sqrt{cx^2 + bx + a}e^{(cx^2+bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out] integral((2\*c^3\*x^5 + 5\*b\*c^2\*x^4 + 4\*(b^2\*c + a\*c^2)\*x^3 + a^2\*b + (b^3 + 6\*a\*b\*c)\*x^2 + 2\*(a\*b^2 + a^2\*c)\*x)\*sqrt(c\*x^2 + b\*x + a)\*e^(c\*x^2 + b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{5}{2}}(2cx + b)e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)^(5/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)



$$3.629 \quad \int e^{a+bx+cx^2} (b + 2cx) (a + bx + cx^2)^{3/2} dx$$

**Optimal.** Leaf size=82

$$\frac{3}{4}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) + e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} - \frac{3}{2}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}$$

[Out]  $(-3E^{(a+bx+cx^2)}\sqrt{a+bx+cx^2})/2 + E^{(a+bx+cx^2)}(a+bx+cx^2)^{3/2} + (3\sqrt{\pi}\operatorname{Erfi}[\sqrt{a+bx+cx^2}])/4$

**Rubi [A]** time = 0.362943, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2176, 2180, 2204}

$$\frac{3}{4}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) + e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} - \frac{3}{2}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a+bx+cx^2)}(b+2cx)(a+bx+cx^2)^{3/2}, x]$

[Out]  $(-3E^{(a+bx+cx^2)}\sqrt{a+bx+cx^2})/2 + E^{(a+bx+cx^2)}(a+bx+cx^2)^{3/2} + (3\sqrt{\pi}\operatorname{Erfi}[\sqrt{a+bx+cx^2}])/4$

#### Rule 6707

$\operatorname{Int}[(F_)^{(v_)}(u_)(w_)^{(m_)}, x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \operatorname{FreeQ}[\{F, m\}, x] \ \&\& \operatorname{EqQ}[w, v]$

#### Rule 2176

$\operatorname{Int}[(b_)(F_)^{(g_)((e_)+(f_)(x_))}^{(n_)}((c_)+(d_)(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m(bF^{(g(e+f*x)))})^n/(f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}(bF^{(g(e+f*x)))})^n, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \operatorname{GtQ}[m, 0] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& \text{!UseGamma} === \operatorname{True}$

#### Rule 2180

$\operatorname{Int}[(F_)^{(g_)((e_)+(f_)(x_))}/\sqrt{(c_)+(d_)(x_)}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g(e-(c*f)/d)+(f*g*x^2)/d)}, x], x, \sqrt{c+d*x}], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \text{!UseGamma} === \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{(a_)+(b_)((c_)+(d_)(x_))^2}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\sqrt{\pi}\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int e^{a+bx+cx^2} (b+2cx) (a+bx+cx^2)^{3/2} dx &= \text{Subst} \left( \int e^x x^{3/2} dx, x, a+bx+cx^2 \right) \\
&= e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} - \frac{3}{2} \text{Subst} \left( \int e^x \sqrt{x} dx, x, a+bx+cx^2 \right) \\
&= -\frac{3}{2} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} + \frac{3}{4} \text{Subst} \left( \int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{3}{2} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} + \frac{3}{2} \text{Subst} \left( \int e^{x^2} dx, x, a+bx+cx^2 \right) \\
&= -\frac{3}{2} e^{a+bx+cx^2} \sqrt{a+bx+cx^2} + e^{a+bx+cx^2} (a+bx+cx^2)^{3/2} + \frac{3}{4} \sqrt{\pi} \text{erfi} \left( \sqrt{a+bx+cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.109704, size = 47, normalized size = 0.57

$$-\frac{\sqrt{a+x(b+cx)} \Gamma\left(\frac{5}{2}, -a-x(b+cx)\right)}{\sqrt{-a-x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2), x]

[Out] -((Sqrt[a + x\*(b + c\*x)]\*Gamma[5/2, -a - x\*(b + c\*x)])/Sqrt[-a - x\*(b + c\*x)])

**Maple [A]** time = 0.041, size = 69, normalized size = 0.8

$$e^{cx^2+bx+a} (cx^2 + bx + a)^{\frac{3}{2}} + \frac{3\sqrt{\pi}}{4} \text{erfi} \left( \sqrt{cx^2 + bx + a} \right) - \frac{3e^{cx^2+bx+a}}{2} \sqrt{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(3/2), x)

[Out] exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(3/2)+3/4\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-3/2\*exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{3}{2}} (2cx + b) e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(3/2), x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^(3/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (2c^2x^3 + 3bcx^2 + ab + (b^2 + 2ac)x) \sqrt{cx^2 + bx + a} e^{(cx^2+bx+a)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] integral((2\*c^2\*x^3 + 3\*b\*c\*x^2 + a\*b + (b^2 + 2\*a\*c)\*x)\*sqrt(c\*x^2 + b\*x + a)\*e^(c\*x^2 + b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^{\frac{3}{2}}(2cx + b)e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)^(3/2)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

### 3.630 $\int e^{a+bx+cx^2} (b + 2cx) \sqrt{a + bx + cx^2} dx$

**Optimal.** Leaf size=52

$$e^{a+bx+cx^2} \sqrt{a + bx + cx^2} - \frac{1}{2} \sqrt{\pi} \operatorname{Erfi} \left( \sqrt{a + bx + cx^2} \right)$$

[Out]  $E^{(a + b*x + c*x^2)*\operatorname{Sqrt}[a + b*x + c*x^2]} - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*x + c*x^2]])/2$

**Rubi [A]** time = 0.234901, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2176, 2180, 2204}

$$e^{a+bx+cx^2} \sqrt{a + bx + cx^2} - \frac{1}{2} \sqrt{\pi} \operatorname{Erfi} \left( \sqrt{a + bx + cx^2} \right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a + b*x + c*x^2)}*(b + 2*c*x)*\operatorname{Sqrt}[a + b*x + c*x^2], x]$

[Out]  $E^{(a + b*x + c*x^2)*\operatorname{Sqrt}[a + b*x + c*x^2]} - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*x + c*x^2]])/2$

#### Rule 6707

$\operatorname{Int}[(F_)^{(v_*)}*(u_*)*(w_)^{(m_*)}, x\_Symbol] := \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \operatorname{FreeQ}[\{F, m\}, x] \ \&\& \operatorname{EqQ}[w, v]$

#### Rule 2176

$\operatorname{Int}[(b_*)*(F_)^{((g_*)*((e_*) + (f_*)*(x_)))^{(n_*)}*((c_*) + (d_*)*(x_)))^{(m_*)}, x\_Symbol] := \operatorname{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \operatorname{GtQ}[m, 0] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& \text{!}\$UseGamma == True$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_*)*((e_*) + (f_*)*(x_)))/\operatorname{Sqrt}[(c_*) + (d_*)*(x_)]}, x\_Symbol] := \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \text{!}\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_*) + (b_*)*((c_*) + (d_*)*(x_))^2)}, x\_Symbol] := \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2} dx &= \text{Subst} \left( \int e^x \sqrt{x} dx, x, a+bx+cx^2 \right) \\
&= e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \frac{1}{2} \text{Subst} \left( \int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2 \right) \\
&= e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \text{Subst} \left( \int e^{x^2} dx, x, \sqrt{a+bx+cx^2} \right) \\
&= e^{a+bx+cx^2} \sqrt{a+bx+cx^2} - \frac{1}{2} \sqrt{\pi} \text{erfi} \left( \sqrt{a+bx+cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0475016, size = 46, normalized size = 0.88

$$\frac{\sqrt{a+x(b+cx)} \text{Gamma} \left( \frac{3}{2}, -a-x(b+cx) \right)}{\sqrt{-a-x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*Gamma[3/2, -a - x\*(b + c\*x)])/Sqrt[-a - x\*(b + c\*x)]

**Maple [A]** time = 0.04, size = 44, normalized size = 0.9

$$-\frac{\sqrt{\pi}}{2} \text{erfi} \left( \sqrt{cx^2 + bx + a} \right) + e^{cx^2+bx+a} \sqrt{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2), x)

[Out] -1/2\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)+exp(c\*x^2+b\*x+a)\*(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a} (2cx + b) e^{(cx^2+bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \sqrt{cx^2 + bx + a} (2cx + b) e^{(cx^2+bx+a)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a), x)

**Sympy [A]** time = 12.8545, size = 78, normalized size = 1.5

$$\frac{\left(\sqrt{-a - bx - cx^2}e^{a+bx+cx^2} + \frac{\sqrt{\pi} \operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right)}{2}\right)\sqrt{a + bx + cx^2}}{\sqrt{-a - bx - cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] (sqrt(-a - b\*x - c\*x\*\*2)\*exp(a + b\*x + c\*x\*\*2) + sqrt(pi)\*erfc(sqrt(-a - b\*x - c\*x\*\*2))/2)\*sqrt(a + b\*x + c\*x\*\*2)/sqrt(-a - b\*x - c\*x\*\*2)

**Giac [A]** time = 1.25469, size = 63, normalized size = 1.21

$$-\frac{1}{2}\sqrt{\pi}i \operatorname{erf}\left(-\sqrt{cx^2 + bx + ai}\right) + \sqrt{cx^2 + bx + a}e^{(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] -1/2\*sqrt(pi)\*i\*erf(-sqrt(c\*x^2 + b\*x + a)\*i) + sqrt(c\*x^2 + b\*x + a)\*e^(c\*x^2 + b\*x + a)

$$3.631 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=21

$$\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right)$$

[Out] Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]]

**Rubi [A]** time = 0.263932, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6707, 2180, 2204}

$$\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/Sqrt[a + b\*x + c\*x^2], x]

[Out] Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]]

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_), x\_Symbol] :> With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2180

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}} dx &= \operatorname{Subst}\left(\int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2\right) \\ &= 2 \operatorname{Subst}\left(\int e^{x^2} dx, x, \sqrt{a+bx+cx^2}\right) \\ &= \sqrt{\pi} \operatorname{erfi}\left(\sqrt{a+bx+cx^2}\right) \end{aligned}$$

**Mathematica [B]** time = 0.0589681, size = 46, normalized size = 2.19

$$\frac{\sqrt{-a-x(b+cx)} \operatorname{Gamma}\left(\frac{1}{2}, -a-x(b+cx)\right)}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[-a - x\*(b + c\*x)]\*Gamma[1/2, -a - x\*(b + c\*x)])/Sqrt[a + x\*(b + c\*x)]

**Maple [A]** time = 0.043, size = 18, normalized size = 0.9

$$\operatorname{erfi}\left(\sqrt{cx^2 + bx + a}\right)\sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/sqrt(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(2cx + b)e^{(cx^2+bx+a)}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out] integral((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/sqrt(c\*x^2 + b\*x + a), x)

**Sympy [B]** time = 5.72087, size = 49, normalized size = 2.33

$$\frac{\sqrt{\pi}\sqrt{-a - bx - cx^2} \operatorname{erfc}\left(\sqrt{-a - bx - cx^2}\right)}{\sqrt{a + bx + cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] sqrt(pi)*sqrt(-a - b*x - c*x**2)*erfc(sqrt(-a - b*x - c*x**2))/sqrt(a + b*x
+ c*x**2)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/sqrt(c*x^2 + b*x + a), x)
```

$$3.632 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=51

$$2\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}$$

[Out] (-2\*E^(a + b\*x + c\*x^2))/Sqrt[a + b\*x + c\*x^2] + 2\*Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]]

**Rubi [A]** time = 0.314806, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2177, 2180, 2204}

$$2\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (-2\*E^(a + b\*x + c\*x^2))/Sqrt[a + b\*x + c\*x^2] + 2\*Sqrt[Pi]\*Erfi[Sqrt[a + b\*x + c\*x^2]]

#### Rule 6707

Int[(F\_)^(v\_)\*(u\_)\*(w\_)^(m\_), x\_Symbol] := With[{q = DerivativeDivides[v, u, x]}, Dist[q, Subst[Int[x^m\*F^x, x], x, v], x] /; !FalseQ[q]] /; FreeQ[{F, m}, x] && EqQ[w, v]

#### Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2180

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}} dx &= \text{Subst} \left( \int \frac{e^x}{x^{3/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}} + 2 \text{Subst} \left( \int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}} + 4 \text{Subst} \left( \int e^{x^2} dx, x, \sqrt{a+bx+cx^2} \right) \\
&= -\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}} + 2\sqrt{\pi} \text{erfi} \left( \sqrt{a+bx+cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.095154, size = 62, normalized size = 1.22

$$\frac{2\sqrt{-a-x(b+cx)}\Gamma\left(\frac{1}{2}, -a-x(b+cx)\right) - 2e^{a+x(b+cx)}}{\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (-2\*E^(a + x\*(b + c\*x)) + 2\*Sqrt[-a - x\*(b + c\*x)]\*Gamma[1/2, -a - x\*(b + c\*x)]) / Sqrt[a + x\*(b + c\*x)]

**Maple [A]** time = 0.039, size = 45, normalized size = 0.9

$$2 \operatorname{erfi} \left( \sqrt{cx^2 + bx + a} \right) \sqrt{\pi} - 2 \frac{e^{cx^2+bx+a}}{\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(3/2), x)

[Out] 2\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-2\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)e^{(cx^2+bx+a)}}{c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c^2\*x^4 + 2\*b\*c\*x^3 + 2\*a\*b\*x + (b^2 + 2\*a\*c)\*x^2 + a^2), x)

**Sympy [A]** time = 9.46166, size = 80, normalized size = 1.57

$$\frac{\left(-2\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right) + \frac{2e^{a+bx+cx^2}}{\sqrt{-a-bx-cx^2}}\right)(-a-bx-cx^2)^{\frac{3}{2}}}{(a+bx+cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] (-2\*sqrt(pi)\*erfc(sqrt(-a - b\*x - c\*x\*\*2)) + 2\*exp(a + b\*x + c\*x\*\*2)/sqrt(-a - b\*x - c\*x\*\*2))\*(-a - b\*x - c\*x\*\*2)\*\*(3/2)/(a + b\*x + c\*x\*\*2)\*\*(3/2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(3/2), x)

$$3.633 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=85

$$\frac{4}{3}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} - \frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

[Out]  $(-2E^{(a + b*x + c*x^2)})/(3*(a + b*x + c*x^2)^{(3/2)}) - (4E^{(a + b*x + c*x^2)})/(3*\operatorname{Sqrt}[a + b*x + c*x^2]) + (4*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*x + c*x^2]])/3$

**Rubi [A]** time = 0.348027, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2177, 2180, 2204}

$$\frac{4}{3}\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} - \frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{(a + b*x + c*x^2)}*(b + 2*c*x))/(a + b*x + c*x^2)^{(5/2)}, x]$

[Out]  $(-2E^{(a + b*x + c*x^2)})/(3*(a + b*x + c*x^2)^{(3/2)}) - (4E^{(a + b*x + c*x^2)})/(3*\operatorname{Sqrt}[a + b*x + c*x^2]) + (4*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*x + c*x^2]])/3$

#### Rule 6707

$\operatorname{Int}[(F_)^{(v_)}*(u_)*(w_)^{(m_)}, x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m * F^x, x], x, v], x] /; \text{!FalseQ}[q]] /; \operatorname{FreeQ}[\{F, m\}, x] \&\& \operatorname{EqQ}[w, v]$

#### Rule 2177

$\operatorname{Int}[(b_)*(F_)^{((g_)*((e_.) + (f_)*(x_)))^{(n_)}*((c_.) + (d_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*(b * F^{(g*(e + f*x))})^n / (d*(m + 1)), x] - \operatorname{Dist}[(f * g * n * \operatorname{Log}[F]) / (d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b * F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& \text{!}\$UseGamma == True$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_)*((e_.) + (f_)*(x_))) / \operatorname{Sqrt}[(c_.) + (d_)*(x_)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_)*((c_.) + (d_)*(x_))^{2})}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]) / (2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}} dx &= \text{Subst} \left( \int \frac{e^x}{x^{5/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}} + \frac{2}{3} \text{Subst} \left( \int \frac{e^x}{x^{3/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} + \frac{4}{3} \text{Subst} \left( \int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} + \frac{8}{3} \text{Subst} \left( \int e^{x^2} dx, x, \sqrt{a+bx+cx^2} \right) \\
&= -\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} + \frac{4}{3} \sqrt{\pi} \text{erfi} \left( \sqrt{a+bx+cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.127352, size = 77, normalized size = 0.91

$$\frac{2 \left( 2(-a - x(b + cx))^{3/2} \text{Gamma} \left( \frac{1}{2}, -a - x(b + cx) \right) + e^{a+x(b+cx)}(2(a + x(b + cx)) + 1) \right)}{3(a + x(b + cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*(E^(a + x\*(b + c\*x))\*(1 + 2\*(a + x\*(b + c\*x))) + 2\*(-a - x\*(b + c\*x))^(3/2)\*Gamma[1/2, -a - x\*(b + c\*x)])/(3\*(a + x\*(b + c\*x))^(3/2))

**Maple [A]** time = 0.041, size = 70, normalized size = 0.8

$$-\frac{2e^{cx^2+bx+a}}{3} (cx^2 + bx + a)^{-\frac{3}{2}} + \frac{4\sqrt{\pi}}{3} \text{erfi} \left( \sqrt{cx^2 + bx + a} \right) - \frac{4e^{cx^2+bx+a}}{3} \frac{1}{\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(5/2), x)

[Out] -2/3\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(3/2)+4/3\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-4/3\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)e^{(cx^2+bx+a)}}{c^3x^6 + 3bc^2x^5 + 3(b^2c + ac^2)x^4 + 3a^2bx + (b^3 + 6abc)x^3 + a^3 + 3(ab^2 + a^2c)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c^3\*x^6 + 3\*b\*c^2\*x^5 + 3\*(b^2\*c + a\*c^2)\*x^4 + 3\*a^2\*b\*x + (b^3 + 6\*a\*b\*c)\*x^3 + a^3 + 3\*(a\*b^2 + a^2\*c)\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*(5/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(5/2), x)

$$3.634 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}} dx$$

**Optimal.** Leaf size=115

$$\frac{8}{15} \sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} - \frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}$$

[Out]  $(-2 * E^{(a + b * x + c * x^2)}) / (5 * (a + b * x + c * x^2)^{(5/2)}) - (4 * E^{(a + b * x + c * x^2)}) / (15 * (a + b * x + c * x^2)^{(3/2)}) - (8 * E^{(a + b * x + c * x^2)}) / (15 * \operatorname{Sqrt}[a + b * x + c * x^2]) + (8 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[a + b * x + c * x^2]]) / 15$

**Rubi [A]** time = 0.35466, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2177, 2180, 2204}

$$\frac{8}{15} \sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} - \frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{(a + b * x + c * x^2)} * (b + 2 * c * x)) / (a + b * x + c * x^2)^{(7/2)}, x]$

[Out]  $(-2 * E^{(a + b * x + c * x^2)}) / (5 * (a + b * x + c * x^2)^{(5/2)}) - (4 * E^{(a + b * x + c * x^2)}) / (15 * (a + b * x + c * x^2)^{(3/2)}) - (8 * E^{(a + b * x + c * x^2)}) / (15 * \operatorname{Sqrt}[a + b * x + c * x^2]) + (8 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[a + b * x + c * x^2]]) / 15$

#### Rule 6707

$\operatorname{Int}[(F_)^{(v_)} * (u_) * (w_)^{(m_)}, x\_Symbol] := \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m * F^x, x], x, v], x] /; \operatorname{!FalseQ}[q] /; \operatorname{FreeQ}[\{F, m\}, x] \&\& \operatorname{EqQ}[w, v]$

#### Rule 2177

$\operatorname{Int}[(b_.) * (F_)^{((g_.) * ((e_.) + (f_.) * (x_)))}^{(n_)} * ((c_.) + (d_.) * (x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[(c + d * x)^{(m + 1)} * (b * F^{(g * (e + f * x))})^n / (d * (m + 1)), x] - \operatorname{Dist}[(f * g * n * \operatorname{Log}[F]) / (d * (m + 1)), \operatorname{Int}[(c + d * x)^{(m + 1)} * (b * F^{(g * (e + f * x))})^n, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2 * m] \&\& \operatorname{!UseGamma} == \operatorname{True}$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_)))} / \operatorname{Sqrt}[(c_.) + (d_.) * (x_)], x\_Symbol] := \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g * (e - (c * f) / d) + (f * g * x^2) / d)}, x], x, \operatorname{Sqrt}[c + d * x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \&\& \operatorname{!UseGamma} == \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{2})}, x\_Symbol] := \operatorname{Simp}[(F^{a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d * x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]]}) / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rubi steps



$$\begin{aligned}
\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}} dx &= \text{Subst} \left( \int \frac{e^x}{x^{7/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}} + \frac{2}{5} \text{Subst} \left( \int \frac{e^x}{x^{5/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} + \frac{4}{15} \text{Subst} \left( \int \frac{e^x}{x^{3/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} - \frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}} + \frac{8}{15} \text{Subst} \left( \int \frac{e^x}{\sqrt{x}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} - \frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}} + \frac{16}{15} \text{Subst} \left( \int e^{x^2} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}} - \frac{4e^{a+bx+cx^2}}{15(a+bx+cx^2)^{3/2}} - \frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}} + \frac{8}{15} \sqrt{\pi} \text{erfi} \left( \sqrt{a+bx+cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.151417, size = 91, normalized size = 0.79

$$\frac{8(-a-x(b+cx))^{5/2} \text{Gamma}\left(\frac{1}{2}, -a-x(b+cx)\right) - 2e^{a+x(b+cx)}(4(a+x(b+cx))^2 + 2(a+x(b+cx)) + 3)}{15(a+x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^(7/2), x]

[Out] (-2\*E^(a + x\*(b + c\*x))\*(3 + 2\*(a + x\*(b + c\*x)) + 4\*(a + x\*(b + c\*x))^2) + 8\*(-a - x\*(b + c\*x))^(5/2)\*Gamma[1/2, -a - x\*(b + c\*x)]/(15\*(a + x\*(b + c\*x))^(5/2))

**Maple [A]** time = 0.051, size = 95, normalized size = 0.8

$$-\frac{2e^{cx^2+bx+a}}{5}(cx^2+bx+a)^{-\frac{5}{2}} - \frac{4e^{cx^2+bx+a}}{15}(cx^2+bx+a)^{-\frac{3}{2}} + \frac{8\sqrt{\pi}}{15}\text{erfi}\left(\sqrt{cx^2+bx+a}\right) - \frac{8e^{cx^2+bx+a}}{15}\frac{1}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(7/2), x)

[Out] -2/5\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(5/2)-4/15\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(3/2)+8/15\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-8/15\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(7/2),x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)e^{(cx^2+bx+a)}}{c^4x^8 + 4bc^3x^7 + 2(3b^2c^2 + 2ac^3)x^6 + 4(b^3c + 3abc^2)x^5 + 4a^3bx + (b^4 + 12ab^2c + 6a^2c^2)x^4 + a^4 + 4(ab^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c^4\*x^8 + 4\*b\*c^3\*x^7 + 2\*(3\*b^2\*c^2 + 2\*a\*c^3)\*x^6 + 4\*(b^3\*c + 3\*a\*b\*c^2)\*x^5 + 4\*a^3\*b\*x + (b^4 + 12\*a\*b^2\*c + 6\*a^2\*c^2)\*x^4 + a^4 + 4\*(a\*b^3 + 3\*a^2\*b\*c)\*x^3 + 2\*(3\*a^2\*b^2 + 2\*a^3\*c)\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2+bx+a)}}{(cx^2 + bx + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(7/2),x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(7/2), x)

$$3.635 \quad \int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}} dx$$

**Optimal.** Leaf size=145

$$\frac{16}{105} \sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}}$$

[Out]  $(-2 * E^{(a + b * x + c * x^2)}) / (7 * (a + b * x + c * x^2)^{(7/2)}) - (4 * E^{(a + b * x + c * x^2)}) / (35 * (a + b * x + c * x^2)^{(5/2)}) - (8 * E^{(a + b * x + c * x^2)}) / (105 * (a + b * x + c * x^2)^{(3/2)}) - (16 * E^{(a + b * x + c * x^2)}) / (105 * \operatorname{Sqrt}[a + b * x + c * x^2]) + (16 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[a + b * x + c * x^2]]) / 105$

**Rubi [A]** time = 0.379406, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {6707, 2177, 2180, 2204}

$$\frac{16}{105} \sqrt{\pi} \operatorname{Erfi}\left(\sqrt{a+bx+cx^2}\right) - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{(a + b * x + c * x^2)} * (b + 2 * c * x)) / (a + b * x + c * x^2)^{(9/2)}, x]$

[Out]  $(-2 * E^{(a + b * x + c * x^2)}) / (7 * (a + b * x + c * x^2)^{(7/2)}) - (4 * E^{(a + b * x + c * x^2)}) / (35 * (a + b * x + c * x^2)^{(5/2)}) - (8 * E^{(a + b * x + c * x^2)}) / (105 * (a + b * x + c * x^2)^{(3/2)}) - (16 * E^{(a + b * x + c * x^2)}) / (105 * \operatorname{Sqrt}[a + b * x + c * x^2]) + (16 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[a + b * x + c * x^2]]) / 105$

#### Rule 6707

$\operatorname{Int}[(F\_)^{(v\_)} * (u\_)* (w\_)^{(m\_)}], x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{DerivativeDivides}[v, u, x]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^m * F^x, x], x, v], x] /; \text{!FalseQ}[q] /; \operatorname{FreeQ}[\{F, m\}, x] \&\& \operatorname{EqQ}[w, v]$

#### Rule 2177

$\operatorname{Int}[(b\_)* (F\_)^{((g\_)* ((e\_)* (f\_)* (x\_)))^{(n\_)} * ((c\_)* (d\_)* (x\_))^{(m\_)}], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d * x)^{(m + 1)} * (b * F^{(g * (e + f * x))})^{(n)} / (d * (m + 1)), x] - \operatorname{Dist}[(f * g * n * \operatorname{Log}[F]) / (d * (m + 1)), \operatorname{Int}[(c + d * x)^{(m + 1)} * (b * F^{(g * (e + f * x))})^{(n)}, x], x] /; \operatorname{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2 * m] \&\& \text{!UseGamma} == \text{True}$

#### Rule 2180

$\operatorname{Int}[(F\_)^{((g\_)* ((e\_)* (f\_)* (x\_))) / \operatorname{Sqrt}[(c\_)* (d\_)* (x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g * (e - (c * f) / d) + (f * g * x^2) / d)}, x], x, \operatorname{Sqrt}[c + d * x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \&\& \text{!UseGamma} == \text{True}$

#### Rule 2204

$\operatorname{Int}[(F\_)^{((a\_)* (b\_)* ((c\_)* (d\_)* (x\_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d * x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]]) / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}} dx &= \text{Subst} \left( \int \frac{e^x}{x^{9/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} + \frac{2}{7} \text{Subst} \left( \int \frac{e^x}{x^{7/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} + \frac{4}{35} \text{Subst} \left( \int \frac{e^x}{x^{5/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} + \frac{8}{105} \text{Subst} \left( \int \frac{e^x}{x^{3/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} + \frac{16}{105} \text{Subst} \left( \int \frac{e^x}{x^{1/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} + \frac{32}{105} \text{Subst} \left( \int \frac{e^x}{x^{1/2}} dx, x, a+bx+cx^2 \right) \\
&= -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} + \frac{16}{105} \text{Subst} \left( \int \frac{e^x}{x^{1/2}} dx, x, a+bx+cx^2 \right)
\end{aligned}$$

**Mathematica [A]** time = 0.193919, size = 103, normalized size = 0.71

$$\frac{2 \left( 8(-a-x(b+cx))^{7/2} \text{Gamma} \left( \frac{1}{2}, -a-x(b+cx) \right) + e^{a+x(b+cx)} \left( 8(a+x(b+cx))^3 + 4(a+x(b+cx))^2 + 6(a+x(b+cx)) \right) \right)}{105(a+x(b+cx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a + b\*x + c\*x^2)\*(b + 2\*c\*x))/(a + b\*x + c\*x^2)^(9/2), x]

[Out] (-2\*(E^(a + x\*(b + c\*x)))\*(15 + 6\*(a + x\*(b + c\*x)) + 4\*(a + x\*(b + c\*x))^2 + 8\*(a + x\*(b + c\*x))^3) + 8\*(-a - x\*(b + c\*x))^(7/2)\*Gamma[1/2, -a - x\*(b + c\*x)])/(105\*(a + x\*(b + c\*x))^(7/2))

**Maple [A]** time = 0.041, size = 120, normalized size = 0.8

$$-\frac{2e^{cx^2+bx+a}}{7} (cx^2+bx+a)^{-\frac{7}{2}} - \frac{4e^{cx^2+bx+a}}{35} (cx^2+bx+a)^{-\frac{5}{2}} - \frac{8e^{cx^2+bx+a}}{105} (cx^2+bx+a)^{-\frac{3}{2}} + \frac{16\sqrt{\pi}}{105} \text{erfi} \left( \sqrt{cx^2+bx+a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(9/2), x)

[Out] -2/7\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(7/2)-4/35\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(5/2)-8/105\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(3/2)+16/105\*erfi((c\*x^2+b\*x+a)^(1/2))\*Pi^(1/2)-16/105\*exp(c\*x^2+b\*x+a)/(c\*x^2+b\*x+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2 + bx + a)}}{(cx^2 + bx + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(9/2),x, algorithm="maxima")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(9/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)}{c^5x^{10} + 5bc^4x^9 + 5(2b^2c^3 + ac^4)x^8 + 10(b^3c^2 + 2abc^3)x^7 + 5(b^4c + 6ab^2c^2 + 2a^2c^3)x^6 + 5a^4bx + (b^5 + 2a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(9/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c^5\*x^10 + 5\*b\*c^4\*x^9 + 5\*(2\*b^2\*c^3 + a\*c^4)\*x^8 + 10\*(b^3\*c^2 + 2\*a\*b\*c^3)\*x^7 + 5\*(b^4\*c + 6\*a\*b^2\*c^2 + 2\*a^2\*c^3)\*x^6 + 5\*a^4\*b\*x + (b^5 + 20\*a\*b^3\*c + 30\*a^2\*b\*c^2)\*x^5 + a^5 + 5\*(a\*b^4 + 6\*a^2\*b^2\*c + 2\*a^3\*c^2)\*x^4 + 10\*(a^2\*b^3 + 2\*a^3\*b\*c)\*x^3 + 5\*(2\*a^3\*b^2 + a^4\*c)\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x\*\*2+b\*x+a)\*(2\*c\*x+b)/(c\*x\*\*2+b\*x+a)\*\*(9/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2cx + b)e^{(cx^2 + bx + a)}}{(cx^2 + bx + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*x^2+b\*x+a)\*(2\*c\*x+b)/(c\*x^2+b\*x+a)^(9/2),x, algorithm="giac")

[Out] integrate((2\*c\*x + b)\*e^(c\*x^2 + b\*x + a)/(c\*x^2 + b\*x + a)^(9/2), x)

$$3.636 \quad \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$$

**Optimal.** Leaf size=8

$$-\sin^{-1}(e^{-x})$$

[Out] -ArcSin[E^(-x)]

**Rubi [A]** time = 0.0284385, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2249, 216}

$$-\sin^{-1}(e^{-x})$$

Antiderivative was successfully verified.

[In] Int[1/(E^x\*Sqrt[1 - E^(-2\*x)]),x]

[Out] -ArcSin[E^(-x)]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx &= -\text{Subst} \left( \int \frac{1}{\sqrt{1-x^2}} dx, x, e^{-x} \right) \\ &= -\sin^{-1}(e^{-x}) \end{aligned}$$

**Mathematica [B]** time = 0.017202, size = 42, normalized size = 5.25

$$\frac{e^{-x}\sqrt{e^{2x}-1}\tan^{-1}\left(\sqrt{e^{2x}-1}\right)}{\sqrt{1-e^{-2x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^x\*Sqrt[1 - E^(-2\*x)]),x]

[Out] (Sqrt[-1 + E^(2\*x)]\*ArcTan[Sqrt[-1 + E^(2\*x)]])/(E^x\*Sqrt[1 - E^(-2\*x)])

---

**Maple [B]** time = 0.073, size = 37, normalized size = 4.6

$$-\frac{1}{e^x} \sqrt{(e^x)^2 - 1} \arctan\left(\frac{1}{\sqrt{(e^x)^2 - 1}}\right) \frac{1}{\sqrt{\frac{(e^x)^2 - 1}{(e^x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(x)/(1-1/exp(2\*x))^(1/2), x)

[Out] -1/((exp(x)^2-1)/exp(x)^2)^(1/2)/exp(x)\*(exp(x)^2-1)^(1/2)\*arctan(1/(exp(x)^2-1)^(1/2))

---

**Maxima [B]** time = 1.45803, size = 19, normalized size = 2.38

$$\arctan\left(\sqrt{-e^{(-2x)} + 1}e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(x)/(1-1/exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] arctan(sqrt(-e^(-2\*x) + 1)\*e^x)

---

**Fricas [B]** time = 0.755627, size = 55, normalized size = 6.88

$$2 \arctan\left(\left(\sqrt{-e^{(-2x)} + 1} - 1\right)e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(x)/(1-1/exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] 2\*arctan((sqrt(-e^(-2\*x) + 1) - 1)\*e^x)

---

**Sympy [A]** time = 0.894799, size = 7, normalized size = 0.88

$$-\operatorname{asin}(e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(x)/(1-1/exp(2\*x))\*\*(1/2), x)

[Out] -asin(exp(-x))

---

**Giac [B]** time = 1.3274, size = 19, normalized size = 2.38

$$-\arctan(i) + \arctan\left(\sqrt{e^{(2x)} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1-1/exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(i) + arctan(sqrt(e^(2*x) - 1))
```



$$3.637 \quad \int \frac{e^x}{4+e^{2x}} dx$$

**Optimal.** Leaf size=12

$$\frac{1}{2} \tan^{-1}\left(\frac{e^x}{2}\right)$$

[Out] ArcTan[E^x/2]/2

**Rubi [A]** time = 0.0193594, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 203}

$$\frac{1}{2} \tan^{-1}\left(\frac{e^x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/(4 + E^(2\*x)),x]

[Out] ArcTan[E^x/2]/2

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{4+e^{2x}} dx &= \text{Subst}\left(\int \frac{1}{4+x^2} dx, x, e^x\right) \\ &= \frac{1}{2} \tan^{-1}\left(\frac{e^x}{2}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0026816, size = 12, normalized size = 1.

$$\frac{1}{2} \tan^{-1}\left(\frac{e^x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(4 + E^(2\*x)),x]

[Out] ArcTan[E<sup>x</sup>/2]/2

**Maple [A]** time = 0.02, size = 8, normalized size = 0.7

$$\frac{1}{2} \arctan\left(\frac{e^x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(4+exp(2\*x)),x)

[Out] 1/2\*arctan(1/2\*exp(x))

**Maxima [A]** time = 1.45997, size = 9, normalized size = 0.75

$$\frac{1}{2} \arctan\left(\frac{1}{2} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(4+exp(2\*x)),x, algorithm="maxima")

[Out] 1/2\*arctan(1/2\*e<sup>x</sup>)

**Fricas [A]** time = 0.810531, size = 28, normalized size = 2.33

$$\frac{1}{2} \arctan\left(\frac{1}{2} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(4+exp(2\*x)),x, algorithm="fricas")

[Out] 1/2\*arctan(1/2\*e<sup>x</sup>)

**Sympy [B]** time = 0.107225, size = 15, normalized size = 1.25

$$\text{RootSum}\left(16z^2 + 1, (i \mapsto i \log(8i + e^x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(4+exp(2\*x)),x)

[Out] RootSum(16\*\_z\*\*2 + 1, Lambda(\_i, \_i\*log(8\*\_i + exp(x))))

**Giac [A]** time = 1.23667, size = 9, normalized size = 0.75

$$\frac{1}{2} \arctan\left(\frac{1}{2} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(4+exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/2*e^x)
```

$$3.638 \quad \int \frac{e^x}{1-e^{2x}} dx$$

**Optimal.** Leaf size=4

$$\tanh^{-1}(e^x)$$

[Out] ArcTanh[E^x]

**Rubi [A]** time = 0.0196363, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 206}

$$\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - E^(2\*x)),x]

[Out] ArcTanh[E^x]

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\int \frac{e^x}{1-e^{2x}} dx = \text{Subst} \left( \int \frac{1}{1-x^2} dx, x, e^x \right) = \tanh^{-1}(e^x)$$

**Mathematica [A]** time = 0.0030205, size = 4, normalized size = 1.

$$\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 - E^(2\*x)),x]

[Out] ArcTanh[E^x]

**Maple [A]** time = 0.018, size = 4, normalized size = 1.

$$\operatorname{Artanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1-exp(2*x)),x)`

[Out] `arctanh(exp(x))`

---

**Maxima [B]** time = 0.962192, size = 20, normalized size = 5.

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="maxima")`

[Out] `1/2*log(e^x + 1) - 1/2*log(e^x - 1)`

---

**Fricas [B]** time = 0.735926, size = 50, normalized size = 12.5

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="fricas")`

[Out] `1/2*log(e^x + 1) - 1/2*log(e^x - 1)`

---

**Sympy [B]** time = 0.099919, size = 15, normalized size = 3.75

$$-\frac{\log(e^x - 1)}{2} + \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x)`

[Out] `-log(exp(x) - 1)/2 + log(exp(x) + 1)/2`

---

**Giac [B]** time = 1.27548, size = 22, normalized size = 5.5

$$\frac{1}{2} \log(e^x + 1) - \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x)),x, algorithm="giac")`

[Out] `1/2*log(e^x + 1) - 1/2*log(abs(e^x - 1))`

$$3.639 \quad \int \frac{e^x}{3-4e^{2x}} dx$$

**Optimal.** Leaf size=20

$$\frac{\tanh^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] ArcTanh[(2\*E^x)/Sqrt[3]]/(2\*Sqrt[3])

**Rubi [A]** time = 0.0217914, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 206}

$$\frac{\tanh^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[E^x/(3 - 4\*E^(2\*x)), x]

[Out] ArcTanh[(2\*E^x)/Sqrt[3]]/(2\*Sqrt[3])

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\int \frac{e^x}{3-4e^{2x}} dx = \text{Subst}\left(\int \frac{1}{3-4x^2} dx, x, e^x\right) = \frac{\tanh^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

**Mathematica [A]** time = 0.0063098, size = 20, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(3 - 4\*E^(2\*x)),x]

[Out] ArcTanh[(2\*E^x)/Sqrt[3]]/(2\*Sqrt[3])

**Maple [A]** time = 0.019, size = 14, normalized size = 0.7

$$\frac{\sqrt{3}}{6} \operatorname{Arctanh}\left(\frac{2e^x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(3-4\*exp(2\*x)),x)

[Out] 1/6\*arctanh(2/3\*exp(x)\*3^(1/2))\*3^(1/2)

**Maxima [A]** time = 1.45953, size = 35, normalized size = 1.75

$$-\frac{1}{12} \sqrt{3} \log\left(-\frac{\sqrt{3}-2e^x}{\sqrt{3}+2e^x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(3-4\*exp(2\*x)),x, algorithm="maxima")

[Out] -1/12\*sqrt(3)\*log(-(sqrt(3) - 2\*e^x)/(sqrt(3) + 2\*e^x))

**Fricas [B]** time = 0.708965, size = 90, normalized size = 4.5

$$\frac{1}{12} \sqrt{3} \log\left(\frac{4\sqrt{3}e^x + 4e^{(2x)} + 3}{4e^{(2x)} - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(3-4\*exp(2\*x)),x, algorithm="fricas")

[Out] 1/12\*sqrt(3)\*log((4\*sqrt(3)\*e^x + 4\*e^(2\*x) + 3)/(4\*e^(2\*x) - 3))

**Sympy [A]** time = 0.116251, size = 15, normalized size = 0.75

$$\operatorname{RootSum}\left(48z^2 - 1, (i \mapsto i \log(6i + e^x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(3-4\*exp(2\*x)),x)

[Out] RootSum(48\*\_z\*\*2 - 1, Lambda(\_i, \_i\*log(6\*\_i + exp(x))))

**Giac [B]** time = 1.31441, size = 41, normalized size = 2.05

$$\frac{1}{12} \sqrt{3} \log\left(\frac{1}{2} \sqrt{3} + e^x\right) - \frac{1}{12} \sqrt{3} \log\left(\left|-\frac{1}{2} \sqrt{3} + e^x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(3-4*exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(1/2*sqrt(3) + e^x) - 1/12*sqrt(3)*log(abs(-1/2*sqrt(3) + e^x))
```



### 3.640 $\int e^x \sqrt{3 - 4e^{2x}} dx$

**Optimal.** Leaf size=36

$$\frac{1}{2}e^x\sqrt{3 - 4e^{2x}} + \frac{3}{4}\sin^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)$$

[Out]  $(E^x*\text{Sqrt}[3 - 4*E^{(2*x)}])/2 + (3*\text{ArcSin}[(2*E^x)/\text{Sqrt}[3]])/4$

**Rubi [A]** time = 0.0288832, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2249, 195, 216}

$$\frac{1}{2}e^x\sqrt{3 - 4e^{2x}} + \frac{3}{4}\sin^{-1}\left(\frac{2e^x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^x*\text{Sqrt}[3 - 4*E^{(2*x)}], x]$

[Out]  $(E^x*\text{Sqrt}[3 - 4*E^{(2*x)}])/2 + (3*\text{ArcSin}[(2*E^x)/\text{Sqrt}[3]])/4$

#### Rule 2249

$\text{Int}[(a_ + (b_)*(F_)^{(e_)*((c_)+(d_)*(x_))})^{(p_)}*(G_)^{(h_)*((f_)+(g_)*(x_))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(d*E*\text{Log}[F])/(g*h*\text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*\text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m]-1)}*(a + b*F^{(c*e - (d*e*f)/g})*x^{\text{Numerator}[m]})^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \|\| \text{GtQ}[m, 1]] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 195

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[2*p] \|\| (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4*p]) \|\| (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3*p]) \|\| \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

#### Rubi steps

$$\begin{aligned} \int e^x \sqrt{3 - 4e^{2x}} dx &= \text{Subst}\left(\int \sqrt{3 - 4x^2} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{3 - 4e^{2x}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3 - 4x^2}} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{3 - 4e^{2x}} + \frac{3}{4}\sin^{-1}\left(\frac{2e^x}{\sqrt{3}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0145496, size = 36, normalized size = 1.

$$\frac{1}{4} \left( 2e^x \sqrt{3 - 4e^{2x}} + 3 \sin^{-1} \left( \frac{2e^x}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sqrt[3 - 4\*E^(2\*x)], x]

[Out] (2\*E^x\*Sqrt[3 - 4\*E^(2\*x)] + 3\*ArcSin[(2\*E^x)/Sqrt[3]])/4

**Maple [A]** time = 0.059, size = 26, normalized size = 0.7

$$\frac{e^x}{2} \sqrt{3 - 4(e^x)^2} + \frac{3}{4} \arcsin \left( \frac{2e^x \sqrt{3}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(3-4\*exp(2\*x))^(1/2), x)

[Out] 1/2\*exp(x)\*(3-4\*exp(x)^2)^(1/2)+3/4\*arcsin(2/3\*exp(x)\*3^(1/2))

**Maxima [A]** time = 1.46051, size = 34, normalized size = 0.94

$$\frac{1}{2} \sqrt{-4e^{(2x)} + 3e^x} + \frac{3}{4} \arcsin \left( \frac{2}{3} \sqrt{3e^x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(3-4\*exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] 1/2\*sqrt(-4\*e^(2\*x) + 3)\*e^x + 3/4\*arcsin(2/3\*sqrt(3)\*e^x)

**Fricas [A]** time = 0.824951, size = 103, normalized size = 2.86

$$\frac{1}{2} \sqrt{-4e^{(2x)} + 3e^x} - \frac{3}{4} \arctan \left( \frac{1}{2} \sqrt{-4e^{(2x)} + 3e^{(-x)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(3-4\*exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] 1/2\*sqrt(-4\*e^(2\*x) + 3)\*e^x - 3/4\*arctan(1/2\*sqrt(-4\*e^(2\*x) + 3)\*e^(-x))

**Sympy [A]** time = 1.48305, size = 42, normalized size = 1.17

$$\left\{ \frac{\sqrt{3-4e^{2x}}}{2} + \frac{3 \operatorname{asin} \left( \frac{2\sqrt{3}e^x}{3} \right)}{4} \quad \text{for } e^x < \log \left( \frac{\sqrt{3}}{2} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(3-4\*exp(2\*x))\*\*(1/2),x)

[Out] Piecewise((sqrt(3 - 4\*exp(2\*x))\*exp(x)/2 + 3\*asin(2\*sqrt(3)\*exp(x)/3)/4, exp(x) < log(sqrt(3)/2))

**Giac [A]** time = 1.21404, size = 34, normalized size = 0.94

$$\frac{1}{2} \sqrt{-4e^{2x} + 3} + \frac{3}{4} \arcsin\left(\frac{2}{3} \sqrt{3}e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(3-4\*exp(2\*x))^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(-4\*e^(2\*x) + 3)\*e^x + 3/4\*arcsin(2/3\*sqrt(3)\*e^x)

### 3.641 $\int e^{x^2} x^3 dx$

**Optimal.** Leaf size=22

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

[Out]  $-E^{x^2}/2 + (E^{x^2}*x^2)/2$

**Rubi [A]** time = 0.021128, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2212, 2209}

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E<sup>x^2</sup>\*x<sup>3</sup>,x]

[Out]  $-E^{x^2}/2 + (E^{x^2}*x^2)/2$

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned} \int e^{x^2} x^3 dx &= \frac{1}{2}e^{x^2}x^2 - \int e^{x^2} x dx \\ &= -\frac{e^{x^2}}{2} + \frac{1}{2}e^{x^2}x^2 \end{aligned}$$

**Mathematica [A]** time = 0.0020051, size = 14, normalized size = 0.64

$$\frac{1}{2}e^{x^2}(x^2 - 1)$$

Antiderivative was successfully verified.

[In] Integrate[E<sup>x^2</sup>\*x<sup>3</sup>,x]

[Out]  $(E^{x^2}*(-1 + x^2))/2$

---

**Maple [A]** time = 0.019, size = 12, normalized size = 0.6

$$\frac{(x^2 - 1)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)*x^3,x)`

[Out]  $1/2*(x^2-1)*exp(x^2)$

---

**Maxima [A]** time = 0.97308, size = 15, normalized size = 0.68

$$\frac{1}{2}(x^2 - 1)e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x^3,x, algorithm="maxima")`

[Out]  $1/2*(x^2 - 1)*e^{(x^2)}$

---

**Fricas [A]** time = 0.721474, size = 31, normalized size = 1.41

$$\frac{1}{2}(x^2 - 1)e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x^3,x, algorithm="fricas")`

[Out]  $1/2*(x^2 - 1)*e^{(x^2)}$

---

**Sympy [A]** time = 0.083672, size = 10, normalized size = 0.45

$$\frac{(x^2 - 1)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)*x**3,x)`

[Out]  $(x**2 - 1)*exp(x**2)/2$

---

**Giac [A]** time = 1.14152, size = 15, normalized size = 0.68

$$\frac{1}{2}(x^2 - 1)e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

### 3.642 $\int e^x \sqrt{1 - e^{2x}} dx$

**Optimal.** Leaf size=29

$$\frac{1}{2}e^x\sqrt{1 - e^{2x}} + \frac{1}{2}\sin^{-1}(e^x)$$

[Out] (E^x\*Sqrt[1 - E^(2\*x)])/2 + ArcSin[E^x]/2

**Rubi [A]** time = 0.026527, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2249, 195, 216}

$$\frac{1}{2}e^x\sqrt{1 - e^{2x}} + \frac{1}{2}\sin^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x\*Sqrt[1 - E^(2\*x)], x]

[Out] (E^x\*Sqrt[1 - E^(2\*x)])/2 + ArcSin[E^x]/2

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int e^x \sqrt{1 - e^{2x}} dx &= \text{Subst} \left( \int \sqrt{1 - x^2} dx, x, e^x \right) \\ &= \frac{1}{2}e^x\sqrt{1 - e^{2x}} + \frac{1}{2}\text{Subst} \left( \int \frac{1}{\sqrt{1 - x^2}} dx, x, e^x \right) \\ &= \frac{1}{2}e^x\sqrt{1 - e^{2x}} + \frac{1}{2}\sin^{-1}(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0115649, size = 26, normalized size = 0.9

$$\frac{1}{2} \left( e^x \sqrt{1 - e^{2x}} + \sin^{-1}(e^x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sqrt[1 - E^(2\*x)],x]

[Out] (E^x\*Sqrt[1 - E^(2\*x)] + ArcSin[E^x])/2

**Maple [A]** time = 0.054, size = 21, normalized size = 0.7

$$\frac{e^x}{2} \sqrt{1 - (e^x)^2} + \frac{\arcsin(e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(1-exp(2\*x))^(1/2),x)

[Out] 1/2\*exp(x)\*(1-exp(x)^2)^(1/2)+1/2\*arcsin(exp(x))

**Maxima [A]** time = 1.45959, size = 27, normalized size = 0.93

$$\frac{1}{2} \sqrt{-e^{(2x)} + 1} e^x + \frac{1}{2} \arcsin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1-exp(2\*x))^(1/2),x, algorithm="maxima")

[Out] 1/2\*sqrt(-e^(2\*x) + 1)\*e^x + 1/2\*arcsin(e^x)

**Fricas [A]** time = 0.892407, size = 95, normalized size = 3.28

$$\frac{1}{2} \sqrt{-e^{(2x)} + 1} e^x - \arctan\left(\left(\sqrt{-e^{(2x)} + 1} - 1\right) e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1-exp(2\*x))^(1/2),x, algorithm="fricas")

[Out] 1/2\*sqrt(-e^(2\*x) + 1)\*e^x - arctan((sqrt(-e^(2\*x) + 1) - 1)\*e^(-x))

**Sympy [A]** time = 1.27658, size = 24, normalized size = 0.83

$$\begin{cases} \frac{\sqrt{1 - e^{2x}} e^x}{2} + \frac{\operatorname{asin}(e^x)}{2} & \text{for } e^x < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1-exp(2\*x))\*\*(1/2),x)

[Out] Piecewise((sqrt(1 - exp(2\*x))\*exp(x)/2 + asin(exp(x))/2, exp(x) < 0))



---

**Giac [A]** time = 1.2158, size = 27, normalized size = 0.93

$$\frac{1}{2} \sqrt{-e^{(2x)} + 1} e^x + \frac{1}{2} \arcsin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1-exp(2\*x))^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(-e^(2\*x) + 1)\*e^x + 1/2\*arcsin(e^x)

$$3.643 \quad \int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} dx$$

**Optimal.** Leaf size=14

$$\sinh^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)$$

[Out] ArcSinh[(1 + 2\*E^x)/Sqrt[3]]

**Rubi [A]** time = 0.0362688, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 619, 215}

$$\sinh^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[1 + E^x + E^(2\*x)], x]

[Out] ArcSinh[(1 + 2\*E^x)/Sqrt[3]]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

#### Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}} dx, x, e^x\right) \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+2e^x\right)}{\sqrt{3}} \\ &= \sinh^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0095732, size = 14, normalized size = 1.

$$\sinh^{-1}\left(\frac{2e^x + 1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/Sqrt[1 + E^x + E^(2\*x)], x]

[Out] ArcSinh[(1 + 2\*E^x)/Sqrt[3]]

**Maple [A]** time = 0.062, size = 11, normalized size = 0.8

$$\operatorname{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(e^x + \frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(1+exp(x)+exp(2\*x))^(1/2), x)

[Out] arcsinh(2/3\*3^(1/2)\*(exp(x)+1/2))

**Maxima [A]** time = 1.44264, size = 16, normalized size = 1.14

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2e^x + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1+exp(x)+exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] arcsinh(1/3\*sqrt(3)\*(2\*e^x + 1))

**Fricas [A]** time = 0.881631, size = 61, normalized size = 4.36

$$-\log\left(2\sqrt{e^{2x} + e^x + 1} - 2e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1+exp(x)+exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] -log(2\*sqrt(e^(2\*x) + e^x + 1) - 2\*e^x - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\sqrt{e^{2x} + e^x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)+exp(2*x))**(1/2),x)
```

```
[Out] Integral(exp(x)/sqrt(exp(2*x) + exp(x) + 1), x)
```

---

**Giac [A]** time = 1.28356, size = 28, normalized size = 2.

$$-\log\left(2\sqrt{e^{2x} + e^x + 1} - 2e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(2*sqrt(e^(2*x) + e^x + 1) - 2*e^x - 1)
```

$$3.644 \quad \int \frac{e^x}{-4+e^{2x}} dx$$

**Optimal.** Leaf size=12

$$-\frac{1}{2} \tanh^{-1}\left(\frac{e^x}{2}\right)$$

[Out] -ArcTanh[E^x/2]/2

**Rubi [A]** time = 0.0198792, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 207}

$$-\frac{1}{2} \tanh^{-1}\left(\frac{e^x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/(-4 + E^(2\*x)),x]

[Out] -ArcTanh[E^x/2]/2

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{-4+e^{2x}} dx &= \text{Subst}\left(\int \frac{1}{-4+x^2} dx, x, e^x\right) \\ &= -\frac{1}{2} \tanh^{-1}\left(\frac{e^x}{2}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0027812, size = 12, normalized size = 1.

$$-\frac{1}{2} \tanh^{-1}\left(\frac{e^x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(-4 + E^(2\*x)),x]

[Out]  $-\text{ArcTanh}[E^x/2]/2$

---

**Maple [B]** time = 0.027, size = 16, normalized size = 1.3

$$-\frac{\ln(2 + e^x)}{4} + \frac{\ln(-2 + e^x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(-4+exp(2*x)),x)`

[Out]  $-1/4*\ln(2+\exp(x))+1/4*\ln(-2+\exp(x))$

---

**Maxima [B]** time = 0.961811, size = 20, normalized size = 1.67

$$-\frac{1}{4} \log(e^x + 2) + \frac{1}{4} \log(e^x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-4+exp(2*x)),x, algorithm="maxima")`

[Out]  $-1/4*\log(e^x + 2) + 1/4*\log(e^x - 2)$

---

**Fricas [B]** time = 0.740969, size = 51, normalized size = 4.25

$$-\frac{1}{4} \log(e^x + 2) + \frac{1}{4} \log(e^x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-4+exp(2*x)),x, algorithm="fricas")`

[Out]  $-1/4*\log(e^x + 2) + 1/4*\log(e^x - 2)$

---

**Sympy [A]** time = 0.101817, size = 15, normalized size = 1.25

$$\frac{\log(e^x - 2)}{4} - \frac{\log(e^x + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-4+exp(2*x)),x)`

[Out]  $\log(\exp(x) - 2)/4 - \log(\exp(x) + 2)/4$

---

**Giac [B]** time = 1.24851, size = 22, normalized size = 1.83

$$-\frac{1}{4} \log(e^x + 2) + \frac{1}{4} \log(|e^x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-4+exp(2*x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^x + 2) + 1/4*log(abs(e^x - 2))
```

### 3.645 $\int e^{2-x^2} x dx$

**Optimal.** Leaf size=13

$$-\frac{1}{2}e^{2-x^2}$$

[Out]  $-E^{(2 - x^2)}/2$

**Rubi [A]** time = 0.0093858, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2209}

$$-\frac{1}{2}e^{2-x^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2 - x^2)\*x,x]

[Out]  $-E^{(2 - x^2)}/2$

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x]
/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\int e^{2-x^2} x dx = -\frac{1}{2}e^{2-x^2}$$

**Mathematica [A]** time = 0.0020969, size = 13, normalized size = 1.

$$-\frac{1}{2}e^{2-x^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2 - x^2)\*x,x]

[Out]  $-E^{(2 - x^2)}/2$

**Maple [A]** time = 0.018, size = 11, normalized size = 0.9

$$-\frac{e^{-x^2+2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(exp(-x^2+2)*x,x)`

[Out] `-1/2*exp(-x^2+2)`

**Maxima [A]** time = 0.962511, size = 14, normalized size = 1.08

$$-\frac{1}{2}e^{(-x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-x^2+2)*x,x, algorithm="maxima")`

[Out] `-1/2*e^(-x^2 + 2)`

**Fricas [A]** time = 0.762508, size = 26, normalized size = 2.

$$-\frac{1}{2}e^{(-x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-x^2+2)*x,x, algorithm="fricas")`

[Out] `-1/2*e^(-x^2 + 2)`

**Sympy [A]** time = 0.083658, size = 8, normalized size = 0.62

$$-\frac{e^{2-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-x**2+2)*x,x)`

[Out] `-exp(2 - x**2)/2`

**Giac [A]** time = 1.26808, size = 14, normalized size = 1.08

$$-\frac{1}{2}e^{(-x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-x^2+2)*x,x, algorithm="giac")`

[Out] `-1/2*e^(-x^2 + 2)`

### 3.646 $\int (e^x - x^e) dx$

**Optimal.** Leaf size=16

$$e^x - \frac{x^{1+e}}{1+e}$$

[Out]  $E^x - x^{(1 + E)/(1 + E)}$

**Rubi [A]** time = 0.004769, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2194}

$$e^x - \frac{x^{1+e}}{1+e}$$

Antiderivative was successfully verified.

[In] Int[E^x - x^E, x]

[Out]  $E^x - x^{(1 + E)/(1 + E)}$

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int (e^x - x^e) dx &= -\frac{x^{1+e}}{1+e} + \int e^x dx \\ &= e^x - \frac{x^{1+e}}{1+e} \end{aligned}$$

**Mathematica [A]** time = 0.0053215, size = 16, normalized size = 1.

$$e^x - \frac{x^{1+e}}{1+e}$$

Antiderivative was successfully verified.

[In] Integrate[E^x - x^E, x]

[Out]  $E^x - x^{(1 + E)/(1 + E)}$

**Maple [A]** time = 0.036, size = 16, normalized size = 1.

$$e^x - \frac{x^{1+E}}{1+E}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)-x^E,x)`

[Out] `exp(x)-x^(1+E)/(1+E)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)-x^E,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.86158, size = 43, normalized size = 2.69

$$-\frac{xx^E - (E + 1)e^x}{E + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)-x^E,x, algorithm="fricas")`

[Out] `-(x*x^E - (E + 1)*e^x)/(E + 1)`

**Sympy [A]** time = 0.080443, size = 10, normalized size = 0.62

$$-\frac{x^{1+e}}{1+e} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)-x**E,x)`

[Out] `-x**(1 + E)/(1 + E) + exp(x)`

**Giac [A]** time = 1.16478, size = 20, normalized size = 1.25

$$-\frac{x^{E+1}}{E+1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)-x^E,x, algorithm="giac")`

[Out] `-x^(E + 1)/(E + 1) + e^x`

$$3.647 \quad \int \frac{-1+e^{2x}}{3+e^{2x}} dx$$

**Optimal.** Leaf size=18

$$\frac{2}{3} \log(e^{2x} + 3) - \frac{x}{3}$$

[Out]  $-x/3 + (2*\text{Log}[3 + E^{(2*x)}])/3$

**Rubi [A]** time = 0.0258383, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2282, 72}

$$\frac{2}{3} \log(e^{2x} + 3) - \frac{x}{3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(-1 + E^{(2*x)})/(3 + E^{(2*x)}), x]$

[Out]  $-x/3 + (2*\text{Log}[3 + E^{(2*x)}])/3$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_)^(m_)) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{-1+e^{2x}}{3+e^{2x}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{-1+x}{x(3+x)} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{1}{3x} + \frac{4}{3(3+x)} \right) dx, x, e^{2x} \right) \\ &= -\frac{x}{3} + \frac{2}{3} \log(3 + e^{2x}) \end{aligned}$$

**Mathematica [A]** time = 0.0086721, size = 18, normalized size = 1.

$$\frac{2}{3} \log(e^{2x} + 3) - \frac{x}{3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(-1 + E^{(2*x)})/(3 + E^{(2*x)}), x]$

[Out]  $-x/3 + (2*\text{Log}[3 + E^{(2*x)}])/3$

---

**Maple [A]** time = 0.025, size = 18, normalized size = 1.

$$-\frac{\ln(e^{2x})}{6} + \frac{2 \ln(3 + e^{2x})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+exp(2*x))/(3+exp(2*x)),x)`

[Out]  $-1/6*\ln(\exp(2*x))+2/3*\ln(3+\exp(2*x))$

---

**Maxima [A]** time = 0.957969, size = 18, normalized size = 1.

$$-\frac{1}{3}x + \frac{2}{3} \log(e^{(2x)} + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+exp(2*x))/(3+exp(2*x)),x, algorithm="maxima")`

[Out]  $-1/3*x + 2/3*\log(e^{(2*x)} + 3)$

---

**Fricas [A]** time = 0.946902, size = 42, normalized size = 2.33

$$-\frac{1}{3}x + \frac{2}{3} \log(e^{(2x)} + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+exp(2*x))/(3+exp(2*x)),x, algorithm="fricas")`

[Out]  $-1/3*x + 2/3*\log(e^{(2*x)} + 3)$

---

**Sympy [A]** time = 0.087024, size = 14, normalized size = 0.78

$$-\frac{x}{3} + \frac{2 \log(e^{2x} + 3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+exp(2*x))/(3+exp(2*x)),x)`

[Out]  $-x/3 + 2*\log(\exp(2*x) + 3)/3$

---

**Giac [A]** time = 1.2009, size = 18, normalized size = 1.

$$-\frac{1}{3}x + \frac{2}{3}\log(e^{2x} + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(2\*x))/(3+exp(2\*x)),x, algorithm="giac")

[Out] -1/3\*x + 2/3\*log(e^(2\*x) + 3)

$$3.648 \quad \int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

**Optimal.** Leaf size=4

$$\sin^{-1}(e^x)$$

[Out] ArcSin[E^x]

**Rubi [A]** time = 0.0226324, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2249, 216}

$$\sin^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[1 - E^(2\*x)], x]

[Out] ArcSin[E^x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \text{Subst} \left( \int \frac{1}{\sqrt{1-x^2}} dx, x, e^x \right) = \sin^{-1}(e^x)$$

**Mathematica [A]** time = 0.0041817, size = 4, normalized size = 1.

$$\sin^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/Sqrt[1 - E^(2\*x)], x]

[Out] ArcSin[E^x]

**Maple [A]** time = 0.058, size = 4, normalized size = 1.

$$\arcsin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1-exp(2*x))^(1/2),x)`

[Out] `arcsin(exp(x))`

---

**Maxima [A]** time = 1.4503, size = 4, normalized size = 1.

$$\arcsin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="maxima")`

[Out] `arcsin(e^x)`

---

**Fricas [B]** time = 0.76985, size = 59, normalized size = 14.75

$$-2 \arctan\left(\left(\sqrt{-e^{2x} + 1} - 1\right)e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="fricas")`

[Out] `-2*arctan((sqrt(-e^(2*x) + 1) - 1)*e^(-x))`

---

**Sympy [A]** time = 0.671693, size = 3, normalized size = 0.75

$$\operatorname{asin}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x))**(1/2),x)`

[Out] `asin(exp(x))`

---

**Giac [A]** time = 1.36482, size = 4, normalized size = 1.

$$\arcsin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="giac")`

[Out] `arcsin(e^x)`



$$3.649 \quad \int \frac{e^{2x}}{1+e^{4x}} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{2} \tan^{-1}(e^{2x})$$

[Out] ArcTan[E^(2\*x)]/2

**Rubi [A]** time = 0.0204029, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 203}

$$\frac{1}{2} \tan^{-1}(e^{2x})$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(1 + E^(4\*x)),x]

[Out] ArcTan[E^(2\*x)]/2

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^(h\_.)\*((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^{4x}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \tan^{-1}(e^{2x}) \end{aligned}$$

**Mathematica [A]** time = 0.0030139, size = 10, normalized size = 1.

$$\frac{1}{2} \tan^{-1}(e^{2x})$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(1 + E^(4\*x)),x]

[Out] ArcTan[E^(2\*x)]/2

**Maple [A]** time = 0.02, size = 8, normalized size = 0.8

$$\frac{\arctan\left((e^x)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(1+exp(4\*x)),x)

[Out] 1/2\*arctan(exp(x)^2)

**Maxima [A]** time = 1.49731, size = 9, normalized size = 0.9

$$\frac{1}{2} \arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(4\*x)),x, algorithm="maxima")

[Out] 1/2\*arctan(e^(2\*x))

**Fricas [A]** time = 0.751055, size = 28, normalized size = 2.8

$$\frac{1}{2} \arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(4\*x)),x, algorithm="fricas")

[Out] 1/2\*arctan(e^(2\*x))

**Sympy [B]** time = 0.109118, size = 17, normalized size = 1.7

$$\text{RootSum}\left(16z^2 + 1, \left(i \mapsto i \log\left(4i + e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(4\*x)),x)

[Out] RootSum(16\*\_z\*\*2 + 1, Lambda(\_i, \_i\*log(4\*\_i + exp(2\*x))))

**Giac [A]** time = 1.17685, size = 9, normalized size = 0.9

$$\frac{1}{2} \arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(e^(2*x))
```

$$3.650 \quad \int \frac{1}{-3e^x + e^{2x}} dx$$

**Optimal.** Leaf size=27

$$-\frac{x}{9} + \frac{e^{-x}}{3} + \frac{1}{9} \log(3 - e^x)$$

[Out] 1/(3\*E^x) - x/9 + Log[3 - E^x]/9

**Rubi [A]** time = 0.0188416, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 44}

$$-\frac{x}{9} + \frac{e^{-x}}{3} + \frac{1}{9} \log(3 - e^x)$$

Antiderivative was successfully verified.

[In] Int[(-3\*E^x + E^(2\*x))^(-1), x]

[Out] 1/(3\*E^x) - x/9 + Log[3 - E^x]/9

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{-3e^x + e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{(-3 + x)x^2} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{1}{9(-3 + x)} - \frac{1}{3x^2} - \frac{1}{9x} \right) dx, x, e^x \right) \\ &= \frac{e^{-x}}{3} - \frac{x}{9} + \frac{1}{9} \log(3 - e^x) \end{aligned}$$

**Mathematica [A]** time = 0.018038, size = 23, normalized size = 0.85

$$\frac{1}{9} (-x + 3e^{-x} + \log(3 - e^x))$$

Antiderivative was successfully verified.

[In] Integrate[(-3\*E^x + E^(2\*x))^(-1), x]

[Out]  $(3/E^x - x + \text{Log}[3 - E^x])/9$

**Maple [A]** time = 0.027, size = 20, normalized size = 0.7

$$\frac{\ln(e^x - 3)}{9} + \frac{1}{3e^x} - \frac{\ln(e^x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3*exp(x)+exp(2*x)),x)`

[Out] `1/9*ln(exp(x)-3)+1/3/exp(x)-1/9*ln(exp(x))`

**Maxima [A]** time = 0.975396, size = 23, normalized size = 0.85

$$-\frac{1}{9}x + \frac{1}{3}e^{(-x)} + \frac{1}{9}\log(e^x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="maxima")`

[Out] `-1/9*x + 1/3*e^(-x) + 1/9*log(e^x - 3)`

**Fricas [A]** time = 0.773875, size = 59, normalized size = 2.19

$$-\frac{1}{9}(xe^x - e^x \log(e^x - 3) - 3)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="fricas")`

[Out] `-1/9*(x*e^x - e^x*log(e^x - 3) - 3)*e^(-x)`

**Sympy [A]** time = 0.099519, size = 17, normalized size = 0.63

$$-\frac{x}{9} + \frac{\log(e^x - 3)}{9} + \frac{e^{-x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*exp(x)+exp(2*x)),x)`

[Out] `-x/9 + log(exp(x) - 3)/9 + exp(-x)/3`

**Giac [A]** time = 1.24845, size = 24, normalized size = 0.89

$$-\frac{1}{9}x + \frac{1}{3}e^{(-x)} + \frac{1}{9}\log(|e^x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] -1/9*x + 1/3*e^(-x) + 1/9*log(abs(e^x - 3))
```

$$3.651 \quad \int \frac{e^x(-2+e^x)}{1+e^x} dx$$

**Optimal.** Leaf size=12

$$e^x - 3 \log(e^x + 1)$$

[Out]  $E^x - 3 \text{Log}[1 + E^x]$

**Rubi [A]** time = 0.0351896, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2282, 43}

$$e^x - 3 \log(e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^x*(-2 + E^x))/(1 + E^x), x]$

[Out]  $E^x - 3 \text{Log}[1 + E^x]$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x(-2+e^x)}{1+e^x} dx &= \text{Subst} \left( \int \frac{-2+x}{1+x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( 1 - \frac{3}{1+x} \right) dx, x, e^x \right) \\ &= e^x - 3 \log(1 + e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0103021, size = 12, normalized size = 1.

$$e^x - 3 \log(e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(E^x*(-2 + E^x))/(1 + E^x), x]$

[Out]  $E^x - 3 \text{Log}[1 + E^x]$

---

**Maple [A]** time = 0.02, size = 11, normalized size = 0.9

$$e^x - 3 \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(-2+exp(x))/(1+exp(x)),x)`

[Out] `exp(x)-3*ln(1+exp(x))`

---

**Maxima [A]** time = 1.00554, size = 14, normalized size = 1.17

$$e^x - 3 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="maxima")`

[Out] `e^x - 3*log(e^x + 1)`

---

**Fricas [A]** time = 0.770717, size = 30, normalized size = 2.5

$$e^x - 3 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="fricas")`

[Out] `e^x - 3*log(e^x + 1)`

---

**Sympy [A]** time = 0.092809, size = 10, normalized size = 0.83

$$e^x - 3 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x)`

[Out] `exp(x) - 3*log(exp(x) + 1)`

---

**Giac [A]** time = 1.28569, size = 14, normalized size = 1.17

$$e^x - 3 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="giac")`

[Out] `e^x - 3*log(e^x + 1)`



$$3.652 \quad \int \frac{e^x}{-1+e^{2x}} dx$$

**Optimal.** Leaf size=6

$$-\tanh^{-1}(e^x)$$

[Out] -ArcTanh[E^x]

**Rubi [A]** time = 0.0188114, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 207}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(-1 + E^(2\*x)), x]

[Out] -ArcTanh[E^x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^(h\_.)\*((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{-1+e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{-1+x^2} dx, x, e^x \right) \\ &= -\tanh^{-1}(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0020784, size = 6, normalized size = 1.

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(-1 + E^(2\*x)), x]

[Out] -ArcTanh[E^x]

**Maple [A]** time = 0.018, size = 6, normalized size = 1.

$$-\operatorname{Artanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(-1+exp(2*x)),x)`

[Out] `-arctanh(exp(x))`

---

**Maxima [B]** time = 0.977553, size = 20, normalized size = 3.33

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")`

[Out] `-1/2*log(e^x + 1) + 1/2*log(e^x - 1)`

---

**Fricas [B]** time = 0.909077, size = 51, normalized size = 8.5

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")`

[Out] `-1/2*log(e^x + 1) + 1/2*log(e^x - 1)`

---

**Sympy [B]** time = 0.101805, size = 15, normalized size = 2.5

$$\frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x)`

[Out] `log(exp(x) - 1)/2 - log(exp(x) + 1)/2`

---

**Giac [B]** time = 1.23276, size = 22, normalized size = 3.67

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")`

[Out] `-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))`

$$3.653 \quad \int \frac{e^x}{1+e^{2x}} dx$$

**Optimal.** Leaf size=4

$$\tan^{-1}(e^x)$$

[Out] ArcTan[E^x]

**Rubi [A]** time = 0.0190307, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2249, 203}

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + E^(2\*x)), x]

[Out] ArcTan[E^x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\int \frac{e^x}{1+e^{2x}} dx = \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, e^x\right) = \tan^{-1}(e^x)$$

**Mathematica [A]** time = 0.0025029, size = 4, normalized size = 1.

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + E^(2\*x)), x]

[Out] ArcTan[E^x]

**Maple [A]** time = 0.02, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+exp(2*x)),x)`

[Out] `arctan(exp(x))`

---

**Maxima [A]** time = 1.88194, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")`

[Out] `arctan(e^x)`

---

**Fricas [A]** time = 0.869882, size = 18, normalized size = 4.5

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")`

[Out] `arctan(e^x)`

---

**Sympy [B]** time = 0.104633, size = 15, normalized size = 3.75

$$\text{RootSum}\left(4z^2 + 1, (i \mapsto i \log(2i + e^x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x)`

[Out] `RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))`

---

**Giac [A]** time = 1.25289, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")`

[Out] `arctan(e^x)`

$$3.654 \quad \int \frac{e^{-x} + e^x}{-e^{-x} + e^x} dx$$

**Optimal.** Leaf size=12

$$\log(e^{-x} - e^x)$$

[Out] Log[E<sup>^</sup>(-x) - E<sup>^</sup>x]

**Rubi [A]** time = 0.0398882, antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2282, 446, 72}

$$\log(1 - e^{2x}) - x$$

Antiderivative was successfully verified.

[In] Int[(E<sup>^</sup>(-x) + E<sup>^</sup>x)/(-E<sup>^</sup>(-x) + E<sup>^</sup>x), x]

[Out] -x + Log[1 - E<sup>^</sup>(2\*x)]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 72

```
Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-x} + e^x}{-e^{-x} + e^x} dx &= \text{Subst} \left( \int \frac{-1 - x^2}{x(1 - x^2)} dx, x, e^x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{-1 - x}{(1 - x)x} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{2}{-1 + x} - \frac{1}{x} \right) dx, x, e^{2x} \right) \\ &= -x + \log(1 - e^{2x}) \end{aligned}$$

**Mathematica [A]** time = 0.0082017, size = 14, normalized size = 1.17

$$\log(1 - e^{2x}) - x$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-x) + E^x)/(-E^(-x) + E^x), x]

[Out] -x + Log[1 - E^(2\*x)]

**Maple [A]** time = 0.027, size = 17, normalized size = 1.4

$$\ln(-1 + e^x) - \ln(e^x) + \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-x)+exp(x))/(-1/exp(x)+exp(x)), x)

[Out] ln(-1+exp(x))-ln(exp(x))+ln(1+exp(x))

**Maxima [A]** time = 0.963803, size = 14, normalized size = 1.17

$$\log(e^{(-x)} - e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))/(-1/exp(x)+exp(x)), x, algorithm="maxima")

[Out] log(e^(-x) - e^x)

**Fricas [A]** time = 0.961889, size = 31, normalized size = 2.58

$$-x + \log(e^{(2x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))/(-1/exp(x)+exp(x)), x, algorithm="fricas")

[Out] -x + log(e^(2\*x) - 1)

**Sympy [A]** time = 0.092038, size = 8, normalized size = 0.67

$$-x + \log(e^{2x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))/(-1/exp(x)+exp(x)), x)

[Out]  $-x + \log(\exp(2*x) - 1)$

---

**Giac [A]** time = 1.26354, size = 16, normalized size = 1.33

$$-x + \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((exp(-x)+exp(x))/(-1/exp(x)+exp(x)),x, algorithm="giac")`

[Out]  $-x + \log(\text{abs}(e^{2*x} - 1))$

$$3.655 \quad \int \frac{-e^{-x} + e^x}{e^{-x} + e^x} dx$$

**Optimal.** Leaf size=10

$$\log(e^{-x} + e^x)$$

[Out] Log[E<sup>^-x</sup> + E<sup>^x</sup>]

**Rubi [A]** time = 0.0372131, antiderivative size = 12, normalized size of antiderivative = 1.2, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2282, 446, 72}

$$\log(e^{2x} + 1) - x$$

Antiderivative was successfully verified.

[In] Int[(-E<sup>^-x</sup> + E<sup>^x</sup>)/(E<sup>^-x</sup> + E<sup>^x</sup>), x]

[Out] -x + Log[1 + E<sup>^(2\*x)</sup>]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{-e^{-x} + e^x}{e^{-x} + e^x} dx &= \text{Subst} \left( \int \frac{-1 + x^2}{x(1 + x^2)} dx, x, e^x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{-1 + x}{x(1 + x)} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -\frac{1}{x} + \frac{2}{1 + x} \right) dx, x, e^{2x} \right) \\ &= -x + \log(1 + e^{2x}) \end{aligned}$$



**Mathematica [A]** time = 0.0079775, size = 12, normalized size = 1.2

$$\log(e^{2x} + 1) - x$$

Antiderivative was successfully verified.

[In] Integrate[(-E^(-x) + E^x)/(E^(-x) + E^x), x]

[Out] -x + Log[1 + E^(2\*x)]

---

**Maple [A]** time = 0.026, size = 14, normalized size = 1.4

$$\ln(1 + (e^x)^2) - \ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/exp(x)+exp(x))/(exp(-x)+exp(x)), x)

[Out] ln(1+exp(x)^2)-ln(exp(x))

---

**Maxima [A]** time = 0.978719, size = 11, normalized size = 1.1

$$\log(e^{(-x)} + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))/(exp(-x)+exp(x)), x, algorithm="maxima")

[Out] log(e^(-x) + e^x)

---

**Fricas [A]** time = 0.799678, size = 31, normalized size = 3.1

$$-x + \log(e^{(2*x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))/(exp(-x)+exp(x)), x, algorithm="fricas")

[Out] -x + log(e^(2\*x) + 1)

---

**Sympy [A]** time = 0.090436, size = 8, normalized size = 0.8

$$-x + \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))/(exp(-x)+exp(x)), x)

[Out]  $-x + \log(\exp(2*x) + 1)$

---

**Giac** [A] time = 1.33075, size = 15, normalized size = 1.5

$$-x + \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))/(exp(-x)+exp(x)),x, algorithm="giac")`

[Out]  $-x + \log(e^{2*x} + 1)$

$$3.656 \quad \int \frac{e^{-2x} + e^{2x}}{-e^{-2x} + e^{2x}} dx$$

**Optimal.** Leaf size=18

$$\frac{1}{2} \log(1 - e^{4x}) - x$$

[Out] -x + Log[1 - E^(4\*x)]/2

**Rubi [A]** time = 0.0444535, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2282, 446, 72}

$$\frac{1}{2} \log(1 - e^{4x}) - x$$

Antiderivative was successfully verified.

[In] Int[(E^(-2\*x) + E^(2\*x))/(-E^(-2\*x) + E^(2\*x)), x]

[Out] -x + Log[1 - E^(4\*x)]/2

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-2x} + e^{2x}}{-e^{-2x} + e^{2x}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{-1 - x^2}{x(1 - x^2)} dx, x, e^{2x} \right) \\ &= \frac{1}{4} \text{Subst} \left( \int \frac{-1 - x}{(1 - x)x} dx, x, e^{4x} \right) \\ &= \frac{1}{4} \text{Subst} \left( \int \left( \frac{2}{-1 + x} - \frac{1}{x} \right) dx, x, e^{4x} \right) \\ &= -x + \frac{1}{2} \log(1 - e^{4x}) \end{aligned}$$

**Mathematica [A]** time = 0.008311, size = 18, normalized size = 1.

$$\frac{1}{2} \log(1 - e^{4x}) - x$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-2\*x) + E^(2\*x))/(-E^(-2\*x) + E^(2\*x)), x]

[Out] -x + Log[1 - E^(4\*x)]/2

**Maple [A]** time = 0.03, size = 30, normalized size = 1.7

$$\frac{\ln(1 + (e^x)^2)}{2} + \frac{\ln(-1 + e^x)}{2} - \ln(e^x) + \frac{\ln(1 + e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-2\*x)+exp(2\*x))/(-1/exp(2\*x)+exp(2\*x)), x)

[Out] 1/2\*ln(1+exp(x)^2)+1/2\*ln(-1+exp(x))-ln(exp(x))+1/2\*ln(1+exp(x))

**Maxima [A]** time = 0.98155, size = 19, normalized size = 1.06

$$\frac{1}{2} \log(e^{2x} - e^{-2x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-2\*x)+exp(2\*x))/(-1/exp(2\*x)+exp(2\*x)), x, algorithm="maxima")

[Out] 1/2\*log(e^(2\*x) - e^(-2\*x))

**Fricas [A]** time = 0.852482, size = 36, normalized size = 2.

$$-x + \frac{1}{2} \log(e^{4x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-2\*x)+exp(2\*x))/(-1/exp(2\*x)+exp(2\*x)), x, algorithm="fricas")

[Out] -x + 1/2\*log(e^(4\*x) - 1)

**Sympy [A]** time = 0.095284, size = 10, normalized size = 0.56

$$-x + \frac{\log(e^{4x} - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(-2*x)+exp(2*x))/(-1/exp(2*x)+exp(2*x)),x)
```

```
[Out] -x + log(exp(4*x) - 1)/2
```

---

**Giac [A]** time = 1.20685, size = 19, normalized size = 1.06

$$-x + \frac{1}{2} \log(|e^{4x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(-2*x)+exp(2*x))/(-1/exp(2*x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] -x + 1/2*log(abs(e^(4*x) - 1))
```

$$3.657 \quad \int \frac{e^x}{\sqrt{1+e^{2x}}} dx$$

**Optimal.** Leaf size=4

$$\sinh^{-1}(e^x)$$

[Out] ArcSinh[E^x]

**Rubi [A]** time = 0.0214806, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 215}

$$\sinh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[1 + E^(2\*x)], x]

[Out] ArcSinh[E^x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rubi steps

$$\int \frac{e^x}{\sqrt{1+e^{2x}}} dx = \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, e^x \right) = \sinh^{-1}(e^x)$$

**Mathematica [A]** time = 0.0037465, size = 4, normalized size = 1.

$$\sinh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/Sqrt[1 + E^(2\*x)], x]

[Out] ArcSinh[E^x]

**Maple [A]** time = 0.065, size = 4, normalized size = 1.

$$\operatorname{Arcsinh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+exp(2*x))^(1/2),x)`

[Out] `arcsinh(exp(x))`

---

**Maxima [A]** time = 1.45739, size = 4, normalized size = 1.

$$\operatorname{arsinh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="maxima")`

[Out] `arcsinh(e^x)`

---

**Fricas [B]** time = 0.770909, size = 42, normalized size = 10.5

$$-\log\left(\sqrt{e^{2x}+1}-e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="fricas")`

[Out] `-log(sqrt(e^(2*x) + 1) - e^x)`

---

**Sympy [A]** time = 0.620196, size = 3, normalized size = 0.75

$$\operatorname{asinh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x))**(1/2),x)`

[Out] `asinh(exp(x))`

---

**Giac [B]** time = 1.22555, size = 22, normalized size = 5.5

$$-\log\left(\sqrt{e^{2x}+1}-e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="giac")`

[Out] `-log(sqrt(e^(2*x) + 1) - e^x)`

$$3.658 \quad \int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}} dx$$

**Optimal.** Leaf size=11

$$2e^{\sqrt{x+4}}$$

[Out] 2\*E^Sqrt[4 + x]

**Rubi [A]** time = 0.0237167, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2209}

$$2e^{\sqrt{x+4}}$$

Antiderivative was successfully verified.

[In] Int[E^Sqrt[4 + x]/Sqrt[4 + x],x]

[Out] 2\*E^Sqrt[4 + x]

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n)) / (b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}} dx = 2e^{\sqrt{4+x}}$$

**Mathematica [A]** time = 0.0033913, size = 11, normalized size = 1.

$$2e^{\sqrt{x+4}}$$

Antiderivative was successfully verified.

[In] Integrate[E^Sqrt[4 + x]/Sqrt[4 + x],x]

[Out] 2\*E^Sqrt[4 + x]

**Maple [A]** time = 0.024, size = 9, normalized size = 0.8

$$2e^{\sqrt{4+x}}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(exp((4+x)^(1/2))/(4+x)^(1/2),x)
```

```
[Out] 2*exp((4+x)^(1/2))
```

---

**Maxima [A]** time = 0.967904, size = 11, normalized size = 1.

$$2e^{(\sqrt{x+4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*e^(sqrt(x + 4))
```

---

**Fricas [A]** time = 0.678837, size = 26, normalized size = 2.36

$$2e^{(\sqrt{x+4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*e^(sqrt(x + 4))
```

---

**Sympy [A]** time = 0.193108, size = 8, normalized size = 0.73

$$2e^{\sqrt{x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((4+x)**(1/2))/(4+x)**(1/2),x)
```

```
[Out] 2*exp(sqrt(x + 4))
```

---

**Giac [A]** time = 1.25851, size = 11, normalized size = 1.

$$2e^{(\sqrt{x+4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*e^(sqrt(x + 4))
```

$$3.659 \quad \int \frac{x}{\sqrt{-1+e^{2x^2}}} dx$$

**Optimal.** Leaf size=18

$$\frac{1}{2} \tan^{-1}(\sqrt{e^{2x^2}-1})$$

[Out] ArcTan[Sqrt[-1 + E^(2\*x^2)]]/2

**Rubi [A]** time = 0.0605359, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6715, 2282, 63, 203}

$$\frac{1}{2} \tan^{-1}(\sqrt{e^{2x^2}-1})$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[-1 + E^(2\*x^2)],x]

[Out] ArcTan[Sqrt[-1 + E^(2\*x^2)]]/2

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-1+e^{2x^2}}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{\sqrt{-1+e^{2x}}} dx, x, x^2 \right) \\
&= \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{-1+xx}} dx, x, e^{2x^2} \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, \sqrt{-1+e^{2x^2}} \right) \\
&= \frac{1}{2} \tan^{-1} \left( \sqrt{-1+e^{2x^2}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0162912, size = 18, normalized size = 1.

$$\frac{1}{2} \tan^{-1} \left( \sqrt{e^{2x^2} - 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[-1 + E^(2\*x^2)], x]

[Out] ArcTan[Sqrt[-1 + E^(2\*x^2)]]/2

**Maple [A]** time = 0.06, size = 14, normalized size = 0.8

$$\frac{1}{2} \arctan \left( \sqrt{-1 + e^{2x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-1+exp(2\*x^2))^(1/2), x)

[Out] 1/2\*arctan((-1+exp(2\*x^2))^(1/2))

**Maxima [A]** time = 1.45893, size = 18, normalized size = 1.

$$\frac{1}{2} \arctan \left( \sqrt{e^{(2x^2)} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+exp(2\*x^2))^(1/2), x, algorithm="maxima")

[Out] 1/2\*arctan(sqrt(e^(2\*x^2) - 1))

**Fricas [A]** time = 0.789069, size = 45, normalized size = 2.5

$$\frac{1}{2} \arctan \left( \sqrt{e^{(2x^2)} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+exp(2*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(sqrt(e^(2*x^2) - 1))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(e^{x^2} - 1)(e^{x^2} + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+exp(2*x**2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((exp(x**2) - 1)*(exp(x**2) + 1)), x)
```

---

**Giac [A]** time = 1.23556, size = 18, normalized size = 1.

$$\frac{1}{2} \arctan\left(\sqrt{e^{(2x^2)} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+exp(2*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(e^(2*x^2) - 1))
```

### 3.660 $\int e^x \sqrt{9 + e^{2x}} dx$

**Optimal.** Leaf size=31

$$\frac{1}{2}e^x\sqrt{e^{2x}+9} + \frac{9}{2}\sinh^{-1}\left(\frac{e^x}{3}\right)$$

[Out]  $(E^x*\text{Sqrt}[9 + E^{(2*x)}])/2 + (9*\text{ArcSinh}[E^x/3])/2$

**Rubi [A]** time = 0.0250647, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 195, 215}

$$\frac{1}{2}e^x\sqrt{e^{2x}+9} + \frac{9}{2}\sinh^{-1}\left(\frac{e^x}{3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^x*\text{Sqrt}[9 + E^{(2*x)}], x]$

[Out]  $(E^x*\text{Sqrt}[9 + E^{(2*x)}])/2 + (9*\text{ArcSinh}[E^x/3])/2$

#### Rule 2249

$\text{Int}[(a_ + (b_)*(F_)^{(e_)*((c_)+(d_)*(x_))})^{(p_)}*(G_)^{(h_)*((f_)+(g_)*(x_))}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(d*e*\text{Log}[F])/(g*h*\text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*\text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m]-1)}*(a + b*F^{(c*e - (d*e*f)/g})*x^{\text{Numerator}[m]})^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \|\ \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 195

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[2*p] \|\ (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4*p]) \|\ (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3*p]) \|\ \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

#### Rule 215

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2], x\_Symbol] := \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int e^x \sqrt{9 + e^{2x}} dx &= \text{Subst}\left(\int \sqrt{9 + x^2} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{9 + e^{2x}} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9 + x^2}} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{9 + e^{2x}} + \frac{9}{2}\sinh^{-1}\left(\frac{e^x}{3}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0104095, size = 30, normalized size = 0.97

$$\frac{1}{2} \left( e^x \sqrt{e^{2x} + 9} + 9 \sinh^{-1} \left( \frac{e^x}{3} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sqrt[9 + E^(2\*x)], x]

[Out] (E^x\*Sqrt[9 + E^(2\*x)] + 9\*ArcSinh[E^x/3])/2

**Maple [A]** time = 0.059, size = 21, normalized size = 0.7

$$\frac{e^x}{2} \sqrt{9 + (e^x)^2} + \frac{9}{2} \operatorname{Arcsinh} \left( \frac{e^x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(9+exp(2\*x))^(1/2), x)

[Out] 1/2\*exp(x)\*(9+exp(x)^2)^(1/2)+9/2\*arcsinh(1/3\*exp(x))

**Maxima [A]** time = 1.46023, size = 27, normalized size = 0.87

$$\frac{1}{2} \sqrt{e^{(2x)} + 9e^x} + \frac{9}{2} \operatorname{arsinh} \left( \frac{1}{3} e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(9+exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] 1/2\*sqrt(e^(2\*x) + 9)\*e^x + 9/2\*arcsinh(1/3\*e^x)

**Fricas [A]** time = 0.777622, size = 84, normalized size = 2.71

$$\frac{1}{2} \sqrt{e^{(2x)} + 9e^x} - \frac{9}{2} \log \left( \sqrt{e^{(2x)} + 9} - e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(9+exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] 1/2\*sqrt(e^(2\*x) + 9)\*e^x - 9/2\*log(sqrt(e^(2\*x) + 9) - e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e^{2x} + 9e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(9+exp(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(exp(2*x) + 9)*exp(x), x)
```

**Giac [A]** time = 1.24677, size = 39, normalized size = 1.26

$$\frac{1}{2} \sqrt{e^{(2x)} + 9} e^x - \frac{9}{2} \log\left(\sqrt{e^{(2x)} + 9} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(9+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(e^(2*x) + 9)*e^x - 9/2*log(sqrt(e^(2*x) + 9) - e^x)
```

### 3.661 $\int e^x \sqrt{1 + e^{2x}} dx$

**Optimal.** Leaf size=27

$$\frac{1}{2}e^x\sqrt{e^{2x}+1} + \frac{1}{2}\sinh^{-1}(e^x)$$

[Out] (E^x\*Sqrt[1 + E^(2\*x)])/2 + ArcSinh[E^x]/2

**Rubi [A]** time = 0.022784, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 195, 215}

$$\frac{1}{2}e^x\sqrt{e^{2x}+1} + \frac{1}{2}\sinh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x\*Sqrt[1 + E^(2\*x)], x]

[Out] (E^x\*Sqrt[1 + E^(2\*x)])/2 + ArcSinh[E^x]/2

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int e^x \sqrt{1 + e^{2x}} dx &= \text{Subst} \left( \int \sqrt{1 + x^2} dx, x, e^x \right) \\ &= \frac{1}{2}e^x\sqrt{1 + e^{2x}} + \frac{1}{2} \text{Subst} \left( \int \frac{1}{\sqrt{1 + x^2}} dx, x, e^x \right) \\ &= \frac{1}{2}e^x\sqrt{1 + e^{2x}} + \frac{1}{2}\sinh^{-1}(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0093585, size = 24, normalized size = 0.89

$$\frac{1}{2} \left( e^x \sqrt{e^{2x} + 1} + \sinh^{-1}(e^x) \right)$$



Antiderivative was successfully verified.

[In] Integrate[E^x\*Sqrt[1 + E^(2\*x)], x]

[Out] (E^x\*Sqrt[1 + E^(2\*x)] + ArcSinh[E^x])/2

**Maple [A]** time = 0.058, size = 19, normalized size = 0.7

$$\frac{e^x \sqrt{1 + (e^x)^2}}{2} + \frac{\operatorname{Arcsinh}(e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(1+exp(2\*x))^(1/2), x)

[Out] 1/2\*exp(x)\*(1+exp(x)^2)^(1/2)+1/2\*arcsinh(exp(x))

**Maxima [A]** time = 1.48817, size = 24, normalized size = 0.89

$$\frac{1}{2} \sqrt{e^{(2x)} + 1} e^x + \frac{1}{2} \operatorname{arsinh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1+exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] 1/2\*sqrt(e^(2\*x) + 1)\*e^x + 1/2\*arcsinh(e^x)

**Fricas [A]** time = 0.657948, size = 84, normalized size = 3.11

$$\frac{1}{2} \sqrt{e^{(2x)} + 1} e^x - \frac{1}{2} \log\left(\sqrt{e^{(2x)} + 1} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1+exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] 1/2\*sqrt(e^(2\*x) + 1)\*e^x - 1/2\*log(sqrt(e^(2\*x) + 1) - e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e^{2x} + 1} e^x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1+exp(2\*x))\*\*(1/2), x)

[Out] Integral(sqrt(exp(2\*x) + 1)\*exp(x), x)

---

**Giac [A]** time = 1.22607, size = 39, normalized size = 1.44

$$\frac{1}{2} \sqrt{e^{(2x)} + 1} e^x - \frac{1}{2} \log\left(\sqrt{e^{(2x)} + 1} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(1+exp(2\*x))^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(e^(2\*x) + 1)\*e^x - 1/2\*log(sqrt(e^(2\*x) + 1) - e^x)

$$3.662 \quad \int \frac{e^{x^2} x}{1+e^{2x^2}} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{2} \tan^{-1}(e^{x^2})$$

[Out] ArcTan[E^x^2]/2

**Rubi [A]** time = 0.126492, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6715, 2249, 203}

$$\frac{1}{2} \tan^{-1}(e^{x^2})$$

Antiderivative was successfully verified.

[In] Int[(E^x^2\*x)/(1 + E^(2\*x^2)),x]

[Out] ArcTan[E^x^2]/2

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{x^2} x}{1+e^{2x^2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{e^x}{1+e^{2x}} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, e^{x^2} \right) \\ &= \frac{1}{2} \tan^{-1}(e^{x^2}) \end{aligned}$$

**Mathematica [A]** time = 0.0158131, size = 10, normalized size = 1.

$$\frac{1}{2} \tan^{-1}(e^{x^2})$$

Antiderivative was successfully verified.

[In] Integrate[(E^x^2\*x)/(1 + E^(2\*x^2)),x]

[Out] ArcTan[E^x^2]/2

---

**Maple [A]** time = 0.021, size = 8, normalized size = 0.8

$$\frac{\arctan(e^{x^2})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)\*x/(1+exp(2\*x^2)),x)

[Out] 1/2\*arctan(exp(x^2))

---

**Maxima [A]** time = 1.49493, size = 9, normalized size = 0.9

$$\frac{1}{2} \arctan(e^{(x^2)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*x/(1+exp(2\*x^2)),x, algorithm="maxima")

[Out] 1/2\*arctan(e^(x^2))

---

**Fricas [A]** time = 0.814956, size = 28, normalized size = 2.8

$$\frac{1}{2} \arctan(e^{(x^2)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*x/(1+exp(2\*x^2)),x, algorithm="fricas")

[Out] 1/2\*arctan(e^(x^2))

---

**Sympy [B]** time = 0.129271, size = 17, normalized size = 1.7

$$\text{RootSum}\left(16z^2 + 1, \left(i \mapsto i \log(4i + e^{x^2})\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x/(1+exp(2*x**2)),x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x**2))))
```

---

**Giac [A]** time = 1.25936, size = 9, normalized size = 0.9

$$\frac{1}{2} \arctan\left(e^{(x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x/(1+exp(2*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(e^(x^2))
```

### 3.663 $\int e^{x^{3/2}} x^2 dx$

**Optimal.** Leaf size=28

$$\frac{2}{3}e^{x^{3/2}}x^{3/2} - \frac{2e^{x^{3/2}}}{3}$$

[Out]  $(-2E^{x^{3/2}})/3 + (2E^{x^{3/2}}*x^{3/2})/3$

**Rubi [A]** time = 0.0423121, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2216, 2212, 2209}

$$\frac{2}{3}e^{x^{3/2}}x^{3/2} - \frac{2e^{x^{3/2}}}{3}$$

Antiderivative was successfully verified.

[In] Int[E<sup>x<sup>3/2</sup></sup>\*x<sup>2</sup>,x]

[Out]  $(-2E^{x^{3/2}})/3 + (2E^{x^{3/2}}*x^{3/2})/3$

#### Rule 2216

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*F^(a + b*x^(k*n)), x], x, (c + d*x)^(1/k)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && !IntegerQ[n]
```

#### Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\begin{aligned} \int e^{x^{3/2}} x^2 dx &= 2 \operatorname{Subst} \left( \int e^{x^3} x^5 dx, x, \sqrt{x} \right) \\ &= \frac{2}{3} e^{x^{3/2}} x^{3/2} - 2 \operatorname{Subst} \left( \int e^{x^3} x^2 dx, x, \sqrt{x} \right) \\ &= -\frac{2}{3} e^{x^{3/2}} + \frac{2}{3} e^{x^{3/2}} x^{3/2} \end{aligned}$$

**Mathematica [C]** time = 0.0013322, size = 13, normalized size = 0.46

$$-\frac{2}{3}\text{Gamma}\left(2, -x^{3/2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^(3/2)\*x^2,x]

[Out] (-2\*Gamma[2, -x^(3/2)])/3

---

**Maple [A]** time = 0.016, size = 17, normalized size = 0.6

$$-\frac{2}{3}e^{x^{\frac{3}{2}}} + \frac{2}{3}e^{x^{\frac{3}{2}}}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^(3/2))\*x^2,x)

[Out] -2/3\*exp(x^(3/2))+2/3\*exp(x^(3/2))\*x^(3/2)

---

**Maxima [A]** time = 0.968502, size = 15, normalized size = 0.54

$$\frac{2}{3}\left(x^{\frac{3}{2}} - 1\right)e^{\left(x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^(3/2))\*x^2,x, algorithm="maxima")

[Out] 2/3\*(x^(3/2) - 1)\*e^(x^(3/2))

---

**Fricas [A]** time = 0.695311, size = 42, normalized size = 1.5

$$\frac{2}{3}\left(x^{\frac{3}{2}} - 1\right)e^{\left(x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^(3/2))\*x^2,x, algorithm="fricas")

[Out] 2/3\*(x^(3/2) - 1)\*e^(x^(3/2))

---

**Sympy [A]** time = 4.29522, size = 24, normalized size = 0.86

$$\frac{2x^{\frac{3}{2}}e^{x^{\frac{3}{2}}}}{3} - \frac{2e^{x^{\frac{3}{2}}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(3/2))*x**2,x)
```

```
[Out] 2*x**(3/2)*exp(x**(3/2))/3 - 2*exp(x**(3/2))/3
```

---

**Giac [A]** time = 1.24377, size = 15, normalized size = 0.54

$$\frac{2}{3} \left( x^{\frac{3}{2}} - 1 \right) e^{\left( x^{\frac{3}{2}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(3/2))*x^2,x, algorithm="giac")
```

```
[Out] 2/3*(x^(3/2) - 1)*e^(x^(3/2))
```



$$3.664 \quad \int \frac{e^x}{\sqrt{-3+e^{2x}}} dx$$

**Optimal.** Leaf size=16

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-3}}\right)$$

[Out] ArcTanh[E^x/Sqrt[-3 + E^(2\*x)]]

**Rubi [A]** time = 0.0244502, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2249, 217, 206}

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[-3 + E^(2\*x)],x]

[Out] ArcTanh[E^x/Sqrt[-3 + E^(2\*x)]]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*(f\_.) + (g\_.)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{\sqrt{-3+e^{2x}}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-3+x^2}} dx, x, e^x\right) \\ &= \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{e^x}{\sqrt{-3+e^{2x}}}\right) \\ &= \tanh^{-1}\left(\frac{e^x}{\sqrt{-3+e^{2x}}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0042846, size = 16, normalized size = 1.

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/Sqrt[-3 + E^(2\*x)], x]

[Out] ArcTanh[E^x/Sqrt[-3 + E^(2\*x)]]

**Maple [A]** time = 0.06, size = 13, normalized size = 0.8

$$\ln\left(e^x + \sqrt{-3 + (e^x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(-3+exp(2\*x))^(1/2), x)

[Out] ln(exp(x)+(-3+exp(x)^2)^(1/2))

**Maxima [A]** time = 0.966175, size = 22, normalized size = 1.38

$$\log\left(2\sqrt{e^{(2x)}-3} + 2e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-3+exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] log(2\*sqrt(e^(2\*x) - 3) + 2\*e^x)

**Fricas [A]** time = 0.737928, size = 42, normalized size = 2.62

$$-\log\left(\sqrt{e^{(2x)}-3} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-3+exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] -log(sqrt(e^(2\*x) - 3) - e^x)

**Sympy [A]** time = 0.80179, size = 10, normalized size = 0.62

$$\operatorname{acosh}\left(\frac{\sqrt{3}e^x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-3+exp(2*x))**(1/2),x)
```

```
[Out] acosh(sqrt(3)*exp(x)/3)
```

---

**Giac [A]** time = 1.22365, size = 22, normalized size = 1.38

$$-\log\left(-\sqrt{e^{2x}-3}+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-3+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(e^(2*x) - 3) + e^x)
```

$$3.665 \quad \int \frac{e^x}{16 - e^{2x}} dx$$

**Optimal.** Leaf size=12

$$\frac{1}{4} \tanh^{-1} \left( \frac{e^x}{4} \right)$$

[Out] ArcTanh[E<sup>x</sup>/4]/4

**Rubi [A]** time = 0.0221998, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 206}

$$\frac{1}{4} \tanh^{-1} \left( \frac{e^x}{4} \right)$$

Antiderivative was successfully verified.

[In] Int[E<sup>x</sup>/(16 - E<sup>(2\*x)</sup>), x]

[Out] ArcTanh[E<sup>x</sup>/4]/4

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{16 - e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{16 - x^2} dx, x, e^x \right) \\ &= \frac{1}{4} \tanh^{-1} \left( \frac{e^x}{4} \right) \end{aligned}$$

**Mathematica [A]** time = 0.0031379, size = 12, normalized size = 1.

$$\frac{1}{4} \tanh^{-1} \left( \frac{e^x}{4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E<sup>x</sup>/(16 - E<sup>(2\*x)</sup>), x]

[Out] ArcTanh[E<sup>x</sup>/4]/4

**Maple [B]** time = 0.023, size = 16, normalized size = 1.3

$$\frac{\ln(e^x + 4)}{8} - \frac{\ln(-4 + e^x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(16-exp(2\*x)),x)

[Out] 1/8\*ln(exp(x)+4)-1/8\*ln(-4+exp(x))

**Maxima [B]** time = 0.966717, size = 20, normalized size = 1.67

$$\frac{1}{8} \log(e^x + 4) - \frac{1}{8} \log(e^x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(16-exp(2\*x)),x, algorithm="maxima")

[Out] 1/8\*log(e<sup>x</sup> + 4) - 1/8\*log(e<sup>x</sup> - 4)

**Fricas [B]** time = 0.7882, size = 50, normalized size = 4.17

$$\frac{1}{8} \log(e^x + 4) - \frac{1}{8} \log(e^x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(16-exp(2\*x)),x, algorithm="fricas")

[Out] 1/8\*log(e<sup>x</sup> + 4) - 1/8\*log(e<sup>x</sup> - 4)

**Sympy [B]** time = 0.105618, size = 15, normalized size = 1.25

$$-\frac{\log(e^x - 4)}{8} + \frac{\log(e^x + 4)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(16-exp(2\*x)),x)

[Out] -log(exp(x) - 4)/8 + log(exp(x) + 4)/8

**Giac [B]** time = 1.21672, size = 22, normalized size = 1.83

$$\frac{1}{8} \log(e^x + 4) - \frac{1}{8} \log(|e^x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(16-exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/8*log(e^x + 4) - 1/8*log(abs(e^x - 4))
```

$$3.666 \quad \int \frac{e^{5x}}{1+e^{10x}} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{5} \tan^{-1}(e^{5x})$$

[Out] ArcTan[E^(5\*x)]/5

**Rubi [A]** time = 0.0208617, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2249, 203}

$$\frac{1}{5} \tan^{-1}(e^{5x})$$

Antiderivative was successfully verified.

[In] Int[E^(5\*x)/(1 + E^(10\*x)),x]

[Out] ArcTan[E^(5\*x)]/5

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{5x}}{1+e^{10x}} dx &= \frac{1}{5} \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, e^{5x} \right) \\ &= \frac{1}{5} \tan^{-1}(e^{5x}) \end{aligned}$$

**Mathematica [A]** time = 0.0031279, size = 10, normalized size = 1.

$$\frac{1}{5} \tan^{-1}(e^{5x})$$

Antiderivative was successfully verified.

[In] Integrate[E^(5\*x)/(1 + E^(10\*x)),x]

[Out] ArcTan[E^(5\*x)]/5

**Maple [A]** time = 0.02, size = 8, normalized size = 0.8

$$\frac{\arctan\left((e^x)^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(5\*x)/(1+exp(10\*x)),x)

[Out] 1/5\*arctan(exp(x)^5)

**Maxima [A]** time = 1.45586, size = 9, normalized size = 0.9

$$\frac{1}{5} \arctan\left(e^{(5x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(5\*x)/(1+exp(10\*x)),x, algorithm="maxima")

[Out] 1/5\*arctan(e^(5\*x))

**Fricas [A]** time = 0.85661, size = 28, normalized size = 2.8

$$\frac{1}{5} \arctan\left(e^{(5x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(5\*x)/(1+exp(10\*x)),x, algorithm="fricas")

[Out] 1/5\*arctan(e^(5\*x))

**Sympy [B]** time = 0.112689, size = 17, normalized size = 1.7

$$\text{RootSum}\left(100z^2 + 1, \left(i \mapsto i \log\left(10i + e^{5x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(5\*x)/(1+exp(10\*x)),x)

[Out] RootSum(100\*\_z\*\*2 + 1, Lambda(\_i, \_i\*log(10\*\_i + exp(5\*x))))

**Giac [A]** time = 1.73921, size = 9, normalized size = 0.9

$$\frac{1}{5} \arctan\left(e^{(5x)}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(5*x)/(1+exp(10*x)),x, algorithm="giac")
```

```
[Out] 1/5*arctan(e^(5*x))
```

$$3.667 \quad \int \frac{e^{4x}}{\sqrt{16+e^{8x}}} dx$$

**Optimal.** Leaf size=14

$$\frac{1}{4} \sinh^{-1} \left( \frac{e^{4x}}{4} \right)$$

[Out] ArcSinh[E^(4\*x)/4]/4

**Rubi [A]** time = 0.0234841, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2249, 215}

$$\frac{1}{4} \sinh^{-1} \left( \frac{e^{4x}}{4} \right)$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x)/Sqrt[16 + E^(8\*x)], x]

[Out] ArcSinh[E^(4\*x)/4]/4

#### Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

#### Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{\sqrt{16+e^{8x}}} dx &= \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{16+x^2}} dx, x, e^{4x} \right) \\ &= \frac{1}{4} \sinh^{-1} \left( \frac{e^{4x}}{4} \right) \end{aligned}$$

**Mathematica [A]** time = 0.0044799, size = 14, normalized size = 1.

$$\frac{1}{4} \sinh^{-1} \left( \frac{e^{4x}}{4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x)/Sqrt[16 + E^(8\*x)], x]

[Out] ArcSinh[E^(4\*x)/4]/4

---

**Maple [F]** time = 0.122, size = 0, normalized size = 0.

$$\int e^{4x} \frac{1}{\sqrt{16 + e^{8x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x)/(16+exp(8\*x))^(1/2), x)

[Out] int(exp(4\*x)/(16+exp(8\*x))^(1/2), x)

---

**Maxima [A]** time = 1.45731, size = 12, normalized size = 0.86

$$\frac{1}{4} \operatorname{arsinh}\left(\frac{1}{4} e^{(4x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(16+exp(8\*x))^(1/2), x, algorithm="maxima")

[Out] 1/4\*arcsinh(1/4\*e^(4\*x))

---

**Fricas [A]** time = 0.597388, size = 54, normalized size = 3.86

$$-\frac{1}{4} \log\left(\sqrt{e^{(8x)} + 16} - e^{(4x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(16+exp(8\*x))^(1/2), x, algorithm="fricas")

[Out] -1/4\*log(sqrt(e^(8\*x) + 16) - e^(4\*x))

---

**Sympy [A]** time = 0.946482, size = 8, normalized size = 0.57

$$\frac{\operatorname{asinh}\left(\frac{e^{4x}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(16+exp(8\*x))\*\*(1/2), x)

[Out] asinh(exp(4\*x)/4)/4

---

**Giac [A]** time = 1.23226, size = 24, normalized size = 1.71

$$-\frac{1}{4} \log\left(\sqrt{e^{(8x)} + 16} - e^{(4x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x)/(16+exp(8\*x))^(1/2),x, algorithm="giac")

[Out] -1/4\*log(sqrt(e^(8\*x) + 16) - e^(4\*x))

### 3.668 $\int e^{4x^3} x^2 \cos(7x^3) dx$

**Optimal.** Leaf size=35

$$\frac{7}{195}e^{4x^3} \sin(7x^3) + \frac{4}{195}e^{4x^3} \cos(7x^3)$$

[Out]  $(4E^{(4*x^3)}*Cos[7*x^3])/195 + (7E^{(4*x^3)}*Sin[7*x^3])/195$

**Rubi [A]** time = 0.17855, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {6715, 4433}

$$\frac{7}{195}e^{4x^3} \sin(7x^3) + \frac{4}{195}e^{4x^3} \cos(7x^3)$$

Antiderivative was successfully verified.

[In] Int[E^(4\*x^3)\*x^2\*Cos[7\*x^3],x]

[Out]  $(4E^{(4*x^3)}*Cos[7*x^3])/195 + (7E^{(4*x^3)}*Sin[7*x^3])/195$

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 4433

Int[Cos[(d\_) + (e\_)\*(x\_)]\*(F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))), x\_Symbol] := Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Cos[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] + Simp[(e\*F^(c\*(a + b\*x))\*Sin[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\begin{aligned} \int e^{4x^3} x^2 \cos(7x^3) dx &= \frac{1}{3} \text{Subst} \left( \int e^{4x} \cos(7x) dx, x, x^3 \right) \\ &= \frac{4}{195}e^{4x^3} \cos(7x^3) + \frac{7}{195}e^{4x^3} \sin(7x^3) \end{aligned}$$

**Mathematica [A]** time = 0.0534506, size = 28, normalized size = 0.8

$$\frac{1}{195}e^{4x^3} (7 \sin(7x^3) + 4 \cos(7x^3))$$

Antiderivative was successfully verified.

[In] Integrate[E^(4\*x^3)\*x^2\*Cos[7\*x^3],x]

[Out]  $(E^{(4*x^3)}*(4*Cos[7*x^3] + 7*Sin[7*x^3]))/195$

**Maple [A]** time = 0.052, size = 53, normalized size = 1.5

$$\left( \frac{14 e^{4x^3}}{195} \tan\left(\frac{7x^3}{2}\right) - \frac{4 e^{4x^3}}{195} \left( \tan\left(\frac{7x^3}{2}\right) \right)^2 + \frac{4 e^{4x^3}}{195} \left( 1 + \left( \tan\left(\frac{7x^3}{2}\right) \right)^2 \right)^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4\*x^3)\*x^2\*cos(7\*x^3), x)

[Out] (14/195\*exp(4\*x^3)\*tan(7/2\*x^3)-4/195\*exp(4\*x^3)\*tan(7/2\*x^3)^2+4/195\*exp(4\*x^3))/(1+tan(7/2\*x^3)^2)

**Maxima [A]** time = 0.968881, size = 39, normalized size = 1.11

$$\frac{4}{195} \cos(7x^3) e^{(4x^3)} + \frac{7}{195} e^{(4x^3)} \sin(7x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x^3)\*x^2\*cos(7\*x^3), x, algorithm="maxima")

[Out] 4/195\*cos(7\*x^3)\*e^(4\*x^3) + 7/195\*e^(4\*x^3)\*sin(7\*x^3)

**Fricas [A]** time = 0.868176, size = 77, normalized size = 2.2

$$\frac{4}{195} \cos(7x^3) e^{(4x^3)} + \frac{7}{195} e^{(4x^3)} \sin(7x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x^3)\*x^2\*cos(7\*x^3), x, algorithm="fricas")

[Out] 4/195\*cos(7\*x^3)\*e^(4\*x^3) + 7/195\*e^(4\*x^3)\*sin(7\*x^3)

**Sympy [A]** time = 2.24314, size = 32, normalized size = 0.91

$$\frac{7e^{4x^3} \sin(7x^3)}{195} + \frac{4e^{4x^3} \cos(7x^3)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4\*x\*\*3)\*x\*\*2\*cos(7\*x\*\*3), x)

[Out] 7\*exp(4\*x\*\*3)\*sin(7\*x\*\*3)/195 + 4\*exp(4\*x\*\*3)\*cos(7\*x\*\*3)/195

**Giac [A]** time = 1.23861, size = 34, normalized size = 0.97

$$\frac{1}{195} (4 \cos(7x^3) + 7 \sin(7x^3)) e^{(4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x^3)*x^2*cos(7*x^3),x, algorithm="giac")
```

```
[Out] 1/195*(4*cos(7*x^3) + 7*sin(7*x^3))*e^(4*x^3)
```

### 3.669 $\int e^{1+x^2} x dx$

**Optimal.** Leaf size=11

$$\frac{e^{x^2+1}}{2}$$

[Out]  $E^{(1 + x^2)}/2$

**Rubi [A]** time = 0.008696, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2209}

$$\frac{e^{x^2+1}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^(1 + x^2)\*x,x]

[Out] E^(1 + x^2)/2

#### Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x]
;/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

#### Rubi steps

$$\int e^{1+x^2} x dx = \frac{e^{1+x^2}}{2}$$

**Mathematica [A]** time = 0.0015886, size = 11, normalized size = 1.

$$\frac{e^{x^2+1}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(1 + x^2)\*x,x]

[Out] E^(1 + x^2)/2

**Maple [A]** time = 0.019, size = 9, normalized size = 0.8

$$\frac{e^{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(exp(x^2+1)*x,x)`

[Out] `1/2*exp(x^2+1)`

---

**Maxima [A]** time = 0.968426, size = 11, normalized size = 1.

$$\frac{1}{2} e^{(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2+1)*x,x, algorithm="maxima")`

[Out] `1/2*e^(x^2 + 1)`

---

**Fricas [A]** time = 0.865666, size = 23, normalized size = 2.09

$$\frac{1}{2} e^{(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2+1)*x,x, algorithm="fricas")`

[Out] `1/2*e^(x^2 + 1)`

---

**Sympy [A]** time = 0.0847, size = 7, normalized size = 0.64

$$\frac{e^{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2+1)*x,x)`

[Out] `exp(x**2 + 1)/2`

---

**Giac [A]** time = 1.26829, size = 11, normalized size = 1.

$$\frac{1}{2} e^{(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2+1)*x,x, algorithm="giac")`

[Out] `1/2*e^(x^2 + 1)`

$$3.670 \quad \int e^{1+x^3} x^2 dx$$

**Optimal.** Leaf size=11

$$\frac{e^{x^3+1}}{3}$$

[Out]  $E^{(1 + x^3)}/3$

**Rubi [A]** time = 0.0155205, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2209}

$$\frac{e^{x^3+1}}{3}$$

Antiderivative was successfully verified.

[In] Int[E^(1 + x^3)\*x^2,x]

[Out]  $E^{(1 + x^3)}/3$

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int e^{1+x^3} x^2 dx = \frac{e^{1+x^3}}{3}$$

**Mathematica [A]** time = 0.0015736, size = 11, normalized size = 1.

$$\frac{e^{x^3+1}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[E^(1 + x^3)\*x^2,x]

[Out]  $E^{(1 + x^3)}/3$

**Maple [A]** time = 0.019, size = 9, normalized size = 0.8

$$\frac{e^{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^3+1)*x^2,x)`

[Out] `1/3*exp(x^3+1)`

**Maxima [A]** time = 0.956121, size = 11, normalized size = 1.

$$\frac{1}{3} e^{(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^3+1)*x^2,x, algorithm="maxima")`

[Out] `1/3*e^(x^3 + 1)`

**Fricas [A]** time = 0.862101, size = 23, normalized size = 2.09

$$\frac{1}{3} e^{(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^3+1)*x^2,x, algorithm="fricas")`

[Out] `1/3*e^(x^3 + 1)`

**Sympy [A]** time = 0.084111, size = 7, normalized size = 0.64

$$\frac{e^{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**3+1)*x**2,x)`

[Out] `exp(x**3 + 1)/3`

**Giac [A]** time = 1.25404, size = 11, normalized size = 1.

$$\frac{1}{3} e^{(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^3+1)*x^2,x, algorithm="giac")`

[Out] `1/3*e^(x^3 + 1)`

$$3.671 \quad \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=9

$$2e^{\sqrt{x}}$$

[Out] 2\*E^Sqrt[x]

**Rubi [A]** time = 0.0103662, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[E^Sqrt[x]/Sqrt[x],x]

[Out] 2\*E^Sqrt[x]

Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n \* F^(a + b\*(c + d\*x)^n)) / (b\*f\*n\*(c + d\*x)^n \* Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}}$$

**Mathematica [A]** time = 0.0017917, size = 9, normalized size = 1.

$$2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[E^Sqrt[x]/Sqrt[x],x]

[Out] 2\*E^Sqrt[x]

**Maple [A]** time = 0.023, size = 7, normalized size = 0.8

$$2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^(1/2))/x^(1/2),x)

[Out]  $2\exp(x^{1/2})$

**Maxima [A]** time = 0.961795, size = 8, normalized size = 0.89

$$2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/2))/x^(1/2),x, algorithm="maxima")`

[Out]  $2e^{\sqrt{x}}$

**Fricas [A]** time = 0.819512, size = 18, normalized size = 2.

$$2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/2))/x^(1/2),x, algorithm="fricas")`

[Out]  $2e^{\sqrt{x}}$

**Sympy [A]** time = 0.191567, size = 7, normalized size = 0.78

$$2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**(1/2))/x**(1/2),x)`

[Out]  $2\exp(\sqrt{x})$

**Giac [A]** time = 1.18495, size = 8, normalized size = 0.89

$$2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/2))/x^(1/2),x, algorithm="giac")`

[Out]  $2e^{\sqrt{x}}$

$$3.672 \quad \int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} dx$$

**Optimal.** Leaf size=9

$$3e^{\sqrt[3]{x}}$$

[Out]  $3E^{x^{1/3}}$

**Rubi [A]** time = 0.0109865, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$3e^{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[E<sup>x<sup>1/3</sup></sup>/x<sup>2/3</sup>, x]

[Out]  $3E^{x^{1/3}}$

Rule 2209

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

Rubi steps

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} dx = 3e^{\sqrt[3]{x}}$$

**Mathematica [A]** time = 0.0018391, size = 9, normalized size = 1.

$$3e^{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[E<sup>x<sup>1/3</sup></sup>/x<sup>2/3</sup>, x]

[Out]  $3E^{x^{1/3}}$

**Maple [A]** time = 0.02, size = 7, normalized size = 0.8

$$3e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x<sup>1/3</sup>)/x<sup>2/3</sup>, x)

[Out]  $3\exp(x^{1/3})$

---

**Maxima [A]** time = 0.95909, size = 8, normalized size = 0.89

$$3e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/3))/x^(2/3),x, algorithm="maxima")`

[Out]  $3e^{x^{1/3}}$

---

**Fricas [A]** time = 0.803634, size = 20, normalized size = 2.22

$$3e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/3))/x^(2/3),x, algorithm="fricas")`

[Out]  $3e^{x^{1/3}}$

---

**Sympy [A]** time = 0.372059, size = 7, normalized size = 0.78

$$3e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**(1/3))/x**(2/3),x)`

[Out]  $3\exp(x^{1/3})$

---

**Giac [A]** time = 1.27028, size = 8, normalized size = 0.89

$$3e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^(1/3))/x^(2/3),x, algorithm="giac")`

[Out]  $3e^{x^{1/3}}$

### 3.673 $\int e^{3x} (-8 + 2x^3 + x^5) dx$

**Optimal.** Leaf size=68

$$\frac{1}{3}e^{3x}x^5 - \frac{5}{9}e^{3x}x^4 + \frac{38}{27}e^{3x}x^3 - \frac{38}{27}e^{3x}x^2 + \frac{76}{81}e^{3x}x - \frac{724e^{3x}}{243}$$

[Out]  $(-724E^{(3*x)})/243 + (76E^{(3*x)*x})/81 - (38E^{(3*x)*x^2})/27 + (38E^{(3*x)*x^3})/27 - (5E^{(3*x)*x^4})/9 + (E^{(3*x)*x^5})/3$

**Rubi [A]** time = 0.107394, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2196, 2194, 2176}

$$\frac{1}{3}e^{3x}x^5 - \frac{5}{9}e^{3x}x^4 + \frac{38}{27}e^{3x}x^3 - \frac{38}{27}e^{3x}x^2 + \frac{76}{81}e^{3x}x - \frac{724e^{3x}}{243}$$

Antiderivative was successfully verified.

[In] Int[E^(3\*x)\*(-8 + 2\*x^3 + x^5),x]

[Out]  $(-724E^{(3*x)})/243 + (76E^{(3*x)*x})/81 - (38E^{(3*x)*x^2})/27 + (38E^{(3*x)*x^3})/27 - (5E^{(3*x)*x^4})/9 + (E^{(3*x)*x^5})/3$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rubi steps



$$\begin{aligned}
\int e^{3x}(-8 + 2x^3 + x^5) dx &= \int (-8e^{3x} + 2e^{3x}x^3 + e^{3x}x^5) dx \\
&= 2 \int e^{3x}x^3 dx - 8 \int e^{3x} dx + \int e^{3x}x^5 dx \\
&= -\frac{8e^{3x}}{3} + \frac{2}{3}e^{3x}x^3 + \frac{1}{3}e^{3x}x^5 - \frac{5}{3} \int e^{3x}x^4 dx - 2 \int e^{3x}x^2 dx \\
&= -\frac{8e^{3x}}{3} - \frac{2}{3}e^{3x}x^2 + \frac{2}{3}e^{3x}x^3 - \frac{5}{9}e^{3x}x^4 + \frac{1}{3}e^{3x}x^5 + \frac{4}{3} \int e^{3x}x dx + \frac{20}{9} \int e^{3x}x^3 dx \\
&= -\frac{8e^{3x}}{3} + \frac{4}{9}e^{3x}x - \frac{2}{3}e^{3x}x^2 + \frac{38}{27}e^{3x}x^3 - \frac{5}{9}e^{3x}x^4 + \frac{1}{3}e^{3x}x^5 - \frac{4}{9} \int e^{3x} dx - \frac{20}{9} \int e^{3x}x^2 dx \\
&= -\frac{76e^{3x}}{27} + \frac{4}{9}e^{3x}x - \frac{38}{27}e^{3x}x^2 + \frac{38}{27}e^{3x}x^3 - \frac{5}{9}e^{3x}x^4 + \frac{1}{3}e^{3x}x^5 + \frac{40}{27} \int e^{3x}x dx \\
&= -\frac{76e^{3x}}{27} + \frac{76}{81}e^{3x}x - \frac{38}{27}e^{3x}x^2 + \frac{38}{27}e^{3x}x^3 - \frac{5}{9}e^{3x}x^4 + \frac{1}{3}e^{3x}x^5 - \frac{40}{81} \int e^{3x} dx \\
&= -\frac{724e^{3x}}{243} + \frac{76}{81}e^{3x}x - \frac{38}{27}e^{3x}x^2 + \frac{38}{27}e^{3x}x^3 - \frac{5}{9}e^{3x}x^4 + \frac{1}{3}e^{3x}x^5
\end{aligned}$$

**Mathematica [A]** time = 0.0212306, size = 34, normalized size = 0.5

$$\frac{1}{243}e^{3x}(81x^5 - 135x^4 + 342x^3 - 342x^2 + 228x - 724)$$

Antiderivative was successfully verified.

[In] Integrate[E^(3\*x)\*(-8 + 2\*x^3 + x^5), x]

[Out] (E^(3\*x)\*(-724 + 228\*x - 342\*x^2 + 342\*x^3 - 135\*x^4 + 81\*x^5))/243

**Maple [A]** time = 0.024, size = 32, normalized size = 0.5

$$\frac{e^{3x}(81x^5 - 135x^4 + 342x^3 - 342x^2 + 228x - 724)}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3\*x)\*(x^5+2\*x^3-8), x)

[Out] 1/243\*exp(3\*x)\*(81\*x^5-135\*x^4+342\*x^3-342\*x^2+228\*x-724)

**Maxima [A]** time = 0.983796, size = 80, normalized size = 1.18

$$\frac{1}{243}(81x^5 - 135x^4 + 180x^3 - 180x^2 + 120x - 40)e^{(3x)} + \frac{2}{27}(9x^3 - 9x^2 + 6x - 2)e^{(3x)} - \frac{8}{3}e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)\*(x^5+2\*x^3-8), x, algorithm="maxima")

[Out] 1/243\*(81\*x^5 - 135\*x^4 + 180\*x^3 - 180\*x^2 + 120\*x - 40)\*e^(3\*x) + 2/27\*(9\*x^3 - 9\*x^2 + 6\*x - 2)\*e^(3\*x) - 8/3\*e^(3\*x)

**Fricas [A]** time = 0.853035, size = 92, normalized size = 1.35

$$\frac{1}{243} (81x^5 - 135x^4 + 342x^3 - 342x^2 + 228x - 724)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)\*(x^5+2\*x^3-8),x, algorithm="fricas")

[Out] 1/243\*(81\*x^5 - 135\*x^4 + 342\*x^3 - 342\*x^2 + 228\*x - 724)\*e^(3\*x)

---

**Sympy [A]** time = 0.101229, size = 31, normalized size = 0.46

$$\frac{(81x^5 - 135x^4 + 342x^3 - 342x^2 + 228x - 724)e^{3x}}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)\*(x\*\*5+2\*x\*\*3-8),x)

[Out] (81\*x\*\*5 - 135\*x\*\*4 + 342\*x\*\*3 - 342\*x\*\*2 + 228\*x - 724)\*exp(3\*x)/243

---

**Giac [A]** time = 1.2537, size = 42, normalized size = 0.62

$$\frac{1}{243} (81x^5 - 135x^4 + 342x^3 - 342x^2 + 228x - 724)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)\*(x^5+2\*x^3-8),x, algorithm="giac")

[Out] 1/243\*(81\*x^5 - 135\*x^4 + 342\*x^3 - 342\*x^2 + 228\*x - 724)\*e^(3\*x)

### 3.674 $\int (e^x + x)^2 dx$

**Optimal.** Leaf size=28

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

[Out]  $-2E^x + E^{(2*x)}/2 + 2E^x*x + x^3/3$

**Rubi [A]** time = 0.023869, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6742, 2194, 2176}

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[(E^x + x)^2,x]

[Out]  $-2E^x + E^{(2*x)}/2 + 2E^x*x + x^3/3$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int (e^x + x)^2 dx &= \int (e^{2x} + 2e^x x + x^2) dx \\ &= \frac{x^3}{3} + 2 \int e^x x dx + \int e^{2x} dx \\ &= \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} - 2 \int e^x dx \\ &= -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} \end{aligned}$$

**Mathematica [A]** time = 0.014661, size = 26, normalized size = 0.93

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + e^x(2x - 2)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x + x)^2,x]

[Out] E^(2\*x)/2 + x^3/3 + E^x\*(-2 + 2\*x)

**Maple [A]** time = 0.019, size = 22, normalized size = 0.8

$$\frac{x^3}{3} + \frac{(e^x)^2}{2} + 2e^x x - 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)+x)^2,x)

[Out] 1/3\*x^3+1/2\*exp(x)^2+2\*exp(x)\*x-2\*exp(x)

**Maxima [A]** time = 0.98071, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="maxima")

[Out] 1/3\*x^3 + 2\*(x - 1)\*e^x + 1/2\*e^(2\*x)

**Fricas [A]** time = 0.863153, size = 53, normalized size = 1.89

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="fricas")

[Out] 1/3\*x^3 + 2\*(x - 1)\*e^x + 1/2\*e^(2\*x)

**Sympy [A]** time = 0.096342, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(x)+x)\*\*2,x)

[Out] x\*\*3/3 + (4\*x - 4)\*exp(x)/2 + exp(2\*x)/2

---

**Giac [A]** time = 1.23513, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="giac")

[Out] 1/3\*x^3 + 2\*(x - 1)\*e^x + 1/2\*e^(2\*x)

$$3.675 \quad \int e^{-4x} (e^x + e^{2x} + e^{3x}) dx$$

**Optimal.** Leaf size=26

$$-\frac{1}{3}e^{-3x} - \frac{e^{-2x}}{2} - e^{-x}$$

[Out]  $-1/(3E^{(3*x)}) - 1/(2E^{(2*x)}) - E^{(-x)}$

**Rubi [A]** time = 0.0220473, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2282, 14}

$$-\frac{1}{3}e^{-3x} - \frac{e^{-2x}}{2} - e^{-x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^x + E^{(2*x)} + E^{(3*x)})/E^{(4*x)}, x]$

[Out]  $-1/(3E^{(3*x)}) - 1/(2E^{(2*x)}) - E^{(-x)}$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rubi steps

$$\begin{aligned} \int e^{-4x} (e^x + e^{2x} + e^{3x}) dx &= \text{Subst} \left( \int \frac{1+x+x^2}{x^4} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{1}{x^4} + \frac{1}{x^3} + \frac{1}{x^2} \right) dx, x, e^x \right) \\ &= -\frac{1}{3}e^{-3x} - \frac{e^{-2x}}{2} - e^{-x} \end{aligned}$$

**Mathematica [A]** time = 0.0111108, size = 23, normalized size = 0.88

$$-\frac{1}{6}e^{-3x} (3e^x + 6e^{2x} + 2)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(E^x + E^{(2*x)} + E^{(3*x)})/E^{(4*x)}, x]$

[Out]  $-(2 + 3E^x + 6E^{(2*x)})/(6E^{(3*x)})$

**Maple [A]** time = 0.021, size = 20, normalized size = 0.8

$$-\frac{1}{3(e^x)^3} - \frac{1}{2(e^x)^2} - (e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((exp(x)+exp(2*x)+exp(3*x))/exp(4*x), x)`

[Out]  $-1/3/\exp(x)^3 - 1/2/\exp(x)^2 - 1/\exp(x)$

**Maxima [A]** time = 0.976115, size = 26, normalized size = 1.

$$-e^{(-x)} - \frac{1}{2}e^{(-2x)} - \frac{1}{3}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x), x, algorithm="maxima")`

[Out]  $-e^{(-x)} - 1/2*e^{(-2*x)} - 1/3*e^{(-3*x)}$

**Fricas [A]** time = 0.759663, size = 53, normalized size = 2.04

$$-\frac{1}{6}(6e^{(2x)} + 3e^x + 2)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x), x, algorithm="fricas")`

[Out]  $-1/6*(6*e^{(2*x)} + 3*e^x + 2)*e^{(-3*x)}$

**Sympy [A]** time = 0.104578, size = 22, normalized size = 0.85

$$-e^{-x} - \frac{e^{-2x}}{2} - \frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x), x)`

[Out]  $-\exp(-x) - \exp(-2*x)/2 - \exp(-3*x)/3$

**Giac [A]** time = 1.21308, size = 24, normalized size = 0.92

$$-\frac{1}{6}(6e^{(2x)} + 3e^x + 2)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x, algorithm="giac")
```

```
[Out] -1/6*(6*e^(2*x) + 3*e^x + 2)*e^(-3*x)
```



$$3.676 \quad \int \frac{e^x}{1+2e^x+e^{2x}} dx$$

**Optimal.** Leaf size=9

$$-\frac{1}{e^x + 1}$$

[Out]  $-(1 + E^x)^{-1}$

**Rubi [A]** time = 0.0241844, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2282, 32}

$$-\frac{1}{e^x + 1}$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + 2\*E^x + E^(2\*x)),x]

[Out]  $-(1 + E^x)^{-1}$

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{e^x}{1+2e^x+e^{2x}} dx = \text{Subst} \left( \int \frac{1}{(1+x)^2} dx, x, e^x \right) \\ = -\frac{1}{1+e^x}$$

**Mathematica [A]** time = 0.0074786, size = 9, normalized size = 1.

$$-\frac{1}{e^x + 1}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + 2\*E^x + E^(2\*x)),x]

[Out]  $-(1 + E^x)^{-1}$

---

**Maple [A]** time = 0.023, size = 9, normalized size = 1.

$$-(1 + e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+2*exp(x)+exp(2*x)),x)`

[Out] `-1/(1+exp(x))`

---

**Maxima [A]** time = 0.971287, size = 11, normalized size = 1.22

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")`

[Out] `-1/(e^x + 1)`

---

**Fricas [A]** time = 0.806071, size = 19, normalized size = 2.11

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")`

[Out] `-1/(e^x + 1)`

---

**Sympy [A]** time = 0.079009, size = 7, normalized size = 0.78

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x)`

[Out] `-1/(exp(x) + 1)`

---

**Giac [A]** time = 1.24297, size = 11, normalized size = 1.22

$$-\frac{1}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] -1/(e^x + 1)
```

### 3.677 $\int e^{-x} \cos(3x) dx$

**Optimal.** Leaf size=27

$$\frac{3}{10}e^{-x} \sin(3x) - \frac{1}{10}e^{-x} \cos(3x)$$

[Out]  $-\text{Cos}[3*x]/(10*E^x) + (3*\text{Sin}[3*x])/(10*E^x)$

**Rubi [A]** time = 0.0103131, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4433}

$$\frac{3}{10}e^{-x} \sin(3x) - \frac{1}{10}e^{-x} \cos(3x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[3*x]/E^x, x]$

[Out]  $-\text{Cos}[3*x]/(10*E^x) + (3*\text{Sin}[3*x])/(10*E^x)$

#### Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x]
  + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

#### Rubi steps

$$\int e^{-x} \cos(3x) dx = -\frac{1}{10}e^{-x} \cos(3x) + \frac{3}{10}e^{-x} \sin(3x)$$

**Mathematica [A]** time = 0.026826, size = 20, normalized size = 0.74

$$-\frac{1}{10}e^{-x}(\cos(3x) - 3 \sin(3x))$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Cos}[3*x]/E^x, x]$

[Out]  $-(\text{Cos}[3*x] - 3*\text{Sin}[3*x])/(10*E^x)$

**Maple [A]** time = 0.03, size = 22, normalized size = 0.8

$$-\frac{e^{-x} \cos(3x)}{10} + \frac{3e^{-x} \sin(3x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(3*x)/exp(x),x)`

[Out] `-1/10*exp(-x)*cos(3*x)+3/10*exp(-x)*sin(3*x)`

**Maxima [A]** time = 0.96265, size = 23, normalized size = 0.85

$$-\frac{1}{10}(\cos(3x) - 3 \sin(3x))e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)/exp(x),x, algorithm="maxima")`

[Out] `-1/10*(cos(3*x) - 3*sin(3*x))*e^(-x)`

**Fricas [A]** time = 0.823648, size = 62, normalized size = 2.3

$$-\frac{1}{10} \cos(3x) e^{(-x)} + \frac{3}{10} e^{(-x)} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)/exp(x),x, algorithm="fricas")`

[Out] `-1/10*cos(3*x)*e^(-x) + 3/10*e^(-x)*sin(3*x)`

**Sympy [A]** time = 0.483834, size = 20, normalized size = 0.74

$$\frac{3e^{-x} \sin(3x)}{10} - \frac{e^{-x} \cos(3x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)/exp(x),x)`

[Out] `3*exp(-x)*sin(3*x)/10 - exp(-x)*cos(3*x)/10`

**Giac [A]** time = 1.24043, size = 23, normalized size = 0.85

$$-\frac{1}{10}(\cos(3x) - 3 \sin(3x))e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)/exp(x),x, algorithm="giac")`

[Out] `-1/10*(cos(3*x) - 3*sin(3*x))*e^(-x)`

$$3.678 \quad \int \frac{e^{2x}}{2+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=17

$$2 \log(e^x + 2) - \log(e^x + 1)$$

[Out] -Log[1 + E^x] + 2\*Log[2 + E^x]

**Rubi [A]** time = 0.0320087, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2282, 632, 31}

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(2 + 3\*E^x + E^(2\*x)),x]

[Out] -Log[1 + E^x] + 2\*Log[2 + E^x]

#### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :=> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{2+3e^x+e^{2x}} dx &= \text{Subst} \left( \int \frac{x}{2+3x+x^2} dx, x, e^x \right) \\ &= 2 \text{Subst} \left( \int \frac{1}{2+x} dx, x, e^x \right) - \text{Subst} \left( \int \frac{1}{1+x} dx, x, e^x \right) \\ &= -\log(1 + e^x) + 2 \log(2 + e^x) \end{aligned}$$

**Mathematica [A]** time = 0.013487, size = 17, normalized size = 1.

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

**Maple [A]** time = 0.026, size = 16, normalized size = 0.9

$$-\ln(1 + e^x) + 2 \ln(2 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

**Maxima [A]** time = 0.979573, size = 20, normalized size = 1.18

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

**Fricas [A]** time = 0.808695, size = 42, normalized size = 2.47

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

**Sympy [A]** time = 0.12145, size = 14, normalized size = 0.82

$$-\log(e^x + 1) + 2 \log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -log(exp(x) + 1) + 2*log(exp(x) + 2)
```

**Giac [A]** time = 1.26502, size = 20, normalized size = 1.18

$$2 \log(e^x + 2) - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```



$$3.679 \quad \int \frac{e^{2x}}{1+e^x} dx$$

**Optimal.** Leaf size=12

$$e^x - \log(e^x + 1)$$

[Out] E<sup>x</sup> - Log[1 + E<sup>x</sup>]

**Rubi [A]** time = 0.0224585, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E<sup>(2\*x)</sup>/(1 + E<sup>x</sup>), x]

[Out] E<sup>x</sup> - Log[1 + E<sup>x</sup>]

#### Rule 2248

Int[((a\_) + (b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(p\_)\*(G\_)^((h\_)\*((f\_) + (g\_)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^x} dx &= \text{Subst} \left( \int \frac{x}{1+x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( 1 + \frac{1}{-1-x} \right) dx, x, e^x \right) \\ &= e^x - \log(1 + e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0078046, size = 12, normalized size = 1.

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[E<sup>(2\*x)</sup>/(1 + E<sup>x</sup>), x]

[Out]  $E^x - \text{Log}[1 + E^x]$

---

**Maple [A]** time = 0.02, size = 11, normalized size = 0.9

$$e^x - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)/(1+exp(x)),x)`

[Out] `exp(x)-ln(1+exp(x))`

---

**Maxima [A]** time = 0.977899, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")`

[Out] `e^x - log(e^x + 1)`

---

**Fricas [A]** time = 0.871764, size = 27, normalized size = 2.25

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")`

[Out] `e^x - log(e^x + 1)`

---

**Sympy [A]** time = 0.085678, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x)`

[Out] `exp(x) - log(exp(x) + 1)`

---

**Giac [A]** time = 1.22499, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

### 3.680 $\int e^{3x} \cos(5x) dx$

**Optimal.** Leaf size=27

$$\frac{5}{34}e^{3x} \sin(5x) + \frac{3}{34}e^{3x} \cos(5x)$$

[Out]  $(3 * E^{(3 * x)} * \text{Cos}[5 * x]) / 34 + (5 * E^{(3 * x)} * \text{Sin}[5 * x]) / 34$

**Rubi [A]** time = 0.0107157, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4433}

$$\frac{5}{34}e^{3x} \sin(5x) + \frac{3}{34}e^{3x} \cos(5x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(3 * x)} * \text{Cos}[5 * x], x]$

[Out]  $(3 * E^{(3 * x)} * \text{Cos}[5 * x]) / 34 + (5 * E^{(3 * x)} * \text{Sin}[5 * x]) / 34$

#### Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x]
  + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

#### Rubi steps

$$\int e^{3x} \cos(5x) dx = \frac{3}{34}e^{3x} \cos(5x) + \frac{5}{34}e^{3x} \sin(5x)$$

**Mathematica [A]** time = 0.0270256, size = 22, normalized size = 0.81

$$\frac{1}{34}e^{3x}(5 \sin(5x) + 3 \cos(5x))$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[E^{(3 * x)} * \text{Cos}[5 * x], x]$

[Out]  $(E^{(3 * x)} * (3 * \text{Cos}[5 * x] + 5 * \text{Sin}[5 * x])) / 34$

**Maple [A]** time = 0.026, size = 22, normalized size = 0.8

$$\frac{3 e^{3x} \cos(5x)}{34} + \frac{5 e^{3x} \sin(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(3*x)*cos(5*x),x)`

[Out] `3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)`

**Maxima [A]** time = 0.985301, size = 26, normalized size = 0.96

$$\frac{1}{34} (3 \cos(5x) + 5 \sin(5x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*cos(5*x),x, algorithm="maxima")`

[Out] `1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)`

**Fricas [A]** time = 0.921112, size = 63, normalized size = 2.33

$$\frac{3}{34} \cos(5x)e^{(3x)} + \frac{5}{34} e^{(3x)} \sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*cos(5*x),x, algorithm="fricas")`

[Out] `3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)`

**Sympy [A]** time = 0.307838, size = 26, normalized size = 0.96

$$\frac{5e^{3x} \sin(5x)}{34} + \frac{3e^{3x} \cos(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*cos(5*x),x)`

[Out] `5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34`

**Giac [A]** time = 1.2394, size = 26, normalized size = 0.96

$$\frac{1}{34} (3 \cos(5x) + 5 \sin(5x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*cos(5*x),x, algorithm="giac")`

[Out] `1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)`

### 3.681 $\int e^x \operatorname{sech}(e^x) dx$

**Optimal.** Leaf size=5

$$\tan^{-1}(\sinh(e^x))$$

[Out] ArcTan[Sinh[E^x]]

**Rubi [A]** time = 0.013739, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2282, 3770}

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

[In] Int[E^x\*Sech[E^x],x]

[Out] ArcTan[Sinh[E^x]]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

#### Rubi steps

$$\begin{aligned} \int e^x \operatorname{sech}(e^x) dx &= \operatorname{Subst}\left(\int \operatorname{sech}(x) dx, x, e^x\right) \\ &= \tan^{-1}(\sinh(e^x)) \end{aligned}$$

**Mathematica [A]** time = 0.0056137, size = 5, normalized size = 1.

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sech[E^x],x]

[Out] ArcTan[Sinh[E^x]]

**Maple [A]** time = 0.02, size = 5, normalized size = 1.

$$\arctan(\sinh(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*sech(exp(x)),x)`

[Out] `arctan(sinh(exp(x)))`

**Maxima [A]** time = 0.96951, size = 5, normalized size = 1.

`arctan(sinh(ex))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sech(exp(x)),x, algorithm="maxima")`

[Out] `arctan(sinh(ex))`

**Fricas [B]** time = 0.90772, size = 82, normalized size = 16.4

`2 arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sech(exp(x)),x, algorithm="fricas")`

[Out] `2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sech(exp(x)),x)`

[Out] `Integral(exp(x)*sech(exp(x)), x)`

**Giac [A]** time = 1.2071, size = 8, normalized size = 1.6

`2 arctan(e(ex))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sech(exp(x)),x, algorithm="giac")`

[Out] `2*arctan(e(ex))`

$$3.682 \quad \int \frac{e^{-x}}{1+2e^x} dx$$

**Optimal.** Leaf size=21

$$-2x - e^{-x} + 2 \log(2e^x + 1)$$

[Out]  $-E^{-x} - 2*x + 2*Log[1 + 2*E^x]$

**Rubi [A]** time = 0.0252346, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 44}

$$-2x - e^{-x} + 2 \log(2e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(E^x*(1 + 2*E^x)), x]$

[Out]  $-E^{-x} - 2*x + 2*Log[1 + 2*E^x]$

#### Rule 2248

$\text{Int}[(a_ + (b_)*(F_)^((e_)*((c_ + (d_)*(x_))))^((p_)*(G_)^((h_)*((f_ + (g_)*(x_))))), x\_Symbol] :> \text{With}[\{m = \text{FullSimplify}[(g*h*Log[G])/(d*e*Log[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d})/(\text{d*e*Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] || \text{GeQ}[m, 1]] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 44

$\text{Int}[(a_ + (b_)*(x_))^{(m_)*((c_ + (d_)*(x_))^{(n_)}), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \& \& \text{NeQ}[b*c - a*d, 0] \& \& \text{ILtQ}[m, 0] \& \& \text{IntegerQ}[n] \& \& !(\text{IGtQ}[n, 0] \& \& \text{LtQ}[m + n + 2, 0])]$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-x}}{1+2e^x} dx &= \text{Subst} \left( \int \frac{1}{x^2(1+2x)} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{1}{x^2} - \frac{2}{x} + \frac{4}{1+2x} \right) dx, x, e^x \right) \\ &= -e^{-x} - 2x + 2 \log(1 + 2e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0152523, size = 21, normalized size = 1.

$$-2x - e^{-x} + 2 \log(2e^x + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(E^x*(1 + 2*E^x)), x]$



[Out]  $-E^{(-x)} - 2*x + 2*\text{Log}[1 + 2*E^x]$

---

**Maple [A]** time = 0.025, size = 22, normalized size = 1.1

$$-(e^x)^{-1} - 2 \ln(e^x) + 2 \ln(1 + 2e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/exp(x)/(1+2*exp(x)),x)`

[Out]  $-1/\exp(x) - 2*\ln(\exp(x)) + 2*\ln(1+2*\exp(x))$

---

**Maxima [A]** time = 0.976428, size = 22, normalized size = 1.05

$$-e^{(-x)} + 2 \log(e^{(-x)} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="maxima")`

[Out]  $-e^{(-x)} + 2*\log(e^{(-x)} + 2)$

---

**Fricas [A]** time = 0.958201, size = 62, normalized size = 2.95

$$-(2xe^x - 2e^x \log(2e^x + 1) + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="fricas")`

[Out]  $-(2*x*e^x - 2*e^x*\log(2*e^x + 1) + 1)*e^{(-x)}$

---

**Sympy [A]** time = 0.097492, size = 17, normalized size = 0.81

$$-2x + 2 \log\left(e^x + \frac{1}{2}\right) - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+2*exp(x)),x)`

[Out]  $-2*x + 2*\log(\exp(x) + 1/2) - \exp(-x)$

---

**Giac [A]** time = 1.19674, size = 26, normalized size = 1.24

$$-2x - e^{(-x)} + 2 \log(2e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="giac")
```

```
[Out] -2*x - e^(-x) + 2*log(2*e^x + 1)
```

### 3.683 $\int e^x \cos(4 + 3x) dx$

**Optimal.** Leaf size=27

$$\frac{3}{10}e^x \sin(3x + 4) + \frac{1}{10}e^x \cos(3x + 4)$$

[Out] (E^x\*Cos[4 + 3\*x])/10 + (3\*E^x\*Sin[4 + 3\*x])/10

**Rubi [A]** time = 0.0099718, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4433}

$$\frac{3}{10}e^x \sin(3x + 4) + \frac{1}{10}e^x \cos(3x + 4)$$

Antiderivative was successfully verified.

[In] Int[E^x\*Cos[4 + 3\*x],x]

[Out] (E^x\*Cos[4 + 3\*x])/10 + (3\*E^x\*Sin[4 + 3\*x])/10

#### Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_.)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

#### Rubi steps

$$\int e^x \cos(4 + 3x) dx = \frac{1}{10}e^x \cos(4 + 3x) + \frac{3}{10}e^x \sin(4 + 3x)$$

**Mathematica [A]** time = 0.0420207, size = 22, normalized size = 0.81

$$\frac{1}{10}e^x(3 \sin(3x + 4) + \cos(3x + 4))$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Cos[4 + 3\*x],x]

[Out] (E^x\*(Cos[4 + 3\*x] + 3\*Sin[4 + 3\*x]))/10

**Maple [A]** time = 0.023, size = 22, normalized size = 0.8

$$\frac{e^x \cos(4 + 3x)}{10} + \frac{3 e^x \sin(4 + 3x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*cos(4+3*x),x)`

[Out] `1/10*exp(x)*cos(4+3*x)+3/10*exp(x)*sin(4+3*x)`

---

**Maxima [A]** time = 0.972004, size = 26, normalized size = 0.96

$$\frac{1}{10} (\cos(3x + 4) + 3 \sin(3x + 4))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*cos(4+3*x),x, algorithm="maxima")`

[Out] `1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x`

---

**Fricas [A]** time = 0.679479, size = 63, normalized size = 2.33

$$\frac{1}{10} \cos(3x + 4) e^x + \frac{3}{10} e^x \sin(3x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*cos(4+3*x),x, algorithm="fricas")`

[Out] `1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)`

---

**Sympy [A]** time = 0.311352, size = 24, normalized size = 0.89

$$\frac{3e^x \sin(3x + 4)}{10} + \frac{e^x \cos(3x + 4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*cos(4+3*x),x)`

[Out] `3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10`

---

**Giac [A]** time = 1.22541, size = 26, normalized size = 0.96

$$\frac{1}{10} (\cos(3x + 4) + 3 \sin(3x + 4))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*cos(4+3*x),x, algorithm="giac")`

[Out] `1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x`

### 3.684 $\int e^x \sec^3(1 - e^x) dx$

**Optimal.** Leaf size=34

$$-\frac{1}{2} \tanh^{-1}(\sin(1 - e^x)) - \frac{1}{2} \tan(1 - e^x) \sec(1 - e^x)$$

[Out] -ArcTanh[Sin[1 - E^x]]/2 - (Sec[1 - E^x]\*Tan[1 - E^x])/2

**Rubi [A]** time = 0.0298723, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2282, 3768, 3770}

$$-\frac{1}{2} \tanh^{-1}(\sin(1 - e^x)) - \frac{1}{2} \tan(1 - e^x) \sec(1 - e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x\*Sec[1 - E^x]^3,x]

[Out] -ArcTanh[Sin[1 - E^x]]/2 - (Sec[1 - E^x]\*Tan[1 - E^x])/2

#### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :=> -Simp[(b\*Cos[c + d\*x]^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int e^x \sec^3(1 - e^x) dx &= \text{Subst} \left( \int \sec^3(1 - x) dx, x, e^x \right) \\ &= -\frac{1}{2} \sec(1 - e^x) \tan(1 - e^x) + \frac{1}{2} \text{Subst} \left( \int \sec(1 - x) dx, x, e^x \right) \\ &= -\frac{1}{2} \tanh^{-1}(\sin(1 - e^x)) - \frac{1}{2} \sec(1 - e^x) \tan(1 - e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0163438, size = 34, normalized size = 1.

$$-\frac{1}{2} \tanh^{-1}(\sin(1 - e^x)) - \frac{1}{2} \tan(1 - e^x) \sec(1 - e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sec[1 - E^x]^3,x]

[Out] -ArcTanh[Sin[1 - E^x]]/2 - (Sec[1 - E^x]\*Tan[1 - E^x])/2

**Maple [A]** time = 0.305, size = 28, normalized size = 0.8

$$\frac{\sec(-1 + e^x) \tan(-1 + e^x)}{2} + \frac{\ln(\sec(-1 + e^x) + \tan(-1 + e^x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*sec(-1+exp(x))^3,x)

[Out] 1/2\*sec(-1+exp(x))\*tan(-1+exp(x))+1/2\*ln(sec(-1+exp(x))+tan(-1+exp(x)))

**Maxima [A]** time = 0.964896, size = 53, normalized size = 1.56

$$-\frac{\sin(e^x - 1)}{2(\sin(e^x - 1)^2 - 1)} + \frac{1}{4} \log(\sin(e^x - 1) + 1) - \frac{1}{4} \log(\sin(e^x - 1) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*sec(-1+exp(x))^3,x, algorithm="maxima")

[Out] -1/2\*sin(e^x - 1)/(sin(e^x - 1)^2 - 1) + 1/4\*log(sin(e^x - 1) + 1) - 1/4\*log(sin(e^x - 1) - 1)

**Fricas [B]** time = 0.735651, size = 157, normalized size = 4.62

$$\frac{\cos(e^x - 1)^2 \log(\sin(e^x - 1) + 1) - \cos(e^x - 1)^2 \log(-\sin(e^x - 1) + 1) + 2 \sin(e^x - 1)}{4 \cos(e^x - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*sec(-1+exp(x))^3,x, algorithm="fricas")

[Out] 1/4\*(cos(e^x - 1)^2\*log(sin(e^x - 1) + 1) - cos(e^x - 1)^2\*log(-sin(e^x - 1) + 1) + 2\*sin(e^x - 1))/cos(e^x - 1)^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int e^x \sec^3(e^x - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*sec(-1+exp(x))\*\*3,x)

[Out] Integral(exp(x)\*sec(exp(x) - 1)\*\*3, x)

---

**Giac [A]** time = 1.20622, size = 55, normalized size = 1.62

$$-\frac{\sin(e^x - 1)}{2(\sin(e^x - 1)^2 - 1)} + \frac{1}{4} \log(\sin(e^x - 1) + 1) - \frac{1}{4} \log(-\sin(e^x - 1) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*sec(-1+exp(x))^3,x, algorithm="giac")

[Out] -1/2\*sin(e^x - 1)/(sin(e^x - 1)^2 - 1) + 1/4\*log(sin(e^x - 1) + 1) - 1/4\*log(-sin(e^x - 1) + 1)

### 3.685 $\int (e^{-x} + e^x) x dx$

**Optimal.** Leaf size=26

$$-e^{-x}x + e^x x - e^{-x} - e^x$$

[Out]  $-E^{(-x)} - E^x - x/E^x + E^x*x$

**Rubi [A]** time = 0.0174877, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {14, 2176, 2194}

$$-e^{-x}x + e^x x - e^{-x} - e^x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{(-x)} + E^x)*x, x]$

[Out]  $-E^{(-x)} - E^x - x/E^x + E^x*x$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_))] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

#### Rule 2176

$\text{Int}[(b_)*(F_)^{((g_)*((e_)+(f_)*(x_)))^{(n_)}*((c_)+(d_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Simp}[(c+d*x)^m*(b*F^{(g*(e+f*x)))^n}/(f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(f*g*n*\text{Log}[F]), \text{Int}[(c+d*x)^{(m-1)}*(b*F^{(g*(e+f*x)))^n}, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == True$

#### Rule 2194

$\text{Int}[(F_)^{((c_)*((a_)+(b_)*(x_)))^{(n_)}], x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a+b*x)))^n}/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int (e^{-x} + e^x) x dx &= \int (e^{-x}x + e^x x) dx \\ &= \int e^{-x}x dx + \int e^x x dx \\ &= -e^{-x}x + e^x x + \int e^{-x} dx - \int e^x dx \\ &= -e^{-x} - e^x - e^{-x}x + e^x x \end{aligned}$$

**Mathematica [A]** time = 0.0208862, size = 20, normalized size = 0.77

$$e^{-x} (e^{2x}(x-1) - x - 1)$$

Antiderivative was successfully verified.



[In] Integrate[(E^(-x) + E^x)\*x,x]

[Out] (-1 + E^(2\*x))\*(-1 + x) - x)/E^x

**Maple [A]** time = 0.021, size = 23, normalized size = 0.9

$$-(e^x)^{-1} - e^x - \frac{x}{e^x} + e^x x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-x)+exp(x))\*x,x)

[Out] -1/exp(x)-exp(x)-x/exp(x)+exp(x)\*x

**Maxima [A]** time = 0.965757, size = 22, normalized size = 0.85

$$-(x + 1)e^{(-x)} + (x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))\*x,x, algorithm="maxima")

[Out] -(x + 1)\*e^(-x) + (x - 1)\*e^x

**Fricas [A]** time = 0.847601, size = 46, normalized size = 1.77

$$\left((x - 1)e^{(2*x)} - x - 1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))\*x,x, algorithm="fricas")

[Out] ((x - 1)\*e^(2\*x) - x - 1)\*e^(-x)

**Sympy [A]** time = 0.09574, size = 14, normalized size = 0.54

$$(-x - 1)e^{-x} + (x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))\*x,x)

[Out] (-x - 1)\*exp(-x) + (x - 1)\*exp(x)

**Giac [A]** time = 1.21823, size = 22, normalized size = 0.85

$$-(x + 1)e^{(-x)} + (x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(-x)+exp(x))*x,x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + (x - 1)*e^x
```

$$3.686 \quad \int \frac{e^x}{2+3e^x+e^{2x}} dx$$

**Optimal.** Leaf size=15

$$\log(e^x + 1) - \log(e^x + 2)$$

[Out] Log[1 + E^x] - Log[2 + E^x]

**Rubi [A]** time = 0.0289803, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 616, 31}

$$\log(e^x + 1) - \log(e^x + 2)$$

Antiderivative was successfully verified.

[In] Int[E^x/(2 + 3\*E^x + E^(2\*x)), x]

[Out] Log[1 + E^x] - Log[2 + E^x]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{2+3e^x+e^{2x}} dx &= \text{Subst} \left( \int \frac{1}{2+3x+x^2} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \frac{1}{1+x} dx, x, e^x \right) - \text{Subst} \left( \int \frac{1}{2+x} dx, x, e^x \right) \\ &= \log(1 + e^x) - \log(2 + e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0065474, size = 10, normalized size = 0.67

$$-2 \tanh^{-1}(2e^x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(2 + 3\*E^x + E^(2\*x)),x]

[Out] -2\*ArcTanh[3 + 2\*E^x]

**Maple [A]** time = 0.026, size = 14, normalized size = 0.9

$$\ln(1 + e^x) - \ln(2 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(2+3\*exp(x)+exp(2\*x)),x)

[Out] ln(1+exp(x))-ln(2+exp(x))

**Maxima [A]** time = 0.972564, size = 18, normalized size = 1.2

$$-\log(e^x + 2) + \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3\*exp(x)+exp(2\*x)),x, algorithm="maxima")

[Out] -log(e^x + 2) + log(e^x + 1)

**Fricas [A]** time = 0.896429, size = 41, normalized size = 2.73

$$-\log(e^x + 2) + \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3\*exp(x)+exp(2\*x)),x, algorithm="fricas")

[Out] -log(e^x + 2) + log(e^x + 1)

**Sympy [A]** time = 0.111125, size = 12, normalized size = 0.8

$$\log(e^x + 1) - \log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3\*exp(x)+exp(2\*x)),x)

[Out] log(exp(x) + 1) - log(exp(x) + 2)

**Giac [A]** time = 1.21116, size = 18, normalized size = 1.2

$$-\log(e^x + 2) + \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] -log(e^x + 2) + log(e^x + 1)
```

$$3.687 \quad \int \frac{e^{2x}}{\sqrt[3]{1+e^x}} dx$$

**Optimal.** Leaf size=27

$$\frac{3}{5}(e^x + 1)^{5/3} - \frac{3}{2}(e^x + 1)^{2/3}$$

[Out]  $(-3*(1 + E^x)^{(2/3)})/2 + (3*(1 + E^x)^{(5/3)})/5$

**Rubi [A]** time = 0.0259199, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{3}{5}(e^x + 1)^{5/3} - \frac{3}{2}(e^x + 1)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(2*x)}/(1 + E^x)^{(1/3)}, x]$

[Out]  $(-3*(1 + E^x)^{(2/3)})/2 + (3*(1 + E^x)^{(5/3)})/5$

#### Rule 2248

$\text{Int}[(a_.) + (b_.)*(F_.)^{((e_.)*((c_.) + (d_.)*(x_)))}^{(p_.)}*(G_.)^{((h_.)*((f_.) + (g_.)*(x_)))}, x\_Symbol] :> \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d})/(\text{d}*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] || \text{GeQ}[m, 1]] /; \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{\sqrt[3]{1+e^x}} dx &= \text{Subst} \left( \int \frac{x}{\sqrt[3]{1+x}} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -\frac{1}{\sqrt[3]{1+x}} + (1+x)^{2/3} \right) dx, x, e^x \right) \\ &= -\frac{3}{2}(1+e^x)^{2/3} + \frac{3}{5}(1+e^x)^{5/3} \end{aligned}$$

**Mathematica [A]** time = 0.0101943, size = 20, normalized size = 0.74

$$\frac{3}{10}(e^x + 1)^{2/3}(2e^x - 3)$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(1 + E^x)^(1/3), x]

[Out] (3\*(1 + E^x)^(2/3)\*(-3 + 2\*E^x))/10

**Maple [A]** time = 0.02, size = 18, normalized size = 0.7

$$-\frac{3}{2}(1 + e^x)^{\frac{2}{3}} + \frac{3}{5}(1 + e^x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(1+exp(x))^(1/3), x)

[Out] -3/2\*(1+exp(x))^(2/3)+3/5\*(1+exp(x))^(5/3)

**Maxima [A]** time = 0.991878, size = 23, normalized size = 0.85

$$\frac{3}{5}(e^x + 1)^{\frac{5}{3}} - \frac{3}{2}(e^x + 1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))^(1/3), x, algorithm="maxima")

[Out] 3/5\*(e^x + 1)^(5/3) - 3/2\*(e^x + 1)^(2/3)

**Fricas [A]** time = 0.78088, size = 46, normalized size = 1.7

$$\frac{3}{10}(2e^x - 3)(e^x + 1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))^(1/3), x, algorithm="fricas")

[Out] 3/10\*(2\*e^x - 3)\*(e^x + 1)^(2/3)

**Sympy [A]** time = 1.89628, size = 22, normalized size = 0.81

$$\frac{3(e^x + 1)^{\frac{5}{3}}}{5} - \frac{3(e^x + 1)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))\*\*(1/3), x)

[Out] 3\*(exp(x) + 1)\*\*(5/3)/5 - 3\*(exp(x) + 1)\*\*(2/3)/2

**Giac [A]** time = 1.19501, size = 23, normalized size = 0.85

$$\frac{3}{5}(e^x + 1)^{\frac{5}{3}} - \frac{3}{2}(e^x + 1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))^(1/3),x, algorithm="giac")

[Out] 3/5\*(e^x + 1)^(5/3) - 3/2\*(e^x + 1)^(2/3)



$$3.688 \quad \int \frac{e^{2x}}{\sqrt[4]{1+e^x}} dx$$

**Optimal.** Leaf size=27

$$\frac{4}{7}(e^x + 1)^{7/4} - \frac{4}{3}(e^x + 1)^{3/4}$$

[Out]  $(-4*(1 + E^x)^{(3/4)})/3 + (4*(1 + E^x)^{(7/4)})/7$

**Rubi [A]** time = 0.0258179, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2248, 43}

$$\frac{4}{7}(e^x + 1)^{7/4} - \frac{4}{3}(e^x + 1)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[E^(2\*x)/(1 + E^x)^(1/4), x]

[Out]  $(-4*(1 + E^x)^{(3/4)})/3 + (4*(1 + E^x)^{(7/4)})/7$

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{\sqrt[4]{1+e^x}} dx &= \text{Subst} \left( \int \frac{x}{\sqrt[4]{1+x}} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -\frac{1}{\sqrt[4]{1+x}} + (1+x)^{3/4} \right) dx, x, e^x \right) \\ &= -\frac{4}{3}(1+e^x)^{3/4} + \frac{4}{7}(1+e^x)^{7/4} \end{aligned}$$

**Mathematica [A]** time = 0.0099306, size = 20, normalized size = 0.74

$$\frac{4}{21}(e^x + 1)^{3/4}(3e^x - 4)$$

Antiderivative was successfully verified.

[In] Integrate[E^(2\*x)/(1 + E^x)^(1/4), x]

[Out] (4\*(1 + E^x)^(3/4)\*(-4 + 3\*E^x))/21

**Maple [A]** time = 0.021, size = 18, normalized size = 0.7

$$-\frac{4}{3}(1 + e^x)^{\frac{3}{4}} + \frac{4}{7}(1 + e^x)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2\*x)/(1+exp(x))^(1/4), x)

[Out] -4/3\*(1+exp(x))^(3/4)+4/7\*(1+exp(x))^(7/4)

**Maxima [A]** time = 0.959522, size = 23, normalized size = 0.85

$$\frac{4}{7}(e^x + 1)^{\frac{7}{4}} - \frac{4}{3}(e^x + 1)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))^(1/4), x, algorithm="maxima")

[Out] 4/7\*(e^x + 1)^(7/4) - 4/3\*(e^x + 1)^(3/4)

**Fricas [A]** time = 0.771953, size = 46, normalized size = 1.7

$$\frac{4}{21}(3e^x - 4)(e^x + 1)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))^(1/4), x, algorithm="fricas")

[Out] 4/21\*(3\*e^x - 4)\*(e^x + 1)^(3/4)

**Sympy [A]** time = 2.62772, size = 22, normalized size = 0.81

$$\frac{4(e^x + 1)^{\frac{7}{4}}}{7} - \frac{4(e^x + 1)^{\frac{3}{4}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2\*x)/(1+exp(x))\*\*(1/4), x)

[Out] 4\*(exp(x) + 1)\*\*(7/4)/7 - 4\*(exp(x) + 1)\*\*(3/4)/3

**Giac [A]** time = 1.23547, size = 23, normalized size = 0.85

$$\frac{4}{7}(e^x + 1)^{\frac{7}{4}} - \frac{4}{3}(e^x + 1)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x))^(1/4),x, algorithm="giac")
```

```
[Out] 4/7*(e^x + 1)^(7/4) - 4/3*(e^x + 1)^(3/4)
```

$$3.689 \quad \int \frac{-e^x + 2e^{2x}}{\sqrt{-1 - 6e^x + 3e^{2x}}} dx$$

**Optimal.** Leaf size=62

$$\frac{2}{3} \sqrt{-6e^x + 3e^{2x} - 1} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-e^x)}{\sqrt{-6e^x + 3e^{2x} - 1}}\right)}{\sqrt{3}}$$

[Out] (2\*Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)])/3 - ArcTanh[(Sqrt[3]\*(1 - E^x))/Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)]]/Sqrt[3]

**Rubi [A]** time = 0.052227, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2282, 640, 621, 206}

$$\frac{2}{3} \sqrt{-6e^x + 3e^{2x} - 1} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-e^x)}{\sqrt{-6e^x + 3e^{2x} - 1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-E^x + 2\*E^(2\*x))/Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)], x]

[Out] (2\*Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)])/3 - ArcTanh[(Sqrt[3]\*(1 - E^x))/Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)]]/Sqrt[3]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

#### Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{-e^x + 2e^{2x}}{\sqrt{-1 - 6e^x + 3e^{2x}}} dx &= \text{Subst} \left( \int \frac{-1 + 2x}{\sqrt{-1 - 6x + 3x^2}} dx, x, e^x \right) \\
&= \frac{2}{3} \sqrt{-1 - 6e^x + 3e^{2x}} + \text{Subst} \left( \int \frac{1}{\sqrt{-1 - 6x + 3x^2}} dx, x, e^x \right) \\
&= \frac{2}{3} \sqrt{-1 - 6e^x + 3e^{2x}} + 2 \text{Subst} \left( \int \frac{1}{12 - x^2} dx, x, \frac{-6 + 6e^x}{\sqrt{-1 - 6e^x + 3e^{2x}}} \right) \\
&= \frac{2}{3} \sqrt{-1 - 6e^x + 3e^{2x}} - \frac{\tanh^{-1} \left( \frac{\sqrt{3}(1-e^x)}{\sqrt{-1-6e^x+3e^{2x}}} \right)}{\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.0406603, size = 54, normalized size = 0.87

$$\frac{2}{3} \sqrt{-6e^x + 3e^{2x} - 1} + \frac{\tanh^{-1} \left( \frac{e^x - 1}{\sqrt{-2e^x + e^{2x} - \frac{1}{3}}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-E^x + 2\*E^(2\*x))/Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)], x]

[Out] (2\*Sqrt[-1 - 6\*E^x + 3\*E^(2\*x)])/3 + ArcTanh[(-1 + E^x)/Sqrt[-1/3 - 2\*E^x + E^(2\*x)]]/Sqrt[3]

**Maple [A]** time = 0.068, size = 50, normalized size = 0.8

$$\frac{\sqrt{3}}{3} \ln \left( \frac{(-3 + 3e^x)\sqrt{3}}{3} + \sqrt{-1 - 6e^x + 3(e^x)^2} \right) + \frac{2}{3} \sqrt{-1 - 6e^x + 3(e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-exp(x)+2\*exp(2\*x))/(-1-6\*exp(x)+3\*exp(2\*x))^(1/2), x)

[Out] 1/3\*ln(1/3\*(-3+3\*exp(x))\*3^(1/2)+(-1-6\*exp(x)+3\*exp(x)^2)^(1/2))\*3^(1/2)+2/3\*(-1-6\*exp(x)+3\*exp(x)^2)^(1/2)

**Maxima [A]** time = 1.45963, size = 65, normalized size = 1.05

$$\frac{1}{3} \sqrt{3} \log \left( 2 \sqrt{3} \sqrt{3e^{(2x)} - 6e^x - 1} + 6e^x - 6 \right) + \frac{2}{3} \sqrt{3e^{(2x)} - 6e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-exp(x)+2\*exp(2\*x))/(-1-6\*exp(x)+3\*exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] 1/3\*sqrt(3)\*log(2\*sqrt(3)\*sqrt(3\*e^(2\*x) - 6\*e^x - 1) + 6\*e^x - 6) + 2/3\*sqrt(3\*e^(2\*x) - 6\*e^x - 1)

**Fricas [A]** time = 0.819745, size = 173, normalized size = 2.79

$$\frac{1}{6} \sqrt{3} \log\left(\left(\sqrt{3}e^x - \sqrt{3}\right)\sqrt{3e^{2x} - 6e^x - 1} + 3e^{2x} - 6e^x + 1\right) + \frac{2}{3} \sqrt{3e^{2x} - 6e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-exp(x)+2\*exp(2\*x))/(-1-6\*exp(x)+3\*exp(2\*x))^(1/2),x, algorithm="fricas")

[Out] 1/6\*sqrt(3)\*log((sqrt(3)\*e^x - sqrt(3))\*sqrt(3\*e^(2\*x) - 6\*e^x - 1) + 3\*e^(2\*x) - 6\*e^x + 1) + 2/3\*sqrt(3\*e^(2\*x) - 6\*e^x - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2e^x - 1)e^x}{\sqrt{3e^{2x} - 6e^x - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-exp(x)+2\*exp(2\*x))/(-1-6\*exp(x)+3\*exp(2\*x))\*\*(1/2),x)

[Out] Integral((2\*exp(x) - 1)\*exp(x)/sqrt(3\*exp(2\*x) - 6\*exp(x) - 1), x)

**Giac [A]** time = 1.20756, size = 66, normalized size = 1.06

$$-\frac{1}{3} \sqrt{3} \log\left(\left|-\sqrt{3}e^x + \sqrt{3} + \sqrt{3e^{2x} - 6e^x - 1}\right|\right) + \frac{2}{3} \sqrt{3e^{2x} - 6e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-exp(x)+2\*exp(2\*x))/(-1-6\*exp(x)+3\*exp(2\*x))^(1/2),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*log(abs(-sqrt(3)\*e^x + sqrt(3) + sqrt(3\*e^(2\*x) - 6\*e^x - 1))) + 2/3\*sqrt(3\*e^(2\*x) - 6\*e^x - 1)

### 3.690 $\int e^x (-5x + x^2) dx$

**Optimal.** Leaf size=19

$$e^x x^2 - 7e^x x + 7e^x$$

[Out] 7\*E^x - 7\*E^x\*x + E^x\*x^2

**Rubi [A]** time = 0.0429315, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {1593, 2196, 2176, 2194}

$$e^x x^2 - 7e^x x + 7e^x$$

Antiderivative was successfully verified.

[In] Int[E^x\*(-5\*x + x^2),x]

[Out] 7\*E^x - 7\*E^x\*x + E^x\*x^2

#### Rule 1593

Int[(u\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 2196

Int[(F\_)^((c\_)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma == True

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

Int[(F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned}
\int e^x(-5x + x^2) dx &= \int e^x(-5 + x)x dx \\
&= \int (-5e^x x + e^x x^2) dx \\
&= -\left(5 \int e^x x dx\right) + \int e^x x^2 dx \\
&= -5e^x x + e^x x^2 - 2 \int e^x x dx + 5 \int e^x dx \\
&= 5e^x - 7e^x x + e^x x^2 + 2 \int e^x dx \\
&= 7e^x - 7e^x x + e^x x^2
\end{aligned}$$

**Mathematica [A]** time = 0.0269181, size = 12, normalized size = 0.63

$$e^x(x^2 - 7x + 7)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*(-5\*x + x^2), x]

[Out] E^x\*(7 - 7\*x + x^2)

**Maple [A]** time = 0.022, size = 12, normalized size = 0.6

$$e^x(x^2 - 7x + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(x^2-5\*x), x)

[Out] exp(x)\*(x^2-7\*x+7)

**Maxima [A]** time = 0.980786, size = 26, normalized size = 1.37

$$(x^2 - 2x + 2)e^x - 5(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x^2-5\*x), x, algorithm="maxima")

[Out] (x^2 - 2\*x + 2)\*e^x - 5\*(x - 1)\*e^x

**Fricas [A]** time = 0.721356, size = 28, normalized size = 1.47

$$(x^2 - 7x + 7)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x^2-5\*x), x, algorithm="fricas")



[Out]  $(x^2 - 7x + 7)e^x$

---

**Sympy [A]** time = 0.0871, size = 10, normalized size = 0.53

$$(x^2 - 7x + 7)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(x**2-5*x),x)`

[Out]  $(x^2 - 7x + 7)\exp(x)$

---

**Giac [A]** time = 1.26269, size = 15, normalized size = 0.79

$$(x^2 - 7x + 7)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(x^2-5*x),x, algorithm="giac")`

[Out]  $(x^2 - 7x + 7)e^x$

### 3.691 $\int e^{3x} (-x + x^2) dx$

**Optimal.** Leaf size=32

$$\frac{1}{3}e^{3x}x^2 - \frac{5}{9}e^{3x}x + \frac{5e^{3x}}{27}$$

[Out]  $(5 * E^{(3 * x)}) / 27 - (5 * E^{(3 * x) * x}) / 9 + (E^{(3 * x) * x^2}) / 3$

**Rubi [A]** time = 0.0506393, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {1593, 2196, 2176, 2194}

$$\frac{1}{3}e^{3x}x^2 - \frac{5}{9}e^{3x}x + \frac{5e^{3x}}{27}$$

Antiderivative was successfully verified.

[In] Int[E^(3\*x)\*(-x + x^2),x]

[Out]  $(5 * E^{(3 * x)}) / 27 - (5 * E^{(3 * x) * x}) / 9 + (E^{(3 * x) * x^2}) / 3$

#### Rule 1593

Int[(u\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 2196

Int[(F\_)^((c\_)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rule 2194

Int[(F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned}
\int e^{3x}(-x+x^2) dx &= \int e^{3x}(-1+x)x dx \\
&= \int (-e^{3x}x + e^{3x}x^2) dx \\
&= -\int e^{3x}x dx + \int e^{3x}x^2 dx \\
&= -\frac{1}{3}e^{3x}x + \frac{1}{3}e^{3x}x^2 + \frac{1}{3}\int e^{3x} dx - \frac{2}{3}\int e^{3x}x dx \\
&= \frac{e^{3x}}{9} - \frac{5}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2 + \frac{2}{9}\int e^{3x} dx \\
&= \frac{5e^{3x}}{27} - \frac{5}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2
\end{aligned}$$

**Mathematica [A]** time = 0.0284713, size = 19, normalized size = 0.59

$$\frac{1}{27}e^{3x}(9x^2 - 15x + 5)$$

Antiderivative was successfully verified.

[In] Integrate[E^(3\*x)\*(-x + x^2),x]

[Out] (E^(3\*x)\*(5 - 15\*x + 9\*x^2))/27

**Maple [A]** time = 0.022, size = 17, normalized size = 0.5

$$\frac{e^{3x}(9x^2 - 15x + 5)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3\*x)\*(x^2-x),x)

[Out] 1/27\*exp(3\*x)\*(9\*x^2-15\*x+5)

**Maxima [A]** time = 0.975857, size = 38, normalized size = 1.19

$$\frac{1}{27}(9x^2 - 6x + 2)e^{(3x)} - \frac{1}{9}(3x - 1)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)\*(x^2-x),x, algorithm="maxima")

[Out] 1/27\*(9\*x^2 - 6\*x + 2)\*e^(3\*x) - 1/9\*(3\*x - 1)\*e^(3\*x)

**Fricas [A]** time = 0.808133, size = 45, normalized size = 1.41

$$\frac{1}{27}(9x^2 - 15x + 5)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*(x^2-x),x, algorithm="fricas")`

[Out]  $1/27*(9*x^2 - 15*x + 5)*e^{(3*x)}$

**Sympy [A]** time = 0.096386, size = 15, normalized size = 0.47

$$\frac{(9x^2 - 15x + 5)e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*(x**2-x),x)`

[Out]  $(9*x**2 - 15*x + 5)*exp(3*x)/27$

**Giac [A]** time = 1.2162, size = 22, normalized size = 0.69

$$\frac{1}{27}(9x^2 - 15x + 5)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*(x^2-x),x, algorithm="giac")`

[Out]  $1/27*(9*x^2 - 15*x + 5)*e^{(3*x)}$

$$3.692 \quad \int e^{x^x} x^{2x} (1 + \log(x)) dx$$

**Optimal.** Leaf size=11

$$e^{x^x} (x^x - 1)$$

[Out]  $E^{x^x}(-1 + x^x)$

**Rubi [F]** time = 0.145014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int e^{x^x} x^{2x} (1 + \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[ $E^{x^x} x^{2x} (1 + \text{Log}[x])$ ], x]

[Out] Defer[Int][ $E^{x^x} x^{2x}$ ], x] + Log[x]\*Defer[Int][ $E^{x^x} x^{2x}$ ], x] - Defer[Int][Defer[Int][ $E^{x^x} x^{2x}$ ], x]/x, x]

Rubi steps

$$\begin{aligned} \int e^{x^x} x^{2x} (1 + \log(x)) dx &= \int (e^{x^x} x^{2x} + e^{x^x} x^{2x} \log(x)) dx \\ &= \int e^{x^x} x^{2x} dx + \int e^{x^x} x^{2x} \log(x) dx \\ &= \log(x) \int e^{x^x} x^{2x} dx + \int e^{x^x} x^{2x} dx - \int \frac{\int e^{x^x} x^{2x} dx}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.0425892, size = 11, normalized size = 1.

$$e^{x^x} (x^x - 1)$$

Antiderivative was successfully verified.

[In] Integrate[ $E^{x^x} x^{2x} (1 + \text{Log}[x])$ ], x]

[Out]  $E^{x^x}(-1 + x^x)$

**Maple [B]** time = 0.04, size = 22, normalized size = 2.

$$e^{\ln(x)x} e^{e^{\ln(x)x}} - e^{e^{\ln(x)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^x)\*x^(2\*x)\*(1+ln(x)), x)

[Out] exp(ln(x)\*x)\*exp(exp(ln(x)\*x))-exp(exp(ln(x)\*x))

---

**Maxima [A]** time = 1.14235, size = 14, normalized size = 1.27

$$(x^x - 1)e^{(x^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^x)\*x^(2\*x)\*(1+log(x)),x, algorithm="maxima")

[Out] (x^x - 1)\*e^(x^x)

---

**Fricas [A]** time = 0.82625, size = 26, normalized size = 2.36

$$(x^x - 1)e^{(x^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^x)\*x^(2\*x)\*(1+log(x)),x, algorithm="fricas")

[Out] (x^x - 1)\*e^(x^x)

---

**Sympy [A]** time = 0.516164, size = 8, normalized size = 0.73

$$(x^x - 1)e^{x^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x\*\*x)\*x\*\*(2\*x)\*(1+ln(x)),x)

[Out] (x\*\*x - 1)\*exp(x\*\*x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{2x}(\log(x) + 1)e^{(x^x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^x)\*x^(2\*x)\*(1+log(x)),x, algorithm="giac")

[Out] integrate(x^(2\*x)\*(log(x) + 1)\*e^(x^x), x)

$$3.693 \quad \int \frac{e^{5x} + e^{7x}}{e^{-x} + e^x} dx$$

**Optimal.** Leaf size=9

$$\frac{e^{6x}}{6}$$

[Out] E^(6\*x)/6

**Rubi [A]** time = 0.0258115, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2282, 30}

$$\frac{e^{6x}}{6}$$

Antiderivative was successfully verified.

[In] Int[(E^(5\*x) + E^(7\*x))/(E^(-x) + E^x), x]

[Out] E^(6\*x)/6

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{e^{5x} + e^{7x}}{e^{-x} + e^x} dx = \text{Subst} \left( \int x^5 dx, x, e^x \right) = \frac{e^{6x}}{6}$$

**Mathematica [A]** time = 0.0010617, size = 9, normalized size = 1.

$$\frac{e^{6x}}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(5\*x) + E^(7\*x))/(E^(-x) + E^x), x]

[Out] E^(6\*x)/6

---

**Maple [A]** time = 0.023, size = 7, normalized size = 0.8

$$\frac{(e^x)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(5\*x)+exp(7\*x))/(exp(-x)+exp(x)),x)

[Out] 1/6\*exp(x)^6

---

**Maxima [A]** time = 0.989968, size = 8, normalized size = 0.89

$$\frac{1}{6} e^{(6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(5\*x)+exp(7\*x))/(exp(-x)+exp(x)),x, algorithm="maxima")

[Out] 1/6\*e^(6\*x)

---

**Fricas [A]** time = 0.822916, size = 18, normalized size = 2.

$$\frac{1}{6} e^{(6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(5\*x)+exp(7\*x))/(exp(-x)+exp(x)),x, algorithm="fricas")

[Out] 1/6\*e^(6\*x)

---

**Sympy [A]** time = 0.116089, size = 5, normalized size = 0.56

$$\frac{e^{6x}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(5\*x)+exp(7\*x))/(exp(-x)+exp(x)),x)

[Out] exp(6\*x)/6

---

**Giac [A]** time = 1.21292, size = 8, normalized size = 0.89

$$\frac{1}{6} e^{(6x)}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x, algorithm="giac")
```

```
[Out] 1/6*e^(6*x)
```

$$3.694 \quad \int x^{-2-\frac{1}{x}}(1 - \log(x)) dx$$

**Optimal.** Leaf size=9

$$-x^{-1/x}$$

[Out]  $-x^{(-x^{-1})}$

**Rubi [F]** time = 0.0726652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^{-2-\frac{1}{x}}(1 - \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x<sup>(-2 - x<sup>(-1)</sup>)</sup>\*(1 - Log[x]), x]

[Out] Defer[Int][x<sup>(-2 - x<sup>(-1)</sup>)</sup>, x] - Log[x]\*Defer[Int][x<sup>(-2 - x<sup>(-1)</sup>)</sup>, x] + Defer[Int][Defer[Int][x<sup>(-2 - x<sup>(-1)</sup>)</sup>, x]/x, x]

Rubi steps

$$\begin{aligned} \int x^{-2-\frac{1}{x}}(1 - \log(x)) dx &= \int \left( x^{-2-\frac{1}{x}} - x^{-2-\frac{1}{x}} \log(x) \right) dx \\ &= \int x^{-2-\frac{1}{x}} dx - \int x^{-2-\frac{1}{x}} \log(x) dx \\ &= -\left( \log(x) \int x^{-2-\frac{1}{x}} dx \right) + \int x^{-2-\frac{1}{x}} dx + \int \frac{\int x^{-2-\frac{1}{x}} dx}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.0199442, size = 9, normalized size = 1.

$$-x^{-1/x}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-2 - x<sup>(-1)</sup>)</sup>\*(1 - Log[x]), x]

[Out]  $-x^{(-x^{-1})}$

**Maple [A]** time = 0.032, size = 18, normalized size = 2.

$$-x^2 x^{-\frac{1+2x}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-2-1/x)</sup>\*(1-ln(x)), x)

[Out]  $-x^2*x^{-(1+2*x)/x}$

---

**Maxima [A]** time = 1.16794, size = 12, normalized size = 1.33

$$-\frac{1}{x^{\left(\frac{1}{x}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="maxima")`

[Out]  $-1/x^{(1/x)}$

---

**Fricas [A]** time = 0.815074, size = 30, normalized size = 3.33

$$-\frac{x^2}{x^{\frac{2x+1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="fricas")`

[Out]  $-x^2/x^{((2*x + 1)/x)}$

---

**Sympy [A]** time = 0.338044, size = 12, normalized size = 1.33

$$-x^2x^{-2-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-2-1/x)*(1-ln(x)),x)`

[Out]  $-x**2*x**(-2 - 1/x)$

---

**Giac [A]** time = 1.29806, size = 22, normalized size = 2.44

$$-xe^{\left(-\frac{x\log(x)+\log(x)}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="giac")`

[Out]  $-x*e^{-(x*\log(x) + \log(x))/x}$

### 3.695 $\int (a + be^x)^2 dx$

**Optimal.** Leaf size=25

$$a^2x + 2abe^x + \frac{1}{2}b^2e^{2x}$$

[Out] 2\*a\*b\*E^x + (b^2\*E^(2\*x))/2 + a^2\*x

**Rubi [A]** time = 0.0143958, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2282, 43}

$$a^2x + 2abe^x + \frac{1}{2}b^2e^{2x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x)^2,x]

[Out] 2\*a\*b\*E^x + (b^2\*E^(2\*x))/2 + a^2\*x

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int (a + be^x)^2 dx &= \text{Subst} \left( \int \frac{(a + bx)^2}{x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( 2ab + \frac{a^2}{x} + b^2x \right) dx, x, e^x \right) \\ &= 2abe^x + \frac{1}{2}b^2e^{2x} + a^2x \end{aligned}$$

**Mathematica [A]** time = 0.0094567, size = 25, normalized size = 1.

$$a^2x + 2abe^x + \frac{1}{2}b^2e^{2x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x)^2,x]

[Out]  $2*a*b*E^x + (b^2*E^{(2*x)})/2 + a^2*x$

---

**Maple [A]** time = 0.037, size = 24, normalized size = 1.

$$\frac{(e^x)^2 b^2}{2} + 2 a b e^x + a^2 \ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*exp(x))^2,x)`

[Out]  $1/2*\exp(x)^2*b^2+2*a*b*\exp(x)+a^2*\ln(\exp(x))$

---

**Maxima [A]** time = 0.968326, size = 28, normalized size = 1.12

$$a^2x + \frac{1}{2} b^2 e^{(2x)} + 2 a b e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))^2,x, algorithm="maxima")`

[Out]  $a^2*x + 1/2*b^2*e^{(2*x)} + 2*a*b*e^x$

---

**Fricas [A]** time = 0.831677, size = 50, normalized size = 2.

$$a^2x + \frac{1}{2} b^2 e^{(2x)} + 2 a b e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))^2,x, algorithm="fricas")`

[Out]  $a^2*x + 1/2*b^2*e^{(2*x)} + 2*a*b*e^x$

---

**Sympy [A]** time = 0.118013, size = 22, normalized size = 0.88

$$a^2x + 2 a b e^x + \frac{b^2 e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))**2,x)`

[Out]  $a**2*x + 2*a*b*\exp(x) + b**2*\exp(2*x)/2$

---

**Giac [A]** time = 1.30014, size = 28, normalized size = 1.12

$$a^2x + \frac{1}{2} b^2 e^{(2x)} + 2 a b e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(x))^2,x, algorithm="giac")
```

```
[Out] a^2*x + 1/2*b^2*e^(2*x) + 2*a*b*e^x
```

### 3.696 $\int (a + be^x)^3 dx$

**Optimal.** Leaf size=40

$$3a^2be^x + a^3x + \frac{3}{2}ab^2e^{2x} + \frac{1}{3}b^3e^{3x}$$

[Out]  $3*a^2*b*E^x + (3*a*b^2*E^{(2*x)})/2 + (b^3*E^{(3*x)})/3 + a^3*x$

**Rubi [A]** time = 0.0195108, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2282, 43}

$$3a^2be^x + a^3x + \frac{3}{2}ab^2e^{2x} + \frac{1}{3}b^3e^{3x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x)^3, x]

[Out]  $3*a^2*b*E^x + (3*a*b^2*E^{(2*x)})/2 + (b^3*E^{(3*x)})/3 + a^3*x$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int (a + be^x)^3 dx &= \text{Subst} \left( \int \frac{(a + bx)^3}{x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( 3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2 \right) dx, x, e^x \right) \\ &= 3a^2be^x + \frac{3}{2}ab^2e^{2x} + \frac{1}{3}b^3e^{3x} + a^3x \end{aligned}$$

**Mathematica [A]** time = 0.0124967, size = 40, normalized size = 1.

$$3a^2be^x + a^3x + \frac{3}{2}ab^2e^{2x} + \frac{1}{3}b^3e^{3x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x)^3, x]

[Out]  $3a^2bE^x + (3ab^2E^{(2x)})/2 + (b^3E^{(3x)})/3 + a^3x$

**Maple [A]** time = 0.039, size = 36, normalized size = 0.9

$$\frac{b^3 (e^x)^3}{3} + \frac{3ab^2 (e^x)^2}{2} + 3a^2be^x + a^3 \ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*exp(x))^3,x)`

[Out]  $1/3b^3\exp(x)^3 + 3/2a*b^2*\exp(x)^2 + 3a^2*b*\exp(x) + a^3*\ln(\exp(x))$

**Maxima [A]** time = 0.955045, size = 45, normalized size = 1.12

$$a^3x + \frac{1}{3}b^3e^{(3x)} + \frac{3}{2}ab^2e^{(2x)} + 3a^2be^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))^3,x, algorithm="maxima")`

[Out]  $a^3x + 1/3b^3e^{(3x)} + 3/2a*b^2*e^{(2x)} + 3a^2*b*e^x$

**Fricas [A]** time = 0.768507, size = 80, normalized size = 2.

$$a^3x + \frac{1}{3}b^3e^{(3x)} + \frac{3}{2}ab^2e^{(2x)} + 3a^2be^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))^3,x, algorithm="fricas")`

[Out]  $a^3x + 1/3b^3e^{(3x)} + 3/2a*b^2*e^{(2x)} + 3a^2*b*e^x$

**Sympy [A]** time = 0.141257, size = 37, normalized size = 0.92

$$a^3x + 3a^2be^x + \frac{3ab^2e^{2x}}{2} + \frac{b^3e^{3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))**3,x)`

[Out]  $a**3*x + 3*a**2*b*\exp(x) + 3*a*b**2*\exp(2*x)/2 + b**3*\exp(3*x)/3$

**Giac [A]** time = 1.2367, size = 45, normalized size = 1.12

$$a^3x + \frac{1}{3}b^3e^{(3x)} + \frac{3}{2}ab^2e^{(2x)} + 3a^2be^x$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(x))^3,x, algorithm="giac")
```

```
[Out] a^3*x + 1/3*b^3*e^(3*x) + 3/2*a*b^2*e^(2*x) + 3*a^2*b*e^x
```

### 3.697 $\int (a + be^x)^4 dx$

**Optimal.** Leaf size=53

$$3a^2b^2e^{2x} + 4a^3be^x + a^4x + \frac{4}{3}ab^3e^{3x} + \frac{1}{4}b^4e^{4x}$$

[Out]  $4a^3bE^x + 3a^2b^2E^{(2x)} + (4a^3b^3E^{(3x)})/3 + (b^4E^{(4x)})/4 + a^{4x}$

**Rubi [A]** time = 0.0264042, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2282, 43}

$$3a^2b^2e^{2x} + 4a^3be^x + a^4x + \frac{4}{3}ab^3e^{3x} + \frac{1}{4}b^4e^{4x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x)^4,x]

[Out]  $4a^3bE^x + 3a^2b^2E^{(2x)} + (4a^3b^3E^{(3x)})/3 + (b^4E^{(4x)})/4 + a^{4x}$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int (a + be^x)^4 dx &= \text{Subst} \left( \int \frac{(a + bx)^4}{x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( 4a^3b + \frac{a^4}{x} + 6a^2b^2x + 4ab^3x^2 + b^4x^3 \right) dx, x, e^x \right) \\ &= 4a^3be^x + 3a^2b^2e^{2x} + \frac{4}{3}ab^3e^{3x} + \frac{1}{4}b^4e^{4x} + a^4x \end{aligned}$$

**Mathematica [A]** time = 0.0165201, size = 53, normalized size = 1.

$$3a^2b^2e^{2x} + 4a^3be^x + a^4x + \frac{4}{3}ab^3e^{3x} + \frac{1}{4}b^4e^{4x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x)^4,x]

[Out]  $4a^3bE^x + 3a^2b^2E^{(2x)} + (4ab^3E^{(3x)})/3 + (b^4E^{(4x)})/4 + a^{4x}$

**Maple [A]** time = 0.036, size = 48, normalized size = 0.9

$$\frac{b^4 (e^x)^4}{4} + \frac{4ab^3 (e^x)^3}{3} + 3b^2a^2 (e^x)^2 + 4a^3be^x + a^4 \ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(x))^4,x)

[Out]  $1/4*b^4*exp(x)^4 + 4/3*a*b^3*exp(x)^3 + 3*b^2*a^2*exp(x)^2 + 4*a^3*b*exp(x) + a^4*\ln(exp(x))$

**Maxima [A]** time = 0.9777, size = 61, normalized size = 1.15

$$a^4x + \frac{1}{4}b^4e^{(4x)} + \frac{4}{3}ab^3e^{(3x)} + 3a^2b^2e^{(2x)} + 4a^3be^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^4,x, algorithm="maxima")

[Out]  $a^4*x + 1/4*b^4*e^{(4*x)} + 4/3*a*b^3*e^{(3*x)} + 3*a^2*b^2*e^{(2*x)} + 4*a^3*b*e^{x}$

**Fricas [A]** time = 0.865348, size = 107, normalized size = 2.02

$$a^4x + \frac{1}{4}b^4e^{(4x)} + \frac{4}{3}ab^3e^{(3x)} + 3a^2b^2e^{(2x)} + 4a^3be^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^4,x, algorithm="fricas")

[Out]  $a^4*x + 1/4*b^4*e^{(4*x)} + 4/3*a*b^3*e^{(3*x)} + 3*a^2*b^2*e^{(2*x)} + 4*a^3*b*e^{x}$

**Sympy [A]** time = 0.162058, size = 51, normalized size = 0.96

$$a^4x + 4a^3be^x + 3a^2b^2e^{2x} + \frac{4ab^3e^{3x}}{3} + \frac{b^4e^{4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*\*4,x)

[Out]  $a^{4x} + 4a^3b \exp(x) + 3a^2b^2 \exp(2x) + 4ab^3 \exp(3x)/3 + b^4 \exp(4x)/4$

**Giac [A]** time = 1.55905, size = 61, normalized size = 1.15

$$a^4x + \frac{1}{4}b^4e^{(4x)} + \frac{4}{3}ab^3e^{(3x)} + 3a^2b^2e^{(2x)} + 4a^3be^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))^4,x, algorithm="giac")`

[Out]  $a^{4x} + 1/4*b^4*e^{(4x)} + 4/3*a*b^3*e^{(3x)} + 3*a^2*b^2*e^{(2x)} + 4*a^3*b*e^{(x)}$

$$3.698 \quad \int \frac{1}{\sqrt{a+be^{c+dx}}} dx$$

**Optimal.** Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

[Out]  $(-2*\text{ArcTanh}[\text{Sqrt}[a + b*E^{(c + d*x)}]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d)$

**Rubi [A]** time = 0.0273548, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2282, 63, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/\text{Sqrt}[a + b*E^{(c + d*x)}], x]$

[Out]  $(-2*\text{ArcTanh}[\text{Sqrt}[a + b*E^{(c + d*x)}]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d)$

#### Rule 2282

$\text{Int}[u, x\_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_)})^{(m\_)} /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c\_)*(a\_)+(b\_)*x)}]*(F\_)[v\_] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

#### Rule 63

$\text{Int}[(a\_)+(b\_)*(x\_)^{(m_)*((c\_)+(d\_)*(x\_)^{(n_)}), x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n}, x], x, (a+b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\text{Int}[(a\_)+(b\_)*(x\_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

#### Rubi steps

$$\int \frac{1}{\sqrt{a + be^{c+dx}}} dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, e^{c+dx}\right)}{d}$$

$$= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + be^{c+dx}}\right)}{bd}$$

$$= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

**Mathematica [A]** time = 0.0127239, size = 32, normalized size = 1.

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b\*E^(c + d\*x)], x]

[Out] (-2\*ArcTanh[Sqrt[a + b\*E^(c + d\*x)]/Sqrt[a]])/(Sqrt[a]\*d)

**Maple [A]** time = 0.189, size = 26, normalized size = 0.8

$$-2 \frac{1}{d\sqrt{a}} \text{Artanh}\left(\frac{\sqrt{a + be^{dx+c}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*exp(d\*x+c))^(1/2), x)

[Out] -2\*arctanh((a+b\*exp(d\*x+c))^(1/2)/a^(1/2))/d/a^(1/2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.908636, size = 208, normalized size = 6.5

$$\left[ \frac{\log\left(\left(\frac{be^{(dx+c)} - 2\sqrt{be^{(dx+c)} + a}\sqrt{a} + 2a\right)e^{-dx-c}}{\sqrt{ad}}\right)}{\sqrt{ad}}, \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{be^{(dx+c)} + a}\sqrt{-a}}{a}\right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [log((b\*e^(d\*x + c) - 2\*sqrt(b\*e^(d\*x + c) + a)\*sqrt(a) + 2\*a)\*e^(-d\*x - c))/(sqrt(a)\*d), 2\*sqrt(-a)\*arctan(sqrt(b\*e^(d\*x + c) + a)\*sqrt(-a)/a)/(a\*d)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + be^{c+dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))\*\*(1/2),x)

[Out] Integral(1/sqrt(a + b\*exp(c + d\*x)), x)

**Giac [A]** time = 1.23535, size = 39, normalized size = 1.22

$$\frac{2 \arctan\left(\frac{\sqrt{be^{(dx+c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^(1/2),x, algorithm="giac")

[Out] 2\*arctan(sqrt(b\*e^(d\*x + c) + a)/sqrt(-a))/(sqrt(-a)\*d)

$$3.699 \quad \int \frac{1}{\sqrt{-a+be^{c+dx}}} dx$$

**Optimal.** Leaf size=34

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{be^{c+dx}-a}}{\sqrt{a}} \right)}{\sqrt{ad}}$$

[Out] (2\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/(Sqrt[a]\*d)

**Rubi [A]** time = 0.0294162, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 63, 205}

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{be^{c+dx}-a}}{\sqrt{a}} \right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-a + b\*E^(c + d\*x)],x]

[Out] (2\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/(Sqrt[a]\*d)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (a_.)*(v_)^(n_)^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rubi steps



$$\begin{aligned} \int \frac{1}{\sqrt{-a + be^{c+dx}}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, e^{c+dx}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + be^{c+dx}}\right)}{bd} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{ad}} \end{aligned}$$

**Mathematica [A]** time = 0.0132134, size = 34, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{be^{c+dx}-a}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-a + b\*E^(c + d\*x)], x]

[Out] (2\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/(Sqrt[a]\*d)

**Maple [A]** time = 0.191, size = 28, normalized size = 0.8

$$2 \frac{1}{d\sqrt{a}} \arctan\left(\frac{\sqrt{-a + be^{dx+c}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a+b\*exp(d\*x+c))^(1/2), x)

[Out] 2\*arctan((-a+b\*exp(d\*x+c))^(1/2)/a^(1/2))/d/a^(1/2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a+b\*exp(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.815585, size = 207, normalized size = 6.09

$$\left[ -\frac{\sqrt{-a} \log\left(\left(b e^{(dx+c)} - 2 \sqrt{b e^{(dx+c)} - a} \sqrt{-a} - 2 a\right) e^{(-dx-c)}\right)}{ad}, \frac{2 \arctan\left(\frac{\sqrt{b e^{(dx+c)} - a}}{\sqrt{a}}\right)}{\sqrt{ad}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a+b\*exp(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [-sqrt(-a)\*log((b\*e^(d\*x + c) - 2\*sqrt(b\*e^(d\*x + c) - a)\*sqrt(-a) - 2\*a)\*e^(-d\*x - c))/(a\*d), 2\*arctan(sqrt(b\*e^(d\*x + c) - a)/sqrt(a))/(sqrt(a)\*d)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a + be^{c+dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a+b\*exp(d\*x+c))\*\*(1/2),x)

[Out] Integral(1/sqrt(-a + b\*exp(c + d\*x)), x)

**Giac [A]** time = 1.31754, size = 36, normalized size = 1.06

$$\frac{2 \arctan\left(\frac{\sqrt{be^{(dx+c)}-a}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a+b\*exp(d\*x+c))^(1/2),x, algorithm="giac")

[Out] 2\*arctan(sqrt(b\*e^(d\*x + c) - a)/sqrt(a))/(sqrt(a)\*d)

### 3.700 $\int \sqrt{a + be^{c+dx}} dx$

**Optimal.** Leaf size=53

$$\frac{2\sqrt{a + be^{c+dx}}}{d} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

[Out] (2\*Sqrt[a + b\*E^(c + d\*x)])/d - (2\*Sqrt[a]\*ArcTanh[Sqrt[a + b\*E^(c + d\*x)]/Sqrt[a]])/d

**Rubi [A]** time = 0.0348304, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2282, 50, 63, 208}

$$\frac{2\sqrt{a + be^{c+dx}}}{d} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*E^(c + d\*x)], x]

[Out] (2\*Sqrt[a + b\*E^(c + d\*x)])/d - (2\*Sqrt[a]\*ArcTanh[Sqrt[a + b\*E^(c + d\*x)]/Sqrt[a]])/d

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 50

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + be^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, e^{c+dx}\right)}{d} \\
&= \frac{2\sqrt{a + be^{c+dx}}}{d} + \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, e^{c+dx}\right)}{d} \\
&= \frac{2\sqrt{a + be^{c+dx}}}{d} + \frac{(2a) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + be^{c+dx}}\right)}{bd} \\
&= \frac{2\sqrt{a + be^{c+dx}}}{d} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0165842, size = 51, normalized size = 0.96

$$\frac{2\sqrt{a + be^{c+dx}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*E^(c + d\*x)], x]

[Out] (2\*Sqrt[a + b\*E^(c + d\*x)] - 2\*Sqrt[a]\*ArcTanh[Sqrt[a + b\*E^(c + d\*x)]/Sqrt[a]])/d

**Maple [A]** time = 0.182, size = 42, normalized size = 0.8

$$\frac{1}{d} \left( 2\sqrt{a + be^{dx+c}} - 2\sqrt{a} \text{Artanh}\left(\frac{\sqrt{a + be^{dx+c}}}{\sqrt{a}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(d\*x+c))^(1/2), x)

[Out] 1/d\*(2\*(a+b\*exp(d\*x+c))^(1/2)-2\*a^(1/2)\*arctanh((a+b\*exp(d\*x+c))^(1/2)/a^(1/2)))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.79088, size = 278, normalized size = 5.25

$$\left[ \frac{\sqrt{a} \log\left(\left(b e^{(dx+c)} - 2\sqrt{b e^{(dx+c)} + a}\sqrt{a} + 2a\right)e^{(-dx-c)}\right) + 2\sqrt{b e^{(dx+c)} + a}}{d}, \frac{2\left(\sqrt{-a} \arctan\left(\frac{\sqrt{b e^{(dx+c)} + a}\sqrt{-a}}{a}\right) + \sqrt{b e^{(dx+c)} + a}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(a)\*log((b\*e^(d\*x + c) - 2\*sqrt(b\*e^(d\*x + c) + a)\*sqrt(a) + 2\*a)\*e^(-d\*x - c)) + 2\*sqrt(b\*e^(d\*x + c) + a))/d, 2\*(sqrt(-a)\*arctan(sqrt(b\*e^(d\*x + c) + a)\*sqrt(-a)/a) + sqrt(b\*e^(d\*x + c) + a))/d]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b e^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(d\*x+c))\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*exp(c + d\*x)), x)

**Giac [A]** time = 1.30406, size = 59, normalized size = 1.11

$$\frac{2\left(\frac{a \arctan\left(\frac{\sqrt{b e^{(dx+c)} + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b e^{(dx+c)} + a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(d\*x+c))^(1/2),x, algorithm="giac")

[Out] 2\*(a\*arctan(sqrt(b\*e^(d\*x + c) + a)/sqrt(-a))/sqrt(-a) + sqrt(b\*e^(d\*x + c) + a))/d

### 3.701 $\int \sqrt{-a + be^{c+dx}} dx$

**Optimal.** Leaf size=57

$$\frac{2\sqrt{be^{c+dx} - a}}{d} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{be^{c+dx} - a}}{\sqrt{a}}\right)}{d}$$

[Out] (2\*Sqrt[-a + b\*E^(c + d\*x)]/d - (2\*Sqrt[a]\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/d

**Rubi [A]** time = 0.0355086, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2282, 50, 63, 205}

$$\frac{2\sqrt{be^{c+dx} - a}}{d} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{be^{c+dx} - a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-a + b\*E^(c + d\*x)], x]

[Out] (2\*Sqrt[-a + b\*E^(c + d\*x)]/d - (2\*Sqrt[a]\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/d

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-a + be^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x} dx, x, e^{c+dx}\right)}{d} \\
&= \frac{2\sqrt{-a + be^{c+dx}}}{d} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, e^{c+dx}\right)}{d} \\
&= \frac{2\sqrt{-a + be^{c+dx}}}{d} - \frac{(2a) \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + be^{c+dx}}\right)}{bd} \\
&= \frac{2\sqrt{-a + be^{c+dx}}}{d} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0174209, size = 55, normalized size = 0.96

$$\frac{2\sqrt{be^{c+dx} - a} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{be^{c+dx} - a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-a + b\*E^(c + d\*x)], x]

[Out] (2\*Sqrt[-a + b\*E^(c + d\*x)] - 2\*Sqrt[a]\*ArcTan[Sqrt[-a + b\*E^(c + d\*x)]/Sqrt[a]])/d

**Maple [A]** time = 0.155, size = 48, normalized size = 0.8

$$-2 \frac{\sqrt{a}}{d} \arctan\left(\frac{\sqrt{-a + be^{dx+c}}}{\sqrt{a}}\right) + 2 \frac{\sqrt{-a + be^{dx+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*exp(d\*x+c))^(1/2), x)

[Out] -2\*arctan((-a+b\*exp(d\*x+c))^(1/2)/a^(1/2))\*a^(1/2)/d+2\*(-a+b\*exp(d\*x+c))^(1/2)/d

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*exp(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.786176, size = 277, normalized size = 4.86

$$\left[ \frac{\sqrt{-a} \log\left(\left(b e^{(dx+c)} - 2 \sqrt{b e^{(dx+c)} - a} \sqrt{-a} - 2a\right) e^{-dx-c}\right) + 2 \sqrt{b e^{(dx+c)} - a}}{d}, -\frac{2\left(\sqrt{a} \arctan\left(\frac{\sqrt{b e^{(dx+c)} - a}}{\sqrt{a}}\right) - \sqrt{b e^{(dx+c)} - a}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*exp(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(-a)\*log((b\*e^(d\*x + c) - 2\*sqrt(b\*e^(d\*x + c) - a)\*sqrt(-a) - 2\*a)\*e^(-d\*x - c)) + 2\*sqrt(b\*e^(d\*x + c) - a))/d, -2\*(sqrt(a)\*arctan(sqrt(b\*e^(d\*x + c) - a)/sqrt(a)) - sqrt(b\*e^(d\*x + c) - a))/d]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a + b e^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*exp(d\*x+c))\*\*(1/2),x)

[Out] Integral(sqrt(-a + b\*exp(c + d\*x)), x)

**Giac [A]** time = 1.25716, size = 61, normalized size = 1.07

$$-\frac{2\left(\sqrt{a} \arctan\left(\frac{\sqrt{b e^{(dx+c)} - a}}{\sqrt{a}}\right) - \sqrt{b e^{(dx+c)} - a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*exp(d\*x+c))^(1/2),x, algorithm="giac")

[Out] -2\*(sqrt(a)\*arctan(sqrt(b\*e^(d\*x + c) - a)/sqrt(a)) - sqrt(b\*e^(d\*x + c) - a))/d



### 3.702 $\int e^{6x} \sin(3x) dx$

**Optimal.** Leaf size=27

$$\frac{2}{15}e^{6x} \sin(3x) - \frac{1}{15}e^{6x} \cos(3x)$$

[Out]  $-(E^{(6*x)*Cos[3*x]})/15 + (2*E^{(6*x)*Sin[3*x]})/15$

**Rubi [A]** time = 0.011199, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4432}

$$\frac{2}{15}e^{6x} \sin(3x) - \frac{1}{15}e^{6x} \cos(3x)$$

Antiderivative was successfully verified.

[In] Int[E^(6\*x)\*Sin[3\*x],x]

[Out]  $-(E^{(6*x)*Cos[3*x]})/15 + (2*E^{(6*x)*Sin[3*x]})/15$

#### Rule 4432

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*Sin[(d\_.) + (e\_.)\*(x\_)], x\_Symbol] :=  
 Simp[(b\*c\*Log[F]\*F^(c\*(a + b\*x))\*Sin[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x]  
 - Simp[(e\*F^(c\*(a + b\*x))\*Cos[d + e\*x]/(e^2 + b^2\*c^2\*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2\*c^2\*Log[F]^2, 0]

#### Rubi steps

$$\int e^{6x} \sin(3x) dx = -\frac{1}{15}e^{6x} \cos(3x) + \frac{2}{15}e^{6x} \sin(3x)$$

**Mathematica [A]** time = 0.0228407, size = 20, normalized size = 0.74

$$-\frac{1}{15}e^{6x}(\cos(3x) - 2\sin(3x))$$

Antiderivative was successfully verified.

[In] Integrate[E^(6\*x)\*Sin[3\*x],x]

[Out]  $-(E^{(6*x)*(Cos[3*x] - 2*Sin[3*x])})/15$

**Maple [A]** time = 0.022, size = 22, normalized size = 0.8

$$-\frac{e^{6x} \cos(3x)}{15} + \frac{2e^{6x} \sin(3x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(6*x)*sin(3*x),x)`

[Out] `-1/15*exp(6*x)*cos(3*x)+2/15*exp(6*x)*sin(3*x)`

**Maxima [A]** time = 0.956844, size = 23, normalized size = 0.85

$$-\frac{1}{15}(\cos(3x) - 2 \sin(3x))e^{(6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(6*x)*sin(3*x),x, algorithm="maxima")`

[Out] `-1/15*(cos(3*x) - 2*sin(3*x))*e^(6*x)`

**Fricas [A]** time = 0.917258, size = 65, normalized size = 2.41

$$-\frac{1}{15} \cos(3x) e^{(6x)} + \frac{2}{15} e^{(6x)} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(6*x)*sin(3*x),x, algorithm="fricas")`

[Out] `-1/15*cos(3*x)*e^(6*x) + 2/15*e^(6*x)*sin(3*x)`

**Sympy [A]** time = 0.318589, size = 24, normalized size = 0.89

$$\frac{2e^{6x} \sin(3x)}{15} - \frac{e^{6x} \cos(3x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(6*x)*sin(3*x),x)`

[Out] `2*exp(6*x)*sin(3*x)/15 - exp(6*x)*cos(3*x)/15`

**Giac [A]** time = 1.2606, size = 23, normalized size = 0.85

$$-\frac{1}{15}(\cos(3x) - 2 \sin(3x))e^{(6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(6*x)*sin(3*x),x, algorithm="giac")`

[Out] `-1/15*(cos(3*x) - 2*sin(3*x))*e^(6*x)`

$$3.703 \quad \int \frac{e^{3x}}{1+e^{2x}} dx$$

**Optimal.** Leaf size=10

$$e^x - \tan^{-1}(e^x)$$

[Out] E<sup>x</sup> - ArcTan[E<sup>x</sup>]

**Rubi [A]** time = 0.0231388, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2248, 321, 203}

$$e^x - \tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E<sup>(3\*x)</sup>/(1 + E<sup>(2\*x)</sup>), x]

[Out] E<sup>x</sup> - ArcTan[E<sup>x</sup>]

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(p\_.)\*(G\_)^(h\_.)\*((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{3x}}{1+e^{2x}} dx &= \text{Subst} \left( \int \frac{x^2}{1+x^2} dx, x, e^x \right) \\ &= e^x - \text{Subst} \left( \int \frac{1}{1+x^2} dx, x, e^x \right) \\ &= e^x - \tan^{-1}(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0064293, size = 10, normalized size = 1.

$$e^x - \tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^(3\*x)/(1 + E^(2\*x)),x]

[Out] E^x - ArcTan[E^x]

**Maple [A]** time = 0.021, size = 9, normalized size = 0.9

$$e^x - \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3\*x)/(1+exp(2\*x)),x)

[Out] exp(x)-arctan(exp(x))

**Maxima [A]** time = 1.44897, size = 11, normalized size = 1.1

$$-\arctan(e^x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(1+exp(2\*x)),x, algorithm="maxima")

[Out] -arctan(e^x) + e^x

**Fricas [A]** time = 0.857106, size = 27, normalized size = 2.7

$$-\arctan(e^x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(1+exp(2\*x)),x, algorithm="fricas")

[Out] -arctan(e^x) + e^x

**Sympy [B]** time = 0.121135, size = 19, normalized size = 1.9

$$e^x + \text{RootSum}\left(4z^2 + 1, (i \mapsto i \log(-2i + e^x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(1+exp(2\*x)),x)

[Out] exp(x) + RootSum(4\*\_z\*\*2 + 1, Lambda(\_i, \_i\*log(-2\*\_i + exp(x))))

**Giac [A]** time = 1.19609, size = 11, normalized size = 1.1

$$-\arctan(e^x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(1+exp(2\*x)),x, algorithm="giac")

[Out] -arctan(e^x) + e^x

$$3.704 \quad \int \frac{e^{3x}}{-1+e^{2x}} dx$$

**Optimal.** Leaf size=10

$$e^x - \tanh^{-1}(e^x)$$

[Out] E<sup>x</sup> - ArcTanh[E<sup>x</sup>]

**Rubi [A]** time = 0.0243592, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2248, 321, 207}

$$e^x - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^(3\*x)/(-1 + E^(2\*x)), x]

[Out] E<sup>x</sup> - ArcTanh[E<sup>x</sup>]

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{3x}}{-1+e^{2x}} dx &= \text{Subst} \left( \int \frac{x^2}{-1+x^2} dx, x, e^x \right) \\ &= e^x + \text{Subst} \left( \int \frac{1}{-1+x^2} dx, x, e^x \right) \\ &= e^x - \tanh^{-1}(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0062246, size = 10, normalized size = 1.

$$e^x - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^(3\*x)/(-1 + E^(2\*x)), x]

[Out] E^x - ArcTanh[E^x]

**Maple [B]** time = 0.022, size = 18, normalized size = 1.8

$$e^x + \frac{\ln(-1 + e^x)}{2} - \frac{\ln(1 + e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3\*x)/(-1+exp(2\*x)), x)

[Out] exp(x)+1/2\*ln(-1+exp(x))-1/2\*ln(1+exp(x))

**Maxima [B]** time = 0.959045, size = 23, normalized size = 2.3

$$e^x - \frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(-1+exp(2\*x)), x, algorithm="maxima")

[Out] e^x - 1/2\*log(e^x + 1) + 1/2\*log(e^x - 1)

**Fricas [B]** time = 0.870644, size = 58, normalized size = 5.8

$$e^x - \frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(-1+exp(2\*x)), x, algorithm="fricas")

[Out] e^x - 1/2\*log(e^x + 1) + 1/2\*log(e^x - 1)

**Sympy [B]** time = 0.116369, size = 19, normalized size = 1.9

$$e^x + \frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(-1+exp(2\*x)), x)

[Out] exp(x) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2

---

**Giac [B]** time = 1.22216, size = 24, normalized size = 2.4

$$e^x - \frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(-1+exp(2\*x)),x, algorithm="giac")

[Out] e^x - 1/2\*log(e^x + 1) + 1/2\*log(abs(e^x - 1))



$$3.705 \quad \int \frac{e^{-x}}{\sqrt{1+e^{2x}}} dx$$

**Optimal.** Leaf size=18

$$-e^{-x}\sqrt{e^{2x}+1}$$

[Out] -(Sqrt[1 + E^(2\*x)]/E^x)

**Rubi [A]** time = 0.0255804, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2249, 191}

$$-e^{-x}\sqrt{e^{2x}+1}$$

Antiderivative was successfully verified.

[In] Int[1/(E^x\*Sqrt[1 + E^(2\*x)]),x]

[Out] -(Sqrt[1 + E^(2\*x)]/E^x)

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{-x}}{\sqrt{1+e^{2x}}} dx &= -\text{Subst} \left( \int \frac{1}{\sqrt{1+\frac{1}{x^2}}} dx, x, e^{-x} \right) \\ &= -e^{-x}\sqrt{1+e^{2x}} \end{aligned}$$

**Mathematica [A]** time = 0.0121807, size = 18, normalized size = 1.

$$-e^{-x}\sqrt{e^{2x}+1}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^x\*Sqrt[1 + E^(2\*x)]),x]

[Out] -(Sqrt[1 + E^(2\*x)]/E^x)

---

**Maple [A]** time = 0.06, size = 15, normalized size = 0.8

$$-\frac{1}{e^x} \sqrt{1 + (e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/exp(x)/(1+exp(2*x))^(1/2),x)`

[Out] `-1/exp(x)*(1+exp(x)^2)^(1/2)`

---

**Maxima [A]** time = 0.999682, size = 19, normalized size = 1.06

$$-\sqrt{e^{(2x)} + 1}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="maxima")`

[Out] `-sqrt(e^(2*x) + 1)*e^(-x)`

---

**Fricas [A]** time = 0.919925, size = 28, normalized size = 1.56

$$-\sqrt{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="fricas")`

[Out] `-sqrt(e^(-2*x) + 1)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-x}}{\sqrt{e^{2x} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(x)/(1+exp(2*x))**(1/2),x)`

[Out] `Integral(exp(-x)/sqrt(exp(2*x) + 1), x)`

---

**Giac [A]** time = 1.31232, size = 28, normalized size = 1.56

$$\frac{2}{\left(\sqrt{e^{(2x)} + 1} - e^x\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(e^(2*x) + 1) - e^x)^2 - 1)
```

$$3.706 \quad \int \frac{e^x}{-1-8e^x+e^{2x}} dx$$

**Optimal.** Leaf size=20

$$\frac{\tanh^{-1}\left(\frac{4-e^x}{\sqrt{17}}\right)}{\sqrt{17}}$$

[Out] ArcTanh[(4 - E^x)/Sqrt[17]]/Sqrt[17]

**Rubi [A]** time = 0.0430682, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{4-e^x}{\sqrt{17}}\right)}{\sqrt{17}}$$

Antiderivative was successfully verified.

[In] Int[E^x/(-1 - 8\*E^x + E^(2\*x)),x]

[Out] ArcTanh[(4 - E^x)/Sqrt[17]]/Sqrt[17]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{e^x}{-1-8e^x+e^{2x}} dx &= \text{Subst}\left(\int \frac{1}{-1-8x+x^2} dx, x, e^x\right) \\ &= -\left(2 \text{Subst}\left(\int \frac{1}{68-x^2} dx, x, -8+2e^x\right)\right) \\ &= \frac{\tanh^{-1}\left(\frac{4-e^x}{\sqrt{17}}\right)}{\sqrt{17}} \end{aligned}$$

**Mathematica [A]** time = 0.0121242, size = 19, normalized size = 0.95

$$\frac{\tanh^{-1}\left(\frac{e^x-4}{\sqrt{17}}\right)}{\sqrt{17}}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(-1 - 8\*E^x + E^(2\*x)), x]

[Out] -(ArcTanh[(-4 + E^x)/Sqrt[17]]/Sqrt[17])

**Maple [A]** time = 0.021, size = 18, normalized size = 0.9

$$-\frac{\sqrt{17}}{17} \operatorname{Artanh}\left(\frac{(2e^x - 8)\sqrt{17}}{34}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(-1-8\*exp(x)+exp(2\*x)), x)

[Out] -1/17\*17^(1/2)\*arctanh(1/34\*(2\*exp(x)-8)\*17^(1/2))

**Maxima [A]** time = 1.45928, size = 35, normalized size = 1.75

$$\frac{1}{34} \sqrt{17} \log\left(-\frac{\sqrt{17} - e^x + 4}{\sqrt{17} + e^x - 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-1-8\*exp(x)+exp(2\*x)), x, algorithm="maxima")

[Out] 1/34\*sqrt(17)\*log(-(sqrt(17) - e^x + 4)/(sqrt(17) + e^x - 4))

**Fricas [B]** time = 0.916552, size = 127, normalized size = 6.35

$$\frac{1}{34} \sqrt{17} \log\left(-\frac{2(\sqrt{17} + 4)e^x - 8\sqrt{17} - e^{2x} - 33}{e^{2x} - 8e^x - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-1-8\*exp(x)+exp(2\*x)), x, algorithm="fricas")

[Out] 1/34\*sqrt(17)\*log(-(2\*(sqrt(17) + 4)\*e^x - 8\*sqrt(17) - e^(2\*x) - 33)/(e^(2\*x) - 8\*e^x - 1))

**Sympy [A]** time = 0.13306, size = 17, normalized size = 0.85

$$\operatorname{RootSum}\left(68z^2 - 1, (i \mapsto i \log(-34i + e^x - 4))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-1-8\*exp(x)+exp(2\*x)),x)

[Out] RootSum(68\*\_z\*\*2 - 1, Lambda(\_i, \_i\*log(-34\*\_i + exp(x) - 4)))

**Giac [B]** time = 1.27252, size = 45, normalized size = 2.25

$$\frac{1}{34} \sqrt{17} \log \left( \frac{|-2\sqrt{17} + 2e^x - 8|}{|2\sqrt{17} + 2e^x - 8|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-1-8\*exp(x)+exp(2\*x)),x, algorithm="giac")

[Out] 1/34\*sqrt(17)\*log(abs(-2\*sqrt(17) + 2\*e^x - 8)/abs(2\*sqrt(17) + 2\*e^x - 8))

### 3.707 $\int e^{7x} x^3 dx$

**Optimal.** Leaf size=44

$$\frac{1}{7}e^{7x}x^3 - \frac{3}{49}e^{7x}x^2 + \frac{6}{343}e^{7x}x - \frac{6e^{7x}}{2401}$$

[Out]  $(-6E^{(7*x)})/2401 + (6E^{(7*x)*x})/343 - (3E^{(7*x)*x^2})/49 + (E^{(7*x)*x^3})/7$

**Rubi [A]** time = 0.0348359, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2176, 2194}

$$\frac{1}{7}e^{7x}x^3 - \frac{3}{49}e^{7x}x^2 + \frac{6}{343}e^{7x}x - \frac{6e^{7x}}{2401}$$

Antiderivative was successfully verified.

[In] Int[E^(7\*x)\*x^3,x]

[Out]  $(-6E^{(7*x)})/2401 + (6E^{(7*x)*x})/343 - (3E^{(7*x)*x^2})/49 + (E^{(7*x)*x^3})/7$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !UseGamma == True
```

#### Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned} \int e^{7x} x^3 dx &= \frac{1}{7}e^{7x}x^3 - \frac{3}{7} \int e^{7x} x^2 dx \\ &= -\frac{3}{49}e^{7x}x^2 + \frac{1}{7}e^{7x}x^3 + \frac{6}{49} \int e^{7x} x dx \\ &= \frac{6}{343}e^{7x}x - \frac{3}{49}e^{7x}x^2 + \frac{1}{7}e^{7x}x^3 - \frac{6}{343} \int e^{7x} dx \\ &= -\frac{6e^{7x}}{2401} + \frac{6}{343}e^{7x}x - \frac{3}{49}e^{7x}x^2 + \frac{1}{7}e^{7x}x^3 \end{aligned}$$

**Mathematica [A]** time = 0.0073774, size = 24, normalized size = 0.55

$$\frac{e^{7x} (343x^3 - 147x^2 + 42x - 6)}{2401}$$

Antiderivative was successfully verified.

[In] Integrate[E^(7\*x)\*x^3,x]

[Out] (E^(7\*x)\*(-6 + 42\*x - 147\*x^2 + 343\*x^3))/2401

**Maple [A]** time = 0.021, size = 22, normalized size = 0.5

$$\frac{(343x^3 - 147x^2 + 42x - 6)e^{7x}}{2401}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(7\*x)\*x^3,x)

[Out] 1/2401\*(343\*x^3-147\*x^2+42\*x-6)\*exp(7\*x)

**Maxima [A]** time = 0.964457, size = 28, normalized size = 0.64

$$\frac{1}{2401} (343x^3 - 147x^2 + 42x - 6)e^{(7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(7\*x)\*x^3,x, algorithm="maxima")

[Out] 1/2401\*(343\*x^3 - 147\*x^2 + 42\*x - 6)\*e^(7\*x)

**Fricas [A]** time = 0.807989, size = 63, normalized size = 1.43

$$\frac{1}{2401} (343x^3 - 147x^2 + 42x - 6)e^{(7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(7\*x)\*x^3,x, algorithm="fricas")

[Out] 1/2401\*(343\*x^3 - 147\*x^2 + 42\*x - 6)\*e^(7\*x)

**Sympy [A]** time = 0.088803, size = 20, normalized size = 0.45

$$\frac{(343x^3 - 147x^2 + 42x - 6)e^{7x}}{2401}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(7\*x)\*x\*\*3,x)

[Out] (343\*x\*\*3 - 147\*x\*\*2 + 42\*x - 6)\*exp(7\*x)/2401



**Giac [A]** time = 1.40771, size = 28, normalized size = 0.64

$$\frac{1}{2401} (343x^3 - 147x^2 + 42x - 6)e^{(7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(7*x)*x^3,x, algorithm="giac")
```

```
[Out] 1/2401*(343*x^3 - 147*x^2 + 42*x - 6)*e^(7*x)
```

### 3.708 $\int e^{8-2x} x^3 dx$

**Optimal.** Leaf size=52

$$-\frac{1}{2}e^{8-2x}x^3 - \frac{3}{4}e^{8-2x}x^2 - \frac{3}{4}e^{8-2x}x - \frac{3}{8}e^{8-2x}$$

[Out]  $(-3E^{(8 - 2*x)})/8 - (3E^{(8 - 2*x)*x})/4 - (3E^{(8 - 2*x)*x^2})/4 - (E^{(8 - 2*x)*x^3})/2$

**Rubi [A]** time = 0.0392965, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2176, 2194}

$$-\frac{1}{2}e^{8-2x}x^3 - \frac{3}{4}e^{8-2x}x^2 - \frac{3}{4}e^{8-2x}x - \frac{3}{8}e^{8-2x}$$

Antiderivative was successfully verified.

[In] Int[E^(8 - 2\*x)\*x^3,x]

[Out]  $(-3E^{(8 - 2*x)})/8 - (3E^{(8 - 2*x)*x})/4 - (3E^{(8 - 2*x)*x^2})/4 - (E^{(8 - 2*x)*x^3})/2$

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int e^{8-2x} x^3 dx &= -\frac{1}{2}e^{8-2x}x^3 + \frac{3}{2} \int e^{8-2x} x^2 dx \\ &= -\frac{3}{4}e^{8-2x}x^2 - \frac{1}{2}e^{8-2x}x^3 + \frac{3}{2} \int e^{8-2x} x dx \\ &= -\frac{3}{4}e^{8-2x}x - \frac{3}{4}e^{8-2x}x^2 - \frac{1}{2}e^{8-2x}x^3 + \frac{3}{4} \int e^{8-2x} dx \\ &= -\frac{3}{8}e^{8-2x} - \frac{3}{4}e^{8-2x}x - \frac{3}{4}e^{8-2x}x^2 - \frac{1}{2}e^{8-2x}x^3 \end{aligned}$$

**Mathematica [A]** time = 0.0097179, size = 26, normalized size = 0.5

$$-\frac{1}{8}e^{8-2x}(4x^3 + 6x^2 + 6x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[E^(8 - 2\*x)\*x^3,x]

[Out] -(E^(8 - 2\*x)\*(3 + 6\*x + 6\*x^2 + 4\*x^3))/8

**Maple [A]** time = 0.02, size = 24, normalized size = 0.5

$$-\frac{(4x^3 + 6x^2 + 6x + 3)e^{8-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(8-2\*x)\*x^3,x)

[Out] -1/8\*(4\*x^3+6\*x^2+6\*x+3)\*exp(8-2\*x)

**Maxima [A]** time = 0.962147, size = 41, normalized size = 0.79

$$-\frac{1}{8}(4x^3e^8 + 6x^2e^8 + 6xe^8 + 3e^8)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(8-2\*x)\*x^3,x, algorithm="maxima")

[Out] -1/8\*(4\*x^3\*e^8 + 6\*x^2\*e^8 + 6\*x\*e^8 + 3\*e^8)\*e^(-2\*x)

**Fricas [A]** time = 0.829967, size = 61, normalized size = 1.17

$$-\frac{1}{8}(4x^3 + 6x^2 + 6x + 3)e^{(-2x+8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(8-2\*x)\*x^3,x, algorithm="fricas")

[Out] -1/8\*(4\*x^3 + 6\*x^2 + 6\*x + 3)\*e^(-2\*x + 8)

**Sympy [A]** time = 0.093912, size = 24, normalized size = 0.46

$$\frac{(-4x^3 - 6x^2 - 6x - 3)e^{8-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(8-2\*x)\*x\*\*3,x)

[Out] (-4\*x\*\*3 - 6\*x\*\*2 - 6\*x - 3)\*exp(8 - 2\*x)/8

**Giac [A]** time = 1.27472, size = 31, normalized size = 0.6

$$-\frac{1}{8}(4x^3 + 6x^2 + 6x + 3)e^{(-2x+8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(8-2\*x)\*x^3,x, algorithm="giac")

[Out] -1/8\*(4\*x^3 + 6\*x^2 + 6\*x + 3)\*e^(-2\*x + 8)

### 3.709 $\int e^x \sqrt{9 - e^{2x}} dx$

**Optimal.** Leaf size=33

$$\frac{1}{2}e^x\sqrt{9 - e^{2x}} + \frac{9}{2}\sin^{-1}\left(\frac{e^x}{3}\right)$$

[Out]  $(E^x*\text{Sqrt}[9 - E^{(2*x)}])/2 + (9*\text{ArcSin}[E^x/3])/2$

**Rubi [A]** time = 0.0255834, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2249, 195, 216}

$$\frac{1}{2}e^x\sqrt{9 - e^{2x}} + \frac{9}{2}\sin^{-1}\left(\frac{e^x}{3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^x*\text{Sqrt}[9 - E^{(2*x)}], x]$

[Out]  $(E^x*\text{Sqrt}[9 - E^{(2*x)}])/2 + (9*\text{ArcSin}[E^x/3])/2$

#### Rule 2249

$\text{Int}[(a_ + (b_)*(F_)^{(e_)*((c_ + (d_)*(x_))})^{(p_)}*(G_)^{(h_)*((f_ + (g_)*(x_))})}, x\_Symbol] := \text{With}[\{m = \text{FullSimplify}[(d*e*\text{Log}[F])/(g*h*\text{Log}[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*\text{Log}[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)}*(a + b*F^{(c*e - (d*e*f)/g})*x^{\text{Numerator}[m]}]^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \|\ \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 195

$\text{Int}[(a_ + (b_)*(x_)^{(n_)} )^{(p_)}, x\_Symbol] := \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p - 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[2*p] \|\ (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4*p]) \|\ (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3*p]) \|\ \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2], x\_Symbol] := \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

#### Rubi steps

$$\begin{aligned} \int e^x \sqrt{9 - e^{2x}} dx &= \text{Subst}\left(\int \sqrt{9 - x^2} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{9 - e^{2x}} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9 - x^2}} dx, x, e^x\right) \\ &= \frac{1}{2}e^x\sqrt{9 - e^{2x}} + \frac{9}{2}\sin^{-1}\left(\frac{e^x}{3}\right) \end{aligned}$$

**Mathematica [A]** time = 0.011567, size = 32, normalized size = 0.97

$$\frac{1}{2} \left( e^x \sqrt{9 - e^{2x}} + 9 \sin^{-1} \left( \frac{e^x}{3} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*Sqrt[9 - E^(2\*x)],x]

[Out] (E^x\*Sqrt[9 - E^(2\*x)] + 9\*ArcSin[E^x/3])/2

**Maple [A]** time = 0.057, size = 23, normalized size = 0.7

$$\frac{e^x}{2} \sqrt{9 - (e^x)^2} + \frac{9}{2} \arcsin \left( \frac{e^x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(9-exp(2\*x))^(1/2),x)

[Out] 1/2\*exp(x)\*(9-exp(x)^2)^(1/2)+9/2\*arcsin(1/3\*exp(x))

**Maxima [A]** time = 1.4395, size = 30, normalized size = 0.91

$$\frac{1}{2} \sqrt{-e^{(2x)} + 9} e^x + \frac{9}{2} \arcsin \left( \frac{1}{3} e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(9-exp(2\*x))^(1/2),x, algorithm="maxima")

[Out] 1/2\*sqrt(-e^(2\*x) + 9)\*e^x + 9/2\*arcsin(1/3\*e^x)

**Fricas [A]** time = 0.713558, size = 97, normalized size = 2.94

$$\frac{1}{2} \sqrt{-e^{(2x)} + 9} e^x - 9 \arctan \left( \left( \sqrt{-e^{(2x)} + 9} - 3 \right) e^{(-x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(9-exp(2\*x))^(1/2),x, algorithm="fricas")

[Out] 1/2\*sqrt(-e^(2\*x) + 9)\*e^x - 9\*arctan((sqrt(-e^(2\*x) + 9) - 3)\*e^(-x))

**Sympy [A]** time = 1.35304, size = 29, normalized size = 0.88

$$\begin{cases} \frac{\sqrt{9-e^{2x}}e^x}{2} + \frac{9 \operatorname{asin}\left(\frac{e^x}{3}\right)}{2} & \text{for } e^x < \log(3) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(9-exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((sqrt(9 - exp(2*x))*exp(x)/2 + 9*asin(exp(x)/3)/2, exp(x) < log(3
))
```

**Giac [A]** time = 1.28686, size = 30, normalized size = 0.91

$$\frac{1}{2} \sqrt{-e^{(2x)} + 9e^x} + \frac{9}{2} \arcsin\left(\frac{1}{3} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(9-exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-e^(2*x) + 9)*e^x + 9/2*arcsin(1/3*e^x)
```

### 3.710 $\int e^{6x} \sqrt{9 - e^{2x}} dx$

**Optimal.** Leaf size=50

$$-\frac{1}{7}(9 - e^{2x})^{7/2} + \frac{18}{5}(9 - e^{2x})^{5/2} - 27(9 - e^{2x})^{3/2}$$

[Out]  $-27*(9 - E^{(2*x)})^{(3/2)} + (18*(9 - E^{(2*x)})^{(5/2)})/5 - (9 - E^{(2*x)})^{(7/2)}/7$

**Rubi [A]** time = 0.0362776, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2248, 43}

$$-\frac{1}{7}(9 - e^{2x})^{7/2} + \frac{18}{5}(9 - e^{2x})^{5/2} - 27(9 - e^{2x})^{3/2}$$

Antiderivative was successfully verified.

[In] Int[E^(6\*x)\*Sqrt[9 - E^(2\*x)],x]

[Out]  $-27*(9 - E^{(2*x)})^{(3/2)} + (18*(9 - E^{(2*x)})^{(5/2)})/5 - (9 - E^{(2*x)})^{(7/2)}/7$

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int e^{6x} \sqrt{9 - e^{2x}} dx &= \frac{1}{2} \text{Subst} \left( \int \sqrt{9 - xx^2} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (81\sqrt{9 - x} - 18(9 - x)^{3/2} + (9 - x)^{5/2}) dx, x, e^{2x} \right) \\ &= -27(9 - e^{2x})^{3/2} + \frac{18}{5}(9 - e^{2x})^{5/2} - \frac{1}{7}(9 - e^{2x})^{7/2} \end{aligned}$$

**Mathematica [A]** time = 0.0196505, size = 33, normalized size = 0.66

$$-\frac{1}{35}(9 - e^{2x})^{3/2}(36e^{2x} + 5e^{4x} + 216)$$

Antiderivative was successfully verified.



[In] Integrate[E^(6\*x)\*Sqrt[9 - E^(2\*x)], x]

[Out] -((9 - E^(2\*x))^(3/2)\*(216 + 36\*E^(2\*x) + 5\*E^(4\*x)))/35

**Maple [A]** time = 0.021, size = 46, normalized size = 0.9

$$-\frac{(e^x)^4}{7} (9 - (e^x)^2)^{\frac{3}{2}} - \frac{36 (e^x)^2}{35} (9 - (e^x)^2)^{\frac{3}{2}} - \frac{216}{35} (9 - (e^x)^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(6\*x)\*(9-exp(2\*x))^(1/2), x)

[Out] -1/7\*exp(x)^4\*(9-exp(x)^2)^(3/2)-36/35\*exp(x)^2\*(9-exp(x)^2)^(3/2)-216/35\*(9-exp(x)^2)^(3/2)

**Maxima [A]** time = 0.974887, size = 50, normalized size = 1.

$$-\frac{1}{7} (-e^{(2x)} + 9)^{\frac{7}{2}} + \frac{18}{5} (-e^{(2x)} + 9)^{\frac{5}{2}} - 27 (-e^{(2x)} + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)\*(9-exp(2\*x))^(1/2), x, algorithm="maxima")

[Out] -1/7\*(-e^(2\*x) + 9)^(7/2) + 18/5\*(-e^(2\*x) + 9)^(5/2) - 27\*(-e^(2\*x) + 9)^(3/2)

**Fricas [A]** time = 0.872713, size = 95, normalized size = 1.9

$$\frac{1}{35} (5 e^{(6x)} - 9 e^{(4x)} - 108 e^{(2x)} - 1944) \sqrt{-e^{(2x)} + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)\*(9-exp(2\*x))^(1/2), x, algorithm="fricas")

[Out] 1/35\*(5\*e^(6\*x) - 9\*e^(4\*x) - 108\*e^(2\*x) - 1944)\*sqrt(-e^(2\*x) + 9)

**Sympy [A]** time = 3.56376, size = 41, normalized size = 0.82

$$\begin{cases} -\frac{(9-e^{2x})^{\frac{7}{2}}}{7} + \frac{18(9-e^{2x})^{\frac{5}{2}}}{5} - 27(9-e^{2x})^{\frac{3}{2}} & \text{for } e^x < \log(3) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)\*(9-exp(2\*x))^(1/2), x)

[Out] Piecewise((-9 - exp(2\*x))\*\*(7/2)/7 + 18\*(9 - exp(2\*x))\*\*(5/2)/5 - 27\*(9 - exp(2\*x))\*\*(3/2), exp(x) < log(3))

**Giac [A]** time = 1.31294, size = 72, normalized size = 1.44

$$\frac{1}{7} (e^{2x} - 9)^3 \sqrt{-e^{2x} + 9} + \frac{18}{5} (e^{2x} - 9)^2 \sqrt{-e^{2x} + 9} - 27 (-e^{2x} + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)\*(9-exp(2\*x))^(1/2),x, algorithm="giac")

[Out] 1/7\*(e^(2\*x) - 9)^3\*sqrt(-e^(2\*x) + 9) + 18/5\*(e^(2\*x) - 9)^2\*sqrt(-e^(2\*x) + 9) - 27\*(-e^(2\*x) + 9)^(3/2)

$$3.711 \quad \int \frac{e^{6x}}{(9-e^x)^{5/2}} dx$$

**Optimal.** Leaf size=81

$$\frac{2}{7}(9-e^x)^{7/2} - 18(9-e^x)^{5/2} + 540(9-e^x)^{3/2} - 14580\sqrt{9-e^x} - \frac{65610}{\sqrt{9-e^x}} + \frac{39366}{(9-e^x)^{3/2}}$$

[Out] 39366/(9 - E^x)^(3/2) - 65610/Sqrt[9 - E^x] - 14580\*Sqrt[9 - E^x] + 540\*(9 - E^x)^(3/2) - 18\*(9 - E^x)^(5/2) + (2\*(9 - E^x)^(7/2))/7

**Rubi [A]** time = 0.0451012, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2248, 43}

$$\frac{2}{7}(9-e^x)^{7/2} - 18(9-e^x)^{5/2} + 540(9-e^x)^{3/2} - 14580\sqrt{9-e^x} - \frac{65610}{\sqrt{9-e^x}} + \frac{39366}{(9-e^x)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[E^(6\*x)/(9 - E^x)^(5/2), x]

[Out] 39366/(9 - E^x)^(3/2) - 65610/Sqrt[9 - E^x] - 14580\*Sqrt[9 - E^x] + 540\*(9 - E^x)^(3/2) - 18\*(9 - E^x)^(5/2) + (2\*(9 - E^x)^(7/2))/7

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{e^{6x}}{(9-e^x)^{5/2}} dx &= \text{Subst} \left( \int \frac{x^5}{(9-x)^{5/2}} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( \frac{59049}{(9-x)^{5/2}} - \frac{32805}{(9-x)^{3/2}} + \frac{7290}{\sqrt{9-x}} - 810\sqrt{9-x} + 45(9-x)^{3/2} - (9-x)^{5/2} \right) dx, x, e^x \right) \\ &= \frac{39366}{(9-e^x)^{3/2}} - \frac{65610}{\sqrt{9-e^x}} - 14580\sqrt{9-e^x} + 540(9-e^x)^{3/2} - 18(9-e^x)^{5/2} + \frac{2}{7}(9-e^x)^{7/2} \end{aligned}$$

**Mathematica [A]** time = 0.0268025, size = 48, normalized size = 0.59

$$\frac{2(-839808e^x + 23328e^{2x} + 432e^{3x} + 18e^{4x} + e^{5x} + 5038848)}{7(9-e^x)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(6\*x)/(9 - E^x)^(5/2),x]

[Out] (-2\*(5038848 - 839808\*E^x + 23328\*E^(2\*x) + 432\*E^(3\*x) + 18\*E^(4\*x) + E^(5\*x)))/(7\*(9 - E^x)^(3/2))

**Maple [A]** time = 0.057, size = 62, normalized size = 0.8

$$39366 (9 - e^x)^{-3/2} + 540 (9 - e^x)^{3/2} - 18 (9 - e^x)^{5/2} + \frac{2}{7} (9 - e^x)^{7/2} - 65610 \frac{1}{\sqrt{9 - e^x}} - 14580 \sqrt{9 - e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(6\*x)/(9-exp(x))^(5/2),x)

[Out] 39366/(9-exp(x))^(3/2)+540\*(9-exp(x))^(3/2)-18\*(9-exp(x))^(5/2)+2/7\*(9-exp(x))^(7/2)-65610/(9-exp(x))^(1/2)-14580\*(9-exp(x))^(1/2)

**Maxima [A]** time = 0.983339, size = 82, normalized size = 1.01

$$\frac{2}{7}(-e^x + 9)^{7/2} - 18(-e^x + 9)^{5/2} + 540(-e^x + 9)^{3/2} - 14580 \sqrt{-e^x + 9} - \frac{65610}{\sqrt{-e^x + 9}} + \frac{39366}{(-e^x + 9)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)/(9-exp(x))^(5/2),x, algorithm="maxima")

[Out] 2/7\*(-e^x + 9)^(7/2) - 18\*(-e^x + 9)^(5/2) + 540\*(-e^x + 9)^(3/2) - 14580\*sqrt(-e^x + 9) - 65610/sqrt(-e^x + 9) + 39366/(-e^x + 9)^(3/2)

**Fricas [A]** time = 0.863059, size = 163, normalized size = 2.01

$$\frac{2(e^{5x} + 18e^{4x} + 432e^{3x} + 23328e^{2x} - 839808e^x + 5038848)\sqrt{-e^x + 9}}{7(e^{2x} - 18e^x + 81)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)/(9-exp(x))^(5/2),x, algorithm="fricas")

[Out] -2/7\*(e^(5\*x) + 18\*e^(4\*x) + 432\*e^(3\*x) + 23328\*e^(2\*x) - 839808\*e^x + 5038848)\*sqrt(-e^x + 9)/(e^(2\*x) - 18\*e^x + 81)

**Sympy [A]** time = 60.0249, size = 61, normalized size = 0.75

$$\frac{2(9 - e^x)^{7/2}}{7} - 18(9 - e^x)^{5/2} + 540(9 - e^x)^{3/2} - 14580\sqrt{9 - e^x} - \frac{65610}{\sqrt{9 - e^x}} + \frac{39366}{(9 - e^x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)/(9-exp(x))\*\*(5/2),x)

[Out]  $2*(9 - \exp(x))^{7/2}/7 - 18*(9 - \exp(x))^{5/2} + 540*(9 - \exp(x))^{3/2} - 14580*\sqrt{9 - \exp(x)} - 65610/\sqrt{9 - \exp(x)} + 39366/(9 - \exp(x))^{3/2}$

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**Giac [A]** time = 1.28495, size = 101, normalized size = 1.25

$$-\frac{2}{7}(e^x - 9)^3\sqrt{-e^x + 9} - 18(e^x - 9)^2\sqrt{-e^x + 9} + 540(-e^x + 9)^{\frac{3}{2}} - 14580\sqrt{-e^x + 9} - \frac{13122(5e^x - 42)}{(e^x - 9)\sqrt{-e^x + 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6\*x)/(9-exp(x))^(5/2),x, algorithm="giac")

[Out]  $-2/7*(e^x - 9)^3*\sqrt{-e^x + 9} - 18*(e^x - 9)^2*\sqrt{-e^x + 9} + 540*(-e^x + 9)^{3/2} - 14580*\sqrt{-e^x + 9} - 13122*(5*e^x - 42)/((e^x - 9)*\sqrt{-e^x + 9})$

### 3.712 $\int (2 - 7e^{x^4})^5 x^3 dx$

**Optimal.** Leaf size=55

$$8x^4 - 140e^{x^4} + 490e^{2x^4} - \frac{3430e^{3x^4}}{3} + \frac{12005e^{4x^4}}{8} - \frac{16807e^{5x^4}}{20}$$

[Out]  $-140 * E^{x^4} + 490 * E^{(2 * x^4)} - (3430 * E^{(3 * x^4)}) / 3 + (12005 * E^{(4 * x^4)}) / 8 - (16807 * E^{(5 * x^4)}) / 20 + 8 * x^4$

**Rubi [A]** time = 0.0857752, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6715, 2282, 43}

$$8x^4 - 140e^{x^4} + 490e^{2x^4} - \frac{3430e^{3x^4}}{3} + \frac{12005e^{4x^4}}{8} - \frac{16807e^{5x^4}}{20}$$

Antiderivative was successfully verified.

[In] Int[(2 - 7 \* E^x^4)^5 \* x^3, x]

[Out]  $-140 * E^{x^4} + 490 * E^{(2 * x^4)} - (3430 * E^{(3 * x^4)}) / 3 + (12005 * E^{(4 * x^4)}) / 8 - (16807 * E^{(5 * x^4)}) / 20 + 8 * x^4$

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int (2 - 7e^{x^4})^5 x^3 dx &= \frac{1}{4} \text{Subst} \left( \int (2 - 7e^x)^5 dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left( \int \frac{(2 - 7x)^5}{x} dx, x, e^{x^4} \right) \\ &= \frac{1}{4} \text{Subst} \left( \int \left( -560 + \frac{32}{x} + 3920x - 13720x^2 + 24010x^3 - 16807x^4 \right) dx, x, e^{x^4} \right) \\ &= -140e^{x^4} + 490e^{2x^4} - \frac{3430e^{3x^4}}{3} + \frac{12005e^{4x^4}}{8} - \frac{16807e^{5x^4}}{20} + 8x^4 \end{aligned}$$

**Mathematica [A]** time = 0.0289462, size = 55, normalized size = 1.

$$8x^4 - 140e^{x^4} + 490e^{2x^4} - \frac{3430e^{3x^4}}{3} + \frac{12005e^{4x^4}}{8} - \frac{16807e^{5x^4}}{20}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 7\*E^x^4)^5\*x^3,x]

[Out] -140\*E^x^4 + 490\*E^(2\*x^4) - (3430\*E^(3\*x^4))/3 + (12005\*E^(4\*x^4))/8 - (16807\*E^(5\*x^4))/20 + 8\*x^4

**Maple [A]** time = 0.021, size = 47, normalized size = 0.9

$$-\frac{16807 (e^{x^4})^5}{20} + \frac{12005 (e^{x^4})^4}{8} - \frac{3430 (e^{x^4})^3}{3} + 490 (e^{x^4})^2 - 140 e^{x^4} + 8 \ln (e^{x^4})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-7\*exp(x^4))^5\*x^3,x)

[Out] -16807/20\*exp(x^4)^5+12005/8\*exp(x^4)^4-3430/3\*exp(x^4)^3+490\*exp(x^4)^2-140\*exp(x^4)+8\*ln(exp(x^4))

**Maxima [A]** time = 0.974356, size = 59, normalized size = 1.07

$$8x^4 - \frac{16807}{20} e^{(5x^4)} + \frac{12005}{8} e^{(4x^4)} - \frac{3430}{3} e^{(3x^4)} + 490 e^{(2x^4)} - 140 e^{(x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-7\*exp(x^4))^5\*x^3,x, algorithm="maxima")

[Out] 8\*x^4 - 16807/20\*e^(5\*x^4) + 12005/8\*e^(4\*x^4) - 3430/3\*e^(3\*x^4) + 490\*e^(2\*x^4) - 140\*e^(x^4)

**Fricas [A]** time = 0.925768, size = 131, normalized size = 2.38

$$8x^4 - \frac{16807}{20} e^{(5x^4)} + \frac{12005}{8} e^{(4x^4)} - \frac{3430}{3} e^{(3x^4)} + 490 e^{(2x^4)} - 140 e^{(x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-7\*exp(x^4))^5\*x^3,x, algorithm="fricas")

[Out] 8\*x^4 - 16807/20\*e^(5\*x^4) + 12005/8\*e^(4\*x^4) - 3430/3\*e^(3\*x^4) + 490\*e^(2\*x^4) - 140\*e^(x^4)

**Sympy [A]** time = 0.141958, size = 49, normalized size = 0.89

$$8x^4 - \frac{16807e^{5x^4}}{20} + \frac{12005e^{4x^4}}{8} - \frac{3430e^{3x^4}}{3} + 490e^{2x^4} - 140e^{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-7\*exp(x\*\*4))\*\*5\*x\*\*3,x)

[Out] 8\*x\*\*4 - 16807\*exp(5\*x\*\*4)/20 + 12005\*exp(4\*x\*\*4)/8 - 3430\*exp(3\*x\*\*4)/3 + 490\*exp(2\*x\*\*4) - 140\*exp(x\*\*4)

**Giac [A]** time = 1.26594, size = 59, normalized size = 1.07

$$8x^4 - \frac{16807}{20}e^{(5x^4)} + \frac{12005}{8}e^{(4x^4)} - \frac{3430}{3}e^{(3x^4)} + 490e^{(2x^4)} - 140e^{(x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-7\*exp(x^4))^5\*x^3,x, algorithm="giac")

[Out] 8\*x^4 - 16807/20\*e^(5\*x^4) + 12005/8\*e^(4\*x^4) - 3430/3\*e^(3\*x^4) + 490\*e^(2\*x^4) - 140\*e^(x^4)



### 3.713 $\int e^{x^2} \sqrt{1 - e^{2x^2}} x dx$

**Optimal.** Leaf size=35

$$\frac{1}{4} e^{x^2} \sqrt{1 - e^{2x^2}} + \frac{1}{4} \sin^{-1}(e^{x^2})$$

[Out]  $(E^{x^2} \sqrt{1 - E^{(2*x^2)}})/4 + \text{ArcSin}[E^{x^2}]/4$

**Rubi [A]** time = 0.160398, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {6715, 2249, 195, 216}

$$\frac{1}{4} e^{x^2} \sqrt{1 - e^{2x^2}} + \frac{1}{4} \sin^{-1}(e^{x^2})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{x^2} \sqrt{1 - E^{(2*x^2)}} * x, x]$

[Out]  $(E^{x^2} \sqrt{1 - E^{(2*x^2)}})/4 + \text{ArcSin}[E^{x^2}]/4$

#### Rule 6715

$\text{Int}[(u_*)(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x], x, x^{(m + 1)}], x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{FunctionOfQ}[x^{(m + 1)}, u, x]$

#### Rule 2249

$\text{Int}[(a_*) + (b_*)(F_)^{((e_*)((c_*) + (d_*)(x_)))})^{(p_*)} (G_)^{((h_*)((f_*) + (g_*)(x_)))}, x\_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(d*e*Log[F])/(g*h*Log[G])]\}, \text{Dist}[\text{Denominator}[m]/(g*h*Log[G]), \text{Subst}[\text{Int}[x^{(\text{Denominator}[m] - 1)} * (a + b * F^{(c*e - (d*e*f)/g}) * x^{\text{Numerator}[m]})^p, x], x, G^{((h*(f + g*x))/\text{Denominator}[m])}], x] /; \text{LtQ}[m, -1] \ || \ \text{GtQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

#### Rule 195

$\text{Int}[(a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p - 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[2*p] \ || \ (\text{EqQ}[n, 2] \ \&\& \ \text{IntegerQ}[4*p]) \ || \ (\text{EqQ}[n, 2] \ \&\& \ \text{IntegerQ}[3*p]) \ || \ \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)(x_)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int e^{x^2} \sqrt{1 - e^{2x^2}} x dx &= \frac{1}{2} \text{Subst} \left( \int e^x \sqrt{1 - e^{2x}} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \sqrt{1 - x^2} dx, x, e^{x^2} \right) \\
&= \frac{1}{4} e^{x^2} \sqrt{1 - e^{2x^2}} + \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{1 - x^2}} dx, x, e^{x^2} \right) \\
&= \frac{1}{4} e^{x^2} \sqrt{1 - e^{2x^2}} + \frac{1}{4} \sin^{-1} (e^{x^2})
\end{aligned}$$

**Mathematica [A]** time = 0.0275902, size = 32, normalized size = 0.91

$$\frac{1}{4} \left( e^{x^2} \sqrt{1 - e^{2x^2}} + \sin^{-1} (e^{x^2}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2\*Sqrt[1 - E^(2\*x^2)]\*x,x]

[Out] (E^x^2\*Sqrt[1 - E^(2\*x^2)] + ArcSin[E^x^2])/4

**Maple [A]** time = 0.058, size = 27, normalized size = 0.8

$$\frac{e^{x^2}}{4} \sqrt{1 - (e^{x^2})^2} + \frac{\arcsin(e^{x^2})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)\*x\*(1-exp(2\*x^2))^(1/2),x)

[Out] 1/4\*exp(x^2)\*(1-exp(x^2)^2)^(1/2)+1/4\*arcsin(exp(x^2))

**Maxima [A]** time = 1.46789, size = 35, normalized size = 1.

$$\frac{1}{4} \sqrt{-e^{(2x^2)} + 1} e^{(x^2)} + \frac{1}{4} \arcsin(e^{(x^2)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*x\*(1-exp(2\*x^2))^(1/2),x, algorithm="maxima")

[Out] 1/4\*sqrt(-e^(2\*x^2) + 1)\*e^(x^2) + 1/4\*arcsin(e^(x^2))

**Fricas [A]** time = 0.932436, size = 113, normalized size = 3.23

$$\frac{1}{4} \sqrt{-e^{(2x^2)} + 1} e^{(x^2)} - \frac{1}{2} \arctan \left( \left( \sqrt{-e^{(2x^2)} + 1} - 1 \right) e^{(-x^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(1-exp(2*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-e^(2*x^2) + 1)*e^(x^2) - 1/2*arctan((sqrt(-e^(2*x^2) + 1) - 1)*e^
(-x^2))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x*(1-exp(2*x**2))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.30599, size = 35, normalized size = 1.

$$\frac{1}{4} \sqrt{-e^{(2x^2)} + 1} e^{(x^2)} + \frac{1}{4} \arcsin\left(e^{(x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(1-exp(2*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-e^(2*x^2) + 1)*e^(x^2) + 1/4*arcsin(e^(x^2))
```

$$3.714 \quad \int e^{x^3} (1 - e^{4x^3})^2 x^2 dx$$

**Optimal.** Leaf size=32

$$\frac{e^{x^3}}{3} - \frac{2e^{5x^3}}{15} + \frac{e^{9x^3}}{27}$$

[Out]  $E^{x^3}/3 - (2*E^{(5*x^3)})/15 + E^{(9*x^3)}/27$

**Rubi [A]** time = 0.208785, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {6715, 2249, 194}

$$\frac{e^{x^3}}{3} - \frac{2e^{5x^3}}{15} + \frac{e^{9x^3}}{27}$$

Antiderivative was successfully verified.

[In] Int[E<sup>x^3</sup>\*(1 - E<sup>(4\*x^3)</sup>)<sup>2</sup>\*x<sup>2</sup>,x]

[Out]  $E^{x^3}/3 - (2*E^{(5*x^3)})/15 + E^{(9*x^3)}/27$

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

#### Rule 2249

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(d\*e\*Log[F])/(g\*h\*Log[G])]}, Dist[Denominator[m]/(g\*h\*Log[G]), Subst[Int[x^(Denominator[m] - 1)\*(a + b\*F^(c\*e - (d\*e\*f)/g)\*x^Numerator[m]]^p, x], x, G^((h\*(f + g\*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int e^{x^3} (1 - e^{4x^3})^2 x^2 dx &= \frac{1}{3} \text{Subst} \left( \int e^x (1 - e^{4x})^2 dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left( \int (1 - x^4)^2 dx, x, e^{x^3} \right) \\ &= \frac{1}{3} \text{Subst} \left( \int (1 - 2x^4 + x^8) dx, x, e^{x^3} \right) \\ &= \frac{e^{x^3}}{3} - \frac{2e^{5x^3}}{15} + \frac{e^{9x^3}}{27} \end{aligned}$$

**Mathematica [A]** time = 0.0238624, size = 29, normalized size = 0.91

$$\frac{1}{135}e^{x^3}(-18e^{4x^3} + 5e^{8x^3} + 45)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^3\*(1 - E^(4\*x^3))^2\*x^2,x]

[Out] (E^x^3\*(45 - 18\*E^(4\*x^3) + 5\*E^(8\*x^3)))/135

**Maple [A]** time = 0.021, size = 24, normalized size = 0.8

$$\frac{(e^{x^3})^9}{27} - \frac{2(e^{x^3})^5}{15} + \frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^3)\*(1-exp(4\*x^3))^2\*x^2,x)

[Out] 1/27\*exp(x^3)^9-2/15\*exp(x^3)^5+1/3\*exp(x^3)

**Maxima [A]** time = 0.958564, size = 31, normalized size = 0.97

$$\frac{1}{27}e^{(9x^3)} - \frac{2}{15}e^{(5x^3)} + \frac{1}{3}e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^3)\*(1-exp(4\*x^3))^2\*x^2,x, algorithm="maxima")

[Out] 1/27\*e^(9\*x^3) - 2/15\*e^(5\*x^3) + 1/3\*e^(x^3)

**Fricas [A]** time = 0.869861, size = 63, normalized size = 1.97

$$\frac{1}{27}e^{(9x^3)} - \frac{2}{15}e^{(5x^3)} + \frac{1}{3}e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^3)\*(1-exp(4\*x^3))^2\*x^2,x, algorithm="fricas")

[Out] 1/27\*e^(9\*x^3) - 2/15\*e^(5\*x^3) + 1/3\*e^(x^3)

**Sympy [A]** time = 0.143512, size = 24, normalized size = 0.75

$$\frac{e^{9x^3}}{27} - \frac{2e^{5x^3}}{15} + \frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**3)*(1-exp(4*x**3))**2*x**2,x)
```

```
[Out] exp(9*x**3)/27 - 2*exp(5*x**3)/15 + exp(x**3)/3
```

---

**Giac [A]** time = 1.25431, size = 31, normalized size = 0.97

$$\frac{1}{27} e^{(9x^3)} - \frac{2}{15} e^{(5x^3)} + \frac{1}{3} e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*(1-exp(4*x^3))^2*x^2,x, algorithm="giac")
```

```
[Out] 1/27*e^(9*x^3) - 2/15*e^(5*x^3) + 1/3*e^(x^3)
```

### 3.715 $\int e^{e^x+x} dx$

**Optimal.** Leaf size=5

$$e^{e^x}$$

[Out] E^E^x

**Rubi [A]** time = 0.0053152, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2282, 2194}

$$e^{e^x}$$

Antiderivative was successfully verified.

[In] Int[E^(E^x + x), x]

[Out] E^E^x

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\int e^{e^x+x} dx = \text{Subst}\left(\int e^x dx, x, e^x\right) = e^{e^x}$$

**Mathematica [A]** time = 0.0047743, size = 5, normalized size = 1.

$$e^{e^x}$$

Antiderivative was successfully verified.

[In] Integrate[E^(E^x + x), x]

[Out] E^E^x

**Maple [A]** time = 0.017, size = 4, normalized size = 0.8

$$e^{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(exp(x)+x),x)`

[Out] `exp(exp(x))`

---

**Maxima [A]** time = 0.963657, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x+exp(x)),x, algorithm="maxima")`

[Out] `e^(e^x)`

---

**Fricas [A]** time = 0.797798, size = 12, normalized size = 2.4

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x+exp(x)),x, algorithm="fricas")`

[Out] `e^(e^x)`

---

**Sympy [A]** time = 0.638304, size = 3, normalized size = 0.6

$$e^{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(exp(x)+x),x)`

[Out] `exp(exp(x))`

---

**Giac [A]** time = 1.29512, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x+exp(x)),x, algorithm="giac")`

[Out] `e^(e^x)`



$$3.716 \quad \int e^{e^{e^x} + e^x + x} dx$$

**Optimal.** Leaf size=7

$$e^{e^{e^x}}$$

[Out] E^E^E^x

**Rubi [A]** time = 0.0149321, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2282, 2194}

$$e^{e^{e^x}}$$

Antiderivative was successfully verified.

[In] Int[E^(E^E^x + E^x + x), x]

[Out] E^E^E^x

#### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_), x_Symbol] :=> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned} \int e^{e^{e^x} + e^x + x} dx &= \text{Subst}\left(\int e^{e^x + x} dx, x, e^x\right) \\ &= \text{Subst}\left(\int e^x dx, x, e^{e^x}\right) \\ &= e^{e^{e^x}} \end{aligned}$$

**Mathematica [A]** time = 0.0109893, size = 7, normalized size = 1.

$$e^{e^{e^x}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(E^E^x + E^x + x), x]

[Out] E^E^E^x

**Maple [A]** time = 0.021, size = 5, normalized size = 0.7

$$e^{e^{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(exp(exp(x))+exp(x)+x),x)

[Out] exp(exp(exp(x)))

**Maxima [A]** time = 0.963808, size = 5, normalized size = 0.71

$$e^{(e^{(e^x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(x))+exp(x)+x),x, algorithm="maxima")

[Out] e^(e^(e^x))

**Fricas [A]** time = 0.839971, size = 18, normalized size = 2.57

$$e^{(e^{(e^x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(x))+exp(x)+x),x, algorithm="fricas")

[Out] e^(e^(e^x))

**Sympy [A]** time = 0.975098, size = 5, normalized size = 0.71

$$e^{e^{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(x))+exp(x)+x),x)

[Out] exp(exp(exp(x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(x+e^x+e^{(e^x)})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(exp(exp(x))+exp(x)+x),x, algorithm="giac")

[Out] integrate(e^(x + e^x + e^(e^x)), x)

### 3.717 $\int (e^{-x} + e^x)^2 dx$

**Optimal.** Leaf size=22

$$2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

[Out]  $-1/(2E^{(2*x)}) + E^{(2*x)}/2 + 2*x$

**Rubi [A]** time = 0.0181731, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2282, 266, 43}

$$2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[(E^(-x) + E^x)^2,x]

[Out]  $-1/(2E^{(2*x)}) + E^{(2*x)}/2 + 2*x$

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int (e^{-x} + e^x)^2 dx &= \text{Subst} \left( \int \frac{(1+x^2)^2}{x^3} dx, x, e^x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(1+x)^2}{x^2} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( 1 + \frac{1}{x^2} + \frac{2}{x} \right) dx, x, e^{2x} \right) \\ &= -\frac{1}{2} e^{-2x} + \frac{e^{2x}}{2} + 2x \end{aligned}$$

**Mathematica [A]** time = 0.012073, size = 20, normalized size = 0.91

$$\frac{1}{2}(4x - e^{-2x} + e^{2x})$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-x) + E^x)^2,x]

[Out] (-E^(-2\*x) + E^(2\*x) + 4\*x)/2

---

**Maple [A]** time = 0.02, size = 17, normalized size = 0.8

$$2x - \frac{1}{2(e^x)^2} + \frac{(e^x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-x)+exp(x))^2,x)

[Out] 2\*x-1/2/exp(x)^2+1/2\*exp(x)^2

---

**Maxima [A]** time = 0.968346, size = 22, normalized size = 1.

$$2x + \frac{1}{2}e^{(2x)} - \frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))^2,x, algorithm="maxima")

[Out] 2\*x + 1/2\*e^(2\*x) - 1/2\*e^(-2\*x)

---

**Fricas [A]** time = 0.737353, size = 57, normalized size = 2.59

$$\frac{1}{2}(4xe^{(2x)} + e^{(4x)} - 1)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(-x)+exp(x))^2,x, algorithm="fricas")

[Out] 1/2\*(4\*x\*e^(2\*x) + e^(4\*x) - 1)\*e^(-2\*x)

---

**Sympy [A]** time = 0.099853, size = 17, normalized size = 0.77

$$2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(-x)+exp(x))**2,x)
```

```
[Out] 2*x + exp(2*x)/2 - exp(-2*x)/2
```

---

**Giac [A]** time = 1.2042, size = 32, normalized size = 1.45

$$-\frac{1}{2} \left( 2e^{2x} + 1 \right) e^{-2x} + 2x + \frac{1}{2} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(-x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*e^(2*x) + 1)*e^(-2*x) + 2*x + 1/2*e^(2*x)
```

$$3.718 \quad \int \frac{1}{e^{-x}+e^x} dx$$

**Optimal.** Leaf size=4

$$\tan^{-1}(e^x)$$

[Out] ArcTan[E^x]

**Rubi [A]** time = 0.0106405, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2282, 203}

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(E^(-x) + E^x)^(-1), x]

[Out] ArcTan[E^x]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\int \frac{1}{e^{-x}+e^x} dx = \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, e^x\right) = \tan^{-1}(e^x)$$

**Mathematica [A]** time = 0.0026222, size = 4, normalized size = 1.

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-x) + E^x)^(-1), x]

[Out] ArcTan[E^x]

**Maple [A]** time = 0.02, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(exp(-x)+exp(x)),x)

[Out] arctan(exp(x))

---

**Maxima [B]** time = 1.45054, size = 9, normalized size = 2.25

$$-\arctan(e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(exp(-x)+exp(x)),x, algorithm="maxima")

[Out] -arctan(e^(-x))

---

**Fricas [A]** time = 0.760628, size = 18, normalized size = 4.5

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(exp(-x)+exp(x)),x, algorithm="fricas")

[Out] arctan(e^x)

---

**Sympy [B]** time = 0.10334, size = 15, normalized size = 3.75

$$\text{RootSum}(4z^2 + 1, (i \mapsto i \log(2i + e^x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(exp(-x)+exp(x)),x)

[Out] RootSum(4\*\_z\*\*2 + 1, Lambda(\_i, \_i\*log(2\*\_i + exp(x))))

---

**Giac [A]** time = 1.222, size = 4, normalized size = 1.

$$\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(exp(-x)+exp(x)),x, algorithm="giac")

[Out] arctan(e^x)

$$3.719 \quad \int \frac{1}{(e^{-x}+e^x)^2} dx$$

**Optimal.** Leaf size=13

$$-\frac{1}{2(e^{2x}+1)}$$

[Out] -1/(2\*(1 + E^(2\*x)))

**Rubi [A]** time = 0.0124029, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2282, 261}

$$-\frac{1}{2(e^{2x}+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^(-x) + E^x)^(-2), x]

[Out] -1/(2\*(1 + E^(2\*x)))

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

#### Rubi steps

$$\int \frac{1}{(e^{-x}+e^x)^2} dx = \text{Subst} \left( \int \frac{x}{(1+x^2)^2} dx, x, e^x \right) = -\frac{1}{2(1+e^{2x})}$$

**Mathematica [A]** time = 0.0086088, size = 13, normalized size = 1.

$$-\frac{1}{2e^{2x}+2}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-x) + E^x)^(-2), x]



[Out]  $-(2 + 2 * E^{(2 * x)})^{-1}$

---

**Maple [A]** time = 0.022, size = 11, normalized size = 0.9

$$-\frac{1}{2 + 2 (e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(exp(-x)+exp(x))^2,x)`

[Out]  $-1/2/(1+\exp(x)^2)$

---

**Maxima [A]** time = 0.965802, size = 14, normalized size = 1.08

$$\frac{1}{2(e^{(-2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(exp(-x)+exp(x))^2,x, algorithm="maxima")`

[Out]  $1/2/(e^{(-2*x)} + 1)$

---

**Fricas [A]** time = 0.79847, size = 27, normalized size = 2.08

$$-\frac{1}{2(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(exp(-x)+exp(x))^2,x, algorithm="fricas")`

[Out]  $-1/2/(e^{(2*x)} + 1)$

---

**Sympy [A]** time = 0.075101, size = 10, normalized size = 0.77

$$-\frac{1}{2e^{2x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(exp(-x)+exp(x))**2,x)`

[Out]  $-1/(2*\exp(2*x) + 2)$

---

**Giac [A]** time = 1.31566, size = 14, normalized size = 1.08

$$-\frac{1}{2(e^{2x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(exp(-x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] -1/2/(e^(2*x) + 1)
```

$$3.720 \quad \int \frac{1}{-e^{-x} + e^x} dx$$

**Optimal.** Leaf size=6

$$-\tanh^{-1}(e^x)$$

[Out] -ArcTanh[E^x]

**Rubi [A]** time = 0.0103198, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 207}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(-E^(-x) + E^x)^(-1), x]

[Out] -ArcTanh[E^x]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\int \frac{1}{-e^{-x} + e^x} dx = \text{Subst} \left( \int \frac{1}{-1 + x^2} dx, x, e^x \right) \\ = -\tanh^{-1}(e^x)$$

**Mathematica [A]** time = 0.002364, size = 6, normalized size = 1.

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[(-E^(-x) + E^x)^(-1), x]

[Out] -ArcTanh[E^x]

**Maple [A]** time = 0.02, size = 6, normalized size = 1.

$$-\operatorname{Artanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1/exp(x)+exp(x)),x)`

[Out] `-arctanh(exp(x))`

---

**Maxima [B]** time = 0.967801, size = 26, normalized size = 4.33

$$-\frac{1}{2} \log(e^{-x} + 1) + \frac{1}{2} \log(e^{-x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x)),x, algorithm="maxima")`

[Out] `-1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)`

---

**Fricas [B]** time = 0.795371, size = 51, normalized size = 8.5

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x)),x, algorithm="fricas")`

[Out] `-1/2*log(e^x + 1) + 1/2*log(e^x - 1)`

---

**Sympy [B]** time = 0.101279, size = 15, normalized size = 2.5

$$\frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x)),x)`

[Out] `log(exp(x) - 1)/2 - log(exp(x) + 1)/2`

---

**Giac [B]** time = 1.27, size = 22, normalized size = 3.67

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x)),x, algorithm="giac")`

[Out] `-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))`

$$3.721 \quad \int \frac{1}{(-e^{-x} + e^x)^2} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{2(1 - e^{2x})}$$

[Out] 1/(2\*(1 - E^(2\*x)))

**Rubi [A]** time = 0.0139409, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 261}

$$\frac{1}{2(1 - e^{2x})}$$

Antiderivative was successfully verified.

[In] Int[(-E^(-x) + E^x)^(-2), x]

[Out] 1/(2\*(1 - E^(2\*x)))

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

#### Rubi steps

$$\int \frac{1}{(-e^{-x} + e^x)^2} dx = \text{Subst} \left( \int \frac{x}{(1 - x^2)^2} dx, x, e^x \right) \\ = \frac{1}{2(1 - e^{2x})}$$

**Mathematica [A]** time = 0.0112566, size = 11, normalized size = 0.73

$$\frac{1}{2 - 2e^{2x}}$$

Antiderivative was successfully verified.

[In] Integrate[(-E^(-x) + E^x)^(-2), x]

[Out]  $(2 - 2E^{(2*x)})^{-1}$

---

**Maple [A]** time = 0.018, size = 11, normalized size = 0.7

$$-\frac{1}{2(e^x)^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1/exp(x)+exp(x))^2,x)`

[Out]  $-1/2/(\exp(x)^2-1)$

---

**Maxima [A]** time = 0.961202, size = 14, normalized size = 0.93

$$\frac{1}{2(e^{-2x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x))^2,x, algorithm="maxima")`

[Out]  $1/2/(e^{-2*x} - 1)$

---

**Fricas [A]** time = 0.889163, size = 27, normalized size = 1.8

$$-\frac{1}{2(e^{2x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x))^2,x, algorithm="fricas")`

[Out]  $-1/2/(e^{2*x} - 1)$

---

**Sympy [A]** time = 0.074859, size = 10, normalized size = 0.67

$$-\frac{1}{2e^{2x} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1/exp(x)+exp(x))**2,x)`

[Out]  $-1/(2*\exp(2*x) - 2)$

---

**Giac [A]** time = 1.29891, size = 14, normalized size = 0.93

$$-\frac{1}{2(e^{2x}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] -1/2/(e^(2*x) - 1)
```

$$3.722 \quad \int e^x (-e^{-x} + e^x)^2 dx$$

**Optimal.** Leaf size=22

$$-e^{-x} - 2e^x + \frac{e^{3x}}{3}$$

[Out]  $-E^{-x} - 2E^x + E^{(3*x)}/3$

**Rubi [A]** time = 0.0277052, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2282, 14}

$$-e^{-x} - 2e^x + \frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

[In] Int[E^x\*(-E^(-x) + E^x)^2,x]

[Out]  $-E^{-x} - 2E^x + E^{(3*x)}/3$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

#### Rubi steps

$$\begin{aligned} \int e^x (-e^{-x} + e^x)^2 dx &= \text{Subst} \left( \int \frac{\frac{1}{x} - 2x + x^3}{x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -2 + \frac{1}{x^2} + x^2 \right) dx, x, e^x \right) \\ &= -e^{-x} - 2e^x + \frac{e^{3x}}{3} \end{aligned}$$

**Mathematica [A]** time = 0.007386, size = 22, normalized size = 1.

$$-e^{-x} - 2e^x + \frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*(-E^(-x) + E^x)^2,x]



[Out]  $-E^{(-x)} - 2*E^x + E^{(3*x)}/3$

**Maple [A]** time = 0.023, size = 18, normalized size = 0.8

$$\frac{(e^x)^3}{3} - 2e^x - (e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(-1/exp(x)+exp(x))^2,x)`

[Out]  $1/3*\exp(x)^3-2*\exp(x)-1/\exp(x)$

**Maxima [A]** time = 0.967162, size = 28, normalized size = 1.27

$$-\frac{1}{3} \left( 6e^{(-2x)} - 1 \right) e^{(3x)} - e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))^2,x, algorithm="maxima")`

[Out]  $-1/3*(6*e^{(-2*x)} - 1)*e^{(3*x)} - e^{(-x)}$

**Fricas [A]** time = 0.842875, size = 51, normalized size = 2.32

$$\frac{1}{3} \left( e^{(4x)} - 6e^{(2x)} - 3 \right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))^2,x, algorithm="fricas")`

[Out]  $1/3*(e^{(4*x)} - 6*e^{(2*x)} - 3)*e^{(-x)}$

**Sympy [A]** time = 0.110703, size = 15, normalized size = 0.68

$$\frac{e^{3x}}{3} - 2e^x - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))**2,x)`

[Out]  $\exp(3*x)/3 - 2*\exp(x) - \exp(-x)$

**Giac [A]** time = 1.30317, size = 23, normalized size = 1.05

$$\frac{1}{3} e^{(3x)} - e^{(-x)} - 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*e^(3*x) - e^(-x) - 2*e^x
```

$$3.723 \quad \int e^x (-e^{-x} + e^x)^3 dx$$

**Optimal.** Leaf size=31

$$3x + \frac{e^{-2x}}{2} - \frac{3e^{2x}}{2} + \frac{e^{4x}}{4}$$

[Out] 1/(2\*E^(2\*x)) - (3\*E^(2\*x))/2 + E^(4\*x)/4 + 3\*x

**Rubi [A]** time = 0.0366155, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 266, 43}

$$3x + \frac{e^{-2x}}{2} - \frac{3e^{2x}}{2} + \frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

[In] Int[E^x\*(-E^(-x) + E^x)^3,x]

[Out] 1/(2\*E^(2\*x)) - (3\*E^(2\*x))/2 + E^(4\*x)/4 + 3\*x

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int e^x (-e^{-x} + e^x)^3 dx &= \text{Subst} \left( \int \frac{(-1 + x^2)^3}{x^3} dx, x, e^x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{(-1 + x)^3}{x^2} dx, x, e^{2x} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \left( -3 - \frac{1}{x^2} + \frac{3}{x} + x \right) dx, x, e^{2x} \right) \\ &= \frac{e^{-2x}}{2} - \frac{3e^{2x}}{2} + \frac{e^{4x}}{4} + 3x \end{aligned}$$

**Mathematica [A]** time = 0.0126826, size = 29, normalized size = 0.94

$$\frac{1}{2} \left( 6x + e^{-2x} - 3e^{2x} + \frac{e^{4x}}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*(-E^(-x) + E^x)^3,x]

[Out] (E^(-2\*x) - 3\*E^(2\*x) + E^(4\*x))/2 + 6\*x)/2

**Maple [A]** time = 0.026, size = 25, normalized size = 0.8

$$\frac{(e^x)^4}{4} - \frac{3(e^x)^2}{2} + 3 \ln(e^x) + \frac{1}{2(e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(-1/exp(x)+exp(x))^3,x)

[Out] 1/4\*exp(x)^4-3/2\*exp(x)^2+3\*ln(exp(x))+1/2/exp(x)^2

**Maxima [A]** time = 0.968184, size = 32, normalized size = 1.03

$$-\frac{1}{4} (6e^{(-2x)} - 1)e^{(4x)} + 3x + \frac{1}{2} e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(-1/exp(x)+exp(x))^3,x, algorithm="maxima")

[Out] -1/4\*(6\*e^(-2\*x) - 1)\*e^(4\*x) + 3\*x + 1/2\*e^(-2\*x)

**Fricas [A]** time = 0.856181, size = 74, normalized size = 2.39

$$\frac{1}{4} (12xe^{(2x)} + e^{(6x)} - 6e^{(4x)} + 2)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(-1/exp(x)+exp(x))^3,x, algorithm="fricas")

[Out] 1/4\*(12\*x\*e^(2\*x) + e^(6\*x) - 6\*e^(4\*x) + 2)\*e^(-2\*x)

**Sympy [A]** time = 0.133643, size = 26, normalized size = 0.84

$$3x + \frac{e^{4x}}{4} - \frac{3e^{2x}}{2} + \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(-1/exp(x)+exp(x))\*\*3,x)

[Out] 3\*x + exp(4\*x)/4 - 3\*exp(2\*x)/2 + exp(-2\*x)/2

**Giac [A]** time = 1.20512, size = 41, normalized size = 1.32

$$-\frac{1}{2}(3e^{2x} - 1)e^{-2x} + 3x + \frac{1}{4}e^{4x} - \frac{3}{2}e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(-1/exp(x)+exp(x))^3,x, algorithm="giac")

[Out] -1/2\*(3\*e^(2\*x) - 1)\*e^(-2\*x) + 3\*x + 1/4\*e^(4\*x) - 3/2\*e^(2\*x)

$$3.724 \quad \int \frac{1+4^x}{1+2^x} dx$$

**Optimal.** Leaf size=22

$$x - \frac{2 \log(2^x + 1)}{\log(2)} + \frac{2^x}{\log(2)}$$

[Out]  $x + 2^x/\text{Log}[2] - (2*\text{Log}[1 + 2^x])/\text{Log}[2]$

**Rubi [A]** time = 0.0267354, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 894}

$$x - \frac{2 \log(2^x + 1)}{\log(2)} + \frac{2^x}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + 4^x)/(1 + 2^x), x]$

[Out]  $x + 2^x/\text{Log}[2] - (2*\text{Log}[1 + 2^x])/\text{Log}[2]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$   $\text{FunctionOfExponentialQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_)})^{(m\_)} /;$   $\text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ \text{!MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))*}$   $(F\_)[v_] /;$   $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

#### Rule 894

$\text{Int}[((d\_)+(e\_)*(x\_))^{(m\_)*((f\_)+(g\_)*(x\_))^{(n\_)*((a\_)+(c\_)*(x\_)^2)^{(p\_)}}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /;$   $\text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ ((\text{EqQ}[p, 1] \ \&\& \ \text{IntegersQ}[m, n]) \ || \ (\text{ILtQ}[m, 0] \ \&\& \ \text{ILtQ}[n, 0]))$

#### Rubi steps

$$\begin{aligned} \int \frac{1+4^x}{1+2^x} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x(1+x)} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(1 + \frac{1}{x} - \frac{2}{1+x}\right) dx, x, 2^x\right)}{\log(2)} \\ &= x + \frac{2^x}{\log(2)} - \frac{2 \log(1+2^x)}{\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0122895, size = 21, normalized size = 0.95

$$\frac{2^x + x \log(2) - 2 \log(2^x + 1)}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4^x)/(1 + 2^x),x]

[Out] (2^x + x\*Log[2] - 2\*Log[1 + 2^x])/Log[2]

**Maple [A]** time = 0.027, size = 27, normalized size = 1.2

$$x + \frac{e^{x \ln(2)}}{\ln(2)} - 2 \frac{\ln(1 + e^{x \ln(2)})}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4^x)/(1+2^x),x)

[Out] x+1/ln(2)\*exp(x\*ln(2))-2/ln(2)\*ln(1+exp(x\*ln(2)))

**Maxima [A]** time = 1.45288, size = 30, normalized size = 1.36

$$x + \frac{2^x}{\log(2)} - \frac{2 \log(2^x + 1)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4^x)/(1+2^x),x, algorithm="maxima")

[Out] x + 2^x/log(2) - 2\*log(2^x + 1)/log(2)

**Fricas [A]** time = 0.651464, size = 57, normalized size = 2.59

$$\frac{x \log(2) + 2^x - 2 \log(2^x + 1)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4^x)/(1+2^x),x, algorithm="fricas")

[Out] (x\*log(2) + 2^x - 2\*log(2^x + 1))/log(2)

**Sympy [A]** time = 0.289959, size = 29, normalized size = 1.32

$$x + \frac{e^{\frac{x \log(4)}{2}}}{\log(2)} - \frac{2 \log\left(e^{\frac{x \log(4)}{2}} + 1\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4\*\*x)/(1+2\*\*x),x)

[Out]  $x + \frac{\exp(x \cdot \log(4)/2)}{\log(2)} - \frac{2 \cdot \log(\exp(x \cdot \log(4)/2) + 1)}{\log(2)}$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x + 1}{2^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+4^x)/(1+2^x),x, algorithm="giac")`

[Out] `integrate((4^x + 1)/(2^x + 1), x)`



$$3.725 \quad \int \frac{1+4^x}{1+2^{-x}} dx$$

**Optimal.** Leaf size=34

$$\frac{2 \log(2^x + 1)}{\log(2)} - \frac{2^x}{\log(2)} + \frac{2^{2x-1}}{\log(2)}$$

[Out]  $-(2^x/\text{Log}[2]) + 2^{(-1 + 2*x)}/\text{Log}[2] + (2*\text{Log}[1 + 2^x])/ \text{Log}[2]$

**Rubi [A]** time = 0.0312803, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2282, 697}

$$\frac{2 \log(2^x + 1)}{\log(2)} - \frac{2^x}{\log(2)} + \frac{2^{2x-1}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + 4^x)/(1 + 2^{-x}), x]$

[Out]  $-(2^x/\text{Log}[2]) + 2^{(-1 + 2*x)}/\text{Log}[2] + (2*\text{Log}[1 + 2^x])/ \text{Log}[2]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$   $\text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_)})^{(m\_)} /;$   $\text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))} (F\_)[v_] /;$   $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

#### Rule 697

$\text{Int}[(d + (e_)*(x_))^{(m_)*((a_) + (c_)*(x_)^2)^{(p_)}], x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(a + c*x^2)^p, x], x] /;$   $\text{FreeQ}[\{a, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{1+4^x}{1+2^{-x}} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x} dx, x, 2^x\right)}{\log(2)} \\ &= \frac{\text{Subst}\left(\int \left(-1 + x + \frac{2}{1+x}\right) dx, x, 2^x\right)}{\log(2)} \\ &= -\frac{2^x}{\log(2)} + \frac{2^{-1+2x}}{\log(2)} + \frac{2 \log(1+2^x)}{\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0214097, size = 23, normalized size = 0.68

$$\frac{2^x(2^x - 2) + 4 \log(2^x + 1)}{\log(4)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4^x)/(1 + 2^(-x)),x]

[Out] (2^x\*(-2 + 2^x) + 4\*Log[1 + 2^x])/Log[4]

**Maple [A]** time = 0.03, size = 40, normalized size = 1.2

$$-\frac{e^{x \ln(2)}}{\ln(2)} + \frac{(e^{x \ln(2)})^2}{2 \ln(2)} + 2 \frac{\ln(1 + e^{x \ln(2)})}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4^x)/(1+1/(2^x)),x)

[Out] -1/ln(2)\*exp(x\*ln(2))+1/2/ln(2)\*exp(x\*ln(2))^2+2/ln(2)\*ln(1+exp(x\*ln(2)))

**Maxima [A]** time = 1.45459, size = 54, normalized size = 1.59

$$2x - \frac{2^{2x-1}(2^{-x+1} - 1)}{\log(2)} + \frac{2 \log\left(\frac{1}{2^x} + 1\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4^x)/(1+1/(2^x)),x, algorithm="maxima")

[Out] 2\*x - 2^(2\*x - 1)\*(2^(-x + 1) - 1)/log(2) + 2\*log(1/2^x + 1)/log(2)

**Fricas [A]** time = 0.843412, size = 63, normalized size = 1.85

$$\frac{2^{2x} - 2 \cdot 2^x + 4 \log(2^x + 1)}{2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4^x)/(1+1/(2^x)),x, algorithm="fricas")

[Out] 1/2\*(2^(2\*x) - 2\*2^x + 4\*log(2^x + 1))/log(2)

**Sympy [A]** time = 0.316153, size = 42, normalized size = 1.24

$$\frac{4^x \log(2) - 2e^{\frac{x \log(4)}{2}} \log(2)}{2 \log(2)^2} + \frac{2 \log\left(e^{\frac{x \log(4)}{2}} + 1\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4\*\*x)/(1+1/(2\*\*x)),x)

[Out]  $(4^{x+1} \log(2) - 2 \exp(x \log(4)/2) \log(2)) / (2 \log(2)^2) + 2 \log(\exp(x \log(4)/2) + 1) / \log(2)$

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{4^x + 1}{\frac{1}{2^x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4^x)/(1+1/(2^x)),x, algorithm="giac")

[Out] integrate((4^x + 1)/(1/2^x + 1), x)

$$3.726 \quad \int \left( \frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x} \right) dx$$

**Optimal.** Leaf size=23

$$\sqrt{\pi} \operatorname{Erfi}(a+x) - \frac{e^{(a+x)^2}}{x}$$

[Out]  $-(E^{(a+x)^2}/x) + \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[a+x]$

**Rubi [A]** time = 0.0481148, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2220, 2204}

$$\sqrt{\pi} \operatorname{Erfi}(a+x) - \frac{e^{(a+x)^2}}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(a+x)^2}/x^2 - (2*a*E^{(a+x)^2})/x, x]$

[Out]  $-(E^{(a+x)^2}/x) + \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[a+x]$

#### Rule 2220

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(f*(e + f*x)^{(m+1)}*F^{(a + b*(c + d*x)^2})/((m+1)*f^2), x] + (-\operatorname{Dist}[(2*b*d^2*\operatorname{Log}[F])/(f^2*(m+1)), \operatorname{Int}[(e + f*x)^{(m+2)}*F^{(a + b*(c + d*x)^2)}, x], x] + \operatorname{Dist}[(2*b*d*(d*e - c*f)*\operatorname{Log}[F])/(f^2*(m+1)), \operatorname{Int}[(e + f*x)^{(m+1)}*F^{(a + b*(c + d*x)^2)}, x], x]) /; \operatorname{FreeQ}\{F, a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[d*e - c*f, 0] \&\& \operatorname{LtQ}[m, -1]$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] := \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \left( \frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x} \right) dx &= - \left( (2a) \int \frac{e^{(a+x)^2}}{x} dx \right) + \int \frac{e^{(a+x)^2}}{x^2} dx \\ &= -\frac{e^{(a+x)^2}}{x} + 2 \int e^{(a+x)^2} dx \\ &= -\frac{e^{(a+x)^2}}{x} + \sqrt{\pi} \operatorname{erfi}(a+x) \end{aligned}$$

**Mathematica [A]** time = 0.0752959, size = 23, normalized size = 1.

$$\sqrt{\pi} \operatorname{Erfi}(a+x) - \frac{e^{(a+x)^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + x)^2/x^2 - (2\*a\*E^(a + x)^2)/x,x]

[Out] -(E^(a + x)^2/x) + Sqrt[Pi]\*Erfi[a + x]

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int \frac{e^{(a+x)^2}}{x^2} - 2 \frac{ae^{(a+x)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((a+x)^2)/x^2-2\*a\*exp((a+x)^2)/x,x)

[Out] int(exp((a+x)^2)/x^2-2\*a\*exp((a+x)^2)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{2ae^{(a+x)^2}}{x} + \frac{e^{(a+x)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((a+x)^2)/x^2-2\*a\*exp((a+x)^2)/x,x, algorithm="maxima")

[Out] integrate(-2\*a\*e^((a + x)^2)/x + e^((a + x)^2)/x^2, x)

**Fricas [A]** time = 0.733647, size = 70, normalized size = 3.04

$$\frac{\sqrt{\pi}x \operatorname{erfi}(a + x) - e^{(a^2+2ax+x^2)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((a+x)^2)/x^2-2\*a\*exp((a+x)^2)/x,x, algorithm="fricas")

[Out] (sqrt(pi)\*x\*erfi(a + x) - e^(a^2 + 2\*a\*x + x^2))/x

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\left(\int -\frac{e^{x^2}e^{2ax}}{x^2} dx + \int \frac{2ae^{x^2}e^{2ax}}{x} dx\right)e^{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((a+x)\*\*2)/x\*\*2-2\*a\*exp((a+x)\*\*2)/x,x)

[Out] -(Integral(-exp(x\*\*2)\*exp(2\*a\*x)/x\*\*2, x) + Integral(2\*a\*exp(x\*\*2)\*exp(2\*a\*x)/x, x))\*exp(a\*\*2)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{2ae^{(a+x)^2}}{x} + \frac{e^{(a+x)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(-2*a*e^((a + x)^2)/x + e^((a + x)^2)/x^2, x)
```

$$3.727 \quad \int e^{-x^2} (x^4 + x^6 + x^8) dx$$

**Optimal.** Leaf size=66

$$\frac{147}{32}\sqrt{\pi}\operatorname{Erf}(x) - \frac{1}{2}e^{-x^2}x^7 - \frac{9}{4}e^{-x^2}x^5 - \frac{49}{8}e^{-x^2}x^3 - \frac{147}{16}e^{-x^2}x$$

[Out]  $(-147*x)/(16*E^{x^2}) - (49*x^3)/(8*E^{x^2}) - (9*x^5)/(4*E^{x^2}) - x^7/(2*E^{x^2}) + (147*sqrt[\pi]*Erf[x])/32$

**Rubi [A]** time = 0.183538, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1594, 2226, 2212, 2205}

$$\frac{147}{32}\sqrt{\pi}\operatorname{Erf}(x) - \frac{1}{2}e^{-x^2}x^7 - \frac{9}{4}e^{-x^2}x^5 - \frac{49}{8}e^{-x^2}x^3 - \frac{147}{16}e^{-x^2}x$$

Antiderivative was successfully verified.

[In] Int[(x^4 + x^6 + x^8)/E^x^2,x]

[Out]  $(-147*x)/(16*E^{x^2}) - (49*x^3)/(8*E^{x^2}) - (9*x^5)/(4*E^{x^2}) - x^7/(2*E^{x^2}) + (147*sqrt[\pi]*Erf[x])/32$

#### Rule 1594

Int[(u\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_) + (c\_)\*(x\_)^(r\_))^(n\_), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p) + c\*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

#### Rule 2226

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^(n\_)))\*(u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2212

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^(n\_)))\*((c\_) + (d\_)\*(x\_)^(m\_)), x\_Symbol] :> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rule 2205

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*sqrt[\pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int e^{-x^2} (x^4 + x^6 + x^8) dx &= \int e^{-x^2} x^4 (1 + x^2 + x^4) dx \\
&= \int (e^{-x^2} x^4 + e^{-x^2} x^6 + e^{-x^2} x^8) dx \\
&= \int e^{-x^2} x^4 dx + \int e^{-x^2} x^6 dx + \int e^{-x^2} x^8 dx \\
&= -\frac{1}{2} e^{-x^2} x^3 - \frac{1}{2} e^{-x^2} x^5 - \frac{1}{2} e^{-x^2} x^7 + \frac{3}{2} \int e^{-x^2} x^2 dx + \frac{5}{2} \int e^{-x^2} x^4 dx + \frac{7}{2} \int e^{-x^2} x^6 dx \\
&= -\frac{3}{4} e^{-x^2} x - \frac{7}{4} e^{-x^2} x^3 - \frac{9}{4} e^{-x^2} x^5 - \frac{1}{2} e^{-x^2} x^7 + \frac{3}{4} \int e^{-x^2} dx + \frac{15}{4} \int e^{-x^2} x^2 dx + \frac{35}{4} \int e^{-x^2} x^4 dx \\
&= -\frac{21}{8} e^{-x^2} x - \frac{49}{8} e^{-x^2} x^3 - \frac{9}{4} e^{-x^2} x^5 - \frac{1}{2} e^{-x^2} x^7 + \frac{3}{8} \sqrt{\pi} \operatorname{erf}(x) + \frac{15}{8} \int e^{-x^2} dx + \frac{105}{8} \int e^{-x^2} x^2 dx \\
&= -\frac{147}{16} e^{-x^2} x - \frac{49}{8} e^{-x^2} x^3 - \frac{9}{4} e^{-x^2} x^5 - \frac{1}{2} e^{-x^2} x^7 + \frac{21}{16} \sqrt{\pi} \operatorname{erf}(x) + \frac{105}{16} \int e^{-x^2} dx \\
&= -\frac{147}{16} e^{-x^2} x - \frac{49}{8} e^{-x^2} x^3 - \frac{9}{4} e^{-x^2} x^5 - \frac{1}{2} e^{-x^2} x^7 + \frac{147}{32} \sqrt{\pi} \operatorname{erf}(x)
\end{aligned}$$

**Mathematica [A]** time = 0.0229439, size = 41, normalized size = 0.62

$$\frac{1}{32} \left( 147 \sqrt{\pi} \operatorname{Erf}(x) - 2e^{-x^2} x (8x^6 + 36x^4 + 98x^2 + 147) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4 + x^6 + x^8)/E^x^2,x]

[Out] ((-2\*x\*(147 + 98\*x^2 + 36\*x^4 + 8\*x^6))/E^x^2 + 147\*Sqrt[Pi]\*Erf[x])/32

**Maple [A]** time = 0.025, size = 51, normalized size = 0.8

$$-\frac{147x}{16e^{x^2}} - \frac{49x^3}{8e^{x^2}} - \frac{9x^5}{4e^{x^2}} - \frac{x^7}{2e^{x^2}} + \frac{147\operatorname{Erf}(x)\sqrt{\pi}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8+x^6+x^4)/exp(x^2),x)

[Out] -147/16\*x/exp(x^2)-49/8\*x^3/exp(x^2)-9/4\*x^5/exp(x^2)-1/2\*x^7/exp(x^2)+147/32\*erf(x)\*Pi^(1/2)

**Maxima [A]** time = 0.975671, size = 100, normalized size = 1.52

$$-\frac{1}{16} (8x^7 + 28x^5 + 70x^3 + 105x)e^{(-x^2)} - \frac{1}{8} (4x^5 + 10x^3 + 15x)e^{(-x^2)} - \frac{1}{4} (2x^3 + 3x)e^{(-x^2)} + \frac{147}{32} \sqrt{\pi} \operatorname{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="maxima")

[Out] -1/16\*(8\*x^7 + 28\*x^5 + 70\*x^3 + 105\*x)\*e^(-x^2) - 1/8\*(4\*x^5 + 10\*x^3 + 15\*x)\*e^(-x^2) - 1/4\*(2\*x^3 + 3\*x)\*e^(-x^2) + 147/32\*sqrt(pi)\*erf(x)



**Fricas [A]** time = 0.754979, size = 101, normalized size = 1.53

$$-\frac{1}{16} (8x^7 + 36x^5 + 98x^3 + 147x)e^{(-x^2)} + \frac{147}{32} \sqrt{\pi} \operatorname{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="fricas")

[Out] -1/16\*(8\*x^7 + 36\*x^5 + 98\*x^3 + 147\*x)\*e^(-x^2) + 147/32\*sqrt(pi)\*erf(x)

**Sympy [A]** time = 158.37, size = 54, normalized size = 0.82

$$-\frac{x^7 e^{-x^2}}{2} - \frac{9x^5 e^{-x^2}}{4} - \frac{49x^3 e^{-x^2}}{8} - \frac{147x e^{-x^2}}{16} + \frac{147\sqrt{\pi} \operatorname{erf}(x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*8+x\*\*6+x\*\*4)/exp(x\*\*2),x)

[Out] -x\*\*7\*exp(-x\*\*2)/2 - 9\*x\*\*5\*exp(-x\*\*2)/4 - 49\*x\*\*3\*exp(-x\*\*2)/8 - 147\*x\*exp(-x\*\*2)/16 + 147\*sqrt(pi)\*erf(x)/32

**Giac [A]** time = 1.22418, size = 47, normalized size = 0.71

$$-\frac{1}{16} (8x^7 + 36x^5 + 98x^3 + 147x)e^{(-x^2)} + \frac{147}{32} \sqrt{\pi} \operatorname{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="giac")

[Out] -1/16\*(8\*x^7 + 36\*x^5 + 98\*x^3 + 147\*x)\*e^(-x^2) + 147/32\*sqrt(pi)\*erf(x)

$$3.728 \quad \int \frac{1}{-e^x + e^{3x}} dx$$

**Optimal.** Leaf size=12

$$e^{-x} - \tanh^{-1}(e^x)$$

[Out] E<sup>^(-x)</sup> - ArcTanh[E<sup>^x</sup>]

**Rubi [A]** time = 0.0133264, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2282, 325, 207}

$$e^{-x} - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(-E<sup>^x</sup> + E<sup>^(3\*x)</sup>)<sup>^(-1)</sup>, x]

[Out] E<sup>^(-x)</sup> - ArcTanh[E<sup>^x</sup>]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{-e^x + e^{3x}} dx &= \text{Subst} \left( \int \frac{1}{x^2(-1 + x^2)} dx, x, e^x \right) \\ &= e^{-x} + \text{Subst} \left( \int \frac{1}{-1 + x^2} dx, x, e^x \right) \\ &= e^{-x} - \tanh^{-1}(e^x) \end{aligned}$$

**Mathematica [C]** time = 0.0059167, size = 19, normalized size = 1.58

$$e^{-x} {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; e^{2x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-E^x + E^(3\*x))^(-1), x]

[Out] Hypergeometric2F1[-1/2, 1, 1/2, E^(2\*x)]/E^x

**Maple [A]** time = 0.026, size = 20, normalized size = 1.7

$$\frac{\ln(-1 + e^x)}{2} + (e^x)^{-1} - \frac{\ln(1 + e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-exp(x)+exp(3\*x)), x)

[Out] 1/2\*ln(-1+exp(x))+1/exp(x)-1/2\*ln(1+exp(x))

**Maxima [A]** time = 0.967865, size = 26, normalized size = 2.17

$$e^{(-x)} - \frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-exp(x)+exp(3\*x)), x, algorithm="maxima")

[Out] e^(-x) - 1/2\*log(e^x + 1) + 1/2\*log(e^x - 1)

**Fricas [B]** time = 0.821906, size = 74, normalized size = 6.17

$$-\frac{1}{2} (e^x \log(e^x + 1) - e^x \log(e^x - 1) - 2)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-exp(x)+exp(3\*x)), x, algorithm="fricas")

[Out] -1/2\*(e^x\*log(e^x + 1) - e^x\*log(e^x - 1) - 2)\*e^(-x)

**Sympy [B]** time = 0.111915, size = 20, normalized size = 1.67

$$\frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2} + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-exp(x)+exp(3\*x)), x)

[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)

---

**Giac [A]** time = 1.22295, size = 27, normalized size = 2.25

$$e^{(-x)} - \frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-exp(x)+exp(3\*x)),x, algorithm="giac")

[Out] e^(-x) - 1/2\*log(e^x + 1) + 1/2\*log(abs(e^x - 1))

$$3.729 \quad \int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx$$

**Optimal.** Leaf size=16

$$e^x - \frac{3e^x}{1-x}$$

[Out]  $E^x - (3E^x)/(1 - x)$

**Rubi [A]** time = 0.0742178, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2199, 2194, 2177, 2178}

$$e^x - \frac{3e^x}{1-x}$$

Antiderivative was successfully verified.

[In] Int[(E^x\*(-5 + x + x^2))/(-1 + x)^2,x]

[Out]  $E^x - (3E^x)/(1 - x)$

#### Rule 2199

Int[(F\_)^((c\_)\*(v\_))\*(u\_)^(m\_)\*(w\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx &= \int \left( e^x - \frac{3e^x}{(-1+x)^2} + \frac{3e^x}{-1+x} \right) dx \\
&= -\left( 3 \int \frac{e^x}{(-1+x)^2} dx \right) + 3 \int \frac{e^x}{-1+x} dx + \int e^x dx \\
&= e^x - \frac{3e^x}{1-x} + 3e\text{Ei}(-1+x) - 3 \int \frac{e^x}{-1+x} dx \\
&= e^x - \frac{3e^x}{1-x}
\end{aligned}$$

**Mathematica [A]** time = 0.0329099, size = 12, normalized size = 0.75

$$\frac{e^x(x+2)}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x\*(-5 + x + x^2))/(-1 + x)^2,x]

[Out] (E^x\*(2 + x))/(-1 + x)

**Maple [A]** time = 0.019, size = 12, normalized size = 0.8

$$\frac{(2+x)e^x}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*(x^2+x-5)/(x-1)^2,x)

[Out] 1/(x-1)\*(2+x)\*exp(x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(x^2+x)e^x}{x^2-2x+1} + \frac{5eE_2(-x+1)}{x-1} + \int \frac{(3x+1)e^x}{x^3-3x^2+3x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x^2+x-5)/(-1+x)^2,x, algorithm="maxima")

[Out] (x^2 + x)\*e^x/(x^2 - 2\*x + 1) + 5\*e\*exp\_integral\_e(2, -x + 1)/(x - 1) + integrate((3\*x + 1)\*e^x/(x^3 - 3\*x^2 + 3\*x - 1), x)

**Fricas [A]** time = 0.788438, size = 28, normalized size = 1.75

$$\frac{(x+2)e^x}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x^2+x-5)/(-1+x)^2,x, algorithm="fricas")

[Out] (x + 2)\*e^x/(x - 1)

**Sympy [A]** time = 0.094451, size = 8, normalized size = 0.5

$$\frac{(x + 2)e^x}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x\*\*2+x-5)/(-1+x)\*\*2,x)

[Out] (x + 2)\*exp(x)/(x - 1)

**Giac [B]** time = 1.24843, size = 43, normalized size = 2.69

$$\frac{3e^{\left((x-1)\left(\frac{1}{x-1}+1\right)\right)}}{x-1} + e^{\left((x-1)\left(\frac{1}{x-1}+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)\*(x^2+x-5)/(-1+x)^2,x, algorithm="giac")

[Out] 3\*e^((x - 1)\*(1/(x - 1) + 1))/(x - 1) + e^((x - 1)\*(1/(x - 1) + 1))

$$3.730 \quad \int \frac{e^{x^2} x^3}{(1+x^2)^2} dx$$

**Optimal.** Leaf size=16

$$\frac{e^{x^2}}{2(x^2+1)}$$

[Out] E<sup>x^2</sup>/(2\*(1 + x^2))

**Rubi [A]** time = 0.0576866, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2289}

$$\frac{e^{x^2}}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[(E<sup>x^2</sup>\*x<sup>3</sup>)/(1 + x<sup>2</sup>)<sup>2</sup>,x]

[Out] E<sup>x^2</sup>/(2\*(1 + x^2))

#### Rule 2289

```
Int[(F_)^(u_)*(v_)^(n_.)*(w_), x_Symbol] := With[{z = Log[F]*v*D[u, x] + (n + 1)*D[v, x]}, Simp[(Coefficient[w, x, Exponent[w, x]]*F^u*v^(n + 1))/Coefficient[z, x, Exponent[z, x]], x] /; EqQ[Exponent[w, x], Exponent[z, x]] && EqQ[w*Coefficient[z, x, Exponent[z, x]], z*Coefficient[w, x, Exponent[w, x]]] /; FreeQ[{F, n}, x] && PolynomialQ[u, x] && PolynomialQ[v, x] && PolynomialQ[w, x]
```

#### Rubi steps

$$\int \frac{e^{x^2} x^3}{(1+x^2)^2} dx = \frac{e^{x^2}}{2(1+x^2)}$$

**Mathematica [A]** time = 0.0345679, size = 16, normalized size = 1.

$$\frac{e^{x^2}}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(E<sup>x^2</sup>\*x<sup>3</sup>)/(1 + x<sup>2</sup>)<sup>2</sup>,x]

[Out] E<sup>x^2</sup>/(2\*(1 + x^2))



**Maple [A]** time = 0.02, size = 14, normalized size = 0.9

$$\frac{e^{x^2}}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)\*x^3/(x^2+1)^2,x)

[Out] 1/2\*exp(x^2)/(x^2+1)

---

**Maxima [A]** time = 0.975302, size = 18, normalized size = 1.12

$$\frac{e^{(x^2)}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*x^3/(x^2+1)^2,x, algorithm="maxima")

[Out] 1/2\*e^(x^2)/(x^2 + 1)

---

**Fricas [A]** time = 0.883296, size = 31, normalized size = 1.94

$$\frac{e^{(x^2)}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*x^3/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*e^(x^2)/(x^2 + 1)

---

**Sympy [A]** time = 0.091789, size = 10, normalized size = 0.62

$$\frac{e^{x^2}}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x\*\*2)\*x\*\*3/(x\*\*2+1)\*\*2,x)

[Out] exp(x\*\*2)/(2\*x\*\*2 + 2)

---

**Giac [A]** time = 1.3054, size = 18, normalized size = 1.12

$$\frac{e^{(x^2)}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)/(x^2 + 1)
```

$$3.731 \quad \int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx$$

**Optimal.** Leaf size=33

$$\frac{1}{32}e^x\sqrt{16e^{2x}+25}-\frac{25}{128}\sinh^{-1}\left(\frac{4e^x}{5}\right)$$

[Out] (E^x\*Sqrt[25 + 16\*E^(2\*x)])/32 - (25\*ArcSinh[(4\*E^x)/5])/128

**Rubi [A]** time = 0.032377, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2248, 321, 215}

$$\frac{1}{32}e^x\sqrt{16e^{2x}+25}-\frac{25}{128}\sinh^{-1}\left(\frac{4e^x}{5}\right)$$

Antiderivative was successfully verified.

[In] Int[E^(3\*x)/Sqrt[25 + 16\*E^(2\*x)], x]

[Out] (E^x\*Sqrt[25 + 16\*E^(2\*x)])/32 - (25\*ArcSinh[(4\*E^x)/5])/128

#### Rule 2248

Int[((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.)\*(G\_)^((h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := With[{m = FullSimplify[(g\*h\*Log[G])/(d\*e\*Log[F])]}, Dist[(Denominator[m]\*G^(f\*h - (c\*g\*h)/d))/(d\*e\*Log[F]), Subst[Int[x^(Numerator[m] - 1)\*(a + b\*x^Denominator[m])^p, x], x, F^((e\*(c + d\*x))/Denominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx &= \text{Subst}\left(\int \frac{x^2}{\sqrt{25+16x^2}} dx, x, e^x\right) \\ &= \frac{1}{32}e^x\sqrt{25+16e^{2x}} - \frac{25}{32}\text{Subst}\left(\int \frac{1}{\sqrt{25+16x^2}} dx, x, e^x\right) \\ &= \frac{1}{32}e^x\sqrt{25+16e^{2x}} - \frac{25}{128}\sinh^{-1}\left(\frac{4e^x}{5}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0156171, size = 33, normalized size = 1.

$$\frac{1}{32}e^x\sqrt{16e^{2x}+25}-\frac{25}{128}\sinh^{-1}\left(\frac{4e^x}{5}\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(3\*x)/Sqrt[25 + 16\*E^(2\*x)],x]

[Out] (E^x\*Sqrt[25 + 16\*E^(2\*x)])/32 - (25\*ArcSinh[(4\*E^x)/5])/128

**Maple [A]** time = 0.065, size = 23, normalized size = 0.7

$$\frac{e^x}{32}\sqrt{25+16(e^x)^2}-\frac{25}{128}\operatorname{Arcsinh}\left(\frac{4e^x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3\*x)/(25+16\*exp(2\*x))^(1/2),x)

[Out] 1/32\*exp(x)\*(25+16\*exp(x)^2)^(1/2)-25/128\*arcsinh(4/5\*exp(x))

**Maxima [B]** time = 1.01677, size = 100, normalized size = 3.03

$$\frac{25\sqrt{16e^{(2x)}+25e^{(-x)}}}{32((16e^{(2x)}+25)e^{(-2x)}-16)}-\frac{25}{256}\log\left(\sqrt{16e^{(2x)}+25e^{(-x)}}+4\right)+\frac{25}{256}\log\left(\sqrt{16e^{(2x)}+25e^{(-x)}}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(25+16\*exp(2\*x))^(1/2),x, algorithm="maxima")

[Out] 25/32\*sqrt(16\*e^(2\*x) + 25)\*e^(-x)/((16\*e^(2\*x) + 25)\*e^(-2\*x) - 16) - 25/256\*log(sqrt(16\*e^(2\*x) + 25)\*e^(-x) + 4) + 25/256\*log(sqrt(16\*e^(2\*x) + 25)\*e^(-x) - 4)

**Fricas [A]** time = 0.886718, size = 103, normalized size = 3.12

$$\frac{1}{32}\sqrt{16e^{(2x)}+25}e^x+\frac{25}{128}\log\left(\sqrt{16e^{(2x)}+25}-4e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(25+16\*exp(2\*x))^(1/2),x, algorithm="fricas")

[Out] 1/32\*sqrt(16\*e^(2\*x) + 25)\*e^x + 25/128\*log(sqrt(16\*e^(2\*x) + 25) - 4\*e^x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{3x}}{\sqrt{16e^{2x}+25}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(25+16\*exp(2\*x))\*\*(1/2),x)

[Out] Integral(exp(3\*x)/sqrt(16\*exp(2\*x) + 25), x)

**Giac [A]** time = 1.24746, size = 45, normalized size = 1.36

$$\frac{1}{32} \sqrt{16e^{(2x)} + 25}e^x + \frac{25}{128} \log\left(\sqrt{16e^{(2x)} + 25} - 4e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3\*x)/(25+16\*exp(2\*x))^(1/2),x, algorithm="giac")

[Out] 1/32\*sqrt(16\*e^(2\*x) + 25)\*e^x + 25/128\*log(sqrt(16\*e^(2\*x) + 25) - 4\*e^x)

$$3.732 \quad \int \frac{1+e^x}{\sqrt{e^x+x}} dx$$

**Optimal.** Leaf size=11

$$2\sqrt{x+e^x}$$

[Out] 2\*Sqrt[E^x + x]

**Rubi [A]** time = 0.0252986, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {6686}

$$2\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Int[(1 + E^x)/Sqrt[E^x + x], x]

[Out] 2\*Sqrt[E^x + x]

**Rule 6686**

Int[(u\_)\*(y\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[(q\*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{1+e^x}{\sqrt{e^x+x}} dx = 2\sqrt{e^x+x}$$

**Mathematica [A]** time = 0.0075249, size = 11, normalized size = 1.

$$2\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + E^x)/Sqrt[E^x + x], x]

[Out] 2\*Sqrt[E^x + x]

**Maple [A]** time = 0.066, size = 9, normalized size = 0.8

$$2\sqrt{e^x+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+exp(x))/(exp(x)+x)^(1/2), x)

[Out] 2\*(exp(x)+x)^(1/2)

---

**Maxima [A]** time = 0.96614, size = 11, normalized size = 1.

$$2\sqrt{x + e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))/(x+exp(x))^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(x + e^x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))/(x+exp(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0.147997, size = 8, normalized size = 0.73

$$2\sqrt{x + e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))/(exp(x)+x)\*\*(1/2),x)

[Out] 2\*sqrt(x + exp(x))

---

**Giac [A]** time = 1.28738, size = 11, normalized size = 1.

$$2\sqrt{x + e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))/(x+exp(x))^(1/2),x, algorithm="giac")

[Out] 2\*sqrt(x + e^x)

$$3.733 \quad \int \frac{1+e^x}{e^x+x} dx$$

**Optimal.** Leaf size=6

$$\log(x + e^x)$$

[Out] Log[E<sup>x</sup> + x]

**Rubi [A]** time = 0.0195129, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {6684}

$$\log(x + e^x)$$

Antiderivative was successfully verified.

[In] Int[(1 + E<sup>x</sup>)/(E<sup>x</sup> + x), x]

[Out] Log[E<sup>x</sup> + x]

**Rule 6684**

Int[(u\_)/(y\_), x\_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q\*Log[RemoveContent[y, x]], x] /; !FalseQ[q]]

**Rubi steps**

$$\int \frac{1 + e^x}{e^x + x} dx = \log(e^x + x)$$

**Mathematica [A]** time = 0.0298571, size = 6, normalized size = 1.

$$\log(x + e^x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + E<sup>x</sup>)/(E<sup>x</sup> + x), x]

[Out] Log[E<sup>x</sup> + x]

**Maple [A]** time = 0.017, size = 6, normalized size = 1.

$$\ln(e^x + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+exp(x))/(exp(x)+x), x)

[Out] ln(exp(x)+x)



---

**Maxima [A]** time = 0.980101, size = 7, normalized size = 1.17

$$\log(x + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(x+exp(x)),x, algorithm="maxima")
```

```
[Out] log(x + e^x)
```

---

**Fricas [A]** time = 0.89915, size = 19, normalized size = 3.17

$$\log(x + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(x+exp(x)),x, algorithm="fricas")
```

```
[Out] log(x + e^x)
```

---

**Sympy [A]** time = 0.095375, size = 5, normalized size = 0.83

$$\log(x + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(exp(x)+x),x)
```

```
[Out] log(x + exp(x))
```

---

**Giac [A]** time = 1.2249, size = 7, normalized size = 1.17

$$\log(x + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(x+exp(x)),x, algorithm="giac")
```

```
[Out] log(x + e^x)
```

$$3.734 \quad \int \frac{e^{x^2}}{x^2} dx$$

**Optimal.** Leaf size=19

$$\sqrt{\pi}\operatorname{Erfi}(x) - \frac{e^{x^2}}{x}$$

[Out]  $-(E^{x^2}/x) + \operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[x]$

**Rubi [A]** time = 0.0152378, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2214, 2204}

$$\sqrt{\pi}\operatorname{Erfi}(x) - \frac{e^{x^2}}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{x^2}/x^2, x]$

[Out]  $-(E^{x^2}/x) + \operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[x]$

#### Rule 2214

$\operatorname{Int}[(F_)^{\wedge}((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{\wedge}(n_))*((c_.) + (d_.)*(x_))^{\wedge}(m_.), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{\wedge}(m + 1)*F^{\wedge}(a + b*(c + d*x)^{\wedge}n)/(d*(m + 1)), x] - \operatorname{Dist}[(b*n*\operatorname{Log}[F])/(m + 1), \operatorname{Int}[(c + d*x)^{\wedge}(m + n)*F^{\wedge}(a + b*(c + d*x)^{\wedge}n), x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{IntegerQ}[(2*(m + 1))/n] \ \&\& \ \operatorname{LtQ}[-4, (m + 1)/n, 5] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ ((\operatorname{GtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m, -1]) \ || \ (\operatorname{GtQ}[-n, 0] \ \&\& \ \operatorname{LeQ}[-n, m + 1]))$

#### Rule 2204

$\operatorname{Int}[(F_)^{\wedge}((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{\wedge}2), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{\wedge}a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned} \int \frac{e^{x^2}}{x^2} dx &= -\frac{e^{x^2}}{x} + 2 \int e^{x^2} dx \\ &= -\frac{e^{x^2}}{x} + \sqrt{\pi}\operatorname{erfi}(x) \end{aligned}$$

**Mathematica [A]** time = 0.0043091, size = 19, normalized size = 1.

$$\sqrt{\pi}\operatorname{Erfi}(x) - \frac{e^{x^2}}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[E^{x^2}/x^2, x]$

[Out]  $-(E^{x^2}/x) + \text{Sqrt}[\text{Pi}] * \text{Erfi}[x]$

**Maple [A]** time = 0.022, size = 17, normalized size = 0.9

$$-\frac{e^{x^2}}{x} + \text{erfi}(x) \sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)/x^2,x)`

[Out]  $-\exp(x^2)/x + \text{erfi}(x) * \text{Pi}^{(1/2)}$

**Maxima [A]** time = 1.05658, size = 26, normalized size = 1.37

$$-\frac{\sqrt{-x^2} \Gamma\left(-\frac{1}{2}, -x^2\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)/x^2,x, algorithm="maxima")`

[Out]  $-1/2 * \text{sqrt}(-x^2) * \text{gamma}(-1/2, -x^2) / x$

**Fricas [A]** time = 0.801298, size = 46, normalized size = 2.42

$$\frac{\sqrt{\pi x} \text{erfi}(x) - e^{(x^2)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)/x^2,x, algorithm="fricas")`

[Out]  $(\text{sqrt}(\text{pi}) * x * \text{erfi}(x) - e^{(x^2)}) / x$

**Sympy [A]** time = 0.744633, size = 14, normalized size = 0.74

$$\sqrt{\pi} \text{erfi}(x) - \frac{e^{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)/x**2,x)`

[Out]  $\text{sqrt}(\text{pi}) * \text{erfi}(x) - \exp(x**2) / x$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(x^2)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)/x^2, x)
```

$$3.735 \quad \int \frac{e^{x^2}(1+4x^4)}{x^2} dx$$

**Optimal.** Leaf size=19

$$2e^{x^2}x - \frac{e^{x^2}}{x}$$

[Out]  $-(E^{x^2}/x) + 2E^{x^2}x$

**Rubi [A]** time = 0.100237, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6742, 2214, 2204, 2212}

$$2e^{x^2}x - \frac{e^{x^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[(E^x^2\*(1 + 4\*x^4))/x^2,x]

[Out]  $-(E^{x^2}/x) + 2E^{x^2}x$

#### Rule 6742

Int[u\_, x\_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2214

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[((c + d\*x)^(m + 1)\*F^(a + b\*(c + d\*x)^n))/(d\*(m + 1)), x] - Dist[(b\*n\*Log[F])/(m + 1), Int[(c + d\*x)^(m + n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n, 0] && LeQ[-n, m + 1]))

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :=> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2212

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[((c + d\*x)^(m - n + 1)\*F^(a + b\*(c + d\*x)^n))/(b\*d\*n\*Log[F]), x] - Dist[(m - n + 1)/(b\*n\*Log[F]), Int[(c + d\*x)^(m - n)\*F^(a + b\*(c + d\*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2\*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

#### Rubi steps

$$\begin{aligned}\int \frac{e^{x^2}(1+4x^4)}{x^2} dx &= \int \left( \frac{e^{x^2}}{x^2} + 4e^{x^2}x^2 \right) dx \\ &= 4 \int e^{x^2}x^2 dx + \int \frac{e^{x^2}}{x^2} dx \\ &= -\frac{e^{x^2}}{x} + 2e^{x^2}x\end{aligned}$$

**Mathematica [A]** time = 0.0137588, size = 16, normalized size = 0.84

$$\frac{e^{x^2}(2x^2-1)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x^2\*(1 + 4\*x^4))/x^2,x]

[Out] (E^x^2\*(-1 + 2\*x^2))/x

**Maple [A]** time = 0.019, size = 16, normalized size = 0.8

$$\frac{e^{x^2}(2x^2-1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)\*(4\*x^4+1)/x^2,x)

[Out] exp(x^2)\*(2\*x^2-1)/x

**Maxima [C]** time = 1.05857, size = 49, normalized size = 2.58

$$2xe^{(x^2)} + i\sqrt{\pi}\operatorname{erf}(ix) - \frac{\sqrt{-x^2}\Gamma\left(-\frac{1}{2}, -x^2\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)\*(4\*x^4+1)/x^2,x, algorithm="maxima")

[Out] 2\*x\*e^(x^2) + I\*sqrt(pi)\*erf(I\*x) - 1/2\*sqrt(-x^2)\*gamma(-1/2, -x^2)/x

**Fricas [A]** time = 0.854392, size = 31, normalized size = 1.63

$$\frac{(2x^2-1)e^{(x^2)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(4*x^4+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x^2 - 1)*e^(x^2)/x
```

**Sympy [A]** time = 0.08812, size = 12, normalized size = 0.63

$$\frac{(2x^2 - 1)e^{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(4*x**4+1)/x**2,x)
```

```
[Out] (2*x**2 - 1)*exp(x**2)/x
```

**Giac [A]** time = 1.24531, size = 27, normalized size = 1.42

$$\frac{2x^2e^{(x^2)} - e^{(x^2)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(4*x^4+1)/x^2,x, algorithm="giac")
```

```
[Out] (2*x^2*e^(x^2) - e^(x^2))/x
```

### 3.736 $\int \sqrt{f^x}(a + bx)^2 dx$

**Optimal.** Leaf size=56

$$-\frac{8b\sqrt{f^x}(a + bx)}{\log^2(f)} + \frac{2\sqrt{f^x}(a + bx)^2}{\log(f)} + \frac{16b^2\sqrt{f^x}}{\log^3(f)}$$

[Out]  $(16*b^2*\text{Sqrt}[f^x])/Log[f]^3 - (8*b*\text{Sqrt}[f^x]*(a + b*x))/Log[f]^2 + (2*\text{Sqrt}[f^x]*(a + b*x)^2)/Log[f]$

**Rubi [A]** time = 0.0407702, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$-\frac{8b\sqrt{f^x}(a + bx)}{\log^2(f)} + \frac{2\sqrt{f^x}(a + bx)^2}{\log(f)} + \frac{16b^2\sqrt{f^x}}{\log^3(f)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f^x]*(a + b*x)^2, x]$

[Out]  $(16*b^2*\text{Sqrt}[f^x])/Log[f]^3 - (8*b*\text{Sqrt}[f^x]*(a + b*x))/Log[f]^2 + (2*\text{Sqrt}[f^x]*(a + b*x)^2)/Log[f]$

#### Rule 2176

$\text{Int}[(b_*)*(F_)^{((g_*)*((e_*) + (f_*)*(x_)))})^{(n_*)}*((c_*) + (d_*)*(x_))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n}/(f*g*n*Log[F]), x] - \text{Dist}[(d*m)/(f*g*n*Log[F]), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !$UseGamma == True$

#### Rule 2194

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))})^{(n_*)}, x\_Symbol] :> \text{Simp}[(F^{(c*(a + b*x)))^n}/(b*c*n*Log[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int \sqrt{f^x}(a + bx)^2 dx &= \frac{2\sqrt{f^x}(a + bx)^2}{\log(f)} - \frac{(4b) \int \sqrt{f^x}(a + bx) dx}{\log(f)} \\ &= -\frac{8b\sqrt{f^x}(a + bx)}{\log^2(f)} + \frac{2\sqrt{f^x}(a + bx)^2}{\log(f)} + \frac{(8b^2) \int \sqrt{f^x} dx}{\log^2(f)} \\ &= \frac{16b^2\sqrt{f^x}}{\log^3(f)} - \frac{8b\sqrt{f^x}(a + bx)}{\log^2(f)} + \frac{2\sqrt{f^x}(a + bx)^2}{\log(f)} \end{aligned}$$

**Mathematica [A]** time = 0.038849, size = 41, normalized size = 0.73

$$\frac{2\sqrt{f^x}(\log^2(f)(a + bx)^2 - 4b \log(f)(a + bx) + 8b^2)}{\log^3(f)}$$

Antiderivative was successfully verified.



[In] Integrate[Sqrt[f^x]\*(a + b\*x)^2,x]

[Out]  $(2\sqrt{f^x}*(8b^2 - 4b*(a + b*x)*\text{Log}[f] + (a + b*x)^2*\text{Log}[f]^2))/\text{Log}[f]^3$

**Maple [A]** time = 0.043, size = 60, normalized size = 1.1

$$\frac{(b^2x^2(\ln(f))^2 + 2(\ln(f))^2abx + (\ln(f))^2a^2 - 4\ln(f)b^2x - 4\ln(f)ba + 8b^2)\sqrt{f^x}}{(\ln(f))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*(f^x)^(1/2),x)

[Out]  $2*(b^2*x^2*\ln(f)^2 + 2*\ln(f)^2*a*b*x + \ln(f)^2*a^2 - 4*\ln(f)*b^2*x - 4*\ln(f)*b*a + 8*b^2)*(f^x)^(1/2)/\ln(f)^3$

**Maxima [A]** time = 1.00998, size = 85, normalized size = 1.52

$$\frac{4(x \log(f) - 2)ab\sqrt{f^x}}{\log(f)^2} + \frac{2a^2\sqrt{f^x}}{\log(f)} + \frac{2(x^2 \log(f)^2 - 4x \log(f) + 8)b^2\sqrt{f^x}}{\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(f^x)^(1/2),x, algorithm="maxima")

[Out]  $4*(x*\log(f) - 2)*a*b*\text{sqrt}(f^x)/\log(f)^2 + 2*a^2*\text{sqrt}(f^x)/\log(f) + 2*(x^2*\log(f)^2 - 4*x*\log(f) + 8)*b^2*\text{sqrt}(f^x)/\log(f)^3$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(f^x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0.147631, size = 94, normalized size = 1.68

$$\begin{cases} \frac{(2a^2 \log(f)^2 + 4abx \log(f)^2 - 8ab \log(f) + 2b^2x^2 \log(f)^2 - 8b^2x \log(f) + 16b^2)\sqrt{f^x}}{\log(f)^3} & \text{for } \log(f)^3 \neq 0 \\ a^2x + abx^2 + \frac{b^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(f**x)**(1/2),x)
```

```
[Out] Piecewise(((2*a**2*log(f)**2 + 4*a*b*x*log(f)**2 - 8*a*b*log(f) + 2*b**2*x*
*2*log(f)**2 - 8*b**2*x*log(f) + 16*b**2)*sqrt(f**x)/log(f)**3, Ne(log(f)**
3, 0)), (a**2*x + a*b*x**2 + b**2*x**3/3, True))
```

**Giac [B]** time = 1.30601, size = 1910, normalized size = 34.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f^x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*((2*(pi*b^2*x^2*log(abs(f))*sgn(f) - pi*b^2*x^2*log(abs(f)) + 2*pi*a*b*x
*log(abs(f))*sgn(f) - 2*pi*a*b*x*log(abs(f)) - 2*pi*b^2*x*sgn(f) + pi*a^2*log
(abs(f))*sgn(f) + 2*pi*b^2*x - pi*a^2*log(abs(f)) - 2*pi*a*b*sgn(f) + 2*pi
i*a*b)*(pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f))^2
)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f))^2)^2
+ (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3)^2) - (
pi^2*b^2*x^2*sgn(f) - pi^2*b^2*x^2 + 2*b^2*x^2*log(abs(f))^2 + 2*pi^2*a*b*x
*sgn(f) - 2*pi^2*a*b*x + 4*a*b*x*log(abs(f))^2 + pi^2*a^2*sgn(f) - pi^2*a^2
- 8*b^2*x*log(abs(f)) + 2*a^2*log(abs(f))^2 - 8*a*b*log(abs(f)) + 16*b^2)*
(3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3)/((pi^3*sg
n(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f))^2)^2 + (3*pi^2*
log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3)^2))*cos(-1/4*pi*
x*sgn(f) + 1/4*pi*x) - ((pi^2*b^2*x^2*sgn(f) - pi^2*b^2*x^2 + 2*b^2*x^2*log
(abs(f))^2 + 2*pi^2*a*b*x*sgn(f) - 2*pi^2*a*b*x + 4*a*b*x*log(abs(f))^2 + p
i^2*a^2*sgn(f) - pi^2*a^2 - 8*b^2*x*log(abs(f)) + 2*a^2*log(abs(f))^2 - 8*a
*b*log(abs(f)) + 16*b^2)*(pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 +
3*pi*log(abs(f))^2)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi
*log(abs(f))^2)^2 + (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log
(abs(f))^3)^2) + 2*(pi*b^2*x^2*log(abs(f))*sgn(f) - pi*b^2*x^2*log(abs(f))
+ 2*pi*a*b*x*log(abs(f))*sgn(f) - 2*pi*a*b*x*log(abs(f)) - 2*pi*b^2*x*sgn(f
) + pi*a^2*log(abs(f))*sgn(f) + 2*pi*b^2*x - pi*a^2*log(abs(f)) - 2*pi*a*b*
sgn(f) + 2*pi*a*b)*(3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(
abs(f))^3)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(
f))^2)^2 + (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^
3)^2))*sin(-1/4*pi*x*sgn(f) + 1/4*pi*x)*abs(f)^(1/2*x) + abs(f)^(1/2*x)*((
pi^2*b^2*i*x^2*sgn(f) - pi^2*b^2*i*x^2 + 2*b^2*i*x^2*log(abs(f))^2 + 2*pi^2
*a*b*i*x*sgn(f) - 2*pi*b^2*x^2*log(abs(f))*sgn(f) - 2*pi^2*a*b*i*x + 2*pi*b
^2*x^2*log(abs(f)) + 4*a*b*i*x*log(abs(f))^2 + pi^2*a^2*i*sgn(f) - 4*pi*a*b
*x*log(abs(f))*sgn(f) - pi^2*a^2*i + 4*pi*a*b*x*log(abs(f)) - 8*b^2*i*x*log
(abs(f)) + 2*a^2*i*log(abs(f))^2 + 4*pi*b^2*x*sgn(f) - 2*pi*a^2*log(abs(f))
*sgn(f) - 4*pi*b^2*x + 2*pi*a^2*log(abs(f)) - 8*a*b*i*log(abs(f)) + 4*pi*a*
b*sgn(f) - 4*pi*a*b + 16*b^2*i)*e^(1/4*pi*i*x*(sgn(f) - 1))/(pi^3*i*sgn(f)
- 3*pi*i*log(abs(f))^2*sgn(f) - pi^3*i + 3*pi*i*log(abs(f))^2 - 3*pi^2*log(
abs(f))*sgn(f) + 3*pi^2*log(abs(f)) - 2*log(abs(f))^3) + (pi^2*b^2*i*x^2*sg
n(f) - pi^2*b^2*i*x^2 + 2*b^2*i*x^2*log(abs(f))^2 + 2*pi^2*a*b*i*x*sgn(f) +
2*pi*b^2*x^2*log(abs(f))*sgn(f) - 2*pi^2*a*b*i*x - 2*pi*b^2*x^2*log(abs(f)
) + 4*a*b*i*x*log(abs(f))^2 + pi^2*a^2*i*sgn(f) + 4*pi*a*b*x*log(abs(f))*sg
n(f) - pi^2*a^2*i - 4*pi*a*b*x*log(abs(f)) - 8*b^2*i*x*log(abs(f)) + 2*a^2*
i*log(abs(f))^2 - 4*pi*b^2*x*sgn(f) + 2*pi*a^2*log(abs(f))*sgn(f) + 4*pi*b^
2*x - 2*pi*a^2*log(abs(f)) - 8*a*b*i*log(abs(f)) - 4*pi*a*b*sgn(f) + 4*pi*a
*b + 16*b^2*i)*e^(-1/4*pi*i*x*(sgn(f) - 1))/(pi^3*i*sgn(f) - 3*pi*i*log(abs
(f))^2*sgn(f) - pi^3*i + 3*pi*i*log(abs(f))^2 + 3*pi^2*log(abs(f))*sgn(f) -
```

$$3\pi^2 \log(\text{abs}(f)) + 2\log(\text{abs}(f))^3)/i$$

$$3.737 \quad \int 3^{1+x^2} x dx$$

**Optimal.** Leaf size=15

$$\frac{3^{x^2+1}}{2\log(3)}$$

[Out]  $3^{(1 + x^2)}/(2*\text{Log}[3])$

**Rubi [A]** time = 0.0094084, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2209}

$$\frac{3^{x^2+1}}{2\log(3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[3^{(1 + x^2)}*x, x]$

[Out]  $3^{(1 + x^2)}/(2*\text{Log}[3])$

**Rule 2209**

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^n * F^{(a + b*(c + d*x)^n)} / (b*f*n*(c + d*x)^n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int 3^{1+x^2} x dx = \frac{3^{1+x^2}}{2\log(3)}$$

**Mathematica [A]** time = 0.0020117, size = 12, normalized size = 0.8

$$\frac{3^{x^2+1}}{\log(9)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[3^{(1 + x^2)}*x, x]$

[Out]  $3^{(1 + x^2)}/\text{Log}[9]$

**Maple [A]** time = 0.024, size = 14, normalized size = 0.9

$$\frac{3^{x^2+1}}{2 \ln(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(3^(x^2+1)*x,x)`

[Out]  $1/2*3^{(x^2+1)}/\ln(3)$

**Maxima [A]** time = 0.96544, size = 18, normalized size = 1.2

$$\frac{3^{x^2+1}}{2 \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3^(x^2+1)*x,x, algorithm="maxima")`

[Out]  $1/2*3^{(x^2 + 1)}/\log(3)$

**Fricas [A]** time = 0.761449, size = 32, normalized size = 2.13

$$\frac{3^{x^2+1}}{2 \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3^(x^2+1)*x,x, algorithm="fricas")`

[Out]  $1/2*3^{(x^2 + 1)}/\log(3)$

**Sympy [A]** time = 0.093969, size = 10, normalized size = 0.67

$$\frac{3^{x^2+1}}{2 \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3**(x**2+1)*x,x)`

[Out]  $3^{(x**2 + 1)}/(2*\log(3))$

**Giac [A]** time = 1.19112, size = 18, normalized size = 1.2

$$\frac{3^{x^2+1}}{2 \log(3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3^(x^2+1)*x,x, algorithm="giac")`

[Out]  $1/2*3^{(x^2 + 1)}/\log(3)$

$$3.738 \quad \int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

[Out]  $2^{(1 + \text{Sqrt}[x])}/\text{Log}[2]$

**Rubi [A]** time = 0.0105539, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{\text{Sqrt}[x]}/\text{Sqrt}[x], x]$

[Out]  $2^{(1 + \text{Sqrt}[x])}/\text{Log}[2]$

#### Rule 2209

$\text{Int}[(F_)^{\text{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[\frac{(e + f*x)^n * F^{(a + b*(c + d*x)^n)}}{(b*f*n*(c + d*x)^n * \text{Log}[F])}, x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rubi steps

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

**Mathematica [A]** time = 0.002815, size = 14, normalized size = 1.

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[2^{\text{Sqrt}[x]}/\text{Sqrt}[x], x]$

[Out]  $2^{(1 + \text{Sqrt}[x])}/\text{Log}[2]$

**Maple [A]** time = 0.018, size = 12, normalized size = 0.9

$$2 \frac{2^{\sqrt{x}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2^(x^(1/2))/x^(1/2),x)`

[Out]  $2/\ln(2)*2^{(x^{1/2})}$

**Maxima [A]** time = 0.963074, size = 16, normalized size = 1.14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")`

[Out]  $2^{(\text{sqrt}(x) + 1)}/\log(2)$

**Fricas [A]** time = 0.648334, size = 27, normalized size = 1.93

$$\frac{2 \cdot 2^{(\sqrt{x})}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")`

[Out]  $2*2^{\text{sqrt}(x)}/\log(2)$

**Sympy [A]** time = 0.138863, size = 10, normalized size = 0.71

$$\frac{2 \cdot 2^{\sqrt{x}}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2**(x**(1/2))/x**(1/2),x)`

[Out]  $2*2^{(\text{sqrt}(x))}/\log(2)$

**Giac [A]** time = 1.22954, size = 15, normalized size = 1.07

$$\frac{2 \cdot 2^{(\sqrt{x})}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")`

[Out]  $2*2^{\text{sqrt}(x)}/\log(2)$

$$3.739 \quad \int \frac{2^{\frac{1}{x}}}{x^2} dx$$

**Optimal.** Leaf size=11

$$-\frac{2^{\frac{1}{x}}}{\log(2)}$$

[Out]  $-(2^x)^{-1}/\text{Log}[2]$

**Rubi [A]** time = 0.0109216, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2209}

$$-\frac{2^{\frac{1}{x}}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^x^{-1}/x^2, x]$

[Out]  $-(2^x)^{-1}/\text{Log}[2]$

**Rule 2209**

$\text{Int}[(F_)^{\wedge}((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{\wedge}(n_))*((e_.) + (f_.)*(x_))^{\wedge}(m_.), x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^{\wedge}n * F^{\wedge}(a + b*(c + d*x)^{\wedge}n) / (b*f*n*(c + d*x)^{\wedge}n * \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{2^{\frac{1}{x}}}{x^2} dx = -\frac{2^{\frac{1}{x}}}{\log(2)}$$

**Mathematica [A]** time = 0.0015678, size = 11, normalized size = 1.

$$-\frac{2^{\frac{1}{x}}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[2^x^{-1}/x^2, x]$

[Out]  $-(2^x)^{-1}/\text{Log}[2]$

**Maple [A]** time = 0.022, size = 12, normalized size = 1.1

$$-\frac{\sqrt[3]{2}}{\ln(2)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2^(1/x)/x^2,x)`

[Out]  $-2^{(1/x)}/\ln(2)$

**Maxima [A]** time = 0.97364, size = 15, normalized size = 1.36

$$-\frac{2^{\left(\frac{1}{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(1/x)/x^2,x, algorithm="maxima")`

[Out]  $-2^{(1/x)}/\log(2)$

**Fricas [A]** time = 0.721701, size = 23, normalized size = 2.09

$$-\frac{2^{\left(\frac{1}{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(1/x)/x^2,x, algorithm="fricas")`

[Out]  $-2^{(1/x)}/\log(2)$

**Sympy [A]** time = 0.098418, size = 8, normalized size = 0.73

$$-\frac{2^{\frac{1}{x}}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2**(1/x)/x**2,x)`

[Out]  $-2^{(1/x)}/\log(2)$

**Giac [A]** time = 1.29599, size = 15, normalized size = 1.36

$$-\frac{2^{\left(\frac{1}{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2^(1/x)/x^2,x, algorithm="giac")`

[Out]  $-2^{(1/x)}/\log(2)$

### 3.740 $\int (2^{-x} + 2^x) dx$

**Optimal.** Leaf size=20

$$\frac{2^x}{\log(2)} - \frac{2^{-x}}{\log(2)}$$

[Out]  $-(1/(2^x \cdot \text{Log}[2])) + 2^x/\text{Log}[2]$

**Rubi [A]** time = 0.0062042, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2194}

$$\frac{2^x}{\log(2)} - \frac{2^{-x}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[2^{(-x)} + 2^x, x]$

[Out]  $-(1/(2^x \cdot \text{Log}[2])) + 2^x/\text{Log}[2]$

#### Rule 2194

$\text{Int}[(F_)^{((c_.) * ((a_.) + (b_.) * (x_)))}^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c * (a + b * x))})^n / (b * c * n * \text{Log}[F]), x] / ; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int (2^{-x} + 2^x) dx &= \int 2^{-x} dx + \int 2^x dx \\ &= -\frac{2^{-x}}{\log(2)} + \frac{2^x}{\log(2)} \end{aligned}$$

**Mathematica [A]** time = 0.0031962, size = 20, normalized size = 1.

$$\frac{2^x}{\log(2)} - \frac{2^{-x}}{\log(2)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[2^{(-x)} + 2^x, x]$

[Out]  $-(1/(2^x \cdot \text{Log}[2])) + 2^x/\text{Log}[2]$

**Maple [A]** time = 0.018, size = 21, normalized size = 1.1

$$-\frac{1}{2^x \ln(2)} + \frac{2^x}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2^x)+2^x,x)`

[Out]  $-1/(2^x)/\ln(2)+2^x/\ln(2)$

**Maxima [A]** time = 0.979951, size = 27, normalized size = 1.35

$$\frac{2^x}{\log(2)} - \frac{1}{2^x \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2^x)+2^x,x, algorithm="maxima")`

[Out]  $2^x/\log(2) - 1/(2^x*\log(2))$

**Fricas [A]** time = 0.795417, size = 38, normalized size = 1.9

$$\frac{2^{2x} - 1}{2^x \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2^x)+2^x,x, algorithm="fricas")`

[Out]  $(2^{(2*x)} - 1)/(2^x*\log(2))$

**Sympy [A]** time = 0.113027, size = 17, normalized size = 0.85

$$\frac{2^x \log(2) - 2^{-x} \log(2)}{\log(2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2**x)+2**x,x)`

[Out]  $(2**x*\log(2) - 2**(-x)*\log(2))/\log(2)**2$

**Giac [A]** time = 1.20912, size = 27, normalized size = 1.35

$$\frac{2^x}{\log(2)} - \frac{1}{2^x \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2^x)+2^x,x, algorithm="giac")`

[Out]  $2^x/\log(2) - 1/(2^x*\log(2))$

### 3.741 $\int e^{-4x} (2 - 3x + x^2) dx$

**Optimal.** Leaf size=32

$$-\frac{1}{4}e^{-4x}x^2 + \frac{5}{8}e^{-4x}x - \frac{11e^{-4x}}{32}$$

[Out]  $-11/(32*E^{(4*x)}) + (5*x)/(8*E^{(4*x)}) - x^2/(4*E^{(4*x)})$

**Rubi [A]** time = 0.0439118, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2196, 2194, 2176}

$$-\frac{1}{4}e^{-4x}x^2 + \frac{5}{8}e^{-4x}x - \frac{11e^{-4x}}{32}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3\*x + x^2)/E^(4\*x), x]

[Out]  $-11/(32*E^{(4*x)}) + (5*x)/(8*E^{(4*x)}) - x^2/(4*E^{(4*x)})$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rubi steps

$$\begin{aligned} \int e^{-4x} (2 - 3x + x^2) dx &= \int (2e^{-4x} - 3e^{-4x}x + e^{-4x}x^2) dx \\ &= 2 \int e^{-4x} dx - 3 \int e^{-4x}x dx + \int e^{-4x}x^2 dx \\ &= -\frac{1}{2}e^{-4x} + \frac{3}{4}e^{-4x}x - \frac{1}{4}e^{-4x}x^2 + \frac{1}{2} \int e^{-4x}x dx - \frac{3}{4} \int e^{-4x} dx \\ &= -\frac{5}{16}e^{-4x} + \frac{5}{8}e^{-4x}x - \frac{1}{4}e^{-4x}x^2 + \frac{1}{8} \int e^{-4x} dx \\ &= -\frac{11}{32}e^{-4x} + \frac{5}{8}e^{-4x}x - \frac{1}{4}e^{-4x}x^2 \end{aligned}$$

**Mathematica [A]** time = 0.0194691, size = 19, normalized size = 0.59

$$-\frac{1}{32}e^{-4x}(8x^2 - 20x + 11)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3\*x + x^2)/E^(4\*x), x]

[Out] -(11 - 20\*x + 8\*x^2)/(32\*E^(4\*x))

---

**Maple [A]** time = 0.019, size = 19, normalized size = 0.6

$$-\frac{8x^2 - 20x + 11}{32e^{4x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-3\*x+2)/exp(4\*x), x)

[Out] -1/32\*(8\*x^2-20\*x+11)/exp(4\*x)

---

**Maxima [A]** time = 1.00812, size = 46, normalized size = 1.44

$$-\frac{1}{32}(8x^2 + 4x + 1)e^{(-4x)} + \frac{3}{16}(4x + 1)e^{(-4x)} - \frac{1}{2}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3\*x+2)/exp(4\*x), x, algorithm="maxima")

[Out] -1/32\*(8\*x^2 + 4\*x + 1)\*e^(-4\*x) + 3/16\*(4\*x + 1)\*e^(-4\*x) - 1/2\*e^(-4\*x)

---

**Fricas [A]** time = 0.701215, size = 49, normalized size = 1.53

$$-\frac{1}{32}(8x^2 - 20x + 11)e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3\*x+2)/exp(4\*x), x, algorithm="fricas")

[Out] -1/32\*(8\*x^2 - 20\*x + 11)\*e^(-4\*x)

---

**Sympy [A]** time = 0.09161, size = 15, normalized size = 0.47

$$\frac{(-8x^2 + 20x - 11)e^{-4x}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)/exp(4*x),x)
```

```
[Out] (-8*x**2 + 20*x - 11)*exp(-4*x)/32
```

---

**Giac [A]** time = 1.2642, size = 22, normalized size = 0.69

$$-\frac{1}{32}(8x^2 - 20x + 11)e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/exp(4*x),x, algorithm="giac")
```

```
[Out] -1/32*(8*x^2 - 20*x + 11)*e^(-4*x)
```

$$3.742 \quad \int \left( k^{x/2} + x^{\sqrt{k}} \right) dx$$

**Optimal.** Leaf size=33

$$\frac{2k^{x/2}}{\log(k)} + \frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}$$

[Out]  $x^{(1 + \text{Sqrt}[k])}/(1 + \text{Sqrt}[k]) + (2*k^{(x/2)})/\text{Log}[k]$

**Rubi [A]** time = 0.0090667, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2194}

$$\frac{2k^{x/2}}{\log(k)} + \frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}$$

Antiderivative was successfully verified.

[In] Int[k^(x/2) + x^Sqrt[k], x]

[Out]  $x^{(1 + \text{Sqrt}[k])}/(1 + \text{Sqrt}[k]) + (2*k^{(x/2)})/\text{Log}[k]$

**Rule 2194**

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

**Rubi steps**

$$\begin{aligned} \int \left( k^{x/2} + x^{\sqrt{k}} \right) dx &= \frac{x^{1+\sqrt{k}}}{1+\sqrt{k}} + \int k^{x/2} dx \\ &= \frac{x^{1+\sqrt{k}}}{1+\sqrt{k}} + \frac{2k^{x/2}}{\log(k)} \end{aligned}$$

**Mathematica [A]** time = 0.0160845, size = 33, normalized size = 1.

$$\frac{2k^{x/2}}{\log(k)} + \frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}$$

Antiderivative was successfully verified.

[In] Integrate[k^(x/2) + x^Sqrt[k], x]

[Out]  $x^{(1 + \text{Sqrt}[k])}/(1 + \text{Sqrt}[k]) + (2*k^{(x/2)})/\text{Log}[k]$

**Maple [A]** time = 0.037, size = 28, normalized size = 0.9

$$2 \frac{k^{x/2}}{\ln(k)} + x^{1+\sqrt{k}} (1 + \sqrt{k})^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(k^(1/2*x)+x^(k^(1/2)),x)`

[Out]  $2*k^{(1/2*x)}/\ln(k)+x^{(1+k^{(1/2)})}/(1+k^{(1/2)})$

**Maxima [A]** time = 0.976703, size = 36, normalized size = 1.09

$$\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1} + \frac{2k^{\frac{1}{2}x}}{\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="maxima")`

[Out]  $x^{(\text{sqrt}(k) + 1)}/(\text{sqrt}(k) + 1) + 2*k^{(1/2*x)}/\log(k)$

**Fricas [A]** time = 0.792184, size = 111, normalized size = 3.36

$$\frac{2(k-1)k^{\frac{1}{2}x} + (\sqrt{k}x \log(k) - x \log(k))x^{(\sqrt{k})}}{(k-1)\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="fricas")`

[Out]  $(2*(k - 1)*k^{(1/2*x)} + (\text{sqrt}(k)*x*\log(k) - x*\log(k))*x^{\text{sqrt}(k)})/((k - 1)*\log(k))$

**Sympy [A]** time = 0.101513, size = 36, normalized size = 1.09

$$\begin{cases} \frac{2k^{\frac{x}{2}}}{\log(k)} & \text{for } \log(k) \neq 0 \\ x & \text{otherwise} \end{cases} + \begin{cases} \frac{x^{\sqrt{k}+1}}{\sqrt{k}+1} & \text{for } \sqrt{k} \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(k**(1/2*x)+x**(k**(1/2)),x)`

[Out] `Piecewise((2*k**(x/2)/log(k), Ne(log(k), 0)), (x, True)) + Piecewise((x**(sqrt(k) + 1)/(sqrt(k) + 1), Ne(sqrt(k), -1)), (log(x), True))`

**Giac [A]** time = 1.27404, size = 36, normalized size = 1.09

$$\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1} + \frac{2k^{\frac{1}{2}x}}{\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="giac")
```

```
[Out] x^(sqrt(k) + 1)/(sqrt(k) + 1) + 2*k^(1/2*x)/log(k)
```

$$3.743 \quad \int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=21

$$\frac{2^{\sqrt{x}+1}5^{\sqrt{x}}}{\log(10)}$$

[Out] (2^(1 + Sqrt[x])\*5^Sqrt[x])/Log[10]

**Rubi [A]** time = 0.0114011, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2209}

$$\frac{2^{\sqrt{x}+1}5^{\sqrt{x}}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Int[10^Sqrt[x]/Sqrt[x],x]

[Out] (2^(1 + Sqrt[x])\*5^Sqrt[x])/Log[10]

**Rule 2209**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^n\*F^(a + b\*(c + d\*x)^n))/(b\*f\*n\*(c + d\*x)^n\*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d\*e - c\*f, 0]

**Rubi steps**

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx = \frac{2^{1+\sqrt{x}}5^{\sqrt{x}}}{\log(10)}$$

**Mathematica [A]** time = 0.0028632, size = 21, normalized size = 1.

$$\frac{2^{\sqrt{x}+1}5^{\sqrt{x}}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Integrate[10^Sqrt[x]/Sqrt[x],x]

[Out] (2^(1 + Sqrt[x])\*5^Sqrt[x])/Log[10]

**Maple [A]** time = 0.019, size = 12, normalized size = 0.6

$$2 \frac{10^{\sqrt{x}}}{\ln(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(10^(x^(1/2))/x^(1/2),x)`

[Out] `2/ln(10)*10^(x^(1/2))`

**Maxima [A]** time = 0.949181, size = 15, normalized size = 0.71

$$\frac{2 \cdot 10^{(\sqrt{x})}}{\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10^(x^(1/2))/x^(1/2),x, algorithm="maxima")`

[Out] `2*10^sqrt(x)/log(10)`

**Fricas [A]** time = 0.883467, size = 30, normalized size = 1.43

$$\frac{2 \cdot 10^{(\sqrt{x})}}{\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10^(x^(1/2))/x^(1/2),x, algorithm="fricas")`

[Out] `2*10^sqrt(x)/log(10)`

**Sympy [A]** time = 0.139031, size = 10, normalized size = 0.48

$$\frac{2 \cdot 10^{\sqrt{x}}}{\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10**(x**(1/2))/x**(1/2),x)`

[Out] `2*10**(sqrt(x))/log(10)`

**Giac [A]** time = 1.21473, size = 15, normalized size = 0.71

$$\frac{2 \cdot 10^{(\sqrt{x})}}{\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10^(x^(1/2))/x^(1/2),x, algorithm="giac")`

[Out] `2*10^sqrt(x)/log(10)`

$$3.744 \quad \int \left( \frac{1}{\sqrt{e^x+x}} + \frac{e^x}{\sqrt{e^x+x}} \right) dx$$

**Optimal.** Leaf size=11

$$2\sqrt{x+e^x}$$

[Out] 2\*Sqrt[E^x + x]

**Rubi [A]** time = 0.0408029, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {2261}

$$2\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[E^x + x] + E^x/Sqrt[E^x + x], x]

[Out] 2\*Sqrt[E^x + x]

**Rule 2261**

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) \* ((b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) + (a\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int \left( \frac{1}{\sqrt{e^x+x}} + \frac{e^x}{\sqrt{e^x+x}} \right) dx &= \int \frac{1}{\sqrt{e^x+x}} dx + \int \frac{e^x}{\sqrt{e^x+x}} dx \\ &= 2\sqrt{e^x+x} \end{aligned}$$

**Mathematica [A]** time = 0.0047456, size = 11, normalized size = 1.

$$2\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[E^x + x] + E^x/Sqrt[E^x + x], x]

[Out] 2\*Sqrt[E^x + x]

**Maple [A]** time = 0.042, size = 9, normalized size = 0.8

$$2\sqrt{e^x+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(exp(x)+x)^(1/2)+1/(exp(x)+x)^(1/2),x)`

[Out] `2*(exp(x)+x)^(1/2)`

**Maxima [A]** time = 1.09435, size = 11, normalized size = 1.

$$2\sqrt{x+e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(x+exp(x))^(1/2)+1/(x+exp(x))^(1/2),x, algorithm="maxima")`

[Out] `2*sqrt(x + e^x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(x+exp(x))^(1/2)+1/(x+exp(x))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x + 1}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(exp(x)+x)**(1/2)+1/(exp(x)+x)**(1/2),x)`

[Out] `Integral((exp(x) + 1)/sqrt(x + exp(x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\sqrt{x+e^x}} + \frac{1}{\sqrt{x+e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(x+exp(x))^(1/2)+1/(x+exp(x))^(1/2),x, algorithm="giac")`

[Out] `integrate(e^x/sqrt(x + e^x) + 1/sqrt(x + e^x), x)`

$$3.745 \quad \int \left( \frac{(1+e^x)x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx$$

**Optimal.** Leaf size=12

$$2x\sqrt{x+e^x}$$

[Out] 2\*x\*Sqrt[E^x + x]

**Rubi [A]** time = 0.258604, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {6742, 2273, 2262}

$$2x\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Int[((1 + E^x)\*x)/Sqrt[E^x + x] + 2\*Sqrt[E^x + x], x]

[Out] 2\*x\*Sqrt[E^x + x]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2273

Int[(x\_)^(m\_)\*(E^(x\_) + (x\_)^(m\_))^(n\_), x\_Symbol] := -Simp[(E^x + x^m)^(n+1)/(n+1), x] + (Dist[m, Int[x^(m-1)\*(E^x + x^m)^n, x], x] + Int[(E^x + x^m)^(n+1), x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[n, -1]

#### Rule 2262

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*(x\_)^(m\_)\*((b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))) + (a\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^m\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p+1))/(b\*d\*e\*(p+1)\*Log[F]), x] + (-Dist[m/(b\*d\*e\*(p+1)\*Log[F]), Int[x^(m-1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p+1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m+n-1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \left( \frac{(1+e^x)x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx &= 2 \int \sqrt{e^x+x} dx + \int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx \\ &= 2 \int \sqrt{e^x+x} dx + \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx \\ &= 2 \int \sqrt{e^x+x} dx + \int \frac{x}{\sqrt{e^x+x}} dx + \int \frac{e^x x}{\sqrt{e^x+x}} dx \\ &= -2\sqrt{e^x+x} + 2x\sqrt{e^x+x} + \int \frac{1}{\sqrt{e^x+x}} dx - \int \frac{x}{\sqrt{e^x+x}} dx + \int \sqrt{e^x+x} dx \\ &= 2x\sqrt{e^x+x} \end{aligned}$$

**Mathematica [A]** time = 0.0923953, size = 12, normalized size = 1.

$$2x\sqrt{x + e^x}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + E^x)\*x)/Sqrt[E^x + x] + 2\*Sqrt[E^x + x], x]

[Out] 2\*x\*Sqrt[E^x + x]

**Maple [A]** time = 0.048, size = 10, normalized size = 0.8

$$2x\sqrt{e^x + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+exp(x))\*x/(exp(x)+x)^(1/2)+2\*(exp(x)+x)^(1/2), x)

[Out] 2\*x\*(exp(x)+x)^(1/2)

**Maxima [A]** time = 1.13824, size = 22, normalized size = 1.83

$$\frac{2(x^2 + xe^x)}{\sqrt{x + e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))\*x/(x+exp(x))^(1/2)+2\*(x+exp(x))^(1/2), x, algorithm="maxima")

[Out] 2\*(x^2 + x\*e^x)/sqrt(x + e^x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))\*x/(x+exp(x))^(1/2)+2\*(x+exp(x))^(1/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x + 3x + 2e^x}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))\*x/(exp(x)+x)\*\*(1/2)+2\*(exp(x)+x)\*\*(1/2),x)

[Out] Integral((x\*exp(x) + 3\*x + 2\*exp(x))/sqrt(x + exp(x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e^x + 1)}{\sqrt{x + e^x}} + 2\sqrt{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))\*x/(x+exp(x))^(1/2)+2\*(x+exp(x))^(1/2),x, algorithm="giac")

[Out] integrate(x\*(e^x + 1)/sqrt(x + e^x) + 2\*sqrt(x + e^x), x)



$$3.746 \quad \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx$$

**Optimal.** Leaf size=12

$$2x\sqrt{x+e^x}$$

[Out] 2\*x\*Sqrt[E^x + x]

**Rubi [A]** time = 0.130266, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$ , Rules used = {2273, 2262}

$$2x\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[E^x + x] + (E^x\*x)/Sqrt[E^x + x] + 2\*Sqrt[E^x + x],x]

[Out] 2\*x\*Sqrt[E^x + x]

Rule 2273

Int[(x\_)^(m\_)\*(E^(x\_) + (x\_)^(m\_))^(n\_), x\_Symbol] :> -Simp[(E^x + x^m)^(n+1)/(n+1), x] + (Dist[m, Int[x^(m-1)\*(E^x + x^m)^n, x], x] + Int[(E^x + x^m)^(n+1), x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[n, -1]

Rule 2262

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*(x\_)^(m\_)\*((b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))) + (a\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^m\*(a\*x^n + b\*F^(e\*(c+d\*x)))^(p+1))/(b\*d\*e\*(p+1)\*Log[F], x] + (-Dist[m/(b\*d\*e\*(p+1)\*Log[F]), Int[x^(m-1)\*(a\*x^n + b\*F^(e\*(c+d\*x)))^(p+1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m+n-1)\*(a\*x^n + b\*F^(e\*(c+d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} + 2\sqrt{e^x+x} \right) dx &= 2 \int \sqrt{e^x+x} dx + \int \frac{x}{\sqrt{e^x+x}} dx + \int \frac{e^x x}{\sqrt{e^x+x}} dx \\ &= -2\sqrt{e^x+x} + 2x\sqrt{e^x+x} + \int \frac{1}{\sqrt{e^x+x}} dx - \int \frac{x}{\sqrt{e^x+x}} dx + \int \sqrt{e^x+x} dx \\ &= 2x\sqrt{e^x+x} \end{aligned}$$

**Mathematica [A]** time = 0.0446522, size = 12, normalized size = 1.

$$2x\sqrt{x+e^x}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[E^x + x] + (E^x\*x)/Sqrt[E^x + x] + 2\*Sqrt[E^x + x],x]

[Out]  $2x\sqrt{e^x + x}$

**Maple [A]** time = 0.039, size = 10, normalized size = 0.8

$$2x\sqrt{e^x + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x)`

[Out]  $2x*(\exp(x)+x)^{(1/2)}$

**Maxima [A]** time = 1.08326, size = 12, normalized size = 1.

$$2\sqrt{x + e^x}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2)+2*(x+exp(x))^(1/2),x, algorithm="maxima")`

[Out]  $2*\text{sqrt}(x + e^x)*x$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2)+2*(x+exp(x))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x + 3x + 2e^x}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(exp(x)+x)**(1/2)+exp(x)*x/(exp(x)+x)**(1/2)+2*(exp(x)+x)**(1/2),x)`

[Out] `Integral((x*exp(x) + 3*x + 2*exp(x))/sqrt(x + exp(x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x+e^x}} + 2\sqrt{x+e^x} + \frac{x}{\sqrt{x+e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2)+2*(x+exp(x))^(1/2), x  
, algorithm="giac")
```

```
[Out] integrate(x*e^x/sqrt(x + e^x) + 2*sqrt(x + e^x) + x/sqrt(x + e^x), x)
```

$$3.747 \quad \int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx$$

**Optimal.** Leaf size=26

$$2x\sqrt{x+e^x} - 2\text{CannotIntegrate}(\sqrt{x+e^x}, x)$$

[Out]  $2*x*\text{Sqrt}[E^x + x] - 2*\text{CannotIntegrate}[\text{Sqrt}[E^x + x], x]$

**Rubi [A]** time = 0.18311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[(1 + E^x)*x/\text{Sqrt}[E^x + x], x]$

[Out]  $2*x*\text{Sqrt}[E^x + x] - 2*\text{Defer}[\text{Int}[\text{Sqrt}[E^x + x], x]$

Rubi steps

$$\begin{aligned} \int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx &= \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx \\ &= \int \frac{x}{\sqrt{e^x+x}} dx + \int \frac{e^x x}{\sqrt{e^x+x}} dx \\ &= -2\sqrt{e^x+x} + 2x\sqrt{e^x+x} - 2 \int \sqrt{e^x+x} dx + \int \frac{1}{\sqrt{e^x+x}} dx - \int \frac{x}{\sqrt{e^x+x}} dx + \int \sqrt{e^x+x} dx \\ &= 2x\sqrt{e^x+x} - 2 \int \sqrt{e^x+x} dx \end{aligned}$$

**Mathematica [A]** time = 0.121931, size = 0, normalized size = 0.

$$\int \frac{(1+e^x)x}{\sqrt{e^x+x}} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[(1 + E^x)*x/\text{Sqrt}[E^x + x], x]$

[Out]  $\text{Integrate}[(1 + E^x)*x/\text{Sqrt}[E^x + x], x]$

**Maple [A]** time = 0.033, size = 0, normalized size = 0.

$$\int (1+e^x)x \frac{1}{\sqrt{e^x+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+exp(x))*x/(exp(x)+x)^(1/2),x)`

[Out] `int((1+exp(x))*x/(exp(x)+x)^(1/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e^x + 1)}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+exp(x))*x/(x+exp(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x*(e^x + 1)/sqrt(x + e^x), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+exp(x))*x/(x+exp(x))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e^x + 1)}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+exp(x))*x/(exp(x)+x)**(1/2),x)`

[Out] `Integral(x*(exp(x) + 1)/sqrt(x + exp(x)), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e^x + 1)}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+exp(x))*x/(x+exp(x))^(1/2),x, algorithm="giac")`

[Out] `integrate(x*(e^x + 1)/sqrt(x + e^x), x)`

$$3.748 \quad \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx$$

**Optimal.** Leaf size=26

$$2x\sqrt{x+e^x} - 2\text{CannotIntegrate}(\sqrt{x+e^x}, x)$$

[Out]  $2*x*\text{Sqrt}[E^x + x] - 2*\text{CannotIntegrate}[\text{Sqrt}[E^x + x], x]$

**Rubi [A]** time = 0.117417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x/\text{Sqrt}[E^x + x] + (E^x*x)/\text{Sqrt}[E^x + x], x]$

[Out]  $2*x*\text{Sqrt}[E^x + x] - 2*\text{Defer}[\text{Int}][\text{Sqrt}[E^x + x], x]$

Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx &= \int \frac{x}{\sqrt{e^x+x}} dx + \int \frac{e^x x}{\sqrt{e^x+x}} dx \\ &= -2\sqrt{e^x+x} + 2x\sqrt{e^x+x} - 2 \int \sqrt{e^x+x} dx + \int \frac{1}{\sqrt{e^x+x}} dx - \int \frac{x}{\sqrt{e^x+x}} dx + \int \sqrt{e^x+x} dx \\ &= 2x\sqrt{e^x+x} - 2 \int \sqrt{e^x+x} dx \end{aligned}$$

**Mathematica [A]** time = 0.0704123, size = 0, normalized size = 0.

$$\int \left( \frac{x}{\sqrt{e^x+x}} + \frac{e^x x}{\sqrt{e^x+x}} \right) dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[x/\text{Sqrt}[E^x + x] + (E^x*x)/\text{Sqrt}[E^x + x], x]$

[Out]  $\text{Integrate}[x/\text{Sqrt}[E^x + x] + (E^x*x)/\text{Sqrt}[E^x + x], x]$

**Maple [A]** time = 0.032, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{e^x+x}} + e^x x \frac{1}{\sqrt{e^x+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x/(\exp(x)+x)^{(1/2)}+\exp(x)*x/(\exp(x)+x)^{(1/2)}, x)$

[Out] `int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x+e^x}} + \frac{x}{\sqrt{x+e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x*e^x/sqrt(x + e^x) + x/sqrt(x + e^x), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e^x + 1)}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(exp(x)+x)**(1/2)+exp(x)*x/(exp(x)+x)**(1/2),x)`

[Out] `Integral(x*(exp(x) + 1)/sqrt(x + exp(x)), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x+e^x}} + \frac{x}{\sqrt{x+e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x+exp(x))^(1/2)+exp(x)*x/(x+exp(x))^(1/2),x, algorithm="giac")`

[Out] `integrate(x*e^x/sqrt(x + e^x) + x/sqrt(x + e^x), x)`

$$3.749 \quad \int \frac{e^x x}{\sqrt{e^x + x}} dx$$

**Optimal.** Leaf size=50

$$-\text{CannotIntegrate}\left(\frac{1}{\sqrt{x+e^x}}, x\right) - 3\text{CannotIntegrate}\left(\sqrt{x+e^x}, x\right) + 2\sqrt{x+e^x}x + 2\sqrt{x+e^x}$$

[Out] 2\*Sqrt[E^x + x] + 2\*x\*Sqrt[E^x + x] - CannotIntegrate[1/Sqrt[E^x + x], x] - 3\*CannotIntegrate[Sqrt[E^x + x], x]

**Rubi [A]** time = 0.0810275, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^x x}{\sqrt{e^x + x}} dx$$

Verification is Not applicable to the result.

[In] Int[(E^x\*x)/Sqrt[E^x + x], x]

[Out] 2\*Sqrt[E^x + x] + 2\*x\*Sqrt[E^x + x] - Defer[Int][1/Sqrt[E^x + x], x] - 3\*Defer[Int][Sqrt[E^x + x], x]

Rubi steps

$$\begin{aligned} \int \frac{e^x x}{\sqrt{e^x + x}} dx &= 2x\sqrt{e^x + x} - 2 \int \sqrt{e^x + x} dx - \int \frac{x}{\sqrt{e^x + x}} dx \\ &= 2\sqrt{e^x + x} + 2x\sqrt{e^x + x} - 2 \int \sqrt{e^x + x} dx - \int \frac{1}{\sqrt{e^x + x}} dx - \int \sqrt{e^x + x} dx \end{aligned}$$

**Mathematica [A]** time = 0.0970847, size = 0, normalized size = 0.

$$\int \frac{e^x x}{\sqrt{e^x + x}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(E^x\*x)/Sqrt[E^x + x], x]

[Out] Integrate[(E^x\*x)/Sqrt[E^x + x], x]

**Maple [A]** time = 0.029, size = 0, normalized size = 0.

$$\int e^x x \frac{1}{\sqrt{e^x + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*x/(exp(x)+x)^(1/2), x)



[Out] `int(exp(x)*x/(exp(x)+x)^(1/2), x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(x+exp(x))^(1/2), x, algorithm="maxima")`

[Out] `integrate(x*e^x/sqrt(x + e^x), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(x+exp(x))^(1/2), x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(exp(x)+x)**(1/2), x)`

[Out] `Integral(x*exp(x)/sqrt(x + exp(x)), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^x}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(x+exp(x))^(1/2), x, algorithm="giac")`

[Out] `integrate(x*e^x/sqrt(x + e^x), x)`

$$3.750 \quad \int \left( \frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}} + \frac{4}{5}x\sqrt{5e^x+x^3} \right) dx$$

Optimal. Leaf size=20

$$\frac{2}{5}x^2\sqrt{x^3+5e^x}$$

[Out] (2\*x^2\*Sqrt[5\*E^x + x^3])/5

**Rubi [A]** time = 0.597482, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {6742, 2262}

$$\frac{2}{5}x^2\sqrt{x^3+5e^x}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(5\*E^x + 3\*x^2))/(5\*Sqrt[5\*E^x + x^3]) + (4\*x\*Sqrt[5\*E^x + x^3])/5, x]

[Out] (2\*x^2\*Sqrt[5\*E^x + x^3])/5

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2262

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))\*(x\_)^(m\_.)\*((b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) + (a\_.)\*(x\_)^(n\_.))^((p\_.), x\_Symbol] := Simp[(x^m\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1))/(b\*d\*e\*(p + 1)\*Log[F]), x] + (-Dist[m/(b\*d\*e\*(p + 1)\*Log[F]), Int[x^(m - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m + n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \left( \frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}} + \frac{4}{5}x\sqrt{5e^x+x^3} \right) dx &= \frac{1}{5} \int \frac{x^2(5e^x+3x^2)}{\sqrt{5e^x+x^3}} dx + \frac{4}{5} \int x\sqrt{5e^x+x^3} dx \\ &= \frac{1}{5} \int \left( \frac{5e^x x^2}{\sqrt{5e^x+x^3}} + \frac{3x^4}{\sqrt{5e^x+x^3}} \right) dx + \frac{4}{5} \int x\sqrt{5e^x+x^3} dx \\ &= \frac{3}{5} \int \frac{x^4}{\sqrt{5e^x+x^3}} dx + \frac{4}{5} \int x\sqrt{5e^x+x^3} dx + \int \frac{e^x x^2}{\sqrt{5e^x+x^3}} dx \\ &= \frac{2}{5}x^2\sqrt{5e^x+x^3} \end{aligned}$$

**Mathematica [A]** time = 0.161067, size = 20, normalized size = 1.

$$\frac{2}{5}x^2\sqrt{x^3+5e^x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5*E^x + 3*x^2))/(5*Sqrt[5*E^x + x^3]) + (4*x*Sqrt[5*E^x + x^3])/5,x]
```

```
[Out] (2*x^2*Sqrt[5*E^x + x^3])/5
```

**Maple [A]** time = 0.085, size = 16, normalized size = 0.8

$$\frac{2x^2}{5}\sqrt{5e^x + x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)^(1/2),x)
```

```
[Out] 2/5*x^2*(5*exp(x)+x^3)^(1/2)
```

**Maxima [A]** time = 1.13466, size = 31, normalized size = 1.55

$$\frac{2(x^5 + 5x^2e^x)}{5\sqrt{x^3 + 5e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x^5 + 5*x^2*e^x)/sqrt(x^3 + 5*e^x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{7x^4}{\sqrt{x^3+5e^x}} dx + \int \frac{20xe^x}{\sqrt{x^3+5e^x}} dx + \int \frac{5x^2e^x}{\sqrt{x^3+5e^x}} dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/5*x**2*(5*exp(x)+3*x**2)/(5*exp(x)+x**3)**(1/2)+4/5*x*(5*exp(x)
+x**3)**(1/2),x)
```

```
[Out] (Integral(7*x**4/sqrt(x**3 + 5*exp(x)), x) + Integral(20*x*exp(x)/sqrt(x**3
+ 5*exp(x)), x) + Integral(5*x**2*exp(x)/sqrt(x**3 + 5*exp(x)), x))/5
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2 + 5e^x)x^2}{5\sqrt{x^3 + 5e^x}} + \frac{4}{5}\sqrt{x^3 + 5e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/5*(3*x^2 + 5*e^x)*x^2/sqrt(x^3 + 5*e^x) + 4/5*sqrt(x^3 + 5*e^x)
*x, x)
```

$$3.751 \quad \int \frac{e^x x^2}{\sqrt{5e^x + x^3}} dx$$

**Optimal.** Leaf size=65

$$-\frac{3}{5} \text{CannotIntegrate}\left(\frac{x^4}{\sqrt{x^3 + 5e^x}}, x\right) - \frac{4}{5} \text{CannotIntegrate}\left(x\sqrt{x^3 + 5e^x}, x\right) + \frac{2}{5} \sqrt{x^3 + 5e^x} x^2$$

[Out] (2\*x^2\*Sqrt[5\*E^x + x^3])/5 - (3\*CannotIntegrate[x^4/Sqrt[5\*E^x + x^3], x])/5 - (4\*CannotIntegrate[x\*Sqrt[5\*E^x + x^3], x])/5

**Rubi [A]** time = 0.165871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}} dx$$

Verification is Not applicable to the result.

[In] Int[(E^x\*x^2)/Sqrt[5\*E^x + x^3], x]

[Out] (2\*x^2\*Sqrt[5\*E^x + x^3])/5 - (3\*Defer[Int][x^4/Sqrt[5\*E^x + x^3], x])/5 - (4\*Defer[Int][x\*Sqrt[5\*E^x + x^3], x])/5

Rubi steps

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}} dx = \frac{2}{5} x^2 \sqrt{5e^x + x^3} - \frac{3}{5} \int \frac{x^4}{\sqrt{5e^x + x^3}} dx - \frac{4}{5} \int x \sqrt{5e^x + x^3} dx$$

**Mathematica [A]** time = 0.187728, size = 0, normalized size = 0.

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(E^x\*x^2)/Sqrt[5\*E^x + x^3], x]

[Out] Integrate[(E^x\*x^2)/Sqrt[5\*E^x + x^3], x]

**Maple [A]** time = 0.041, size = 0, normalized size = 0.

$$\int e^x x^2 \frac{1}{\sqrt{5e^x + x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)\*x^2/(5\*exp(x)+x^3)^(1/2), x)

[Out] `int(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^x}{\sqrt{x^3 + 5e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2*e^x/sqrt(x^3 + 5*e^x), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^x}{\sqrt{x^3 + 5e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x**2/(5*exp(x)+x**3)**(1/2),x)`

[Out] `Integral(x**2*exp(x)/sqrt(x**3 + 5*exp(x)), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 e^x}{\sqrt{x^3 + 5e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^2*e^x/sqrt(x^3 + 5*e^x), x)`

$$3.752 \quad \int -\frac{1+e^x}{\sqrt[3]{e^x+x}} dx$$

**Optimal.** Leaf size=13

$$-\frac{3}{2}(x+e^x)^{2/3}$$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Rubi [A]** time = 0.024423, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {6686}

$$-\frac{3}{2}(x+e^x)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[-((1 + E^x)/(E^x + x)^{(1/3)}), x]$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Rule 6686**

$\text{Int}[(u_)*(y_)^{(m_.)}, x\_Symbol] := \text{With}[\{q = \text{DerivativeDivides}[y, u, x]\}, \text{Simp}[(q*y^{(m+1)})/(m+1), x] /; \text{!FalseQ}[q] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

**Rubi steps**

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}} dx = -\frac{3}{2}(e^x+x)^{2/3}$$

**Mathematica [A]** time = 0.0086212, size = 13, normalized size = 1.

$$-\frac{3}{2}(x+e^x)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[-((1 + E^x)/(E^x + x)^{(1/3)}), x]$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Maple [A]** time = 0.028, size = 9, normalized size = 0.7

$$-\frac{3}{2}(e^x+x)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-1-\exp(x))/(\exp(x)+x)^{(1/3)}, x)$

[Out]  $-3/2*(\exp(x)+x)^{(2/3)}$

---

**Maxima [A]** time = 0.947088, size = 11, normalized size = 0.85

$$-\frac{3}{2}(x + e^x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1-exp(x))/(x+exp(x))^(1/3),x, algorithm="maxima")`

[Out]  $-3/2*(x + e^x)^{(2/3)}$

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1-exp(x))/(x+exp(x))^(1/3),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0.220586, size = 12, normalized size = 0.92

$$-\frac{3(x + e^x)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1-exp(x))/(exp(x)+x)**(1/3),x)`

[Out]  $-3*(x + \exp(x))^{(2/3)}/2$

---

**Giac [A]** time = 1.23833, size = 11, normalized size = 0.85

$$-\frac{3}{2}(x + e^x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1-exp(x))/(x+exp(x))^(1/3),x, algorithm="giac")`

[Out]  $-3/2*(x + e^x)^{(2/3)}$



$$3.753 \quad \int \left( -\frac{1}{\sqrt[3]{e^x+x}} + \frac{x}{\sqrt[3]{e^x+x}} - (e^x + x)^{2/3} \right) dx$$

**Optimal.** Leaf size=13

$$-\frac{3}{2}(x + e^x)^{2/3}$$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Rubi [A]** time = 0.0657407, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2273}

$$-\frac{3}{2}(x + e^x)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[-(E^x + x)^{-1/3} + x/(E^x + x)^{1/3} - (E^x + x)^{2/3}, x]$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Rule 2273**

$\text{Int}[(x_)^{(m_.)}*(E^{(x_)} + (x_)^{(m_.)})^{(n_)}, x\_Symbol] \rightarrow -\text{Simp}[(E^x + x^m)^{(n+1)}/(n+1), x] + (\text{Dist}[m, \text{Int}[x^{(m-1)}*(E^x + x^m)^n, x], x] + \text{Int}[(E^x + x^m)^{(n+1)}, x]) /; \text{RationalQ}[m, n] \&\& \text{GtQ}[m, 0] \&\& \text{LtQ}[n, 0] \&\& \text{NeQ}[n, -1]$

**Rubi steps**

$$\begin{aligned} \int \left( -\frac{1}{\sqrt[3]{e^x+x}} + \frac{x}{\sqrt[3]{e^x+x}} - (e^x + x)^{2/3} \right) dx &= -\int \frac{1}{\sqrt[3]{e^x+x}} dx + \int \frac{x}{\sqrt[3]{e^x+x}} dx - \int (e^x + x)^{2/3} dx \\ &= -\frac{3}{2}(e^x + x)^{2/3} \end{aligned}$$

**Mathematica [A]** time = 0.0048584, size = 13, normalized size = 1.

$$-\frac{3}{2}(x + e^x)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[-(E^x + x)^{-1/3} + x/(E^x + x)^{1/3} - (E^x + x)^{2/3}, x]$

[Out]  $(-3*(E^x + x)^{(2/3)})/2$

**Maple [A]** time = 0.042, size = 9, normalized size = 0.7

$$-\frac{3}{2}(e^x + x)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x)
```

```
[Out] -3/2*(exp(x)+x)^(2/3)
```

**Maxima [A]** time = 1.10009, size = 11, normalized size = 0.85

$$-\frac{3}{2}(x + e^x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(x+exp(x))^(1/3)+x/(x+exp(x))^(1/3)-(x+exp(x))^(2/3),x, algorithm="maxima")
```

```
[Out] -3/2*(x + e^x)^(2/3)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(x+exp(x))^(1/3)+x/(x+exp(x))^(1/3)-(x+exp(x))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\sqrt[3]{x + e^x}} dx - \int \frac{1}{\sqrt[3]{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(exp(x)+x)**(1/3)+x/(exp(x)+x)**(1/3)-(exp(x)+x)**(2/3),x)
```

```
[Out] -Integral(exp(x)/(x + exp(x))**(1/3), x) - Integral((x + exp(x))**(-1/3), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -(x + e^x)^{\frac{2}{3}} + \frac{x}{(x + e^x)^{\frac{1}{3}}} - \frac{1}{(x + e^x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(x+exp(x))^(1/3)+x/(x+exp(x))^(1/3)-(x+exp(x))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(x + e^x)^(2/3) + x/(x + e^x)^(1/3) - 1/(x + e^x)^(1/3), x)
```

$$3.754 \quad \int \frac{x}{\sqrt[3]{e^x+x}} dx$$

**Optimal.** Leaf size=36

$$\text{CannotIntegrate}\left(\frac{1}{\sqrt[3]{x+e^x}}, x\right) + \text{CannotIntegrate}\left((x+e^x)^{2/3}, x\right) - \frac{3}{2}(x+e^x)^{2/3}$$

[Out]  $(-3*(E^x + x)^{(2/3)})/2 + \text{CannotIntegrate}[(E^x + x)^{(-1/3)}, x] + \text{CannotIntegrate}[(E^x + x)^{(2/3)}, x]$

**Rubi [A]** time = 0.0396365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x}{\sqrt[3]{e^x+x}} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x/(E^x + x)^{(1/3)}, x]$

[Out]  $(-3*(E^x + x)^{(2/3)})/2 + \text{Defer}[\text{Int}][(E^x + x)^{(-1/3)}, x] + \text{Defer}[\text{Int}][(E^x + x)^{(2/3)}, x]$

Rubi steps

$$\int \frac{x}{\sqrt[3]{e^x+x}} dx = -\frac{3}{2}(e^x+x)^{2/3} + \int \frac{1}{\sqrt[3]{e^x+x}} dx + \int (e^x+x)^{2/3} dx$$

**Mathematica [A]** time = 0.0477577, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{e^x+x}} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[x/(E^x + x)^{(1/3)}, x]$

[Out]  $\text{Integrate}[x/(E^x + x)^{(1/3)}, x]$

**Maple [A]** time = 0.017, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt[3]{e^x+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x/(\exp(x)+x)^{(1/3)}, x)$

[Out]  $\text{int}(x/(\exp(x)+x)^{(1/3)}, x)$

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x + e^x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x))^(1/3),x, algorithm="maxima")

[Out] integrate(x/(x + e^x)^(1/3), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x))^(1/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(exp(x)+x)\*\*(1/3),x)

[Out] Integral(x/(x + exp(x))\*\*(1/3), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x + e^x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x))^(1/3),x, algorithm="giac")

[Out] integrate(x/(x + e^x)^(1/3), x)

$$3.755 \quad \int \frac{5x + e^x(3+2x)}{\sqrt[3]{e^x+x}} dx$$

**Optimal.** Leaf size=12

$$3x(x + e^x)^{2/3}$$

[Out] 3\*x\*(E^x + x)^(2/3)

**Rubi [A]** time = 0.336884, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {6742, 2273, 2261, 2262}

$$3x(x + e^x)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(5\*x + E^x\*(3 + 2\*x))/(E^x + x)^(1/3), x]

[Out] 3\*x\*(E^x + x)^(2/3)

#### Rule 6742

Int[u\_, x\_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]

#### Rule 2273

Int[(x\_)^(m\_)\*(E^(x\_) + (x\_)^(m\_))^(n\_), x\_Symbol] :=> -Simp[(E^x + x^m)^(n + 1)/(n + 1), x] + (Dist[m, Int[x^(m - 1)\*(E^x + x^m)^n, x], x] + Int[(E^x + x^m)^(n + 1), x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[n, -1]

#### Rule 2261

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*((b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))) + (a\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :=> Simp[(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x] && NeQ[p, -1]

#### Rule 2262

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*(x\_)^(m\_)\*((b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))) + (a\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :=> Simp[(x^m\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] + (-Dist[m/(b\*d\*e\*(p + 1)\*Log[F]), Int[x^(m - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m + n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{5x + e^x(3 + 2x)}{\sqrt[3]{e^x + x}} dx &= \int \left( \frac{5x}{\sqrt[3]{e^x + x}} + \frac{e^x(3 + 2x)}{\sqrt[3]{e^x + x}} \right) dx \\
&= 5 \int \frac{x}{\sqrt[3]{e^x + x}} dx + \int \frac{e^x(3 + 2x)}{\sqrt[3]{e^x + x}} dx \\
&= -\frac{15}{2} (e^x + x)^{2/3} + 5 \int \frac{1}{\sqrt[3]{e^x + x}} dx + 5 \int (e^x + x)^{2/3} dx + \int \left( \frac{3e^x}{\sqrt[3]{e^x + x}} + \frac{2e^x x}{\sqrt[3]{e^x + x}} \right) dx \\
&= -\frac{15}{2} (e^x + x)^{2/3} + 2 \int \frac{e^x x}{\sqrt[3]{e^x + x}} dx + 3 \int \frac{e^x}{\sqrt[3]{e^x + x}} dx + 5 \int \frac{1}{\sqrt[3]{e^x + x}} dx + 5 \int (e^x + x)^{2/3} dx \\
&= -3(e^x + x)^{2/3} + 3x(e^x + x)^{2/3} - 2 \int \frac{x}{\sqrt[3]{e^x + x}} dx - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx - 3 \int (e^x + x)^{2/3} dx + 5 \int (e^x + x)^{2/3} dx \\
&= 3x(e^x + x)^{2/3} - 2 \int \frac{1}{\sqrt[3]{e^x + x}} dx - 2 \int (e^x + x)^{2/3} dx - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx - 3 \int (e^x + x)^{2/3} dx
\end{aligned}$$

**Mathematica [A]** time = 0.0917841, size = 12, normalized size = 1.

$$3x(x + e^x)^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(5\*x + E^x\*(3 + 2\*x))/(E^x + x)^(1/3), x]

[Out] 3\*x\*(E^x + x)^(2/3)

**Maple [A]** time = 0.029, size = 10, normalized size = 0.8

$$3x(e^x + x)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x+exp(x)\*(3+2\*x))/(exp(x)+x)^(1/3), x)

[Out] 3\*x\*(exp(x)+x)^(2/3)

**Maxima [A]** time = 1.08398, size = 22, normalized size = 1.83

$$\frac{3(x^2 + xe^x)}{(x + e^x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x+exp(x)\*(3+2\*x))/(x+exp(x))^(1/3), x, algorithm="maxima")

[Out] 3\*(x^2 + x\*e^x)/(x + e^x)^(1/3)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x+exp(x)*(3+2*x))/(x+exp(x))^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2xe^x + 5x + 3e^x}{\sqrt[3]{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x+exp(x)*(3+2*x))/(exp(x)+x)**(1/3),x)
```

```
[Out] Integral((2*x*exp(x) + 5*x + 3*exp(x))/(x + exp(x))**(1/3), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x + 3)e^x + 5x}{(x + e^x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x+exp(x)*(3+2*x))/(x+exp(x))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((2*x + 3)*e^x + 5*x)/(x + e^x)^(1/3), x)
```



$$3.756 \quad \int \left( \frac{2x}{\sqrt[3]{e^x+x}} + \frac{2e^x x}{\sqrt[3]{e^x+x}} + 3(e^x+x)^{2/3} \right) dx$$

**Optimal.** Leaf size=12

$$3x(x+e^x)^{2/3}$$

[Out] 3\*x\*(E^x + x)^(2/3)

**Rubi [A]** time = 0.143765, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {2273, 2262}

$$3x(x+e^x)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(2\*x)/(E^x + x)^(1/3) + (2\*E^x\*x)/(E^x + x)^(1/3) + 3\*(E^x + x)^(2/3), x]

[Out] 3\*x\*(E^x + x)^(2/3)

#### Rule 2273

Int[(x\_)^(m\_)\*(E^(x\_) + (x\_)^(m\_))^(n\_), x\_Symbol] :> -Simp[(E^x + x^m)^(n + 1)/(n + 1), x] + (Dist[m, Int[x^(m - 1)\*(E^x + x^m)^n, x], x] + Int[(E^x + x^m)^(n + 1), x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[n, -1]

#### Rule 2262

Int[(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))\*(x\_)^(m\_)\*((b\_)\*(F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))) + (a\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^m\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] + (-Dist[m/(b\*d\*e\*(p + 1)\*Log[F]), Int[x^(m - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m + n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \left( \frac{2x}{\sqrt[3]{e^x+x}} + \frac{2e^x x}{\sqrt[3]{e^x+x}} + 3(e^x+x)^{2/3} \right) dx &= 2 \int \frac{x}{\sqrt[3]{e^x+x}} dx + 2 \int \frac{e^x x}{\sqrt[3]{e^x+x}} dx + 3 \int (e^x+x)^{2/3} dx \\ &= -3(e^x+x)^{2/3} + 3x(e^x+x)^{2/3} + 2 \int \frac{1}{\sqrt[3]{e^x+x}} dx - 2 \int \frac{x}{\sqrt[3]{e^x+x}} dx + 2 \int \dots \\ &= 3x(e^x+x)^{2/3} \end{aligned}$$

**Mathematica [A]** time = 0.0470589, size = 12, normalized size = 1.

$$3x(x+e^x)^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(2\*x)/(E^x + x)^(1/3) + (2\*E^x\*x)/(E^x + x)^(1/3) + 3\*(E^x + x)^(2/3),x]

[Out] 3\*x\*(E^x + x)^(2/3)

**Maple [A]** time = 0.043, size = 10, normalized size = 0.8

$$3x(e^x + x)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2\*x/(exp(x)+x)^(1/3)+2\*exp(x)\*x/(exp(x)+x)^(1/3)+3\*(exp(x)+x)^(2/3),x)

[Out] 3\*x\*(exp(x)+x)^(2/3)

**Maxima [A]** time = 1.1051, size = 22, normalized size = 1.83

$$\frac{3(x^2 + xe^x)}{(x + e^x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*x/(x+exp(x))^(1/3)+2\*exp(x)\*x/(x+exp(x))^(1/3)+3\*(x+exp(x))^(2/3),x, algorithm="maxima")

[Out] 3\*(x^2 + x\*e^x)/(x + e^x)^(1/3)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*x/(x+exp(x))^(1/3)+2\*exp(x)\*x/(x+exp(x))^(1/3)+3\*(x+exp(x))^(2/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2xe^x + 5x + 3e^x}{\sqrt[3]{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*x/(exp(x)+x)\*\*(1/3)+2\*exp(x)\*x/(exp(x)+x)\*\*(1/3)+3\*(exp(x)+x)\*\*(2/3),x)

[Out] Integral((2\*x\*exp(x) + 5\*x + 3\*exp(x))/(x + exp(x))\*\*(1/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2xe^x}{(x+e^x)^{\frac{1}{3}}} + 3(x+e^x)^{\frac{2}{3}} + \frac{2x}{(x+e^x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2\*x/(x+exp(x))^(1/3)+2\*exp(x)\*x/(x+exp(x))^(1/3)+3\*(x+exp(x))^(2/3),x, algorithm="giac")

[Out] integrate(2\*x\*e^x/(x + e^x)^(1/3) + 3\*(x + e^x)^(2/3) + 2\*x/(x + e^x)^(1/3), x)

$$3.757 \quad \int e^x (-e^{-x} + e^x) (e^{-x} + e^x)^2 dx$$

**Optimal.** Leaf size=31

$$-x + \frac{e^{-2x}}{2} + \frac{e^{2x}}{2} + \frac{e^{4x}}{4}$$

[Out] 1/(2\*E^(2\*x)) + E^(2\*x)/2 + E^(4\*x)/4 - x

**Rubi [A]** time = 0.0493072, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2282, 14}

$$-x + \frac{e^{-2x}}{2} + \frac{e^{2x}}{2} + \frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

[In] Int[E^x\*(-E^(-x) + E^x)\*(E^(-x) + E^x)^2,x]

[Out] 1/(2\*E^(2\*x)) + E^(2\*x)/2 + E^(4\*x)/4 - x

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

#### Rubi steps

$$\begin{aligned} \int e^x (-e^{-x} + e^x) (e^{-x} + e^x)^2 dx &= \text{Subst} \left( \int \frac{-1 - \frac{1}{x^2} + x^2 + x^4}{x} dx, x, e^x \right) \\ &= \text{Subst} \left( \int \left( -\frac{1}{x^3} - \frac{1}{x} + x + x^3 \right) dx, x, e^x \right) \\ &= \frac{e^{-2x}}{2} + \frac{e^{2x}}{2} + \frac{e^{4x}}{4} - x \end{aligned}$$

**Mathematica [A]** time = 0.0082503, size = 31, normalized size = 1.

$$-x + \frac{e^{-2x}}{2} + \frac{e^{2x}}{2} + \frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

[In] Integrate[E^x\*(-E^(-x) + E^x)\*(E^(-x) + E^x)^2,x]

[Out]  $1/(2E^{(2*x)}) + E^{(2*x)}/2 + E^{(4*x)}/4 - x$

---

**Maple [A]** time = 0.02, size = 23, normalized size = 0.7

$$-x + \frac{(e^x)^2}{2} + \frac{(e^x)^4}{4} + \frac{1}{2(e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(-1/exp(x)+exp(x))*(exp(-x)+exp(x))^2,x)`

[Out]  $-x+1/2*\exp(x)^2+1/4*\exp(x)^4+1/2/\exp(x)^2$

---

**Maxima [A]** time = 0.97658, size = 32, normalized size = 1.03

$$\frac{1}{4}(2e^{(-2x)} + 1)e^{(4x)} - x + \frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="maxima")`

[Out]  $1/4*(2*e^{(-2*x)} + 1)*e^{(4*x)} - x + 1/2*e^{(-2*x)}$

---

**Fricas [A]** time = 0.67376, size = 74, normalized size = 2.39

$$-\frac{1}{4}(4xe^{(2x)} - e^{(6x)} - 2e^{(4x)} - 2)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="fricas")`

[Out]  $-1/4*(4*x*e^{(2*x)} - e^{(6*x)} - 2*e^{(4*x)} - 2)*e^{(-2*x)}$

---

**Sympy [A]** time = 0.117177, size = 22, normalized size = 0.71

$$-x + \frac{e^{4x}}{4} + \frac{e^{2x}}{2} + \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(-1/exp(x)+exp(x))*(exp(-x)+exp(x))**2,x)`

[Out]  $-x + \exp(4*x)/4 + \exp(2*x)/2 + \exp(-2*x)/2$

---

**Giac [A]** time = 1.2568, size = 38, normalized size = 1.23

$$\frac{1}{2} (e^{2x} + 1)e^{-2x} - x + \frac{1}{4} e^{4x} + \frac{1}{2} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-1/exp(x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-2*x) - x + 1/4*e^(4*x) + 1/2*e^(2*x)
```

$$3.758 \quad \int \frac{x}{e^x+x} dx$$

**Optimal.** Leaf size=11

$$\text{CannotIntegrate}\left(\frac{x}{x+e^x}, x\right)$$

[Out] CannotIntegrate[x/(E^x + x), x]

**Rubi [A]** time = 0.0290672, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x}{e^x+x} dx$$

Verification is Not applicable to the result.

[In] Int[x/(E^x + x), x]

[Out] Defer[Int][x/(E^x + x), x]

Rubi steps

$$\int \frac{x}{e^x+x} dx = \int \frac{x}{e^x+x} dx$$

**Mathematica [A]** time = 2.35447, size = 0, normalized size = 0.

$$\int \frac{x}{e^x+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(E^x + x), x]

[Out] Integrate[x/(E^x + x), x]

**Maple [A]** time = 0.024, size = 0, normalized size = 0.

$$\int \frac{x}{e^x+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(exp(x)+x), x)

[Out] int(x/(exp(x)+x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x)),x, algorithm="maxima")

[Out] integrate(x/(x + e^x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{x + e^x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x)),x, algorithm="fricas")

[Out] integral(x/(x + e^x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(exp(x)+x),x)

[Out] Integral(x/(x + exp(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+exp(x)),x, algorithm="giac")

[Out] integrate(x/(x + e^x), x)



$$3.759 \quad \int \frac{x^2}{\sqrt{e^x+x}} dx$$

**Optimal.** Leaf size=15

$$\text{CannotIntegrate}\left(\frac{x^2}{\sqrt{x+e^x}}, x\right)$$

[Out] CannotIntegrate[x^2/Sqrt[E^x + x], x]

**Rubi [A]** time = 0.0572397, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^2}{\sqrt{e^x+x}} dx$$

Verification is Not applicable to the result.

[In] Int[x^2/Sqrt[E^x + x], x]

[Out] Defer[Int][x^2/Sqrt[E^x + x], x]

Rubi steps

$$\int \frac{x^2}{\sqrt{e^x+x}} dx = \int \frac{x^2}{\sqrt{e^x+x}} dx$$

**Mathematica [A]** time = 0.0483567, size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{e^x+x}} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/Sqrt[E^x + x], x]

[Out] Integrate[x^2/Sqrt[E^x + x], x]

**Maple [A]** time = 0.028, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{e^x+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(exp(x)+x)^(1/2), x)

[Out] int(x^2/(exp(x)+x)^(1/2), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x+exp(x))^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(x + e^x), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x+exp(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(exp(x)+x)\*\*(1/2),x)

[Out] Integral(x\*\*2/sqrt(x + exp(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x+exp(x))^(1/2),x, algorithm="giac")

[Out] integrate(x^2/sqrt(x + e^x), x)

$$3.760 \quad \int \frac{e^x}{e^x+x} dx$$

**Optimal.** Leaf size=13

$$\text{CannotIntegrate}\left(\frac{e^x}{x+e^x}, x\right)$$

[Out] CannotIntegrate[E^x/(E^x + x), x]

**Rubi [A]** time = 0.0341178, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^x}{e^x+x} dx$$

Verification is Not applicable to the result.

[In] Int[E^x/(E^x + x), x]

[Out] Defer[Int][E^x/(E^x + x), x]

Rubi steps

$$\int \frac{e^x}{e^x+x} dx = \int \frac{e^x}{e^x+x} dx$$

**Mathematica [A]** time = 0.0178615, size = 0, normalized size = 0.

$$\int \frac{e^x}{e^x+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^x/(E^x + x), x]

[Out] Integrate[E^x/(E^x + x), x]

**Maple [A]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{e^x}{e^x+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(exp(x)+x), x)

[Out] int(exp(x)/(exp(x)+x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$x - \int \frac{x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(x+exp(x)),x, algorithm="maxima")

[Out] x - integrate(x/(x + e^x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^x}{x + e^x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(x+exp(x)),x, algorithm="fricas")

[Out] integral(e^x/(x + e^x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(exp(x)+x),x)

[Out] Integral(exp(x)/(x + exp(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{x + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(x+exp(x)),x, algorithm="giac")

[Out] integrate(e^x/(x + e^x), x)

$$3.761 \quad \int \frac{e^x}{e^x + x^2} dx$$

**Optimal.** Leaf size=15

$$\text{CannotIntegrate}\left(\frac{e^x}{x^2 + e^x}, x\right)$$

[Out] CannotIntegrate[E^x/(E^x + x^2), x]

**Rubi [A]** time = 0.0392725, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{e^x}{e^x + x^2} dx$$

Verification is Not applicable to the result.

[In] Int[E^x/(E^x + x^2), x]

[Out] Defer[Int][E^x/(E^x + x^2), x]

Rubi steps

$$\int \frac{e^x}{e^x + x^2} dx = \int \frac{e^x}{e^x + x^2} dx$$

**Mathematica [A]** time = 0.0333633, size = 0, normalized size = 0.

$$\int \frac{e^x}{e^x + x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[E^x/(E^x + x^2), x]

[Out] Integrate[E^x/(E^x + x^2), x]

**Maple [A]** time = 0.028, size = 0, normalized size = 0.

$$\int \frac{e^x}{e^x + x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(exp(x)+x^2), x)

[Out] int(exp(x)/(exp(x)+x^2), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$x - \int \frac{x^2}{x^2 + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(exp(x)+x^2),x, algorithm="maxima")

[Out] x - integrate(x^2/(x^2 + e^x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^x}{x^2 + e^x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(exp(x)+x^2),x, algorithm="fricas")

[Out] integral(e^x/(x^2 + e^x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{x^2 + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(exp(x)+x\*\*2),x)

[Out] Integral(exp(x)/(x\*\*2 + exp(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{x^2 + e^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(exp(x)+x^2),x, algorithm="giac")

[Out] integrate(e^x/(x^2 + e^x), x)

$$3.762 \quad \int \frac{F0(x)}{x+F0(x)} dx$$

**Optimal.** Leaf size=14

$$x - \text{CannotIntegrate}\left(\frac{x}{F0(x) + x}, x\right)$$

[Out] x - CannotIntegrate[x/(x + F0[x]), x]

**Rubi [A]** time = 0.0455445, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F0(x)}{x + F0(x)} dx$$

Verification is Not applicable to the result.

[In] Int[F0[x]/(x + F0[x]), x]

[Out] x - Defer[Int][x/(x + F0[x]), x]

Rubi steps

$$\begin{aligned} \int \frac{F0(x)}{x + F0(x)} dx &= \int \left(1 - \frac{x}{x + F0(x)}\right) dx \\ &= x - \int \frac{x}{x + F0(x)} dx \end{aligned}$$

**Mathematica [A]** time = 0.0379292, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{x + F0(x)} dx$$

Verification is Not applicable to the result.

[In] Integrate[F0[x]/(x + F0[x]), x]

[Out] Integrate[F0[x]/(x + F0[x]), x]

**Maple [A]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{x + F0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F0(x)/(x+F0(x)), x)

[Out] int(F0(x)/(x+F0(x)), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x)),x, algorithm="maxima")

[Out] integrate(F0(x)/(x + F0(x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F_0(x)}{x + F_0(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x)),x, algorithm="fricas")

[Out] integral(F0(x)/(x + F0(x)), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x)),x)

[Out] Integral(F0(x)/(x + F0(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x)),x, algorithm="giac")

[Out] integrate(F0(x)/(x + F0(x)), x)



$$3.763 \quad \int \frac{F0(x)}{x^2+F0(x)} dx$$

**Optimal.** Leaf size=18

$$x - \text{CannotIntegrate}\left(\frac{x^2}{F0(x) + x^2}, x\right)$$

[Out] x - CannotIntegrate[x^2/(x^2 + F0[x]), x]

**Rubi [A]** time = 0.0699767, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F0(x)}{x^2 + F0(x)} dx$$

Verification is Not applicable to the result.

[In] Int[F0[x]/(x^2 + F0[x]), x]

[Out] x - Defer[Int][x^2/(x^2 + F0[x]), x]

Rubi steps

$$\begin{aligned} \int \frac{F0(x)}{x^2 + F0(x)} dx &= \int \left(1 - \frac{x^2}{x^2 + F0(x)}\right) dx \\ &= x - \int \frac{x^2}{x^2 + F0(x)} dx \end{aligned}$$

**Mathematica [A]** time = 0.0439654, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{x^2 + F0(x)} dx$$

Verification is Not applicable to the result.

[In] Integrate[F0[x]/(x^2 + F0[x]), x]

[Out] Integrate[F0[x]/(x^2 + F0[x]), x]

**Maple [A]** time = 0.021, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{x^2 + F0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F0(x)/(x^2+F0(x)), x)

[Out] int(F0(x)/(x^2+F0(x)), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x^2 + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x^2+F0(x)),x, algorithm="maxima")

[Out] integrate(F0(x)/(x^2 + F0(x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F_0(x)}{x^2 + F_0(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x^2+F0(x)),x, algorithm="fricas")

[Out] integral(F0(x)/(x^2 + F0(x)), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x^2 + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x\*\*2+F0(x)),x)

[Out] Integral(F0(x)/(x\*\*2 + F0(x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{x^2 + F_0(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x^2+F0(x)),x, algorithm="giac")

[Out] integrate(F0(x)/(x^2 + F0(x)), x)

$$3.764 \quad \int \frac{F0(x)}{(x+F0(x))^2} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate}\left(\frac{1}{F0(x)+x}, x\right) - \text{CannotIntegrate}\left(\frac{x}{(F0(x)+x)^2}, x\right)$$

[Out] -CannotIntegrate[x/(x + F0[x])^2, x] + CannotIntegrate[(x + F0[x])^(-1), x]

**Rubi [A]** time = 0.0561132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F0(x)}{(x+F0(x))^2} dx$$

Verification is Not applicable to the result.

[In] Int[F0[x]/(x + F0[x])^2,x]

[Out] -Defer[Int][x/(x + F0[x])^2, x] + Defer[Int][(x + F0[x])^(-1), x]

Rubi steps

$$\begin{aligned} \int \frac{F0(x)}{(x+F0(x))^2} dx &= \int \left( -\frac{x}{(x+F0(x))^2} + \frac{1}{x+F0(x)} \right) dx \\ &= -\int \frac{x}{(x+F0(x))^2} dx + \int \frac{1}{x+F0(x)} dx \end{aligned}$$

**Mathematica [A]** time = 0.0169089, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{(x+F0(x))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F0[x]/(x + F0[x])^2,x]

[Out] Integrate[F0[x]/(x + F0[x])^2, x]

**Maple [A]** time = 0.02, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{(x+F0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F0(x)/(x+F0(x))^2,x)

[Out] int(F0(x)/(x+F0(x))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x))^2,x, algorithm="maxima")

[Out] integrate(F0(x)/(x + F0(x))^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F_0(x)}{x^2 + 2x F_0(x) + F_0(x)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x))^2,x, algorithm="fricas")

[Out] integral(F0(x)/(x^2 + 2\*x\*F0(x) + F0(x)^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x))\*\*2,x)

[Out] Integral(F0(x)/(x + F0(x))\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F0(x)/(x+F0(x))^2,x, algorithm="giac")

[Out] integrate(F0(x)/(x + F0(x))^2, x)

$$3.765 \quad \int \frac{F0(x)}{(x^2 + F0(x))^2} dx$$

**Optimal.** Leaf size=27

$$\text{CannotIntegrate}\left(\frac{1}{F0(x) + x^2}, x\right) - \text{CannotIntegrate}\left(\frac{x^2}{(F0(x) + x^2)^2}, x\right)$$

[Out] -CannotIntegrate[x^2/(x^2 + F0[x])^2, x] + CannotIntegrate[(x^2 + F0[x])^(-1), x]

**Rubi [A]** time = 0.0806389, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{F0(x)}{(x^2 + F0(x))^2} dx$$

Verification is Not applicable to the result.

[In] Int[F0[x]/(x^2 + F0[x])^2, x]

[Out] -Defer[Int][x^2/(x^2 + F0[x])^2, x] + Defer[Int][(x^2 + F0[x])^(-1), x]

Rubi steps

$$\begin{aligned} \int \frac{F0(x)}{(x^2 + F0(x))^2} dx &= \int \left( -\frac{x^2}{(x^2 + F0(x))^2} + \frac{1}{x^2 + F0(x)} \right) dx \\ &= -\int \frac{x^2}{(x^2 + F0(x))^2} dx + \int \frac{1}{x^2 + F0(x)} dx \end{aligned}$$

**Mathematica [A]** time = 0.0187016, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{(x^2 + F0(x))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F0[x]/(x^2 + F0[x])^2, x]

[Out] Integrate[F0[x]/(x^2 + F0[x])^2, x]

**Maple [A]** time = 0.022, size = 0, normalized size = 0.

$$\int \frac{F0(x)}{(x^2 + F0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F0(x)/(x^2+F0(x))^2,x)`

[Out] `int(F0(x)/(x^2+F0(x))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x^2 + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="maxima")`

[Out] `integrate(F0(x)/(x^2 + F0(x))^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F_0(x)}{x^4 + 2x^2F_0(x) + F_0(x)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="fricas")`

[Out] `integral(F0(x)/(x^4 + 2*x^2*F0(x) + F0(x)^2), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x^2 + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F0(x)/(x**2+F0(x))**2,x)`

[Out] `Integral(F0(x)/(x**2 + F0(x))**2, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F_0(x)}{(x^2 + F_0(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="giac")`

[Out] `integrate(F0(x)/(x^2 + F0(x))^2, x)`

$$3.766 \quad \int (aF^{c+dx})^m (bF^{e+fx})^n dx$$

**Optimal.** Leaf size=36

$$\frac{(aF^{c+dx})^m (bF^{e+fx})^n}{\log(F)(dm + fn)}$$

[Out]  $((aF^{(c + d*x)})^m*(bF^{(e + f*x)})^n)/((d*m + f*n)*\text{Log}[F])$

**Rubi [A]** time = 0.0986903, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2281, 2227, 2194}

$$\frac{(aF^{c+dx})^m (bF^{e+fx})^n}{\log(F)(dm + fn)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(aF^{(c + d*x)})^m*(bF^{(e + f*x)})^n, x]$

[Out]  $((aF^{(c + d*x)})^m*(bF^{(e + f*x)})^n)/((d*m + f*n)*\text{Log}[F])$

#### Rule 2281

$\text{Int}[(u_*)*((a_*)*(F_)^{(v_)})^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(aF^v)^n/F^{(n*v)}, \text{Int}[uF^{(n*v)}, x], x] /; \text{FreeQ}\{F, a, n\}, x] \ \&\& \ !\text{IntegerQ}[n]$

#### Rule 2227

$\text{Int}[(u_*)*(F_)^{((a_*) + (b_*)*(v_))}, x\_Symbol] \rightarrow \text{Int}[uF^{(a + b*NormalizePowerOfLinear[v, x])}, x] /; \text{FreeQ}\{F, a, b\}, x] \ \&\& \ \text{PolynomialQ}[u, x] \ \&\& \ \text{PowerOfLinearQ}[v, x] \ \&\& \ !\text{PowerOfLinearMatchQ}[v, x]$

#### Rule 2194

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int (aF^{c+dx})^m (bF^{e+fx})^n dx &= \left( F^{-m(c+dx)} (aF^{c+dx})^m \right) \int F^{m(c+dx)} (bF^{e+fx})^n dx \\ &= \left( F^{-m(c+dx)-n(e+fx)} (aF^{c+dx})^m (bF^{e+fx})^n \right) \int F^{m(c+dx)+n(e+fx)} dx \\ &= \left( F^{-m(c+dx)-n(e+fx)} (aF^{c+dx})^m (bF^{e+fx})^n \right) \int F^{cm+en+(dm+fn)x} dx \\ &= \frac{(aF^{c+dx})^m (bF^{e+fx})^n}{(dm + fn) \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0464839, size = 36, normalized size = 1.

$$\frac{(aF^{c+dx})^m (bF^{e+fx})^n}{dm \log(F) + fn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*F^(c + d\*x))^m\*(b\*F^(e + f\*x))^n,x]

[Out] ((a\*F^(c + d\*x))^m\*(b\*F^(e + f\*x))^n)/(d\*m\*Log[F] + f\*n\*Log[F])

**Maple [A]** time = 0.038, size = 37, normalized size = 1.

$$\frac{(aF^{dx+c})^m (bF^{fx+e})^n}{\ln(F)(md + fn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*F^(d\*x+c))^m\*(b\*F^(f\*x+e))^n,x)

[Out] (a\*F^(d\*x+c))^m\*(b\*F^(f\*x+e))^n/(d\*m+f\*n)/ln(F)

**Maxima [A]** time = 1.1545, size = 88, normalized size = 2.44

$$\frac{(F^e)^n a^m b^n e^{\left(m \log(F^{dx+c}) + n \log\left(F^{dx+c} \frac{f}{d}\right)\right)}}{(dm + fn) \left(F \frac{cf}{d}\right)^n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*F^(d\*x+c))^m\*(b\*F^(f\*x+e))^n,x, algorithm="maxima")

[Out] (F^e)^n\*a^m\*b^n\*e^(m\*log(F^(d\*x + c)) + n\*log((F^(d\*x + c))^(f/d)))/((d\*m + f\*n)\*(F^(c\*f/d))^n\*log(F))

**Fricas [A]** time = 0.913505, size = 124, normalized size = 3.44

$$\frac{e^{((dmx+cm)\log(F)+(fnx+en)\log(F)+m\log(a)+n\log(b))}}{(dm + fn)\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*F^(d\*x+c))^m\*(b\*F^(f\*x+e))^n,x, algorithm="fricas")

[Out] e^((d\*m\*x + c\*m)\*log(F) + (f\*n\*x + e\*n)\*log(F) + m\*log(a) + n\*log(b))/((d\*m + f\*n)\*log(F))



**Sympy [A]** time = 65.8399, size = 143, normalized size = 3.97

$$\begin{cases} a^m b^n x & \text{for } F = 1 \wedge \left( F = 1 \vee d = -\frac{fn}{m} \right) \\ a^m b^n x (F^c)^m (F^e)^n (F^{fx})^n \left( F^{-\frac{fnx}{m}} \right)^m + \frac{a^m b^n (F^c)^m (F^e)^n (F^{fx})^n \left( F^{-\frac{fnx}{m}} \right)^m}{fn \log(F)} & \text{for } d = -\frac{fn}{m} \\ \frac{a^m b^n (F^c)^m (F^e)^n (F^{dx})^m (F^{fx})^n}{dm \log(F) + fn \log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*F\*\*(d\*x+c))\*m\*(b\*F\*\*(f\*x+e))\*n,x)

[Out] Piecewise((a\*\*m\*b\*\*n\*x, Eq(F, 1) & (Eq(F, 1) | Eq(d, -f\*n/m))), (a\*\*m\*b\*\*n\*x\*(F\*\*c)\*\*m\*(F\*\*e)\*\*n\*(F\*\*(f\*x))\*\*n\*(F\*\*(-f\*n\*x/m))\*\*m + a\*\*m\*b\*\*n\*(F\*\*c)\*\*m\*(F\*\*e)\*\*n\*(F\*\*(f\*x))\*\*n\*(F\*\*(-f\*n\*x/m))\*\*m/(f\*n\*log(F)), Eq(d, -f\*n/m)), (a\*\*m\*b\*\*n\*(F\*\*c)\*\*m\*(F\*\*e)\*\*n\*(F\*\*(d\*x))\*\*m\*(F\*\*(f\*x))\*\*n/(d\*m\*log(F) + f\*n\*log(F)), True))

**Giac [A]** time = 1.68147, size = 63, normalized size = 1.75

$$\frac{e^{(dmx \log(F) + fnx \log(F) + cm \log(F) + ne \log(F) + m \log(a) + n \log(b))}}{dm \log(F) + fn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*F^(d\*x+c))^m\*(b\*F^(f\*x+e))^n,x, algorithm="giac")

[Out] e^(d\*m\*x\*log(F) + f\*n\*x\*log(F) + c\*m\*log(F) + n\*e\*log(F) + m\*log(a) + n\*log(b))/(d\*m\*log(F) + f\*n\*log(F))

$$3.767 \quad \int e^{a+c+bx^n+dx^n} dx$$

**Optimal.** Leaf size=37

$$\frac{x e^{a+c} (-(b+d)x^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -(b+d)x^n\right)}{n}$$

[Out]  $-\left(\left(E^{a+c} x \text{Gamma}[n^{(-1)}, -((b+d)x^n)]\right) / \left(n \left(-((b+d)x^n)\right)^{n^{(-1)}}\right)\right)$

**Rubi [A]** time = 0.0351137, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {6741, 2208}

$$\frac{x e^{a+c} (-(b+d)x^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -(b+d)x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[E^(a + c + b\*x^n + d\*x^n), x]

[Out]  $-\left(\left(E^{a+c} x \text{Gamma}[n^{(-1)}, -((b+d)x^n)]\right) / \left(n \left(-((b+d)x^n)\right)^{n^{(-1)}}\right)\right)$

#### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^n)), x\_Symbol] := -Simp[(F^a \*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]]) / (d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\begin{aligned} \int e^{a+c+bx^n+dx^n} dx &= \int e^{a+c+(b+d)x^n} dx \\ &= \frac{e^{a+c} x (-(b+d)x^n)^{-1/n} \Gamma\left(\frac{1}{n}, -(b+d)x^n\right)}{n} \end{aligned}$$

**Mathematica [A]** time = 0.01572, size = 37, normalized size = 1.

$$\frac{x e^{a+c} (-(b+d)x^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -(b+d)x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + c + b\*x^n + d\*x^n), x]

[Out]  $-\left(\left(E^{a+c} x \text{Gamma}[n^{(-1)}, -((b+d)x^n)]\right) / \left(n \left(-((b+d)x^n)\right)^{n^{(-1)}}\right)\right)$

---

**Maple [C]** time = 0.24, size = 241, normalized size = 6.5

$$\frac{e^{a+c}}{n} (-b-d)^{-n-1} \left( \frac{n^2 x^{1-n} (-b-d)^{n-1-1} (nx^n (-b-d) + n + 1)}{(1+n)(1+2n)} (x^n (-b-d))^{-\frac{1+n}{2n}} e^{-\frac{x^n(-b-d)}{2}} M_{n-1-\frac{1+n}{2n}, \frac{1+n}{2n}+\frac{1}{2}}(x^n (-b-d)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a+c+b\*x^n+d\*x^n), x)

[Out] exp(a+c)/n\*(-b-d)^(-1/n)\*(n^2\*x^(1-n)\*(-b-d)^(1/n-1)\*(n\*x^n\*(-b-d)+n+1)/(1+n)/(1+2\*n)\*(x^n\*(-b-d))^(-1/2\*(1+n)/n)\*exp(-1/2\*x^n\*(-b-d))\*WhittakerM(1/n-1/2\*(1+n)/n, 1/2\*(1+n)/n+1/2, x^n\*(-b-d))+n\*x^(1-n)\*(-b-d)^(1/n-1)\*(1+n)/(1+2\*n)\*(x^n\*(-b-d))^(-1/2\*(1+n)/n)\*exp(-1/2\*x^n\*(-b-d))\*WhittakerM(1/n-1/2\*(1+n)/n+1, 1/2\*(1+n)/n+1/2, x^n\*(-b-d))

---

**Maxima [A]** time = 1.09025, size = 49, normalized size = 1.32

$$\frac{x e^{(a+c)} \Gamma\left(\frac{1}{n}, -(b+d)x^n\right)}{(-(b+d)x^n)^{\left(\frac{1}{n}\right)} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a+c+b\*x^n+d\*x^n), x, algorithm="maxima")

[Out] -x\*e^(a + c)\*gamma(1/n, -(b + d)\*x^n)/((-b + d)\*x^n)^(1/n)\*n

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{((b+d)x^n+a+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a+c+b\*x^n+d\*x^n), x, algorithm="fricas")

[Out] integral(e^((b + d)\*x^n + a + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^a e^c \int e^{bx^n} e^{dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a+c+b\*x\*\*n+d\*x\*\*n), x)

[Out] exp(a)\*exp(c)\*Integral(exp(b\*x\*\*n)\*exp(d\*x\*\*n), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int e^{(bx^n+dx^n+a+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a+c+b*x^n+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x^n + d*x^n + a + c), x)
```

### 3.768 $\int f^{a+bx^n} g^{c+dx^n} dx$

**Optimal.** Leaf size=50

$$\frac{x f^a g^c \left(-x^n(b \log(f) + d \log(g))\right)^{-1/n} \Gamma\left(\frac{1}{n}, -x^n(b \log(f) + d \log(g))\right)}{n}$$

[Out]  $-\left(\left(f^a g^c x \Gamma\left[n^{(-1)}, -\left(x^n(b \log[f] + d \log[g])\right)\right]\right) / \left(n \left(-\left(x^n(b \log[f] + d \log[g])\right)\right)^{n^{(-1)}}\right)\right)$

**Rubi [A]** time = 0.0444385, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2287, 2208}

$$\frac{x f^a g^c \left(-x^n(b \log(f) + d \log(g))\right)^{-1/n} \Gamma\left(\frac{1}{n}, -x^n(b \log(f) + d \log(g))\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x^n)\*g^(c + d\*x^n), x]

[Out]  $-\left(\left(f^a g^c x \Gamma\left[n^{(-1)}, -\left(x^n(b \log[f] + d \log[g])\right)\right]\right) / \left(n \left(-\left(x^n(b \log[f] + d \log[g])\right)\right)^{n^{(-1)}}\right)\right)$

#### Rule 2287

Int[(u\_.)\*(F\_)^(v\_)\*(G\_)^(w\_), x\_Symbol] := With[{z = v\*Log[F] + w\*Log[G]}, Int[u\*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^n)), x\_Symbol] := -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rubi steps

$$\begin{aligned} \int f^{a+bx^n} g^{c+dx^n} dx &= \int \exp\left(a \log(f) + c \log(g) + x^n(b \log(f) + d \log(g))\right) dx \\ &= \frac{f^a g^c x \Gamma\left(\frac{1}{n}, -x^n(b \log(f) + d \log(g))\right) \left(-x^n(b \log(f) + d \log(g))\right)^{-1/n}}{n} \end{aligned}$$

**Mathematica [A]** time = 0.0216559, size = 50, normalized size = 1.

$$\frac{x f^a g^c \left(-x^n(b \log(f) + d \log(g))\right)^{-1/n} \Gamma\left(\frac{1}{n}, -x^n(b \log(f) + d \log(g))\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x^n)\*g^(c + d\*x^n), x]

[Out]  $-\left(\left(f^a g^c x \Gamma\left[n^(-1), -(x^n(b \log[f] + d \log[g]))\right]\right)\right) / \left(n \left(-\left(x^n(b \log[f] + d \log[g])\right)\right)^{n^(-1)}\right)$

**Maple [F]** time = 0.352, size = 0, normalized size = 0.

$$\int f^{a+bx^n} g^{c+dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(f^(a+b*x^n)*g^(c+d*x^n), x)`

[Out] `int(f^(a+b*x^n)*g^(c+d*x^n), x)`

**Maxima [A]** time = 1.24461, size = 68, normalized size = 1.36

$$\frac{f^a g^c x \Gamma\left(\frac{1}{n}, -(b \log(f) + d \log(g)) x^n\right)}{\left(-\left(b \log(f) + d \log(g)\right) x^n\right)^{\left(\frac{1}{n}\right) n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*g^(c+d*x^n), x, algorithm="maxima")`

[Out] `-f^a*g^c*x*gamma(1/n, -(b*log(f) + d*log(g))*x^n)/((-b*log(f) + d*log(g))*x^n)^(1/n)*n`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(f^{bx^n+a} g^{dx^n+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f^(a+b*x^n)*g^(c+d*x^n), x, algorithm="fricas")`

[Out] `integral(f^(b*x^n + a)*g^(d*x^n + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(a+b*x**n)*g**(c+d*x**n), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{bx^n+a} g^{dx^n+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(a+b*x^n)*g^(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(f^(b*x^n + a)*g^(d*x^n + c), x)
```

### 3.769 $\int e^{x^n} x^m dx$

**Optimal.** Leaf size=37

$$-\frac{x^{m+1} (-x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -x^n\right)}{n}$$

[Out]  $-\left(x^{(1+m)} \text{Gamma}\left[\frac{(1+m)}{n}, -x^n\right]\right) / \left(n \left(-x^n\right)^{\left(\frac{(1+m)}{n}\right)}\right)$

**Rubi [A]** time = 0.0169732, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2218}

$$-\frac{x^{m+1} (-x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[E^x^n\*x^m,x]

[Out]  $-\left(x^{(1+m)} \text{Gamma}\left[\frac{(1+m)}{n}, -x^n\right]\right) / \left(n \left(-x^n\right)^{\left(\frac{(1+m)}{n}\right)}\right)$

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)))\*((e\_.) + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^(n)\*Log[F])]) / (f\*n\*(-(b\*(c + d\*x)^(n)\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int e^{x^n} x^m dx = -\frac{x^{1+m} (-x^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -x^n\right)}{n}$$

**Mathematica [A]** time = 0.007955, size = 37, normalized size = 1.

$$-\frac{x^{m+1} (-x^n)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -x^n\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^n\*x^m,x]

[Out]  $-\left(x^{(1+m)} \text{Gamma}\left[\frac{(1+m)}{n}, -x^n\right]\right) / \left(n \left(-x^n\right)^{\left(\frac{(1+m)}{n}\right)}\right)$

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int e^{x^n} x^m dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^n)*x^m,x)`

[Out] `int(exp(x^n)*x^m,x)`

**Maxima [A]** time = 1.04909, size = 51, normalized size = 1.38

$$-\frac{x^{m+1}\Gamma\left(\frac{m+1}{n}, -x^n\right)}{n(-x^n)^{\frac{m+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^n)*x^m,x, algorithm="maxima")`

[Out] `-x^(m + 1)*gamma((m + 1)/n, -x^n)/(n*(-x^n)^((m + 1)/n))`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m e^{x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^n)*x^m,x, algorithm="fricas")`

[Out] `integral(x^m*e^(x^n), x)`

**Sympy [C]** time = 1.87454, size = 105, normalized size = 2.84

$$\frac{m e^{-\frac{i\pi}{n}} e^{-\frac{i\pi m}{n}} \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) \gamma\left(\frac{m}{n} + \frac{1}{n}, x^n e^{i\pi}\right)}{n^2 \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)} + \frac{e^{-\frac{i\pi}{n}} e^{-\frac{i\pi m}{n}} \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) \gamma\left(\frac{m}{n} + \frac{1}{n}, x^n e^{i\pi}\right)}{n^2 \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**n)*x**m,x)`

[Out] `m*exp(-I*pi/n)*exp(-I*pi*m/n)*gamma(m/n + 1/n)*lowergamma(m/n + 1/n, x**n*exp_polar(I*pi))/(n**2*gamma(m/n + 1 + 1/n)) + exp(-I*pi/n)*exp(-I*pi*m/n)*gamma(m/n + 1/n)*lowergamma(m/n + 1/n, x**n*exp_polar(I*pi))/(n**2*gamma(m/n + 1 + 1/n))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m e^{x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^n)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(x^n), x)
```

### 3.770 $\int f^{x^n} x^m dx$

**Optimal.** Leaf size=41

$$\frac{x^{m+1} (\log(f) (-x^n))^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, \log(f) (-x^n)\right)}{n}$$

[Out] -((x^(1 + m)\*Gamma[(1 + m)/n, -(x^n\*Log[f])])/(n\*(-(x^n\*Log[f]))^((1 + m)/n)))

**Rubi [A]** time = 0.015585, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2218}

$$\frac{x^{m+1} (\log(f) (-x^n))^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, \log(f) (-x^n)\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[f^x^n\*x^m,x]

[Out] -((x^(1 + m)\*Gamma[(1 + m)/n, -(x^n\*Log[f])])/(n\*(-(x^n\*Log[f]))^((1 + m)/n)))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{x^n} x^m dx = -\frac{x^{1+m} \Gamma\left(\frac{1+m}{n}, -x^n \log(f)\right) (-x^n \log(f))^{-\frac{1+m}{n}}}{n}$$

**Mathematica [A]** time = 0.0084263, size = 41, normalized size = 1.

$$\frac{x^{m+1} (\log(f) (-x^n))^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, \log(f) (-x^n)\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[f^x^n\*x^m,x]

[Out] -((x^(1 + m)\*Gamma[(1 + m)/n, -(x^n\*Log[f])])/(n\*(-(x^n\*Log[f]))^((1 + m)/n)))

**Maple [F]** time = 0.181, size = 0, normalized size = 0.

$$\int f^{x^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(x^n)\*x^m,x)

[Out] int(f^(x^n)\*x^m,x)

---

**Maxima [A]** time = 1.11877, size = 57, normalized size = 1.39

$$\frac{x^{m+1} \Gamma\left(\frac{m+1}{n}, -x^n \log(f)\right)}{(-x^n \log(f))^{\frac{m+1}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(x^n)\*x^m,x, algorithm="maxima")

[Out] -x^(m + 1)\*gamma((m + 1)/n, -x^n\*log(f))/((-x^n\*log(f))^(m + 1)/n)\*n

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(f^{(x^n)}x^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(x^n)\*x^m,x, algorithm="fricas")

[Out] integral(f^(x^n)\*x^m, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{x^n} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*(x\*\*n)\*x\*\*m,x)

[Out] Integral(f\*\*(x\*\*n)\*x\*\*m, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(x^n)}x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(x^n)*x^m,x, algorithm="giac")
```

```
[Out] integrate(f^(x^n)*x^m, x)
```

$$3.771 \quad \int e^{(a+bx)^n} (a+bx)^m dx$$

Optimal. Leaf size=52

$$-\frac{(a+bx)^{m+1} (-a+bx)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -(a+bx)^n\right)}{bn}$$

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -(a + b\*x)^n])/(b\*n\*(-(a + b\*x)^n)^(1 + m)/n)))

**Rubi [A]** time = 0.0263118, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2218}

$$-\frac{(a+bx)^{m+1} (-a+bx)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -(a+bx)^n\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x)^n\*(a + b\*x)^m,x]

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -(a + b\*x)^n])/(b\*n\*(-(a + b\*x)^n)^(1 + m)/n)))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x))^n\*Log[F]])/(f\*n\*(-(b\*(c + d\*x))^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int e^{(a+bx)^n} (a+bx)^m dx = -\frac{(a+bx)^{1+m} (-a+bx)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -(a+bx)^n\right)}{bn}$$

**Mathematica [A]** time = 0.0139617, size = 52, normalized size = 1.

$$-\frac{(a+bx)^{m+1} (-a+bx)^{-\frac{m+1}{n}} \text{Gamma}\left(\frac{m+1}{n}, -(a+bx)^n\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x)^n\*(a + b\*x)^m,x]

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -(a + b\*x)^n])/(b\*n\*(-(a + b\*x)^n)^(1 + m)/n)))

**Maple [F]** time = 0.294, size = 0, normalized size = 0.

$$\int e^{(bx+a)^n} (bx+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)^n)\*(b\*x+a)^m,x)

[Out] int(exp((b\*x+a)^n)\*(b\*x+a)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m e^{(bx+a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="maxima")

[Out] integrate((b\*x + a)^m\*e^((b\*x + a)^n), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^m e^{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="fricas")

[Out] integral((b\*x + a)^m\*e^((b\*x + a)^n), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a+bx)^m e^{(a+bx)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*\*n)\*(b\*x+a)\*\*m,x)

[Out] Integral((a + b\*x)\*\*m\*exp((a + b\*x)\*\*n), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m e^{(bx+a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="giac")

[Out] integrate((b\*x + a)^m\*e^((b\*x + a)^n), x)

$$3.772 \quad \int f^{(a+bx)^n} (a+bx)^m dx$$

**Optimal.** Leaf size=56

$$\frac{(a+bx)^{m+1} (\log(f) (-(a+bx)^n))^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, \log(f) (-(a+bx)^n)\right)}{bn}$$

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -((a + b\*x)^n\*Log[f])])/(b\*n\*(-((a + b\*x)^n\*Log[f]))^((1 + m)/n)))

**Rubi [A]** time = 0.0253398, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2218}

$$\frac{(a+bx)^{m+1} (\log(f) (-(a+bx)^n))^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, \log(f) (-(a+bx)^n)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b\*x)^n\*(a + b\*x)^m,x]

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -((a + b\*x)^n\*Log[f])])/(b\*n\*(-((a + b\*x)^n\*Log[f]))^((1 + m)/n)))

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F])])/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\int f^{(a+bx)^n} (a+bx)^m dx = -\frac{(a+bx)^{1+m} \Gamma\left(\frac{1+m}{n}, -(a+bx)^n \log(f)\right) (-(a+bx)^n \log(f))^{-\frac{1+m}{n}}}{bn}$$

**Mathematica [A]** time = 0.0137881, size = 56, normalized size = 1.

$$\frac{(a+bx)^{m+1} (\log(f) (-(a+bx)^n))^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, \log(f) (-(a+bx)^n)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b\*x)^n\*(a + b\*x)^m,x]

[Out] -(((a + b\*x)^(1 + m)\*Gamma[(1 + m)/n, -((a + b\*x)^n\*Log[f])])/(b\*n\*(-((a + b\*x)^n\*Log[f]))^((1 + m)/n)))



**Maple [F]** time = 0.34, size = 0, normalized size = 0.

$$\int f^{(bx+a)^n} (bx+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^((b\*x+a)^n)\*(b\*x+a)^m,x)

[Out] int(f^((b\*x+a)^n)\*(b\*x+a)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m f^{(bx+a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="maxima")

[Out] integrate((b\*x + a)^m\*f^((b\*x + a)^n), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)^m f^{(bx+a)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="fricas")

[Out] integral((b\*x + a)^m\*f^((b\*x + a)^n), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int f^{(a+bx)^n} (a+bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f\*\*((b\*x+a)\*\*n)\*(b\*x+a)\*\*m,x)

[Out] Integral(f\*\*((a + b\*x)\*\*n)\*(a + b\*x)\*\*m, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^m f^{(bx+a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^((b\*x+a)^n)\*(b\*x+a)^m,x, algorithm="giac")

[Out] integrate((b\*x + a)^m\*f^((b\*x + a)^n), x)

### 3.773 $\int e^{(a+bx)^3} x dx$

**Optimal.** Leaf size=80

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2(-(a+bx)^3)^{2/3}}$$

[Out] (a\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(1/3)) - ((a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

**Rubi [A]** time = 0.0436073, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2226, 2208, 2218}

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2(-(a+bx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b\*x)^3\*x, x]

[Out] (a\*(a + b\*x)\*Gamma[1/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(1/3)) - ((a + b\*x)^2\*Gamma[2/3, -(a + b\*x)^3])/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

#### Rule 2226

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* (u\_), x\_Symbol] :> Int[ExpandLinearProduct[F^(a + b\*(c + d\*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

#### Rule 2208

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> -Simp[(F^a\*(c + d\*x)\*Gamma[1/n, -(b\*(c + d\*x)^n\*Log[F]])]/(d\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

#### Rule 2218

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\* ((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^a\*(e + f\*x)^(m + 1)\*Gamma[(m + 1)/n, -(b\*(c + d\*x)^n\*Log[F]])]/(f\*n\*(-(b\*(c + d\*x)^n\*Log[F]))^(m + 1)/n), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int e^{(a+bx)^3} x dx &= \int \left( -\frac{ae^{(a+bx)^3}}{b} + \frac{e^{(a+bx)^3}(a+bx)}{b} \right) dx \\ &= \frac{\int e^{(a+bx)^3}(a+bx) dx}{b} - \frac{a \int e^{(a+bx)^3} dx}{b} \\ &= \frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2(-(a+bx)^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.0380104, size = 74, normalized size = 0.92

$$\frac{(a + bx) \left( a \sqrt[3]{-(a + bx)^3} \Gamma\left(\frac{1}{3}, -(a + bx)^3\right) - (a + bx) \Gamma\left(\frac{2}{3}, -(a + bx)^3\right) \right)}{3b^2 (-(a + bx)^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a + b\*x)^3\*x,x]

[Out] ((a + b\*x)\*(a\*(-(a + b\*x)^3)^(1/3)\*Gamma[1/3, -(a + b\*x)^3] - (a + b\*x)\*Gamma[2/3, -(a + b\*x)^3]))/(3\*b^2\*(-(a + b\*x)^3)^(2/3))

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int e^{(bx+a)^3} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((b\*x+a)^3)\*x,x)

[Out] int(exp((b\*x+a)^3)\*x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x e^{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^3)\*x,x, algorithm="maxima")

[Out] integrate(x\*e^((b\*x + a)^3), x)

**Fricas [A]** time = 0.754021, size = 203, normalized size = 2.54

$$\frac{(-b^3)^{\frac{2}{3}} a \Gamma\left(\frac{1}{3}, -b^3 x^3 - 3 a b^2 x^2 - 3 a^2 b x - a^3\right) - (-b^3)^{\frac{1}{3}} b \Gamma\left(\frac{2}{3}, -b^3 x^3 - 3 a b^2 x^2 - 3 a^2 b x - a^3\right)}{3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^3)\*x,x, algorithm="fricas")

[Out] -1/3\*((-b^3)^(2/3)\*a\*gamma(1/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3) - (-b^3)^(1/3)\*b\*gamma(2/3, -b^3\*x^3 - 3\*a\*b^2\*x^2 - 3\*a^2\*b\*x - a^3))/b^4

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{a^3} \int x e^{b^3 x^3} e^{3 a b^2 x^2} e^{3 a^2 b x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)\*\*3)\*x,x)

[Out] exp(a\*\*3)\*Integral(x\*exp(b\*\*3\*x\*\*3)\*exp(3\*a\*b\*\*2\*x\*\*2)\*exp(3\*a\*\*2\*b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x e^{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp((b\*x+a)^3)\*x,x, algorithm="giac")

[Out] integrate(x\*e^((b\*x + a)^3), x)

$$3.774 \quad \int \frac{5x^2 + 3\sqrt[3]{e^x + x} + e^x(3x + 2x^2)}{x\sqrt[3]{e^x + x}} dx$$

**Optimal.** Leaf size=17

$$3(x + e^x)^{2/3}x + 3\log(x)$$

[Out] 3\*x\*(E^x + x)^(2/3) + 3\*Log[x]

**Rubi [A]** time = 0.635028, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$ , Rules used = {6742, 2261, 2273, 2262}

$$3(x + e^x)^{2/3}x + 3\log(x)$$

Antiderivative was successfully verified.

[In] Int[(5\*x^2 + 3\*(E^x + x)^(1/3) + E^x\*(3\*x + 2\*x^2))/(x\*(E^x + x)^(1/3)),x]

[Out] 3\*x\*(E^x + x)^(2/3) + 3\*Log[x]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2261

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) \* ((b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) + (a\_.)\*(x\_)^(n\_.))^((p\_.), x\_Symbol] := Simp[(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x] && NeQ[p, -1]

#### Rule 2273

Int[(x\_)^(m\_.)\*(E^(x\_) + (x\_)^(m\_.))^((n\_.), x\_Symbol] := -Simp[(E^x + x^m)^(n + 1)/(n + 1), x] + (Dist[m, Int[x^(m - 1)\*(E^x + x^m)^n, x], x] + Int[(E^x + x^m)^(n + 1), x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[n, -1]

#### Rule 2262

Int[(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) \* (x\_)^(m\_.) \* ((b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))) + (a\_.)\*(x\_)^(n\_.))^((p\_.), x\_Symbol] := Simp[(x^m\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1)/(b\*d\*e\*(p + 1)\*Log[F]), x] + (-Dist[m/(b\*d\*e\*(p + 1)\*Log[F]), Int[x^(m - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^(p + 1), x], x] - Dist[(a\*n)/(b\*d\*e\*Log[F]), Int[x^(m + n - 1)\*(a\*x^n + b\*F^(e\*(c + d\*x)))^p, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{5x^2 + 3\sqrt[3]{e^x + x} + e^x(3x + 2x^2)}{x\sqrt[3]{e^x + x}} dx &= \int \left( \frac{3}{x} + \frac{3e^x}{\sqrt[3]{e^x + x}} + \frac{(5 + 2e^x)x}{\sqrt[3]{e^x + x}} \right) dx \\
&= 3 \log(x) + 3 \int \frac{e^x}{\sqrt[3]{e^x + x}} dx + \int \frac{(5 + 2e^x)x}{\sqrt[3]{e^x + x}} dx \\
&= \frac{9}{2} (e^x + x)^{2/3} + 3 \log(x) - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx + \int \left( \frac{5x}{\sqrt[3]{e^x + x}} + \frac{2e^x x}{\sqrt[3]{e^x + x}} \right) dx \\
&= \frac{9}{2} (e^x + x)^{2/3} + 3 \log(x) + 2 \int \frac{e^x x}{\sqrt[3]{e^x + x}} dx - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx + 5 \int \frac{x}{\sqrt[3]{e^x + x}} dx \\
&= -3 (e^x + x)^{2/3} + 3x (e^x + x)^{2/3} + 3 \log(x) - 2 \int \frac{x}{\sqrt[3]{e^x + x}} dx - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx - 3 \\
&= 3x (e^x + x)^{2/3} + 3 \log(x) - 2 \int \frac{1}{\sqrt[3]{e^x + x}} dx - 2 \int (e^x + x)^{2/3} dx - 3 \int \frac{1}{\sqrt[3]{e^x + x}} dx
\end{aligned}$$

**Mathematica [A]** time = 0.220314, size = 17, normalized size = 1.

$$3(x + e^x)^{2/3} x + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5\*x^2 + 3\*(E^x + x)^(1/3) + E^x\*(3\*x + 2\*x^2))/(x\*(E^x + x)^(1/3)), x]

[Out] 3\*x\*(E^x + x)^(2/3) + 3\*Log[x]

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x} (5x^2 + 3\sqrt[3]{e^x + x} + e^x(2x^2 + 3x)) \frac{1}{\sqrt[3]{e^x + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+3\*(exp(x)+x)^(1/3)+exp(x)\*(2\*x^2+3\*x))/x/(exp(x)+x)^(1/3), x)

[Out] int((5\*x^2+3\*(exp(x)+x)^(1/3)+exp(x)\*(2\*x^2+3\*x))/x/(exp(x)+x)^(1/3), x)

**Maxima [A]** time = 1.09234, size = 28, normalized size = 1.65

$$\frac{3(x^2 + xe^x)}{(x + e^x)^{1/3}} + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*(x+exp(x))^(1/3)+exp(x)\*(2\*x^2+3\*x))/x/(x+exp(x))^(1/3), x, algorithm="maxima")

[Out] 3\*(x^2 + x\*e^x)/(x + e^x)^(1/3) + 3\*log(x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*(x+exp(x))^(1/3)+exp(x)\*(2\*x^2+3\*x))/x/(x+exp(x))^(1/3), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2e^x + 5x^2 + 3xe^x + 3\sqrt[3]{x + e^x}}{x\sqrt[3]{x + e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+3\*(exp(x)+x)\*\*(1/3)+exp(x)\*(2\*x\*\*2+3\*x))/x/(exp(x)+x)\*\*(1/3), x)

[Out] Integral((2\*x\*\*2\*exp(x) + 5\*x\*\*2 + 3\*x\*exp(x) + 3\*(x + exp(x))\*\*(1/3))/(x\*(x + exp(x))\*\*(1/3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + (2x^2 + 3x)e^x + 3(x + e^x)^{\frac{1}{3}}}{(x + e^x)^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+3\*(x+exp(x))^(1/3)+exp(x)\*(2\*x^2+3\*x))/x/(x+exp(x))^(1/3), x, algorithm="giac")

[Out] integrate((5\*x^2 + (2\*x^2 + 3\*x)\*e^x + 3\*(x + e^x)^(1/3))/((x + e^x)^(1/3)\*x), x)





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+`') or type(expn,'*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```



```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```