

# Computer algebra independent integration tests

## 0-Independent-test-suites/Welz-Problems

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 116 ]. This is test number [ 11 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 86.21 ( 100 )	% 13.79 ( 16 )
Mathematica	% 81.9 ( 95 )	% 18.1 ( 21 )
Maple	% 51.72 ( 60 )	% 48.28 ( 56 )
Maxima	% 16.38 ( 19 )	% 83.62 ( 97 )
Fricas	% 75.86 ( 88 )	% 24.14 ( 28 )
Sympy	% 25. ( 29 )	% 75. ( 87 )
Giac	% 14.66 ( 17 )	% 85.34 ( 99 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

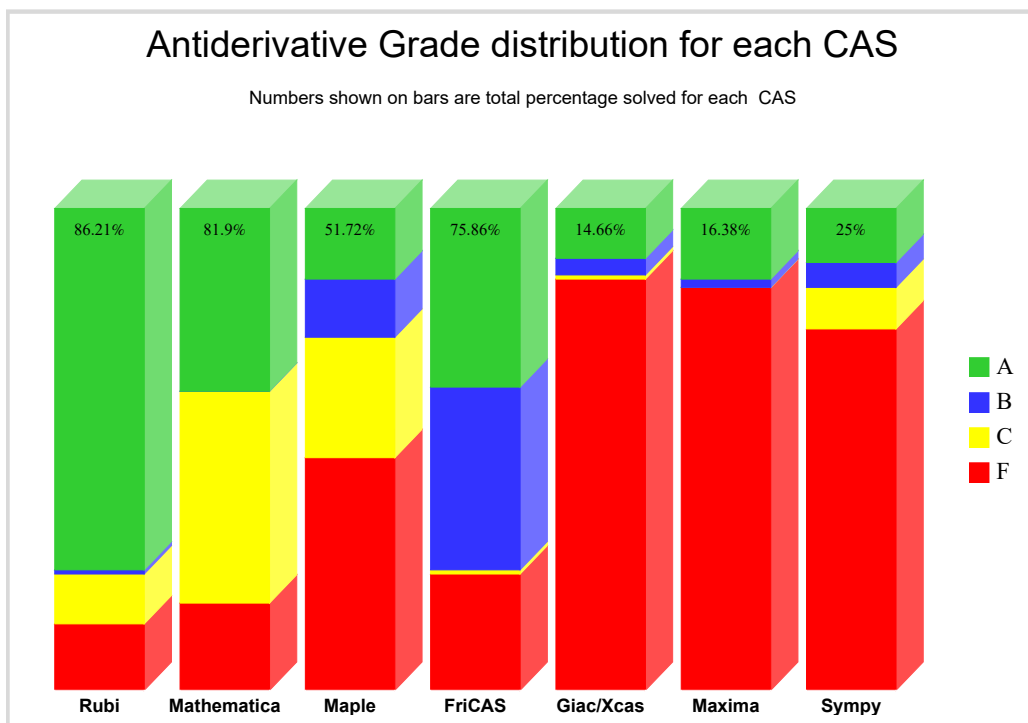
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

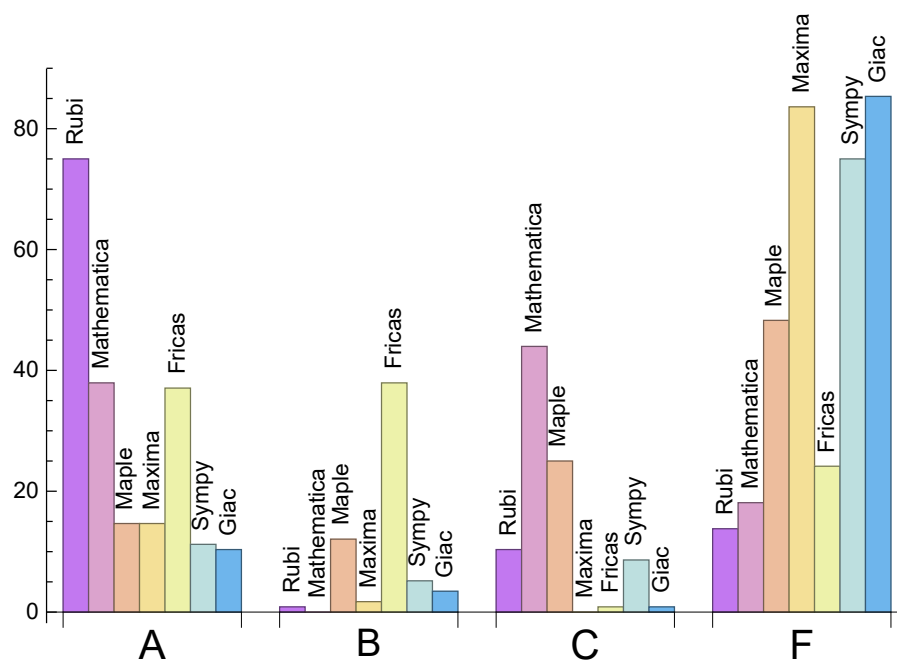
System	% A grade	% B grade	% C grade	% F grade
Rubi	75.	0.86	10.34	13.79
Mathematica	37.93	0.	43.97	18.1
Maple	14.66	12.07	25.	48.28
Maxima	14.66	1.72	0.	83.62
Fricas	37.07	37.93	0.86	24.14
Sympy	11.21	5.17	8.62	75.
Giac	10.34	3.45	0.86	85.34



The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	144.1	6.45	81.	1.
Mathematica	0.71	211.54	3.69	86.	0.99
Maple	0.12	2061.15	11.33	144.5	1.56
Maxima	1.21	108.53	1.57	84.	1.41
Fricas	13.72	2890.17	17.35	491.	5.07
Sympy	3.27	202.38	4.97	37.	0.82
Giac	1.09	100.71	1.95	86.	1.48

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {9, 10, 82, 98, 113, 114, 116}

**Mathematica** {4, 9, 10, 39, 40, 51, 53, 54, 55, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

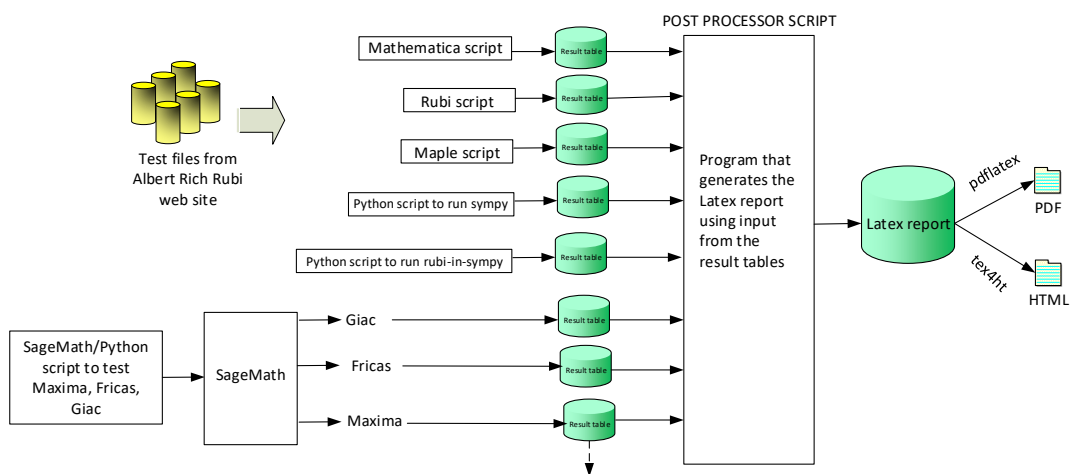
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

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# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 105, 106, 107 }

B grade: { 10 }

C grade: { 2, 46, 52, 82, 83, 98, 100, 101, 102, 113, 114, 116 }

F grade: { 43, 44, 45, 58, 59, 60, 61, 95, 103, 104, 108, 109, 110, 111, 112, 115 }

#### 2.1.2 Mathematica

A grade: { 1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 57, 62, 85, 94, 96, 97, 99, 103, 104, 105, 107 }

B grade: { }

C grade: { 2, 4, 24, 39, 40, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116 }

F grade: { 12, 13, 37, 38, 44, 45, 46, 58, 59, 60, 61, 92, 93, 95, 100, 102, 108, 109, 110, 111, 112 }

#### 2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 63, 64, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 52, 56, 57, 66, 67, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 106, 107 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 58, 59, 60, 61, 69, 70, 71, 72, 77, 78, 79, 80, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

#### 2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 99, 107 }

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 47, 48, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 29, 38, 44, 45, 46, 49, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 94, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

## 2.1.6 Sympy

A grade: { 1, 2, 14, 15, 20, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

## 2.1.7 Giac

A grade: { 1, 8, 16, 21, 22, 23, 33, 34, 36, 57, 62, 107 }

B grade: { 2, 3, 4, 7 }

C grade: { 24 }

F grade: { 5, 6, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 25, 26, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$



Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	28	12	18
normalized size	1	1.	1.	0.93	1.2	1.87	0.8	1.2
time (sec)	N/A	0.001	0.004	0.002	0.925	2.09	0.055	1.072

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	55	4	42	55
normalized size	1	3.47	2.47	2.27	3.67	0.27	2.8	3.67
time (sec)	N/A	0.056	0.026	0.003	0.948	2.141	57.851	1.071

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	370	0	251	0	239
normalized size	1	1.	0.84	4.51	0.	3.06	0.	2.91
time (sec)	N/A	0.076	0.108	0.041	0.	2.081	0.	1.111

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	167	172	0	205	0	127
normalized size	1	1.	3.88	4.	0.	4.77	0.	2.95
time (sec)	N/A	0.012	3.397	0.047	0.	2.048	0.	1.087

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	126	115	0	309	0	0
normalized size	1	1.	1.7	1.55	0.	4.18	0.	0.
time (sec)	N/A	0.056	0.132	0.014	0.	2.128	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	59	125	72	267	0	0
normalized size	1	1.	0.92	1.95	1.12	4.17	0.	0.
time (sec)	N/A	0.026	0.059	0.025	1.429	2.192	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	75	45	0	215	0	136
normalized size	1	1.	1.56	0.94	0.	4.48	0.	2.83
time (sec)	N/A	0.013	0.107	0.017	0.	2.017	0.	1.088

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	0	55	53	27
normalized size	1	1.	1.	0.7	0.	1.83	1.77	0.9
time (sec)	N/A	0.082	0.041	0.003	0.	2.008	0.818	1.071

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	220	365	340	902	0	1287	0	0
normalized size	1	1.66	1.55	4.1	0.	5.85	0.	0.
time (sec)	N/A	0.506	0.688	0.123	0.	2.291	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	220	541	311	1542	0	1287	0	0
normalized size	1	2.46	1.41	7.01	0.	5.85	0.	0.
time (sec)	N/A	0.751	0.649	0.021	0.	2.36	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	125	278	0	678	0	0
normalized size	1	1.	0.91	2.01	0.	4.91	0.	0.
time (sec)	N/A	0.075	0.199	0.02	0.	2.281	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	1068	0	0
normalized size	1	1.	0.	0.	0.	8.54	0.	0.
time (sec)	N/A	0.185	0.239	0.026	0.	19.344	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	998	0	0
normalized size	1	1.	0.	0.	0.	12.32	0.	0.
time (sec)	N/A	0.162	0.176	0.022	0.	38.964	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	162	15	0
normalized size	1	1.	1.	0.	0.	5.23	0.48	0.
time (sec)	N/A	0.054	0.008	0.014	0.	4.026	1.281	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	0	85	15	0
normalized size	1	1.	1.	0.67	0.	2.58	0.45	0.
time (sec)	N/A	0.063	0.011	0.033	0.	4.074	0.706	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	78	56	20
normalized size	1	1.	1.	0.84	1.05	4.11	2.95	1.05
time (sec)	N/A	0.27	0.017	0.002	0.934	1.972	6.489	1.097

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	74	2149	0
normalized size	1	1.	0.83	2.31	0.	1.42	41.33	0.
time (sec)	N/A	0.023	0.039	0.026	0.	2.204	2.733	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	76	0	0
normalized size	1	1.	0.89	0.	0.	1.36	0.	0.
time (sec)	N/A	0.022	0.063	0.023	0.	2.155	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	34	313	0
normalized size	1	1.	1.	0.	0.	2.	18.41	0.
time (sec)	N/A	0.054	0.007	0.019	0.	1.992	3.679	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	35	36	0
normalized size	1	1.	1.	0.	0.	1.75	1.8	0.
time (sec)	N/A	0.06	0.006	0.028	0.	1.985	2.868	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	48	54	124	36	55
normalized size	1	1.	0.86	1.14	1.29	2.95	0.86	1.31
time (sec)	N/A	0.033	0.042	0.01	0.947	2.076	0.132	1.08

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	43	20	26
normalized size	1	1.	1.	0.95	1.23	1.95	0.91	1.18
time (sec)	N/A	0.024	0.02	0.002	0.935	1.956	0.103	1.075

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	69	135	51	100
normalized size	1	1.	0.79	0.82	1.11	2.18	0.82	1.61
time (sec)	N/A	0.091	0.065	0.012	0.955	2.189	0.158	1.079

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	961	454	0	1095	0	424
normalized size	1	1.	11.17	5.28	0.	12.73	0.	4.93
time (sec)	N/A	0.084	2.38	0.034	0.	2.264	0.	1.137

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	39	15	0
normalized size	1	1.	1.	0.	0.	2.05	0.79	0.
time (sec)	N/A	0.065	0.01	0.027	0.	2.025	0.206	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	47	27	0
normalized size	1	1.	1.	0.	0.	1.81	1.04	0.
time (sec)	N/A	0.097	0.017	0.015	0.	1.975	0.879	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	487	46	0
normalized size	1	1.	0.89	0.	0.	7.73	0.73	0.
time (sec)	N/A	0.219	0.192	0.019	0.	2.422	1.593	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	552	51	0
normalized size	1	1.	1.55	0.3	0.	6.73	0.62	0.
time (sec)	N/A	0.072	0.057	0.008	0.	2.562	1.438	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	606	679	412	0	699	0	0	0
normalized size	1	1.12	0.68	0.	1.15	0.	0.	0.
time (sec)	N/A	4.799	0.244	0.072	1.019	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	34	313	0
normalized size	1	1.	1.	0.	0.	2.	18.41	0.
time (sec)	N/A	0.055	0.009	0.026	0.	2.1	2.814	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	74	2149	0
normalized size	1	1.	0.88	2.31	0.	1.42	41.33	0.
time (sec)	N/A	0.024	0.06	0.022	0.	2.249	2.653	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	65	109	0	0
normalized size	1	1.	0.97	1.29	1.91	3.21	0.	0.
time (sec)	N/A	0.044	0.118	0.039	1.138	2.183	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	65	84	198	36	86
normalized size	1	1.	0.98	1.12	1.45	3.41	0.62	1.48
time (sec)	N/A	0.036	0.015	0.036	1.417	1.964	0.928	1.074

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	81	48	84	200	37	86
normalized size	1	1.	1.4	0.83	1.45	3.45	0.64	1.48
time (sec)	N/A	0.035	0.012	0.041	1.419	2.159	0.95	1.066

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	105	225	29	0
normalized size	1	1.	1.76	0.24	2.14	4.59	0.59	0.
time (sec)	N/A	0.005	0.04	0.023	1.44	2.032	0.894	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	84	198	32	85
normalized size	1	1.	1.	1.18	1.53	3.6	0.58	1.55
time (sec)	N/A	0.033	0.012	0.037	1.434	2.026	0.915	1.085

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	788	0	0
normalized size	1	1.	0.	0.	0.	8.12	0.	0.
time (sec)	N/A	0.044	0.076	0.033	0.	37.366	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.078	0.033	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	176	59	0	0	828	0	0
normalized size	1	1.6	0.54	0.	0.	7.53	0.	0.
time (sec)	N/A	0.025	0.025	0.074	0.	18.574	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	131	85	0	0	410	0	0
normalized size	1	1.62	1.05	0.	0.	5.06	0.	0.
time (sec)	N/A	0.073	0.014	0.011	0.	4.684	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	117	127	0	0	1153	0	0
normalized size	1	1.77	1.92	0.	0.	17.47	0.	0.
time (sec)	N/A	0.056	0.082	0.042	0.	13.516	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	145	140	0	0	1866	0	0
normalized size	1	1.84	1.77	0.	0.	23.62	0.	0.
time (sec)	N/A	0.098	0.167	0.012	0.	12.361	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	118	0	55	0	0	3509	0	0
normalized size	1	0.	0.47	0.	0.	29.74	0.	0.
time (sec)	N/A	21.937	0.193	0.014	0.	167.475	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	1.635	0.038	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.462	0.669	0.039	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	493	576	0	0	0	0	0	0
normalized size	1	1.17	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.846	0.382	0.177	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	198	584	0	5156	0	0
normalized size	1	1.	0.49	1.43	0.	12.67	0.	0.
time (sec)	N/A	0.676	2.075	0.089	0.	3.179	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	610	719	0	11992	0	0
normalized size	1	1.	0.94	1.11	0.	18.51	0.	0.
time (sec)	N/A	1.158	6.087	0.053	0.	3.69	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	989	0	0	0	0
normalized size	1	1.	1.04	0.93	0.	0.	0.	0.
time (sec)	N/A	2.49	6.17	0.08	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	342	21028	0	11537	0	0
normalized size	1	1.	0.9	55.63	0.	30.52	0.	0.
time (sec)	N/A	0.774	6.027	0.533	0.	4.008	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	30714	0	0
normalized size	1	1.	2.24	136.04	0.	48.14	0.	0.
time (sec)	N/A	1.397	11.52	3.525	0.	6.015	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	1272	0	0
normalized size	1	3.09	3.23	19.32	0.	19.27	0.	0.
time (sec)	N/A	1.234	1.148	0.094	0.	3.062	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.234	0.033	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.245	0.046	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	111	0	0	495	0	0
normalized size	1	1.	1.14	0.	0.	5.1	0.	0.
time (sec)	N/A	0.013	0.06	0.166	0.	20.738	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	107	20	69	142	262	32	0
normalized size	1	1.47	0.27	0.95	1.95	3.59	0.44	0.
time (sec)	N/A	0.044	0.003	0.046	1.428	2.053	1.106	0.



Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	90	49	96	225	37	97
normalized size	1	1.	1.34	0.73	1.43	3.36	0.55	1.45
time (sec)	N/A	0.037	0.024	0.033	1.465	1.953	0.945	1.146

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-2)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	482	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.355	0.048	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	280	0	0	0	0	10386	0	0
normalized size	1	0.	0.	0.	0.	37.09	0.	0.
time (sec)	N/A	0.235	0.107	0.086	0.	51.38	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-2)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.343	0.048	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.138	0.115	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	63	22	31
normalized size	1	1.	1.	0.96	1.24	2.52	0.88	1.24
time (sec)	N/A	0.023	0.008	0.001	0.937	2.302	0.097	1.078

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	158	73	0
normalized size	1	1.	1.46	0.88	0.	2.68	1.24	0.
time (sec)	N/A	0.032	0.018	0.024	0.	1.979	0.16	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	213	100	0
normalized size	1	1.	1.27	0.96	0.	2.73	1.28	0.
time (sec)	N/A	0.077	0.02	0.018	0.	1.942	0.167	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	138	0	0
normalized size	1	1.	2.24	2.04	0.	2.82	0.	0.
time (sec)	N/A	0.007	0.097	0.013	0.	2.516	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	173	0	0
normalized size	1	1.	2.04	6.89	0.	3.26	0.	0.
time (sec)	N/A	0.015	0.091	0.024	0.	2.422	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	5727	1512	0	957	0	0
normalized size	1	1.	76.36	20.16	0.	12.76	0.	0.
time (sec)	N/A	0.094	7.084	0.071	0.	2.568	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	6283	0	0
normalized size	1	1.	1.88	2.67	0.	36.74	0.	0.
time (sec)	N/A	0.08	0.432	0.072	0.	25.347	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0
normalized size	1	1.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.261	0.058	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.251	0.029	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.203	0.048	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.175	0.026	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	3299	0	0
normalized size	1	1.	0.22	1.29	0.	25.98	0.	0.
time (sec)	N/A	0.019	0.022	0.099	0.	5.567	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	4625	0	0
normalized size	1	1.	0.34	1.53	0.	29.46	0.	0.
time (sec)	N/A	0.033	0.029	0.106	0.	5.796	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	48	286	0	1447	0	0
normalized size	1	1.	0.65	3.86	0.	19.55	0.	0.
time (sec)	N/A	0.156	0.019	0.115	0.	3.586	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	1214	0	0
normalized size	1	1.	0.31	3.72	0.	11.79	0.	0.
time (sec)	N/A	0.305	0.029	0.16	0.	4.282	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	5180	0	0
normalized size	1	1.	1.56	0.	0.	63.95	0.	0.
time (sec)	N/A	0.011	0.1	0.048	0.	19.52	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	921	0	0
normalized size	1	1.	1.56	0.	0.	11.37	0.	0.
time (sec)	N/A	0.011	0.095	0.026	0.	15.508	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	5544	0	0
normalized size	1	1.	1.04	0.	0.	49.06	0.	0.
time (sec)	N/A	0.014	0.072	0.033	0.	11.962	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	4852	0	0
normalized size	1	1.	1.14	0.	0.	44.51	0.	0.
time (sec)	N/A	0.012	0.063	0.036	0.	9.774	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	193	0	0
normalized size	1	1.	1.83	2.37	0.	2.22	0.	0.
time (sec)	N/A	0.874	0.916	0.034	0.	2.36	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	C	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	1	529	127	317	0	150	0	0
normalized size	1	529.	127.	317.	0.	150.	0.	0.
time (sec)	N/A	1.669	0.586	0.041	0.	2.283	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	144	0	0
normalized size	1	3.91	2.89	11.65	0.	3.13	0.	0.
time (sec)	N/A	1.492	1.072	0.047	0.	2.315	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0
normalized size	1	1.	10.09	8.06	0.	0.	0.	0.
time (sec)	N/A	0.096	0.449	0.059	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	117	0	0
normalized size	1	1.	2.	10.43	0.	5.09	0.	0.
time (sec)	N/A	0.054	0.007	0.021	0.	2.202	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	47	353	0	27647	0	0
normalized size	1	1.	0.22	1.62	0.	126.82	0.	0.
time (sec)	N/A	0.041	0.056	0.195	0.	49.077	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	50	350	0	28331	0	0
normalized size	1	1.	0.24	1.67	0.	134.91	0.	0.
time (sec)	N/A	0.032	0.07	0.173	0.	49.042	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	65	349	0	28004	0	0
normalized size	1	1.	0.29	1.57	0.	126.14	0.	0.
time (sec)	N/A	0.03	0.066	0.173	0.	44.341	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	68	350	0	28069	0	0
normalized size	1	1.	0.32	1.64	0.	131.16	0.	0.
time (sec)	N/A	0.029	0.053	0.169	0.	41.119	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	953	0	0
normalized size	1	1.	10.54	5.03	0.	14.66	0.	0.
time (sec)	N/A	0.129	3.205	0.132	0.	3.467	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	301	0	0
normalized size	1	1.	13.9	4.94	0.	4.78	0.	0.
time (sec)	N/A	0.128	7.546	0.129	0.	2.834	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.243	0.029	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	0	0	0	1422	0	0
normalized size	1	1.	0.	0.	0.	13.17	0.	0.
time (sec)	N/A	0.093	0.057	0.023	0.	9.281	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	135	120	0	0	0	0	0
normalized size	1	1.38	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.155	0.046	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.288	0.088	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	80	0	0	983	0	0
normalized size	1	1.	0.83	0.	0.	10.24	0.	0.
time (sec)	N/A	0.078	0.081	0.044	0.	2.351	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	122	112	0	0	660	0	0
normalized size	1	1.39	1.27	0.	0.	7.5	0.	0.
time (sec)	N/A	0.062	0.143	0.037	0.	26.951	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	1049	0	0
normalized size	1	0.11	0.11	0.	0.	4.5	0.	0.
time (sec)	N/A	0.011	0.027	0.036	0.	23.868	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	116	282	0	0
normalized size	1	1.	0.89	0.	1.41	3.44	0.	0.
time (sec)	N/A	0.058	0.06	0.039	1.4	2.12	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	409	0	0	0	882	0	0
normalized size	1	3.03	0.	0.	0.	6.53	0.	0.
time (sec)	N/A	0.337	0.19	0.079	0.	111.657	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	409	150	0	0	882	0	0
normalized size	1	3.03	1.11	0.	0.	6.53	0.	0.
time (sec)	N/A	0.301	0.441	0.036	0.	99.462	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	399	0	0	0	737	0	0
normalized size	1	3.35	0.	0.	0.	6.19	0.	0.
time (sec)	N/A	0.299	0.163	0.069	0.	93.427	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0
normalized size	1	0.	1.	0.79	0.	0.	0.	0.
time (sec)	N/A	0.425	0.151	0.036	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0
normalized size	1	0.	1.	0.79	0.	0.	0.	0.
time (sec)	N/A	0.22	0.075	0.027	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0
normalized size	1	1.	1.1	0.87	0.	0.	0.	0.
time (sec)	N/A	0.029	0.018	0.013	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	142	258	31	0
normalized size	1	1.	1.51	0.18	2.12	3.85	0.46	0.
time (sec)	N/A	0.01	0.103	0.025	1.397	1.599	1.16	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	66	99	230	41	100
normalized size	1	1.	0.93	0.94	1.41	3.29	0.59	1.43
time (sec)	N/A	0.038	0.038	0.035	1.396	1.559	1.026	1.097

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.345	0.053	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.415	0.452	0.091	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	5191	0	0
normalized size	1	0.	0.	0.	0.	26.09	0.	0.
time (sec)	N/A	0.945	0.348	0.088	0.	14.505	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.39	0.041	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.15	0.011	0.	0.	0.	0.



Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	132	21	111	0	0	532	0	0
normalized size	1	0.16	0.84	0.	0.	4.03	0.	0.
time (sec)	N/A	0.008	0.094	0.038	0.	1.677	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	250	26	26	0	0	0	0	0
normalized size	1	0.1	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.014	0.037	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	383	0	138	0	0	0	0	0
normalized size	1	0.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	0.153	0.039	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	272	21	109	0	0	902	0	0
normalized size	1	0.08	0.4	0.	0.	3.32	0.	0.
time (sec)	N/A	0.008	0.088	0.038	0.	21.83	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [ 2.143 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	10	0.1
2	C	5	2	3.47	37	0.054
3	A	9	6	1.	15	0.4
4	A	3	3	1.	19	0.158
5	A	8	7	1.	17	0.412
6	A	6	6	1.	17	0.353
7	A	3	3	1.	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	4	2	1.	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.	45	0.067
12	A	7	4	1.	32	0.125
13	A	5	3	1.	32	0.094
14	A	2	2	1.	27	0.074
15	A	2	2	1.	29	0.069
16	A	2	1	1.	30	0.033
17	A	3	2	1.	13	0.154
18	A	3	2	1.	15	0.133
19	A	2	2	1.	23	0.087
20	A	2	2	1.	25	0.08
21	A	3	2	1.	11	0.182
22	A	2	2	1.	18	0.111
23	A	6	6	1.	20	0.3
24	A	6	5	1.	31	0.161
25	A	2	2	1.	29	0.069
26	A	2	2	1.	35	0.057
27	A	5	5	1.	32	0.156
28	A	6	6	1.	21	0.286
29	A	359	30	1.12	14	2.143
30	A	2	2	1.	23	0.087
31	A	3	2	1.	13	0.154
32	A	2	2	1.	33	0.061
33	A	5	5	1.	15	0.333
34	A	5	5	1.	15	0.333
35	A	1	1	1.	11	0.091
36	A	5	5	1.	15	0.333
37	A	1	1	1.	17	0.059
38	A	3	3	1.	18	0.167
39	A	2	2	1.6	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.17	32	0.094
47	A	19	9	1.	20	0.45
48	A	29	9	1.	20	0.45
49	A	49	9	1.	20	0.45
50	A	14	6	1.	23	0.261
51	A	24	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	C	9	8	3.09	48	0.167
53	A	7	7	1.	24	0.292
54	A	7	7	1.	24	0.292
55	A	1	1	1.	18	0.056
56	A	8	8	1.47	13	0.615
57	A	6	6	1.	15	0.4
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	F	0	0	N/A	0	N/A
61	F	0	0	N/A	0	N/A
62	A	1	1	1.	38	0.026
63	A	1	1	1.	33	0.03
64	A	2	2	1.	39	0.051
65	A	1	1	1.	19	0.053
66	A	4	4	1.	19	0.21
67	A	4	4	1.	24	0.167
68	A	1	1	1.	24	0.042
69	A	7	7	1.	24	0.292
70	A	7	7	1.	24	0.292
71	A	3	3	1.	26	0.115
72	A	3	3	1.	22	0.136
73	A	1	1	1.	22	0.045
74	A	1	1	1.	23	0.043
75	A	8	8	1.	18	0.444
76	A	8	7	1.	23	0.304
77	A	1	1	1.	21	0.048
78	A	1	1	1.	19	0.053
79	A	1	1	1.	19	0.053
80	A	1	1	1.	19	0.053
81	A	4	4	1.	34	0.118
82	C	5	5	529.	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.	29	0.069
85	A	2	2	1.	18	0.111
86	A	1	1	1.	25	0.04
87	A	1	1	1.	25	0.04
88	A	1	1	1.	25	0.04
89	A	1	1	1.	25	0.04
90	A	2	2	1.	40	0.05
91	A	2	2	1.	40	0.05
92	A	1	1	1.	18	0.056
93	A	3	3	1.	15	0.2
94	A	7	7	1.38	21	0.333
95	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	5	1.	24	0.208
97	A	7	7	1.39	19	0.368
98	C	1	1	0.11	20	0.05
99	A	5	5	1.	22	0.227
100	C	4	2	3.03	25	0.08
101	C	5	3	3.03	24	0.125
102	C	4	2	3.35	23	0.087
103	F	0	0	N/A	0	N/A
104	F	0	0	N/A	0	N/A
105	A	3	3	1.	19	0.158
106	A	2	2	1.	11	0.182
107	A	6	6	1.	15	0.4
108	F	0	0	N/A	0	N/A
109	F	0	0	N/A	0	N/A
110	F	0	0	N/A	0	N/A
111	F	0	0	N/A	0	N/A
112	F	0	0	N/A	0	N/A
113	C	1	1	0.16	19	0.053
114	C	1	1	0.1	20	0.05
115	F	0	0	N/A	0	N/A
116	C	1	1	0.08	19	0.053

# Chapter 3

## Listing of integrals

### 3.1 $\int \frac{1}{\sqrt{1-ax}} dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] (-2\*Sqrt[1 - a\*x])/a

---

**Rubi [A]** time = 0.0014936, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {32}

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - a\*x],x]

[Out] (-2\*Sqrt[1 - a\*x])/a

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rubi steps

$$\int \frac{1}{\sqrt{1-ax}} dx = -\frac{2\sqrt{1-ax}}{a}$$

**Mathematica [A]** time = 0.004261, size = 15, normalized size = 1.

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - a\*x],x]

[Out] (-2\*Sqrt[1 - a\*x])/a

**Maple [A]** time = 0.002, size = 14, normalized size = 0.9

$$-2 \frac{\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a\*x+1)^(1/2),x)

[Out] -2\*(-a\*x+1)^(1/2)/a

**Maxima [A]** time = 0.924617, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x+1)^(1/2),x, algorithm="maxima")

[Out] -2\*sqrt(-a\*x + 1)/a

**Fricas [A]** time = 2.0903, size = 28, normalized size = 1.87

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x+1)^(1/2),x, algorithm="fricas")

[Out] -2\*sqrt(-a\*x + 1)/a

**Sympy [A]** time = 0.055326, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x+1)\*\*(1/2),x)

[Out] -2\*sqrt(-a\*x + 1)/a

---

**Giac [A]** time = 1.07191, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x+1)^(1/2),x, algorithm="giac")

[Out] -2\*sqrt(-a\*x + 1)/a

$$3.2 \quad \int \frac{-2 \log(-\sqrt{-1+ax}) + \log(-1+ax)}{2\pi\sqrt{-1+ax}} dx$$

**Optimal.** Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] (-2\*Sqrt[1 - a\*x])/a

**Rubi [C]** time = 0.0558809, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$ , Rules used = {12, 2295}

$$\frac{\sqrt{ax-1} \log(ax-1)}{\pi a} - \frac{2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a}$$

Antiderivative was successfully verified.

[In] Int[(-2\*Log[-Sqrt[-1 + a\*x]] + Log[-1 + a\*x])/(2\*Pi\*Sqrt[-1 + a\*x]),x]

[Out] (-2\*Sqrt[-1 + a\*x]\*Log[-Sqrt[-1 + a\*x]])/(a\*Pi) + (Sqrt[-1 + a\*x]\*Log[-1 + a\*x])/(a\*Pi)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{-2 \log(-\sqrt{-1+ax}) + \log(-1+ax)}{2\pi\sqrt{-1+ax}} dx &= \frac{\int \frac{-2 \log(-\sqrt{-1+ax}) + \log(-1+ax)}{\sqrt{-1+ax}} dx}{2\pi} \\ &= \frac{\text{Subst}\left(\int (-2 \log(-x) + \log(x^2)) dx, x, \sqrt{-1+ax}\right)}{a\pi} \\ &= \frac{\text{Subst}\left(\int \log(x^2) dx, x, \sqrt{-1+ax}\right)}{a\pi} - \frac{2 \text{Subst}\left(\int \log(-x) dx, x, \sqrt{-1+ax}\right)}{a\pi} \\ &= -\frac{2\sqrt{-1+ax} \log(-\sqrt{-1+ax})}{a\pi} + \frac{\sqrt{-1+ax} \log(-1+ax)}{a\pi} \end{aligned}$$

**Mathematica [C]** time = 0.025668, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1} (\log(ax-1) - 2 \log(-\sqrt{ax-1}))}{\pi a}$$

Antiderivative was successfully verified.



[In] Integrate[(-2\*Log[-Sqrt[-1 + a\*x]] + Log[-1 + a\*x])/(2\*Pi\*Sqrt[-1 + a\*x]),x]

[Out] (Sqrt[-1 + a\*x]\*(-2\*Log[-Sqrt[-1 + a\*x]] + Log[-1 + a\*x]))/(a\*Pi)

**Maple [C]** time = 0.003, size = 34, normalized size = 2.3

$$\frac{1}{a\pi} \sqrt{ax-1} \left( \ln(ax-1) - 2 \ln\left(-\sqrt{ax-1}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2\*(ln(a\*x-1)-2\*ln(-(a\*x-1)^(1/2)))/Pi/(a\*x-1)^(1/2),x)

[Out] (a\*x-1)^(1/2)\*(ln(a\*x-1)-2\*ln(-(a\*x-1)^(1/2)))/a/Pi

**Maxima [B]** time = 0.947855, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1} \log(ax-1) - 2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2\*(log(a\*x-1)-2\*log(-(a\*x-1)^(1/2)))/pi/(a\*x-1)^(1/2),x, algorithm="maxima")

[Out] (sqrt(a\*x - 1)\*log(a\*x - 1) - 2\*sqrt(a\*x - 1)\*log(-sqrt(a\*x - 1)))/(pi\*a)

**Fricas [A]** time = 2.14124, size = 4, normalized size = 0.27

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2\*(log(a\*x-1)-2\*log(-(a\*x-1)^(1/2)))/pi/(a\*x-1)^(1/2),x, algorithm="fricas")

[Out] 0

**Sympy [A]** time = 57.8506, size = 42, normalized size = 2.8

$$\frac{\begin{cases} \frac{-2\sqrt{ax-1} \log(-\sqrt{ax-1}) + \sqrt{ax-1} \log(ax-1)}{a} & \text{for } a \neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2\*(ln(a\*x-1)-2\*ln(-(a\*x-1)\*\*(1/2)))/pi/(a\*x-1)\*\*(1/2),x)

```
[Out] Piecewise((( -2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x - 1))/a, Ne(a, 0)), (pi*x, True))/pi
```

---

**Giac [B]** time = 1.07061, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1} \log(ax-1) - 2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

$$3.3 \quad \int \frac{1}{(2x + \sqrt{1+x^2})^2} dx$$

**Optimal.** Leaf size=82

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}}$$

[Out] (4\*x)/(3\*(1 - 3\*x^2)) - (2\*Sqrt[1 + x^2])/(3\*(1 - 3\*x^2)) - ArcTanh[Sqrt[3]\*x]/(3\*Sqrt[3]) + ArcTanh[(Sqrt[3]\*Sqrt[1 + x^2])/2]/(3\*Sqrt[3])

**Rubi [A]** time = 0.0758864, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6742, 199, 207, 444, 47, 63}

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2\*x + Sqrt[1 + x^2])^(-2), x]

[Out] (4\*x)/(3\*(1 - 3\*x^2)) - (2\*Sqrt[1 + x^2])/(3\*(1 - 3\*x^2)) - ArcTanh[Sqrt[3]\*x]/(3\*Sqrt[3]) + ArcTanh[(Sqrt[3]\*Sqrt[1 + x^2])/2]/(3\*Sqrt[3])

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 199**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

**Rule 207**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

**Rule 444**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

**Rule 47**

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^n/(b\*(m + 1)), x] - Dist[(d\*n)/(b\*(m + 1)), I

```

nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]

```

### Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(2x + \sqrt{1+x^2})^2} dx &= \int \left( \frac{8}{3(-1+3x^2)^2} - \frac{4x\sqrt{1+x^2}}{(-1+3x^2)^2} + \frac{5}{3(-1+3x^2)} \right) dx \\
&= \frac{5}{3} \int \frac{1}{-1+3x^2} dx + \frac{8}{3} \int \frac{1}{(-1+3x^2)^2} dx - 4 \int \frac{x\sqrt{1+x^2}}{(-1+3x^2)^2} dx \\
&= \frac{4x}{3(1-3x^2)} - \frac{5 \tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} - \frac{4}{3} \int \frac{1}{-1+3x^2} dx - 2 \operatorname{Subst} \left( \int \frac{\sqrt{1+x}}{(-1+3x)^2} dx, x, x^2 \right) \\
&= \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} - \frac{1}{3} \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+x}(-1+3x)} dx, x, x^2 \right) \\
&= \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} - \frac{2}{3} \operatorname{Subst} \left( \int \frac{1}{-4+3x^2} dx, x, \sqrt{1+x^2} \right) \\
&= \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.107544, size = 69, normalized size = 0.84

$$\frac{1}{9} \left( \frac{6(\sqrt{x^2+1}-2x)}{3x^2-1} + \sqrt{3} \tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right) - \sqrt{3} \tanh^{-1}(\sqrt{3}x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2), x]
```

```
[Out] ((6*(-2*x + Sqrt[1 + x^2]))/(-1 + 3*x^2) - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2])/9
```

**Maple [B]** time = 0.041, size = 370, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2\*x+(x^2+1)^(1/2))^2,x)

[Out] 
$$\begin{aligned} & -1/2*x/(3*x^2-1)-1/9*\operatorname{arctanh}(x*3^{(1/2)})*3^{(1/2)}-5/18*x/(x^2-1/3)-3^{(1/2)}*(- \\ & 1/12/(x-1/3*3^{(1/2)})*((x-1/3*3^{(1/2)})^2+2/3*3^{(1/2)}*(x-1/3*3^{(1/2)})+4/3)^{(3/2)}+1/36*3^{(1/2)}*(1/3*(9*(x-1/3*3^{(1/2)})^2+6*3^{(1/2)}*(x-1/3*3^{(1/2)})+12)^{(1/2)}+1/3*3^{(1/2)}*\operatorname{arcsinh}(x)-2/3*3^{(1/2)}*\operatorname{arctanh}(3/4*(8/3+2/3*3^{(1/2)}*(x-1/3*3^{(1/2)}))) *3^{(1/2)}/(9*(x-1/3*3^{(1/2)})^2+6*3^{(1/2)}*(x-1/3*3^{(1/2)})+12)^{(1/2)}) \\ & +1/12*x*((x-1/3*3^{(1/2)})^2+2/3*3^{(1/2)}*(x-1/3*3^{(1/2)})+4/3)^{(1/2)}+1/12*\operatorname{arcsinh}(x))+3^{(1/2)}*(-1/12/(x+1/3*3^{(1/2)})*((x+1/3*3^{(1/2)})^2-2/3*3^{(1/2)}*(x+1/3*3^{(1/2)})+4/3)^{(3/2)}-1/36*3^{(1/2)}*(1/3*(9*(x+1/3*3^{(1/2)})^2-6*3^{(1/2)}*(x+1/3*3^{(1/2)})+12)^{(1/2)}-1/3*3^{(1/2)}*\operatorname{arcsinh}(x)-2/3*3^{(1/2)}*\operatorname{arctanh}(3/4*(8/3-2/3*3^{(1/2)}*(x+1/3*3^{(1/2)}))) *3^{(1/2)}/(9*(x+1/3*3^{(1/2)})^2-6*3^{(1/2)}*(x+1/3*3^{(1/2)})+12)^{(1/2)}) \\ & +1/12*x*((x+1/3*3^{(1/2)})^2-2/3*3^{(1/2)}*(x+1/3*3^{(1/2)})+4/3)^{(1/2)}+1/12*\operatorname{arcsinh}(x)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + \sqrt{x^2 + 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((2\*x + sqrt(x^2 + 1))^( -2), x)

**Fricas [A]** time = 2.08059, size = 251, normalized size = 3.06

$$\frac{\sqrt{3}(3x^2 - 1) \log\left(\frac{3x^2 - 2\sqrt{3}x + 1}{3x^2 - 1}\right) + \sqrt{3}(3x^2 - 1) \log\left(\frac{3x^2 + 4\sqrt{3}\sqrt{x^2 + 1} + 7}{3x^2 - 1}\right) - 24x + 12\sqrt{x^2 + 1}}{18(3x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/18*(\operatorname{sqrt}(3)*(3*x^2 - 1)*\log((3*x^2 - 2*\operatorname{sqrt}(3)*x + 1)/(3*x^2 - 1)) + \operatorname{sqrt}(3) \\ & *(3*x^2 - 1)*\log((3*x^2 + 4*\operatorname{sqrt}(3)*\operatorname{sqrt}(x^2 + 1) + 7)/(3*x^2 - 1)) - 24*x + 12*\operatorname{sqrt}(x^2 + 1))/(3*x^2 - 1) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + \sqrt{x^2 + 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x+(x\*\*2+1)\*\*(1/2))\*\*2,x)

[Out] Integral((2\*x + sqrt(x\*\*2 + 1))\*\*(-2), x)

**Giac [B]** time = 1.11115, size = 239, normalized size = 2.91

$$\frac{1}{18} \sqrt{3} \log\left(\frac{|6x - 2\sqrt{3}|}{|6x + 2\sqrt{3}|}\right) - \frac{1}{18} \sqrt{3} \log\left(\frac{\left| -6x - 8\sqrt{3} + 6\sqrt{x^2 + 1} - \frac{6}{x - \sqrt{x^2 + 1}} \right|}{2\left(3x - 4\sqrt{3} - 3\sqrt{x^2 + 1} + \frac{3}{x - \sqrt{x^2 + 1}}\right)}\right) - \frac{4\left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2 + 1}}\right)}{3\left(3\left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2 + 1}}\right)^2 - 16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2\*x+(x^2+1)^(1/2))^2,x, algorithm="giac")

[Out] 1/18\*sqrt(3)\*log(abs(6\*x - 2\*sqrt(3))/abs(6\*x + 2\*sqrt(3))) - 1/18\*sqrt(3)\*log(-1/2\*abs(-6\*x - 8\*sqrt(3) + 6\*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3\*x - 4\*sqrt(3) - 3\*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3\*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3\*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))))^2 - 16) - 4/3\*x/(3\*x^2 - 1)

$$3.4 \quad \int \frac{1}{\sqrt{-1+x^2}(-4+3x^2)^2} dx$$

**Optimal.** Leaf size=43

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)} + \frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

[Out] (3\*x\*Sqrt[-1 + x^2])/(8\*(4 - 3\*x^2)) + (5\*ArcTanh[x/(2\*Sqrt[-1 + x^2])])/16

**Rubi [A]** time = 0.0123968, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {382, 377, 207}

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)} + \frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x^2]\*(-4 + 3\*x^2)^2), x]

[Out] (3\*x\*Sqrt[-1 + x^2])/(8\*(4 - 3\*x^2)) + (5\*ArcTanh[x/(2\*Sqrt[-1 + x^2])])/16

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x^2}(-4+3x^2)^2} dx &= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8} \int \frac{1}{\sqrt{-1+x^2}(-4+3x^2)} dx \\ &= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8} \text{Subst}\left(\int \frac{1}{-4+x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\ &= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} + \frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right) \end{aligned}$$

**Mathematica [C]** time = 3.39706, size = 167, normalized size = 3.88

$$\frac{x\sqrt{x^2-1} \left( \frac{8x^2(x^2-1) {}_2F_1\left(2, 3; \frac{7}{2}; \frac{x^2}{4-3x^2}\right)}{45x^2-60} - \frac{x^2(2x^2-3)\sqrt{\frac{x^2-1}{3x^2-4}} \left( 2\sqrt{\frac{x^2-x^4}{(4-3x^2)^2}} - \sin^{-1}\left(\sqrt{\frac{x^2}{4-3x^2}}\right) \right)}{4\left(\frac{x^2}{4-3x^2}\right)^{5/2}(x^2-1)} \right)}{16\left(1 - \frac{3x^2}{4}\right)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[-1 + x^2]\*(-4 + 3\*x^2)^2), x]

[Out]  $-(x\sqrt{-1+x^2}*(-(x^2*(-3+2x^2)*\sqrt{(-1+x^2)/(-4+3x^2)})*(2*\sqrt{(x^2-x^4)/(4-3x^2)^2}-\text{ArcSin}[\sqrt{x^2/(4-3x^2)}]))/(4*(x^2/(4-3x^2))^{5/2}*(-1+x^2))+(8*x^2*(-1+x^2)*\text{Hypergeometric2F1}[2, 3, 7/2, x^2/(4-3x^2)]/(-60+45*x^2)))/(16*(1-(3*x^2)/4)^2)$

**Maple [B]** time = 0.047, size = 172, normalized size = 4.

$$-\frac{1}{16}\sqrt{\left(x + \frac{2\sqrt{3}}{3}\right)^2 - \frac{4\sqrt{3}}{3}\left(x + \frac{2\sqrt{3}}{3}\right) + \frac{1}{3}\left(x + \frac{2\sqrt{3}}{3}\right)^{-1}} - \frac{5}{32}\text{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3} - \frac{4\sqrt{3}}{3}\left(x + \frac{2\sqrt{3}}{3}\right)\right)\right) \frac{1}{\sqrt{9\left(x + \frac{2\sqrt{3}}{3}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3\*x^2-4)^2/(x^2-1)^(1/2), x)

[Out]  $-1/16/(x+2/3*3^{(1/2)})*((x+2/3*3^{(1/2)})^2-4/3*3^{(1/2)}*(x+2/3*3^{(1/2)})+1/3)^{(1/2)}-5/32*\text{arctanh}(3/2*(2/3-4/3*3^{(1/2)}*(x+2/3*3^{(1/2)}))*3^{(1/2)}/(9*(x+2/3*3^{(1/2)})^2-12*3^{(1/2)}*(x+2/3*3^{(1/2)})+3)^{(1/2)})-1/16/(x-2/3*3^{(1/2)})*((x-2/3*3^{(1/2)})^2+4/3*3^{(1/2)}*(x-2/3*3^{(1/2)})+1/3)^{(1/2)}+5/32*\text{arctanh}(3/2*(2/3+4/3*3^{(1/2)}*(x-2/3*3^{(1/2)}))*3^{(1/2)}/(9*(x-2/3*3^{(1/2)})^2+12*3^{(1/2)}*(x-2/3*3^{(1/2)})+3)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-4)^2\sqrt{x^2-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x^2-4)^2/(x^2-1)^(1/2), x, algorithm="maxima")

[Out] integrate(1/((3\*x^2-4)^2\*sqrt(x^2-1)), x)

**Fricas [B]** time = 2.04815, size = 205, normalized size = 4.77

$$\frac{12x^2 + 5(3x^2 - 4)\log\left(3x^2 - 3\sqrt{x^2-1}x - 2\right) - 5(3x^2 - 4)\log\left(x^2 - \sqrt{x^2-1}x - 2\right) + 12\sqrt{x^2-1}x - 16}{32(3x^2 - 4)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")

[Out]  $-1/32*(12*x^2 + 5*(3*x^2 - 4)*\log(3*x^2 - 3*\sqrt{x^2 - 1}*x - 2) - 5*(3*x^2 - 4)*\log(x^2 - \sqrt{x^2 - 1}*x - 2) + 12*\sqrt{x^2 - 1}*x - 16)/(3*x^2 - 4)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}(3x^2-4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x\*\*2-4)\*\*2/(x\*\*2-1)\*\*(1/2),x)

[Out] Integral(1/(sqrt((x - 1)\*(x + 1))\*(3\*x\*\*2 - 4)\*\*2), x)

**Giac [B]** time = 1.08749, size = 127, normalized size = 2.95

$$\frac{5(x - \sqrt{x^2 - 1})^2 - 3}{4(3(x - \sqrt{x^2 - 1})^4 - 10(x - \sqrt{x^2 - 1})^2 + 3)} - \frac{5}{32} \log\left(\left|3(x - \sqrt{x^2 - 1})^2 - 1\right|\right) + \frac{5}{32} \log\left(\left|(x - \sqrt{x^2 - 1})^2 - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")

[Out]  $1/4*(5*(x - \sqrt{x^2 - 1})^2 - 3)/(3*(x - \sqrt{x^2 - 1})^4 - 10*(x - \sqrt{x^2 - 1})^2 + 3) - 5/32*\log(\text{abs}(3*(x - \sqrt{x^2 - 1})^2 - 1)) + 5/32*\log(\text{abs}((x - \sqrt{x^2 - 1})^2 - 3))$

$$3.5 \quad \int \frac{1}{(2\sqrt{x} + \sqrt{1+x})^2} dx$$

**Optimal.** Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)} + \frac{8}{9(1-3x)} + \frac{5}{9} \log(1-3x) - \frac{8}{9} \sinh^{-1}(\sqrt{x}) + \frac{10}{9} \tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

[Out] 8/(9\*(1 - 3\*x)) - (4\*Sqrt[x]\*Sqrt[1 + x])/(3\*(1 - 3\*x)) - (8\*ArcSinh[Sqrt[x]])/9 + (10\*ArcTanh[(2\*Sqrt[x])/Sqrt[1 + x]])/9 + (5\*Log[1 - 3\*x])/9

**Rubi [A]** time = 0.0559415, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {6742, 97, 157, 54, 215, 93, 207}

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)} + \frac{8}{9(1-3x)} + \frac{5}{9} \log(1-3x) - \frac{8}{9} \sinh^{-1}(\sqrt{x}) + \frac{10}{9} \tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2\*Sqrt[x] + Sqrt[1 + x])^(-2), x]

[Out] 8/(9\*(1 - 3\*x)) - (4\*Sqrt[x]\*Sqrt[1 + x])/(3\*(1 - 3\*x)) - (8\*ArcSinh[Sqrt[x]])/9 + (10\*ArcTanh[(2\*Sqrt[x])/Sqrt[1 + x]])/9 + (5\*Log[1 - 3\*x])/9

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 97

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p)/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p - 1)\*Simp[d\*e\*n + c\*f\*p + d\*f\*(n + p)\*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

#### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

#### Rule 54

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_)]\*Sqrt[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b\*c - a\*d + d\*x^2], x], x, Sqrt[a + b\*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b\*c - a\*d, 0] && GtQ[b, 0]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(2\sqrt{x} + \sqrt{1+x})^2} dx &= \int \left( \frac{8}{3(-1+3x)^2} - \frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2} + \frac{5}{3(-1+3x)} \right) dx \\ &= \frac{8}{9(1-3x)} + \frac{5}{9} \log(1-3x) - 4 \int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2} dx \\ &= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9} \log(1-3x) - \frac{4}{3} \int \frac{\frac{1}{2} + x}{\sqrt{x}\sqrt{1+x}(-1+3x)} dx \\ &= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9} \log(1-3x) - \frac{4}{9} \int \frac{1}{\sqrt{x}\sqrt{1+x}} dx - \frac{10}{9} \int \frac{1}{\sqrt{x}\sqrt{1+x}(-1+3x)} dx \\ &= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9} \log(1-3x) - \frac{8}{9} \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x} \right) - \frac{20}{9} \text{Subst} \left( \int \frac{1}{\sqrt{x}\sqrt{1+x}(-1+3x)} dx, x, \sqrt{x} \right) \\ &= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} - \frac{8}{9} \sinh^{-1}(\sqrt{x}) + \frac{10}{9} \tanh^{-1} \left( \frac{2\sqrt{x}}{\sqrt{1+x}} \right) + \frac{5}{9} \log(1-3x) \end{aligned}$$

**Mathematica [A]** time = 0.132287, size = 126, normalized size = 1.7

$$\frac{12x^{3/2} + 12\sqrt{x} - 8\sqrt{x+1} + 15\sqrt{x+1}x \log(1-3x) - 5\sqrt{x+1} \log(1-3x) + 10\sqrt{-x-1}(3x-1) \tan^{-1} \left( \frac{2\sqrt{x}}{\sqrt{-x-1}} \right) - 8\sqrt{x+1}}{9\sqrt{x+1}(3x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2), x]
```

```
[Out] (12*Sqrt[x] + 12*x^(3/2) - 8*Sqrt[1 + x] - 8*Sqrt[1 + x]*(-1 + 3*x)*ArcSinh[Sqrt[x]] + 10*Sqrt[-1 - x]*(-1 + 3*x)*ArcTan[(2*Sqrt[x])/Sqrt[-1 - x]] - 5*Sqrt[1 + x]*Log[1 - 3*x] + 15*x*Sqrt[1 + x]*Log[1 - 3*x])/(9*Sqrt[1 + x]*(-1 + 3*x))
```

**Maple [B]** time = 0.014, size = 115, normalized size = 1.6

$$-\frac{8}{27x-9} + \frac{5 \ln(3x-1)}{9} - \frac{1}{27x-9} \sqrt{x}\sqrt{1+x} \left( 12 \ln \left( \frac{1}{2} + x + \sqrt{x(1+x)} \right) x - 15 \operatorname{Artanh} \left( \frac{1}{4} \frac{1+5x}{\sqrt{x(1+x)}} \right) x - 4 \ln \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x)`

[Out] 
$$-8/9/(3*x-1)+5/9*\ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*\ln(1/2+x+(x*(1+x))^(1/2))*x-15*\operatorname{arctanh}(1/4*(1+5*x)/(x*(1+x))^(1/2))*x-4*\ln(1/2+x+(x*(1+x))^(1/2)))+5*\operatorname{arctanh}(1/4*(1+5*x)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2)/(x*(1+x))^(1/2)/(3*x-1)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sqrt{x+1} + 2\sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")`

[Out] `integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)`

**Fricas [A]** time = 2.12849, size = 309, normalized size = 4.18

$$\frac{5(3x-1)\log(3\sqrt{x+1}\sqrt{x}-3x-1)-4(3x-1)\log(2\sqrt{x+1}\sqrt{x}-2x-1)-5(3x-1)\log(\sqrt{x+1}\sqrt{x}-x+1)-5(3x-1)\log(\sqrt{x+1}\sqrt{x}-x-1)}{9(3x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")`

[Out] 
$$-1/9*(5*(3*x-1)*\log(3*\sqrt{x+1}*\sqrt{x}-3*x-1)-4*(3*x-1)*\log(2*\sqrt{x+1}*\sqrt{x}-2*x-1)-5*(3*x-1)*\log(\sqrt{x+1}*\sqrt{x}-x+1)-5*(3*x-1)*\log(\sqrt{x+1}*\sqrt{x}-x-1))-12*\sqrt{x+1}*\sqrt{x}-12*x+12)/(3*x-1)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\sqrt{x} + \sqrt{x+1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)`

[Out] `Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)`

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

### 3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} dx$

**Optimal.** Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

[Out] Sqrt[-1 + x^2]/(I - x) - (I\*ArcTan[(1 - I\*x)/(Sqrt[2]\*Sqrt[-1 + x^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]

**Rubi [A]** time = 0.0260134, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {733, 844, 217, 206, 725, 204}

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^2]/(-I + x)^2, x]

[Out] Sqrt[-1 + x^2]/(I - x) - (I\*ArcTan[(1 - I\*x)/(Sqrt[2]\*Sqrt[-1 + x^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]

#### Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)),
Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1])
&& NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

[{a, c, d, e}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1+x^2}}{(-i+x)^2} dx &= \frac{\sqrt{-1+x^2}}{i-x} + \int \frac{x}{(-i+x)\sqrt{-1+x^2}} dx \\ &= \frac{\sqrt{-1+x^2}}{i-x} + i \int \frac{1}{(-i+x)\sqrt{-1+x^2}} dx + \int \frac{1}{\sqrt{-1+x^2}} dx \\ &= \frac{\sqrt{-1+x^2}}{i-x} - i \operatorname{Subst}\left(\int \frac{1}{-2-x^2} dx, x, \frac{-1+ix}{\sqrt{-1+x^2}}\right) + \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\ &= \frac{\sqrt{-1+x^2}}{i-x} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0594532, size = 59, normalized size = 0.92

$$-\frac{\sqrt{x^2-1}}{x-i} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right) - \frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2, x]

[Out] -(Sqrt[-1 + x^2]/(-I + x)) + ArcTanh[x/Sqrt[-1 + x^2]] - ArcTanh[(I + x)/(Sqrt[2]\*Sqrt[-1 + x^2])]/Sqrt[2]

**Maple [B]** time = 0.025, size = 125, normalized size = 2.

$$\frac{1}{2x-2i} \left( (x-i)^2 - 2 + 2i(x-i) \right)^{\frac{3}{2}} + \ln \left( x + \sqrt{(x-i)^2 - 2 + 2i(x-i)} \right) + \frac{i}{2} \sqrt{2} \arctan \left( \frac{(-4 + 2i(x-i)) \sqrt{2}}{4 \sqrt{(x-i)^2 - 2 + 2i(x-i)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)^(1/2)/(x-I)^2, x)

[Out] 1/2/(x-I)\*((x-I)^2-2+2\*I\*(x-I))^(3/2)+ln(x+((x-I)^2-2+2\*I\*(x-I))^(1/2))+1/2\*I\*2^(1/2)\*arctan(1/4\*(-4+2\*I\*(x-I))\*2^(1/2)/((x-I)^2-2+2\*I\*(x-I))^(1/2))-1/2\*I\*((x-I)^2-2+2\*I\*(x-I))^(1/2)-1/2\*x\*((x-I)^2-2+2\*I\*(x-I))^(1/2)

**Maxima [A]** time = 1.429, size = 72, normalized size = 1.12

$$\frac{1}{2}i\sqrt{2} \arcsin\left(\frac{ix}{|x-i|} - \frac{1}{|x-i|}\right) - \frac{\sqrt{x^2-1}}{x-i} + \log\left(2x + 2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")

[Out] 1/2\*I\*sqrt(2)\*arcsin(I\*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I) + log(2\*x + 2\*sqrt(x^2 - 1))

**Fricas [A]** time = 2.19194, size = 267, normalized size = 4.17

$$\frac{\sqrt{2}(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+(2x-2i)\log\left(-x+\sqrt{x^2-1}\right)+2x}{2x-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")

[Out] -(sqrt(2)\*(x - I)\*log(-x + I\*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)\*(x - I) \*log(-x - I\*sqrt(2) + sqrt(x^2 - 1) + I) + (2\*x - 2\*I)\*log(-x + sqrt(x^2 - 1)) + 2\*x + 2\*sqrt(x^2 - 1) - 2\*I)/(2\*x - 2\*I)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2-1)\*\*(1/2)/(-I+x)\*\*2,x)

[Out] Integral(sqrt((x - 1)\*(x + 1))/(x - I)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}}{(x-i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")

[Out] integrate(sqrt(x^2 - 1)/(x - I)^2, x)



$$3.7 \quad \int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx$$

**Optimal.** Leaf size=48

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

[Out]  $-(x\sqrt{-1+x^2})/(4*(1+x^2)) + (3*\text{ArcTanh}[(\text{Sqrt}[2]*x)/\text{Sqrt}[-1+x^2]])/(4*\text{Sqrt}[2])$

**Rubi [A]** time = 0.0134643, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {382, 377, 206}

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[-1+x^2]*(1+x^2)^2), x]$

[Out]  $-(x\sqrt{-1+x^2})/(4*(1+x^2)) + (3*\text{ArcTanh}[(\text{Sqrt}[2]*x)/\text{Sqrt}[-1+x^2]])/(4*\text{Sqrt}[2])$

#### Rule 382

$\text{Int}[(a_+ + (b_+)(x_+)^{n_+})^{p_+}((c_+ + (d_+)(x_+)^{n_+})^{q_+}), x\_Symbol]$   
 $\rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q, x], x] /;$   $\text{FreeQ}[\{a, b, c, d, n, q\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[n*(p+q+2)+1, 0]$  &&  $(\text{LtQ}[p, -1] \mid \mid \text{!LtQ}[q, -1])$  &&  $\text{NeQ}[p, -1]$

#### Rule 377

$\text{Int}[(a_+ + (b_+)(x_+)^{n_+})^{p_+}/((c_+ + (d_+)(x_+)^{n_+})^{q_+}), x\_Symbol]$   $\rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /;$   $\text{FreeQ}[\{a, b, c, d\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[n*p + 1, 0]$  &&  $\text{IntegerQ}[n]$

#### Rule 206

$\text{Int}[(a_+ + (b_+)(x_+)^2)^{-1}], x\_Symbol]$   $\rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$   $\text{FreeQ}[\{a, b\}, x]$  &&  $\text{NegQ}[a/b]$  &&  $(\text{GtQ}[a, 0] \mid \mid \text{LtQ}[b, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)} + \frac{3}{4} \int \frac{1}{\sqrt{-1+x^2}(1+x^2)} dx \\
&= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)} + \frac{3}{4} \text{Subst} \left( \int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt{-1+x^2}} \right) \\
&= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)} + \frac{3 \tanh^{-1} \left( \frac{\sqrt{2}x}{\sqrt{-1+x^2}} \right)}{4\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.106882, size = 75, normalized size = 1.56

$$\frac{\sqrt{x^2-1} \left( 3\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}(x^2+1) \tanh^{-1} \left( \sqrt{2}\sqrt{\frac{x^2}{x^2-1}} \right) - 2x^2 \right)}{8(x^3+x)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + x^2]\*(1 + x^2)^2),x]

[Out] (Sqrt[-1 + x^2]\*(-2\*x^2 + 3\*Sqrt[2]\*Sqrt[x^2/(-1 + x^2)]\*(1 + x^2)\*ArcTanh[Sqrt[2]\*Sqrt[x^2/(-1 + x^2)]])/(8\*(x + x^3))

**Maple [A]** time = 0.017, size = 45, normalized size = 0.9

$$-\frac{x}{8} \frac{1}{\sqrt{x^2-1}} \left( \frac{x^2}{x^2-1} - \frac{1}{2} \right)^{-1} + \frac{3\sqrt{2}}{8} \text{Artanh} \left( x\sqrt{2} \frac{1}{\sqrt{x^2-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x)

[Out] -1/8\*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8\*arctanh(x\*2^(1/2)/(x^2-1)^(1/2))\*2^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)^2\sqrt{x^2-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 1)^2\*sqrt(x^2 - 1)), x)

**Fricas [B]** time = 2.01657, size = 215, normalized size = 4.48

$$\frac{3\sqrt{2}(x^2+1)\log\left(\frac{9x^2+2\sqrt{2}(3x^2-1)+2\sqrt{x^2-1}(3\sqrt{2}x+4x)-3}{x^2+1}\right)-4x^2-4\sqrt{x^2-1}x-4}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")

[Out] 1/16\*(3\*sqrt(2)\*(x^2 + 1)\*log((9\*x^2 + 2\*sqrt(2)\*(3\*x^2 - 1) + 2\*sqrt(x^2 - 1)\*(3\*sqrt(2)\*x + 4\*x) - 3)/(x^2 + 1)) - 4\*x^2 - 4\*sqrt(x^2 - 1)\*x - 4)/(x^2 + 1)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2+1)\*\*2/(x\*\*2-1)\*\*(1/2),x)

[Out] Exception raised: ValueError

**Giac [B]** time = 1.08795, size = 136, normalized size = 2.83

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{(x-\sqrt{x^2-1})^2-2\sqrt{2}+3}{(x-\sqrt{x^2-1})^2+2\sqrt{2}+3}\right)-\frac{3(x-\sqrt{x^2-1})^2+1}{2\left((x-\sqrt{x^2-1})^4+6(x-\sqrt{x^2-1})^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")

[Out] -3/16\*sqrt(2)\*log(((x - sqrt(x^2 - 1))^2 - 2\*sqrt(2) + 3)/((x - sqrt(x^2 - 1))^2 + 2\*sqrt(2) + 3)) - 1/2\*(3\*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1))^4 + 6\*(x - sqrt(x^2 - 1))^2 + 1)

$$3.8 \quad \int \frac{1}{(\sqrt{-1+x}+\sqrt{x})^2 \sqrt{-1+x}} dx$$

**Optimal.** Leaf size=30

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

[Out] 2\*Sqrt[-1 + x] + (4\*(-1 + x)^(3/2))/3 - (4\*x^(3/2))/3

**Rubi [A]** time = 0.0824455, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {6689, 43}

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2\*Sqrt[-1 + x]),x]

[Out] 2\*Sqrt[-1 + x] + (4\*(-1 + x)^(3/2))/3 - (4\*x^(3/2))/3

#### Rule 6689

Int[(u\_)\*((e\_)\*Sqrt[(a\_.) + (b\_.)\*(x\_)^(n\_.)] + (f\_.)\*Sqrt[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(m\_), x\_Symbol] := Dist[(a\*e^2 - c\*f^2)^m, Int[ExpandIntegrand[u/(e\*Sqrt[a + b\*x^n] - f\*Sqrt[c + d\*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b\*e^2 - d\*f^2, 0]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(\sqrt{-1+x}+\sqrt{x})^2 \sqrt{-1+x}} dx &= \int \left( -\frac{1}{\sqrt{-1+x}} - 2\sqrt{x} + \frac{2x}{\sqrt{-1+x}} \right) dx \\ &= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2 \int \frac{x}{\sqrt{-1+x}} dx \\ &= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2 \int \left( \frac{1}{\sqrt{-1+x}} + \sqrt{-1+x} \right) dx \\ &= 2\sqrt{-1+x} + \frac{4}{3}(-1+x)^{3/2} - \frac{4x^{3/2}}{3} \end{aligned}$$

**Mathematica [A]** time = 0.0413616, size = 30, normalized size = 1.

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2\*Sqrt[-1 + x]),x]

[Out] 2\*Sqrt[-1 + x] + (4\*(-1 + x)^(3/2))/3 - (4\*x^(3/2))/3

**Maple [A]** time = 0.003, size = 21, normalized size = 0.7

$$\frac{4}{3}(-1+x)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x)

[Out] 4/3\*(-1+x)^(3/2)-4/3\*x^(3/2)+2\*(-1+x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}(\sqrt{x-1} + \sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")

[Out] integrate(1/(sqrt(x - 1)\*(sqrt(x - 1) + sqrt(x))^2), x)

**Fricas [A]** time = 2.00777, size = 55, normalized size = 1.83

$$\frac{2}{3}(2x+1)\sqrt{x-1} - \frac{4}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")

[Out] 2/3\*(2\*x + 1)\*sqrt(x - 1) - 4/3\*x^(3/2)

**Sympy [B]** time = 0.818293, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1} + 6x - 3} - \frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1} + 6x - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)\*\*(1/2)/((-1+x)\*\*(1/2)+x\*\*(1/2))\*\*2,x)

[Out]  $-4\sqrt{x}/(6\sqrt{x}\sqrt{x-1} + 6x - 3) - 2\sqrt{x-1}/(6\sqrt{x}\sqrt{x-1} + 6x - 3)$

---

**Giac [A]** time = 1.07121, size = 27, normalized size = 0.9

$$\frac{4}{3}(x-1)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")`

[Out]  $4/3*(x-1)^{(3/2)} - 4/3*x^{(3/2)} + 2*\sqrt{x-1}$

$$3.9 \quad \int \frac{1}{\sqrt{-1+x^2}(\sqrt{x}+\sqrt{-1+x^2})^2} dx$$

**Optimal.** Leaf size=220

$$\frac{2-4x}{5(\sqrt{x^2-1}+\sqrt{x})} - \frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2\sqrt{x^2-1}}}{2-(1-\sqrt{5})x}\right) - \frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}\sqrt{x^2-1}}}{-\sqrt{5}x-x+2}\right) + \dots$$

```
[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[
(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqr
t[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])*x)])/50 - (Sqrt[110 +
50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*
Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x)]
)/50
```

**Rubi [A]** time = 0.505796, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {6742, 736, 826, 1166, 207, 203, 1018, 1034, 725, 206, 204, 985}

$$\frac{2\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)} + \frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)} - \frac{2}{5}\sqrt{\frac{1}{5}(5\sqrt{5}-2)}\tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)\sqrt{x^2-1}}}\right) + \sqrt{\frac{2}{5(\sqrt{5}-1)}}\tan^{-1}\left(\frac{2-\dots}{\sqrt{2(\dots)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2), x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqr
t[2*(-1 + Sqrt[5]])*Sqrt[-1 + x^2])]) - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5]])*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5]])*Sqrt
[-1 + x^2])]) - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5]])*Sqrt[-1 + x^2])])/5
```

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

**Rule 736**

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1)
), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

**Rule 826**

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

#### Rule 1166

```
Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

#### Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

#### Rule 203

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

#### Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

#### Rule 725

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

#### Rule 206



Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 985

Int[1/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + f\*x^2]), x], x] - Dist[(2\*c)/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x^2}(\sqrt{x}+\sqrt{-1+x^2})^2} dx &= \int \left( -\frac{2\sqrt{x}}{(-1-x+x^2)^2} + \frac{2x}{\sqrt{-1+x^2}(-1-x+x^2)^2} + \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)} \right) dx \\ &= -\left( 2 \int \frac{\sqrt{x}}{(-1-x+x^2)^2} dx \right) + 2 \int \frac{x}{\sqrt{-1+x^2}(-1-x+x^2)^2} dx + \int \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)} dx \\ &= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{2}{5} \int \frac{-\frac{1}{2}-x}{\sqrt{x}(-1-x+x^2)} dx + \frac{2}{5} \int \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)} dx \\ &= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{4}{5} \text{Subst} \left( \int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4} dx, x, \sqrt{x} \right) - \frac{2}{5} \int \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)} dx \\ &= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \tan^{-1} \left( \frac{2-(1-\sqrt{5})}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}} \right) \\ &= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \sqrt{\frac{2}{5}} (-11+5\sqrt{5}) \tan^{-1} \left( \sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{-1+x^2} \right) \end{aligned}$$

**Mathematica [A]** time = 0.687549, size = 340, normalized size = 1.55

$$\frac{2}{5} \left( \frac{\sqrt{x}(1-2x)}{-x^2+x+1} + \frac{\sqrt{x^2-1}(1-2x)}{x^2-x-1} - \frac{1}{2} \sqrt{\frac{5}{2}} (1+\sqrt{5}) \tan^{-1} \left( \frac{-\sqrt{5}x+x-2}{\sqrt{2}(\sqrt{5}-1)\sqrt{x^2-1}} \right) - \sqrt{\sqrt{5}-\frac{2}{5}} \tan^{-1} \left( \frac{(\sqrt{5}-1)x}{\sqrt{2}(\sqrt{5}-1)\sqrt{-1+x^2}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[-1 + x^2]\*(Sqrt[x] + Sqrt[-1 + x^2])^2), x]

[Out] (2\*(((1 - 2\*x)\*Sqrt[x])/(1 + x - x^2) + ((1 - 2\*x)\*Sqrt[-1 + x^2])/(-1 - x + x^2) + Sqrt[(-11 + 5\*Sqrt[5])/10]\*ArcTan[Sqrt[2/(-1 + Sqrt[5])]\*Sqrt[x]] - (Sqrt[(5\*(1 + Sqrt[5]))/2]\*ArcTan[(-2 + x - Sqrt[5]\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[x])])

```
[5]])*Sqrt[-1 + x^2]))/2 - Sqrt[-2/5 + Sqrt[5]]*ArcTan[(2 + (-1 + Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5]])*Sqrt[-1 + x^2])] - Sqrt[(11 + 5*Sqrt[5])/10]*Ar
cTanh[Sqrt[2/(1 + Sqrt[5]])*Sqrt[x]] - Sqrt[5/(2*(1 + Sqrt[5]))]*ArcTanh[(-
2 + x + Sqrt[5]*x)/(Sqrt[2*(1 + Sqrt[5]])*Sqrt[-1 + x^2])] - Sqrt[2/5 + Sqr
t[5]]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5]])*Sqrt[-1 + x^2])
])/5
```

**Maple [B]** time = 0.123, size = 902, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5
^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-
1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1
/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2)
)^(1/2)+2/5/(1/2-1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^
(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2
+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/
2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^
(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+
1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5
^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2
*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1
/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5
^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*ar
ctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*
(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)
)*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-
1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2*5^(1/2))
^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*5^(1/2))^(1/2)*arct
anh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+
4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*
5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}(\sqrt{x^2-1}+\sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

---

**Fricas [B]** time = 2.29054, size = 1287, normalized size = 5.85

$$4\sqrt{5}(x^2 - x - 1)\sqrt{10\sqrt{5} - 22}\arctan\left(\frac{1}{2}\sqrt{2x^2 - \sqrt{x^2 - 1}(2x + \sqrt{5} - 1) + \sqrt{5}x - x}\sqrt{10\sqrt{5} - 22}(\sqrt{5} + 2) + \frac{1}{4}(\sqrt{5}(2x + \sqrt{5} - 1) + \sqrt{5}x - x)\sqrt{10\sqrt{5} - 22}\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")

[Out] 1/50\*(4\*sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) - 22)\*arctan(1/2\*sqrt(2\*x^2 - sqrt(x^2 - 1)\*(2\*x + sqrt(5) - 1) + sqrt(5)\*x - x)\*sqrt(10\*sqrt(5) - 22)\*(sqrt(5) + 2) + 1/4\*(sqrt(5)\*(2\*x + 1) - 2\*sqrt(x^2 - 1)\*(sqrt(5) + 2) + 4\*x + 3)\*sqrt(10\*sqrt(5) - 22)) - 4\*sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) - 22)\*arctan(1/4\*(sqrt(2)\*sqrt(2\*x + sqrt(5) - 1)\*(sqrt(5) + 2) - 2\*sqrt(x)\*(sqrt(5) + 2))\*sqrt(10\*sqrt(5) - 22)) - sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) - 4\*x + 2\*sqrt(5) + 4\*sqrt(x^2 - 1) + 2) + sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) + 4\*sqrt(x)) + sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(-sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) - 4\*x + 2\*sqrt(5) + 4\*sqrt(x^2 - 1) + 2) - sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(-sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) + 4\*sqrt(x)) - 40\*x^2 - 20\*sqrt(x^2 - 1)\*(2\*x - 1) + 20\*(2\*x - 1)\*sqrt(x) + 40\*x + 40)/(x^2 - x - 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}(\sqrt{x} + \sqrt{x^2-1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2-1)\*\*(1/2)/(x\*\*(1/2)+(x\*\*2-1)\*\*(1/2))\*\*2,x)

[Out] Integral(1/(sqrt((x - 1)\*(x + 1))\*(sqrt(x) + sqrt(x\*\*2 - 1))\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}(\sqrt{x^2-1} + \sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")

[Out] integrate(1/(sqrt(x^2 - 1)\*(sqrt(x^2 - 1) + sqrt(x))^2), x)

$$3.10 \quad \int \frac{(\sqrt{x} - \sqrt{-1+x^2})^2}{(1+x-x^2)^2 \sqrt{-1+x^2}} dx$$

**Optimal.** Leaf size=220

$$\frac{2-4x}{5(\sqrt{x^2-1} + \sqrt{x})} - \frac{1}{50} \sqrt{50\sqrt{5}-110} \tan^{-1} \left( \frac{\sqrt{2\sqrt{5}-2\sqrt{x^2-1}}}{2-(1-\sqrt{5})x} \right) - \frac{1}{50} \sqrt{110+50\sqrt{5}} \tanh^{-1} \left( \frac{\sqrt{2+2\sqrt{5}\sqrt{x^2-1}}}{-\sqrt{5}x-x+2} \right) + \frac{1}{25}$$

[Out] (2 - 4\*x)/(5\*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50\*Sqrt[5]]\*ArcTan[(Sqrt[2 + 2\*Sqrt[5]]\*Sqrt[x])/2])/25 - (Sqrt[-110 + 50\*Sqrt[5]]\*ArcTan[(Sqrt[-2 + 2\*Sqrt[5]]\*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])\*x))]/50 - (Sqrt[110 + 50\*Sqrt[5]]\*ArcTanh[(Sqrt[-2 + 2\*Sqrt[5]]\*Sqrt[x])/2])/25 - (Sqrt[110 + 50\*Sqrt[5]]\*ArcTanh[(Sqrt[2 + 2\*Sqrt[5]]\*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]\*x)])/50

**Rubi [B]** time = 0.751087, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 736, 826, 1166, 207, 203, 975, 1034, 725, 206, 204, 1018, 1065}

$$-\frac{\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)} + \frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)} - \frac{(3-x)\sqrt{x^2-1}}{5(-x^2+x+1)} + \frac{(x+2)\sqrt{x^2-1}}{5(-x^2+x+1)} + \frac{1}{5} \sqrt{\frac{1}{5}(2+5\sqrt{5})} \tan^{-1} \left( \frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)\sqrt{x^2-1}}} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2\*Sqrt[-1 + x^2]), x]

[Out] (2\*(1 - 2\*x)\*Sqrt[x])/(5\*(1 + x - x^2)) - ((1 - 2\*x)\*Sqrt[-1 + x^2])/(5\*(1 + x - x^2)) - ((3 - x)\*Sqrt[-1 + x^2])/(5\*(1 + x - x^2)) + ((2 + x)\*Sqrt[-1 + x^2])/(5\*(1 + x - x^2)) + (Sqrt[(2\*(-11 + 5\*Sqrt[5]))/5]\*ArcTan[Sqrt[2/(-1 + Sqrt[5])]\*Sqrt[x]])/5 - (Sqrt[(-11 + 5\*Sqrt[5])/10]\*ArcTan[(2 - (1 - Sqrt[5])\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5\*Sqrt[5])/5]\*ArcTan[(2 - (1 - Sqrt[5])\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5 + (Sqrt[(2 + 5\*Sqrt[5])/5]\*ArcTan[(2 - (1 - Sqrt[5])\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5 - (Sqrt[(2\*(11 + 5\*Sqrt[5]))/5]\*ArcTanh[Sqrt[2/(1 + Sqrt[5])]\*Sqrt[x]])/5 - (Sqrt[(-2 + 5\*Sqrt[5])/5]\*ArcTanh[(2 - (1 + Sqrt[5])\*x)/(Sqrt[2\*(1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5\*Sqrt[5])/5]\*ArcTanh[(2 - (1 + Sqrt[5])\*x)/(Sqrt[2\*(1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5 + (Sqrt[(11 + 5\*Sqrt[5])/10]\*ArcTanh[(2 - (1 + Sqrt[5])\*x)/(Sqrt[2\*(1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/5

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 736

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^m\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^(m - 1)\*(b\*e\*m + 2\*c\*d\*(2\*p + 3) + 2\*c\*e\*(m + 2\*p + 3)\*x)\*(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L

tQ[m, 1] || (ILtQ[m + 2\*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d, e, m, p, x]

### Rule 826

Int[((f\_.) + (g\_.)\*(x\_))/(Sqrt[(d\_.) + (e\_.)\*(x\_)]\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)), x\_Symbol] := Dist[2, Subst[Int[(e\*f - d\*g + g\*x^2)/(c\*d^2 - b\*d\*e + a\*e^2 - (2\*c\*d - b\*e)\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0]

### Rule 1166

Int[((d\_.) + (e\_.)\*(x\_)^2)/((a\_.) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

### Rule 207

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 975

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_.) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((b^3\*f + b\*c\*(c\*d - 3\*a\*f) + c\*(2\*c^2\*d + b^2\*f - c\*(2\*a\*f))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + f\*x^2)^(q + 1))/((b^2 - 4\*a\*c)\*(b^2\*d\*f + (c\*d - a\*f)^2)\*(p + 1)), x] - Dist[1/((b^2 - 4\*a\*c)\*(b^2\*d\*f + (c\*d - a\*f)^2)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + f\*x^2)^q\*Simp[2\*c\*(b^2\*d\*f + (c\*d - a\*f)^2)\*(p + 1) - (2\*c^2\*d + b^2\*f - c\*(2\*a\*f))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) + (2\*f\*(b^3\*f + b\*c\*(c\*d - 3\*a\*f))\*(p + q + 2) - (2\*c^2\*d + b^2\*f - c\*(2\*a\*f))\*(b\*f\*(p + 1)))\*x + c\*f\*(2\*c^2\*d + b^2\*f - c\*(2\*a\*f))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[b^2\*d\*f + (c\*d - a\*f)^2, 0] && (!(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0])

### Rule 1034

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c]

### Rule 725

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(
q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
+ 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
- a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(\sqrt{x} - \sqrt{-1+x^2})^2}{(1+x-x^2)^2 \sqrt{-1+x^2}} dx &= \int \left( -\frac{2\sqrt{x}}{(-1-x+x^2)^2} - \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)^2} + \frac{x}{\sqrt{-1+x^2}(-1-x+x^2)^2} + \frac{1}{\sqrt{-1+x^2}} \right) dx \\
&= -\left( 2 \int \frac{\sqrt{x}}{(-1-x+x^2)^2} dx \right) - \int \frac{1}{\sqrt{-1+x^2}(-1-x+x^2)^2} dx + \int \frac{x}{\sqrt{-1+x^2}(-1-x+x^2)^2} dx + \int \frac{1}{\sqrt{-1+x^2}} dx \\
&= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \int \frac{1}{\sqrt{-1+x^2}} dx \\
&= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{4}{5} \operatorname{Subst} \left( \frac{1}{\sqrt{-1+x^2}}, -1+x^2 \right) \\
&= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{25} (5 - \sqrt{5}) \\
&= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \sqrt{\frac{2}{5}} \left( \frac{1}{\sqrt{5-1}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.648909, size = 311, normalized size = 1.41

$$\frac{1}{25} \left( \frac{-20x^{3/2} + 20\sqrt{x^2-1}x - 10\sqrt{x^2-1} + \sqrt{50\sqrt{5}-110}(-x^2+x+1) \tan^{-1} \left( \sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{x} \right) + \sqrt{10(1+\sqrt{5})}(-x^2+x+1)}{-x^2+x+1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2\*Sqrt[-1 + x^2]),x]

[Out] ((10\*Sqrt[x] - 20\*x^(3/2) - 10\*Sqrt[-1 + x^2] + 20\*x\*Sqrt[-1 + x^2] + Sqrt[-110 + 50\*Sqrt[5]]\*(1 + x - x^2)\*ArcTan[Sqrt[2/(-1 + Sqrt[5])]\*Sqrt[x]] + Sqrt[10\*(1 + Sqrt[5])]\*(1 + x - x^2)\*ArcTan[(-2 + x - Sqrt[5]\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])] + 5\*Sqrt[2/(-1 + Sqrt[5])]\*(-1 - x + x^2)\*ArcTan[(-2 + x - Sqrt[5]\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/(1 + x - x^2) - Sqrt[110 + 50\*Sqrt[5]]\*ArcTanh[Sqrt[2/(1 + Sqrt[5])]\*Sqrt[x]] + Sqrt[2/(1 + Sqrt[5])]\*(5 + 2\*Sqrt[5])\*ArcTanh[(-2 + x + Sqrt[5]\*x)/(Sqrt[2\*(1 + Sqrt[5])]\*Sqrt[-1 + x^2])])]/25

**Maple [B]** time = 0.021, size = 1542, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)

[Out] 2/5\*x^(1/2)/(x-1/2\*5^(1/2)-1/2)-4/5/(2+2\*5^(1/2))^(1/2)\*arctanh(2\*x^(1/2)/(2+2\*5^(1/2))^(1/2))+2/5\*x^(1/2)/(x+1/2\*5^(1/2)-1/2)+4/5/(-2+2\*5^(1/2))^(1/2)\*arctan(2\*x^(1/2)/(-2+2\*5^(1/2))^(1/2))+1/10\*5^(1/2)/(1/2-1/2\*5^(1/2))/(x+

$$\begin{aligned}
& 1/2*5^{(1/2)-1/2}*((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)}-1/10*5^{(1/2)}/(1/2+1/2*5^{(1/2)})/(x-1/2*5^{(1/2)-1/2})*((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)}-2/5/(-2+2*5^{(1/2)})^{(1/2)}*\arctan(2*(1-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2}))/(-2+2*5^{(1/2)})^{(1/2)}/(4*(x+1/2*5^{(1/2)-1/2})^2+4*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+2-2*5^{(1/2)})^{(1/2)})+2/5/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*(1+5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2}))/((2+2*5^{(1/2)})^{(1/2)}/(4*(x-1/2*5^{(1/2)-1/2})^2+4*(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+2+2*5^{(1/2)})^{(1/2)})+1/25*5^{(1/2)}*(4*(x+1/2*5^{(1/2)-1/2})^2+4*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+2-2*5^{(1/2)})^{(1/2)}+1/10/(1/2-1/2*5^{(1/2)})*\ln(x+((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)})+1/20/(1/2-1/2*5^{(1/2)})*(4*(x+1/2*5^{(1/2)-1/2})^2+4*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+2-2*5^{(1/2)})^{(1/2)}+1/25*\ln(x+((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)})*5^{(1/2)}+1/10/(1/2+1/2*5^{(1/2)})*\ln(x+((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)})+1/20/(1/2+1/2*5^{(1/2)})*(4*(x-1/2*5^{(1/2)-1/2})^2+4*(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+2+2*5^{(1/2)})^{(1/2)}-1/25*5^{(1/2)}*(4*(x-1/2*5^{(1/2)-1/2})^2+4*(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+2+2*5^{(1/2)})^{(1/2)}-1/25*\ln(x+((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)})*5^{(1/2)}-1/10/(1/2-1/2*5^{(1/2)})*\ln(x+((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)})*5^{(1/2)}-1/5/(1/2-1/2*5^{(1/2)})/(x+1/2*5^{(1/2)-1/2})*((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(3/2)}+1/20/(1/2+1/2*5^{(1/2)})*5^{(1/2)}*(4*(x-1/2*5^{(1/2)-1/2})^2+4*(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+2+2*5^{(1/2)})^{(1/2)}+1/10/(1/2+1/2*5^{(1/2)})*\ln(x+((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)})*5^{(1/2)}-1/5/(1/2+1/2*5^{(1/2)})/(x-1/2*5^{(1/2)-1/2})*((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(3/2)}+1/5/(1/2+1/2*5^{(1/2)})*x*((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)}+1/5/(1/2-1/2*5^{(1/2)})*x*((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)}-1/20/(1/2-1/2*5^{(1/2)})*5^{(1/2)}*(4*(x+1/2*5^{(1/2)-1/2})^2+4*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+2-2*5^{(1/2)})^{(1/2)}-1/5*\ln(x+((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)})-1/5*\ln(x+((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)})+4/25*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*(1+5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2}))/((2+2*5^{(1/2)})^{(1/2)}/(4*(x-1/2*5^{(1/2)-1/2})^2+4*(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+2+2*5^{(1/2)})^{(1/2)})-1/10/(1/2-1/2*5^{(1/2)})/(x+1/2*5^{(1/2)-1/2})*((x+1/2*5^{(1/2)-1/2})^2+(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+1/2-1/2*5^{(1/2)})^{(1/2)}-1/10/(1/2+1/2*5^{(1/2)})/(x-1/2*5^{(1/2)-1/2})*((x-1/2*5^{(1/2)-1/2})^2+(5^{(1/2)+1})*(x-1/2*5^{(1/2)-1/2})+1/2+1/2*5^{(1/2)})^{(1/2)}+4/25*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\arctan(2*(1-5^{(1/2)+1})*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2}))/(-2+2*5^{(1/2)})^{(1/2)}/(4*(x+1/2*5^{(1/2)-1/2})^2+4*(-5^{(1/2)+1})*(x+1/2*5^{(1/2)-1/2})+2-2*5^{(1/2)})^{(1/2)})-8/25/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*x^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)})*5^{(1/2)}-8/25/(-2+2*5^{(1/2)})^{(1/2)}*\arctan(2*x^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)})*5^{(1/2)}
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2\left(x^{\frac{5}{2}}-3x^{\frac{3}{2}}\right)}{5\left(x^2-x-1\right)}+\int \frac{x^{\frac{3}{2}}+\sqrt{x}}{5\left(x^2-x-1\right)} dx+\int \frac{x^2+x-1}{\left(x^4-2x^3-x^2+2x+1\right)\sqrt{x+1}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")

[Out] -2/5\*(x^(5/2) - 3\*x^(3/2))/(x^2 - x - 1) + integrate(1/5\*(x^(3/2) + sqrt(x))/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2\*x^3 - x^2 + 2\*x + 1



) $\sqrt{x + 1}\sqrt{x - 1}$ ), x)

**Fricas [B]** time = 2.35997, size = 1287, normalized size = 5.85

$$4\sqrt{5}(x^2 - x - 1)\sqrt{10\sqrt{5} - 22}\arctan\left(\frac{1}{2}\sqrt{2x^2 - \sqrt{x^2 - 1}(2x + \sqrt{5} - 1) + \sqrt{5}x - x}\sqrt{10\sqrt{5} - 22}(\sqrt{5} + 2) + \frac{1}{4}(\sqrt{5}(2x + 1) + \sqrt{5} - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")

[Out] 1/50\*(4\*sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) - 22)\*arctan(1/2\*sqrt(2\*x^2 - sqrt(x^2 - 1)\*(2\*x + sqrt(5) - 1) + sqrt(5)\*x - x)\*sqrt(10\*sqrt(5) - 22)\*(sqrt(5) + 2) + 1/4\*(sqrt(5)\*(2\*x + 1) - 2\*sqrt(x^2 - 1)\*(sqrt(5) + 2) + 4\*x + 3)\*sqrt(10\*sqrt(5) - 22)) - 4\*sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) - 22)\*arctan(1/4\*(sqrt(2)\*sqrt(2\*x + sqrt(5) - 1)\*(sqrt(5) + 2) - 2\*sqrt(x)\*(sqrt(5) + 2))\*sqrt(10\*sqrt(5) - 22)) - sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) - 4\*x + 2\*sqrt(5) + 4\*sqrt(x^2 - 1) + 2) + sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) + 4\*sqrt(x)) + sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(-sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) - 4\*x + 2\*sqrt(5) + 4\*sqrt(x^2 - 1) + 2) - sqrt(5)\*(x^2 - x - 1)\*sqrt(10\*sqrt(5) + 22)\*log(-sqrt(10\*sqrt(5) + 22)\*(sqrt(5) - 3) + 4\*sqrt(x)) - 40\*x^2 - 20\*sqrt(x^2 - 1)\*(2\*x - 1) + 20\*(2\*x - 1)\*sqrt(x) + 40\*x + 40)/(x^2 - x - 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*(1/2)-(x\*\*2-1)\*\*(1/2))\*\*2/(-x\*\*2+x+1)\*\*2/(x\*\*2-1)\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sqrt{x^2 - 1} - \sqrt{x})^2}{(x^2 - x - 1)^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm="giac")

[Out] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2\*sqrt(x^2 - 1)), x)

$$3.11 \quad \int \left( \frac{1}{\sqrt{2}(1+x)^2 \sqrt{-i+x^2}} + \frac{1}{\sqrt{2}(1+x)^2 \sqrt{i+x^2}} \right) dx$$

**Optimal.** Leaf size=138

$$-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{x^2 - i}}{\sqrt{2}(x+1)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sqrt{x^2 + i}}{\sqrt{2}(x+1)} + \frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

[Out]  $((-1/2 - I/2)*\text{Sqrt}[-I + x^2])/(\text{Sqrt}[2]*(1 + x)) - ((1/2 - I/2)*\text{Sqrt}[I + x^2])/(\text{Sqrt}[2]*(1 + x)) + \text{ArcTanh}[(I + x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[-I + x^2])]/((1 - I)^{3/2}*\text{Sqrt}[2]) - \text{ArcTanh}[(I - x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[I + x^2])]/((1 + I)^{3/2}*\text{Sqrt}[2])$

**Rubi [A]** time = 0.0750856, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {731, 725, 206}

$$-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{x^2 - i}}{\sqrt{2}(x+1)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sqrt{x^2 + i}}{\sqrt{2}(x+1)} + \frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[2]*(1 + x)^2*\text{Sqrt}[-I + x^2]) + 1/(\text{Sqrt}[2]*(1 + x)^2*\text{Sqrt}[I + x^2]), x]$

[Out]  $((-1/2 - I/2)*\text{Sqrt}[-I + x^2])/(\text{Sqrt}[2]*(1 + x)) - ((1/2 - I/2)*\text{Sqrt}[I + x^2])/(\text{Sqrt}[2]*(1 + x)) + \text{ArcTanh}[(I + x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[-I + x^2])]/((1 - I)^{3/2}*\text{Sqrt}[2]) - \text{ArcTanh}[(I - x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[I + x^2])]/((1 + I)^{3/2}*\text{Sqrt}[2])$

#### Rule 731

$\text{Int}[\left((d_) + (e_.)*(x_)^m\right)*\left((a_) + (c_.)*(x_)^2\right)^p, x\_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[(c*d)/(c*d^2 + a*e^2), \text{Int}[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c, d, e, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m + 2*p + 3, 0]$

#### Rule 725

$\text{Int}[1/\left(\left((d_) + (e_.)*(x_*)\right)*\text{Sqrt}[(a_) + (c_.)*(x_)^2]\right), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}[\{a, c, d, e\}, x]$

#### Rule 206

$\text{Int}[\left((a_) + (b_.)*(x_)^2\right)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rubi steps

$$\begin{aligned}
\int \left( \frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}} + \frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}} \right) dx &= \frac{\int \frac{1}{(1+x)^2\sqrt{-i+x^2}} dx}{\sqrt{2}} + \frac{\int \frac{1}{(1+x)^2\sqrt{i+x^2}} dx}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \int \frac{1}{(1+x)\sqrt{i+x^2}} dx}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + -\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{i+x^2}} dx\right)}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.19884, size = 125, normalized size = 0.91

$$\frac{i\left((1+i)\left(i\sqrt{x^2-i} + \sqrt{x^2+i}\right) + \sqrt{1-i}(x+1)\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right) + \sqrt{1+i}(x+1)\tanh^{-1}\left(\frac{(1+i)^{3/2}(1+ix)}{2\sqrt{x^2+i}}\right)\right)}{2\sqrt{2}(x+1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[2]\*(1+x)^2\*Sqrt[-I+x^2]) + 1/(Sqrt[2]\*(1+x)^2\*Sqrt[I+x^2]),x]

[Out] ((I/2)\*((1+I)\*(I\*Sqrt[-I+x^2] + Sqrt[I+x^2])) + Sqrt[1-I]\*(1+x)\*ArcTanh[(I+x)/(Sqrt[1-I]\*Sqrt[-I+x^2])] + Sqrt[1+I]\*(1+x)\*ArcTanh[(1+I)^(3/2)\*(1+I\*x)/(2\*Sqrt[I+x^2])])/(Sqrt[2]\*(1+x))

**Maple [B]** time = 0.02, size = 278, normalized size = 2.

$$-\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1-i-2x} - \frac{i\sqrt{2}}{1+x}\sqrt{(1+x)^2-1-i-2x} - \frac{\sqrt{2}}{4\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1-i-2x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2/(1+x)^2\*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2\*2^(1/2)/(I+x^2)^(1/2),x)

[Out] -1/4\*2^(1/2)/(1+x)\*((1+x)^2-1-I-2\*x)^(1/2)-1/4\*I\*2^(1/2)/(1+x)\*((1+x)^2-1-I-2\*x)^(1/2)-1/4\*2^(1/2)/(1-I)^(1/2)\*ln((-2\*I-2\*x+2\*(1-I)^(1/2)\*((1+x)^2-1-I-2\*x)^(1/2))/(1+x))-1/4\*I\*2^(1/2)/(1-I)^(1/2)\*ln((-2\*I-2\*x+2\*(1-I)^(1/2)\*((1+x)^2-1-I-2\*x)^(1/2))/(1+x))-1/4\*2^(1/2)/(1+x)\*((1+x)^2-1+I-2\*x)^(1/2)+1/4\*I\*2^(1/2)/(1+x)\*((1+x)^2-1+I-2\*x)^(1/2)-1/4\*2^(1/2)/(1+I)^(1/2)\*ln((2\*I-2\*x+2\*(1+I)^(1/2)\*((1+x)^2-1+I-2\*x)^(1/2))/(1+x))+1/4\*I\*2^(1/2)/(1+I)^(1/2)\*ln((2\*I-2\*x+2\*(1+I)^(1/2)\*((1+x)^2-1+I-2\*x)^(1/2))/(1+x))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [A]** time = 2.28061, size = 678, normalized size = 4.91

$$\sqrt{-\frac{1}{2}i + \frac{1}{2}}(-i-1)x - i + 1 \log\left(\sqrt{2}\sqrt{-\frac{1}{2}i + \frac{1}{2}} - x + \sqrt{x^2 - i - 1}\right) + \sqrt{-\frac{1}{2}i + \frac{1}{2}}((i-1)x + i - 1) \log\left(-\sqrt{2}\sqrt{-\frac{1}{2}i + \frac{1}{2}} - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqrt(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I) - I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.12 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{(1+x)^2 \sqrt{1+x^4}} dx$$

**Optimal.** Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)} - \frac{\sqrt{1+ix^2}}{2(x+1)} - \frac{1}{4}(1-i)^{3/2} \tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{4}(1+i)^{3/2} \tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

[Out] -Sqrt[1 - I\*x^2]/(2\*(1 + x)) - Sqrt[1 + I\*x^2]/(2\*(1 + x)) - ((1 - I)^(3/2)\*ArcTanh[(1 + I\*x)/(Sqrt[1 - I]\*Sqrt[1 - I\*x^2])])/4 - ((1 + I)^(3/2)\*ArcTanh[(1 - I\*x)/(Sqrt[1 + I]\*Sqrt[1 + I\*x^2])])/4

**Rubi [A]** time = 0.185294, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2133, 731, 725, 206}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)} - \frac{\sqrt{1+ix^2}}{2(x+1)} - \frac{1}{4}(1-i)^{3/2} \tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{4}(1+i)^{3/2} \tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2\*Sqrt[1 + x^4]),x]

[Out] -Sqrt[1 - I\*x^2]/(2\*(1 + x)) - Sqrt[1 + I\*x^2]/(2\*(1 + x)) - ((1 - I)^(3/2)\*ArcTanh[(1 + I\*x)/(Sqrt[1 - I]\*Sqrt[1 - I\*x^2])])/4 - ((1 + I)^(3/2)\*ArcTanh[(1 - I\*x)/(Sqrt[1 + I]\*Sqrt[1 + I\*x^2])])/4

#### Rule 2133

Int[(((c\_) + (d\_)\*(x\_))^(m\_)\*Sqrt[(b\_)\*(x\_)^2 + Sqrt[(a\_) + (e\_)\*(x\_)^4]])/Sqrt[(a\_) + (e\_)\*(x\_)^4], x\_Symbol] := Dist[(1 - I)/2, Int[(c + d\*x)^m/Sqrt[Sqrt[a] - I\*b\*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d\*x)^m/Sqrt[Sqrt[a] + I\*b\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ[a, 0]

#### Rule 731

Int[(((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 3, 0]

#### Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{(1 + x)^2 \sqrt{1 + x^4}} dx &= \left(\frac{1}{2} - \frac{i}{2}\right) \int \frac{1}{(1 + x)^2 \sqrt{1 - ix^2}} dx + \left(\frac{1}{2} + \frac{i}{2}\right) \int \frac{1}{(1 + x)^2 \sqrt{1 + ix^2}} dx \\
&= -\frac{\sqrt{1 - ix^2}}{2(1 + x)} - \frac{\sqrt{1 + ix^2}}{2(1 + x)} - \frac{1}{2}i \int \frac{1}{(1 + x)\sqrt{1 - ix^2}} dx + \frac{1}{2}i \int \frac{1}{(1 + x)\sqrt{1 + ix^2}} dx \\
&= -\frac{\sqrt{1 - ix^2}}{2(1 + x)} - \frac{\sqrt{1 + ix^2}}{2(1 + x)} + \frac{1}{2}i \operatorname{Subst} \left( \int \frac{1}{(1 - i) - x^2} dx, x, \frac{1 + ix}{\sqrt{1 - ix^2}} \right) - \frac{1}{2}i \operatorname{Subst} \left( \int \frac{1}{(1 + i) - x^2} dx, x, \frac{1 + ix}{\sqrt{1 + ix^2}} \right) \\
&= -\frac{\sqrt{1 - ix^2}}{2(1 + x)} - \frac{\sqrt{1 + ix^2}}{2(1 + x)} - \frac{1}{4}(1 - i)^{3/2} \tanh^{-1} \left( \frac{1 + ix}{\sqrt{1 - i}\sqrt{1 - ix^2}} \right) - \frac{1}{4}(1 + i)^{3/2} \tanh^{-1} \left( \frac{1 - ix}{\sqrt{1 + i}\sqrt{1 + ix^2}} \right)
\end{aligned}$$

**Mathematica [F]** time = 0.239471, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{(1 + x)^2 \sqrt{1 + x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2\*Sqrt[1 + x^4]), x]

[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2\*Sqrt[1 + x^4]), x]

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{(1 + x)^2} \sqrt{x^2 + \sqrt{x^4 + 1}} \frac{1}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2), x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)\*(x + 1)^2), x)

**Fricas [B]** time = 19.3436, size = 1068, normalized size = 8.54

$$4(x+1)\sqrt{\sqrt{2}+1}\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}(x^3+1)+\sqrt{x^4+1}(\sqrt{2}x-x-1)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\left(2x^2-\sqrt{2}(x^2+1)+2\sqrt{x^4+1}(\sqrt{2}-1)+2\right)\sqrt{2\sqrt{2}+2}\sqrt{x^2-2x+1}}{2(x^2-2x+1)}\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] 1/8\*(4\*(x + 1)\*sqrt(sqrt(2) + 1)\*arctan(1/2\*(2\*(x^3 + x^2 - sqrt(2)\*(x^3 + 1) + sqrt(x^4 + 1)\*(sqrt(2)\*x - x - 1) - x + 1)\*sqrt(x^2 + sqrt(x^4 + 1))\*sqrt(sqrt(2) + 1) + (2\*x^2 - sqrt(2)\*(x^2 + 1) + 2\*sqrt(x^4 + 1)\*(sqrt(2) - 1) + 2)\*sqrt(2\*sqrt(2) + 2)\*sqrt(sqrt(2) + 1))/(x^2 - 2\*x + 1)) + (x + 1)\*sqrt(sqrt(2) - 1)\*log(-((2\*x^3 - sqrt(2)\*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)\*(sqrt(2)\*(x - 1) - 2\*x) - 2)\*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)\*(x^2 + 1) + 2\*sqrt(x^4 + 1))\*sqrt(sqrt(2) - 1))/(x^2 + 2\*x + 1)) - (x + 1)\*sqrt(sqrt(2) - 1)\*log(-((2\*x^3 - sqrt(2)\*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)\*(sqrt(2)\*(x - 1) - 2\*x) - 2)\*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)\*(x^2 + 1) + 2\*sqrt(x^4 + 1))\*sqrt(sqrt(2) - 1))/(x^2 + 2\*x + 1)) + 4\*sqrt(x^2 + sqrt(x^4 + 1))\*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)^2 \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+(x\*\*4+1)\*\*(1/2))\*\*(1/2)/(1+x)\*\*2/(x\*\*4+1)\*\*(1/2),x)

[Out] Integral(sqrt(x\*\*2 + sqrt(x\*\*4 + 1))/((x + 1)\*\*2\*sqrt(x\*\*4 + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)\*(x + 1)^2), x)

$$3.13 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}} dx$$

**Optimal.** Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

[Out] -(Sqrt[1 - I]\*ArcTanh[(1 + I\*x)/(Sqrt[1 - I]\*Sqrt[1 - I\*x^2])])/2 - (Sqrt[1 + I]\*ArcTanh[(1 - I\*x)/(Sqrt[1 + I]\*Sqrt[1 + I\*x^2])])/2

**Rubi [A]** time = 0.16247, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2133, 725, 206}

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)\*Sqrt[1 + x^4]), x]

[Out] -(Sqrt[1 - I]\*ArcTanh[(1 + I\*x)/(Sqrt[1 - I]\*Sqrt[1 - I\*x^2])])/2 - (Sqrt[1 + I]\*ArcTanh[(1 - I\*x)/(Sqrt[1 + I]\*Sqrt[1 + I\*x^2])])/2

#### Rule 2133

Int[(((c\_.) + (d\_.)\*(x\_)^(m\_.))\*Sqrt[(b\_.)\*(x\_)^2 + Sqrt[(a\_) + (e\_.)\*(x\_)^4]])/Sqrt[(a\_) + (e\_.)\*(x\_)^4], x\_Symbol] := Dist[(1 - I)/2, Int[(c + d\*x)^m/Sqrt[Sqrt[a] - I\*b\*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d\*x)^m/Sqrt[Sqrt[a] + I\*b\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ[a, 0]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}} dx &= \left(\frac{1}{2} - \frac{i}{2}\right) \int \frac{1}{(1+x)\sqrt{1-ix^2}} dx + \left(\frac{1}{2} + \frac{i}{2}\right) \int \frac{1}{(1+x)\sqrt{1+ix^2}} dx \\ &= \left(-\frac{1}{2} - \frac{i}{2}\right) \text{Subst}\left(\int \frac{1}{(1+i)-x^2} dx, x, \frac{1-ix}{\sqrt{1+ix^2}}\right) + \left(-\frac{1}{2} + \frac{i}{2}\right) \text{Subst}\left(\int \frac{1}{(1-i)-x^2} dx, x, \frac{1+ix}{\sqrt{1-ix^2}}\right) \\ &= -\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right) \end{aligned}$$



**Mathematica [F]** time = 0.175985, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{(1 + x)\sqrt{1 + x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)\*Sqrt[1 + x^4]),x]

[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)\*Sqrt[1 + x^4]), x]

**Maple [F]** time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \sqrt{x^2 + \sqrt{x^4 + 1}} \frac{1}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)\*(x + 1)), x)

**Fricas [B]** time = 38.9638, size = 998, normalized size = 12.32

$$\frac{1}{2} \sqrt{2\sqrt{2}-2} \arctan \left( \frac{\left(2x^2 - \sqrt{2}(x^3 - x^2 + x + 1) + \sqrt{x^4 + 1}(\sqrt{2}(x - 1) - 2) - 2x\right) \sqrt{x^2 + \sqrt{x^4 + 1}} \sqrt{2\sqrt{2}-2} + (x^2 + \sqrt{x^4 + 1})(\sqrt{2}(x - 1) - 2) - 2x}{2(x^2 - 2x + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] 1/2\*sqrt(2\*sqrt(2) - 2)\*arctan(1/2\*((2\*x^2 - sqrt(2)\*(x^3 - x^2 + x + 1) + sqrt(x^4 + 1)\*(sqrt(2)\*(x - 1) - 2) - 2\*x)\*sqrt(x^2 + sqrt(x^4 + 1))\*sqrt(2\*sqrt(2) - 2) + (x^2 + sqrt(2)\*sqrt(x^4 + 1) + 1)\*sqrt(2\*sqrt(2) + 2)\*sqrt(

$$\frac{2\sqrt{2} - 2}{x^2 - 2x + 1} - \frac{1}{8}\sqrt{2\sqrt{2} + 2}\log\left(-\frac{(2x^3 - \sqrt{2}(x^3 - x^2 - x - 1) + \sqrt{x^4 + 1})(\sqrt{2}(x - 1) - 2x) - 2}{\sqrt{x^2 + \sqrt{x^4 + 1}} + (x^2 - \sqrt{2}(x^2 + 1) + \sqrt{x^4 + 1})(\sqrt{2} - 2) + 1}\sqrt{2\sqrt{2} + 2}\right)}{x^2 + 2x + 1} + \frac{1}{8}\sqrt{2\sqrt{2} + 2}\log\left(-\frac{(2x^3 - \sqrt{2}(x^3 - x^2 - x - 1) + \sqrt{x^4 + 1})(\sqrt{2}(x - 1) - 2x) - 2}{\sqrt{x^2 + \sqrt{x^4 + 1}} - (x^2 - \sqrt{2}(x^2 + 1) + \sqrt{x^4 + 1})(\sqrt{2} - 2) + 1}\sqrt{2\sqrt{2} + 2}\right)}{x^2 + 2x + 1}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+(x\*\*4+1)\*\*(1/2))\*\*(1/2)/(1+x)/(x\*\*4+1)\*\*(1/2),x)

[Out] Integral(sqrt(x\*\*2 + sqrt(x\*\*4 + 1))/((x + 1)\*sqrt(x\*\*4 + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)\*(x + 1)), x)

$$3.14 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx$$

**Optimal.** Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2 + \sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

[Out] ArcTanh[(Sqrt[2]\*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

**Rubi [A]** time = 0.0544915, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2 + \sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4], x]

[Out] ArcTanh[(Sqrt[2]\*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

#### Rule 2132

Int[Sqrt[(c\_.)\*(x\_)^2 + (d\_.)\*Sqrt[(a\_) + (b\_.)\*(x\_)^4]]/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[d, Subst[Int[1/(1 - 2\*c\*x^2), x], x, x/Sqrt[c\*x^2 + d\*Sqrt[a + b\*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b\*d^2, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx = \text{Subst} \left( \int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt{x^2 + \sqrt{1+x^4}}} \right)$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2 + \sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

**Mathematica [A]** time = 0.0083347, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2 + \sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4], x]

[Out] ArcTanh[(Sqrt[2]\*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

**Maple [F]** time = 0.014, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2), x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)

**Fricas [B]** time = 4.026, size = 162, normalized size = 5.23

$$\frac{1}{4} \sqrt{2} \log \left( 4x^4 + 4\sqrt{x^4 + 1}x^2 + 2 \left( \sqrt{2}x^3 + \sqrt{2}\sqrt{x^4 + 1}x \right) \sqrt{x^2 + \sqrt{x^4 + 1}} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2), x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*log(4\*x^4 + 4\*sqrt(x^4 + 1)\*x^2 + 2\*(sqrt(2)\*x^3 + sqrt(2)\*sqrt(x^4 + 1)\*x)\*sqrt(x^2 + sqrt(x^4 + 1)) + 1)

**Sympy [A]** time = 1.28076, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2} \left( \begin{matrix} 1, 1 \\ \frac{1}{4}, \frac{3}{4} \end{matrix} \middle| \frac{1}{2} x^4 \right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

$$3.15 \quad \int \frac{\sqrt{-x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx$$

**Optimal.** Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

[Out] ArcTan[(Sqrt[2]\*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

**Rubi [A]** time = 0.0626, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4], x]

[Out] ArcTan[(Sqrt[2]\*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

#### Rule 2132

Int[Sqrt[(c\_.)\*(x\_)^2 + (d\_.)\*Sqrt[(a\_) + (b\_.)\*(x\_)^4]]/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] :> Dist[d, Subst[Int[1/(1 - 2\*c\*x^2), x], x, x/Sqrt[c\*x^2 + d\*Sqrt[a + b\*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b\*d^2, 0]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\int \frac{\sqrt{-x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx = \text{Subst} \left( \int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt{-x^2 + \sqrt{1+x^4}}} \right) \\ = \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^2 + \sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

**Mathematica [A]** time = 0.0113053, size = 33, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] ArcTan[(Sqrt[2]\*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

**Maple [C]** time = 0.033, size = 22, normalized size = 0.7

$$-\frac{\sqrt{2}}{4x^2} {}_3F_2\left(\frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{3}{2}, \frac{3}{2}; -x^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)

[Out] -1/4\*2^(1/2)/x^2\*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)

**Fricas [A]** time = 4.07396, size = 85, normalized size = 2.58

$$-\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-x^2 + \sqrt{x^4 + 1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*sqrt(-x^2 + sqrt(x^4 + 1))/x)

**Sympy [A]** time = 0.705757, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\frac{1}{2}, 1, 1 \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```



$$3.16 \quad \int \frac{(-1+x)^{3/2} + (1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx$$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]

**Rubi [A]** time = 0.270052, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$ , Rules used = {6688}

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)\*(1 + x)^(3/2)), x]

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]

Rule 6688

Int[u\_, x\_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned} \int \frac{(-1+x)^{3/2} + (1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx &= \int \left( \frac{1}{(-1+x)^{3/2}} + \frac{1}{(1+x)^{3/2}} \right) dx \\ &= -\frac{2}{\sqrt{-1+x}} - \frac{2}{\sqrt{1+x}} \end{aligned}$$

**Mathematica [A]** time = 0.0174567, size = 19, normalized size = 1.

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)\*(1 + x)^(3/2)), x]

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]

**Maple [A]** time = 0.002, size = 16, normalized size = 0.8

$$-2 \frac{1}{\sqrt{-1+x}} - 2 \frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((−1+x)^(3/2)+(1+x)^(3/2))/(−1+x)^(3/2)/(1+x)^(3/2),x)`

[Out] `−2/(−1+x)^(1/2)−2/(1+x)^(1/2)`

**Maxima [A]** time = 0.933505, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((−1+x)^(3/2)+(1+x)^(3/2))/(−1+x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")`

[Out] `−2/sqrt(x + 1) − 2/sqrt(x − 1)`

**Fricas [A]** time = 1.97151, size = 78, normalized size = 4.11

$$-\frac{2\left((x+1)\sqrt{x-1} + \sqrt{x+1}(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((−1+x)^(3/2)+(1+x)^(3/2))/(−1+x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")`

[Out] `−2*((x + 1)*sqrt(x − 1) + sqrt(x + 1)*(x − 1))/(x^2 − 1)`

**Sympy [B]** time = 6.48919, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1} - \frac{2x\sqrt{x+1}}{x^2-1} - \frac{2\sqrt{x-1}}{x^2-1} + \frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((−1+x)**(3/2)+(1+x)**(3/2))/(−1+x)**(3/2)/(1+x)**(3/2),x)`

[Out] `−2*x*sqrt(x − 1)/(x**2 − 1) − 2*x*sqrt(x + 1)/(x**2 − 1) − 2*sqrt(x − 1)/(x**2 − 1) + 2*sqrt(x + 1)/(x**2 − 1)`

**Giac [A]** time = 1.09706, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

### 3.17 $\int (x + \sqrt{a + x^2})^b dx$

**Optimal.** Leaf size=52

$$\frac{(\sqrt{a+x^2}+x)^{b+1}}{2(b+1)} - \frac{a(\sqrt{a+x^2}+x)^{b-1}}{2(1-b)}$$

[Out]  $-(a*(x + \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x + \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

**Rubi [A]** time = 0.0232867, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2117, 14}

$$\frac{(\sqrt{a+x^2}+x)^{b+1}}{2(b+1)} - \frac{a(\sqrt{a+x^2}+x)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x + \text{Sqrt}[a + x^2])^b, x]$

[Out]  $-(a*(x + \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x + \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

#### Rule 2117

$\text{Int}[(g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*\text{Sqrt}[(a_.) + (c_.)*(x_.)^2])^{(n_.)}], x\_Symbol] \rightarrow \text{Dist}[1/(2*e), \text{Subst}[\text{Int}[(g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2)/(d - x)^2, x], x, d + e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, n\}, x] \&\& \text{EqQ}[e^2 - c*f^2, 0] \&\& \text{IntegerQ}[p]$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a_ + (b_)*(v_)] /; \text{FreeQ}\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

#### Rubi steps

$$\begin{aligned} \int (x + \sqrt{a + x^2})^b dx &= \frac{1}{2} \text{Subst} \left( \int x^{-2+b} (a + x^2) dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (ax^{-2+b} + x^b) dx, x, x + \sqrt{a + x^2} \right) \\ &= -\frac{a(x + \sqrt{a + x^2})^{-1+b}}{2(1-b)} + \frac{(x + \sqrt{a + x^2})^{1+b}}{2(1+b)} \end{aligned}$$

**Mathematica [A]** time = 0.0386577, size = 43, normalized size = 0.83

$$\frac{(\sqrt{a+x^2}+x)^{b-1} \left( (b-1)x(\sqrt{a+x^2}+x) + ab \right)}{b^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^b, x]

[Out] ((x + Sqrt[a + x^2])^(-1 + b)\*(a\*b + (-1 + b)\*x\*(x + Sqrt[a + x^2])))/(-1 + b^2)

**Maple [B]** time = 0.026, size = 120, normalized size = 2.3

$$\frac{b}{4\sqrt{\pi}} a^{\frac{b}{2} + \frac{1}{2}} \left( 8 \frac{\sqrt{\pi} x^{1+b} a^{-b/2-1/2}}{(1+b)b(-2+2b)} \left( \frac{ab}{x^2} + b - 1 \right) \left( \sqrt{1 + \frac{a}{x^2}} + 1 \right)^{-1+b} + 4 \frac{\sqrt{\pi} x^{1+b} a^{-b/2-1/2}}{(1+b)b} \sqrt{1 + \frac{a}{x^2}} \left( \sqrt{1 + \frac{a}{x^2}} + 1 \right)^{-1+b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^b, x)

[Out] 1/4\*a^(1/2\*b+1/2)/Pi^(1/2)\*b\*(8\*Pi^(1/2)/(1+b)/b\*x^(1+b)\*a^(-1/2\*b-1/2)\*(1/x^2\*a\*b+b-1)/(-2+2\*b)\*((1+1/x^2\*a)^(1/2)+1)^(-1+b)+4\*Pi^(1/2)/(1+b)/b\*x^(1+b)\*a^(-1/2\*b-1/2)\*(1+1/x^2\*a)^(1/2)\*((1+1/x^2\*a)^(1/2)+1)^(-1+b))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b, x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^b, x)

**Fricas [A]** time = 2.2043, size = 74, normalized size = 1.42

$$\frac{(\sqrt{x^2 + ab} - x)(x + \sqrt{x^2 + a})^b}{b^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b, x, algorithm="fricas")

[Out] (sqrt(x^2 + a)\*b - x)\*(x + sqrt(x^2 + a))^b/(b^2 - 1)

**Sympy [B]** time = 2.73252, size = 2149, normalized size = 41.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^b, x)

### 3.18 $\int (x - \sqrt{a + x^2})^b dx$

**Optimal.** Leaf size=56

$$\frac{(x - \sqrt{a + x^2})^{b+1}}{2(b+1)} - \frac{a(x - \sqrt{a + x^2})^{b-1}}{2(1-b)}$$

[Out]  $-(a*(x - \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x - \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

**Rubi [A]** time = 0.0224871, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2117, 14}

$$\frac{(x - \sqrt{a + x^2})^{b+1}}{2(b+1)} - \frac{a(x - \sqrt{a + x^2})^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x - \text{Sqrt}[a + x^2])^b, x]$

[Out]  $-(a*(x - \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x - \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

#### Rule 2117

$\text{Int}[(g + (h + (d + (e + (f + \text{Sqrt}[a + (c + x^2)]))^n))^p, x\_Symbol] := \text{Dist}[1/(2*e), \text{Subst}[\text{Int}[(g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2)/(d - x)^2, x], x, d + e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, n\}, x] \&\& \text{EqQ}[e^2 - c*f^2, 0] \&\& \text{IntegerQ}[p]$

#### Rule 14

$\text{Int}[(u + (c + x)^m), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a + (b + x)^v)] /; \text{FreeQ}\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

#### Rubi steps

$$\begin{aligned} \int (x - \sqrt{a + x^2})^b dx &= \frac{1}{2} \text{Subst} \left( \int x^{-2+b} (a + x^2) dx, x, x - \sqrt{a + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (ax^{-2+b} + x^b) dx, x, x - \sqrt{a + x^2} \right) \\ &= -\frac{a(x - \sqrt{a + x^2})^{-1+b}}{2(1-b)} + \frac{(x - \sqrt{a + x^2})^{1+b}}{2(1+b)} \end{aligned}$$

**Mathematica [A]** time = 0.0633378, size = 50, normalized size = 0.89

$$\frac{1}{2} (x - \sqrt{a + x^2})^{b-1} \left( \frac{(x - \sqrt{a + x^2})^2}{b+1} + \frac{a}{b-1} \right)$$



Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^b, x]

[Out] ((x - Sqrt[a + x^2])^(-1 + b)\*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))/2

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^b,x)

[Out] int((x-(x^2+a)^(1/2))^b,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^b, x)

**Fricas [A]** time = 2.1555, size = 76, normalized size = 1.36

$$\frac{(\sqrt{x^2 + a} + x)(x - \sqrt{x^2 + a})^b}{b^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")

[Out] -(sqrt(x^2 + a)\*b + x)\*(x - sqrt(x^2 + a))^b/(b^2 - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{a + x^2})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x\*\*2+a)\*\*(1/2))\*\*b,x)

```
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

$$3.19 \quad \int \frac{(x + \sqrt{a + x^2})^b}{\sqrt{a + x^2}} dx$$

**Optimal.** Leaf size=17

$$\frac{(\sqrt{a + x^2} + x)^b}{b}$$

[Out] (x + Sqrt[a + x^2])^b/b

**Rubi [A]** time = 0.0535661, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2122, 30}

$$\frac{(\sqrt{a + x^2} + x)^b}{b}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2], x]

[Out] (x + Sqrt[a + x^2])^b/b

#### Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_.), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{(x + \sqrt{a + x^2})^b}{\sqrt{a + x^2}} dx = \text{Subst} \left( \int x^{-1+b} dx, x, x + \sqrt{a + x^2} \right) \\ = \frac{(x + \sqrt{a + x^2})^b}{b}$$

**Mathematica [A]** time = 0.0067221, size = 17, normalized size = 1.

$$\frac{(\sqrt{a + x^2} + x)^b}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^b/b

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + a}\right)^b \frac{1}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

**Fricas [A]** time = 1.99166, size = 34, normalized size = 2.

$$\frac{\left(x + \sqrt{x^2 + a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + a))^b/b

**Sympy [B]** time = 3.67894, size = 313, normalized size = 18.41

$$\left\{ \begin{array}{l} \frac{\sqrt{a} a^{\frac{b}{2}} \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b x \sqrt{\frac{a}{x^2} + 1}} - \frac{2 a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \Gamma\left(1 - \frac{b}{2}\right)}{b^2 \Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}} x \cosh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b} - \frac{a^{\frac{b}{2}} x \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b \sqrt{\frac{a}{x^2} + 1}} \\ \frac{a^{\frac{b}{2}} \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b \sqrt{1 + \frac{x^2}{a}}} - \frac{2 a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \Gamma\left(1 - \frac{b}{2}\right)}{b^2 \Gamma\left(-\frac{b}{2}\right)} - \frac{a^{\frac{b}{2}} x^2 \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{a b \sqrt{1 + \frac{x^2}{a}}} + \frac{a^{\frac{b}{2}} x \cosh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/
(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*b) - a**(b/2)*x*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2)/Abs(a) > 1), (-a**(b/2)*sinh(-b*asinh(
x/sqrt(a)) + asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asi
nh(x/sqrt(a)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh
(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + \sqrt{x^2 + a})^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

$$3.20 \quad \int \frac{(x - \sqrt{a+x^2})^b}{\sqrt{a+x^2}} dx$$

**Optimal.** Leaf size=20

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

[Out] -((x - Sqrt[a + x^2])^b/b)

**Rubi [A]** time = 0.0598107, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2122, 30}

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2], x]

[Out] -((x - Sqrt[a + x^2])^b/b)

#### Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned} \int \frac{(x - \sqrt{a+x^2})^b}{\sqrt{a+x^2}} dx &= -\text{Subst}\left(\int x^{-1+b} dx, x, x - \sqrt{a+x^2}\right) \\ &= -\frac{(x - \sqrt{a+x^2})^b}{b} \end{aligned}$$

**Mathematica [A]** time = 0.00611, size = 20, normalized size = 1.

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^b/b)

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b \frac{1}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x - \sqrt{x^2 + a})^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

**Fricas [A]** time = 1.98498, size = 35, normalized size = 1.75

$$-\frac{(x - \sqrt{x^2 + a})^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] -(x - sqrt(x^2 + a))^b/b

**Sympy [A]** time = 2.86755, size = 36, normalized size = 1.8

$$\begin{cases} \frac{(x - \sqrt{a + x^2})^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x\*\*2+a)\*\*(1/2))\*\*b/(x\*\*2+a)\*\*(1/2),x)

[Out] Piecewise((-x - sqrt(a + x\*\*2))\*\*b/b, Ne(b, 0)), (Piecewise((asinh(x\*sqrt(1/a)), a > 0), (acosh(x\*sqrt(-1/a)), a < 0)), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x - \sqrt{x^2 + a})^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)



$$3.21 \quad \int \frac{1}{(a+be^{px})^2} dx$$

**Optimal.** Leaf size=42

$$-\frac{\log(a+be^{px})}{a^2p} + \frac{x}{a^2} + \frac{1}{ap(a+be^{px})}$$

[Out] 1/(a\*(a + b\*E^(p\*x))\*p) + x/a^2 - Log[a + b\*E^(p\*x)]/(a^2\*p)

**Rubi [A]** time = 0.0334166, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2282, 44}

$$-\frac{\log(a+be^{px})}{a^2p} + \frac{x}{a^2} + \frac{1}{ap(a+be^{px})}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(p\*x))^(-2), x]

[Out] 1/(a\*(a + b\*E^(p\*x))\*p) + x/a^2 - Log[a + b\*E^(p\*x)]/(a^2\*p)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+be^{px})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{px}\right)}{p} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, e^{px}\right)}{p} \\ &= \frac{1}{a(a+be^{px})p} + \frac{x}{a^2} - \frac{\log(a+be^{px})}{a^2p} \end{aligned}$$

**Mathematica [A]** time = 0.0415607, size = 36, normalized size = 0.86

$$\frac{\frac{a}{a+be^{px}} - \log(a+be^{px}) + px}{a^2p}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(p\*x))^(-2),x]

[Out] (a/(a + b\*E^(p\*x)) + p\*x - Log[a + b\*E^(p\*x)])/(a^2\*p)

**Maple [A]** time = 0.01, size = 48, normalized size = 1.1

$$\frac{\ln(e^{px})}{pa^2} - \frac{\ln(a + be^{px})}{pa^2} + \frac{1}{a(a + be^{px})p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*exp(p\*x))^2,x)

[Out] 1/p/a^2\*ln(exp(p\*x))-ln(a+b\*exp(p\*x))/a^2/p+1/a/(a+b\*exp(p\*x))/p

**Maxima [A]** time = 0.947001, size = 54, normalized size = 1.29

$$\frac{x}{a^2} + \frac{1}{(abe^{(px)} + a^2)p} - \frac{\log\left( be^{(px)} + a \right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(p\*x))^2,x, algorithm="maxima")

[Out] x/a^2 + 1/((a\*b\*e^(p\*x) + a^2)\*p) - log(b\*e^(p\*x) + a)/(a^2\*p)

**Fricas [A]** time = 2.0758, size = 124, normalized size = 2.95

$$\frac{bpxe^{(px)} + apx - (be^{(px)} + a)\log\left( be^{(px)} + a \right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(p\*x))^2,x, algorithm="fricas")

[Out] (b\*p\*x\*e^(p\*x) + a\*p\*x - (b\*e^(p\*x) + a)\*log(b\*e^(p\*x) + a) + a)/(a^2\*b\*p\*e^(p\*x) + a^3\*p)

**Sympy [A]** time = 0.131613, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left( \frac{a}{b} + e^{px} \right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(p\*x))\*\*2,x)

[Out] 1/(a\*\*2\*p + a\*b\*p\*exp(p\*x)) + x/a\*\*2 - log(a/b + exp(p\*x))/(a\*\*2\*p)

**Giac [A]** time = 1.07974, size = 55, normalized size = 1.31

$$\frac{x}{a^2} - \frac{\log\left(\left|be^{(px)} + a\right|\right)}{a^2p} + \frac{1}{\left(be^{(px)} + a\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(p\*x))^2,x, algorithm="giac")

[Out] x/a^2 - log(abs(b\*e^(p\*x) + a))/(a^2\*p) + 1/((b\*e^(p\*x) + a)\*a\*p)

$$3.22 \quad \int \frac{1}{(be^{-px} + ae^{px})^2} dx$$

**Optimal.** Leaf size=22

$$-\frac{1}{2ap(ae^{2px} + b)}$$

[Out] -1/(2\*a\*(b + a\*E^(2\*p\*x))\*p)

**Rubi [A]** time = 0.0235268, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2282, 261}

$$-\frac{1}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] Int[(b/E^(p\*x) + a\*E^(p\*x))^(-2), x]

[Out] -1/(2\*a\*(b + a\*E^(2\*p\*x))\*p)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

#### Rubi steps

$$\int \frac{1}{(be^{-px} + ae^{px})^2} dx = \frac{\text{Subst}\left(\int \frac{x}{(b+ax^2)^2} dx, x, e^{px}\right)}{p} = -\frac{1}{2a(b + ae^{2px})p}$$

**Mathematica [A]** time = 0.0202344, size = 22, normalized size = 1.

$$-\frac{1}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(b/E^(p\*x) + a\*E^(p\*x))^(2), x]

[Out] -1/(2\*a\*(b + a\*E^(2\*p\*x))\*p)

**Maple [A]** time = 0.002, size = 21, normalized size = 1.

$$-\frac{1}{2pa(a(e^{px})^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/exp(p\*x)+a\*exp(p\*x))^2, x)

[Out] -1/2/p/a/(a\*exp(p\*x)^2+b)

**Maxima [A]** time = 0.935151, size = 27, normalized size = 1.23

$$\frac{1}{2(b^2e^{(-2px)} + ab)p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(p\*x)+a\*exp(p\*x))^2, x, algorithm="maxima")

[Out] 1/2/((b^2\*e^(-2\*p\*x) + a\*b)\*p)

**Fricas [A]** time = 1.95579, size = 43, normalized size = 1.95

$$-\frac{1}{2(a^2pe^{(2px)} + abp)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(p\*x)+a\*exp(p\*x))^2, x, algorithm="fricas")

[Out] -1/2/(a^2\*p\*e^(2\*p\*x) + a\*b\*p)

**Sympy [A]** time = 0.102672, size = 20, normalized size = 0.91

$$\frac{1}{2abp + 2b^2pe^{-2px}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(p\*x)+a\*exp(p\*x))\*\*2, x)

[Out] 1/(2\*a\*b\*p + 2\*b\*\*2\*p\*exp(-2\*p\*x))

---

**Giac [A]** time = 1.07501, size = 26, normalized size = 1.18

$$-\frac{1}{2(ae^{(2px)} + b)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(p\*x)+a\*exp(p\*x))^2,x, algorithm="giac")

[Out] -1/2/((a\*e^(2\*p\*x) + b)\*a\*p)

$$3.23 \quad \int \frac{x}{(be^{-px} + ae^{px})^2} dx$$

**Optimal.** Leaf size=62

$$-\frac{\log(ae^{2px} + b)}{4abp^2} + \frac{x}{2abp} - \frac{x}{2ap(ae^{2px} + b)}$$

[Out] x/(2\*a\*b\*p) - x/(2\*a\*(b + a\*E^(2\*p\*x))\*p) - Log[b + a\*E^(2\*p\*x)]/(4\*a\*b\*p^2)

**Rubi [A]** time = 0.0907172, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{\log(ae^{2px} + b)}{4abp^2} + \frac{x}{2abp} - \frac{x}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] Int[x/(b/E^(p\*x) + a\*E^(p\*x))^2,x]

[Out] x/(2\*a\*b\*p) - x/(2\*a\*(b + a\*E^(2\*p\*x))\*p) - Log[b + a\*E^(2\*p\*x)]/(4\*a\*b\*p^2)

#### Rule 2283

Int[(u\_.)\*((a\_.)\*(F\_)^(v\_) + (b\_.)\*(F\_)^(w\_))^(n\_), x\_Symbol] := Int[u\*F^(n\*v)\*(a + b\*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ[n, 0] && LinearQ[{v, w}, x]

#### Rule 2191

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{(be^{-px} + ae^{px})^2} dx &= \int \frac{e^{2px}x}{(b + ae^{2px})^2} dx \\
 &= -\frac{x}{2a(b + ae^{2px})p} + \frac{\int \frac{1}{b+ae^{2px}} dx}{2ap} \\
 &= -\frac{x}{2a(b + ae^{2px})p} + \frac{\text{Subst}\left(\int \frac{1}{x(b+ax)} dx, x, e^{2px}\right)}{4ap^2} \\
 &= -\frac{x}{2a(b + ae^{2px})p} - \frac{\text{Subst}\left(\int \frac{1}{b+ax} dx, x, e^{2px}\right)}{4bp^2} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, e^{2px}\right)}{4abp^2} \\
 &= \frac{x}{2abp} - \frac{x}{2a(b + ae^{2px})p} - \frac{\log(b + ae^{2px})}{4abp^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0648487, size = 49, normalized size = 0.79

$$\frac{\frac{2pxe^{2px}}{ae^{2px}+b} - \frac{\log(ae^{2px}+b)}{a}}{4bp^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b/E^(p\*x) + a\*E^(p\*x))^2,x]

[Out] ((2\*E^(2\*p\*x)\*p\*x)/(b + a\*E^(2\*p\*x)) - Log[b + a\*E^(2\*p\*x)]/a)/(4\*b\*p^2)

**Maple [A]** time = 0.012, size = 51, normalized size = 0.8

$$-\frac{\ln(a(e^{px})^2 + b)}{4p^2ba} + \frac{x(e^{px})^2}{2bp(a(e^{px})^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b/exp(p\*x)+a\*exp(p\*x))^2,x)

[Out] -1/4/p^2/b/a\*ln(a\*exp(p\*x)^2+b)+1/2/p\*x\*exp(p\*x)^2/b/(a\*exp(p\*x)^2+b)



**Maxima [A]** time = 0.955416, size = 69, normalized size = 1.11

$$\frac{xe^{(2px)}}{2(abpe^{(2px)} + b^2p)} - \frac{\log\left(\frac{ae^{(2px)}+b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/exp(p\*x)+a\*exp(p\*x))^2,x, algorithm="maxima")

[Out] 1/2\*x\*e^(2\*p\*x)/(a\*b\*p\*e^(2\*p\*x) + b^2\*p) - 1/4\*log((a\*e^(2\*p\*x) + b)/a)/(a\*b\*p^2)

**Fricas [A]** time = 2.1886, size = 135, normalized size = 2.18

$$\frac{2apxe^{(2px)} - (ae^{(2px)} + b)\log(ae^{(2px)} + b)}{4(a^2bp^2e^{(2px)} + ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/exp(p\*x)+a\*exp(p\*x))^2,x, algorithm="fricas")

[Out] 1/4\*(2\*a\*p\*x\*e^(2\*p\*x) - (a\*e^(2\*p\*x) + b)\*log(a\*e^(2\*p\*x) + b))/(a^2\*b\*p^2\*e^(2\*p\*x) + a\*b^2\*p^2)

**Sympy [A]** time = 0.158276, size = 51, normalized size = 0.82

$$\frac{x}{2abp + 2b^2pe^{-2px}} - \frac{x}{2abp} - \frac{\log\left(\frac{a}{b} + e^{-2px}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/exp(p\*x)+a\*exp(p\*x))\*\*2,x)

[Out] x/(2\*a\*b\*p + 2\*b\*\*2\*p\*exp(-2\*p\*x)) - x/(2\*a\*b\*p) - log(a/b + exp(-2\*p\*x))/(4\*a\*b\*p\*\*2)

**Giac [A]** time = 1.07893, size = 100, normalized size = 1.61

$$\frac{2apxe^{(2px)} - ae^{(2px)}\log(-ae^{(2px)} - b) - b\log(-ae^{(2px)} - b)}{4(a^2bp^2e^{(2px)} + ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b/exp(p\*x)+a\*exp(p\*x))^2,x, algorithm="giac")

[Out] 1/4\*(2\*a\*p\*x\*e^(2\*p\*x) - a\*e^(2\*p\*x)\*log(-a\*e^(2\*p\*x) - b) - b\*log(-a\*e^(2\*p\*x) - b))/(a^2\*b\*p^2\*e^(2\*p\*x) + a\*b^2\*p^2)

$$3.24 \quad \int \frac{1-x+3x^2}{\sqrt{1-x+x^2}(1+x+x^2)^2} dx$$

**Optimal.** Leaf size=86

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

[Out] ((1 + x)\*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]\*ArcTan[(Sqrt[2]\*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]\*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[6]

**Rubi [A]** time = 0.0839424, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3\*x^2)/(Sqrt[1 - x + x^2]\*(1 + x + x^2)^2), x]

[Out] ((1 + x)\*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]\*ArcTan[(Sqrt[2]\*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]\*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[6]

### Rule 1060

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> Simp[((a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q + 1)\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) + c\*(A\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)) - B\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f) + C\*(b^2\*d - a\*b\*e - 2\*a\*(c\*d - a\*f)))\*x)/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), x] + Dist[1/((b^2 - 4\*a\*c)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q\*Simp[(b\*B - 2\*A\*c - 2\*a\*C)\*((c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f))\*(p + 1) + (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(a\*f\*(p + 1) - c\*d\*(p + 2)) - e\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (2\*f\*((A\*c - a\*C)\*(2\*a\*c\*e - b\*(c\*d + a\*f)) + (A\*b - a\*B)\*(2\*c^2\*d + b^2\*f - c\*(b\*e + 2\*a\*f)))\*(p + q + 2) - (b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(b\*f\*(p + 1) - c\*e\*(2\*p + q + 4)))\*x - c\*f\*(b^2\*(C\*d + A\*f) - b\*(B\*c\*d + A\*c\*e + a\*C\*e + a\*B\*f) + 2\*(A\*c\*(c\*d - a\*f) - a\*(c\*C\*d - B\*c\*e - a\*C\*f)))\*(2\*p + 2\*q + 5)\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && NeQ[(c\*d - a\*f)^2 - (b\*d - a\*e)\*(c\*e - b\*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{1-x+3x^2}{\sqrt{1-x+x^2}(1+x+x^2)^2} dx &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12} \int \frac{18-6x}{\sqrt{1-x+x^2}(1+x+x^2)} dx \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48} \int \frac{24+24x}{\sqrt{1-x+x^2}(1+x+x^2)} dx - \frac{1}{48} \int \frac{-48+48x}{\sqrt{1-x+x^2}(1+x+x^2)} dx \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24 \operatorname{Subst}\left(\int \frac{1}{1728-2x^2} dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) + 288 \operatorname{Subst}\left(\int \frac{1}{1728-2x^2} dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}} \end{aligned}$$

**Mathematica [C]** time = 2.38024, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \frac{(7-i\sqrt{3}) \tan^{-1}\left(\frac{3((-21-4i\sqrt{3})x^4+14(7-2i\sqrt{3})x^3+(-103-36i\sqrt{3})x^2+(9-2i\sqrt{3})x+1))}{(84i-113\sqrt{3})x^4+2(52\sqrt{3}-3i\sqrt{3}\sqrt{x^2-x+1}+21\sqrt{3}+138i)x^3+(52\sqrt{3}-3i\sqrt{3}\sqrt{x^2-x+1}-59\sqrt{3}-180i)x^2+2(2\sqrt{3}-3i\sqrt{3}\sqrt{x^2-x+1}+138i)x+138i}\right)}{4\sqrt{3-3i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x + 3\*x^2)/(Sqrt[1 - x + x^2]\*(1 + x + x^2)^2), x]

[Out] 
$$\frac{((1+x)\sqrt{1-x+x^2})/(1+x+x^2) + ((7-I\sqrt{3})\text{ArcTan}[(3*(-17 - (64I)\sqrt{3} + (94 + (32I)\sqrt{3}))x + (-103 - (36I)\sqrt{3})x^2 + 14*(7 - (2I)\sqrt{3})x^3 + (-21 - (4I)\sqrt{3})x^4)]/(96I + 67\sqrt{3}) + (84I - 113\sqrt{3})x^4 - 52\sqrt{3 - (3I)\sqrt{3}}\sqrt{1-x+x^2} + 2*x*(132I - 69\sqrt{3} + 26\sqrt{3 - (3I)\sqrt{3}})\sqrt{1-x+x^2}) + x^2*(-180I - 59\sqrt{3} + 52\sqrt{3 - (3I)\sqrt{3}})\sqrt{1-x+x^2}) + 2*x^3*(138I + 21\sqrt{3} + 52\sqrt{3 - (3I)\sqrt{3}})\sqrt{1-x+x^2})]/(4*\sqrt{3 - (3I)\sqrt{3}}) - ((I/4)*(-7*I + \sqrt{3})*\text{ArcTan}[(3*(-17 + (64I)\sqrt{3} + (94 - (32I)\sqrt{3}))x + (-103 + (36I)\sqrt{3})x^2 + 14*(7 + (2I)\sqrt{3})x^3 + (-21 + (4I)\sqrt{3})x^4)]/(96I - 67\sqrt{3}) + (84I + 113\sqrt{3})x^4 + 52\sqrt{3 + (3I)\sqrt{3}}\sqrt{1-x+x^2} + x^2*(-180I + 59\sqrt{3} - 52\sqrt{3 + (3I)\sqrt{3}})\sqrt{1-x+x^2}) + x*(264I + 138\sqrt{3} - 52\sqrt{3 + (3I)\sqrt{3}})\sqrt{1-x+x^2}) - 2*x^3*(-138I + 21\sqrt{3} + 52\sqrt{3 + (3I)\sqrt{3}})\sqrt{1-x+x^2})]/\sqrt{3 + (3I)\sqrt{3}} - ((-7*I + \sqrt{3})*\text{Log}[16*(1+x+x^2)^2])/(8*\sqrt{3 + (3I)\sqrt{3}}) - ((7*I + \sqrt{3})*\text{Log}[16*(1+x+x^2)^2])/(8*\sqrt{3 - (3I)\sqrt{3}}) + ((7*I + \sqrt{3})*\text{Log}[(1+x+x^2)*(11I + 4\sqrt{3} + (11I + 4\sqrt{3})x^2 + (10I)\sqrt{1-I\sqrt{3}})\sqrt{1-x+x^2} - x*(17I + 4\sqrt{3} + (8I)\sqrt{1-I\sqrt{3}})\sqrt{1-x+x^2}]))/(8*\sqrt{3 - (3I)\sqrt{3}}) + ((-7*I + \sqrt{3})*\text{Log}[(1+x+x^2)*(-11I + 4\sqrt{3} + (-11I + 4\sqrt{3})x^2 - (10I)\sqrt{1+I\sqrt{3}})\sqrt{1-x+x^2} + x*(17I - 4\sqrt{3} + (8I)\sqrt{1+I\sqrt{3}})\sqrt{1-x+x^2}]))/(8*\sqrt{3 + (3I)\sqrt{3}})$$

**Maple [B]** time = 0.034, size = 454, normalized size = 5.3

$$\frac{1}{6} \left( 6 \frac{\sqrt{6}(1+x)^2}{(1-x)^2} \text{Arctanh} \left( \frac{1}{4} \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3\sqrt{6}} \right) \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} - 9 \frac{\sqrt{2}(1+x)^2}{(1-x)^2} \arctan \left( 2 \frac{(1+x)\sqrt{2}}{1-x} \frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2} + 3}} \right) \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2), x)

[Out] 
$$\frac{1}{6} * (6 * 6^{(1/2)} * \text{arctanh}(1/4 * ((1+x)^2/(1-x)^2+3)^{(1/2)} * 6^{(1/2)}) * ((1+x)^2/(1-x)^2+3)^{(1/2)} * (1+x)^2/(1-x)^2 - 9 * \text{arctan}(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^{(1/2)} * 2^{(1/2)}) * 2^{(1/2)} * ((1+x)^2/(1-x)^2+3)^{(1/2)} * (1+x)^2/(1-x)^2 + 2 * 6^{(1/2)} * \text{arctanh}(1/4 * ((1+x)^2/(1-x)^2+3)^{(1/2)} * 6^{(1/2)}) * ((1+x)^2/(1-x)^2+3)^{(1/2)} - 3 * 2^{(1/2)} * \text{arctan}(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^{(1/2)} * 2^{(1/2)}) * ((1+x)^2/(1-x)^2+3)^{(1/2)} + 12 * (1+x)^3/(1-x)^3 + 36 * (1+x)/(1-x) / (((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x)))^{(1/2)} / (1+(1+x)/(1-x)) / (3*(1+x)^2/(1-x)^2+1) - 1/2 * ((1+x)^2/(1-x)^2+3)^{(1/2)} * (6^{(1/2)} * \text{arctanh}(1/4 * ((1+x)^2/(1-x)^2+3)^{(1/2)} * 6^{(1/2)}) - 3 * 2^{(1/2)} * \text{arctan}(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^{(1/2)} * 2^{(1/2)})) / (1+(1+x)/(1-x)) / (((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x)))^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{(x^2 + x + 1)^2 \sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")

[Out] integrate((3\*x^2 - x + 1)/((x^2 + x + 1)^2\*sqrt(x^2 - x + 1)), x)

**Fricas [B]** time = 2.26406, size = 1095, normalized size = 12.73

$$8\sqrt{6}\sqrt{3}(x^2+x+1)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}(2x-\sqrt{6}+1)-\sqrt{6}(x+1)+4(\sqrt{6}\sqrt{3}+3\sqrt{3})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")

[Out] -1/12\*(8\*sqrt(6)\*sqrt(3)\*(x^2+x+1)\*arctan(2/3\*sqrt(6)\*sqrt(3)\*(x-1)+2/3\*sqrt(2\*x^2-sqrt(x^2-x+1)\*(2\*x-sqrt(6)+1)-sqrt(6)\*(x+1)+4\*(sqrt(6)\*sqrt(3)+3\*sqrt(3))-2/3\*sqrt(x^2-x+1)\*(sqrt(6)\*sqrt(3)+3\*sqrt(3))+sqrt(3)\*(2\*x-1))+8\*sqrt(6)\*sqrt(3)\*(x^2+x+1)\*arctan(2/3\*sqrt(6)\*sqrt(3)\*(x-1)+2/3\*sqrt(2\*x^2-sqrt(x^2-x+1)\*(2\*x+sqrt(6)+1)+sqrt(6)\*(x+1)+4\*(sqrt(6)\*sqrt(3)-3\*sqrt(3))-2/3\*sqrt(x^2-x+1)\*(sqrt(6)\*sqrt(3)-3\*sqrt(3))-sqrt(3)\*(2\*x-1))-sqrt(6)\*(x^2+x+1)\*log(12168\*x^2-6084\*sqrt(x^2-x+1)\*(2\*x+sqrt(6)+1)+6084\*sqrt(6)\*(x+1)+24336)+sqrt(6)\*(x^2+x+1)\*log(12168\*x^2-6084\*sqrt(x^2-x+1)\*(2\*x-sqrt(6)+1)-6084\*sqrt(6)\*(x+1)+24336)-12\*x^2-12\*sqrt(x^2-x+1)\*(x+1)-12\*x-12)/(x^2+x+1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2-x+1}{\sqrt{x^2-x+1}(x^2+x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2-x+1)/(x\*\*2+x+1)\*\*2/(x\*\*2-x+1)\*\*(1/2),x)

[Out] Integral((3\*x\*\*2 - x + 1)/(sqrt(x\*\*2 - x + 1)\*(x\*\*2 + x + 1)\*\*2), x)

**Giac [C]** time = 1.13719, size = 424, normalized size = 4.93

$$-\frac{1}{12}\sqrt{6}(2i\sqrt{3}+1)\log\left(2i\sqrt{6}\sqrt{3}-12x+6\sqrt{6}-6i\sqrt{3}+12\sqrt{x^2-x+1}-6\right)+\frac{1}{12}\sqrt{6}(-2i\sqrt{3}+1)\log\left(2i\sqrt{6}\sqrt{3}-12x+6\sqrt{6}+6i\sqrt{3}+12\sqrt{x^2-x+1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")

[Out] -1/12\*sqrt(6)\*(2\*I\*sqrt(3)+1)\*log(2\*I\*sqrt(6)\*sqrt(3)-12\*x+6\*sqrt(6)-6\*I\*sqrt(3)+12\*sqrt(x^2-x+1)-6)+1/12\*sqrt(6)\*(-2\*I\*sqrt(3)+1)\*log(2\*I\*sqrt(6)\*sqrt(3)-12\*x-6\*sqrt(6)+6\*I\*sqrt(3)+12\*sqrt(x^2-x+1)-6)

$$\begin{aligned}
& + 1) - 6) - 1/12*\sqrt{6}*(-2*I*\sqrt{3} + 1)*\log(-2*I*\sqrt{6}*\sqrt{3} - 12* \\
& x + 6*\sqrt{6} + 6*I*\sqrt{3} + 12*\sqrt{x^2 - x + 1} - 6) + 1/12*\sqrt{6}*(2*I \\
& *\sqrt{3} + 1)*\log(-2*I*\sqrt{6}*\sqrt{3} - 12*x - 6*\sqrt{6} - 6*I*\sqrt{3} + 1 \\
& 2*\sqrt{x^2 - x + 1} - 6) + ((x - \sqrt{x^2 - x + 1})^3 + 4*(x - \sqrt{x^2 - x \\
& + 1))^2 - 10*x + 10*\sqrt{x^2 - x + 1} + 5)/((x - \sqrt{x^2 - x + 1})^4 + 2* \\
& (x - \sqrt{x^2 - x + 1})^3 + (x - \sqrt{x^2 - x + 1})^2 - 6*x + 6*\sqrt{x^2 - \\
& x + 1} + 3)
\end{aligned}$$

$$3.25 \quad \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx$$

**Optimal.** Leaf size=19

$$2\sqrt{\sqrt{a^2 + x^2} + x}$$

[Out] 2\*Sqrt[x + Sqrt[a^2 + x^2]]

**Rubi [A]** time = 0.0651093, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2122, 30}

$$2\sqrt{\sqrt{a^2 + x^2} + x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]

[Out] 2\*Sqrt[x + Sqrt[a^2 + x^2]]

#### Rule 2122

Int[((g\_) + (i\_)\*(x\_)^2)^(m\_)\*((d\_) + (e\_)\*(x\_) + (f\_)\*Sqrt[(a\_) + (c\_)\*(x\_)^2])^(n\_), x\_Symbol] := Dist[(1\*(i/c)^m)/(2^(2\*m + 1)\*e\*f^(2\*m)), Subst[Int[(x^n\*(d^2 + a\*f^2 - 2\*d\*x + x^2)^(2\*m + 1))/(-d + x)^(2\*(m + 1)), x], x, d + e\*x + f\*Sqrt[a + c\*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c\*f^2, 0] && EqQ[c\*g - a\*i, 0] && IntegerQ[2\*m] && (IntegerQ[m] || GtQ[i/c, 0])

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx &= \text{Subst} \left( \int \frac{1}{\sqrt{x}} dx, x, x + \sqrt{a^2 + x^2} \right) \\ &= 2\sqrt{x + \sqrt{a^2 + x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.0100317, size = 19, normalized size = 1.

$$2\sqrt{\sqrt{a^2 + x^2} + x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]

[Out] 2\*Sqrt[x + Sqrt[a^2 + x^2]]

---

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt{x + \sqrt{a^2 + x^2}} \frac{1}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)

[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)

---

**Fricas [A]** time = 2.02462, size = 39, normalized size = 2.05

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")

[Out] 2\*sqrt(x + sqrt(a^2 + x^2))

---

**Sympy [A]** time = 0.206048, size = 15, normalized size = 0.79

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a\*\*2+x\*\*2)\*\*(1/2))\*\*(1/2)/(a\*\*2+x\*\*2)\*\*(1/2),x)

[Out] 2\*sqrt(x + sqrt(a\*\*2 + x\*\*2))

---



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

$$3.26 \quad \int \frac{\sqrt{bx + \sqrt{a + b^2x^2}}}{\sqrt{a + b^2x^2}} dx$$

**Optimal.** Leaf size=26

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

[Out] (2\*Sqrt[b\*x + Sqrt[a + b^2\*x^2]])/b

**Rubi [A]** time = 0.0970069, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$ , Rules used = {2122, 30}

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*x + Sqrt[a + b^2\*x^2]]/Sqrt[a + b^2\*x^2], x]

[Out] (2\*Sqrt[b\*x + Sqrt[a + b^2\*x^2]])/b

#### Rule 2122

```
Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)
*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

#### Rubi steps

$$\int \frac{\sqrt{bx + \sqrt{a + b^2x^2}}}{\sqrt{a + b^2x^2}} dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, bx + \sqrt{a + b^2x^2}\right)}{b} = \frac{2\sqrt{bx + \sqrt{a + b^2x^2}}}{b}$$

**Mathematica [A]** time = 0.0170815, size = 26, normalized size = 1.

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*x + Sqrt[a + b^2\*x^2]]/Sqrt[a + b^2\*x^2], x]

[Out] (2\*Sqrt[b\*x + Sqrt[a + b^2\*x^2]])/b

**Maple [F]** time = 0.015, size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+(b^2\*x^2+a)^(1/2))^(1/2)/(b^2\*x^2+a)^(1/2), x)

[Out] int((b\*x+(b^2\*x^2+a)^(1/2))^(1/2)/(b^2\*x^2+a)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+(b^2\*x^2+a)^(1/2))^(1/2)/(b^2\*x^2+a)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b\*x + sqrt(b^2\*x^2 + a))/sqrt(b^2\*x^2 + a), x)

**Fricas [A]** time = 1.97501, size = 47, normalized size = 1.81

$$\frac{2\sqrt{bx + \sqrt{b^2x^2 + a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+(b^2\*x^2+a)^(1/2))^(1/2)/(b^2\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] 2\*sqrt(b\*x + sqrt(b^2\*x^2 + a))/b

**Sympy [A]** time = 0.879158, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx + \sqrt{a + b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+(b\*\*2\*x\*\*2+a)\*\*(1/2))\*\*(1/2)/(b\*\*2\*x\*\*2+a)\*\*(1/2), x)

[Out] Piecewise((2\*sqrt(b\*x + sqrt(a + b\*\*2\*x\*\*2))/b, Ne(b, 0)), (x/a\*\*(1/4), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+(b^2\*x^2+a)^(1/2))^(1/2)/(b^2\*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*x + sqrt(b^2\*x^2 + a))/sqrt(b^2\*x^2 + a), x)

$$3.27 \quad \int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}} dx$$

**Optimal.** Leaf size=63

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out]  $(-2*\text{ArcTan}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)} - (2*\text{ArcTanh}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi [A]** time = 0.219029, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {2120, 329, 212, 206, 203}

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Sqrt}[a^2 + x^2]*\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]), x]$

[Out]  $(-2*\text{ArcTan}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)} - (2*\text{ArcTanh}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)}$

#### Rule 2120

$\text{Int}[(x_)^{(p_)}*((g_) + (i_)*(x_)^2)^{(m_)}*((e_)*(x_) + (f_)*\text{Sqrt}[(a_) + (c_)*(x_)^2])^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(1*(i/c)^m)/(2^{(2*m + p + 1)}*e^{(p + 1)*f^{(2*m)}}), \text{Subst}[\text{Int}[x^{(n - 2*m - p - 2)}*(-(a*f^2) + x^2)^p*(a*f^2 + x^2)^{(2*m + 1)}, x], x, e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, e, f, g, i, n\}, x] \&\& \text{EqQ}[e^2 - c*f^2, 0] \&\& \text{EqQ}[c*g - a*i, 0] \&\& \text{IntegersQ}[p, 2*m] \& \& (\text{IntegerQ}[m] \parallel \text{GtQ}[i/c, 0])$

#### Rule 329

$\text{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)}))/c^n]^p, x], x, (c*x)^{(1/k)}], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 212

$\text{Int}[(a_) + (b_)*(x_)^4)^{(-1)}, x\_Symbol] \rightarrow \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a/b, 0]$

#### Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

**Rule 203**

$\text{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTan}[\text{Rt}[b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] / ; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}} dx &= 2 \text{Subst} \left( \int \frac{1}{\sqrt{x}(-a^2+x^2)} dx, x, x + \sqrt{a^2+x^2} \right) \\ &= 4 \text{Subst} \left( \int \frac{1}{-a^2+x^4} dx, x, \sqrt{x+\sqrt{a^2+x^2}} \right) \\ &= \frac{2 \text{Subst} \left( \int \frac{1}{a-x^2} dx, x, \sqrt{x+\sqrt{a^2+x^2}} \right) - 2 \text{Subst} \left( \int \frac{1}{a+x^2} dx, x, \sqrt{x+\sqrt{a^2+x^2}} \right)}{a} \\ &= \frac{2 \tan^{-1} \left( \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}} \right) - 2 \tanh^{-1} \left( \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}} \right)}{a^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.192124, size = 56, normalized size = 0.89

$$\frac{2 \left( \tan^{-1} \left( \frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}} \right) + \tanh^{-1} \left( \frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}} \right) \right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[a^2 + x^2]\*Sqrt[x + Sqrt[a^2 + x^2]]), x]

[Out] (-2\*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]))/a^(3/2)

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{x} \frac{1}{\sqrt{a^2+x^2}} \frac{1}{\sqrt{x+\sqrt{a^2+x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2), x)

[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a^2 + x^2)\*sqrt(x + sqrt(a^2 + x^2)))\*x, x)

**Fricas [A]** time = 2.42215, size = 487, normalized size = 7.73

$$\left[ \frac{2\sqrt{a} \arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - \sqrt{a} \log\left(\frac{a^2 + \sqrt{a^2+x^2}a - ((a-x)\sqrt{a+\sqrt{a^2+x^2}}\sqrt{a})\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right) - \sqrt{-a}}{a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] [-(2\*sqrt(a)\*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)\*log((a^2 + sqrt(a^2 + x^2)\*a - ((a - x)\*sqrt(a) + sqrt(a^2 + x^2)\*sqrt(a))\*sqrt(x + sqrt(a^2 + x^2))))/x)/a^2, (2\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt(x + sqrt(a^2 + x^2)))/a - sqrt(-a)\*log(-(a^2 - sqrt(a^2 + x^2)\*a - (sqrt(-a)\*(a + x) - sqrt(a^2 + x^2)\*sqrt(-a))\*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]

**Sympy [C]** time = 1.59298, size = 46, normalized size = 0.73

$$\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right) {}_3F_2\left(\begin{matrix} \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}} \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a\*\*2+x\*\*2)\*\*(1/2)/(x+(a\*\*2+x\*\*2)\*\*(1/2))\*\*(1/2),x)

[Out] -gamma(3/4)\*\*2\*gamma(5/4)\*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a\*\*2\*exp\_polar(I\*pi)/x\*\*2)/(pi\*x\*\*(3/2)\*gamma(7/4))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + x^2} \sqrt{x + \sqrt{a^2 + x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a^2 + x^2)\*sqrt(x + sqrt(a^2 + x^2)))\*x, x)

$$3.28 \quad \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

**Optimal.** Leaf size=82

$$2\sqrt{\sqrt{a^2 + x^2} + x} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}}\right) - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}}\right)$$

[Out] 2\*Sqrt[x + Sqrt[a^2 + x^2]] - 2\*Sqrt[a]\*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2\*Sqrt[a]\*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]

**Rubi [A]** time = 0.0719809, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2 + x^2} + x} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}}\right) - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x, x]

[Out] 2\*Sqrt[x + Sqrt[a^2 + x^2]] - 2\*Sqrt[a]\*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2\*Sqrt[a]\*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]

#### Rule 2119

Int[((g\_.) + (h\_.)\*(x\_)^(m\_.))\*((e\_.)\*(x\_) + (f\_.)\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2])^(n\_.), x\_Symbol] :> Dist[1/(2^(m + 1)\*e^(m + 1)), Subst[Int[x^(n - m - 2)\*(a\*f^2 + x^2)\*(-(a\*f^2\*h) + 2\*e\*g\*x + h\*x^2)^m, x], x, e\*x + f\*Sqrt[a + c\*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c\*f^2, 0] && IntegerQ[m]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 212

Int[((a\_.) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ



[a/b, 0]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 203**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rubi steps**

$$\begin{aligned}
 \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx &= \text{Subst} \left( \int \frac{a^2 + x^2}{\sqrt{x}(-a^2 + x^2)} dx, x, x + \sqrt{a^2 + x^2} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} + (2a^2) \text{Subst} \left( \int \frac{1}{\sqrt{x}(-a^2 + x^2)} dx, x, x + \sqrt{a^2 + x^2} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} + (4a^2) \text{Subst} \left( \int \frac{1}{-a^2 + x^4} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} - (2a) \text{Subst} \left( \int \frac{1}{a - x^2} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) - (2a) \text{Subst} \left( \int \frac{1}{a + x^2} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} - 2\sqrt{a} \tan^{-1} \left( \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right) - 2\sqrt{a} \tanh^{-1} \left( \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0573668, size = 127, normalized size = 1.55

$$\frac{2\sqrt{a^2 + x^2} \left( \sqrt{a^2 + x^2} + x \right) \left( -\sqrt{\sqrt{a^2 + x^2} + x} + \sqrt{a} \tan^{-1} \left( \frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}} \right) + \sqrt{a} \tanh^{-1} \left( \frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}} \right) \right)}{x \left( \sqrt{a^2 + x^2} + x \right) + a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]

[Out] (-2\*Sqrt[a^2 + x^2]\*(x + Sqrt[a^2 + x^2])\*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqrt[a]\*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]\*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x\*(x + Sqrt[a^2 + x^2]))

**Maple [C]** time = 0.008, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}F_2(-1/4, -1/4, 1/4; 1/2, 3/4; -\frac{a^2}{x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)`

[Out] `2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)`

**Fricas [A]** time = 2.56245, size = 552, normalized size = 6.73

$$\left[ -2\sqrt{a} \arctan\left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}}\right) + \sqrt{a} \log\left(\frac{a^2 + \sqrt{a^2 + x^2}a - \left((a-x)\sqrt{a} + \sqrt{a^2 + x^2}\sqrt{a}\right)\sqrt{x + \sqrt{a^2 + x^2}}}{x}\right) + 2\sqrt{x + \sqrt{a^2 + x^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")`

[Out] `[-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 + sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2))]`

**Sympy [C]** time = 1.43801, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_3F_2\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)`

[Out] `sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

### 3.29 $\int x^3 \log^3(2+x) \log(3+x) dx$

**Optimal.** Leaf size=606

$$-\frac{5609}{96}\text{PolyLog}(2, -x-2) - \frac{563}{8}\text{PolyLog}(3, -x-2) - \frac{195}{2}\text{PolyLog}(4, -x-2) - \frac{195}{4}\log^2(x+2)\text{PolyLog}(2, -x-2) +$$

[Out]  $(-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2+x)^2)/64 - (71*(2+x)^3)/216 + (3*(2+x)^4)/256 + (2069*\text{Log}[2+x])/144 - (187*x^2*\text{Log}[2+x])/64 + (83*x^3*\text{Log}[2+x])/288 - (3*x^4*\text{Log}[2+x])/128 + (6733*(2+x)*\text{Log}[2+x])/32 - (377*(2+x)^2*\text{Log}[2+x])/32 + (71*(2+x)^3*\text{Log}[2+x])/72 - (3*(2+x)^4*\text{Log}[2+x])/64 - (43*\text{Log}[2+x]^2)/12 - (17*x^3*\text{Log}[2+x]^2)/48 + (3*x^4*\text{Log}[2+x]^2)/64 - (1251*(2+x)*\text{Log}[2+x]^2)/16 + (273*(2+x)^2*\text{Log}[2+x]^2)/32 - (3*(2+x)^3*\text{Log}[2+x]^2)/4 + (3*(2+x)^4*\text{Log}[2+x]^2)/64 + (65*(2+x)*\text{Log}[2+x]^3)/4 - (33*(2+x)^2*\text{Log}[2+x]^3)/8 + (3*(2+x)^3*\text{Log}[2+x]^3)/4 - ((2+x)^4*\text{Log}[2+x]^3)/16 + (3891*\text{Log}[3+x])/128 - (115*x^2*\text{Log}[3+x])/48 + (37*x^3*\text{Log}[3+x])/144 - (3*x^4*\text{Log}[3+x])/128 + (415*(3+x)*\text{Log}[3+x])/12 - (4083*\text{Log}[2+x]*\text{Log}[3+x])/32 - 25*x*\text{Log}[2+x]*\text{Log}[3+x] + (13*x^2*\text{Log}[2+x]*\text{Log}[3+x])/4 - (7*x^3*\text{Log}[2+x]*\text{Log}[3+x])/12 + (3*x^4*\text{Log}[2+x]*\text{Log}[3+x])/32 + (963*\text{Log}[2+x]^2*\text{Log}[3+x])/16 + 6*x*\text{Log}[2+x]^2*\text{Log}[3+x] - (3*x^2*\text{Log}[2+x]^2*\text{Log}[3+x])/2 + (x^3*\text{Log}[2+x]^2*\text{Log}[3+x])/2 - (3*x^4*\text{Log}[2+x]^2*\text{Log}[3+x])/16 - (81*\text{Log}[2+x]^3*\text{Log}[3+x])/4 + (x^4*\text{Log}[2+x]^3*\text{Log}[3+x])/4 - (5609*\text{PolyLog}[2, -2-x])/96 + (563*\text{Log}[2+x]*\text{PolyLog}[2, -2-x])/8 - (195*\text{Log}[2+x]^2*\text{PolyLog}[2, -2-x])/4 - (563*\text{PolyLog}[3, -2-x])/8 + (195*\text{Log}[2+x]*\text{PolyLog}[3, -2-x])/2 - (195*\text{PolyLog}[4, -2-x])/2$

**Rubi [A]** time = 4.79891, antiderivative size = 679, normalized size of antiderivative = 1.12, number of steps used = 359, number of rules used = 30, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.143$ , Rules used = {2439, 2416, 2389, 2296, 2295, 2401, 2390, 2305, 2304, 2396, 2433, 2374, 2383, 6589, 2411, 2346, 2302, 30, 2330, 2319, 43, 2334, 2301, 6742, 2430, 2393, 2391, 2394, 2395, 2398}

$$-\frac{5609}{96}\text{PolyLog}(2, -x-2) - \frac{563}{8}\text{PolyLog}(3, -x-2) - \frac{195}{2}\text{PolyLog}(4, -x-2) - \frac{195}{4}\log^2(x+2)\text{PolyLog}(2, -x-2) +$$

Antiderivative was successfully verified.

[In] Int[x^3\*Log[2+x]^3\*Log[3+x], x]

[Out]  $(-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2+x)^2)/64 - (71*(2+x)^3)/216 + (3*(2+x)^4)/256 + (2069*\text{Log}[2+x])/144 - (187*x^2*\text{Log}[2+x])/64 + (83*x^3*\text{Log}[2+x])/288 - (3*x^4*\text{Log}[2+x])/128 + (6365*(2+x)*\text{Log}[2+x])/32 - (273*(2+x)^2*\text{Log}[2+x])/32 + ((2+x)^3*\text{Log}[2+x])/2 - (3*(2+x)^4*\text{Log}[2+x])/128 + ((384*(2+x) - 144*(2+x)^2 + 32*(2+x)^3 - 3*(2+x)^4 - 192*\text{Log}[2+x])*\text{Log}[2+x])/128 + (17*(36*(2+x) - 9*(2+x)^2 + (2+x)^3 - 24*\text{Log}[2+x])*\text{Log}[2+x])/72 + (43*\text{Log}[2+x]^2)/12 - (17*x^3*\text{Log}[2+x]^2)/48 + (3*x^4*\text{Log}[2+x]^2)/64 - (1251*(2+x)*\text{Log}[2+x]^2)/16 + (273*(2+x)^2*\text{Log}[2+x]^2)/32 - (3*(2+x)^3*\text{Log}[2+x]^2)/4 + (3*(2+x)^4*\text{Log}[2+x]^2)/64 + (65*(2+x)*\text{Log}[2+x]^3)/4 - (33*(2+x)^2*\text{Log}[2+x]^3)/8 + (3*(2+x)^3*\text{Log}[2+x]^3)/4 - ((2+x)^4*\text{Log}[2+x]^3)/16 + (3891*\text{Log}[3+x])/128 - (115*x^2*\text{Log}[3+x])/48 + (37*x^3*\text{Log}[3+x])/144 - (3*x^4*\text{Log}[3+x])/128 + (415*(3+x)*\text{Log}[3+x])/12 - (4083*\text{Log}[2+x]*\text{Log}[3+x])/32 - 25*x*\text{Log}[2+x]*\text{Log}[3+x] + (13*x^2*\text{Log}[2+x]*\text{Log}[3+x])/4 - (7*x^3*\text{Log}[2+x]*\text{Log}[3+x])/12 + (3*x^4*\text{Log}[2+x]*\text{Log}[3+x])/32 + (963*\text{Log}[2+x]^2*\text{Log}[3+x])/16 + 6*x*\text{Log}[2+x]^2*\text{Log}[3+x] - (3*x^2*\text{Log}[2+x]^2*\text{Log}[3+x])/2 + (x^3*\text{Log}[2+x]^2*\text{Log}[3+x])/2 - (3*x^4*\text{Log}[2+x]^2*\text{Log}[3+x])/16 - (81*\text{Log}[2+x]^3*\text{Log}[3+x])/4 + (x^4*\text{Log}[2+x]^3*\text{Log}[3+x])/4 - (5609*\text{PolyLog}[2, -2-x])/96 + (563*\text{Log}[2+x]*\text{PolyLog}[2, -2-x])/8 - (195*\text{Log}[2+x]^2*\text{PolyLog}[2, -2-x])/4 - (563*\text{PolyLog}[3, -2-x])/8 + (195*\text{Log}[2+x]*\text{PolyLog}[3, -2-x])/2 - (195*\text{PolyLog}[4, -2-x])/2$

$$\begin{aligned} &^2 \text{Log}[3 + x] - (3x^2 \text{Log}[2 + x]^2 \text{Log}[3 + x])/2 + (x^3 \text{Log}[2 + x]^2 \text{Log}[3 + x])/2 \\ &- (3x^4 \text{Log}[2 + x]^2 \text{Log}[3 + x])/16 - (81 \text{Log}[2 + x]^3 \text{Log}[3 + x])/4 + (x^4 \text{Log}[2 + x]^3 \text{Log}[3 + x])/4 \\ &- (5609 \text{PolyLog}[2, -2 - x])/96 + (563 \text{Log}[2 + x] \text{PolyLog}[2, -2 - x])/8 - (195 \text{Log}[2 + x]^2 \text{PolyLog}[2, -2 - x])/4 \\ &- (563 \text{PolyLog}[3, -2 - x])/8 + (195 \text{Log}[2 + x] \text{PolyLog}[3, -2 - x])/2 - (195 \text{PolyLog}[4, -2 - x])/2 \end{aligned}$$
**Rule 2439**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c(d + e x)^n] b)^p (f + \text{Log}[h(i + j x)^m]) (h(i + j x)^m + (j x)^m) (g x)^r, x\_Symbol] \\ &:\> \text{Simp}[(x^{r+1} (a + b \text{Log}[c(d + e x)^n])^p (f + g \text{Log}[h(i + j x)^m])) / (r + 1), x] \\ &+ (-\text{Dist}[(g j^m) / (r + 1), \text{Int}[(x^{r+1} (a + b \text{Log}[c(d + e x)^n])^p] / (i + j x), x], x] \\ &- \text{Dist}[(b e^n p) / (r + 1), \text{Int}[(x^{r+1} (a + b \text{Log}[c(d + e x)^n])^{p-1} (f + g \text{Log}[h(i + j x)^m])) / (d + e x), x], x]) /; \\ &\text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, m, n, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[r] \&\& (\text{EqQ}[p, 1] \parallel \text{GtQ}[r, 0]) \&\& \text{NeQ}[r, -1] \end{aligned}$$
**Rule 2416**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c(d + e x)^n] b)^p (h x)^m (f + (g x)^r)^q, x\_Symbol] \\ &:\> \text{Int}[\text{ExpandIntegrand}[(a + b \text{Log}[c(d + e x)^n])^p, (h x)^m (f + g x^r)^q, x], x] /; \\ &\text{FreeQ}\{a, b, c, d, e, f, g, h, m, n, p, q, r, x\} \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[q] \end{aligned}$$
**Rule 2389**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c(d + e x)^n] b)^p, x\_Symbol] \\ &:\> \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b \text{Log}[c x^n])^p, x], x, d + e x], x] /; \\ &\text{FreeQ}\{a, b, c, d, e, n, p, x\} \end{aligned}$$
**Rule 2296**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c x^n] b)^p, x\_Symbol] \\ &:\> \text{Simp}[x (a + b \text{Log}[c x^n])^p, x] - \text{Dist}[b^n p, \text{Int}[(a + b \text{Log}[c x^n])^{p-1}, x], x] /; \\ &\text{FreeQ}\{a, b, c, n, x\} \&\& \text{GtQ}[p, 0] \&\& \text{IntegerQ}[2 p] \end{aligned}$$
**Rule 2295**

$$\begin{aligned} &\text{Int}[\text{Log}[c x^n], x\_Symbol] \\ &:\> \text{Simp}[x \text{Log}[c x^n], x] - \text{Simp}[n x, x] /; \\ &\text{FreeQ}\{c, n, x\} \end{aligned}$$
**Rule 2401**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c(d + e x)^n] b)^p (f + (g x)^m) (f + g x)^q (a + b \text{Log}[c(d + e x)^n])^p, x\_Symbol] \\ &:\> \text{Int}[\text{ExpandIntegrand}[(f + g x)^q (a + b \text{Log}[c(d + e x)^n])^p, x], x] /; \\ &\text{FreeQ}\{a, b, c, d, e, f, g, n, p, x\} \&\& \text{NeQ}[e f - d g, 0] \&\& \text{IGtQ}[q, 0] \end{aligned}$$
**Rule 2390**

$$\begin{aligned} &\text{Int}[(a + \text{Log}[c(d + e x)^n] b)^p (f + (g x)^m) (f x)^q (a + b \text{Log}[c x^n])^p, x\_Symbol] \\ &:\> \text{Dist}[1/e, \text{Subst}[\text{Int}[(f x)/d]^q (a + b \text{Log}[c x^n])^p, x], x, d + e x], x] /; \\ &\text{FreeQ}\{a, b, c, d, e, f, g, n, p, q, x\} \&\& \text{EqQ}[e f - d g, 0] \end{aligned}$$
**Rule 2305**

$$\text{Int}[(a + \text{Log}[c x^n] b)^p (d x)^m, x\_Symbol]$$

```
1] :=> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

#### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :=>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p)/((f_.) + (g_.
)*(x_)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :=> Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^p)/(x_), x_Symbol] :=> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

#### Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p)*PolyLog[k_, (e_.)*(x_)^(q_
.))]/(x_), x_Symbol] :=> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :=> Dist[1/e, Subst[Int
[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :=> Dist[d, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p]/x,
```

$x]$ ,  $x]$  + Dist[e, Int[(d + e\*x)^(q - 1)\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2\*q]

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2330

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*x^n])^p, (d + e\*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))

#### Rule 2319

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/(e\*(q + 1)), x] - Dist[(b\*n\*p)/(e\*(q + 1)), Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2334

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := With[{u = IntHide[x^m\*(d + e\*x^r)^q, x]}, Simp[u\*(a + b\*Log[c\*x^n]), x] - Dist[b\*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.)), x\_Symbol] := Simp[x\*(a + b\*Log[c

```
*(d + e*x)^n]]^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
  b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
  og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m])]/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^p)*((f_.) + (g_.
)*(x_)^q), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

### Rubi steps



$$\begin{aligned}
\int x^3 \log^3(2+x) \log(3+x) dx &= \frac{1}{4} x^4 \log^3(2+x) \log(3+x) - \frac{1}{4} \int \frac{x^4 \log^3(2+x)}{3+x} dx - \frac{3}{4} \int \frac{x^4 \log^2(2+x) \log(3+x)}{2+x} dx \\
&= \frac{1}{4} x^4 \log^3(2+x) \log(3+x) - \frac{1}{4} \int \left( -27 \log^3(2+x) + 9x \log^3(2+x) - 3x^2 \log^3(2+x) \right) dx \\
&= \frac{1}{4} x^4 \log^3(2+x) \log(3+x) - \frac{1}{4} \int x^3 \log^3(2+x) dx + \frac{3}{4} \int x^2 \log^3(2+x) dx - \frac{3}{4} \int x \log^3(2+x) dx \\
&= 6x \log^2(2+x) \log(3+x) - \frac{3}{2} x^2 \log^2(2+x) \log(3+x) + \frac{1}{2} x^3 \log^2(2+x) \log(3+x) - \frac{1}{2} x^4 \log^2(2+x) \log(3+x) \\
&= \frac{27}{4} (2+x) \log^3(2+x) + 6x \log^2(2+x) \log(3+x) - \frac{3}{2} x^2 \log^2(2+x) \log(3+x) + \frac{1}{2} x^3 \log^2(2+x) \log(3+x) \\
&= -\frac{81}{4} (2+x) \log^2(2+x) + \frac{27}{4} (2+x) \log^3(2+x) + 6x \log^2(2+x) \log(3+x) - \frac{3}{2} x^2 \log^2(2+x) \log(3+x) \\
&= -\frac{81x}{2} + \frac{81}{2} (2+x) \log(2+x) - \frac{17}{48} x^3 \log^2(2+x) + \frac{3}{64} x^4 \log^2(2+x) - \frac{81}{4} (2+x) \log(3+x) \\
&= -\frac{81x}{2} + \frac{81}{2} (2+x) \log(2+x) - \frac{17}{48} x^3 \log^2(2+x) + \frac{3}{64} x^4 \log^2(2+x) - \frac{765}{16} (2+x) \log(3+x) \\
&= -\frac{765x}{8} + \frac{27}{32} (2+x)^2 - \frac{1}{6} (2+x)^3 + \frac{3}{512} (2+x)^4 + \frac{765}{8} (2+x) \log(2+x) - \frac{27}{16} (2+x) \log(3+x) \\
&= -\frac{857x}{8} + \frac{79}{32} (2+x)^2 - \frac{71}{216} (2+x)^3 + \frac{3}{256} (2+x)^4 - \frac{75}{64} x^2 \log(2+x) + \frac{83}{288} x^3 \log(2+x) \\
&= -\frac{16463x}{96} + \frac{185}{64} (2+x)^2 - \frac{71}{216} (2+x)^3 + \frac{3}{256} (2+x)^4 - \frac{75}{64} x^2 \log(2+x) + \frac{83}{288} x^3 \log(2+x) \\
&= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64} (2+x)^2 - \frac{71}{216} (2+x)^3 + \frac{3}{256} (2+x)^4 +
\end{aligned}$$

**Mathematica [A]** time = 0.243863, size = 412, normalized size = 0.68

$$-224640 \text{PolyLog}(4, -x-2) - 24(4680 \log^2(x+2) - 6756 \log(x+2) + 5609) \text{PolyLog}(2, -x-2) + 288(780 \log(x+2) - 5609) \text{PolyLog}(3, -x-2) - 224640 \text{PolyLog}(4, -x-2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[2+x]^3\*Log[3+x],x]

[Out] (-195984 - 558290\*x + 17705\*x^2 - 1050\*x^3 + 54\*x^4 + 910528\*Log[2+x] + 400008\*x\*Log[2+x] - 22836\*x^2\*Log[2+x] + 2072\*x^3\*Log[2+x] - 162\*x^4\*Log[2+x] - 302016\*Log[2+x]^2 - 118800\*x\*Log[2+x]^2 + 11880\*x^2\*Log[2+x]^2 - 1680\*x^3\*Log[2+x]^2 + 216\*x^4\*Log[2+x]^2 + 48384\*Log[2+x]^3 + 15552\*x\*Log[2+x]^3 - 2592\*x^2\*Log[2+x]^3 + 576\*x^3\*Log[2+x]^3 - 144\*x^4\*Log[2+x]^3 + 309078\*Log[3+x] + 79680\*x\*Log[3+x] - 5520\*x^2\*Log[3+x] + 592\*x^3\*Log[3+x] - 54\*x^4\*Log[3+x] - 293976\*Log[2+x]\*Log[3+x] - 57600\*x\*Log[2+x]\*Log[3+x] + 7488\*x^2\*Log[2+x]\*Log[3+x] - 1344\*x^3\*Log[2+x]\*Log[3+x] + 216\*x^4\*Log[2+x]\*Log[3+x] + 138672\*Log[2+x]^2\*Log[3+x] + 13824\*x\*Log[2+x]^2\*Log[3+x] - 3456\*x^2\*Log[2+x]^2\*Log[3+x] + 1152\*x^3\*Log[2+x]^2\*Log[3+x] - 432\*x^4\*Log[2+x]^2\*Log[3+x] - 46656\*Log[2+x]^3\*Log[3+x] + 576\*x^4\*Log[2+x]^3\*Log[3+x] - 24\*(5609 - 6756\*Log[2+x] + 4680\*Log[2+x]^2)\*PolyLog[2, -2-x] + 288\*(-563 + 780\*Log[2+x])\*PolyLog[3, -2-x] - 224640\*PolyLog[4, -2-x])/2304

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int x^3 (\ln(2+x))^3 \ln(3+x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(2+x)^3\*ln(3+x),x)

[Out] int(x^3\*ln(2+x)^3\*ln(3+x),x)

**Maxima [A]** time = 1.01933, size = 699, normalized size = 1.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(2+x)^3\*log(3+x),x, algorithm="maxima")

[Out] 3/128\*x^4 + 1/16\*(4\*x^4\*log(x + 3) - x^4 + 4\*x^3 - 18\*x^2 + 108\*x - 324\*log(x + 3))\*log(x + 2)^3 - 65/4\*log(x + 3)\*log(x + 2)^3 + 195/4\*log(x + 3)\*log(x + 2)^2\*log(-x - 2) - 175/384\*x^3 + 1/96\*(9\*x^4 - 70\*x^3 + 495\*x^2 - 6\*(3\*x^4 - 8\*x^3 + 24\*x^2 - 96\*x)\*log(x + 3) + 4680\*log(x + 3)\*log(-x - 2) - 4950\*x + 4680\*dilog(x + 3) + 5778\*log(x + 3) + 6048\*log(x + 2))\*log(x + 2)^2 + 195/4\*dilog(x + 3)\*log(x + 2)^2 - 195/4\*dilog(-x - 2)\*log(x + 2)^2 + 563/16\*log(x + 3)\*log(x + 2)^2 + 21\*log(x + 2)^3 + 17705/2304\*x^2 + 1/8\*(780\*log(x + 2)^2 - 563\*log(x + 2))\*dilog(-x - 2) - 1/1152\*(27\*x^4 - 296\*x^3 - 18720\*log(x + 2)^3 + 2760\*x^2 + 40536\*log(x + 2)^2 - 39840\*x - 67308\*log(x + 2))\*log(x + 3) - 1/1152\*(81\*x^4 - 1036\*x^3 + 56160\*log(x + 3)\*log(x + 2)^2 + 112320\*log(x + 3)\*log(x + 2)\*log(-x - 2) + 11418\*x^2 - 12\*(9\*x^4 - 56\*x^3 + 312\*x^2 + 4680\*log(x + 2)^2 - 2400\*x - 6756\*log(x + 2))\*log(x + 3) + 112320\*dilog(x + 3)\*log(x + 2) + 112320\*dilog(-x - 2)\*log(x + 2) - 81072\*log(x + 3)\*log(x + 2) + 72576\*log(x + 2)^2 - 200004\*x - 81072\*dilog(-x - 2) + 146988\*log(x + 3) + 302016\*log(x + 2) - 112320\*polylog(3, -x - 2))\*log(x + 2) + 563/8\*dilog(-x - 2)\*log(x + 2) - 5609/96\*log(x + 3)\*log(x + 2) + 1573/12\*log(x + 2)^2 - 279145/1152\*x - 5609/96\*dilog(-x - 2) + 17171/128\*log(x + 3) + 14227/36\*log(x + 2) - 195/2\*polylog(4, -x - 2) - 563/8\*polylog(3, -x - 2)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \log(x+3) \log(x+2)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(2+x)^3\*log(3+x),x, algorithm="fricas")

[Out] integral(x^3\*log(x + 3)\*log(x + 2)^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(x+3) \log(x+2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

$$3.30 \quad \int \frac{(x + \sqrt{b+x^2})^a}{\sqrt{b+x^2}} dx$$

**Optimal.** Leaf size=17

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

[Out] (x + Sqrt[b + x^2])^a/a

**Rubi [A]** time = 0.0545937, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2122, 30}

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2], x]

[Out] (x + Sqrt[b + x^2])^a/a

#### Rule 2122

```
Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

#### Rubi steps

$$\int \frac{(x + \sqrt{b+x^2})^a}{\sqrt{b+x^2}} dx = \text{Subst} \left( \int x^{-1+a} dx, x, x + \sqrt{b+x^2} \right) \\ = \frac{(x + \sqrt{b+x^2})^a}{a}$$

**Mathematica [A]** time = 0.009478, size = 17, normalized size = 1.

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2], x]

[Out] (x + Sqrt[b + x^2])^a/a

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + b})^a \frac{1}{\sqrt{x^2 + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2), x)

[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + \sqrt{x^2 + b})^a}{\sqrt{x^2 + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2), x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)

**Fricas [A]** time = 2.10029, size = 34, normalized size = 2.

$$\frac{(x + \sqrt{x^2 + b})^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2), x, algorithm="fricas")

[Out] (x + sqrt(x^2 + b))^a/a

**Sympy [B]** time = 2.81428, size = 313, normalized size = 18.41

$$\left\{ \begin{array}{l} \frac{\sqrt{b} b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax \sqrt{\frac{b}{x^2} + 1}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b}} - \frac{b^{\frac{a}{2}} x \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b} \sqrt{\frac{b}{x^2} + 1}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a^2 \Gamma\left(-\frac{a}{2}\right)} \\ \frac{b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{1 + \frac{x^2}{b}}} - \frac{b^{\frac{a}{2}} x^2 \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab \sqrt{1 + \frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a \sqrt{b}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a^2 \Gamma\left(-\frac{a}{2}\right)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x\*\*2+b)\*\*(1/2))\*\*a/(x\*\*2+b)\*\*(1/2),x)

[Out] Piecewise((-sqrt(b)\*b\*\*(a/2)\*sinh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*x\*sqrt(b/x\*\*2 + 1)) + b\*\*(a/2)\*x\*cosh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*sqrt(b)) - b\*\*(a/2)\*x\*sinh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*sqrt(b)\*sqrt(b/x\*\*2 + 1)) - 2\*b\*\*(a/2)\*cosh(a\*asinh(x/sqrt(b)))\*gamma(1 - a/2)/(a\*\*2\*gamma(-a/2)), Abs(x\*\*2)/Abs(b) > 1), (-b\*\*(a/2)\*sinh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*sqrt(1 + x\*\*2/b)) - b\*\*(a/2)\*x\*\*2\*sinh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*b\*sqrt(1 + x\*\*2/b)) + b\*\*(a/2)\*x\*cosh(-a\*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a\*sqrt(b)) - 2\*b\*\*(a/2)\*cosh(a\*asinh(x/sqrt(b)))\*gamma(1 - a/2)/(a\*\*2\*gamma(-a/2)), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + \sqrt{x^2 + b})^a}{\sqrt{x^2 + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)

### 3.31 $\int (x + \sqrt{b + x^2})^a dx$

**Optimal.** Leaf size=52

$$\frac{(\sqrt{b+x^2}+x)^{a+1}}{2(a+1)} - \frac{b(\sqrt{b+x^2}+x)^{a-1}}{2(1-a)}$$

[Out]  $-(b*(x + \text{Sqrt}[b + x^2])^{(-1 + a)})/(2*(1 - a)) + (x + \text{Sqrt}[b + x^2])^{(1 + a)}/(2*(1 + a))$

**Rubi [A]** time = 0.0239267, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2117, 14}

$$\frac{(\sqrt{b+x^2}+x)^{a+1}}{2(a+1)} - \frac{b(\sqrt{b+x^2}+x)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x + \text{Sqrt}[b + x^2])^a, x]$

[Out]  $-(b*(x + \text{Sqrt}[b + x^2])^{(-1 + a)})/(2*(1 - a)) + (x + \text{Sqrt}[b + x^2])^{(1 + a)}/(2*(1 + a))$

#### Rule 2117

$\text{Int}[(g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*\text{Sqrt}[(a_.) + (c_.)*(x_.)^2])^{(n_.)}]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(2*e), \text{Subst}[\text{Int}[(g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2)/(d - x)^2, x], x, d + e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, n\}, x] \&\& \text{EqQ}[e^2 - c*f^2, 0] \&\& \text{IntegerQ}[p]$

#### Rule 14

$\text{Int}[(u_.)*((c_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \&\& \text{SumQ}[u] \&\& \text{!LinearQ}[u, x] \&\& \text{!MatchQ}[u, (a_.) + (b_.)*(v_.)] /; \text{FreeQ}\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

#### Rubi steps

$$\begin{aligned} \int (x + \sqrt{b + x^2})^a dx &= \frac{1}{2} \text{Subst} \left( \int x^{-2+a} (b + x^2) dx, x, x + \sqrt{b + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int (bx^{-2+a} + x^a) dx, x, x + \sqrt{b + x^2} \right) \\ &= -\frac{b(x + \sqrt{b + x^2})^{-1+a}}{2(1-a)} + \frac{(x + \sqrt{b + x^2})^{1+a}}{2(1+a)} \end{aligned}$$

**Mathematica [A]** time = 0.0595495, size = 46, normalized size = 0.88

$$\frac{1}{2} (\sqrt{b+x^2}+x)^{a-1} \left( \frac{(\sqrt{b+x^2}+x)^2}{a+1} + \frac{b}{a-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[b + x^2])^a,x]

[Out] ((x + Sqrt[b + x^2])^(-1 + a)\*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))/2

**Maple [B]** time = 0.022, size = 120, normalized size = 2.3

$$\frac{a}{4\sqrt{\pi}}b^{\frac{a}{2}+\frac{1}{2}}\left(8\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)a(2a-2)}\left(\frac{ab}{x^2}+a-1\right)\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}+4\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)a}\sqrt{1+\frac{b}{x^2}}\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+b)^(1/2))^a,x)

[Out] 1/4\*b^(1/2\*a+1/2)/Pi^(1/2)\*a\*(8\*Pi^(1/2)/(1+a)/a\*x^(1+a)\*b^(-1/2\*a-1/2)\*(1/x^2\*a\*b+a-1)/(2\*a-2)\*((1+1/x^2\*b)^(1/2)+1)^(a-1)+4\*Pi^(1/2)/(1+a)/a\*x^(1+a)\*b^(-1/2\*a-1/2)\*(1+1/x^2\*b)^(1/2)\*((1+1/x^2\*b)^(1/2)+1)^(a-1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + b})^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + b))^a, x)

**Fricas [A]** time = 2.24924, size = 74, normalized size = 1.42

$$\frac{(\sqrt{x^2 + b}a - x)(x + \sqrt{x^2 + b})^a}{a^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")

[Out] (sqrt(x^2 + b)\*a - x)\*(x + sqrt(x^2 + b))^a/(a^2 - 1)

**Sympy [B]** time = 2.65297, size = 2149, normalized size = 41.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.





---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + b})^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

$$3.32 \quad \int (6 + 3x^a + 2x^{2a})^{\frac{1}{a}} (x^a + x^{2a} + x^{3a}) dx$$

**Optimal.** Leaf size=34

$$\frac{x^{a+1} (3x^a + 2x^{2a} + 6)^{\frac{1}{a}+1}}{6(a+1)}$$

[Out] (x^(1 + a)\*(6 + 3\*x^a + 2\*x^(2\*a))^(1 + a^(-1)))/(6\*(1 + a))

**Rubi [A]** time = 0.0436392, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {1594, 1747}

$$\frac{x^{a+1} (3x^a + 2x^{2a} + 6)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

[In] Int[(6 + 3\*x^a + 2\*x^(2\*a))^a^(-1)\*(x^a + x^(2\*a) + x^(3\*a)), x]

[Out] (x^(1 + a)\*(6 + 3\*x^a + 2\*x^(2\*a))^(1 + a^(-1)))/(6\*(1 + a))

**Rule 1594**

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.) + (c\_.)\*(x\_)^(r\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p) + c\*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

**Rule 1747**

Int[((g\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.) + (c\_.)\*(x\_)^(n2\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.) + (f\_.)\*(x\_)^(n2\_.)), x\_Symbol] :> Simp[(d\*(g\*x)^(m + 1)\*(a + b\*x^n + c\*x^(2\*n))^(p + 1))/(a\*g\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[n2, 2\*n] && EqQ[a\*e\*(m + 1) - b\*d\*(m + n\*(p + 1) + 1), 0] && EqQ[a\*f\*(m + 1) - c\*d\*(m + 2\*n\*(p + 1) + 1), 0] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int (6 + 3x^a + 2x^{2a})^{\frac{1}{a}} (x^a + x^{2a} + x^{3a}) dx &= \int x^a (1 + x^a + x^{2a}) (6 + 3x^a + 2x^{2a})^{\frac{1}{a}} dx \\ &= \frac{x^{1+a} (6 + 3x^a + 2x^{2a})^{1+\frac{1}{a}}}{6(1+a)} \end{aligned}$$

**Mathematica [A]** time = 0.117796, size = 33, normalized size = 0.97

$$\frac{x^{a+1} (3x^a + 2x^{2a} + 6)^{\frac{1}{a}+1}}{6a + 6}$$

Antiderivative was successfully verified.

[In] Integrate[(6 + 3\*x^a + 2\*x^(2\*a))^a^(-1)\*(x^a + x^(2\*a) + x^(3\*a)),x]

[Out] (x^(1 + a)\*(6 + 3\*x^a + 2\*x^(2\*a))^(1 + a^(-1)))/(6 + 6\*a)

**Maple [A]** time = 0.039, size = 44, normalized size = 1.3

$$\frac{xx^a (6 + 3x^a + 2(x^a)^2) \sqrt[a]{6 + 3x^a + 2(x^a)^2}}{6 + 6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6+3\*x^a+2\*x^(2\*a))^(1/a)\*(x^a+x^(2\*a)+x^(3\*a)),x)

[Out] 1/6\*x\*x^a\*(6+3\*x^a+2\*(x^a)^2)/(1+a)\*(6+3\*x^a+2\*(x^a)^2)^(1/a)

**Maxima [A]** time = 1.13808, size = 65, normalized size = 1.91

$$\frac{(2xx^{3a} + 3xx^{2a} + 6xx^a)(2x^{2a} + 3x^a + 6)^{\left(\frac{1}{a}\right)}}{6(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6+3\*x^a+2\*x^(2\*a))^(1/a)\*(x^a+x^(2\*a)+x^(3\*a)),x, algorithm="maxima")

[Out] 1/6\*(2\*x\*x^(3\*a) + 3\*x\*x^(2\*a) + 6\*x\*x^a)\*(2\*x^(2\*a) + 3\*x^a + 6)^(1/a)/(a + 1)

**Fricas [A]** time = 2.1829, size = 109, normalized size = 3.21

$$\frac{(2xx^{3a} + 3xx^{2a} + 6xx^a)(2x^{2a} + 3x^a + 6)^{\left(\frac{1}{a}\right)}}{6(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6+3\*x^a+2\*x^(2\*a))^(1/a)\*(x^a+x^(2\*a)+x^(3\*a)),x, algorithm="fricas")

[Out] 1/6\*(2\*x\*x^(3\*a) + 3\*x\*x^(2\*a) + 6\*x\*x^a)\*(2\*x^(2\*a) + 3\*x^a + 6)^(1/a)/(a + 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6+3\*x\*\*a+2\*x\*\*(2\*a))\*\*(1/a)\*(x\*\*a+x\*\*(2\*a)+x\*\*(3\*a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (2x^{2a} + 3x^a + 6)^{\left(\frac{1}{a}\right)} (x^{3a} + x^{2a} + x^a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6+3\*x^a+2\*x^(2\*a))^(1/a)\*(x^a+x^(2\*a)+x^(3\*a)),x, algorithm="gias")

[Out] integrate((2\*x^(2\*a) + 3\*x^a + 6)^(1/a)\*(x^(3\*a) + x^(2\*a) + x^a), x)

### 3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}} dx$

**Optimal.** Leaf size=58

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

[Out] (Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3\*Log[1 - (1 - x^2)^(1/3)])/4

**Rubi [A]** time = 0.0357832, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {266, 55, 618, 204, 31}

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1 - x^2)^(1/3)),x]

[Out] (Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3\*Log[1 - (1 - x^2)^(1/3)])/4

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(1/3))), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[
Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(1/n), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{\sqrt[3]{1-xx}} dx, x, x^2 \right) \\
&= -\frac{\log(x)}{2} - \frac{3}{4} \text{Subst} \left( \int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^2} \right) + \frac{3}{4} \text{Subst} \left( \int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1-x^2} \right) \\
&= -\frac{\log(x)}{2} + \frac{3}{4} \log \left( 1 - \sqrt[3]{1-x^2} \right) - \frac{3}{2} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1-x^2} \right) \\
&= \frac{1}{2} \sqrt{3} \tan^{-1} \left( \frac{1 + 2\sqrt[3]{1-x^2}}{\sqrt{3}} \right) - \frac{\log(x)}{2} + \frac{3}{4} \log \left( 1 - \sqrt[3]{1-x^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.014505, size = 57, normalized size = 0.98

$$\frac{1}{2} \left( \frac{3}{2} \log \left( 1 - \sqrt[3]{1-x^2} \right) + \sqrt{3} \tan^{-1} \left( \frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}} \right) - \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1 - x^2)^(1/3)), x]

[Out] (Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x] + (3\*Log[1 - (1 - x^2)^(1/3)]))/2)/2

**Maple [C]** time = 0.036, size = 65, normalized size = 1.1

$$\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{4\pi} \left( \frac{2\pi\sqrt{3}}{3\Gamma(2/3)} \left( -\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 2\ln(x) + i\pi \right) + \frac{2\pi\sqrt{3}x^2}{9\Gamma(2/3)} {}_3F_2\left(1, 1, \frac{4}{3}; 2, 2; x^2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+1)^(1/3), x)

[Out] 1/4/Pi\*3^(1/2)\*GAMMA(2/3)\*(2/3\*(-1/6\*Pi\*3^(1/2)-3/2\*ln(3)+2\*ln(x)+I\*Pi)\*Pi\*3^(1/2)/GAMMA(2/3)+2/9\*Pi\*3^(1/2)/GAMMA(2/3)\*x^2\*hypergeom([1, 1, 4/3], [2, 2], x^2))

**Maxima [A]** time = 1.41737, size = 84, normalized size = 1.45

$$\frac{1}{2} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} \left( 2(-x^2 + 1)^{\frac{1}{3}} + 1 \right) \right) - \frac{1}{4} \log \left( (-x^2 + 1)^{\frac{2}{3}} + (-x^2 + 1)^{\frac{1}{3}} + 1 \right) + \frac{1}{2} \log \left( (-x^2 + 1)^{\frac{1}{3}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(1/3), x, algorithm="maxima")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^2 + 1)^(1/3) + 1)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log((-x^2 + 1)^(1/3) - 1)

---

**Fricas [A]** time = 1.96424, size = 198, normalized size = 3.41

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3}(-x^2 + 1)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) - \frac{1}{4} \log\left(\left(-x^2 + 1\right)^{\frac{2}{3}} + \left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2} \log\left(\left(-x^2 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")

[Out] 1/2\*sqrt(3)\*arctan(2/3\*sqrt(3)\*(-x^2 + 1)^(1/3) + 1/3\*sqrt(3)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log((-x^2 + 1)^(1/3) - 1)

---

**Sympy [C]** time = 0.927993, size = 36, normalized size = 0.62

$$\frac{e^{-\frac{i\pi}{3}} \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{1}{x^2}\right)}{2x^{\frac{2}{3}} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x\*\*2+1)\*\*(1/3),x)

[Out] -exp(-I\*pi/3)\*gamma(1/3)\*hyper((1/3, 1/3), (4/3,), x\*\*(-2))/(2\*x\*\*(2/3)\*gamma(4/3))

---

**Giac [A]** time = 1.07427, size = 86, normalized size = 1.48

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}\left(2\left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4} \log\left(\left(-x^2 + 1\right)^{\frac{2}{3}} + \left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2} \log\left(-\left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^2 + 1)^(1/3) + 1)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log(-(-x^2 + 1)^(1/3) + 1)



$$3.34 \quad \int \frac{1}{x(1-x^2)^{2/3}} dx$$

**Optimal.** Leaf size=58

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) - \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

[Out] -(Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3\*Log[1 - (1 - x^2)^(1/3)])/4

**Rubi [A]** time = 0.0346961, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {266, 57, 618, 204, 31}

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) - \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1 - x^2)^(2/3)),x]

[Out] -(Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3\*Log[1 - (1 - x^2)^(1/3)])/4

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 57

Int[1/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(2/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q^2), x] + (-Dist[3/(2\*b\*q), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q^2), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x]]) /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-x^2)^{2/3}} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{(1-x)^{2/3} x} dx, x, x^2 \right) \\
&= -\frac{\log(x)}{2} - \frac{3}{4} \text{Subst} \left( \int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^2} \right) - \frac{3}{4} \text{Subst} \left( \int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1-x^2} \right) \\
&= -\frac{\log(x)}{2} + \frac{3}{4} \log \left( 1 - \sqrt[3]{1-x^2} \right) + \frac{3}{2} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1-x^2} \right) \\
&= -\frac{1}{2} \sqrt{3} \tan^{-1} \left( \frac{1 + 2\sqrt[3]{1-x^2}}{\sqrt{3}} \right) - \frac{\log(x)}{2} + \frac{3}{4} \log \left( 1 - \sqrt[3]{1-x^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0124386, size = 81, normalized size = 1.4

$$\frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^2} \right) - \frac{1}{4} \log \left( (1-x^2)^{2/3} + \sqrt[3]{1-x^2} + 1 \right) - \frac{1}{2} \sqrt{3} \tan^{-1} \left( \frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1 - x^2)^(2/3)),x]

[Out] -(Sqrt[3]\*ArcTan[(1 + 2\*(1 - x^2)^(1/3))/Sqrt[3]])/2 + Log[1 - (1 - x^2)^(1/3)]/2 - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/4

**Maple [C]** time = 0.041, size = 48, normalized size = 0.8

$$\frac{1}{2\Gamma(2/3)} \left( \left( \frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 2\ln(x) + i\pi \right) \Gamma\left(\frac{2}{3}\right) + \frac{2\Gamma(2/3)x^2}{3} {}_3F_2\left(1, 1, \frac{5}{3}; 2, 2; x^2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+1)^(2/3),x)

[Out] 1/2/GAMMA(2/3)\*((1/6\*Pi\*3^(1/2)-3/2\*ln(3)+2\*ln(x)+I\*Pi)\*GAMMA(2/3)+2/3\*GAMMA(2/3)\*x^2\*hypergeom([1,1,5/3],[2,2],x^2))

**Maxima [A]** time = 1.41876, size = 84, normalized size = 1.45

$$-\frac{1}{2} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} \left( 2(-x^2+1)^{1/3} + 1 \right) \right) - \frac{1}{4} \log \left( (-x^2+1)^{2/3} + (-x^2+1)^{1/3} + 1 \right) + \frac{1}{2} \log \left( (-x^2+1)^{1/3} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")

[Out] -1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^2 + 1)^(1/3) + 1)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log((-x^2 + 1)^(1/3) - 1)

**Fricas [A]** time = 2.15924, size = 200, normalized size = 3.45

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")

[Out] -1/2\*sqrt(3)\*arctan(2/3\*sqrt(3)\*(-x^2 + 1)^(1/3) + 1/3\*sqrt(3)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log((-x^2 + 1)^(1/3) - 1)

**Sympy [C]** time = 0.949689, size = 37, normalized size = 0.64

$$\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x\*\*2+1)\*\*(2/3),x)

[Out] -exp(-2\*I\*pi/3)\*gamma(2/3)\*hyper((2/3, 2/3), (5/3,), x\*\*(-2))/(2\*x\*\*(4/3)\*gamma(5/3))

**Giac [A]** time = 1.06623, size = 86, normalized size = 1.48

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")

[Out] -1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^2 + 1)^(1/3) + 1)) - 1/4\*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2\*log(-(-x^2 + 1)^(1/3) + 1)

$$3.35 \quad \int \frac{1}{\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=49

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -(ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2

**Rubi [A]** time = 0.0050845, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {239}

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(-1/3), x]

[Out] -(ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] :> Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}} dx = -\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log\left(x + \sqrt[3]{1-x^3}\right)$$

**Mathematica [A]** time = 0.0398614, size = 86, normalized size = 1.76

$$-\frac{1}{6} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{1}{3} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}} - 1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(-1/3), x]

[Out] ArcTan[(-1 + (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3

**Maple [C]** time = 0.023, size = 12, normalized size = 0.2

$$x {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)^(1/3),x)

[Out] x\*hypergeom([1/3,1/3],[4/3],x^3)

**Maxima [A]** time = 1.43961, size = 105, normalized size = 2.14

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")

[Out] -1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3)/x - 1)) + 1/3\*log((-x^3 + 1)^(1/3)/x + 1) - 1/6\*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)

**Fricas [B]** time = 2.03169, size = 225, normalized size = 4.59

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")

[Out] -1/3\*sqrt(3)\*arctan(-1/3\*(sqrt(3)\*x - 2\*sqrt(3)\*(-x^3 + 1)^(1/3))/x) + 1/3\*log((x + (-x^3 + 1)^(1/3))/x) - 1/6\*log((x^2 - (-x^3 + 1)^(1/3)\*x + (-x^3 + 1)^(2/3))/x^2)

**Sympy [C]** time = 0.893634, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x\*\*3+1)\*\*(1/3),x)

[Out] x\*gamma(1/3)\*hyper((1/3, 1/3), (4/3,), x\*\*3\*exp\_polar(2\*I\*pi))/(3\*gamma(4/3))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(-1/3), x)

$$3.36 \quad \int \frac{1}{x \sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=55

$$\frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

**Rubi [A]** time = 0.0332979, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1 - x^3)^(1/3)),x]

[Out] ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 55

Int[1/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(1/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(1/n), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{\sqrt[3]{1-xx}} dx, x, x^3 \right) \\
&= -\frac{\log(x)}{2} - \frac{1}{2} \text{Subst} \left( \int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) + \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1-x^3} \right) \\
&= -\frac{\log(x)}{2} + \frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^3} \right) - \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1-x^3} \right) \\
&= \frac{\tan^{-1} \left( \frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^3} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.012324, size = 55, normalized size = 1.

$$\frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^3} \right) + \frac{\tan^{-1} \left( \frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1 - x^3)^(1/3)), x]

[Out] ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

**Maple [C]** time = 0.037, size = 65, normalized size = 1.2

$$\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{6\pi} \left( \frac{2\pi\sqrt{3}}{3\Gamma(2/3)} \left( -\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 3\ln(x) + i\pi \right) + \frac{2\pi\sqrt{3}x^3}{9\Gamma(2/3)} {}_3F_2\left(1, 1, \frac{4}{3}; 2, 2; x^3\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+1)^(1/3), x)

[Out] 1/6/Pi\*3^(1/2)\*GAMMA(2/3)\*(2/3\*(-1/6\*Pi\*3^(1/2)-3/2\*ln(3)+3\*ln(x)+I\*Pi)\*Pi\*3^(1/2)/GAMMA(2/3)+2/9\*Pi\*3^(1/2)/GAMMA(2/3)\*x^3\*hypergeom([1, 1, 4/3], [2, 2], x^3))

**Maxima [A]** time = 1.43444, size = 84, normalized size = 1.53

$$\frac{1}{3} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} \left( 2(-x^3 + 1)^{\frac{1}{3}} + 1 \right) \right) - \frac{1}{6} \log \left( (-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1 \right) + \frac{1}{3} \log \left( (-x^3 + 1)^{\frac{1}{3}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3), x, algorithm="maxima")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3) + 1)) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)



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**Fricas [A]** time = 2.02553, size = 198, normalized size = 3.6

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3}(-x^3 + 1)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) - \frac{1}{6} \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left((-x^3 + 1)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(2/3\*sqrt(3)\*(-x^3 + 1)^(1/3) + 1/3\*sqrt(3)) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)

---

**Sympy [C]** time = 0.914684, size = 32, normalized size = 0.58

$$\frac{e^{-\frac{i\pi}{3}} \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{4}{3}, \frac{1}{x^3}\right)}{3x \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x\*\*3+1)\*\*(1/3),x)

[Out] -exp(-I\*pi/3)\*gamma(1/3)\*hyper((1/3, 1/3), (4/3,), x\*\*(-3))/(3\*x\*gamma(4/3))

---

**Giac [A]** time = 1.08466, size = 85, normalized size = 1.55

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}\left(2(-x^3 + 1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{6} \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left(\left|(-x^3 + 1)^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3) + 1)) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log(abs((-x^3 + 1)^(1/3) - 1))

$$3.37 \quad \int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=97

$$\frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

[Out]  $-(\text{Sqrt}[3]*\text{ArcTan}[(1 + (2^{1/3})*(1 - x))/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/(2*2^{(1/3)}) - \text{Log}[(1 - x)*(1 + x)^2]/(4*2^{(1/3)}) + (3*\text{Log}[-1 + x + 2^{(2/3)}*(1 - x^3)^{(1/3)}])/(4*2^{(1/3)})$

**Rubi [A]** time = 0.0438348, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2148}

$$\frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)\*(1 - x^3)^(1/3)), x]

[Out]  $-(\text{Sqrt}[3]*\text{ArcTan}[(1 + (2^{1/3})*(1 - x))/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/(2*2^{(1/3)}) - \text{Log}[(1 - x)*(1 + x)^2]/(4*2^{(1/3)}) + (3*\text{Log}[-1 + x + 2^{(2/3)}*(1 - x^3)^{(1/3)}])/(4*2^{(1/3)})$

#### Rule 2148

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] :> Simp[(Sqrt[3]\*ArcTan[(1 - (2^(1/3)\*Rt[b, 3]\*(c - d\*x))/(d\*(a + b\*x^3)^(1/3)))/Sqrt[3]])/(2^(4/3)\*Rt[b, 3]\*c), x] + (Simp[Log[(c + d\*x)^2\*(c - d\*x)]/(2^(7/3)\*Rt[b, 3]\*c), x] - Simp[(3\*Log[Rt[b, 3]\*(c - d\*x) + 2^(2/3)\*d\*(a + b\*x^3)^(1/3)]/(2^(7/3)\*Rt[b, 3]\*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c^3 + a\*d^3, 0]

#### Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx = -\frac{\sqrt{3} \tan^{-1}\left(\frac{1 + \frac{\sqrt[3]{2(1-x)}}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3 \log\left(-1 + x + 2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

**Mathematica [F]** time = 0.0762007, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 + x)\*(1 - x^3)^(1/3)), x]

[Out] Integrate[1/((1 + x)\*(1 - x^3)^(1/3)), x]

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(-x^3+1)^(1/3), x)

[Out] int(1/(1+x)/(-x^3+1)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(-x^3+1)^(1/3), x, algorithm="maxima")

[Out] integrate(1/((-x^3 + 1)^(1/3)\*(x + 1)), x)

**Fricas [B]** time = 37.3663, size = 788, normalized size = 8.12

$$\frac{1}{12} \sqrt{32^{\frac{2}{3}}} \arctan \left( \frac{\sqrt{3} 2^{\frac{1}{6}} \left( 2^{\frac{5}{6}} (13x^6 + 2x^5 + 19x^4 - 4x^3 + 19x^2 + 2x + 13) - 4\sqrt{2}(5x^5 - 5x^4 + 6x^3 - 6x^2 + 5x - 5) \right)}{6(3x^6 - 18x^5 - 3x^4 - 28x^3 - 3x^2 - 18x + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(-x^3+1)^(1/3), x, algorithm="fricas")

[Out] 1/12\*sqrt(3)\*2^(2/3)\*arctan(1/6\*sqrt(3)\*2^(1/6)\*(2^(5/6)\*(13\*x^6 + 2\*x^5 + 19\*x^4 - 4\*x^3 + 19\*x^2 + 2\*x + 13) - 4\*sqrt(2)\*(5\*x^5 - 5\*x^4 + 6\*x^3 - 6\*x^2 + 5\*x - 5)\*(-x^3 + 1)^(1/3) + 16\*2^(1/6)\*(x^4 + 2\*x^3 + 2\*x^2 + 2\*x + 1)\*(-x^3 + 1)^(2/3))/(3\*x^6 - 18\*x^5 - 3\*x^4 - 28\*x^3 - 3\*x^2 - 18\*x + 3)) - 1/24\*2^(2/3)\*log((4\*2^(2/3)\*(-x^3 + 1)^(2/3)\*(x^2 + 1) + 2^(1/3)\*(5\*x^4 + 6\*x^2 + 5) - 2\*(3\*x^3 - x^2 + x - 3)\*(-x^3 + 1)^(1/3))/(x^4 + 4\*x^3 + 6\*x^2 + 4\*x + 1)) + 1/12\*2^(2/3)\*log((2^(2/3)\*(x^2 + 2\*x + 1) - 2\*2^(1/3)\*(-x^3 + 1)^(1/3)\*(x - 1) - 4\*(-x^3 + 1)^(2/3))/(x^2 + 2\*x + 1))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(-x\*\*3+1)\*\*(1/3),x)

[Out] Integral(1/((-x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)\*(x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate(1/((-x^3 + 1)^(1/3)\*(x + 1)), x)

$$3.38 \quad \int \frac{x}{(1+x)\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=145

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

[Out] (Sqrt[3]\*ArcTan[(1 + (2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/(2\*2^(1/3)) - ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)\*(1 + x)^2]/(4\*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3\*Log[-1 + x + 2^(2/3)\*(1 - x^3)^(1/3)])/(4\*2^(1/3))

**Rubi [A]** time = 0.107369, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2152, 239, 2148}

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)\*(1 - x^3)^(1/3)), x]

[Out] (Sqrt[3]\*ArcTan[(1 + (2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/(2\*2^(1/3)) - ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)\*(1 + x)^2]/(4\*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3\*Log[-1 + x + 2^(2/3)\*(1 - x^3)^(1/3)])/(4\*2^(1/3))

#### Rule 2152

Int[((e\_.) + (f\_.)\*(x\_))/(((c\_.) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] := Dist[f/d, Int[1/(a + b\*x^3)^(1/3), x], x] + Dist[(d\*e - c\*f)/d, Int[1/((c + d\*x)\*(a + b\*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 2148

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] := Simp[(Sqrt[3]\*ArcTan[(1 - (2^(1/3)\*Rt[b, 3]\*(c - d\*x))/(d\*(a + b\*x^3)^(1/3))]/Sqrt[3])/(2^(4/3)\*Rt[b, 3]\*c), x] + (Simp[Log[(c + d\*x)^2\*(c - d\*x)]/(2^(7/3)\*Rt[b, 3]\*c), x] - Simp[(3\*Log[Rt[b, 3]\*(c - d\*x) + 2^(2/3)\*d\*(a + b\*x^3)^(1/3)]/(2^(7/3)\*Rt[b, 3]\*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c^3 + a\*d^3, 0]

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}} dx = \int \frac{1}{\sqrt[3]{1-x^3}} dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx$$

$$= \frac{\sqrt{3} \tan^{-1}\left(\frac{1+\sqrt[3]{2(1-x)}}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} + \frac{\log((1-x)(1+x)^2)}{4\sqrt[3]{2}} + \frac{1}{2} \log\left(x + \sqrt[3]{1-x^3}\right) - \frac{3 \log\left(\dots\right)}{\dots}$$

**Mathematica [F]** time = 0.0784828, size = 0, normalized size = 0.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/((1 + x)\*(1 - x^3)^(1/3)), x]

[Out] Integrate[x/((1 + x)\*(1 - x^3)^(1/3)), x]

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{1+x} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+x)/(-x^3+1)^(1/3), x)

[Out] int(x/(1+x)/(-x^3+1)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-x^3+1)^(1/3), x, algorithm="maxima")

[Out] integrate(x/((-x^3 + 1)^(1/3)\*(x + 1)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-x^3+1)^(1/3), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-x\*\*3+1)\*\*(1/3),x)

[Out] Integral(x/((-x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)\*(x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate(x/((-x^3 + 1)^(1/3)\*(x + 1)), x)

$$3.39 \quad \int \frac{1}{x \sqrt[3]{2-3x+x^2}} dx$$

**Optimal.** Leaf size=110

$$\frac{3 \log\left(-2^{2/3} \sqrt[3]{x^2-3x+2} - x + 2\right)}{4 \sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(2-x)}}{\sqrt{3} \sqrt[3]{x^2-3x+2}} + \frac{1}{\sqrt{3}}\right)}{2 \sqrt[3]{2}} - \frac{\log(2-x)}{4 \sqrt[3]{2}} - \frac{\log(x)}{2 \sqrt[3]{2}}$$

[Out] -(Sqrt[3]\*ArcTan[1/Sqrt[3] + (2^(1/3)\*(2 - x))/(Sqrt[3]\*(2 - 3\*x + x^2)^(1/3))])/(2\*2^(1/3)) - Log[2 - x]/(4\*2^(1/3)) - Log[x]/(2\*2^(1/3)) + (3\*Log[2 - x - 2^(2/3)\*(2 - 3\*x + x^2)^(1/3)])/(4\*2^(1/3))

**Rubi [A]** time = 0.0249725, antiderivative size = 176, normalized size of antiderivative = 1.6, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {755, 123}

$$\frac{3 \sqrt[3]{x-2} \sqrt[3]{x-1} \log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}} - \sqrt[3]{2} \sqrt[3]{x-1}\right)}{4 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}} - \frac{\sqrt[3]{x-2} \sqrt[3]{x-1} \log(x)}{2 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}} - \frac{\sqrt{3} \sqrt[3]{x-2} \sqrt[3]{x-1} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{\sqrt[3]{2(x-2)^{2/3}}}{\sqrt{3} \sqrt[3]{x-1}}\right)}{2 \sqrt[3]{2} \sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(2 - 3\*x + x^2)^(1/3)), x]

[Out] -(Sqrt[3]\*(-2 + x)^(1/3)\*(-1 + x)^(1/3)\*ArcTan[1/Sqrt[3] - (2^(1/3)\*(-2 + x)^(2/3))/(Sqrt[3]\*(-1 + x)^(1/3))])/(2\*2^(1/3)\*(2 - 3\*x + x^2)^(1/3)) + (3\*(-2 + x)^(1/3)\*(-1 + x)^(1/3)\*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)\*(-1 + x)^(1/3)])/(4\*2^(1/3)\*(2 - 3\*x + x^2)^(1/3)) - ((-2 + x)^(1/3)\*(-1 + x)^(1/3)\*Log[x])/(2\*2^(1/3)\*(2 - 3\*x + x^2)^(1/3))

#### Rule 755

Int[1/(((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(1/3)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[((b + q + 2\*c\*x)^(1/3)\*(b - q + 2\*c\*x)^(1/3))/(a + b\*x + c\*x^2)^(1/3), Int[1/((d + e\*x)\*(b + q + 2\*c\*x)^(1/3)\*(b - q + 2\*c\*x)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c^2\*d^2 - b\*c\*d\*e - 2\*b^2\*e^2 + 9\*a\*c\*e^2, 0]

#### Rule 123

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_)^(1/3))\*((e\_.) + (f\_.)\*(x\_)^(1/3))), x\_Symbol] := With[{q = Rt[(b\*(b\*e - a\*f))/(b\*c - a\*d)^2, 3]}, -Simp[Log[a + b\*x]/(2\*q\*(b\*c - a\*d)), x] + (-Simp[(Sqrt[3]\*ArcTan[1/Sqrt[3] + (2\*q\*(c + d\*x)^(2/3))/(Sqrt[3]\*(e + f\*x)^(1/3))])/(2\*q\*(b\*c - a\*d)), x] + Simp[(3\*Log[q\*(c + d\*x)^(2/3) - (e + f\*x)^(1/3)])/(4\*q\*(b\*c - a\*d)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - b\*c\*f - a\*d\*f, 0]

#### Rubi steps

$$\int \frac{1}{x \sqrt[3]{2-3x+x^2}} dx = \frac{(\sqrt[3]{-4+2x\sqrt[3]{-2+2x}}) \int \frac{1}{x \sqrt[3]{-4+2x\sqrt[3]{-2+2x}}} dx}{\sqrt[3]{2-3x+x^2}} = -\frac{\sqrt{3} \sqrt[3]{-2+x} \sqrt[3]{-1+x} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{\sqrt[3]{2(-2+x)^{2/3}}}{\sqrt{3} \sqrt[3]{-1+x}}\right)}{2 \sqrt[3]{2} \sqrt[3]{2-3x+x^2}} + \frac{3 \sqrt[3]{-2+x} \sqrt[3]{-1+x} \log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}} - \sqrt[3]{2} \sqrt[3]{-1+x}\right)}{4 \sqrt[3]{2} \sqrt[3]{2-3x+x^2}}$$



**Mathematica [C]** time = 0.0252075, size = 59, normalized size = 0.54

$$\frac{3\sqrt[3]{1-\frac{2}{x}}\sqrt[3]{1-\frac{1}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x\*(2 - 3\*x + x^2)^(1/3)),x]

[Out] (-3\*(1 - 2/x)^(1/3)\*(1 - x^(-1))^(1/3)\*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1), 2/x])/(2\*(2 - 3\*x + x^2)^(1/3))

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{x^2-3x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^2-3\*x+2)^(1/3),x)

[Out] int(1/x/(x^2-3\*x+2)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-3x+2)^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-3\*x+2)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((x^2 - 3\*x + 2)^(1/3)\*x), x)

**Fricas [B]** time = 18.5736, size = 828, normalized size = 7.53

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+36x^5-612x^4+2880x^3-5760x^2+5184x-1728)+12\sqrt{2}(x^5-38x^4+252x^3-648x^2+720x-288)\right)}{6(x^6-108x^5+972x^4-3456x^3+6084x^2-4320x+1296)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-3\*x+2)^(1/3),x, algorithm="fricas")

[Out] -1/12\*sqrt(3)\*2^(2/3)\*arctan(1/6\*sqrt(3)\*2^(1/6)\*(2^(5/6)\*(x^6 + 36\*x^5 - 612\*x^4 + 2880\*x^3 - 5760\*x^2 + 5184\*x - 1728) + 12\*sqrt(2)\*(x^5 - 38\*x^4 + 252\*x^3 - 648\*x^2 + 720\*x - 288))\*(x^2 - 3\*x + 2)^(1/3) + 48\*2^(1/6)\*(x^4 -

$$6x^3 + 6x^2)(x^2 - 3x + 2)^{2/3})/(x^6 - 108x^5 + 972x^4 - 3456x^3 + 6048x^2 - 5184x + 1728)) + 1/12 \cdot 2^{2/3} \cdot \log((2^{2/3} \cdot x^2 + 6 \cdot 2^{1/3} \cdot (x^2 - 3x + 2)^{1/3} \cdot (x - 2) + 12 \cdot (x^2 - 3x + 2)^{2/3})/x^2) - 1/24 \cdot 2^{2/3} \cdot \log((12 \cdot 2^{2/3} \cdot (x^2 - 3x + 2)^{2/3} \cdot (x^2 - 6x + 6) + 2^{1/3} \cdot (x^4 - 36x^3 + 180x^2 - 288x + 144) - 6 \cdot (x^3 - 14x^2 + 36x - 24) \cdot (x^2 - 3x + 2)^{1/3})/x^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{(x-2)(x-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x\*\*2-3\*x+2)\*\*(1/3),x)

[Out] Integral(1/(x\*((x - 2)\*(x - 1))\*\*(1/3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 3x + 2)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-3\*x+2)^(1/3),x, algorithm="giac")

[Out] integrate(1/((x^2 - 3\*x + 2)^(1/3)\*x), x)

$$3.40 \quad \int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}} dx$$

**Optimal.** Leaf size=81

$$-\frac{3}{4} \log\left(\sqrt[3]{x^3-3x^2+7x-5}-x+1\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3-3x^2+7x-5}} + \frac{1}{\sqrt{3}}\right) + \frac{1}{4} \log(1-x)$$

[Out] (Sqrt[3]\*ArcTan[1/Sqrt[3] + (2\*(-1 + x))/(Sqrt[3]\*(-5 + 7\*x - 3\*x^2 + x^3)^(1/3))])/2 + Log[1 - x]/4 - (3\*Log[1 - x + (-5 + 7\*x - 3\*x^2 + x^3)^(1/3)])/4

**Rubi [A]** time = 0.0728112, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1} \tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}+1}{\sqrt[3]{(x-1)^2+4}}}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}} - \frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1} \log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 7\*x - 3\*x^2 + x^3)^(-1/3), x]

[Out] (Sqrt[3]\*(4 + (-1 + x)^2)^(1/3)\*(-1 + x)^(1/3)\*ArcTan[(1 + (2\*(-1 + x)^(2/3)))/(4 + (-1 + x)^2)^(1/3)]/Sqrt[3])/(2\*(4\*(-1 + x) + (-1 + x)^3)^(1/3)) - (3\*(4 + (-1 + x)^2)^(1/3)\*(-1 + x)^(1/3)\*Log[-(4 + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4\*(4\*(-1 + x) + (-1 + x)^3)^(1/3))

#### Rule 2067

Int[(P3\_)^(p\_), x\_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2\*c^3 - 9\*b\*c\*d + 27\*a\*d^2)/(27\*d^2) - ((c^2 - 3\*b\*d)\*x)/(3\*d) + d\*x^3, x]^p, x], x, x + c/(3\*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]

#### Rule 2011

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[(a\*x^j + b\*x^n)^FracPart[p]/(x^(j\*FracPart[p])\*(a + b\*x^(n-j))^FracPart[p]), Int[x^(j\*p)\*(a + b\*x^(n-j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n-j]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := With[{k = GCD[m+1, n]}, Dist[1/k, Subst[Int[x^((m+1)/k-1)\*(a + b\*x^(n/k))]^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

**Rule 239**

$\text{Int}[(a_ + (b_ .)*(x_ )^3)^{-1/3}, x\_Symbol] := \text{Simp}[\text{ArcTan}[(1 + (2*\text{Rt}[b, 3]*x)/(a + b*x^3)^{1/3})/\text{Sqrt}[3]]/(\text{Sqrt}[3]*\text{Rt}[b, 3]), x] - \text{Simp}[\text{Log}[(a + b*x^3)^{1/3} - \text{Rt}[b, 3]*x]/(2*\text{Rt}[b, 3]), x] /; \text{FreeQ}[\{a, b\}, x]$

**Rubi steps**

$$\begin{aligned} \int \frac{1}{\sqrt[3]{-5 + 7x - 3x^2 + x^3}} dx &= \text{Subst} \left( \int \frac{1}{\sqrt[3]{4x + x^3}} dx, x, -1 + x \right) \\ &= \frac{(\sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x}) \text{Subst} \left( \int \frac{1}{\sqrt[3]{x} \sqrt[3]{4+x^2}} dx, x, -1 + x \right)}{\sqrt[3]{4(-1 + x) + (-1 + x)^3}} \\ &= \frac{(3\sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x}) \text{Subst} \left( \int \frac{x}{\sqrt[3]{4+x^6}} dx, x, \sqrt[3]{-1 + x} \right)}{\sqrt[3]{4(-1 + x) + (-1 + x)^3}} \\ &= \frac{(3\sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x}) \text{Subst} \left( \int \frac{1}{\sqrt[3]{4+x^3}} dx, x, (-1 + x)^{2/3} \right)}{2\sqrt[3]{4(-1 + x) + (-1 + x)^3}} \\ &= \frac{\sqrt{3} \sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x} \tan^{-1} \left( \frac{1 + \frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}} \right)}{2\sqrt[3]{-4(1-x) + (-1+x)^3}} - \frac{3\sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x} \log(\sqrt[3]{4 + (-1 + x)^2} \sqrt[3]{-1 + x})}{4\sqrt[3]{-4(1-x) + (-1+x)^3}} \end{aligned}$$

**Mathematica [C]** time = 0.0139155, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix + (2-i)}\sqrt[3]{i(x-1)}(x - (1-2i))F_1\left(\frac{2}{3}; \frac{1}{3}, \frac{1}{3}, \frac{5}{3}; -\frac{1}{4}i(x - (1-2i)), -\frac{1}{2}i(x - (1-2i))\right)}{4\sqrt[3]{x^3 - 3x^2 + 7x - 5}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(-5 + 7\*x - 3\*x^2 + x^3)^(-1/3), x]

[Out] (3\*((2 - I) + I\*x)^(1/3)\*(I\*(-1 + x))^(1/3)\*((-1 + 2\*I) + x)\*AppellF1[2/3, 1/3, 1/3, 5/3, (-I/4)\*((-1 + 2\*I) + x), (-I/2)\*((-1 + 2\*I) + x)]/(4\*(-5 + 7\*x - 3\*x^2 + x^3)^(1/3))

**Maple [F]** time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3 - 3x^2 + 7x - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3-3\*x^2+7\*x-5)^(1/3), x)

[Out] int(1/(x^3-3\*x^2+7\*x-5)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 3x^2 + 7x - 5)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-3\*x^2+7\*x-5)^(1/3),x, algorithm="maxima")

[Out] integrate((x^3 - 3\*x^2 + 7\*x - 5)^(-1/3), x)

**Fricas [A]** time = 4.68439, size = 410, normalized size = 5.06

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}(x^3 - 3x^2 + 7x - 5)^{\frac{1}{3}}(x - 1) + \sqrt{3}(20389537x^2 - 40779074x + 53222437) + 17987998}{7204617x^2 - 14409234x - 20666867}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-3\*x^2+7\*x-5)^(1/3),x, algorithm="fricas")

[Out] -1/2\*sqrt(3)\*arctan((22791076\*sqrt(3)\*(x^3 - 3\*x^2 + 7\*x - 5)^(1/3)\*(x - 1) + sqrt(3)\*(20389537\*x^2 - 40779074\*x + 53222437) + 17987998\*sqrt(3)\*(x^3 - 3\*x^2 + 7\*x - 5)^(2/3))/(7204617\*x^2 - 14409234\*x - 20666867)) - 1/4\*log(3\*(x^3 - 3\*x^2 + 7\*x - 5)^(1/3)\*(x - 1) - 3\*(x^3 - 3\*x^2 + 7\*x - 5)^(2/3) + 4)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3 - 3x^2 + 7x - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*3-3\*x\*\*2+7\*x-5)\*\*(1/3),x)

[Out] Integral((x\*\*3 - 3\*x\*\*2 + 7\*x - 5)\*\*(-1/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 3x^2 + 7x - 5)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-3\*x^2+7\*x-5)^(1/3),x, algorithm="giac")

[Out] integrate((x^3 - 3\*x^2 + 7\*x - 5)^(-1/3), x)

$$3.41 \quad \int \frac{1}{\sqrt[3]{x(-q+x^2)}} dx$$

**Optimal.** Leaf size=66

$$-\frac{3}{4} \log\left(\sqrt[3]{x(x^2-q)}-x\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x(x^2-q)}} + \frac{1}{\sqrt{3}}\right) + \frac{\log(x)}{4}$$

[Out] (Sqrt[3]\*ArcTan[1/Sqrt[3] + (2\*x)/(Sqrt[3]\*(x\*(-q + x^2))^(1/3))])/2 + Log[x]/4 - (3\*Log[-x + (x\*(-q + x^2))^(1/3)])/4

**Rubi [A]** time = 0.0564275, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {1979, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q} \tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}} - \frac{3\sqrt[3]{x}\sqrt[3]{x^2-q} \log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(-q + x^2))^(-1/3),x]

[Out] (Sqrt[3]\*x^(1/3)\*(-q + x^2)^(1/3)\*ArcTan[(1 + (2\*x^(2/3))/(-q + x^2)^(1/3))/Sqrt[3]])/(2\*(-(q\*x) + x^3)^(1/3)) - (3\*x^(1/3)\*(-q + x^2)^(1/3)\*Log[x^(2/3) - (-q + x^2)^(1/3)])/(4\*(-(q\*x) + x^3)^(1/3))

#### Rule 1979

Int[(u\_)^(p\_), x\_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

#### Rule 2011

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[(a\*x^j + b\*x^n)^FracPart[p]/(x^(j\*FracPart[p])\*(a + b\*x^(n-j))^FracPart[p]), Int[x^(j\*p)\*(a + b\*x^(n-j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n-j]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := With[{k = GCD[m+1, n]}, Dist[1/k, Subst[Int[x^((m+1)/k-1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt[3]{x(-q+x^2)}} dx &= \int \frac{1}{\sqrt[3]{-qx+x^3}} dx \\
 &= \frac{(\sqrt[3]{x}\sqrt[3]{-q+x^2}) \int \frac{1}{\sqrt[3]{x}\sqrt[3]{-q+x^2}} dx}{\sqrt[3]{-qx+x^3}} \\
 &= \frac{(3\sqrt[3]{x}\sqrt[3]{-q+x^2}) \text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}} dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}} \\
 &= \frac{(3\sqrt[3]{x}\sqrt[3]{-q+x^2}) \text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}} dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}} \\
 &= \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-q+x^2} \tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\sqrt[3]{-q+x^2} \log(x^{2/3} - \sqrt[3]{-q+x^2})}{4\sqrt[3]{-qx+x^3}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0822427, size = 127, normalized size = 1.92

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-q} \left( -2 \log\left(1 - \frac{x^{2/3}}{\sqrt[3]{x^2-q}}\right) + \log\left(\frac{x^{4/3}}{(x^2-q)^{2/3}} + \frac{x^{2/3}}{\sqrt[3]{x^2-q}} + 1\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}} + 1}{\sqrt{3}}\right) \right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(-q + x^2))^(1/3), x]

[Out] (x^(1/3)\*(-q + x^2)^(1/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*x^(2/3)))/(-q + x^2)^(1/3)]/Sqrt[3]) - 2\*Log[1 - x^(2/3)/(-q + x^2)^(1/3)] + Log[1 + x^(4/3)/(-q + x^2)^(2/3) + x^(2/3)/(-q + x^2)^(1/3)])/(4\*(-(q\*x) + x^3)^(1/3))

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x(x^2-q)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(x^2-q))^(1/3), x)

[Out] int(1/(x\*(x^2-q))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - q)x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*(x^2-q))^(1/3),x, algorithm="maxima")

[Out] integrate(((x^2 - q)\*x)^(-1/3), x)

---

**Fricas [B]** time = 13.5156, size = 1153, normalized size = 17.47

$$\frac{1}{2} \sqrt{3} \arctan \left( \frac{4 \sqrt{3} (q^{12} - 15 q^{10} + 90 q^8 - 351 q^6 + 810 q^4 - 1215 q^2 + 729) (x^3 - qx)^{\frac{1}{3}} x - 2 \sqrt{3} (q^{12} + 6 q^{11} - 15 q^{10} - 54 q^9 + 90 q^8 + 270 q^7 - 351 q^6 - 810 q^5 + 810 q^4 + 1458 q^3 - 1215 q^2 - 1458 q + 729) (x^3 - qx)^{\frac{2}{3}} - \sqrt{3} (q^{13} + 10 q^{12} - 15 q^{11} - 282 q^{10} + 90 q^9 + 2178 q^8 - 351 q^7 - 6534 q^6 + 810 q^5 + 7614 q^4 - 1215 q^3 - (q^{12} - 6 q^{11} - 15 q^{10} + 54 q^9 + 90 q^8 - 270 q^7 - 351 q^6 + 810 q^5 + 810 q^4 - 1458 q^3 - 1215 q^2 + 1458 q + 729) x^2 - 2430 q^2 + 729 q)}{q^{13} + 18 q^{12} + 81 q^{11} - 162 q^{10} - 1350 q^9 + 810 q^8 + 6561 q^7 - 2430 q^6 - 12150 q^5 + 4374 q^4 + 6561 q^3 - 9 (q^{12} + 2 q^{11} - 15 q^{10} - 18 q^9 + 90 q^8 + 90 q^7 - 351 q^6 - 270 q^5 + 810 q^4 + 486 q^3 - 1215 q^2 - 486 q + 729) x^2 - 4374 q^2 + 729 q)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*(x^2-q))^(1/3),x, algorithm="fricas")

[Out] 1/2\*sqrt(3)\*arctan((4\*sqrt(3)\*(q^12 - 15\*q^10 + 90\*q^8 - 351\*q^6 + 810\*q^4 - 1215\*q^2 + 729)\*(x^3 - q\*x)^(1/3)\*x - 2\*sqrt(3)\*(q^12 + 6\*q^11 - 15\*q^10 - 54\*q^9 + 90\*q^8 + 270\*q^7 - 351\*q^6 - 810\*q^5 + 810\*q^4 + 1458\*q^3 - 1215\*q^2 - 1458\*q + 729)\*(x^3 - q\*x)^(2/3) - sqrt(3)\*(q^13 + 10\*q^12 - 15\*q^11 - 282\*q^10 + 90\*q^9 + 2178\*q^8 - 351\*q^7 - 6534\*q^6 + 810\*q^5 + 7614\*q^4 - 1215\*q^3 - (q^12 - 6\*q^11 - 15\*q^10 + 54\*q^9 + 90\*q^8 - 270\*q^7 - 351\*q^6 + 810\*q^5 + 810\*q^4 - 1458\*q^3 - 1215\*q^2 + 1458\*q + 729)\*x^2 - 2430\*q^2 + 729\*q))/(q^13 + 18\*q^12 + 81\*q^11 - 162\*q^10 - 1350\*q^9 + 810\*q^8 + 6561\*q^7 - 2430\*q^6 - 12150\*q^5 + 4374\*q^4 + 6561\*q^3 - 9\*(q^12 + 2\*q^11 - 15\*q^10 - 18\*q^9 + 90\*q^8 + 90\*q^7 - 351\*q^6 - 270\*q^5 + 810\*q^4 + 486\*q^3 - 1215\*q^2 - 486\*q + 729)\*x^2 - 4374\*q^2 + 729\*q)) - 1/4\*log(-3\*(x^3 - q\*x)^(1/3)\*x + q + 3\*(x^3 - q\*x)^(2/3))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*(x\*\*2-q))\*\*(1/3),x)

[Out] Integral((x\*(-q + x\*\*2))\*\*(-1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - q)x)^{\frac{1}{3}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

$$3.42 \quad \int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}} dx$$

**Optimal.** Leaf size=79

$$-\frac{3}{4} \log\left(\sqrt[3]{(x-1)(q+x^2-2x)} - x + 1\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)(q+x^2-2x)}} + \frac{1}{\sqrt{3}}\right) + \frac{1}{4} \log(1-x)$$

[Out] (Sqrt[3]\*ArcTan[1/Sqrt[3] + (2\*(-1 + x))/(Sqrt[3]\*((-1 + x)\*(q - 2\*x + x^2))^(1/3))])/2 + Log[1 - x]/4 - (3\*Log[1 - x + ((-1 + x)\*(q - 2\*x + x^2))^(1/3)]))/4

**Rubi [A]** time = 0.0978022, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1} \tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}+1}{\sqrt[3]{q+(x-1)^2-1}}}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}} - \frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1} \log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)\*(q - 2\*x + x^2))^(1/3), x]

[Out] (Sqrt[3]\*(-1 + q + (-1 + x)^2)^(1/3)\*(-1 + x)^(1/3)\*ArcTan[(1 + (2\*(-1 + x)^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2\*(-((1 - q)\*(-1 + x)) + (-1 + x)^3)^(1/3)) - (3\*(-1 + q + (-1 + x)^2)^(1/3)\*(-1 + x)^(1/3)\*Log[-(-1 + q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4\*(-((1 - q)\*(-1 + x)) + (-1 + x)^3)^(1/3))

#### Rule 2067

Int[(P3\_)^(p\_), x\_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2\*c^3 - 9\*b\*c\*d + 27\*a\*d^2)/(27\*d^2) - ((c^2 - 3\*b\*d)\*x)/(3\*d) + d\*x^3, x]^p, x], x, x + c/(3\*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]

#### Rule 2011

Int[((a\_.)\*(x\_)^(j\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Dist[(a\*x^j + b\*x^n)^FracPart[p]/(x^(j\*FracPart[p])\*(a + b\*x^(n-j))^FracPart[p]), Int[x^(j\*p)\*(a + b\*x^(n-j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 275

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 239

Int[((a\_) + (b\_)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}} dx &= \text{Subst} \left( \int \frac{1}{\sqrt[3]{-(1-q)x+x^3}} dx, x, -1+x \right) \\ &= \frac{(\sqrt[3]{-1+q+(-1+x)^2\sqrt[3]{-1+x}}) \text{Subst} \left( \int \frac{1}{\sqrt[3]{x\sqrt[3]{-1+q+x^2}}} dx, x, -1+x \right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}} \\ &= \frac{(3\sqrt[3]{-1+q+(-1+x)^2\sqrt[3]{-1+x}}) \text{Subst} \left( \int \frac{x}{\sqrt[3]{-1+q+x^6}} dx, x, \sqrt[3]{-1+x} \right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}} \\ &= \frac{(3\sqrt[3]{-1+q+(-1+x)^2\sqrt[3]{-1+x}}) \text{Subst} \left( \int \frac{1}{\sqrt[3]{-1+q+x^3}} dx, x, (-1+x)^{2/3} \right)}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}} \\ &= \frac{\sqrt{3}\sqrt[3]{-1+q+(-1+x)^2\sqrt[3]{-1+x}} \tan^{-1} \left( \frac{1 + \frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}} \right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{-1+q+(-1+x)^2\sqrt[3]{-1+x}}}{4\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} \end{aligned}$$

**Mathematica [A]** time = 0.166759, size = 140, normalized size = 1.77

$$\frac{\sqrt[3]{x-1}\sqrt[3]{q+(x-2)x} \left( -2 \log \left( 1 - \frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}} \right) + \log \left( \frac{(x-1)^{4/3}}{(q+(x-2)x)^{2/3}} + \frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}} + 1}{\sqrt{3}} \right) \right)}{4\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)\*(q - 2\*x + x^2))^(-1/3), x]

[Out] ((-1 + x)^(1/3)\*(q + (-2 + x)\*x)^(1/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(-1 + x)^(2/3)))/(q + (-2 + x)\*x)^(1/3)]/Sqrt[3]] - 2\*Log[1 - (-1 + x)^(2/3)/(q + (-2 + x)\*x)^(1/3)] + Log[1 + (-1 + x)^(4/3)/(q + (-2 + x)\*x)^(2/3) + (-1 + x)^(2/3)/(q + (-2 + x)\*x)^(1/3)])/(4\*((-1 + x)\*(q + (-2 + x)\*x))^(1/3))

**Maple [F]** time = 0.012, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)(x^2+q-2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)`

[Out] `int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + q - 2x)(x - 1))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")`

[Out] `integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)`

**Fricas [B]** time = 12.361, size = 1866, normalized size = 23.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")`

[Out]  $\frac{1}{2}\sqrt{3}\arctan\left(\frac{(2\sqrt{3}(q^{12} - 18q^{11} + 117q^{10} - 346q^9 + 414q^8 - 18q^7 + 69q^6 - 774q^5 - 234q^4 + 1058q^3 + 621q^2 + 378q - 539)(x^3 + (q + 2)x - 3x^2 - q)^{2/3} + 4\sqrt{3}(q^{12} - 12q^{11} + 51q^{10} - 70q^9 - 90q^8 + 288q^7 - 57q^6 + 54q^5 - 810q^4 + 320q^3 + 291q^2 - (q^{12} - 12q^{11} + 51q^{10} - 70q^9 - 90q^8 + 288q^7 - 57q^6 + 54q^5 - 810q^4 + 320q^3 + 291q^2 + 714q + 49)x + 714q + 49)(x^3 + (q + 2)x - 3x^2 - q)^{1/3} - \sqrt{3}(q^{13} - 22q^{12} + 177q^{11} - 514q^{10} - 434q^9 + 5346q^8 - 8247q^7 - 4542q^6 + 19638q^5 - 8050q^4 - 10343q^3 + (q^{12} - 6q^{11} - 15q^{10} + 206q^9 - 594q^8 + 594q^7 - 183q^6 + 882q^5 - 1386q^4 - 418q^3 - 39q^2 + 1050q + 637)x^2 + 6186q^2 - 2(q^{12} - 6q^{11} - 15q^{10} + 206q^9 - 594q^8 + 594q^7 - 183q^6 + 882q^5 - 1386q^4 - 418q^3 - 39q^2 + 1050q + 637)x + 1501q + 32)}{(q^{13} - 22q^{12} + 249q^{11} - 1546q^{10} + 4702q^9 - 4230q^8 - 10623q^7 + 25338q^6 - 3546q^5 - 31306q^4 + 18817q^3 + 9(q^{12} - 14q^{11} + 73q^{10} - 162q^9 + 78q^8 + 186q^7 - 15q^6 - 222q^5 - 618q^4 + 566q^3 + 401q^2 + 602q - 147)x^2 + 9714q^2 - 18(q^{12} - 14q^{11} + 73q^{10} - 162q^9 + 78q^8 + 186q^7 - 15q^6 - 222q^5 - 618q^4 + 566q^3 + 401q^2 + 602q - 147)x - 995q + 8)}\right) - \frac{1}{4}\log(3(x^3 + (q + 2)x - 3x^2 - q)^{1/3}(x - 1) + q - 3(x^3 + (q + 2)x - 3x^2 - q)^{2/3} - 1)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + q - 2x)(x - 1))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)\*(x^2+q-2\*x))^(1/3),x, algorithm="giac")

[Out] integrate(((x^2 + q - 2\*x)\*(x - 1))^(-1/3), x)

$$3.43 \quad \int \frac{1}{x \sqrt[3]{(-1+x)(q-2qx+x^2)}} dx$$

**Optimal.** Leaf size=118

$$\frac{3 \log\left(\sqrt[3]{(x-1)(-2qx+q+x^2)} - \sqrt[3]{q}(x-1)\right)}{4\sqrt[3]{q}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{q}(x-1)}{\sqrt{3}\sqrt[3]{(x-1)(-2qx+q+x^2)}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{q}} + \frac{\log(1-x)}{4\sqrt[3]{q}} + \frac{\log(x)}{2\sqrt[3]{q}}$$

[Out] (Sqrt[3]\*ArcTan[1/Sqrt[3] + (2\*q^(1/3)\*(-1 + x))/(Sqrt[3]\*((-1 + x)\*(q - 2\*q\*x + x^2))^(1/3))]/(2\*q^(1/3)) + Log[1 - x]/(4\*q^(1/3)) + Log[x]/(2\*q^(1/3)) - (3\*Log[-(q^(1/3)\*(-1 + x)) + ((-1 + x)\*(q - 2\*q\*x + x^2))^(1/3)])/ (4\*q^(1/3)))

**Rubi [F]** time = 21.9369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x \sqrt[3]{(-1+x)(q-2qx+x^2)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*((-1 + x)\*(q - 2\*q\*x + x^2))^(1/3)), x]

[Out] ((-1 - 2\*q - (1 - 5\*q + 4\*q^2 + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[-((-1 + q)^3\*q)]))^(2/3))/(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[-((-1 + q)^3\*q)]))^(1/3) + 3\*x^(1/3)\*(-1 + 5\*q - 4\*q^2 + ((1 - 4\*q)^2\*(1 - q)^2)/(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3) + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3) + (3\*(1 - 5\*q + 4\*q^2 + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3))\*((-1 - 2\*q)/3 + x))/(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(1/3) + 9\*((-1 - 2\*q)/3 + x)^2^(1/3)\*Defer[Subst][Defer[Int][1/(((1 + 2\*q)/3 + x)\*(-1 - 5\*q + 4\*q^2 + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3))/(3\*(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(1/3)) + x]^(1/3)\*((-1 + 5\*q - 4\*q^2 + ((1 - 4\*q)^2\*(1 - q)^2)/(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3) + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3))/9 + ((1 - 5\*q + 4\*q^2 + (1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(2/3))\*x]/(3\*(1 + 6\*q - 15\*q^2 + 8\*q^3 + 3\*Sqrt[3]\*Sqrt[(1 - q)^3\*q]))^(1/3)) + x^2^(1/3)), x], x, (-1 - 2\*q)/3 + x]]/(3\*(-q + 3\*q\*x + (-1 - 2\*q)\*x^2 + x^3)^(1/3))

Rubi steps

$$\int \frac{1}{x^3 \sqrt{(-1+x)(q-2qx+x^2)}} dx = \text{Subst} \left( \int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right) \sqrt{-\frac{2}{27}(1-q)^2(1+8q) - \frac{1}{3}(1-4q)(1-q)x + x^3}} dx, x, \frac{1}{3} \right.$$

$$\left. \left( \sqrt[3]{-1-2q - \frac{1-5q+4q^2+(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q})^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}} + 3x \sqrt[3]{-1+5q-4q^2 + \frac{1-5q+4q^2+(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q})^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}} \right)} \right.$$

$$= \frac{3((x-1)(-2qx+q+x^2))^{2/3} {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

**Mathematica [C]** time = 0.19341, size = 55, normalized size = 0.47

$$\frac{3((x-1)(-2qx+q+x^2))^{2/3} {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*((-1+x)\*(q-2\*q\*x+x^2))^(1/3)),x]

[Out] (3\*((-1+x)\*(q-2\*q\*x+x^2))^(2/3)\*Hypergeometric2F1[2/3, 1, 5/3, (q-2\*q\*x+x^2)/(q\*(-1+x)^2)])/(4\*q\*(-1+x)^2)

**Maple [F]** time = 0.014, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{(-1+x)(-2qx+x^2+q)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((-1+x)\*(-2\*q\*x+x^2+q))^(1/3),x)

[Out] int(1/x/((-1+x)\*(-2\*q\*x+x^2+q))^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(2qx-x^2-q)(x-1))^{1/3} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((-1+x)\*(-2\*q\*x+x^2+q))^(1/3),x, algorithm="maxima")

[Out] integrate(1/((-2\*q\*x-x^2-q)\*(x-1))^(1/3)\*x, x)

**Fricas [B]** time = 167.475, size = 3509, normalized size = 29.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((-1+x)\*(-2\*q\*x+x^2+q))^(1/3),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/12*(\sqrt{3})q\sqrt{(-q)^{1/3}/q}*\log(-((q^3 - 30q^2 - 51q - 1)*x^6 + 5 \\ & 4*(q^3 + 6q^2 + 2q)*x^5 - 27*(17q^3 + 26q^2 + 2q)*x^4 + 486q^3*x + 54 \\ & 0*(2q^3 + q^2)*x^3 - 81q^3 - 135*(8q^3 + q^2)*x^2 + 9*((2q^2 - q - 1)*x \\ & ^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3} \\ & *(-q)^{1/3} + 9*((q^2 + 7q + 1)*x^5 - (19q^2 + 25q + 1)*x^4 + 9*(7q^2 + 3q)*x^3 \\ & + 45q^2*x - 9*(9q^2 + q)*x^2 - 9q^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & *(-q)^{2/3} + \sqrt{3}*(3*((4q^2 + 13q + 1)*x^4 - 6*(7q^2 + 5q)*x^3 - 72q^2*x \\ & + 3*(31q^2 + 5q)*x^2 + 18q^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3} \\ & *(-q)^{2/3} + 3*((q^3 - 5q^2 - 5q)*x^5 + 5*(q^3 + 7q^2 + q)*x^4 - 45q^3*x \\ & - 45*(q^3 + q^2)*x^3 + 9q^3 + 15*(5q^3 + q^2)*x^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & + ((q^3 + 24q^2 + 3q - 1)*x^6 - 54*(q^3 + 2q^2)*x^5 + 81*(3q^3 + 2q^2)*x^4 - 162q^3*x - 108*(4q^3 \\ & + q^2)*x^3 + 27q^3 + 27*(14q^3 + q^2)*x^2)*(-q)^{1/3})*\sqrt{(-q)^{1/3}/q} \\ & /x^6) - 2*(-q)^{2/3}*\log(((q - 1)*x^2 + 3*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & *(q*x - q)*(-q)^{1/3} + 3*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3})*q/x^2) \\ & + (-q)^{2/3}*\log((3*((2q + 1)*x^2 - 6q*x + 3q)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3} \\ & *(-q)^{2/3} + 3*((q^2 + 2q)*x^3 + 9q^2*x - (7q^2 + 2q)*x^2 - 3q^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & - ((q^2 + 7q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36q^2*x + 9*(5q^2 + q)*x^2 + 9q^2)*(-q)^{1/3} \\ & /x^4))/q, 1/12*(2*\sqrt{3})q\sqrt{(-q)^{1/3}/q}*\arctan(1/3*\sqrt{3}*(6*((2q^2 - q - 1)*x^4 \\ & - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3} \\ & *(-q)^{2/3} - 6*((q^3 + 7q^2 + q)*x^5 - (19q^3 + 25q^2 + q)*x^4 + 45q^3*x + 9*(7q^3 + 3q^2)*x^3 \\ & - 9q^3 - 9*(9q^3 + q^2)*x^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} - ((q^3 - 12q^2 - 15q - 1)*x^6 \\ & + 18*(q^3 + 6q^2 + 2q)*x^5 - 9*(17q^3 + 26q^2 + 2q)*x^4 + 162q^3*x + 180*(2q^3 + q^2)*x^3 \\ & - 27q^3 - 45*(8q^3 + q^2)*x^2)*(-q)^{1/3})*\sqrt{(-q)^{1/3}/q} / ((q^3 + 24q^2 + 3q - 1)*x^6 \\ & - 54*(q^3 + 2q^2)*x^5 + 81*(3q^3 + 2q^2)*x^4 - 162q^3*x - 108*(4q^3 + q^2)*x^3 + 27q^3 \\ & + 27*(14q^3 + q^2)*x^2)) - 2*(-q)^{2/3}*\log(((q - 1)*x^2 + 3*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & *(q*x - q)*(-q)^{1/3} + 3*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3})*q/x^2) \\ & + (-q)^{2/3}*\log((3*((2q + 1)*x^2 - 6q*x + 3q)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{2/3} \\ & *(-q)^{2/3} + 3*((q^2 + 2q)*x^3 + 9q^2*x - (7q^2 + 2q)*x^2 - 3q^2)*(-(2q + 1)*x^2 + x^3 + 3q*x - q)^{1/3} \\ & - ((q^2 + 7q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36q^2*x + 9*(5q^2 + q)*x^2 + 9q^2)*(-q)^{1/3} \\ & /x^4))/q] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((-1+x)\*(-2\*q\*x+x\*\*2+q))\*\*(1/3),x)

[Out] Timed out



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(- (2qx - x^2 - q)(x - 1))^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((-1+x)\*(-2\*q\*x+x^2+q))^(1/3),x, algorithm="giac")

[Out] integrate(1/((-2\*q\*x - x^2 - q)\*(x - 1))^(1/3)\*x, x)

$$3.44 \quad \int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)} dx$$

**Optimal.** Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}} + \frac{\log(1-(k+1)x)}{2\sqrt[3]{k}} - \frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)} - \sqrt[3]{kx}\right)}{2\sqrt[3]{k}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{kx} + 1}{\frac{\sqrt[3]{(1-x)x(1-kx)}}{\sqrt{3}}}\right)}{\sqrt[3]{k}}$$

[Out] (Sqrt[3]\*ArcTan[(1 + (2\*k^(1/3)\*x)/((1 - x)\*x\*(1 - k\*x))^(1/3))/Sqrt[3]])/k^(1/3) + Log[x]/(2\*k^(1/3)) + Log[1 - (1 + k)\*x]/(2\*k^(1/3)) - (3\*Log[-(k^(1/3)\*x) + ((1 - x)\*x\*(1 - k\*x))^(1/3)])/ (2\*k^(1/3))

**Rubi [F]** time = 0.610747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)} dx$$

Verification is Not applicable to the result.

[In] Int[(2 - (1 + k)\*x)/(((1 - x)\*x\*(1 - k\*x))^(1/3)\*(1 - (1 + k)\*x)), x]

[Out] (3\*(1 - x)^(1/3)\*x\*(1 - k\*x)^(1/3)\*AppellF1[2/3, 1/3, 1/3, 5/3, x, k\*x])/ (2\*((1 - x)\*x\*(1 - k\*x))^(1/3)) + ((1 - x)^(1/3)\*x^(1/3)\*(1 - k\*x)^(1/3)\*Defe r[Int][1/((1 - x)^(1/3)\*x^(1/3)\*(1 + (-1 - k)\*x)\*(1 - k\*x)^(1/3)), x])/((1 - x)\*x\*(1 - k\*x))^(1/3)

Rubi steps

$$\begin{aligned} \int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)} dx &= \frac{(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}) \int \frac{2-(1+k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}(1-(1+k)x)} dx}{\sqrt[3]{(1-x)x(1-kx)}} \\ &= \frac{(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}) \int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}} dx}{\sqrt[3]{(1-x)x(1-kx)}} + \frac{(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}) \int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)x)}}{ \sqrt[3]{(1-x)x(1-kx)}} \\ &= \frac{3\sqrt[3]{1-x}\sqrt[3]{1-kx}F_1\left(\frac{2}{3}; \frac{1}{3}, \frac{1}{3}; \frac{5}{3}; x, kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}} + \frac{(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}) \int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)x)}}{ \sqrt[3]{(1-x)x(1-kx)}} \end{aligned}$$

**Mathematica [F]** time = 1.63514, size = 0, normalized size = 0.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(2 - (1 + k)\*x)/(((1 - x)\*x\*(1 - k\*x))^(1/3)\*(1 - (1 + k)\*x)), x]

[Out] Integrate[(2 - (1 + k)\*x)/(((1 - x)\*x\*(1 - k\*x))^(1/3)\*(1 - (1 + k)\*x)), x]

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{2 - (1+k)x}{1 - (1+k)x} \frac{1}{\sqrt[3]{(1-x)x(-kx+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-(1+k)\*x)/((1-x)\*x\*(-k\*x+1))^(1/3)/(1-(1+k)\*x), x)

[Out] int((2-(1+k)\*x)/((1-x)\*x\*(-k\*x+1))^(1/3)/(1-(1+k)\*x), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x - 2}{((kx-1)(x-1)x)^{\frac{1}{3}} ((k+1)x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(1+k)\*x)/((1-x)\*x\*(-k\*x+1))^(1/3)/(1-(1+k)\*x), x, algorithm="maxima")

[Out] integrate(((k + 1)\*x - 2)/(((k\*x - 1)\*(x - 1)\*x)^(1/3)\*((k + 1)\*x - 1)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(1+k)\*x)/((1-x)\*x\*(-k\*x+1))^(1/3)/(1-(1+k)\*x), x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(1+k)\*x)/((1-x)\*x\*(-k\*x+1))^(1/3)/(1-(1+k)\*x), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x - 2}{((kx-1)(x-1)x)^{\frac{1}{3}} ((k+1)x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi  
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

$$3.45 \quad \int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}} dx$$

**Optimal.** Leaf size=176

$$\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}} + \frac{\log(1-kx)}{2 \cdot 2^{2/3}\sqrt[3]{1-k}} - \frac{3 \log(kx + 2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)} - 1)}{2 \cdot 2^{2/3}\sqrt[3]{1-k}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-kx)}}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}} + 1\right)}{2^{2/3}\sqrt[3]{1-k}}$$

[Out] -((Sqrt[3]\*ArcTan[(1 + (2^(1/3))\*(1 - k\*x))/((1 - k)^(1/3)\*((1 - x)\*x\*(1 - k\*x))^(1/3))]/Sqrt[3])/(2^(2/3)\*(1 - k)^(1/3))) + Log[1 - (2 - k)\*x]/(2^(2/3)\*(1 - k)^(1/3)) + Log[1 - k\*x]/(2\*2^(2/3)\*(1 - k)^(1/3)) - (3\*Log[-1 + k\*x + 2^(2/3)\*(1 - k)^(1/3)\*((1 - x)\*x\*(1 - k\*x))^(1/3)])/(2\*2^(2/3)\*(1 - k)^(1/3))

**Rubi [F]** time = 0.461683, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - k\*x)/((1 + (-2 + k)\*x)\*((1 - x)\*x\*(1 - k\*x))^(2/3)), x]

[Out] ((1 - x)^(2/3)\*x^(2/3)\*(1 - k\*x)^(2/3)\*Defer[Int][(1 - k\*x)^(1/3)/((1 - x)^(2/3)\*x^(2/3)\*(1 + (-2 + k)\*x)), x])/((1 - x)\*x\*(1 - k\*x))^(2/3)

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}} dx = \frac{((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}) \int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)} dx}{((1-x)x(1-kx))^{2/3}}$$

**Mathematica [F]** time = 0.668614, size = 0, normalized size = 0.

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - k\*x)/((1 + (-2 + k)\*x)\*((1 - x)\*x\*(1 - k\*x))^(2/3)), x]

[Out] Integrate[(1 - k\*x)/((1 + (-2 + k)\*x)\*((1 - x)\*x\*(1 - k\*x))^(2/3)), x]

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{-kx + 1}{1 + (-2 + k)x} ((1 - x)x(-kx + 1))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)`

[Out] `int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{kx - 1}{((kx - 1)(x - 1)x)^{\frac{2}{3}} ((k - 2)x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxima")`

[Out] `-integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{kx - 1}{((kx - 1)(x - 1)x)^{\frac{2}{3}} ((k - 2)x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac")`

[Out] `integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)`

$$3.46 \quad \int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=493

$$\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{(a+b)\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{(a+b)\tan^{-1}\left(\frac{1-x}{\sqrt[3]{2}\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}\sqrt[3]{1-x^3}}$$

```
[Out] ((a + b)*ArcTan[(1 - (2*2^(1/3))*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/(2^(1/3)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3))*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x))/(1 - x^3)^(1/3)]/Sqrt[3])/Sqrt[3] - ((a - c)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3)]/Sqrt[3])/(2^(1/3)*Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)]/Sqrt[3])/(2^(1/3)*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - ((a - c)*Log[1 + x^3]/(6*2^(1/3)) - ((b + c)*Log[1 + x^3]/(6*2^(1/3)) + ((a + b)*Log[1 + (2^(2/3)*(1 - x)^2]/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)))/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + ((a - c)*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + (c*Log[x + (1 - x^3)^(1/3)]/2 - ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)))
```

**Rubi [C]** time = 0.846166, antiderivative size = 576, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {6728, 239, 2148}

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x - i\sqrt{3} + 1\right)(3ib - \sqrt{3}(2a + b - i\sqrt{3}c - c))}{4\sqrt[3]{2}(\sqrt{3} + i)} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x + i\sqrt{3} + 1\right)(\sqrt{3}(2a + b - i\sqrt{3}c - c))}{4\sqrt[3]{2}(-\sqrt{3} + i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

```
[Out] -((c*ArcTan[(1 - (2*x))/(1 - x^3)^(1/3)]/Sqrt[3])/Sqrt[3]) - ((2*a + b - I*Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3)]/(2*Sqrt[3]))/(2*2^(1/3)*(I + Sqrt[3])) + ((2*a + b + I*Sqrt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(2 - (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3)]/(2*Sqrt[3]))/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*(2*a + b - c - I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] + 2*x))]/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))]/(12*2^(1/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)]/2 - (((3*I)*b - Sqrt[3]*(2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)*(I + Sqrt[3])) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqrt[3]*c))*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)*(I - Sqrt[3]))
```

**Rule 6728**

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

**Rule 239**

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] := Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

**Rule 2148**

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] := Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sqr
t[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^(
1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

**Rubi steps**

$$\begin{aligned} \int \frac{a + bx + cx^2}{(1 - x + x^2)\sqrt[3]{1 - x^3}} dx &= \int \left( \frac{c}{\sqrt[3]{1 - x^3}} + \frac{a - c + (b + c)x}{(1 - x + x^2)\sqrt[3]{1 - x^3}} \right) dx \\ &= c \int \frac{1}{\sqrt[3]{1 - x^3}} dx + \int \frac{a - c + (b + c)x}{(1 - x + x^2)\sqrt[3]{1 - x^3}} dx \\ &= -\frac{c \tan^{-1} \left( \frac{1 - \frac{2x}{\sqrt[3]{1 - x^3}}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{2} c \log \left( x + \sqrt[3]{1 - x^3} \right) + \int \left( \frac{b - \frac{i(2a + b - c)}{\sqrt{3}} + c}{(-1 - i\sqrt{3} + 2x)\sqrt[3]{1 - x^3}} + \frac{b + \frac{i(2a + b - c)}{\sqrt{3}}}{(-1 + i\sqrt{3} + 2x)\sqrt[3]{1 - x^3}} \right) dx \\ &= -\frac{c \tan^{-1} \left( \frac{1 - \frac{2x}{\sqrt[3]{1 - x^3}}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{2} c \log \left( x + \sqrt[3]{1 - x^3} \right) + \frac{1}{3} (3b - i\sqrt{3}(2a + b - c) + 3c) \int \frac{1}{(-1 - i\sqrt{3} + 2x)\sqrt[3]{1 - x^3}} dx \\ &= -\frac{c \tan^{-1} \left( \frac{1 - \frac{2x}{\sqrt[3]{1 - x^3}}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{(2a + b - i\sqrt{3}b - c - i\sqrt{3}c) \tan^{-1} \left( \frac{2 - \frac{\sqrt[3]{2}(1 - i\sqrt{3} + 2x)}{\sqrt[3]{1 - x^3}}}{2\sqrt{3}} \right)}{2\sqrt[3]{2}(i + \sqrt{3})} + \frac{(2a + b + i\sqrt{3}b - c + i\sqrt{3}c) \tan^{-1} \left( \frac{2 - \frac{\sqrt[3]{2}(1 + i\sqrt{3} + 2x)}{\sqrt[3]{1 - x^3}}}{2\sqrt{3}} \right)}{2\sqrt[3]{2}(i + \sqrt{3})} \end{aligned}$$

**Mathematica [F]** time = 0.382454, size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{(1 - x + x^2)\sqrt[3]{1 - x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*x + c\*x^2)/((1 - x + x^2)\*(1 - x^3)^(1/3)), x]

[Out] Integrate[(a + b\*x + c\*x^2)/((1 - x + x^2)\*(1 - x^3)^(1/3)), x]

**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{x^2 - x + 1} \frac{1}{\sqrt[3]{-x^3 + 1}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)`

[Out] `int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{(-x^3 + 1)^{\frac{1}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt[3]{-(x-1)(x^2+x+1)}(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)`

[Out] `Integral((a + b*x + c*x**2)/((-x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{(-x^3 + 1)^{\frac{1}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)`

$$3.47 \quad \int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5} dx$$

**Optimal.** Leaf size=407

$$\frac{x}{28(3-2x)^{9/2}(2x^2+x+1)^4} + \frac{5(4377x+3049)}{153664(3-2x)^{9/2}(2x^2+x+1)} + \frac{3049x+1387}{32928(3-2x)^{9/2}(2x^2+x+1)^2} + \frac{73x+23}{1176(3-2x)^{9/2}(2x^2+x+1)^3}$$

[Out] -19255/(395136\*(3 - 2\*x)^(9/2)) - 462025/(30118144\*(3 - 2\*x)^(7/2)) - 38491/(8605184\*(3 - 2\*x)^(5/2)) - 141045/(120472576\*(3 - 2\*x)^(3/2)) - 38225/(240945152\*Sqrt[3 - 2\*x]) + x/(28\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^4) + (23 + 73\*x)/(1176\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^3) + (1387 + 3049\*x)/(32928\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^2) + (5\*(3049 + 4377\*x))/(153664\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)) + (5\*Sqrt[(149046503977 + 40815066112\*Sqrt[14])/2]\*ArcTan[(Sqrt[7 + 2\*Sqrt[14]] - 2\*Sqrt[3 - 2\*x])/Sqrt[-7 + 2\*Sqrt[14]])]/3373232128 - (5\*Sqrt[(149046503977 + 40815066112\*Sqrt[14])/2]\*ArcTan[(Sqrt[7 + 2\*Sqrt[14]] + 2\*Sqrt[3 - 2\*x])/Sqrt[-7 + 2\*Sqrt[14]])]/3373232128 + (5\*Sqrt[(-149046503977 + 40815066112\*Sqrt[14])/2]\*Log[3 + Sqrt[14] - Sqrt[7 + 2\*Sqrt[14]]]\*Sqrt[3 - 2\*x] - 2\*x)/6746464256 - (5\*Sqrt[(-149046503977 + 40815066112\*Sqrt[14])/2]\*Log[3 + Sqrt[14] + Sqrt[7 + 2\*Sqrt[14]]]\*Sqrt[3 - 2\*x] - 2\*x)/6746464256

**Rubi [A]** time = 0.676273, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$ , Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{x}{28(3-2x)^{9/2}(2x^2+x+1)^4} + \frac{5(4377x+3049)}{153664(3-2x)^{9/2}(2x^2+x+1)} + \frac{3049x+1387}{32928(3-2x)^{9/2}(2x^2+x+1)^2} + \frac{73x+23}{1176(3-2x)^{9/2}(2x^2+x+1)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2\*x)^(11/2)\*(1 + x + 2\*x^2)^5), x]

[Out] -19255/(395136\*(3 - 2\*x)^(9/2)) - 462025/(30118144\*(3 - 2\*x)^(7/2)) - 38491/(8605184\*(3 - 2\*x)^(5/2)) - 141045/(120472576\*(3 - 2\*x)^(3/2)) - 38225/(240945152\*Sqrt[3 - 2\*x]) + x/(28\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^4) + (23 + 73\*x)/(1176\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^3) + (1387 + 3049\*x)/(32928\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^2) + (5\*(3049 + 4377\*x))/(153664\*(3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)) + (5\*Sqrt[(149046503977 + 40815066112\*Sqrt[14])/2]\*ArcTan[(Sqrt[7 + 2\*Sqrt[14]] - 2\*Sqrt[3 - 2\*x])/Sqrt[-7 + 2\*Sqrt[14]])]/3373232128 - (5\*Sqrt[(149046503977 + 40815066112\*Sqrt[14])/2]\*ArcTan[(Sqrt[7 + 2\*Sqrt[14]] + 2\*Sqrt[3 - 2\*x])/Sqrt[-7 + 2\*Sqrt[14]])]/3373232128 + (5\*Sqrt[(-149046503977 + 40815066112\*Sqrt[14])/2]\*Log[3 + Sqrt[14] - Sqrt[7 + 2\*Sqrt[14]]]\*Sqrt[3 - 2\*x] - 2\*x)/6746464256 - (5\*Sqrt[(-149046503977 + 40815066112\*Sqrt[14])/2]\*Log[3 + Sqrt[14] + Sqrt[7 + 2\*Sqrt[14]]]\*Sqrt[3 - 2\*x] - 2\*x)/6746464256

**Rule 740**

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(b\*c\*d - b^2\*e + 2\*a\*c\*e + c\*(2\*c\*d - b\*e)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)\*(c\*d^2 - b\*d\*e + a\*e

```

^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

### Rule 822

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
+ b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

### Rule 828

```

Int((((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]

```

### Rule 826

```

Int(((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]

```

### Rule 1169

```

Int(((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

```

### Rule 634

```

Int(((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

### Rule 618

```

Int(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-2x)^{11/2} (1+x+2x^2)^5} dx &= \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{1}{784} \int \frac{700-644x}{(3-2x)^{11/2} (1+x+2x^2)^4} dx \\
&= \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{23+73x}{1176(3-2x)^{9/2} (1+x+2x^2)^3} + \frac{\int \frac{325752-543704x}{(3-2x)^{11/2} (1+x+2x^2)^4} dx}{460992} \\
&= \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{23+73x}{1176(3-2x)^{9/2} (1+x+2x^2)^3} + \frac{1387+3}{32928(3-2x)^{9/2}} \\
&= \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{23+73x}{1176(3-2x)^{9/2} (1+x+2x^2)^3} + \frac{1387+3}{32928(3-2x)^{9/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{23+73x}{1176(3-2x)^{9/2} (1+x+2x^2)^3} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2} (1+x+2x^2)^4} + \frac{1387+3}{1176(3-2x)^{9/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{1387+3}{28(3-2x)^{9/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} \\
&= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 2.07546, size = 198, normalized size = 0.49

$$\frac{56(-88070400x^{12}+677249280x^{11}-1873554048x^{10}+2443779648x^9-2343370048x^8+3106712560x^7-2888625656x^6+1470758860x^5-1627773523x^4+1073855x^3-141045x^2+141045x-56)}{(3-2x)^{9/2}(2x^2+x+1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2\*x)^(11/2)\*(1 + x + 2\*x^2)^5),x]

[Out] ((56\*(-40289347 + 429812744\*x - 135202154\*x^2 + 1073855156\*x^3 - 1627773523\*x^4 + 1470758860\*x^5 - 2888625656\*x^6 + 3106712560\*x^7 - 2343370048\*x^8 + 2443779648\*x^9 - 1873554048\*x^10 + 677249280\*x^11 - 88070400\*x^12))/((3 - 2\*x)^(9/2)\*(1 + x + 2\*x^2)^4) + (45\*I)\*Sqrt[14 - (2\*I)\*Sqrt[7]]\*(115739\*I + 146319\*Sqrt[7])\*ArcTanh[Sqrt[6 - 4\*x]/Sqrt[7 - I\*Sqrt[7]]] - (45\*I)\*Sqrt[14 + (2\*I)\*Sqrt[7]]\*(-115739\*I + 146319\*Sqrt[7])\*ArcTanh[Sqrt[6 - 4\*x]/Sqrt[7 + I\*Sqrt[7]])/121436356608

**Maple [A]** time = 0.089, size = 584, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3-2\*x)^(11/2)/(2\*x^2+x+1)^5,x)

[Out] 1/151263/(3-2\*x)^(9/2)+5/235298/(3-2\*x)^(7/2)+19/470596/(3-2\*x)^(5/2)+185/2823576/(3-2\*x)^(3/2)+505/3294172/(3-2\*x)^(1/2)+1/6588344\*(567651623/32\*(3-2\*x)^(1/2)-6194606411/192\*(3-2\*x)^(3/2)+9801432515/384\*(3-2\*x)^(5/2)-8763772549/768\*(3-2\*x)^(7/2)+149630663/48\*(3-2\*x)^(9/2)-200063633/384\*(3-2\*x)^(11/2)+18969965/384\*(3-2\*x)^(13/2)-526135/256\*(3-2\*x)^(15/2))/((3-2\*x)^2-7+14\*x)^4+731595/13492928512\*ln(3-2\*x+14^(1/2)+(3-2\*x)^(1/2)\*(7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))^(1/2)\*14^(1/2)-1424965/6746464256\*ln(3-2\*x+14^(1/2)+(3-2\*x)^(1/2)\*(7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))^(1/2)-731595/6746464256/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)+(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))\*14^(1/2)+1424965/3373232128/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)+(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))-578695/3373232128/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)+(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*14^(1/2)-731595/13492928512\*ln(3-2\*x+14^(1/2)-(3-2\*x)^(1/2)\*(7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))^(1/2)\*14^(1/2)+1424965/6746464256\*ln(3-2\*x+14^(1/2)-(3-2\*x)^(1/2)\*(7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))^(1/2)-731595/6746464256/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)-(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))\*14^(1/2)+1424965/3373232128/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)-(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))-578695/3373232128/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)-(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*14^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5 (-2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(11/2)/(2\*x^2+x+1)^5,x, algorithm="maxima")

[Out] integrate(1/((2\*x^2 + x + 1)^5\*(-2\*x + 3)^(11/2)), x)

**Fricas [B]** time = 3.1791, size = 5156, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(11/2)/(2\*x^2+x+1)^5,x, algorithm="fricas")

[Out] 
$$\frac{1}{852282865707923134247251378176} \cdot (2263908918780 \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot \sqrt{14} \cdot \sqrt{7} \cdot (512x^{13} - 2816x^{12} + 5632x^{11} - 5888x^{10} + 6848x^9 - 8992x^8 + 6112x^7 - 4240x^6 + 4994x^5 - 1707x^4 + 936x^3 - 1242x^2 - 162x - 243) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) \cdot \arctan\left(\frac{1}{10052187156951869469526908685753437228729401815040} \cdot 22241759018113166^{3/4} \cdot \sqrt{12577271771} \cdot \sqrt{79716926} \cdot \sqrt{-2089731384934400} \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224\right) \cdot (7645 \cdot \sqrt{14} - 115739) - 4190418993502514995568679111884800x + 2095209496751257497784339555942400 \cdot \sqrt{14} + 6285628490253772493353018667827200) \cdot (115739 \cdot \sqrt{14} \cdot \sqrt{7} - 107030 \cdot \sqrt{7}) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) - \frac{1}{1958184534851295802906658902} \cdot 22241759018113166^{3/4} \cdot \sqrt{79716926} \cdot (115739 \cdot \sqrt{14} \cdot \sqrt{7} - 107030 \cdot \sqrt{7}) \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) - \frac{2}{7} \cdot \sqrt{14} \cdot \sqrt{7} - \sqrt{7}) + 2263908918780 \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot \sqrt{14} \cdot \sqrt{7} \cdot (512x^{13} - 2816x^{12} + 5632x^{11} - 5888x^{10} + 6848x^9 - 8992x^8 + 6112x^7 - 4240x^6 + 4994x^5 - 1707x^4 + 936x^3 - 1242x^2 - 162x - 243) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) \cdot \arctan\left(\frac{1}{24628619072593968384668700756050455442} \cdot 22241759018113166^{3/4} \cdot \sqrt{12577271771} \cdot \sqrt{22241759018113166^{1/4}} \cdot \sqrt{79716926} \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224\right) \cdot (7645 \cdot \sqrt{14} - 115739) - 2005242886101391892x + 1002621443050695946 \cdot \sqrt{14} + 3007864329152087838) \cdot (115739 \cdot \sqrt{14} \cdot \sqrt{7} - 107030 \cdot \sqrt{7}) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) - \frac{1}{1958184534851295802906658902} \cdot 22241759018113166^{3/4} \cdot \sqrt{79716926} \cdot (115739 \cdot \sqrt{14} \cdot \sqrt{7} - 107030 \cdot \sqrt{7}) \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) + \frac{2}{7} \cdot \sqrt{14} \cdot \sqrt{7} + \sqrt{7}) + 315 \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot (41794627698688x^{13} - 229870452342784x^{12} + 459740904685568x^{11} - 480638218534912x^{10} + 559003145469952x^9 - 734018148958208x^8 + 498923368153088x^7 - 346111760629760x^6 + 407660880326656x^5 - 139342635706368x^4 + 76405803761664x^3 - 10138462422208x^2 - 21292357711 \cdot \sqrt{14}) \cdot (512x^{13} - 2816x^{12} + 5632x^{11} - 5888x^{10} + 6848x^9 - 8992x^8 + 6112x^7 - 4240x^6 + 4994x^5 - 1707x^4 + 936x^3 - 1242x^2 - 162x - 243) - 13224081420288x - 19836122130432) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) \cdot \log\left(\frac{2089731384934400}{12577271771} \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224\right) \cdot (7645 \cdot \sqrt{14} - 115739) - 333173924345386159308800x + 166586962172693079654400 \cdot \sqrt{14} + 499760886518079238963200) - 315 \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot (41794627698688x^{13} - 229870452342784x^{12} + 459740904685568x^{11} - 480638218534912x^{10} + 559003145469952x^9 - 734018148958208x^8 + 498923368153088x^7 - 346111760629760x^6 + 407660880326656x^5 - 139342635706368x^4 + 76405803761664x^3 - 10138462422208x^2 - 21292357711 \cdot \sqrt{14}) \cdot (512x^{13} - 2816x^{12} + 5632x^{11} - 5888x^{10} + 6848x^9 - 8992x^8 + 6112x^7 - 4240x^6 + 4994x^5 - 1707x^4 + 936x^3 - 1242x^2 - 162x - 243) - 13224081420288x - 19836122130432) \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224) \cdot \log\left(\frac{-2089731384934400}{12577271771} \cdot 22241759018113166^{1/4} \cdot \sqrt{79716926} \cdot \sqrt{-2x+3} \cdot \sqrt{21292357711} \cdot \sqrt{14} + 81630132224\right) \cdot (7645 \cdot \sqrt{14} - 115739) - 333173924345386159308800x + 166586962172693079654400 \cdot \sqrt{14} + 499760886518079238963200) + 393027605675872810832 \cdot (88070400x^{12} - 677249280x^{11} + 1873554048x^{10} - 2443779648x^9 + 2343370048x^8 - 3106712560x^7 + 2888625656x^6 - 1470758860x^5 + 1627773523x^4 - 1073855156x^3 + 135202154x^2 - 429812744x + 40289347) \cdot \sqrt{-2x+3}) / (512x^{13} - 2816x^{12} + 5632x^{11} - 5888x^{10} + 6848x^9 - 8992x^8 + 6112x^7 -$$

$4240x^6 + 4994x^5 - 1707x^4 + 936x^3 - 1242x^2 - 162x - 243$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)\*\*(11/2)/(2\*x\*\*2+x+1)\*\*5,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5 (-2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(11/2)/(2\*x^2+x+1)^5,x, algorithm="giac")

[Out] integrate(1/((2\*x^2 + x + 1)^5\*(-2\*x + 3)^(11/2)), x)



$$3.48 \quad \int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}} dx$$

**Optimal.** Leaf size=648

result too large to display

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
- 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
+ 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
+ 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]])]/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

---

**Rubi [A]** time = 1.1581, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$ , Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{11275(14627273 - 35058731x)}{3966920982528(3 - 2x)^{19/2}(2x^2 + x + 1)} + \frac{451(14627273x + 28962039)}{283351498752(3 - 2x)^{19/2}(2x^2 + x + 1)^2} + \frac{451(998691x + 8110)}{10119696384(3 - 2x)^{19/2}(2x^2 + x + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10), x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
- 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
```

$$\begin{aligned}
& + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^{(19/2)}*(1 + \\
& x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 - \\
& 2*x)^{(19/2)}*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]* (9756589235 \\
& + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr \\
& t[-7 + 2*Sqrt[14]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2] \\
& ]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[ \\
& 3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])/318603459702399434752 + (11275*(975658923 \\
& 5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[ \\
& 7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x))/637206919404798869504 - (11275*(97565 \\
& 89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S \\
& qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x))/637206919404798869504
\end{aligned}$$

#### Rule 740

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

#### Rule 822

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

#### Rule 828

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]]/(a + b*x + c*x^2), x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]

```

#### Rule 826

```

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

```

#### Rule 1169

```

Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :

```

```
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rubi steps



**Mathematica [C]** time = 6.08693, size = 610, normalized size = 0.94

$$\frac{x}{63(3-2x)^{19/2}(2x^2+x+1)^9} + \frac{67816x+20776}{1568(3-2x)^{19/2}(2x^2+x+1)^8} + \frac{117492592x+46521776}{1372(3-2x)^{19/2}(2x^2+x+1)^7} + \frac{164128134240x+74020332960}{1176(3-2x)^{19/2}(2x^2+x+1)^6} + \frac{184316990760000x+9420954905}{980(3-2x)^{19/2}(2x^2+x+1)^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2\*x)^(21/2)\*(1 + x + 2\*x^2)^10), x]

[Out]  $x/(63*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^5) + ((95476201213680000 + 157747397367934080*x)/(784*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^4) + ((72879297583985544960 + 89735798552133000960*x)/(588*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^3) + ((36432734212165998389760 + 18400346379541577848320*x)/(392*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2)^2) + ((6440121232839552246912000 - 15435719146659136558464000*x)/(196*(3 - 2*x)^{(19/2)}*(1 + x + 2*x^2))) + (39479926882545221954112000/(19*(3 - 2*x)^{(19/2)})) + (-908021664138480966930240000/(17*(3 - 2*x)^{(17/2)})) + (-19105520493023248582746201600/(3 - 2*x)^{(15/2)} + (-2684955743553723946588431072000/(13*(3 - 2*x)^{(13/2)})) + (-150994423858598796539274120000000/(3 - 2*x)^{(11/2)} + (-8237718113587514139784976619840000/(3 - 2*x)^{(9/2)} + (-338389312036560466460044072847040000/(3 - 2*x)^{(7/2)} + (-10135305528576510550836394515648960000/(3 - 2*x)^{(5/2)} + (-204334375738495648812805956791073600000/(3 - 2*x)^{(3/2)} + (-2230994866519889796828561036406228800000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 - (71750597240923349846054347713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) + (Sqrt[(7 + I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 + (71750597240923349846054347713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/196)/392)/588)/784)/980)/1176)/1372)/1568)/1764$

**Maple [A]** time = 0.053, size = 719, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3-2\*x)^(21/2)/(2\*x^2+x+1)^10, x)

[Out]  $1/5367029731/(3-2*x)^{(19/2)}+5/4802079233/(3-2*x)^{(17/2)}+73/23727920916/(3-2*x)^{(15/2)}+165/25705247659/(3-2*x)^{(13/2)}+2365/221460595216/(3-2*x)^{(11/2)}+30349/1993145356944/(3-2*x)^{(9/2)}+854095/43406276662336/(3-2*x)^{(7/2)}+75933/3100448333024/(3-2*x)^{(5/2)}+8519225/260437659974016/(3-2*x)^{(3/2)}+891605/12401793332096/(3-2*x)^{(1/2)}+1/86812553324672*(-43462358811134257841/1179648*(3-2*x)^{(27/2)}+192384852501874197/65536*(3-2*x)^{(29/2)}-1352841099712333/81$

$92*(3-2*x)^{(31/2)}+4606702222670185/786432*(3-2*x)^{(33/2)}-25865320405815/262$   
 $144*(3-2*x)^{(35/2)}+544765170330150812273/1024*(3-2*x)^{(1/2)}-347698778390586$   
 $0258979/1536*(3-2*x)^{(3/2)}+9364999706478908741137/2048*(3-2*x)^{(5/2)}-238519$   
 $05772903279054347/4096*(3-2*x)^{(7/2)}+192983613795383541041317/36864*(3-2*x)$   
 $^{(9/2)}-57758421475348449750643/16384*(3-2*x)^{(11/2)}+60333035869584695411551$   
 $/32768*(3-2*x)^{(13/2)}-149770885083493978040723/196608*(3-2*x)^{(15/2)}+662568$   
 $99944582155696811/262144*(3-2*x)^{(17/2)}-17729978841543630405471/262144*(3-2$   
 $*x)^{(19/2)}+2869878271121283060373/196608*(3-2*x)^{(21/2)}-1655749892113878944$   
 $81/65536*(3-2*x)^{(23/2)}+45406001689183688581/131072*(3-2*x)^{(25/2))}/((3-2*x$   
 $)^{-2-7+14*x})^9+206922416016525/1274413838809597739008*\ln(3-2*x+14^{(1/2)}+(3-2$   
 $*x)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)}*14^{(1/2)}-3896156139350$   
 $75/637206919404798869504*\ln(3-2*x+14^{(1/2)}+(3-2*x)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)}$   
 $)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)}-206922416016525/637206919404798869504/(-7+2*14^{(1/2)}$   
 $)^{(1/2)}*\arctan((2*(3-2*x)^{(1/2)}+(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})$   
 $*(7+2*14^{(1/2)})^{(1/2)}*14^{(1/2)}+389615613935075/318603459702399434752/(-7+2*14^{(1/2)}$   
 $)^{(1/2)}*\arctan((2*(3-2*x)^{(1/2)}+(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})$   
 $*(7+2*14^{(1/2)})^{(1/2)}-110005543624625/318603459702399434752/(-7+2*14^{(1/2)})^{(1/2)}$   
 $*14^{(1/2)}-206922416016525/1274413838809597739008*\ln(3-2*x+14^{(1/2)}-(3-2*x)^{(1/2)}$   
 $*(7+2*14^{(1/2)})^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)}*14^{(1/2)}+389615613935075/6$   
 $37206919404798869504*\ln(3-2*x+14^{(1/2)}-(3-2*x)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)})$   
 $*(7+2*14^{(1/2)})^{(1/2)}-206922416016525/637206919404798869504/(-7+2*14^{(1/2)})^{(1/2)}$   
 $*\arctan((2*(3-2*x)^{(1/2)}-(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})$   
 $*(7+2*14^{(1/2)})^{(1/2)}*14^{(1/2)}+389615613935075/318603459702399434752/(-7+2*14^{(1/2)}$   
 $)^{(1/2)}*\arctan((2*(3-2*x)^{(1/2)}-(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})$   
 $*(7+2*14^{(1/2)})^{(1/2)}-110005543624625/318603459702399434752/(-7+2*14^{(1/2)})^{(1/2)}$   
 $*\arctan((2*(3-2*x)^{(1/2)}-(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})*14^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10} (-2x + 3)^{\frac{21}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(21/2)/(2\*x^2+x+1)^10,x, algorithm="maxima")

[Out] integrate(1/((2\*x^2 + x + 1)^10\*(-2\*x + 3)^(21/2)), x)

**Fricas [B]** time = 3.68979, size = 11992, normalized size = 18.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(21/2)/(2\*x^2+x+1)^10,x, algorithm="fricas")

[Out] 1/1094755373086200603246995644663447631605361478665641987670016\*(4732002380  
085251586622550100\*4787936175075825342943147314686^(1/4)\*sqrt(1169607525756  
986)\*sqrt(14)\*sqrt(7)\*(524288\*x^28 - 5505024\*x^27 + 24772608\*x^26 - 6468403  
2\*x^25 + 119734272\*x^24 - 194052096\*x^23 + 295206912\*x^22 - 386777088\*x^21  
+ 449261568\*x^20 - 515594240\*x^19 + 540503040\*x^18 - 496581120\*x^17 + 46771  
2000\*x^16 - 411828480\*x^15 + 303534720\*x^14 - 248434368\*x^13 + 186495624\*x^

$12 - 105219828x^{11} + 83621482x^{10} - 49793667x^9 + 19105065x^8 - 20036484x^7 + 5497632x^6 - 2235114x^5 + 3276126x^4 + 734832x^3 + 826686x^2 + 137781x + 59049) \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} \cdot \arctan\left(\frac{1}{36562170851931970248855340113387035354417457241870626866024945379489008832725311219252 \cdot 4787936175075825342943147314686^{3/4}} \cdot \sqrt{2776387167632535361} \cdot \sqrt{12865682783326846} \cdot \sqrt{1169607525756986} \cdot \sqrt{4787936175075825342943147314686^{1/4}} \cdot \sqrt{1169607525756986} \cdot \sqrt{-2x + 3} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936}\right) \cdot (2148932869 \cdot \sqrt{14} - 9756589235) - 71440233164918992209696826631202812x + 28280279689505005187146 \cdot \sqrt{22335021272086100802556094} + 107160349747378488314545239946804218) \cdot (9756589235 \cdot \sqrt{14} \cdot \sqrt{7} - 30085060166 \cdot \sqrt{7}) \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} - \frac{1}{1023573670806157676669100144258228441327447900096742 \cdot 4787936175075825342943147314686^{3/4}} \cdot \sqrt{1169607525756986} \cdot (9756589235 \cdot \sqrt{14} \cdot \sqrt{7} - 30085060166 \cdot \sqrt{7}) \cdot \sqrt{-2x + 3} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} + \frac{2}{7} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} \cdot \sqrt{7} + \sqrt{7}) + 4732002380085251586622550100 \cdot 4787936175075825342943147314686^{1/4} \cdot \sqrt{1169607525756986} \cdot \sqrt{14} \cdot \sqrt{7} \cdot (524288x^{28} - 5505024x^{27} + 24772608x^{26} - 64684032x^{25} + 119734272x^{24} - 194052096x^{23} + 295206912x^{22} - 386777088x^{21} + 449261568x^{20} - 515594240x^{19} + 540503040x^{18} - 496581120x^{17} + 467712000x^{16} - 411828480x^{15} + 303534720x^{14} - 248434368x^{13} + 186495624x^{12} - 105219828x^{11} + 83621482x^{10} - 49793667x^9 + 19105065x^8 - 20036484x^7 + 5497632x^6 - 2235114x^5 + 3276126x^4 + 734832x^3 + 826686x^2 + 137781x + 59049) \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} \cdot \arctan\left(\frac{1}{39296670234816303076555330542603297083388480635973027797585697454399143598928370335464344780800 \cdot 4787936175075825342943147314686^{3/4}} \cdot \sqrt{2776387167632535361} \cdot \sqrt{1169607525756986} \cdot \sqrt{-14862107440409842545228890767360000 \cdot 4787936175075825342943147314686^{1/4}} \cdot \sqrt{1169607525756986} \cdot \sqrt{-2x + 3} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936}\right) \cdot (2148932869 \cdot \sqrt{14} - 9756589235) - 1061752420864956548109093061495542399038192585561809435358469816320000x + 420304555190263689316852795001664341102416628348354560000 \cdot \sqrt{22335021272086100802556094} + 1592628631297434822163639592243313598557288878342714153037704724480000) \cdot (9756589235 \cdot \sqrt{14} \cdot \sqrt{7} - 30085060166 \cdot \sqrt{7}) \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} - \frac{1}{1023573670806157676669100144258228441327447900096742 \cdot 4787936175075825342943147314686^{3/4}} \cdot \sqrt{1169607525756986} \cdot (9756589235 \cdot \sqrt{14} \cdot \sqrt{7} - 30085060166 \cdot \sqrt{7}) \cdot \sqrt{-2x + 3} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} - \frac{2}{7} \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936} \cdot \sqrt{14} \cdot \sqrt{7} - \sqrt{7}) + 271150425 \cdot 4787936175075825342943147314686^{1/4} \cdot \sqrt{1169607525756986} \cdot (642998537252061761731821568x^{28} - 6751484641146648498184126464x^{27} + 30381680885159918241828569088x^{26} - 79329944533473119853663485952x^{25} + 146844790944939604835504750592x^{24} - 237989833600419359560990457856x^{23} + 362048363881489025715123781632x^{22} - 474352077153419437787597242368x^{21} + 550984441886077267281495195648x^{20} - 632336315413643784471854448640x^{19} + 662885025215707070319757885440x^{18} - 609018199514371017360613048320x^{17} + 573612464628670331388690432000x^{16} - 505075664975624031448627937280x^{15} + 372261773996761581935835217920x^{14} - 304685469106942025132773736448x^{13} + 228722407218762404519491928064x^{12} - 129043951976611196927641387008x^{11} + 102555257051181053298083889152x^{10} - 61068067637283818105902989312x^9 + 23430879305087206538965155840x^8 - 24573192412708929931548033024x^7 + 6742418926906827559038615552x^6 - 2741193833525857491515080704x^5 + 4017914249140640432768679936x^4 + 901214411022199723237834752x^3 + 1013866212399974688642564096x^2 - 327571850528462403199 \cdot \sqrt{14} \cdot (524288x^{28} - 5505024x^{27} + 24772608x^{26} - 64684032x^{25} + 119734272x^{24} - 194052096x^{23} + 295206912x^{22} - 386777088x^{21} + 449261568x^{20} - 515594240x^{19} + 540503040x^{18} - 496581120x^{17} + 467712000x^{16} - 411828480x^{15} + 303534720x^{14} - 248434368x^{13} + 186495624x^{12} - 105219828x^{11} + 83621482x^{10} - 49793667x^9 + 19105065x^8 - 20036484x^7 + 5497632x^6 - 2235114x^5 + 3276126x^4 + 734832x^3 + 826686x^2 + 137781x + 59049) + 168977702066662448107094016x + 72419015171426763474468864) \cdot \sqrt{327571850528462403199 \cdot \sqrt{14} + 1226422380928157351936}$

```

rt(327571850528462403199*sqrt(14) + 1226422380928157351936)*log(14862107440
409842545228890767360000/2776387167632535361*478793617507582534294314731468
6^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sq
rt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 38242
2319640069460132720868272698184789257093120000*x + 151385426388014656165701
481356328960000*sqrt(22335021272086100802556094) + 573633479460104190199081
302409047277183885639680000) - 271150425*4787936175075825342943147314686^(1
/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 675148464114
6648498184126464*x^27 + 30381680885159918241828569088*x^26 - 79329944533473
119853663485952*x^25 + 146844790944939604835504750592*x^24 - 23798983360041
9359560990457856*x^23 + 362048363881489025715123781632*x^22 - 4743520771534
19437787597242368*x^21 + 550984441886077267281495195648*x^20 - 632336315413
643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 60901819951
4371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 5050756649
75624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 304685469
106942025132773736448*x^13 + 228722407218762404519491928064*x^12 - 12904395
1976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 6106806
7637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 2457319241
2708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 27411938335258
57491515080704*x^5 + 4017914249140640432768679936*x^4 + 9012144110221997232
37834752*x^3 + 1013866212399974688642564096*x^2 - 327571850528462403199*sq
rt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 1197342
72*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20
- 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9) + 168977702066662448107094016*x + 72419015171426763474468864)*sqrt(32757
1850528462403199*sqrt(14) + 1226422380928157351936)*log(-148621074404098425
45228890767360000/2776387167632535361*4787936175075825342943147314686^(1/4)
)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14)
+ 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640
069460132720868272698184789257093120000*x + 1513854263880146561657014813563
28960000*sqrt(22335021272086100802556094) + 5736334794601041901990813024090
47277183885639680000) + 1272935063665829315736416183610522832*(240031204937
714427494400*x^27 - 2621948941596237063782400*x^26 + 1236504505589681110548
4800*x^25 - 33969890064381284111155200*x^24 + 65360120291258796757811200*x^
23 - 106701725825102321939251200*x^22 + 162290307223249502039654400*x^21 -
216634228326470609547509760*x^20 + 253788172995391086570485760*x^19 - 28727
9159180291305208156160*x^18 + 304010591010966811155955200*x^17 - 2826446645
39994827031006720*x^16 + 258819256815163249845447936*x^15 - 229408132984166
521977166336*x^14 + 172649692294614969274168896*x^13 - 13331254137724638611
5890240*x^12 + 102031573634317834547976132*x^11 - 5979110268149411757214917
6*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8 +
10718131725916893151555068*x^7 - 8685973988079840377705700*x^6 + 3673303058
277822225386926*x^5 - 809990362095044210054958*x^4 + 1362587089603925431664
856*x^3 + 111926768697602999806116*x^2 + 205702452014540322797289*x - 48844
17100172357749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x
^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 38
6777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120
*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 +
186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*
x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 +
826686*x^2 + 137781*x + 59049)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)\*\*(21/2)/(2\*x\*\*2+x+1)\*\*10,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10} (-2x + 3)^{\frac{21}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(21/2)/(2\*x^2+x+1)^10,x, algorithm="giac")

[Out] integrate(1/((2\*x^2 + x + 1)^10\*(-2\*x + 3)^(21/2)), x)

$$3.49 \quad \int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}} dx$$

**Optimal.** Leaf size=1058

result too large to display

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^(3
9/2)) - 3948194343291401740321996415/(202881463139404195937734623232*(3 - 2
*x)^(37/2)) - 304688229262620222736480811/(537361713180043545997243056128*(
3 - 2*x)^(35/2)) + 2124315846756567455653862925/(16888510985658511445627638
90688*(3 - 2*x)^(33/2)) + 47657515074514118796095929535/(666328524343253997
03658138959872*(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(124667917
457770102671360389021696*(3 - 2*x)^(29/2)) + 149066309808794760843017404825
/(1624981820656451683095663001731072*(3 - 2*x)^(27/2)) + 158486139641690665
43734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168
222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^(23/2))
+ 14011818498091020272474956375/(10110997995195699361484125344104448*(3 -
2*x)^(21/2)) + 173441368149804378661935869705/(8965084889073520100515924471
77261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 101190274412779618678573275245/(
3963511214116714149701777134888943616*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(34350430522344855964082068502370844672*(3 - 2*x)^(13/2)) - 2
211619588790911794826342607495/(406920484649315986036049119181931544576*(3
- 2*x)^(11/2)) - 14340146755077247627940437025/(73985542663511997461099839
851260280832*(3 - 2*x)^(9/2)) - 4611053278117143010907562317585/(7250583181
024175751187784305423507521536*(3 - 2*x)^(7/2)) - 4059653724406305107209268
90227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 49866814
79187781853417316522775/(87006998172290109014253411665082090258432*(3 - 2*x
)^(3/2)) - 927027754781476746208047620505/(58004665448193406009502274443388
060172288*sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
+ (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
+ (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
*(919498192874055581221 + 908287136092467468517*x))/(1576711628592227945545
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 27513723463194262383705*x))/(20018492580021161284337664*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673
415406553789342019*x))/(76434244396444433994743808*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(88411609113007981044643 - 5712269536245152162963*x))/(1
25891696652967303050166272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
1347681225760814815 + 965934812839019490346107*x))/(19583152812683802696692
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*sqrt[(7 + 2*sqrt[14])/2])*(302
97118912219360725028693061 + 8061110911143276053983022787*sqrt[14])*ArcTan[
(sqrt[7 + 2*sqrt[14]] - 2*sqrt[3 - 2*x])/sqrt[-7 + 2*sqrt[14]]]/8120653162
74707684133031842207432842412032 - (115*sqrt[(7 + 2*sqrt[14])/2])*(302971189
```

12219360725028693061 + 8061110911143276053983022787\* $\sqrt{14}$ )\*ArcTan[( $\sqrt{7 + 2*\sqrt{14}} + 2*\sqrt{3 - 2*x}$ )/ $\sqrt{-7 + 2*\sqrt{14}}$ ])/8120653162747076  
 84133031842207432842412032 + (115\*(30297118912219360725028693061 - 80611109  
 11143276053983022787\* $\sqrt{14}$ ))\* $\sqrt{(-7 + 2*\sqrt{14})/2}$ \*Log[3 +  $\sqrt{14}$  -  
 $\sqrt{7 + 2*\sqrt{14}}$ ]\* $\sqrt{3 - 2*x} - 2*x$ )/1624130632549415368266063684414  
 865684824064 - (115\*(30297118912219360725028693061 - 8061110911143276053983  
 022787\* $\sqrt{14}$ ))\* $\sqrt{(-7 + 2*\sqrt{14})/2}$ \*Log[3 +  $\sqrt{14}$  +  $\sqrt{7 + 2*\sqrt{14}}$ ]  
 $\sqrt{14}$ ]\* $\sqrt{3 - 2*x} - 2*x$ )/1624130632549415368266063684414865684824064

**Rubi [A]** time = 2.48972, antiderivative size = 1058, normalized size of antiderivative =  
 1., number of steps used = 49, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}}$   
 = 0.45, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((3 - 2\*x)^(41/2)\*(1 + x + 2\*x^2)^20), x]

[Out] 
$$\begin{aligned} & -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^{(39/2)}) - 3948194343291401740321996415/(202881463139404195937734623232*(3 - 2*x)^{(37/2)}) - 304688229262620222736480811/(537361713180043545997243056128*(3 - 2*x)^{(35/2)}) + 2124315846756567455653862925/(1688851098565851144562763890688*(3 - 2*x)^{(33/2)}) + 47657515074514118796095929535/(66632852434325399703658138959872*(3 - 2*x)^{(31/2)}) + 34911619993974714062172751985/(124667917457770102671360389021696*(3 - 2*x)^{(29/2)}) + 149066309808794760843017404825/(1624981820656451683095663001731072*(3 - 2*x)^{(27/2)}) + 15848613964169066543734380171/(601845118761648771516912222863360*(3 - 2*x)^{(25/2)}) + 11155168222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^{(23/2)}) + 14011818498091020272474956375/(10110997995195699361484125344104448*(3 - 2*x)^{(21/2)}) + 173441368149804378661935869705/(896508488907352010051592447177261056*(3 - 2*x)^{(19/2)}) - 22724090823469905152713519545/(1604278348571050965355481221264572416*(3 - 2*x)^{(17/2)}) - 101190274412779618678573275245/(3963511214116714149701777134888943616*(3 - 2*x)^{(15/2)}) - 460503190416958283087439337135/(34350430522344855964082068502370844672*(3 - 2*x)^{(13/2)}) - 2211619588790911794826342607495/(406920484649315986036049119181931544576*(3 - 2*x)^{(11/2)}) - 143401467550777247627940437025/(73985542663511997461099839851260280832*(3 - 2*x)^{(9/2)}) - 4611053278117143010907562317585/(7250583181024175751187784305423507521536*(3 - 2*x)^{(7/2)}) - 405965372440630510720926890227/(2071595194578335928910795515835287863296*(3 - 2*x)^{(5/2)}) - 4986681479187781853417316522775/(87006998172290109014253411665082090258432*(3 - 2*x)^{(3/2)}) - 927027754781476746208047620505/(58004665448193406009502274443388060172288*\sqrt{3 - 2*x}) + x/(133*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/(595601664*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)/(25015269888*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^15) + (41652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(297448235814912*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^13) + (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(1099368679571914752*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^11) + (45187921585208601 + 78752911037377255*x)/(3420258114223734784*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^10) + (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^9) + (691833601144925854831 + 919498192874055581221*x)/(48266682507925345271808*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^8) + (23*(919498192874055581221 + 908287136092467468517*x))/(1576711628592227945545 \end{aligned}$$

```

728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 27513723463194262383705*x))/(20018492580021161284337664*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673
415406553789342019*x))/(76434244396444433994743808*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(88411609113007981044643 - 5712269536245152162963*x))/(1
25891696652967303050166272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
1347681225760814815 + 965934812839019490346107*x))/(19583152812683802696692
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
97118912219360725028693061 + 8061110911143276053983022787*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/8120653162
74707684133031842207432842412032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
12219360725028693061 + 8061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/8120653162747076
84133031842207432842412032 + (115*(30297118912219360725028693061 - 80611109
11143276053983022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414
865684824064 - (115*(30297118912219360725028693061 - 8061110911143276053983
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064

```

#### Rule 740

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

#### Rule 822

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
+ b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

#### Rule 828

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]]/(a + b*x + c*x^
2), x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]

```

#### Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

#### Rule 1169

```
Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rubi steps



**Mathematica [C]** time = 6.17009, size = 1100, normalized size = 1.04

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2\*x)^(41/2)\*(1 + x + 2\*x^2)^20), x]

[Out]  $x/(133*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{19}) + ((44296 + 146216*x)/(3528*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{18}) + ((223125616 + 589021552*x)/(3332*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{17}) + ((865861681440 + 2110519336800*x)/(3136*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{16}) + ((2984274342235200 + 6928434268875840*x)/(2940*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{15}) + ((9408813737133390720 + 20924013532366815360*x)/(2744*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{14}) + ((27243065619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{13}) + ((72110377354780278913835520 + 145295342948683106164016640*x)/(2352*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{12}) + ((172901458108932896335179801600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{11}) + ((370557652515461812186329087129600 + 645802967231886306826540424448000*x)/(1960*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^{10}) + ((696175598675973438759010577554944000 + 1088028437838790621809440473088716800*x)/(1764*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 1410229454280293592108580217248432347955200*x)/(1372*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^7) + ((1283308803395067168818807997696073436639232000 + 421439161286999121770135584246204836237312000*x)/(1176*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^6) + ((359909043739097249991695788946258930146664448000 - 1443636121324398194831693460992758930913796096000*x)/(980*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^5) + ((-1152021624816869759475691381872221626869209284608000 - 3040089329780519199031170166260953381570260254720000*x)/(784*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^4) + ((-2255746282697145245681128263365627409125133109002240000 - 2609695511325529255410382651665073470845732989009920000*x)/(588*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^3) + ((-1790251120769313069211522499042240401000172830460805120000 + 115668033214143596894295804604678509924267822733393920000*x)/(392*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^2) + ((72870860924910466043406356900947461252288728322038169600000 + 2464467090087282692969213073458776810025190662610343034880000*x)/(196*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)) + (-530550566665897087493026465460148012491929957574880460800000/(3 - 2*x)^{(39/2)} + (-1708089006242241264480481073293611769771298388785813753364480000/(37*(3 - 2*x)^{(37/2)}) + (-696740950089909200017539783692427216704271188038402697920512000/(3 - 2*x)^{(35/2)} + (757366667762147355602446006474261151597409525795681824661504000000/(3 - 2*x)^{(33/2)} + (6167726649054233403507372547934021941920835094010816556282758758400000/(31*(3 - 2*x)^{(31/2)}) + (98044550412701599247213819645778610361943940861637274650890661068800000/(29*(3 - 2*x)^{(29/2)}) + (4496423323436580179825935667807239175646629240803415910250222313472000000/(3 - 2*x)^{(27/2)} + (487904184130260773926886832047572655461484781443782543411352841560457216000/(3 - 2*x)^{(25/2)} + (42926886721523802306414887155091882259902542088067698170622802545418240000000/(3 - 2*x)^{(23/2)} + (2893692593980364723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^{(21/2)} + (118767476492930264374166633243140666046068763101817907661320807641190359040000000/(3 - 2*x)^{(19/2)} + (-2313064137166228597053737241416368284722516912423159767489332810437803253760000000/(3 - 2*x)^{(17/2)} + (-992239519653790860422623948957964852355985846800936213338418761762097950023680000000/(3 - 2*x)^{(15/2)} + (-109415183151546322431572415879018096250836012099731766901467841654602614755123200000000/(3 - 2*x)^{(13/2)} + (-8073268485314233063840337934095431560069216535225849300748018943930634745621913600000000/(3 - 2*x)^{(11/2)} + (-44337987226211231305207361494572283981715203938096393248399$

```

666651183999754721382400000000/(3 - 2*x)^(9/2) + (-18330190892216697744173
706790143700087358561576136178754174544727578117325359791923200000000/(3 -
2*x)^(7/2) + (-553541210002735957048844214716028245499086746401723523324780
660557661668413725058949120000000/(3 - 2*x)^(5/2) + (-113323856633918397403
43974428370683887566771471384841151672642393999283182139266339840000000000/
(3 - 2*x)^(3/2) + (-1327220262908131487403839635355234271426655189754352930
64356777236410088640362467513344000000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-1858108368071384082365375489497327979997317265656094102900994881
30974124096507454518681600000000 - (38534140062781031467679876224014966993
363355592186583754201688526589748283311569009254400000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-185810836807138408236537548949732797999731726565
609410290099488130974124096507454518681600000000 + (3853414006278103146767
9876224014966993363355592186583754201688526589748283311569009254400000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724

```

**Maple [A]** time = 0.08, size = 989, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x)
```

```

[Out] 7192279694031133468210490184035/3248261265098830736532127368829731369648128
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*7+2*14^(1/2))^(1/2)
*14^(1/2)+13457531633280790190212932747565/81206531627470768413303184220743
2842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-3484168674905226483378299702015/8
12065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(
3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)-71922796
94031133468210490184035/3248261265098830736532127368829731369648128*ln(3-2*
x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2
)+13457531633280790190212932747565/8120653162747076841330318422074328424120
32/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+
2*14^(1/2))^(1/2))*(7+2*14^(1/2))-3484168674905226483378299702015/812065316
274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(
1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)-1345753163328079
0190212932747565/1624130632549415368266063684414865684824064*ln(3-2*x+14^(1
/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)+13457531633280
790190212932747565/1624130632549415368266063684414865684824064*ln(3-2*x+14^(
1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)+683151246370
725/30145677658996078082575630336/(3-2*x)^(1/2)+10/2952313853011644037/(3-2
*x)^(37/2)+143/7819642097165976098/(3-2*x)^(35/2)+355/5266289575642392066/(
3-2*x)^(33/2)+52865/277038748585308867472/(3-2*x)^(31/2)+14333/323956601168
30472406/(3-2*x)^(29/2)+1478345/1689042692987850837168/(3-2*x)^(27/2)+47538
7/312785683886639043920/(3-2*x)^(25/2)+16575515/7006399319060714583808/(3-2
*x)^(23/2)+246866015/73567192850137503129984/(3-2*x)^(21/2)+1/3111898385606
868039/(3-2*x)^(39/2)+8972680075/1667523037936450070946304/(3-2*x)^(17/2)+1
02495360575/16479051198430800701116416/(3-2*x)^(15/2)+122484655975/17852305
464966700759542784/(3-2*x)^(13/2)+10815878546425/14803680993257002629836247
04/(3-2*x)^(11/2)+320421783064625/30145677658996078082575630336/(3-2*x)^(3/
2)+8192823353/1863702218870150079292928/(3-2*x)^(19/2)+769045155125/1009341
88590388654294338048/(3-2*x)^(9/2)+838467657280275/105509871806486273289014

```



706176/(3-2\*x)^(7/2)+9270470094105/1076631344964145645806272512/(3-2\*x)^(5/2)-7192279694031133468210490184035/1624130632549415368266063684414865684824064/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)+(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))\*14^(1/2)-7192279694031133468210490184035/1624130632549415368266063684414865684824064/(-7+2\*14^(1/2))^(1/2)\*arctan((2\*(3-2\*x)^(1/2)-(7+2\*14^(1/2))^(1/2))/(-7+2\*14^(1/2))^(1/2))\*(7+2\*14^(1/2))\*14^(1/2)+1/30145677658996078082575630336\*(807597736492641378942268937217995835353849465/1048576\*(3-2\*x)^(1/2)+490738543064879423955077165987434152441563270473/1002342287671296\*(3-2\*x)^(53/2)-55011835288361289002011693179378316699033102675/1002342287671296\*(3-2\*x)^(55/2)+1808668971148992206490172102870787954874541181/334114095890432\*(3-2\*x)^(57/2)-11968977253082880651292892111395530933265219/25701084299264\*(3-2\*x)^(59/2)+339556544641293541759958988614814460549873/9826885173248\*(3-2\*x)^(61/2)-64243396719140374998473027009027485263697/29480655519744\*(3-2\*x)^(63/2)+129886852748727110357425618672922324659/1133871366144\*(3-2\*x)^(65/2)-503502693505289734438057515605193725/103079215104\*(3-2\*x)^(67/2)+133883313322119397348791732981953297/824633720832\*(3-2\*x)^(69/2)-3254850748003483429666738850178379/824633720832\*(3-2\*x)^(71/2)+360433340020130123942335063779145/5772436045824\*(3-2\*x)^(73/2)-928342237074576734557978321305/1924145348608\*(3-2\*x)^(75/2)-44796329357069082297154473725670903546220392558695/9070970929152\*(3-2\*x)^(43/2)+28607223317693223698395672584150593863016075796143/29480655519744\*(3-2\*x)^(45/2)-5059022664167725408892162874688680417923742003781/29480655519744\*(3-2\*x)^(47/2)+73012476452577571533836489036461787385135079265/2680059592704\*(3-2\*x)^(49/2)-1939242920901534821454026903132433081580221023737/501171143835648\*(3-2\*x)^(51/2)-1006304725834560333245233940167063186576585913370455/10720238370816\*(3-2\*x)^(39/2)+13805722741822612586258592099428566280191230197271405/39307540692992\*(3-2\*x)^(37/2)-22397546321209486953062074374795737299957063565/3145728\*(3-2\*x)^(3/2)+404531566689883337048499233527781983599187634017/12582912\*(3-2\*x)^(5/2)-1188598027552254830082683218064697188605612952419/12582912\*(3-2\*x)^(7/2)+3831583379166294091823572953989993625772471445345/18874368\*(3-2\*x)^(9/2)+9977850126168010187169130424774568330973123412551261/21592276992\*(3-2\*x)^(13/2)-1255696718499588580979726331572072320357969297077745/2399141888\*(3-2\*x)^(15/2)+2672239984790337844292019294315182385216573077301785/117922622078976\*(3-2\*x)^(41/2)+1186323846453826237212517196312193819452761764018822545/3915399561216\*(3-2\*x)^(21/2)-17650942358963262675871173166229809316744939271143/51904512\*(3-2\*x)^(11/2)-6886173809894005543994516442461871486007042005189775/125627793408\*(3-2\*x)^(27/2)+136329987967245395141848253765147208279814148352958009/5527622909952\*(3-2\*x)^(29/2)-55066091420817590167865401986871791412011888132876913/5527622909952\*(3-2\*x)^(31/2)+2737487528928439357869138774910126923363791747141675/755914244096\*(3-2\*x)^(33/2)-11664572170215876884203668230743495214488310113371105/9826885173248\*(3-2\*x)^(35/2)+12646629333382722716904430763732665179119615389552413/25098715136\*(3-2\*x)^(17/2)-2593673203685044441695042001860835122939346700333136537/6199382638592\*(3-2\*x)^(19/2)-755930116404682856570195190192032441294632160945523631/3915399561216\*(3-2\*x)^(23/2)+853508502207214511987093866021124087908041634697244059/7830799122432\*(3-2\*x)^(25/2))/((3-2\*x)^2-7+14\*x)^19

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{20} (-2x + 3)^{\frac{41}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(41/2)/(2\*x^2+x+1)^20,x, algorithm="maxima")

[Out] integrate(1/((2\*x^2 + x + 1)^20\*(-2\*x + 3)^(41/2)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(41/2)/(2\*x^2+x+1)^20,x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)\*\*(41/2)/(2\*x\*\*2+x+1)\*\*20,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{20} (-2x + 3)^{\frac{41}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2\*x)^(41/2)/(2\*x^2+x+1)^20,x, algorithm="giac")

[Out] integrate(1/((2\*x^2 + x + 1)^20\*(-2\*x + 3)^(41/2)), x)

$$3.50 \quad \int \frac{1}{(3-2x+x^2)^{11/2} (1+x+2x^2)^5} dx$$

**Optimal.** Leaf size=378

$$\frac{63043297 - 29625922x}{4116000000(x^2 - 2x + 3)^{3/2}} - \frac{31(7434109 - 3088870x)}{41160000000\sqrt{x^2 - 2x + 3}} + \frac{3(8233x + 8822)}{343000(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)} + \frac{1}{117600(x^2 - 2x + 3)}$$

[Out]  $-(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^{(9/2)}) - (4878869 - 2578034*x)/(411600000*(3 - 2*x + x^2)^{(7/2)}) - (30316369 - 15043110*x)/(68600000*(3 - 2*x + x^2)^{(5/2)}) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x + x^2)^{(3/2)}) - (31*(7434109 - 3088870*x))/(41160000000*\text{Sqrt}[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)) + (\text{Sqrt}[(151363871237318045 + 110320475741093888*\text{Sqrt}[2])/70]*\text{ArcTan}[(\text{Sqrt}[5/(7*(151363871237318045 + 110320475741093888*\text{Sqrt}[2]))])*(308108167 + 312239803*\text{Sqrt}[2] + (932587773 + 620347970*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - 2*x + x^2])]/13720000000 - (\text{Sqrt}[(-151363871237318045 + 110320475741093888*\text{Sqrt}[2])/70]*\text{ArcTanh}[(\text{Sqrt}[5/(7*(-151363871237318045 + 110320475741093888*\text{Sqrt}[2]))])*(308108167 - 312239803*\text{Sqrt}[2] + (932587773 - 620347970*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - 2*x + x^2])]/13720000000$

**Rubi [A]** time = 0.773819, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{63043297 - 29625922x}{4116000000(x^2 - 2x + 3)^{3/2}} - \frac{31(7434109 - 3088870x)}{41160000000\sqrt{x^2 - 2x + 3}} + \frac{3(8233x + 8822)}{343000(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)} + \frac{1}{117600(x^2 - 2x + 3)}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2\*x + x^2)^(11/2)\*(1 + x + 2\*x^2)^5), x]

[Out]  $-(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^{(9/2)}) - (4878869 - 2578034*x)/(411600000*(3 - 2*x + x^2)^{(7/2)}) - (30316369 - 15043110*x)/(68600000*(3 - 2*x + x^2)^{(5/2)}) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x + x^2)^{(3/2)}) - (31*(7434109 - 3088870*x))/(41160000000*\text{Sqrt}[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2*x + x^2)^{(9/2)}*(1 + x + 2*x^2)) + (\text{Sqrt}[(151363871237318045 + 110320475741093888*\text{Sqrt}[2])/70]*\text{ArcTan}[(\text{Sqrt}[5/(7*(151363871237318045 + 110320475741093888*\text{Sqrt}[2]))])*(308108167 + 312239803*\text{Sqrt}[2] + (932587773 + 620347970*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - 2*x + x^2])]/13720000000 - (\text{Sqrt}[(-151363871237318045 + 110320475741093888*\text{Sqrt}[2])/70]*\text{ArcTanh}[(\text{Sqrt}[5/(7*(-151363871237318045 + 110320475741093888*\text{Sqrt}[2]))])*(308108167 - 312239803*\text{Sqrt}[2] + (932587773 - 620347970*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - 2*x + x^2])]/13720000000$

Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

### Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

### Rule 1035

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

```

### Rule 1029

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ

```

$[b*d - a*e, 0] \ \&\& \ \text{EqQ}[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]$

#### Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x\_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 204

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x\_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5} dx &= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} - \frac{\int \frac{-1235+1335x-800x^2}{(3-2x+x^2)^{11/2}(1+x+2x^2)^4} dx}{1400} \\
&= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} - \frac{1}{123480000(3-2x+x^2)^{9/2}} \\
&= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} + \frac{1}{123480000(3-2x+x^2)^{9/2}} \\
&= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} + \frac{1}{123480000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{1}{1050(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{1}{280(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}} \\
&= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-151515151x}{6860000000(3-2x+x^2)^{9/2}}
\end{aligned}$$

**Mathematica [C]** time = 6.02736, size = 342, normalized size = 0.9

$$560(4596238560x^{17} - 38639385552x^{16} + 188603773872x^{15} - 606785954952x^{14} + 1459208021718x^{13} - 267914387048x^{12} + 381111111111x^{11} - 381111111111x^{10} + 381111111111x^9 - 381111111111x^8 + 381111111111x^7 - 381111111111x^6 + 381111111111x^5 - 381111111111x^4 + 381111111111x^3 - 381111111111x^2 + 381111111111x - 381111111111)$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2\*x + x^2)^(11/2)\*(1 + x + 2\*x^2)^5), x]

```
[Out] (560*(-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3
- 1409335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952
*x^7 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 3999656
132532*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 +
188603773872*x^15 - 38639385552*x^16 + 4596238560*x^17) - (9*I)*Sqrt[50 +
(10*I)*Sqrt[7]]*(-299844895*I + 932587773*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 +
x + 5*x^2 - 3*x^3 + 2*x^4)^4*ArcTanh[(13 + I*Sqrt[7] + (-5 - I*Sqrt[7])*x)
/(Sqrt[50 + (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])] + 9*Sqrt[50 - (10*I)*Sqrt
[7]]*(299844895 - (932587773*I)*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 + x + 5*x^2
- 3*x^3 + 2*x^4)^4*ArcTanh[(-13 + I*Sqrt[7] + (5 - I*Sqrt[7])*x)/(Sqrt[50
- (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])]/(69148800000000*(3 - 2*x + x^2)^(
9/2)*(1 + x + 2*x^2)^4)
```

**Maple [B]** time = 0.533, size = 21028, normalized size = 55.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5 (x^2 - 2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

**Fricas [B]** time = 4.00831, size = 11537, normalized size = 30.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/710865244472321675802807529502400000000*(2646020608687651230198155914607
4412800*x^18 - 211681648695012098415852473168595302400*x^17 + 1018717934344
745723626290027123864892800*x^16 - 321491503955496244690759436248041155200
*x^15 + 7688343631118056605744516779381246569200*x^14 - 1398091139115337718
7559506313807067863200*x^13 + 20977982138251784909414754860497120398000*x^1
2 - 25712705264922250829450580100197810638400*x^11 + 2875728272779347952619
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 180458605512497813899514233
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
```

$4096608462190931685680490685615200*x^5 - 3602042876982878244*33780221308347$   
 $3608^{(1/4)}*\sqrt{205487899}*\sqrt{35}*\sqrt{2}*(16*x^{18} - 128*x^{17} + 616*x^{16}$   
 $- 1944*x^{15} + 4649*x^{14} - 8454*x^{13} + 12685*x^{12} - 15548*x^{11} + 17389*x^{10}$   
 $- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*$   
 $x^3 + 1647*x^2 + 162*x + 243)*\sqrt{151363871237318045*\sqrt{2} + 22064095148$   
 $2187776)*\arctan(1/964393622349963919677467835514205441102895152270484353118$   
 $304*\sqrt{205487899}*(12071210867722009415131100925112940*\sqrt{4167294734812$   
 $9)*\sqrt{7}*\sqrt{2}*(10*\sqrt{2} + 9) + \sqrt{205487899}*(5*337802213083473608$   
 $^{(3/4)}*\sqrt{41672947348129}*\sqrt{35}*(534678000*\sqrt{2} - 573381349) + 2876$   
 $830586*337802213083473608^{(1/4)}*\sqrt{41672947348129}*\sqrt{35}*(201502465*sq$   
 $rt(2) + 108532744))*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776) +$   
 $2414242173544401883026220185022588*\sqrt{41672947348129}*\sqrt{7}*(125*\sqrt{2}$   
 $+ 172))*\sqrt{164483605088694913184970968*x^2 + \sqrt{205487899}*(33780221$   
 $3083473608^{(1/4)}*\sqrt{35}*\sqrt{7}*\sqrt{x^2 - 2*x + 3}*(89801606*\sqrt{2} - 4$   
 $2834985) - 337802213083473608^{(1/4)}*\sqrt{35}*\sqrt{7}*(\sqrt{2}*(89801606*x -$   
 $132636591) - 42834985*x + 222438197))*\sqrt{151363871237318045*\sqrt{2} + 22$   
 $0640951482187776) - 41120901272173728296242742*\sqrt{x^2 - 2*x + 3}*(4*x + 1$   
 $) - 123362703816521184888728226*x + 205604506360868641481213710*\sqrt{2} + 2$   
 $87846308905216098073699194) + 5/476*\sqrt{7}*\sqrt{2}*(\sqrt{2}*(10*x - 19) +$   
 $9*x - 29) + 1/1149179274607135296320480808070751888*\sqrt{205487899}*(5*3378$   
 $02213083473608^{(3/4)}*\sqrt{35}*(\sqrt{2}*(534678000*x + 38703349) - 573381349$   
 $*x - 495974651) + 2876830586*337802213083473608^{(1/4)}*\sqrt{35}*(\sqrt{2}*(20$   
 $1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^{($   
 $3/4)}*\sqrt{35}*(534678000*\sqrt{2} - 573381349) + 2876830586*3378022130834736$   
 $08^{(1/4)}*\sqrt{35}*(201502465*\sqrt{2} + 108532744))*\sqrt{x^2 - 2*x + 3}*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776) - 1/476*\sqrt{x^2 - 2*x + 3}*(5*\sqrt{7}*\sqrt{2}*(10*\sqrt{2} + 9) + \sqrt{7}*(125*\sqrt{2} + 172)) + 1/476*\sqrt{7}*(25*\sqrt{2}*(5*x - 1) + 172*x - 82)) - 3602042876982878244*337802213083473608^{(1/4)}*\sqrt{205487899}*\sqrt{35}*\sqrt{2}*(16*x^{18} - 128*x^{17} + 616*x^{16} - 1944*x^{15} + 4649*x^{14} - 8454*x^{13} + 12685*x^{12} - 15548*x^{11} + 17389*x^{10} - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776)*\arctan(-1/964393622349963919677467835514205441102895152270484353118304*\sqrt{205487899}*(12071210867722009415131100925112940*\sqrt{41672947348129}*\sqrt{7}*\sqrt{2}*(10*\sqrt{2} + 9) - \sqrt{205487899}*(5*337802213083473608^{(3/4)}*\sqrt{41672947348129}*\sqrt{35}*(534678000*\sqrt{2} - 573381349) + 2876830586*337802213083473608^{(1/4)}*\sqrt{41672947348129}*\sqrt{35}*(201502465*\sqrt{2} + 108532744))*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776) + 2414242173544401883026220185022588*\sqrt{41672947348129}*\sqrt{7}*(125*\sqrt{2} + 172))*\sqrt{164483605088694913184970968*x^2 - \sqrt{205487899}*(337802213083473608^{(1/4)}*\sqrt{35}*\sqrt{7}*\sqrt{x^2 - 2*x + 3}*(89801606*\sqrt{2} - 42834985) - 337802213083473608^{(1/4)}*\sqrt{35}*\sqrt{7}*(\sqrt{2}*(89801606*x - 132636591) - 42834985*x + 222438197))*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776) - 41120901272173728296242742*\sqrt{x^2 - 2*x + 3}*(4*x + 1) - 123362703816521184888728226*x + 205604506360868641481213710*\sqrt{2} + 287846308905216098073699194) - 5/476*\sqrt{7}*\sqrt{2}*(\sqrt{2}*(10*x - 19) + 9*x - 29) + 1/1149179274607135296320480808070751888*\sqrt{205487899}*(5*337802213083473608^{(3/4)}*\sqrt{35}*(\sqrt{2}*(534678000*x + 38703349) - 573381349*x - 495974651) + 2876830586*337802213083473608^{(1/4)}*\sqrt{35}*(\sqrt{2}*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^{(3/4)}*\sqrt{35}*(534678000*\sqrt{2} - 573381349) + 2876830586*337802213083473608^{(1/4)}*\sqrt{35}*(201502465*\sqrt{2} + 108532744))*\sqrt{x^2 - 2*x + 3})*\sqrt{151363871237318045*\sqrt{2} + 220640951482187776) + 1/476*\sqrt{x^2 - 2*x + 3}*(5*\sqrt{7}*\sqrt{2}*(10*\sqrt{2} + 9) + \sqrt{7}*(125*\sqrt{2} + 172)) - 1/476*\sqrt{7}*(25*\sqrt{2}*(5*x - 1) + 172*x - 82)) + 9*337802213083473608^{(1/4)}*\sqrt{205487899}*\sqrt{35}*\sqrt{7}*(3530255223715004416*x^{18} - 28242041789720035328*x^{17} + 135914826113027670016*x^{16} - 428926009681373036544*x^{15} + 1025759783440690970624*x^{14} - 1865298603830415458304*x^{13} + 2798830469551551938560*x^{12} - 3430525513645055541248*x^{11} + 3836725505323763236$



$$\begin{aligned}
& 864x^{10} - 3640134417553133928448x^9 + 3410447187060176453632x^8 - 2407634062573633011712x^7 + 2028793548878716600320x^6 - 969496340812733087744x^5 \\
& + 972364673182001528832x^4 - 87373816786946359296x^3 + 363395647091163267072x^2 - 151363871237318045\sqrt{2}(16x^{18} - 128x^{17} + 616x^{16} - 1944x^{15} \\
& + 4649x^{14} - 8454x^{13} + 12685x^{12} - 15548x^{11} + 17389x^{10} - 16498x^9 + 15457x^8 - 10912x^7 + 9195x^6 - 4394x^5 + 4407x^4 - 396x^3 \\
& + 1647x^2 + 162x + 243) + 35743834140114419712x + 53615751210171629568)\sqrt{151363871237318045\sqrt{2} + 220640951482187776} \cdot \log(19083512352618334937598521302939860992x^2 + 236911417693579806112743424/2041974420058321\sqrt{205487899}) \\
& \cdot (337802213083473608^{1/4})\sqrt{35}\sqrt{7}\sqrt{x^2 - 2x + 3} \cdot (89801606\sqrt{2} - 42834985) - 337802213083473608^{1/4})\sqrt{35}\sqrt{7} \\
& \cdot (\sqrt{2}(89801606x - 132636591) - 42834985x + 222438197))\sqrt{151363871237318045\sqrt{2} + 220640951482187776} - 4770878088154583734399630325734965248\sqrt{x^2 - 2x + 3} \\
& \cdot (4x + 1) - 14312634264463751203198890977204895744x + 23854390440772918671998151628674826240\sqrt{2} + 33396146617082086140797412280144756736) \\
& - 9 \cdot 337802213083473608^{1/4})\sqrt{205487899})\sqrt{35}\sqrt{7} \cdot (3530255223715004416x^{18} - 28242041789720035328x^{17} + 135914826113027670016x^{16} \\
& - 428926009681373036544x^{15} + 1025759783440690970624x^{14} - 1865298603830415458304x^{13} + 2798830469551551938560x^{12} - 3430525513645055541248x^{11} \\
& + 3836725505323763236864x^{10} - 3640134417553133928448x^9 + 3410447187060176453632x^8 - 2407634062573633011712x^7 + 2028793548878716600320x^6 \\
& - 969496340812733087744x^5 + 972364673182001528832x^4 - 87373816786946359296x^3 + 363395647091163267072x^2 - 151363871237318045\sqrt{2} \\
& \cdot (16x^{18} - 128x^{17} + 616x^{16} - 1944x^{15} + 4649x^{14} - 8454x^{13} + 12685x^{12} - 15548x^{11} + 17389x^{10} - 16498x^9 + 15457x^8 - 10912x^7 + 9195x^6 \\
& - 4394x^5 + 4407x^4 - 396x^3 + 1647x^2 + 162x + 243) + 35743834140114419712x + 53615751210171629568)\sqrt{151363871237318045\sqrt{2} + 220640951482187776} \\
& \cdot \log(19083512352618334937598521302939860992x^2 - 236911417693579806112743424/2041974420058321\sqrt{205487899}) \cdot (337802213083473608^{1/4})\sqrt{35}\sqrt{7}\sqrt{x^2 - 2x + 3} \\
& \cdot (89801606\sqrt{2} - 42834985) - 337802213083473608^{1/4})\sqrt{35}\sqrt{7} \cdot (\sqrt{2}(89801606x - 132636591) - 42834985x + 222438197))\sqrt{151363871237318045\sqrt{2} + 220640951482187776} \\
& - 4770878088154583734399630325734965248\sqrt{x^2 - 2x + 3} \cdot (4x + 1) - 14312634264463751203198890977204895744x + 23854390440772918671998151628674826240\sqrt{2} \\
& + 33396146617082086140797412280144756736) + 7288133014054049357177045697296871075600x^4 - 654890100650193679474043588865341716800x^3 + 2723747464067850985085226744599034867600x^2 \\
& + 5756926178104321961473983880 \cdot (4596238560x^{17} - 38639385552x^{16} + 188603773872x^{15} - 606785954952x^{14} + 1459208021718x^{13} - 2679143870481x^{12} + 3999656132532x^{11} - 4915797913008x^{10} \\
& + 5380603084494x^9 - 5134334619701x^8 + 4591320676952x^7 - 3359813871472x^6 + 2503427226914x^5 - 1409335257371x^4 + 1002897791524x^3 - 266966654968x^2 + 261702502714x - 53205422447)\sqrt{x^2 - 2x + 3} + 267909586629624687057563286354003429600x \\
& + 401864379944437030586344929531005144400)/(16x^{18} - 128x^{17} + 616x^{16} - 1944x^{15} + 4649x^{14} - 8454x^{13} + 12685x^{12} - 15548x^{11} + 17389x^{10} - 16498x^9 + 15457x^8 - 10912x^7 + 9195x^6 - 4394x^5 + 4407x^4 - 396x^3 + 1647x^2 + 162x + 243)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2-2\*x+3)\*\*(11/2)/(2\*x\*\*2+x+1)\*\*5,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")`

[Out] Timed out

$$3.51 \quad \int \frac{1}{(3-2x+x^2)^{21/2} (1+x+2x^2)^{10}} dx$$

**Optimal.** Leaf size=638

result too large to display

```
[Out] (37358055634422583 - 14024622879097678*x)/(184012447920000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*
x)/(1564105807320000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 7813986379726516886*x))/(406667509903200000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(9384634843920000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(31282116146400000000000000*(3 - 2*
x + x^2)^(7/2)) - (6551405511565449301689 - 3127298559983309301910*x)/(5213
686024400000000000000000*(3 - 2*x + x^2)^(5/2)) - (4179039782398459850819 -
1886993445589652402694*x)/(104273720488000000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
000000000000000000000*sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
))*(1 + x + 2*x^2)^9 + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2))*(1 + x
+ 2*x^2)^8 + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2))*(1 + x + 2*
x^2)^7 + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2))*(1 + x +
2*x^2)^6 + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2))*(1 + x + 2*x^2)^4 + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2))*(1 + x + 2*x^2)^3 + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2))*(1 + x + 2*x^2)^2 -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2))*(1 + x + 2*x^2) + (sqrt[(81042225921274689605478944797800854846405 + 57
305922523001707126026363878666500308992*sqrt[2])/70]*ArcTan[(sqrt[5/(7*(810
42225921274689605478944797800854846405 + 5730592252300170712602636387866650
0308992*sqrt[2]))]*(272944589523248381749 + 191941026386645109841*sqrt[2] +
(656826642296538601431 + 464885615909893491590*sqrt[2])*x))/sqrt[3 - 2*x +
x^2]])/32282885600000000000000000 - (sqrt[(-810422259212746896054789447978
00854846405 + 57305922523001707126026363878666500308992*sqrt[2])/70]*ArcTan
h[(sqrt[5/(7*(-81042225921274689605478944797800854846405 + 5730592252300170
7126026363878666500308992*sqrt[2]))]*(272944589523248381749 - 1919410263866
45109841*sqrt[2] + (656826642296538601431 - 464885615909893491590*sqrt[2])*
x))/sqrt[3 - 2*x + x^2]])/32282885600000000000000000000
```

**Rubi [A]** time = 1.39713, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{12105495874518671061833 - 5117656435043679338190x}{10427372048800000000000000000000\sqrt{x^2 - 2x + 3}} - \frac{146548895467025x + 37857197792117}{2421216420000000(x^2 - 2x + 3)^{19/2}(2x^2 + x + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10), x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
```

```

00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*x)
/(15641058073200000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 7813986379726516886*x))/(40666750990320000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(93846348439200000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(312821161464000000000000*(3 - 2*
x + x^2)^(7/2)) - (6551405511565449301689 - 3127298559983309301910*x)/(5213
6860244000000000000000*(3 - 2*x + x^2)^(5/2)) - (4179039782398459850819 -
1886993445589652402694*x)/(10427372048800000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
00000000000000*sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2)
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (sqrt[(81042225921274689605478944797800854846405 + 57
305922523001707126026363878666500308992*sqrt[2])/70]*ArcTan[(sqrt[5/(7*(810
42225921274689605478944797800854846405 + 5730592252300170712602636387866650
0308992*sqrt[2]))])*(272944589523248381749 + 191941026386645109841*sqrt[2] +
(656826642296538601431 + 464885615909893491590*sqrt[2])*x)/sqrt[3 - 2*x +
x^2]])/3228288560000000000000000000000000 - (sqrt[(-810422259212746896054789447978
00854846405 + 57305922523001707126026363878666500308992*sqrt[2])/70]*ArcTan
h[(sqrt[5/(7*(-81042225921274689605478944797800854846405 + 5730592252300170
7126026363878666500308992*sqrt[2]))])*(272944589523248381749 - 1919410263866
45109841*sqrt[2] + (656826642296538601431 - 464885615909893491590*sqrt[2])*
x)/sqrt[3 - 2*x + x^2]])/3228288560000000000000000000000000

```

#### Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp
[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

#### Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x
_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*

```

```
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

### Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps









[Out] 1/1434466423992337676564216536036272863645522031860857110126813332618112898  
720320000000000000000000\*(36045960776272236628083717974972055111190660172853  
358396135728761934386631817942748278579200\*x<sup>38</sup> - 5587123920322196677352976  
28612066854223455232679227055140103795809982992793178112598317977600\*x<sup>37</sup> +  
48121357636323435898491763496587693573439531330759233458841197897182406153  
47695356895190323200\*x<sup>36</sup> - 28710607758300836474268681367065241896063360827  
677699962522107958880738952242991399003888332800\*x<sup>35</sup> + 1315091821471322213  
079849345669386715662902248081338714385016894218223678278587868742508613888  
00\*x<sup>34</sup> - 48615412586213217241775435956082154303807288316753404802358233229  
5256693402493082467454965081600\*x<sup>33</sup> + 150125118590038017914558770715112989  
4284646412544039883821859865409233818038611634065993336166400\*x<sup>32</sup> - 396012  
076807241950819334539073291504431090617269457971847700920669696894988016137  
7086262197766400\*x<sup>31</sup> + 909142000002142860704234021157216421398787616081889  
7418150894645435329015717575267031369642428000\*x<sup>30</sup> - 184247649298721582706  
989900442437618212098383039369354048250008452972140026730578162474910272628  
00\*x<sup>29</sup> + 33413073756673638925333625169011170445598811516221975115590200411  
293389434416479555356860509015600\*x<sup>28</sup> - 5481653256044929545971751700338269  
9673242410936114304344629842103656622934490247108012261346586400\*x<sup>27</sup> + 822  
459830940635186677366276046635475885728402385815973257367014937498803836507  
49401133206999014400\*x<sup>26</sup> - 11372284806763969440259273586264909409387404544  
3618754295078471234595964240139128161766283626302000\*x<sup>25</sup> + 146086574413322  
248286514192550522624098477614094095488624493581512454991258074867544318895  
241990800\*x<sup>24</sup> - 1750270940810010216829737529974120232517363052261271442728  
11232619626419165679723993392477178363200\*x<sup>23</sup> + 19688729160578415943345565  
4443374481739030277196290989156609388218395099469530751149958413044135200\*x<sup>22</sup>  
- 208068683375682167383215047521697995267539026087882795784482813901791  
360434798005710722616487282000\*x<sup>21</sup> + 2081714449184784825196181653920157303  
47012009814583465001141378703189206795143605224483243158516400\*x<sup>20</sup> - 19622  
755618454040835316742234157685550832000179582185155831117699557408106901596  
9836642878534431200\*x<sup>19</sup> + 176534941677723459681422280024952573032106299529  
482816321219585323399086976471958310981405494523200\*x<sup>18</sup> - 1491362557380113  
805569548293989292587370076152040747303305658872207307833829238226195713407  
37358000\*x<sup>17</sup> + 12189081448358772438901196169673375625310538365442623433615  
0913799569962877883235263704480534144400\*x<sup>16</sup> - 919831860532221296355370692  
78588580392985745730700928388526309371776740142438834607398588992195200\*x<sup>15</sup>  
+ 69317814132471559316390137037592557060398996838342232414889371690271398  
098098738643314402130954400\*x<sup>14</sup> - 4574307084113250024797073972709329687876  
5897323708593659902862883667237249390700654758574610918000\*x<sup>13</sup> + 329969655  
216763949298031215090491433294517890491697894556446151291991903089176733485  
18481311574800\*x<sup>12</sup> - 17770083757788737971933739892049927033484890029804651  
938270182161740937851280707834822272274354400\*x<sup>11</sup> + 1354422526745145970196  
036923825637435189936268397849855148372985225665526414709333739259622802880  
0\*x<sup>10</sup> - 481375973272848865172866855106995818624092546697867179982576756873  
2599092879797201593187475517200\*x<sup>9</sup> + 5091181133639025216832620106123280320  
347641869015804163342220634415255665812683873707564839486000\*x<sup>8</sup> - 46421311  
850305640075834899457188406077339946202653776901797199608409580382714283736  
3184426478400\*x<sup>7</sup> + 1771233883264782126042267141811413849986971398265032235  
916172889879027134542752439323372429279200\*x<sup>6</sup> + 23911503454316320991841103  
2521665649750496447867853609069487786445410804754998849116452338787600\*x<sup>5</sup>  
+ 79817891129994413353362937273464455099835468\*1264938752804265123815574105  
117799608149057272418<sup>(1/4)</sup>\*sqrt(1590558865810545927822094)\*sqrt(35)\*sqrt(2  
)\* (512\*x<sup>38</sup> - 7936\*x<sup>37</sup> + 68352\*x<sup>36</sup> - 407808\*x<sup>35</sup> + 1867968\*x<sup>34</sup> - 6905376  
\*x<sup>33</sup> + 21323904\*x<sup>32</sup> - 56249904\*x<sup>31</sup> + 129135330\*x<sup>30</sup> - 261706983\*x<sup>29</sup> + 4  
74602241\*x<sup>28</sup> - 778618854\*x<sup>27</sup> + 1168229184\*x<sup>26</sup> - 1615329345\*x<sup>25</sup> + 207502  
6563\*x<sup>24</sup> - 2486100252\*x<sup>23</sup> + 2796604422\*x<sup>22</sup> - 2955425895\*x<sup>21</sup> + 295688552  
9\*x<sup>20</sup> - 2787233482\*x<sup>19</sup> + 2507517852\*x<sup>18</sup> - 2118344505\*x<sup>17</sup> + 1731347859\*x<sup>16</sup>  
- 1306537272\*x<sup>15</sup> + 984596334\*x<sup>14</sup> - 649738605\*x<sup>13</sup> + 468691803\*x<sup>12</sup> -  
252407834\*x<sup>11</sup> + 192383368\*x<sup>10</sup> - 68375067\*x<sup>9</sup> + 72315585\*x<sup>8</sup> - 6593724\*x<sup>7</sup>  
+ 25158762\*x<sup>6</sup> + 3396411\*x<sup>5</sup> + 6720651\*x<sup>4</sup> + 1325322\*x<sup>3</sup> + 1023516\*x<sup>2</sup> + 1

$$\begin{aligned}
& 37781x + 59049) \sqrt{81042225921274689605478944797800854846405} \sqrt{2} + 1 \\
& 14611845046003414252052727757333000617984) \arctan(1/54206850781156887023310 \\
& 518673090274966005685838243268724684064391985051350175945649154733957770247 \\
& 43167351056637371274953501437271981836435236061968 \sqrt{7952794329052729639} \\
& 11047) \cdot (9939513250523192816422116593216797292815016511001378462170679301990 \\
& \sqrt{11005224487862873621128239642490888848098}) \sqrt{288886807671054271567} \\
& 2947094311) \sqrt{7} \cdot (10 \sqrt{2} + 9) + \sqrt{1590558865810545927822094} \cdot (5 \cdot 1 \\
& 264938752804265123815574105117799608149057272418^{(3/4)} \sqrt{288886807671054} \\
& 2715672947094311) \sqrt{35} \cdot (340613697110906370000 \sqrt{2} - 483753219647003 \\
& 202703) + 5566956030336910747377329 \cdot 126493875280426512381557410511779960814 \\
& 9057272418^{(1/4)} \sqrt{2888868076710542715672947094311} \sqrt{35} \cdot (4373478266 \\
& 4604992355 \sqrt{2} - 66269826580994560232) \sqrt{81042225921274689605478944} \\
& 797800854846405 \sqrt{2} + 114611845046003414252052727757333000617984) + 147 \\
& 46181254044456871569661311413855791035947867693704217232597372869522935182 \\
& 724790786 \sqrt{2888868076710542715672947094311} \sqrt{7} \cdot (125 \sqrt{2} + 172) \\
& ) \sqrt{5191798731734901573730421875012971256390643826285581511813805064} x^2 \\
& + \sqrt{1590558865810545927822094} \cdot (126493875280426512381557410511779960814 \\
& 9057272418^{(1/4)} \sqrt{35} \sqrt{7} \sqrt{x^2 - 2x + 3} \cdot (43268355662383849682 \\
& \sqrt{2} - 62135959399493560795) - 1264938752804265123815574105117799608149 \\
& 057272418^{(1/4)} \sqrt{35} \sqrt{7} \cdot (\sqrt{2} \cdot (43268355662383849682 \cdot x - 1054043 \\
& 15061877410477) - 62135959399493560795 \cdot x + 148672670724261260159) \sqrt{810} \\
& 42225921274689605478944797800854846405 \sqrt{2} + 11461184504600341425205272 \\
& 7757333000617984) - 1297949682933725393432605468753242814097660956571395377 \\
& 953451266 \sqrt{x^2 - 2x + 3} \cdot (4x + 1) - 389384904880117618029781640625972 \\
& 8442292982869714186133860353798 \cdot x + 874869761179272589826814757400740628067 \\
& 45190 \sqrt{11005224487862873621128239642490888848098} + 9085647780536077754 \\
& 028238281272699698683626695999767645674158862) + 5/35309486994022006419332 \cdot \\
& \sqrt{11005224487862873621128239642490888848098} \sqrt{7} \cdot (\sqrt{2} \cdot (10x - 19 \\
& ) + 9x - 29) + 1/701918227692516147086715878423299535653311118502220320740 \\
& 26984349485892917146977000414136 \sqrt{1590558865810545927822094} \cdot (5 \cdot 1264938 \\
& 752804265123815574105117799608149057272418^{(3/4)} \sqrt{35} \cdot (\sqrt{2} \cdot (3406136 \\
& 97110906370000 \cdot x + 143139522536096832703) - 483753219647003202703 \cdot x - 19747 \\
& 4174574809537297) + 5566956030336910747377329 \cdot 12649387528042651238155741051 \\
& 17799608149057272418^{(1/4)} \sqrt{35} \cdot (\sqrt{2} \cdot (43734782664604992355 \cdot x + 2253 \\
& 5043916389567877) - 66269826580994560232 \cdot x - 21199738748215424478) - (5 \cdot 126 \\
& 4938752804265123815574105117799608149057272418^{(3/4)} \sqrt{35} \cdot (340613697110 \\
& 906370000 \sqrt{2} - 483753219647003202703) + 5566956030336910747377329 \cdot 1264 \\
& 938752804265123815574105117799608149057272418^{(1/4)} \sqrt{35} \cdot (4373478266460 \\
& 4992355 \sqrt{2} - 66269826580994560232) \sqrt{x^2 - 2x + 3} \sqrt{81042225} \\
& 921274689605478944797800854846405 \sqrt{2} + 1146118450460034142520527277573 \\
& 33000617984) - 1/35309486994022006419332 \sqrt{x^2 - 2x + 3} \cdot (5 \sqrt{110052} \\
& 24487862873621128239642490888848098) \sqrt{7} \cdot (10 \sqrt{2} + 9) + 74179594525 \\
& 256316007 \sqrt{7} \cdot (125 \sqrt{2} + 172) + 1/476 \sqrt{7} \cdot (25 \sqrt{2} \cdot (5x - 1 \\
& ) + 172x - 82) + 79817891129994413353362937273464455099835468 \cdot 12649387528 \\
& 04265123815574105117799608149057272418^{(1/4)} \sqrt{1590558865810545927822094} \\
& ) \sqrt{35} \sqrt{2} \cdot (512x^{38} - 7936x^{37} + 68352x^{36} - 407808x^{35} + 18679 \\
& 68x^{34} - 6905376x^{33} + 21323904x^{32} - 56249904x^{31} + 129135330x^{30} - 2 \\
& 61706983x^{29} + 474602241x^{28} - 778618854x^{27} + 1168229184x^{26} - 1615329 \\
& 345x^{25} + 2075026563x^{24} - 2486100252x^{23} + 2796604422x^{22} - 2955425895 \\
& \cdot x^{21} + 2956885529x^{20} - 2787233482x^{19} + 2507517852x^{18} - 2118344505x^{17} \\
& + 1731347859x^{16} - 1306537272x^{15} + 984596334x^{14} - 649738605x^{13} + \\
& 468691803x^{12} - 252407834x^{11} + 192383368x^{10} - 68375067x^9 + 72315585 \cdot \\
& x^8 - 6593724x^7 + 25158762x^6 + 3396411x^5 + 6720651x^4 + 1325322x^3 \\
& + 1023516x^2 + 137781x + 59049) \sqrt{810422259212746896054789447978008548} \\
& 46405 \sqrt{2} + 114611845046003414252052727757333000617984) \arctan(-1/54206 \\
& 850781156887023310518673090274966005685838243268724684064391985051350175945 \\
& 64915473395777024743167351056637371274953501437271981836435236061968 \sqrt{7} \\
& 95279432905272963911047) \cdot (9939513250523192816422116593216797292815016511001 \\
& 378462170679301990 \sqrt{11005224487862873621128239642490888848098}) \sqrt{288}
\end{aligned}$$

$8868076710542715672947094311) \cdot \sqrt{7} \cdot (10 \cdot \sqrt{2} + 9) - \sqrt{(1590558865810545927822094) \cdot (5 \cdot 1264938752804265123815574105117799608149057272418^{3/4}) \cdot \sqrt{(2888868076710542715672947094311) \cdot \sqrt{35} \cdot (340613697110906370000 \cdot \sqrt{2} - 483753219647003202703) + 5566956030336910747377329 \cdot 1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{(2888868076710542715672947094311) \cdot \sqrt{35} \cdot (43734782664604992355 \cdot \sqrt{2} - 66269826580994560232)} \cdot \sqrt{(81042225921274689605478944797800854846405 \cdot \sqrt{2} + 114611845046003414252052727757333000617984) + 147461812540444568715696613114138557910359478676937042172325597372869522935182724790786 \cdot \sqrt{(2888868076710542715672947094311) \cdot \sqrt{7} \cdot (125 \cdot \sqrt{2} + 172)) \cdot \sqrt{(5191798731734901573730421875012971256390643826285581511813805064 \cdot x^2 - \sqrt{(1590558865810545927822094) \cdot (1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{35} \cdot \sqrt{7} \cdot \sqrt{x^2 - 2x + 3}) \cdot (43268355662383849682 \cdot x - 105404315061877410477) - 62135959399493560795) - 1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{35} \cdot \sqrt{7} \cdot (\sqrt{2} \cdot (43268355662383849682 \cdot x - 105404315061877410477) - 62135959399493560795 \cdot x + 148672670724261260159)} \cdot \sqrt{(81042225921274689605478944797800854846405 \cdot \sqrt{2} + 114611845046003414252052727757333000617984) - 1297949682933725393432605468753242814097660956571395377953451266 \cdot \sqrt{x^2 - 2x + 3}) \cdot (4x + 1) - 3893849048801176180297816406259728442292982869714186133860353798 \cdot x + 87486976117927258982681475740074062806745190 \cdot \sqrt{(11005224487862873621128239642490888848098) + 9085647780536077754028238281272699698683626695999767645674158862) - 5/35309486994022006419332 \cdot \sqrt{(11005224487862873621128239642490888848098) \cdot \sqrt{7} \cdot (\sqrt{2} \cdot (10x - 19) + 9x - 29) + 1/7019182276925161470867158784232995356533111850222032074026984349485892917146977000414136 \cdot \sqrt{(1590558865810545927822094) \cdot (5 \cdot 1264938752804265123815574105117799608149057272418^{3/4}) \cdot \sqrt{35} \cdot (\sqrt{2} \cdot (340613697110906370000 \cdot x + 143139522536096832703) - 483753219647003202703 \cdot x - 197474174574809537297) + 5566956030336910747377329 \cdot 1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{35} \cdot (\sqrt{2} \cdot (43734782664604992355 \cdot x + 22535043916389567877) - 66269826580994560232 \cdot x - 21199738748215424478) - (5 \cdot 1264938752804265123815574105117799608149057272418^{3/4}) \cdot \sqrt{35} \cdot (340613697110906370000 \cdot \sqrt{2} - 483753219647003202703) + 5566956030336910747377329 \cdot 1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{35} \cdot (43734782664604992355 \cdot \sqrt{2} - 66269826580994560232)} \cdot \sqrt{x^2 - 2x + 3)} \cdot \sqrt{(81042225921274689605478944797800854846405 \cdot \sqrt{2} + 114611845046003414252052727757333000617984) + 1/35309486994022006419332 \cdot \sqrt{x^2 - 2x + 3}) \cdot (5 \cdot \sqrt{(11005224487862873621128239642490888848098) \cdot \sqrt{7} \cdot (10 \cdot \sqrt{2} + 9) + 74179594525256316007 \cdot \sqrt{7} \cdot (125 \cdot \sqrt{2} + 172)) - 1/476 \cdot \sqrt{7} \cdot (25 \cdot \sqrt{2} \cdot (5x - 1) + 172x - 82)) + 24453 \cdot 1264938752804265123815574105117799608149057272418^{1/4}) \cdot \sqrt{(1590558865810545927822094) \cdot \sqrt{35} \cdot \sqrt{7} \cdot (58681264663553748097050996611754496316407808 \cdot x^{38} - 909559602285083095504290447482194692904321024 \cdot x^{37} + 7833948832584425370956308047669225258240442368 \cdot x^{36} - 46739627304520560359301118801262456316018819072 \cdot x^{35} + 214091258966892905713578429763409810498374336512 \cdot x^{34} - 791437884096390872694182856990021126475411881984 \cdot x^{33} + 2443971981023852389183004169635504201209831489536 \cdot x^{32} - 6446905281100567635350197739288116580793540673536 \cdot x^{31} + 14800438431924516080565532176343356954693567774720 \cdot x^{30} - 29994720183053049751603920938291975718069728182272 \cdot x^{29} + 54395038503977968497675563443793146276549591302144 \cdot x^{28} - 89238943444544755685020562033988611037555993870336 \cdot x^{27} + 133892902214827011092889528472923281328218944045056 \cdot x^{26} - 185135876587402190011531997633716034835132689940480 \cdot x^{25} + 237822622904897041561702187373073394318812215508992 \cdot x^{24} - 284936536851054039764888677994792967684286178131968 \cdot x^{23} + 320523992669231941724388481023319612454782787125248 \cdot x^{22} - 338726814722685956738928688519407226164440916295680 \cdot x^{21} + 338894106068517834886487069250634573161464946753536 \cdot x^{20} - 319449971946016546489637350034669310345971696140288 \cdot x^{19} + 287391247503511322489973442496808422958321912250368 \cdot x^{18} - 242787372161112804792074580815007335314268010577920 \cdot x^{17} + 198432972536437767771981576557768362169992275296256 \cdot x^{16} - 149744647365292015359562891324224536622995129499648 \cdot x^{15} + 112846402465271023024054467724570114025686780870656 \cdot x^{14} - 74467740316666419201365857719494322341993062072320 \cdot x^{13} + 5371$

7632299767958169950489423652550531043035185152\*x<sup>12</sup> - 289289275588053521479  
 65359067020100302706002886656\*x<sup>11</sup> + 22049412762644251773309104679542809356  
 433843290112\*x<sup>10</sup> - 7836592584014101531712860147940403658565697404928\*x<sup>9</sup> +  
 8288222622431088813634530458617273979494874480640\*x<sup>8</sup> - 755718873364113816  
 635702120278992782166815932416\*x<sup>7</sup> + 28834921318932789503548025890975347172  
 93712375808\*x<sup>6</sup> + 389268931244541502203228657135011133961927655424\*x<sup>5</sup> + 77  
 0266211020267891996472416855047787936254787584\*x<sup>4</sup> + 1518975997000593369833  
 59025256804087045027790848\*x<sup>3</sup> + 117307057194105230541603999703274443460516  
 511744\*x<sup>2</sup> - 81042225921274689605478944797800854846405\*sqrt(2)\*(512\*x<sup>38</sup> -  
 7936\*x<sup>37</sup> + 68352\*x<sup>36</sup> - 407808\*x<sup>35</sup> + 1867968\*x<sup>34</sup> - 6905376\*x<sup>33</sup> + 213239  
 04\*x<sup>32</sup> - 56249904\*x<sup>31</sup> + 129135330\*x<sup>30</sup> - 261706983\*x<sup>29</sup> + 474602241\*x<sup>28</sup>  
 - 778618854\*x<sup>27</sup> + 1168229184\*x<sup>26</sup> - 1615329345\*x<sup>25</sup> + 2075026563\*x<sup>24</sup> - 24  
 86100252\*x<sup>23</sup> + 2796604422\*x<sup>22</sup> - 2955425895\*x<sup>21</sup> + 2956885529\*x<sup>20</sup> - 27872  
 33482\*x<sup>19</sup> + 2507517852\*x<sup>18</sup> - 2118344505\*x<sup>17</sup> + 1731347859\*x<sup>16</sup> - 13065372  
 72\*x<sup>15</sup> + 984596334\*x<sup>14</sup> - 649738605\*x<sup>13</sup> + 468691803\*x<sup>12</sup> - 252407834\*x<sup>11</sup>  
 + 192383368\*x<sup>10</sup> - 68375067\*x<sup>9</sup> + 72315585\*x<sup>8</sup> - 6593724\*x<sup>7</sup> + 25158762\*x<sup>6</sup>  
 + 3396411\*x<sup>5</sup> + 6720651\*x<sup>4</sup> + 1325322\*x<sup>3</sup> + 1023516\*x<sup>2</sup> + 137781\*x + 5904  
 9) + 15791334622283396419062076883133098158146453504\*x + 676771483812145560  
 8169461521342756353491337216)\*sqrt(8104222592127468960547894479780085484640  
 5\*sqrt(2) + 114611845046003414252052727757333000617984)\*log(514926300974084  
 6168871608737947327093513510106682349523414420454231938660554455908352\*x<sup>2</sup>  
 + 16517307604525632141069927349727551216675979497245715202048/1665374957748  
 9013357854121082231147111\*sqrt(1590558865810545927822094)\*(1264938752804265  
 123815574105117799608149057272418<sup>(1/4)</sup>\*sqrt(35)\*sqrt(7)\*sqrt(x<sup>2</sup> - 2\*x + 3  
 )\*(43268355662383849682\*sqrt(2) - 62135959399493560795) - 12649387528042651  
 23815574105117799608149057272418<sup>(1/4)</sup>\*sqrt(35)\*sqrt(7)\*(sqrt(2)\*(432683556  
 62383849682\*x - 105404315061877410477) - 62135959399493560795\*x + 148672670  
 724261260159))\*sqrt(81042225921274689605478944797800854846405\*sqrt(2) + 114  
 611845046003414252052727757333000617984) - 12873157524352115422179021844868  
 31773378377526670587380853605113557984665138613977088\*sqrt(x<sup>2</sup> - 2\*x + 3)\*(  
 4\*x + 1) - 3861947257305634626653706553460495320135132580011762142560815340  
 673953995415841931264\*x + 8677020686577845807036123864705024753105175633308  
 5943213766737920\*sqrt(11005224487862873621128239642490888848098) + 90112102  
 670464807955253152914078224136486426866941116659752357949058926559702978396  
 16) - 24453\*1264938752804265123815574105117799608149057272418<sup>(1/4)</sup>\*sqrt(15  
 90558865810545927822094)\*sqrt(35)\*sqrt(7)\*(58681264663553748097050996611754  
 496316407808\*x<sup>38</sup> - 909559602285083095504290447482194692904321024\*x<sup>37</sup> + 78  
 33948832584425370956308047669225258240442368\*x<sup>36</sup> - 46739627304520560359301  
 118801262456316018819072\*x<sup>35</sup> + 2140912589668929057135784297634098104983743  
 36512\*x<sup>34</sup> - 791437884096390872694182856990021126475411881984\*x<sup>33</sup> + 244397  
 1981023852389183004169635504201209831489536\*x<sup>32</sup> - 644690528110056763535019  
 7739288116580793540673536\*x<sup>31</sup> + 148004384319245160805655321763433569546935  
 67774720\*x<sup>30</sup> - 29994720183053049751603920938291975718069728182272\*x<sup>29</sup> + 5  
 4395038503977968497675563443793146276549591302144\*x<sup>28</sup> - 892389434445447556  
 85020562033988611037555993870336\*x<sup>27</sup> + 13389290221482701109288952847292328  
 1328218944045056\*x<sup>26</sup> - 185135876587402190011531997633716034835132689940480  
 \*x<sup>25</sup> + 237822622904897041561702187373073394318812215508992\*x<sup>24</sup> - 28493653  
 6851054039764888677994792967684286178131968\*x<sup>23</sup> + 320523992669231941724388  
 481023319612454782787125248\*x<sup>22</sup> - 3387268147226859567389286885194072261644  
 40916295680\*x<sup>21</sup> + 338894106068517834886487069250634573161464946753536\*x<sup>20</sup>  
 - 319449971946016546489637350034669310345971696140288\*x<sup>19</sup> + 2873912475035  
 11322489973442496808422958321912250368\*x<sup>18</sup> - 24278737216111280479207458081  
 5007335314268010577920\*x<sup>17</sup> + 198432972536437767771981576557768362169992275  
 296256\*x<sup>16</sup> - 149744647365292015359562891324224536622995129499648\*x<sup>15</sup> + 11  
 2846402465271023024054467724570114025686780870656\*x<sup>14</sup> - 744677403166664192  
 01365857719494322341993062072320\*x<sup>13</sup> + 53717632299767958169950489423652550  
 531043035185152\*x<sup>12</sup> - 28928927558805352147965359067020100302706002886656\*x  
<sup>11</sup> + 22049412762644251773309104679542809356433843290112\*x<sup>10</sup> - 78365925840  
 14101531712860147940403658565697404928\*x<sup>9</sup> + 828822262243108881363453045861

$7273979494874480640*x^8 - 755718873364113816635702120278992782166815932416*x^7 + 2883492131893278950354802589097534717293712375808*x^6 + 389268931244541502203228657135011133961927655424*x^5 + 770266211020267891996472416855047787936254787584*x^4 + 151897599700059336983359025256804087045027790848*x^3 + 117307057194105230541603999703274443460516511744*x^2 - 81042225921274689605478944797800854846405*\sqrt{2}*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 15791334622283396419062076883133098158146453504*x + 6767714838121455608169461521342756353491337216)*\sqrt{81042225921274689605478944797800854846405*\sqrt{2} + 114611845046003414252052727757333000617984)*\log(5149263009740846168871608737947327093513510106682349523414420454231938660554455908352*x^2 - 16517307604525632141069927349727551216675979497245715202048/16653749577489013357854121082231147111*\sqrt{1590558865810545927822094}*(1264938752804265123815574105117799608149057272418^(1/4)*\sqrt{35}*\sqrt{7}*\sqrt{x^2 - 2*x + 3}*(43268355662383849682*\sqrt{2} - 62135959399493560795) - 1264938752804265123815574105117799608149057272418^(1/4)*\sqrt{35}*\sqrt{7}*(\sqrt{2}*(43268355662383849682*x - 105404315061877410477) - 62135959399493560795*x + 148672670724261260159))*\sqrt{81042225921274689605478944797800854846405*\sqrt{2} + 114611845046003414252052727757333000617984) - 1287315752435211542217902184486831773378377526670587380853605113557984665138613977088*\sqrt{x^2 - 2*x + 3}*(4*x + 1) - 3861947257305634626653706553460495320135132580011762142560815340673953995415841931264*x + 86770206865778458070361238647050247531051756333085943213766737920*\sqrt{11005224487862873621128239642490888848098} + 9011210267046480795525315291407822413648642686694111665975235794905892655970297839616) + 473149067064481998763217709555105306943512932580756046793648401639888862209988063963205432771600*x^4 + 93305673492052096078916346238331863328268414300012419250553508426219541327449647498113404575200*x^3 + 72057846855248153407479950560295894451534022924762066351912610467773507163772995097552926305600*x^2 + 10688997388865973828268515625026705527863090230587961936087245720*(3372249001933422237824271360*x^37 - 53502205399640031394796147712*x^36 + 469149394082989701729494575872*x^35 - 2847499220912667753383035299072*x^34 + 13254252261100740556512388253568*x^33 - 49770080058525077628064229832576*x^32 + 156010734937008739388220889457760*x^31 - 417516398850754397130111919794336*x^30 + 971538171913365251873706873353652*x^29 - 1993653213575521837888601204380228*x^28 + 365553471852957606257345414140031*x^27 - 6054769996581738503753686155104785*x^26 + 9155494158513869230271529746307221*x^25 - 12740106677685048178693605103009787*x^24 + 16442770202470076313197215936814318*x^23 - 19772569734288744720189854470201506*x^22 + 22286437617621909921609206629636086*x^21 - 23584986647560742443188031208946882*x^20 + 23579397211179175240196614296051673*x^19 - 22218747553941794885903840542461607*x^18 + 19912295454080246583636391613811979*x^17 - 16801760806053390242995145349148613*x^16 + 13613407965006475288139078599341572*x^15 - 10279305650733178669223634020962076*x^14 + 7606288378303449524327938977040824*x^13 - 5069838234992751929471190426115248*x^12 + 3507425970596197680016078213030977*x^11 - 1974814483061344405275851094534735*x^10 + 1357002388430055881833293557852283*x^9 - 566969010759169461615951049236597*x^8 + 458426000073846882432457044306894*x^7 - 94704557665253489332536549937026*x^6 + 135183920426913231415208872303230*x^5 - 1023095318901774638403186272874*x^4 + 29398041153524973343917601742151*x^3 + 1933957195570062708781629134823*x^2 + 3397462350398947848063583843461*x - 8003871087155316861345369643)*\sqrt{x^2 - 2*x + 3} + 9700094768975712958699224113885985791552656932179508931988236024507972118200210878516740079600*x + 41571834724181626965853817630939939106654243995055038279949582962177023363715189479357$

```
45748400)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
+ 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2-2\*x+3)\*\*(21/2)/(2\*x\*\*2+x+1)\*\*10,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2\*x+3)^(21/2)/(2\*x^2+x+1)^10,x, algorithm="giac")

[Out] Timed out

$$3.52 \quad \int \frac{-a - \sqrt{1+a^2} + x}{(-a + \sqrt{1+a^2} + x)\sqrt{(-a+x)(1+x^2)}} dx$$

**Optimal.** Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a(x-a)}}{\sqrt{(x^2+1)(x-a)}}\right)$$

[Out] -(Sqrt[2]\*Sqrt[a + Sqrt[1 + a^2]]\*ArcTan[(Sqrt[2]\*Sqrt[-a + Sqrt[1 + a^2]]\*(-a + x))/Sqrt[(-a + x)\*(1 + x^2)]])

**Rubi [C]** time = 1.23427, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6719, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\Pi\left(\frac{2}{1-i(a-\sqrt{a^2+1})}; \sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2}{1-ia}\right)}{(1-i(a-\sqrt{a^2+1}))\sqrt{(x^2+1)(-a-x)}} + \frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2}{1-ia}\right)}{\sqrt{(x^2+1)(-a-x)}}$$

Antiderivative was successfully verified.

[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)\*Sqrt[(-a + x)\*(1 + x^2)]), x]

[Out] (((2\*I)\*Sqrt[(a - x)/(I + a)]\*Sqrt[1 + x^2]\*EllipticF[ArcSin[Sqrt[1 - I\*x]/Sqrt[2]], 2/(1 - I\*a)]/Sqrt[-((a - x)\*(1 + x^2))] + (4\*Sqrt[1 + a^2]\*Sqrt[(a - x)/(I + a)]\*Sqrt[1 + x^2]\*EllipticPi[2/(1 - I\*(a - Sqrt[1 + a^2]))], ArcSin[Sqrt[1 - I\*x]/Sqrt[2]], 2/(1 - I\*a)]/((1 - I\*(a - Sqrt[1 + a^2]))\*Sqrt[-((a - x)\*(1 + x^2))])

#### Rule 6719

Int[(u\_.)\*((a\_.)\*(v\_)^(m\_.)\*(w\_)^(n\_.))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a\*v^m\*w^n)^FracPart[p])/(v^(m\*FracPart[p])\*w^(n\*FracPart[p]))], Int[u\*v^(m\*p)\*w^(n\*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 719

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 932

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)
]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rubi steps



$$\begin{aligned}
\int \frac{-a - \sqrt{1+a^2} + x}{(-a + \sqrt{1+a^2} + x)\sqrt{(-a+x)(1+x^2)}} dx &= \frac{(\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)\sqrt{1+x^2}} dx}{\sqrt{(-a+x)(1+x^2)}} \\
&= \frac{(\sqrt{-a+x}\sqrt{1+x^2}) \int \left( \frac{1}{\sqrt{-a+x}\sqrt{1+x^2}} - \frac{2\sqrt{1+a^2}}{\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)\sqrt{1+x^2}} \right) dx}{\sqrt{(-a+x)(1+x^2)}} \\
&= \frac{(\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{1}{\sqrt{-a+x}\sqrt{1+x^2}} dx}{\sqrt{(-a+x)(1+x^2)}} - \frac{(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{1}{\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)\sqrt{1+x^2}} dx}{\sqrt{(-a+x)(1+x^2)}} \\
&= -\frac{(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)} dx}{\sqrt{(-a+x)(1+x^2)}} + \frac{(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)} dx}{\sqrt{(-a+x)(1+x^2)}} \\
&= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)(1+x^2)}} + \frac{(4\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}) \int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)} dx}{\sqrt{-(a-x)(1+x^2)}} \\
&= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)(1+x^2)}} + \frac{(4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}) \int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}(-a+\sqrt{1+a^2}+x)} dx}{\sqrt{-(a-x)(1+x^2)}} \\
&= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)(1+x^2)}} + \frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\Pi\left(\frac{1}{1-ia}\right)}{(1-i)(a-\sqrt{1+a^2})}
\end{aligned}$$

**Mathematica [C]** time = 1.14754, size = 213, normalized size = 3.23

$$\frac{2\sqrt{\frac{a-x}{a+i}}\left(2i\sqrt{a^2+1}\sqrt{1-ix}\sqrt{x^2+1}\Pi\left(\frac{2i}{a-\sqrt{a^2+1}+i};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2i}{a+i}\right)-\left(\sqrt{a^2+1}-a-i\right)\sqrt{1+ix}(x+i)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\middle|\frac{2i}{a+i}\right)\right)}{\left(-\sqrt{a^2+1}+a+i\right)\sqrt{1-ix}\sqrt{\left(x^2+1\right)(x-a)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)\*Sqrt[(-a + x)\*(1 + x^2)]), x]

[Out] (2\*Sqrt[(a - x)/(I + a)]\*(-((-I - a + Sqrt[1 + a^2])\*Sqrt[1 + I\*x]\*(I + x)\*EllipticF[ArcSin[Sqrt[1 - I\*x]/Sqrt[2]], (2\*I)/(I + a)]) + (2\*I)\*Sqrt[1 + a^2]\*Sqrt[1 - I\*x]\*Sqrt[1 + x^2]\*EllipticPi[(2\*I)/(I + a - Sqrt[1 + a^2]), ArcSin[Sqrt[1 - I\*x]/Sqrt[2]], (2\*I)/(I + a)]))/((I + a - Sqrt[1 + a^2])\*Sqrt[1 - I\*x]\*Sqrt[(-a + x)\*(1 + x^2)])

**Maple [C]** time = 0.094, size = 1275, normalized size = 19.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x)`

[Out]  $2*(-a-I)*((-a+x)/(-a-I))^{1/2}*((x-I)/(a-I))^{1/2}*((x+I)/(a+I))^{1/2}/(-a*x^2+x^3-a+x)^{1/2}*EllipticF(((-a+x)/(-a-I))^{1/2},((a+I)/(a-I))^{1/2})-2*(a^2+1)^{1/2}*((-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1/(a-I)*x-I/(a-I))^{1/2}*(1/(a+I)*x+I/(a+I))^{1/2}/(-a*x^2+x^3-a+x)^{1/2}/(a^2+1)^{1/2}*EllipticPi(((-a+x)/(-a-I))^{1/2},-(a+I)/(a^2+1)^{1/2},((a+I)/(a-I))^{1/2})*a+I*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1/(a-I)*x-I/(a-I))^{1/2}*(1/(a+I)*x+I/(a+I))^{1/2}/(-a*x^2+x^3-a+x)^{1/2}/(a^2+1)^{1/2}*EllipticPi(((a+I)/(a-I))^{1/2},-(a+I)/(a^2+1)^{1/2},((a+I)/(a-I))^{1/2})-(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1/(a-I)*x-I/(a-I))^{1/2}*(1/(a+I)*x+I/(a+I))^{1/2}/(-a*x^2+x^3-a+x)^{1/2}/(a^2+1)^{1/2}*EllipticPi(((a+I)/(a-I))^{1/2},(a+I)/(a^2+1)^{1/2},((a+I)/(a-I))^{1/2})*a-I*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1/(a-I)*x-I/(a-I))^{1/2}*(1/(a+I)*x+I/(a+I))^{1/2}/(-a*x^2+x^3-a+x)^{1/2}/(a^2+1)^{1/2}*EllipticPi(((a+I)/(a-I))^{1/2},(a+I)/(a^2+1)^{1/2},((a+I)/(a-I))^{1/2})-I/(a^2+1)^{1/2}*(1-I*x)^{1/2}*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1+I*x)^{1/2}/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^{1/2}/(-I-a-(a^2+1)^{1/2})*EllipticPi(1/2*2^{1/2}*(-I*(x+I))^{1/2},-2*I/(-I-a-(a^2+1)^{1/2}),2^{1/2}*(-I/(-a-I))^{1/2})*a^2-I/(a^2+1)^{1/2}*(1-I*x)^{1/2}*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1+I*x)^{1/2}/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^{1/2}/(-I-a-(a^2+1)^{1/2})*EllipticPi(1/2*2^{1/2}*(-I*(x+I))^{1/2},-2*I/(-I-a-(a^2+1)^{1/2}),2^{1/2}*(-I/(-a-I))^{1/2}))+I/(a^2+1)^{1/2}*(1-I*x)^{1/2}*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1+I*x)^{1/2}/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^{1/2}/(-I-a+(a^2+1)^{1/2})*EllipticPi(1/2*2^{1/2}*(-I*(x+I))^{1/2},-2*I/(-I-a+(a^2+1)^{1/2}),2^{1/2}*(-I/(-a-I))^{1/2})*a^2+I/(a^2+1)^{1/2}*(1-I*x)^{1/2}*(-1/(-a-I)*a+1/(-a-I)*x)^{1/2}*(1+I*x)^{1/2}/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^{1/2}/(-I-a+(a^2+1)^{1/2})*EllipticPi(1/2*2^{1/2}*(-I*(x+I))^{1/2},-2*I/(-I-a+(a^2+1)^{1/2}),2^{1/2}*(-I/(-a-I))^{1/2})),2^{1/2}*(-I/(-a-I))^{1/2}))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-(x^2+1)}(a-x)(a-x-\sqrt{a^2+1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x, algorithm="maxima")`

[Out] `integrate((a-x+sqrt(a^2+1))/(sqrt(-(x^2+1))*(a-x))*(a-x-sqrt(a^2+1))),x)`

**Fricas [A]** time = 3.06192, size = 1272, normalized size = 19.27

$$\left[ \frac{1}{4} \sqrt{-2a-2\sqrt{a^2+1}} \log \left( -\frac{8ax^7+x^8+4(2a^2+15)x^6-8(4a^3+15a)x^5+2(8a^4+80a^2+67)x^4+64a^4-8(20a^3}{\dots} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x, algorithm="fricas")`

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
- a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1)))/(a*x^2 - x^3 + a - x))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}(a - x - \sqrt{a^2 + 1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

### 3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)} dx$

**Optimal.** Leaf size=198

$$\frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{b \log(x^2+3)}{4 \cdot 2^{2/3}} + \frac{3b \log\left(2^{2/3} - \sqrt[3]{1-x^2}\right)}{4 \cdot 2^{2/3}}$$

[Out] (a\*ArcTan[Sqrt[3]/x])/(2\*2^(2/3)\*Sqrt[3]) + (Sqrt[3]\*b\*ArcTan[(1 + (2 - 2\*x^2)^(1/3))/Sqrt[3]])/(2\*2^(2/3)) + (a\*ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*(1 - x^2)^(1/3)))/x])/(2\*2^(2/3)\*Sqrt[3]) - (a\*ArcTanh[x])/(6\*2^(2/3)) + (a\*ArcTanh[x/(1 + 2^(1/3)\*(1 - x^2)^(1/3))])/(2\*2^(2/3)) - (b\*Log[3 + x^2])/(4\*2^(2/3)) + (3\*b\*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4\*2^(2/3))

**Rubi [A]** time = 0.0971639, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {1010, 393, 444, 55, 617, 204, 31}

$$\frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{b \log(x^2+3)}{4 \cdot 2^{2/3}} + \frac{3b \log\left(2^{2/3} - \sqrt[3]{1-x^2}\right)}{4 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((1 - x^2)^(1/3)\*(3 + x^2)), x]

[Out] (a\*ArcTan[Sqrt[3]/x])/(2\*2^(2/3)\*Sqrt[3]) + (Sqrt[3]\*b\*ArcTan[(1 + (2 - 2\*x^2)^(1/3))/Sqrt[3]])/(2\*2^(2/3)) + (a\*ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*(1 - x^2)^(1/3)))/x])/(2\*2^(2/3)\*Sqrt[3]) - (a\*ArcTanh[x])/(6\*2^(2/3)) + (a\*ArcTanh[x/(1 + 2^(1/3)\*(1 - x^2)^(1/3))])/(2\*2^(2/3)) - (b\*Log[3 + x^2])/(4\*2^(2/3)) + (3\*b\*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4\*2^(2/3))

#### Rule 1010

Int[((g\_) + (h\_)\*(x\_)^2)\*((a\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] :> Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

#### Rule 393

Int[1/(((a\_) + (b\_)\*(x\_)^2)^(1/3)\*((c\_) + (d\_)\*(x\_)^2)), x\_Symbol] :> With[{q = Rt[-(b/a), 2]}, Simp[(q\*ArcTan[Sqrt[3]/(q\*x)])/(2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x] + (Simp[(q\*ArcTanh[(a^(1/3)\*q\*x)/(a^(1/3) + 2^(1/3)\*(a + b\*x^2)^(1/3))])/(2\*2^(2/3)\*a^(1/3)\*d), x] - Simp[(q\*ArcTanh[q\*x])/(6\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTan[(Sqrt[3]\*(a^(1/3) - 2^(1/3)\*(a + b\*x^2)^(1/3)))/(a^(1/3)\*q\*x)])/(2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c + 3\*a\*d, 0] && NegQ[b/a]

#### Rule 444

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n +

1, 0]

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x))] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1/3), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a + bx}{\sqrt[3]{1-x^2}(3+x^2)} dx = a \int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} dx + b \int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)} dx$$

$$= \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2 \cdot 2^{2/3}} + \frac{1}{2} b \operatorname{Subst}\left[\int \frac{1}{\sqrt[3]{1-x^2}} dx, x, \frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right]$$

$$= \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{b \log\left(3 + \frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{4 \cdot 2^{2/3}}$$

$$= \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{b \log\left(3 + \frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{4 \cdot 2^{2/3}}$$

$$= \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} + \frac{\sqrt{3} b \tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3} \sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2 \cdot 2^{2/3}}$$

**Mathematica [C]** time = 0.234498, size = 145, normalized size = 0.73

$$\frac{1}{6} b x^2 F_1\left(1; \frac{1}{3}, 1; 2; x^2, -\frac{x^2}{3}\right) - \frac{9 a x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}(x^2+3) \left(2 x^2 \left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; x^2, -\frac{x^2}{3}\right) - F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; x^2, -\frac{x^2}{3}\right)\right) - 9 F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((1 - x^2)^(1/3)\*(3 + x^2)),x]

[Out] (b\*x^2\*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/6 - (9\*a\*x\*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)\*(3 + x^2)\*(-9\*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2\*x^2\*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 3} \frac{1}{\sqrt[3]{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)

[Out] int((b\*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")

[Out] integrate((b\*x + a)/((x^2 + 3)\*(-x^2 + 1)^(1/3)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[3]{-(x-1)(x+1)}(x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x\*\*2+1)\*\*(1/3)/(x\*\*2+3),x)

```
[Out] Integral((a + b*x)/((-x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

$$3.54 \quad \int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}} dx$$

**Optimal.** Leaf size=198

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{x^2+1})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{b \log(3-x^2)}{4 \cdot 2^{2/3}} - \frac{3b \log\left(2^{2/3} - \sqrt[3]{x^2+1}\right)}{4 \cdot 2^{2/3}}$$

[Out]  $-(a \cdot \text{ArcTan}[x]) / (6 \cdot 2^{(2/3)}) + (a \cdot \text{ArcTan}[x / (1 + 2^{(1/3)} \cdot (1 + x^2)^{(1/3)})]) / (2 \cdot 2^{(2/3)}) - (\text{Sqrt}[3] \cdot b \cdot \text{ArcTan}[(1 + 2^{(1/3)} \cdot (1 + x^2)^{(1/3)}) / \text{Sqrt}[3]]) / (2 \cdot 2^{(2/3)}) - (a \cdot \text{ArcTanh}[\text{Sqrt}[3] / x]) / (2 \cdot 2^{(2/3)} \cdot \text{Sqrt}[3]) - (a \cdot \text{ArcTanh}[(\text{Sqrt}[3] \cdot (1 - 2^{(1/3)} \cdot (1 + x^2)^{(1/3)})] / x]) / (2 \cdot 2^{(2/3)} \cdot \text{Sqrt}[3]) + (b \cdot \text{Log}[3 - x^2]) / (4 \cdot 2^{(2/3)}) - (3 \cdot b \cdot \text{Log}[2^{(2/3)} - (1 + x^2)^{(1/3)}]) / (4 \cdot 2^{(2/3)})$

**Rubi [A]** time = 0.0905163, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {1010, 392, 444, 55, 617, 204, 31}

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{x^2+1})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{b \log(3-x^2)}{4 \cdot 2^{2/3}} - \frac{3b \log\left(2^{2/3} - \sqrt[3]{x^2+1}\right)}{4 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((3 - x^2)\*(1 + x^2)^(1/3)), x]

[Out]  $-(a \cdot \text{ArcTan}[x]) / (6 \cdot 2^{(2/3)}) + (a \cdot \text{ArcTan}[x / (1 + 2^{(1/3)} \cdot (1 + x^2)^{(1/3)})]) / (2 \cdot 2^{(2/3)}) - (\text{Sqrt}[3] \cdot b \cdot \text{ArcTan}[(1 + 2^{(1/3)} \cdot (1 + x^2)^{(1/3)}) / \text{Sqrt}[3]]) / (2 \cdot 2^{(2/3)}) - (a \cdot \text{ArcTanh}[\text{Sqrt}[3] / x]) / (2 \cdot 2^{(2/3)} \cdot \text{Sqrt}[3]) - (a \cdot \text{ArcTanh}[(\text{Sqrt}[3] \cdot (1 - 2^{(1/3)} \cdot (1 + x^2)^{(1/3)})] / x]) / (2 \cdot 2^{(2/3)} \cdot \text{Sqrt}[3]) + (b \cdot \text{Log}[3 - x^2]) / (4 \cdot 2^{(2/3)}) - (3 \cdot b \cdot \text{Log}[2^{(2/3)} - (1 + x^2)^{(1/3)}]) / (4 \cdot 2^{(2/3)})$

#### Rule 1010

Int[((g\_) + (h\_)\*(x\_)^2)\*((a\_) + (c\_)\*(x\_)^2)^(p\_)\*((d\_) + (f\_)\*(x\_)^2)^(q\_), x\_Symbol] :> Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

#### Rule 392

Int[1/(((a\_) + (b\_)\*(x\_)^2)^(1/3)\*((c\_) + (d\_)\*(x\_)^2)), x\_Symbol] :> With[{q = Rt[b/a, 2]}, Simp[(q\*ArcTanh[Sqrt[3]/(q\*x)]) / (2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x] + (-Simp[(q\*ArcTan[(a^(1/3)\*q\*x)/(a^(1/3) + 2^(1/3)\*(a + b\*x^2)^(1/3))]) / (2\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTan[q\*x]) / (6\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTanh[(Sqrt[3]\*(a^(1/3) - 2^(1/3)\*(a + b\*x^2)^(1/3))]) / (a^(1/3)\*q\*x)] / (2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c + 3\*a\*d, 0] && PosQ[b/a]

#### Rule 444

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n +



1, 0]

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a + bx}{(3 - x^2)\sqrt[3]{1 + x^2}} dx = a \int \frac{1}{(3 - x^2)\sqrt[3]{1 + x^2}} dx + b \int \frac{x}{(3 - x^2)\sqrt[3]{1 + x^2}} dx$$

$$= -\frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{x}{1 + \sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1 - \sqrt[3]{2}\sqrt[3]{1+x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{1}{2} b \operatorname{Su}$$

$$= -\frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{x}{1 + \sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1 - \sqrt[3]{2}\sqrt[3]{1+x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{b \log}{4}$$

$$= -\frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{x}{1 + \sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1 - \sqrt[3]{2}\sqrt[3]{1+x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{b \log}{4}$$

$$= -\frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{x}{1 + \sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + \sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}(1 - \sqrt[3]{2}\sqrt[3]{1+x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

**Mathematica [C]** time = 0.244593, size = 153, normalized size = 0.77

$$\frac{1}{6} b x^2 F_1\left(1; \frac{1}{3}, 1; 2; -x^2, \frac{x^2}{3}\right) - \frac{9 a x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}{(x^2 - 3)\sqrt[3]{x^2 + 1} \left(2 x^2 \left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; -x^2, \frac{x^2}{3}\right) - F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; -x^2, \frac{x^2}{3}\right)\right) + 9 F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((3 - x^2)\*(1 + x^2)^(1/3)),x]

[Out] (b\*x^2\*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9\*a\*x\*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)\*(1 + x^2)^(1/3)\*(9\*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2\*x^2\*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 3} \frac{1}{\sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)

[Out] int((b\*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{bx + a}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")

[Out] -integrate((b\*x + a)/((x^2 + 1)^(1/3)\*(x^2 - 3)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{a}{x^2 \sqrt[3]{x^2 + 1} - 3 \sqrt[3]{x^2 + 1}} dx - \int \frac{bx}{x^2 \sqrt[3]{x^2 + 1} - 3 \sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x\*\*2+3)/(x\*\*2+1)\*\*(1/3),x)

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

$$3.55 \quad \int \frac{1}{x \sqrt[3]{4-6x+3x^2}} dx$$

**Optimal.** Leaf size=97

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2 \cdot 2^{2/3}}$$

[Out]  $-(\text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(2 - x))/(\text{Sqrt}[3]*(4 - 6*x + 3*x^2)^{(1/3)})]/(2^{(2/3)*\text{Sqrt}[3]}) - \text{Log}[x]/(2*2^{(2/3)}) + \text{Log}[6 - 3*x - 3*2^{(1/3)}*(4 - 6*x + 3*x^2)^{(1/3)}]/(2*2^{(2/3)})$

**Rubi [A]** time = 0.0133008, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {750}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(4 - 6\*x + 3\*x^2)^(1/3)),x]

[Out]  $-(\text{ArcTan}[1/\text{Sqrt}[3] + (2^{(2/3)}*(2 - x))/(\text{Sqrt}[3]*(4 - 6*x + 3*x^2)^{(1/3)})]/(2^{(2/3)*\text{Sqrt}[3]}) - \text{Log}[x]/(2*2^{(2/3)}) + \text{Log}[6 - 3*x - 3*2^{(1/3)}*(4 - 6*x + 3*x^2)^{(1/3)}]/(2*2^{(2/3)})$

### Rule 750

Int[1/(((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(1/3)), x\_Symbol] := With[{q = Rt[3\*c\*e^2\*(2\*c\*d - b\*e), 3]}, -Simp[(Sqrt[3]\*c\*e\*ArcTan[1/Sqrt[3] + (2\*(c\*d - b\*e - c\*e\*x))/(Sqrt[3]\*q\*(a + b\*x + c\*x^2)^(1/3))]/q^2, x] + (-Simp[(3\*c\*e\*Log[d + e\*x])/(2\*q^2), x] + Simp[(3\*c\*e\*Log[c\*d - b\*e - c\*e\*x - q\*(a + b\*x + c\*x^2)^(1/3)]/(2\*q^2), x))]/; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && EqQ[c^2\*d^2 - b\*c\*d\*e + b^2\*e^2 - 3\*a\*c\*e^2, 0] && PosQ[c\*e^2\*(2\*c\*d - b\*e)]

### Rubi steps

$$\int \frac{1}{x \sqrt[3]{4-6x+3x^2}} dx = -\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2 \cdot 2^{2/3}} + \frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2 \cdot 2^{2/3}}$$

**Mathematica [C]** time = 0.0602103, size = 111, normalized size = 1.14

$$-\frac{\sqrt[3]{\frac{3x+i\sqrt{3}-3}{x}} \sqrt[3]{\frac{9x-3i\sqrt{3}-9}{x}} F_1\left(\frac{2}{3}; \frac{1}{3}, \frac{1}{3}, \frac{5}{3}; \frac{3-i\sqrt{3}}{3x}, \frac{3+i\sqrt{3}}{3x}\right)}{2\sqrt[3]{3x^2-6x+4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x\*(4 - 6\*x + 3\*x^2)^(1/3)),x]

[Out] -((( -3 + I\*Sqrt[3] + 3\*x)/x)^(1/3)\*((-9 - (3\*I)\*Sqrt[3] + 9\*x)/x)^(1/3)\*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I\*Sqrt[3])/(3\*x), (3 + I\*Sqrt[3])/(3\*x)])/(2\*(4 - 6\*x + 3\*x^2)^(1/3))

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{3x^2 - 6x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(3\*x^2-6\*x+4)^(1/3),x)

[Out] int(1/x/(3\*x^2-6\*x+4)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 6x + 4)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(3\*x^2-6\*x+4)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((3\*x^2 - 6\*x + 4)^(1/3)\*x), x)

**Fricas [B]** time = 20.7382, size = 495, normalized size = 5.1

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan \left( \frac{4^{\frac{1}{6}} \sqrt{3} \left( 4^{\frac{1}{3}} x^3 + 2 \cdot 4^{\frac{2}{3}} (3x^2 - 6x + 4)^{\frac{2}{3}} (x - 2) + 4 (3x^2 - 6x + 4)^{\frac{1}{3}} (x^2 - 4x + 4) \right)}{6(x^3 - 12x^2 + 24x - 16)} \right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log \left( \frac{4^{\frac{1}{3}} (x^3 - 12x^2 + 24x - 16) + 4 (3x^2 - 6x + 4)^{\frac{1}{3}} (x^2 - 4x + 4)}{4^{\frac{1}{3}} (x^3 - 12x^2 + 24x - 16)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(3\*x^2-6\*x+4)^(1/3),x, algorithm="fricas")

[Out] -1/6\*4^(1/6)\*sqrt(3)\*arctan(1/6\*4^(1/6)\*sqrt(3)\*(4^(1/3)\*x^3 + 2\*4^(2/3)\*(3\*x^2 - 6\*x + 4)^(2/3)\*(x - 2) + 4\*(3\*x^2 - 6\*x + 4)^(1/3)\*(x^2 - 4\*x + 4))/(x^3 - 12\*x^2 + 24\*x - 16)) + 1/12\*4^(2/3)\*log((4^(1/3)\*(x^3 - 12\*x^2 + 24\*x - 16) + 4\*(3\*x^2 - 6\*x + 4)^(1/3)\*(x^2 - 4\*x + 4))/4^(1/3)\*(x^3 - 12\*x^2 + 24\*x - 16)) - 1/24\*4^(2/3)\*log((4^(2/3)\*(3\*x^2 - 6\*x + 4)^(2/3) + 4^(1/3)\*(x^2 - 4\*x + 4) - 2\*(3\*x^2 - 6\*x + 4)^(1/3)\*(x - 2))/x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{3x^2 - 6x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(3\*x\*\*2-6\*x+4)\*\*(1/3),x)

[Out] Integral(1/(x\*(3\*x\*\*2 - 6\*x + 4)\*\*(1/3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 6x + 4)^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(3\*x^2-6\*x+4)^(1/3),x, algorithm="giac")

[Out] integrate(1/((3\*x^2 - 6\*x + 4)^(1/3)\*x), x)

### 3.56 $\int x \sqrt[3]{1-x^3} dx$

**Optimal.** Leaf size=73

$$\frac{1}{3} \sqrt[3]{1-x^3} x^2 - \frac{1}{6} \log\left(-\sqrt[3]{1-x^3} - x\right) - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out]  $(x^2*(1-x^3)^{(1/3)})/3 - \text{ArcTan}[(1-(2*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) - \text{Log}[-x - (1-x^3)^{(1/3)}/6]$

**Rubi [A]** time = 0.0435833, antiderivative size = 107, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$ , Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3} \sqrt[3]{1-x^3} x^2 + \frac{1}{18} \log\left(\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{1}{9} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x\*(1-x^3)^(1/3),x]

[Out]  $(x^2*(1-x^3)^{(1/3)})/3 - \text{ArcTan}[(1-(2*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[1+x^2/(1-x^3)^{(2/3)} - x/(1-x^3)^{(1/3)}/18 - \text{Log}[1+x/(1-x^3)^{(1/3)}/9]$

#### Rule 279

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^p)/(c\*(m+n\*p+1)), x] + Dist[(a\*n\*p)/(m+n\*p+1), Int[(c\*x)^m\*(a+b\*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 331

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[a^(p+(m+1)/n), Subst[Int[x^m/(1-b\*x^n)^(p+(m+1)/n+1), x], x, x/(a+b\*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegersQ[m, p+(m+1)/n]

#### Rule 292

Int[(x\_)/((a\_.) + (b\_.)\*(x\_)^3), x\_Symbol] := -Dist[(3\*Rt[a, 3]\*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]\*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a+b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[3]{1-x^3} dx &= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3} \int \frac{x}{(1-x^3)^{2/3}} dx \\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3} \text{Subst}\left(\int \frac{x}{1+x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9} \text{Subst}\left(\int \frac{1}{1+x} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9} \text{Subst}\left(\int \frac{1+x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9} \log\left(1 + \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{18} \text{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{6} \text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{18} \log\left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9} \log\left(1 + \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{18} \log\left(1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9} \log\left(1 + \frac{x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.0026731, size = 20, normalized size = 0.27

$$\frac{1}{2}x^2 {}_2F_1\left(-\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(1 - x^3)^(1/3), x]

[Out] (x^2\*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3])/2



---

**Maple [C]** time = 0.046, size = 69, normalized size = 1.

$$-\frac{x^2(x^3-1)}{3}(-x^3+1)^{-\frac{2}{3}} + \frac{x^2}{6}(x^3-1)^{\frac{2}{3}}(-\text{signum}(x^3-1))^{\frac{2}{3}} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) (\text{signum}(x^3-1))^{-\frac{2}{3}} (-x^3+1)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-x^3+1)^(1/3),x)

[Out] -1/3\*x^2\*(x^3-1)/(-x^3+1)^(2/3)+1/6\*(x^3-1)^(2/3)/signum(x^3-1)^(2/3)\*(-signum(x^3-1))^(2/3)\*x^2\*hypergeom([2/3,2/3],[5/3],x^3)/(-x^3+1)^(2/3)

---

**Maxima [A]** time = 1.42811, size = 142, normalized size = 1.95

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right) - \frac{(-x^3+1)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)} - \frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right) + \frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x} + \frac{(-x^3-1)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-x^3+1)^(1/3),x, algorithm="maxima")

[Out] -1/9\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3)/x - 1)) - 1/3\*(-x^3 + 1)^(1/3)/(x\*((x^3 - 1)/x^3 - 1)) - 1/9\*log((-x^3 + 1)^(1/3)/x + 1) + 1/18\*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)

---

**Fricas [A]** time = 2.05284, size = 262, normalized size = 3.59

$$\frac{1}{3}(-x^3+1)^{\frac{1}{3}}x^2 - \frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right) - \frac{1}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{18}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-x^3+1)^(1/3),x, algorithm="fricas")

[Out] 1/3\*(-x^3 + 1)^(1/3)\*x^2 - 1/9\*sqrt(3)\*arctan(-1/3\*(sqrt(3)\*x - 2\*sqrt(3))\*(-x^3 + 1)^(1/3)/x) - 1/9\*log((x + (-x^3 + 1)^(1/3))/x) + 1/18\*log((x^2 - (-x^3 + 1)^(1/3)\*x + (-x^3 + 1)^(2/3))/x^2)

---

**Sympy [C]** time = 1.10618, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(5/3))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-x^3 + 1)^{\frac{1}{3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

$$3.57 \quad \int \frac{\sqrt[3]{1-x^3}}{x} dx$$

**Optimal.** Leaf size=67

$$\sqrt[3]{1-x^3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

**Rubi [A]** time = 0.0372041, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {266, 50, 57, 618, 204, 31}

$$\sqrt[3]{1-x^3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(1/3)/x,x]

[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^n/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 57

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(2/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q^2), x] + (-Dist[3/(2\*b\*q), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q^2), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x^3}}{x} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{\sqrt[3]{1-x}}{x} dx, x, x^3 \right) \\
&= \sqrt[3]{1-x^3} + \frac{1}{3} \text{Subst} \left( \int \frac{1}{(1-x)^{2/3} x} dx, x, x^3 \right) \\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2} \text{Subst} \left( \int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) - \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1-x^3} \right) \\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2} \log(1 - \sqrt[3]{1-x^3}) + \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1-x^3} \right) \\
&= \sqrt[3]{1-x^3} - \frac{\tan^{-1} \left( \frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2} \log(1 - \sqrt[3]{1-x^3})
\end{aligned}$$

**Mathematica [A]** time = 0.0236412, size = 90, normalized size = 1.34

$$\sqrt[3]{1-x^3} + \frac{1}{3} \log(1 - \sqrt[3]{1-x^3}) - \frac{1}{6} \log\left((1-x^3)^{2/3} + \sqrt[3]{1-x^3} + 1\right) - \frac{\tan^{-1} \left( \frac{2\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x, x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 - (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

**Maple [C]** time = 0.033, size = 49, normalized size = 0.7

$$-\frac{1}{9\Gamma(2/3)} \left( -3 \left( 3 + 1/6 \pi \sqrt{3} - 3/2 \ln(3) + 3 \ln(x) + i\pi \right) \Gamma(2/3) + \Gamma\left(\frac{2}{3}\right) x^3 {}_3F_2\left(\frac{2}{3}, 1, 1; 2, 2; x^3\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/x, x)
```

```
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GAMMA(2/3)*x^3*hypergeom([2/3, 1, 1], [2, 2], x^3))
```

**Maxima [A]** time = 1.46498, size = 96, normalized size = 1.43

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)+(-x^3+1)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")

[Out] -1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)

**Fricas [A]** time = 1.95305, size = 225, normalized size = 3.36

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+(-x^3+1)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")

[Out] -1/3\*sqrt(3)\*arctan(2/3\*sqrt(3)\*(-x^3 + 1)^(1/3) + 1/3\*sqrt(3)) + (-x^3 + 1)^(1/3) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)

**Sympy [C]** time = 0.944826, size = 37, normalized size = 0.55

$$\frac{x e^{\frac{i\pi}{3}} \Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix} \middle| \frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(1/3)/x,x)

[Out] -x\*exp(I\*pi/3)\*gamma(-1/3)\*hyper((-1/3, -1/3), (2/3,), x\*\*(-3))/(3\*gamma(2/3))

**Giac [A]** time = 1.14603, size = 97, normalized size = 1.45

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)+(-x^3+1)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log(abs((-x^3 + 1)^(1/3) - 1))

$$3.58 \quad \int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

**Optimal.** Leaf size=482

$$\sqrt[3]{1-x^3} - \frac{1}{3}\sqrt[3]{2} \log(x^3+1) + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} + \frac{1}{3}\sqrt[3]{2} \log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\log\left(\frac{(1-x)}{(1-x^3)^{1/3}}\right)}{3}$$

```
[Out] (1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2
^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(1/3)*Lo
g[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1
+ (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/
(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/3 - Log[
2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/
(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (1 - x^3)^(
1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

**Rubi [F]** time = 0.0520396, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x), x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx = \int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

**Mathematica [F]** time = 0.35477, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \sqrt[3]{-x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(1/3)/(1+x),x)

[Out] int((-x^3+1)^(1/3)/(1+x),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(1/3)/(1+x),x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1)\*\*(1/3)/(x + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```



$$3.59 \quad \int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

**Optimal.** Leaf size=280

$$-\frac{\log(-3(x-1)(x^2-x+1))}{2 \cdot 2^{2/3}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2 \cdot 2^{2/3}} + \frac{3 \log(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1))}{2 \cdot 2^{2/3}} + \frac{1}{2} \log(\sqrt[3]{1-x^3}+x) - \frac{\log(\sqrt[3]{1-x^3})}{2}$$

[Out] (Sqrt[3]\*ArcTan[(1 + (2\*2^(1/3))\*(-1 + x))/(1 - x^3)^(1/3)]/Sqrt[3])/2^(2/3) + ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 - (2\*2^(1/3)\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)\*Sqrt[3]) - ArcTan[(1 + 2^(2/3)\*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)\*Sqrt[3]) - Log[-3\*(-1 + x)\*(1 - x + x^2)]/(2\*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2\*2^(2/3)) + (3\*Log[-(2^(1/3)\*(-1 + x)) + (1 - x^3)^(1/3)])/(2\*2^(2/3)) + Log[x + (1 - x^3)^(1/3)]/2 - Log[2^(1/3)\*x + (1 - x^3)^(1/3)]/(2\*2^(2/3))

**Rubi [F]** time = 0.235482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2), x]

[Out] ((2\*I)\*Defer[Int][(1 - x^3)^(1/3)/(1 + I\*Sqrt[3] - 2\*x), x])/Sqrt[3] + ((2\*I)\*Defer[Int][(1 - x^3)^(1/3)/(-1 + I\*Sqrt[3] + 2\*x), x])/Sqrt[3]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx &= \int \left( \frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}(1+i\sqrt{3}-2x)} + \frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}(-1+i\sqrt{3}+2x)} \right) dx \\ &= \frac{(2i) \int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x} dx}{\sqrt{3}} + \frac{(2i) \int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x} dx}{\sqrt{3}} \end{aligned}$$

**Mathematica [F]** time = 0.106666, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]

[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]

---

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 - x + 1} \sqrt[3]{-x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(1/3)/(x^2-x+1),x)

[Out] int((-x^3+1)^(1/3)/(x^2-x+1),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)

---

**Fricas [B]** time = 51.3796, size = 10386, normalized size = 37.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/9*\sqrt{3}*2^{(1/3)}*\arctan(1/3*(26795748*\sqrt{3}*2^{(2/3)}*(586745*x^{11} - 70 \\ & 6109*x^{10} - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 + \\ & 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^{(1/3)} + 26795748*\sqrt{3} \\ & *2^{(1/3)}*(340975*x^{10} + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^6 \\ & + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^{(2/3)} \\ & + 7*\sqrt{273426}*2^{(1/6)}*(6*\sqrt{3}*2^{(2/3)}*(338078915*x^{10} - 459916473 \\ & *x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448 \\ & 15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^{(2/3)} + \sqrt{3} \\ & *2^{(1/3)}*(2332175065*x^{12} - 3283524318*x^{11} + 1882024851*x^{10} - 39193009 \\ & 70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11 \\ & 45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*\sqrt{3} \\ & *(493920245*x^{11} - 452201839*x^{10} - 276972599*x^9 - 661557480*x^8 + 13 \\ & 75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3 \\ & - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^{(1/3)}) - 3*\sqrt{3}*(2247079524645*x^{12} \\ & - 5276442179264*x^{11} + 3816306322874*x^{10} - 3280399521884*x^9 + 6278089 \\ & 258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25 \\ & 58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515 \\ & 2977792))/(18230538112975*x^{12} - 14115716188440*x^{11} - 20854883745366*x^{10} \\ & + 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059 \\ & 560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296 \\ & 4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*\sqrt{3}*2^{(1/3)}*\arctan(- \\ & 1/3*(13397874*\sqrt{3}*2^{(2/3)}*(18803*x^{11} - 25367*x^{10} - 203754*x^9 + 40802 \end{aligned}$$

$$\begin{aligned}
&1x^8 - 139829x^7 + 7128x^6 - 233871x^5 + 225275x^4 - 47094x^3 - 10225 \\
&*x^2 + 2921x)*(-x^3 + 1)^{(1/3)} + 26795748*\sqrt{3}*2^{(1/3)}*(10589x^{10} - 73 \\
&935x^9 + 63883x^8 + 142959x^7 - 173613x^6 - 31588x^5 + 79410x^4 - 437 \\
&7x^3 - 13328x^2 + 2921x)*(-x^3 + 1)^{(2/3)} - 7*\sqrt{273426}*(6*\sqrt{3})*2^{(2/3)} \\
&(309683372x^{10} - 328552599x^9 - 24698630x^8 - 422031122x^7 + 7021 \\
&64163x^6 - 95703451x^5 - 206316094x^4 + 60985482x^3 + 11167816x^2 - 37 \\
&33038x)*(-x^3 + 1)^{(2/3)} + \sqrt{3}*2^{(1/3)}*(2345654785x^{12} - 2502234618x^{11} \\
&- 252041853x^{10} - 4416416426x^9 + 6899968311x^8 - 1680852528x^7 + 1 \\
&576960038x^6 - 2990585436x^5 + 642930363x^4 + 528479914x^3 - 117963261x^2 \\
&- 38399466x + 8532241) - 6*\sqrt{3}*(491687266x^{11} - 516958230x^{10} - \\
&69305552x^9 - 808934094x^8 + 1418391515x^7 - 385704187x^6 - 112721241x^5 \\
&- 69510422x^4 + 47121139x^3 + 11465929x^2 - 4799203x)*(-x^3 + 1)^{(1/3)} \\
&)*\sqrt{(6*2^{(2/3)}*(4x^{10} - 27x^9 + 32x^8 + 6x^7 + 12x^6 - 65x^5 + 4 \\
&8x^4 - 6x^3 - 4x^2 + x)*(-x^3 + 1)^{(2/3)} - 2^{(1/3)}*(35x^{12} - 66x^{11} - \\
&201x^{10} + 338x^9 + 90x^8 - 90x^7 - 249x^6 - 18x^5 + 306x^4 - 166x^3 \\
&+ 15x^2 + 6x - 1) - 6*(x^{11} + 29x^{10} - 93x^9 + 66x^8 - 19x^7 + 87x^6 \\
&- 99x^5 + 10x^4 + 27x^3 - 11x^2 + x)*(-x^3 + 1)^{(1/3)})/(x^{12} - 6x^{11} \\
&+ 21x^{10} - 50x^9 + 90x^8 - 126x^7 + 141x^6 - 126x^5 + 90x^4 - 50x^3 \\
&+ 21x^2 - 6x + 1) - 3*\sqrt{3}*(2995162579x^{12} + 315959718008x^{11} - 8 \\
&49682072424x^{10} + 177300060912x^9 - 508006765899x^8 + 3583876884636x^7 \\
&- 3031033916540x^6 - 1410763301208x^5 + 2375077456341x^4 - 546587071308x^3 \\
&- 175036021936x^2 + 63861157012x - 3114267965))/(367648430113x^{12} - \\
&1408582980384x^{11} - 1269375810828x^{10} + 5714713216048x^9 - 1087485936795 \\
&*x^8 - 126379999188x^7 - 10319650860540x^6 + 10854292018608x^5 - 1383220 \\
&291365x^4 - 1828745373668x^3 + 426327416076x^2 + 93479232396x - 2492267 \\
&5961) - 1/18*\sqrt{3}*2^{(1/3)}*\arctan(1/3*(13397874*\sqrt{3})*2^{(2/3)}*(17344x^{11} \\
&- 120304x^{10} + 110610x^9 + 203214x^8 - 213415x^7 - 96387x^6 + 3010 \\
&2x^5 + 157561x^4 - 101868x^3 + 15151x^2 + 913x)*(-x^3 + 1)^{(1/3)} - 267 \\
&95748*\sqrt{3}*2^{(1/3)}*(1277x^{10} + 57510x^9 - 189677x^8 + 108972x^7 + 10 \\
&2426x^6 - 47461x^5 - 82155x^4 + 56409x^3 - 7301x^2 - 913x)*(-x^3 + 1) \\
&^{(2/3)} + 7*\sqrt{273426}*(6*\sqrt{3})*2^{(2/3)}*(8733539x^{10} - 122586360x^9 + \\
&269810944x^8 - 28009538x^7 - 316185126x^6 + 161786897x^5 + 95479640x^4 \\
&- 80193978x^3 + 11163982x^2 + 1166814x)*(-x^3 + 1)^{(2/3)} - \sqrt{3}*2^{(1/3)} \\
&*(1971824x^{12} - 78264612x^{11} + 705529692x^{10} - 1556393152x^9 + 93384 \\
&9120x^8 + 135726408x^7 - 213906684x^6 + 446158968x^5 - 582881445x^4 + \\
&182390318x^3 + 31120185x^2 - 12999294x - 833569) + 6*\sqrt{3}*(12965988x^{11} \\
&- 175265260x^{10} + 270273662x^9 + 299814882x^8 - 663644613x^7 + 7755 \\
&3085x^6 + 286893603x^5 - 82332150x^4 - 33723265x^3 + 10863861x^2 + 333 \\
&245x)*(-x^3 + 1)^{(1/3)})*\sqrt{(6*2^{(2/3)}*(143x^{10} - 177x^9 - 2x^8 - 54x^7 \\
&+ 141x^6 - 31x^5 - 18x^4 - 6x^3 + 7x^2 - x)*(-x^3 + 1)^{(2/3)} + 2^{(1/3)} \\
&*(1081x^{12} - 1338x^{11} - 15x^{10} - 1130x^9 + 1962x^8 - 234x^7 + 33x^6 \\
&- 630x^5 + 234x^4 + 58x^3 - 15x^2 - 6x + 1) - 6*(227x^{11} - 281x^{10} \\
&- 3x^9 - 162x^8 + 319x^7 - 51x^6 - 21x^5 - 58x^4 + 33x^3 - x^2 - x) \\
&)*(-x^3 + 1)^{(1/3)})/(x^{12} - 6x^{11} + 21x^{10} - 50x^9 + 90x^8 - 126x^7 + \\
&141x^6 - 126x^5 + 90x^4 - 50x^3 + 21x^2 - 6x + 1) - 3*\sqrt{3}*(67113 \\
&679084x^{12} - 61534090748x^{11} - 1006807736260x^{10} + 1996201310444x^9 + 1 \\
&93806523788x^8 - 2673973669800x^7 + 775957356356x^6 + 2110159119756x^5 \\
&- 1821028473882x^4 + 377014646048x^3 + 67410900094x^2 - 19835743048x - \\
&1369553867))/(168032067092x^{12} - 2318893136652x^{11} + 4401905935020x^{10} + \\
&1550444734940x^9 - 6210007783092x^8 - 1634341806144x^7 + 6341768478444x^6 \\
&- 948091553244x^5 - 2281774840272x^4 + 1036207535072x^3 - 5948022808 \\
&2x^2 - 20085678624x - 761048497) + 1/3*\sqrt{3}*\arctan((4*\sqrt{3})*(-x^3 + \\
&1)^{(1/3)}*x^2 + 2*\sqrt{3}*(-x^3 + 1)^{(2/3)}*x - \sqrt{3}*(x^3 - 1))/(9x^3 - \\
&1) + 1/48*2^{(1/3)}*\log(7717175424*(6*2^{(2/3)}*(143x^{10} - 177x^9 - 2x^8 - \\
&54x^7 + 141x^6 - 31x^5 - 18x^4 - 6x^3 + 7x^2 - x)*(-x^3 + 1)^{(2/3)} + \\
&2^{(1/3)}*(1081x^{12} - 1338x^{11} - 15x^{10} - 1130x^9 + 1962x^8 - 234x^7 + \\
&33x^6 - 630x^5 + 234x^4 + 58x^3 - 15x^2 - 6x + 1) - 6*(227x^{11} - 281 \\
&*x^10 - 3x^9 - 162x^8 + 319x^7 - 51x^6 - 21x^5 - 58x^4 + 33x^3 - x^2 \\
&- x)*(-x^3 + 1)^{(1/3)})/(x^{12} - 6x^{11} + 21x^{10} - 50x^9 + 90x^8 - 126x^7
\end{aligned}$$

$7 + 141x^6 - 126x^5 + 90x^4 - 50x^3 + 21x^2 - 6x + 1) + 1/48 \cdot 2^{(1/3)}$   
 $\cdot \log(1929293856 \cdot (6 \cdot 2^{(2/3)}) \cdot (143x^{10} - 177x^9 - 2x^8 - 54x^7 + 141x^6 -$   
 $31x^5 - 18x^4 - 6x^3 + 7x^2 - x) \cdot (-x^3 + 1)^{(2/3)} + 2^{(1/3)} \cdot (1081x^{12}$   
 $- 1338x^{11} - 15x^{10} - 1130x^9 + 1962x^8 - 234x^7 + 33x^6 - 630x^5 +$   
 $234x^4 + 58x^3 - 15x^2 - 6x + 1) - 6 \cdot (227x^{11} - 281x^{10} - 3x^9 - 16$   
 $2x^8 + 319x^7 - 51x^6 - 21x^5 - 58x^4 + 33x^3 - x^2 - x) \cdot (-x^3 + 1)^{(1/3)}) / (x^{12} - 6x^{11} + 21x^{10} - 50x^9 + 90x^8 - 126x^7 + 141x^6 - 126x^5 + 90x^4 - 50x^3 + 21x^2 - 6x + 1) - 1/48 \cdot 2^{(1/3)} \cdot \log(7717175424 \cdot (6 \cdot 2^{(2/3)}) \cdot (4x^{10} - 27x^9 + 32x^8 + 6x^7 + 12x^6 - 65x^5 + 48x^4 - 6x^3 - 4x^2 + x) \cdot (-x^3 + 1)^{(2/3)} - 2^{(1/3)} \cdot (35x^{12} - 66x^{11} - 201x^{10} + 338x^9 + 90x^8 - 90x^7 - 249x^6 - 18x^5 + 306x^4 - 166x^3 + 15x^2 + 6x - 1) - 6 \cdot (x^{11} + 29x^{10} - 93x^9 + 66x^8 - 19x^7 + 87x^6 - 99x^5 + 10x^4 + 27x^3 - 11x^2 + x) \cdot (-x^3 + 1)^{(1/3)}) / (x^{12} - 6x^{11} + 21x^{10} - 50x^9 + 90x^8 - 126x^7 + 141x^6 - 126x^5 + 90x^4 - 50x^3 + 21x^2 - 6x + 1) - 1/48 \cdot 2^{(1/3)} \cdot \log(1929293856 \cdot (6 \cdot 2^{(2/3)}) \cdot (4x^{10} - 27x^9 + 32x^8 + 6x^7 + 12x^6 - 65x^5 + 48x^4 - 6x^3 - 4x^2 + x) \cdot (-x^3 + 1)^{(2/3)}) - 2^{(1/3)} \cdot (35x^{12} - 66x^{11} - 201x^{10} + 338x^9 + 90x^8 - 90x^7 - 249x^6 - 18x^5 + 306x^4 - 166x^3 + 15x^2 + 6x - 1) - 6 \cdot (x^{11} + 29x^{10} - 93x^9 + 66x^8 - 19x^7 + 87x^6 - 99x^5 + 10x^4 + 27x^3 - 11x^2 + x) \cdot (-x^3 + 1)^{(1/3)}) / (x^{12} - 6x^{11} + 21x^{10} - 50x^9 + 90x^8 - 126x^7 + 141x^6 - 126x^5 + 90x^4 - 50x^3 + 21x^2 - 6x + 1) + 1/6 \cdot \log(3 \cdot (-x^3 + 1)^{(1/3)} \cdot x^2 + 3 \cdot (-x^3 + 1)^{(2/3)} \cdot x + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(1/3)/(x\*\*2-x+1),x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)/(x\*\*2 - x + 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{x^2-x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)

$$3.60 \quad \int \frac{\sqrt[3]{1-x^3}}{2+x} dx$$

**Optimal.** Leaf size=232

$$\frac{1}{2}x {}_2F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -\frac{x^3}{8}\right) + \sqrt[3]{1-x^3} - \frac{\log(x^3+8)}{\sqrt[3]{3}} + \frac{1}{2}3^{2/3} \log\left(3^{2/3} - \sqrt[3]{1-x^3}\right) - \log\left(-\sqrt[3]{1-x^3} - x\right) + \frac{1}{2}3^{2/3} \log\left(\dots\right)$$

[Out]  $(1 - x^3)^{1/3} + (x \text{AppellF1}[1/3, -1/3, 1, 4/3, x^3, -x^3/8])/2 - (2 \text{ArcTan}[(1 - (2*x)/(1 - x^3)^{1/3})/\text{Sqrt}[3]])/\text{Sqrt}[3] + 3^{1/6} \text{ArcTan}[(1 - (3^{2/3}*x)/(1 - x^3)^{1/3})/\text{Sqrt}[3]] - 3^{1/6} \text{ArcTan}[1/\text{Sqrt}[3] + (2*(1 - x^3)^{1/3})/(3*3^{1/6})] - \text{Log}[8 + x^3]/3^{1/3} + (3^{2/3} \text{Log}[3^{2/3} - (1 - x^3)^{1/3}])/2 - \text{Log}[-x - (1 - x^3)^{1/3}] + (3^{2/3} \text{Log}[-(3^{2/3}*x)/2 - (1 - x^3)^{1/3}])/2$

**Rubi [F]** time = 0.0499624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(1/3)/(2 + x), x]

[Out] Defer[Int][(1 - x^3)^(1/3)/(2 + x), x]

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} dx = \int \frac{\sqrt[3]{1-x^3}}{2+x} dx$$

**Mathematica [F]** time = 0.343464, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(1/3)/(2 + x), x]

[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{2+x} \sqrt[3]{-x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^3+1)^(1/3)/(2+x),x)`

[Out] `int((-x^3+1)^(1/3)/(2+x),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x + 2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)**(1/3)/(2+x),x)`

[Out] `Integral((-x - 1)*(x**2 + x + 1)**(1/3)/(x + 2), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x + 2), x)`

$$3.61 \quad \int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}} dx$$

**Optimal.** Leaf size=168

$$-\frac{x^2 F_1\left(\frac{2}{3}; 1, \frac{1}{3}; \frac{5}{3}; x^3, -\frac{x^3}{2}\right)}{2\sqrt[3]{2}} + \frac{\log(1-x^3)}{6\sqrt[3]{3}} + \frac{\log(\sqrt[3]{3}-\sqrt[3]{x^3+2})}{2\sqrt[3]{3}} - \frac{\log(\sqrt[3]{3}x-\sqrt[3]{x^3+2})}{\sqrt[3]{3}} + \frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{3}x+1}{\sqrt[3]{x^3+2}}\right)}{3^{5/6}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{3}x+1}{\sqrt[3]{x^3+2}}\right)}{3^{5/6}}$$

[Out]  $-(x^2 \text{AppellF1}[2/3, 1, 1/3, 5/3, x^3, -x^3/2])/(2*2^{(1/3)}) + (2*\text{ArcTan}[(1 + (2*3^{(1/3)}*x)/(2 + x^3)^{(1/3)})/\text{Sqrt}[3]])/3^{(5/6)} + \text{ArcTan}[(3^{(1/3)} + 2*(2 + x^3)^{(1/3)})/3^{(5/6)}]/3^{(5/6)} + \text{Log}[1 - x^3]/(6*3^{(1/3)}) + \text{Log}[3^{(1/3)} - (2 + x^3)^{(1/3)}]/(2*3^{(1/3)}) - \text{Log}[3^{(1/3)}*x - (2 + x^3)^{(1/3)}]/3^{(1/3)}$

**Rubi [F]** time = 0.204262, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Int[(2 + x)/((1 + x + x^2)\*(2 + x^3)^(1/3)), x]

[Out] (1 - I\*Sqrt[3])\*Defer[Int][1/((1 - I\*Sqrt[3] + 2\*x)\*(2 + x^3)^(1/3)), x] + (1 + I\*Sqrt[3])\*Defer[Int][1/((1 + I\*Sqrt[3] + 2\*x)\*(2 + x^3)^(1/3)), x]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}} dx &= \int \left( \frac{1-i\sqrt{3}}{(1-i\sqrt{3}+2x)\sqrt[3]{2+x^3}} + \frac{1+i\sqrt{3}}{(1+i\sqrt{3}+2x)\sqrt[3]{2+x^3}} \right) dx \\ &= (1-i\sqrt{3}) \int \frac{1}{(1-i\sqrt{3}+2x)\sqrt[3]{2+x^3}} dx + (1+i\sqrt{3}) \int \frac{1}{(1+i\sqrt{3}+2x)\sqrt[3]{2+x^3}} dx \end{aligned}$$

**Mathematica [F]** time = 0.137935, size = 0, normalized size = 0.

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(2 + x)/((1 + x + x^2)\*(2 + x^3)^(1/3)), x]

[Out] Integrate[(2 + x)/((1 + x + x^2)\*(2 + x^3)^(1/3)), x]

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{2+x}{x^2+x+1} \frac{1}{\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)

[Out] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")

[Out] integrate((x + 2)/((x^3 + 2)^(1/3)\*(x^2 + x + 1)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^3+2)^{\frac{2}{3}}(x+2)}{x^5+x^4+x^3+2x^2+2x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")

[Out] integral((x^3 + 2)^(2/3)\*(x + 2)/(x^5 + x^4 + x^3 + 2\*x^2 + 2\*x + 2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{\sqrt[3]{x^3+2}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x\*\*2+x+1)/(x\*\*3+2)\*\*(1/3),x)

[Out] Integral((x + 2)/((x\*\*3 + 2)\*\*(1/3)\*(x\*\*2 + x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

$$3.62 \quad \int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4} dx$$

**Optimal.** Leaf size=25

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

[Out] Log[9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4]/8

**Rubi [A]** time = 0.0226836, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1587}

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Antiderivative was successfully verified.

[In] Int[(3 - 3\*x + 30\*x^2 + 160\*x^3)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4), x]

[Out] Log[9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4]/8

Rule 1587

Int[(Pp\_)/(Qq\_), x\_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]\*Log[RemoveContent[Qq, x]])/(q\*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]\*D[Qq, x])/(q\*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4} dx = \frac{1}{8} \log(9+24x-12x^2+80x^3+320x^4)$$

**Mathematica [A]** time = 0.0081255, size = 25, normalized size = 1.

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 3\*x + 30\*x^2 + 160\*x^3)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4), x]

[Out] Log[9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4]/8

**Maple [A]** time = 0.001, size = 24, normalized size = 1.

$$\frac{\ln(320x^4 + 80x^3 - 12x^2 + 24x + 9)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)`

[Out]  $1/8*\ln(320*x^4+80*x^3-12*x^2+24*x+9)$

---

**Maxima [A]** time = 0.936682, size = 31, normalized size = 1.24

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxima")`

[Out]  $1/8*\log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)$

---

**Fricas [A]** time = 2.30158, size = 63, normalized size = 2.52

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fricas")`

[Out]  $1/8*\log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)$

---

**Sympy [A]** time = 0.097344, size = 22, normalized size = 0.88

$$\frac{\log(320x^4 + 80x^3 - 12x^2 + 24x + 9)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)`

[Out]  $\log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8$

---

**Giac [A]** time = 1.07841, size = 31, normalized size = 1.24

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac")`

[Out]  $1/8*\log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)$

$$3.63 \quad \int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx$$

**Optimal.** Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}} - \frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

[Out] -ArcTan[(7 - 40\*x)/(5\*Sqrt[11])]/(2\*Sqrt[11]) + ArcTan[(57 + 30\*x - 40\*x^2 + 800\*x^3)/(6\*Sqrt[11])]/(2\*Sqrt[11])

**Rubi [A]** time = 0.031717, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$ , Rules used = {2090}

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}} - \frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 12\*x + 20\*x^2)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4), x]

[Out] -ArcTan[(7 - 40\*x)/(5\*Sqrt[11])]/(2\*Sqrt[11]) + ArcTan[(57 + 30\*x - 40\*x^2 + 800\*x^3)/(6\*Sqrt[11])]/(2\*Sqrt[11])

#### Rule 2090

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2 + (d\_.)\*(x\_)^3 + (e\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-(C\*(2\*e\*(B\*d - 4\*A\*e) + C\*(d^2 - 4\*c\*e))), 2]}, Simp[(2\*C^2\*ArcTan[(C\*d - B\*e + 2\*C\*e\*x)/q])/q, x] - Simp[(2\*C^2\*ArcTan[(C\*(4\*B\*c\*C - 3\*B^2\*d - 4\*A\*C\*d + 12\*A\*B\*e + 4\*C\*(2\*c\*C - B\*d + 2\*A\*e)\*x + 4\*C\*(2\*C\*d - B\*e)\*x^2 + 8\*C^2\*e\*x^3))/(q\*(B^2 - 4\*A\*C))]/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2\*d + 2\*C\*(b\*C + A\*d) - 2\*B\*(c\*C + 2\*A\*e), 0] && EqQ[2\*B^2\*c\*C - 8\*a\*C^3 - B^3\*d - 4\*A\*B\*C\*d + 4\*A\*(B^2 + 2\*A\*C)\*e, 0] && NegQ[C\*(2\*e\*(B\*d - 4\*A\*e) + C\*(d^2 - 4\*c\*e))]

#### Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx = -\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}} + \frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

**Mathematica [C]** time = 0.0184698, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[320\#1^4 + 80\#1^3 - 12\#1^2 + 24\#1 + 9\&, \frac{20\#1^2 \log(x - \#1) + 12\#1 \log(x - \#1) + 3 \log(x - \#1)}{160\#1^3 + 30\#1^2 - 3\#1 + 3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 12\*x + 20\*x^2)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4), x]

[Out] RootSum[9 + 24\*#1 - 12\*#1^2 + 80\*#1^3 + 320\*#1^4 & , (3\*Log[x - #1] + 12\*Log[x - #1]\*#1 + 20\*Log[x - #1]\*#1^2)/(3 - 3\*#1 + 30\*#1^2 + 160\*#1^3) & ]/8

**Maple [A]** time = 0.024, size = 52, normalized size = 0.9

$$\frac{\sqrt{11}}{22} \arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right) + \frac{\sqrt{11}}{22} \arctan\left(-\frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} + \frac{400\sqrt{11}x^3}{33}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((20\*x^2+12\*x+3)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9), x)

[Out] 1/22\*11^(1/2)\*arctan(1/55\*(40\*x-7)\*11^(1/2))+1/22\*11^(1/2)\*arctan(-20/33\*11^(1/2)\*x^2+5/11\*11^(1/2)\*x+19/22\*11^(1/2)+400/33\*11^(1/2)\*x^3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{20x^2 + 12x + 3}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((20\*x^2+12\*x+3)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9), x, algorithm="maxima")

[Out] integrate((20\*x^2 + 12\*x + 3)/(320\*x^4 + 80\*x^3 - 12\*x^2 + 24\*x + 9), x)

**Fricas [A]** time = 1.9788, size = 158, normalized size = 2.68

$$\frac{1}{22} \sqrt{11} \arctan\left(\frac{1}{66} \sqrt{11}(800x^3 - 40x^2 + 30x + 57)\right) + \frac{1}{22} \sqrt{11} \arctan\left(\frac{1}{55} \sqrt{11}(40x - 7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((20\*x^2+12\*x+3)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9), x, algorithm="fricas")

[Out] 1/22\*sqrt(11)\*arctan(1/66\*sqrt(11)\*(800\*x^3 - 40\*x^2 + 30\*x + 57)) + 1/22\*sqrt(11)\*arctan(1/55\*sqrt(11)\*(40\*x - 7))

**Sympy [A]** time = 0.160272, size = 73, normalized size = 1.24

$$\frac{\sqrt{11} \left( 2 \operatorname{atan}\left(\frac{8\sqrt{11}x}{11} - \frac{7\sqrt{11}}{55}\right) + 2 \operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33} - \frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22}\right) \right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((20\*x\*\*2+12\*x+3)/(320\*x\*\*4+80\*x\*\*3-12\*x\*\*2+24\*x+9), x)

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**  
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac  
")
```

```
[Out] sage0*x
```

$$3.64 \quad \int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4} dx$$

**Optimal.** Leaf size=78

$$2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9) - 2\sqrt{11} \tan^{-1}\left(\frac{800x^3 - 40x^2 + 30x + 57}{6\sqrt{11}}\right) + 2\sqrt{11} \tan^{-1}\left(\frac{7 - 40x}{5\sqrt{11}}\right)$$

[Out] 2\*Sqrt[11]\*ArcTan[(7 - 40\*x)/(5\*Sqrt[11])] - 2\*Sqrt[11]\*ArcTan[(57 + 30\*x - 40\*x^2 + 800\*x^3)/(6\*Sqrt[11])] + 2\*Log[9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4]

**Rubi [A]** time = 0.07667, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {2100, 2090}

$$2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9) - 2\sqrt{11} \tan^{-1}\left(\frac{800x^3 - 40x^2 + 30x + 57}{6\sqrt{11}}\right) + 2\sqrt{11} \tan^{-1}\left(\frac{7 - 40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

[In] Int[-((84 + 576\*x + 400\*x^2 - 2560\*x^3)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4)), x]

[Out] 2\*Sqrt[11]\*ArcTan[(7 - 40\*x)/(5\*Sqrt[11])] - 2\*Sqrt[11]\*ArcTan[(57 + 30\*x - 40\*x^2 + 800\*x^3)/(6\*Sqrt[11])] + 2\*Log[9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4]

#### Rule 2100

Int[(Pm\_)/(Qn\_), x\_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[(Coeff[Pm, x, m]\*Log[Qn])/(n\*Coeff[Qn, x, n]), x] + Dist[1/(n\*Coeff[Qn, x, n]), Int[ExpandToSum[n\*Coeff[Qn, x, n]\*Pm - Coeff[Pm, x, m]\*D[Qn, x], x]/Qn, x], x] /; EqQ[m, n - 1] /; PolyQ[Pm, x] && PolyQ[Qn, x]

#### Rule 2090

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2 + (d\_.)\*(x\_)^3 + (e\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-(C\*(2\*e\*(B\*d - 4\*A\*e) + C\*(d^2 - 4\*c\*e))), 2]}, Simp[(2\*C^2\*ArcTan[(C\*d - B\*e + 2\*C\*e\*x)/q])/q, x] - Simp[(2\*C^2\*ArcTan[(C\*(4\*B\*c\*C - 3\*B^2\*d - 4\*A\*C\*d + 12\*A\*B\*e + 4\*C\*(2\*c\*C - B\*d + 2\*A\*e)\*x + 4\*C\*(2\*C\*d - B\*e)\*x^2 + 8\*C^2\*e\*x^3))/(q\*(B^2 - 4\*A\*C))]/q, x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2\*d + 2\*C\*(b\*C + A\*d) - 2\*B\*(c\*C + 2\*A\*e), 0] && EqQ[2\*B^2\*c\*C - 8\*a\*C^3 - B^3\*d - 4\*A\*B\*C\*d + 4\*A\*(B^2 + 2\*A\*C)\*e, 0] && NegQ[C\*(2\*e\*(B\*d - 4\*A\*e) + C\*(d^2 - 4\*c\*e))]

#### Rubi steps

$$\int -\frac{84 + 576x + 400x^2 - 2560x^3}{9 + 24x - 12x^2 + 80x^3 + 320x^4} dx = 2 \log(9 + 24x - 12x^2 + 80x^3 + 320x^4) - \frac{\int \frac{168960 + 675840x + 1126400x^2}{9 + 24x - 12x^2 + 80x^3 + 320x^4} dx}{1280} \\ = 2\sqrt{11} \tan^{-1}\left(\frac{7 - 40x}{5\sqrt{11}}\right) - 2\sqrt{11} \tan^{-1}\left(\frac{57 + 30x - 40x^2 + 800x^3}{6\sqrt{11}}\right) + 2 \log(9$$

**Mathematica [C]** time = 0.0201547, size = 99, normalized size = 1.27

$$\frac{1}{2} \text{RootSum} \left[ 320\#1^4 + 80\#1^3 - 12\#1^2 + 24\#1 + 9 \&, \frac{640\#1^3 \log(x - \#1) - 100\#1^2 \log(x - \#1) - 144\#1 \log(x - \#1) - 21}{160\#1^3 + 30\#1^2 - 3\#1 + 3} \right]$$

Antiderivative was successfully verified.

[In] Integrate[-((84 + 576\*x + 400\*x^2 - 2560\*x^3)/(9 + 24\*x - 12\*x^2 + 80\*x^3 + 320\*x^4)),x]

[Out] RootSum[9 + 24\*#1 - 12\*#1^2 + 80\*#1^3 + 320\*#1^4 & , (-21\*Log[x - #1] - 144\*Log[x - #1]\*#1 - 100\*Log[x - #1]\*#1^2 + 640\*Log[x - #1]\*#1^3)/(3 - 3\*#1 + 30\*#1^2 + 160\*#1^3) & ]/2

**Maple [A]** time = 0.018, size = 75, normalized size = 1.

$$2 \ln(6400x^4 + 1600x^3 - 240x^2 + 480x + 180) - 2\sqrt{11} \arctan\left(-\frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} + \frac{400\sqrt{11}x^3}{33}\right) - 2\sqrt{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2560\*x^3-400\*x^2-576\*x-84)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9),x)

[Out] 2\*ln(6400\*x^4+1600\*x^3-240\*x^2+480\*x+180)-2\*11^(1/2)\*arctan(-20/33\*11^(1/2)\*x^2+5/11\*11^(1/2)\*x+19/22\*11^(1/2)+400/33\*11^(1/2)\*x^3)-2\*11^(1/2)\*arctan(1/55\*(40\*x-7)\*11^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4 \int \frac{640x^3 - 100x^2 - 144x - 21}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2560\*x^3-400\*x^2-576\*x-84)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9),x, algorithm="maxima")

[Out] 4\*integrate((640\*x^3 - 100\*x^2 - 144\*x - 21)/(320\*x^4 + 80\*x^3 - 12\*x^2 + 24\*x + 9), x)

**Fricas [A]** time = 1.94244, size = 213, normalized size = 2.73

$$-2\sqrt{11} \arctan\left(\frac{1}{66}\sqrt{11}(800x^3 - 40x^2 + 30x + 57)\right) - 2\sqrt{11} \arctan\left(\frac{1}{55}\sqrt{11}(40x - 7)\right) + 2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2560\*x^3-400\*x^2-576\*x-84)/(320\*x^4+80\*x^3-12\*x^2+24\*x+9),x, algorithm="fricas")



```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

**Sympy [A]** time = 0.16712, size = 100, normalized size = 1.28

$$\sqrt{11} \left( -2 \operatorname{atan} \left( \frac{8\sqrt{11}x}{11} - \frac{7\sqrt{11}}{55} \right) - 2 \operatorname{atan} \left( \frac{400\sqrt{11}x^3}{33} - \frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} \right) \right) + 2 \log \left( x^4 + \frac{x^3}{4} - \frac{3x^2}{80} + \frac{3x}{40} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$\operatorname{sage}_0 x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.65 \quad \int \frac{\sqrt{1-x^4}}{1+x^4} dx$$

**Optimal.** Leaf size=49

$$\frac{1}{2} \tan^{-1} \left( \frac{x(x^2+1)}{\sqrt{1-x^4}} \right) + \frac{1}{2} \tanh^{-1} \left( \frac{x(1-x^2)}{\sqrt{1-x^4}} \right)$$

[Out] ArcTan[(x\*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x\*(1 - x^2))/Sqrt[1 - x^4]]/2

**Rubi [A]** time = 0.0072942, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {405}

$$\frac{1}{2} \tan^{-1} \left( \frac{x(x^2+1)}{\sqrt{1-x^4}} \right) + \frac{1}{2} \tanh^{-1} \left( \frac{x(1-x^2)}{\sqrt{1-x^4}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^4]/(1 + x^4), x]

[Out] ArcTan[(x\*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x\*(1 - x^2))/Sqrt[1 - x^4]]/2

#### Rule 405

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^4]/((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[-(a\*b), 4]}, Simp[(a\*ArcTan[(q\*x\*(a + q^2\*x^2))/(a\*Sqrt[a + b\*x^4]])]/(2\*c\*q), x] + Simp[(a\*ArcTanh[(q\*x\*(a - q^2\*x^2))/(a\*Sqrt[a + b\*x^4]])]/(2\*c\*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && NegQ[a\*b]

#### Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4} dx = \frac{1}{2} \tan^{-1} \left( \frac{x(1+x^2)}{\sqrt{1-x^4}} \right) + \frac{1}{2} \tanh^{-1} \left( \frac{x(1-x^2)}{\sqrt{1-x^4}} \right)$$

**Mathematica [C]** time = 0.0968655, size = 110, normalized size = 2.24

$$\frac{5x\sqrt{1-x^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; x^4, -x^4\right)}{(x^4+1)\left(2x^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; x^4, -x^4\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; x^4, -x^4\right)\right) - 5F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; x^4, -x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[1 - x^4]/(1 + x^4), x]

[Out] (-5\*x\*Sqrt[1 - x^4]\*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)\*(-5\*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^4, -x^4])))

---

**Maple [B]** time = 0.013, size = 100, normalized size = 2.

$$\frac{1}{4} \arctan\left(-\frac{1}{x}\sqrt{-x^4+1}+1\right) - \frac{1}{8} \ln\left(\left(\frac{-x^4+1}{2x^2} - \frac{1}{x}\sqrt{-x^4+1}+1\right)\left(\frac{-x^4+1}{2x^2} + \frac{1}{x}\sqrt{-x^4+1}+1\right)^{-1}\right) - \frac{1}{4} \arctan\left(\frac{1}{x}\sqrt{-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^(1/2)/(x^4+1), x)

[Out] 1/4\*arctan(-(-x^4+1)^(1/2)/x+1)-1/8\*ln((1/2\*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1)/(1/2\*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x+1))-1/4\*arctan((-x^4+1)^(1/2)/x+1)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^4+1), x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)

---

**Fricas [A]** time = 2.51574, size = 138, normalized size = 2.82

$$-\frac{1}{2} \arctan\left(\frac{\sqrt{-x^4+1}x}{x^2-1}\right) + \frac{1}{4} \log\left(-\frac{x^4-2x^2-2\sqrt{-x^4+1}x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^4+1), x, algorithm="fricas")

[Out] -1/2\*arctan(sqrt(-x^4 + 1)\*x/(x^2 - 1)) + 1/4\*log(-x^4 - 2\*x^2 - 2\*sqrt(-x^4 + 1)\*x - 1)/(x^4 + 1))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)\*\*(1/2)/(x\*\*4+1), x)

[Out] Integral(sqrt(-(x - 1)\*(x + 1)\*(x\*\*2 + 1))/(x\*\*4 + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

$$3.66 \quad \int \frac{\sqrt{1+x^4}}{1-x^4} dx$$

**Optimal.** Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

[Out] ArcTan[(Sqrt[2]\*x)/Sqrt[1 + x^4]]/(2\*Sqrt[2]) + ArcTanh[(Sqrt[2]\*x)/Sqrt[1 + x^4]]/(2\*Sqrt[2])

**Rubi [A]** time = 0.0145405, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^4]/(1 - x^4), x]

[Out] ArcTan[(Sqrt[2]\*x)/Sqrt[1 + x^4]]/(2\*Sqrt[2]) + ArcTanh[(Sqrt[2]\*x)/Sqrt[1 + x^4]]/(2\*Sqrt[2])

#### Rule 404

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^4]/((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] := Dist[a/c, Subst[Int[1/(1 - 4\*a\*b\*x^4), x], x, x/Sqrt[a + b\*x^4]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && PosQ[a\*b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x^4}}{1-x^4} dx &= \text{Subst} \left( \int \frac{1}{1-4x^4} dx, x, \frac{x}{\sqrt{1+x^4}} \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt{1+x^4}} \right) + \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt{1+x^4}} \right) \\ &= \frac{\tan^{-1} \left( \frac{\sqrt{2}x}{\sqrt{1+x^4}} \right)}{2\sqrt{2}} + \frac{\tanh^{-1} \left( \frac{\sqrt{2}x}{\sqrt{1+x^4}} \right)}{2\sqrt{2}} \end{aligned}$$

**Mathematica [C]** time = 0.0909969, size = 108, normalized size = 2.04

$$\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{(x^4-1)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]

[Out] (-5\*x\*Sqrt[1 + x^4]\*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)\*(5\*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))

**Maple [C]** time = 0.024, size = 365, normalized size = 6.9

$$-\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)\sqrt{1-ix^2}\sqrt{1+ix^2}}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\frac{1}{\sqrt{x^4+1}}-\frac{\frac{i}{2}\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)\sqrt{1-ix^2}\sqrt{1+ix^2}}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\frac{1}{\sqrt{x^4+1}}-\frac{i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)^(1/2)/(-x^4+1),x)

[Out] -1/(1/2\*2^(1/2)+1/2\*I\*2^(1/2))\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*EllipticF(x\*(1/2\*2^(1/2)+1/2\*I\*2^(1/2)),I)-1/2\*I/(1/2\*2^(1/2)+1/2\*I\*2^(1/2))\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*EllipticE(x\*(1/2\*2^(1/2)+1/2\*I\*2^(1/2)),I)-1/2\*I/(1/2\*2^(1/2)+1/2\*I\*2^(1/2))\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*(EllipticF(x\*(1/2\*2^(1/2)+1/2\*I\*2^(1/2)),I)-EllipticE(x\*(1/2\*2^(1/2)+1/2\*I\*2^(1/2)),I))-(-1)^(3/4)\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*EllipticPi((-1)^(1/4)\*x,I,(-I)^(1/2)/(-1)^(1/4))+1/2\*I/(1/2\*2^(1/2)+1/2\*I\*2^(1/2))\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*EllipticF(x\*(1/2\*2^(1/2)+1/2\*I\*2^(1/2)),I)-(-1)^(3/4)\*(1-I\*x^2)^(1/2)\*(1+I\*x^2)^(1/2)/(x^4+1)^(1/2)\*EllipticPi((-1)^(1/4)\*x,-I,(-I)^(1/2)/(-1)^(1/4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")

[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)

**Fricas [A]** time = 2.42213, size = 173, normalized size = 3.26

$$\frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) + \frac{1}{8} \sqrt{2} \log\left(\frac{x^4 + 2\sqrt{2}\sqrt{x^4+1}x + 2x^2 + 1}{x^4 - 2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")

[Out] 1/4\*sqrt(2)\*arctan(sqrt(2)\*x/sqrt(x^4 + 1)) + 1/8\*sqrt(2)\*log((x^4 + 2\*sqrt(2)\*sqrt(x^4 + 1)\*x + 2\*x^2 + 1)/(x^4 - 2\*x^2 + 1))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)\*\*(1/2)/(-x\*\*4+1),x)

[Out] -Integral(sqrt(x\*\*4 + 1)/(x\*\*4 - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4+1}}{x^4-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")

[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)

$$3.67 \quad \int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$$

**Optimal.** Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-px}}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

[Out] (Sqrt[2 - p]\*ArcTan[(Sqrt[2 - p]\*x)/Sqrt[1 + p\*x^2 + x^4]])/4 + (Sqrt[2 + p]\*ArcTanh[(Sqrt[2 + p]\*x)/Sqrt[1 + p\*x^2 + x^4]])/4

**Rubi [A]** time = 0.0944013, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2071, 1093, 205, 208}

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-px}}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + p\*x^2 + x^4]/(1 - x^4),x]

[Out] (Sqrt[2 - p]\*ArcTan[(Sqrt[2 - p]\*x)/Sqrt[1 + p\*x^2 + x^4]])/4 + (Sqrt[2 + p]\*ArcTanh[(Sqrt[2 + p]\*x)/Sqrt[1 + p\*x^2 + x^4]])/4

#### Rule 2071

Int[Sqrt[v\_]/((d\_) + (e\_)\*(x\_)^4), x\_Symbol] := With[{a = Coeff[v, x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2\*b\*x^2 + (b^2 - 4\*a\*c)\*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c\*d + a\*e, 0] && PosQ[a\*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]

#### Rule 1093

Int[((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c\*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx &= \text{Subst} \left( \int \frac{1}{1-2px^2 + (-4+p^2)x^4} dx, x, \frac{x}{\sqrt{1+px^2+x^4}} \right) \\
&= \frac{1}{4}(-4+p^2) \text{Subst} \left( \int \frac{1}{-2-p + (-4+p^2)x^2} dx, x, \frac{x}{\sqrt{1+px^2+x^4}} \right) - \frac{1}{4}(-4+p^2) \text{Subst} \left( \int \frac{1}{-2-p + (-4+p^2)x^2} dx, x, \frac{x}{\sqrt{1+px^2+x^4}} \right) \\
&= \frac{1}{4}\sqrt{2-p} \tan^{-1} \left( \frac{\sqrt{2-px}}{\sqrt{1+px^2+x^4}} \right) + \frac{1}{4}\sqrt{2+p} \tanh^{-1} \left( \frac{\sqrt{2+px}}{\sqrt{1+px^2+x^4}} \right)
\end{aligned}$$

**Mathematica [C]** time = 7.08417, size = 5727, normalized size = 76.36

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + p\*x^2 + x^4]/(1 - x^4), x]

[Out] Result too large to show

**Maple [C]** time = 0.071, size = 1512, normalized size = 20.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+p\*x^2+1)^(1/2)/(-x^4+1), x)

[Out]  $1/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} * \text{EllipticF}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)}) * p-1/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} * \text{EllipticF}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)}) - 2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} / (p+(p^2-4)^{(1/2)}) * \text{EllipticF}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)}) + 2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} / (p+(p^2-4)^{(1/2)}) * \text{EllipticE}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)}) + 1/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} * \text{EllipticPi}((-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)} * x, -1/(-1/2*p+1/2*(p^2-4)^{(1/2)}))^{(1/2)}, (-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)} / (-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}) - 1/2*p / (-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} * (1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} * \text{EllipticPi}((-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)} * x, -1/(-1/2*p+1/2*(p^2-4)^{(1/2)}))^{(1/2)}, (-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)} / (-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}) + 1/2 * (-1-p) / (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1-(-1/2*p+1/2*(p^2-4)^{(1/2)}) * x^2)^{(1/2)} * (1-(-1/2*p-1/2*(p^2-4)^{(1/2)}) * x^2)^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} * \text{EllipticF}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)}) + 2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)} * (1-(-1/2*p+1/2*(p^2-4)^{(1/2)}) * x^2)^{(1/2)} * (1-(-1/2*p-1/2*(p^2-4)^{(1/2)}) * x^2)^{(1/2)} / (x^4+p*x^2+1)^{(1/2)} / (p+(p^2-4)^{(1/2)}) * (\text{EllipticF}(1/2*x * (-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)}))^{(1/2)})$

$$\begin{aligned} & 1/2)) - \text{EllipticE}(1/2 * x * (-2 * p + 2 * (p^2 - 4)^{1/2})^{1/2}, (-1 - p * (-1/2 * p - 1/2 * (p^2 - 4)^{1/2}))^{1/2}))^{1/2} \\ & + 1/4 * (2 + p) * (-1/2 / (2 + p))^{1/2} * \text{arctanh}(1/2 * (p * x^2 + 2 * x^2 + p + 2) / (2 + p)^{1/2} / (x^4 + p * x^2 + 1)^{1/2}) \\ & + 1 / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2} * (1 - (-1/2 * p + 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} * (1 - (-1/2 * p - 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} \\ & / (x^4 + p * x^2 + 1)^{1/2} * \text{EllipticPi}((-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2} * x, 1 / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2}))^{1/2}, \\ & (-1/2 * p - 1/2 * (p^2 - 4)^{1/2})^{1/2} / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2}))^{1/2} \\ & - 1/2 * (1 + p) / (-2 * p + 2 * (p^2 - 4)^{1/2})^{1/2} * (1 - (-1/2 * p + 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} * (1 - (-1/2 * p - 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} \\ & / (x^4 + p * x^2 + 1)^{1/2} * \text{EllipticF}(1/2 * x * (-2 * p + 2 * (p^2 - 4)^{1/2})^{1/2}, (-1 - p * (-1/2 * p - 1/2 * (p^2 - 4)^{1/2}))^{1/2}))^{1/2} \\ & - 1/4 * (2 + p) * (-1/2 / (2 + p))^{1/2} * \text{arctanh}(1/2 * (p * x^2 + 2 * x^2 + p + 2) / (2 + p)^{1/2} / (x^4 + p * x^2 + 1)^{1/2}) \\ & - 1 / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2} * (1 - (-1/2 * p + 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} * (1 - (-1/2 * p - 1/2 * (p^2 - 4)^{1/2}) * x^2)^{1/2} \\ & / (x^4 + p * x^2 + 1)^{1/2} * \text{EllipticPi}((-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2} * x, 1 / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2}))^{1/2}, \\ & (-1/2 * p - 1/2 * (p^2 - 4)^{1/2})^{1/2} / (-1/2 * p + 1/2 * (p^2 - 4)^{1/2})^{1/2}))^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+p\*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")

[Out] -integrate(sqrt(x^4 + p\*x^2 + 1)/(x^4 - 1), x)

**Fricas [A]** time = 2.56811, size = 957, normalized size = 12.76

$$\left[ \frac{1}{8} \sqrt{p-2} \log \left( \frac{x^4 + 2(p-1)x^2 - 2\sqrt{x^4 + px^2 + 1}\sqrt{p-2}x + 1}{x^4 + 2x^2 + 1} \right) + \frac{1}{8} \sqrt{p+2} \log \left( \frac{x^4 + 2(p+1)x^2 + 2\sqrt{x^4 + px^2 + 1}\sqrt{p+2}x + 1}{x^4 - 2x^2 + 1} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+p\*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")

[Out] [1/8\*sqrt(p - 2)\*log((x^4 + 2\*(p - 1)\*x^2 - 2\*sqrt(x^4 + p\*x^2 + 1)\*sqrt(p - 2)\*x + 1)/(x^4 + 2\*x^2 + 1)) + 1/8\*sqrt(p + 2)\*log((x^4 + 2\*(p + 1)\*x^2 + 2\*sqrt(x^4 + p\*x^2 + 1)\*sqrt(p + 2)\*x + 1)/(x^4 - 2\*x^2 + 1)), 1/4\*sqrt(-p + 2)\*arctan(sqrt(-p + 2)\*x/sqrt(x^4 + p\*x^2 + 1)) + 1/8\*sqrt(p + 2)\*log((x^4 + 2\*(p + 1)\*x^2 + 2\*sqrt(x^4 + p\*x^2 + 1)\*sqrt(p + 2)\*x + 1)/(x^4 - 2\*x^2 + 1)), -1/4\*sqrt(-p - 2)\*arctan(sqrt(x^4 + p\*x^2 + 1)\*sqrt(-p - 2)/((p + 2)\*x)) + 1/8\*sqrt(p - 2)\*log((x^4 + 2\*(p - 1)\*x^2 - 2\*sqrt(x^4 + p\*x^2 + 1)\*sqrt(p - 2)\*x + 1)/(x^4 + 2\*x^2 + 1)), 1/4\*sqrt(-p + 2)\*arctan(sqrt(-p + 2)\*x/sqrt(x^4 + p\*x^2 + 1)) - 1/4\*sqrt(-p - 2)\*arctan(sqrt(x^4 + p\*x^2 + 1)\*sqrt(-p - 2)/((p + 2)\*x))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

$$3.68 \quad \int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx$$

**Optimal.** Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p} \tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-px(\sqrt{p^2+4+p-2x^2})}}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{\sqrt{p^2+4}+p} \tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4+px}(-\sqrt{p^2+4+p-2x^2})}}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

[Out]  $-(\text{Sqrt}[p + \text{Sqrt}[4 + p^2]] * \text{ArcTan}[(\text{Sqrt}[p + \text{Sqrt}[4 + p^2]]) * x * (p - \text{Sqrt}[4 + p^2] - 2 * x^2)] / (2 * \text{Sqrt}[2] * \text{Sqrt}[1 + p * x^2 - x^4])) / (2 * \text{Sqrt}[2]) + (\text{Sqrt}[-p + \text{Sqrt}[4 + p^2]] * \text{ArcTanh}[(\text{Sqrt}[-p + \text{Sqrt}[4 + p^2]]) * x * (p + \text{Sqrt}[4 + p^2] - 2 * x^2)] / (2 * \text{Sqrt}[2] * \text{Sqrt}[1 + p * x^2 - x^4])) / (2 * \text{Sqrt}[2])$

**Rubi [A]** time = 0.0800218, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2072}

$$\frac{\sqrt{\sqrt{p^2+4}-p} \tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-px(\sqrt{p^2+4+p-2x^2})}}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{\sqrt{p^2+4}+p} \tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4+px}(-\sqrt{p^2+4+p-2x^2})}}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[1 + p * x^2 - x^4] / (1 + x^4), x]$

[Out]  $-(\text{Sqrt}[p + \text{Sqrt}[4 + p^2]] * \text{ArcTan}[(\text{Sqrt}[p + \text{Sqrt}[4 + p^2]]) * x * (p - \text{Sqrt}[4 + p^2] - 2 * x^2)] / (2 * \text{Sqrt}[2] * \text{Sqrt}[1 + p * x^2 - x^4])) / (2 * \text{Sqrt}[2]) + (\text{Sqrt}[-p + \text{Sqrt}[4 + p^2]] * \text{ArcTanh}[(\text{Sqrt}[-p + \text{Sqrt}[4 + p^2]]) * x * (p + \text{Sqrt}[4 + p^2] - 2 * x^2)] / (2 * \text{Sqrt}[2] * \text{Sqrt}[1 + p * x^2 - x^4])) / (2 * \text{Sqrt}[2])$

**Rule 2072**

$\text{Int}[\text{Sqrt}[(a_) + (b_.) * (x_)^2 + (c_.) * (x_)^4] / ((d_) + (e_.) * (x_)^4), x\_Symbol] :> \text{With}[\{q = \text{Sqrt}[b^2 - 4 * a * c]\}, -\text{Simp}[(a * \text{Sqrt}[b + q] * \text{ArcTan}[(\text{Sqrt}[b + q] * x * (b - q + 2 * c * x^2)] / (2 * \text{Sqrt}[2] * \text{Rt}[-(a * c), 2] * \text{Sqrt}[a + b * x^2 + c * x^4]))] / (2 * \text{Sqrt}[2] * \text{Rt}[-(a * c), 2] * d), x] + \text{Simp}[(a * \text{Sqrt}[-b + q] * \text{ArcTanh}[(\text{Sqrt}[-b + q] * x * (b + q + 2 * c * x^2)] / (2 * \text{Sqrt}[2] * \text{Rt}[-(a * c), 2] * \text{Sqrt}[a + b * x^2 + c * x^4]))] / (2 * \text{Sqrt}[2] * \text{Rt}[-(a * c), 2] * d), x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c * d + a * e, 0] \&\& \text{NegQ}[a * c]$

**Rubi steps**

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx = -\frac{\sqrt{p+\sqrt{4+p^2}} \tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}x(p-\sqrt{4+p^2}-2x^2)}}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+p^2}} \tanh^{-1}\left(\frac{\sqrt{-p+\sqrt{4+p^2}x(p+\sqrt{4+p^2}-2x^2)}}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}$$

**Mathematica [C]** time = 0.432111, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4-p}} + 2\sqrt{1 - \frac{2x^2}{\sqrt{p^2+4+p}}}} \left(2i \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4-p}}}x\right), \frac{p-\sqrt{p^2+4}}{\sqrt{p^2+4+p}}\right) - (p+2i)\Pi\left(\frac{1}{2}i(p-\sqrt{p^2+4}); i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4-p}}}x\right)\right)\right)}{4\sqrt{\frac{1}{\sqrt{p^2+4-p}}}\sqrt{px^2-x^4+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + p\*x^2 - x^4]/(1 + x^4),x]

[Out] (Sqrt[2 + (4\*x^2)/(-p + Sqrt[4 + p^2])]\*Sqrt[1 - (2\*x^2)/(p + Sqrt[4 + p^2])])\*((2\*I)\*EllipticF[I\*ArcSinh[Sqrt[2]\*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]]\*x], (p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])) - (2\*I + p)\*EllipticPi[(I/2)\*(p - Sqrt[4 + p^2]), I\*ArcSinh[Sqrt[2]\*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]]\*x], (p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])] + (-2\*I + p)\*EllipticPi[(I/2)\*(-p + Sqrt[4 + p^2]), I\*ArcSinh[Sqrt[2]\*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]]\*x], (p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])])/(4\*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]\*Sqrt[1 + p\*x^2 - x^4])

**Maple [B]** time = 0.072, size = 456, normalized size = 2.7

$$\frac{\sqrt{2}}{32} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{p^2 + 4} \ln \left( \frac{-x^4 + px^2 + 1}{x^2} + \frac{\sqrt{2}}{x} \sqrt{-x^4 + px^2 + 1} \sqrt{p + \sqrt{p^2 + 4}} + \sqrt{p^2 + 4} \right) - \frac{\sqrt{2}}{4} \arctan \left( \frac{1}{2} \left( 2 \sqrt{p + \sqrt{p^2 + 4}} \sqrt{p^2 + 4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+p\*x^2+1)^(1/2)/(x^4+1),x)

[Out] 1/32\*2^(1/2)\*(p+(p^2+4)^(1/2))^(1/2)\*(p^2+4)^(1/2)\*ln((-x^4+p\*x^2+1)/x^2+(-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x\*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/4\*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)\*arctan(1/2\*(2\*(-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x+2\*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/32\*2^(1/2)\*(p+(p^2+4)^(1/2))^(1/2)\*p\*ln((-x^4+p\*x^2+1)/x^2+(-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x\*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/32\*2^(1/2)\*(p+(p^2+4)^(1/2))^(1/2)\*(p^2+4)^(1/2)\*ln((-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x\*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p\*x^2+1)/x^2-(p^2+4)^(1/2))+1/4\*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)\*arctan(1/2\*(2\*(p+(p^2+4)^(1/2))^(1/2)-2\*(-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x)/(-p+(p^2+4)^(1/2))^(1/2))+1/32\*2^(1/2)\*(p+(p^2+4)^(1/2))^(1/2)\*p\*ln((-x^4+p\*x^2+1)^(1/2)\*2^(1/2)/x\*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p\*x^2+1)/x^2-(p^2+4)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+p\*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + p\*x^2 + 1)/(x^4 + 1), x)

**Fricas [B]** time = 25.3474, size = 6283, normalized size = 36.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+p\*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")

[Out] 
$$-1/32*(8*\sqrt{2}*\sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*(p^2 + 4)^{3/4}*\arctan(1/4$$

$$*(2*(p^3 + 4*p)*x^{12} - 2*(p^4 - 2*p^2 - 24)*x^{10} - 20*(p^3 + 4*p)*x^8 + 2*($$

$$3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +$$

$$4)*x^{12} - (p^3 + 4*p)*x^{10} - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^{12} - (p$$

$$^2 - 6)*x^{10} - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*\sqrt{p^2 + 4})$$

$$*\sqrt{p^2 + 4} + 2*((p^2 + 4)*x^{12} - (p^3 + 4*p)*x^{10} - (p^3 + 4*p)*x^6 - ($$

$$p^2 + 4)*x^4)*\sqrt{p^2 + 4} + \sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*(2*(\sqrt{2})*($$

$$x^9 - p*x^7 - x^5)*\sqrt{-x^4 + p*x^2 + 1}*\sqrt{p^2 + 4} + \sqrt{2}*(x^{11} - 2$$

$$*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*\sqrt{-x^4 + p*x^2 + 1})*(p^2 + 4)^{3/4}$$

$$- (\sqrt{2})*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*\sqrt{-x^4 + p*x$$

$$^2 + 1}*\sqrt{p^2 + 4} + \sqrt{2}*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 +$$

$$4*p)*x^3 - (p^2 + 4)*x)*\sqrt{-x^4 + p*x^2 + 1})*(p^2 + 4)^{1/4}) - (2*((p^$$

$$3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*\sqrt{-x^4 + p*x^2 + 1}*\sqrt{$$

$$p^2 + 4} + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 -$$

$$32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*\sqrt{-x^4 + p*x^2 + 1} - 2*((p*x^$$

$$10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*\sqrt{-x^4 + p*x^2 + 1}$$

$$)*\sqrt{p^2 + 4} + ((p^2 + 4)*x^{10} - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^$$

$$3 + 4*p)*x^4 + (p^2 + 4)*x^2)*\sqrt{-x^4 + p*x^2 + 1})*\sqrt{p^2 + 4} - \sqrt{($$

$$p^2 + \sqrt{p^2 + 4}}*p + 4)*((\sqrt{2})*(x^{11} - p*x^9 - p*x^5 - x^3)*\sqrt{p^2$$

$$+ 4} + \sqrt{2}*(2*x^{13} - 5*p*x^{11} + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*$$

$$x^5 - p*x^3))*(p^2 + 4)^{3/4} - (\sqrt{2})*(p*x^{11} - (p^2 - 6)*x^9 - 10*p*x^7$$

$$+ (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*\sqrt{p^2 + 4} + \sqrt{2}*((p^2 + 4)*x^{11}$$

$$- (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^{1/4}))*\sqrt{$$

$$-((p^2 + 4)*x^4 - (p^2 + 4)^{3/2}*x^2 - \sqrt{2}*\sqrt{-x^4 + p*x^2 + 1})*\sqrt{$$

$$p^2 + \sqrt{p^2 + 4}}*p + 4)*(p^2 + 4)^{3/4}*x - (p^3 + 4*p)*x^2 - p^2 -$$

$$4)/((p^2 + 4)*x^4 + p^2 + 4))/((p^2 + 4)*x^{12} - 3*(p^3 + 4*p)*x^{10} + (2*p^$$

$$4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 +$$

$$4*p)*x^2 - p^2 - 4)) + 8*\sqrt{2}*\sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*(p^2 + 4)^{3/4}$$

$$*\arctan(-1/4*(2*(p^3 + 4*p)*x^{12} - 2*(p^4 - 2*p^2 - 24)*x^{10} - 20*(p^3$$

$$+ 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)$$

$$*x^2 - 2*((p^2 + 4)*x^{12} - (p^3 + 4*p)*x^{10} - (p^3 + 4*p)*x^6 - (p^2 + 4)*x$$

$$^4 + (p*x^{12} - (p^2 - 6)*x^{10} - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^$$

$$2)*\sqrt{p^2 + 4})*\sqrt{p^2 + 4} + 2*((p^2 + 4)*x^{12} - (p^3 + 4*p)*x^{10} - (p$$

$$^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*\sqrt{p^2 + 4} - \sqrt{p^2 + \sqrt{p^2 + 4}}*p +$$

$$4)*(2*(\sqrt{2})*(x^9 - p*x^7 - x^5)*\sqrt{-x^4 + p*x^2 + 1}*\sqrt{p^2 + 4} +$$

$$\sqrt{2}*(x^{11} - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*\sqrt{-x^4 + p*x^2$$

$$+ 1})*(p^2 + 4)^{3/4} - (\sqrt{2})*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x$$

$$)*\sqrt{-x^4 + p*x^2 + 1}*\sqrt{p^2 + 4} + \sqrt{2}*((p^2 + 4)*x^9 + 4*(p^2 +$$

$$4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*\sqrt{-x^4 + p*x^2 + 1})*(p^2 + 4)$$

$$^{1/4}) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*\sqrt{-x^$$

$$4 + p*x^2 + 1}*\sqrt{p^2 + 4} + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6$$

$$- (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*\sqrt{-x^4 + p*x^$$

$$2 + 1} - 2*((p*x^{10} - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*\sqrt{$$

$$-x^4 + p*x^2 + 1}*\sqrt{p^2 + 4} + ((p^2 + 4)*x^{10} - (p^3 + 4*p)*x^8 - 2*(p$$

$$^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*\sqrt{-x^4 + p*x^2 + 1})*\sqrt{$$

$$p^2 + 4} + \sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*((\sqrt{2})*(x^{11} - p*x^9 - p*x^5$$

$$- x^3)*\sqrt{p^2 + 4} + \sqrt{2}*(2*x^{13} - 5*p*x^{11} + (3*p^2 - 8)*x^9 + 10*p$$

$$*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^{3/4} - (\sqrt{2})*(p*x^{11} - (p^2 -$$

$$6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*\sqrt{p^2 + 4} + \sqrt{2}$$

$$)*((p^2 + 4)*x^{11} - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^$$

$$2 + 4)^{1/4}))*\sqrt{-((p^2 + 4)*x^4 - (p^2 + 4)^{3/2}*x^2 + \sqrt{2}*\sqrt{-x$$

$$^4 + p*x^2 + 1})*\sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*(p^2 + 4)^{3/4}*x - (p^3 +$$

$$4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4))/((p^2 + 4)*x^{12} - 3*(p^3 +$$

$$4*p)*x^{10} + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)$$

$$*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (\sqrt{2}*\sqrt{p^2 + 4})*p - \sqrt{2}*($$

$$p^2 + 4)*\sqrt{p^2 + \sqrt{p^2 + 4}}*p + 4)*(p^2 + 4)^{1/4}*\log(-((p^2 + 4)*$$

$$x^4 - (p^2 + 4)^{3/2}*x^2 + \sqrt{2}*\sqrt{-x^4 + p*x^2 + 1})*\sqrt{p^2 + \sqrt{p^2 +$$

$$p^2 + 4)p + 4)(p^2 + 4)^{3/4}x - (p^3 + 4p)x^2 - p^2 - 4)/((p^2 + 4)x^4 + p^2 + 4)) + (\sqrt{2})\sqrt{p^2 + 4})p - \sqrt{2})(p^2 + 4))\sqrt{p^2 + 4})\sqrt{p^2 + 4})p + 4)(p^2 + 4)^{1/4})\log(-((p^2 + 4)x^4 - (p^2 + 4)^{3/2})x^2 - \sqrt{2})\sqrt{-x^4 + px^2 + 1})\sqrt{p^2 + 4})\sqrt{p^2 + 4})p + 4)(p^2 + 4)^{3/4}x - (p^3 + 4p)x^2 - p^2 - 4)/((p^2 + 4)x^4 + p^2 + 4)))/(p^2 + 4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+p\*x\*\*2+1)\*\*(1/2)/(x\*\*4+1),x)

[Out] Integral(sqrt(p\*x\*\*2 - x\*\*4 + 1)/(x\*\*4 + 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+p\*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + p\*x^2 + 1)/(x^4 + 1), x)

$$3.69 \quad \int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}} dx$$

**Optimal.** Leaf size=80

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} - b \tan^{-1}\left(\sqrt[4]{x^2-1}\right) + b \tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

[Out] (a\*ArcTan[x/(Sqrt[2]\*(-1 + x^2)^(1/4))])/(2\*Sqrt[2]) - b\*ArcTan[(-1 + x^2)^(1/4)] + (a\*ArcTanh[x/(Sqrt[2]\*(-1 + x^2)^(1/4))])/(2\*Sqrt[2]) + b\*ArcTanh[(-1 + x^2)^(1/4)]

**Rubi [A]** time = 0.0398388, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} - b \tan^{-1}\left(\sqrt[4]{x^2-1}\right) + b \tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((2 - x^2)\*(-1 + x^2)^(1/4)),x]

[Out] (a\*ArcTan[x/(Sqrt[2]\*(-1 + x^2)^(1/4))])/(2\*Sqrt[2]) - b\*ArcTan[(-1 + x^2)^(1/4)] + (a\*ArcTanh[x/(Sqrt[2]\*(-1 + x^2)^(1/4))])/(2\*Sqrt[2]) + b\*ArcTanh[(-1 + x^2)^(1/4)]

#### Rule 1010

Int[((g\_) + (h\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

#### Rule 398

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[-(b^2/a), 4]}, Simp[(b\*ArcTan[(q\*x)/(Sqrt[2]\*(a + b\*x^2)^(1/4))])]/(2\*Sqrt[2]\*a\*d\*q), x] + Simp[(b\*ArcTanh[(q\*x)/(Sqrt[2]\*(a + b\*x^2)^(1/4))])]/(2\*Sqrt[2]\*a\*d\*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] && NegQ[b^2/a]

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ



[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{a + bx}{(2 - x^2)\sqrt[4]{-1 + x^2}} dx &= a \int \frac{1}{(2 - x^2)\sqrt[4]{-1 + x^2}} dx + b \int \frac{x}{(2 - x^2)\sqrt[4]{-1 + x^2}} dx \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b \text{Subst}\left(\int \frac{1}{(2-x)\sqrt[4]{-1+x}} dx, x, x^2\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b) \text{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt[4]{-1+x^2}\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[4]{-1+x^2}\right) - b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[4]{-1+x^2}\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b \tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b \tanh^{-1}\left(\sqrt[4]{-1+x^2}\right) \end{aligned}$$

**Mathematica [C]** time = 0.260884, size = 157, normalized size = 1.96

$$\frac{x \left( bx\sqrt[4]{1-x^2}(x^2-2) F_1\left(1; \frac{1}{4}, 1; 2; x^2, \frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2}, \frac{1}{4}, 1; \frac{3}{2}; x^2, \frac{x^2}{2}\right)}{x^2 \left( 2F_1\left(\frac{3}{2}, \frac{1}{4}, 2; \frac{5}{2}; x^2, \frac{x^2}{2}\right) + F_1\left(\frac{3}{2}, \frac{5}{4}, 1; \frac{5}{2}; x^2, \frac{x^2}{2}\right) \right) + 6F_1\left(\frac{1}{2}, \frac{1}{4}, 1; \frac{3}{2}; x^2, \frac{x^2}{2}\right)} \right)}{4(x^2-2)\sqrt[4]{x^2-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((2 - x^2)\*(-1 + x^2)^(1/4)), x]

[Out] (x\*(b\*x\*(1 - x^2)^(1/4)\*(-2 + x^2)\*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24\*a\*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2]))/(6\*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2\*(2\*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5

/4, 1, 5/2, x^2, x^2/2]])))/(4\*(-2 + x^2)\*(-1 + x^2)^(1/4))

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 2} \frac{1}{\sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)

[Out] int((b\*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{bx + a}{(x^2 - 1)^{\frac{1}{4}}(x^2 - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")

[Out] -integrate((b\*x + a)/((x^2 - 1)^(1/4)\*(x^2 - 2)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{a}{x^2 \sqrt[4]{x^2 - 1} - 2 \sqrt[4]{x^2 - 1}} dx - \int \frac{bx}{x^2 \sqrt[4]{x^2 - 1} - 2 \sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x\*\*2+2)/(x\*\*2-1)\*\*(1/4),x)

[Out] -Integral(a/(x\*\*2\*(x\*\*2 - 1)\*\*(1/4) - 2\*(x\*\*2 - 1)\*\*(1/4)), x) - Integral(b\*x/(x\*\*2\*(x\*\*2 - 1)\*\*(1/4) - 2\*(x\*\*2 - 1)\*\*(1/4)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 - 1)^{\frac{1}{4}}(x^2 - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

$$3.70 \quad \int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} dx$$

**Optimal.** Leaf size=88

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + b \tan^{-1}\left(\sqrt[4]{-x^2-1}\right) - b \tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

[Out] (a\*ArcTan[x/(Sqrt[2]\*(-1 - x^2)^(1/4))])/(2\*Sqrt[2]) + b\*ArcTan[(-1 - x^2)^(1/4)] + (a\*ArcTanh[x/(Sqrt[2]\*(-1 - x^2)^(1/4))])/(2\*Sqrt[2]) - b\*ArcTanh[(-1 - x^2)^(1/4)]

**Rubi [A]** time = 0.0475753, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + b \tan^{-1}\left(\sqrt[4]{-x^2-1}\right) - b \tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((-1 - x^2)^(1/4)\*(2 + x^2)),x]

[Out] (a\*ArcTan[x/(Sqrt[2]\*(-1 - x^2)^(1/4))])/(2\*Sqrt[2]) + b\*ArcTan[(-1 - x^2)^(1/4)] + (a\*ArcTanh[x/(Sqrt[2]\*(-1 - x^2)^(1/4))])/(2\*Sqrt[2]) - b\*ArcTanh[(-1 - x^2)^(1/4)]

#### Rule 1010

Int[((g\_) + (h\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

#### Rule 398

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[-(b^2/a), 4]}, Simp[(b\*ArcTan[(q\*x)/(Sqrt[2]\*(a + b\*x^2)^(1/4))])]/(2\*Sqrt[2]\*a\*d\*q), x] + Simp[(b\*ArcTanh[(q\*x)/(Sqrt[2]\*(a + b\*x^2)^(1/4))])]/(2\*Sqrt[2]\*a\*d\*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] & NegQ[b^2/a]

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} dx &= a \int \frac{1}{\sqrt[4]{-1-x^2}(2+x^2)} dx + b \int \frac{x}{\sqrt[4]{-1-x^2}(2+x^2)} dx \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b \operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{-1-x}(2+x)} dx, x, x^2\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b) \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt[4]{-1-x^2}\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[4]{-1-x^2}\right) + b \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt[4]{-1-x^2}\right) \\ &= \frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b \tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b \tanh^{-1}\left(\sqrt[4]{-1-x^2}\right) \end{aligned}$$

**Mathematica [C]** time = 0.251017, size = 162, normalized size = 1.84

$$\frac{x \left( bx \sqrt[4]{x^2+1} F_1\left(1; \frac{1}{4}, 1; 2; -x^2, -\frac{x^2}{2}\right) - \frac{24a F_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}{(x^2+2) \left( x^2 \left( 2F_1\left(\frac{3}{2}; \frac{1}{4}, 2; \frac{5}{2}; -x^2, -\frac{x^2}{2}\right) + F_1\left(\frac{3}{2}; \frac{5}{4}, 1; \frac{5}{2}; -x^2, -\frac{x^2}{2}\right) \right) - 6F_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)} \right)}{4\sqrt[4]{-x^2-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((-1 - x^2)^(1/4)\*(2 + x^2)), x]

[Out] (x\*(b\*x\*(1 + x^2)^(1/4)\*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2] - (24\*a\*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2]))/((2 + x^2)\*(-6\*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2\*(2\*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -x^2/2] + Appell

F1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])))))/(4\*(-1 - x^2)^(1/4))

---

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 2} \frac{1}{\sqrt[4]{-x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)

[Out] int((b\*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(-x^2 - 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")

[Out] integrate((b\*x + a)/((x^2 + 2)\*(-x^2 - 1)^(1/4)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[4]{-x^2 - 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x\*\*2-1)\*\*(1/4)/(x\*\*2+2),x)

[Out] Integral((a + b\*x)/((-x\*\*2 - 1)\*\*(1/4)\*(x\*\*2 + 2)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(-x^2 - 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

$$3.71 \quad \int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)} dx$$

**Optimal.** Leaf size=149

$$\frac{1}{2}a \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right) + \frac{b \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

[Out] (b\*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]\*(1 - x^2)^(1/4))])/Sqrt[2] + (a\*ArcTan[(1 - Sqrt[1 - x^2])/(x\*(1 - x^2)^(1/4))])/2 + (b\*ArcTanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]\*(1 - x^2)^(1/4))])/Sqrt[2] + (a\*ArcTanh[(1 + Sqrt[1 - x^2])/(x\*(1 - x^2)^(1/4))])/2

**Rubi [A]** time = 0.0450532, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1010, 397, 439}

$$\frac{1}{2}a \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right) + \frac{b \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((1 - x^2)^(1/4)\*(2 - x^2)), x]

[Out] (b\*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]\*(1 - x^2)^(1/4))])/Sqrt[2] + (a\*ArcTan[(1 - Sqrt[1 - x^2])/(x\*(1 - x^2)^(1/4))])/2 + (b\*ArcTanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]\*(1 - x^2)^(1/4))])/Sqrt[2] + (a\*ArcTanh[(1 + Sqrt[1 - x^2])/(x\*(1 - x^2)^(1/4))])/2

### Rule 1010

Int[((g\_) + (h\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

### Rule 397

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[b^2/a, 4]}, -Simp[(b\*ArcTan[(b + q^2\*Sqrt[a + b\*x^2])/(q^3\*x\*(a + b\*x^2)^(1/4))])]/(2\*a\*d\*q), x] - Simp[(b\*ArcTanh[(b - q^2\*Sqrt[a + b\*x^2])/(q^3\*x\*(a + b\*x^2)^(1/4))])]/(2\*a\*d\*q), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] && PosQ[b^2/a]

### Rule 439

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := -Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b\*x^2])/(Sqrt[2]\*Rt[a, 4]\*(a + b\*x^2)^(1/4))])]/(Sqrt[2]\*Rt[a, 4]\*d), x] - Simp[(1\*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b\*x^2])/(Sqrt[2]\*Rt[a, 4]\*(a + b\*x^2)^(1/4))])]/(Sqrt[2]\*Rt[a, 4]\*d), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] && PosQ[a]

### Rubi steps



$$\int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)} dx = a \int \frac{1}{\sqrt[4]{1-x^2}(2-x^2)} dx + b \int \frac{x}{\sqrt[4]{1-x^2}(2-x^2)} dx$$

$$= \frac{b \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b \tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a \tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)$$

**Mathematica [C]** time = 0.203389, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1; \frac{1}{4}, 1; 2; x^2, \frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; x^2, \frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}(x^2-2)\left(x^2\left(2F_1\left(\frac{3}{2}; \frac{1}{4}, 2; \frac{5}{2}; x^2, \frac{x^2}{2}\right) + F_1\left(\frac{3}{2}; \frac{5}{4}, 1; \frac{5}{2}; x^2, \frac{x^2}{2}\right)\right) + 6F_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; x^2, \frac{x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((1 - x^2)^(1/4)\*(2 - x^2)),x]

[Out] (b\*x^2\*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6\*a\*x\*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)\*(-2 + x^2)\*(6\*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2\*(2\*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])))

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+2} \frac{1}{\sqrt[4]{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)

[Out] int((b\*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")

[Out] -integrate((b\*x + a)/((x^2 - 2)\*(-x^2 + 1)^(1/4)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2 \sqrt[4]{1-x^2} - 2\sqrt[4]{1-x^2}} dx - \int \frac{bx}{x^2 \sqrt[4]{1-x^2} - 2\sqrt[4]{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 - 2)(-x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

$$3.72 \quad \int \frac{a+bx}{\sqrt[4]{1+x^2}(2+x^2)} dx$$

**Optimal.** Leaf size=135

$$-\frac{1}{2}a \tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right) - \frac{1}{2}a \tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right) - \frac{b \tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

[Out]  $-\left(\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2}\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}\right) - \left(\frac{a \operatorname{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x\left(1+x^2\right)^{1/4}}\right]}{2}\right) - \left(\frac{a \operatorname{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x\left(1+x^2\right)^{1/4}}\right]}{2}\right) - \left(\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2}\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}\right)$

**Rubi [A]** time = 0.035622, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1010, 397, 439}

$$-\frac{1}{2}a \tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right) - \frac{1}{2}a \tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right) - \frac{b \tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)/((1 + x^2)^(1/4)\*(2 + x^2)), x]

[Out]  $-\left(\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2}\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}\right) - \left(\frac{a \operatorname{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x\left(1+x^2\right)^{1/4}}\right]}{2}\right) - \left(\frac{a \operatorname{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x\left(1+x^2\right)^{1/4}}\right]}{2}\right) - \left(\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2}\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}\right)$

#### Rule 1010

Int[((g\_) + (h\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_)\*((d\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Dist[g, Int[(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] + Dist[h, Int[x\*(a + c\*x^2)^p\*(d + f\*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

#### Rule 397

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[b^2/a, 4]}, -Simp[(b\*ArcTan[(b + q^2\*Sqrt[a + b\*x^2])/(q^3\*x\*(a + b\*x^2)^(1/4))])]/(2\*a\*d\*q), x] - Simp[(b\*ArcTanh[(b - q^2\*Sqrt[a + b\*x^2])/(q^3\*x\*(a + b\*x^2)^(1/4))])]/(2\*a\*d\*q), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] && PosQ[b^2/a]

#### Rule 439

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^2)^(1/4)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := -Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b\*x^2])/(Sqrt[2]\*Rt[a, 4]\*(a + b\*x^2)^(1/4))])]/(Sqrt[2]\*Rt[a, 4]\*d), x] - Simp[(1\*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b\*x^2])/(Sqrt[2]\*Rt[a, 4]\*(a + b\*x^2)^(1/4))])]/(Sqrt[2]\*Rt[a, 4]\*d), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c - 2\*a\*d, 0] && PosQ[a]

#### Rubi steps

$$\int \frac{a + bx}{\sqrt[4]{1+x^2}(2+x^2)} dx = a \int \frac{1}{\sqrt[4]{1+x^2}(2+x^2)} dx + b \int \frac{x}{\sqrt[4]{1+x^2}(2+x^2)} dx$$

$$= -\frac{b \tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a \tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a \tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b \tanh^{-1}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}$$

**Mathematica [C]** time = 0.174923, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1; \frac{1}{4}, 1; 2; -x^2, -\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}{\sqrt[4]{x^2+1}(x^2+2)\left(x^2\left(2F_1\left(\frac{3}{2}; \frac{1}{4}, 2; \frac{5}{2}; -x^2, -\frac{x^2}{2}\right) + F_1\left(\frac{3}{2}; \frac{5}{4}, 1; \frac{5}{2}; -x^2, -\frac{x^2}{2}\right)\right) - 6F_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x)/((1 + x^2)^(1/4)\*(2 + x^2)), x]

[Out] (b\*x^2\*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/4 - (6\*a\*x\*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/((1 + x^2)^(1/4)\*(2 + x^2)\*(-6\*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2\*(2\*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])))

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 2} \frac{1}{\sqrt[4]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)/(x^2+1)^(1/4)/(x^2+2), x)

[Out] int((b\*x+a)/(x^2+1)^(1/4)/(x^2+2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)/(x^2+1)^(1/4)/(x^2+2), x, algorithm="maxima")

[Out] integrate((b\*x + a)/((x^2 + 2)\*(x^2 + 1)^(1/4)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[4]{x^2 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

$$3.73 \quad \int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx$$

**Optimal.** Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2x})}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2x+1}}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

[Out] -ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*x))/Sqrt[1 - x^3]]/(3\*2^(2/3)\*Sqrt[3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3\*2^(2/3)\*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)\*x)/Sqrt[1 - x^3]]/(3\*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9\*2^(2/3))

**Rubi [A]** time = 0.0192024, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2x})}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2x+1}}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 - x^3]\*(4 - x^3)),x]

[Out] -ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*x))/Sqrt[1 - x^3]]/(3\*2^(2/3)\*Sqrt[3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3\*2^(2/3)\*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)\*x)/Sqrt[1 - x^3]]/(3\*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9\*2^(2/3))

#### Rule 484

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^3)\*Sqrt[(c\_) + (d\_.)\*(x\_)^3]), x\_Symbol] :> With[{q = Rt[d/c, 3]}, Simp[(q\*ArcTanh[Sqrt[c + d\*x^3]/Rt[c, 2]])/(9\*2^(2/3)\*b\*Rt[c, 2]), x] + (-Simp[(q\*ArcTanh[(Rt[c, 2]\*(1 - 2^(1/3)\*q\*x))/Sqrt[c + d\*x^3]])/(3\*2^(2/3)\*b\*Rt[c, 2]), x] + Simp[(q\*ArcTan[Sqrt[c + d\*x^3]/(Sqrt[3]\*Rt[c, 2])])/(3\*2^(2/3)\*Sqrt[3]\*b\*Rt[c, 2]), x] - Simp[(q\*ArcTan[(Sqrt[3]\*Rt[c, 2]\*(1 + 2^(1/3)\*q\*x))/Sqrt[c + d\*x^3]])/(3\*2^(2/3)\*Sqrt[3]\*b\*Rt[c, 2]), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[4\*b\*c - a\*d, 0] && PosQ[c]

#### Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2x})}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} - \frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2x}}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

**Mathematica [C]** time = 0.0217669, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 - x^3]\*(4 - x^3)),x]

[Out] (x^2\*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8

**Maple [C]** time = 0.099, size = 164, normalized size = 1.3

$$\frac{i}{36}\sqrt{2}\sum_{\alpha=\text{RootOf}(Z^3-4)}\alpha^2(-2\alpha^2+\alpha+1+i\sqrt{3}(1-\alpha))\sqrt{\frac{i}{2}(2x+1-i\sqrt{3})}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)

[Out] 1/36\*I\*2^(1/2)\*sum(\_alpha^2\*(1/2\*I\*(2\*x+1-I\*3^(1/2)))^(1/2)\*((-1+x)/(I\*3^(1/2)-3))^(1/2)\*(-1/2\*I\*(2\*x+1+I\*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)\*(-2\*\_alpha^2+\_alpha+1+I\*3^(1/2)\*(1-\_alpha))\*EllipticPi(1/3\*3^(1/2)\*(I\*(x+1/2-1/2\*I\*3^(1/2))\*3^(1/2))^(1/2),1/2\*\_alpha-1/3\*I\*\_alpha^2\*3^(1/2)-1/2+1/6\*I\*\_alpha\*3^(1/2)+1/6\*I\*3^(1/2),(I\*3^(1/2)/(-3/2+1/2\*I\*3^(1/2)))^(1/2),\_alpha=RootOf(\_Z^3-4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{(x^3-4)\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x/((x^3 - 4)\*sqrt(-x^3 + 1)), x)

**Fricas [B]** time = 5.56681, size = 3299, normalized size = 25.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] -1/31104\*432^(5/6)\*sqrt(3)\*log(144\*(36\*x^9 - 8208\*x^6 + 9504\*x^3 - 648\*2^(2/3)\*(x^8 - 5\*x^5 + 4\*x^2) + (2592\*x^6 - 2592\*x^3 - 432^(5/6)\*sqrt(3)\*(x^7 - 26\*x^4 + 16\*x) - 216\*432^(1/6)\*sqrt(3)\*(7\*x^5 - 4\*x^2))\*sqrt(-x^3 + 1) + 3888\*2^(1/3)\*(x^7 - x^4) - 2304)/(x^9 - 12\*x^6 + 48\*x^3 - 64)) - 1/31104\*432^(5/6)\*sqrt(3)\*log(36\*(36\*x^9 - 8208\*x^6 + 9504\*x^3 - 648\*2^(2/3)\*(x^8 - 5\*x^5 + 4\*x^2) + (2592\*x^6 - 2592\*x^3 - 432^(5/6)\*sqrt(3)\*(x^7 - 26\*x^4 + 16\*x) - 216\*432^(1/6)\*sqrt(3)\*(7\*x^5 - 4\*x^2))\*sqrt(-x^3 + 1) + 3888\*2^(1/3)\*(x^7 - x^4) - 2304)/(x^9 - 12\*x^6 + 48\*x^3 - 64)) + 1/31104\*432^(5/6)\*sqrt(3)\*log(144\*(36\*x^9 - 8208\*x^6 + 9504\*x^3 - 648\*2^(2/3)\*(x^8 - 5\*x^5 + 4\*x^2)

$$\begin{aligned}
& - (2592x^6 - 2592x^3 - 432^{5/6}\sqrt{3})(x^7 - 26x^4 + 16x) - 216 \cdot 432^{1/6} \sqrt{3} (7x^5 - 4x^2) \sqrt{-x^3 + 1} + 3888 \cdot 2^{1/3} (x^7 - x^4 - 2304) / (x^9 - 12x^6 + 48x^3 - 64) + 1/31104 \cdot 432^{5/6} \sqrt{3} \log(36 \cdot (36x^9 - 8208x^6 + 9504x^3 - 648 \cdot 2^{2/3})(x^8 - 5x^5 + 4x^2) - (2592x^6 - 2592x^3 - 432^{5/6}\sqrt{3})(x^7 - 26x^4 + 16x) - 216 \cdot 432^{1/6} \sqrt{3} (7x^5 - 4x^2) \sqrt{-x^3 + 1} + 3888 \cdot 2^{1/3} (x^7 - x^4 - 2304) / (x^9 - 12x^6 + 48x^3 - 64)) - 1/1944 \cdot 432^{5/6} \arctan(1/216 \sqrt{-x^3 + 1} (72 \cdot 432^{1/6} x^2 + 432^{5/6} x + 72 \sqrt{3})) / (2x^3 - 1) + 1/3888 \cdot 432^{5/6} \arctan(-1/648 (6 \sqrt{-x^3 + 1} (432^{5/6} (x^4 + 2x) - 36 \sqrt{3} (x^3 - 4) + 18 \cdot 432^{1/6} (x^5 + 8x^2)) + (108 \sqrt{3} \cdot 2^{2/3} (x^5 - x^2) - 216 \sqrt{3} \cdot 2^{1/3} (x^4 - x) - 108 \sqrt{3} (x^6 - x^3) - \sqrt{-x^3 + 1} (432^{5/6} (2x^4 + x) - 36 \sqrt{3} (5x^3 - 8) - 18 \cdot 432^{1/6} (x^5 - 10x^2)))) \sqrt{(36x^9 - 8208x^6 + 9504x^3 - 648 \cdot 2^{2/3})(x^8 - 5x^5 + 4x^2) + (2592x^6 - 2592x^3 - 432^{5/6}\sqrt{3})(x^7 - 26x^4 + 16x) - 216 \cdot 432^{1/6} \sqrt{3} (7x^5 - 4x^2) \sqrt{-x^3 + 1} + 3888 \cdot 2^{1/3} (x^7 - x^4 - 2304) / (x^9 - 12x^6 + 48x^3 - 64)) / (x^6 + 3x^3 - 4) + 1/3888 \cdot 432^{5/6} \arctan(-1/648 (6 \sqrt{-x^3 + 1} (432^{5/6} (x^4 + 2x) - 36 \sqrt{3} (x^3 - 4) + 18 \cdot 432^{1/6} (x^5 + 8x^2)) - (108 \sqrt{3} \cdot 2^{2/3} (x^5 - x^2) - 216 \sqrt{3} \cdot 2^{1/3} (x^4 - x) - 108 \sqrt{3} (x^6 - x^3) + \sqrt{-x^3 + 1} (432^{5/6} (2x^4 + x) - 36 \sqrt{3} (5x^3 - 8) - 18 \cdot 432^{1/6} (x^5 - 10x^2)))) \sqrt{(36x^9 - 8208x^6 + 9504x^3 - 648 \cdot 2^{2/3})(x^8 - 5x^5 + 4x^2) - (2592x^6 - 2592x^3 - 432^{5/6}\sqrt{3})(x^7 - 26x^4 + 16x) - 216 \cdot 432^{1/6} \sqrt{3} (7x^5 - 4x^2) \sqrt{-x^3 + 1} + 3888 \cdot 2^{1/3} (x^7 - x^4 - 2304) / (x^9 - 12x^6 + 48x^3 - 64)) / (x^6 + 3x^3 - 4)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3 \sqrt{1-x^3} - 4 \sqrt{1-x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x\*\*3+4)/(-x\*\*3+1)\*\*(1/2),x)

[Out] -Integral(x/(x\*\*3\*sqrt(1 - x\*\*3) - 4\*sqrt(1 - x\*\*3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{(x^3 - 4)\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)\*sqrt(-x^3 + 1)), x)



$$3.74 \quad \int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}} dx$$

**Optimal.** Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{dx+1}}{\sqrt{dx^3-1}}\right)}{3 \cdot 2^{2/3} d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9 \cdot 2^{2/3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{dx})}{\sqrt{dx^3-1}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}}$$

[Out] -ArcTan[(1 + 2^(1/3)\*d^(1/3)\*x)/Sqrt[-1 + d\*x^3]]/(3\*2^(2/3)\*d^(2/3)) - ArcTan[Sqrt[-1 + d\*x^3]]/(9\*2^(2/3)\*d^(2/3)) - ArcTanh[(Sqrt[3]\*(1 - 2^(1/3)\*d^(1/3)\*x))/Sqrt[-1 + d\*x^3]]/(3\*2^(2/3)\*Sqrt[3]\*d^(2/3)) - ArcTanh[Sqrt[-1 + d\*x^3]/Sqrt[3]]/(3\*2^(2/3)\*Sqrt[3]\*d^(2/3))

**Rubi [A]** time = 0.0329981, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {485}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{dx+1}}{\sqrt{dx^3-1}}\right)}{3 \cdot 2^{2/3} d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9 \cdot 2^{2/3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{dx})}{\sqrt{dx^3-1}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((4 - d\*x^3)\*Sqrt[-1 + d\*x^3]),x]

[Out] -ArcTan[(1 + 2^(1/3)\*d^(1/3)\*x)/Sqrt[-1 + d\*x^3]]/(3\*2^(2/3)\*d^(2/3)) - ArcTan[Sqrt[-1 + d\*x^3]]/(9\*2^(2/3)\*d^(2/3)) - ArcTanh[(Sqrt[3]\*(1 - 2^(1/3)\*d^(1/3)\*x))/Sqrt[-1 + d\*x^3]]/(3\*2^(2/3)\*Sqrt[3]\*d^(2/3)) - ArcTanh[Sqrt[-1 + d\*x^3]/Sqrt[3]]/(3\*2^(2/3)\*Sqrt[3]\*d^(2/3))

#### Rule 485

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^3)\*Sqrt[(c\_) + (d\_.)\*(x\_)^3]), x\_Symbol] := With[{q = Rt[d/c, 3]}, -Simp[(q\*ArcTan[Sqrt[c + d\*x^3]/Rt[-c, 2]])/(9\*2^(2/3)\*b\*Rt[-c, 2]), x] + (-Simp[(q\*ArcTan[(Rt[-c, 2]\*(1 - 2^(1/3)\*q\*x))/Sqrt[c + d\*x^3]])/(3\*2^(2/3)\*b\*Rt[-c, 2]), x] - Simp[(q\*ArcTanh[Sqrt[c + d\*x^3]/(Sqrt[3]\*Rt[-c, 2])])/(3\*2^(2/3)\*Sqrt[3]\*b\*Rt[-c, 2]), x] - Simp[(q\*ArcTanh[(Sqrt[3]\*Rt[-c, 2]\*(1 + 2^(1/3)\*q\*x))/Sqrt[c + d\*x^3]])/(3\*2^(2/3)\*Sqrt[3]\*b\*Rt[-c, 2]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[4\*b\*c - a\*d, 0] && NegQ[c]

#### Rubi steps

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}} dx = -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{dx}}{\sqrt{-1+dx^3}}\right)}{3 \cdot 2^{2/3} d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9 \cdot 2^{2/3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{dx})}{\sqrt{-1+dx^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3} d^{2/3}}$$

**Mathematica [C]** time = 0.0290417, size = 54, normalized size = 0.34

$$\frac{x^2 \sqrt{1-dx^3} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; dx^3, \frac{dx^3}{4}\right)}{8\sqrt{dx^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((4 - d\*x^3)\*Sqrt[-1 + d\*x^3]),x]

[Out] (x^2\*Sqrt[1 - d\*x^3]\*AppellF1[2/3, 1/2, 1, 5/3, d\*x^3, (d\*x^3)/4])/(8\*Sqrt[-1 + d\*x^3])

**Maple [C]** time = 0.106, size = 240, normalized size = 1.5

$$-\frac{i}{9}\sqrt{2} \sum_{\alpha=\text{RootOf}(dZ^3-4)} \frac{1}{-\alpha} \sqrt{-\frac{i}{2}\left(2x + \frac{1}{\sqrt[3]{d}} + i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right) \sqrt[3]{d} \sqrt{\left(x - \frac{1}{\sqrt[3]{d}}\right)\left(-3\frac{1}{\sqrt[3]{d}} - i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)^{-1}}} \sqrt{\frac{i}{2}\left(2x + \frac{1}{\sqrt[3]{d}} - i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-d\*x^3+4)/(d\*x^3-1)^(1/2),x)

[Out] -1/9\*I\*2^(1/2)\*sum(1/\_alpha/d^(4/3)\*(-1/2\*I\*(2\*x+1/d^(1/3)+I\*3^(1/2)/d^(1/3))\*d^(1/3))^(1/2)\*((x-1/d^(1/3))/(-3/d^(1/3)-I\*3^(1/2)/d^(1/3)))^(1/2)\*(1/2\*I\*(2\*x+1/d^(1/3)-I\*3^(1/2)/d^(1/3))\*d^(1/3))^(1/2)/(d\*x^3-1)^(1/2)\*(-2\*\_alpha^2\*d+I\*3^(1/2)\*\_alpha\*d^(2/3)-I\*3^(1/2)\*d^(1/3)+\_alpha\*d^(2/3)+d^(1/3))\*EllipticPi(1/3\*3^(1/2)\*(-I\*(x+1/2/d^(1/3)+1/2\*I\*3^(1/2)/d^(1/3))\*3^(1/2)\*d^(1/3))^(1/2),1/3\*I\*3^(1/2)\*\_alpha^2\*d^(2/3)-1/6\*I\*3^(1/2)\*\_alpha\*d^(1/3)-1/6\*I\*3^(1/2)+1/2\*\_alpha\*d^(1/3)-1/2,(-I\*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2\*I\*3^(1/2)/d^(1/3)))^(1/2)),\_alpha=RootOf(\_Z^3\*d-4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3-1}(dx^3-4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x^3+4)/(d\*x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(d\*x^3 - 1)\*(d\*x^3 - 4)), x)

**Fricas [B]** time = 5.7955, size = 4625, normalized size = 29.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x^3+4)/(d\*x^3-1)^(1/2),x, algorithm="fricas")

[Out] -1/9\*sqrt(3)\*(1/432)^(1/6)\*(d^(-4))^(1/6)\*arctan(1/3\*(3\*(sqrt(3)\*sqrt(1/3)\*d^2\*sqrt(d^(-4))\*x + 2\*sqrt(3)\*(1/432)^(1/6)\*d\*(d^(-4))^(1/6)\*x^2 - 24\*sqrt(3)\*(1/432)^(5/6)\*(d^4\*x^3 - 4\*d^3)\*(d^(-4))^(5/6))\*sqrt(d\*x^3 - 1) + (2\*sqrt(3)\*(1/2)^(1/3)\*(d^2\*x^3 - d)\*(d^(-4))^(1/3) + sqrt(3)\*(d\*x^4 - x) + 3\*(sqrt(3)\*sqrt(1/3)\*d^2\*sqrt(d^(-4))\*x + 2\*sqrt(3)\*(1/432)^(1/6)\*d\*(d^(-4))^(1/6)\*x^2 + 24\*sqrt(3)\*(1/432)^(5/6)\*(d^4\*x^3 + 2\*d^3)\*(d^(-4))^(5/6))\*sqrt(d

```

*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
+ 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
+ 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
- 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
+ sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
- (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3 \sqrt{dx^3 - 1} - 4\sqrt{dx^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x\*\*3+4)/(d\*x\*\*3-1)\*\*(1/2),x)

[Out] -Integral(x/(d\*x\*\*3\*sqrt(d\*x\*\*3 - 1) - 4\*sqrt(d\*x\*\*3 - 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3 - 1}(dx^3 - 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

$$3.75 \quad \int \frac{x}{\sqrt{-1+x^3}(8+x^3)} dx$$

**Optimal.** Leaf size=74

$$\frac{1}{18} \tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right) + \frac{1}{18} \tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

[Out] ArcTan[(1 - x)^2/(3\*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - ArcTanh[(Sqrt[3]\*(1 - x))/Sqrt[-1 + x^3]]/(6\*Sqrt[3])

**Rubi [A]** time = 0.155523, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {486, 444, 63, 204, 2138, 203, 2145, 206}

$$\frac{1}{18} \tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right) + \frac{1}{18} \tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1 + x^3]\*(8 + x^3)), x]

[Out] ArcTan[(1 - x)^2/(3\*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - ArcTanh[(Sqrt[3]\*(1 - x))/Sqrt[-1 + x^3]]/(6\*Sqrt[3])

#### Rule 486

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^3)\*Sqrt[(c\_) + (d\_.)\*(x\_)^3]), x\_Symbol] := With[{q = Rt[d/c, 3]}, Dist[(d\*q)/(4\*b), Int[x^2/((8\*c - d\*x^3)\*Sqrt[c + d\*x^3]), x], x] + (-Dist[q^2/(12\*b), Int[(1 + q\*x)/((2 - q\*x)\*Sqrt[c + d\*x^3]), x], x] + Dist[1/(12\*b\*c), Int[(2\*c\*q^2 - 2\*d\*x - d\*q\*x^2)/((4 + 2\*q\*x + q^2\*x^2)\*Sqrt[c + d\*x^3]), x], x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[8\*b\*c + a\*d, 0]

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{-1+x^3}(8+x^3)} dx &= -\left(\frac{1}{12} \int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx\right) - \frac{1}{12} \int \frac{-2-2x+x^2}{(4-2x+x^2)\sqrt{-1+x^3}} dx - \frac{1}{4} \int \frac{x^2}{(-8-x^3)\sqrt{-1+x^3}} dx \\ &= -\left(\frac{1}{12} \text{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}} dx, x, x^3\right)\right) + \frac{1}{6} \text{Subst}\left(\int \frac{1}{9+x^2} dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right) - \frac{1}{3} \text{Subst}\left(\int \frac{x^2}{-8-x^3} dx, x, \sqrt{-1+x^3}\right) \\ &= \frac{1}{18} \tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}} - \frac{1}{6} \text{Subst}\left(\int \frac{1}{-9-x^2} dx, x, \sqrt{-1+x^3}\right) \\ &= \frac{1}{18} \tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18} \tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}} \end{aligned}$$

**Mathematica [C]** time = 0.0194046, size = 48, normalized size = 0.65

$$\frac{x^2 \sqrt{1-x^3} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{x^3}{8}\right)}{16\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)), x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8])/(16*Sqrt[-1 + x^3])
```

**Maple [C]** time = 0.115, size = 286, normalized size = 3.9

$$-\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{9}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\text{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}+\frac{1}{2},\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^3+8)/(x^3-1)^(1/2),x)

[Out]  $-1/9*(-3/2-1/2*I*3^{(1/2)})*((-1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2-1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2+1/2*I*3^{(1/2)})/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3-1)^{(1/2)}*\text{EllipticPi}(((x+1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},1/6*I*3^{(1/2)}+1/2,((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}+1/36*2^{(1/2)}*\text{sum}((2-\alpha)*(\alpha-1)*(-I*3^{(1/2)}-3)*((-1+x)/(-I*3^{(1/2)}-3))^{(1/2)}*((2*x+1-I*3^{(1/2)})/(-I*3^{(1/2)}+3))^{(1/2)}*((2*x+1+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)}/(x^3-1)^{(1/2)}*\text{EllipticPi}(((x+1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},1/6*I*\alpha*3^{(1/2)}+1/2*\alpha-1/6*I*3^{(1/2)}-1/2,((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}),\alpha=\text{RootOf}(\_Z^2-2*\_Z+4))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+8)\sqrt{x^3-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 8)\*sqrt(x^3 - 1)), x)

**Fricas [B]** time = 3.58602, size = 1447, normalized size = 19.55

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}(x^4+12x^3+12x^2-16x)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)-\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}(x^4+12x^3+12x^2-16x)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")

[Out]  $1/216*\text{sqrt}(3)*\log(4*(x^6+48*x^5+186*x^4-56*x^3+6*\text{sqrt}(3)*(x^4+12*x^3+12*x^2-16*x)*\text{sqrt}(x^3-1)-120*x^2-96*x+64)/(x^6-6*x^5+24*x^4-56*x^3+96*x^2-96*x+64))-1/216*\text{sqrt}(3)*\log(4*(x^6+48*x^5+186*x^4-56*x^3-6*\text{sqrt}(3)*(x^4+12*x^3+12*x^2-16*x)*\text{sqrt}(x^3-1)-120*x^2-96*x+64)/(x^6-6*x^5+24*x^4-56*x^3+96*x^2-96*x+64))+1/54*\text{arctan}(1/6*(x^3-12*x^2-6*x-10)*\text{sqrt}(x^3-1)/(x^4-x^3-x+1))-1/54*\text{arctan}(-1/3*(\text{sqrt}(x^3-1)*(x^2-8*x+10)+(3*\text{sqrt}(3)*(x^3+x^2-2*x)-\text{sqrt}(x^3-1)*(x^2+10*x-8)))*\text{sqrt}((x^6+48*x^5+186*x^4-56*x^3+6*\text{sqrt}(3)*(x^4+12*x^3+12*x^2-16*x)*\text{sqrt}(x^3-1)-120*x^2-96*x+64)/(x^6-6*x^5+24*x^4-56*x^3+96*x^2-96*x+64)))/(x^3-3*x^2+2))-1/54*\text{arctan}(-1/3*(\text{sqrt}(x^3-1)*(x^2-8*x+10)-(3*\text{sqrt}(3)$

$$\frac{(x^3 + x^2 - 2x) + \sqrt{x^3 - 1}(x^2 + 10x - 8)\sqrt{(x^6 + 48x^5 + 186x^4 - 56x^3 - 6\sqrt{3}(x^4 + 12x^3 + 12x^2 - 16x)\sqrt{x^3 - 1} - 120x^2 - 96x + 64)}}{(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64)}$$

$$\frac{)}{(x^3 - 3x^2 + 2)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x+2)(x^2-2x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x\*\*3+8)/(x\*\*3-1)\*\*(1/2),x)

[Out] Integral(x/(sqrt((x - 1)\*(x\*\*2 + x + 1))\*(x + 2)\*(x\*\*2 - 2\*x + 4)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 8)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(x/((x^3 + 8)\*sqrt(x^3 - 1)), x)



$$3.76 \quad \int \frac{x}{(8-dx^3)\sqrt{1+dx^3}} dx$$

**Optimal.** Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{dx+1}\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{dx+1}\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

[Out] -ArcTan[(Sqrt[3]\*(1 + d^(1/3)\*x))/Sqrt[1 + d\*x^3]]/(6\*Sqrt[3]\*d^(2/3)) + ArcTanh[(1 + d^(1/3)\*x)^2/(3\*Sqrt[1 + d\*x^3])]/(18\*d^(2/3)) - ArcTanh[Sqrt[1 + d\*x^3]/3]/(18\*d^(2/3))

**Rubi [A]** time = 0.304998, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{dx+1}\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{dx+1}\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((8 - d\*x^3)\*Sqrt[1 + d\*x^3]),x]

[Out] -ArcTan[(Sqrt[3]\*(1 + d^(1/3)\*x))/Sqrt[1 + d\*x^3]]/(6\*Sqrt[3]\*d^(2/3)) + ArcTanh[(1 + d^(1/3)\*x)^2/(3\*Sqrt[1 + d\*x^3])]/(18\*d^(2/3)) - ArcTanh[Sqrt[1 + d\*x^3]/3]/(18\*d^(2/3))

#### Rule 486

Int[(x\_)/(((a\_) + (b\_.)\*(x\_)^3)\*Sqrt[(c\_) + (d\_.)\*(x\_)^3]), x\_Symbol] := With[{q = Rt[d/c, 3]}, Dist[(d\*q)/(4\*b), Int[x^2/((8\*c - d\*x^3)\*Sqrt[c + d\*x^3]), x], x] + (-Dist[q^2/(12\*b), Int[(1 + q\*x)/((2 - q\*x)\*Sqrt[c + d\*x^3]), x], x] + Dist[1/(12\*b\*c), Int[(2\*c\*q^2 - 2\*d\*x - d\*q\*x^2)/((4 + 2\*q\*x + q^2\*x^2)\*Sqrt[c + d\*x^3]), x], x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[8\*b\*c + a\*d, 0]

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

### Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{(8-dx^3)\sqrt{1+dx^3}} dx &= -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{(4+2\sqrt[3]{dx+d^{2/3}x^2})\sqrt{1+dx^3}} dx}{12d} + \frac{\int \frac{1+\sqrt[3]{dx}}{(2-\sqrt[3]{dx})\sqrt{1+dx^3}} dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d} \int \frac{x^2}{(8-dx^3)\sqrt{1+dx^3}} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{9-x^2} dx, x, \frac{(1+\sqrt[3]{dx})^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d} \text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}} dx, x, x^3\right) + \frac{1}{3}d^{4/3} \text{Subst} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{3}(1+\sqrt[3]{dx})}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{(1+\sqrt[3]{dx})^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2} dx, x, \sqrt{1+dx^3}\right)}{6d^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{3}(1+\sqrt[3]{dx})}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{(1+\sqrt[3]{dx})^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}} \end{aligned}$$

**Mathematica [C]** time = 0.0292129, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -dx^3, \frac{dx^3}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

**Maple [C]** time = 0.16, size = 383, normalized size = 3.7

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3} \sum_{\alpha=\text{RootOf}(dZ^3-8)} \frac{1}{-\alpha} \sqrt[3]{-d^2} \sqrt{\frac{i}{2}d \left( 2x + \frac{1}{d} \left( -i\sqrt{3}\sqrt[3]{-d^2} + \sqrt[3]{-d^2} \right) \right)} \frac{1}{\sqrt[3]{-d^2}} \sqrt{d \left( x - \frac{1}{d} \sqrt[3]{-d^2} \right) \left( -3\sqrt[3]{-d^2} + \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-d\*x^3+8)/(d\*x^3+1)^(1/2),x)

[Out]  $-1/27*I/d^3*2^{(1/2)}*sum(1/_alpha*(-d^2)^{(1/3)}*(1/2*I*d*(2*x+1/d*(-I*3^{(1/2)}*(-d^2)^{(1/3)}+(-d^2)^{(1/3)})))/(-d^2)^{(1/3)}^{(1/2)}*(d*(x-1/d*(-d^2)^{(1/3)})/(-3*(-d^2)^{(1/3)}+I*3^{(1/2)}*(-d^2)^{(1/3)})^{(1/2)}*(-1/2*I*d*(2*x+1/d*(I*3^{(1/2)}*(-d^2)^{(1/3)}+(-d^2)^{(1/3)})))/(-d^2)^{(1/3)}^{(1/2)}/(d*x^3+1)^{(1/2)}*(I*(-d^2)^{(1/3)}*_alpha*3^{(1/2)}*d-I*3^{(1/2)}*(-d^2)^{(2/3)}+2*_alpha^2*d^2-(-d^2)^{(1/3)}*_alpha*d-(-d^2)^{(2/3)})*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2/d*(-d^2)^{(1/3)}-1/2*I*3^{(1/2)}/d*(-d^2)^{(1/3)})*3^{(1/2)}*d/(-d^2)^{(1/3)})^{(1/2)},-1/18/d*(2*I*3^{(1/2)}*(-d^2)^{(1/3)}*_alpha^2*d-I*3^{(1/2)}*(-d^2)^{(2/3)}*_alpha+I*3^{(1/2)}*d-3*(-d^2)^{(2/3)}*_alpha-3*d), (I*3^{(1/2)}/d*(-d^2)^{(1/3)})/(-3/2/d*(-d^2)^{(1/3)}+1/2*I*3^{(1/2)}/d*(-d^2)^{(1/3)})^{(1/2)},_alpha=\text{RootOf}(_Z^3*d-8))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3+1}(dx^3-8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x^3+8)/(d\*x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(d\*x^3 + 1)\*(d\*x^3 - 8)), x)

**Fricas [B]** time = 4.28187, size = 1214, normalized size = 11.79

$$2\sqrt{3}(d^2)^{\frac{1}{6}}d \arctan\left(\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}(d^2x^6-40dx^3-32)(d^2)^{\frac{2}{3}}+3\sqrt{3}(5d^2x^4+8dx)(d^2)^{\frac{1}{3}}\right)\sqrt{dx^3+1}(d^2)^{\frac{1}{6}}}{9(d^4x^7-7d^3x^4-8d^2x)}}\right)+2(d^2)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318d^3x^6+1200d^2x^3+18(5d^2x^7+64dx^4+32x)(d^2)^{(2/3)}+6(7d^3x^6+152d^2x^3+(d^2x^7+80dx^4+160x)(d^2)^{(2/3)}+6(5d^2x^5+32dx^2)(d^2)^{(1/3)}+64d)\sqrt{dx^3+1}+18(d^3x^8+38d^2x^5+64dx^2)(d^2)^{(1/3)}+640d)/(d^3x^9-24d^2x^6+192dx^3-512)}{(d^4x^9-276d^3x^6-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x^3+8)/(d\*x^3+1)^(1/2),x, algorithm="fricas")

[Out]  $1/108*(2*\text{sqrt}(3)*(d^2)^{(1/6)}*d*\text{arctan}(-1/9*(9*\text{sqrt}(3)*d^3*x^5 - \text{sqrt}(3)*(d^2*x^6 - 40*d*x^3 - 32)*(d^2)^{(2/3)} + 3*\text{sqrt}(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^{(1/3)})*\text{sqrt}(d*x^3 + 1)*(d^2)^{(1/6)})/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)^{(2/3)}*\log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4 + 32*x)*(d^2)^{(2/3)} + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 160*x)*(d^2)^{(2/3)} + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^{(1/3)} + 64*d)*\text{sqrt}(d*x^3 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^{(1/3)} + 640*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^{(2/3)}*\log((d^4*x^9 - 276*d^3*x^6 -$

$$\frac{1608d^2x^3 - 18(d^2x^7 - 52dx^4 - 80x)(d^2)^{2/3} - 6(4d^3x^6 + 164d^2x^3 + (d^2x^7 - 28dx^4 - 272x)(d^2)^{2/3} - 24(d^2x^5 + dx^2)(d^2)^{1/3} + 160d)\sqrt{dx^3 + 1} + 18(d^3x^8 + 20d^2x^5 - 8dx^2)(d^2)^{1/3} - 1088d}{(d^3x^9 - 24d^2x^6 + 192dx^3 - 512)} \cdot \frac{1}{d^2}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x\*\*3+8)/(d\*x\*\*3+1)\*\*(1/2),x)

[Out] -Integral(x/(d\*x\*\*3\*sqrt(d\*x\*\*3 + 1) - 8\*sqrt(d\*x\*\*3 + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d\*x^3+8)/(d\*x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x/(sqrt(d\*x^3 + 1)\*(d\*x^3 - 8)), x)

$$3.77 \quad \int \frac{1}{\sqrt[3]{1-3x^2}(3-x^2)} dx$$

**Optimal.** Leaf size=81

$$\frac{1}{4} \tan^{-1} \left( \frac{1 - \sqrt[3]{1-3x^2}}{x} \right) - \frac{\tanh^{-1} \left( \frac{(1 - \sqrt[3]{1-3x^2})^2}{3\sqrt{3}x} \right)}{4\sqrt{3}} + \frac{\tanh^{-1} \left( \frac{x}{\sqrt{3}} \right)}{4\sqrt{3}}$$

[Out] ArcTan[(1 - (1 - 3\*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4\*Sqrt[3]) - ArcTanh[(1 - (1 - 3\*x^2)^(1/3))^2/(3\*Sqrt[3]\*x)]/(4\*Sqrt[3])

**Rubi [A]** time = 0.0114325, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {395}

$$\frac{1}{4} \tan^{-1} \left( \frac{1 - \sqrt[3]{1-3x^2}}{x} \right) - \frac{\tanh^{-1} \left( \frac{(1 - \sqrt[3]{1-3x^2})^2}{3\sqrt{3}x} \right)}{4\sqrt{3}} + \frac{\tanh^{-1} \left( \frac{x}{\sqrt{3}} \right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - 3\*x^2)^(1/3)\*(3 - x^2)), x]

[Out] ArcTan[(1 - (1 - 3\*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4\*Sqrt[3]) - ArcTanh[(1 - (1 - 3\*x^2)^(1/3))^2/(3\*Sqrt[3]\*x)]/(4\*Sqrt[3])

### Rule 395

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/3)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[-(b/a), 2]}, -Simp[(q\*ArcTanh[(q\*x)/3])/(12\*Rt[a, 3]\*d), x] + (Simp[(q\*ArcTanh[(Rt[a, 3] - (a + b\*x^2)^(1/3))^2/(3\*Rt[a, 3]^2\*q\*x)])/(12\*Rt[a, 3]\*d), x] - Simp[(q\*ArcTan[(Sqrt[3]\*(Rt[a, 3] - (a + b\*x^2)^(1/3)))/(Rt[a, 3]\*q\*x)])/(4\*Sqrt[3]\*Rt[a, 3]\*d), x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c - 9\*a\*d, 0] && NegQ[b/a]

### Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}(3-x^2)} dx = \frac{1}{4} \tan^{-1} \left( \frac{1 - \sqrt[3]{1-3x^2}}{x} \right) + \frac{\tanh^{-1} \left( \frac{x}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{\tanh^{-1} \left( \frac{(1 - \sqrt[3]{1-3x^2})^2}{3\sqrt{3}x} \right)}{4\sqrt{3}}$$

**Mathematica [C]** time = 0.100266, size = 126, normalized size = 1.56

$$\frac{9x F_1 \left( \frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; 3x^2, \frac{x^2}{3} \right)}{\sqrt[3]{1-3x^2}(x^2-3) \left( 2x^2 \left( F_1 \left( \frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; 3x^2, \frac{x^2}{3} \right) + 3F_1 \left( \frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; 3x^2, \frac{x^2}{3} \right) \right) + 9F_1 \left( \frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; 3x^2, \frac{x^2}{3} \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1 - 3\*x^2)^(1/3)\*(3 - x^2)),x]

[Out] (-9\*x\*AppellF1[1/2, 1/3, 1, 3/2, 3\*x^2, x^2/3])/((1 - 3\*x^2)^(1/3)\*(-3 + x^2)\*(9\*AppellF1[1/2, 1/3, 1, 3/2, 3\*x^2, x^2/3] + 2\*x^2\*(AppellF1[3/2, 1/3, 2, 5/2, 3\*x^2, x^2/3] + 3\*AppellF1[3/2, 4/3, 1, 5/2, 3\*x^2, x^2/3])))

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2 + 3} \frac{1}{\sqrt[3]{-3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3\*x^2+1)^(1/3)/(-x^2+3),x)

[Out] int(1/(-3\*x^2+1)^(1/3)/(-x^2+3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2 - 3)(-3x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3\*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")

[Out] -integrate(1/((x^2 - 3)\*(-3\*x^2 + 1)^(1/3)), x)

**Fricas [B]** time = 19.5199, size = 5180, normalized size = 63.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3\*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")

[Out] 1/72\*sqrt(6)\*sqrt(3)\*sqrt(2)\*arctan(1/9\*(36\*sqrt(6)\*sqrt(3)\*sqrt(2)\*(3\*x^11 - 1117\*x^9 + 3918\*x^7 - 1866\*x^5 + 255\*x^3 - 9\*x) + sqrt(3)\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^12 + 2184\*x^10 - 211215\*x^8 + 94152\*x^6 - 13581\*x^4 + 432\*x^2 + 27) + 12\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^10 - 107\*x^8 - 7262\*x^6 + 2322\*x^4 - 243\*x^2 + 9) - 48\*sqrt(3)\*(5\*x^9 - 245\*x^7 + 183\*x^5 - 15\*x^3))\*(-3\*x^2 + 1)^(2/3) - 12\*sqrt(3)\*(29\*x^11 + 293\*x^9 - 2670\*x^7 + 4986\*x^5 - 1215\*x^3 + 81\*x) - 6\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(49\*x^10 - 5043\*x^8 + 3658\*x^6 + 378\*x^4 - 171\*x^2 + 9) - 2\*sqrt(3)\*(x^11 + 917\*x^9 - 40566\*x^7 + 15786\*x^5 - 2043\*x^3 + 81\*x))\*(-3\*x^2 + 1)^(1/3))\*sqrt((x^6 - 93\*x^4 + 4\*sqrt(6)\*sqrt(2)\*(x^5 + 13\*x^3) - 117\*x^2 - 2\*(4\*sqrt(6)\*sqrt(2)\*x^3 - 3\*x^4 - 18\*x^2 + 9))\*(-3\*x^2 + 1)^(2/3) + (6\*x^4 - sqrt(6)\*sqrt(2)\*(x^5 - 10\*x^3 - 27\*x) - 108\*x^2 - 18))\*(-3\*x^2 + 1)^(1/3) + 9)/(x^6 - 9\*x^4 + 27\*x^2 - 27)) + 12\*(2\*sqrt(6)\*sqrt(3)\*sqrt(2)\*(35\*x^9 - 4860\*x^7 + 2106\*x^5 - 396\*x^3 + 27\*x) - 3\*sqrt(3)\*(x^10 + 589\*x^8 + 3946\*x^6 - 774\*x^4 - 27\*x^2 + 9))\*(-3\*x^2 + 1)^(2/3) - 3\*sqrt(3)\*(x^12 + 3150\*x^10 + 77991\*x^8 + 4260\*x^6 - 14337\*x^4 + 2862\*x^2 -

135) - 6\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^11 - 1591\*x^9 + 42426\*x^7 - 15102\*x^5 + 1269\*x^3 - 27\*x) - 6\*sqrt(3)\*(27\*x^10 + 2307\*x^8 + 4574\*x^6 - 2538\*x^4 + 279\*x^2 - 9))\*(-3\*x^2 + 1)^(1/3))/(x^12 - 4986\*x^10 + 327519\*x^8 - 159660\*x^6 + 25839\*x^4 - 2106\*x^2 + 81)) + 1/72\*sqrt(6)\*sqrt(3)\*sqrt(2)\*arctan(1/9\*(36\*sqrt(6)\*sqrt(3)\*sqrt(2)\*(3\*x^11 - 1117\*x^9 + 3918\*x^7 - 1866\*x^5 + 255\*x^3 - 9\*x) + sqrt(3)\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^12 + 2184\*x^10 - 211215\*x^8 + 94152\*x^6 - 13581\*x^4 + 432\*x^2 + 27) + 12\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^10 - 107\*x^8 - 7262\*x^6 + 2322\*x^4 - 243\*x^2 + 9) + 48\*sqrt(3)\*(5\*x^9 - 245\*x^7 + 183\*x^5 - 15\*x^3)))\*(-3\*x^2 + 1)^(2/3) + 12\*sqrt(3)\*(29\*x^11 + 293\*x^9 - 2670\*x^7 + 4986\*x^5 - 1215\*x^3 + 81\*x) - 6\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(49\*x^10 - 5043\*x^8 + 3658\*x^6 + 378\*x^4 - 171\*x^2 + 9) + 2\*sqrt(3)\*(x^11 + 917\*x^9 - 40566\*x^7 + 15786\*x^5 - 2043\*x^3 + 81\*x))\*(-3\*x^2 + 1)^(1/3))\*sqrt((x^6 - 93\*x^4 - 4\*sqrt(6)\*sqrt(2)\*(x^5 + 13\*x^3) - 117\*x^2 + 2\*(4\*sqrt(6)\*sqrt(2)\*x^3 + 3\*x^4 + 18\*x^2 - 9))\*(-3\*x^2 + 1)^(2/3) + (6\*x^4 + sqrt(6)\*sqrt(2)\*(x^5 - 10\*x^3 - 27\*x) - 108\*x^2 - 18))\*(-3\*x^2 + 1)^(1/3) + 9)/(x^6 - 9\*x^4 + 27\*x^2 - 27)) + 12\*(2\*sqrt(6)\*sqrt(3)\*sqrt(2)\*(35\*x^9 - 4860\*x^7 + 2106\*x^5 - 396\*x^3 + 27\*x) + 3\*sqrt(3)\*(x^10 + 589\*x^8 + 3946\*x^6 - 774\*x^4 - 27\*x^2 + 9))\*(-3\*x^2 + 1)^(2/3) + 3\*sqrt(3)\*(x^12 + 3150\*x^10 + 77991\*x^8 + 4260\*x^6 - 14337\*x^4 + 2862\*x^2 - 135) - 6\*(sqrt(6)\*sqrt(3)\*sqrt(2)\*(x^11 - 1591\*x^9 + 42426\*x^7 - 15102\*x^5 + 1269\*x^3 - 27\*x) + 6\*sqrt(3)\*(27\*x^10 + 2307\*x^8 + 4574\*x^6 - 2538\*x^4 + 279\*x^2 - 9))\*(-3\*x^2 + 1)^(1/3))/(x^12 - 4986\*x^10 + 327519\*x^8 - 159660\*x^6 + 25839\*x^4 - 2106\*x^2 + 81)) - 1/288\*sqrt(6)\*sqrt(2)\*log(12\*(x^6 - 93\*x^4 + 4\*sqrt(6)\*sqrt(2)\*(x^5 + 13\*x^3) - 117\*x^2 - 2\*(4\*sqrt(6)\*sqrt(2)\*x^3 + 3\*x^4 + 18\*x^2 + 9))\*(-3\*x^2 + 1)^(2/3) + (6\*x^4 - sqrt(6)\*sqrt(2)\*(x^5 - 10\*x^3 - 27\*x) - 108\*x^2 - 18))\*(-3\*x^2 + 1)^(1/3) + 9)/(x^6 - 9\*x^4 + 27\*x^2 - 27)) + 1/288\*sqrt(6)\*sqrt(2)\*log(12\*(x^6 - 93\*x^4 - 4\*sqrt(6)\*sqrt(2)\*(x^5 + 13\*x^3) - 117\*x^2 + 2\*(4\*sqrt(6)\*sqrt(2)\*x^3 + 3\*x^4 + 18\*x^2 - 9))\*(-3\*x^2 + 1)^(2/3) + (6\*x^4 + sqrt(6)\*sqrt(2)\*(x^5 - 10\*x^3 - 27\*x) - 108\*x^2 - 18))\*(-3\*x^2 + 1)^(1/3) + 9)/(x^6 - 9\*x^4 + 27\*x^2 - 27)) + 1/72\*sqrt(3)\*log(-(x^12 + 2598\*x^10 + 55143\*x^8 + 14228\*x^6 - 22113\*x^4 - 7290\*x^2 + 8\*(3\*x^10 + 576\*x^8 + 5598\*x^6 + 5832\*x^4 - 729\*x^2 - sqrt(3)\*(41\*x^9 + 1368\*x^7 + 4482\*x^5 + 864\*x^3 - 243\*x)))\*(-3\*x^2 + 1)^(2/3) - 4\*sqrt(3)\*(25\*x^11 + 2359\*x^9 + 15426\*x^7 + 6966\*x^5 - 4347\*x^3 + 243\*x) - 4\*(84\*x^10 + 4536\*x^8 + 20880\*x^6 + 5832\*x^4 - 2916\*x^2 - sqrt(3)\*(x^11 + 521\*x^9 + 7362\*x^7 + 10746\*x^5 - 1971\*x^3 - 243\*x))\*(-3\*x^2 + 1)^(1/3) + 729)/(x^12 - 18\*x^10 + 135\*x^8 - 540\*x^6 + 1215\*x^4 - 1458\*x^2 + 729))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2 \sqrt[3]{1-3x^2} - 3\sqrt[3]{1-3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3\*x\*\*2+1)\*\*(1/3)/(-x\*\*2+3),x)

[Out] -Integral(1/(x\*\*2\*(1 - 3\*x\*\*2)\*\*(1/3) - 3\*(1 - 3\*x\*\*2)\*\*(1/3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2 - 3)(-3x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```



$$3.78 \quad \int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}} dx$$

**Optimal.** Leaf size=81

$$\frac{\tan^{-1}\left(\frac{(1-\sqrt[3]{3x^2+1})^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4} \tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

[Out] ArcTan[x/Sqrt[3]]/(4\*Sqrt[3]) + ArcTan[(1 - (1 + 3\*x^2)^(1/3))^2/(3\*Sqrt[3]\*x)]/(4\*Sqrt[3]) - ArcTanh[(1 - (1 + 3\*x^2)^(1/3))/x]/4

**Rubi [A]** time = 0.0107141, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{(1-\sqrt[3]{3x^2+1})^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4} \tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 + x^2)\*(1 + 3\*x^2)^(1/3)),x]

[Out] ArcTan[x/Sqrt[3]]/(4\*Sqrt[3]) + ArcTan[(1 - (1 + 3\*x^2)^(1/3))^2/(3\*Sqrt[3]\*x)]/(4\*Sqrt[3]) - ArcTanh[(1 - (1 + 3\*x^2)^(1/3))/x]/4

#### Rule 394

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/3)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[b/a, 2]}, Simp[(q\*ArcTan[(q\*x)/3])/(12\*Rt[a, 3]\*d), x] + (Simp[(q\*ArcTan[(Rt[a, 3] - (a + b\*x^2)^(1/3))^2/(3\*Rt[a, 3]^2\*q\*x)])/(12\*Rt[a, 3]\*d), x] - Simp[(q\*ArcTanh[(Sqrt[3]\*(Rt[a, 3] - (a + b\*x^2)^(1/3)))/(Rt[a, 3]\*q\*x)])/(4\*Sqrt[3]\*Rt[a, 3]\*d), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c - 9\*a\*d, 0] && PosQ[b/a]

#### Rubi steps

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}} dx = \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{(1-\sqrt[3]{1+3x^2})^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4} \tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

**Mathematica [C]** time = 0.0950302, size = 126, normalized size = 1.56

$$\frac{9x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -3x^2, -\frac{x^2}{3}\right)}{(x^2 + 3)\sqrt[3]{3x^2 + 1}} \left( 2x^2 \left( F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; -3x^2, -\frac{x^2}{3}\right) + 3F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; -3x^2, -\frac{x^2}{3}\right) \right) - 9F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -3x^2, -\frac{x^2}{3}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((3 + x^2)\*(1 + 3\*x^2)^(1/3)),x]

[Out] (-9\*x\*AppellF1[1/2, 1/3, 1, 3/2, -3\*x^2, -x^2/3])/((3 + x^2)\*(1 + 3\*x^2)^(1/3))\*(-9\*AppellF1[1/2, 1/3, 1, 3/2, -3\*x^2, -x^2/3] + 2\*x^2\*(AppellF1[3/2, 1/3, 2, 5/2, -3\*x^2, -x^2/3] + 3\*AppellF1[3/2, 4/3, 1, 5/2, -3\*x^2, -x^2/3]))

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 + 3} \frac{1}{\sqrt[3]{3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+3)/(3\*x^2+1)^(1/3),x)

[Out] int(1/(x^2+3)/(3\*x^2+1)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 1)^{\frac{1}{3}}(x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3)/(3\*x^2+1)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((3\*x^2 + 1)^(1/3)\*(x^2 + 3)), x)

**Fricas [B]** time = 15.5078, size = 921, normalized size = 11.37

$$\frac{1}{36} \sqrt{3} \arctan \left( \frac{4\sqrt{3}(3x^4 - 10x^3 - 36x^2 + 18x + 9)(3x^2 + 1)^{\frac{2}{3}} - 4\sqrt{3}(x^5 + 15x^4 - 26x^3 - 54x^2 + 9x - 9)(3x^2 + 1)^{\frac{1}{3}}}{x^6 + 126x^5 - 225x^4 - 828x^3 - 81x^2 - 162x + 81} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3)/(3\*x^2+1)^(1/3),x, algorithm="fricas")

[Out] 1/36\*sqrt(3)\*arctan((4\*sqrt(3)\*(3\*x^4 - 10\*x^3 - 36\*x^2 + 18\*x + 9)\*(3\*x^2 + 1)^(2/3) - 4\*sqrt(3)\*(x^5 + 15\*x^4 - 26\*x^3 - 54\*x^2 + 9\*x - 9)\*(3\*x^2 + 1)^(1/3) + sqrt(3)\*(x^6 - 2\*x^5 - 105\*x^4 - 28\*x^3 + 63\*x^2 + 126\*x + 9))/(x^6 + 126\*x^5 - 225\*x^4 - 828\*x^3 - 81\*x^2 - 162\*x + 81)) - 1/36\*sqrt(3)\*arctan(2\*(2\*sqrt(3)\*(23\*x^3 + 9\*x)\*(3\*x^2 + 1)^(2/3) + sqrt(3)\*(x^5 - 80\*x^3 - 9\*x)\*(3\*x^2 + 1)^(1/3) + sqrt(3)\*(11\*x^5 + 10\*x^3 - 9\*x))/(x^6 - 657\*x^4 - 189\*x^2 - 27)) + 1/24\*log((x^6 + 108\*x^5 + 549\*x^4 + 360\*x^3 + 99\*x^2 + 6\*(3\*x^4 + 32\*x^3 + 42\*x^2 + 3)\*(3\*x^2 + 1)^(2/3) + 6\*(x^5 + 27\*x^4 + 70\*x^3 + 18\*x^2 + 9\*x + 3)\*(3\*x^2 + 1)^(1/3) + 108\*x - 9)/(x^6 + 9\*x^4 + 27\*x^2 + 27))

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 3)\sqrt[3]{3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2+3)/(3\*x\*\*2+1)\*\*(1/3), x)

[Out] Integral(1/((x\*\*2 + 3)\*(3\*x\*\*2 + 1)\*\*(1/3)), x)

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 1)^{\frac{1}{3}}(x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3)/(3\*x^2+1)^(1/3), x, algorithm="giac")

[Out] integrate(1/((3\*x^2 + 1)^(1/3)\*(x^2 + 3)), x)

$$3.79 \quad \int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} dx$$

**Optimal.** Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

[Out] ArcTan[Sqrt[3]/x]/(2\*2^(2/3)\*Sqrt[3]) + ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*(1 - x^2)^(1/3)))/x]/(2\*2^(2/3)\*Sqrt[3]) - ArcTanh[x]/(6\*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)\*(1 - x^2)^(1/3))]/(2\*2^(2/3))

**Rubi [A]** time = 0.0136163, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^2)^(1/3)\*(3 + x^2)),x]

[Out] ArcTan[Sqrt[3]/x]/(2\*2^(2/3)\*Sqrt[3]) + ArcTan[(Sqrt[3]\*(1 - 2^(1/3)\*(1 - x^2)^(1/3)))/x]/(2\*2^(2/3)\*Sqrt[3]) - ArcTanh[x]/(6\*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)\*(1 - x^2)^(1/3))]/(2\*2^(2/3))

### Rule 393

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/3)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] := With[{q = Rt[-(b/a), 2]}, Simp[(q\*ArcTan[Sqrt[3]/(q\*x)])/((2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x] + (Simp[(q\*ArcTanh[(a^(1/3)\*q\*x)/(a^(1/3) + 2^(1/3)\*(a + b\*x^2)^(1/3))])/((2\*2^(2/3)\*a^(1/3)\*d), x] - Simp[(q\*ArcTanh[q\*x])/((6\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTan[(Sqrt[3]\*(a^(1/3) - 2^(1/3)\*(a + b\*x^2)^(1/3)))/(a^(1/3)\*q\*x)])/((2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x)]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c + 3\*a\*d, 0] && NegQ[b/a]

### Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2 \cdot 2^{2/3}}$$

**Mathematica [C]** time = 0.0717142, size = 118, normalized size = 1.04

$$\frac{9x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; x^2, -\frac{x^2}{3}\right) - F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; x^2, -\frac{x^2}{3}\right)\right) - 9F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1 - x^2)^(1/3)\*(3 + x^2)),x]

[Out]  $(-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 + 3} \frac{1}{\sqrt[3]{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)

[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 3)\*(-x^2 + 1)^(1/3)), x)

**Fricas [B]** time = 11.9618, size = 5544, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")

[Out]  $-1/20736*432^{(5/6)}*\sqrt{3}*\log(10368*(6*2^{(2/3)}*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432^{(1/6)}*\sqrt{3}*(x^5 - x^3) + (432^{(5/6)}*\sqrt{3}*(7*x^3 - 3*x) + 216*2^{(1/3)}*(x^4 + 3*x^2))*(-x^2 + 1)^{(2/3)} - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^{(1/3)})/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432^{(5/6)}*\sqrt{3}*\log(2592*(6*2^{(2/3)}*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432^{(1/6)}*\sqrt{3}*(x^5 - x^3) + (432^{(5/6)}*\sqrt{3}*(7*x^3 - 3*x) + 216*2^{(1/3)}*(x^4 + 3*x^2))*(-x^2 + 1)^{(2/3)} - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^{(1/3)})/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^{(5/6)}*\sqrt{3}*\log(10368*(6*2^{(2/3)}*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^{(1/6)}*\sqrt{3}*(x^5 - x^3) - (432^{(5/6)}*\sqrt{3}*(7*x^3 - 3*x) - 216*2^{(1/3)}*(x^4 + 3*x^2))*(-x^2 + 1)^{(2/3)} + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^{(1/3)})/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^{(5/6)}*\sqrt{3}*\log(2592*(6*2^{(2/3)}*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^{(1/6)}*\sqrt{3}*(x^5 - x^3$

) - (432<sup>(5/6)</sup>\*sqrt(3)\*(7\*x<sup>3</sup> - 3\*x) - 216\*2<sup>(1/3)</sup>\*(x<sup>4</sup> + 3\*x<sup>2</sup>))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> + 72\*(x<sup>5</sup> - 18\*x<sup>4</sup> + 24\*x<sup>3</sup> + 18\*x<sup>2</sup> - 9\*x)\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup>)/(x<sup>6</sup> + 9\*x<sup>4</sup> + 27\*x<sup>2</sup> + 27)) - 1/1296\*432<sup>(5/6)</sup>\*arctan(1/36\*(432<sup>(5/6)</sup>\*(x<sup>5</sup> - 18\*x<sup>3</sup> + 9\*x)\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup> + sqrt(3)\*2<sup>(1/3)</sup>\*(432<sup>(5/6)</sup>\*(x<sup>4</sup> + 9\*x<sup>2</sup>))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> - 288\*sqrt(3)\*(2\*x<sup>4</sup> - 3\*x<sup>2</sup>))\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup> + 6\*432<sup>(1/6)</sup>\*(x<sup>6</sup> + 141\*x<sup>4</sup> - 153\*x<sup>2</sup> + 27)) - 648\*432<sup>(1/6)</sup>\*(3\*x<sup>3</sup> - x)\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup>) - 72\*sqrt(3)\*(7\*x<sup>5</sup> + 6\*x<sup>3</sup> - 9\*x))/(x<sup>6</sup> - 225\*x<sup>4</sup> + 243\*x<sup>2</sup> - 27)) - 1/2592\*432<sup>(5/6)</sup>\*arctan(-1/18\*(sqrt(2)\*(18\*sqrt(3)\*2<sup>(2/3)</sup>\*(29\*x<sup>11</sup> + 879\*x<sup>9</sup> - 12078\*x<sup>7</sup> + 10638\*x<sup>5</sup> - 3807\*x<sup>3</sup> + 243\*x) - 2\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup>\*(432<sup>(5/6)</sup>\*(x<sup>10</sup> + 153\*x<sup>8</sup> - 1701\*x<sup>6</sup> + 459\*x<sup>4</sup>) - 216\*sqrt(3)\*2<sup>(1/3)</sup>\*(31\*x<sup>9</sup> - 297\*x<sup>7</sup> - 27\*x<sup>5</sup> - 27\*x<sup>3</sup>)) - 36\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup>\*(sqrt(3)\*(x<sup>11</sup> + 1167\*x<sup>9</sup> - 13158\*x<sup>7</sup> + 17550\*x<sup>5</sup> - 4779\*x<sup>3</sup> + 243\*x) - 8\*sqrt(3)\*(13\*x<sup>10</sup> - 6\*x<sup>8</sup> - 1404\*x<sup>6</sup> + 1350\*x<sup>4</sup> - 81\*x<sup>2</sup>)) - 3\*432<sup>(1/6)</sup>\*(x<sup>12</sup> + 7620\*x<sup>10</sup> - 92115\*x<sup>8</sup> + 169776\*x<sup>6</sup> - 109269\*x<sup>4</sup> + 16524\*x<sup>2</sup> - 729))\*sqrt((6\*2<sup>(2/3)</sup>\*(x<sup>6</sup> + 225\*x<sup>4</sup> - 189\*x<sup>2</sup> + 27) + 144\*432<sup>(1/6)</sup>\*sqrt(3)\*(x<sup>5</sup> - x<sup>3</sup>) + (432<sup>(5/6)</sup>\*sqrt(3)\*(7\*x<sup>3</sup> - 3\*x) + 216\*2<sup>(1/3)</sup>\*(x<sup>4</sup> + 3\*x<sup>2</sup>))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> - 72\*(x<sup>5</sup> + 18\*x<sup>4</sup> + 24\*x<sup>3</sup> - 18\*x<sup>2</sup> - 9\*x)\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup>)/(x<sup>6</sup> + 9\*x<sup>4</sup> + 27\*x<sup>2</sup> + 27)) - 216\*(sqrt(3)\*2<sup>(2/3)</sup>\*(x<sup>10</sup> + 144\*x<sup>8</sup> - 918\*x<sup>6</sup> + 2808\*x<sup>4</sup> - 243\*x<sup>2</sup>) - 3\*432<sup>(1/6)</sup>\*(31\*x<sup>9</sup> - 568\*x<sup>7</sup> + 1710\*x<sup>5</sup> - 432\*x<sup>3</sup> + 27\*x))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> - 18\*sqrt(3)\*(x<sup>12</sup> - 366\*x<sup>10</sup> + 14535\*x<sup>8</sup> - 42660\*x<sup>6</sup> + 58239\*x<sup>4</sup> - 14094\*x<sup>2</sup> + 729) + 144\*sqrt(3)\*(11\*x<sup>11</sup> - 807\*x<sup>9</sup> + 4518\*x<sup>7</sup> - 5238\*x<sup>5</sup> + 3807\*x<sup>3</sup> - 243\*x) - (-x<sup>2</sup> + 1)<sup>(1/3)</sup>\*(432<sup>(5/6)</sup>\*(x<sup>11</sup> - 1215\*x<sup>9</sup> + 11754\*x<sup>7</sup> - 21006\*x<sup>5</sup> + 5589\*x<sup>3</sup> - 243\*x) - 432\*sqrt(3)\*2<sup>(1/3)</sup>\*(13\*x<sup>10</sup> - 120\*x<sup>8</sup> + 1242\*x<sup>6</sup> - 1728\*x<sup>4</sup> + 81\*x<sup>2</sup>)))/(x<sup>12</sup> - 8334\*x<sup>10</sup> + 110727\*x<sup>8</sup> - 301860\*x<sup>6</sup> + 187839\*x<sup>4</sup> - 21870\*x<sup>2</sup> + 729)) - 1/2592\*432<sup>(5/6)</sup>\*arctan(1/18\*(sqrt(2)\*(18\*sqrt(3)\*2<sup>(2/3)</sup>\*(29\*x<sup>11</sup> + 879\*x<sup>9</sup> - 12078\*x<sup>7</sup> + 10638\*x<sup>5</sup> - 3807\*x<sup>3</sup> + 243\*x) + 2\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup>\*(432<sup>(5/6)</sup>\*(x<sup>10</sup> + 153\*x<sup>8</sup> - 1701\*x<sup>6</sup> + 459\*x<sup>4</sup>) + 216\*sqrt(3)\*2<sup>(1/3)</sup>\*(31\*x<sup>9</sup> - 297\*x<sup>7</sup> - 27\*x<sup>5</sup> - 27\*x<sup>3</sup>)) - 36\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup>\*(sqrt(3)\*(x<sup>11</sup> + 1167\*x<sup>9</sup> - 13158\*x<sup>7</sup> + 17550\*x<sup>5</sup> - 4779\*x<sup>3</sup> + 243\*x) + 8\*sqrt(3)\*(13\*x<sup>10</sup> - 6\*x<sup>8</sup> - 1404\*x<sup>6</sup> + 1350\*x<sup>4</sup> - 81\*x<sup>2</sup>)) + 3\*432<sup>(1/6)</sup>\*(x<sup>12</sup> + 7620\*x<sup>10</sup> - 92115\*x<sup>8</sup> + 169776\*x<sup>6</sup> - 109269\*x<sup>4</sup> + 16524\*x<sup>2</sup> - 729))\*sqrt((6\*2<sup>(2/3)</sup>\*(x<sup>6</sup> + 225\*x<sup>4</sup> - 189\*x<sup>2</sup> + 27) - 144\*432<sup>(1/6)</sup>\*sqrt(3)\*(x<sup>5</sup> - x<sup>3</sup>) - (432<sup>(5/6)</sup>\*sqrt(3)\*(7\*x<sup>3</sup> - 3\*x) - 216\*2<sup>(1/3)</sup>\*(x<sup>4</sup> + 3\*x<sup>2</sup>))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> + 72\*(x<sup>5</sup> - 18\*x<sup>4</sup> + 24\*x<sup>3</sup> + 18\*x<sup>2</sup> - 9\*x)\*(-x<sup>2</sup> + 1)<sup>(1/3)</sup>)/(x<sup>6</sup> + 9\*x<sup>4</sup> + 27\*x<sup>2</sup> + 27)) - 216\*(sqrt(3)\*2<sup>(2/3)</sup>\*(x<sup>10</sup> + 144\*x<sup>8</sup> - 918\*x<sup>6</sup> + 2808\*x<sup>4</sup> - 243\*x<sup>2</sup>) + 3\*432<sup>(1/6)</sup>\*(31\*x<sup>9</sup> - 568\*x<sup>7</sup> + 1710\*x<sup>5</sup> - 432\*x<sup>3</sup> + 27\*x))\*(-x<sup>2</sup> + 1)<sup>(2/3)</sup> - 18\*sqrt(3)\*(x<sup>12</sup> - 366\*x<sup>10</sup> + 14535\*x<sup>8</sup> - 42660\*x<sup>6</sup> + 58239\*x<sup>4</sup> - 14094\*x<sup>2</sup> + 729) - 144\*sqrt(3)\*(11\*x<sup>11</sup> - 807\*x<sup>9</sup> + 4518\*x<sup>7</sup> - 5238\*x<sup>5</sup> + 3807\*x<sup>3</sup> - 243\*x) + (-x<sup>2</sup> + 1)<sup>(1/3)</sup>\*(432<sup>(5/6)</sup>\*(x<sup>11</sup> - 1215\*x<sup>9</sup> + 11754\*x<sup>7</sup> - 21006\*x<sup>5</sup> + 5589\*x<sup>3</sup> - 243\*x) + 432\*sqrt(3)\*2<sup>(1/3)</sup>\*(13\*x<sup>10</sup> - 120\*x<sup>8</sup> + 1242\*x<sup>6</sup> - 1728\*x<sup>4</sup> + 81\*x<sup>2</sup>)))/(x<sup>12</sup> - 8334\*x<sup>10</sup> + 110727\*x<sup>8</sup> - 301860\*x<sup>6</sup> + 187839\*x<sup>4</sup> - 21870\*x<sup>2</sup> + 729))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}(x^2+3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x\*\*2+1)\*\*(1/3)/(x\*\*2+3)),x)

[Out] Integral(1/((-x - 1)\*(x + 1)\*\*(1/3)\*(x\*\*2 + 3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

$$3.80 \quad \int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}} dx$$

**Optimal.** Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{x^2+1})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

[Out] -ArcTan[x]/(6\*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)\*(1 + x^2)^(1/3))]/(2\*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2\*2^(2/3)\*Sqrt[3]) - ArcTanh[(Sqrt[3]\*(1 - 2^(1/3)\*(1 + x^2)^(1/3)))/x]/(2\*2^(2/3)\*Sqrt[3])

**Rubi [A]** time = 0.0121913, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{x^2+1})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x^2)\*(1 + x^2)^(1/3)),x]

[Out] -ArcTan[x]/(6\*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)\*(1 + x^2)^(1/3))]/(2\*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2\*2^(2/3)\*Sqrt[3]) - ArcTanh[(Sqrt[3]\*(1 - 2^(1/3)\*(1 + x^2)^(1/3)))/x]/(2\*2^(2/3)\*Sqrt[3])

#### Rule 392

Int[1/(((a\_) + (b\_.)\*(x\_)^2)^(1/3)\*((c\_) + (d\_.)\*(x\_)^2)), x\_Symbol] :> With[{q = Rt[b/a, 2]}, Simp[(q\*ArcTanh[Sqrt[3]/(q\*x)])/(2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x] + (-Simp[(q\*ArcTan[(a^(1/3)\*q\*x)/(a^(1/3) + 2^(1/3)\*(a + b\*x^2)^(1/3))])/(2\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTan[q\*x])/(6\*2^(2/3)\*a^(1/3)\*d), x] + Simp[(q\*ArcTanh[(Sqrt[3]\*(a^(1/3) - 2^(1/3)\*(a + b\*x^2)^(1/3)))/(a^(1/3)\*q\*x)])/(2\*2^(2/3)\*Sqrt[3]\*a^(1/3)\*d), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b\*c + 3\*a\*d, 0] && PosQ[b/a]

#### Rubi steps

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}} dx = -\frac{\tan^{-1}(x)}{6 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1+x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

**Mathematica [C]** time = 0.0633664, size = 124, normalized size = 1.14

$$\frac{9x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1} \left(2x^2 \left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; -x^2, \frac{x^2}{3}\right) - F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; -x^2, \frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}\right)$$



Warning: Unable to verify antiderivative.

[In] Integrate[1/((3 - x^2)\*(1 + x^2)^(1/3)),x]

[Out]  $(-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^{(1/3)}*(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2 + 3} \frac{1}{\sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")

[Out] -integrate(1/((x^2 + 1)^(1/3)\*(x^2 - 3)), x)

**Fricas [B]** time = 9.7741, size = 4852, normalized size = 44.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")

[Out]  $\frac{1}{2592} \cdot 432^{5/6} \cdot \sqrt{3} \cdot \arctan\left(\frac{-1}{54} \cdot (2592x^{11} - 393984x^9 - 699840x^7 - 373248x^5 - 69984x^3 - \sqrt{6} \cdot (18\sqrt{3}) \cdot 2^{2/3} \cdot (19x^{11} + 111x^9 + 6030x^7 + 7182x^5 + 2511x^3 + 243x)) + 3 \cdot 432^{1/6} \cdot \sqrt{3} \cdot (x^{12} + 924x^{10} - 33363x^8 - 60912x^6 - 36693x^4 - 8748x^2 - 729) + (432^{5/6}) \cdot \sqrt{3} \cdot (x^{10} - 78x^8 - 720x^6 - 594x^4 - 81x^2) + 432 \cdot \sqrt{3} \cdot 2^{1/3} \cdot (13x^9 - 177x^7 - 153x^5 - 27x^3)\right) \cdot (x^2 + 1)^{2/3} + 36 \cdot (96x^{10} - 4032x^8 - 2592x^6 + \sqrt{3} \cdot (x^{11} + 369x^9 - 3654x^7 - 5454x^5 - 2187x^3 - 243x)) \cdot (x^2 + 1)^{1/3} \cdot \sqrt{((2 \cdot 2^{2/3}) \cdot (x^6 - 57x^4 - 117x^2 - 27) + (x^2 + 1)^{2/3}) \cdot (432^{5/6}) \cdot (x^3 + x) + 24 \cdot 2^{1/3} \cdot (x^4 + 9x^2)} - 8 \cdot (6x^4 - 18x^2 + \sqrt{3} \cdot (x^5 - 9x)) \cdot (x^2 + 1)^{1/3} - 8 \cdot 432^{1/6} \cdot (x^5 + 18x^3 + 9x)) / (x^6 - 9x^4 + 27x^2 - 27) + 216 \cdot (\sqrt{3}) \cdot 2^{2/3} \cdot (x^{10} + 276x^8 + 1206x^6 + 756x^4 + 81x^2) + 432^{1/6} \cdot \sqrt{3} \cdot (31x^9 - 1620x^7 - 2070x^5 - 756x^3 - 81x) \cdot (x^2 + 1)^{2/3} + 18 \cdot \sqrt{3} \cdot (x^{12} + 1422x^{10} + 2$

```

1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
+ 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
))*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
- 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))

```

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2 \sqrt[3]{x^2+1} - 3\sqrt[3]{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x\*\*2+3)/(x\*\*2+1)\*\*(1/3),x)

[Out] -Integral(1/(x\*\*2\*(x\*\*2 + 1)\*\*(1/3) - 3\*(x\*\*2 + 1)\*\*(1/3)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

$$3.81 \quad \int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}} dx$$

**Optimal.** Leaf size=87

$$\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

[Out]  $(-2*\text{Sqrt}[x]*\text{Sqrt}[a^2 - (1 + a^2)*x + x^2]*\text{ArcTan}[\frac{(1 - a)*\text{Sqrt}[x]}{\text{Sqrt}[a^2 - (1 + a^2)*x + x^2]}])/((1 - a)*\text{Sqrt}[a^2*x - (1 + a^2)*x^2 + x^3])$

**Rubi [A]** time = 0.873863, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2056, 6733, 1698, 205}

$$\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + x)/((-a + x)*\text{Sqrt}[a^2*x - (1 + a^2)*x^2 + x^3]), x]$

[Out]  $(-2*\text{Sqrt}[x]*\text{Sqrt}[a^2 - (1 + a^2)*x + x^2]*\text{ArcTan}[\frac{(1 - a)*\text{Sqrt}[x]}{\text{Sqrt}[a^2 - (1 + a^2)*x + x^2]}])/((1 - a)*\text{Sqrt}[a^2*x - (1 + a^2)*x^2 + x^3])$

#### Rule 2056

$\text{Int}[(u_.)*(P_)^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{m = \text{MinimumMonomialExponent}[P, x]\}, \text{Dist}[P^{\text{FracPart}[p]}/(x^{(m*\text{FracPart}[p])}*\text{Distrib}[1/x^m, P]^{\text{FracPart}[p]}), \text{Int}[u*x^{(m*p)}*\text{Distrib}[1/x^m, P]^p, x], x]] /; \text{FreeQ}[p, x] \&\& \text{!IntegerQ}[p] \&\& \text{SumQ}[P] \&\& \text{EveryQ}[\text{BinomialQ}[\#1, x] \&, P] \&\& \text{!PolyQ}[P, x, 2]$

#### Rule 6733

$\text{Int}[(u_)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(u /. x \rightarrow x^k), x], x, x^{(1/k)}], x]] /; \text{FractionQ}[m]$

#### Rule 1698

$\text{Int}[(A_.) + (B_.)*(x_)^2)/(((d_.) + (e_.)*(x_)^2)*\text{Sqrt}[(a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x\_Symbol] \rightarrow \text{Dist}[A, \text{Subst}[\text{Int}[1/(d - (b*d - 2*a*e)*x^2), x], x, x/\text{Sqrt}[a + b*x^2 + c*x^4]], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{EqQ}[B*d + A*e, 0]$

#### Rule 205

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}} dx &= \frac{\left(\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right) \int \frac{a+x}{\sqrt{x(-a+x)}\sqrt{a^2-(1+a^2)x+x^2}} dx}{\sqrt{a^2x-(1+a^2)x^2+x^3}} \\
&= \frac{\left(2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right) \text{Subst}\left(\int \frac{a+x^2}{(-a+x^2)\sqrt{a^2+(-1-a^2)x^2+x^4}} dx, x, \sqrt{x}\right)}{\sqrt{a^2x-(1+a^2)x^2+x^3}} \\
&= \frac{\left(2a\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right) \text{Subst}\left(\int \frac{1}{-a-(-2a^2-a(-1-a^2))x^2} dx, x, \frac{\sqrt{x}}{\sqrt{a^2-(1+a^2)x}}\right)}{\sqrt{a^2x-(1+a^2)x^2+x^3}} \\
&= -\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2} \tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)x^2+x^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.915921, size = 159, normalized size = 1.83

$$\frac{2i(a^2-x)^{3/2} \sqrt{\frac{x-1}{x-a^2}} \sqrt{\frac{x}{x-a^2}} \left( (a+1) \text{EllipticF}\left(i \sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right), 1-\frac{1}{a^2}\right) - 2\Pi\left(\frac{a-1}{a}; i \sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right) \middle| 1-\frac{1}{a^2}\right) \right)}{(a-1)\sqrt{-a^2} \sqrt{(x-1)x(x-a^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + x)/((-a + x)\*Sqrt[a^2\*x - (1 + a^2)\*x^2 + x^3]),x]

[Out] ((-2\*I)\*(a^2 - x)^(3/2)\*Sqrt[(-1 + x)/(-a^2 + x)]\*Sqrt[x/(-a^2 + x)]\*((1 + a)\*EllipticF[I\*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2\*EllipticPi[i[(-1 + a)/a, I\*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)\*Sqrt[-a^2]\*Sqrt[(-1 + x)\*x\*(-a^2 + x)])

**Maple [C]** time = 0.034, size = 206, normalized size = 2.4

$$-2 \frac{a^2}{\sqrt{-a^2x^2+a^2x+x^3-x^2}} \sqrt{\frac{-a^2+x}{a^2}} \sqrt{\frac{-1+x}{a^2-1}} \sqrt{\frac{x}{a^2}} \text{EllipticF}\left(\sqrt{\frac{-a^2+x}{a^2}}, \sqrt{\frac{a^2}{a^2-1}}\right) - 4 \frac{a^3}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+x)/(-a+x)/(a^2\*x-(a^2+1)\*x^2+x^3)^(1/2),x)

[Out] -2\*a^2\*(-(-a^2+x)/a^2)^(1/2)\*((-1+x)/(a^2-1))^(1/2)\*(x/a^2)^(1/2)/(-a^2\*x^2+a^2\*x+x^3-x^2)^(1/2)\*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-4\*a^3\*(-(-a^2+x)/a^2)^(1/2)\*((-1+x)/(a^2-1))^(1/2)\*(x/a^2)^(1/2)/(-a^2\*x^2+a^2\*x+x^3-x^2)^(1/2)/(a^2-a)\*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(a^2/(a^2-1))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a+x}{\sqrt{a^2x - (a^2+1)x^2 + x^3(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x)/(-a+x)/(a^2\*x-(a^2+1)\*x^2+x^3)^(1/2),x, algorithm="maxima")

[Out] -integrate((a+x)/(sqrt(a^2\*x - (a^2+1)\*x^2 + x^3)\*(a-x)), x)

**Fricas [A]** time = 2.35953, size = 193, normalized size = 2.22

$$\frac{\arctan\left(\frac{\sqrt{a^2x - (a^2+1)x^2 + x^3(a^2 - 2(a^2 - a + 1)x + x^2)}}{2((a-1)x^3 - (a^3 - a^2 + a - 1)x^2 + (a^3 - a^2)x)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x)/(-a+x)/(a^2\*x-(a^2+1)\*x^2+x^3)^(1/2),x, algorithm="fricas")

[Out] arctan(1/2\*sqrt(a^2\*x - (a^2 + 1)\*x^2 + x^3)\*(a^2 - 2\*(a^2 - a + 1)\*x + x^2)/((a - 1)\*x^3 - (a^3 - a^2 + a - 1)\*x^2 + (a^3 - a^2)\*x))/(a - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a+x}{\sqrt{x(-a^2+x)(x-1)(-a+x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x)/(-a+x)/(a\*\*2\*x-(a\*\*2+1)\*x\*\*2+x\*\*3)\*\*(1/2),x)

[Out] Integral((a+x)/(sqrt(x\*(-a\*\*2+x)\*(x-1))\*(-a+x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{a+x}{\sqrt{a^2x - (a^2+1)x^2 + x^3(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x)/(-a+x)/(a^2\*x-(a^2+1)\*x^2+x^3)^(1/2),x, algorithm="giac")

[Out] integrate(-(a+x)/(sqrt(a^2\*x - (a^2+1)\*x^2 + x^3)\*(a-x)), x)

$$3.82 \quad \int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}} dx$$

**Optimal.** Leaf size=1

0

[Out] 0

**Rubi [C]** time = 1.66858, antiderivative size = 529, normalized size of antiderivative = 529., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2056, 6733, 1708, 1103, 1706}

$$\frac{2(1-a)\sqrt{x}\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}} + \frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a}}}{a\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(-2 + a + x)/((-a + x)\*Sqrt[(2 - a)\*a\*x + (-1 - 2\*a + a^2)\*x^2 + x^3]], x]

[Out] (2\*(1 - a)\*Sqrt[x]\*Sqrt[(2 - a)\*a - (1 + 2\*a - a^2)\*x + x^2]\*ArcTan[(Sqrt[-1 + 2\*a - a^2]\*Sqrt[x])/Sqrt[(2 - a)\*a - (1 + 2\*a - a^2)\*x + x^2]]/(a\*Sqrt[-1 + 2\*a - a^2]\*Sqrt[(2 - a)\*a\*x - (1 + 2\*a - a^2)\*x^2 + x^3]) + (((2 - a)\*a)^(3/4)\*Sqrt[x]\*(1 + x/Sqrt[(2 - a)\*a])\*Sqrt[((2 - a)\*a - (1 + 2\*a - a^2)\*x + x^2]/((2 - a)\*a\*(1 + x/Sqrt[(2 - a)\*a])^2))\*EllipticF[2\*ArcTan[Sqrt[x]/((2 - a)\*a)^(1/4)], (2 + (1 + 2\*a - a^2)/Sqrt[(2 - a)\*a])/4]/(a\*Sqrt[(2 - a)\*a\*x - (1 + 2\*a - a^2)\*x^2 + x^3]) + ((2 - a)\*(1 - Sqrt[(2 - a)\*a])\*Sqrt[x]\*(1 + x/Sqrt[(2 - a)\*a])\*Sqrt[((2 - a)\*a - (1 + 2\*a - a^2)\*x + x^2]/((2 - a)\*a\*(1 + x/Sqrt[(2 - a)\*a])^2))\*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4\*Sqrt[(2 - a)\*a]), 2\*ArcTan[Sqrt[x]/((2 - a)\*a)^(1/4)], (2 + (1 + 2\*a - a^2)/Sqrt[(2 - a)\*a])/4]/(((2 - a)\*a)^(3/4)\*Sqrt[(2 - a)\*a\*x - (1 + 2\*a - a^2)\*x^2 + x^3])

#### Rule 2056

Int[(u\_.)\*(P\_)^(p\_.), x\_Symbol] := With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m\*FracPart[p]))\*Distrib[1/x^m, P]^FracPart[p]], Int[u\*x^(m\*p)\*Distrib[1/x^m, P]^p, x], x] /; FreeQ[p, x] && !IntegerQ[p] && SumQ[P] && EveryQ[BinomialQ[#1, x] &, P] && !PolyQ[P, x, 2]

#### Rule 6733

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(u /. x -> x^k), x], x, x^(1/k)], x] /; FractionQ[m]

#### Rule 1708

Int[((A\_.) + (B\_.)\*(x\_)^2)/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(A\*(c\*d + a\*e\*q) - a\*B\*(e + d\*q))/(c\*d^2 - a\*e^2), Int[1/Sqrt[a + b\*x^2 + c\*x^4], x], x] + Dist[(a\*(B\*d - A\*e)\*(e + d\*q))/(c\*d^2 - a\*e^2), Int[(1 + q\*x^2)/((d + e\*x^2)\*Sqrt[a + b\*x^2 + c\*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0

] && PosQ[c/a] && NeQ[c\*A^2 - a\*B^2, 0]

### Rule 1103

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4], x\_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2\*x^2)\*Sqrt[(a + b\*x^2 + c\*x^4)/(a\*(1 + q^2\*x^2)^2)]\*EllipticF[2\*ArcTan[q\*x], 1/2 - (b\*q^2)/(4\*c)])/(2\*q\*Sqrt[a + b\*x^2 + c\*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[c/a]

### Rule 1706

Int[((A\_) + (B\_)\*(x\_)^2)/(((d\_) + (e\_)\*(x\_)^2)\*Sqrt[(a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B\*d - A\*e)\*ArcTan[(Rt[-b + (c\*d)/e + (a\*e)/d, 2]\*x)/Sqrt[a + b\*x^2 + c\*x^4]])/(2\*d\*e\*Rt[-b + (c\*d)/e + (a\*e)/d, 2]), x] + Simp[((B\*d + A\*e)\*(A + B\*x^2)\*Sqrt[(A^2\*(a + b\*x^2 + c\*x^4))/(a\*(A + B\*x^2)^2)]\*EllipticPi[Cancel[-((B\*d - A\*e)^2/(4\*d\*e\*A\*B))], 2\*ArcTan[q\*x], 1/2 - (b\*A)/(4\*a\*B)])/(4\*d\*e\*A\*q\*Sqrt[a + b\*x^2 + c\*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[c/a] && EqQ[c\*A^2 - a\*B^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax + (-1-2a+a^2)x^2 + x^3}} dx &= \frac{\left(\sqrt{x}\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}\right) \int \frac{-2+a+x}{\sqrt{x(-a+x)}\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}}}{\sqrt{(2-a)ax + (-1-2a+a^2)x^2 + x^3}} \\ &= \frac{\left(2\sqrt{x}\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}\right) \text{Subst}\left(\int \frac{-2+a}{(-a+x^2)\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}}\right)}{\sqrt{(2-a)ax + (-1-2a+a^2)x^2 + x^3}} \\ &= \frac{\left(2\sqrt{(2-a)a}\sqrt{x}\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{(2-a)a + (-1-2a+a^2)x + x^2}}\right)}{a\sqrt{(2-a)ax + (-1-2a+a^2)x^2 + x^3}} \\ &= \frac{2(1-a)\sqrt{x}\sqrt{(2-a)a - (1+2a-a^2)x + x^2} \tan^{-1}\left(\frac{\sqrt{-1+2a-a^2}\sqrt{x}}{\sqrt{(2-a)a - (1+2a-a^2)x + x^2}}\right)}{a\sqrt{-1+2a-a^2}\sqrt{(2-a)ax - (1+2a-a^2)x^2 + x^3}} \end{aligned}$$

**Mathematica [C]** time = 0.585636, size = 127, normalized size = 127.

$$\frac{2\sqrt{\frac{1}{x-1} + 1}(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1} + 1}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right), \frac{1}{(a-1)^2}\right) - 2\Pi\left(\frac{1}{1-a}; \sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right) \middle| \frac{1}{(a-1)^2}\right)\right)}{\sqrt{-(a-1)^2}\sqrt{(x-1)x(a^2 - 2a + x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + a + x)/((-a + x)\*Sqrt[(2 - a)\*a\*x + (-1 - 2\*a + a^2)\*x^2 + x^3]), x]

[Out] (2\*Sqrt[1 + (-1 + x)^(-1)]\*Sqrt[1 + (-1 + a)^2/(-1 + x)]\*(-1 + x)^(3/2)\*(EllipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2\*Elliptic



$\text{Pi}[(1 - a)^{-1}, \text{ArcSin}[\text{Sqrt}[-(-1 + a)^2]/\text{Sqrt}[-1 + x]], (-1 + a)^{-2}]]/(\text{Sqrt}[-(-1 + a)^2]*\text{Sqrt}[(-1 + x)*x*(-2*a + a^2 + x)])$

**Maple [C]** time = 0.041, size = 317, normalized size = 317.

$$2 \frac{a^2 - 2a}{\sqrt{a^2x^2 - a^2x - 2ax^2 + x^3 + 2ax - x^2}} \sqrt{\frac{a^2 - 2a + x}{a^2 - 2a}} \sqrt{\frac{-1 + x}{-a^2 + 2a - 1}} \sqrt{\frac{x}{-a^2 + 2a}} \text{EllipticF}\left(\sqrt{\frac{a^2 - 2a + x}{a^2 - 2a}}, \sqrt{\frac{-a^2 + x}{-a^2 + 2a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2+a+x)/(-a+x)/((2-a)\*a\*x+(a^2-2\*a-1)\*x^2+x^3)^(1/2),x)

[Out]  $2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^{1/2}*((-1+x)/(-a^2+2*a-1))^{1/2}*(x/(-a^2+2*a))^{1/2}/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^{1/2}*\text{EllipticF}((a^2-2*a+x)/(a^2-2*a))^{1/2},((-a^2+2*a)/(-a^2+2*a-1))^{1/2}+2*(2*a-2)*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^{1/2}*((-1+x)/(-a^2+2*a-1))^{1/2}*(x/(-a^2+2*a))^{1/2}/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^{1/2}/(-a^2+a)*\text{EllipticPi}((a^2-2*a+x)/(a^2-2*a))^{1/2},(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a+x-2}{\sqrt{-(a-2)ax + (a^2-2a-1)x^2 + x^3(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+a+x)/(-a+x)/((2-a)\*a\*x+(a^2-2\*a-1)\*x^2+x^3)^(1/2),x, algorithm="maxima")

[Out] -integrate((a+x-2)/(sqrt(-(a-2)\*a\*x+(a^2-2\*a-1)\*x^2+x^3)\*(a-x)),x)

**Fricas [C]** time = 2.28269, size = 150, normalized size = 150.

$$\frac{\log\left(\frac{a^2-2(a^2-a)x-x^2+2\sqrt{(a^2-2a-1)x^2+x^3-(a^2-2a)xa}}{a^2-2ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+a+x)/(-a+x)/((2-a)\*a\*x+(a^2-2\*a-1)\*x^2+x^3)^(1/2),x, algorithm="fricas")

[Out]  $\log(-(a^2-2*(a^2-a)*x-x^2+2*\text{sqrt}((a^2-2*a-1)*x^2+x^3-(a^2-2*a)*x)*a)/(a^2-2*a*x+x^2))/a$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + x - 2}{\sqrt{x(x-1)(a^2 - 2a + x)}(-a + x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+a+x)/(-a+x)/((2-a)\*a\*x+(a\*\*2-2\*a-1)\*x\*\*2+x\*\*3)\*\*(1/2),x)

[Out] Integral((a + x - 2)/(sqrt(x\*(x - 1)\*(a\*\*2 - 2\*a + x))\*(-a + x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{a + x - 2}{\sqrt{-(a-2)ax + (a^2 - 2a - 1)x^2 + x^3}(a - x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+a+x)/(-a+x)/((2-a)\*a\*x+(a^2-2\*a-1)\*x^2+x^3)^(1/2),x, algorithm="giac")

[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)\*a\*x + (a^2 - 2\*a - 1)\*x^2 + x^3)\*(a - x)), x)

$$3.83 \quad \int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}} dx$$

**Optimal.** Leaf size=46

$$\log \left( \frac{-2 \left( \sqrt{(1-x)x(a^2-2ax+x)} + x \right) - a^2 + 2ax + x^2}{(a-x)^2} \right)$$

[Out] Log[(-a^2 + 2\*a\*x + x^2 - 2\*(x + Sqrt[(1 - x)\*x\*(a^2 + x - 2\*a\*x)]))]/(a - x)^2]

**Rubi [C]** time = 1.49236, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$ , Rules used = {2056, 6733, 1710, 1104, 419, 1220, 537}

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}} + 1\Pi\left(\frac{1}{a}; \sin^{-1}(\sqrt{x}) \mid -\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-(1-2a)x^3}} - \frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}} + 1F\left(\sin^{-1}(\sqrt{x}) \mid -\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

[In] Int[(-a + (-1 + 2\*a)\*x)/((-a + x)\*Sqrt[a^2\*x - (-1 + 2\*a + a^2)\*x^2 + (-1 + 2\*a)\*x^3]), x]

[Out] (-2\*(1 - 2\*a)\*Sqrt[1 - x]\*Sqrt[x]\*Sqrt[1 + ((1 - 2\*a)\*x)/a^2]\*EllipticF[ArcSin[Sqrt[x]], -((1 - 2\*a)/a^2)]/Sqrt[a^2\*x + (1 - 2\*a - a^2)\*x^2 - (1 - 2\*a)\*x^3] + (4\*(1 - a)\*Sqrt[1 - x]\*Sqrt[x]\*Sqrt[1 + ((1 - 2\*a)\*x)/a^2]\*EllipticPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2\*a)/a^2)]/Sqrt[a^2\*x + (1 - 2\*a - a^2)\*x^2 - (1 - 2\*a)\*x^3]

#### Rule 2056

Int[(u\_.)\*(P\_)^(p\_.), x\_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m\*FracPart[p]))\*Distrib[1/x^m, P]^FracPart[p]], Int[u\*x^(m\*p)\*Distrib[1/x^m, P]^p, x], x] /; FreeQ[p, x] && !IntegerQ[p] && SumQ[P] && EveryQ[BinomialQ[#1, x] &, P] && !PolyQ[P, x, 2]

#### Rule 6733

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(u /. x -> x^k), x], x, x^(1/k)], x] /; FractionQ[m]

#### Rule 1710

Int[((A\_.) + (B\_.)\*(x\_)^2)/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4]), x\_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b\*x^2 + c\*x^4], x], x] + Dist[(e\*A - d\*B)/e, Int[1/((d + e\*x^2)\*Sqrt[a + b\*x^2 + c\*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[c/a]

#### Rule 1104

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4], x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(Sqrt[1 + (2\*c\*x^2)/(b - q)]\*Sqrt[1 + (2\*c\*x^2)/(b + q)])/Sqrt[a + b\*x^2 + c\*x^4], Int[1/(Sqrt[1 + (2\*c\*x^2)/(b - q)]\*Sqrt[1 + (

$2*c*x^2)/(b + q)]), x], x]] /; FreeQ[\{a, b, c\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&$   
 $\& NegQ[c/a]$

Rule 419

$Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x\_Symbol] :> S$   
 $imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt$   
 $[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d\}, x] \&\& NegQ[d/c] \&\& GtQ[c, 0] \&\& GtQ$   
 $[a, 0] \&\& !(NegQ[b/a] \&\& SimplerSqrtQ[-(b/a), -(d/c)])]$

Rule 1220

$Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x\_S$   
 $ymbol] :> With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[\{a, b, c, d, e\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NegQ[c/a]$

Rule 537

$Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x\_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d, e, f\}, x] \&\& !GtQ[d/c, 0] \&\& GtQ[c, 0] \&\& GtQ[e, 0] \&\& !( !GtQ[f/e, 0] \&\& SimplerSqrtQ[-(f/e), -(d/c)])]$

Rubi steps

$$\int \frac{-a + (-1 + 2a)x}{(-a + x)\sqrt{a^2x - (-1 + 2a + a^2)x^2 + (-1 + 2a)x^3}} dx = \frac{\left(\sqrt{x}\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}\right) \int \frac{-a+(-1+2a)x}{\sqrt{x}(-a+x)\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}} dx}{\sqrt{a^2x - (-1 + 2a + a^2)x^2 + (-1 + 2a)x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}\right) \text{Subst}\left(\int \frac{-a+(-1+2a)x}{(-a+x^2)\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}} dx, x, \sqrt{x}\right)}{\sqrt{a^2x - (-1 + 2a + a^2)x^2 + (-1 + 2a)x^3}}$$

$$= \frac{\left(4(1 - a)a\sqrt{x}\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}\right) \text{Subst}\left(\int \frac{-a+(-1+2a)x}{(-a+x^2)\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}} dx, x, \sqrt{x}\right)}{\sqrt{a^2x - (-1 + 2a + a^2)x^2 + (-1 + 2a)x^3}}$$

$$= \frac{\left(4(1 - a)a\sqrt{1 - x}\sqrt{x}\sqrt{1 + \frac{(1-2a)x}{a^2}}\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}\right) \text{Subst}\left(\int \frac{-a+(-1+2a)x}{(-a+x^2)\sqrt{a^2 - (-1 + 2a + a^2)x + (-1 + 2a)x^2}} dx, x, \sqrt{x}\right)}{\sqrt{a^2x - (-1 + 2a + a^2)x^2 + (-1 + 2a)x^3}}$$

$$= \frac{2(1 - 2a)\sqrt{1 - x}\sqrt{x}\sqrt{1 + \frac{(1-2a)x}{a^2}}F\left(\sin^{-1}\left(\sqrt{x}\right) \middle| -\frac{1-2a}{a^2}\right)}{\sqrt{a^2x + (1 - 2a - a^2)x^2 - (1 - 2a)x^3}} + \frac{4(1 - a)a\sqrt{1 - x}\sqrt{x}\sqrt{1 + \frac{(1-2a)x}{a^2}}}{\sqrt{a^2x + (1 - 2a - a^2)x^2 - (1 - 2a)x^3}}$$

**Mathematica [C]** time = 1.07234, size = 133, normalized size = 2.89

$$\frac{2i(x - 1)^{3/2} \sqrt{\frac{x}{x-1}} \sqrt{\frac{a^2 - 2ax + x}{(2a-1)(x-1)}} \left(2a\Pi\left(1 - a; i \sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right) \middle| -\frac{(a-1)^2}{2a-1}\right) - \text{EllipticF}\left(i \sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right), -\frac{(a-1)^2}{2a-1}\right)\right)}{\sqrt{-(x - 1)x(a^2 - 2ax + x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a))] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a))])/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

**Maple [C]** time = 0.047, size = 536, normalized size = 11.7

$$2 \frac{a^2}{(-1 + 2a) \sqrt{-a^2 x^2 + 2 a x^3 + a^2 x - 2 a x^2 - x^3 + x^2}} \sqrt{-\frac{-1 + 2a}{a^2} \left(x - \frac{a^2}{-1 + 2a}\right)} \sqrt{(-1 + x) \left(\frac{a^2}{-1 + 2a} - 1\right)^{-1}} \sqrt{\frac{(-1 + 2a)}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x)
```

```
[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(2a-1)x-a}{\sqrt{(2a-1)x^3+a^2x-(a^2+2a-1)x^2(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a - x)), x)
```

**Fricas [A]** time = 2.31541, size = 144, normalized size = 3.13

$$\log \left( \frac{a^2 - 2(a-1)x - x^2 + 2\sqrt{(2a-1)x^3 + a^2x - (a^2 + 2a - 1)x^2}}{a^2 - 2ax + x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
1)*x^2)))/(a^2 - 2*a*x + x^2))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2ax - a - x}{\sqrt{x(x-1)(-a^2 + 2ax - x)}(-a + x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2a-1)x-a}{\sqrt{(2a-1)x^3+a^2x-(a^2+2a-1)x^2(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

$$3.84 \quad \int \frac{1 - \sqrt[3]{2x}}{(2^{2/3} + x)\sqrt{1+x^3}} dx$$

**Optimal.** Leaf size=32

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{3}(\sqrt[3]{2x+1})}{\sqrt{x^3+1}} \right)}{\sqrt{3}}$$

[Out] (2\*ArcTan[(Sqrt[3]\*(1 + 2^(1/3)\*x))/Sqrt[1 + x^3]])/Sqrt[3]

**Rubi [A]** time = 0.0963157, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2137, 203}

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{3}(\sqrt[3]{2x+1})}{\sqrt{x^3+1}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2^(1/3)\*x)/((2^(2/3) + x)\*Sqrt[1 + x^3]), x]

[Out] (2\*ArcTan[(Sqrt[3]\*(1 + 2^(1/3)\*x))/Sqrt[1 + x^3]])/Sqrt[3]

#### Rule 2137

Int[((e\_) + (f\_)\*(x\_))/(((c\_) + (d\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_)^3]), x\_Symbol] :> Dist[(2\*e)/d, Subst[Int[1/(1 + 3\*a\*x^2), x], x, (1 + (2\*d\*x)/c)/Sqrt[a + b\*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && EqQ[b\*c^3 - 4\*a\*d^3, 0] && EqQ[2\*d\*e + c\*f, 0]

#### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1 - \sqrt[3]{2x}}{(2^{2/3} + x)\sqrt{1+x^3}} dx &= 2 \text{Subst} \left( \int \frac{1}{1 + 3x^2} dx, x, \frac{1 + \sqrt[3]{2x}}{\sqrt{1+x^3}} \right) \\ &= \frac{2 \tan^{-1} \left( \frac{\sqrt{3}(1 + \sqrt[3]{2x})}{\sqrt{1+x^3}} \right)}{\sqrt{3}} \end{aligned}$$

**Mathematica [C]** time = 0.448681, size = 323, normalized size = 10.09

$$2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3+3i}}}\left(\sqrt{2ix + \sqrt{3} - i}\left((-3i\sqrt{2} + 4\sqrt{3} + \sqrt[3]{2}\sqrt{3}\right)x + \sqrt[3]{2}\sqrt{3} - 2\sqrt{3} + 3i\sqrt[3]{2} + 6i\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-2ix + \sqrt{3} + i}}{\sqrt{2}\sqrt[4]{3}}\right), \sqrt{3}\right)\right)$$

$$(1 + 2 \cdot 2^{2/3} - i\sqrt{3})\sqrt{-2ix + \sqrt{3} + i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - 2^(1/3)\*x)/((2^(2/3) + x)\*Sqrt[1 + x^3]),x]

[Out] (-2\*Sqrt[2/3]\*Sqrt[(I\*(1 + x))/(3\*I + Sqrt[3])])\*(Sqrt[-I + Sqrt[3] + (2\*I)\*x]\*(6\*I + (3\*I)\*2^(1/3) - 2\*Sqrt[3] + 2^(1/3)\*Sqrt[3] + ((-3\*I)\*2^(1/3) + 4\*Sqrt[3] + 2^(1/3)\*Sqrt[3])\*x)\*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2\*I)\*x]/(Sqrt[2]\*3^(1/4))], (2\*Sqrt[3])/(3\*I + Sqrt[3])] - (6\*I)\*Sqrt[3]\*Sqrt[I + Sqrt[3] - (2\*I)\*x]\*Sqrt[1 - x + x^2]\*EllipticPi[(2\*Sqrt[3])/(I + (2\*I)\*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2\*I)\*x]/(Sqrt[2]\*3^(1/4))], (2\*Sqrt[3])/(3\*I + Sqrt[3])])/(1 + 2\*2^(2/3) - I\*Sqrt[3])\*Sqrt[I + Sqrt[3] - (2\*I)\*x]\*Sqrt[1 + x^3]

**Maple [C]** time = 0.059, size = 258, normalized size = 8.1

$$-2 \frac{\sqrt[3]{2} (3/2 - i/2\sqrt{3})}{\sqrt{x^3 + 1}} \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x-1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x-1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \text{EllipticF} \left( \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \right) + 6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-2^(1/3)\*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)

[Out] -2\*2^(1/3)\*(3/2-1/2\*I\*3^(1/2))\*((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2-1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2+1/2\*I\*3^(1/2))/(-3/2+1/2\*I\*3^(1/2)))^(1/2)/(x^3+1)^(1/2)\*EllipticF(((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2), ((-3/2+1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2))+6\*(3/2-1/2\*I\*3^(1/2))\*((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2-1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2+1/2\*I\*3^(1/2))/(-3/2+1/2\*I\*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)\*EllipticPi(((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2), (-3/2+1/2\*I\*3^(1/2))/(2^(2/3)-1), ((-3/2+1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{1}{3}}x - 1}{\sqrt{x^3 + 1} \left(x + 2^{\frac{2}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2^(1/3)\*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((2^(1/3)\*x - 1)/(sqrt(x^3 + 1)\*(x + 2^(2/3))), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2^(1/3)\*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")



[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1} + 2^{\frac{2}{3}}\sqrt{x^3+1}} dx - \int -\frac{1}{x\sqrt{x^3+1} + 2^{\frac{2}{3}}\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2\*\*(1/3)\*x)/(2\*\*(2/3)+x)/(x\*\*3+1)\*\*(1/2),x)

[Out] -Integral(2\*\*(1/3)\*x/(x\*sqrt(x\*\*3 + 1) + 2\*\*(2/3)\*sqrt(x\*\*3 + 1)), x) - Integral(-1/(x\*sqrt(x\*\*3 + 1) + 2\*\*(2/3)\*sqrt(x\*\*3 + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{2^{\frac{1}{3}}x - 1}{\sqrt{x^3+1}\left(x + 2^{\frac{2}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2^(1/3)\*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(2^(1/3)\*x - 1)/(sqrt(x^3 + 1)\*(x + 2^(2/3))), x)

$$3.85 \quad \int \frac{1+x}{(-2+x)\sqrt{1+x^3}} dx$$

**Optimal.** Leaf size=23

$$-\frac{2}{3} \tanh^{-1} \left( \frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

[Out]  $(-2*\text{ArcTanh}[(1+x)^2/(3*\text{Sqrt}[1+x^3])])/3$

**Rubi [A]** time = 0.0539309, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2138, 206}

$$-\frac{2}{3} \tanh^{-1} \left( \frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1+x)/((-2+x)\*Sqrt[1+x^3]),x]

[Out]  $(-2*\text{ArcTanh}[(1+x)^2/(3*\text{Sqrt}[1+x^3])])/3$

#### Rule 2138

Int[((e\_) + (f\_.)\*(x\_))/(((c\_) + (d\_.)\*(x\_))\*Sqrt[(a\_) + (b\_.)\*(x\_)^3]), x\_Symbol] :> Dist[(-2\*e)/d, Subst[Int[1/(9 - a\*x^2), x], x, (1 + (f\*x)/e)^2/Sqrt[a + b\*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d\*e - c\*f, 0] && EqQ[b\*c^3 + 8\*a\*d^3, 0] && EqQ[2\*d\*e + c\*f, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1+x}{(-2+x)\sqrt{1+x^3}} dx &= - \left( 2 \text{Subst} \left( \int \frac{1}{9-x^2} dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}} \right) \right) \\ &= -\frac{2}{3} \tanh^{-1} \left( \frac{(1+x)^2}{3\sqrt{1+x^3}} \right) \end{aligned}$$

**Mathematica [A]** time = 0.0072606, size = 46, normalized size = 2.

$$\frac{1}{3} \log \left( 3 - \frac{(x+1)^2}{\sqrt{x^3+1}} \right) - \frac{1}{3} \log \left( \frac{(x+1)^2}{\sqrt{x^3+1}} + 3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1+x)/((-2+x)\*Sqrt[1+x^3]),x]

[Out]  $\text{Log}[3 - (1 + x)^2/\text{Sqrt}[1 + x^3]]/3 - \text{Log}[3 + (1 + x)^2/\text{Sqrt}[1 + x^3]]/3$

**Maple [C]** time = 0.021, size = 240, normalized size = 10.4

$$2 \frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3 + 1}} \sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \text{EllipticF}\left(\sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}}\right) - 2 \frac{3/2}{\sqrt{x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((1+x)/(-2+x)/(x^3+1)^{(1/2)}, x)$

[Out]  $2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*\text{EllipticF}(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}, ((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)})-2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*\text{EllipticPi}(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}, -1/6*I*3^{(1/2)}+1/2, ((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((1+x)/(-2+x)/(x^3+1)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((x + 1)/(\text{sqrt}(x^3 + 1)*(x - 2)), x)$

**Fricas [B]** time = 2.20194, size = 117, normalized size = 5.09

$$\frac{1}{3} \log\left(\frac{x^3 + 12x^2 - 6\sqrt{x^3+1}(x+1) - 6x + 10}{x^3 - 6x^2 + 12x - 8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((1+x)/(-2+x)/(x^3+1)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out]  $1/3*\log((x^3 + 12*x^2 - 6*\text{sqrt}(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 + 12*x - 8))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)(x^2-x+1)}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-2+x)/(x\*\*3+1)\*\*(1/2),x)

[Out] Integral((x + 1)/(sqrt((x + 1)\*(x\*\*2 - x + 1))\*(x - 2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((x + 1)/(sqrt(x^3 + 1)\*(x - 2)), x)

$$3.86 \quad \int \frac{x}{\sqrt{1+x^3}(10+6\sqrt{3}+x^3)} dx$$

**Optimal.** Leaf size=218

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(-2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}$$

[Out] -((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 + Sqrt[3])\*(1 + x))/(Sqrt[2]\*Sqrt[1 + x^3]])/(2\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTan[((1 - Sqrt[3])\*Sqrt[1 + x^3])/(Sqrt[2]\*3^(3/4))])/(3\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 + Sqrt[3] - 2\*x))/(Sqrt[2]\*Sqrt[1 + x^3])])/(3\*Sqrt[2]\*3^(1/4)) - ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 - Sqrt[3])\*(1 + x))/(Sqrt[2]\*Sqrt[1 + x^3])])/(6\*Sqrt[2]\*3^(1/4))

**Rubi [A]** time = 0.0409792, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {487}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(-2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x^3]\*(10 + 6\*Sqrt[3] + x^3)),x]

[Out] -((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 + Sqrt[3])\*(1 + x))/(Sqrt[2]\*Sqrt[1 + x^3]])/(2\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTan[((1 - Sqrt[3])\*Sqrt[1 + x^3])/(Sqrt[2]\*3^(3/4))])/(3\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 + Sqrt[3] - 2\*x))/(Sqrt[2]\*Sqrt[1 + x^3])])/(3\*Sqrt[2]\*3^(1/4)) - ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 - Sqrt[3])\*(1 + x))/(Sqrt[2]\*Sqrt[1 + x^3])])/(6\*Sqrt[2]\*3^(1/4))

**Rule 487**

Int[(x\_)/(Sqrt[(a\_) + (b\_.)\*(x\_)^3]\*((c\_) + (d\_.)\*(x\_)^3)), x\_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b\*c - 10\*a\*d)/(6\*a\*d)]}, -Simp[(q\*(2 - r)\*ArcTan[((1 - r)\*Sqrt[a + b\*x^3])/(Sqrt[2]\*Rt[a, 2]\*r^(3/2))])/(3\*Sqrt[2]\*Rt[a, 2]\*d\*r^(3/2)), x] + (-Simp[(q\*(2 - r)\*ArcTan[(Rt[a, 2]\*Sqrt[r]\*(1 + r)\*(1 + q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(2\*Sqrt[2]\*Rt[a, 2]\*d\*r^(3/2)), x] - Simp[(q\*(2 - r)\*ArcTanh[(Rt[a, 2]\*Sqrt[r]\*(1 + r - 2\*q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(3\*Sqrt[2]\*Rt[a, 2]\*d\*Sqrt[r]), x] - Simp[(q\*(2 - r)\*ArcTanh[(Rt[a, 2]\*(1 - r)\*Sqrt[r]\*(1 + q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(6\*Sqrt[2]\*Rt[a, 2]\*d\*Sqrt[r]), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b^2\*c^2 - 20\*a\*b\*c\*d - 8\*a^2\*d^2, 0] && PosQ[a]

**Rubi steps**

$$\int \frac{x}{\sqrt{1+x^3}(10+6\sqrt{3}+x^3)} dx = -\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(-2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}}$$

**Mathematica [C]** time = 0.0563565, size = 47, normalized size = 0.22

$$\frac{x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -x^3, -\frac{x^3}{10+6\sqrt{3}}\right)}{20 + 12\sqrt{3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 + x^3]\*(10 + 6\*Sqrt[3] + x^3)),x]

[Out] (x^2\*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6\*Sqrt[3]))])/(20 + 12\*Sqrt[3])

**Maple [C]** time = 0.195, size = 353, normalized size = 1.6

$$-\frac{\sqrt{2}}{18} \sum_{\alpha=\text{RootOf}(\_Z^2+(-1-\sqrt{3})\_Z+2\sqrt{3}+4)} \frac{(-\sqrt{3}\alpha + \alpha - 2)(-i\sqrt{3} + 3)(-1 + 2\alpha - \sqrt{3}\alpha)}{-\sqrt{3} + 2\alpha - 1} \sqrt{\frac{1 + \dots}{-i\sqrt{3} - \dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(10+x^3+6\*3^(1/2))/(x^3+1)^(1/2),x)

[Out] -1/18\*2^(1/2)\*sum((-3^(1/2)\*\_alpha+\_alpha-2)/(-3^(1/2)+2\*\_alpha-1)\*(-I\*3^(1/2)+3)\*((1+x)/(-I\*3^(1/2)+3))^(1/2)\*((2\*x-1-I\*3^(1/2))/(-I\*3^(1/2)-3))^(1/2)\*((2\*x-1+I\*3^(1/2))/(I\*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)\*(-1+2\*\_alpha-3^(1/2))\*\_alpha)\*EllipticPi(((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2),-1/2\*I\*\_alpha+1/3\*I\*\_alpha\*3^(1/2)+1/2\*3^(1/2)\*\_alpha-\_alpha-1/6\*I\*3^(1/2)+1/2,((-3/2+1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2),\_alpha=RootOf(\_Z^2+(-1-3^(1/2))\*\_Z+2\*3^(1/2)+4))+1/9\*(-1-3^(1/2))/(2+3^(1/2))\*(3/2-1/2\*I\*3^(1/2))\*((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2-1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x-1/2+1/2\*I\*3^(1/2))/(-3/2+1/2\*I\*3^(1/2)))^(1/2)/(x^3+1)^(1/2)\*3^(1/2)\*EllipticPi(((1+x)/(3/2-1/2\*I\*3^(1/2)))^(1/2),1/3\*(-3/2+1/2\*I\*3^(1/2))\*3^(1/2),((-3/2+1/2\*I\*3^(1/2))/(-3/2-1/2\*I\*3^(1/2)))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3+6\*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 6\*sqrt(3) + 10)\*sqrt(x^3 + 1)), x)

**Fricas [B]** time = 49.077, size = 27647, normalized size = 126.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3+6\*3^(1/2)))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/432*\sqrt{-2*(7*\sqrt{3} + 12)*\sqrt{-672*\sqrt{3} + 1164} + 24}*(56*\sqrt{3} \\ & + 97)*\sqrt{-56*\sqrt{3} + 97}*(-672*\sqrt{3} + 1164)^{3/4}*\arctan(-1/1296*(6 \\ & *\sqrt{x^3 + 1}*((459*x^{16} - 13425*x^{15} - 33201*x^{14} + 950652*x^{13} - 997302* \\ & x^{12} - 14760972*x^{11} + 47069892*x^{10} - 49762248*x^9 - 8212536*x^8 + 8437780 \\ & 8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979 \\ & 2*x^2 + \sqrt{3}*(265*x^{16} - 7751*x^{15} - 19167*x^{14} + 548864*x^{13} - 575818*x \\ & ^{12} - 8522268*x^{11} + 27175852*x^{10} - 28730312*x^9 - 4741560*x^8 + 48715600* \\ & x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x \\ & ^2 - 2488832*x - 1889792) + (3691*x^{16} - 6128*x^{15} - 537864*x^{14} + 1586477* \\ & x^{13} + 16210952*x^{12} - 77181756*x^{11} + 84218362*x^{10} + 71018320*x^9 - 25445 \\ & 5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 + \\ & 78464672*x^3 - 78514944*x^2 + \sqrt{3}*(2131*x^{16} - 3538*x^{15} - 310536*x^{14} \\ & + 915953*x^{13} + 9359398*x^{12} - 44560908*x^{11} + 48623494*x^{10} + 41002448*x^9 \\ & - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*x \\ & ^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13 \\ & 160704)*\sqrt{-672*\sqrt{3} + 1164} - 4310784*x - 3273216)*(-672*\sqrt{3} + 11 \\ & 64)^{3/4} + 3*(984*x^{15} - 30612*x^{14} + 164676*x^{13} - 205368*x^{12} - 289200*x \\ & ^{11} + 183720*x^{10} + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 15 \\ & 58464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*\sqrt{3}*(142*x^{15} - 44 \\ & 19*x^{14} + 23781*x^{13} - 29608*x^{12} - 41940*x^{11} + 26454*x^{10} + 128152*x^9 - \\ & 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 + \\ & 108288*x^2 - 96000*x - 33920) + (4945*x^{15} - 88617*x^{14} + 738528*x^{13} - 186 \\ & 0046*x^{12} - 784596*x^{11} + 7668708*x^{10} - 6570680*x^9 - 6903864*x^8 + 154441 \\ & 44*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 \\ & + \sqrt{3}*(2855*x^{15} - 51163*x^{14} + 426388*x^{13} - 1073898*x^{12} - 452980*x^ \\ & 11 + 4427548*x^{10} - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - \\ & 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) \\ & + 2674176*x + 944896)*\sqrt{-672*\sqrt{3} + 1164} - 665088*x - 235008)*(-672* \\ & \sqrt{3} + 1164)^{1/4})*\sqrt{-2*(7*\sqrt{3} + 12)*\sqrt{-672*\sqrt{3} + 1164} + 24} \\ & *\sqrt{-56*\sqrt{3} + 97} + 36*(144*x^{17} - 5976*x^{16} + 5544*x^{15} + 299664 \\ & *x^{14} - 1062360*x^{13} + 116712*x^{12} + 3600000*x^{11} - 4761216*x^{10} - 1046592* \\ & x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 - \\ & 1861632*x^3 + 1640448*x^2 + 12*\sqrt{3}*(7*x^{17} - 286*x^{16} + 238*x^{15} + 142 \\ & 55*x^{14} - 50390*x^{13} + 5942*x^{12} + 171808*x^{11} - 226888*x^{10} - 48920*x^9 + \\ & 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3 \\ & + 78784*x^2 - 39040*x - 18176) - (1164*x^{17} - 6276*x^{16} - 26052*x^{15} + 3328 \\ & 44*x^{14} - 1632156*x^{13} + 4149132*x^{12} - 5805024*x^{11} + 318696*x^{10} + 126210 \\ & 72*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 1093612 \\ & 8*x^4 + 6434304*x^3 - 6426240*x^2 + 24*\sqrt{3}*(28*x^{17} - 151*x^{16} - 626*x^ \\ & 15 + 8006*x^{14} - 39266*x^{13} + 99812*x^{12} - 139652*x^{11} + 7661*x^{10} + 303610 \\ & *x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154 \\ & 784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^{17} - 96354*x^{16} + 84798*x \\ & ^{15} + 4817124*x^{14} - 17052930*x^{13} + 1941678*x^{12} + 57963744*x^{11} - 7660368 \\ & 0*x^{10} - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 1132 \\ & 69536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + \sqrt{3}*(1351*x^{17} \\ & - 55630*x^{16} + 48958*x^{15} + 2781167*x^{14} - 9845510*x^{13} + 1121030*x^{12} + 3 \\ & 3465376*x^{11} - 44227144*x^{10} - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - \\ & 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - \\ & 7571584*x - 3526400) - 13114368*x - 6107904)*\sqrt{-672*\sqrt{3} + 1164} + 32 \\ & 61696*x + 1519104)*\sqrt{-672*\sqrt{3} + 1164} + 12*(97*x^{17} - 523*x^{16} - 217 \\ & 1*x^{15} + 27737*x^{14} - 136013*x^{13} + 345761*x^{12} - 483752*x^{11} + 26558*x^{10} \\ & + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 9113 \\ & 44*x^4 + 536192*x^3 - 535520*x^2 + 2*\sqrt{3}*(28*x^{17} - 151*x^{16} - 626*x^{15} \\ & + 8006*x^{14} - 39266*x^{13} + 99812*x^{12} - 139652*x^{11} + 7661*x^{10} + 303610*x \\ & ^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 15478 \\ & 4*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*\sqrt{-672*\sqrt{3} \\ & (3 + 1164) - 811008*x - 377856)*\sqrt{-56*\sqrt{3} + 97} - (\sqrt{x^3 + 1}*((45 \end{aligned}$$

$$\begin{aligned}
& 9x^{16} - 1557x^{15} - 26415x^{14} - 1449954x^{13} + 4677912x^{12} + 12651948x^{11} - 55684800x^{10} + 62834256x^9 + 8526168x^8 - 105313392x^7 + 99605088x^6 \\
& - 18897984x^5 - 42499296x^4 + 37357632x^3 - 8256960x^2 + \sqrt{3} \cdot (265x^{16} - 899x^{15} - 15249x^{14} - 837130x^{13} + 2700776x^{12} + 7304604x^{11} \\
& - 32149640x^{10} + 36277360x^9 + 4922568x^8 - 60802736x^7 + 57507040x^6 - 10910784x^5 \\
& - 24536992x^4 + 21568448x^3 - 4767168x^2 + 1207168x + 1383424) + (3691x^{16} + 17731x^{15} - 951114x^{14} + 450359x^{13} + 4370159x^{12} \\
& + 30318522x^{11} - 78096668x^{10} + 9429316x^9 + 146877876x^8 - 197107784x^7 - 30834152x^6 \\
& + 185125776x^5 - 132260896x^4 - 45545344x^3 + 69517536x^2 + \sqrt{3} \cdot (2131x^{16} + 10237x^{15} - 549126x^{14} + 260015x^{13} + 2523 \\
& 113x^{12} + 17504406x^{11} - 45089132x^{10} + 5444020x^9 + 84799980x^8 - 113800232x^7 - 17802104x^6 \\
& + 106882416x^5 - 76360864x^4 - 26295616x^3 + 40135968x^2 - 7907648x - 5562368) - 13696448x - 9634304) \cdot \sqrt{-672 \cdot \sqrt{3}} \\
& + 1164) + 2090880x + 2396160) \cdot (-672 \cdot \sqrt{3} + 1164)^{(3/4)} + 3 \cdot (984x^{15} - 14712x^{14} \\
& - 53940x^{13} + 411732x^{12} - 280248x^{11} - 324624x^{10} + 180816x^9 - 518544x^8 + 974304x^7 \\
& - 887136x^6 - 1404096x^5 + 1843584x^4 + 135936x^3 - 696192x^2 + 4 \cdot \sqrt{3} \cdot (142x^{15} - 2124x^{14} - 7773x^{13} + 594 \\
& 47x^{12} - 40626x^{11} - 46860x^{10} + 26308x^9 - 75276x^8 + 140472x^7 - 127784x^6 - 202896x^5 \\
& + 266016x^4 + 19712x^3 - 100512x^2 + 62400x + 24832) + (4945x^{15} - 37473x^{14} - 490698x^{13} \\
& + 2249468x^{12} + 474132x^{11} - 8423784x^{10} + 5853520x^9 + 8451720x^8 - 15320016x^7 + 768064x^6 + 1040 \\
& 5056x^5 - 6627744x^4 - 700480x^3 + 2799552x^2 + \sqrt{3} \cdot (2855x^{15} - 21635x^{14} - 283306x^{13} \\
& + 1298732x^{12} + 273748x^{11} - 4863472x^{10} + 3379536x^9 + 4879608x^8 - 8845008x^7 \\
& + 443456x^6 + 6007360x^5 - 3826528x^4 - 404416x^3 + 1616320x^2 - 1003648x - 399360) - 1738368x - 691712) \cdot \sqrt{-672 \cdot \sqrt{3}} \\
& + 1164) + 432384x + 172032) \cdot (-672 \cdot \sqrt{3} + 1164)^{(1/4)} \cdot \sqrt{-2 \cdot (7 \cdot \sqrt{3} + 12) \cdot \sqrt{-672 \cdot \sqrt{3}} + 1164} + 24) \cdot \sqrt{-56 \cdot \sqrt{3} + 97} \\
& - 6 \cdot (4680x^{16} - 60552x^{15} + 89856x^{14} + 278280x^{13} + 64440x^{12} - 1285200x^{11} - 255600x^{10} \\
& + 3098880x^9 - 1770336x^8 - 3614400x^7 + 3895488x^6 + 1199232x^5 - 2905344x^4 + 681984x^3 + 649728x^2 + 108 \cdot \sqrt{3} \cdot (2 \\
& 5x^{16} - 324x^{15} + 489x^{14} + 1482x^{13} + 316x^{12} - 6984x^{11} - 1312x^{10} + 16624x^9 - 9792x^8 \\
& - 19328x^7 + 20976x^6 + 6240x^5 - 15552x^4 + 3712x^3 + 3456x^2 - 4096x - 1280) + (1164x^{17} + 1248x^{16} - 246120x^{15} \\
& + 518172x^{14} + 2607528x^{13} - 8301144x^{12} + 7017600x^{11} + 6258120x^{10} - 21360336x^9 + 16998960x^8 \\
& + 966336x^7 - 18216672x^6 + 15860544x^5 - 4720704x^4 - 6023424x^3 + 5362176x^2 + 48 \cdot \sqrt{3} \cdot (14x^{17} + 15x^{16} - 296 \\
& 0x^{15} + 6232x^{14} + 31362x^{13} - 99844x^{12} + 84404x^{11} + 75267x^{10} - 256916x^9 + 204458x^8 \\
& + 11616x^7 - 219104x^6 + 190768x^5 - 56784x^4 - 72448x^3 + 64496x^2 - 24480x - 13376) + (2340x^{17} - 35850x^{16} - 106410x^{15} \\
& - 2064744x^{14} + 11945946x^{13} - 1710042x^{12} - 46293732x^{11} + 59161524x^{10} + 18480192x^9 \\
& - 122366520x^8 + 81203856x^7 + 45222000x^6 - 100598112x^5 + 42207168x^4 + 29609472x^3 \\
& - 22458240x^2 + \sqrt{3} \cdot (1351x^{17} - 20698x^{16} - 61436x^{15} - 1192081x^{14} + 6896998x^{13} - 987292x^{12} - 26 \\
& 727704x^{11} + 34156928x^{10} + 10669552x^9 - 70648352x^8 + 46883072x^7 + 26108944x^6 - 58080352x^5 \\
& + 24368320x^4 + 17095040x^3 - 12966272x^2 + 4724480x + 2581504) + 8183040x + 4471296) \cdot \sqrt{-672 \cdot \sqrt{3}} \\
& + 1164) - 2035200x - 1112064) \cdot \sqrt{-672 \cdot \sqrt{3}} + 1164) + 24 \cdot (627x^{16} - 14286x^{15} + 39762x^{14} \\
& + 50142x^{13} - 216816x^{12} + 112284x^{11} + 325707x^{10} - 586326x^9 - 3294x^8 + 631752x^7 \\
& - 539220x^6 - 184392x^5 + 483816x^4 - 115296x^3 - 108576x^2 + 2 \cdot \sqrt{3} \cdot (181x^{16} - 4124x^{15} + 11478x^{14} \\
& + 14474x^{13} - 62584x^{12} + 32412x^{11} + 94021x^{10} - 169244x^9 - 954x^8 + 182368x^7 - 155648x^6 \\
& - 53232x^5 + 139664x^4 - 33280x^3 - 31344x^2 + 37024x + 11584) + 128256x + 40128) \cdot \sqrt{-672 \cdot \sqrt{3}} \\
& + 1164) - 764928x - 239616) \cdot \sqrt{-56 \cdot \sqrt{3} + 97}) \cdot \sqrt{(36x^8 + 72x^7 + 1656x^6 + 720x^5 + 1440x^4 \\
& + 2016x^3 + (60x^6 + 324x^5 + 576x^4 + 696x^3 + 432x^2 + 36 \cdot \sqrt{3}) \cdot (x^6 + 5x^5 + 10x^4 + 10x^3 \\
& + 8x^2 + 4x) + (123x^6 + 2016x^5 + 2214x^4 + 2064x^3 + 396x^2 + \sqrt{3} \cdot (71x^6 + 1164x^5 + 1278x^4 + 1192x^3 \\
& + 228x^2 - 112) - 192) \cdot \sqrt{-672 \cdot \sqrt{3}} + 1164) + 144x + 96) \cdot \sqrt{x^3 + 1) \cdot \sqrt{-2 \cdot (7 \cdot \sqrt{3} + 12) \cdot \sqrt{-672 \cdot \sqrt{3}} + 1164} + 24) \cdot (-672 \cdot \sqrt{3}}
\end{aligned}$$



$$\begin{aligned}
& ) + 1164)^{(1/4)} - 288*x^2 + 144*\sqrt{3}*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 \\
& + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 \\
& + \sqrt{3}*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*\sqrt{(-672*\sqrt{3} + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16 \\
& *x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^{17} + 13*x^{16} - 5 \\
& 22*x^{15} + 1742*x^{14} + 3008*x^{13} - 16884*x^{12} + 11656*x^{11} + 23944*x^{10} - 42 \\
& 336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*x^3 \\
& - 1664*x^2 + 256*x) - 1/432*\sqrt{-2*(7*\sqrt{3} + 12)*\sqrt{(-672*\sqrt{3} + 1164) + 24}*(56*\sqrt{3} + 97)*\sqrt{(-56*\sqrt{3} + 97)*(-672*\sqrt{3} + 1164)} \\
& ^{(3/4)}*\arctan(-1/1296*(6*\sqrt{x^3 + 1})*((459*x^{16} - 13425*x^{15} - 33201*x^{14} \\
& + 950652*x^{13} - 997302*x^{12} - 14760972*x^{11} + 47069892*x^{10} - 49762248*x^9 \\
& - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - \\
& 36433344*x^3 + 12609792*x^2 + \sqrt{3}*(265*x^{16} - 7751*x^{15} - 19167*x^{14} + \\
& 548864*x^{13} - 575818*x^{12} - 8522268*x^{11} + 27175852*x^{10} - 28730312*x^9 - \\
& 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 2 \\
& 1034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) + (3691*x^{16} - 6128*x^{15} - \\
& 537864*x^{14} + 1586477*x^{13} + 16210952*x^{12} - 77181756*x^{11} + 84218362*x^{10} \\
& + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864 \\
& *x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 + \sqrt{3}*(2131*x^{16} - 3 \\
& 538*x^{15} - 310536*x^{14} + 915953*x^{13} + 9359398*x^{12} - 44560908*x^{11} + 48623 \\
& 494*x^{10} + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 14 \\
& 7990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 759 \\
& 8336) + 21204736*x + 13160704)*\sqrt{(-672*\sqrt{3} + 1164) - 4310784*x - 3273 \\
& 216)*(-672*\sqrt{3} + 1164)^{(3/4)} + 3*(984*x^{15} - 30612*x^{14} + 164676*x^{13} - \\
& 205368*x^{12} - 289200*x^{11} + 183720*x^{10} + 886752*x^9 - 71568*x^8 - 1960992 \\
& *x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4 \\
& *\sqrt{3}*(142*x^{15} - 4419*x^{14} + 23781*x^{13} - 29608*x^{12} - 41940*x^{11} + 264 \\
& 54*x^{10} + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 3 \\
& 57936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^{15} - 88617*x \\
& ^{14} + 738528*x^{13} - 1860046*x^{12} - 784596*x^{11} + 7668708*x^{10} - 6570680*x^9 \\
& - 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 1 \\
& 55968*x^3 - 3016704*x^2 + \sqrt{3}*(2855*x^{15} - 51163*x^{14} + 426388*x^{13} - 1 \\
& 073898*x^{12} - 452980*x^{11} + 4427548*x^{10} - 3793592*x^9 - 3985944*x^8 + 8916 \\
& 720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 \\
& + 1543936*x + 545536) + 2674176*x + 944896)*\sqrt{(-672*\sqrt{3} + 1164) - 66 \\
& 5088*x - 235008)*(-672*\sqrt{3} + 1164)^{(1/4)}*\sqrt{-2*(7*\sqrt{3} + 12)*\sqrt{(-672*\sqrt{3} + 1164) + 24}*\sqrt{(-56*\sqrt{3} + 97) - 36*(144*x^{17} - 5976*x^{16} \\
& + 5544*x^{15} + 299664*x^{14} - 1062360*x^{13} + 116712*x^{12} + 3600000*x^{11} - \\
& 4761216*x^{10} - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016 \\
& 832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*\sqrt{3}*(7*x^{17} - 28 \\
& 6*x^{16} + 238*x^{15} + 14255*x^{14} - 50390*x^{13} + 5942*x^{12} + 171808*x^{11} - 226 \\
& 888*x^{10} - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - \\
& 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^{17} - 6276*x \\
& ^{16} - 26052*x^{15} + 332844*x^{14} - 1632156*x^{13} + 4149132*x^{12} - 5805024*x^{11} \\
& + 318696*x^{10} + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - \\
& 20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*\sqrt{3}*(28*x \\
& ^{17} - 151*x^{16} - 626*x^{15} + 8006*x^{14} - 39266*x^{13} + 99812*x^{12} - 139652*x^{11} \\
& + 7661*x^{10} + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176 \\
& *x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^{17} \\
& - 96354*x^{16} + 84798*x^{15} + 4817124*x^{14} - 17052930*x^{13} + 1941678*x^{12} + \\
& 57963744*x^{11} - 76603680*x^{10} - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 \\
& - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 \\
& + \sqrt{3}*(1351*x^{17} - 55630*x^{16} + 48958*x^{15} + 2781167*x^{14} - 9845510*x^{13} \\
& + 1121030*x^{12} + 33465376*x^{11} - 44227144*x^{10} - 9629336*x^9 + 8078428 \\
& 0*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 1733536 \\
& 0*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*\sqrt{(-6 \\
& 72*\sqrt{3} + 1164) + 3261696*x + 1519104)*\sqrt{(-672*\sqrt{3} + 1164) + 12*(9 \\
& 7*x^{17} - 523*x^{16} - 2171*x^{15} + 27737*x^{14} - 136013*x^{13} + 345761*x^{12} - 48 \\
& 3752*x^{11} + 26558*x^{10} + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x
\end{aligned}$$

$$\begin{aligned}
& ^6 - 1680688x^5 + 911344x^4 + 536192x^3 - 535520x^2 + 2\sqrt{3}(28x^1 \\
& 7 - 151x^{16} - 626x^{15} + 8006x^{14} - 39266x^{13} + 99812x^{12} - 139652x^{11} \\
& + 7661x^{10} + 303610x^9 - 478214x^8 + 231392x^7 + 321412x^6 - 485176x \\
& ^5 + 263080x^4 + 154784x^3 - 154592x^2 + 78464x + 36544) + 271808x + 1 \\
& 26592)\sqrt{-672\sqrt{3} + 1164} - 811008x - 377856)\sqrt{-56\sqrt{3} + 97} \\
& ) - (\sqrt{x^3 + 1})((459x^{16} - 1557x^{15} - 26415x^{14} - 1449954x^{13} + 467 \\
& 7912x^{12} + 12651948x^{11} - 55684800x^{10} + 62834256x^9 + 8526168x^8 - 10 \\
& 5313392x^7 + 99605088x^6 - 18897984x^5 - 42499296x^4 + 37357632x^3 - 8 \\
& 256960x^2 + \sqrt{3}(265x^{16} - 899x^{15} - 15249x^{14} - 837130x^{13} + 2700 \\
& 776x^{12} + 7304604x^{11} - 32149640x^{10} + 36277360x^9 + 4922568x^8 - 6080 \\
& 2736x^7 + 57507040x^6 - 10910784x^5 - 24536992x^4 + 21568448x^3 - 4767 \\
& 168x^2 + 1207168x + 1383424) + (3691x^{16} + 17731x^{15} - 951114x^{14} + 45 \\
& 0359x^{13} + 4370159x^{12} + 30318522x^{11} - 78096668x^{10} + 9429316x^9 + 14 \\
& 6877876x^8 - 197107784x^7 - 30834152x^6 + 185125776x^5 - 132260896x^4 \\
& - 45545344x^3 + 69517536x^2 + \sqrt{3}(2131x^{16} + 10237x^{15} - 549126x^{14} \\
& 14 + 260015x^{13} + 2523113x^{12} + 17504406x^{11} - 45089132x^{10} + 5444020x \\
& ^9 + 84799980x^8 - 113800232x^7 - 17802104x^6 + 106882416x^5 - 76360864 \\
& *x^4 - 26295616x^3 + 40135968x^2 - 7907648x - 5562368) - 13696448x - 96 \\
& 34304)\sqrt{-672\sqrt{3} + 1164} + 2090880x + 2396160)(-672\sqrt{3} + 116 \\
& 4)^{(3/4)} + 3*(984x^{15} - 14712x^{14} - 53940x^{13} + 411732x^{12} - 280248x^{11} \\
& 1 - 324624x^{10} + 180816x^9 - 518544x^8 + 974304x^7 - 887136x^6 - 14040 \\
& 96x^5 + 1843584x^4 + 135936x^3 - 696192x^2 + 4\sqrt{3}(142x^{15} - 2124 \\
& *x^{14} - 7773x^{13} + 59447x^{12} - 40626x^{11} - 46860x^{10} + 26308x^9 - 7527 \\
& 6x^8 + 140472x^7 - 127784x^6 - 202896x^5 + 266016x^4 + 19712x^3 - 100 \\
& 512x^2 + 62400x + 24832) + (4945x^{15} - 37473x^{14} - 490698x^{13} + 224946 \\
& 8x^{12} + 474132x^{11} - 8423784x^{10} + 5853520x^9 + 8451720x^8 - 15320016x \\
& x^7 + 768064x^6 + 10405056x^5 - 6627744x^4 - 700480x^3 + 2799552x^2 + \\
& \sqrt{3}(2855x^{15} - 21635x^{14} - 283306x^{13} + 1298732x^{12} + 273748x^{11} \\
& - 4863472x^{10} + 3379536x^9 + 4879608x^8 - 8845008x^7 + 443456x^6 + 600 \\
& 7360x^5 - 3826528x^4 - 404416x^3 + 1616320x^2 - 1003648x - 399360) - 1 \\
& 738368x - 691712)\sqrt{-672\sqrt{3} + 1164} + 432384x + 172032)(-672\sqrt{3} + 1164)^{(1/4)} \\
& )\sqrt{-2*(7\sqrt{3} + 12)}\sqrt{-672\sqrt{3} + 1164} + 24) \\
& )\sqrt{-56\sqrt{3} + 97} + 6*(4680x^{16} - 60552x^{15} + 89856x^{14} + 278280x \\
& x^{13} + 64440x^{12} - 1285200x^{11} - 255600x^{10} + 3098880x^9 - 1770336x^8 \\
& - 3614400x^7 + 3895488x^6 + 1199232x^5 - 2905344x^4 + 681984x^3 + 6497 \\
& 28x^2 + 108\sqrt{3}(25x^{16} - 324x^{15} + 489x^{14} + 1482x^{13} + 316x^{12} \\
& - 6984x^{11} - 1312x^{10} + 16624x^9 - 9792x^8 - 19328x^7 + 20976x^6 + 62 \\
& 40x^5 - 15552x^4 + 3712x^3 + 3456x^2 - 4096x - 1280) + (1164x^{17} + 12 \\
& 48x^{16} - 246120x^{15} + 518172x^{14} + 2607528x^{13} - 8301144x^{12} + 7017600 \\
& *x^{11} + 6258120x^{10} - 21360336x^9 + 16998960x^8 + 966336x^7 - 18216672x \\
& x^6 + 15860544x^5 - 4720704x^4 - 6023424x^3 + 5362176x^2 + 48\sqrt{3}( \\
& 14x^{17} + 15x^{16} - 2960x^{15} + 6232x^{14} + 31362x^{13} - 99844x^{12} + 84404 \\
& *x^{11} + 75267x^{10} - 256916x^9 + 204458x^8 + 11616x^7 - 219104x^6 + 190 \\
& 768x^5 - 56784x^4 - 72448x^3 + 64496x^2 - 24480x - 13376) + (2340x^{17} \\
& - 35850x^{16} - 106410x^{15} - 2064744x^{14} + 11945946x^{13} - 1710042x^{12} - \\
& 46293732x^{11} + 59161524x^{10} + 18480192x^9 - 122366520x^8 + 81203856x^7 \\
& 7 + 45222000x^6 - 100598112x^5 + 42207168x^4 + 29609472x^3 - 22458240x \\
& ^2 + \sqrt{3}(1351x^{17} - 20698x^{16} - 61436x^{15} - 1192081x^{14} + 6896998x \\
& x^{13} - 987292x^{12} - 26727704x^{11} + 34156928x^{10} + 10669552x^9 - 7064835 \\
& 2x^8 + 46883072x^7 + 26108944x^6 - 58080352x^5 + 24368320x^4 + 1709504 \\
& 0x^3 - 12966272x^2 + 4724480x + 2581504) + 8183040x + 4471296)\sqrt{-67} \\
& 2\sqrt{3} + 1164) - 2035200x - 1112064)\sqrt{-672\sqrt{3} + 1164} + 24*(62 \\
& 7x^{16} - 14286x^{15} + 39762x^{14} + 50142x^{13} - 216816x^{12} + 112284x^{11} + \\
& 325707x^{10} - 586326x^9 - 3294x^8 + 631752x^7 - 539220x^6 - 184392x^5 \\
& + 483816x^4 - 115296x^3 - 108576x^2 + 2\sqrt{3}(181x^{16} - 4124x^{15} + \\
& 11478x^{14} + 14474x^{13} - 62584x^{12} + 32412x^{11} + 94021x^{10} - 169244x^9 \\
& - 954x^8 + 182368x^7 - 155648x^6 - 53232x^5 + 139664x^4 - 33280x^3 \\
& - 31344x^2 + 37024x + 11584) + 128256x + 40128)\sqrt{-672\sqrt{3} + 1164} \\
& ) - 764928x - 239616)\sqrt{-56\sqrt{3} + 97})\sqrt{(36x^8 + 72x^7 + 1656
\end{aligned}$$

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*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 324*x^5 + 576*x^4 + 696*x^
3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (1
23*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x
^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164)
+ 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6
+ 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x +
2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16
)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656
*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5
+ 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) + 1/5184*((7*sqrt(3) + 12)*sq
rt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 + 72*x^7 + 1656*x^6
+ 720*x^5 + 1440*x^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 +
432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x
^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5
+ 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 1
44*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164)
+ 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6
*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46
*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^
3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304
)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) -
1/5184*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3)
) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/3
6*(36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 3
24*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 1
0*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2
+ sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*
sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) +
12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 +
144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(
26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*
x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*s
qrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32
*x^3 + 64*x^2 + 32*x + 16)) + 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2
+ sqrt(3)*(x^2 - 10*x - 8) - 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1
))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}(x^3+10+6\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x\*\*3+6\*3\*\*(1/2))/(x\*\*3+1)\*\*(1/2),x)

[Out] Integral(x/(sqrt((x + 1)\*(x\*\*2 - x + 1))\*(x\*\*3 + 10 + 6\*sqrt(3))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

$$3.87 \quad \int \frac{x}{\sqrt{1+x^3}(10-6\sqrt{3}+x^3)} dx$$

**Optimal.** Leaf size=210

$$\frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(-2x - \sqrt{3} + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 - \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{2\sqrt{2}3^{3/4}}$$

[Out]  $-\left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3] - 2x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(3 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) - \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(6 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(2 \cdot \text{Sqrt}[2] \cdot 3^{3/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(2 \cdot \text{Sqrt}[2] \cdot 3^{3/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(6 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) - \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3] - 2x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(3 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right)$

**Rubi [A]** time = 0.0317457, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {487}

$$\frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(-2x - \sqrt{3} + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 - \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}}\right)}{2\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x^3]\*(10 - 6\*Sqrt[3] + x^3)), x]

[Out]  $-\left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3] - 2x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(3 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) - \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(6 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(2 \cdot \text{Sqrt}[2] \cdot 3^{3/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(2 \cdot \text{Sqrt}[2] \cdot 3^{3/4}\right) + \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTanh}\left[\left(3^{1/4}\right) \cdot (1 + \text{Sqrt}[3]) \cdot (1 + x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(6 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right) - \left((2 + \text{Sqrt}[3]) \cdot \text{ArcTan}\left[\left(3^{1/4}\right) \cdot (1 - \text{Sqrt}[3] - 2x)\right] / \left(\text{Sqrt}[2] \cdot \text{Sqrt}[1 + x^3]\right)\right) / \left(3 \cdot \text{Sqrt}[2] \cdot 3^{1/4}\right)$

**Rule 487**

Int[(x\_)/(Sqrt[(a\_) + (b\_.)\*(x\_)^3]\*((c\_) + (d\_.)\*(x\_)^3)), x\_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b\*c - 10\*a\*d)/(6\*a\*d)]}, -Simp[(q\*(2 - r) \* ArcTan[((1 - r)\*Sqrt[a + b\*x^3])/(Sqrt[2]\*Rt[a, 2]\*r^(3/2))])/(3\*Sqrt[2]\*Rt[a, 2]\*d\*r^(3/2)), x] + (-Simp[(q\*(2 - r)\*ArcTan[(Rt[a, 2]\*Sqrt[r]\*(1 + r)\*(1 + q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(2\*Sqrt[2]\*Rt[a, 2]\*d\*r^(3/2)), x] - Simp[(q\*(2 - r)\*ArcTanh[(Rt[a, 2]\*Sqrt[r]\*(1 + r - 2\*q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(3\*Sqrt[2]\*Rt[a, 2]\*d\*Sqrt[r]), x] - Simp[(q\*(2 - r)\*ArcTanh[(Rt[a, 2]\*(1 - r)\*Sqrt[r]\*(1 + q\*x))/(Sqrt[2]\*Sqrt[a + b\*x^3])])/(6\*Sqrt[2]\*Rt[a, 2]\*d\*Sqrt[r]), x]]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b^2\*c^2 - 20\*a\*b\*c\*d - 8\*a^2\*d^2, 0] && PosQ[a]

**Rubi steps**

$$\int \frac{x}{\sqrt{1+x^3}(10-6\sqrt{3}+x^3)} dx = -\frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1 - \sqrt{3} - 2x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2 + \sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 - \sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2 + \sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}}$$

**Mathematica [C]** time = 0.0703071, size = 50, normalized size = 0.24

$$\frac{x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -x^3, \frac{1}{4}(5 + 3\sqrt{3})x^3\right)}{4(3\sqrt{3} - 5)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 + x^3]\*(10 - 6\*Sqrt[3] + x^3)), x]

[Out]  $-(x^2 \text{AppellF1}[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3\sqrt{3})x^3)/4]) / (4(-5 + 3\sqrt{3}))$

**Maple [C]** time = 0.173, size = 350, normalized size = 1.7

$$-\frac{\sqrt{2}}{18} \sum_{\alpha=\text{RootOf}(\_Z^2+(\sqrt{3}-1)\_Z-2\sqrt{3}+4)} \frac{(-\sqrt{3}\alpha - \alpha + 2)(-i\sqrt{3} + 3)(-1 + 2\alpha + \sqrt{3}\alpha)}{-\sqrt{3} - 2\alpha + 1} \sqrt{\frac{1+x}{-i\sqrt{3} + \dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(10+x^3-6\*3^(1/2))/(x^3+1)^(1/2), x)

[Out]  $-1/18 \cdot 2^{1/2} \cdot \text{sum}((-3^{1/2} \alpha - \alpha + 2) / (-3^{1/2} - 2\alpha + 1) \cdot (-i \cdot 3^{1/2} + 3) \cdot ((1+x) / (-i \cdot 3^{1/2} + 3))^{1/2} \cdot ((2x-1-i \cdot 3^{1/2}) / (-i \cdot 3^{1/2} - 3))^{1/2} \cdot ((2x-1+i \cdot 3^{1/2}) / (i \cdot 3^{1/2} - 3))^{1/2} / (x^3+1)^{1/2} \cdot (-1+2\alpha+3^{1/2}\alpha) \cdot \text{EllipticPi}(((1+x)/(3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2}, 1/2 \cdot i \cdot \alpha + 1/3 \cdot i \cdot \alpha \cdot 3^{1/2} - 1/2 \cdot 3^{1/2} \cdot \alpha - \alpha - 1/6 \cdot i \cdot 3^{1/2} + 1/2, ((-3/2+1/2 \cdot i \cdot 3^{1/2}) / (-3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2}), \alpha = \text{RootOf}(\_Z^2+(3^{1/2}-1)\_Z-2 \cdot 3^{1/2}+4)) + 1/9 \cdot (3^{1/2}-1) / (-2+3^{1/2}) \cdot (3/2-1/2 \cdot i \cdot 3^{1/2}) \cdot ((1+x)/(3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2} \cdot ((x-1/2-1/2 \cdot i \cdot 3^{1/2}) / (-3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2} \cdot ((x-1/2+1/2 \cdot i \cdot 3^{1/2}) / (-3/2+1/2 \cdot i \cdot 3^{1/2}))^{1/2} / (x^3+1)^{1/2} \cdot 3^{1/2} \cdot \text{EllipticPi}(((1+x)/(3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2}, -1/3 \cdot (-3/2+1/2 \cdot i \cdot 3^{1/2}) \cdot 3^{1/2}, ((-3/2+1/2 \cdot i \cdot 3^{1/2}) / (-3/2-1/2 \cdot i \cdot 3^{1/2}))^{1/2}))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3-6\*3^(1/2))/(x^3+1)^(1/2), x, algorithm="maxima")

[Out] integrate(x/((x^3 - 6\*sqrt(3) + 10)\*sqrt(x^3 + 1)), x)

**Fricas [B]** time = 49.0419, size = 28331, normalized size = 134.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3-6\*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 
$$-1/108*\sqrt{3}*\sqrt{\sqrt{3}*\sqrt{56*\sqrt{3} + 97}*(7*\sqrt{3} - 12) + 6}*(672*\sqrt{3} + 1164)^{1/4}*(56*\sqrt{3} + 97)*(56*\sqrt{3} - 97)*\arctan(1/324*(216*\sqrt{3}*(97*x^{17} - 523*x^{16} - 2171*x^{15} + 27737*x^{14} - 136013*x^{13} + 345761*x^{12} - 483752*x^{11} + 26558*x^{10} + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*\sqrt{3}*(28*x^{17} - 151*x^{16} - 626*x^{15} + 8006*x^{14} - 39266*x^{13} + 99812*x^{12} - 139652*x^{11} + 7661*x^{10} + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*(56*\sqrt{3} + 97) - 36*\sqrt{3}*(\sqrt{3}*(2340*x^{17} - 96354*x^{16} + 84798*x^{15} + 4817124*x^{14} - 17052930*x^{13} + 1941678*x^{12} + 57963744*x^{11} - 76603680*x^{10} - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 - \sqrt{3}*(1351*x^{17} - 55630*x^{16} + 48958*x^{15} + 2781167*x^{14} - 9845510*x^{13} + 1121030*x^{12} + 33465376*x^{11} - 44227144*x^{10} - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*\sqrt{3} + 97) + 6*(97*x^{17} - 523*x^{16} - 2171*x^{15} + 27737*x^{14} - 136013*x^{13} + 345761*x^{12} - 483752*x^{11} + 26558*x^{10} + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*\sqrt{3}*(28*x^{17} - 151*x^{16} - 626*x^{15} + 8006*x^{14} - 39266*x^{13} + 99812*x^{12} - 139652*x^{11} + 7661*x^{10} + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*\sqrt{56*\sqrt{3} + 97})*\sqrt{56*\sqrt{3} + 97} + 3*\sqrt{\sqrt{3}*\sqrt{56*\sqrt{3} + 97}*(7*\sqrt{3} - 12) + 6}*((2*\sqrt{3}*(3691*x^{16} - 6128*x^{15} - 537864*x^{14} + 1586477*x^{13} + 16210952*x^{12} - 77181756*x^{11} + 84218362*x^{10} + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - \sqrt{3}*(2131*x^{16} - 3538*x^{15} - 310536*x^{14} + 915953*x^{13} + 9359398*x^{12} - 44560908*x^{11} + 48623494*x^{10} + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13160704)*\sqrt{x^3 + 1}*(56*\sqrt{3} + 97) + (459*x^{16} - 13425*x^{15} - 33201*x^{14} + 950652*x^{13} - 997302*x^{12} - 14760972*x^{11} + 47069892*x^{10} - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - \sqrt{3}*(265*x^{16} - 7751*x^{15} - 19167*x^{14} + 548864*x^{13} - 575818*x^{12} - 8522268*x^{11} + 27175852*x^{10} - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) - 4310784*x - 3273216)*\sqrt{x^3 + 1}*\sqrt{56*\sqrt{3} + 97})*(672*\sqrt{3} + 1164)^{3/4} + 6*(\sqrt{3}*(4945*x^{15} - 88617*x^{14} + 738528*x^{13} - 1860046*x^{12} - 784596*x^{11} + 7668708*x^{10} - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 - \sqrt{3}*(2855*x^{15} - 51163*x^{14} + 426388*x^{13} - 1073898*x^{12} - 452980*x^{11} + 4427548*x^{10} - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896)*\sqrt{x^3 + 1}*(56*\sqrt{3} + 97) + 2*(246*x^{15} - 7653*x^{14} + 41169*x^{13} - 51342*x^{12} - 72300*x^{11} + 45930*x^{10} + 221688*x^9 - 17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3 + 187584*x^2 - \sqrt{3}*(142*x^{15} - 4419*x^{14} + 23781*x^{13} - 29608*x^{12} - 41940*x^{11} + 26454*x^{10} + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272*x - 58752)*\sqrt{x^3 + 1}*\sqrt{56*\sqrt{3} + 97})*(672*\sqrt{3} + 1164)^{1/4}) + 108*(12*x^{17} - 498*x^{16} + 462*x^{15} + 24972*x^{14} - 88530*x^{13} + 9726*x^{12} + 300000*x^{11} - 396768*x^{10} - 87216*x^9 + 723072*x^8 - 549408*x^7 - 220128*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - \sqrt{3}*(7*x^{17} - 286*x^{16} + 238*x^{15} + 14255*x^{14} - 50390*x^{13} + 5942*x^{12} + 171808*x^{11}$$

$$\begin{aligned}
& - 226888x^{10} - 48920x^9 + 415384x^8 - 315088x^7 - 125600x^6 + 336608x^5 \\
& - 177344x^4 - 89152x^3 + 78784x^2 - 39040x - 18176) - 67584x - 3148 \\
& 8) \sqrt{56\sqrt{3} + 97} + (144\sqrt{3})(627x^{16} - 14286x^{15} + 39762x^{14} \\
& + 50142x^{13} - 216816x^{12} + 112284x^{11} + 325707x^{10} - 586326x^9 - 3294 \\
& *x^8 + 631752x^7 - 539220x^6 - 184392x^5 + 483816x^4 - 115296x^3 - 108 \\
& 576x^2 - 2\sqrt{3})(181x^{16} - 4124x^{15} + 11478x^{14} + 14474x^{13} - 62584 \\
& *x^{12} + 32412x^{11} + 94021x^{10} - 169244x^9 - 954x^8 + 182368x^7 - 15564 \\
& 8x^6 - 53232x^5 + 139664x^4 - 33280x^3 - 31344x^2 + 37024x + 11584) + \\
& 128256x + 40128)(56\sqrt{3} + 97) + 12\sqrt{3}(\sqrt{3})(2340x^{17} - 358 \\
& 50x^{16} - 106410x^{15} - 2064744x^{14} + 11945946x^{13} - 1710042x^{12} - 46293 \\
& 732x^{11} + 59161524x^{10} + 18480192x^9 - 122366520x^8 + 81203856x^7 + 45 \\
& 222000x^6 - 100598112x^5 + 42207168x^4 + 29609472x^3 - 22458240x^2 - s \\
& \text{qrt}(3)(1351x^{17} - 20698x^{16} - 61436x^{15} - 1192081x^{14} + 6896998x^{13} - \\
& 987292x^{12} - 26727704x^{11} + 34156928x^{10} + 10669552x^9 - 70648352x^8 \\
& + 46883072x^7 + 26108944x^6 - 58080352x^5 + 24368320x^4 + 17095040x^3 \\
& - 12966272x^2 + 4724480x + 2581504) + 8183040x + 4471296)(56\sqrt{3} + \\
& 97) + 6(97x^{17} + 104x^{16} - 20510x^{15} + 43181x^{14} + 217294x^{13} - 69176 \\
& 2x^{12} + 584800x^{11} + 521510x^{10} - 1780028x^9 + 1416580x^8 + 80528x^7 \\
& - 1518056x^6 + 1321712x^5 - 393392x^4 - 501952x^3 + 446848x^2 - 4\sqrt{3} \\
& (3)(14x^{17} + 15x^{16} - 2960x^{15} + 6232x^{14} + 31362x^{13} - 99844x^{12} + \\
& 84404x^{11} + 75267x^{10} - 256916x^9 + 204458x^8 + 11616x^7 - 219104x^6 \\
& + 190768x^5 - 56784x^4 - 72448x^3 + 64496x^2 - 24480x - 13376) - 16960 \\
& 0x - 92672)\sqrt{56\sqrt{3} + 97})\sqrt{56\sqrt{3} + 97} - \sqrt{\sqrt{3}\sqrt{56\sqrt{3} + 97}} \\
& \sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) + 6)((2\sqrt{3})(3691x^{16} + 17731x^{15} \\
& - 951114x^{14} + 450359x^{13} + 4370159x^{12} + 30318522x^{11} - 78096668x^{10} \\
& + 9429316x^9 + 146877876x^8 - 197107784x^7 - 30834152x^6 + 185125776 \\
& *x^5 - 132260896x^4 - 45545344x^3 + 69517536x^2 - \sqrt{3})(2131x^{16} + 1 \\
& 0237x^{15} - 549126x^{14} + 260015x^{13} + 2523113x^{12} + 17504406x^{11} - 4508 \\
& 9132x^{10} + 5444020x^9 + 84799980x^8 - 113800232x^7 - 17802104x^6 + 106 \\
& 882416x^5 - 76360864x^4 - 26295616x^3 + 40135968x^2 - 7907648x - 55623 \\
& 68) - 13696448x - 9634304)\sqrt{x^3 + 1})(56\sqrt{3} + 97) + (459x^{16} - 1 \\
& 557x^{15} - 26415x^{14} - 1449954x^{13} + 4677912x^{12} + 12651948x^{11} - 55684 \\
& 800x^{10} + 62834256x^9 + 8526168x^8 - 105313392x^7 + 99605088x^6 - 1889 \\
& 7984x^5 - 42499296x^4 + 37357632x^3 - 8256960x^2 - \sqrt{3})(265x^{16} - \\
& 899x^{15} - 15249x^{14} - 837130x^{13} + 2700776x^{12} + 7304604x^{11} - 3214964 \\
& 0x^{10} + 36277360x^9 + 4922568x^8 - 60802736x^7 + 57507040x^6 - 1091078 \\
& 4x^5 - 24536992x^4 + 21568448x^3 - 4767168x^2 + 1207168x + 1383424) + \\
& 2090880x + 2396160)\sqrt{x^3 + 1})\sqrt{56\sqrt{3} + 97})(672\sqrt{3} + 11 \\
& 64)^{(3/4)} + 6(\sqrt{3})(4945x^{15} - 37473x^{14} - 490698x^{13} + 2249468x^{12} \\
& + 474132x^{11} - 8423784x^{10} + 5853520x^9 + 8451720x^8 - 15320016x^7 + \\
& 768064x^6 + 10405056x^5 - 6627744x^4 - 700480x^3 + 2799552x^2 - \sqrt{3} \\
& )(2855x^{15} - 21635x^{14} - 283306x^{13} + 1298732x^{12} + 273748x^{11} - 4863 \\
& 472x^{10} + 3379536x^9 + 4879608x^8 - 8845008x^7 + 443456x^6 + 6007360x^5 \\
& - 3826528x^4 - 404416x^3 + 1616320x^2 - 1003648x - 399360) - 1738368 \\
& *x - 691712)\sqrt{x^3 + 1})(56\sqrt{3} + 97) + 2(246x^{15} - 3678x^{14} - 13 \\
& 485x^{13} + 102933x^{12} - 70062x^{11} - 81156x^{10} + 45204x^9 - 129636x^8 + \\
& 243576x^7 - 221784x^6 - 351024x^5 + 460896x^4 + 33984x^3 - 174048x^2 \\
& - \sqrt{3})(142x^{15} - 2124x^{14} - 7773x^{13} + 59447x^{12} - 40626x^{11} - 46 \\
& 860x^{10} + 26308x^9 - 75276x^8 + 140472x^7 - 127784x^6 - 202896x^5 + 2 \\
& 66016x^4 + 19712x^3 - 100512x^2 + 62400x + 24832) + 108096x + 43008)\sqrt{x^3 + 1})\sqrt{56\sqrt{3} + 97})(672\sqrt{3} + 1164)^{(1/4)} + 108(130x^{16} \\
& - 1682x^{15} + 2496x^{14} + 7730x^{13} + 1790x^{12} - 35700x^{11} - 7100x^{10} \\
& + 86080x^9 - 49176x^8 - 100400x^7 + 108208x^6 + 33312x^5 - 80704x^4 \\
& + 18944x^3 + 18048x^2 - 3\sqrt{3})(25x^{16} - 324x^{15} + 489x^{14} + 1482 \\
& *x^{13} + 316x^{12} - 6984x^{11} - 1312x^{10} + 16624x^9 - 9792x^8 - 19328x^7 \\
& + 20976x^6 + 6240x^5 - 15552x^4 + 3712x^3 + 3456x^2 - 4096x - 1280) \\
& - 21248x - 6656)\sqrt{56\sqrt{3} + 97})\sqrt{(9x^8 + 18x^7 + 414x^6 + 1 \\
& 80x^5 + 360x^4 + 504x^3 - 72x^2 + 36\sqrt{3})(26x^7 + 38x^6 + 42x^5 \\
& + 46x^4 + 46x^3 + 42x^2 - \sqrt{3})(15x^7 + 22x^6 + 24x^5 + 27x^4 + 2
\end{aligned}$$



$$\begin{aligned}
& 6x^3 + 24x^2 + 12x + 4) + 20x + 8) \sqrt{56\sqrt{3} + 97} + (\sqrt{3})(12 \\
& 3x^6 + 2016x^5 + 2214x^4 + 2064x^3 + 396x^2 - \sqrt{3})(71x^6 + 1164x \\
& ^5 + 1278x^4 + 1192x^3 + 228x^2 - 112) - 192) \sqrt{x^3 + 1} \sqrt{56\sqrt{3} \\
& (3) + 97} + 6(5x^6 + 27x^5 + 48x^4 + 58x^3 + 36x^2 - 3\sqrt{3})(x^6 + \\
& 5x^5 + 10x^4 + 10x^3 + 8x^2 + 4x) + 12x + 8) \sqrt{x^3 + 1}) \sqrt{(\sqrt{3} \\
& \sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) + 6)(672\sqrt{3} + 1164)^{(1/4)} \\
& - 36\sqrt{3})(x^7 + 4x^6 + 6x^5 + 5x^4 - 4x^3 + 6x^2 + 4x - 8) - 144 \\
& *x + 576)/(x^8 - 4x^7 + 16x^6 - 16x^5 + 28x^4 + 32x^3 + 64x^2 + 32x \\
& + 16))/(x^{17} + 13x^{16} - 522x^{15} + 1742x^{14} + 3008x^{13} - 16884x^{12} + 1 \\
& 1656x^{11} + 23944x^{10} - 42336x^9 + 9136x^8 + 36256x^7 - 27360x^6 - 256 \\
& *x^5 + 13376x^4 - 5760x^3 - 1664x^2 + 256x) - 1/108\sqrt{3}\sqrt{(\sqrt{3} \\
& (3)\sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) + 6)(672\sqrt{3} + 1164)^{(1/4)}*( \\
& 56\sqrt{3} + 97)(56\sqrt{3} - 97)\arctan(-1/324*(216\sqrt{3})(97x^{17} - 52 \\
& 3x^{16} - 2171x^{15} + 27737x^{14} - 136013x^{13} + 345761x^{12} - 483752x^{11} + \\
& 26558x^{10} + 1051756x^9 - 1656560x^8 + 801584x^7 + 1113424x^6 - 168068 \\
& 8x^5 + 911344x^4 + 536192x^3 - 535520x^2 - 2\sqrt{3})(28x^{17} - 151x^{16} \\
& 6 - 626x^{15} + 8006x^{14} - 39266x^{13} + 99812x^{12} - 139652x^{11} + 7661x^{10} \\
& 0 + 303610x^9 - 478214x^8 + 231392x^7 + 321412x^6 - 485176x^5 + 263080 \\
& *x^4 + 154784x^3 - 154592x^2 + 78464x + 36544) + 271808x + 126592)*(56* \\
& \sqrt{3} + 97) - 36\sqrt{3}(\sqrt{3})(2340x^{17} - 96354x^{16} + 84798x^{15} + \\
& 4817124x^{14} - 17052930x^{13} + 1941678x^{12} + 57963744x^{11} - 76603680x^{10} \\
& - 16678512x^9 + 139922496x^8 - 106227360x^7 - 42453216x^6 + 113269536* \\
& x^5 - 59694624x^4 - 30025728x^3 + 26496000x^2 - \sqrt{3})(1351x^{17} - 556 \\
& 30x^{16} + 48958x^{15} + 2781167x^{14} - 9845510x^{13} + 1121030x^{12} + 3346537 \\
& 6x^{11} - 44227144x^{10} - 9629336x^9 + 80784280x^8 - 61330384x^7 - 245103 \\
& 68x^6 + 65396192x^5 - 34464704x^4 - 17335360x^3 + 15297472x^2 - 757158 \\
& 4x - 3526400) - 13114368x - 6107904)*(56\sqrt{3} + 97) + 6(97x^{17} - 523 \\
& *x^{16} - 2171x^{15} + 27737x^{14} - 136013x^{13} + 345761x^{12} - 483752x^{11} + \\
& 26558x^{10} + 1051756x^9 - 1656560x^8 + 801584x^7 + 1113424x^6 - 1680688 \\
& *x^5 + 911344x^4 + 536192x^3 - 535520x^2 - 2\sqrt{3})(28x^{17} - 151x^{16} \\
& - 626x^{15} + 8006x^{14} - 39266x^{13} + 99812x^{12} - 139652x^{11} + 7661x^{10} \\
& + 303610x^9 - 478214x^8 + 231392x^7 + 321412x^6 - 485176x^5 + 263080* \\
& x^4 + 154784x^3 - 154592x^2 + 78464x + 36544) + 271808x + 126592)\sqrt{(\sqrt{3} \\
& (3)\sqrt{56\sqrt{3} + 97})\sqrt{56\sqrt{3} + 97} - 3\sqrt{(\sqrt{3})\sqrt{56\sqrt{3} + \\
& 97})(7\sqrt{3} - 12) + 6)((2\sqrt{3})(3691x^{16} - 6128x^{15} - 537864x^{14} \\
& + 1586477x^{13} + 16210952x^{12} - 77181756x^{11} + 84218362x^{10} + 71018320x \\
& ^9 - 254455812x^8 + 196076008x^7 + 120105208x^6 - 256326864x^5 + 134645 \\
& 168x^4 + 78464672x^3 - 78514944x^2 - \sqrt{3})(2131x^{16} - 3538x^{15} - 31 \\
& 0536x^{14} + 915953x^{13} + 9359398x^{12} - 44560908x^{11} + 48623494x^{10} + 41 \\
& 002448x^9 - 146910132x^8 + 113204536x^7 + 69342776x^6 - 147990384x^5 + \\
& 77737424x^4 + 45301600x^3 - 45330624x^2 + 12242560x + 7598336) + 21204 \\
& 736x + 13160704)\sqrt{x^3 + 1}(56\sqrt{3} + 97) + (459x^{16} - 13425x^{15} \\
& - 33201x^{14} + 950652x^{13} - 997302x^{12} - 14760972x^{11} + 47069892x^{10} - \\
& 49762248x^9 - 8212536x^8 + 84377808x^7 - 88427328x^6 + 25613856x^5 + 2 \\
& 7458496x^4 - 36433344x^3 + 12609792x^2 - \sqrt{3})(265x^{16} - 7751x^{15} - \\
& 19167x^{14} + 548864x^{13} - 575818x^{12} - 8522268x^{11} + 27175852x^{10} - 28 \\
& 730312x^9 - 4741560x^8 + 48715600x^7 - 51053600x^6 + 14788128x^5 + 158 \\
& 53184x^4 - 21034816x^3 + 7280256x^2 - 2488832x - 1889792) - 4310784x - \\
& 3273216)\sqrt{x^3 + 1}\sqrt{56\sqrt{3} + 97})(672\sqrt{3} + 1164)^{(3/4)} + \\
& 6(\sqrt{3})(4945x^{15} - 88617x^{14} + 738528x^{13} - 1860046x^{12} - 784596x \\
& ^{11} + 7668708x^{10} - 6570680x^9 - 6903864x^8 + 15444144x^7 - 4312832x^6 \\
& - 9559200x^5 + 9359808x^4 - 155968x^3 - 3016704x^2 - \sqrt{3})(2855x^{15} \\
& 5 - 51163x^{14} + 426388x^{13} - 1073898x^{12} - 452980x^{11} + 4427548x^{10} - \\
& 3793592x^9 - 3985944x^8 + 8916720x^7 - 2490016x^6 - 5519008x^5 + 54039 \\
& 04x^4 - 90048x^3 - 1741696x^2 + 1543936x + 545536) + 2674176x + 944896 \\
& )\sqrt{x^3 + 1}(56\sqrt{3} + 97) + 2(246x^{15} - 7653x^{14} + 41169x^{13} - \\
& 51342x^{12} - 72300x^{11} + 45930x^{10} + 221688x^9 - 17892x^8 - 490248x^7 \\
& + 462360x^6 + 389616x^5 - 619728x^4 + 16608x^3 + 187584x^2 - \sqrt{3})( \\
& 142x^{15} - 4419x^{14} + 23781x^{13} - 29608x^{12} - 41940x^{11} + 26454x^{10} +
\end{aligned}$$

$128152x^9 - 10692x^8 - 283320x^7 + 267064x^6 + 224784x^5 - 357936x^4 + 9632x^3 + 108288x^2 - 96000x - 33920) - 166272x - 58752) \cdot \sqrt{x^3 + 1} \cdot \sqrt{56\sqrt{3} + 97}) \cdot (672\sqrt{3} + 1164)^{(1/4)} + 108 \cdot (12x^{17} - 498x^{16} + 462x^{15} + 24972x^{14} - 88530x^{13} + 9726x^{12} + 300000x^{11} - 396768x^{10} - 87216x^9 + 723072x^8 - 549408x^7 - 220128x^6 + 584736x^5 - 308256x^4 - 155136x^3 + 136704x^2 - \sqrt{3} \cdot (7x^{17} - 286x^{16} + 238x^{15} + 14255x^{14} - 50390x^{13} + 5942x^{12} + 171808x^{11} - 226888x^{10} - 48920x^9 + 415384x^8 - 315088x^7 - 125600x^6 + 336608x^5 - 177344x^4 - 89152x^3 + 78784x^2 - 39040x - 18176) - 67584x - 31488) \cdot \sqrt{56\sqrt{3} + 97} + (144\sqrt{3}) \cdot (627x^{16} - 14286x^{15} + 39762x^{14} + 50142x^{13} - 216816x^{12} + 112284x^{11} + 325707x^{10} - 586326x^9 - 3294x^8 + 631752x^7 - 539220x^6 - 184392x^5 + 483816x^4 - 115296x^3 - 108576x^2 - 2\sqrt{3}) \cdot (181x^{16} - 4124x^{15} + 11478x^{14} + 14474x^{13} - 62584x^{12} + 32412x^{11} + 94021x^{10} - 169244x^9 - 954x^8 + 182368x^7 - 155648x^6 - 53232x^5 + 139664x^4 - 33280x^3 - 31344x^2 + 37024x + 11584) + 128256x + 40128) \cdot (56\sqrt{3} + 97) + 12\sqrt{3} \cdot (\sqrt{3}) \cdot (2340x^{17} - 35850x^{16} - 106410x^{15} - 2064744x^{14} + 11945946x^{13} - 1710042x^{12} - 46293732x^{11} + 59161524x^{10} + 18480192x^9 - 122366520x^8 + 81203856x^7 + 45222000x^6 - 100598112x^5 + 42207168x^4 + 29609472x^3 - 22458240x^2 - \sqrt{3}) \cdot (1351x^{17} - 20698x^{16} - 61436x^{15} - 1192081x^{14} + 6896998x^{13} - 987292x^{12} - 26727704x^{11} + 34156928x^{10} + 10669552x^9 - 70648352x^8 + 46883072x^7 + 26108944x^6 - 58080352x^5 + 24368320x^4 + 17095040x^3 - 12966272x^2 + 4724480x + 2581504) + 8183040x + 4471296) \cdot (56\sqrt{3} + 97) + 6 \cdot (97x^{17} + 104x^{16} - 20510x^{15} + 43181x^{14} + 217294x^{13} - 691762x^{12} + 584800x^{11} + 521510x^{10} - 1780028x^9 + 1416580x^8 + 80528x^7 - 1518056x^6 + 1321712x^5 - 393392x^4 - 501952x^3 + 446848x^2 - 4\sqrt{3}) \cdot (14x^{17} + 15x^{16} - 2960x^{15} + 6232x^{14} + 31362x^{13} - 99844x^{12} + 84404x^{11} + 75267x^{10} - 256916x^9 + 204458x^8 + 11616x^7 - 219104x^6 + 190768x^5 - 56784x^4 - 72448x^3 + 64496x^2 - 24480x - 13376) - 169600x - 92672) \cdot \sqrt{56\sqrt{3} + 97}) \cdot \sqrt{56\sqrt{3} + 97} + \sqrt{(\sqrt{3}) \cdot \sqrt{56\sqrt{3} + 97}) \cdot (7\sqrt{3} - 12) + 6} \cdot ((2\sqrt{3}) \cdot (3691x^{16} + 17731x^{15} - 951114x^{14} + 450359x^{13} + 4370159x^{12} + 30318522x^{11} - 78096668x^{10} + 9429316x^9 + 146877876x^8 - 197107784x^7 - 30834152x^6 + 185125776x^5 - 132260896x^4 - 45545344x^3 + 69517536x^2 - \sqrt{3}) \cdot (2131x^{16} + 10237x^{15} - 549126x^{14} + 260015x^{13} + 2523113x^{12} + 17504406x^{11} - 45089132x^{10} + 5444020x^9 + 84799980x^8 - 113800232x^7 - 17802104x^6 + 106882416x^5 - 76360864x^4 - 26295616x^3 + 40135968x^2 - 7907648x - 5562368) - 13696448x - 9634304) \cdot \sqrt{x^3 + 1} \cdot \sqrt{56\sqrt{3} + 97} + (459x^{16} - 1557x^{15} - 26415x^{14} - 1449954x^{13} + 4677912x^{12} + 12651948x^{11} - 55684800x^{10} + 62834256x^9 + 8526168x^8 - 105313392x^7 + 99605088x^6 - 18897984x^5 - 42499296x^4 + 37357632x^3 - 8256960x^2 - \sqrt{3}) \cdot (265x^{16} - 899x^{15} - 15249x^{14} - 837130x^{13} + 2700776x^{12} + 7304604x^{11} - 32149640x^{10} + 36277360x^9 + 4922568x^8 - 60802736x^7 + 57507040x^6 - 10910784x^5 - 24536992x^4 + 21568448x^3 - 4767168x^2 + 1207168x + 1383424) + 2090880x + 2396160) \cdot \sqrt{x^3 + 1} \cdot \sqrt{56\sqrt{3} + 97}) \cdot (672\sqrt{3} + 1164)^{(3/4)} + 6 \cdot (\sqrt{3}) \cdot (4945x^{15} - 37473x^{14} - 490698x^{13} + 2249468x^{12} + 474132x^{11} - 8423784x^{10} + 5853520x^9 + 8451720x^8 - 15320016x^7 + 768064x^6 + 10405056x^5 - 6627744x^4 - 700480x^3 + 2799552x^2 - \sqrt{3}) \cdot (2855x^{15} - 21635x^{14} - 283306x^{13} + 1298732x^{12} + 273748x^{11} - 4863472x^{10} + 3379536x^9 + 4879608x^8 - 8845008x^7 + 443456x^6 + 6007360x^5 - 3826528x^4 - 404416x^3 + 1616320x^2 - 1003648x - 399360) - 1738368x - 691712) \cdot \sqrt{x^3 + 1} \cdot \sqrt{56\sqrt{3} + 97} + 2 \cdot (246x^{15} - 3678x^{14} - 13485x^{13} + 102933x^{12} - 70062x^{11} - 81156x^{10} + 45204x^9 - 129636x^8 + 243576x^7 - 221784x^6 - 351024x^5 + 460896x^4 + 33984x^3 - 174048x^2 - \sqrt{3}) \cdot (142x^{15} - 2124x^{14} - 7773x^{13} + 59447x^{12} - 40626x^{11} - 46860x^{10} + 26308x^9 - 75276x^8 + 140472x^7 - 127784x^6 - 202896x^5 + 266016x^4 + 19712x^3 - 100512x^2 + 62400x + 24832) + 108096x + 43008) \cdot \sqrt{x^3 + 1} \cdot \sqrt{56\sqrt{3} + 97}) \cdot (672\sqrt{3} + 1164)^{(1/4)} + 108 \cdot (130x^{16} - 1682x^{15} + 2496x^{14} + 7730x^{13} + 1790x^{12} - 35700x^{11} - 7100x^{10} + 86080x^9 - 49176x$

$$\begin{aligned}
&^8 - 100400x^7 + 108208x^6 + 33312x^5 - 80704x^4 + 18944x^3 + 18048x^2 \\
&- 3\sqrt{3}(25x^{16} - 324x^{15} + 489x^{14} + 1482x^{13} + 316x^{12} - 6984x^{11} \\
&- 1312x^{10} + 16624x^9 - 9792x^8 - 19328x^7 + 20976x^6 + 6240x^5 \\
&- 15552x^4 + 3712x^3 + 3456x^2 - 4096x - 1280) - 21248x - 6656)\sqrt{(56\sqrt{3} + 97)} \\
&\sqrt{(9x^8 + 18x^7 + 414x^6 + 180x^5 + 360x^4 + 504x^3 - 72x^2 + 36\sqrt{3}(26x^7 \\
&+ 38x^6 + 42x^5 + 46x^4 + 46x^3 + 42x^2 - \sqrt{3}(15x^7 + 22x^6 + 24x^5 + 27x^4 \\
&+ 26x^3 + 24x^2 + 12x + 4) + 20x + 8)\sqrt{56\sqrt{3} + 97} - (\sqrt{3}(123x^6 + 2016x^5 + 2214x^4 \\
&+ 2064x^3 + 396x^2 - \sqrt{3}(71x^6 + 1164x^5 + 1278x^4 + 1192x^3 + 228x^2 - 112) \\
&- 192)\sqrt{x^3 + 1})\sqrt{56\sqrt{3} + 97} + 6(5x^6 + 27x^5 + 48x^4 + 58x^3 + 36x^2 \\
&- 3\sqrt{3}(x^6 + 5x^5 + 10x^4 + 10x^3 + 8x^2 + 4x) + 12x + 8)\sqrt{x^3 + 1})\sqrt{(\sqrt{3}\sqrt{56\sqrt{3} + 97} \\
&+ 97)(7\sqrt{3} - 12) + 6)(672\sqrt{3} + 1164)^{1/4} - 36\sqrt{3}(x^7 + 4x^6 + 6x^5 + 5x^4 \\
&- 4x^3 + 6x^2 + 4x - 8) - 144x + 576)/(x^8 - 4x^7 + 16x^6 - 16x^5 + 28x^4 \\
&+ 32x^3 + 64x^2 + 32x + 16)))/(x^{17} + 13x^{16} - 522x^{15} + 1742x^{14} + 3008x^{13} \\
&- 16884x^{12} + 11656x^{11} + 23944x^{10} - 42336x^9 + 9136x^8 + 36256x^7 - 27360x^6 \\
&- 256x^5 + 13376x^4 - 5760x^3 - 1664x^2 + 256x) - 1/1296\sqrt{(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} \\
&- 12) + 6)(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) - 6)(672\sqrt{3} + 1164)^{1/4} \\
&\log(1/9(9x^8 + 18x^7 + 414x^6 + 180x^5 + 360x^4 + 504x^3 - 72x^2 + 36\sqrt{3}(26x^7 \\
&+ 38x^6 + 42x^5 + 46x^4 + 46x^3 + 42x^2 - \sqrt{3}(15x^7 + 22x^6 + 24x^5 + 27x^4 \\
&+ 26x^3 + 24x^2 + 12x + 4) + 20x + 8)\sqrt{56\sqrt{3} + 97} + (\sqrt{3}(123x^6 + 2016x^5 \\
&+ 2214x^4 + 2064x^3 + 396x^2 - \sqrt{3}(71x^6 + 1164x^5 + 1278x^4 + 1192x^3 + 228x^2 \\
&- 112) - 192)\sqrt{x^3 + 1})\sqrt{56\sqrt{3} + 97} + 6(5x^6 + 27x^5 + 48x^4 + 58x^3 \\
&+ 36x^2 - 3\sqrt{3}(x^6 + 5x^5 + 10x^4 + 10x^3 + 8x^2 + 4x) + 12x + 8)\sqrt{x^3 + 1}) \\
&\sqrt{(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) + 6)(672\sqrt{3} + 1164)^{1/4} - 36\sqrt{3}(x^7 \\
&+ 4x^6 + 6x^5 + 5x^4 - 4x^3 + 6x^2 + 4x - 8) - 144x + 576)/(x^8 - 4x^7 + 16x^6 \\
&- 16x^5 + 28x^4 + 32x^3 + 64x^2 + 32x + 16)) + 1/1296\sqrt{(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} \\
&- 12) + 6)(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) - 6)(672\sqrt{3} + 1164)^{1/4} \\
&\log(1/9(9x^8 + 18x^7 + 414x^6 + 180x^5 + 360x^4 + 504x^3 - 72x^2 + 36\sqrt{3}(26x^7 \\
&+ 38x^6 + 42x^5 + 46x^4 + 46x^3 + 42x^2 - \sqrt{3}(15x^7 + 22x^6 + 24x^5 + 27x^4 \\
&+ 26x^3 + 24x^2 + 12x + 4) + 20x + 8)\sqrt{56\sqrt{3} + 97} - (\sqrt{3}(123x^6 + 2016x^5 \\
&+ 2214x^4 + 2064x^3 + 396x^2 - \sqrt{3}(71x^6 + 1164x^5 + 1278x^4 + 1192x^3 + 228x^2 \\
&- 112) - 192)\sqrt{x^3 + 1})\sqrt{56\sqrt{3} + 97} + 6(5x^6 + 27x^5 + 48x^4 + 58x^3 \\
&+ 36x^2 - 3\sqrt{3}(x^6 + 5x^5 + 10x^4 + 10x^3 + 8x^2 + 4x) + 12x + 8)\sqrt{x^3 + 1}) \\
&\sqrt{(\sqrt{3}\sqrt{56\sqrt{3} + 97})(7\sqrt{3} - 12) + 6)(672\sqrt{3} + 1164)^{1/4} - 36\sqrt{3}(x^7 \\
&+ 4x^6 + 6x^5 + 5x^4 - 4x^3 + 6x^2 + 4x - 8) - 144x + 576)/(x^8 - 4x^7 + 16x^6 \\
&- 16x^5 + 28x^4 + 32x^3 + 64x^2 + 32x + 16)) + 1/72\sqrt{(14\sqrt{3} + 24)}\log((x^8 - 16x^7 \\
&+ 112x^6 - 16x^5 + 112x^4 + 224x^3 + 64x^2 + 2(5x^6 - 54x^5 + 96x^4 - 56x^3 \\
&- 36x^2 - 3\sqrt{3}(x^6 - 10x^5 + 20x^4 - 8x^3 - 4x^2 + 8x) + 24x - 16)\sqrt{x^3 + 1}) \\
&\sqrt{(14\sqrt{3} + 24)} + 16\sqrt{3}(x^7 - 2x^6 + 6x^5 + 5x^4 + 2x^3 + 6x^2 + 4x + 4) \\
&+ 128x + 112)/(x^8 + 8x^7 + 16x^6 - 16x^5 - 56x^4 + 32x^3 + 64x^2 - 64x + 16))
\end{aligned}$$


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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)(x^3-6\sqrt{3}+10)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x\*\*3-6\*3\*\*(1/2))/(x\*\*3+1)\*\*(1/2),x)

[Out] Integral(x/(sqrt((x + 1)\*(x\*\*2 - x + 1))\*(x\*\*3 - 6\*sqrt(3) + 10)), x)

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(10+x^3-6\*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(x/((x^3 - 6\*sqrt(3) + 10)\*sqrt(x^3 + 1)), x)

$$3.88 \quad \int \frac{x}{\sqrt{-1+x^3}(-10-6\sqrt{3}+x^3)} dx$$

**Optimal.** Leaf size=222

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}+2x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

[Out] ((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 - Sqrt[3]))\*(1 - x)]/(Sqrt[2]\*Sqrt[-1 + x^3]))/(6\*Sqrt[2]\*3^(1/4)) + ((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 + Sqrt[3] + 2\*x))/(Sqrt[2]\*Sqrt[-1 + x^3])])/(3\*Sqrt[2]\*3^(1/4)) + ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 + Sqrt[3]))\*(1 - x)]/(Sqrt[2]\*Sqrt[-1 + x^3]))/(2\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTanh[((1 - Sqrt[3])\*Sqrt[-1 + x^3])/(Sqrt[2]\*3^(3/4))])/(3\*Sqrt[2]\*3^(3/4))

**Rubi [A]** time = 0.0295865, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {488}

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}+2x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1 + x^3]\*(-10 - 6\*Sqrt[3] + x^3)), x]

[Out] ((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 - Sqrt[3]))\*(1 - x)]/(Sqrt[2]\*Sqrt[-1 + x^3]))/(6\*Sqrt[2]\*3^(1/4)) + ((2 - Sqrt[3])\*ArcTan[(3^(1/4)\*(1 + Sqrt[3] + 2\*x))/(Sqrt[2]\*Sqrt[-1 + x^3])])/(3\*Sqrt[2]\*3^(1/4)) + ((2 - Sqrt[3])\*ArcTanh[(3^(1/4)\*(1 + Sqrt[3]))\*(1 - x)]/(Sqrt[2]\*Sqrt[-1 + x^3]))/(2\*Sqrt[2]\*3^(3/4)) - ((2 - Sqrt[3])\*ArcTanh[((1 - Sqrt[3])\*Sqrt[-1 + x^3])/(Sqrt[2]\*3^(3/4))])/(3\*Sqrt[2]\*3^(3/4))

#### Rule 488

Int[(x\_)/(Sqrt[(a\_) + (b\_.)\*(x\_)^3]\*((c\_) + (d\_.)\*(x\_)^3)), x\_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b\*c - 10\*a\*d)/(6\*a\*d)]}, Simp[(q\*(2 - r)\*ArcTanh[((1 - r)\*Sqrt[a + b\*x^3])/(Sqrt[2]\*Rt[-a, 2]\*r^(3/2))])/(3\*Sqrt[2]\*Rt[-a, 2]\*d\*r^(3/2)), x] + (-Simp[(q\*(2 - r)\*ArcTanh[(Rt[-a, 2]\*Sqrt[r]\*(1 + r)\*(1 + q\*x)]/(Sqrt[2]\*Sqrt[a + b\*x^3]))]/(2\*Sqrt[2]\*Rt[-a, 2]\*d\*r^(3/2)), x] - Simp[(q\*(2 - r)\*ArcTan[(Rt[-a, 2]\*Sqrt[r]\*(1 + r - 2\*q\*x)]/(Sqrt[2]\*Sqrt[a + b\*x^3]))]/(3\*Sqrt[2]\*Rt[-a, 2]\*d\*Sqrt[r]), x] - Simp[(q\*(2 - r)\*ArcTan[(Rt[-a, 2]\*(1 - r)\*Sqrt[r]\*(1 + q\*x)]/(Sqrt[2]\*Sqrt[a + b\*x^3]))]/(6\*Sqrt[2]\*Rt[-a, 2]\*d\*Sqrt[r]), x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b^2\*c^2 - 20\*a\*b\*c\*d - 8\*a^2\*d^2, 0] && NegQ[a]

#### Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}(-10-6\sqrt{3}+x^3)} dx = \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

**Mathematica [C]** time = 0.0663756, size = 65, normalized size = 0.29

$$\frac{x^2 \sqrt{1-x^3} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{10+6\sqrt{3}}\right)}{(20+12\sqrt{3})\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-1 + x^3]\*(-10 - 6\*Sqrt[3] + x^3)), x]

[Out] -((x^2\*Sqrt[1 - x^3]\*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6\*Sqrt[3])]) / ((20 + 12\*Sqrt[3])\*Sqrt[-1 + x^3]))

**Maple [C]** time = 0.173, size = 349, normalized size = 1.6

$$-\frac{\sqrt{2}}{18} \sum_{\alpha=\text{RootOf}(Z^2+(1+\sqrt{3})Z+2\sqrt{3}+4)} \frac{(-\sqrt{3}\alpha + \alpha + 2)(-i\sqrt{3}-3)(1+2\alpha - \sqrt{3}\alpha)}{-\sqrt{3}-2\alpha-1} \sqrt{\frac{-1+x}{-i\sqrt{3}-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-10+x^3-6\*3^(1/2))/(x^3-1)^(1/2), x)

[Out] -1/18\*2^(1/2)\*sum((-3^(1/2)\*\_alpha+\_alpha+2)/(-3^(1/2)-2\*\_alpha-1)\*(-I\*3^(1/2)-3)\*((-1+x)/(-I\*3^(1/2)-3))^(1/2)\*((2\*x+1-I\*3^(1/2))/(-I\*3^(1/2)+3))^(1/2)\*((2\*x+1+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)\*(1+2\*\_alpha-3^(1/2))\*\_alpha)\*EllipticPi(((1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2), -1/2\*I\*\_alpha+1/3\*I\*\_alpha\*3^(1/2)-1/2\*3^(1/2)\*\_alpha+\_alpha+1/6\*I\*3^(1/2)+1/2, ((3/2+1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2)), \_alpha=RootOf(\_Z^2+(1+3^(1/2))\*\_Z+2\*3^(1/2)+4))+1/9\*(-1-3^(1/2))/(2+3^(1/2))\*(-3/2-1/2\*I\*3^(1/2))\*((-1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x+1/2-1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x+1/2+1/2\*I\*3^(1/2))/(3/2+1/2\*I\*3^(1/2)))^(1/2)/(x^3-1)^(1/2)\*3^(1/2)\*EllipticPi(((1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2), -1/3\*(3/2+1/2\*I\*3^(1/2))\*3^(1/2), ((3/2+1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x^3-6\*3^(1/2))/(x^3-1)^(1/2), x, algorithm="maxima")

[Out] integrate(x/((x^3 - 6\*sqrt(3) - 10)\*sqrt(x^3 - 1)), x)

**Fricas [B]** time = 44.3407, size = 28004, normalized size = 126.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x^3-6\*3^(1/2)))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{432} \sqrt{2(7\sqrt{3} + 12)} \sqrt{-672\sqrt{3} + 1164} + 24) (56\sqrt{3} + 97) \sqrt{-56\sqrt{3} + 97} (-672\sqrt{3} + 1164)^{3/4} \arctan\left(\frac{1}{1296} (6\sqrt{3} \sqrt{x^3 - 1} + (459x^{16} + 13425x^{15} - 33201x^{14} - 950652x^{13} - 997302x^{12} + 14760972x^{11} + 47069892x^{10} + 49762248x^9 - 8212536x^8 - 84377808x^7 - 88427328x^6 - 25613856x^5 + 27458496x^4 + 36433344x^3 + 12609792x^2 + \sqrt{3}(265x^{16} + 7751x^{15} - 19167x^{14} - 548864x^{13} - 575818x^{12} + 8522268x^{11} + 27175852x^{10} + 28730312x^9 - 4741560x^8 - 48715600x^7 - 51053600x^6 - 14788128x^5 + 15853184x^4 + 21034816x^3 + 7280256x^2 + 2488832x - 1889792) - (3691x^{16} + 6128x^{15} - 537864x^{14} - 1586477x^{13} + 16210952x^{12} + 77181756x^{11} + 84218362x^{10} - 71018320x^9 - 254455812x^8 - 196076008x^7 + 120105208x^6 + 256326864x^5 + 134645168x^4 - 78464672x^3 - 78514944x^2 + \sqrt{3}(2131x^{16} + 3538x^{15} - 310536x^{14} - 915953x^{13} + 9359398x^{12} + 44560908x^{11} + 48623494x^{10} - 41002448x^9 - 146910132x^8 - 113204536x^7 + 69342776x^6 + 147990384x^5 + 77737424x^4 - 45301600x^3 - 45330624x^2 - 12242560x + 7598336) - 21204736x + 13160704) \sqrt{-672\sqrt{3} + 1164} + 4310784x - 3273216) (-672\sqrt{3} + 1164)^{3/4} + 3(984x^{15} + 30612x^{14} + 164676x^{13} + 205368x^{12} - 289200x^{11} - 183720x^{10} + 886752x^9 + 71568x^8 - 1960992x^7 - 1849440x^6 + 1558464x^5 + 2478912x^4 + 66432x^3 - 750336x^2 + 4\sqrt{3}(142x^{15} + 4419x^{14} + 23781x^{13} + 29608x^{12} - 41940x^{11} - 26454x^{10} + 128152x^9 + 10692x^8 - 283320x^7 - 267064x^6 + 224784x^5 + 357936x^4 + 9632x^3 - 108288x^2 - 96000x + 33920) - (4945x^{15} + 88617x^{14} + 738528x^{13} + 1860046x^{12} - 784596x^{11} - 7668708x^{10} - 6570680x^9 + 6903864x^8 + 15444144x^7 + 4312832x^6 - 9559200x^5 - 9359808x^4 - 155968x^3 + 3016704x^2 + \sqrt{3}(2855x^{15} + 51163x^{14} + 426388x^{13} + 1073898x^{12} - 452980x^{11} - 4427548x^{10} - 3793592x^9 + 3985944x^8 + 8916720x^7 + 2490016x^6 - 5519008x^5 - 5403904x^4 - 90048x^3 + 1741696x^2 + 1543936x - 545536) + 2674176x - 944896) \sqrt{-672\sqrt{3} + 1164} - 665088x + 235008) (-672\sqrt{3} + 1164)^{1/4} \sqrt{2(7\sqrt{3} + 12)} \sqrt{-672\sqrt{3} + 1164} + 24) \sqrt{-56\sqrt{3} + 97} + 36(144x^{17} + 5976x^{16} + 5544x^{15} - 299664x^{14} - 1062360x^{13} - 116712x^{12} + 3600000x^{11} + 4761216x^{10} - 1046592x^9 - 8676864x^8 - 6592896x^7 + 2641536x^6 + 7016832x^5 + 3699072x^4 - 1861632x^3 - 1640448x^2 + 12\sqrt{3}(7x^{17} + 286x^{16} + 238x^{15} - 14255x^{14} - 50390x^{13} - 5942x^{12} + 171808x^{11} + 226888x^{10} - 48920x^9 - 415384x^8 - 315088x^7 + 125600x^6 + 336608x^5 + 177344x^4 - 89152x^3 - 78784x^2 - 39040x + 18176) + (1164x^{17} + 6276x^{16} - 26052x^{15} - 332844x^{14} - 1632156x^{13} - 4149132x^{12} - 5805024x^{11} - 318696x^{10} + 12621072x^9 + 19878720x^8 + 9619008x^7 - 13361088x^6 - 20168256x^5 - 10936128x^4 + 6434304x^3 + 6426240x^2 + 24\sqrt{3}(28x^{17} + 151x^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 99812x^{12} - 139652x^{11} - 7661x^{10} + 303610x^9 + 478214x^8 + 231392x^7 - 321412x^6 - 485176x^5 - 263080x^4 + 154784x^3 + 154592x^2 + 78464x - 36544) - (2340x^{17} + 96354x^{16} + 84798x^{15} - 4817124x^{14} - 17052930x^{13} - 1941678x^{12} + 57963744x^{11} + 76603680x^{10} - 16678512x^9 - 139922496x^8 - 106227360x^7 + 42453216x^6 + 113269536x^5 + 59694624x^4 - 30025728x^3 - 26496000x^2 + \sqrt{3}(1351x^{17} + 55630x^{16} + 48958x^{15} - 2781167x^{14} - 9845510x^{13} - 1121030x^{12} + 33465376x^{11} + 44227144x^{10} - 9629336x^9 - 80784280x^8 - 61330384x^7 + 24510368x^6 + 65396192x^5 + 34464704x^4 - 17335360x^3 - 15297472x^2 - 7571584x + 3526400) - 13114368x + 6107904) \sqrt{-672\sqrt{3} + 1164} + 3261696x - 1519104) \sqrt{-672\sqrt{3} + 1164} - 12(97x^{17} + 523x^{16} - 2171x^{15} - 27737x^{14} - 136013x^{13} - 345761x^{12} - 483752x^{11} - 26558x^{10} + 1051756x^9 + 1656560x^8 + 801584x^7 - 1113424x^6 - 1680688x^5 - 911344x^4 + 536192x^3 + 535520x^2 + 2\sqrt{3}(28x^{17} + 151x^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 99812x^{12} - 139652x^{11} - 7661x^{10} + 303610x^9 + 478214x^8 + 231392x^7 - 321412x^6 - 485176x^5 - 263080x^4 + 154784x^3$

$$\begin{aligned}
& 3 + 154592x^2 + 78464x - 36544) + 271808x - 126592) \sqrt{-672\sqrt{3} + 1164} - 811008x + 377856) \sqrt{-56\sqrt{3} + 97} - (\sqrt{x^3 - 1}) \cdot ((459x^{16} + 1557x^{15} - 26415x^{14} + 1449954x^{13} + 4677912x^{12} - 12651948x^{11} - 55684800x^{10} - 62834256x^9 + 8526168x^8 + 105313392x^7 + 99605088x^6 + 18897984x^5 - 42499296x^4 - 37357632x^3 - 8256960x^2 + \sqrt{3})(265x^{16} + 899x^{15} - 15249x^{14} + 837130x^{13} + 2700776x^{12} - 7304604x^{11} - 32149640x^{10} - 36277360x^9 + 4922568x^8 + 60802736x^7 + 57507040x^6 + 10910784x^5 - 24536992x^4 - 21568448x^3 - 4767168x^2 - 1207168x + 1383424) - (3691x^{16} - 17731x^{15} - 951114x^{14} - 450359x^{13} + 4370159x^{12} - 30318522x^{11} - 78096668x^{10} - 9429316x^9 + 146877876x^8 + 197107784x^7 - 30834152x^6 - 185125776x^5 - 132260896x^4 + 45545344x^3 + 69517536x^2 + \sqrt{3})(2131x^{16} - 10237x^{15} - 549126x^{14} - 260015x^{13} + 2523113x^{12} - 17504406x^{11} - 45089132x^{10} - 5444020x^9 + 84799980x^8 + 113800232x^7 - 17802104x^6 - 106882416x^5 - 76360864x^4 + 26295616x^3 + 40135968x^2 + 7907648x - 5562368) + 13696448x - 9634304) \sqrt{-672\sqrt{3} + 1164} - 2090880x + 2396160) \cdot (-672\sqrt{3} + 1164)^{3/4} + 3 \cdot (984x^{15} + 14712x^{14} - 53940x^{13} - 411732x^{12} - 280248x^{11} + 324624x^{10} + 180816x^9 + 518544x^8 + 974304x^7 + 887136x^6 - 1404096x^5 - 1843584x^4 + 135936x^3 + 696192x^2 + 4\sqrt{3})(142x^{15} + 2124x^{14} - 7773x^{13} - 59447x^{12} - 40626x^{11} + 46860x^{10} + 26308x^9 + 75276x^8 + 140472x^7 + 127784x^6 - 202896x^5 - 266016x^4 + 19712x^3 + 100512x^2 + 62400x - 24832) - (4945x^{15} + 37473x^{14} - 490698x^{13} - 2249468x^{12} + 474132x^{11} + 8423784x^{10} + 5853520x^9 - 8451720x^8 - 15320016x^7 - 768064x^6 + 10405056x^5 + 6627744x^4 - 700480x^3 - 2799552x^2 + \sqrt{3})(2855x^{15} + 21635x^{14} - 283306x^{13} - 1298732x^{12} + 273748x^{11} + 4863472x^{10} + 3379536x^9 - 4879608x^8 - 8845008x^7 - 443456x^6 + 6007360x^5 + 3826528x^4 - 404416x^3 - 1616320x^2 - 1003648x + 399360) - 1738368x + 691712) \sqrt{-672\sqrt{3} + 1164} + 432384x - 172032) \cdot (-672\sqrt{3} + 1164)^{1/4}) \sqrt{2(7\sqrt{3} + 12) \sqrt{-672\sqrt{3} + 1164} + 24} \sqrt{-56\sqrt{3} + 97} + 6 \cdot (4680x^{16} + 60552x^{15} + 89856x^{14} - 278280x^{13} + 64440x^{12} + 1285200x^{11} - 255600x^{10} - 3098880x^9 - 1770336x^8 + 3614400x^7 + 3895488x^6 - 1199232x^5 - 2905344x^4 - 681984x^3 + 649728x^2 + 108\sqrt{3})(25x^{16} + 324x^{15} + 489x^{14} - 1482x^{13} + 316x^{12} + 6984x^{11} - 1312x^{10} - 16624x^9 - 9792x^8 + 19328x^7 + 20976x^6 - 6240x^5 - 15552x^4 - 3712x^3 + 3456x^2 + 4096x - 1280) + (1164x^{17} - 1248x^{16} - 246120x^{15} - 518172x^{14} + 2607528x^{13} + 8301144x^{12} + 7017600x^{11} - 6258120x^{10} - 21360336x^9 - 16998960x^8 + 966336x^7 + 18216672x^6 + 15860544x^5 + 4720704x^4 - 6023424x^3 - 5362176x^2 + 48\sqrt{3})(14x^{17} - 15x^{16} - 2960x^{15} - 6232x^{14} + 31362x^{13} + 99844x^{12} + 84404x^{11} - 75267x^{10} - 256916x^9 - 204458x^8 + 11616x^7 + 219104x^6 + 190768x^5 + 56784x^4 - 72448x^3 - 64496x^2 - 24480x + 13376) - (2340x^{17} + 35850x^{16} - 106410x^{15} + 2064744x^{14} + 11945946x^{13} + 1710042x^{12} - 46293732x^{11} - 59161524x^{10} + 18480192x^9 + 122366520x^8 + 81203856x^7 - 45222000x^6 - 100598112x^5 - 42207168x^4 + 29609472x^3 + 22458240x^2 + \sqrt{3})(1351x^{17} + 20698x^{16} - 61436x^{15} + 1192081x^{14} + 6896998x^{13} + 987292x^{12} - 26727704x^{11} - 34156928x^{10} + 10669552x^9 + 70648352x^8 + 46883072x^7 - 26108944x^6 - 58080352x^5 - 24368320x^4 + 17095040x^3 + 12966272x^2 + 4724480x - 2581504) + 8183040x - 4471296) \sqrt{-672\sqrt{3} + 1164} - 2035200x + 1112064) \sqrt{-672\sqrt{3} + 1164} - 24 \cdot (627x^{16} + 14286x^{15} + 39762x^{14} - 50142x^{13} - 216816x^{12} - 112284x^{11} + 325707x^{10} + 586326x^9 - 3294x^8 - 631752x^7 - 539220x^6 + 184392x^5 + 483816x^4 + 115296x^3 - 108576x^2 + 2\sqrt{3})(181x^{16} + 4124x^{15} + 11478x^{14} - 14474x^{13} - 62584x^{12} - 32412x^{11} + 94021x^{10} + 169244x^9 - 954x^8 - 182368x^7 - 155648x^6 + 53232x^5 + 139664x^4 + 33280x^3 - 31344x^2 - 37024x + 11584) - 128256x + 40128) \sqrt{-672\sqrt{3} + 1164} + 764928x - 239616) \sqrt{-56\sqrt{3} + 97}) \sqrt{(36x^8 - 72x^7 + 1656x^6 - 720x^5 + 1440x^4 - 2016x^3 + (60x^6 - 324x^5 + 576x^4 - 696x^3 + 432x^2 + 36\sqrt{3})(x^6 - 5x^5 + 10x^4 - 10x^3 + 8x^2 - 4x) - (123x^6 - 2016x^5 + 2214x^4 - 2064x^3 + 396x^2 + \sqrt{3})(71x^6 - 1164x^5 + 1278x^4 - 1192x^3 +
\end{aligned}$$



$$\begin{aligned}
& 228x^2 - 112) - 192) \sqrt{-672\sqrt{3} + 1164} - 144x + 96) \sqrt{x^3 - 1} \\
& \sqrt{2(7\sqrt{3} + 12) \sqrt{-672\sqrt{3} + 1164} + 24} (-672\sqrt{3} + 1164)^{1/4} - 288x^2 - 144\sqrt{3} (x^7 - 4x^6 + 6x^5 - 5x^4 - 4x^3 - 6x^2 + 4x + 8) + 72(26x^7 - 38x^6 + 42x^5 - 46x^4 + 46x^3 - 42x^2 + \sqrt{3}(15x^7 - 22x^6 + 24x^5 - 27x^4 + 26x^3 - 24x^2 + 12x - 4) + 20x - 8) \sqrt{-672\sqrt{3} + 1164} + 576x + 2304) / (x^8 + 4x^7 + 16x^6 + 16x^5 + 28x^4 - 32x^3 + 64x^2 - 32x + 16)) / (x^{17} - 13x^{16} - 522x^{15} - 1742x^{14} + 3008x^{13} + 16884x^{12} + 11656x^{11} - 23944x^{10} - 42336x^9 - 9136x^8 + 36256x^7 + 27360x^6 - 256x^5 - 13376x^4 - 5760x^3 + 1664x^2 + 256x) + 1/432 \sqrt{2(7\sqrt{3} + 12) \sqrt{-672\sqrt{3} + 1164} + 24} (56\sqrt{3} + 97) \sqrt{-56\sqrt{3} + 97} (-672\sqrt{3} + 1164)^{3/4} \operatorname{arctan}(1/1296(6\sqrt{x^3 - 1}((459x^{16} + 13425x^{15} - 33201x^{14} - 950652x^{13} - 997302x^{12} + 14760972x^{11} + 47069892x^{10} + 49762248x^9 - 8212536x^8 - 84377808x^7 - 88427328x^6 - 25613856x^5 + 27458496x^4 + 36433344x^3 + 12609792x^2 + \sqrt{3}(265x^{16} + 7751x^{15} - 19167x^{14} - 548864x^{13} - 575818x^{12} + 8522268x^{11} + 27175852x^{10} + 28730312x^9 - 4741560x^8 - 48715600x^7 - 51053600x^6 - 14788128x^5 + 15853184x^4 + 21034816x^3 + 7280256x^2 + 2488832x - 1889792) - (3691x^{16} + 6128x^{15} - 537864x^{14} - 1586477x^{13} + 16210952x^{12} + 77181756x^{11} + 84218362x^{10} - 71018320x^9 - 254455812x^8 - 196076008x^7 + 120105208x^6 + 256326864x^5 + 134645168x^4 - 78464672x^3 - 78514944x^2 + \sqrt{3}(2131x^{16} + 3538x^{15} - 310536x^{14} - 915953x^{13} + 9359398x^{12} + 44560908x^{11} + 48623494x^{10} - 41002448x^9 - 146910132x^8 - 113204536x^7 + 69342776x^6 + 147990384x^5 + 77737424x^4 - 45301600x^3 - 45330624x^2 - 12242560x + 7598336) - 21204736x + 13160704) \sqrt{-672\sqrt{3} + 1164} + 4310784x - 3273216) (-672\sqrt{3} + 1164)^{3/4} + 3(984x^{15} + 30612x^{14} + 164676x^{13} + 205368x^{12} - 289200x^{11} - 183720x^{10} + 886752x^9 + 71568x^8 - 1960992x^7 - 1849440x^6 + 1558464x^5 + 2478912x^4 + 66432x^3 - 750336x^2 + 4\sqrt{3})(142x^{15} + 4419x^{14} + 23781x^{13} + 29608x^{12} - 41940x^{11} - 26454x^{10} + 128152x^9 + 10692x^8 - 283320x^7 - 267064x^6 + 224784x^5 + 357936x^4 + 9632x^3 - 108288x^2 - 96000x + 33920) - (4945x^{15} + 88617x^{14} + 738528x^{13} + 1860046x^{12} - 784596x^{11} - 7668708x^{10} - 6570680x^9 + 6903864x^8 + 15444144x^7 + 4312832x^6 - 9559200x^5 - 9359808x^4 - 155968x^3 + 3016704x^2 + \sqrt{3}(2855x^{15} + 51163x^{14} + 426388x^{13} + 1073898x^{12} - 452980x^{11} - 4427548x^{10} - 3793592x^9 + 3985944x^8 + 8916720x^7 + 2490016x^6 - 5519008x^5 - 5403904x^4 - 90048x^3 + 1741696x^2 + 1543936x - 545536) + 2674176x - 944896) \sqrt{-672\sqrt{3} + 1164} - 665088x + 235008) (-672\sqrt{3} + 1164)^{1/4}) \sqrt{2(7\sqrt{3} + 12) \sqrt{-672\sqrt{3} + 1164} + 24} \sqrt{-56\sqrt{3} + 97} - 36(144x^{17} + 5976x^{16} + 5544x^{15} - 299664x^{14} - 1062360x^{13} - 116712x^{12} + 3600000x^{11} + 4761216x^{10} - 1046592x^9 - 8676864x^8 - 6592896x^7 + 2641536x^6 + 7016832x^5 + 3699072x^4 - 1861632x^3 - 1640448x^2 + 12\sqrt{3}(7x^{17} + 286x^{16} + 238x^{15} - 14255x^{14} - 50390x^{13} - 5942x^{12} + 171808x^{11} + 226888x^{10} - 48920x^9 - 415384x^8 - 315088x^7 + 125600x^6 + 336608x^5 + 177344x^4 - 89152x^3 - 78784x^2 - 39040x + 18176) + (1164x^{17} + 6276x^{16} - 26052x^{15} - 332844x^{14} - 1632156x^{13} - 4149132x^{12} - 5805024x^{11} - 318696x^{10} + 12621072x^9 + 19878720x^8 + 9619008x^7 - 13361088x^6 - 20168256x^5 - 10936128x^4 + 6434304x^3 + 6426240x^2 + 24\sqrt{3}(28x^{17} + 151x^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 99812x^{12} - 139652x^{11} - 7661x^{10} + 303610x^9 + 478214x^8 + 231392x^7 - 321412x^6 - 485176x^5 - 263080x^4 + 154784x^3 + 154592x^2 + 78464x - 36544) - (2340x^{17} + 96354x^{16} + 84798x^{15} - 4817124x^{14} - 17052930x^{13} - 1941678x^{12} + 57963744x^{11} + 76603680x^{10} - 16678512x^9 - 139922496x^8 - 106227360x^7 + 42453216x^6 + 113269536x^5 + 59694624x^4 - 30025728x^3 - 26496000x^2 + \sqrt{3}(1351x^{17} + 55630x^{16} + 48958x^{15} - 2781167x^{14} - 9845510x^{13} - 1121030x^{12} + 33465376x^{11} + 44227144x^{10} - 9629336x^9 - 80784280x^8 - 61330384x^7 + 24510368x^6 + 65396192x^5 + 34464704x^4 - 17335360x^3 - 15297472x^2 - 7571584x + 3526400) - 13114368x + 6107904) \sqrt{-672\sqrt{3} + 1164} + 3261696x - 1519104) \sqrt{-672\sqrt{3} + 1164} - 12(97x^{17} +
\end{aligned}$$

$$\begin{aligned}
& 523x^{16} - 2171x^{15} - 27737x^{14} - 136013x^{13} - 345761x^{12} - 483752x^{11} \\
& - 26558x^{10} + 1051756x^9 + 1656560x^8 + 801584x^7 - 1113424x^6 - 1680 \\
& 688x^5 - 911344x^4 + 536192x^3 + 535520x^2 + 2\sqrt{3}(28x^{17} + 151x \\
& ^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 99812x^{12} - 139652x^{11} - 7661x \\
& ^{10} + 303610x^9 + 478214x^8 + 231392x^7 - 321412x^6 - 485176x^5 - 2630 \\
& 80x^4 + 154784x^3 + 154592x^2 + 78464x - 36544) + 271808x - 126592)\sqrt{3} \\
& \sqrt{-672\sqrt{3} + 1164} - 811008x + 377856)\sqrt{-56\sqrt{3} + 97} - (\sqrt{3} \\
& (x^3 - 1)((459x^{16} + 1557x^{15} - 26415x^{14} + 1449954x^{13} + 4677912x^{12} \\
& - 12651948x^{11} - 55684800x^{10} - 62834256x^9 + 8526168x^8 + 105313392x \\
& ^7 + 99605088x^6 + 18897984x^5 - 42499296x^4 - 37357632x^3 - 8256960x^2 \\
& + \sqrt{3}(265x^{16} + 899x^{15} - 15249x^{14} + 837130x^{13} + 2700776x^{12} \\
& - 7304604x^{11} - 32149640x^{10} - 36277360x^9 + 4922568x^8 + 60802736x^7 \\
& + 57507040x^6 + 10910784x^5 - 24536992x^4 - 21568448x^3 - 4767168x^2 - \\
& 1207168x + 1383424) - (3691x^{16} - 17731x^{15} - 951114x^{14} - 450359x^{13} \\
& + 4370159x^{12} - 30318522x^{11} - 78096668x^{10} - 9429316x^9 + 146877876x \\
& ^8 + 197107784x^7 - 30834152x^6 - 185125776x^5 - 132260896x^4 + 4554534 \\
& 4x^3 + 69517536x^2 + \sqrt{3}(2131x^{16} - 10237x^{15} - 549126x^{14} - 2600 \\
& 15x^{13} + 2523113x^{12} - 17504406x^{11} - 45089132x^{10} - 5444020x^9 + 8479 \\
& 9980x^8 + 113800232x^7 - 17802104x^6 - 106882416x^5 - 76360864x^4 + 26 \\
& 295616x^3 + 40135968x^2 + 7907648x - 5562368) + 13696448x - 9634304)\sqrt{3} \\
& \sqrt{-672\sqrt{3} + 1164} - 2090880x + 2396160)(-672\sqrt{3} + 1164)^{(3/4)} \\
& + 3*(984x^{15} + 14712x^{14} - 53940x^{13} - 411732x^{12} - 280248x^{11} + 32462 \\
& 4x^{10} + 180816x^9 + 518544x^8 + 974304x^7 + 887136x^6 - 1404096x^5 - \\
& 1843584x^4 + 135936x^3 + 696192x^2 + 4\sqrt{3}(142x^{15} + 2124x^{14} - 7 \\
& 773x^{13} - 59447x^{12} - 40626x^{11} + 46860x^{10} + 26308x^9 + 75276x^8 + 1 \\
& 40472x^7 + 127784x^6 - 202896x^5 - 266016x^4 + 19712x^3 + 100512x^2 + \\
& 62400x - 24832) - (4945x^{15} + 37473x^{14} - 490698x^{13} - 2249468x^{12} + \\
& 474132x^{11} + 8423784x^{10} + 5853520x^9 - 8451720x^8 - 15320016x^7 - 768 \\
& 064x^6 + 10405056x^5 + 6627744x^4 - 700480x^3 - 2799552x^2 + \sqrt{3}( \\
& 2855x^{15} + 21635x^{14} - 283306x^{13} - 1298732x^{12} + 273748x^{11} + 4863472 \\
& *x^{10} + 3379536x^9 - 4879608x^8 - 8845008x^7 - 443456x^6 + 6007360x^5 \\
& + 3826528x^4 - 404416x^3 - 1616320x^2 - 1003648x + 399360) - 1738368x \\
& + 691712)\sqrt{-672\sqrt{3} + 1164} + 432384x - 172032)(-672\sqrt{3} + 11 \\
& 64)^{(1/4)}\sqrt{2*(7\sqrt{3} + 12)\sqrt{-672\sqrt{3} + 1164} + 24}\sqrt{-56 \\
& *\sqrt{3} + 97} - 6*(4680x^{16} + 60552x^{15} + 89856x^{14} - 278280x^{13} + 644 \\
& 40x^{12} + 1285200x^{11} - 255600x^{10} - 3098880x^9 - 1770336x^8 + 3614400* \\
& x^7 + 3895488x^6 - 1199232x^5 - 2905344x^4 - 681984x^3 + 649728x^2 + 1 \\
& 08*\sqrt{3}(25x^{16} + 324x^{15} + 489x^{14} - 1482x^{13} + 316x^{12} + 6984x^{11} \\
& 1 - 1312x^{10} - 16624x^9 - 9792x^8 + 19328x^7 + 20976x^6 - 6240x^5 - 1 \\
& 5552x^4 - 3712x^3 + 3456x^2 + 4096x - 1280) + (1164x^{17} - 1248x^{16} - \\
& 246120x^{15} - 518172x^{14} + 2607528x^{13} + 8301144x^{12} + 7017600x^{11} - 62 \\
& 58120x^{10} - 21360336x^9 - 16998960x^8 + 966336x^7 + 18216672x^6 + 1586 \\
& 0544x^5 + 4720704x^4 - 6023424x^3 - 5362176x^2 + 48*\sqrt{3}(14x^{17} - \\
& 15x^{16} - 2960x^{15} - 6232x^{14} + 31362x^{13} + 99844x^{12} + 84404x^{11} - 75 \\
& 267x^{10} - 256916x^9 - 204458x^8 + 11616x^7 + 219104x^6 + 190768x^5 + \\
& 56784x^4 - 72448x^3 - 64496x^2 - 24480x + 13376) - (2340x^{17} + 35850x \\
& ^{16} - 106410x^{15} + 2064744x^{14} + 11945946x^{13} + 1710042x^{12} - 46293732* \\
& x^{11} - 59161524x^{10} + 18480192x^9 + 122366520x^8 + 81203856x^7 - 452220 \\
& 00x^6 - 100598112x^5 - 42207168x^4 + 29609472x^3 + 22458240x^2 + \sqrt{3}( \\
& 3)(1351x^{17} + 20698x^{16} - 61436x^{15} + 1192081x^{14} + 6896998x^{13} + 987 \\
& 292x^{12} - 26727704x^{11} - 34156928x^{10} + 10669552x^9 + 70648352x^8 + 46 \\
& 883072x^7 - 26108944x^6 - 58080352x^5 - 24368320x^4 + 17095040x^3 + 12 \\
& 966272x^2 + 4724480x - 2581504) + 8183040x - 4471296)\sqrt{-672\sqrt{3} \\
& + 1164} - 2035200x + 1112064)\sqrt{-672\sqrt{3} + 1164} - 24*(627x^{16} + 1 \\
& 4286x^{15} + 39762x^{14} - 50142x^{13} - 216816x^{12} - 112284x^{11} + 325707x^ \\
& 10 + 586326x^9 - 3294x^8 - 631752x^7 - 539220x^6 + 184392x^5 + 483816* \\
& x^4 + 115296x^3 - 108576x^2 + 2*\sqrt{3}(181x^{16} + 4124x^{15} + 11478x^{14} \\
& 4 - 14474x^{13} - 62584x^{12} - 32412x^{11} + 94021x^{10} + 169244x^9 - 954x^ \\
& 8 - 182368x^7 - 155648x^6 + 53232x^5 + 139664x^4 + 33280x^3 - 31344x^
\end{aligned}$$

```

2 - 37024*x + 11584) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928
*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720
*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^
2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2
016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278
*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x +
96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*
(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 -
5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*
x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8
+ 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 -
13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 239
44*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^
4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-67
2*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*
(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 +
1440*x^4 - 2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36
*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*s
qrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*
sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3
- 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 +
12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^
7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/5184*sqrt(
2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-
672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72
*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x
^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2
- 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*
x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt
(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*s
qrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x
^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6
+ 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 -
27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164)
+ 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 -
32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*log((x^8 + 16*x^7 + 112*x^6 + 16*
x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^6 + 54*x^5 + 96*x^4 + 56*x^3 - 36
*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 + 8*x^3 - 4*x^2 - 8*x) - 24*x - 16)
*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*
x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^
5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x^3-6\sqrt{3}-10)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x\*\*3-6\*3\*\*(1/2))/(x\*\*3-1)\*\*(1/2), x)

[Out] Integral(x/(sqrt((x - 1)\*(x\*\*2 + x + 1))\*(x\*\*3 - 6\*sqrt(3) - 10)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

$$3.89 \quad \int \frac{x}{\sqrt{-1+x^3}(-10+6\sqrt{3}+x^3)} dx$$

**Optimal.** Leaf size=214

$$-\frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

[Out]  $-\left((2+\sqrt{3})\text{ArcTan}\left[\frac{3^{1/4}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(2\sqrt{2}3^{3/4}) + \left((2+\sqrt{3})\text{ArcTan}\left[\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right]\right)/(3\sqrt{2}3^{3/4}) + \left((2+\sqrt{3})\text{ArcTanh}\left[\frac{3^{1/4}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(6\sqrt{2}3^{1/4}) + \left((2+\sqrt{3})\text{ArcTanh}\left[\frac{3^{1/4}(1-\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(3\sqrt{2}3^{1/4})$

**Rubi [A]** time = 0.0290123, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {488}

$$-\frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1+x^3]\*(-10+6\*Sqrt[3]+x^3)),x]

[Out]  $-\left((2+\sqrt{3})\text{ArcTan}\left[\frac{3^{1/4}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(2\sqrt{2}3^{3/4}) + \left((2+\sqrt{3})\text{ArcTan}\left[\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right]\right)/(3\sqrt{2}3^{3/4}) + \left((2+\sqrt{3})\text{ArcTanh}\left[\frac{3^{1/4}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(6\sqrt{2}3^{1/4}) + \left((2+\sqrt{3})\text{ArcTanh}\left[\frac{3^{1/4}(1-\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right]\right)/(3\sqrt{2}3^{1/4})$

#### Rule 488

Int[(x\_)/(Sqrt[(a\_)+(b\_.)\*(x\_)^3]\*((c\_)+(d\_.)\*(x\_)^3)), x\_Symbol] := With[{q = Rt[b/a, 3], r = Simplify[(b\*c - 10\*a\*d)/(6\*a\*d)]}, Simp[(q\*(2-r)\*ArcTanh[((1-r)\*Sqrt[a+b\*x^3])/(Sqrt[2]\*Rt[-a, 2]\*r^(3/2))])/(3\*Sqrt[2]\*Rt[-a, 2]\*d\*r^(3/2)), x] + (-Simp[(q\*(2-r)\*ArcTanh[(Rt[-a, 2]\*Sqrt[r]\*(1+r)\*(1+q\*x))/(Sqrt[2]\*Sqrt[a+b\*x^3])])/(2\*Sqrt[2]\*Rt[-a, 2]\*d\*r^(3/2)), x] - Simp[(q\*(2-r)\*ArcTan[(Rt[-a, 2]\*Sqrt[r]\*(1+r-2\*q\*x))/(Sqrt[2]\*Sqrt[a+b\*x^3])])/(3\*Sqrt[2]\*Rt[-a, 2]\*d\*Sqrt[r]), x] - Simp[(q\*(2-r)\*ArcTan[(Rt[-a, 2]\*(1-r)\*Sqrt[r]\*(1+q\*x))/(Sqrt[2]\*Sqrt[a+b\*x^3])])/(6\*Sqrt[2]\*Rt[-a, 2]\*d\*Sqrt[r]), x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[b^2\*c^2 - 20\*a\*b\*c\*d - 8\*a^2\*d^2, 0] && NegQ[a]

#### Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}(-10+6\sqrt{3}+x^3)} dx = -\frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

**Mathematica [C]** time = 0.0531543, size = 68, normalized size = 0.32

$$\frac{x^2 \sqrt{1-x^3} F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{x^3}{-10+6\sqrt{3}}\right)}{4(3\sqrt{3}-5)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-1 + x^3]\*(-10 + 6\*Sqrt[3] + x^3)),x]

[Out] (x^2\*Sqrt[1 - x^3]\*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6\*Sqrt[3]))])/ (4\*(-5 + 3\*Sqrt[3])\*Sqrt[-1 + x^3])

**Maple [C]** time = 0.169, size = 350, normalized size = 1.6

$$\frac{(\sqrt{3}-1)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{-18+9\sqrt{3}} \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}} \sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)} \sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)} \text{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-10+x^3+6\*3^(1/2))/(x^3-1)^(1/2),x)

[Out] 1/9\*(3^(1/2)-1)/(-2+3^(1/2))\*(-3/2-1/2\*I\*3^(1/2))\*((-1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x+1/2-1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2)\*((x+1/2+1/2\*I\*3^(1/2))/(3/2+1/2\*I\*3^(1/2)))^(1/2)/(x^3-1)^(1/2)\*3^(1/2)\*EllipticPi(((1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2), 1/3\*(3/2+1/2\*I\*3^(1/2))\*3^(1/2), ((3/2+1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2))-1/18\*2^(1/2)\*sum((-3^(1/2)\*\_alpha-\_alpha-2)/(-3^(1/2)+2\*\_alpha+1)\*(-I\*3^(1/2)-3)\*((-1+x)/(-I\*3^(1/2)-3))^(1/2)\*((2\*x+1-I\*3^(1/2))/(-I\*3^(1/2)+3))^(1/2)\*((2\*x+1+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)\*(1+2\*\_alpha+3^(1/2)\*\_alpha)\*EllipticPi(((1+x)/(-3/2-1/2\*I\*3^(1/2)))^(1/2), 1/2\*I\*\_alpha+1/3\*I\*\_alpha\*3^(1/2)+1/2\*3^(1/2)\*\_alpha+\_alpha+1/6\*I\*3^(1/2)+1/2, ((3/2+1/2\*I\*3^(1/2))/(3/2-1/2\*I\*3^(1/2)))^(1/2)), \_alpha=RootOf(\_Z^2+(1-3^(1/2))\*\_Z-2\*3^(1/2)+4))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x^3+6\*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 6\*sqrt(3) - 10)\*sqrt(x^3 - 1)), x)

**Fricas [B]** time = 41.119, size = 28069, normalized size = 131.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x^3+6\*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{216} \sqrt{3} \sqrt{-4\sqrt{3}\sqrt{56\sqrt{3}+97}} (7\sqrt{3}-12)+24) \cdot (672\sqrt{3}+1164)^{1/4} (56\sqrt{3}+97) (56\sqrt{3}-97) \arctan\left(\frac{-1/64}{8(432\sqrt{3}(97x^{17}+523x^{16}-2171x^{15}-27737x^{14}-136013x^{13}-345761x^{12}-483752x^{11}-26558x^{10}+1051756x^9+1656560x^8+801584x^7-1113424x^6-1680688x^5-911344x^4+536192x^3+535520x^2-2\sqrt{3}(28x^{17}+151x^{16}-626x^{15}-8006x^{14}-39266x^{13}-99812x^{12}-139652x^{11}-7661x^{10}+303610x^9+478214x^8+231392x^7-321412x^6-485176x^5-263080x^4+154784x^3+154592x^2+78464x-36544)+271808x-126592)}{(56\sqrt{3}+97)}+72\sqrt{3}(\sqrt{3}(2340x^{17}+96354x^{16}+84798x^{15}-4817124x^{14}-17052930x^{13}-1941678x^{12}+57963744x^{11}+76603680x^{10}-16678512x^9-139922496x^8-106227360x^7+42453216x^6+113269536x^5+59694624x^4-30025728x^3-26496000x^2-2\sqrt{3}(1351x^{17}+55630x^{16}+48958x^{15}-2781167x^{14}-9845510x^{13}-1121030x^{12}+33465376x^{11}+44227144x^{10}-9629336x^9-80784280x^8-61330384x^7+24510368x^6+65396192x^5+34464704x^4-17335360x^3-15297472x^2-7571584x+3526400)-13114368x+6107904)}{(56\sqrt{3}+97)}-6(97x^{17}+523x^{16}-2171x^{15}-27737x^{14}-136013x^{13}-345761x^{12}-483752x^{11}-26558x^{10}+1051756x^9+1656560x^8+801584x^7-1113424x^6-1680688x^5-911344x^4+536192x^3+535520x^2-2\sqrt{3}(28x^{17}+151x^{16}-626x^{15}-8006x^{14}-39266x^{13}-99812x^{12}-139652x^{11}-7661x^{10}+303610x^9+478214x^8+231392x^7-321412x^6-485176x^5-263080x^4+154784x^3+154592x^2+78464x-36544)+271808x-126592)\sqrt{56\sqrt{3}+97}}{\sqrt{56\sqrt{3}+97}}-\sqrt{1/2}(288\sqrt{3}(627x^{16}+14286x^{15}+39762x^{14}-50142x^{13}-216816x^{12}-112284x^{11}+325707x^{10}+586326x^9-3294x^8-631752x^7-539220x^6+184392x^5+483816x^4+115296x^3-108576x^2-2\sqrt{3}(181x^{16}+4124x^{15}+11478x^{14}-14474x^{13}-62584x^{12}-32412x^{11}+94021x^{10}+169244x^9-954x^8-182368x^7-155648x^6+53232x^5+139664x^4+33280x^3-31344x^2-37024x+11584)-128256x+40128)(56\sqrt{3}+97)+24\sqrt{3}(\sqrt{3}(2340x^{17}+35850x^{16}-106410x^{15}+2064744x^{14}+11945946x^{13}+1710042x^{12}-46293732x^{11}-59161524x^{10}+18480192x^9+122366520x^8+81203856x^7-45222000x^6-100598112x^5-42207168x^4+29609472x^3+22458240x^2-\sqrt{3}(1351x^{17}+20698x^{16}-61436x^{15}+1192081x^{14}+6896998x^{13}+987292x^{12}-26727704x^{11}-34156928x^{10}+10669552x^9+70648352x^8+46883072x^7-26108944x^6-58080352x^5-24368320x^4+17095040x^3+12966272x^2+4724480x-2581504)+8183040x-4471296)(56\sqrt{3}+97)-6(97x^{17}-104x^{16}-20510x^{15}-43181x^{14}+217294x^{13}+691762x^{12}+584800x^{11}-521510x^{10}-1780028x^9-1416580x^8+80528x^7+1518056x^6+1321712x^5+393392x^4-501952x^3-446848x^2-4\sqrt{3}(14x^{17}-15x^{16}-2960x^{15}-6232x^{14}+31362x^{13}+99844x^{12}+84404x^{11}-75267x^{10}-256916x^9-204458x^8+11616x^7+219104x^6+190768x^5+56784x^4-72448x^3-64496x^2-24480x+13376)-169600x+92672)\sqrt{56\sqrt{3}+97}}{\sqrt{56\sqrt{3}+97}}-\sqrt{-4\sqrt{3}\sqrt{56\sqrt{3}+97}}(7\sqrt{3}-12)+24)\cdot((2\sqrt{3}(3691x^{16}-17731x^{15}-951114x^{14}-450359x^{13}+4370159x^{12}-30318522x^{11}-78096668x^{10}-9429316x^9+146877876x^8+197107784x^7-30834152x^6-185125776x^5-132260896x^4+45545344x^3+69517536x^2-\sqrt{3}(2131x^{16}-10237x^{15}-549126x^{14}-260015x^{13}+2523113x^{12}-17504406x^{11}-45089132x^{10}-5444020x^9+84799980x^8+113800232x^7-17802104x^6-106882416x^5-76360864x^4+26295616x^3+40135968x^2+7907648x-5562368)+13696448x-9634304)\sqrt{x^3-1}(56\sqrt{3}+97)-(459x^{16}+1557x^{15}-26415x^{14}+1449954x^{13}+4677912x^{12}-12651948x^{11}-55684800x^{10}-62834256x^9+8526168x^8+105313392x^7+99605088x^6+18897984x^5-42499296x^4-37357632x^3-8256960x^2-\sqrt{3}(265x^{16}+899x^{15}-15249x$

$$\begin{aligned}
& ^{14} + 837130x^{13} + 2700776x^{12} - 7304604x^{11} - 32149640x^{10} - 36277360x^9 \\
& + 4922568x^8 + 60802736x^7 + 57507040x^6 + 10910784x^5 - 24536992x^4 \\
& - 21568448x^3 - 4767168x^2 - 1207168x + 1383424) - 2090880x + 2396160) \\
& * \sqrt{x^3 - 1} * \sqrt{56\sqrt{3} + 97}) * (672\sqrt{3} + 1164)^{(3/4)} + 6 * (\sqrt{3} * (4945x^{15} \\
& + 37473x^{14} - 490698x^{13} - 2249468x^{12} + 474132x^{11} + 8423784x^{10} + 5853520x^9 \\
& - 8451720x^8 - 15320016x^7 - 768064x^6 + 10405056x^5 + 6627744x^4 - 700480x^3 \\
& - 2799552x^2 - \sqrt{3} * (2855x^{15} + 21635x^{14} - 283306x^{13} - 1298732x^{12} \\
& + 273748x^{11} + 4863472x^{10} + 3379536x^9 - 4879608x^8 - 8845008x^7 - 443456x^6 \\
& + 6007360x^5 + 3826528x^4 - 404416x^3 - 1616320x^2 - 1003648x + 399360) - 1738368x + 691712) \\
& * \sqrt{x^3 - 1} * (56\sqrt{3} + 97) - 2 * (246x^{15} + 3678x^{14} - 13485x^{13} - 102933x^{12} \\
& - 70062x^{11} + 81156x^{10} + 45204x^9 + 129636x^8 + 243576x^7 + 221784x^6 - 351024x^5 \\
& - 460896x^4 + 33984x^3 + 174048x^2 - \sqrt{3} * (142x^{15} + 2124x^{14} - 7773x^{13} - 59447x^{12} \\
& - 40626x^{11} + 46860x^{10} + 26308x^9 + 75276x^8 + 140472x^7 + 127784x^6 - 202896x^5 - 266016x^4 + 19712x^3 \\
& + 100512x^2 + 62400x - 24832) + 108096x - 43008) * \sqrt{x^3 - 1} * \sqrt{56\sqrt{3} + 97}) \\
& * (672\sqrt{3} + 1164)^{(1/4)}) - 216 * (130x^{16} + 1682x^{15} + 2496x^{14} - 7730x^{13} + 1790x^{12} \\
& + 35700x^{11} - 7100x^{10} - 86080x^9 - 49176x^8 + 100400x^7 + 108208x^6 - 33312x^5 - 80704x^4 - 18944x^3 \\
& + 18048x^2 - 3\sqrt{3} * (25x^{16} + 324x^{15} + 489x^{14} - 1482x^{13} + 316x^{12} + 6984x^{11} - 1312x^{10} \\
& - 16624x^9 - 9792x^8 + 19328x^7 + 20976x^6 - 6240x^5 - 15552x^4 - 3712x^3 + 3456x^2 \\
& + 4096x - 1280) + 21248x - 6656) * \sqrt{56\sqrt{3} + 97}) * \sqrt{(18x^8 - 36x^7 + 828x^6 - 360x^5 \\
& + 720x^4 - 1008x^3 - 144x^2 + 72\sqrt{3} * (26x^7 - 38x^6 + 42x^5 - 46x^4 + 46x^3 - 42x^2 - \sqrt{3} * (15x^7 \\
& - 22x^6 + 24x^5 - 27x^4 + 26x^3 - 24x^2 + 12x - 4) + 20x - 8) * \sqrt{56\sqrt{3} + 97}) + (\sqrt{3} * (123x^6 \\
& - 2016x^5 + 2214x^4 - 2064x^3 + 396x^2 - \sqrt{3} * (71x^6 - 1164x^5 + 1278x^4 - 1192x^3 + 228x^2 \\
& - 112) - 192) * \sqrt{x^3 - 1} * \sqrt{56\sqrt{3} + 97}) - 6 * (5x^6 - 27x^5 + 48x^4 - 58x^3 + 36x^2 \\
& - 3\sqrt{3} * (x^6 - 5x^5 + 10x^4 - 10x^3 + 8x^2 - 4x) - 12x + 8) * \sqrt{x^3 - 1}) * \sqrt{-4\sqrt{3} * \sqrt{56\sqrt{3} + 97}) \\
& * (7\sqrt{3} - 12) + 24) * (672\sqrt{3} + 1164)^{(1/4)} + 72\sqrt{3} * (x^7 - 4x^6 + 6x^5 - 5x^4 - 4x^3 - 6x^2 \\
& + 4x + 8) + 288x + 1152) / (x^8 + 4x^7 + 16x^6 + 16x^5 + 28x^4 - 32x^3 + 64x^2 - 32x + 16)) - 3 \\
& * \sqrt{-4\sqrt{3} * \sqrt{56\sqrt{3} + 97}) * (7\sqrt{3} - 12) + 24) * ((2\sqrt{3} * (3691x^{16} + 6128x^{15} - 537864x^{14} \\
& - 1586477x^{13} + 16210952x^{12} + 77181756x^{11} + 84218362x^{10} - 71018320x^9 - 254455812x^8 - 196076008x^7 + 120105208x^6 \\
& + 256326864x^5 + 134645168x^4 - 78464672x^3 - 78514944x^2 - \sqrt{3} * (2131x^{16} + 3538x^{15} - 310536x^{14} \\
& - 915953x^{13} + 9359398x^{12} + 44560908x^{11} + 48623494x^{10} - 41002448x^9 - 146910132x^8 - 113204536x^7 \\
& + 69342776x^6 + 147990384x^5 + 77737424x^4 - 45301600x^3 - 45330624x^2 - 12242560x + 7598336) \\
& - 21204736x + 13160704) * \sqrt{x^3 - 1} * (56\sqrt{3} + 97) - (459x^{16} + 13425x^{15} - 33201x^{14} \\
& - 950652x^{13} - 997302x^{12} + 14760972x^{11} + 47069892x^{10} + 49762248x^9 - 8212536x^8 - 84377808x^7 \\
& - 88427328x^6 - 25613856x^5 + 27458496x^4 + 36433344x^3 + 12609792x^2 - \sqrt{3} * (265x^{16} + 7751x^{15} \\
& - 19167x^{14} - 548864x^{13} - 575818x^{12} + 8522268x^{11} + 27175852x^{10} + 28730312x^9 - 4741560x^8 - 48715600x^7 \\
& - 51053600x^6 - 14788128x^5 + 15853184x^4 + 21034816x^3 + 7280256x^2 + 2488832x - 1889792) \\
& + 4310784x - 3273216) * \sqrt{x^3 - 1} * \sqrt{56\sqrt{3} + 97}) * (672\sqrt{3} + 1164)^{(3/4)} + 6 * (\sqrt{3} * (4945x^{15} \\
& + 88617x^{14} + 738528x^{13} + 1860046x^{12} - 784596x^{11} - 7668708x^{10} - 6570680x^9 + 6903864x^8 \\
& + 15444144x^7 + 4312832x^6 - 9559200x^5 - 9359808x^4 - 155968x^3 + 3016704x^2 - \sqrt{3} * (2855x^{15} \\
& + 51163x^{14} + 426388x^{13} + 1073898x^{12} - 452980x^{11} - 4427548x^{10} - 3793592x^9 + 3985944x^8 + 8916720x^7 + 2490016x^6 \\
& - 5519008x^5 - 5403904x^4 - 90048x^3 + 1741696x^2 + 1543936x - 545536) + 2674176x - 944896) * \sqrt{x^3 - 1} * (56\sqrt{3} + 97) - 2 * (246x^{15} \\
& + 7653x^{14} + 41169x^{13} + 51342x^{12} - 72300x^{11} - 45930x^{10} + 221688x^9 + 17892x^8 - 490248x^7 \\
& - 462360x^6 + 389616x^5 + 619728x^4 + 16608x^3 - 187584x^2 - \sqrt{3} * (142x^{15} + 4419x^{14} + 23781x^{13} + 29608x^{12} \\
& - 41940x^{11} - 26454x^{10} + 128152x^9 + 10692x^8 - 283320x^7 - 2
\end{aligned}$$



$$\begin{aligned}
& 67064x^6 + 224784x^5 + 357936x^4 + 9632x^3 - 108288x^2 - 96000x + 339 \\
& 20) - 166272x + 58752) \cdot \sqrt{x^3 - 1} \cdot \sqrt{(56\sqrt{3} + 97)} \cdot (672\sqrt{3} + \\
& 1164)^{1/4}) - 216 \cdot (12x^{17} + 498x^{16} + 462x^{15} - 24972x^{14} - 88530x^{13} \\
& - 9726x^{12} + 300000x^{11} + 396768x^{10} - 87216x^9 - 723072x^8 - 549408 \\
& *x^7 + 220128x^6 + 584736x^5 + 308256x^4 - 155136x^3 - 136704x^2 - \sqrt{3} \\
& * (7x^{17} + 286x^{16} + 238x^{15} - 14255x^{14} - 50390x^{13} - 5942x^{12} + \\
& 171808x^{11} + 226888x^{10} - 48920x^9 - 415384x^8 - 315088x^7 + 125600x^6 \\
& + 336608x^5 + 177344x^4 - 89152x^3 - 78784x^2 - 39040x + 18176) - 67 \\
& 584x + 31488) \cdot \sqrt{(56\sqrt{3} + 97)} / (x^{17} - 13x^{16} - 522x^{15} - 1742x^{14} \\
& + 3008x^{13} + 16884x^{12} + 11656x^{11} - 23944x^{10} - 42336x^9 - 9136x^8 \\
& + 36256x^7 + 27360x^6 - 256x^5 - 13376x^4 - 5760x^3 + 1664x^2 + 256x) \\
& + 1/216 \cdot \sqrt{3} \cdot \sqrt{-4\sqrt{3}} \cdot \sqrt{(56\sqrt{3} + 97)} \cdot (7\sqrt{3} - 12) \\
& + 24) \cdot (672\sqrt{3} + 1164)^{1/4} \cdot (56\sqrt{3} + 97) \cdot (56\sqrt{3} - 97) \cdot \arctan \\
& (1/648 \cdot (432\sqrt{3}) \cdot (97x^{17} + 523x^{16} - 2171x^{15} - 27737x^{14} - 136013x^{13} \\
& - 345761x^{12} - 483752x^{11} - 26558x^{10} + 1051756x^9 + 1656560x^8 + \\
& 801584x^7 - 1113424x^6 - 1680688x^5 - 911344x^4 + 536192x^3 + 535520x^2 \\
& - 2\sqrt{3}) \cdot (28x^{17} + 151x^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 99 \\
& 812x^{12} - 139652x^{11} - 7661x^{10} + 303610x^9 + 478214x^8 + 231392x^7 - \\
& 321412x^6 - 485176x^5 - 263080x^4 + 154784x^3 + 154592x^2 + 78464x - \\
& 36544) + 271808x - 126592) \cdot (56\sqrt{3} + 97) + 72\sqrt{3} \cdot (\sqrt{3}) \cdot (2340x^{17} \\
& + 96354x^{16} + 84798x^{15} - 4817124x^{14} - 17052930x^{13} - 1941678x^{12} \\
& + 57963744x^{11} + 76603680x^{10} - 16678512x^9 - 139922496x^8 - 10622736 \\
& 0x^7 + 42453216x^6 + 113269536x^5 + 59694624x^4 - 30025728x^3 - 264960 \\
& 00x^2 - \sqrt{3}) \cdot (1351x^{17} + 55630x^{16} + 48958x^{15} - 2781167x^{14} - 9845 \\
& 510x^{13} - 1121030x^{12} + 33465376x^{11} + 44227144x^{10} - 9629336x^9 - 807 \\
& 84280x^8 - 61330384x^7 + 24510368x^6 + 65396192x^5 + 34464704x^4 - 173 \\
& 35360x^3 - 15297472x^2 - 7571584x + 3526400) - 13114368x + 6107904) \cdot (56 \\
& * \sqrt{3} + 97) - 6 \cdot (97x^{17} + 523x^{16} - 2171x^{15} - 27737x^{14} - 136013x^{13} \\
& - 345761x^{12} - 483752x^{11} - 26558x^{10} + 1051756x^9 + 1656560x^8 + 8 \\
& 01584x^7 - 1113424x^6 - 1680688x^5 - 911344x^4 + 536192x^3 + 535520x^2 \\
& - 2\sqrt{3}) \cdot (28x^{17} + 151x^{16} - 626x^{15} - 8006x^{14} - 39266x^{13} - 998 \\
& 12x^{12} - 139652x^{11} - 7661x^{10} + 303610x^9 + 478214x^8 + 231392x^7 - \\
& 321412x^6 - 485176x^5 - 263080x^4 + 154784x^3 + 154592x^2 + 78464x - \\
& 36544) + 271808x - 126592) \cdot \sqrt{(56\sqrt{3} + 97)} \cdot \sqrt{(56\sqrt{3} + 97)} - \\
& \sqrt{1/2} \cdot (288\sqrt{3}) \cdot (627x^{16} + 14286x^{15} + 39762x^{14} - 50142x^{13} - 2 \\
& 16816x^{12} - 112284x^{11} + 325707x^{10} + 586326x^9 - 3294x^8 - 631752x^7 \\
& - 539220x^6 + 184392x^5 + 483816x^4 + 115296x^3 - 108576x^2 - 2\sqrt{3}) \cdot \\
& (181x^{16} + 4124x^{15} + 11478x^{14} - 14474x^{13} - 62584x^{12} - 32412x^{11} \\
& + 94021x^{10} + 169244x^9 - 954x^8 - 182368x^7 - 155648x^6 + 53232x^5 \\
& + 139664x^4 + 33280x^3 - 31344x^2 - 37024x + 11584) - 128256x + 40128) \\
& \cdot (56\sqrt{3} + 97) + 24\sqrt{3} \cdot (\sqrt{3}) \cdot (2340x^{17} + 35850x^{16} - 106410x^{15} \\
& + 2064744x^{14} + 11945946x^{13} + 1710042x^{12} - 46293732x^{11} - 591615 \\
& 24x^{10} + 18480192x^9 + 122366520x^8 + 81203856x^7 - 45222000x^6 - 1005 \\
& 98112x^5 - 42207168x^4 + 29609472x^3 + 22458240x^2 - \sqrt{3}) \cdot (1351x^{17} \\
& + 20698x^{16} - 61436x^{15} + 1192081x^{14} + 6896998x^{13} + 987292x^{12} - 26 \\
& 727704x^{11} - 34156928x^{10} + 10669552x^9 + 70648352x^8 + 46883072x^7 - \\
& 26108944x^6 - 58080352x^5 - 24368320x^4 + 17095040x^3 + 12966272x^2 + \\
& 4724480x - 2581504) + 8183040x - 4471296) \cdot (56\sqrt{3} + 97) - 6 \cdot (97x^{17} \\
& - 104x^{16} - 20510x^{15} - 43181x^{14} + 217294x^{13} + 691762x^{12} + 584800x^{11} \\
& - 521510x^{10} - 1780028x^9 - 1416580x^8 + 80528x^7 + 1518056x^6 + 1 \\
& 321712x^5 + 393392x^4 - 501952x^3 - 446848x^2 - 4\sqrt{3}) \cdot (14x^{17} - 15 \\
& *x^{16} - 2960x^{15} - 6232x^{14} + 31362x^{13} + 99844x^{12} + 84404x^{11} - 7526 \\
& 7x^{10} - 256916x^9 - 204458x^8 + 11616x^7 + 219104x^6 + 190768x^5 + 56 \\
& 784x^4 - 72448x^3 - 64496x^2 - 24480x + 13376) - 169600x + 92672) \cdot \sqrt{3} \\
& \cdot \sqrt{(56\sqrt{3} + 97)} \cdot \sqrt{(56\sqrt{3} + 97)} + \sqrt{-4\sqrt{3}} \cdot \sqrt{(56\sqrt{3} + 97)} \\
& + 97) \cdot (7\sqrt{3} - 12) + 24) \cdot ((2\sqrt{3}) \cdot (3691x^{16} - 17731x^{15} - 951114x^{14} \\
& - 450359x^{13} + 4370159x^{12} - 30318522x^{11} - 78096668x^{10} - 9429316x^9 \\
& + 146877876x^8 + 197107784x^7 - 30834152x^6 - 185125776x^5 - 132260 \\
& 896x^4 + 45545344x^3 + 69517536x^2 - \sqrt{3}) \cdot (2131x^{16} - 10237x^{15} - 5
\end{aligned}$$

$$\begin{aligned}
& 49126x^{14} - 260015x^{13} + 2523113x^{12} - 17504406x^{11} - 45089132x^{10} - 5 \\
& 444020x^9 + 84799980x^8 + 113800232x^7 - 17802104x^6 - 106882416x^5 - \\
& 76360864x^4 + 26295616x^3 + 40135968x^2 + 7907648x - 5562368) + 1369644 \\
& 8x - 9634304) \sqrt{x^3 - 1} (56\sqrt{3} + 97) - (459x^{16} + 1557x^{15} - 26 \\
& 415x^{14} + 1449954x^{13} + 4677912x^{12} - 12651948x^{11} - 55684800x^{10} - 62 \\
& 834256x^9 + 8526168x^8 + 105313392x^7 + 99605088x^6 + 18897984x^5 - 42 \\
& 499296x^4 - 37357632x^3 - 8256960x^2 - \sqrt{3}(265x^{16} + 899x^{15} - 15 \\
& 249x^{14} + 837130x^{13} + 2700776x^{12} - 7304604x^{11} - 32149640x^{10} - 3627 \\
& 7360x^9 + 4922568x^8 + 60802736x^7 + 57507040x^6 + 10910784x^5 - 24536 \\
& 992x^4 - 21568448x^3 - 4767168x^2 - 1207168x + 1383424) - 2090880x + 2 \\
& 396160) \sqrt{x^3 - 1} \sqrt{56\sqrt{3} + 97} (672\sqrt{3} + 1164)^{3/4} + 6 \\
& (\sqrt{3}(4945x^{15} + 37473x^{14} - 490698x^{13} - 2249468x^{12} + 474132x^{11} \\
& 1 + 8423784x^{10} + 5853520x^9 - 8451720x^8 - 15320016x^7 - 768064x^6 + \\
& 10405056x^5 + 6627744x^4 - 700480x^3 - 2799552x^2 - \sqrt{3}(2855x^{15} \\
& + 21635x^{14} - 283306x^{13} - 1298732x^{12} + 273748x^{11} + 4863472x^{10} + 33 \\
& 79536x^9 - 4879608x^8 - 8845008x^7 - 443456x^6 + 6007360x^5 + 3826528x \\
& x^4 - 404416x^3 - 1616320x^2 - 1003648x + 399360) - 1738368x + 691712) * \\
& \sqrt{x^3 - 1} (56\sqrt{3} + 97) - 2(246x^{15} + 3678x^{14} - 13485x^{13} - 10 \\
& 2933x^{12} - 70062x^{11} + 81156x^{10} + 45204x^9 + 129636x^8 + 243576x^7 + \\
& 221784x^6 - 351024x^5 - 460896x^4 + 33984x^3 + 174048x^2 - \sqrt{3}(1 \\
& 42x^{15} + 2124x^{14} - 7773x^{13} - 59447x^{12} - 40626x^{11} + 46860x^{10} + 26 \\
& 308x^9 + 75276x^8 + 140472x^7 + 127784x^6 - 202896x^5 - 266016x^4 + 1 \\
& 9712x^3 + 100512x^2 + 62400x - 24832) + 108096x - 43008) \sqrt{x^3 - 1} * \\
& \sqrt{56\sqrt{3} + 97} (672\sqrt{3} + 1164)^{1/4} - 216(130x^{16} + 1682x^{15} \\
& + 2496x^{14} - 7730x^{13} + 1790x^{12} + 35700x^{11} - 7100x^{10} - 86080x^9 \\
& - 49176x^8 + 100400x^7 + 108208x^6 - 33312x^5 - 80704x^4 - 18944x^3 \\
& + 18048x^2 - 3\sqrt{3}(25x^{16} + 324x^{15} + 489x^{14} - 1482x^{13} + 316x^{12} \\
& + 6984x^{11} - 1312x^{10} - 16624x^9 - 9792x^8 + 19328x^7 + 20976x^6 \\
& - 6240x^5 - 15552x^4 - 3712x^3 + 3456x^2 + 4096x - 1280) + 21248x - 6 \\
& 656) \sqrt{56\sqrt{3} + 97} \sqrt{(18x^8 - 36x^7 + 828x^6 - 360x^5 + 720 \\
& x^4 - 1008x^3 - 144x^2 + 72\sqrt{3}(26x^7 - 38x^6 + 42x^5 - 46x^4 + \\
& 46x^3 - 42x^2 - \sqrt{3}(15x^7 - 22x^6 + 24x^5 - 27x^4 + 26x^3 - 24 \\
& x^2 + 12x - 4) + 20x - 8) \sqrt{56\sqrt{3} + 97} - (\sqrt{3}(123x^6 - 20 \\
& 16x^5 + 2214x^4 - 2064x^3 + 396x^2 - \sqrt{3}(71x^6 - 1164x^5 + 1278x \\
& x^4 - 1192x^3 + 228x^2 - 112) - 192) \sqrt{x^3 - 1} \sqrt{56\sqrt{3} + 97} \\
& - 6(5x^6 - 27x^5 + 48x^4 - 58x^3 + 36x^2 - 3\sqrt{3}(x^6 - 5x^5 + 1 \\
& 0x^4 - 10x^3 + 8x^2 - 4x) - 12x + 8) \sqrt{x^3 - 1} \sqrt{-4\sqrt{3} \sqrt{56\sqrt{3} + 97} \\
& (7\sqrt{3} - 12) + 24) (672\sqrt{3} + 1164)^{1/4} + 72\sqrt{3}(x^7 - 4x^6 + 6x^5 - 5x^4 - 4x^3 - 6x^2 + 4x + 8) \\
& + 288x + 152) / (x^8 + 4x^7 + 16x^6 + 16x^5 + 28x^4 - 32x^3 + 64x^2 - 32x + 16) \\
& ) + 3\sqrt{-4\sqrt{3} \sqrt{56\sqrt{3} + 97} (7\sqrt{3} - 12) + 24} ((2\sqrt{3} \\
& (3691x^{16} + 6128x^{15} - 537864x^{14} - 1586477x^{13} + 16210952x^{12} + 7 \\
& 7181756x^{11} + 84218362x^{10} - 71018320x^9 - 254455812x^8 - 196076008x^7 \\
& + 120105208x^6 + 256326864x^5 + 134645168x^4 - 78464672x^3 - 78514944x^2 \\
& - \sqrt{3}(2131x^{16} + 3538x^{15} - 310536x^{14} - 915953x^{13} + 9359398x^{12} \\
& + 44560908x^{11} + 48623494x^{10} - 41002448x^9 - 146910132x^8 - 11320 \\
& 4536x^7 + 69342776x^6 + 147990384x^5 + 77737424x^4 - 45301600x^3 - 453 \\
& 30624x^2 - 12242560x + 7598336) - 21204736x + 13160704) \sqrt{x^3 - 1} (5 \\
& 6\sqrt{3} + 97) - (459x^{16} + 13425x^{15} - 33201x^{14} - 950652x^{13} - 99730 \\
& 2x^{12} + 14760972x^{11} + 47069892x^{10} + 49762248x^9 - 8212536x^8 - 84377 \\
& 808x^7 - 88427328x^6 - 25613856x^5 + 27458496x^4 + 36433344x^3 + 12609 \\
& 792x^2 - \sqrt{3}(265x^{16} + 7751x^{15} - 19167x^{14} - 548864x^{13} - 575818 \\
& x^{12} + 8522268x^{11} + 27175852x^{10} + 28730312x^9 - 4741560x^8 - 4871560 \\
& 0x^7 - 51053600x^6 - 14788128x^5 + 15853184x^4 + 21034816x^3 + 7280256 \\
& x^2 + 2488832x - 1889792) + 4310784x - 3273216) \sqrt{x^3 - 1} \sqrt{56\sqrt{3} + 97} \\
& (672\sqrt{3} + 1164)^{3/4} + 6(\sqrt{3}(4945x^{15} + 88617x^{14} + 738528x^{13} + 1860046x^{12} \\
& - 784596x^{11} - 7668708x^{10} - 6570680x^9 + 6903864x^8 + 15444144x^7 + 4312832x^6 \\
& - 9559200x^5 - 9359808x^4 - 155968x^3 + 3016704x^2 - \sqrt{3}(2855x^{15} + 51163x^{14} + 426388x^{13} + 107
\end{aligned}$$

```

3898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 891672
0*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 +
1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97)
- 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^
10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728
*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13
+ 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^
7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x
+ 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt
(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 8853
0*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 5
49408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2
- sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^
12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 1256
00*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176)
- 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 174
2*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 913
6*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 +
256*x) + 1/2592*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt
(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 116
4)^(1/4)*log(1/18*(18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3
- 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^
2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4
) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x
^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3
+ 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27
*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3
+ 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) +
97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 -
4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/2592*(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt(-4*sqrt(3)*sqrt(56*sqr
t(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*log(1/18*(18*
x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3
)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 -
22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*
sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^
2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192
)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3
+ 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x
+ 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/36*sqrt(14*sqrt(3) + 24)*arctan
(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8) + 18*x - 12)*sqrt(14*sqrt(3) + 24)
/sqrt(x^3 - 1))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)(x^3-10+6\sqrt{3})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x\*\*3+6\*3\*\*(1/2))/(x\*\*3-1)\*\*(1/2),x)

[Out] Integral(x/(sqrt((x - 1)\*(x\*\*2 + x + 1))\*(x\*\*3 - 10 + 6\*sqrt(3))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x^3+6\*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(x/((x^3 + 6\*sqrt(3) - 10)\*sqrt(x^3 - 1)), x)

$$3.90 \quad \int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-4 + 4\sqrt{3}x^2 + x^4}} dx$$

**Optimal.** Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x - \sqrt{3} + 1)^2}{\sqrt{3}(2\sqrt{3}-3)\sqrt{x^4 + 4\sqrt{3}x^2 - 4}}\right)$$

[Out] (Sqrt[-3 + 2\*Sqrt[3]]\*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3\*(-3 + 2\*Sqrt[3]])\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4])])/3

**Rubi [A]** time = 0.128968, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x - \sqrt{3} + 1)^2}{\sqrt{3}(2\sqrt{3}-3)\sqrt{x^4 + 4\sqrt{3}x^2 - 4}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]), x]

[Out] (Sqrt[-3 + 2\*Sqrt[3]]\*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3\*(-3 + 2\*Sqrt[3]])\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4])])/3

#### Rule 1740

Int[((A\_) + (B\_.)\*(x\_))/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4]), x\_Symbol] :> -Dist[(A^2\*(B\*d + A\*e))/e, Subst[Int[1/(6\*A^3\*B\*d + 3\*A^4\*e - a\*e\*x^2), x], x, (A + B\*x)^2/Sqrt[a + b\*x^2 + c\*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B\*d - A\*e, 0] && EqQ[c^2\*d^6 + a\*e^4\*(13\*c\*d^2 + b\*e^2), 0] && EqQ[b^2\*e^4 - 12\*c\*d^2\*(c\*d^2 - b\*e^2), 0] && EqQ[4\*A\*c\*d\*e + B\*(2\*c\*d^2 - b\*e^2), 0]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-4 + 4\sqrt{3}x^2 + x^4}} dx &= -\left(4(2 - \sqrt{3})\right) \text{Subst} \left[ \int \frac{1}{3(1 - \sqrt{3})^4 + 6(1 - \sqrt{3})^3(1 + \sqrt{3}) + 4x^2} dx, x, \right. \\ &= \left. \frac{1}{3}\sqrt{-3 + 2\sqrt{3}} \tanh^{-1}\left(\frac{(1 - \sqrt{3} + x)^2}{\sqrt{3}(-3 + 2\sqrt{3})\sqrt{-4 + 4\sqrt{3}x^2 + x^4}}\right) \right] \end{aligned}$$

**Mathematica [C]** time = 3.20489, size = 685, normalized size = 10.54

$$(x + \sqrt{3} - 1)^2 \sqrt{-x^3 + (\sqrt{3} - 1)x^2 - 2(2 + \sqrt{3})x + 2(1 + \sqrt{3})} \sqrt{\frac{-\frac{4}{x + \sqrt{3} - 1} + \sqrt{3} + 1}{3 + \sqrt{3} + i\sqrt{2(2 + \sqrt{3})}}} \left( \frac{2(2i\sqrt{3} - \sqrt{2(2 + \sqrt{3})} + \sqrt{6(2 + \sqrt{3})})}{x + \sqrt{3} - 1} + i(-1 + \sqrt{3}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]),x]

[Out] ((-1 + Sqrt[3] + x)^2\*Sqrt[2\*(1 + Sqrt[3]) - 2\*(2 + Sqrt[3])\*x + (-1 + Sqrt[3])\*x^2 - x^3]\*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]])\*(I\*(-1 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]) + (2\*(2\*I)\*Sqrt[3] - Sqrt[2\*(2 + Sqrt[3])]) + Sqrt[6\*(2 + Sqrt[3])])]/(-1 + Sqrt[3] + x)\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] + I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]\*EllipticF[ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])]) + 2\*Sqrt[6]\*Sqrt[(4 + 2\*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]\*EllipticPi[(2\*Sqrt[2\*(2 + Sqrt[3])])/(Sqrt[2\*(2 + Sqrt[3])] + I\*(3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[2\*(2 + Sqrt[3])]] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)\*(2 + Sqrt[3])^(1/4))], ((2\*I)\*Sqrt[2\*(2 + Sqrt[3])])/(3 + Sqrt[3] + I\*Sqrt[2\*(2 + Sqrt[3])])])/(Sqrt[2\*(2 + Sqrt[3])] + I\*(3 + Sqrt[3]))\*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])\*x + ((-1 + Sqrt[3])\*x^2)/2 - x^3/2]\*Sqrt[-4 + 4\*Sqrt[3]\*x^2 + x^4]\*Sqrt[Sqrt[2\*(2 + Sqrt[3])] - I\*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])

**Maple [C]** time = 0.132, size = 327, normalized size = 5.

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2}\sqrt{3} - \frac{i}{2}\right), i\sqrt{1 + 4\sqrt{3}(1 + 1/2\sqrt{3})}\right)}{\frac{i}{2}\sqrt{3} - \frac{i}{2}} \sqrt{1 - \left(-1 + \frac{\sqrt{3}}{2}\right)x^2} \sqrt{1 - \left(1 + \frac{\sqrt{3}}{2}\right)x^2} \frac{1}{\sqrt{-4 + x^4 + 4\sqrt{3}x^2}} - 2\sqrt{3} \left( - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2),x)

[Out] 1/(1/2\*I\*3^(1/2)-1/2\*I)\*(1-(-1+1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(1+1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2)\*EllipticF(x\*(1/2\*I\*3^(1/2)-1/2\*I), I\*(1+4\*3^(1/2)\*(1+1/2\*3^(1/2)))^(1/2))-2\*3^(1/2)\*(-1/2/((-1-3^(1/2))^4+4\*3^(1/2)\*(-1-3^(1/2))^2-4)^(1/2)\*arctanh(1/2\*(4\*3^(1/2)\*(-1-3^(1/2))^2-8+4\*3^(1/2))\*x^2+2\*x^2\*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4\*3^(1/2)\*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2))-1/(-1+1/2\*3^(1/2))^(1/2)/(-1-3^(1/2))\*(1-(-1+1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(1+1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4+4\*3^(1/2)\*x^2)^(1/2)\*EllipticPi((-1+1/2\*3^(1/2))^(1/2)\*x, 1/(-1+1/2\*3^(1/2))/(-1-3^(1/2))^2, (1+1/2\*3^(1/2))^(1/2)/(-1+1/2\*3^(1/2))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4(x + \sqrt{3} + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x)
```

**Fricas [B]** time = 3.4669, size = 953, normalized size = 14.66

$$\frac{1}{12} \sqrt{2\sqrt{3}-3} \log \left( -\frac{37x^{12} - 204x^{11} + 804x^{10} - 2408x^9 + 3708x^8 - 5472x^7 + 6432x^6 + 10944x^5 + 14832x^4 + 19264x^3 + 12864x^2 + (54x^{10} - 300x^9 + 1026x^8 - 2232x^7 + 3024x^6 - 3024x^5 - 1008x^4 - 2016x^3 - 2592x^2 + \sqrt{3})(31x^{10} - 176x^9 + 576x^8 - 1320x^7 + 1848x^6 - 1008x^5 + 1344x^4 + 1632x^3 + 1008x^2 + 832x + 256) - 1152x - 480}{(x^4 + 4\sqrt{3}x^2 - 4)\sqrt{2\sqrt{3}-3} + 3\sqrt{3}(7x^{12} - 40x^{11} + 160x^{10} - 400x^9 + 924x^8 - 960x^7 - 1920x^5 - 3696x^4 - 3200x^3 - 2560x^2 - 1280x - 448) + 6528x + 2368} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 1152*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320*x^3 + 768*x^2 - 384*x + 64))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{(x + 1 + \sqrt{3}) \sqrt{x^4 + 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 - 4)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x)
```



$$3.91 \quad \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx$$

**Optimal.** Leaf size=63

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

[Out]  $-(\text{Sqrt}[3 + 2*\text{Sqrt}[3]]*\text{ArcTan}[(1 + \text{Sqrt}[3] + x)^2/(\text{Sqrt}[3*(3 + 2*\text{Sqrt}[3])]*\text{Sqrt}[-4 - 4*\text{Sqrt}[3]*x^2 + x^4])])/3$

**Rubi [A]** time = 0.12776, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(x + \sqrt{3} + 1)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + \text{Sqrt}[3] + x)/((1 - \text{Sqrt}[3] + x)*\text{Sqrt}[-4 - 4*\text{Sqrt}[3]*x^2 + x^4]), x]$

[Out]  $-(\text{Sqrt}[3 + 2*\text{Sqrt}[3]]*\text{ArcTan}[(1 + \text{Sqrt}[3] + x)^2/(\text{Sqrt}[3*(3 + 2*\text{Sqrt}[3])]*\text{Sqrt}[-4 - 4*\text{Sqrt}[3]*x^2 + x^4])])/3$

#### Rule 1740

$\text{Int}[(A + (B \cdot x))/((d + (e \cdot x))\sqrt{a + (b \cdot x)^2 + c \cdot x^4}), x\_Symbol] \rightarrow -\text{Dist}[(A^2 \cdot (B \cdot d + A \cdot e))/e, \text{Subst}[\text{Int}[1/(6 \cdot A^3 \cdot B \cdot d + 3 \cdot A^4 \cdot e - a \cdot e \cdot x^2), x], x, (A + B \cdot x)^2/\text{Sqrt}[a + b \cdot x^2 + c \cdot x^4]], x] /;$  FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B\*d - A\*e, 0] && EqQ[c^2\*d^6 + a\*e^4\*(13\*c\*d^2 + b\*e^2), 0] && EqQ[b^2\*e^4 - 12\*c\*d^2\*(c\*d^2 - b\*e^2), 0] && EqQ[4\*A\*c\*d\*e + B\*(2\*c\*d^2 - b\*e^2), 0]

#### Rule 203

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} dx &= - \left( (4(2 + \sqrt{3})) \text{Subst} \left[ \int \frac{1}{6(1 - \sqrt{3})(1 + \sqrt{3})^3 + 3(1 + \sqrt{3})^4 + 4x^2} dx, x, \right. \right. \\ & \left. \left. = -\frac{1}{3}\sqrt{3 + 2\sqrt{3}} \tan^{-1} \left( \frac{(1 + \sqrt{3} + x)^2}{\sqrt{3(3 + 2\sqrt{3})}\sqrt{-4 - 4\sqrt{3}x^2 + x^4}} \right) \right] \right) \end{aligned}$$

**Mathematica [C]** time = 7.54576, size = 876, normalized size = 13.9

$$\sqrt{2} \sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}} (-x+\sqrt{3}+1)^2 \left( \frac{2 \left( 2i\sqrt{3}\sqrt{i(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1})+\sqrt{4-2\sqrt{3}+\sqrt{6}}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}+i\sqrt{3}-i+\frac{8i(-2+\sqrt{3})}{-x+\sqrt{3}+1}}+\sqrt{-\frac{2i((-1+\sqrt{3})x-8)}{-x+\sqrt{3}+1}}} \right)}{x-\sqrt{3}-1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)\*Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4]), x]

[Out] -((Sqrt[2]\*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I\*Sqrt[4 - 2\*Sqrt[3]])]\*(1 + Sqrt[3] - x)^2\*((I\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((2\*I)\*(14 - 8\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - x)] + (2\*((2\*I)\*Sqrt[3]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]]) + I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[6]\*Sqrt[-I + I\*Sqrt[3] - Sqrt[12 - 6\*Sqrt[3]] + 2\*Sqrt[4 - 2\*Sqrt[3]] + ((8\*I)\*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2\*Sqrt[12 - 6\*Sqrt[3]] + 4\*Sqrt[4 - 2\*Sqrt[3]] - ((2\*I)\*(14 - 8\*Sqrt[3] + (-1 + Sqrt[3])\*x))/(1 + Sqrt[3] - x)))/(-1 - Sqrt[3] + x))\*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3]))] + 2\*Sqrt[6]\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]\*Sqrt[(4 - 2\*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]\*EllipticPi[(2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)\*(2 - Sqrt[3])^(1/4))], (2\*Sqrt[4 - 2\*Sqrt[3]])/(Sqrt[4 - 2\*Sqrt[3]] + I\*(-3 + Sqrt[3])))]))/((Sqrt[4 - 2\*Sqrt[3]] - I\*(-3 + Sqrt[3]))\*Sqrt[Sqrt[4 - 2\*Sqrt[3]] - I\*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]\*Sqrt[-4 - 4\*Sqrt[3]\*x^2 + x^4]))

**Maple [C]** time = 0.129, size = 311, normalized size = 4.9

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2} + \frac{i}{2}\sqrt{3}\right), i\sqrt{1 - 4\sqrt{3}\left(1 - \frac{1}{2}\sqrt{3}\right)}\right)}{\frac{i}{2} + \frac{i}{2}\sqrt{3}} \sqrt{1 - \left(-1 - \frac{\sqrt{3}}{2}\right)x^2} \sqrt{1 - \left(1 - \frac{\sqrt{3}}{2}\right)x^2} \frac{1}{\sqrt{-4 + x^4 - 4\sqrt{3}x^2}} + 2\sqrt{3} \left(-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2), x)

[Out] 1/(1/2\*I+1/2\*I\*3^(1/2))\*(1-(-1-1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(1-1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2)\*EllipticF(x\*(1/2\*I+1/2\*I\*3^(1/2)), I\*(1-4\*3^(1/2)\*(1-1/2\*3^(1/2)))^(1/2))+2\*3^(1/2)\*(-1/2/((3^(1/2)-1)^4-4\*3^(1/2)\*(3^(1/2)-1)^2-4)^(1/2)\*arctanh(1/2\*(-4\*3^(1/2)\*(3^(1/2)-1)^2-8-4\*3^(1/2))\*x^2+2\*x^2\*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4\*3^(1/2)\*(3^(1/2)-1)^2-4)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2))-1/(-1-1/2\*3^(1/2))^(1/2)/(3^(1/2)-1)\*(1-(1-1/2\*3^(1/2))\*x^2)^(1/2)\*(1-(1-1/2\*3^(1/2))\*x^2)^(1/2)/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2)\*EllipticPi((-1-1/2\*3^(1/2))^(1/2)\*x, 1/(-1-1/2\*3^(1/2))/(3^(1/2)-1)^2, (1-1/2\*3^(1/2))^(1/2)/(-1-1/2\*3^(1/2))^(1/2))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4\*sqrt(3)\*x^2 - 4)\*(x - sqrt(3) + 1)), x)

---

**Fricas [B]** time = 2.83386, size = 301, normalized size = 4.78

$$\frac{1}{6} \sqrt{2\sqrt{3} + 3} \arctan \left( -\frac{(9x^4 - 30x^3 + 18x^2 - 2\sqrt{3}(2x^4 - 10x^3 + 3x^2 - 10x + 2) + 24)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}\sqrt{2\sqrt{3} + 3}}{11x^6 - 42x^5 + 66x^4 - 176x^3 - 132x^2 - 168x - 88} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4\*3^(1/2)\*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6\*sqrt(2\*sqrt(3) + 3)\*arctan(-(9\*x^4 - 30\*x^3 + 18\*x^2 - 2\*sqrt(3)\*(2\*x^4 - 10\*x^3 + 3\*x^2 - 10\*x + 2) + 24)\*sqrt(x^4 - 4\*sqrt(3)\*x^2 - 4)\*sqrt(2\*sqrt(3) + 3)/(11\*x^6 - 42\*x^5 + 66\*x^4 - 176\*x^3 - 132\*x^2 - 168\*x - 88))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1 + \sqrt{3}}{(x - \sqrt{3} + 1)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3\*\*(1/2))/(1+x-3\*\*(1/2))/(-4+x\*\*4-4\*3\*\*(1/2)\*x\*\*2)\*\*(1/2),x)

[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)\*sqrt(x\*\*4 - 4\*sqrt(3)\*x\*\*2 - 4)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x)
```

$$3.92 \quad \int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx$$

**Optimal.** Leaf size=53

$$-\frac{3}{2} \log\left(-\sqrt[3]{x^3+2}+x+2\right) + \sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) + \log(x+1)$$

[Out] Sqrt[3]\*ArcTan[(1+(2\*(2+x))/(2+x^3)^(1/3))/Sqrt[3]]+Log[1+x]-(3\*Log[2+x-(2+x^3)^(1/3)])/2

**Rubi [A]** time = 0.0552897, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2151}

$$-\frac{3}{2} \log\left(-\sqrt[3]{x^3+2}+x+2\right) + \sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1+x)/((1+x)\*(2+x^3)^(1/3)),x]

[Out] Sqrt[3]\*ArcTan[(1+(2\*(2+x))/(2+x^3)^(1/3))/Sqrt[3]]+Log[1+x]-(3\*Log[2+x-(2+x^3)^(1/3)])/2

#### Rule 2151

Int[((e\_)+(f\_.)\*(x\_))/(((c\_)+(d\_.)\*(x\_))\*((a\_)+(b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] :> Simp[(Sqrt[3]\*f\*ArcTan[(1+(2\*Rt[b,3]\*(2\*c+d\*x))/(d\*(a+b\*x^3)^(1/3)))/Sqrt[3]])/(Rt[b,3]\*d),x]+(Simp[(f\*Log[c+d\*x])/(Rt[b,3]\*d),x]-Simp[(3\*f\*Log[Rt[b,3]\*(2\*c+d\*x)-d\*(a+b\*x^3)^(1/3)]]/(2\*Rt[b,3]\*d),x))/; FreeQ[{a,b,c,d,e,f},x]&& EqQ[d\*e+c\*f,0]&& EqQ[2\*b\*c^3-a\*d^3,0]

#### Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx = \sqrt{3} \tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) + \log(1+x) - \frac{3}{2} \log\left(2+x-\sqrt[3]{2+x^3}\right)$$

**Mathematica [F]** time = 0.242957, size = 0, normalized size = 0.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(-1+x)/((1+x)\*(2+x^3)^(1/3)),x]

[Out] Integrate[(-1 + x)/((1 + x)\*(2 + x^3)^(1/3)), x]

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{-1+x}{1+x} \frac{1}{\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)

[Out] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")

[Out] integrate((x - 1)/((x^3 + 2)^(1/3)\*(x + 1)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(1+x)/(x\*\*3+2)\*\*(1/3),x)

[Out] Integral((x - 1)/((x + 1)\*(x\*\*3 + 2)\*\*(1/3)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

$$3.93 \quad \int \frac{1}{(1+x)\sqrt[3]{2+x^3}} dx$$

**Optimal.** Leaf size=108

$$\frac{3}{4} \log\left(-\sqrt[3]{x^3+2}+x+2\right) - \frac{1}{4} \log\left(\sqrt[3]{x^3+2}-x\right) + \frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) - \frac{1}{2} \log(x+1)$$

[Out] ArcTan[(1 + (2\*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2\*Sqrt[3]) - (Sqrt[3]\*ArcTan[(1 + (2\*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3\*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4

**Rubi [A]** time = 0.0925279, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2149, 239, 2151}

$$\frac{3}{4} \log\left(-\sqrt[3]{x^3+2}+x+2\right) - \frac{1}{4} \log\left(\sqrt[3]{x^3+2}-x\right) + \frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) - \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)\*(2 + x^3)^(1/3)),x]

[Out] ArcTan[(1 + (2\*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2\*Sqrt[3]) - (Sqrt[3]\*ArcTan[(1 + (2\*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3\*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4

#### Rule 2149

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] :> Dist[1/(2\*c), Int[1/(a + b\*x^3)^(1/3), x], x] + Dist[1/(2\*c), Int[(c - d\*x)/((c + d\*x)\*(a + b\*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2\*b\*c^3 - a\*d^3, 0]

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] :> Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 2151

Int[((e\_) + (f\_.)\*(x\_))/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] :> Simp[(Sqrt[3]\*f\*ArcTan[(1 + (2\*Rt[b, 3]\*(2\*c + d\*x))/(d\*(a + b\*x^3)^(1/3)))/Sqrt[3]]/(Rt[b, 3]\*d), x] + (Simp[(f\*Log[c + d\*x])/(Rt[b, 3]\*d), x] - Simp[(3\*f\*Log[Rt[b, 3]\*(2\*c + d\*x) - d\*(a + b\*x^3)^(1/3)])/(2\*Rt[b, 3]\*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d\*e + c\*f, 0] && EqQ[2\*b\*c^3 - a\*d^3, 0]

#### Rubi steps



$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}} dx = \frac{1}{2} \int \frac{1}{\sqrt[3]{2+x^3}} dx + \frac{1}{2} \int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}} dx$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2} \log(1+x) + \frac{3}{4} \log\left(2+x-\sqrt[3]{2+x^3}\right) - \frac{1}{4} \log\left(2+x+\sqrt[3]{2+x^3}\right)$$

**Mathematica [F]** time = 0.0572087, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1+x)\*(2+x^3)^(1/3)),x]

[Out] Integrate[1/((1+x)\*(2+x^3)^(1/3)), x]

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \frac{1}{\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(x^3+2)^(1/3),x)

[Out] int(1/(1+x)/(x^3+2)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((x^3+2)^(1/3)\*(x+1)), x)

**Fricas [B]** time = 9.28147, size = 1422, normalized size = 13.17

$$\frac{1}{6} \sqrt{3} \arctan \left( \frac{13910019318573948542 \sqrt{3} (7114781247 x^4 + 13663058416 x^3 - 46178206896 x^2 - 126842559344 x - 126842559344)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
+ sqrt(3)*(93292570833559435663132301885*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 8894265631830874680155802900
48*x^3 + 888876515195959220955879945824*x^2 + 35126059825850824001997196488
0*x - 47674000995597211057816884304))/(78905434814564721745708464883*x^6 +
337746705836458222863347934450*x^5 + 15598952776058587894336070976*x^4 - 89
5430525315100108684787964824*x^3 + 361667862240477028869533375352*x^2 + 254
1802301011632510645972090336*x + 1554815286823334092314485968880)) + 1/12*log((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

$$3.94 \quad \int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}} dx$$

**Optimal.** Leaf size=98

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log(x\sqrt[3]{a+b} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{a+b}} + \frac{\tan^{-1}\left(\frac{2x\sqrt[3]{a+b}+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

[Out] ArcTan[(1 + (2\*(a + b)^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3)) + Log[1 - x^3]/(6\*(a + b)^(1/3)) - Log[(a + b)^(1/3)\*x - (a + b\*x^3)^(1/3)]/(2\*(a + b)^(1/3))

**Rubi [A]** time = 0.093337, antiderivative size = 135, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1 - \frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(\frac{x^2(a+b)^{2/3}}{(a+bx^3)^{2/3}} + \frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}} + 1\right)}{6\sqrt[3]{a+b}} + \frac{\tan^{-1}\left(\frac{2x\sqrt[3]{a+b}+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^3)\*(a + b\*x^3)^(1/3)),x]

[Out] ArcTan[(1 + (2\*(a + b)^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3)) - Log[1 - ((a + b)^(1/3)\*x)/(a + b\*x^3)^(1/3)]/(3\*(a + b)^(1/3)) + Log[1 + ((a + b)^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + ((a + b)^(1/3)\*x)/(a + b\*x^3)^(1/3)]/(6\*(a + b)^(1/3))

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

### Rule 617

`Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

### Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}} dx &= \text{Subst} \left( \int \frac{1}{1-(a+b)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
 &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{1-\sqrt[3]{a+bx}} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) + \frac{1}{3} \text{Subst} \left( \int \frac{2+\sqrt[3]{a+bx}}{1+\sqrt[3]{a+bx}+(a+b)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\
 &= -\frac{\log \left( 1 - \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{a+b}} + \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+\sqrt[3]{a+bx}+(a+b)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) + \frac{\text{Subst} \left( \int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{\sqrt[3]{a+b}} \\
 &= -\frac{\log \left( 1 - \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{a+b}} + \frac{\log \left( 1 + \frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{\sqrt[3]{a+b}} \\
 &= \frac{\tan^{-1} \left( \frac{1 + \frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a+b}} - \frac{\log \left( 1 - \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{3\sqrt[3]{a+b}} + \frac{\log \left( 1 + \frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a+bx^3}} \right)}{6\sqrt[3]{a+b}}
 \end{aligned}$$

**Mathematica [A]** time = 0.155155, size = 120, normalized size = 1.22

$$\frac{-2 \log \left( 1 - \frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}} \right) + \log \left( \frac{x^2(a+b)^{2/3}}{(a+bx^3)^{2/3}} + \frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left( \frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}} + 1 \right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]`

`[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + ((a + b)^(2/3)*x^2)/(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))`

---

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-x^3+1} \frac{1}{\sqrt[3]{bx^3+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)/(b\*x^3+a)^(1/3),x)

[Out] int(1/(-x^3+1)/(b\*x^3+a)^(1/3),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] -integrate(1/((b\*x^3+a)^(1/3)\*(x^3-1)),x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)/(b\*x^3+a)^(1/3),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^3 \sqrt[3]{a+bx^3} - \sqrt[3]{a+bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x\*\*3+1)/(b\*x\*\*3+a)\*\*(1/3),x)

[Out] -Integral(1/(x\*\*3\*(a+b\*x\*\*3)\*\*(1/3)-(a+b\*x\*\*3)\*\*(1/3)),x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

$$3.95 \quad \int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}} dx$$

**Optimal.** Leaf size=154

$$\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} - \frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\tan^{-1}\left(\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

[Out] ArcTan[(1 + (2\*(a + b)^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3)) + ArcTan[(1 + (2\*(a + b\*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b\*x^3)^(1/3)]/(2\*(a + b)^(1/3)) - Log[(a + b)^(1/3)\*x - (a + b\*x^3)^(1/3)]/(2\*(a + b)^(1/3))

**Rubi [F]** time = 0.299582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}} dx$$

Verification is Not applicable to the result.

[In] Int[(1 + x)/((1 + x + x^2)\*(a + b\*x^3)^(1/3)), x]

[Out] ((3 - I\*Sqrt[3])\*Defer[Int][1/((1 - I\*Sqrt[3] + 2\*x)\*(a + b\*x^3)^(1/3)), x])/3 + ((3 + I\*Sqrt[3])\*Defer[Int][1/((1 + I\*Sqrt[3] + 2\*x)\*(a + b\*x^3)^(1/3)), x])/3

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}} dx &= \int \left( \frac{1 - \frac{i}{\sqrt{3}}}{(1 - i\sqrt{3} + 2x)\sqrt[3]{a+bx^3}} + \frac{1 + \frac{i}{\sqrt{3}}}{(1 + i\sqrt{3} + 2x)\sqrt[3]{a+bx^3}} \right) dx \\ &= \frac{1}{3}(3 - i\sqrt{3}) \int \frac{1}{(1 - i\sqrt{3} + 2x)\sqrt[3]{a+bx^3}} dx + \frac{1}{3}(3 + i\sqrt{3}) \int \frac{1}{(1 + i\sqrt{3} + 2x)\sqrt[3]{a+bx^3}} dx \end{aligned}$$

**Mathematica [F]** time = 0.28751, size = 0, normalized size = 0.

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + x)/((1 + x + x^2)\*(a + b\*x^3)^(1/3)), x]

[Out] Integrate[(1 + x)/((1 + x + x^2)\*(a + b\*x^3)^(1/3)), x]

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1+x}{x^2+x+1} \frac{1}{\sqrt[3]{bx^3+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(x^2+x+1)/(b\*x^3+a)^(1/3),x)

[Out] int((1+x)/(x^2+x+1)/(b\*x^3+a)^(1/3),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+x+1)/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] integrate((x + 1)/((b\*x^3 + a)^(1/3)\*(x^2 + x + 1)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+x+1)/(b\*x^3+a)^(1/3),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x\*\*2+x+1)/(b\*x\*\*3+a)\*\*(1/3),x)

[Out] Integral((x + 1)/((a + b\*x\*\*3)\*\*(1/3)\*(x\*\*2 + x + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

$$3.96 \quad \int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}} dx$$

**Optimal.** Leaf size=96

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{a+b}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a+b}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

[Out] -(ArcTan[(1 + (2\*(a + b\*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3))) + Log[1 - x^3]/(6\*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b\*x^3)^(1/3)]/(2\*(a + b)^(1/3))

**Rubi [A]** time = 0.0781431, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {444, 55, 617, 204, 31}

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{a+b}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+1}{\sqrt[3]{a+b}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^3)\*(a + b\*x^3)^(1/3)),x]

[Out] -(ArcTan[(1 + (2\*(a + b\*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]\*(a + b)^(1/3))) + Log[1 - x^3]/(6\*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b\*x^3)^(1/3)]/(2\*(a + b)^(1/3))

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 55

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(1/3)), x\_Symbol] :> With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{(1-x)\sqrt[3]{a+bx}} dx, x, x^3 \right) \\ &= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{1}{2} \text{Subst} \left( \int \frac{1}{(a+b)^{2/3} + \sqrt[3]{a+bx} + x^2} dx, x, \sqrt[3]{a+bx^3} \right) + \frac{\text{Subst} \left( \int \frac{1}{\sqrt[3]{a+b-x}} dx, x, \sqrt[3]{a+bx^3} \right)}{2\sqrt[3]{a+b}} \\ &= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{a+b}} + \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}} \right)}{\sqrt[3]{a+b}} \\ &= -\frac{\tan^{-1} \left( \frac{1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}} \right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3})}{2\sqrt[3]{a+b}} \end{aligned}$$

**Mathematica [A]** time = 0.0808377, size = 80, normalized size = 0.83

$$\frac{-3 \log(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3}) - 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}} + 1}{\sqrt{3}} \right) + \log(1-x^3)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)), x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]] + Log[1 - x^3] - 3*Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^2}{-x^3 + 1} \frac{1}{\sqrt[3]{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3), x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3), x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.35095, size = 983, normalized size = 10.24

$$\left[ 3 \sqrt{\frac{1}{3}}(a+b) \sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}} \log \left( \frac{2bx^3+3 \sqrt{\frac{1}{3}} \left( (bx^3+a)^{\frac{1}{3}}(a+b) - (a+b)(-a-b)^{\frac{1}{3}} - 2(bx^3+a)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}} \right) \sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b} + 3a - 3(bx^3+a)^{\frac{1}{3}}(-a-b)^{\frac{2}{3}} + b}}{x^3-1}} \right) + (-a-b)^{\frac{2}{3}} \right] \\ \frac{\hspace{15em}}{6(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)/(b\*x^3+a)^(1/3),x, algorithm="fricas")

[Out] [1/6\*(3\*sqrt(1/3)\*(a+b)\*sqrt((-a-b)^(1/3)/(a+b))\*log((2\*b\*x^3+3\*sqrt(1/3)\*((b\*x^3+a)^(1/3)\*(a+b)-(a+b)\*(-a-b)^(1/3)-2\*(b\*x^3+a)^(2/3)\*(-a-b)^(2/3))\*sqrt((-a-b)^(1/3)/(a+b))+3\*a-3\*(b\*x^3+a)^(1/3)\*(-a-b)^(2/3)+b)/(x^3-1))+(-a-b)^(2/3)\*log((b\*x^3+a)^(2/3)-(b\*x^3+a)^(1/3)\*(-a-b)^(1/3)+(-a-b)^(2/3))-2\*(-a-b)^(2/3)\*log((b\*x^3+a)^(1/3)+(-a-b)^(1/3)))/(a+b), -1/6\*(6\*sqrt(1/3)\*(a+b)\*sqrt(-(-a-b)^(1/3)/(a+b))\*arctan(sqrt(1/3)\*(2\*(b\*x^3+a)^(1/3)-(-a-b)^(1/3))\*sqrt(-(-a-b)^(1/3)/(a+b)))-(-a-b)^(2/3)\*log((b\*x^3+a)^(2/3)-(b\*x^3+a)^(1/3)\*(-a-b)^(1/3)+(-a-b)^(2/3))+2\*(-a-b)^(2/3)\*log((b\*x^3+a)^(1/3)+(-a-b)^(1/3)))/(a+b)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^3 \sqrt[3]{a+bx^3} - \sqrt[3]{a+bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(-x\*\*3+1)/(b\*x\*\*3+a)\*\*(1/3),x)

[Out] -Integral(x\*\*2/(x\*\*3\*(a+b\*x\*\*3)\*\*(1/3)-(a+b\*x\*\*3)\*\*(1/3)),x)

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.97 \quad \int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

**Optimal.** Leaf size=88

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

[Out] -(ArcTan[(1 - (2\*2^(1/3)\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)\*Sqrt[3])) - Log[1 + x^3]/(6\*2^(1/3)) + Log[-(2^(1/3)\*x) - (1 - x^3)^(1/3)]/(2\*2^(1/3))

**Rubi [A]** time = 0.0618137, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^3)^(1/3)\*(1 + x^3)),x]

[Out] -(ArcTan[(1 - (2\*2^(1/3)\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)\*Sqrt[3])) - Log[1 + (2^(2/3)\*x^2)/(1 - x^3)^(2/3) - (2^(1/3)\*x)/(1 - x^3)^(1/3)]/(6\*2^(1/3)) + Log[1 + (2^(1/3)\*x)/(1 - x^3)^(1/3)]/(3\*2^(1/3))

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \text{Subst} \left( \int \frac{1}{1+2x^3} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\ &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{1+\sqrt[3]{2}x} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) + \frac{1}{3} \text{Subst} \left( \int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) \\ &= \frac{\log \left( 1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left( \int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right) - \frac{\text{Subst} \left( \int \frac{-\sqrt[3]{2}+2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} \\ &= -\frac{\log \left( 1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left( 1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} + \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{\sqrt[3]{2}} \\ &= -\frac{\tan^{-1} \left( \frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log \left( 1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{6\sqrt[3]{2}} + \frac{\log \left( 1 + \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} \right)}{3\sqrt[3]{2}} \end{aligned}$$

**Mathematica [A]** time = 0.142763, size = 112, normalized size = 1.27

$$\frac{-\log \left( \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1 \right) + 2 \log \left( \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} - 1}{\sqrt{3}} \right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^(
1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 + 1} \frac{1}{\sqrt[3]{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] int(1/(-x^3+1)^(1/3)/(x^3+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)\*(-x^3 + 1)^(1/3)), x)

**Fricas [B]** time = 26.9506, size = 660, normalized size = 7.5

$$-\frac{1}{18} \sqrt{6}^{1/6} \arctan \left( \frac{2^{1/6} \left( 6 \sqrt{6}^{2/3} (5x^7 + 4x^4 - x) (-x^3 + 1)^{2/3} - \sqrt{6}^{1/3} (71x^9 - 111x^6 + 33x^3 - 1) + 12 \sqrt{6} (19x^8 - 16x^5 + x^2) \right)}{6 (109x^9 - 105x^6 + 3x^3 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/18\*sqrt(6)\*2^(1/6)\*arctan(1/6\*2^(1/6)\*(6\*sqrt(6)\*2^(2/3)\*(5\*x^7 + 4\*x^4 - x)\*(-x^3 + 1)^(2/3) - sqrt(6)\*2^(1/3)\*(71\*x^9 - 111\*x^6 + 33\*x^3 - 1) + 12\*sqrt(6)\*(19\*x^8 - 16\*x^5 + x^2)\*(-x^3 + 1)^(1/3))/(109\*x^9 - 105\*x^6 + 3\*x^3 + 1)) + 1/18\*2^(2/3)\*log((6\*2^(1/3)\*(-x^3 + 1)^(1/3)\*x^2 + 2^(2/3)\*(x^3 + 1) + 6\*(-x^3 + 1)^(2/3)\*x)/(x^3 + 1)) - 1/36\*2^(2/3)\*log((3\*2^(2/3)\*(5\*x^4 - x)\*(-x^3 + 1)^(2/3) + 2^(1/3)\*(19\*x^6 - 16\*x^3 + 1) - 12\*(2\*x^5 - x^2)\*(-x^3 + 1)^(1/3))/(x^6 + 2\*x^3 + 1))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x\*\*3+1)\*\*(1/3)/(x\*\*3+1),x)



[Out] Integral(1/((-x - 1)\*(x\*\*2 + x + 1)\*\*(1/3)\*(x + 1)\*(x\*\*2 - x + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)\*(-x^3 + 1)^(1/3)), x)

$$3.98 \quad \int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

**Optimal.** Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{2}(1-x)+1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] ArcTan[(1 - (2*2^(1/3))*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3]/(2^(1/3)*Sqrt[3])
+ ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3]/(2*2^(1/3)*Sqrt[3])
+ Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

**Rubi [C]** time = 0.0108413, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

#### Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)} dx = \frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

**Mathematica [C]** time = 0.0269249, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

[Out]  $(x^2 \text{AppellF1}[2/3, 1/3, 1, 5/3, x^3, -x^3])/2$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x}{x^3 + 1} \frac{1}{\sqrt[3]{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-x^3+1)^(1/3)/(x^3+1),x)`

[Out] `int(x/(-x^3+1)^(1/3)/(x^3+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)`

**Fricas [B]** time = 23.8683, size = 1049, normalized size = 4.5

$$-\frac{1}{36} \sqrt{6}^{\frac{1}{6}} (-1)^{\frac{1}{3}} \arctan \left( \frac{2^{\frac{1}{6}} \left( 24 \sqrt{6}^{\frac{2}{3}} (-1)^{\frac{2}{3}} (x^{14} - 2x^{11} - 6x^8 - 2x^5 + x^2) (-x^3 + 1)^{\frac{2}{3}} + 12 \sqrt{6} (-1)^{\frac{1}{3}} (x^{16} - 33x^{13} + \dots) \right)}{6(x^{18} - 102x^{15} + 447x^{12} - 628x^9 + 447x^6 - 102x^3 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")`

[Out] `-1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) - 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) - 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x\*\*3+1)\*\*(1/3)/(x\*\*3+1),x)

[Out] Integral(x/((-x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)\*(x + 1)\*(x\*\*2 - x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 1)(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(x/((x^3 + 1)\*(-x^3 + 1)^(1/3)), x)

$$3.99 \quad \int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

**Optimal.** Leaf size=82

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

[Out] ArcTan[(1 + 2^(2/3)\*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)\*Sqrt[3]) - Log[1 + x^3]/(6\*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2\*2^(1/3))

**Rubi [A]** time = 0.0578244, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2}-\sqrt[3]{1-x^3})}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^3)^(1/3)\*(1 + x^3)), x]

[Out] ArcTan[(1 + 2^(2/3)\*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)\*Sqrt[3]) - Log[1 + x^3]/(6\*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2\*2^(1/3))

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 55

Int[1/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(1/3)), x\_Symbol] :> With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{1}{\sqrt[3]{1-x}(1+x)} dx, x, x^3 \right) \\ &= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2} \text{Subst} \left( \int \frac{1}{2^{2/3} + \sqrt[3]{2}x + x^2} dx, x, \sqrt[3]{1-x^3} \right) - \frac{\text{Subst} \left( \int \frac{1}{\sqrt[3]{2-x}} dx, x, \sqrt[3]{1-x^3} \right)}{2\sqrt[3]{2}} \\ &= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} - \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2^{2/3}\sqrt[3]{1-x^3} \right)}{\sqrt[3]{2}} \\ &= \frac{\tan^{-1} \left( \frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log(\sqrt[3]{2} - \sqrt[3]{1-x^3})}{2\sqrt[3]{2}} \end{aligned}$$

**Mathematica [A]** time = 0.0595163, size = 73, normalized size = 0.89

$$\frac{-\log(x^3 + 1) + 3 \log(\sqrt[3]{2} - \sqrt[3]{1-x^3}) + 2\sqrt{3} \tan^{-1} \left( \frac{2^{2/3}\sqrt[3]{1-x^3} + 1}{\sqrt{3}} \right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 - x^3)^(1/3)\*(1 + x^3)), x]

[Out] (2\*Sqrt[3]\*ArcTan[(1 + 2^(2/3)\*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3\*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6\*2^(1/3))

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^3 + 1} \frac{1}{\sqrt[3]{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1), x)

[Out] int(x^2/(-x^3+1)^(1/3)/(x^3+1), x)

**Maxima [A]** time = 1.40032, size = 116, normalized size = 1.41

$$\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \arctan \left( \frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left( 2^{\frac{1}{3}} + 2(-x^3 + 1)^{\frac{1}{3}} \right) \right) - \frac{1}{12} \cdot 2^{\frac{2}{3}} \log \left( 2^{\frac{2}{3}} + 2^{\frac{1}{3}}(-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}} \right) + \frac{1}{6} \cdot 2^{\frac{2}{3}} \log \left( -2^{\frac{1}{3}} + (-x^3 + 1)^{\frac{1}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1), x, algorithm="maxima")

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**Fricas [A]** time = 2.12013, size = 282, normalized size = 3.44

$$\frac{1}{6} \sqrt{6} 2^{\frac{1}{6}} \arctan\left(\frac{1}{6} \cdot 2^{\frac{1}{6}} \left(\sqrt{6} 2^{\frac{1}{3}} + 2 \sqrt{6} (-x^3 + 1)^{\frac{1}{3}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} (-x^3 + 1)^{\frac{1}{3}} + (-x^3 + 1)^{\frac{2}{3}}\right) + \frac{1}{6} \cdot 2^{\frac{2}{3}} \log\left(-2^{\frac{1}{3}} + (-x^3 + 1)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)(x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-x - 1)*(x**2 + x + 1)**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.100 \quad \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{2\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{\sqrt[3]{2}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

[Out] (Sqrt[3]\*ArcTan[(1 - (2\*2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/2^(1/3) + Log[1 + (2^(2/3)\*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/(2\*2^(1/3)) - Log[1 + (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/2^(1/3)

**Rubi [C]** time = 0.336726, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {6728, 2148}

$$\frac{3(-\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{1-x^3} + 2x - i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(\sqrt{3} + i)} - \frac{3(\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{1-x^3} + 2x + i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(-\sqrt{3} + i)} - \frac{(3 - i\sqrt{3}) \tan^{-1}\left(\frac{2 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}(\sqrt{3} + i)}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((1 - x + x^2)\*(1 - x^3)^(1/3)), x]

[Out] -((3 - I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3))\*(1 - I\*Sqrt[3] + 2\*x))/(1 - x^3)^(1/3)]/(2\*Sqrt[3]))/(2\*2^(1/3)\*(I + Sqrt[3])) + ((3 + I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3)\*(1 + I\*Sqrt[3] + 2\*x))/(1 - x^3)^(1/3)]/(2\*Sqrt[3]))/(2\*2^(1/3)\*(I - Sqrt[3])) + ((I - Sqrt[3])\*Log[-((1 - I\*Sqrt[3] - 2\*x)^2\*(1 - I\*Sqrt[3] + 2\*x))])/(4\*2^(1/3)\*(I + Sqrt[3])) + ((I + Sqrt[3])\*Log[-((1 + I\*Sqrt[3] - 2\*x)^2\*(1 + I\*Sqrt[3] + 2\*x))])/(4\*2^(1/3)\*(I - Sqrt[3])) - (3\*(I - Sqrt[3])\*Log[1 - I\*Sqrt[3] + 2\*x + 2\*2^(2/3)\*(1 - x^3)^(1/3)]/(4\*2^(1/3)\*(I + Sqrt[3])) - (3\*(I + Sqrt[3])\*Log[1 + I\*Sqrt[3] + 2\*x + 2\*2^(2/3)\*(1 - x^3)^(1/3)]/(4\*2^(1/3)\*(I - Sqrt[3]))

#### Rule 6728

Int[(u\_)/((a\_.) + (b\_.)\*(x\_)^(n\_.) + (c\_.)\*(x\_)^(n2\_.)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n + c\*x^(2\*n)), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2\*n] && IGtQ[n, 0]

#### Rule 2148

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] := Simp[Sqrt[3]\*ArcTan[(1 - (2^(1/3)\*Rt[b, 3]\*(c - d\*x))/(d\*(a + b\*x^3)^(1/3))]/Sqrt[3]]/(2^(4/3)\*Rt[b, 3]\*c), x] + (Simp[Log[(c + d\*x)^2\*(c - d\*x)]/(2^(7/3)\*Rt[b, 3]\*c), x] - Simp[(3\*Log[Rt[b, 3]\*(c - d\*x) + 2^(2/3)\*d\*(a + b\*x^3)^(1/3)]/(2^(7/3)\*Rt[b, 3]\*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c^3 + a\*d^3, 0]

#### Rubi steps



$$\begin{aligned} \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}} dx &= \int \left( \frac{1-i\sqrt{3}}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} \right) dx \\ &= (1-i\sqrt{3}) \int \frac{1}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx + (1+i\sqrt{3}) \int \frac{1}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx \\ &= -\frac{(3-i\sqrt{3}) \tan^{-1} \left( \frac{2-\sqrt[3]{2(1-i\sqrt{3}+2x)}}{2\sqrt[3]{1-x^3}} \right)}{2\sqrt[3]{2}(i+\sqrt{3})} + \frac{(3+i\sqrt{3}) \tan^{-1} \left( \frac{2-\sqrt[3]{2(1+i\sqrt{3}+2x)}}{2\sqrt[3]{1-x^3}} \right)}{2\sqrt[3]{2}(i-\sqrt{3})} + \frac{(i-\sqrt{3}) \log(-)}{2\sqrt[3]{2}(i-\sqrt{3})} \end{aligned}$$

**Mathematica [F]** time = 0.19016, size = 0, normalized size = 0.

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + x)/((1 - x + x^2)\*(1 - x^3)^(1/3)), x]

[Out] Integrate[(1 + x)/((1 - x + x^2)\*(1 - x^3)^(1/3)), x]

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1+x}{x^2-x+1} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3), x)

[Out] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3), x, algorithm="maxima")

[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)\*(x^2 - x + 1)), x)

**Fricas [B]** time = 111.657, size = 882, normalized size = 6.53

$$\frac{1}{6} \sqrt{32}^{\frac{2}{3}} (-1)^{\frac{1}{3}} \arctan \left( \frac{\sqrt{32}^{\frac{1}{6}} \left( 4 \cdot 2^{\frac{1}{6}} (-1)^{\frac{2}{3}} (x^4 - 4x^3 + 5x^2 - 4x + 1) (-x^3 + 1)^{\frac{2}{3}} - 4\sqrt{2} (-1)^{\frac{1}{3}} (x^5 - x^4 - 3x^3 + 3x^2 + \dots) \right)}{6(3x^6 - 9x^5 + 6x^4 - x^3 + 6x^2 - \dots)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")

[Out]  $\frac{1}{6}\sqrt{3}2^{2/3}(-1)^{1/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{1/6}\right)\left(4\cdot 2^{1/6}(-1)^{2/3}(x^4 - 4x^3 + 5x^2 - 4x + 1)(-x^3 + 1)^{2/3} - 4\sqrt{2}(-1)^{1/3}(x^5 - x^4 - 3x^3 + 3x^2 + x - 1)(-x^3 + 1)^{1/3} + 2^{5/6}(x^6 - 7x^5 + 10x^4 - 7x^3 + 10x^2 - 7x + 1)\right) / (3x^6 - 9x^5 + 6x^4 - x^3 + 6x^2 - 9x + 3) - \frac{1}{12}2^{2/3}(-1)^{1/3}\log\left(-2^{2/3}(-1)^{1/3}(-x^3 + 1)^{2/3}(x^2 - 3x + 1) + 2^{1/3}(-1)^{2/3}(x^4 - 3x^2 + 1) + 4(-x^3 + 1)^{1/3}(x^2 - x)\right) / (x^4 - 2x^3 + 3x^2 - 2x + 1) + \frac{1}{6}2^{2/3}(-1)^{1/3}\log\left(-2\cdot 2^{1/3}(-1)^{2/3}(-x^3 + 1)^{1/3}(x - 1) + 2^{2/3}(-1)^{1/3}(x^2 - x + 1) - 2(-x^3 + 1)^{2/3}\right) / (x^2 - x + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x\*\*2-x+1)/(-x\*\*3+1)\*\*(1/3),x)

[Out] Integral((x + 1)/((-x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)\*(x\*\*2 - x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)\*(x^2 - x + 1)), x)

$$3.101 \quad \int \frac{(1+x)^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$$

**Optimal.** Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{2\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{\sqrt[3]{2}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

[Out] (Sqrt[3]\*ArcTan[(1 - (2\*2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/2^(1/3) + Log[1 + (2^(2/3)\*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/(2\*2^(1/3)) - Log[1 + (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/2^(1/3)

**Rubi [C]** time = 0.300503, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 5, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1586, 6728, 2148}

$$\frac{3(-\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{1-x^3} + 2x - i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(\sqrt{3} + i)} - \frac{3(\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{1-x^3} + 2x + i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(-\sqrt{3} + i)} - \frac{(3 - i\sqrt{3}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/((1 - x^3)^(1/3)\*(1 + x^3)), x]

[Out] -((3 - I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3)\*(1 - I\*Sqrt[3] + 2\*x))/(1 - x^3)^(1/3))/(2\*Sqrt[3])])/(2\*2^(1/3)\*(I + Sqrt[3])) + ((3 + I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3)\*(1 + I\*Sqrt[3] + 2\*x))/(1 - x^3)^(1/3))/(2\*Sqrt[3])])/(2\*2^(1/3)\*(I - Sqrt[3])) + ((I - Sqrt[3])\*Log[-((1 - I\*Sqrt[3] - 2\*x)^2\*(1 - I\*Sqrt[3] + 2\*x))])/(4\*2^(1/3)\*(I + Sqrt[3])) + ((I + Sqrt[3])\*Log[-((1 + I\*Sqrt[3] - 2\*x)^2\*(1 + I\*Sqrt[3] + 2\*x))])/(4\*2^(1/3)\*(I - Sqrt[3])) - (3\*(I - Sqrt[3])\*Log[1 - I\*Sqrt[3] + 2\*x + 2\*2^(2/3)\*(1 - x^3)^(1/3)])/(4\*2^(1/3)\*(I + Sqrt[3])) - (3\*(I + Sqrt[3])\*Log[1 + I\*Sqrt[3] + 2\*x + 2\*2^(2/3)\*(1 - x^3)^(1/3)])/(4\*2^(1/3)\*(I - Sqrt[3]))

#### Rule 1586

Int[(u\_)\*(Px\_)^(p\_)\*(Qx\_)^(q\_), x\_Symbol] := Int[u\*PolynomialQuotient[Px, Qx, x]^p\*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p, q, 0]

#### Rule 6728

Int[(u\_)/((a\_.) + (b\_.)\*(x\_)^(n\_.) + (c\_.)\*(x\_)^(n2\_.)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n + c\*x^(2\*n)), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2\*n] && IGtQ[n, 0]

#### Rule 2148

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] := Simp[(Sqrt[3]\*ArcTan[(1 - (2^(1/3)\*Rt[b, 3]\*(c - d\*x))/(d\*(a + b\*x^3)^(1/3)))]/Sqrt[3])/((2^(4/3)\*Rt[b, 3]\*c), x] + (Simp[Log[(c + d\*x)^2\*(c - d\*x)]/(2^(7/3)\*Rt[b, 3]\*c), x] - Simp[(3\*Log[Rt[b, 3]\*(c - d\*x) + 2^(2/3)\*d\*(a + b\*x^3)^(1/3)]/2^(1/3), x])

$(1/3)]/(2^{(7/3)}*Rt[b, 3]*c), x)] /; FreeQ[\{a, b, c, d\}, x] \&\& EqQ[b*c^3 + a*d^3, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(1+x)^2}{\sqrt[3]{1-x^3}(1+x^3)} dx &= \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}} dx \\ &= \int \left( \frac{1-i\sqrt{3}}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} \right) dx \\ &= (1-i\sqrt{3}) \int \frac{1}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx + (1+i\sqrt{3}) \int \frac{1}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx \\ &= -\frac{(3-i\sqrt{3}) \tan^{-1}\left(\frac{2-\sqrt[3]{2}(1-i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}(i+\sqrt{3})} + \frac{(3+i\sqrt{3}) \tan^{-1}\left(\frac{2-\sqrt[3]{2}(1+i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}(i-\sqrt{3})} + \frac{(i-\sqrt{3}) \log\left(-\frac{2-\sqrt[3]{2}(1-i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}\right)}{4\sqrt[3]{2}(i+\sqrt{3})} \end{aligned}$$

**Mathematica [C]** time = 0.440747, size = 150, normalized size = 1.11

$$\frac{1}{3}x^3F_1\left(1; \frac{1}{3}, 1; 2; x^3, -x^3\right) + x^2F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right) + \frac{-\log\left(\frac{2^{2/3}x^2}{(x^3-1)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2-\sqrt[3]{2}(1-i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)\*(1 + x^3)), x]

[Out]  $x^2 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, x^3, -x^3\right] + (x^3 \text{AppellF1}\left[1, \frac{1}{3}, 1, 2, x^3, -x^3\right])/3 + (2\sqrt{3} \text{ArcTan}\left[\frac{-1 + (2^{2/3}x^2)/(x^3-1)^{2/3} - \sqrt[3]{2}x/\sqrt[3]{x^3-1}}{-1 + x^3}\right])/3 - \text{Log}\left[1 + \frac{2^{2/3}x^2}{(x^3-1)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right] + 2\log\left[\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right] + 2\sqrt{3}\tan^{-1}\left(\frac{2-\sqrt[3]{2}(1-i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}\right)/(6 \cdot 2^{1/3})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(1+x)^2}{x^3+1} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1), x)

[Out] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Fricas [B]** time = 99.4621, size = 882, normalized size = 6.53

$$\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} (-1)^{\frac{1}{3}} \arctan \left( \frac{\sqrt{3} 2^{\frac{1}{6}} \left( 4 \cdot 2^{\frac{1}{6}} (-1)^{\frac{2}{3}} (x^4 - 4x^3 + 5x^2 - 4x + 1) (-x^3 + 1)^{\frac{2}{3}} - 4\sqrt{2} (-1)^{\frac{1}{3}} (x^5 - x^4 - 3x^3 + 3x^2 + x - 1) (-x^3 + 1)^{\frac{1}{3}} + 2^{\frac{5}{6}} (x^6 - 7x^5 + 10x^4 - 7x^3 + 10x^2 - 7x + 1) \right)}{6(3x^6 - 9x^5 + 6x^4 - x^3 + 6x^2 - 9x + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 + 1)^(1/3)*(x^2 - x)))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((x + 1)/((-x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

$$3.102 \quad \int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}} dx$$

**Optimal.** Leaf size=119

$$-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{(x^3+1)^{2/3}} - \frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}} + 1\right)}{2\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}} + 1\right)}{\sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}\right)}{\sqrt[3]{2}}$$

[Out] -((Sqrt[3]\*ArcTan[(1 - (2\*2^(1/3))\*(1 + x))/(1 + x^3)^(1/3)]/Sqrt[3])/2^(1/3)) - Log[1 + (2^(2/3)\*(1 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)\*(1 + x))/(1 + x^3)^(1/3)]/(2\*2^(1/3)) + Log[1 + (2^(1/3)\*(1 + x))/(1 + x^3)^(1/3)]/2^(1/3)

**Rubi [C]** time = 0.299049, antiderivative size = 399, normalized size of antiderivative = 3.35, number of steps used = 4, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {6728, 2148}

$$\frac{3(-\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{x^3 + 1} - 2x - i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(\sqrt{3} + i)} + \frac{3(\sqrt{3} + i) \log\left(2 \cdot 2^{2/3} \sqrt[3]{x^3 + 1} - 2x + i\sqrt{3} + 1\right)}{4\sqrt[3]{2}(-\sqrt{3} + i)} + \frac{(3 - i\sqrt{3}) \tan^{-1}\left(\frac{2 - \sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}\right)}{2\sqrt[3]{2}(\sqrt{3} + i)}$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/((1 + x + x^2)\*(1 + x^3)^(1/3)), x]

[Out] ((3 - I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3))\*(1 - I\*Sqrt[3] - 2\*x))/(1 + x^3)^(1/3)]/(2\*Sqrt[3]))/(2\*2^(1/3)\*(I + Sqrt[3])) - ((3 + I\*Sqrt[3])\*ArcTan[(2 - (2^(1/3)\*(1 + I\*Sqrt[3] - 2\*x))/(1 + x^3)^(1/3)]/(2\*Sqrt[3]))/(2\*2^(1/3)\*(I - Sqrt[3])) - ((I - Sqrt[3])\*Log[(1 - I\*Sqrt[3] - 2\*x)\*(1 - I\*Sqrt[3] + 2\*x)^2])/(4\*2^(1/3)\*(I + Sqrt[3])) - ((I + Sqrt[3])\*Log[(1 + I\*Sqrt[3] - 2\*x)\*(1 + I\*Sqrt[3] + 2\*x)^2])/(4\*2^(1/3)\*(I - Sqrt[3])) + (3\*(I - Sqrt[3])\*Log[1 - I\*Sqrt[3] - 2\*x + 2\*2^(2/3)\*(1 + x^3)^(1/3)]/(4\*2^(1/3)\*(I + Sqrt[3])) + (3\*(I + Sqrt[3])\*Log[1 + I\*Sqrt[3] - 2\*x + 2\*2^(2/3)\*(1 + x^3)^(1/3)]/(4\*2^(1/3)\*(I - Sqrt[3]))

#### Rule 6728

Int[(u\_)/((a\_.) + (b\_.)\*(x\_)^(n\_.) + (c\_.)\*(x\_)^(n2\_.)), x\_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b\*x^n + c\*x^(2\*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2\*n] && IGtQ[n, 0]

#### Rule 2148

Int[1/(((c\_) + (d\_.)\*(x\_))\*((a\_) + (b\_.)\*(x\_)^3)^(1/3)), x\_Symbol] :> Simp[Sqrt[3]\*ArcTan[(1 - (2^(1/3)\*Rt[b, 3]\*(c - d\*x))/(d\*(a + b\*x^3)^(1/3))]/Sqrt[3]]/(2^(4/3)\*Rt[b, 3]\*c), x] + (Simp[Log[(c + d\*x)^2\*(c - d\*x)]/(2^(7/3)\*Rt[b, 3]\*c), x] - Simp[(3\*Log[Rt[b, 3]\*(c - d\*x) + 2^(2/3)\*d\*(a + b\*x^3)^(1/3)]/(2^(7/3)\*Rt[b, 3]\*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c^3 + a\*d^3, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}} dx &= \int \left( \frac{-1-i\sqrt{3}}{(1-i\sqrt{3}+2x)\sqrt[3]{1+x^3}} + \frac{-1+i\sqrt{3}}{(1+i\sqrt{3}+2x)\sqrt[3]{1+x^3}} \right) dx \\ &= (-1-i\sqrt{3}) \int \frac{1}{(1-i\sqrt{3}+2x)\sqrt[3]{1+x^3}} dx + (-1+i\sqrt{3}) \int \frac{1}{(1+i\sqrt{3}+2x)\sqrt[3]{1+x^3}} dx \\ &= \frac{(3-i\sqrt{3}) \tan^{-1} \left( \frac{2-\sqrt[3]{2(1-i\sqrt{3}-2x)}}{2\sqrt[3]{1+x^3}} \right)}{2\sqrt[3]{2}(i+\sqrt{3})} - \frac{(3+i\sqrt{3}) \tan^{-1} \left( \frac{2-\sqrt[3]{2(1+i\sqrt{3}-2x)}}{2\sqrt[3]{1+x^3}} \right)}{2\sqrt[3]{2}(i-\sqrt{3})} - \frac{(i-\sqrt{3}) \log \left( (1-x) \right)}{4} \end{aligned}$$

**Mathematica [F]** time = 0.162654, size = 0, normalized size = 0.

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x)/((1 + x + x^2)\*(1 + x^3)^(1/3)), x]

[Out] Integrate[(1 - x)/((1 + x + x^2)\*(1 + x^3)^(1/3)), x]

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1-x}{x^2+x+1} \frac{1}{\sqrt[3]{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3), x)

[Out] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3), x, algorithm="maxima")

[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)\*(x^2 + x + 1)), x)

**Fricas [B]** time = 93.4274, size = 737, normalized size = 6.19

$$\frac{1}{6} \sqrt{32}^{\frac{2}{3}} \arctan \left( \frac{\sqrt{32}^{\frac{1}{6}} \left( 2^{\frac{5}{6}} (x^6 + 7x^5 + 10x^4 + 7x^3 + 10x^2 + 7x + 1) - 4\sqrt{2}(x^5 + x^4 - 3x^3 - 3x^2 + x + 1)(x^3 + 1)^{\frac{1}{3}} \right)}{6(3x^6 + 9x^5 + 6x^4 + x^3 + 6x^2 + 9x + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")

[Out]  $\frac{1}{6}\sqrt{3}2^{2/3}\arctan\left(\frac{1}{6}\sqrt{3}2^{1/6}\left(2^{5/6}(x^6 + 7x^5 + 10x^4 + 7x^3 + 10x^2 + 7x + 1) - 4\sqrt{2}(x^5 + x^4 - 3x^3 - 3x^2 + x + 1)(x^3 + 1)^{1/3} + 42^{1/6}(x^4 + 4x^3 + 5x^2 + 4x + 1)(x^3 + 1)^{2/3}\right)\right)}{(3x^6 + 9x^5 + 6x^4 + x^3 + 6x^2 + 9x + 3)} - \frac{1}{12}2^{2/3}\log\left(\frac{2^{2/3}(x^3 + 1)^{2/3}(x^2 + 3x + 1) - 2^{1/3}(x^4 - 3x^2 + 1) - 4(x^3 + 1)^{1/3}(x^2 + x)}{(x^4 + 2x^3 + 3x^2 + 2x + 1)}\right) + \frac{1}{6}2^{2/3}\log\left(\frac{2^{2/3}(x^2 + x + 1) + 22^{1/3}(x^3 + 1)^{1/3}(x + 1) + 2(x^3 + 1)^{2/3}}{(x^2 + x + 1)}\right)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^2\sqrt[3]{x^3+1} + x\sqrt[3]{x^3+1} + \sqrt[3]{x^3+1}} dx - \int -\frac{1}{x^2\sqrt[3]{x^3+1} + x\sqrt[3]{x^3+1} + \sqrt[3]{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x\*\*2+x+1)/(x\*\*3+1)\*\*(1/3),x)

[Out]  $-\text{Integral}\left(\frac{x}{x^2(x^3 + 1)^{1/3} + x(x^3 + 1)^{1/3} + (x^3 + 1)^{1/3}}, x\right) - \text{Integral}\left(-\frac{1}{x^2(x^3 + 1)^{1/3} + x(x^3 + 1)^{1/3} + (x^3 + 1)^{1/3}}, x\right)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)\*(x^2 + x + 1)), x)



$$3.103 \quad \int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx$$

**Optimal.** Leaf size=43

$$x^2 \left( -{}_2F_1 \left( \frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3 \right) \right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

[Out]  $(1 - x^3)^{-1/3} + x/(1 - x^3)^{1/3} - x^2 \text{Hypergeometric2F1}[2/3, 4/3, 5/3, x^3]$

**Rubi [F]** time = 0.425069, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2, x]

[Out]  $(-4 \text{Defer}[\text{Int}][(1 - x^3)^{2/3}/(-1 + I \text{Sqrt}[3] - 2x)^2, x])/3 + (((4I)/3) * \text{Defer}[\text{Int}][(1 - x^3)^{2/3}/(-1 + I \text{Sqrt}[3] - 2x), x])/\text{Sqrt}[3] - (4 \text{Defer}[\text{Int}][(1 - x^3)^{2/3}/(1 + I \text{Sqrt}[3] + 2x)^2, x])/3 + (((4I)/3) * \text{Defer}[\text{Int}][(1 - x^3)^{2/3}/(1 + I \text{Sqrt}[3] + 2x), x])/\text{Sqrt}[3]$

Rubi steps

$$\begin{aligned} \int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx &= \int \left( -\frac{4(1-x^3)^{2/3}}{3(-1+i\sqrt{3}-2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(-1+i\sqrt{3}-2x)} - \frac{4(1-x^3)^{2/3}}{3(1+i\sqrt{3}+2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(1+i\sqrt{3}+2x)} \right) dx \\ &= -\left( \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}-2x)^2} dx \right) - \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(1+i\sqrt{3}+2x)^2} dx + \frac{(4i) \int \frac{(1-x^3)^{2/3}}{-1+i\sqrt{3}-2x} dx}{3\sqrt{3}} + \frac{(4i) \int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}+2x} dx}{3\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.150557, size = 43, normalized size = 1.

$$x^2 {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3 \right) + \frac{(2x+1)(1-x^3)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2, x]

[Out]  $((1 + 2x)(1 - x^3)^{2/3})/(1 + x + x^2) + x^2 \text{Hypergeometric2F1}[1/3, 2/3, 5/3, x^3]$

**Maple [A]** time = 0.036, size = 34, normalized size = 0.8

$$-(-1+x)(1+2x)\frac{1}{\sqrt[3]{-x^3+1}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x)

[Out] -(-1+x)\*(1+2\*x)/(-x^3+1)^(1/3)+x^2\*hypergeom([1/3,2/3],[5/3],x^3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3+1)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2\*x^3 + 3\*x^2 + 2\*x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x^2+x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3)/(x\*\*2+x+1)\*\*2,x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1))\*\*(2/3)/(x\*\*2 + x + 1)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{(x^2 + x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

$$3.104 \quad \int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}} dx$$

**Optimal.** Leaf size=43

$$x^2 \left( -{}_2F_1 \left( \frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3 \right) \right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

[Out]  $(1 - x^3)^{-1/3} + x/(1 - x^3)^{1/3} - x^2 \text{Hypergeometric2F1}[2/3, 4/3, 5/3, x^3]$

**Rubi [F]** time = 0.220072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x)/((1 + x + x^2)\*(1 - x^3)^(1/3)), x]

[Out]  $-((1 + I\sqrt{3})\text{Defer}[\text{Int}[1/((1 - I\sqrt{3} + 2x)*(1 - x^3)^{1/3}), x]) - (1 - I\sqrt{3})\text{Defer}[\text{Int}[1/((1 + I\sqrt{3} + 2x)*(1 - x^3)^{1/3}), x])$

Rubi steps

$$\begin{aligned} \int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}} dx &= \int \left( \frac{-1-i\sqrt{3}}{(1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} + \frac{-1+i\sqrt{3}}{(1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} \right) dx \\ &= (-1-i\sqrt{3}) \int \frac{1}{(1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx + (-1+i\sqrt{3}) \int \frac{1}{(1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}} dx \end{aligned}$$

**Mathematica [A]** time = 0.0745515, size = 43, normalized size = 1.

$$x^2 {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3 \right) + \frac{(2x+1)(1-x^3)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/((1 + x + x^2)\*(1 - x^3)^(1/3)), x]

[Out]  $((1 + 2x)*(1 - x^3)^{2/3})/(1 + x + x^2) + x^2 \text{Hypergeometric2F1}[1/3, 2/3, 5/3, x^3]$

**Maple [A]** time = 0.027, size = 34, normalized size = 0.8

$$-(-1+x)(1+2x) \frac{1}{\sqrt[3]{-x^3+1}} + x^2 {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x)`

[Out] `-(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")`

[Out] `-integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3+1)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")`

[Out] `integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^2\sqrt[3]{1-x^3} + x\sqrt[3]{1-x^3} + \sqrt[3]{1-x^3}} dx - \int \frac{1}{x^2\sqrt[3]{1-x^3} + x\sqrt[3]{1-x^3} + \sqrt[3]{1-x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)`

[Out] `-Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/3)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

$$3.105 \quad \int \frac{(1-x)^2}{(1-x^3)^{4/3}} dx$$

**Optimal.** Leaf size=39

$$x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

[Out] (1 + (1 - 2\*x)\*x)/(1 - x^3)^(1/3) + x^2\*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]

**Rubi [A]** time = 0.0289564, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1854, 12, 364}

$$x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x)^2/(1 - x^3)^(4/3), x]

[Out] (1 + (1 - 2\*x)\*x)/(1 - x^3)^(1/3) + x^2\*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]

#### Rule 1854

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Module[{q = Expon[Pq, x], i}, Simp[((a\*Coeff[Pq, x, q] - b\*x\*ExpandToSum[Pq - Coeff[Pq, x, q]\*x^q, x])\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] + Dist[1/(a\*n\*(p + 1)), Int[Sum[(n\*(p + 1) + i + 1)\*Coeff[Pq, x, i]\*x^i, {i, 0, q - 1}]]\*(a + b\*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{(1-x)^2}{(1-x^3)^{4/3}} dx &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} - \int -\frac{2x}{\sqrt[3]{1-x^3}} dx \\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + 2 \int \frac{x}{\sqrt[3]{1-x^3}} dx \\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) \end{aligned}$$

**Mathematica [A]** time = 0.0182633, size = 43, normalized size = 1.1

$$x^2 \left( -{}_2F_1 \left( \frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3 \right) \right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3), x]

[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2\*Hypergeometric2F1[2/3, 4/3, 5/3, x^3]

**Maple [A]** time = 0.013, size = 34, normalized size = 0.9

$$-(-1+x)(1+2x) \frac{1}{\sqrt[3]{-x^3+1}} + x^2 {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)^2/(-x^3+1)^(4/3), x)

[Out] -(-1+x)\*(1+2\*x)/(-x^3+1)^(1/3)+x^2\*hypergeom([1/3,2/3],[5/3],x^3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x}{(-x^3+1)^{\frac{1}{3}}} - \int \frac{x^2-2x}{(x^3-1)(x^2+x+1)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3), x, algorithm="maxima")

[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2\*x)/((x^3 - 1)\*(x^2 + x + 1)^(1/3)\*(-x + 1)^(1/3)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(-x^3+1)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3), x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2\*x^3 + 3\*x^2 + 2\*x + 1), x)



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^2}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)\*\*2/(-x\*\*3+1)\*\*(4/3), x)

[Out] Integral((x - 1)\*\*2/(-(x - 1)\*(x\*\*2 + x + 1))\*\*(4/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3), x, algorithm="giac")

[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)

### 3.106 $\int (1 - x^3)^{2/3} dx$

**Optimal.** Leaf size=67

$$\frac{1}{3} (1 - x^3)^{2/3} x + \frac{1}{3} \log\left(\sqrt[3]{1 - x^3} + x\right) - \frac{2 \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out]  $(x*(1 - x^3)^{(2/3)})/3 - (2*\text{ArcTan}[(1 - (2*x))/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) + \text{Log}[x + (1 - x^3)^{(1/3)}]/3$

**Rubi [A]** time = 0.0098674, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {195, 239}

$$\frac{1}{3} (1 - x^3)^{2/3} x + \frac{1}{3} \log\left(\sqrt[3]{1 - x^3} + x\right) - \frac{2 \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 - x^3)^{(2/3)}, x]$

[Out]  $(x*(1 - x^3)^{(2/3)})/3 - (2*\text{ArcTan}[(1 - (2*x))/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) + \text{Log}[x + (1 - x^3)^{(1/3)}]/3$

#### Rule 195

$\text{Int}[(a + b*x^n)^p, x\_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{p-1}, x], x] /;$  Free Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

$\text{Int}[(a + b*x^3)^{-1/3}, x\_Symbol] \rightarrow \text{Simp}[\text{ArcTan}[(1 + (2*\text{Rt}[b, 3]*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]]/(\text{Sqrt}[3]*\text{Rt}[b, 3]), x] - \text{Simp}[\text{Log}[(a + b*x^3)^{(1/3)} - \text{Rt}[b, 3]*x]/(2*\text{Rt}[b, 3]), x] /;$  FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int (1 - x^3)^{2/3} dx &= \frac{1}{3} x (1 - x^3)^{2/3} + \frac{2}{3} \int \frac{1}{\sqrt[3]{1 - x^3}} dx \\ &= \frac{1}{3} x (1 - x^3)^{2/3} - \frac{2 \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3} \log\left(x + \sqrt[3]{1 - x^3}\right) \end{aligned}$$

**Mathematica [C]** time = 0.102595, size = 101, normalized size = 1.51

$$\frac{3(x-1)(1-x^3)^{2/3} F_1\left(\frac{5}{3}; -\frac{2}{3}, -\frac{2}{3}, \frac{8}{3}; -\frac{x-1}{1-(-1)^{2/3}}, -\frac{x-1}{1+\sqrt[3]{-1}}\right)}{5\left(\frac{x-1}{1+\sqrt[3]{-1}}+1\right)^{2/3}\left(\frac{x-1}{1-(-1)^{2/3}}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - x^3)^(2/3), x]

[Out] (3\*(-1 + x)\*(1 - x^3)^(2/3)\*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 - (-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5\*(1 + (-1 + x)/(1 + (-1)^(1/3)))^(2/3)\*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))

**Maple [C]** time = 0.025, size = 12, normalized size = 0.2

$$x {}_2F_1\left(-\frac{2}{3}, \frac{1}{3}; \frac{4}{3}; x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3), x)

[Out] x\*hypergeom([-2/3, 1/3], [4/3], x^3)

**Maxima [B]** time = 1.39687, size = 142, normalized size = 2.12

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{1/3}}{x}-1\right)\right)-\frac{(-x^3+1)^{2/3}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\log\left(\frac{(-x^3+1)^{1/3}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{(-x^3+1)^{1/3}}{x}+\frac{(-x^3+1)^{1/3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3), x, algorithm="maxima")

[Out] -2/9\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3)/x - 1)) - 1/3\*(-x^3 + 1)^(2/3)/(x^2\*((x^3 - 1)/x^3 - 1)) + 2/9\*log((-x^3 + 1)^(1/3)/x + 1) - 1/9\*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)

**Fricas [A]** time = 1.59905, size = 258, normalized size = 3.85

$$\frac{1}{3}(-x^3+1)^{2/3}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{1/3}}{3x}\right)+\frac{2}{9}\log\left(\frac{x+(-x^3+1)^{1/3}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{1/3}x+(-x^3+1)^{1/3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3), x, algorithm="fricas")

[Out] 1/3\*(-x^3 + 1)^(2/3)\*x - 2/9\*sqrt(3)\*arctan(-1/3\*(sqrt(3)\*x - 2\*sqrt(3)\*(-x^3 + 1)^(1/3))/x) + 2/9\*log((x + (-x^3 + 1)^(1/3))/x) - 1/9\*log((x^2 - (-x^3 + 1)^(1/3)\*x + (-x^3 + 1)^(1/3))/x^2)

$$3 + 1)^{(1/3)} * x + (-x^3 + 1)^{(2/3)} / x^2$$

**Sympy [C]** time = 1.15963, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| x^3 e^{2i\pi} \right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3),x)

[Out] x\*gamma(1/3)\*hyper((-2/3, 1/3), (4/3,), x\*\*3\*exp\_polar(2\*I\*pi))/(3\*gamma(4/3))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (-x^3 + 1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(2/3), x)

$$3.107 \quad \int \frac{(1-x^3)^{2/3}}{x} dx$$

**Optimal.** Leaf size=70

$$\frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

**Rubi [A]** time = 0.038365, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(2/3)/x,x]

[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2\*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 50

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 55

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(1/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, -Simp[Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(1-x^3)^{2/3}}{x} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{(1-x)^{2/3}}{x} dx, x, x^3 \right) \\ &= \frac{1}{2} (1-x^3)^{2/3} + \frac{1}{3} \text{Subst} \left( \int \frac{1}{\sqrt[3]{1-xx}} dx, x, x^3 \right) \\ &= \frac{1}{2} (1-x^3)^{2/3} - \frac{\log(x)}{2} - \frac{1}{2} \text{Subst} \left( \int \frac{1}{1-x} dx, x, \sqrt[3]{1-x^3} \right) + \frac{1}{2} \text{Subst} \left( \int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1-x^3} \right) \\ &= \frac{1}{2} (1-x^3)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^3} \right) - \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1-x^3} \right) \\ &= \frac{1}{2} (1-x^3)^{2/3} + \frac{\tan^{-1} \left( \frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2} \log \left( 1 - \sqrt[3]{1-x^3} \right) \end{aligned}$$

**Mathematica [A]** time = 0.0384837, size = 65, normalized size = 0.93

$$\frac{1}{2} \left( (1-x^3)^{2/3} + \log \left( 1 - \sqrt[3]{1-x^3} \right) - \log(x) \right) + \frac{\tan^{-1} \left( \frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/x, x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ((1 - x^3)^(2/3) - Log[x] + Log[1 - (1 - x^3)^(1/3)])/2
```

**Maple [C]** time = 0.035, size = 66, normalized size = 0.9

$$-\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{9\pi} \left( -\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)} \left( \frac{3}{2} - \frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 3\ln(x) + i\pi \right) + \frac{2\pi\sqrt{3}x^3}{3\Gamma(2/3)} {}_3F_2\left(\frac{1}{3}, 1, 1; 2, 2; x^3\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/x, x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3, 1, 1], [2, 2]
```

,x^3))

**Maxima [A]** time = 1.39611, size = 99, normalized size = 1.41

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^3 + 1)^{\frac{1}{3}} + 1\right)\right) + \frac{1}{2} (-x^3 + 1)^{\frac{2}{3}} - \frac{1}{6} \log\left(\left(-x^3 + 1\right)^{\frac{2}{3}} + \left(-x^3 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left(\left(-x^3 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(-x^3 + 1)^(1/3) + 1)) + 1/2\*(-x^3 + 1)^(2/3) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)

**Fricas [A]** time = 1.55922, size = 230, normalized size = 3.29

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) + \frac{1}{2} (-x^3 + 1)^{\frac{2}{3}} - \frac{1}{6} \log\left(\left(-x^3 + 1\right)^{\frac{2}{3}} + \left(-x^3 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left(\left(-x^3 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(2/3\*sqrt(3)\*(-x^3 + 1)^(1/3) + 1/3\*sqrt(3)) + 1/2\*(-x^3 + 1)^(2/3) - 1/6\*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3\*log((-x^3 + 1)^(1/3) - 1)

**Sympy [C]** time = 1.02588, size = 41, normalized size = 0.59

$$\frac{x^2 e^{\frac{2i\pi}{3}} \Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix} \middle| \frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3)/x,x)

[Out] -x\*\*2\*exp(2\*I\*pi/3)\*gamma(-2/3)\*hyper((-2/3, -2/3), (1/3,), x\*\*(-3))/(3\*gamma(1/3))

**Giac [A]** time = 1.09656, size = 100, normalized size = 1.43

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^3 + 1)^{\frac{1}{3}} + 1\right)\right) + \frac{1}{2} (-x^3 + 1)^{\frac{2}{3}} - \frac{1}{6} \log\left(\left(-x^3 + 1\right)^{\frac{2}{3}} + \left(-x^3 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left(\left(-x^3 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```



$$3.108 \quad \int \frac{(1-x^3)^{2/3}}{a+bx} dx$$

**Optimal.** Leaf size=384

$$-\frac{x^2 (a^3 + b^3) F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -\frac{b^3 x^3}{a^3}\right)}{2a^2 b^2} - \frac{a^2 \log\left(\sqrt[3]{1-x^3} + x\right)}{2b^3} - \frac{(a^3 + b^3)^{2/3} \log(a^3 + b^3 x^3)}{3b^3} + \frac{(a^3 + b^3)^{2/3} \log\left(-\frac{x \sqrt[3]{a^3}}{a}\right)}{2b^3}$$

```
[Out] (1 - x^3)^(2/3)/(2*b) - ((a^3 + b^3)*x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -(
(b^3*x^3)/a^3)])/(2*a^2*b^2) + (a^2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^3) - ((a^3 + b^3)^(2/3)*ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*x
)/(a*(1 - x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^3) + ((a^3 + b^3)^(2/3)*ArcTan[
(1 + (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^3) + (a*
x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b^2) - ((a^3 + b^3)^(2/3)*Log
[a^3 + b^3*x^3])/(3*b^3) + ((a^3 + b^3)^(2/3)*Log[-((a^3 + b^3)^(1/3)*x)/a
] - (1 - x^3)^(1/3))/(2*b^3) - (a^2*Log[x + (1 - x^3)^(1/3)])/(2*b^3) + ((
a^3 + b^3)^(2/3)*Log[(a^3 + b^3)^(1/3) - b*(1 - x^3)^(1/3)])/(2*b^3)
```

**Rubi [F]** time = 0.0682516, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{a+bx} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x), x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(a + b*x), x]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{a+bx} dx = \int \frac{(1-x^3)^{2/3}}{a+bx} dx$$

**Mathematica [F]** time = 0.34462, size = 0, normalized size = 0.

$$\int \frac{(1-x^3)^{2/3}}{a+bx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a} (-x^3+1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3)/(b\*x+a),x)

[Out] int((-x^3+1)^(2/3)/(b\*x+a),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(b\*x+a),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)/(b\*x + a), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(b\*x+a),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3)/(b\*x+a),x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1)\*\*(2/3)/(a + b\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

$$3.109 \quad \int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

**Optimal.** Leaf size=234

$$\frac{1}{3}x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{2(1-x^3)^{2/3}x^2}{3(x^3+1)} + \frac{(1-x^3)^{2/3}x}{3(x^3+1)} - \frac{(1-x^3)^{2/3}}{3(x^3+1)} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \dots$$

[Out]  $-(1-x^3)^{2/3}/(3(1+x^3)) + (x(1-x^3)^{2/3})/(3(1+x^3)) + (2x^2(1-x^3)^{2/3})/(3(1+x^3)) - (2^{2/3}\text{ArcTan}[(1-(2^{1/3}x)/(1-x^3)^{1/3})/\text{Sqrt}[3]])/(3\text{Sqrt}[3]) - (2^{2/3}\text{ArcTan}[(1+2^{2/3}(1-x^3)^{1/3})/\text{Sqrt}[3]])/(3\text{Sqrt}[3]) + (x^2\text{Hypergeometric2F1}[1/3, 2/3, 5/3, x^3])/3 - \text{Log}[2^{1/3} - (1-x^3)^{1/3}]/(3\cdot 2^{1/3}) + \text{Log}[-(2^{1/3}x) - (1-x^3)^{1/3}]/(3\cdot 2^{1/3})$

**Rubi [F]** time = 0.414851, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]

[Out]  $(-4\text{Defer}[\text{Int}[(1-x^3)^{2/3}/(1+I\text{Sqrt}[3]-2x)^2, x])/3 + (((4I)/3)\text{Defer}[\text{Int}[(1-x^3)^{2/3}/(1+I\text{Sqrt}[3]-2x), x)]/\text{Sqrt}[3] - (4\text{Defer}[\text{Int}[(1-x^3)^{2/3}/(-1+I\text{Sqrt}[3]+2x)^2, x])/3 + (((4I)/3)\text{Defer}[\text{Int}[(1-x^3)^{2/3}/(-1+I\text{Sqrt}[3]+2x), x)]/\text{Sqrt}[3]$

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} dx = \int \left( -\frac{4(1-x^3)^{2/3}}{3(1+i\sqrt{3}-2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(1+i\sqrt{3}-2x)} - \frac{4(1-x^3)^{2/3}}{3(-1+i\sqrt{3}+2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(-1+i\sqrt{3}+2x)} \right) dx$$

$$= -\left( \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(1+i\sqrt{3}-2x)^2} dx \right) - \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}+2x)^2} dx + \frac{(4i) \int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}-2x} dx}{3\sqrt{3}} + \frac{(4i) \int \frac{(1-x^3)^{2/3}}{-1+i\sqrt{3}+2x} dx}{3\sqrt{3}}$$

**Mathematica [F]** time = 0.45237, size = 0, normalized size = 0.

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]

[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^2} (-x^3 + 1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)

[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{(x^2 - x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3 + 1)^{\frac{2}{3}}}{x^4 - 2x^3 + 3x^2 - 2x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2\*x^3 + 3\*x^2 - 2\*x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x^2-x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3)/(x\*\*2-x+1)\*\*2,x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1)\*\*(2/3)/(x\*\*2 - x + 1)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{(x^2 - x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)

$$3.110 \quad \int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

**Optimal.** Leaf size=199

$$\frac{(1-x^3)^{2/3}}{x^2-x+1} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}} + \log\left(\sqrt[3]{1-x^3}+x\right) - \frac{2 \tan^{-1}\left(\frac{1-\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3} \tan^{-1}\left(\frac{1-\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out]  $(1-x^3)^{(2/3)}/(1-x+x^2) - (2*\text{ArcTan}[(1-(2*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]])/\text{Sqrt}[3] + (2^{(2/3)}*\text{ArcTan}[(1-(2*2^{(1/3)}*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]])/\text{Sqrt}[3] + (2^{(2/3)}*\text{ArcTan}[(1+2^{(2/3)}*(1-x^3)^{(1/3)})/\text{Sqrt}[3]])/\text{Sqrt}[3] + \text{Log}[2^{(1/3)}-(1-x^3)^{(1/3)}/2^{(1/3)} - \text{Log}[-(2^{(1/3)}*x)-(1-x^3)^{(1/3)}/2^{(1/3)}] + \text{Log}[x+(1-x^3)^{(1/3)}]$

**Rubi [F]** time = 0.945257, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((1-2\*x)\*(1-x^3)^(2/3))/(1-x+x^2)^2,x]

[Out]  $(-4*\text{Defer}[\text{Int}][(1-x^3)^{(2/3)}/(1+I*\text{Sqrt}[3]-2*x)^2,x])/3 + (4*(1+I*\text{Sqrt}[3])*\text{Defer}[\text{Int}][(1-x^3)^{(2/3)}/(1+I*\text{Sqrt}[3]-2*x)^2,x])/3 - (4*\text{Defer}[\text{Int}][(1-x^3)^{(2/3)}/(-1+I*\text{Sqrt}[3]+2*x)^2,x])/3 + (4*(1-I*\text{Sqrt}[3])*\text{Defer}[\text{Int}][(1-x^3)^{(2/3)}/(-1+I*\text{Sqrt}[3]+2*x)^2,x])/3$

Rubi steps

$$\begin{aligned} \int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx &= \int \left( \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} - \frac{2x(1-x^3)^{2/3}}{(1-x+x^2)^2} \right) dx \\ &= - \left( 2 \int \frac{x(1-x^3)^{2/3}}{(1-x+x^2)^2} dx \right) + \int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} dx \\ &= - \left( 2 \int \left( -\frac{2(1+i\sqrt{3})(1-x^3)^{2/3}}{3(1+i\sqrt{3}-2x)^2} + \frac{2i(1-x^3)^{2/3}}{3\sqrt{3}(1+i\sqrt{3}-2x)} - \frac{2(1-i\sqrt{3})(1-x^3)^{2/3}}{3(-1+i\sqrt{3}+2x)^2} + \frac{2i(1-x^3)^{2/3}}{3\sqrt{3}(-1+i\sqrt{3}+2x)} \right) dx \right) \\ &= - \left( \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(1+i\sqrt{3}-2x)^2} dx \right) - \frac{4}{3} \int \frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}+2x)^2} dx + \frac{1}{3} (4(1-i\sqrt{3})) \int \frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}+2x)} dx \end{aligned}$$

**Mathematica [F]** time = 0.34793, size = 0, normalized size = 0.

$$\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((1 - 2\*x)\*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]

[Out] Integrate[((1 - 2\*x)\*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1-2x}{(x^2-x+1)^2} (-x^3+1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-2\*x)\*(-x^3+1)^(2/3)/(x^2-x+1)^2, x)

[Out] int((1-2\*x)\*(-x^3+1)^(2/3)/(x^2-x+1)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(-x^3+1)^{\frac{2}{3}}(2x-1)}{(x^2-x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2\*x)\*(-x^3+1)^(2/3)/(x^2-x+1)^2, x, algorithm="maxima")

[Out] -integrate((-x^3 + 1)^(2/3)\*(2\*x - 1)/(x^2 - x + 1)^2, x)

**Fricas [B]** time = 14.5052, size = 5191, normalized size = 26.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2\*x)\*(-x^3+1)^(2/3)/(x^2-x+1)^2, x, algorithm="fricas")

[Out] 
$$-1/72*(8*4^{1/3}*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^{2/3}*sqrt(3)*(50*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^{2/3} + 7644*4^{1/3}*sqrt(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^{1/3} - 7*sqrt(39)*(6*4^{1/3}*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898)*(-x^3 + 1)^{2/3} - 4^{2/3}*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 17254*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^{1/3})*sqrt((6*4^{1/3}*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^{2/3} + 4^{2/3}*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^{1/3}))/((x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x - 29747))/((138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2 - 70446*x - 256859)) + 8*4^{1/3}*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^{2/3}*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^{2/3} - 76$$



```

44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
- 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3)
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
- 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1)
*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1} dx - \int \frac{2x(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2\*x)\*(-x\*\*3+1)\*\*(2/3)/(x\*\*2-x+1)\*\*2,x)

[Out] -Integral(-(1 - x\*\*3)\*\*(2/3)/(x\*\*4 - 2\*x\*\*3 + 3\*x\*\*2 - 2\*x + 1), x) - Integ  
ral(2\*x\*(1 - x\*\*3)\*\*(2/3)/(x\*\*4 - 2\*x\*\*3 + 3\*x\*\*2 - 2\*x + 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(-x^3 + 1)^{\frac{2}{3}}(2x - 1)}{(x^2 - x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

$$3.111 \quad \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

**Optimal.** Leaf size=177

$$\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3} + x\right) + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{2\sqrt[3]{2}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \dots$$

[Out]  $(1 - x^3)^{(2/3)}/2 - (\text{Sqrt}[3]*\text{ArcTan}[(1 + (2^{(1/3)}*(1 - x))/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]])/2^{(1/3)} + \text{ArcTan}[(1 - (2*x)/(1 - x^3)^{(1/3)})/\text{Sqrt}[3]]/\text{Sqrt}[3] + (x^2*\text{Hypergeometric2F1}[1/3, 2/3, 5/3, x^3])/2 - \text{Log}[(1 - x)*(1 + x)^2]/(2*2^{(1/3)}) - \text{Log}[x + (1 - x^3)^{(1/3)}]/2 + (3*\text{Log}[-1 + x + 2^{(2/3)}*(1 - x^3)^{(1/3)}])/(2*2^{(1/3)})$

**Rubi [F]** time = 0.0599141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(2/3)/(1 + x), x]

[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx = \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

**Mathematica [F]** time = 0.389881, size = 0, normalized size = 0.

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(2/3)/(1 + x), x]

[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} (-x^3 + 1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^3+1)^(2/3)/(1+x),x)`

[Out] `int((-x^3+1)^(2/3)/(1+x),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(2/3)/(x + 1), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3 + 1)^{\frac{2}{3}}}{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")`

[Out] `integral((-x^3 + 1)^(2/3)/(x + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)**(2/3)/(1+x),x)`

[Out] `Integral((-x - 1)*(x**2 + x + 1)**(2/3)/(x + 1), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

$$3.112 \quad \int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$$

**Optimal.** Leaf size=177

$$\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3} + x\right) + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{2\sqrt[3]{2}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2(1-x)}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \dots$$

[Out]  $(1-x^3)^{2/3}/2 - (\text{Sqrt}[3]*\text{ArcTan}[(1+(2^{1/3}*(1-x))/(1-x^3)^{1/3}))/\text{Sqrt}[3])/2^{1/3} + \text{ArcTan}[(1-(2*x)/(1-x^3)^{1/3}))/\text{Sqrt}[3]]/\text{Sqrt}[3] + (x^2*\text{Hypergeometric2F1}[1/3, 2/3, 5/3, x^3])/2 - \text{Log}[(1-x)*(1+x)^2]/(2*2^{1/3}) - \text{Log}[x+(1-x^3)^{1/3}]/2 + (3*\text{Log}[-1+x+2^{2/3}*(1-x^3)^{1/3}])/(2*2^{1/3})$

**Rubi [F]** time = 0.0706373, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((1-x+x^2)\*(1-x^3)^(2/3))/(1+x^3), x]

[Out] Defer[Int] [(1-x^3)^(2/3)/(1+x), x]

Rubi steps

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx = \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

**Mathematica [F]** time = 0.150094, size = 0, normalized size = 0.

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((1-x+x^2)\*(1-x^3)^(2/3))/(1+x^3), x]

[Out] Integrate[((1-x+x^2)\*(1-x^3)^(2/3))/(1+x^3), x]

**Maple [F]** time = 0.011, size = 0, normalized size = 0.

$$\int \frac{x^2-x+1}{x^3+1} (-x^3+1)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)`

[Out] `int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}(x^2 - x + 1)}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3 + 1)^{\frac{2}{3}}}{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")`

[Out] `integral((-x^3 + 1)^(2/3)/(x + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)`

[Out] `Integral((-x - 1)*(x**2 + x + 1)**(2/3)/(x + 1), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}(x^2 - x + 1)}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```



$$3.113 \quad \int \frac{(1-x^3)^{2/3}}{1+x^3} dx$$

**Optimal.** Leaf size=132

$$\frac{\log(x^3+1)}{3\sqrt[3]{2}} + \frac{\log(-\sqrt[3]{1-x^3}-\sqrt[3]{2x})}{\sqrt[3]{2}} - \frac{1}{2} \log(\sqrt[3]{1-x^3}+x) + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] ArcTan[(1 - (2\*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)\*ArcTan[(1 - (2\*2^(1/3)\*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - Log[1 + x^3]/(3\*2^(1/3)) + Log[-(2^(1/3)\*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2

**Rubi [C]** time = 0.0082518, antiderivative size = 21, normalized size of antiderivative = 0.16, number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {429}

$$x F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[(1 - x^3)^(2/3)/(1 + x^3), x]

[Out] x\*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3]

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x^3} dx = x F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)$$

**Mathematica [C]** time = 0.0937368, size = 111, normalized size = 0.84

$$\frac{4x(1-x^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3}; -\frac{2}{3}, 2; \frac{7}{3}; x^3, -x^3\right) + 2F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; x^3, -x^3\right)\right) - 4F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3), x]

[Out] (-4\*x\*(1 - x^3)^(2/3)\*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)\*(-4\*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3] + x^3\*(3\*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2\*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])))

3, x^3, -x^3] + 2\*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3]))))

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1} (-x^3+1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3)/(x^3+1),x)

[Out] int((-x^3+1)^(2/3)/(x^3+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)

**Fricas [A]** time = 1.6769, size = 532, normalized size = 4.03

$$-\frac{1}{3} \cdot 4^{\frac{1}{3}} \sqrt{3} \arctan\left(-\frac{\sqrt{3}x - 4^{\frac{1}{3}} \sqrt{3} (-x^3+1)^{\frac{1}{3}}}{3x}\right) + \frac{1}{3} \sqrt{3} \arctan\left(-\frac{\sqrt{3}x - 2 \sqrt{3} (-x^3+1)^{\frac{1}{3}}}{3x}\right) + \frac{1}{3} \cdot 4^{\frac{1}{3}} \log\left(\frac{4^{\frac{2}{3}}x + 2(-x^3+1)^{\frac{1}{3}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/3\*4^(1/3)\*sqrt(3)\*arctan(-1/3\*(sqrt(3)\*x - 4^(1/3)\*sqrt(3)\*(-x^3 + 1)^(1/3))/x) + 1/3\*sqrt(3)\*arctan(-1/3\*(sqrt(3)\*x - 2\*sqrt(3)\*(-x^3 + 1)^(1/3))/x) + 1/3\*4^(1/3)\*log((4^(2/3)\*x + 2\*(-x^3 + 1)^(1/3))/x) - 1/6\*4^(1/3)\*log((2\*4^(1/3)\*x^2 - 4^(2/3)\*(-x^3 + 1)^(1/3)\*x + 2\*(-x^3 + 1)^(2/3))/x^2) - 1/3\*log((x + (-x^3 + 1)^(1/3))/x) + 1/6\*log((x^2 - (-x^3 + 1)^(1/3)\*x + (-x^3 + 1)^(2/3))/x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(2/3)/(x\*\*3+1),x)

[Out] Integral((-x - 1)\*(x\*\*2 + x + 1)\*\*(2/3)/((x + 1)\*(x\*\*2 - x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}}}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)

$$3.114 \quad \int \frac{x(1-x^3)^{2/3}}{1+x^3} dx$$

**Optimal.** Leaf size=250

$$-\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3} \log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{2\sqrt[3]{2}} + \frac{2^{2/3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3} + x - 1}{\sqrt[3]{2}}\right)}{2\sqrt[3]{2}}$$

[Out] (2^(2/3)\*ArcTan[(1 - (2\*2^(1/3)\*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)\*Sqrt[3]) - (x^2\*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)\*(1 + x)^2/(6\*2^(1/3)) + Log[1 + (2^(2/3)\*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/(3\*2^(1/3)) - (2^(2/3)\*Log[1 + (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)])]/3 - Log[-1 + x + 2^(2/3)\*(1 - x^3)^(1/3)]/(2\*2^(1/3))

**Rubi [C]** time = 0.0119922, antiderivative size = 26, normalized size of antiderivative = 0.1, number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {510}

$$\frac{1}{2}x^2 F_1\left(\frac{2}{3}; -\frac{2}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Int[(x\*(1 - x^3)^(2/3))/(1 + x^3), x]

[Out] (x^2\*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2

### Rule 510

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(a^p\*c^q\*(e\*x)^(m + 1)\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3} dx = \frac{1}{2}x^2 F_1\left(\frac{2}{3}; -\frac{2}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

**Mathematica [C]** time = 0.0135238, size = 26, normalized size = 0.1

$$\frac{1}{2}x^2 F_1\left(\frac{2}{3}; -\frac{2}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*(1 - x^3)^(2/3))/(1 + x^3), x]

[Out]  $(x^2 \text{AppellF1}[2/3, -2/3, 1, 5/3, x^3, -x^3])/2$

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x}{x^3 + 1} (-x^3 + 1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-x^3+1)^(2/3)/(x^3+1),x)`

[Out] `int(x*(-x^3+1)^(2/3)/(x^3+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}} x}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-x^3 + 1)^{\frac{2}{3}} x}{x^3 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")`

[Out] `integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-x-1)(x^2+x+1)^{\frac{2}{3}}}{(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)`

[Out] `Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{2}{3}} x}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")`

[Out] `integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)`

$$3.115 \quad \int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3} dx$$

**Optimal.** Leaf size=383

$$\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) - \frac{\log(x^3+1)}{3\sqrt[3]{2}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} + \frac{1}{3}2^{2/3} \log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}\right)}{\sqrt[3]{2}}$$

[Out] -((2^(2/3)\*ArcTan[(1 - (2\*2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3])/Sqrt[3]) - ArcTan[(1 + (2^(1/3))\*(1 - x))/(1 - x^3)^(1/3)]/Sqrt[3]/(2^(1/3)\*Sqrt[3]) + ArcTan[(1 - (2\*x))/(1 - x^3)^(1/3)]/Sqrt[3]/Sqrt[3] - (2^(2/3)\*ArcTan[(1 - (2\*2^(1/3)\*x))/(1 - x^3)^(1/3)]/Sqrt[3])/Sqrt[3] + (x^2\*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)\*(1 + x)^2]/(6\*2^(1/3)) - Log[1 + x^3]/(3\*2^(1/3)) - Log[1 + (2^(2/3)\*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)]/(3\*2^(1/3)) + (2^(2/3)\*Log[1 + (2^(1/3)\*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[-(2^(1/3)\*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2 + Log[-1 + x + 2^(2/3)\*(1 - x^3)^(1/3)]/(2\*2^(1/3))

**Rubi [F]** time = 0.398948, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((1 - x)\*(1 - x^3)^(2/3))/(1 + x^3), x]

[Out] (-2\*Defer[Int][(1 - x^3)^(2/3)/(-1 - x), x])/3 - ((1 + (-1)^(2/3))\*Defer[Int][(1 - x^3)^(2/3)/(-1 + (-1)^(1/3)\*x), x])/3 - ((1 - (-1)^(1/3))\*Defer[Int][(1 - x^3)^(2/3)/(-1 - (-1)^(2/3)\*x), x])/3

**Rubi steps**

$$\begin{aligned} \int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3} dx &= \int \left( -\frac{2(1-x^3)^{2/3}}{3(-1-x)} + \frac{(-1-(-1)^{2/3})(1-x^3)^{2/3}}{3(-1+\sqrt[3]{-1}x)} + \frac{(-1+\sqrt[3]{-1})(1-x^3)^{2/3}}{3(-1-(-1)^{2/3}x)} \right) dx \\ &= -\left( \frac{2}{3} \int \frac{(1-x^3)^{2/3}}{-1-x} dx \right) + \frac{1}{3}(-1+\sqrt[3]{-1}) \int \frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}x} dx + \frac{1}{3}(-1-(-1)^{2/3}) \int \frac{(1-x^3)^{2/3}}{-1+\sqrt[3]{-1}x} dx \end{aligned}$$

**Mathematica [C]** time = 0.15278, size = 138, normalized size = 0.36

$$-\frac{1}{2}x^2 {}_2F_1\left(\frac{2}{3}, -\frac{2}{3}, 1; \frac{5}{3}; x^3, -x^3\right) - \frac{4(1-x^3)^{2/3} {}_2F_1\left(\frac{1}{3}, -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}{(x^3+1)\left(x^3\left(3 {}_3F_1\left(\frac{4}{3}, -\frac{2}{3}, 2; \frac{7}{3}; x^3, -x^3\right) + 2 {}_2F_1\left(\frac{4}{3}, \frac{1}{3}, 1; \frac{7}{3}; x^3, -x^3\right)\right) - 4 {}_2F_1\left(\frac{1}{3}, -\frac{2}{3}, 1; \frac{4}{3}; x^3, -x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 - x)\*(1 - x^3)^(2/3))/(1 + x^3),x]

[Out]  $-(x^2 \operatorname{AppellF1}[2/3, -2/3, 1, 5/3, x^3, -x^3])/2 - (4*x*(1 - x^3)^{2/3} \operatorname{AppellF1}[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*\operatorname{AppellF1}[1/3, -2/3, 1, 4/3, x^3, -x^3] + x^3*(3*\operatorname{AppellF1}[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*\operatorname{AppellF1}[4/3, 1/3, 1, 7/3, x^3, -x^3])))$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1-x}{x^3+1} (-x^3+1)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)\*(-x^3+1)^(2/3)/(x^3+1),x)

[Out] int((1-x)\*(-x^3+1)^(2/3)/(x^3+1),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(-x^3+1)^{\frac{2}{3}}(x-1)}{x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)\*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] -integrate((-x^3 + 1)^(2/3)\*(x - 1)/(x^3 + 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(-x^3+1)^{\frac{2}{3}}(x-1)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)\*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] integral(-(-x^3 + 1)^(2/3)\*(x - 1)/(x^3 + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int -\frac{(1-x^3)^{\frac{2}{3}}}{x^3+1} dx - \int \frac{x(1-x^3)^{\frac{2}{3}}}{x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(-x^3 + 1)^{\frac{2}{3}}(x - 1)}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

$$3.116 \quad \int \frac{\sqrt[3]{1-x^3}}{1+x^3} dx$$

**Optimal.** Leaf size=272

$$\frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3 \cdot 2^{2/3}} + \frac{1}{3} \sqrt[3]{2} \log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}} + 2\sqrt[3]{2}\right)}{6 \cdot 2^{2/3}} + \frac{\sqrt[3]{2} \arctan\left(\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
+ ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3)*
(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))
+ (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3) +
(1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3))
```

**Rubi [C]** time = 0.0078488, antiderivative size = 21, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {429}

$$x F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3), x]
```

```
[Out] x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3]
```

#### Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3} dx = x F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)$$

**Mathematica [C]** time = 0.0882679, size = 109, normalized size = 0.4

$$\frac{4x\sqrt[3]{1-x^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; x^3, -x^3\right) + F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; x^3, -x^3\right)\right) - 4F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3), x]
```

[Out]  $(-4*x*(1 - x^3)^{(1/3)}*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 + 1} \sqrt[3]{-x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^3+1)^(1/3)/(x^3+1),x)`

[Out] `int((-x^3+1)^(1/3)/(x^3+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)`

**Fricas [A]** time = 21.8302, size = 902, normalized size = 3.32

$$\frac{1}{18} \sqrt[3]{32} \arctan \left( \frac{6 \sqrt[3]{32} (x^{16} - 33x^{13} + 110x^{10} - 110x^7 + 33x^4 - x)(-x^3 + 1)^{\frac{1}{3}} - 24 \sqrt[3]{32} (x^{14} - 2x^{11} - 6x^8 - 2x^5 + x^2)(-x^3 + 1)^{\frac{2}{3}} - \sqrt{3}(x^{18} + 42x^{15} - 417x^{12} + 812x^9 - 417x^6 + 42x^3 + 1)}{3(x^{18} - 102x^{15} + 447x^{12} - 628x^9 + 447x^6 - 102x^3 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")`

[Out]  $\frac{1}{18} \sqrt{3} * 2^{(1/3)} * \arctan(-1/3 * (6 * \sqrt{3} * 2^{(2/3)} * (x^{16} - 33 * x^{13} + 110 * x^{10} - 110 * x^7 + 33 * x^4 - x) * (-x^3 + 1)^{(1/3)} - 24 * \sqrt{3} * 2^{(1/3)} * (x^{14} - 2 * x^{11} - 6 * x^8 - 2 * x^5 + x^2) * (-x^3 + 1)^{(2/3)} - \sqrt{3} * (x^{18} + 42 * x^{15} - 417 * x^{12} + 812 * x^9 - 417 * x^6 + 42 * x^3 + 1)) / (x^{18} - 102 * x^{15} + 447 * x^{12} - 628 * x^9 + 447 * x^6 - 102 * x^3 + 1)) + 1/18 * 2^{(1/3)} * \log(-12 * (-x^3 + 1)^{(2/3)} * x^2 + 2^{(2/3)} * (x^6 + 2 * x^3 + 1) - 6 * 2^{(1/3)} * (x^4 - x) * (-x^3 + 1)^{(1/3)}) / (x^6 + 2 * x^3 + 1)) - 1/36 * 2^{(1/3)} * \log((12 * 2^{(2/3)} * (x^8 - 4 * x^5 + x^2) * (-x^3 + 1)^{(2/3)} + 2^{(1/3)} * (x^{12} - 32 * x^9 + 78 * x^6 - 32 * x^3 + 1) + 6 * (x^{10} - 11 * x^7 + 11 * x^4 - x) * (-x^3 + 1)^{(1/3)}) / (x^{12} + 4 * x^9 + 6 * x^6 + 4 * x^3 + 1))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{(x+1)(x^2-x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*3+1)\*\*(1/3)/(x\*\*3+1),x)

[Out] Integral((- (x - 1)\*(x\*\*2 + x + 1))\*\*(1/3)/((x + 1)\*(x\*\*2 - x + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34         #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```