Computer algebra independent integration tests

0-Independent-test-suites/Welz-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 86.21 (100)	% 13.79 (16)
Mathematica	% 81.9 (95)	% 18.1 (21)
Maple	% 51.72 (60)	% 48.28 (56)
Maxima	% 16.38 (19)	% 83.62 (97)
Fricas	% 75.86 (88)	% 24.14 (28)
Sympy	% 25. (29)	% 75. (87)
Giac	% 14.66 (17)	% 85.34 (99)

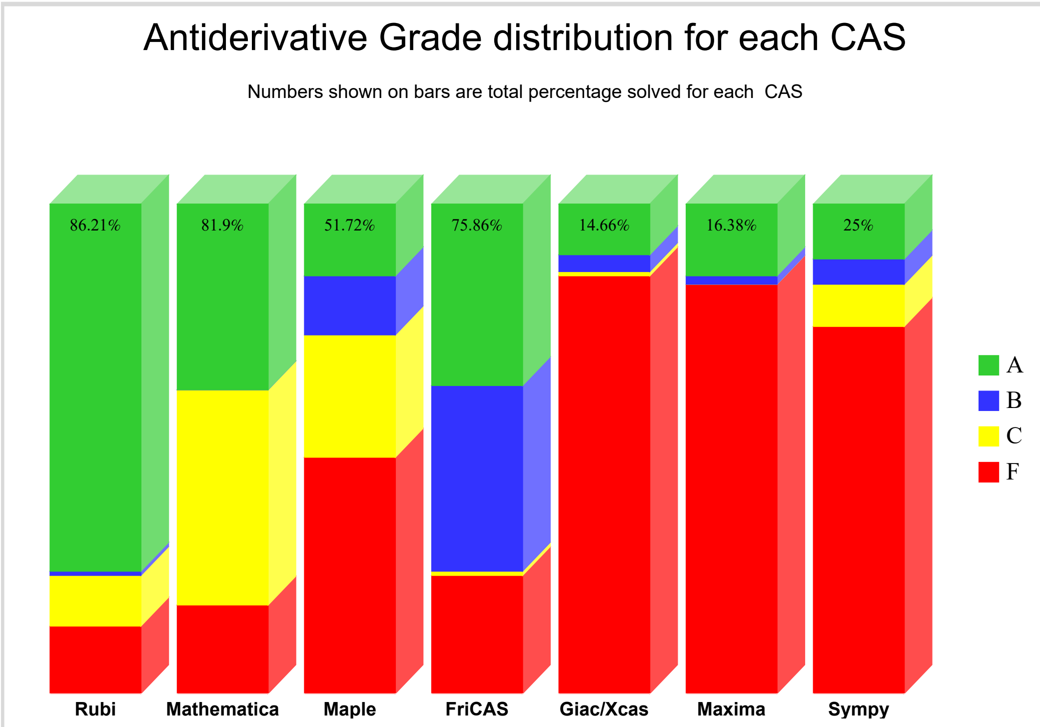
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

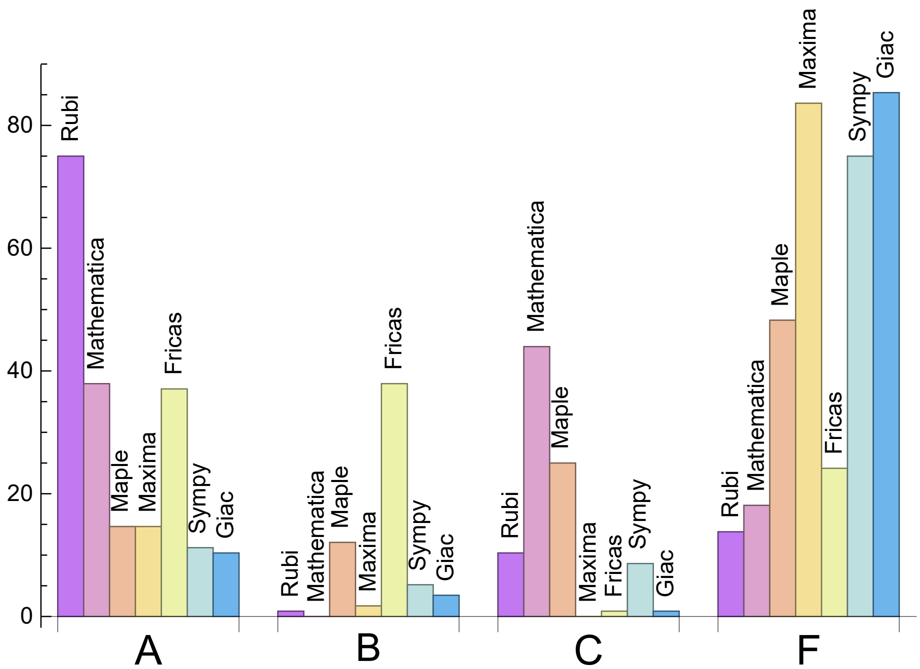
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	75.	0.86	10.34	13.79
Mathematica	37.93	0.	43.97	18.1
Maple	14.66	12.07	25.	48.28
Maxima	14.66	1.72	0.	83.62
Fricas	37.07	37.93	0.86	24.14
Sympy	11.21	5.17	8.62	75.
Giac	10.34	3.45	0.86	85.34

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	144.1	6.45	81.	1.
Mathematica	0.71	211.54	3.69	86.	0.99
Maple	0.12	2061.15	11.33	144.5	1.56
Maxima	1.21	108.53	1.57	84.	1.41
Fricas	13.72	2890.17	17.35	491.	5.07
Sympy	3.27	202.38	4.97	37.	0.82
Giac	1.09	100.71	1.95	86.	1.48

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {9, 10, 82, 98, 113, 114, 116}

Mathematica {4, 9, 10, 39, 40, 51, 53, 54, 55, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

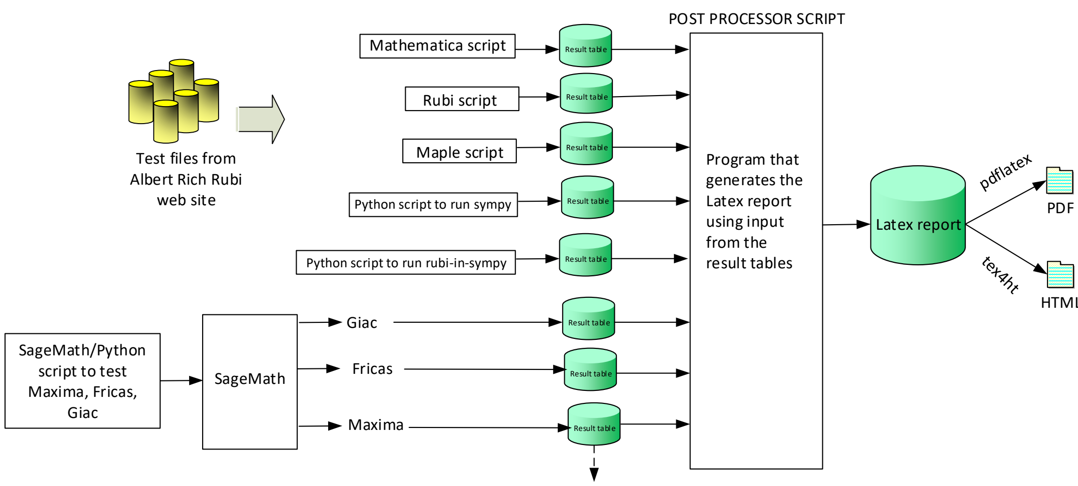
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 105, 106, 107 }

B grade: { 10 }

C grade: { 2, 46, 52, 82, 83, 98, 100, 101, 102, 113, 114, 116 }

F grade: { 43, 44, 45, 58, 59, 60, 61, 95, 103, 104, 108, 109, 110, 111, 112, 115}

2.1.2 Mathematica

A grade: { 1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 57, 62, 85, 94, 96, 97, 99, 103, 104, 105, 107 }

B grade: { }

C grade: { 2, 4, 24, 39, 40, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116 }

F grade: { 12, 13, 37, 38, 44, 45, 46, 58, 59, 60, 61, 92, 93, 95, 100, 102, 108, 109, 110, 111, 112}

2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 63, 64, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 52, 56, 57, 66, 67, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 106, 107 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 58, 59, 60, 61, 69, 70, 71, 72, 77, 78, 79, 80, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 99, 107

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 47, 48, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 29, 38, 44, 45, 46, 49, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 94, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

2.1.6 Sympy

A grade: { 1, 2, 14, 15, 20, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.7 Giac

A grade: { 1, 8, 16, 21, 22, 23, 33, 34, 36, 57, 62, 107 }

B grade: { 2, 3, 4, 7 }

C grade: { 24 }

F grade: { 5, 6, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 25, 26, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	28	12	18
normalized size	1	1.	1.	0.93	1.2	1.87	0.8	1.2
time (sec)	N/A	0.001	0.004	0.002	0.925	2.09	0.055	1.072

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	55	4	42	55
normalized size	1	3.47	2.47	2.27	3.67	0.27	2.8	3.67
time (sec)	N/A	0.056	0.026	0.003	0.948	2.141	57.851	1.071

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	370	0	251	0	239
normalized size	1	1.	0.84	4.51	0.	3.06	0.	2.91
time (sec)	N/A	0.076	0.108	0.041	0.	2.081	0.	1.111

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	167	172	0	205	0	127
normalized size	1	1.	3.88	4.	0.	4.77	0.	2.95
time (sec)	N/A	0.012	3.397	0.047	0.	2.048	0.	1.087

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	126	115	0	309	0	0
normalized size	1	1.	1.7	1.55	0.	4.18	0.	0.
time (sec)	N/A	0.056	0.132	0.014	0.	2.128	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	59	125	72	267	0	0
normalized size	1	1.	0.92	1.95	1.12	4.17	0.	0.
time (sec)	N/A	0.026	0.059	0.025	1.429	2.192	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	75	45	0	215	0	136
normalized size	1	1.	1.56	0.94	0.	4.48	0.	2.83
time (sec)	N/A	0.013	0.107	0.017	0.	2.017	0.	1.088

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	0	55	53	27
normalized size	1	1.	1.	0.7	0.	1.83	1.77	0.9
time (sec)	N/A	0.082	0.041	0.003	0.	2.008	0.818	1.071

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	220	365	340	902	0	1287	0	0
normalized size	1	1.66	1.55	4.1	0.	5.85	0.	0.
time (sec)	N/A	0.506	0.688	0.123	0.	2.291	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	220	541	311	1542	0	1287	0	0
normalized size	1	2.46	1.41	7.01	0.	5.85	0.	0.
time (sec)	N/A	0.751	0.649	0.021	0.	2.36	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	125	278	0	678	0	0
normalized size	1	1.	0.91	2.01	0.	4.91	0.	0.
time (sec)	N/A	0.075	0.199	0.02	0.	2.281	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	1068	0	0
normalized size	1	1.	0.	0.	0.	8.54	0.	0.
time (sec)	N/A	0.185	0.239	0.026	0.	19.344	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	998	0	0
normalized size	1	1.	0.	0.	0.	12.32	0.	0.
time (sec)	N/A	0.162	0.176	0.022	0.	38.964	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	162	15	0
normalized size	1	1.	1.	0.	0.	5.23	0.48	0.
time (sec)	N/A	0.054	0.008	0.014	0.	4.026	1.281	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	0	85	15	0
normalized size	1	1.	1.	0.67	0.	2.58	0.45	0.
time (sec)	N/A	0.063	0.011	0.033	0.	4.074	0.706	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	78	56	20
normalized size	1	1.	1.	0.84	1.05	4.11	2.95	1.05
time (sec)	N/A	0.27	0.017	0.002	0.934	1.972	6.489	1.097

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	74	2149	0
normalized size	1	1.	0.83	2.31	0.	1.42	41.33	0.
time (sec)	N/A	0.023	0.039	0.026	0.	2.204	2.733	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	76	0	0
normalized size	1	1.	0.89	0.	0.	1.36	0.	0.
time (sec)	N/A	0.022	0.063	0.023	0.	2.155	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	34	313	0
normalized size	1	1.	1.	0.	0.	2.	18.41	0.
time (sec)	N/A	0.054	0.007	0.019	0.	1.992	3.679	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	35	36	0
normalized size	1	1.	1.	0.	0.	1.75	1.8	0.
time (sec)	N/A	0.06	0.006	0.028	0.	1.985	2.868	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	48	54	124	36	55
normalized size	1	1.	0.86	1.14	1.29	2.95	0.86	1.31
time (sec)	N/A	0.033	0.042	0.01	0.947	2.076	0.132	1.08

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	43	20	26
normalized size	1	1.	1.	0.95	1.23	1.95	0.91	1.18
time (sec)	N/A	0.024	0.02	0.002	0.935	1.956	0.103	1.075

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	69	135	51	100
normalized size	1	1.	0.79	0.82	1.11	2.18	0.82	1.61
time (sec)	N/A	0.091	0.065	0.012	0.955	2.189	0.158	1.079

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	961	454	0	1095	0	424
normalized size	1	1.	11.17	5.28	0.	12.73	0.	4.93
time (sec)	N/A	0.084	2.38	0.034	0.	2.264	0.	1.137

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	39	15	0
normalized size	1	1.	1.	0.	0.	2.05	0.79	0.
time (sec)	N/A	0.065	0.01	0.027	0.	2.025	0.206	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	47	27	0
normalized size	1	1.	1.	0.	0.	1.81	1.04	0.
time (sec)	N/A	0.097	0.017	0.015	0.	1.975	0.879	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	487	46	0
normalized size	1	1.	0.89	0.	0.	7.73	0.73	0.
time (sec)	N/A	0.219	0.192	0.019	0.	2.422	1.593	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	552	51	0
normalized size	1	1.	1.55	0.3	0.	6.73	0.62	0.
time (sec)	N/A	0.072	0.057	0.008	0.	2.562	1.438	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	606	679	412	0	699	0	0	0
normalized size	1	1.12	0.68	0.	1.15	0.	0.	0.
time (sec)	N/A	4.799	0.244	0.072	1.019	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	34	313	0
normalized size	1	1.	1.	0.	0.	2.	18.41	0.
time (sec)	N/A	0.055	0.009	0.026	0.	2.1	2.814	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	74	2149	0
normalized size	1	1.	0.88	2.31	0.	1.42	41.33	0.
time (sec)	N/A	0.024	0.06	0.022	0.	2.249	2.653	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	65	109	0	0
normalized size	1	1.	0.97	1.29	1.91	3.21	0.	0.
time (sec)	N/A	0.044	0.118	0.039	1.138	2.183	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	65	84	198	36	86
normalized size	1	1.	0.98	1.12	1.45	3.41	0.62	1.48
time (sec)	N/A	0.036	0.015	0.036	1.417	1.964	0.928	1.074

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	81	48	84	200	37	86
normalized size	1	1.	1.4	0.83	1.45	3.45	0.64	1.48
time (sec)	N/A	0.035	0.012	0.041	1.419	2.159	0.95	1.066

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	105	225	29	0
normalized size	1	1.	1.76	0.24	2.14	4.59	0.59	0.
time (sec)	N/A	0.005	0.04	0.023	1.44	2.032	0.894	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	84	198	32	85
normalized size	1	1.	1.	1.18	1.53	3.6	0.58	1.55
time (sec)	N/A	0.033	0.012	0.037	1.434	2.026	0.915	1.085

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	788	0	0
normalized size	1	1.	0.	0.	0.	8.12	0.	0.
time (sec)	N/A	0.044	0.076	0.033	0.	37.366	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.078	0.033	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	176	59	0	0	828	0	0
normalized size	1	1.6	0.54	0.	0.	7.53	0.	0.
time (sec)	N/A	0.025	0.025	0.074	0.	18.574	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	131	85	0	0	410	0	0
normalized size	1	1.62	1.05	0.	0.	5.06	0.	0.
time (sec)	N/A	0.073	0.014	0.011	0.	4.684	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	117	127	0	0	1153	0	0
normalized size	1	1.77	1.92	0.	0.	17.47	0.	0.
time (sec)	N/A	0.056	0.082	0.042	0.	13.516	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	145	140	0	0	1866	0	0
normalized size	1	1.84	1.77	0.	0.	23.62	0.	0.
time (sec)	N/A	0.098	0.167	0.012	0.	12.361	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	118	0	55	0	0	3509	0	0
normalized size	1	0.	0.47	0.	0.	29.74	0.	0.
time (sec)	N/A	21.937	0.193	0.014	0.	167.475	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	1.635	0.038	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.462	0.669	0.039	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	493	576	0	0	0	0	0	0
normalized size	1	1.17	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.846	0.382	0.177	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	198	584	0	5156	0	0
normalized size	1	1.	0.49	1.43	0.	12.67	0.	0.
time (sec)	N/A	0.676	2.075	0.089	0.	3.179	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	610	719	0	11992	0	0
normalized size	1	1.	0.94	1.11	0.	18.51	0.	0.
time (sec)	N/A	1.158	6.087	0.053	0.	3.69	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	989	0	0	0	0
normalized size	1	1.	1.04	0.93	0.	0.	0.	0.
time (sec)	N/A	2.49	6.17	0.08	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	342	21028	0	11537	0	0
normalized size	1	1.	0.9	55.63	0.	30.52	0.	0.
time (sec)	N/A	0.774	6.027	0.533	0.	4.008	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	30714	0	0
normalized size	1	1.	2.24	136.04	0.	48.14	0.	0.
time (sec)	N/A	1.397	11.52	3.525	0.	6.015	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	1272	0	0
normalized size	1	3.09	3.23	19.32	0.	19.27	0.	0.
time (sec)	N/A	1.234	1.148	0.094	0.	3.062	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.234	0.033	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.245	0.046	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	111	0	0	495	0	0
normalized size	1	1.	1.14	0.	0.	5.1	0.	0.
time (sec)	N/A	0.013	0.06	0.166	0.	20.738	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	107	20	69	142	262	32	0
normalized size	1	1.47	0.27	0.95	1.95	3.59	0.44	0.
time (sec)	N/A	0.044	0.003	0.046	1.428	2.053	1.106	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	90	49	96	225	37	97
normalized size	1	1.	1.34	0.73	1.43	3.36	0.55	1.45
time (sec)	N/A	0.037	0.024	0.033	1.465	1.953	0.945	1.146

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-2)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	482	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.355	0.048	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	280	0	0	0	0	10386	0	0
normalized size	1	0.	0.	0.	0.	37.09	0.	0.
time (sec)	N/A	0.235	0.107	0.086	0.	51.38	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-2)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.343	0.048	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.138	0.115	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	63	22	31
normalized size	1	1.	1.	0.96	1.24	2.52	0.88	1.24
time (sec)	N/A	0.023	0.008	0.001	0.937	2.302	0.097	1.078

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	158	73	0
normalized size	1	1.	1.46	0.88	0.	2.68	1.24	0.
time (sec)	N/A	0.032	0.018	0.024	0.	1.979	0.16	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	213	100	0
normalized size	1	1.	1.27	0.96	0.	2.73	1.28	0.
time (sec)	N/A	0.077	0.02	0.018	0.	1.942	0.167	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	138	0	0
normalized size	1	1.	2.24	2.04	0.	2.82	0.	0.
time (sec)	N/A	0.007	0.097	0.013	0.	2.516	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	173	0	0
normalized size	1	1.	2.04	6.89	0.	3.26	0.	0.
time (sec)	N/A	0.015	0.091	0.024	0.	2.422	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	5727	1512	0	957	0	0
normalized size	1	1.	76.36	20.16	0.	12.76	0.	0.
time (sec)	N/A	0.094	7.084	0.071	0.	2.568	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	6283	0	0
normalized size	1	1.	1.88	2.67	0.	36.74	0.	0.
time (sec)	N/A	0.08	0.432	0.072	0.	25.347	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0
normalized size	1	1.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.261	0.058	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.251	0.029	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.203	0.048	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.175	0.026	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	3299	0	0
normalized size	1	1.	0.22	1.29	0.	25.98	0.	0.
time (sec)	N/A	0.019	0.022	0.099	0.	5.567	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	4625	0	0
normalized size	1	1.	0.34	1.53	0.	29.46	0.	0.
time (sec)	N/A	0.033	0.029	0.106	0.	5.796	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	48	286	0	1447	0	0
normalized size	1	1.	0.65	3.86	0.	19.55	0.	0.
time (sec)	N/A	0.156	0.019	0.115	0.	3.586	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	1214	0	0
normalized size	1	1.	0.31	3.72	0.	11.79	0.	0.
time (sec)	N/A	0.305	0.029	0.16	0.	4.282	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	5180	0	0
normalized size	1	1.	1.56	0.	0.	63.95	0.	0.
time (sec)	N/A	0.011	0.1	0.048	0.	19.52	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	921	0	0
normalized size	1	1.	1.56	0.	0.	11.37	0.	0.
time (sec)	N/A	0.011	0.095	0.026	0.	15.508	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	5544	0	0
normalized size	1	1.	1.04	0.	0.	49.06	0.	0.
time (sec)	N/A	0.014	0.072	0.033	0.	11.962	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	4852	0	0
normalized size	1	1.	1.14	0.	0.	44.51	0.	0.
time (sec)	N/A	0.012	0.063	0.036	0.	9.774	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	193	0	0
normalized size	1	1.	1.83	2.37	0.	2.22	0.	0.
time (sec)	N/A	0.874	0.916	0.034	0.	2.36	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	C	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	1	529	127	317	0	150	0	0
normalized size	1	529.	127.	317.	0.	150.	0.	0.
time (sec)	N/A	1.669	0.586	0.041	0.	2.283	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	144	0	0
normalized size	1	3.91	2.89	11.65	0.	3.13	0.	0.
time (sec)	N/A	1.492	1.072	0.047	0.	2.315	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0
normalized size	1	1.	10.09	8.06	0.	0.	0.	0.
time (sec)	N/A	0.096	0.449	0.059	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	117	0	0
normalized size	1	1.	2.	10.43	0.	5.09	0.	0.
time (sec)	N/A	0.054	0.007	0.021	0.	2.202	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	47	353	0	27647	0	0
normalized size	1	1.	0.22	1.62	0.	126.82	0.	0.
time (sec)	N/A	0.041	0.056	0.195	0.	49.077	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	50	350	0	28331	0	0
normalized size	1	1.	0.24	1.67	0.	134.91	0.	0.
time (sec)	N/A	0.032	0.07	0.173	0.	49.042	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	65	349	0	28004	0	0
normalized size	1	1.	0.29	1.57	0.	126.14	0.	0.
time (sec)	N/A	0.03	0.066	0.173	0.	44.341	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	68	350	0	28069	0	0
normalized size	1	1.	0.32	1.64	0.	131.16	0.	0.
time (sec)	N/A	0.029	0.053	0.169	0.	41.119	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	953	0	0
normalized size	1	1.	10.54	5.03	0.	14.66	0.	0.
time (sec)	N/A	0.129	3.205	0.132	0.	3.467	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	301	0	0
normalized size	1	1.	13.9	4.94	0.	4.78	0.	0.
time (sec)	N/A	0.128	7.546	0.129	0.	2.834	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.243	0.029	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	0	0	0	1422	0	0
normalized size	1	1.	0.	0.	0.	13.17	0.	0.
time (sec)	N/A	0.093	0.057	0.023	0.	9.281	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	135	120	0	0	0	0	0
normalized size	1	1.38	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.155	0.046	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.288	0.088	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	80	0	0	983	0	0
normalized size	1	1.	0.83	0.	0.	10.24	0.	0.
time (sec)	N/A	0.078	0.081	0.044	0.	2.351	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	122	112	0	0	660	0	0
normalized size	1	1.39	1.27	0.	0.	7.5	0.	0.
time (sec)	N/A	0.062	0.143	0.037	0.	26.951	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	1049	0	0
normalized size	1	0.11	0.11	0.	0.	4.5	0.	0.
time (sec)	N/A	0.011	0.027	0.036	0.	23.868	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	116	282	0	0
normalized size	1	1.	0.89	0.	1.41	3.44	0.	0.
time (sec)	N/A	0.058	0.06	0.039	1.4	2.12	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	409	0	0	0	882	0	0
normalized size	1	3.03	0.	0.	0.	6.53	0.	0.
time (sec)	N/A	0.337	0.19	0.079	0.	111.657	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	409	150	0	0	882	0	0
normalized size	1	3.03	1.11	0.	0.	6.53	0.	0.
time (sec)	N/A	0.301	0.441	0.036	0.	99.462	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	F	F	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	399	0	0	0	737	0	0
normalized size	1	3.35	0.	0.	0.	6.19	0.	0.
time (sec)	N/A	0.299	0.163	0.069	0.	93.427	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0
normalized size	1	0.	1.	0.79	0.	0.	0.	0.
time (sec)	N/A	0.425	0.151	0.036	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0
normalized size	1	0.	1.	0.79	0.	0.	0.	0.
time (sec)	N/A	0.22	0.075	0.027	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0
normalized size	1	1.	1.1	0.87	0.	0.	0.	0.
time (sec)	N/A	0.029	0.018	0.013	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	142	258	31	0
normalized size	1	1.	1.51	0.18	2.12	3.85	0.46	0.
time (sec)	N/A	0.01	0.103	0.025	1.397	1.599	1.16	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	66	99	230	41	100
normalized size	1	1.	0.93	0.94	1.41	3.29	0.59	1.43
time (sec)	N/A	0.038	0.038	0.035	1.396	1.559	1.026	1.097

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.345	0.053	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.415	0.452	0.091	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	5191	0	0
normalized size	1	0.	0.	0.	0.	26.09	0.	0.
time (sec)	N/A	0.945	0.348	0.088	0.	14.505	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.39	0.041	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.15	0.011	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	132	21	111	0	0	532	0	0
normalized size	1	0.16	0.84	0.	0.	4.03	0.	0.
time (sec)	N/A	0.008	0.094	0.038	0.	1.677	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	250	26	26	0	0	0	0	0
normalized size	1	0.1	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.014	0.037	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	383	0	138	0	0	0	0	0
normalized size	1	0.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	0.153	0.039	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	272	21	109	0	0	902	0	0
normalized size	1	0.08	0.4	0.	0.	3.32	0.	0.
time (sec)	N/A	0.008	0.088	0.038	0.	21.83	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.143]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	10	0.1
2	C	5	2	3.47	37	0.054
3	A	9	6	1.	15	0.4
4	A	3	3	1.	19	0.158
5	A	8	7	1.	17	0.412
6	A	6	6	1.	17	0.353
7	A	3	3	1.	17	0.176
8	A	4	2	1.	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.	45	0.067
12	A	7	4	1.	32	0.125
13	A	5	3	1.	32	0.094
14	A	2	2	1.	27	0.074
15	A	2	2	1.	29	0.069
16	A	2	1	1.	30	0.033
17	A	3	2	1.	13	0.154
18	A	3	2	1.	15	0.133
19	A	2	2	1.	23	0.087
20	A	2	2	1.	25	0.08
21	A	3	2	1.	11	0.182
22	A	2	2	1.	18	0.111
23	A	6	6	1.	20	0.3
24	A	6	5	1.	31	0.161
25	A	2	2	1.	29	0.069
26	A	2	2	1.	35	0.057
27	A	5	5	1.	32	0.156
28	A	6	6	1.	21	0.286
29	A	359	30	1.12	14	2.143
30	A	2	2	1.	23	0.087
31	A	3	2	1.	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	2	2	1.	33	0.061
33	A	5	5	1.	15	0.333
34	A	5	5	1.	15	0.333
35	A	1	1	1.	11	0.091
36	A	5	5	1.	15	0.333
37	A	1	1	1.	17	0.059
38	A	3	3	1.	18	0.167
39	A	2	2	1.6	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.17	32	0.094
47	A	19	9	1.	20	0.45
48	A	29	9	1.	20	0.45
49	A	49	9	1.	20	0.45
50	A	14	6	1.	23	0.261
51	A	24	6	1.	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.	24	0.292
54	A	7	7	1.	24	0.292
55	A	1	1	1.	18	0.056
56	A	8	8	1.47	13	0.615
57	A	6	6	1.	15	0.4
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	F	0	0	N/A	0	N/A
61	F	0	0	N/A	0	N/A
62	A	1	1	1.	38	0.026
63	A	1	1	1.	33	0.03

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	2	2	1.	39	0.051
65	A	1	1	1.	19	0.053
66	A	4	4	1.	19	0.21
67	A	4	4	1.	24	0.167
68	A	1	1	1.	24	0.042
69	A	7	7	1.	24	0.292
70	A	7	7	1.	24	0.292
71	A	3	3	1.	26	0.115
72	A	3	3	1.	22	0.136
73	A	1	1	1.	22	0.045
74	A	1	1	1.	23	0.043
75	A	8	8	1.	18	0.444
76	A	8	7	1.	23	0.304
77	A	1	1	1.	21	0.048
78	A	1	1	1.	19	0.053
79	A	1	1	1.	19	0.053
80	A	1	1	1.	19	0.053
81	A	4	4	1.	34	0.118
82	C	5	5	529.	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.	29	0.069
85	A	2	2	1.	18	0.111
86	A	1	1	1.	25	0.04
87	A	1	1	1.	25	0.04
88	A	1	1	1.	25	0.04
89	A	1	1	1.	25	0.04
90	A	2	2	1.	40	0.05
91	A	2	2	1.	40	0.05
92	A	1	1	1.	18	0.056
93	A	3	3	1.	15	0.2
94	A	7	7	1.38	21	0.333
95	F	0	0	N/A	0	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	5	1.	24	0.208
97	A	7	7	1.39	19	0.368
98	C	1	1	0.11	20	0.05
99	A	5	5	1.	22	0.227
100	C	4	2	3.03	25	0.08
101	C	5	3	3.03	24	0.125
102	C	4	2	3.35	23	0.087
103	F	0	0	N/A	0	N/A
104	F	0	0	N/A	0	N/A
105	A	3	3	1.	19	0.158
106	A	2	2	1.	11	0.182
107	A	6	6	1.	15	0.4
108	F	0	0	N/A	0	N/A
109	F	0	0	N/A	0	N/A
110	F	0	0	N/A	0	N/A
111	F	0	0	N/A	0	N/A
112	F	0	0	N/A	0	N/A
113	C	1	1	0.16	19	0.053
114	C	1	1	0.1	20	0.05
115	F	0	0	N/A	0	N/A
116	C	1	1	0.08	19	0.053

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rubi [A] time = 0.0014936, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {32}

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

Mathematica [A] time = 0.004261, size = 15, normalized size = 1.

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$-2\,\frac{\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/2),x)
```

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Maxima [A] time = 0.924617, size = 18, normalized size = 1.2

$$-\frac{2\,\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Fricas [A] time = 2.0903, size = 28, normalized size = 1.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Sympy [A] time = 0.055326, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Giac [A] time = 1.07191, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

3.2 $\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rubi [C] time = 0.0558809, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 2295}

$$\frac{\sqrt{ax-1}\log(ax-1)}{\pi a}-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 +
a*x])/(a*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx &= \frac{\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{\sqrt{-1+ax}}\,dx}{2\pi} \\
&= \frac{\text{Subst}\left(\int \left(-2\log(-x)+\log\left(x^2\right)\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} \\
&= \frac{\text{Subst}\left(\int \log\left(x^2\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} - \frac{2\,\text{Subst}\left(\int \log(-x)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} \\
&= -\frac{2\sqrt{-1+ax}\log\left(-\sqrt{-1+ax}\right)}{a\pi} + \frac{\sqrt{-1+ax}\log(-1+ax)}{a\pi}
\end{aligned}
$$

Mathematica [C] time = 0.025668, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1}\left(\log(ax-1)-2\log\left(-\sqrt{ax-1}\right)\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]
```

```
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

Maple [C] time = 0.003, size = 34, normalized size = 2.3

$$\frac{1}{a\pi}\sqrt{ax-1}\left(\ln(ax-1)-2\,\ln\left(-\sqrt{ax-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x)
```

```
[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

Maxima [B] time = 0.947855, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1}\log(ax-1)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

Fricas [A] time = 2.14124, size = 4, normalized size = 0.27

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
```

```
[Out] 0
```

Sympy [A] time = 57.8506, size = 42, normalized size = 2.8

$$\frac{\begin{cases} \frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for } a \neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

Giac [B] time = 1.07061, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="giac")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} \, dx$

Optimal. Leaf size=82

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rubi [A] time = 0.0758864, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6742, 199, 207, 444, 47, 63}

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \int \left(\frac{8}{3\left(-1+3x^2\right)^2} - \frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2} + \frac{5}{3\left(-1+3x^2\right)}\right)dx$$

$$= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx + \frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx - 4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx$$

$$= \frac{4x}{3\left(1-3x^2\right)} - \frac{5\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} - \frac{4}{3}\int \frac{1}{-1+3x^2}\,dx - 2\,\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,dx, x, x^2\right)$$

$$= \frac{4x}{3\left(1-3x^2\right)} - \frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x}(-1+3x)}\,dx, x, x^2\right)$$

$$= \frac{4x}{3\left(1-3x^2\right)} - \frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} - \frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{-4+3x^2}\,dx, x, \sqrt{1+x^2}\right)$$

$$= \frac{4x}{3\left(1-3x^2\right)} - \frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.107544, size = 69, normalized size = 0.84

$$\frac{1}{9}\left(\frac{6\left(\sqrt{x^2+1}-2x\right)}{3x^2-1} + \sqrt{3}\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right) - \sqrt{3}\tanh^{-1}\left(\sqrt{3}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] ((6*(-2*x + Sqrt[1 + x^2]))/(-1 + 3*x^2) - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqr
t[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2])/9
```

Maple [B] time = 0.041, size = 370, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(x*3^(1/2))*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

Fricas [A] time = 2.08059, size = 251, normalized size = 3.06

$$\frac{\sqrt{3}\left(3\,x^2-1\right)\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right)+\sqrt{3}\left(3\,x^2-1\right)\log\left(\frac{3\,x^2+4\,\sqrt{3}\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\left(3\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
```

```
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x + \sqrt{x^2 + 1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

Giac [B] time = 1.11115, size = 239, normalized size = 2.91

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x - 2\sqrt{3}\right|}{\left|6x + 2\sqrt{3}\right|}\right) - \frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-6x - 8\sqrt{3} + 6\sqrt{x^2+1} - \frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3x - 4\sqrt{3} - 3\sqrt{x^2+1} + \frac{3}{x-\sqrt{x^2+1}}\right)}\right) - \frac{4\left(x - \sqrt{x^2+1} + \frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x - \sqrt{x^2+1} + \frac{1}{x-\sqrt{x^2+1}}\right)^2 - 16\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$\frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rubi [A] time = 0.0123968, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 377, 207}

$$\frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)}\,dx\\ &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [C] time = 3.39706, size = 167, normalized size = 3.88

$$-\frac{x\sqrt{x^2-1}\left(\frac{8x^2\left(x^2-1\right){}_2F_1\left(2,3;\frac{7}{2};\frac{x^2}{4-3x^2}\right)}{45x^2-60}-\frac{x^2\left(2x^2-3\right)\sqrt{\frac{x^2-1}{3x^2-4}}\left(2\sqrt{\frac{x^2-x^4}{\left(4-3x^2\right)^2}}-\sin^{-1}\left(\sqrt{\frac{x^2}{4-3x^2}}\right)\right)}{4\left(\frac{x^2}{4-3x^2}\right)^{5/2}\left(x^2-1\right)}\right)}{16\left(1-\frac{3x^2}{4}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2]*(-(x^2*(-3 + 2*x^2)*Sqrt[(-1 + x^2)/(-4 + 3*x^2)]*(2*Sqr
t[(x^2 - x^4)/(4 - 3*x^2)^2] - ArcSin[Sqrt[x^2/(4 - 3*x^2)]]))/(4*(x^2/(4 -
 3*x^2))^(5/2)*(-1 + x^2)) + (8*x^2*(-1 + x^2)*Hypergeometric2F1[2, 3, 7/2,
 x^2/(4 - 3*x^2)])/(-60 + 45*x^2)))/(16*(1 - (3*x^2)/4)^2)
```

Maple [B] time = 0.047, size = 172, normalized size = 4.

$$-\frac{1}{16}\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}}{3}\left(x+\frac{2\sqrt{3}}{3}\right)+\frac{1}{3}}\left(x+\frac{2\sqrt{3}}{3}\right)^{-1}-\frac{5}{32}\mathrm{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3}-\frac{4\sqrt{3}}{3}\left(x+\frac{2\sqrt{3}}{3}\right)\right)\frac{1}{\sqrt{9\left(x+2/3\sqrt{3}\right)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^(1/2))+1/3)^(
1/2)-5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3
^(1/2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))-1/16/(x-2/3*3^(1/2))*((x-2/3
*3^(1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/
3*3^(1/2)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3
^(1/2))+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-4\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

Fricas [B] time = 2.04815, size = 205, normalized size = 4.77

$$-\frac{12\,x^2+5\left(3\,x^2-4\right)\log\left(3\,x^2-3\,\sqrt{x^2-1}x-2\right)-5\left(3\,x^2-4\right)\log\left(x^2-\sqrt{x^2-1}x-2\right)+12\,\sqrt{x^2-1}x-16}{32\left(3\,x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(3x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

Giac [B] time = 1.08749, size = 127, normalized size = 2.95

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)}-\frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right)+\frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rubi [A] time = 0.0559415, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 97, 157, 54, 215, 93, 207}

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \left(\frac{8}{3(-1+3x)^2} - \frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2} + \frac{5}{3(-1+3x)}\right) dx$$
$$= \frac{8}{9(1-3x)} + \frac{5}{9}\log(1-3x) - 4\int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}\,dx$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\sqrt{1+x}(-1+3x)}\,dx$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx - \frac{10}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}(-1+3x)}\,d$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{8}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) - \frac{20}{9}\,\text{Subst}\left(\int\right.$$
$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} - \frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right) + \frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right) + \frac{5}{9}\log(1-3x)$$

Mathematica [A] time = 0.132287, size = 126, normalized size = 1.7

$$\frac{12x^{3/2}+12\sqrt{x}-8\sqrt{x+1}+15\sqrt{x+1}x\log(1-3x)-5\sqrt{x+1}\log(1-3x)+10\sqrt{-x-1}(3x-1)\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{-x-1}}\right)-8\sqrt{x+}}{9\sqrt{x+1}(3x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (12*Sqrt[x] + 12*x^(3/2) - 8*Sqrt[1 + x] - 8*Sqrt[1 + x]*(-1 + 3*x)*ArcSinh
[Sqrt[x]] + 10*Sqrt[-1 - x]*(-1 + 3*x)*ArcTan[(2*Sqrt[x])/Sqrt[-1 - x]] - 5
*Sqrt[1 + x]*Log[1 - 3*x] + 15*x*Sqrt[1 + x]*Log[1 - 3*x])/(9*Sqrt[1 + x]*(
-1 + 3*x))
```

Maple [B] time = 0.014, size = 115, normalized size = 1.6

$$-\frac{8}{27x-9}+\frac{5\ln(3x-1)}{9}-\frac{1}{27x-9}\sqrt{x}\sqrt{1+x}\left(12\ln\left(1/2+x+\sqrt{x(1+x)}\right)x-15\,\text{Artanh}\left(1/4\,\frac{1+5x}{\sqrt{x(1+x)}}\right)x-4\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(x*(1+x))^(
1/2))*x-15*arctanh(1/4*(1+5*x)/(x*(1+x))^(1/2))*x-4*ln(1/2+x+(x*(1+x))^(1/2
))+5*arctanh(1/4*(1+5*x)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x))^(1/
2)/(3*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{x+1}+2\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

Fricas [A] time = 2.12849, size = 309, normalized size = 4.18

$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x+1}\sqrt{x}-x+1\right)-5\,(3}{9\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0260134, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {733, 844, 217, 206, 725, 204}

$$\frac{\sqrt{x^2-1}}{-x+i}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}-i\,\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0594532, size = 59, normalized size = 0.92

$$
-\frac{\sqrt{x^2-1}}{x-i}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x^2]/(-I + x)) + ArcTanh[x/Sqrt[-1 + x^2]] - ArcTanh[(I + x)/(S
qrt[2]*Sqrt[-1 + x^2])]/Sqrt[2]
```

Maple [B] time = 0.025, size = 125, normalized size = 2.

$$\frac{1}{2x-2i}\left((x-i)^2-2+2i(x-i)\right)^{\frac{3}{2}}+\ln\left(x+\sqrt{(x-i)^2-2+2i(x-i)}\right)+\frac{i}{2}\sqrt{2}\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4}\frac{}{\sqrt{(x-i)^2-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x)
```

```
[Out] 1/2/(x-I)*((x-I)^2-2+2*I*(x-I))^(3/2)+ln(x+((x-I)^2-2+2*I*(x-I))^(1/2))+1/2
*I*2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)/((x-I)^2-2+2*I*(x-I))^(1/2))-1
/2*I*((x-I)^2-2+2*I*(x-I))^(1/2)-1/2*x*((x-I)^2-2+2*I*(x-I))^(1/2)
```

Maxima [A] time = 1.429, size = 72, normalized size = 1.12

$$\frac{1}{2}i\sqrt{2}\arcsin\left(\frac{ix}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 2.19194, size = 267, normalized size = 4.17

$$-\frac{\sqrt{2}(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+(2x-2i)\log\left(-x+\sqrt{x^2-1}\right)+2x}{2x-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x - I)
*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + (2*x - 2*I)*log(-x + sqrt(x^2 -
1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(2*x - 2*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/(x - I)^2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rubi [A] time = 0.0134643, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {382, 377, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx &= -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)}\,dx\\ &= -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= -\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.106882, size = 75, normalized size = 1.56

$$\frac{\sqrt{x^2-1}\left(3\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\left(x^2+1\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\right)-2x^2\right)}{8\left(x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^2]*(-2*x^2 + 3*Sqrt[2]*Sqrt[x^2/(-1 + x^2)]*(1 + x^2)*ArcTanh[
Sqrt[2]*Sqrt[x^2/(-1 + x^2)]]))/(8*(x + x^3))
```

Maple [A] time = 0.017, size = 45, normalized size = 0.9

$$-\frac{x}{8}\frac{1}{\sqrt{x^2-1}}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)^{-1}+\frac{3\sqrt{2}}{8}\text{Artanh}\left(x\sqrt{2}\frac{1}{\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/8*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))
*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)^2 \sqrt{x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

Fricas [B] time = 2.01657, size = 215, normalized size = 4.48

$$\frac{3\sqrt{2}\left(x^2+1\right)\log\left(\frac{9x^2+2\sqrt{2}\left(3x^2-1\right)+2\sqrt{x^2-1}\left(3\sqrt{2}x+4x\right)-3}{x^2+1}\right)-4x^2-4\sqrt{x^2-1}x-4}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.08795, size = 136, normalized size = 2.83

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}} \, dx$

Optimal. Leaf size=30

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rubi [A] time = 0.0824455, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \int \left(-\frac{1}{\sqrt{-1+x}} - 2\sqrt{x} + \frac{2x}{\sqrt{-1+x}}\right)dx$$
$$= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \frac{x}{\sqrt{-1+x}}\,dx$$
$$= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \left(\frac{1}{\sqrt{-1+x}} + \sqrt{-1+x}\right)dx$$
$$= 2\sqrt{-1+x} + \frac{4}{3}(-1+x)^{3/2} - \frac{4x^{3/2}}{3}$$

Mathematica [A] time = 0.0413616, size = 30, normalized size = 1.

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Maple [A] time = 0.003, size = 21, normalized size = 0.7

$$\frac{4}{3}(-1+x)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x)
```

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

Fricas [A] time = 2.00777, size = 55, normalized size = 1.83

$$\frac{2}{3}(2x+1)\sqrt{x-1}-\frac{4}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

Sympy [B] time = 0.818293, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

```
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

Giac [A] time = 1.07121, size = 27, normalized size = 0.9

$$\frac{4}{3}(x-1)^{\frac{3}{2}}-\frac{4}{3}x^{\frac{3}{2}}+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{x^2-1}}{-\sqrt{5}x-x+2}\right)+\frac{1}{25}$$

```
[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[
(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqr
t[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])*x)])/50 - (Sqrt[110 +
50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*
Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x)])
/50
```

Rubi [A] time = 0.505796, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6742, 736, 826, 1166, 207, 203, 1018, 1034, 725, 206, 204, 985}

$$-\frac{2\sqrt{x^2-1}(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}(1-2x)}{5\left(-x^2+x+1\right)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\frac{2-\left(1\right.}{\sqrt{2\left(\sqrt{5}-\right.}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
```

```
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}+\frac{2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)}\right)dx$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)+2\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int \frac{1}{\sqrt{-1+x^2}\,(-1}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}\,dx+\frac{2}{5}\int \frac{}{\sqrt{-1+x^2}}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}\,\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx,x,\sqrt{x}\right)-\frac{2\,\text{S}}{}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1}}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right)$$

Mathematica [A] time = 0.687549, size = 340, normalized size = 1.55

$$\frac{2}{5}\left(\frac{\sqrt{x}(1-2x)}{-x^2+x+1}+\frac{\sqrt{x^2-1}(1-2x)}{x^2-x-1}-\frac{1}{2}\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+x-2}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\sqrt{\sqrt{5}-\frac{2}{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-1\right)x+}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(((1 - 2*x)*Sqrt[x])/(1 + x - x^2) + ((1 - 2*x)*Sqrt[-1 + x^2])/(-1 - x
+ x^2) + Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]]
- (Sqrt[(5*(1 + Sqrt[5]))/2]*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt
```

```
[5])]*Sqrt[-1 + x^2])])/2 - Sqrt[-2/5 + Sqrt[5]]*ArcTan[(2 + (-1 + Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[(11 + 5*Sqrt[5])/10]*Ar
cTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] - Sqrt[5/(2*(1 + Sqrt[5]))]*ArcTanh[(-
2 + x + Sqrt[5]*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[2/5 + Sqr
t[5]]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])]
))/5
```

Maple [B] time = 0.123, size = 902, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5
^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-
1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1
/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2)
)^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^
(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2
+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/
2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(
1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+
1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5
^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2
*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1
/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5
^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*ar
ctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*
(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))
*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-
1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2*5^(1/2))
^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*5^(1/2))^(1/2)*arct
anh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+
4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*
5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

Fricas [B] time = 2.29054, size = 1287, normalized size = 5.85

$$4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}(2x\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{x^2-1}}{-\sqrt{5}x-x+2}\right)+\frac{1}{25}$$

```
[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[
(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqr
t[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])*x)])/50 - (Sqrt[110 +
50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*
Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x)])
/50
```

Rubi [B] time = 0.751087, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 736, 826, 1166, 207, 203, 975, 1034, 725, 206, 204, 1018, 1065}

$$-\frac{\sqrt{x^2-1}(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}(1-2x)}{5\left(-x^2+x+1\right)}-\frac{(3-x)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{(x+2)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
```

```
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \int\left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}-\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{}{\sqrt{-1+x}}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)-\int \frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x\right.}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\int \frac{}{\sqrt{-1}}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}\text{Subst}\Bigg($$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{25}\left(5-7\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-1\right.}$$

Mathematica [A] time = 0.648909, size = 311, normalized size = 1.41

$$\frac{1}{25}\left(\frac{-20x^{3/2}+20\sqrt{x^2-1}x-10\sqrt{x^2-1}+\sqrt{50\sqrt{5}-110}\left(-x^2+x+1\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)+\sqrt{10\left(1+\sqrt{5}\right)}\left(-x^2+x+1\right.}{-x^2+x+1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((10*Sqrt[x] - 20*x^(3/2) - 10*Sqrt[-1 + x^2] + 20*x*Sqrt[-1 + x^2] + Sqrt[
-110 + 50*Sqrt[5]]*(1 + x - x^2)*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]] + S
qrt[10*(1 + Sqrt[5])]*(1 + x - x^2)*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1
 + Sqrt[5])]*Sqrt[-1 + x^2])] + 5*Sqrt[2/(-1 + Sqrt[5])]*(-1 - x + x^2)*Arc
Tan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/(1 + x -
 x^2) - Sqrt[110 + 50*Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] + Sqr
t[2/(1 + Sqrt[5])]*(5 + 2*Sqrt[5])*ArcTanh[(-2 + x + Sqrt[5]*x)/(Sqrt[2*(1
+ Sqrt[5])]*Sqrt[-1 + x^2])])/25
```

Maple [B] time = 0.021, size = 1542, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] 2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(
2+2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2))^(1/2
)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))+1/10*5^(1/2)/(1/2-1/2*5^(1/2))/(x+
1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/
2-1/2*5^(1/2))^(1/2)-1/10*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x
-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-
2/5/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/
2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(
1/2)-1/2)+2-2*5^(1/2))^(1/2))+2/5/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+
(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)
^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+1/25*5^(1/2)*(4*(x
+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1
/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/
2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+1/20/(1/2-1/2*5^(1/2))*(4*(x+1/2*5^(1/2)-1/
2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/25*ln(x+((x+1/
2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5
^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/
2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))+1/20/(1/2+1/2*5^(1/2))*(4*(x-1/2*5^(
1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)-1/25*5^(1/
2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^
(1/2)-1/25*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+
1/2*5^(1/2))^(1/2))*5^(1/2)-1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2
)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5^(1/2)-1/5/(1
/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+
1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(3/2)+1/20/(1/2+1/2*5^(1/2))*5^(1/2)*(4*(
x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1
/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2)+1/2+1/2*5^(1/2))^(1/2))*5^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)
-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2
))^(3/2)+1/5/(1/2+1/2*5^(1/2))*x*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*
5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/5/(1/2-1/2*5^(1/2))*x*((x+1/2*5^(1/2)
-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/20/(1/2-1
/2*5^(1/2))*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-
1/2)+2-2*5^(1/2))^(1/2)-1/5*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2
*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))-1/5*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1
```

```
/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))+4/25*5^(1/2)/(2+2*5^(1/2
))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2)
)^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1
/2))^(1/2))-1/10/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)
^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/10/(1/2+1/2*5^
(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2)+1/2+1/2*5^(1/2))^(1/2)+4/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1
-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5
^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))-8/25/(
2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)-8/25/(-2+
2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(x^{\frac{5}{2}}-3\,x^{\frac{3}{2}}\right)}{5\left(x^2-x-1\right)}+\int\frac{x^{\frac{3}{2}}+\sqrt{x}}{5\left(x^2-x-1\right)}\,dx+\int\frac{x^2+x-1}{\left(x^4-2\,x^3-x^2+2\,x+1\right)\sqrt{x+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Fricas [B] time = 2.35997, size = 1287, normalized size = 5.85

$$\frac{4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2\,x^2-\sqrt{x^2-1}\left(2\,x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}(2\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
```

```
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\left(x^2-x-1\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)), x)
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rubi [A] time = 0.0750856, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {731, 725, 206}

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int\left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx=\frac{\int\frac{1}{(1+x)^2\sqrt{-i+x^2}}\,dx}{\sqrt{2}}+\frac{\int\frac{1}{(1+x)^2\sqrt{i+x^2}}\,dx}{\sqrt{2}}$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\int\frac{1}{(1+x)\sqrt{i+x^2}}\,dx}{\sqrt{2}}+$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int\frac{1}{(1-i)-x}\right.}{\sqrt{2}}$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\text{ta}}{}$$

Mathematica [A] time = 0.19884, size = 125, normalized size = 0.91

$$\frac{i\left((1+i)\left(i\sqrt{x^2-i}+\sqrt{x^2+i}\right)+\sqrt{1-i}(x+1)\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)+\sqrt{1+i}(x+1)\tanh^{-1}\left(\frac{(1+i)^{3/2}(1+ix)}{2\sqrt{x^2+i}}\right)\right)}{2\sqrt{2}(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((I/2)*((1 + I)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2]) + Sqrt[1 - I]*(1 + x)*Ar
cTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])] + Sqrt[1 + I]*(1 + x)*ArcTanh[(
(1 + I)^(3/2)*(1 + I*x))/(2*Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

Maple [B] time = 0.02, size = 278, normalized size = 2.

$$-\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1-i-2x}-\frac{\frac{i}{4}\sqrt{2}}{1+x}\sqrt{(1+x)^2-1-i-2x}-\frac{\sqrt{2}}{4\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/(1+x)*((1+x)^2-1-I-2*x)^(1/2)-1/4*I*2^(1/2)/(1+x)*((1+x)^2-1-I
-2*x)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-1-I
-2*x)^(1/2))/(1+x))-1/4*I*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((
1+x)^2-1-I-2*x)^(1/2))/(1+x))-1/4*2^(1/2)/(1+x)*((1+x)^2-1+I-2*x)^(1/2)+1/4
*I*2^(1/2)/(1+x)*((1+x)^2-1+I-2*x)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*
x+2*(1+I)^(1/2)*((1+x)^2-1+I-2*x)^(1/2))/(1+x))+1/4*I*2^(1/2)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((1+x)^2-1+I-2*x)^(1/2))/(1+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.28061, size = 678, normalized size = 4.91

$$\sqrt{-\frac{1}{2}i+\frac{1}{2}}(-(i-1)x-i+1)\log\left(\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}((i-1)x+i-1)\log\left(-\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rubi [A] time = 0.185294, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 206}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\right.\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+}}\right.\end{aligned}$$

Mathematica [F] time = 0.239471, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)^2}\sqrt{x^2+\sqrt{x^4+1}}\,\frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

Fricas [B] time = 19.3436, size = 1068, normalized size = 8.54

$$4(x+1)\sqrt{\sqrt{2}+1}\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}\left(x^3+1\right)+\sqrt{x^4+1}\left(\sqrt{2}x-x-1\right)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\left(2x^2-\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{2\sqrt{2}+2}\sqrt{\sqrt{2}\cdots}}{2\left(x^2-2x+1\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
```

```
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)^2 \sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2 - (Sqrt[1
 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rubi [A] time = 0.16247, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2133, 725, 206}

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2 - (Sqrt[1
 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx$$

$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{1-ix}{\sqrt{1+ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx, x, \frac{1+}{\sqrt{1-}}\right.$$

$$= -\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Mathematica [F] time = 0.175985, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\sqrt{x^2+\sqrt{x^4+1}}\frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Fricas [B] time = 38.9638, size = 998, normalized size = 12.32

$$\frac{1}{2}\sqrt{2\sqrt{2} - 2}\arctan\left(\frac{\left(2x^2 - \sqrt{2}\left(x^3 - x^2 + x + 1\right) + \sqrt{x^4 + 1}\left(\sqrt{2}(x - 1) - 2\right) - 2x\right)\sqrt{x^2 + \sqrt{x^4 + 1}}\sqrt{2\sqrt{2} - 2} + \left(x^2 + \right.}{2\left(x^2 - 2x + 1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)\sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] time = 0.0544915, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx = \text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0083347, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt{x^2+\sqrt{x^4+1}} \frac{1}{\sqrt{x^4+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [B] time = 4.026, size = 162, normalized size = 5.23

$$\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] time = 1.28076, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] time = 0.0626, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0113053, size = 33, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Maple [C] time = 0.033, size = 22, normalized size = 0.7

$$-\frac{\sqrt{2}}{4\,x^2}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{5}{4};\,\frac{3}{2},\frac{3}{2};\,-x^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [A] time = 4.07396, size = 85, normalized size = 2.58

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] time = 0.705757, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rubi [A] time = 0.270052, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6688}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right)dx\\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.0174567, size = 19, normalized size = 1.

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$-2\frac{1}{\sqrt{-1+x}}-2\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Maxima [A] time = 0.933505, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Fricas [A] time = 1.97151, size = 78, normalized size = 4.11

$$-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

Sympy [B] time = 6.48919, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

Giac [A] time = 1.09706, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rubi [A] time = 0.0232867, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(x+\sqrt{a+x^2}\right)^b dx = \frac{1}{2}\operatorname{Subst}\left(\int x^{-2+b}\left(a+x^2\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(ax^{-2+b}+x^b\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

Mathematica [A] time = 0.0386577, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b-1}\left((b-1)x\left(\sqrt{a+x^2}+x\right)+ab\right)}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

Maple [B] time = 0.026, size = 120, normalized size = 2.3

$$\frac{b}{4\sqrt{\pi}}a^{\frac{b}{2}+\frac{1}{2}}\left(8\frac{\sqrt{\pi}x^{1+b}a^{-b/2-1/2}}{(1+b)\,b\,(-2+2\,b)}\left(\frac{ab}{x^2}+b-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}+4\frac{\sqrt{\pi}x^{1+b}a^{-b/2-1/2}}{(1+b)\,b}\sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b,x)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/
x^2*a*b+b-1)/(-2+2*b)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+
b)*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

Fricas [A] time = 2.2043, size = 74, normalized size = 1.42

$$\frac{\left(\sqrt{x^2+a}b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [B] time = 2.73252, size = 2149, normalized size = 41.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a
)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2)
 + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) +
a**(9/2)*a**(b/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**(7/2)*a**(b/2)*b**2*x**3*sqrt
(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
```

2*a**(7/2)*x**2*gamma(1 - b/2)) + a**(7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/s
qrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 -
 b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2
)) + 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(
7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**5*a*
*(b/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(
1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 -
b/2)) - 2*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2
)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*ga
mma(1 - b/2)) + 4*a**4*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**4*a**(b/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) - 2*a**4*a**(b/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2
*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 -
b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*
a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*
x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9
/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*
gamma(1 - b/2)), Abs(x**2)/Abs(a) > 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 +
x**2/a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5
/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) + a**(5/2)*a**(b/2)*b*
x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*
a**(5/2)*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2)*x*sqrt(1 + x**2/a)*sinh(b*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1
 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*s
inh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**
(5/2)*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x
/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamm
a(1 - b/2)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt

```
(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
 b/2)) + 2*a**2*a**(b/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*g
amma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)),
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b\,dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rubi [A] time = 0.0224871, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = \frac{1}{2}\,\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

Mathematica [A] time = 0.0633378, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{b-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{b+1}+\frac{a}{b-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b,x)
```

```
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

Fricas [A] time = 2.1555, size = 76, normalized size = 1.36

$$-\frac{\left(\sqrt{x^2+a}b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{a+x^2}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rubi [A] time = 0.0535661, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \text{Subst}\left(\int x^{-1+b}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Mathematica [A] time = 0.0067221, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^b \frac{1}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Fricas [A] time = 1.99166, size = 34, normalized size = 2.

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

Sympy [B] time = 3.67894, size = 313, normalized size = 18.41

$$\begin{cases} -\frac{\sqrt{a}a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)}+\frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b}-\frac{a^{\frac{b}{2}}x\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{a}\right.}{\sqrt{a}b\sqrt{\frac{a}{x^2}+1}} \\ -\frac{a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)}-\frac{a^{\frac{b}{2}}x^2\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asi}\right.}{\sqrt{a}b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/
(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*b) - a**(b/2)*x*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2)/Abs(a) > 1), (-a**(b/2)*sinh(-b*asinh(
x/sqrt(a)) + asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asi
nh(x/sqrt(a)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh
```

```
(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rubi [A] time = 0.0598107, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\mathrm{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Mathematica [A] time = 0.00611, size = 20, normalized size = 1.

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^b \frac{1}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Fricas [A] time = 1.98498, size = 35, normalized size = 1.75

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x - sqrt(x^2 + a))^b/b
```

Sympy [A] time = 2.86755, size = 36, normalized size = 1.8

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.21 $\int \frac{1}{(a+be^{px})^2} \, dx$

Optimal. Leaf size=42

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rubi [A] time = 0.0334166, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 44}

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{px}\right)}{p}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, e^{px}\right)}{p}$$
$$= \frac{1}{a(a+be^{px})p} + \frac{x}{a^2} - \frac{\log(a+be^{px})}{a^2p}$$

Mathematica [A] time = 0.0415607, size = 36, normalized size = 0.86

$$\frac{\frac{a}{a+be^{px}} - \log(a+be^{px}) + px}{a^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
```

```
[Out] (a/(a + b*E^(p*x)) + p*x - Log[a + b*E^(p*x)])/(a^2*p)
```

Maple [A] time = 0.01, size = 48, normalized size = 1.1

$$\frac{\ln(e^{px})}{pa^2} - \frac{\ln(a+be^{px})}{pa^2} + \frac{1}{a(a+be^{px})p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(p*x))^2,x)
```

```
[Out] 1/p/a^2*ln(exp(p*x))-ln(a+b*exp(p*x))/a^2/p+1/a/(a+b*exp(p*x))/p
```

Maxima [A] time = 0.947001, size = 54, normalized size = 1.29

$$\frac{x}{a^2} + \frac{1}{\left(abe^{(px)} + a^2\right)p} - \frac{\log\left(be^{(px)} + a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

Fricas [A] time = 2.0758, size = 124, normalized size = 2.95

$$\frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

Sympy [A] time = 0.131613, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

Giac [A] time = 1.07974, size = 55, normalized size = 1.31

$$\frac{x}{a^2} - \frac{\log\left(\left|be^{(px)} + a\right|\right)}{a^2p} + \frac{1}{\left(be^{(px)} + a\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] x/a^2 - log(abs(b*e^(p*x) + a))/(a^2*p) + 1/((b*e^(p*x) + a)*a*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rubi [A] time = 0.0235268, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 261}

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(b+ax^2\right)^2}\,dx, x, e^{px}\right)}{p}$$
$$= -\frac{1}{2a\left(b+ae^{2px}\right)p}$$

Mathematica [A] time = 0.0202344, size = 22, normalized size = 1.

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Maple [A] time = 0.002, size = 21, normalized size = 1.

$$-\frac{1}{2\,pa\left(a\left(e^{px}\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

Maxima [A] time = 0.935151, size = 27, normalized size = 1.23

$$\frac{1}{2\left(b^2e^{(-2\,px)}+ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

Fricas [A] time = 1.95579, size = 43, normalized size = 1.95

$$-\frac{1}{2\left(a^2pe^{(2px)}+abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

Sympy [A] time = 0.102672, size = 20, normalized size = 0.91

$$\frac{1}{2abp+2b^2pe^{-2px}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] 1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

Giac [A] time = 1.07501, size = 26, normalized size = 1.18

$$-\frac{1}{2\left(ae^{(2px)}+b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2} dx$

Optimal. Leaf size=62

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rubi [A] time = 0.0907172, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rule 2283

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx &= \int \frac{e^{2px}x}{\left(b+ae^{2px}\right)^2}\,dx \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap} \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2} \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\operatorname{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2} \\
&= \frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}
\end{aligned}
$$

Mathematica [A] time = 0.0648487, size = 49, normalized size = 0.79

$$\frac{\frac{2pxe^{2px}}{ae^{2px}+b}-\frac{\log\left(ae^{2px}+b\right)}{a}}{4bp^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

Maple [A] time = 0.012, size = 51, normalized size = 0.8

$$-\frac{\ln\left(a\left(e^{px}\right)^2+b\right)}{4p^2ba}+\frac{x\left(e^{px}\right)^2}{2bp\left(a\left(e^{px}\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/4/p^2/b/a*ln(a*exp(p*x)^2+b)+1/2/p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b)
```

Maxima [A] time = 0.955416, size = 69, normalized size = 1.11

$$\frac{xe^{(2px)}}{2\left(abpe^{(2px)}+b^2p\right)}-\frac{\log\left(\frac{ae^{(2px)}+b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

Fricas [A] time = 2.1886, size = 135, normalized size = 2.18

$$\frac{2\,apxe^{(2\,px)} - \left(ae^{(2\,px)} + b\right)\log\left(ae^{(2\,px)} + b\right)}{4\left(a^2bp^2e^{(2\,px)} + ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)
```

Sympy [A] time = 0.158276, size = 51, normalized size = 0.82

$$\frac{x}{2abp + 2b^2pe^{-2px}} - \frac{x}{2abp} - \frac{\log\left(\frac{a}{b} + e^{-2px}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/(
4*a*b*p**2)
```

Giac [A] time = 1.07893, size = 100, normalized size = 1.61

$$\frac{2\,apxe^{(2\,px)} - ae^{(2\,px)}\log\left(-ae^{(2\,px)} - b\right) - b\log\left(-ae^{(2\,px)} - b\right)}{4\left(a^2bp^2e^{(2\,px)} + ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rubi [A] time = 0.0839424, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
```

```
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\left(1+x+x^2\right)}\,dx$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\left(1+x+x^2\right)}\,dx-\frac{1}{48}\int \frac{-48+48x}{\sqrt{1-x+x^2}\left(1+x \cdots\right.}$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+24\,\mathrm{Subst}\left(\int \frac{1}{1728-2x^2}\,dx,x,\frac{-24+24x}{\sqrt{1-x+x^2}}\right)+288\,\mathrm{Subst}\left(\int \cdots\right.$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

Mathematica [C] time = 2.38024, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}+\frac{\left(7-i\sqrt{3}\right)\tan^{-1}\left(\frac{3\left(\left(-21-4i\sqrt{3}\right)x^4+14\left(7-2i\sqrt{3}\right)x^3+\left(-103-36i\sqrt{3}\right)x^2+\left(94+\cdots\right.\right.}{\left(84i-113\sqrt{3}\right)x^4+2\left(52\sqrt{3-3i\sqrt{3}}\sqrt{x^2-x+1}+21\sqrt{3}+138i\right)x^3+\left(52\sqrt{3-3i\sqrt{3}}\sqrt{x^2-x+1}-59\sqrt{3}-180i\right)x^2+2\left(26\cdots\right.}\right.}{4\sqrt{3-3i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + ((7 - I*Sqrt[3])*ArcTan[(3*(-17
 - (64*I)*Sqrt[3] + (94 + (32*I)*Sqrt[3])*x + (-103 - (36*I)*Sqrt[3])*x^2 +
 14*(7 - (2*I)*Sqrt[3])*x^3 + (-21 - (4*I)*Sqrt[3])*x^4))/(96*I + 67*Sqrt[3
] + (84*I - 113*Sqrt[3])*x^4 - 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]
 + 2*x*(132*I - 69*Sqrt[3] + 26*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ x^2*(-180*I - 59*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ 2*x^3*(138*I + 21*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
)])/(4*Sqrt[3 - (3*I)*Sqrt[3]]) - ((I/4)*(-7*I + Sqrt[3])*ArcTan[(3*(-17 +
(64*I)*Sqrt[3] + (94 - (32*I)*Sqrt[3])*x + (-103 + (36*I)*Sqrt[3])*x^2 + 14
*(7 + (2*I)*Sqrt[3])*x^3 + (-21 + (4*I)*Sqrt[3])*x^4))/(96*I - 67*Sqrt[3] +
 (84*I + 113*Sqrt[3])*x^4 + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] +
x^2*(-180*I + 59*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) +
```

```
x*(264*I + 138*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) - 2*
x^3*(-138*I + 21*Sqrt[3] + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]))])
/Sqrt[3 + (3*I)*Sqrt[3]] - ((-7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sq
rt[3 + (3*I)*Sqrt[3]]) - ((7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sqrt[
3 - (3*I)*Sqrt[3]]) + ((7*I + Sqrt[3])*Log[(1 + x + x^2)*(11*I + 4*Sqrt[3]
+ (11*I + 4*Sqrt[3])*x^2 + (10*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2] - x
*(17*I + 4*Sqrt[3] + (8*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(8*Sqr
t[3 - (3*I)*Sqrt[3]]) + ((-7*I + Sqrt[3])*Log[(1 + x + x^2)*(-11*I + 4*Sqrt
[3] + (-11*I + 4*Sqrt[3])*x^2 - (10*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2
] + x*(17*I - 4*Sqrt[3] + (8*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(
8*Sqrt[3 + (3*I)*Sqrt[3]])
```

Maple [B] time = 0.034, size = 454, normalized size = 5.3

$$\frac{1}{6}\left(6\,\frac{\sqrt{6}\,(1+x)^2}{(1-x)^2}\mathrm{Artanh}\left(1/4\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\sqrt{6}\right)\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}-9\,\frac{\sqrt{2}\,(1+x)^2}{(1-x)^2}\arctan\left(2\,\frac{(1+x)\sqrt{2}}{1-x}\frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}}\right)\sqrt{\frac{(1}{(1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/6*(6*6^(1/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x
)^2+3)^(1/2)*(1+x)^2/(1-x)^2-9*arctan(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^(1/
2)*2^(1/2))*2^(1/2)*((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)^2/(1-x)^2+2*6^(1/2)*arc
tanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x)^2+3)^(1/2)-3*2^(
1/2)*arctan(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^(1/2)*2^(1/2))*((1+x)^2/(1-x)
^2+3)^(1/2)+12*(1+x)^3/(1-x)^3+36*(1+x)/(1-x))/(((1+x)^2/(1-x)^2+3)/(1+(1+x
)/(1-x))^2)^(1/2)/(1+(1+x)/(1-x))/(3*(1+x)^2/(1-x)^2+1)-1/2*((1+x)^2/(1-x)^
2+3)^(1/2)*(6^(1/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))-3*2^(1/2
)*arctan(2*(1+x)/(1-x)/((1+x)^2/(1-x)^2+3)^(1/2)*2^(1/2)))/(1+(1+x)/(1-x))/
(((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2-x+1}{\left(x^2+x+1\right)^2\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

Fricas [B] time = 2.26406, size = 1095, normalized size = 12.73

$$-\frac{8\sqrt{6}\sqrt{3}\left(x^2+x+1\right)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}\left(2x-\sqrt{6}+1\right)-\sqrt{6}(x+1)+4}\left(\sqrt{6}\sqrt{3}+3\sqrt{3}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(2/3*sqrt(6)*sqrt(3)*(x - 1) +
 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) +
 4)*(sqrt(6)*sqrt(3) + 3*sqrt(3)) - 2/3*sqrt(x^2 - x + 1)*(sqrt(6)*sqrt(3)
+ 3*sqrt(3)) + sqrt(3)*(2*x - 1)) + 8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(
2/3*sqrt(6)*sqrt(3)*(x - 1) + 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqr
t(6) + 1) + sqrt(6)*(x + 1) + 4)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - 2/3*sqrt(x
^2 - x + 1)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - sqrt(3)*(2*x - 1)) - sqrt(6)*(x
^2 + x + 1)*log(12168*x^2 - 6084*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 60
84*sqrt(6)*(x + 1) + 24336) + sqrt(6)*(x^2 + x + 1)*log(12168*x^2 - 6084*sq
rt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 6084*sqrt(6)*(x + 1) + 24336) - 12*x^
2 - 12*sqrt(x^2 - x + 1)*(x + 1) - 12*x - 12)/(x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

Giac [C] time = 1.13719, size = 424, normalized size = 4.93

$$-\frac{1}{12}\sqrt{6}\left(2i\sqrt{3}+1\right)\log\left(2i\sqrt{6}\sqrt{3}-12x+6\sqrt{6}-6i\sqrt{3}+12\sqrt{x^2-x+1}-6\right)+\frac{1}{12}\sqrt{6}\left(-2i\sqrt{3}+1\right)\log\left(2i\sqrt{6}\sqrt{3}-12x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*(2*I*sqrt(3) + 1)*log(2*I*sqrt(6)*sqrt(3) - 12*x + 6*sqrt(6)
- 6*I*sqrt(3) + 12*sqrt(x^2 - x + 1) - 6) + 1/12*sqrt(6)*(-2*I*sqrt(3) + 1)
*log(2*I*sqrt(6)*sqrt(3) - 12*x - 6*sqrt(6) + 6*I*sqrt(3) + 12*sqrt(x^2 - x
 + 1) - 6) - 1/12*sqrt(6)*(-2*I*sqrt(3) + 1)*log(-2*I*sqrt(6)*sqrt(3) - 12*
x + 6*sqrt(6) + 6*I*sqrt(3) + 12*sqrt(x^2 - x + 1) - 6) + 1/12*sqrt(6)*(2*I
*sqrt(3) + 1)*log(-2*I*sqrt(6)*sqrt(3) - 12*x - 6*sqrt(6) - 6*I*sqrt(3) + 1
2*sqrt(x^2 - x + 1) - 6) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x
 + 1))^2 - 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*
(x - sqrt(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 -
x + 1) + 3)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

Optimal. Leaf size=19

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rubi [A] time = 0.0651093, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2122, 30}

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.0100317, size = 19, normalized size = 1.

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{a^2+x^2}}\frac{1}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

```
[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

Fricas [A] time = 2.02462, size = 39, normalized size = 2.05

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Sympy [A] time = 0.206048, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rubi [A] time = 0.0970069, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2122, 30}

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}$$

$$= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

Mathematica [A] time = 0.0170815, size = 26, normalized size = 1.

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \sqrt{bx+\sqrt{b^2x^2+a}}\frac{1}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

Fricas [A] time = 1.97501, size = 47, normalized size = 1.81

$$\frac{2\sqrt{bx + \sqrt{b^2x^2 + a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Sympy [A] time = 0.879158, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.219029, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2120, 329, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 4\,\mathrm{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.192124, size = 56, normalized size = 0.89

$$
-\frac{2\left(\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a^2+x^2}}\frac{1}{\sqrt{x+\sqrt{a^2+x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

Fricas [A] time = 2.42215, size = 487, normalized size = 7.73

$$\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)-\sqrt{-a}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

Sympy [C] time = 1.59298, size = 46, normalized size = 0.73

$$-\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4}\end{matrix}\right|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rubi [A] time = 0.0719809, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{\right.$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.0573668, size = 127, normalized size = 1.55

$$-\frac{2\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] (-2*Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqr
t[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[x + S
qrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
```

Maple [C] time = 0.008, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}\,{}_3F_2(-1/4,-1/4,1/4;\,1/2,3/4;\,-\frac{a^2}{x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A] time = 2.56245, size = 552, normalized size = 6.73

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [C] time = 1.43801, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\right|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.29 $\int x^3 \log^3(2+x) \log(3+x)\, dx$

Optimal. Leaf size=606

$$-\frac{5609}{96}\text{PolyLog}(2,-x-2)-\frac{563}{8}\text{PolyLog}(3,-x-2)-\frac{195}{2}\text{PolyLog}(4,-x-2)-\frac{195}{4}\log^2(x+2)\text{PolyLog}(2,-x-2)$$

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6733*(2 + x)*Log[2 + x])/32 - (377*(2 + x)^2*Log[2 + x])/32 + (71*(2
 + x)^3*Log[2 + x])/72 - (3*(2 + x)^4*Log[2 + x])/64 - (43*Log[2 + x]^2)/12
 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (1251*(2 + x)*Log[2
 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/
4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 +
x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]
^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*Log[3 +
x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x])/12 - (4083*Log[
2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*x^2*Log[2 + x]*Log
[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Log[2 + x]*Log[3 + x
])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3
*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*
Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[2 +
 x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[2 + x]*PolyLo
g[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4 - (563*PolyLog[3,
 -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (195*PolyLog[4, -2 -
x])/2
```

Rubi [A] time = 4.79891, antiderivative size = 679, normalized size of antiderivative = 1.12, number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.143, Rules used = {2439, 2416, 2389, 2296, 2295, 2401, 2390, 2305, 2304, 2396, 2433, 2374, 2383, 6589, 2411, 2346, 2302, 30, 2330, 2319, 43, 2334, 2301, 6742, 2430, 2393, 2391, 2394, 2395, 2398}

$$-\frac{5609}{96}\text{PolyLog}(2,-x-2)-\frac{563}{8}\text{PolyLog}(3,-x-2)-\frac{195}{2}\text{PolyLog}(4,-x-2)-\frac{195}{4}\log^2(x+2)\text{PolyLog}(2,-x-2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```


```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
```

```
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6365*(2 + x)*Log[2 + x])/32 - (273*(2 + x)^2*Log[2 + x])/32 + ((2 +
x)^3*Log[2 + x])/2 - (3*(2 + x)^4*Log[2 + x])/128 + ((384*(2 + x) - 144*(2
+ x)^2 + 32*(2 + x)^3 - 3*(2 + x)^4 - 192*Log[2 + x])*Log[2 + x])/128 + (17
*(36*(2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*Log[2 + x])*Log[2 + x])/72 + (4
3*Log[2 + x]^2)/12 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (
1251*(2 + x)*Log[2 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x
)^3*Log[2 + x]^2)/4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x
]^3)/4 - (33*(2 + x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2
 + x)^4*Log[2 + x]^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48
+ (37*x^3*Log[3 + x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x
])/12 - (4083*Log[2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*
x^2*Log[2 + x]*Log[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Lo
g[2 + x]*Log[3 + x])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]
^2*Log[3 + x] - (3*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3
 + x])/2 - (3*x^4*Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x]
)/4 + (x^4*Log[2 + x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563
*Log[2 + x]*PolyLog[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4
 - (563*PolyLog[3, -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (19
5*PolyLog[4, -2 - x])/2
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
```

```
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rubi steps

$$\int x^3\log^3(2+x)\log(3+x)\,dx = \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int\frac{x^4\log^3(2+x)}{3+x}\,dx - \frac{3}{4}\int\frac{x^4\log^2(2+x)\log(3+x)}{2+x}\,d$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int\Big(-27\log^3(2+x) + 9x\log^3(2+x) - 3x^2\log^3(2+x)$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,dx - \frac{3}{4}\int x^3$$

$$= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\log(3+x) -$$

$$= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3$$

$$= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{81}{4}(2+x)\log^2$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{765}{16}(2+x)\log$$

$$= -\frac{765x}{8} + \frac{27}{32}(2+x)^2 - \frac{1}{6}(2+x)^3 + \frac{3}{512}(2+x)^4 + \frac{765}{8}(2+x)\log(2+x) - \frac{27}{16}(2+x)^2$$

$$= -\frac{857x}{8} + \frac{79}{32}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{83}{288}x^3\log(2+$$

$$= -\frac{16463x}{96} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{83}{288}x^3\log$$

$$= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 +$$

Mathematica [A] time = 0.243863, size = 412, normalized size = 0.68

$$\frac{-224640\text{PolyLog}(4,-x-2)-24\left(4680\log^2(x+2)-6756\log(x+2)+5609\right)\text{PolyLog}(2,-x-2)+288(780\log(x+2)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^3 \left(\ln(2+x)\right)^3 \ln(3+x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(2+x)^3*ln(3+x),x)
```

```
[Out] int(x^3*ln(2+x)^3*ln(3+x),x)
```

Maxima [A] time = 1.01933, size = 699, normalized size = 1.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \log(x+3) \log(x+2)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(x+3)\log(x+2)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rubi [A] time = 0.0545937, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)$$
$$= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Mathematica [A] time = 0.009478, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+b}\right)^a \frac{1}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)

[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

Fricas [A] time = 2.10029, size = 34, normalized size = 2.

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

Sympy [B] time = 2.81428, size = 313, normalized size = 18.41

$$\begin{cases} -\frac{\sqrt{b}b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}}+\frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}-\frac{b^{\frac{a}{2}}x\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}}-\frac{2b^{\frac{a}{2}}\cosh\left(a\operatorname{asinh}\left(\right.\right.}{a^2\Gamma\left(-\frac{a}{2}\right.} \\ -\frac{b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}}-\frac{b^{\frac{a}{2}}x^2\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}}+\frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}-\frac{2b^{\frac{a}{2}}\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{}}\right.\right.}{a^2\Gamma\left(-\frac{a}{2}\right)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(
a*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(
b)))/(a*sqrt(b)) - b**(a/2)*x*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/
(a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1
- a/2)/(a**2*gamma(-a/2)), Abs(x**2)/Abs(b) > 1), (-b**(a/2)*sinh(-a*asinh(
x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) - b**(a/2)*x**2*sinh(-a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*c
osh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a
*asinh(x/sqrt(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

```
[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rubi [A] time = 0.0239267, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(x+\sqrt{b+x^2}\right)^a dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+a}\left(b+x^2\right)dx, x, x+\sqrt{b+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(bx^{-2+a}+x^a\right)dx, x, x+\sqrt{b+x^2}\right)$$
$$= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)} + \frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

Mathematica [A] time = 0.0595495, size = 46, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{b+x^2}+x\right)^{a-1}\left(\frac{\left(\sqrt{b+x^2}+x\right)^2}{a+1}+\frac{b}{a-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

Maple [B] time = 0.022, size = 120, normalized size = 2.3

$$\frac{a}{4\sqrt{\pi}}b^{\frac{a}{2}+\frac{1}{2}}\left(8\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)\,a\,(2\,a-2)}\left(\frac{ab}{x^2}+a-1\right)\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}+4\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)\,a}\sqrt{1+\frac{b}{x^2}}\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a,x)
```

```
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(1/
x^2*a*b+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)
*b^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

Fricas [A] time = 2.24924, size = 74, normalized size = 1.42

$$\frac{\left(\sqrt{x^2 + b}a - x\right)\left(x + \sqrt{x^2 + b}\right)^a}{a^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

Sympy [B] time = 2.65297, size = 2149, normalized size = 41.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
a**2*b**(7/2)*b**(a/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(9/2
)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/s
```

```
qrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2
*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) + 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**5*
b**(a/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x
**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 -
a/2)) - 2*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + a
sinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b*
*(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*ga
mma(1 - a/2)) + 4*a*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sq
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x*
*2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a
/2)) - 2*a*b**4*b**(a/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(
7/2)*x**2*gamma(1 - a/2)) - 2*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*as
inh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(
b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*
a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*
x**2*gamma(1 - a/2)) - 2*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x/sqrt(b)) +
 asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)), Abs(x**2)/Abs(b) > 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/
b)*sinh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2
*b**(5/2)*gamma(1 - a/2)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma
(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + a*b**(5/2)*b**(a/2)*x*cosh(a*asinh
(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma
(1 - a/2)) + 2*a*b**3*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*
gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2))
 + 2*a*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*
b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt
(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 -
```

```
 a/2)) + 2*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*g
amma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)),
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx$

Optimal. Leaf size=34

$$\frac{x^{a+1} \left(3x^a+2x^{2a}+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rubi [A] time = 0.0436392, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1594, 1747}

$$\frac{x^{a+1} \left(3x^a+2x^{2a}+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1747

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(d*(g*x)^(
m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*g*(m + 1)), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx = \int x^a \left(1+x^a+x^{2a}\right) \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} dx$$
$$= \frac{x^{1+a} \left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}$$

Mathematica [A] time = 0.117796, size = 33, normalized size = 0.97

$$\frac{x^{a+1} \left(3x^a+2x^{2a}+6\right)^{\frac{1}{a}+1}}{6a+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

Maple [A] time = 0.039, size = 44, normalized size = 1.3

$$\frac{xx^a \left(6+3\,x^a+2\,(x^a)^2\right) \sqrt[a]{6+3\,x^a+2\,(x^a)^2}}{6+6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

Maxima [A] time = 1.13808, size = 65, normalized size = 1.91

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^a\right)\left(2\,x^{2\,a}+3\,x^a+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Fricas [A] time = 2.1829, size = 109, normalized size = 3.21

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a}\right)\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}\left(x^{3\,a}+x^{2\,a}+x^{a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rubi [A] time = 0.0357832, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-xx}}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\
&= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.014505, size = 57, normalized size = 0.98

$$\frac{1}{2}\left(\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right) + \sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right) - \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x] + (3*Log[1 - (1 -
 x^2)^(1/3)])/2)/2
```

Maple [C] time = 0.036, size = 65, normalized size = 1.1

$$\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{4\pi}\left(\frac{2\pi\sqrt{3}}{3\Gamma(2/3)}\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)+\frac{2\pi\sqrt{3}x^2}{9\Gamma(2/3)}{}_3F_2(1,1,\frac{4}{3};2,2;x^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3),x)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3],[2,2],
x^2))
```

Maxima [A] time = 1.41737, size = 84, normalized size = 1.45

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Fricas [A] time = 1.96424, size = 198, normalized size = 3.41

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] time = 0.927993, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))
```

Giac [A] time = 1.07427, size = 86, normalized size = 1.48

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

3.34 $\int \frac{1}{x(1-x^2)^{2/3}} \, dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1
- (1 - x^2)^(1/3)])/4
```

Rubi [A] time = 0.0346961, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1
- (1 - x^2)^(1/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)\\ &= -\frac{\log(x)}{2}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right)-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\ &= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\ &= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0124386, size = 81, normalized size = 1.4

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{4}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 + Log[1 - (1 - x^2)^(1
/3)]/2 - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/4
```

Maple [C] time = 0.041, size = 48, normalized size = 0.8

$$\frac{1}{2\,\Gamma\left(2/3\right)}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{3\,\ln\left(3\right)}{2}+2\,\ln\left(x\right)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{2\,\Gamma\left(2/3\right)x^{2}}{3}{}_{3}F_{2}(1,1,\frac{5}{3};\,2,2;\,x^{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(2/3),x)
```

```
[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAMM
A(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

Maxima [A] time = 1.41876, size = 84, normalized size = 1.45

$$-\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\,\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Fricas [A] time = 2.15924, size = 200, normalized size = 3.45

$$-\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(-x^{2}+1\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-\frac{1}{4}\,\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] time = 0.949689, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*g
amma(5/3))
```

Giac [A] time = 1.06623, size = 86, normalized size = 1.48

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rubi [A] time = 0.0050845, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {239}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] time = 0.0398614, size = 86, normalized size = 1.76

$$-\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

Maple [C] time = 0.023, size = 12, normalized size = 0.2

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

Maxima [A] time = 1.43961, size = 105, normalized size = 2.14

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
```

)

Fricas [B] time = 2.03169, size = 225, normalized size = 4.59

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\,\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\,\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\,\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

Sympy [C] time = 0.893634, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rubi [A] time = 0.0332979, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx,x,x^3\right)\\
&= -\frac{\log(x)}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^3}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)\\
&= -\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.012324, size = 55, normalized size = 1.

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Maple [C] time = 0.037, size = 65, normalized size = 1.2

$$\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{6\pi}\left(\frac{2\pi\sqrt{3}}{3\Gamma(2/3)}\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)+\frac{2\pi\sqrt{3}x^3}{9\Gamma(2/3)}{}_3F_2(1,1,\frac{4}{3};2,2;x^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3),x)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],
x^3))
```

Maxima [A] time = 1.43444, size = 84, normalized size = 1.53

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Fricas [A] time = 2.02553, size = 198, normalized size = 3.6

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [C] time = 0.914684, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

Giac [A] time = 1.08466, size = 85, normalized size = 1.55

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=97

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/
3)) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rubi [A] time = 0.0438348, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2148}

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/
3)) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [F] time = 0.0762007, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^3+1)^(1/3),x)
```

```
[Out] int(1/(1+x)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [B] time = 37.3663, size = 788, normalized size = 8.12

$$\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(13x^6+2x^5+19x^4-4x^3+19x^2+2x+13\right)-4\sqrt{2}\left(5x^5-5x^4+6x^3-6x^2+5x-5\right)\left(-x\right.\right.}{6\left(3x^6-18x^5-3x^4-28x^3-3x^2-18x+3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rubi [A] time = 0.107369, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2152, 239, 2148}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```


```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```


Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{3\log}{}$$

Mathematica [F] time = 0.0784828, size = 0, normalized size = 0.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]

[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{1+x}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(-x^3+1)^(1/3),x)
```

```
[Out] int(x/(1+x)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{3\log\left(-2^{2/3}\sqrt[3]{x^2-3x+2}-x+2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{x^2-3x+2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}$$

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - x))/(Sqrt[3]*(2 - 3*x + x^2)^(1/
3))])/(2*2^(1/3)) - Log[2 - x]/(4*2^(1/3)) - Log[x]/(2*2^(1/3)) + (3*Log[2
- x - 2^(2/3)*(2 - 3*x + x^2)^(1/3)])/(4*2^(1/3))
```

Rubi [A] time = 0.0249725, antiderivative size = 176, normalized size of antiderivative = 1.6, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {755, 123}

$$\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2 + x
)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (3*
(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1 +
 x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^(1
/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 755

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[((b + q + 2*c*x)^(1/3)*(b - q
+ 2*c*x)^(1/3))/(a + b*x + c*x^2)^(1/3), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{\left(\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}$$
$$= -\frac{\sqrt{3}\sqrt[3]{-2+x}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(-2+x)^{2/3}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}+\frac{3\sqrt[3]{-2+x}\sqrt[3]{-1+x}\log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{-1+\cdots}\right.}{4\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}$$

Mathematica [C] time = 0.0252075, size = 59, normalized size = 0.54

$$-\frac{3\sqrt[3]{1-\frac{2}{x}}\sqrt[3]{1-\frac{1}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 - 2/x)^(1/3)*(1 - x^(-1))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1),
 2/x])/(2*(2 - 3*x + x^2)^(1/3))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{x^2-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] int(1/x/(x^2-3*x+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2 - 3\,x + 2\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

Fricas [B] time = 18.5736, size = 828, normalized size = 7.53

$$-\frac{1}{12}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6 + 36\,x^5 - 612\,x^4 + 2880\,x^3 - 5760\,x^2 + 5184\,x - 1728\right) + 12\,\sqrt{2}\left(x^5 - 38\,x^4 + 252\,x^3 - 6\right.\right.}{6\left(x^6 - 108\,x^5 + 972\,x^4 - 3456\,x^3 + 6048\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{(x - 2)\,(x - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2 - 3x + 2\right)^{\frac{1}{3}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{4}\log\left(\sqrt[3]{x^3-3x^2+7x-5}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3-3x^2+7x-5}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 + x))/(Sqrt[3]*(-5 + 7*x - 3*x^2 + x^3)^
(1/3))])/2 + Log[1 - x]/4 - (3*Log[1 - x + (-5 + 7*x - 3*x^2 + x^3)^(1/3)])
/4
```

Rubi [A] time = 0.0728112, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}-\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)\\
&= \frac{\left(\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\sqrt{3}\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\log\left(\sqrt[3]{4+(\cdots\right.}{4\sqrt[3]{-4(1-x)+(-1\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0139155, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix+(2-i)}\sqrt[3]{i(x-1)}(x-(1-2i))F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i(x-(1-2i)),-\frac{1}{2}i(x-(1-2i))\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-I/4)*((-1 + 2*I) + x), (-I/2)*((-1 + 2*I) + x)])/(4*(-5 +
7*x - 3*x^2 + x^3)^(1/3))
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

```
[Out] int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

Fricas [A] time = 4.68439, size = 410, normalized size = 5.06

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}(x-1)+\sqrt{3}\left(20389537x^2-40779074x+53222437\right)+17987998}{7204617x^2-14409234x-20666867}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{4}\log\left(\sqrt[3]{x\left(x^2-q\right)}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x\left(x^2-q\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{\log(x)}{4}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*x)/(Sqrt[3]*(x*(-q + x^2))^(1/3))])/2 + Log[
x]/4 - (3*Log[-x + (x*(-q + x^2))^(1/3)])/4
```

Rubi [A] time = 0.0564275, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx &= \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx \\
&= \frac{\left(\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}} \\
&= \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-q+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\sqrt[3]{-q+x^2}\log\left(x^{2/3}-\sqrt[3]{-q+x^2}\right)}{4\sqrt[3]{-qx+x^3}}
\end{aligned}$$

Mathematica [A] time = 0.0822427, size = 127, normalized size = 1.92

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-q}\left(-2\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-q}}\right)+\log\left(\frac{x^{4/3}}{\left(x^2-q\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-q}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/
3))/Sqrt[3]] - 2*Log[1 - x^(2/3)/(-q + x^2)^(1/3)] + Log[1 + x^(4/3)/(-q +
x^2)^(2/3) + x^(2/3)/(-q + x^2)^(1/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x\left(x^2-q\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-q\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

Fricas [B] time = 13.5156, size = 1153, normalized size = 17.47

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(q^{12}-15\,q^{10}+90\,q^{8}-351\,q^{6}+810\,q^{4}-1215\,q^{2}+729\right)\left(x^{3}-qx\right)^{\frac{1}{3}}x-2\sqrt{3}\left(q^{12}+6\,q^{11}-15\,q^{10}-54\right.}{q^{13}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^{2}-q\right)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{4}\log\left(\sqrt[3]{(x-1)\left(q+x^2-2x\right)}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 + x))/(Sqrt[3]*((-1 + x)*(q - 2*x + x^2)
)^(1/3))])/2 + Log[1 - x]/4 - (3*Log[1 - x + ((-1 + x)*(q - 2*x + x^2))^(1/
3)])/4
```

Rubi [A] time = 0.0978022, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{-(1-q)x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+q+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}}{4\sqrt[3]{(1-q)(1-}}$$

Mathematica [A] time = 0.166759, size = 140, normalized size = 1.77

$$\frac{\sqrt[3]{x-1}\sqrt[3]{q+(x-2)x}\left(-2\log\left(1-\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}\right)+\log\left(\frac{(x-1)^{4/3}}{(q+(x-2)x)^{2/3}}+\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(-1 + x)^(
2/3))/(q + (-2 + x)*x)^(1/3))/Sqrt[3]] - 2*Log[1 - (-1 + x)^(2/3)/(q + (-2
+ x)*x)^(1/3)] + Log[1 + (-1 + x)^(4/3)/(q + (-2 + x)*x)^(2/3) + (-1 + x)^(
2/3)/(q + (-2 + x)*x)^(1/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/3))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(x^2+q-2\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2+q-2\,x\right)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

Fricas [B] time = 12.361, size = 1866, normalized size = 23.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
```

```
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2 + q - 2x\right)(x - 1)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

Optimal. Leaf size=118

$$-\frac{3\log\left(\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}-\sqrt[3]{q}(x-1)\right)}{4\sqrt[3]{q}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{q}(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*q^(1/3)*(-1 + x))/(Sqrt[3]*((-1 + x)*(q - 2*
q*x + x^2))^(1/3))])/(2*q^(1/3)) + Log[1 - x]/(4*q^(1/3)) + Log[x]/(2*q^(1/
3)) - (3*Log[-(q^(1/3)*(-1 + x)) + ((-1 + x)*(q - 2*q*x + x^2))^(1/3)])/(4*
q^(1/3))
```

Rubi [F] time = 21.9369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
 (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q
])^(2/3))/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
```

```
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\,dx, x, \frac{1}{3}(-\right.$$

$$= \frac{\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\ \sqrt[3]{-1+5q-4q^2+\frac{}{(1+6q-}}\right.}{}$$

Mathematica [C] time = 0.19341, size = 55, normalized size = 0.47

$$\frac{3\left((x-1)\left(-2qx+q+x^2\right)\right)^{2/3} {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (3*((-1 + x)*(q - 2*q*x + x^2))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (q - 2
*q*x + x^2)/(q*(-1 + x)^2)])/(4*q*(-1 + x)^2)
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{(-1+x)\left(-2\,qx+x^2+q\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

```
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x-1)\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

Fricas [B] time = 167.475, size = 3509, normalized size = 29.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
```

```
((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x-1)\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$

Optimal. Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(k+1)x)}{2\sqrt[3]{k}}-\frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)}-\sqrt[3]{k}x\right)}{2\sqrt[3]{k}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{k}x}{\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{\sqrt[3]{k}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*k^(1/3)*x)/((1 - x)*x*(1 - k*x))^(1/3))/Sqrt[3]])/k
^(1/3) + Log[x]/(2*k^(1/3)) + Log[1 - (1 + k)*x]/(2*k^(1/3)) - (3*Log[-(k^(
1/3)*x) + ((1 - x)*x*(1 - k*x))^(1/3)])/(2*k^(1/3))
```

Rubi [F] time = 0.610747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)}}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{3\sqrt[3]{1-x}x\sqrt[3]{1-kx}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)x)}}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 1.63514, size = 0, normalized size = 0.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{2-(1+k)\,x}{1-(1+k)\,x}\frac{1}{\sqrt[3]{(1-x)\,x\,(-kx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

```
[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/((((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=176

$$\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(kx+2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}-1\right)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - k*x))/((1 - k)^(1/3)*((1 - x)*x*(1 - k
*x))^(1/3)))/Sqrt[3]])/(2^(2/3)*(1 - k)^(1/3))) + Log[1 - (2 - k)*x]/(2^(2/
3)*(1 - k)^(1/3)) + Log[1 - k*x]/(2*2^(2/3)*(1 - k)^(1/3)) - (3*Log[-1 + k*
x + 2^(2/3)*(1 - k)^(1/3)*((1 - x)*x*(1 - k*x))^(1/3)])/(2*2^(2/3)*(1 - k)^
(1/3))
```

Rubi [F] time = 0.461683, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 0.668614, size = 0, normalized size = 0.

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{-kx+1}{1+(-2+k)\,x}\left((1-x)\,x\,(-kx+1)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}\,((k-2)x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}((k-2)x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{(a+b)\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{(a+b)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{}}{}}{}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] ((a + b)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3
)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3
]])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/S
qrt[3] - ((a - c)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(
1/3)*Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2^
(1/3)*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2])/(12*2^(1/3)) - ((a - c)*L
og[1 + x^3])/(6*2^(1/3)) - ((b + c)*Log[1 + x^3])/(6*2^(1/3)) + ((a + b)*Lo
g[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)])/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/(3*
2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + ((a - c)*
Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + (c*Log[x + (1 - x^3)^(1/
3)])/2 - ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rubi [C] time = 0.846166, antiderivative size = 576, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {6728, 239, 2148}

$$-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)\left(3ib-\sqrt{3}\left(2a+b-i\sqrt{3}c-c\right)\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)\left(\sqrt{3}\left(2a+b+\right.\right.}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) - ((2*a + b - I*
Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/
(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((2*a + b + I*Sq
rt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(2 - (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 -
```

```
x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*
(2*a + b - c - I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] +
 2*x))])/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c + I*
Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(12*2^(1
/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)])/2 - (((3*I)*b - Sqrt[3]*(
2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I + Sqrt[3])) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqr
t[3]*c))*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(
I - Sqrt[3]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx = \int\left(\frac{c}{\sqrt[3]{1-x^3}}+\frac{a-c+(b+c)x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx + \int \frac{a-c+(b+c)x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \int\left(\frac{b-\frac{i(2a+b-c)}{\sqrt{3}}+c}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{b+\frac{i(2a+}{\sqrt{}}}{\left(-1+i\sqrt{3}+\right.}\right.$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{1}{3}\left(3b-i\sqrt{3}(2a+b-c)+3c\right)\int\frac{}{\left(-1-i\sqrt{3}+\right.}$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\left(2a+b-i\sqrt{3}b-c-i\sqrt{3}c\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(2a+b+i\sqrt{3}b-\right.}{}$$

Mathematica [F] time = 0.382454, size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{cx^2+bx+a}{x^2-x+1}\,\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}} \left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt[3]{-\left(x - 1\right) \left(x^2 + x + 1\right)} \left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}}\left(x^2 - x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=407

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x+2}{1176(3-2x)^{9/2}\left(2\right.}$$

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

Rubi [A] time = 0.676273, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x+2}{1176(3-2x)^{9/2}\left(2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```


```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
```

```
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

<u>Rule 740</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

<u>Rule 822</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

<u>Rule 828</u>

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{1}{784}\int \frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{\int \frac{325752-543704x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^3}\,d\cdots}{460992}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049\cdots}{32928(3-2x)^{9/2}\left(1+\cdots\right.}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049\cdots}{32928(3-2x)^{9/2}\left(1+\cdots\right.}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} \cdots$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{\cdots}{1176(3-\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+\cdots\right.}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2\cdots}$$

Mathematica [C] time = 2.07546, size = 198, normalized size = 0.49

$$\frac{56\left(-88070400x^{12}+677249280x^{11}-1873554048x^{10}+2443779648x^{9}-2343370048x^{8}+3106712560x^{7}-2888625656x^{6}+1470758860x^{5}-1627773523x^{4}+107385515\right.}{(3-2x)^{9/2}\left(2x^{2}+x+1\right)^{4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((56*(-40289347 + 429812744*x - 135202154*x^2 + 1073855156*x^3 - 1627773523
*x^4 + 1470758860*x^5 - 2888625656*x^6 + 3106712560*x^7 - 2343370048*x^8 +
2443779648*x^9 - 1873554048*x^10 + 677249280*x^11 - 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) + (45*I)*Sqrt[14 - (2*I)*Sqrt[7]]*(115739*I +
146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7 - I*Sqrt[7]]] - (45*I)*Sqrt[14
 + (2*I)*Sqrt[7]]*(-115739*I + 146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7
 + I*Sqrt[7]]])/121436356608
```

Maple [A] time = 0.089, size = 584, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] 1/151263/(3-2*x)^(9/2)+5/235298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2
823576/(3-2*x)^(3/2)+505/3294172/(3-2*x)^(1/2)+1/6588344*(567651623/32*(3-2
*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+9801432515/384*(3-2*x)^(5/2)-8763772
549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2)-200063633/384*(3-2*x)^(11/
2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*x)^(15/2))/((3-2*x)^2-7+14*x
)^4+731595/13492928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)
)*(7+2*14^(1/2))^(1/2)*14^(1/2)-1424965/6746464256*ln(3-2*x+14^(1/2)+(3-2*x
)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2*
14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2
))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)*
arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*1
4^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)-731595/13492928512*ln(3
-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(
```

```
1/2)+1424965/6746464256*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2
))*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(
3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14
^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+
2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578695/3373232128/
(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*1
4^(1/2))^(1/2))*14^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5(-2\,x+3)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

Fricas [B] time = 3.1791, size = 5156, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/852282865707923134247251378176*(2263908918780*22241759018113166^(1/4)*sqr
t(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10
+ 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
 - 1242*x^2 - 162*x - 243)*sqrt(21292357711*sqrt(14) + 81630132224)*arctan(
1/100521871569518694695269086857534372287294018150 40*22241759018113166^(3/4
)*sqrt(12577271771)*sqrt(79716926)*sqrt(-2089731384934400*22241759018113166
^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 8163013222
4)*(7645*sqrt(14) - 115739) - 4190418993502514995568679111884800*x + 209520
9496751257497784339555942400*sqrt(14) + 6285628490253772493353018667827200)
*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357711*sqrt(14) + 816
30132224) - 1/195818453485129580290665890 2*22241759018113166^(3/4)*sqrt(797
16926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2*x + 3)*sqrt(21292
```

```
357711*sqrt(14) + 81630132224) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 22639089
18780*22241759018113166^(1/4)*sqrt(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2
816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^
6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(2129235771
1*sqrt(14) + 81630132224)*arctan(1/246286190725939683846687007560504554 42*2
2241759018113166^(3/4)*sqrt(12577271771)*sqrt(22241759018113166^(1/4)*sqrt(
79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)*(7645*sqr
t(14) - 115739) - 2005242886101391892*x + 1002621443050695946*sqrt(14) + 30
07864329152087838)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357
711*sqrt(14) + 81630132224) - 1/19581845348512958029066589 02*22241759018113
166^(3/4)*sqrt(79716926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2
*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224) + 2/7*sqrt(14)*sqrt(7) + s
qrt(7)) + 315*22241759018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 -
 229870452342784*x^12 + 459740904685568*x^11 - 480638218534912*x^10 + 55900
3145469952*x^9 - 734018148958208*x^8 + 498923368153088*x^7 - 34611176062976
0*x^6 + 407660880326656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 10
1384624222208*x^2 - 21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11
- 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x
^4 + 936*x^3 - 1242*x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)
*sqrt(21292357711*sqrt(14) + 81630132224)*log(2089731384934400/12577271771*
22241759018113166^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt
(14) + 81630132224)*(7645*sqrt(14) - 115739) - 3331739243453861593088 00*x +
 1665869621726930796544 00*sqrt(14) + 4997608865180792389632 00) - 315*222417
59018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 - 229870452342784*x^1
2 + 459740904685568*x^11 - 480638218534912*x^10 + 559003145469952*x^9 - 734
018148958208*x^8 + 498923368153088*x^7 - 346111760629760*x^6 + 407660880326
656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 101384624222208*x^2 -
21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x
^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*
x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)*sqrt(21292357711*sq
rt(14) + 81630132224)*log(-2089731384934400/12577271771*22241759018113166^(
1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)
*(7645*sqrt(14) - 115739) - 333173924345386159308800*x + 166586962172693079
654400*sqrt(14) + 499760886518079238963200) + 393027605675872810832*(880704
00*x^12 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^
8 - 3106712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107
3855156*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*
x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 -
 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5 (-2\,x+3)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=648

result too large to display

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rubi [A] time = 1.1581, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+81109}{10119696384(3-2x)^{19/2}\left(2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(227573899787428l6768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int \frac{2534672-332298\cdots}{(3-2x)^{21/2}\left(1+x+2\cdots\right.}}{2765952\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+\cdots}{691488(3-2x)^{1\cdots}}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+17\cdots}{7056(3-2x)^{19/2}\left(\cdots\right.}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} + \frac{x}{63(3-2x)^{19/2}\left(1\cdots\right.}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{30295088\cdots}{15550330251509\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{30295088\cdots}{15550330251509\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{30295088\cdots}{15550330251509\cdots}$$

Mathematica [C] time = 6.08693, size = 610, normalized size = 0.94

$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}+\frac{\frac{67816x+20776}{1568(3-2x)^{19/2}\left(2x^2+x+1\right)^8}+\frac{\frac{117492592x+46521776}{1372(3-2x)^{19/2}\left(2x^2+x+1\right)^7}+\frac{\frac{164128134240x+74020332960}{1176(3-2x)^{19/2}\left(2x^2+x+1\right)^6}+\frac{\frac{184316990760000x+94209549053760}{980(3-2x)^{19/2}\left(2x^2+x+1\right)^5}}{}}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 - 2
*x)^(19/2)*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)^(
19/2)*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*x)^
(19/2)*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^5) + ((95476201213680000 + 157747397367934080*
x)/(784*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4) + ((72879297583985544960 + 8973
5798552133000960*x)/(588*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + ((3643273421
2165998389760 + 18400346379541577848320*x)/(392*(3 - 2*x)^(19/2)*(1 + x + 2
*x^2)^2) + ((6440121232839552246912000 - 15435719146659136558464000*x)/(196
*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (39479926882545221954112000/(19*(3 - 2
*x)^(19/2)) + (-908021664138480966930240000/(17*(3 - 2*x)^(17/2)) + (-19105
520493023248582746201600/(3 - 2*x)^(15/2) + (-26849557435537239465884310720
000/(13*(3 - 2*x)^(13/2)) + (-150994423858598796539274120000000/(3 - 2*x)^(
11/2) + (-8237718113587514139784976619840000/(3 - 2*x)^(9/2) + (-3383893120
36560466460044072847040000/(3 - 2*x)^(7/2) + (-1013530552857651055083639451
5648960000/(3 - 2*x)^(5/2) + (-204334375738495648812805956791073600000/(3 -
 2*x)^(3/2) + (-2230994866519889796828561036406228800000/Sqrt[3 - 2*x] + ((
Sqrt[(7 - I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 - (7175
0597240923349846054347713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 -
 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) + (Sqrt[(7 + I*Sqrt[7])/
2]*(-31233928131278457155599854509687203200000 + (7175059724092334984605434
7713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqr
t[7]]])/(-14 - (2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)
/196)/392)/588)/784)/980)/1176)/1372)/1568)/1764
```

Maple [A] time = 0.053, size = 719, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] 1/5367029731/(3-2*x)^(19/2)+5/4802079233/(3-2*x)^(17/2)+73/23727920916/(3-2
*x)^(15/2)+165/25705247659/(3-2*x)^(13/2)+2365/221460595216/(3-2*x)^(11/2)+
30349/1993145356944/(3-2*x)^(9/2)+854095/43406276662336/(3-2*x)^(7/2)+75933
/3100448333024/(3-2*x)^(5/2)+8519225/260437659974016/(3-2*x)^(3/2)+891605/1
2401793332096/(3-2*x)^(1/2)+1/86812553324672*(-43462358811134257841/1179648
*(3-2*x)^(27/2)+192384852501874197/65536*(3-2*x)^(29/2)-1352841099712333/81
92*(3-2*x)^(31/2)+4606702222670185/786432*(3-2*x)^(33/2)-25865320405815/262
144*(3-2*x)^(35/2)+54476517033015O812273/1024*(3-2*x)^(1/2)-347698778390586
0258979/1536*(3-2*x)^(3/2)+93649997064789O8741137/2048*(3-2*x)^(5/2)-238519
05772903279054347/4096*(3-2*x)^(7/2)+192983613795383541041317/36864*(3-2*x)
^(9/2)-57758421475348449750643/16384*(3-2*x)^(11/2)+60333035869584695411551
/32768*(3-2*x)^(13/2)-14977088508349397804O723/196608*(3-2*x)^(15/2)+662568
9994458215569681l/262144*(3-2*x)^(17/2)-17729978841543630405471/262144*(3-2
*x)^(19/2)+286987827112128306O373/196608*(3-2*x)^(21/2)-165574989211387894 4
81/65536*(3-2*x)^(23/2)+45406001689183688581/131072*(3-2*x)^(25/2))/((3-2*x
)^2-7+14*x)^9+206922416016525/127441383880959773900 8*ln(3-2*x+14^(1/2)+(3-2
*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)-3896156139350
75/637206919404798869504*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/
2))*(7+2*14^(1/2))^(1/2)-206922416016525/637206919404798869504/(-7+2*14^(1/
2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/
2))*(7+2*14^(1/2))*14^(1/2)+389615613935075/318603459702399434752/(-7+2*14^
(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^
(1/2))*(7+2*14^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))
^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))
*14^(1/2)-206922416016525/1274413838809597739008*ln(3-2*x+14^(1/2)-(3-2*x)^
(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)+389615613935075/6
37206919404798869504*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*
(7+2*14^(1/2))^(1/2)-206922416016525/637206919404798869504/(-7+2*14^(1/2))^
(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*
(7+2*14^(1/2))*14^(1/2)+389615613935075/318603459702399434752/(-7+2*14^(1/2
))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2
))*(7+2*14^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))^(1/
2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2+x+1\right)^{10}(-2x+3)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

Fricas [B] time = 3.68979, size = 11992, normalized size = 18.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/1094755373086200603246995644663447631605361478665641987670016*(4732002380
085251586622550100*4787936175075825342943147314686^(1/4)*sqrt(1169607525756
986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 6468403
2*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21
+ 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 46771
2000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^
12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 2003648
4*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 +
 137781*x + 59049)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735
1936)*arctan(1/365621708519319702488553401133870353544174572418706268660249
45379489008832725311219252*4787936175075825342943147314686^(3/4)*sqrt(27763
87167632535361)*sqrt(12865682783326846)*sqrt(1169607525756986)*sqrt(4787936
175075825342943147314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14
) - 9756589235) - 71440233164918992209696826631202812*x + 28280279689505005
187146*sqrt(22335021272086100802556094) + 107160349747378488314545239946804
218)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3275718505284
62403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615767666910014
4258228441327447900096742*4787936175075825342943147314686^(3/4)*sqrt(116960
7525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(-2*x +
 3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936) + 2/7*sqr
t(14)*sqrt(7) + sqrt(7)) + 4732002380085251586622550100*4787936175075825342
```

```
943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5
505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^
23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 54
0503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720
*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 -
49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 32
76126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(32757185052846
2403199*sqrt(14) + 1226422380928157351936)*arctan(1/392966702348163030765553
30542603297083388480635973027797585697454399143598928370335464344780800*47
87936175075825342943147314686^(3/4)*sqrt(2776387167632535361)*sqrt(11696075
25756986)*sqrt(-14862107440409842545228890767360000*47879361750758253429431
47314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(32757185052846240
3199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235)
- 1061752420864956548109093061495542399038192585561809435358469816320000*x
+ 420304555190263689316852795001664341102416628348354560000*sqrt(2233502127
2086100802556094) + 1592628631297434822163639592243313598557288878342714153
037704724480000)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615
7676669100144258228441327447900096742*4787936175075825342943147314686^(3/4)
*sqrt(1169607525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))
*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735193
6) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 271150425*47879361750758253429431473
14686^(1/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 6751
484641146648498184126464*x^27 + 30381680885159918241828569088*x^26 - 793299
44533473119853663485952*x^25 + 146844790944939604835504750592*x^24 - 237989
833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 47435
2077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 6323
36315413643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 609
018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 50
5075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 3
04685469106942025132773736448*x^13 + 228722407218762404519491928064*x^12 -
129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 -
 61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24
573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 274119
3833525857491515080704*x^5 + 4017914249140640432768679936*x^4 + 90121441102
2199723237834752*x^3 + 1013866212399974688642564096*x^2 - 32757185052846240
3199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 +
 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261
568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^1
6 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105
219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 +
5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*
x + 59049) + 168977702066662448107094016*x + 72419015171426763474468864)*sq
rt(327571850528462403199*sqrt(14) + 1226422380928157351936)*log(14862107440
409842545228890767360000/2776387167632535361*478793617507582534294314731468
```

6^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sq
rt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 38242
2319640069460132720868272698184789257093120000*x + 1513854263880146561657014
81356328960000*sqrt(22335021272086100802556094) + 573633479460104190199081302409047277183885639680000) - 271150425*4787936175075825342943147314686^(1
/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 675148464114
6648498184126464*x^27 + 30381680885159918241828569088*x^26 - 79329944533473
119853663485952*x^25 + 146844790944939604835504750592*x^24 - 23798983360041
9359560990457856*x^23 + 362048363881489025715123781632*x^22 - 4743520771534
19437787597242368*x^21 + 550984441886077267281495195648*x^20 - 632336315413
643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 60901819951
4371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 5050756649
75624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 304685469
106942025132773736448*x^13 + 228722407218762404519491928064*x^12 - 12904395
1976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 6106806
7637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 2457319241
2708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 27411938335258
57491515080704*x^5 + 4017914249140640432768679936*x^4 + 9012144110221997232
37834752*x^3 + 1013866212399974688642564096*x^2 - 327571850528462403199*sqr
t(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 1197342
72*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20
 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9) + 168977702066662448107094016*x + 72419015171426763474468864)*sqrt(32757
1850528462403199*sqrt(14) + 1226422380928157351936)*log(-148621074404098425
45228890767360000/2776387167632535361*4787936175075825342943147314686^(1/4)
*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14)
+ 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640
069460132720868272698184789257093120000*x + 1513854263880146561657014813563
28960000*sqrt(22335021272086100802556094) + 5736334794601041901990813024090
47277183885639680000) + 1272935063665829315736416183610522832*(240031204937
714427494400*x^27 - 2621948941596237063782400*x^26 + 1236504505589681110548
4800*x^25 - 33969890064381284111155200*x^24 + 65360120291258796757811200*x^
23 - 106701725825102321939251200*x^22 + 162290307223249502039654400*x^21 -
216634228326470609547509760*x^20 + 253788172995391086570485760*x^19 - 28727
9159180291305208156160*x^18 + 304010591010966811155955200*x^17 - 2826446645
39994827031006720*x^16 + 258819256815163249845447936*x^15 - 229408132984166
521977166336*x^14 + 172649692294614969274168896*x^13 - 13331254137724638611
5890240*x^12 + 102031573634317834547976132*x^11 - 5979110268149411757214917
6*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8 +
10718131725916893151555068*x^7 - 8685973988079840377705700*x^6 + 3673303058
277822225386926*x^5 - 809990362095044210054958*x^4 + 1362587089603925431664
856*x^3 + 111926768697602999806116*x^2 + 205702452014540322797289*x - 48844

```
17100172357749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x
^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 38
6777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120
*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 +
 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*
x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 +
 826686*x^2 + 137781*x + 59049)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}(-2\,x+3)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

3.49 $\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx$

Optimal. Leaf size=1058

result too large to display

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^(3
9/2)) - 39481943432914017403219964l5/(20288146313940419593773462323 2*(3 - 2
*x)^(37/2)) - 30468822926262022273648081l/(53736171318004354599724305612 8*(
3 - 2*x)^(35/2)) + 21243158467565674556538629 25/(1688851098565851144562763 8
90688*(3 - 2*x)^(33/2)) + 47657515074514118796095929535/(666328524343253997
03658138959872*(3 - 2*x)^(31/2)) + 349116199939747140621727519 85/(124667917
4577701026713603890216 96*(3 - 2*x)^(29/2)) + 149066309808794760843017404825
/(16249818206564516830956630017310 72*(3 - 2*x)^(27/2)) + 15848613964169066 5
43734380171/(60184511876164877151691222286336 0*(3 - 2*x)^(25/2)) + 11155168
22297077423237689114 5/(1685166332532616560247354224017408*(3 - 2*x)^(23/2))
 + 140118184980910202724749563 75/(101109979951956993614841253441044 48*(3 -
2*x)^(21/2)) + 173441368149804378661935869705/(896508488907352010051592447 1
77261056*(3 - 2*x)^(19/2)) - 227240908234699051527135195 45/(160427834857105
096535548122126457241 6*(3 - 2*x)^(17/2)) - 10119027441277961867857327524 5/(
396351121411671414970177713488894361 6*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(343504305223448559640820685023708446 72*(3 - 2*x)^(13/2)) - 2
21161958879091179482634260749 5/(406920484649315986036049119181931544576*(3
 - 2*x)^(11/2)) - 14340146755077724762794043702 5/(73985542663511997461099839
851260280832*(3 - 2*x)^(9/2)) - 461105327811714301090756231758 5/(7250583181
024175751187784305423507521536*(3 - 2*x)^(7/2)) - 4059653724406305107209268
90227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 49866814
791877818534173165227 75/(870069981722901090142534116650820902584 32*(3 - 2*x
)^(3/2)) - 927027754781476746208047620505/(58004665448193406009502274443388
060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
+ (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
+ (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
```

```
*(919498192874055581221 + 908287136092467468517*x))/(15767116285922279455 45
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(101879828309036267250647 04*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(259931356880226511008 1 - 1042614244862318737918 7*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 275137234631942623837 05*x))/(200184925800211612843376 64*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(265132244281690164788 43 + 30673
415406553789342019*x))/(764342443964444339947438 08*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(884116091130079810446 43 - 5712269536245152162963*x))/(1
2589169665296730305016 6272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
134768122576081481 5 + 96593481283901949034610 7*x))/(1958315281268380269669 2
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
971189122193607250286930 61 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
747076841330318422074328424 12032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
12219360725028693061 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/812065316274707 6
841330318422074328424 12032 + (115*(302971189122193607250286930 61 - 80611109
111432760539830227 87*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/162413063254941536826606368441 4
865684824064 - (115*(302971189122193607250286930 61 - 806111091114327605398 3
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064
```

Rubi [A] time = 2.48972, antiderivative size = 1058, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -1305695962836335553428578 5425/(106924014357253562723941220352*(3 - 2*x)^(3
9/2)) - 3948194343291401740321996415/(202881463139404195937734623232*(3 - 2
*x)^(37/2)) - 30468822926262022273648081 1/(537361713180043545997243056128*(
3 - 2*x)^(35/2)) + 21243158467565674556538629 25/(168885109856585114456276 38
90688*(3 - 2*x)^(33/2)) + 4765751507451411879609592953 5/(66632852434325399 7
03658138959872*(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(124667917
457770102671360389021696*(3 - 2*x)^(29/2)) + 1490663098087947608430174048 25
/(16249818206564516830956630017 31072*(3 - 2*x)^(27/2)) + 158486139641690665
```

43734380171/(6018451187616487715169122228633360*(3 - 2*x)^(25/2)) + 11155168
222970774232376891145/(168516633253261656024735422401740 8*(3 - 2*x)^(23/2))
 + 140118184980910202724749563 75/(101109979951956993614841253441044 48*(3 -
2*x)^(21/2)) + 1734413681498043786619358697 05/(8965084889073520100515924471
77261056*(3 - 2*x)^(19/2)) - 2272409082346990515271351954 5/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 1011902744127796186785732752 45/(
39635112141167141497017771348889436 16*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(3435043052234485596408206850237084467 2*(3 - 2*x)^(13/2)) - 2
2116195887909117948263426074 95/(4069204846493159860360491191819315445 76*(3
 - 2*x)^(11/2)) - 1434014675507772476279404370 25/(739855426635119974610998 39
851260280832*(3 - 2*x)^(9/2)) - 4611053278117143010907562317585/(7250583181
02417575118778430542350752153 6*(3 - 2*x)^(7/2)) - 405965372440630510720926 8
90227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 49866814
7918778185341731652277 5/(870069981722901090142534116650820902584 32*(3 - 2*x
)^(3/2)) - 927027754781476746208047620505/(580046654481934060095022744433 88
060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
 + (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
 + (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
*(919498192874055581221 + 908287136092467468517*x))/(1576711628592227945545
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
28188494452225747*x))/(1018798283090362672506470 4*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 27513723463194262383705*x))/(2001849258002116128433766 4*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673
415406553789342019*x))/(764342443964444339947438 08*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^3) - (115*(8841160911300798104464 3 - 5712269536245152162963*x))/(1
258916966529673030501662 72*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
1347681225760814815 + 96593481283901949034610 7*x))/(1958315281268380269669 2
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
97118912219360725028693061 + 8061110911143276053983022787*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
7470768413303184220743284241203 2 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
12219360725028693061 + 8061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[

```
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
84133031842207432842412032 + (115*(30297118912219360725028693061 - 80611109
11143276053983022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414
865684824064 - (115*(30297118912219360725028693061 - 8061110911143276053983
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{\int \frac{3640-3164x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{19}}\,dx}{3724}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{\int \frac{13067712-1\cdots}{(3-2x)^{41/2}\left(1\cdots\right.}}{13138\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40\cdots}{7976808(3-\cdots}$$

Mathematica [C] time = 6.17009, size = 1100, normalized size = 1.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
- 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720 + 209
2401353236681536 0*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^13) + ((72110377354780278913835520 + 145295342948683106164016640*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((172901458108932896335179 8
01600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((370557652515461812186329087129600 + 645802967231886306826540 4
24448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((696175598675973 4
387590105775549440 00 + 1088028437838790621809440473088716800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 14102294542
802935921085802172484323479552 00*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((1283308803395067168818807997696073436639232000 + 4214391612869991217
701355842462048362373120 00*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(35990904373909724999169578894625893014666444800 0 - 1443636121324398194831 6
934609927589309137960960 00*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-1152021624816869759475691381872221626869209284608000 - 304008932978051919 9
0311701662609533815702602547200 00*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-2255746282697145245681128263365627409125133109002240000 - 260969551
132552925541038265166507347084573298900992000 0*x)/(588*(3 - 2*x)^(39/2)*(1
 + x + 2*x^2)^3) + ((-179025112076931306921152249904224040100017283046080512
0000 + 11566803321414359689429580460467850992426782273339392 0000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((728708609249104660434063569009474612 5
228872832203816960000 0 + 246446709008728269296921307345877681002519066261 03
43034880000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-530550566665897 08
749302646546014801249192995757488046080000 0/(3 - 2*x)^(39/2) + (-1708089006
242241264480481073293611769771298388785813753364480000/(37*(3 - 2*x)^(37/2)
) + (-69674095008990920001753978369242721670427118803840269792051200 0/(3 -
2*x)^(35/2) + (757366667762147355602446006474261151597409525795681824661504
000000/(3 - 2*x)^(33/2) + (616772664905423340350737254793402194192083509401
```

```
0816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196
645778610361943940861637274650890661068800000/(29*(3 - 2*x)^(29/2)) + (4496
42332343658017982593566780723917564662924080341591025022313472000000/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352
841560457216000/(3 - 2*x)^(25/2) + (4292688672152380230641488715509188225990
0254208806769817062280254541824000000/(3 - 2*x)^(23/2) + (2893692593980364
723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (1187674764929302643741666332431406660460687631018179076613208076411
190359040000000/(3 - 2*x)^(19/2) + (-231306413716622859705373724141636828470
2251691242315976748933281043780325376000000/(3 - 2*x)^(17/2) + (-992239519
6537908604226239489579648523559858468009362133384187617620979500236800000000
/(3 - 2*x)^(15/2) + (-1094151831515463224315724158790180962508360120997317660
69014678416546026147551232000000000/(3 - 2*x)^(13/2) + (-807326848531423306384
03379340954315600692165352258493007480189439306347456219136000000000/(3 -
2*x)^(11/2) + (-443379872262112313052073614945722839817152039380963932483996
6666511839997547213824000000000/(3 - 2*x)^(9/2) + (-1833019089221669774417370
67901437000873585615761361787541745447275781173253597919232000000000/(3 -
2*x)^(7/2) + (-55354121000273595704884421471602824549908674640172352332478066
05576616684137250589491200000000/(3 - 2*x)^(5/2) + (-113323856633918397403439
7442837068388756677147138484115167264239399928318213926633984000000000000/
(3 - 2*x)^(3/2) + (-132722026290813148740383963535523427142665518975435293064
35677723641008864036246751334400000000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-18581083680713840823653754894973279799973172656560941029009948813
0974124096507454518681600000000000 - (385341400627810314676798762240149669933
633555592186583754201688526589748283311569009254400000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-185810836807138408236537548949732797999731726565
609410290099488130974124096507454518681600000000000 + (3853414006278103146767
98762240149669933633555592186583754201688526589748283311569009254400000000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724
```

Maple [A] time = 0.08, size = 989, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x)
```

```
[Out] 7192279694031133468210490184035/3248261265098830736532127368829731369648128
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)
*14^(1/2)+1345753163328079019021293274756 5/81206531627470768413303184220743
2842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-3484168674905226483378299702015/8
1206531627470768413303184220743284241 2032/(-7+2*14^(1/2))^(1/2)*arctan((2*(
3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)-71922796
94031133468210490184035/3248261265098830736532127368829731369648128*ln(3-2*
x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2
)+13457531633280790190212932747565/812065316274707684133031842207432842412 0
32/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+
2*14^(1/2))^(1/2))*(7+2*14^(1/2))-3484168674905226483378299702015/812065316
27470768413303184220743284241 2032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(
1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)-1345753163328079
0190212932747565/1624130632549415368266063684414865684824064*ln(3-2*x+14^(1
/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)+13457531633280
790190212932747565/1624130632549415368266063684414865684824064*ln(3-2*x+14^
(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)+683151246370
725/30145677658996078082575630336/(3-2*x)^(1/2)+10/2952313853011644037/(3-2
*x)^(37/2)+143/7819642097165976098/(3-2*x)^(35/2)+355/5266289575642392066/(
3-2*x)^(33/2)+52865/277038748585308867472/(3-2*x)^(31/2)+14333/323956601168
30472406/(3-2*x)^(29/2)+1478345/1689042692987850837168/(3-2*x)^(27/2)+47538
7/312785683886639043920/(3-2*x)^(25/2)+16575515/7006399319060714583808/(3-2
*x)^(23/2)+246866015/73567192850137503129984/(3-2*x)^(21/2)+1/3111898385606
868039/(3-2*x)^(39/2)+8972680075/1667523037936450070946304/(3-2*x)^(17/2)+1
02495360575/16479051198430800701116416/(3-2*x)^(15/2)+122484655975/17852305
464966700759542784/(3-2*x)^(13/2)+10815878546425/1480368099325700262983624 7
04/(3-2*x)^(11/2)+320421783064625/30145677658996078082575630336/(3-2*x)^(3/
2)+8192823353/186370221887015007929 2928/(3-2*x)^(19/2)+769045155125/1009341
885903886542943380 48/(3-2*x)^(9/2)+838467657280275/105509871806486273289014
706176/(3-2*x)^(7/2)+9270470094105/107663134496414564580627 2512/(3-2*x)^(5/
2)-7192279694031133468210490184035/1624130632549415368266063684414865684824
064/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7
+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)-7192279694031133468210490184035
/1624130632549415368266063684414865684824064/(-7+2*14^(1/2))^(1/2)*arctan((
2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))
*14^(1/2)+1/30145677658996078082575630336*(807597736492641378942268937217 99
5835353849465/1048576*(3-2*x)^(1/2)+490738543064879423955077165987434152441
563270473/1002342287671296*(3-2*x)^(53/2)-5501183528836128900201169317937 83
16699033102675/1002342287671296*(3-2*x)^(55/2)+180866897114899220649017210 2
870787954874541181/334114095890432*(3-2*x)^(57/2)-11968977253082880651292 89
2111395530933265219/25701084299264*(3-2*x)^(59/2)+3395565446412935417599589
88614814460549873/9826885173248*(3-2*x)^(61/2)-642433967191403749984730270 0
9027485263697/29480655519744*(3-2*x)^(63/2)+12988685274872711035742561867 29
22324659/1133871366144*(3-2*x)^(65/2)-50350269350528973443805751560519372 5/
```

```
103079215104*(3-2*x)^(67/2)+133883313322119397348791732981953297/8246337208
32*(3-2*x)^(69/2)-3254850748003483429666738850178379/824633720832*(3-2*x)^(
71/2)+360433340020130123942335063779145/5772436045824*(3-2*x)^(73/2)-928342
237074576734557978321305/1924145348608*(3-2*x)^(75/2)-447963293570690822971
54473725670903546220392558695/9070970929152*(3-2*x)^(43/2)+2860722331769322
3698395672584150593863016075796143/29480655519744*(3-2*x)^(45/2)-5059022664
167725408892162874688680417923742003781/29480655519744*(3-2*x)^(47/2)+73012
476452577571533836489036461787385135079265/2680059592704*(3-2*x)^(49/2)-193
9242920901534821454026903132433081580221023737/501171143835648*(3-2*x)^(51/
2)-1006304725834560333245233940167063186576585913370455/10720238370816*(3-2
*x)^(39/2)+13805722741822612586258592099428566280191230197271405/3930754069
2992*(3-2*x)^(37/2)-22397546321209486953062074374795737299957063565/3145728
*(3-2*x)^(3/2)+404531566689883337048499233527781983599187634017/12582912*(3
-2*x)^(5/2)-1188598027552254830082683218064697188605612952419/12582912*(3-2
*x)^(7/2)+3831583379166294091823572953989993625772471445345/18874368*(3-2*x
)^(9/2)+9977850126168010187169130424774568330973123412551261/21592276992*(3
-2*x)^(13/2)-1255696718499588580979726331572072320357969297077745/239914188
8*(3-2*x)^(15/2)+2672239984790337844292019294315182385216573077301785/11792
2622078976*(3-2*x)^(41/2)+1186323846453826237212517196312193819452761764018
822545/3915399561216*(3-2*x)^(21/2)-176509423589632626758711731662298093167
44939271143/51904512*(3-2*x)^(11/2)-688617380989400554399451644246187148600
7042005189775/125627793408*(3-2*x)^(27/2)+136329987967245395141848253765147
208279814148352958009/5527622909952*(3-2*x)^(29/2)-550660914208175901678654
01986871791412011888132876913/5527622909952*(3-2*x)^(31/2)+2737487528928439
357869138774910126923363791747141675/755914244096*(3-2*x)^(33/2)-1166457217
0215876884203668230743495214488310113371105/9826885173248*(3-2*x)^(35/2)+12
646629333382722716904430763732665179119615389552413/25098715136*(3-2*x)^(17
/2)-2593673203685044441695042001860835122939346700333136537/6199382638592*(
3-2*x)^(19/2)-755930116404682856570195190192032441294632160945523631/391539
9561216*(3-2*x)^(23/2)+8535085022072145119870938660211240879080416346972440
59/7830799122432*(3-2*x)^(25/2))/((3-2*x)^2-7+14*x)^19
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^{2} + x + 1\right)^{20} (-2\,x + 3)^{\frac{41}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,x^2+x+1)^{20}(-2\,x+3)^{\frac{41}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

3.50 $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

Optimal. Leaf size=378

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{8}{117600\left(x^2-\right.}$$

```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rubi [A] time = 0.773819, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{8}{117600\left(x^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```


```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
```

```
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
```

```
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx = -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-1235+1335x-800x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^4}\,dx}{1400}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} - \cdots$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \cdots$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \cdots$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{\cdots}{1050\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{1\cdots}{280\,(3-2x+x\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-1\cdots}{6860000000\,(3\cdots}$$

Mathematica [C] time = 6.02736, size = 342, normalized size = 0.9

$$560\left(4596238560x^{17}-38639385552x^{16}+188603773872x^{15}-606785954952x^{14}+1459208021718x^{13}-267914387048\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] (560*(-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3
- 1409335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952
*x^7 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 3999656
132532*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 +
 188603773872*x^15 - 38639385552*x^16 + 4596238560*x^17) - (9*I)*Sqrt[50 +
(10*I)*Sqrt[7]]*(-299844895*I + 932587773*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 +
 x + 5*x^2 - 3*x^3 + 2*x^4)^4*ArcTanh[(13 + I*Sqrt[7] + (-5 - I*Sqrt[7])*x)
/(Sqrt[50 + (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])] + 9*Sqrt[50 - (10*I)*Sqrt
[7]]*(299844895 - (932587773*I)*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 + x + 5*x^2
 - 3*x^3 + 2*x^4)^4*ArcTanh[(-13 + I*Sqrt[7] + (5 - I*Sqrt[7])*x)/(Sqrt[50
- (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])])/(691488000000000*(3 - 2*x + x^2)^(
9/2)*(1 + x + 2*x^2)^4)
```

Maple [B] time = 0.533, size = 21028, normalized size = 55.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5\left(x^2-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

Fricas [B] time = 4.00831, size = 11537, normalized size = 30.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/71086524447232167580280752950240000000000*(264602060868765123019815591460
74412800*x^18 - 21168164869501209841585247316859530240 0*x^17 + 1018717934344
745723626290027123864892800*x^16 - 3214915039555496244690759436248041155200
*x^15 + 7688343631118056605744516779381246569200*x^14 - 1398091139115337718
7559506313807067863200*x^13 + 20977982138251784909414754860497120398000*x^1
2 - 25712705264922250829450580100197810638400*x^11 + 28757282727793479526 19
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 18045860551249781389951 4233
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
4096608462190931685680490685615200*x^5 - 3602042876982878244*33780221308347
3608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16
- 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10
- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*
x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 22064095148
2187776)*arctan(1/964393622349963919677467835514205441102895152270484353118
304*sqrt(205487899)*(120712108677220094151311009251129 40*sqrt(4167294734812
9)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(205487899)*(5*337802213083473608
^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876
830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(201502465*sq
rt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) +
 24142421735444018830262201850 22588*sqrt(41672947348129)*sqrt(7)*(125*sqrt(
2) + 172))*sqrt(164483605088694913184970968*x^2 + sqrt(205487899)*(33780221
3083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 4
2834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x -
 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 22
0640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x + 3)*(4*x + 1
) - 123362703816521184888728226*x + 205604506360868641481213710*sqrt(2) + 2
87846308905216098073699194) + 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10*x - 19) +
```

9*x - 29) + 1/1149179274607135296320480808070751888*sqrt(205487899)*(5*3378
02213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349) - 573381349
*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(sqrt(2)*(20
1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^(
3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*3378022130834736
08^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*x + 3))*sqr
t(151363871237318045*sqrt(2) + 220640951482187776) - 1/476*sqrt(x^2 - 2*x +
 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) + 172)) + 1/
476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) - 3602042876982878244*3378
02213083473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 +
 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 1
7389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*
x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 2
20640951482187776)*arctan(-1/9643936223499639196774678355142054411028951522
70484353118304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(41
672947348129)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) - sqrt(205487899)*(5*3378022
13083473608^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381
349) + 2876830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(2
01502465*sqrt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 2206409514
82187776) + 2414242173544401883026220185022588*sqrt(41672947348129)*sqrt(7)
*(125*sqrt(2) + 172))*sqrt(164483605088694913184970968*x^2 - sqrt(205487899
)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*
sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(8
9801606*x - 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*s
qrt(2) + 220640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x +
3)*(4*x + 1) - 123362703816521184888728226*x + 205604506360868641481213710*
sqrt(2) + 287846308905216098073699194) - 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10
*x - 19) + 9*x - 29) + 1/1149179274607135296320480808070751888*sqrt(2054878
99)*(5*337802213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349)
- 573381349*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(
sqrt(2)*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213
083473608^(3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*33780
2213083473608^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*
x + 3))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) + 1/476*sqrt(
x^2 - 2*x + 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) +
 172)) - 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 9*33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(7)*(3530255223715004416*x^18 -
28242041789720035328*x^17 + 135914826113027670016*x^16 - 428926009681373036
544*x^15 + 1025759783440690970624*x^14 - 1865298603830415458304*x^13 + 2798
830469551551938560*x^12 - 3430525513645055541248*x^11 + 3836725505323763236
864*x^10 - 3640134417553133928448*x^9 + 3410447187060176453632*x^8 - 240763
4062573633011712*x^7 + 2028793548878716600320*x^6 - 969496340812733087744*x
^5 + 972364673182001528832*x^4 - 87373816786946359296*x^3 + 363395647091163
267072*x^2 - 151363871237318045*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16 - 19
44*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16

```
498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3
+ 1647*x^2 + 162*x + 243) + 357438341401144197 12*x + 53615751210171629568)*
sqrt(151363871237318045*sqrt(2) + 220640951482187776)*log(19083512352618334
937598521302939860992*x^2 + 2369114176935798061127434 24/2041974420058321*sq
rt(205487899)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(89801606*sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)
*(sqrt(2)*(89801606*x - 132636591) - 42834985*x + 222438197))*sqrt(15136387
1237318045*sqrt(2) + 220640951482187776) - 477087808815458373439963032573 49
65248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14312634264463751203198890977204895 74
4*x + 238543904407729186719981516286748262 40*sqrt(2) + 333961466170820861 40
797412280144756736) - 9*337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*s
qrt(7)*(3530255223715004416*x^18 - 28242041789720035328*x^17 + 135914826113
027670016*x^16 - 428926009681373036544*x^15 + 1025759783440690970624*x^14 -
 1865298603830415458304*x^13 + 2798830469551551938560*x^12 - 34305255136450
55541248*x^11 + 3836725505323763236864*x^10 - 3640134417553133928448*x^9 +
3410447187060176453632*x^8 - 2407634062573633011712*x^7 + 20287935488787166
00320*x^6 - 969496340812733087744*x^5 + 972364673182001528832*x^4 - 8737381
6786946359296*x^3 + 363395647091163267072*x^2 - 151363871237318045*sqrt(2)*
(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*
x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x
^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243) + 357438341401
14419712*x + 53615751210171629568)*sqrt(151363871237318045*sqrt(2) + 220640
951482187776)*log(19083512352618334937598521302939860992*x^2 - 236911417693
579806112743424/2041974420058321*sqrt(205487899)*(337802213083473608^(1/4)*
sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 42834985) - 337802
213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x - 132636591) - 428
34985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 220640951482187776)
 - 47708780881545837343996303257349 65248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14
312634264463751203198890977204895744*x + 238543904407729186719981516286748 2
6240*sqrt(2) + 333961466170820861407974122801447 56736) + 728813301405404935
717704569729687107 5600*x^4 - 654890100650193679474043588865341716800*x^3 +
272374746406785098508522674459903486 7600*x^2 + 575692617810432196147398388 0
*(4596238560*x^17 - 38639385552*x^16 + 188603773872*x^15 - 606785954952*x^1
4 + 1459208021718*x^13 - 2679143870481*x^12 + 3999656132532*x^11 - 49157979
13008*x^10 + 5380603084494*x^9 - 5134334619701*x^8 + 4591320676952*x^7 - 33
59813871472*x^6 + 2503427226914*x^5 - 1409335257371*x^4 + 1002897791524*x^3
 - 266966654968*x^2 + 261702502714*x - 53205422447)*sqrt(x^2 - 2*x + 3) + 2
679095866296246870575632863540034296 00*x + 4018643799444370305863449295310 0
5144400)/(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13
 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7
 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.51 $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=638

result too large to display

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(104273720488000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 194216499620458423 4*
x)/(156410580732000000000*(3 - 2*x + x^2)^(15/2)) - (11*(750232510630820 1
089 - 781398637972651688 6*x))/(406667509903200000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(6905326851529635901 1 - 4484073619501828600 6*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(93846348439200000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(3128211614640000000000000*(3 - 2*
x + x^2)^(7/2)) - (655140551156544930 1689 - 312729855998330930 1910*x)/(5213
686024400000000000000*(3 - 2*x + x^2)^(5/2)) - (4179039782398459850819 -
1886993445589652402694*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
0000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
 + 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(810422259212746896054789447978008548464 05 + 57
30592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
4222592127468960547894479780085484640 5 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(2729445895232483817 49 + 1919410263866451098 41*Sqrt[2] +
 (6568266422965386014 31 + 4648856159098934915 90*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/32282885600000000000000 - (Sqrt[(-810422259212746896054789447978
00854846405 + 573059225230017071260263638786665003089 92*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 573059225230017 0
71260263638786665003089 92*Sqrt[2]))]*(272944589523248381749 - 1919410263866
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/32282885600000000000000
```

Rubi [A] time = 1.39713, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1210549587451867106183 3-5117656435043679338190x}{104273720488000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)}-417$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(18401244792000000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*
x)/(15641058073200000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 7813986379726516886*x))/(40666750990320000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(3128211614640000000000000*(3 - 2*
x + x^2)^(7/2)) - (655140551165449301689 - 3127298559983309301910*x)/(5213
68602440000000000000000*(3 - 2*x + x^2)^(5/2)) - (4179039782398459850819 -
1886993445589652402694*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
000000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(81042225921274689605478944797800854846405 + 57
305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
42225921274689605478944797800854846405 + 5730592252300170712602636387866650
0308992*Sqrt[2]))]*(272944589523248381749 + 191941026386645109841*Sqrt[2] +
 (656826642296538601431 + 464885615909893491590*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/322828856000000000000000000 - (Sqrt[(-810422259212746896054789447978
00854846405 + 57305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 5730592252300170
```

```
7126026363878666500308992*Sqrt[2]))]*(272944589523248381749 - 1919410263866
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/322828856000000000000000
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{3150}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2 \right.}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x \right.}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{476849951294984711-125}{104273720488000000000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{476849951294984711-125}{104273720488000000000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{476849951294984711-125}{104273720488000000000}$$

Mathematica [C] time = 11.5202, size = 1431, normalized size = 2.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*
x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3 -
 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)^7)
 + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(-731
61291 + 43964675*x))/(90843750000000000*(3 - 2*x + x^2)^5) + (-1340879383 +
 430593031*x)/(181687500000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 11
2950205*x))/(3028125000000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 152
86717673*x))/(36337500000000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277
+ 484788625685*x))/(363375000000000000000*(3 - 2*x + x^2)) + (251943 + 2217
70*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200
0000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000
0000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121010400000000
0000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/
(2964754800000000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 1696303
89653846945*x)/(37059435000000000000000*(1 + x + 2*x^2)^3) + (108242210919
6374795 + 4797048907791526114*x)/(830131344000000000000000*(1 + x + 2*x^2)
^2) + (65571203144429922747 + 367152793968978953465*x)/(3631824630000000000
00000000*(1 + x + 2*x^2))) + ((2324428079549467457950*I + 216341778311919248
41*Sqrt[7])*ArcTan[(-135063738860435016899586558948733259113515 + (18863089
46264666902168552859950458893964 05*I)*Sqrt[7] - 150624136187268800855926877
6761430483700000*x - (1057115009374721927181156513503524479386 80*I)*Sqrt[7]
*x + 4911535405084435870258097898135419857073 60*x^2 - (460764064177139993 39
997510087266331039942 0*I)*Sqrt[7]*x^2 - 18008498514724668919944874526497767
8818020*x^3 + (1978682963779138708638376809534460093968 60*I)*Sqrt[7]*x^3 -
1760048165007618809267744855998310477758 25*x^4 - (207342833228459577163557 0
43035558264835165*I)*Sqrt[7]*x^4 + (18624424819975554815958568260566612600 4
224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (114611845046003414 2
52052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]
+ (30085609324575896241163841036299912662220 8*I)*Sqrt[10*(-5 + I*Sqrt[7])]*
x^2*Sqrt[3 - 2*x + x^2] - (14326480630750426781506590969666625077248 0*I)*Sq
rt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(236877329083883697986467 8
49302388474659482 3*I + 42364294025923873547394266318002595672950 5*Sqrt[7] +
 (18906134860656203017600742185567453116469 36*I)*x + 615057455931122825839 4
328777942059796320*Sqrt[7]*x + (25113002598558229623408930278522391576678 20
```

```
*I)*x^2 - 20278675508011061898677634310942275963200*Sqrt[7]*x^2 - (313421774
62307603571283187974993808123037880*I)*x^3 + 634304316027200432791928669683
69397935660*Sqrt[7]*x^3 + (94474906488662646732838536919046070366969 7*I)*x^4
 + 16381317765107264789462917221030750634835*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/16141442800000000000000000)*(
-2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7])*ArcTan[(35*(436249
42906639466765851862182126076285 95*I + 12104084007406821013541218948000741 6
20843*Sqrt[7] - (40919031596617332707196094500783237405000*I)*x + 175730701
694606521668409393655487422752*Sqrt[7]*x + (264872883292651275777339658533 6
4310310620*I)*x^2 - 579390728800316054247932408884065027 52*Sqrt[7]*x^2 - (1
52388941497528256839248140210078630706 20*I)*x^3 + 18122980457920012365483 67
627667697083876*Sqrt[7]*x^3 - (79583727195997580891324420376561996359 5*I)*x
^4 + 46803765043163613684179763488659287528 1*Sqrt[7]*x^4))/(135063738860435
016899586558948733259113515 + (18863089462646669021685528599504588939640 5*I
)*Sqrt[7] + 15062413618726880085592687767614304837 00000*x - (10571150093747
21927181156513503524479386 80*I)*Sqrt[7]*x - 4911535405084435870258097898135
41985707360*x^2 - (46076406417713999339997510087266331039942 0*I)*Sqrt[7]*x^
2 + 18008498514724668919944874526497767881802 0*x^3 + (197868296377913870863
837680953446009396860*I)*Sqrt[7]*x^3 + 1760048165007618809267744855998310 47
775825*x^4 - (20734283322845957716355704303555826483516 5*I)*Sqrt[7]*x^4 - (
14326480630750426781506590969666625077248*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[
3 - 2*x + x^2] - (14326480630750426781506590969666625077248*I)*Sqrt[70*(5 +
 I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (28652961261500853563013181939333250
154496*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(5 +
I*Sqrt[7])] - ((-23244280795494674579 5*I + 21634177831191924841*Sqrt[7])*Lo
g[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(32282885600000000
000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/32282885600000000000000000)*(2324
42807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*
I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288
5600000000000000000)*(232442807954946745795*I + 21634177831191924841*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I
)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I
*Sqrt[70*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(-5 + I*Sqrt[7]
)] + ((-232442807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(1 + x
+ 2*x^2)*(-163*I + 15*Sqrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*S
qrt[7]*x^2 - (13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sq
rt[10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(32282885600000000000000000
*Sqrt[70*(5 + I*Sqrt[7])])
```

Maple [B] time = 3.525, size = 86793, normalized size = 136.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\left(x^2-2\,x+3\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

Fricas [B] time = 6.01536, size = 30714, normalized size = 48.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/143446642399233767656421653603627286364552203186085711012681333261811 2898
720320000000000000000000*(36045960776272236628083717974972055111190660172853
35839613572876193438663181794274827857920 0*x^38 - 558712392032219667735297 6
28612066854223455232679227055140103795809982992793178112598317977600*x^37 +
 481213576363234358984917634965876935734395313307592334588411978971824061 53
4769535689519032320 0*x^36 - 287106077583008364742686813670652418960633608 27
67769996252210795888073895224299139900388833280 0*x^35 + 1315091821471322213
0798493456693867156629022480813387143850168942182236782785878687425086138 88
00*x^34 - 486154125862132172417754359560821543038072883167534048023582332 29
5256693402493082467454965081600*x^33 + 150125118590038017914558770715112989
42846464125440398838218598654092338180386116340659933361664 00*x^32 - 396012
07680724195081933453907329150443109061726945797184770092066969689498801613 7
708626219776640 0*x^31 + 909142000002142860704234021157216421398787616081889
741815089464543532901571757526703136964242800 0*x^30 - 1842476492987215827 06
```

```
98990044243761821209838303936935404825000845297214002673057816247491027262800*x^29 + 33413073756673638925333625169011170445598811516221975115590200411293389434416479555356860509015600*x^28 - 54816532560449295459717517003382699673242410936114304344629842103656622934490247108012261346586400*x^27 + 82245983094063518667736627604663547588572840238581597325736701493749880383650749401133206999014400*x^26 - 113722848067639694402592735862649094093874045443618754295078471234595964240139128161766283626302000*x^25 + 146086574413322248286514192550522624098477614094095488624493581512454991258074867544318895241990800*x^24 - 175027094081001021682973752997412023251736305226127144272811232619626419165679723993392477178363200*x^23 + 196887291605784159433455654443374481739030277196290989156609388218395099469530751149958413044135200*x^22 - 208068683375682167383215047521697995267539026087882795784482813901791360434798005710722616487282000*x^21 + 208171444918478482519618165392015730347012009814583465001141378703189206795143605224483243158516400*x^20 - 196227556184540408353167422341576855508320001795821851558311176995574081069015969836642878534431200*x^19 + 176534941677723459681422280024952573032106299529482816321219585323399086976471958310981405494523200*x^18 - 149136255738011380556954829398929258737007615204074730330565887220730783382923822619571340737358000*x^17 + 121890814483587724389011961696733756253105383654426234336150913799569962877883235263704480534144400*x^16 - 91983186053222129635537069278588580392985745730700928388526309371776740142438834607398588992195200*x^1
5 + 69317814132471559316390137037592557060398996838342232414889371690271398098098738643314402130954400*x^14 - 45743070841132500247970739727093296878765897323708593659902862883667237249390700654758574610918000*x^13 + 32996965521676394929803121509049143329451789049169789455644615129199190308917673348518481311574800*x^12 - 17770083757788737971933739892049927033484890029804651938270182161740937851280707834822272274354400*x^11 + 13544225267451459701960369238256374351899362683978498551483729852256655264147093337392596228028800*x^10 - 4813759732728488651728668551069958186240925466978671799825767568732599092879797201593187475517200*x^9 + 5091181133639025216832620106123280320347641869015804163342220634415255665812683873707564839486000*x^8 - 464213118503056400758348994571884060773399462026537769017971996084095803827142837363184426478400*x^7 + 1771233883264782126042267141811413849986971398265032235916172889879027134542752439323372429279200*x^6 + 239115034543163209918411032521665649750496447867853609069487786445410804754998849116452338787600*x^5
+ 79817891129994413353362937273464455099835468*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(2
)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376
*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 4
74602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 207502
6563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 295688552
9*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x
^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 -
252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7
 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 1
```

```
37781*x + 59049)*sqrt(810422259212746896054789447978008548464O5*sqrt(2) + 1
14611845046003414252052727757333000617984)*arctan(1/54206850781156887023310
518673090274966005685838243268724684064391985051350175945649154733957770247
4316735105663737127495350143727198183643523606196 8*sqrt(7952794329052729639
11047)*(99395132505231928164221165932167972928150165110013784621706793019 90
*sqrt(11005224487862873621128239642490888848098)*sqrt(288886807671054271567
2947094311)*sqrt(7)*(10*sqrt(2) + 9) + sqrt(15905588658105459278220 94)*(5*1
2649387528042651238155741051177996081490572724 18^(3/4)*sqrt(288886807671054
271567294709431 1)*sqrt(35)*(340613697110906370000*sqrt(2) - 483753219647003
202703) + 556695603033691074737732 9*126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(4373478266
4604992355*sqrt(2) - 66269826580994560232))*sqrt(810422259212746896054789 44
797800854846405*sqrt(2) + 114611845046003414252052727757333000617984) + 147
461812540444568715696613114138557910359478676937042172325597372869522935182
724790786*sqrt(2888868076710542715672947094311)*sqrt(7)*(125*sqrt(2) + 172)
)*sqrt(519179873173490157373042187501297125639064382628558151181380506 4*x^2
 + sqrt(15905588658105459278220 94)*(126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682
*sqrt(2) - 62135959399493560795) - 126493875280426512381557410511779960814 9
057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 1054043
15061877410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(810
422259212746896054789447978008548464O5*sqrt(2) + 114611845046003414252052 72
7757333000617984) - 129794968293372539343260546875324281409766095657139537 7
953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 38938490488011761802978164062597 2
844229298286971418613386035379 8*x + 874869761179272589826814757400740628067
45190*sqrt(11005224487862873621128239642490888848098) + 9085647780536077754
028238281272699698683626695999767645674158862) + 5/35309486994022006419332*
sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(sqrt(2)*(10*x - 19
) + 9*x - 29) + 1/701918227692516147086715878423299535653311118502220320740
269843494858929171469770004141 36*sqrt(15905588658105459278220 94)*(5*1264938
7528042651238155741051177996081490572724 18^(3/4)*sqrt(35)*(sqrt(2)*(3406136
97110906370000*x + 143139522536096832703) - 483753219647003202703*x - 19747
4174574809537297) + 556695603033691074737732 9*126493875280426512381557410 51
177996081490572724 18^(1/4)*sqrt(35)*(sqrt(2)*(43734782664604992355*x + 2253
5043916389567877) - 66269826580994560232*x - 21199738748215424478) - (5*126
49387528042651238155741051177996081490572724 18^(3/4)*sqrt(35)*(340613697110
906370000*sqrt(2) - 483753219647003202703) + 556695603033691074737732 9*1264
938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*(4373478266460
4992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x + 3))*sqrt(81042225
921274689605478944797800854846405*sqrt(2) + 114611845046003414252052727757 3
33000617984) - 1/35309486994022006419332*sqrt(x^2 - 2*x + 3)*(5*sqrt(110052
24487862873621128239642490888848098)*sqrt(7)*(10*sqrt(2) + 9) + 74179594525
256316007*sqrt(7)*(125*sqrt(2) + 172)) + 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1
) + 172*x - 82)) + 79817891129994413353362937273464455099835468*12649387528
04265123815574105117799608149057272418^(1/4)*sqrt(15905588658105459278220 94
```

)*sqrt(35)*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 18679
68*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 2
61706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329
345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895
*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^
17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 +
468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*
x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3
+ 1023516*x^2 + 137781*x + 59049)*sqrt(810422259212746896054789447978008548
46405*sqrt(2) + 114611845046003414252052727757333000617984)*arctan(-1/54206
850781156887023310518673090274966005685838243268724684064391985051350175945
649154733957770247431673510566373712749535014372719818364352360619 68*sqrt(7
952794329052729639110 47)*(993951325052319281642211659321679729281501651100 1
378462170679301990*sqrt(110052244878628736211282396424908888848098)*sqrt(288
88680767105427156729 47094311)*sqrt(7)*(10*sqrt(2) + 9) - sqrt(1590558865810
545927822094)*(5*126493875280426512381557410511779960814905727 2418^(3/4)*sq
rt(28888680767105427156729 47094311)*sqrt(35)*(340613697110906370000*sqrt(2)
 - 483753219647003202703) + 556695603033691074737 7329*126493875280426512381
5574105117799608149057272418^(1/4)*sqrt(28888680767105427156729 47094311)*sq
rt(35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(81042225
92127468960547894479780085484 6405*sqrt(2) + 11461184504600341425205272775 73
33000617984) + 147461812540444568715696613114138557910359478676937042172325
597372869522935182724790786*sqrt(28888680767105427156729 47094311)*sqrt(7)*(
125*sqrt(2) + 172))*sqrt(519179873173490157373042187501297125639064382628 55
81511813805064*x^2 - sqrt(1590558865810545927822094)*(126493875280426512381
5574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43
268355662383849682*sqrt(2) - 62135959399493560795) - 126493875280426512381 5
574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383
849682*x - 105404315061877410477) - 62135959399493560795*x + 14867267072426
1260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 11461184
5046003414252052727757333000617984) - 129794968293372539343260546875324281 4
097660956571395377953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117
618029781640625972844229298286971418613386035379 8*x + 874869761179272589826
814757400740628067451 90*sqrt(110052244878628736211282396424908888848098) + 9
085647780536077754028238281272699698683626695999767645674158862) - 5/353094
86994022006419332*sqrt(110052244878628736211282396424908888848098)*sqrt(7)*(
sqrt(2)*(10*x - 19) + 9*x - 29) + 1/7019182276925161470867158784232995356 53
311118502220320740269843494858929171469770004141 36*sqrt(1590558865810545927
822094)*(5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt(35)
(sqrt(2)(340613697110906370000*x + 143139522536096832703) - 4837532196470
03202703*x - 197474174574809537297) + 556695603033691074737 7329*12649387528
04265123815574105117799608149057272418^(1/4)*sqrt(35)*(sqrt(2)*(43734782664
604992355*x + 22535043916389567877) - 66269826580994560232*x - 211997387482
15424478) - (5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt
(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 556695603033

6910747377329*12649387528042651238155741051177996081490572724 18^(1/4)*sqrt(
35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x +
3))*sqrt(810422259212746896054789447978008548464 05*sqrt(2) + 1146118450460
0341425205272775733300061 7984) + 1/353094869940220064 19332*sqrt(x^2 - 2*x +
3)*(5*sqrt(1100522448786287362112823964249088 8848098)*sqrt(7)*(10*sqrt(2)
+ 9) + 74179594525256316007*sqrt(7)*(125*sqrt(2) + 172)) - 1/476*sqrt(7)*(2
5*sqrt(2)*(5*x - 1) + 172*x - 82)) + 24453*126493875280426512381557410511 77
99608149057272418^(1/4)*sqrt(15905588658105459278 22094)*sqrt(35)*sqrt(7)*(5
8681264663553748097050996611754496316407808*x^38 - 909559602285083095504290
447482194692904321024*x^37 + 783394883258442537095630804766922525824044 2368
*x^36 - 467396273045205603593011188012624563160188 19072*x^35 + 214091258966
892905713578429763409810498374336512*x^34 - 7914378840963908726941828569900
21126475411881984*x^33 + 24439719810238523891830041696355042012 09831489536*
x^32 - 644690528110056763535019773928811658079354067 3536*x^31 + 14800438431
924516080565532176343356954693567774720*x^30 - 2999472018305304975160392093
829197571806972818 2272*x^29 + 543950385039779684976755634437931462765495913
02144*x^28 - 892389434445447556850205620339886110375559938 70336*x^27 + 1338
929022148270110928895284729232813282189440450 56*x^26 - 18513587658740219001
15319976337160348351326899404 80*x^25 + 237822622904897041561702187373073394
318812215508992*x^24 - 2849365368510540397648886779947929676842861781 31968*
x^23 + 320523992669231941724388481023319612454782787125248*x^22 - 338726814
722685956738928688519407226164440916295680*x^21 + 338894106068517834886487 0
69250634573161464946753536*x^20 - 31944997194601654648963735003466931034597
1696140288*x^19 + 287391247503511322489973442496808422958321912250368*x^18
- 242787372161112804792074580815007335314268010577920*x^17 + 19843297253643
776777198157655776836216999227529625 6*x^16 - 1497446473652920153595628913 24
224536622995129499648*x^15 + 112846402465271023024054467724570114025686780 8
70656*x^14 - 744677403166664192013658577194943223419930620 72320*x^13 + 5371
763229976795816995048942365255053104303518515 2*x^12 - 289289275588053521479
653590670201003027060028866 56*x^11 + 220494127626442517733091046795428093 56
433843290112*x^10 - 783659258401410153171286014794040365856569740 4928*x^9 +
828822262243108881363453045861727397949487448064 0*x^8 - 755718873364113816
635702120278992782166815932416*x^7 + 288349213189327895035480258909753471 72
93712375808*x^6 + 38926893124454150220322865713501113396192765542 4*x^5 + 77
026621102026789199647241685504778793625478758 4*x^4 + 151897599700059336983 3
590252568040870450277908 48*x^3 + 117307057194105230541603999703274443460516
511744*x^2 - 810422259212746896054789447978008548464 05*sqrt(2)*(512*x^38 -
7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 213239
04*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28
- 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 24
86100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 27872
33482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 13065372
72*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11
+ 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^
6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 5904

9) + 15791334622283396419062076883133098158146453504*x + 676771483812145560
8169461521342756353491337216)*sqrt(8104222592127468960547894479780085484640
5*sqrt(2) + 11461184504600341425205272757333000617984)*log(514926300974084
6168871608737947327093513510106682349523414420454231938660554455908352*x^2
+ 16517307604525632141069927349727551216675979497245715202048/1665374957748
9013357854121082231147111*sqrt(1590558865810545927822094)*(1264938752804265
123815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(43268355662383849682*sqrt(2) - 62135959399493560795) - 12649387528042651
23815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(432683556
62383849682*x - 105404315061877410477) - 62135959399493560795*x + 148672670
724261260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 114
61184504600341425205272757333000617984) - 12873157524352115422179021844868
31773378377526670587380853605113557984665138613977088*sqrt(x^2 - 2*x + 3)*(
4*x + 1) - 3861947257305634626653706553460495320135132580011762142560815340
673953995415841931264*x + 8677020686577845807036123864705024753105175633308
5943213766737920*sqrt(11005224487862873621128239642490888848098) + 90112102
670464807955253152914078224136486426866941116659752357949058926559702978396
16) - 24453*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(15
90558865810545927822094)*sqrt(35)*sqrt(7)*(58681264663553748097050996611754
496316407808*x^38 - 909559602285083095504290447482194692904321024*x^37 + 78
33948832584425370956308047669225258240442368*x^36 - 46739627304520560359301
118801262456316018819072*x^35 + 2140912589668929057135784297634098104983743
36512*x^34 - 791437884096390872694182856990021126475411881984*x^33 + 244397
1981023852389183004169635504201209831489536*x^32 - 644690528110056763535019
7739288116580793540673536*x^31 + 148004384319245160805655321763433569546935
67774720*x^30 - 29994720183053049751603920938291975718069728182272*x^29 + 5
4395038503977968497675563443793146276549591302144*x^28 - 892389434445447556
85020562033988611037555993870336*x^27 + 13389290221482701109288952847292328
1328218944045056*x^26 - 185135876587402190011531997633716034835132689940480
*x^25 + 2378226222904897041561702187373073394318812215508992*x^24 - 28493653
6851054039764888677994792967684286178131968*x^23 + 320523992669231941724388
481023319612454782787125248*x^22 - 3387268147226859567389286885194072261644
40916295680*x^21 + 338894106068517834886487069250634573161464946753536*x^20
 - 319449971946016546489637350034669310345971696140288*x^19 + 2873912475035
11322489973442496808422958321912250368*x^18 - 24278737216111280479207458081
5007335314268010577920*x^17 + 198432972536437767771981576557768362169992275
296256*x^16 - 14974464736529201535956289132422453662295129499648*x^15 + 11
2846402465271023024054467724570114025686780870656*x^14 - 744677403166664192
01365857719494322341993062072320*x^13 + 53717632299767958169950489423652550
531043035185152*x^12 - 28928927558805352147965359067020100302706002886656*x
^11 + 22049412762644251773309104679542809356433843290112*x^10 - 78365925840
14101531712860147940403658565697404928*x^9 + 828822262243108881363453045861
7273979494874480640*x^8 - 755718873364113816635702120278992782166815932416*
x^7 + 2883492131893278950354802589097534717293712375808*x^6 + 3892689312445
41502203228657135011133961927655424*x^5 + 770266211020267891996472416855047

787936254787584*x^4 + 151897599700059336983359025256804087045027790848*x^3
+ 117307057194105230541603999703274443460516511744*x^2 - 810422259212746896
0547894479780085484 6405*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808
*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 12913
5330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x
^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22
 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 -
2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 6497
38605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^
9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 +
 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 157913346222833964190620 76
883133098158146453504*x + 676771483812145560816946152134275635349133 7216)*s
qrt(810422259212746896054789447978008548 46405*sqrt(2) + 1146118450460034142
520527277573330006 17984)*log(5149263009740846168871608737947327093513510106
682349523414420454231938660554455908352*x^2 - 165173076045256321410699273 49
727551216675979497245715202048/1665374957748901335785412108223114 7111*sqrt(
1590558865810545927822094)*(126493875280426512381557410511779960814905727 24
18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682*sqrt(2)
 - 62135959399493560795) - 1264938752804265123815574105117799608149057272 41
8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877
410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(81042225921
2746896054789447978008548 46405*sqrt(2) + 114611845046003414252052727757333 0
00617984) - 128731575243521154221790218448683177337837752667058738085360511
3557984665138613977088*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 386194725730563462 66
537065534604953201351325800117621425608153406739539954158419312 64*x + 86770
206865778458070361238647050247531051756333085943213766737920*sqrt(110052244
878628736211282396424908888480 98) + 901121026704648079552531529140782241364
864268669411166597523579490589265597029783 9616) + 473149067064481998763217 7
095551053069435129325807560467936484016398888622099880639632054327716 00*x^4
 + 933056734920520960789163462383318633282684143000124192505535084262195413
2744964749811340457520 0*x^3 + 720578468552481534074799505602958944515340229
247620663519126104677735071637729950975529263056 00*x^2 + 106889973888659738
282685156250267055278630902305879619360872457 20*(3372249001933422237824 2713
60*x^37 - 535022053996400313947961477 12*x^36 + 46914939408298970172949457 58
72*x^35 - 28474992209126677533830352990 72*x^34 + 13254252261100740556512388
253568*x^33 - 49770080058525077628064229832576*x^32 + 15601073493700873938 8
220889457760*x^31 - 417516398850754397130111919794336*x^30 + 97153817191336
52518737068733536 52*x^29 - 1993653213575521837888601204380228*x^28 + 365555
34718529576062573454141400 31*x^27 - 6054769996581738503753686155104785*x^26
 + 9155494158513869230271529746307221*x^25 - 127401066776850481786936051030
09787*x^24 + 16442770202470076313197215936814318*x^23 - 1977256973428874472
0189854470201506*x^22 + 22286437617621909921609206629636086*x^21 - 23584986
647560742443188031208946882*x^20 + 23579397211179175240196614296051673*x^19
 - 22218747553941794885903840542461607*x^18 + 199122954540802465836363916 13
811979*x^17 - 16801760806053390242995145349148613*x^16 + 136134079650064752

```
88139078599341572*x^15 - 102793056507331786692236340209620076*x^14 + 7606288
378303449524327938977040824*x^13 - 5069838234992751929471190426115248*x^12
+ 3507425970596197680016078213030977*x^11 - 1974814483061344405275851094534
735*x^10 + 1357002388430055881833293557852283*x^9 - 566969010759169461615951
049236597*x^8 + 458426000073846882432457044306894*x^7 - 947045576652534893
32536549937026*x^6 + 135183920426913231415208872303230*x^5 - 10230953189017
74638403186272874*x^4 + 293980411535249733439176017421 51*x^3 + 193395719557
00627087816291348 23*x^2 + 339746235039894784806358384346 1*x - 8003871087155
5316861345369643)*sqrt(x^2 - 2*x + 3) + 970009476897571295869922411388598 57
915526569321795089319882360245079721182002108785167400796 00*x + 41571834724
1816269658538176309399391066542439950550382799495829621770233637151894793 57
45748400)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
+ 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] Timed out
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}(x-a)}{\sqrt{\left(x^2+1\right)(x-a)}}\right)$$

```
[Out] -(Sqrt[2]*Sqrt[a + Sqrt[1 + a^2]]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*
(-a + x))/Sqrt[(-a + x)*(1 + x^2)]])
```

Rubi [C] time = 1.23427, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6719, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{\left(x^2+1\right)(-(a-x))}}+\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{\left(x^2+1\right)(-(a-x))}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int\left(\frac{1}{\sqrt{-a+x}\sqrt{1+x^2}}-\frac{2\sqrt{1+a^2}}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\right)dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{}{\sqrt{-}}}{\sqrt{(-a+x)\left(1+\right.}}$$

$$= -\frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} + \frac{\left(2i\sqrt{\right.}}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\text{Su}}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\right)\text{Subst}}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\Pi\left(\frac{}{1-i(a-}\right.}{\left(1-i\left(a-\sqrt{1+a^2}\right)\right)}$$

Mathematica [C] time = 1.14754, size = 213, normalized size = 3.23

$$\frac{2\sqrt{\frac{a-x}{a+i}}\left(2i\sqrt{a^2+1}\sqrt{1-ix}\sqrt{x^2+1}\Pi\left(\frac{2i}{a-\sqrt{a^2+1}+i};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2i}{a+i}\right)-\left(\sqrt{a^2+1}-a-i\right)\sqrt{1+ix}(x+i)\text{EllipticF}\left(\sin^{-1}\left(\right.\right.\right.}{\left(-\sqrt{a^2+1}+a+i\right)\sqrt{1-ix}\sqrt{\left(x^2+1\right)(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] (2*Sqrt[(a - x)/(I + a)]*(-((-I - a + Sqrt[1 + a^2])*Sqrt[1 + I*x]*(I + x)*
EllipticF[ArcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]) + (2*I)*Sqrt[1 + a
^2]*Sqrt[1 - I*x]*Sqrt[1 + x^2]*EllipticPi[(2*I)/(I + a - Sqrt[1 + a^2]), A
rcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]))/((I + a - Sqrt[1 + a^2])*Sqr
t[1 - I*x]*Sqrt[(-a + x)*(1 + x^2)])
```

Maple [C] time = 0.094, size = 1275, normalized size = 19.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x)
```

```
[Out] 2*(-a-I)*((-a+x)/(-a-I))^(1/2)*((x-I)/(a-I))^(1/2)*((x+I)/(a+I))^(1/2)/(-a*
x^2+x^3-a+x)^(1/2)*EllipticF(((-a+x)/(-a-I))^(1/2),((a+I)/(a-I))^(1/2))-2*(
a^2+1)^(1/2)*((-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(
a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+
x)/(-a-I))^(1/2),-(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))*a+I*(-1/(-a-I)*a
+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*
x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),-(a+I)/(a
^2+1)^(1/2),((a+I)/(a-I))^(1/2))-(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-
I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/
2)*EllipticPi(((-a+x)/(-a-I))^(1/2),(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2)
)*a-I*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I
/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I
))^(1/2),(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))-I/(a^2+1)^(1/2)*(1-I*x)^(
1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2
*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+
I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))*a^2-I/(a^2+1
```

```
)^(1/2)*(1-I*x)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^
2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/
2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1
/2))+I/(a^2+1)^(1/2)*(1-I*x)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(
1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*
EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(
-I/(-a-I))^(1/2))*a^2+I/(a^2+1)^(1/2)*(1-I*x)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x
)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-
a+(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)
^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)(a-x)}\left(a - x - \sqrt{a^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

Fricas [A] time = 3.06192, size = 1272, normalized size = 19.27

$$\left[\frac{1}{4}\sqrt{-2\,a - 2\,\sqrt{a^2+1}}\log\left(-\frac{8\,ax^7 + x^8 + 4\left(2\,a^2+15\right)x^6 - 8\left(4\,a^3+15\,a\right)x^5 + 2\left(8\,a^4+80\,a^2+67\right)x^4 + 64\,a^4 - 8\left(20\,a^3 \right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
```

```
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-\right.}}{4\ 2^{2/3}}$$

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.0971639, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 393, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-\right.}}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```


Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = a\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} + \frac{1}{2}b\,\text{Subst}\,(\ldots$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \frac{b\log\left(3+\ldots\right.}{4\ 2^{2/3}}\ldots$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \frac{b\log\left(3+\ldots\right.}{4\ 2^{2/3}}\ldots$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\sqrt{3}b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tanh^{-1}\left(\ldots\right.}{2\ldots}\ldots$$

Mathematica [C] time = 0.234498, size = 145, normalized size = 0.73

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,\ldots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3
/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\ 2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2\ldots\right.}}{4\ 2^{2/3}}$$

```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.0905163, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 392, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\ 2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2\ldots\right.}}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = a\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}b\,\mathrm{Sub}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3}{4\ 2}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3}{4\ 2}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}}{2}$$

Mathematica [C] time = 0.244593, size = 153, normalized size = 0.77

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3
/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0133008, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {750}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2\,2^{2/3}} + \frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.0602103, size = 111, normalized size = 1.14

$$-\frac{\sqrt[3]{\frac{3x+i\sqrt{3}-3}{x}}\sqrt[3]{\frac{9x-3i\sqrt{3}-9}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)}{2\sqrt[3]{3x^2-6x+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(((-3 + I*Sqrt[3] + 3*x)/x)^(1/3)*((-9 - (3*I)*Sqrt[3] + 9*x)/x)^(1/3)*App
ellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)])/(2
*(4 - 6*x + 3*x^2)^(1/3))
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{3\,x^2-6\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

```
[Out] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-6\,x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

Fricas [B] time = 20.7382, size = 495, normalized size = 5.1

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}\left(3x^2-6x+4\right)^{\frac{2}{3}}(x-2)+4\left(3x^2-6x+4\right)^{\frac{1}{3}}\left(x^2-4x+4\right)\right)}{6\left(x^3-12x^2+24x-16\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-6x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal. Leaf size=73

$$\frac{1}{3}\sqrt[3]{1-x^3}x^2-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) - Log[-x - (1 - x^3)^(1/3)]/6
```

Rubi [A] time = 0.0435833, antiderivative size = 107, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\sqrt[3]{1-x^3}x^2+\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 - Log[1 + x/(1
 - x^3)^(1/3)]/9
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{1-x^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\int\frac{x}{\left(1-x^3\right)^{2/3}}\,dx\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\,\text{Subst}\left(\int\frac{x}{1+x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{1}{9}\,\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{9}\,\text{Subst}\left(\int\frac{1+x}{1-x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{18}\,\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\,\text{Subst}\left(\int\frac{1}{1-x+x\right.\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{3}\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,\right.\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.0026731, size = 20, normalized size = 0.27

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3])/2
```

Maple [C] time = 0.046, size = 69, normalized size = 1.

$$-\frac{x^2\left(x^3-1\right)}{3}\left(-x^3+1\right)^{-\frac{2}{3}}+\frac{x^2}{6}\left(x^3-1\right)^{\frac{2}{3}}\left(-\text{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}{}_2\text{F}_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^3\right)\left(\text{signum}\left(x^3-1\right)\right)^{-\frac{2}{3}}\left(-x^3+1\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(1/3),x)
```

```
[Out] -1/3*x^2*(x^3-1)/(-x^3+1)^(2/3)+1/6*(x^3-1)^(2/3)/signum(x^3-1)^(2/3)*(-sig
num(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)/(-x^3+1)^(2/3)
```

Maxima [A] time = 1.42811, size = 142, normalized size = 1.95

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Fricas [A] time = 2.05284, size = 262, normalized size = 3.59

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] time = 1.10618, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^3+1\right)^{\frac{1}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rubi [A] time = 0.0372041, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 57, 618, 204, 31}

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1-x^3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3}-\frac{\log(x)}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= \sqrt[3]{1-x^3}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0236412, size = 90, normalized size = 1.34

$$\sqrt[3]{1-x^3}+\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{1}{6}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 -
 (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

Maple [C] time = 0.033, size = 49, normalized size = 0.7

$$-\frac{1}{9\,\Gamma(2/3)}\left(-3\left(3+1/6\,\pi\sqrt{3}-3/2\,\ln(3)+3\,\ln(x)+i\pi\right)\Gamma(2/3)+\Gamma\left(\frac{2}{3}\right)x^3{}_3F_2\left(\frac{2}{3},1,1;\,2,2;\,x^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/x,x)
```

```
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GA
MMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

Maxima [A] time = 1.46498, size = 96, normalized size = 1.43

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

Fricas [A] time = 1.95305, size = 225, normalized size = 3.36

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

Sympy [C] time = 0.944826, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/
3))
```

Giac [A] time = 1.14603, size = 97, normalized size = 1.45

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx$

Optimal. Leaf size=482

$$\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1}{(1-}\right.}{}$$

```
[Out] (1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
 (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2
^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2^(1/3)*Lo
g[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1
+ (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/
(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[
2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/
(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (1 - x^3)^(
1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

Rubi [F] time = 0.0520396, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx = \int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx$$

Mathematica [F] time = 0.35477, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\sqrt[3]{-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(1+x),x)
```

```
[Out] int((-x^3+1)^(1/3)/(1+x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$

Optimal. Leaf size=280

$$-\frac{\log\left(-3(x-1)\left(x^2-x+1\right)\right)}{2\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)}{2\ 2^{2/3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(\sqrt[3]{1-x^3}\right.}{2\ 2^{2}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*(-1 + x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 - (2*2^
(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2^(2/3)*
(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[-3*(-1 + x)*(1 - x + x^2)
]/(2*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[-(2^(1/
3)*(-1 + x)) + (1 - x^3)^(1/3)])/(2*2^(2/3)) + Log[x + (1 - x^3)^(1/3)]/2 -
 Log[2^(1/3)*x + (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [F] time = 0.235482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] + ((2*
I)*Defer[Int][(1 - x^3)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx &= \int \left(\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)}+\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(-1+i\sqrt{3}+2x\right)}\right) dx\\ &= \frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x} dx}{\sqrt{3}}+\frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x} dx}{\sqrt{3}}\end{aligned}$$

Mathematica [F] time = 0.106666, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{1}{x^2-x+1}\sqrt[3]{-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x)
```

```
[Out] int((-x^3+1)^(1/3)/(x^2-x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

Fricas [B] time = 51.3796, size = 10386, normalized size = 37.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(1/3)*arctan(1/3*(26795748*sqrt(3)*2^(2/3)*(586745*x^11 - 70
6109*x^10 - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 +
 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^(1/3) + 26795748*sqr
t(3)*2^(1/3)*(340975*x^10 + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^
6 + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^(2
/3) + 7*sqrt(273426)*2^(1/6)*(6*sqrt(3)*2^(2/3)*(338078915*x^10 - 459916473
*x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448
15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^(2/3) + sqrt
(3)*2^(1/3)*(2332175065*x^12 - 3283524318*x^11 + 1882024851*x^10 - 39193009
70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11
45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*sq
rt(3)*(493920245*x^11 - 452201839*x^10 - 276972599*x^9 - 661557480*x^8 + 13
75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3
 - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^(1/3)) - 3*sqrt(3)*(2247079524645*x
^12 - 5276442179264*x^11 + 3816306322874*x^10 - 3280399521884*x^9 + 6278089
258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25
58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515
2977792))/(18230538112975*x^12 - 14115716188440*x^11 - 20854883745366*x^10
+ 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059
560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296
4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*sqrt(3)*2^(1/3)*arctan(-
1/3*(13397874*sqrt(3)*2^(2/3)*(18803*x^11 - 25367*x^10 - 203754*x^9 + 40802
1*x^8 - 139829*x^7 + 7128*x^6 - 233871*x^5 + 225275*x^4 - 47094*x^3 - 10225
*x^2 + 2921*x)*(-x^3 + 1)^(1/3) + 26795748*sqrt(3)*2^(1/3)*(10589*x^10 - 73
935*x^9 + 63883*x^8 + 142959*x^7 - 173613*x^6 - 31588*x^5 + 79410*x^4 - 437
7*x^3 - 13328*x^2 + 2921*x)*(-x^3 + 1)^(2/3) - 7*sqrt(273426)*(6*sqrt(3)*2^
(2/3)*(309683372*x^10 - 328552599*x^9 - 24698630*x^8 - 422031122*x^7 + 7021
64163*x^6 - 95703451*x^5 - 206316094*x^4 + 60985482*x^3 + 11167816*x^2 - 37
33038*x)*(-x^3 + 1)^(2/3) + sqrt(3)*2^(1/3)*(2345654785*x^12 - 2502234618*x
^11 - 252041853*x^10 - 4416416426*x^9 + 6899968311*x^8 - 1680852528*x^7 + 1
576960038*x^6 - 2990585436*x^5 + 642930363*x^4 + 528479914*x^3 - 117963261*
x^2 - 38399466*x + 8532241) - 6*sqrt(3)*(491687266*x^11 - 516958230*x^10 -
69305552*x^9 - 808934094*x^8 + 1418391515*x^7 - 385704187*x^6 - 112721241*x
^5 - 69510422*x^4 + 47121139*x^3 + 11465929*x^2 - 4799203*x)*(-x^3 + 1)^(1/
3))*sqrt((6*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 4
8*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 -
201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3
 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^
6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11
 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^
```

3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(2995162579*x^12 + 315959718008*x^11 - 849682072424*x^10 + 177300060912*x^9 - 508006765899*x^8 + 3583876884636*x^7 - 3031033916540*x^6 - 1410763301208*x^5 + 2375077456341*x^4 - 546587071308*x^3 - 175036021936*x^2 + 63861157012*x - 3114267965))/(367648430113*x^12 - 1408582980384*x^11 - 1269375810828*x^10 + 5714713216048*x^9 - 1087485936795*x^8 - 126379999188*x^7 - 10319650860540*x^6 + 10854292018608*x^5 - 1383220291365*x^4 - 1828745373668*x^3 + 426327416076*x^2 + 93479232396*x - 24922675961)) - 1/18*sqrt(3)*2^(1/3)*arctan(1/3*(13397874*sqrt(3)*2^(2/3)*(17344*x^11 - 120304*x^10 + 110610*x^9 + 203214*x^8 - 213415*x^7 - 96387*x^6 + 30102*x^5 + 157561*x^4 - 101868*x^3 + 15151*x^2 + 913*x)*(-x^3 + 1)^(1/3) - 26795748*sqrt(3)*2^(1/3)*(1277*x^10 + 57510*x^9 - 189677*x^8 + 108972*x^7 + 102426*x^6 - 47461*x^5 - 82155*x^4 + 56409*x^3 - 7301*x^2 - 913*x)*(-x^3 + 1)^(2/3) + 7*sqrt(273426)*(6*sqrt(3)*2^(2/3)*(8733539*x^10 - 122586360*x^9 + 269810944*x^8 - 28009538*x^7 - 316185126*x^6 + 161786897*x^5 + 95479640*x^4 - 80193978*x^3 + 11163982*x^2 + 1166814*x)*(-x^3 + 1)^(2/3) - sqrt(3)*2^(1/3)*(1971824*x^12 - 78264612*x^11 + 705529692*x^10 - 1556393152*x^9 + 933849120*x^8 + 135726408*x^7 - 213906684*x^6 + 446158968*x^5 - 582881445*x^4 + 182390318*x^3 + 31120185*x^2 - 12999294*x - 833569) + 6*sqrt(3)*(12965988*x^11 - 175265260*x^10 + 270273662*x^9 + 299814882*x^8 - 663644613*x^7 + 77553085*x^6 + 286893603*x^5 - 82332150*x^4 - 33723265*x^3 + 10863861*x^2 + 333245*x)*(-x^3 + 1)^(1/3))*sqrt((6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(67113679084*x^12 - 61534090748*x^11 - 1006807736260*x^10 + 1996201310444*x^9 + 193806523788*x^8 - 2673973669800*x^7 + 775957356356*x^6 + 2110159119756*x^5 - 1821028473882*x^4 + 377014646048*x^3 + 67410900094*x^2 - 19835743048*x - 1369553867))/(168032067092*x^12 - 2318893136652*x^11 + 4401905935020*x^10 + 1550444734940*x^9 - 6210007783092*x^8 - 1634341806144*x^7 + 6341768478444*x^6 - 948091553244*x^5 - 2281774840272*x^4 + 1036207535072*x^3 - 59480228082*x^2 - 20085678624*x - 761048497)) + 1/3*sqrt(3)*arctan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 - 1)) + 1/48*2^(1/3)*log(7717175424*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/48*2^(1/3)*log(1929293856*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 +

```
 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 16
2*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(
1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*
x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 1/48*2^(1/3)*log(7717175424*(6
*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x
^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 +
338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 +
 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5
+ 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10
- 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2
- 6*x + 1)) - 1/48*2^(1/3)*log(1929293856*(6*2^(2/3)*(4*x^10 - 27*x^9 + 32*
x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3
) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249
*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 -
 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)
*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 1
41*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/6*log(3*(-x^3 +
 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$

Optimal. Leaf size=232

$$\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log\left(-\sqrt[3]{}\right.$$

```
[Out] (1 - x^3)^(1/3) + (x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3/8])/2 - (2*ArcTa
n[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + 3^(1/6)*ArcTan[(1 - (3^(2
/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3^(1/6)*ArcTan[1/Sqrt[3] + (2*(1 - x^3)^
(1/3))/(3*3^(1/6))] - Log[8 + x^3]/3^(1/3) + (3^(2/3)*Log[3^(2/3) - (1 - x^
3)^(1/3)])/2 - Log[-x - (1 - x^3)^(1/3)] + (3^(2/3)*Log[-(3^(2/3)*x)/2 - (1
 - x^3)^(1/3)])/2
```

Rubi [F] time = 0.0499624, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(2 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx = \int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Mathematica [F] time = 0.343464, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{2+x}\sqrt[3]{-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(2+x),x)
```

```
[Out] int((-x^3+1)^(1/3)/(2+x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=168

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}+\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\tan^{-1}}{}$$

```
[Out] -(x^2*AppellF1[2/3, 1, 1/3, 5/3, x^3, -x^3/2])/(2*2^(1/3)) + (2*ArcTan[(1 +
 (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]])/3^(5/6) + ArcTan[(3^(1/3) + 2*(2
+ x^3)^(1/3))/3^(5/6)]/3^(5/6) + Log[1 - x^3]/(6*3^(1/3)) + Log[3^(1/3) - (
2 + x^3)^(1/3)]/(2*3^(1/3)) - Log[3^(1/3)*x - (2 + x^3)^(1/3)]/3^(1/3)
```

Rubi [F] time = 0.204262, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] (1 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x] +
(1 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx &= \int \left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}+\frac{1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\right)dx\\ &= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx\end{aligned}$$

Mathematica [F] time = 0.137935, size = 0, normalized size = 0.

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{2+x}{x^2+x+1}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

```
[Out] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^3+2\right)^{\frac{2}{3}}(x+2)}{x^5+x^4+x^3+2\,x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
```

```
[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rubi [A] time = 0.0226836, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1587}

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Mathematica [A] time = 0.0081255, size = 25, normalized size = 1.

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Maple [A] time = 0.001, size = 24, normalized size = 1.

$$\frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Maxima [A] time = 0.936682, size = 31, normalized size = 1.24

$$\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Fricas [A] time = 2.30158, size = 63, normalized size = 2.52

$$\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Sympy [A] time = 0.097344, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4 + 80x^3 - 12x^2 + 24x + 9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

Giac [A] time = 1.07841, size = 31, normalized size = 1.24

$$\frac{1}{8}\log\left(320\,x^4 + 80\,x^3 - 12\,x^2 + 24\,x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="giac")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rubi [A] time = 0.031717, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {2090}

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = -\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}} + \frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

Mathematica [C] time = 0.0184698, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{20\#1^2\log(x-\#1)+12\#1\log(x-\#1)+3\log(x-\#1)}{160\#1^3+30\#1^2-3\#1+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

Maple [A] time = 0.024, size = 52, normalized size = 0.9

$$\frac{\sqrt{11}}{22}\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)+\frac{\sqrt{11}}{22}\arctan\left(-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}x^3}{33}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))+1/22*11^(1/2)*arctan(-20/33*11
^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{20x^2+12x+3}{320x^4+80x^3-12x^2+24x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

Fricas [A] time = 1.9788, size = 158, normalized size = 2.68

$$\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)+\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
```

```
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

Sympy [A] time = 0.160272, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\left(2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)+2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac
")
```

```
[Out] sage0*x
```

3.64 $\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=78

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rubi [A] time = 0.07667, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2100, 2090}

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4)),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2100

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[(Coeff[Pm, x, m]*Log[Qn])/(n*Coeff[Qn, x, n]), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
```

```
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\log\left(9+24x-12x^2+80x^3+320x^4\right) - \frac{\int \frac{168960+675840x+1126400x^2}{9+24x-12x^2+80x^3+320x^4}\,dx}{1280}$$
$$= 2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right) + 2\log\left(9+2\right.$$

Mathematica [C] time = 0.0201547, size = 99, normalized size = 1.27

$$\frac{1}{2}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{640\#1^3\log(x-\#1)-100\#1^2\log(x-\#1)-144\#1\log(x-\#1)-21\right.$$
$$160\#1^3+30\#1^2-3\#1+3$$

Antiderivative was successfully verified.

```
[In] Integrate[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
 320*x^4)),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144
*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 +
30*#1^2 + 160*#1^3) & ]/2
```

Maple [A] time = 0.018, size = 75, normalized size = 1.

$$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\arctan\left(-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}x^3}{33}\right)-2\sqrt{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 2*ln(6400*x^4+1600*x^3-240*x^2+480*x+180)-2*11^(1/2)*arctan(-20/33*11^(1/2)
*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)-2*11^(1/2)*arctan(
```

```
1/55*(40*x-7)*11^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\int \frac{640\,x^3-100\,x^2-144\,x-21}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

Fricas [A] time = 1.94244, size = 213, normalized size = 2.73

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)+2\,\log\left(320\,x^4+80\,x^3-12\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

Sympy [A] time = 0.16712, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\,\mathrm{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)-2\,\mathrm{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="giac")
```

```
[Out] sage0*x
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rubi [A] time = 0.0072942, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {405}

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx=\frac{1}{2}\tan^{-1}\left(\frac{x\left(1+x^2\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Mathematica [C] time = 0.0968655, size = 110, normalized size = 2.24

$$-\frac{5x\sqrt{1-x^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};x^4,-x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (-5*x*Sqrt[1 - x^4]*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)*(-5*
AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^4, -x^4])))
```

Maple [B] time = 0.013, size = 100, normalized size = 2.

$$\frac{1}{4}\arctan\left(-\frac{1}{x}\sqrt{-x^4+1}+1\right)-\frac{1}{8}\ln\left(\left(\frac{-x^4+1}{2\,x^2}-\frac{1}{x}\sqrt{-x^4+1}+1\right)\left(\frac{-x^4+1}{2\,x^2}+\frac{1}{x}\sqrt{-x^4+1}+1\right)^{-1}\right)-\frac{1}{4}\arctan\left(\frac{1}{x}\sqrt{-x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^4+1),x)
```

```
[Out] 1/4*arctan(-(-x^4+1)^(1/2)/x+1)-1/8*ln((1/2*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1
)/(1/2*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x+1))-1/4*arctan((-x^4+1)^(1/2)/x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

Fricas [A] time = 2.51574, size = 138, normalized size = 2.82

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2x^2-2\sqrt{-x^4+1}x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0145405, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0909969, size = 108, normalized size = 2.04

$$-\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] (-5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(5*
AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))
```

Maple [C] time = 0.024, size = 365, normalized size = 6.9

$$-\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}-\frac{\frac{i}{2}\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/
2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+
1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^
2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-Elliptic
E(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1
/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))+1/2*I/(1
/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*Ell
ipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2
)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

Fricas [A] time = 2.42213, size = 173, normalized size = 3.26

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rubi [A] time = 0.0944013, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2071, 1093, 205, 208}

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 2071

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-2px^2+\left(-4+p^2\right)x^4}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)\\ &= \frac{1}{4}\left(-4+p^2\right)\text{Subst}\left(\int \frac{1}{-2-p+\left(-4+p^2\right)x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right) - \frac{1}{4}\left(-4+p^2\right)\text{Subst}\left(\int \frac{}{2}\right.\\ &= \frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\tanh^{-1}\left(\frac{\sqrt{2+p}x}{\sqrt{1+px^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 7.08417, size = 5727, normalized size = 76.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.071, size = 1512, normalized size = 20.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x)
```

```
[Out] 1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1
+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticF(1/2*x
```

```
*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))*p-1/
(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1
/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticF(1/2*x*(
-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))-2/(-2*
p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p
*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*Ell
ipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))
^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^
(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2
-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(
p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x
^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^
2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*p+1/2*(p^
2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/
2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*Ell
ipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-
1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2))+1/2*(-1-p)
/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(
-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticF(1/2*x*(-
2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p
+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p
-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*(Ellip
ticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(
1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4
)^(1/2)))^(1/2)))+1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+2*x^2+p+2)
/(2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1-(-1
/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)
/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,1/(-1/2*
p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(
1/2))^(1/2)))-1/2*(1+p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)
^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(
1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+2*x^2+p+2)/(
2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))-1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2
*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(
x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,1/(-1/2*p+
1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1/
2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Fricas [A] time = 2.56811, size = 957, normalized size = 12.76

$$\left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4 + 2\left(p-1\right)x^2 - 2\sqrt{x^4 + px^2 + 1}\sqrt{p-2}x + 1}{x^4 + 2x^2 + 1}\right) + \frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4 + 2\left(p+1\right)x^2 + 2\sqrt{x^4 + px^2 + 1}\sqrt{p}}{x^4 - 2x^2 + 1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p}\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p
^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rubi [A] time = 0.0800218, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2072}

$$\frac{\sqrt{\sqrt{p^2+4}-p}\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p
^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rule 2072

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, -Simp[(a*Sqrt[b + q]*ArcTan[(Sqrt[b + q
]*x*(b - q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x] + Simp[(a*Sqrt[-b + q]*ArcTanh[(Sqrt[-b + q
]*x*(b + q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d +
a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = -\frac{\sqrt{p+\sqrt{4+p^2}}\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\tanh^{-1}\left(\frac{\sqrt{-p+\sqrt{4+p^2}}x\left(p+\sqrt{4+p^2}-2\right.}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.432111, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4}-p}+2}\sqrt{1-\frac{2x^2}{\sqrt{p^2+4}+p}}\left(2i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4}-p}}x\right),\frac{p-\sqrt{p^2+4}}{\sqrt{p^2+4}+p}\right)-(p+2i)\Pi\left(\frac{1}{2}i\left(p-\sqrt{p^2+4}\right);i\sinh^{-1}\left(\right.\right.\right.}{4\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\sqrt{px^2-x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (Sqrt[2 + (4*x^2)/(-p + Sqrt[4 + p^2])]*Sqrt[1 - (2*x^2)/(p + Sqrt[4 + p^2]
)]*((2*I)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (
p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])] - (2*I + p)*EllipticPi[(I/2)*(p - S
qrt[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - S
qrt[4 + p^2])/(p + Sqrt[4 + p^2])] + (-2*I + p)*EllipticPi[(I/2)*(-p + Sqrt
[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - Sqrt
[4 + p^2])/(p + Sqrt[4 + p^2])]))/(4*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*Sqrt[1
 + p*x^2 - x^4])
```

Maple [B] time = 0.072, size = 456, normalized size = 2.7

$$\frac{\sqrt{2}}{32}\sqrt{p+\sqrt{p^2+4}}\sqrt{p^2+4}\ln\left(\frac{-x^4+px^2+1}{x^2}+\frac{\sqrt{2}}{x}\sqrt{-x^4+px^2+1}\sqrt{p+\sqrt{p^2+4}}+\sqrt{p^2+4}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{1}{2}\left(2\frac{\sqrt{-x^4}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)
```

```
[Out] 1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2+(-
x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/4*2^(
```

```
1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+
2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/32*2^(1/2)*(p+(p^2+4
)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p
^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2
+4)^(1/2)*ln((-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p
*x^2+1)/x^2-(p^2+4)^(1/2))+1/4*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*
(2*(p+(p^2+4)^(1/2))^(1/2)-2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x)/(-p+(p^2+4)^(1
/2))^(1/2))+1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)^(1/2)*
2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p*x^2+1)/x^2-(p^2+4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

Fricas [B] time = 25.3474, size = 6283, normalized size = 36.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(1/4
*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(
3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +
4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p
^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))
*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (
p^2 + 4)*x^4)*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(
x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2
*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(
3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x
^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 +
```

```
4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^
3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sq
rt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 -
 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^
10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1
)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^
3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) - sqrt(
p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2
+ 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*
x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7
 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11
 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4)))*sq
rt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*s
qrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 -
4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^
4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 +
4*p)*x^2 - p^2 - 4)) + 8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^
(3/4)*arctan(-1/4*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3
 + 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)
*x^2 - 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x
^4 + (p*x^12 - (p^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^
2)*sqrt(p^2 + 4))*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p
^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p +
 4)*(2*(sqrt(2)*(x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) +
sqrt(2)*(x^11 - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2
+ 1))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x
)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 +
4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)
^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^
4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6
 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^
2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt
(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p
^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt
(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5
 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p
*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 -
6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2
)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^
2 + 4)^(1/4)))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x
^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 +
4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 +
4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28
)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*
(p^2 + 4))*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*
```

```
x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(
p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x
^4 + p^2 + 4)) + (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*(p^2 + 4))*sqrt(p^2 + s
qrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x
^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 +
4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/(p^2 +
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rubi [A] time = 0.0398388, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
```

```
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx = a\int \frac{1}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{(2-x)\sqrt[4]{-1+x}}\,dx, x, x^2\right)$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1+x^2}\right)$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+x^2}\right) - b\,\text{Subst}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\tanh^{-1}\left(\sqrt[4]{-1+x^2}\right)$$

Mathematica [C] time = 0.260884, size = 157, normalized size = 1.96

$$\frac{x\left(bx\sqrt[4]{1-x^2}\left(x^2-2\right)F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}\right)}{4\left(x^2-2\right)\sqrt[4]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+2}\frac{1}{\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx-\int \frac{bx}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx+a}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}\left(2+x^2\right)} dx$

Optimal. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```

Rubi [A] time = 0.0475753, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```


```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```


Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
```

```
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{-1-x}(2+x)}\,dx, x, x^2\right)$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b)\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1-x^2}\right)$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1-x^2}\right) + b\,\mathrm{Subs}$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\tanh^{-1}\left(\sqrt[4]{-1-x^2}\right)$$

Mathematica [C] time = 0.251017, size = 162, normalized size = 1.84

$$\frac{x\left(bx\sqrt[4]{x^2+1}F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\left(x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)\right)}\right)}{4\sqrt[4]{-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2] - (24*a*Appell
F1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1, 3/
2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -x^2/2] + Appell
F1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])))))/(4*(-1 - x^2)^(1/4))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+2}\frac{1}{\sqrt[4]{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(x^2+2\right)\left(-x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)}\,dx$

Optimal. Leaf size=149

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rubi [A] time = 0.0450532, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1010, 397, 439}

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```


```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```


Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
```

```
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx$$

$$= \frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tanh^{-1}\left(\frac{1+\sqrt{1-}}{x\sqrt[4]{1-x}}\right.$$

Mathematica [C] time = 0.203389, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\left(x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+2}\frac{1}{\sqrt[4]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx-\int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx+a}{\left(x^2-2\right)\left(-x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}(2+x^2)} dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rubi [A] time = 0.035622, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1010, 397, 439}

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
```

```
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx$$

$$= -\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b\tanh^{-1}\left(\frac{1+\sqrt{\cdots}}{\sqrt{2}\sqrt[4]{\cdots}}\right.}{\sqrt{2}}$$

Mathematica [C] time = 0.174923, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{x^2+1}\left(x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4,
1, 3/2, -x^2, -x^2/2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4, 1,
 3/2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -x^2/2] + App
ellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+2}\frac{1}{\sqrt[4]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{\left(x^2+2\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt[4]{x^2+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{\left(x^2 + 2\right)\left(x^2 + 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rubi [A] time = 0.0192024, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\,2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\,2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\,2^{2/3}}$$

Mathematica [C] time = 0.0217669, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Maple [C] time = 0.099, size = 164, normalized size = 1.3

$$\frac{i}{36}\sqrt{2}\sum_{\_alpha=RootOf\left(\_Z^3-4\right)}\_alpha^2\left(-2\_alpha^2+\_alpha+1+i\sqrt{3}\left(1-\_alpha\right)\right)\sqrt{\frac{i}{2}\left(2x+1-i\sqrt{3}\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((-1+x)/(I*3^(1
/2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+
_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/
2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3
-4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [B] time = 5.56681, size = 3299, normalized size = 25.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
```

```
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - Arc
Tan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d
^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1
+ d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rubi [A] time = 0.0329981, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {485}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - Arc
Tan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d
^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1
+ d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 485

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, -Simp[(q*ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]])/(9*2^(2/3)
*b*Rt[-c, 2]), x] + (-Simp[(q*ArcTan[(Rt[-c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c +
 d*x^3]])/(3*2^(2/3)*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[Sqrt[c + d*x^3]/(Sq
rt[3]*Rt[-c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[(S
qrt[3]*Rt[-c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*
Rt[-c, 2]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*
```

```
c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\int \frac{x}{\left(4-dx^3\right)\sqrt{-1+dx^3}}\,dx = -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

Mathematica [C] time = 0.0290417, size = 54, normalized size = 0.34

$$\frac{x^2\sqrt{1-dx^3}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{8\sqrt{dx^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

Maple [C] time = 0.106, size = 240, normalized size = 1.5

$$-\frac{i}{9}\sqrt{2}\sum_{\_alpha=RootOf\left(d\_Z^3-4\right)}\frac{1}{\_alpha}\sqrt{-\frac{i}{2}\left(2x+\frac{1}{\sqrt[3]{d}}+i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)\sqrt[3]{d}}\sqrt{\left(x-\frac{1}{\sqrt[3]{d}}\right)\left(-3\frac{1}{\sqrt[3]{d}}-i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)^{-1}}\sqrt{\frac{i}{2}\left(2x+\frac{1}{\sqrt[3]{d}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+_alpha*d^(2/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*3^(1/2)*_alpha^2*d^(2/3)-1/6*I*3^(1/2)*_alpha*d^(1/3)-1/
```

```
6*I*3^(1/2)+1/2*_alpha*d^(1/3)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

Fricas [B] time = 5.7955, size = 4625, normalized size = 29.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*
d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt
(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1) + (2*sq
rt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))^(1/3) + sqrt(3)*(d*x^4 - x) + 3*(s
qrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1
/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d
*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
```

```
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
 + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
 64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
 - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}(8+x^3)} dx$

Optimal. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rubi [A] time = 0.155523, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {486, 444, 63, 204, 2138, 203, 2145, 206}

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```


```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```


Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)}\,dx = -\left(\frac{1}{12}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right) - \frac{1}{12}\int \frac{-2-2x+x^2}{\left(4-2x+x^2\right)\sqrt{-1+x^3}}\,dx - \frac{1}{4}\int \frac{x^2}{\left(-8-x^3\right)\sqrt{-1+x^3}}$$

$$= -\left(\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}}\,dx, x, x^3\right)\right) + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right) - \frac{1}{3}\,\mathrm{S}$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}} - \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{-9-x^2}\,dx, x, \sqrt{-1+x^3}\right)$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

Mathematica [C] time = 0.0194046, size = 48, normalized size = 0.65

$$\frac{x^2\sqrt{1-x^3}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{16\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8])/(16*Sqrt[-1 + x
^3])
```

Maple [C] time = 0.115, size = 286, normalized size = 3.9

$$-\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{9}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}+\frac{1}{2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x)
```

```
[Out] -1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/
```

```
6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/36*2^(1/
2)*sum((2-_alpha)*(_alpha-1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*(
(2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(
1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*_al
pha*3^(1/2)+1/2*_alpha-1/6*I*3^(1/2)-1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2)),_alpha=RootOf(_Z^2-2*_Z+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

Fricas [B] time = 3.58602, size = 1447, normalized size = 19.55

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)-\frac{1}{216}\sqrt{3}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*(x^4 + 12*
x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24
*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
 + 1/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x +
 1)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) + (3*sqrt(3)*(x^3 +
 x^2 - 2*x) - sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 + 186*x^4
- 56*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2
 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)))/(x^3 -
```

```
 3*x^2 + 2)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) - (3*sqrt(3
)*(x^3 + x^2 - 2*x) + sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
)/(x^3 - 3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + Ar
cTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1
+ d*x^3]/3]/(18*d^(2/3))
```

Rubi [A] time = 0.304998, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + Ar
cTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1
+ d*x^3]/3]/(18*d^(2/3))
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(8-dx^3\right)\sqrt{1+dx^3}}\,dx = -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}x}{\left(2-\sqrt[3]{d}x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d}\int \frac{x^2}{\left(8-dx^3\right)\sqrt{1+dx^3}}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx, x, x^3\right) + \frac{1}{3}d^{4/3}\,\text{Subst}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{1+dx^3}\right)}{6d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

Mathematica [C] time = 0.0292129, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

Maple [C] time = 0.16, size = 383, normalized size = 3.7

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(d\_Z^3-8\right)}\frac{1}{\_alpha}\sqrt[3]{-d^2}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2}+\sqrt[3]{-d^2}\right)\right)\frac{1}{\sqrt[3]{-d^2}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2}\right)\left(-3\sqrt[3]{-d^2}+i\sqrt{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*3^(1/2)
*(-d^2)^(1/3)*_alpha^2*d-I*3^(1/2)*(-d^2)^(2/3)*_alpha+I*3^(1/2)*d-3*(-d^2)
^(2/3)*_alpha-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

Fricas [B] time = 4.28187, size = 1214, normalized size = 11.79

$$2\sqrt{3}\left(d^2\right)^{\frac{1}{6}} d\arctan\left(-\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}\left(d^2x^6-40\,dx^3-32\right)\left(d^2\right)^{\frac{2}{3}}+3\sqrt{3}\left(5\,d^2x^4+8\,dx\right)\left(d^2\right)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\left(d^2\right)^{\frac{1}{6}}}{9\left(d^4x^7-7\,d^3x^4-8\,d^2x\right)}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318\,d^3x^6+1200\,d^2x\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
```

```
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}(3-x^2)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rubi [A] time = 0.0114325, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {395}

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\left(3-x^2\right)}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right) + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.100266, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\left(x^2-3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{-3\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

```
[Out] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 19.5199, size = 5180, normalized size = 63.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11
 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(
3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^2
 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
- 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 +
 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
+ 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*x
^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 204
3*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(
x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-
3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2
 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)
*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(3
)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2 -
 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^5
 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4 +
 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660*
x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9
*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255
*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x
^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x
^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 24
5*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*x
^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(4
9*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 + 9
17*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt
((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)*
```

```
sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sqr
t(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9
*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2
106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
- 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8
 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^1
1 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^1
0 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^1
2 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/2
88*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
- 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2/
3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2
 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/288*sqrt(6)*sqrt(2)*log(1
2*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 + 1
14228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x^
4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-3
*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 43
47*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2 -
 sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x^
2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*x
^2 + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)

[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rubi [A] time = 0.0107141, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] time = 0.0950302, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{(x^2+3)\sqrt[3]{3x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3])/((3 + x^2)*(1 + 3*x^2)^(1
/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1
/3, 2, 5/2, -3*x^2, -x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -x^2/3])
))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

Fricas [B] time = 15.5078, size = 921, normalized size = 11.37

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(3x^4-10x^3-36x^2+18x+9\right)\left(3x^2+1\right)^{\frac{2}{3}}-4\sqrt{3}\left(x^5+15x^4-26x^3-54x^2+9x-9\right)\left(3x^2+1\right)^{\frac{1}{3}}}{x^6+126x^5-225x^4-828x^3-81x^2-162x+81}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.0136163, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.0717142, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(
-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 11.9618, size = 5544, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
```

```
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi [A] time = 0.0121913, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx = -\frac{\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.0633664, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{\left(x^2-3\right)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [B] time = 9.7741, size = 4852, normalized size = 44.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
```

```
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rubi [A] time = 0.873863, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 6733, 1698, 205}

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\int \frac{a+x}{\sqrt{x}(-a+x)\sqrt{a^2-\left(1+a^2\right)x+x^2}}\,dx}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{a+x^2}{\left(-a+x^2\right)\sqrt{a^2+\left(-1-a^2\right)x^2+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{1}{-a-\left(-2a^2-a\left(-1-a^2\right)\right)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a^2-\left(1+a^2\right)x+x^2}}\right)}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-\left(1+a^2\right)x+x^2}}\right)}{(1-a)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

Mathematica [C] time = 0.915921, size = 159, normalized size = 1.83

$$-\frac{2i\left(a^2-x\right)^{3/2}\sqrt{\frac{x-1}{x-a^2}}\sqrt{\frac{x}{x-a^2}}\left((a+1)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right),1-\frac{1}{a^2}\right)-2\Pi\left(\frac{a-1}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(a-1)\sqrt{-a^2}\sqrt{(x-1)x\left(x-a^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 +
a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticP
i[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*
Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])
```

Maple [C] time = 0.034, size = 206, normalized size = 2.4

$$-2\frac{a^2}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}\sqrt{-\frac{-a^2+x}{a^2}}\sqrt{\frac{-1+x}{a^2-1}}\sqrt{\frac{x}{a^2}}\text{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)-4\frac{a^3}{\sqrt{-a^2x^2+a^2x+x^3-x^2}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2
+a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-
4*a^3*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+
a^2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(
a^2/(a^2-1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

Fricas [A] time = 2.35953, size = 193, normalized size = 2.22

$$\frac{\arctan\left(\frac{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\left(a^2-2\left(a^2-a+1\right)x+x^2\right)}{2\left((a-1)x^3-\left(a^3-a^2+a-1\right)x^2+\left(a^3-a^2\right)x\right)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+x}{\sqrt{x\left(-a^2+x\right)(x-1)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [C] time = 1.66858, antiderivative size = 529, normalized size of antiderivative = 529., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 6733, 1708, 1103, 1706}

$$\frac{2(1-a)\sqrt{x}\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}+\frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-\left(-a^2+2a\right.}{(2-a)a}}}{a\sqrt{-\left(-a^2+\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\int \frac{-2+a+x}{\sqrt{x}(-a+x)\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}}}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\mathrm{Subst}\left(\int \frac{-2+a+}{\left(-a+x^2\right)\sqrt{(2-a)a+(}}\right.}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{(2-a}}\right.}{a\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}$$

$$= \frac{2(1-a)\sqrt{x}\sqrt{(2-a)a-\left(1+2a-a^2\right)x+x^2}\tan^{-1}\left(\frac{\sqrt{-1+2a-a^2}\sqrt{}}{\sqrt{(2-a)a-\left(1+2a-a^2\right.}}\right.}{a\sqrt{-1+2a-a^2}\sqrt{(2-a)ax-\left(1+2a-a^2\right)x^2+x^3}}$$

Mathematica [C] time = 0.585636, size = 127, normalized size = 127.

$$\frac{2\sqrt{\frac{1}{x-1}+1}(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1}+1}\left(\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right),\frac{1}{(a-1)^2}\right)-2\Pi\left(\frac{1}{1-a};\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right)|\frac{1}{(a-1)^2}\right)\right)}{\sqrt{-(a-1)^2}\sqrt{(x-1)x\left(a^2-2a+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*(El
lipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2*Elliptic
Pi[(1 - a)^(-1), ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)]))/(
Sqrt[-(-1 + a)^2]*Sqrt[(-1 + x)*x*(-2*a + a^2 + x)])
```

Maple [C] time = 0.041, size = 317, normalized size = 317.

$$2\,\frac{a^2-2\,a}{\sqrt{a^2x^2-a^2x-2\,ax^2+x^3+2\,ax-x^2}}\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a}}\sqrt{\frac{-1+x}{-a^2+2\,a-1}}\sqrt{\frac{x}{-a^2+2\,a}}\mathrm{EllipticF}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a}},\sqrt{\frac{-a^2+2}{-a^2+2\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-
a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2
-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))+2*(2*a-2)*(a^2-2*
a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))
^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a^
2-2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^2-2\,a-1\right)x^2+x^3}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

Fricas [C] time = 2.28269, size = 150, normalized size = 150.

$$\frac{\log\left(-\frac{a^2-2\left(a^2-a\right)x-x^2+2\sqrt{\left(a^2-2\,a-1\right)x^2+x^3-\left(a^2-2\,a\right)x}a}{a^2-2\,ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+x-2}{\sqrt{x\left(x-1\right)\left(a^{2}-2a+x\right)}\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^{2}-2\,a-1\right)x^{2}+x^{3}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}\,dx$

Optimal. Leaf size=46

$$\log\left(\frac{-2\left(\sqrt{(1-x)x\left(a^2-2ax+x\right)}+x\right)-a^2+2ax+x^2}{(a-x)^2}\right)$$

```
[Out] Log[(-a^2 + 2*a*x + x^2 - 2*(x + Sqrt[(1 - x)*x*(a^2 + x - 2*a*x)]))/(a - x
)^2]
```

Rubi [C] time = 1.49236, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {2056, 6733, 1710, 1104, 419, 1220, 537}

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\Pi\left(\frac{1}{a};\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\int \frac{-a+(-}{\sqrt{x}(-a+x)\sqrt{a^2-(-1}}}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int \frac{}{\left(-a+x^2\right.}\right.}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1}}$$

$$= -\frac{\left(4(1-a)a\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+}}$$

$$= -\frac{\left(4(1-a)a\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\sqrt{a^2-\left(-1+2a+a^2\right)x+(\right.}{\sqrt{a^2+\left(1-2a-a^2\right)x+(-1+2}}$$

$$= -\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}F\left(\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{a^2x+\left(1-2a-a^2\right)x^2-(1-2a)x^3}} + \frac{4(}{}$$

Mathematica [C] time = 1.07234, size = 133, normalized size = 2.89

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{-\frac{a^2-2ax+x}{(2a-1)(x-1)}}\left(2a\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|-\frac{(a-1)^2}{2a-1}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right),-\frac{(a-1)^2}{2a-1}\right)\right)}{\sqrt{-(x-1)x\left(a^2-2ax+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)
*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a)
)] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*
a))]))/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

Maple [C] time = 0.047, size = 536, normalized size = 11.7

$$2\frac{a^{2}}{(-1+2a)\sqrt{-a^{2}x^{2}+2ax^{3}+a^{2}x-2ax^{2}-x^{3}+x^{2}}}\sqrt{-\frac{-1+2a}{a^{2}}\left(x-\frac{a^{2}}{-1+2a}\right)}\sqrt{(-1+x)\left(\frac{a^{2}}{-1+2a}-1\right)^{-1}}\sqrt{\frac{(-1+2a)}{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x)
```

```
[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)
-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^
(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(
-1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((
-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*
x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(
a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a
))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/
2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*Elliptic
Pi((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^
2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(2a-1)x-a}{\sqrt{(2a-1)x^{3}+a^{2}x-\left(a^{2}+2a-1\right)x^{2}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

Fricas [A] time = 2.31541, size = 144, normalized size = 3.13

$$\log\left(-\frac{a^{2}-2(a-1)x-x^{2}+2\sqrt{(2a-1)x^{3}+a^{2}x-\left(a^{2}+2a-1\right)x^{2}}}{a^{2}-2ax+x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2ax - a - x}{\sqrt{x\left(x-1\right)\left(-a^2+2ax-x\right)}\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-\left(a^2+2\,a-1\right)x^2}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

3.84 $\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rubi [A] time = 0.0963157, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.448681, size = 323, normalized size = 10.09

$$-\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right),\right.\right.}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*
x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4
*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I +
Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2
/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*S
qrt[3])/(3*I + Sqrt[3])]))/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.059, size = 258, normalized size = 8.1

$$-2\,\frac{\sqrt[3]{2}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int -\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2^{\frac{1}{3}}x - 1}{\sqrt{x^3 + 1}\left(x + 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi [A] time = 0.0539309, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)$$
$$= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

Mathematica [A] time = 0.0072606, size = 46, normalized size = 2.

$$\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)-\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] Log[3 - (1 + x)^2/Sqrt[1 + x^3]]/3 - Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

Maple [C] time = 0.021, size = 240, normalized size = 10.4

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-2\,\frac{3/2-}{\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [B] time = 2.20194, size = 117, normalized size = 5.09

$$\frac{1}{3}\log\left(\frac{x^3+12x^2-6\sqrt{x^3+1}(x+1)-6x+10}{x^3-6x^2+12x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}}{6\sqrt{2}}$$

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x
^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1
/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((
2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(6*Sqrt[2]*3^(1/4))
```

Rubi [A] time = 0.0409792, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x
^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1
/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((
2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(6*Sqrt[2]*3^(1/4))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
```

```
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}}{3\sqrt{2}}$$

Mathematica [C] time = 0.0563565, size = 47, normalized size = 0.22

$$\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

Maple [C] time = 0.195, size = 353, normalized size = 1.6

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(-1-\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\sqrt{3}\_alpha+\_alpha-2\right)\left(-i\sqrt{3}+3\right)\left(-1+2\_alpha-\sqrt{3}\_alpha\right)}{-\sqrt{3}+2\_alpha-1}\sqrt{\frac{1+}{-i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha-2)/(-3^(1/2)+2*_alpha-1)*(-I*3^(1
/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2
```

```
)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-3^(1/2
)*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*
_alpha*3^(1/2)+1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(
1/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticP
i(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [B] time = 49.077, size = 27647, normalized size = 126.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3)
 + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(-1/1296*(6
*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*
x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 8437780
8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979
2*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x
^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*
x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x
^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 - 537864*x^14 + 1586477*
x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x^9 - 25445
5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 +
```

78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273216)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 - 205368*x^12 - 289200*x^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x^14 + 738528*x^13 - 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x - 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 - 5976*x^16 + 5544*x^15 + 299664*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 - 4761216*x^10 - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x^16 - 26052*x^15 + 332844*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11 + 318696*x^10 + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17 - 96354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-672*sqrt(3) + 1164) + 3261696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(-672*sqrt(3

) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 + 1)*((45
9*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^
11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*
x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 + sqrt(3)*(2
65*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11
 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6
 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1
383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 450359*x^13 + 4370159*x^1
2 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 146877876*x^8 - 197107784
*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 45545344*x^3 + 695175
36*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14 + 260015*x^13 + 2523
113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9 + 84799980*x^8 - 113
800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^4 - 26295616*x^3 + 4
0135968*x^2 - 7907648*x - 5562368) - 13696448*x - 9634304)*sqrt(-672*sqrt(3
) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15
 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^11 - 324624*x^10 + 18081
6*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 1404096*x^5 + 1843584*x^4 +
135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 594
47*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 12
7784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 248
32) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 -
8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 1040
5056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 + sqrt(3)*(2855*x^15 - 21
635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 337953
6*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4
 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt
(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqr
t(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97
) - 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*x^13 + 64440*x^12 - 128
5200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8 - 3614400*x^7 + 3895488
*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(2
5*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10
 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 37
12*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 1248*x^16 - 246120*x^15 +
 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600*x^11 + 6258120*x^10 -
21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*x^6 + 15860544*x^5 - 47
20704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(14*x^17 + 15*x^16 - 296
0*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10 - 25
6916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4 - 7
2448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17 - 35850*x^16 - 106410*
x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 591615
24*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 1005
98112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 + sqrt(3)*(1351*x^17
 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26
727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 +

26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 +
4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-672*sqrt(3) + 1164) - 203
5200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(627*x^16 - 14286*x^15 + 3
9762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x
^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*
x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^1
3 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^
7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x +
11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) - 764928*x - 239616)*
sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x
^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3
)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 221
4*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x
^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3
+ 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3
) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^
3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*
x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16
*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16 - 5
22*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 - 42
336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*x^3
- 1664*x^2 + 256*x)) - 1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) +
1164) + 24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^
(3/4)*arctan(-1/1296*(6*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14
+ 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9
- 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 -
36433344*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 +
548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 -
4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 2
1034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 -
537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10
+ 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864
*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3
538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623
494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 14
7990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 759
8336) + 21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273
216)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 -
205368*x^12 - 289200*x^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992
*x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4
sqrt(3)(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 264
54*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 3
57936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x
^14 + 738528*x^13 - 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9

- 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 1
55968*x^3 - 3016704*x^2 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1
073898*x^12 - 452980*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916
720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2
+ 1543936*x + 545536) + 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 66
5088*x - 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt
(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 - 5976*x^
16 + 5544*x^15 + 299664*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 -
4761216*x^10 - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016
832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 28
6*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226
888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 -
177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x
^16 - 26052*x^15 + 332844*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11
+ 318696*x^10 + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 -
20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x
^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^
11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176
*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17
- 96354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 +
57963744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^
7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x
^2 + sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*
x^13 + 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 8078428
0*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 1733536
0*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-6
72*sqrt(3) + 1164) + 3261696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(9
7*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 48
3752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x
^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^1
7 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11
+ 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x
^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 1
26592)*sqrt(-672*sqrt(3) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97
) - (sqrt(x^3 + 1)*((459*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 467
7912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 10
5313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8
256960*x^2 + sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700
776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 6080
2736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767
168*x^2 + 1207168*x + 1383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 45
0359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14
6877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4
- 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^
14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x

^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864
*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96
34304)*sqrt(-672*sqrt(3) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 116
4)^(3/4) + 3*(984*x^15 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^1
1 - 324624*x^10 + 180816*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 14040
96*x^5 + 1843584*x^4 + 135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124
*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7527
6*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100
512*x^2 + 62400*x + 24832) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 224946
8*x^12 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*
x^7 + 768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 +
sqrt(3)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11
- 4863472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 600
7360*x^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1
738368*x - 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24
)*sqrt(-56*sqrt(3) + 97) + 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*
x^13 + 64440*x^12 - 1285200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8
- 3614400*x^7 + 3895488*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 6497
28*x^2 + 108*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12
- 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 62
40*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 12
48*x^16 - 246120*x^15 + 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600
*x^11 + 6258120*x^10 - 21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*
x^6 + 15860544*x^5 - 4720704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(
14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404
*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190
768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17
 - 35850*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 -
 46293732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^
7 + 45222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x
^2 + sqrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*
x^13 - 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 7064835
2*x^8 + 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 1709504
0*x^3 - 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-67
2*sqrt(3) + 1164) - 2035200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(62
7*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 +
 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5
 + 483816*x^4 - 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 +
 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^
9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3
- 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164
) - 764928*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656
*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 324*x^5 + 576*x^4 + 696*x^
3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (1

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23*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*
x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164)
 + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6
 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x +
2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16
)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656
*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5
 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) + 1/5184*((7*sqrt(3) + 12)*sqr
t(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 + 72*x^7 + 1656*x^6
 + 720*x^5 + 1440*x^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 +
432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x
^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5
+ 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 1
44*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164)
 + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6
*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46
*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^
3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304
)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) -
 1/5184*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3
) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/3
6*(36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 3
24*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 1
0*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2
+ sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*
sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) +
12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 +
 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(
26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*
x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*s
qrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32
*x^3 + 64*x^2 + 32*x + 16)) + 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2
 + sqrt(3)*(x^2 - 10*x - 8) - 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\, dx$

Optimal. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh}{3\sqrt{}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 +
 x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/
4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4)
)])/(3*Sqrt[2]*3^(3/4))
```

Rubi [A] time = 0.0317457, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```


```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 +
 x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/
4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4)
)])/(3*Sqrt[2]*3^(3/4))
```


Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
```

```
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\tanh^{-}}{2\sqrt{}}$$

Mathematica [C] time = 0.0703071, size = 50, normalized size = 0.24

$$-\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{4\left(3\sqrt{3}-5\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(4*(-5 + 3
*Sqrt[3]))
```

Maple [C] time = 0.173, size = 350, normalized size = 1.7

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(\sqrt{3}-1\right)\_Z-2\sqrt{3}+4\right)}\frac{\left(-\sqrt{3}\_alpha-\_alpha+2\right)\left(-i\sqrt{3}+3\right)\left(-1+2\_alpha+\sqrt{3}\_alpha\right)}{-\sqrt{3}-2\_alpha+1}\sqrt{\frac{1+}{-i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alpha+2)/(-3^(1/2)-2*_alpha+1)*(-I*3^(1
/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2
```

```
)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+3^(1/2
)*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_
alpha*3^(1/2)-1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/
2)+4))+1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1
/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [B] time = 49.0419, size = 28331, normalized size = 134.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/324*(2
16*sqrt(3)*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345
761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^
7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sq
rt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12
- 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x
^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
 271808*x + 126592)*(56*sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96
354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963
744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 4
```

2453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 -
sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13
+ 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8
 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3
 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3)
+ 97) + 6*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 3457
61*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7
 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqr
t(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 -
 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^
6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
271808*x + 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + 3*sqrt(sq
rt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 -
6128*x^15 - 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84
218362*x^10 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6
- 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(21
31*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x
^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 693427
76*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242
560*x + 7598336) + 21204736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) +
 (459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972
*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 8842732
8*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)
*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x
^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*
x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x
- 1889792) - 4310784*x - 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672
sqrt(3) + 1164)^(3/4) + 6(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 -
 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15
444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704
x^2 - sqrt(3)(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 45298
0*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x
^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 5455
36) + 2674176*x + 944896)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7
653*x^14 + 41169*x^13 - 51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 -
 17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3
+ 187584*x^2 - sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41
940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 +
224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272
*x - 58752)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)
) + 108*(12*x^17 - 498*x^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^1
2 + 300000*x^11 - 396768*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 22012
8*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17
 - 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11
- 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x

^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 3148
8)*sqrt(56*sqrt(3) + 97) + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14
 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294
*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108
576*x^2 - 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584
*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 15564
8*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) +
 128256*x + 40128)*(56*sqrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 358
50*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293
732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45
222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - s
qrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 -
 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8
+ 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3
- 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) +
97) + 6*(97*x^17 + 104*x^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 69176
2*x^12 + 584800*x^11 + 521510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7
- 1518056*x^6 + 1321712*x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt
(3)*(14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 +
84404*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6
+ 190768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 16960
0*x - 92672)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^
15 - 951114*x^14 + 450359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^
10 + 9429316*x^9 + 146877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776
*x^5 - 132260896*x^4 - 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 1
0237*x^15 - 549126*x^14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 4508
9132*x^10 + 5444020*x^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106
882416*x^5 - 76360864*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 55623
68) - 13696448*x - 9634304)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1
557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684
800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 1889
7984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 -
899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11 - 3214964
0*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 1091078
4*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) +
2090880*x + 2396160)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 11
64)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12
 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 +
768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3
)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863
472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x
^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368
*x - 691712)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13
485*x^13 + 102933*x^12 - 70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 +

243576*x^7 - 221784*x^6 - 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2
 - sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46
860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 2
66016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*s
qrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*
x^16 - 1682*x^15 + 2496*x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^
10 + 86080*x^9 - 49176*x^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^
4 + 18944*x^3 + 18048*x^2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482
*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7
 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280)
 - 21248*x - 6656)*sqrt(56*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 1
80*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5
 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(12
3*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x
^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt
(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 +
 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)
 - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144
*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x
 + 16)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 1
1656*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256
*x^5 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) - 1/108*sqrt(3)*sqrt(sqrt(
3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)*(
56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/324*(216*sqrt(3)*(97*x^17 - 52
3*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 168068
8*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^1
6 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^1
0 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080
*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*(56*
sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96354*x^16 + 84798*x^15 +
4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 76603680*x^10
 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*
x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 - sqrt(3)*(1351*x^17 - 556
30*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3346537
6*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 245103
68*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 757158
4*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3) + 97) + 6*(97*x^17 - 523
*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688
*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^16
 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10
 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*

```
x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(
56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - 3*sqrt(sqrt(3)*sqrt(56*sqrt(3) +
97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 - 6128*x^15 - 537864*x^14
+ 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x
^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645
168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(2131*x^16 - 3538*x^15 - 31
0536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41
002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 +
 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204
736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 13425*x^15
- 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 -
49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 2
7458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)*(265*x^16 - 7751*x^15 -
 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28
730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 158
53184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) - 4310784*x -
 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) +
 6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 - 1860046*x^12 - 784596*x
^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6
 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 - sqrt(3)*(2855*x^1
5 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^11 + 4427548*x^10 -
3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 54039
04*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896
)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7653*x^14 + 41169*x^13 -
51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 - 17892*x^8 - 490248*x^7
+ 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3 + 187584*x^2 - sqrt(3)*(
142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 +
128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4
+ 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272*x - 58752)*sqrt(x^3 + 1
)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(12*x^17 - 498*x
^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^12 + 300000*x^11 - 396768
*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 220128*x^6 + 584736*x^5 - 308
256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 +
 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^
9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*
x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 31488)*sqrt(56*sqrt(3) + 97)
 + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x
^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 5392
20*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(181
*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 940
21*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 1396
64*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*(56*s
qrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 35850*x^16 - 106410*x^15 -
2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 59161524*x^10
 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 100598112*x
```

^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - sqrt(3)*(1351*x^17 - 2069
8*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26727704*
x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 + 2610894
4*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 + 4724480
*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) + 97) + 6*(97*x^17 + 104*x
^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 691762*x^12 + 584800*x^11 + 5
21510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7 - 1518056*x^6 + 1321712*
x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt(3)*(14*x^17 + 15*x^16 -
2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10
- 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4
- 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 169600*x - 92672)*sqrt(56*sqr
t(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*s
qrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^15 - 951114*x^14 + 45035
9*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14687
7876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 4
5545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14
+ 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9
+ 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^
4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96343
04)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1557*x^15 - 26415*x^14 -
1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9
+ 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4
+ 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 -
837130*x^13 + 2700776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 +
4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 2
1568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) + 2090880*x + 2396160)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(
4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 - 8423784
*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 10405056*x^
5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3)*(2855*x^15 - 21635*x^1
4 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 3379536*x^9 +
4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4 - 40441
6*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt(x^3 +
1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13485*x^13 + 102933*x^12 -
70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 + 243576*x^7 - 221784*x^6
- 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2 - sqrt(3)*(142*x^15 - 2
124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7
5276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 -
100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*sqrt(x^3 + 1)*sqrt(56*sqr
t(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*x^16 - 1682*x^15 + 2496*
x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^10 + 86080*x^9 - 49176*x
^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^4 + 18944*x^3 + 18048*x^
2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*
x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5
- 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) - 21248*x - 6656)*sqrt(5

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6*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x
^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x
^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*
x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3
 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 2
7*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3
 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 9
7)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x
^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 +
 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16
- 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 -
 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*
x^3 - 1664*x^2 + 256*x)) - 1/1296*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqr
t(3) - 12) + 6)*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*s
qrt(3) + 1164)^(1/4)*log(1/9*(9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4
+ 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3
 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 +
12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 + 2016*x^5
+ 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1
192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*
x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 +
 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt
(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^
7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 -
4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/1296*s
qrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*sqrt(3) + 1164)^(1/4)*log(1/9*(9*x^8
 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26
*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^
6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(
3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - s
qrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36
*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*
sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 +
 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 +
32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(14*sqrt(3) + 24)*log((x^8 - 16*x^
7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 64*x^2 + 2*(5*x^6 - 54*x^5 + 96*
x^4 - 56*x^3 - 36*x^2 - 3*sqrt(3)*(x^6 - 10*x^5 + 20*x^4 - 8*x^3 - 4*x^2 +
8*x) + 24*x - 16)*sqrt(x^3 + 1)*sqrt(14*sqrt(3) + 24) + 16*sqrt(3)*(x^7 - 2
*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7
 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^3-6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}(}{3\sqrt{2}3^{3}}$$

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```

Rubi [A] time = 0.0295865, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}(}{3\sqrt{2}3^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```


```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```


Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
```

```
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh}{2}$$

Mathematica [C] time = 0.0663756, size = 65, normalized size = 0.29

$$-\frac{x^2\sqrt{1-x^3}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

Maple [C] time = 0.173, size = 349, normalized size = 1.6

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(1+\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\sqrt{3}\_alpha+\_alpha+2\right)\left(-i\sqrt{3}-3\right)\left(1+2\_alpha-\sqrt{3}\_alpha\right)}{-\sqrt{3}-2\_alpha-1}\sqrt{\frac{-1+x}{-i\sqrt{3}-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha+2)/(-3^(1/2)-2*_alpha-1)*(-I*3^(1
/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/
```

```
2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha-3^(1/2
)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*
I*_alpha*3^(1/2)-1/2*3^(1/2)*_alpha+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1
/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [B] time = 44.3407, size = 28004, normalized size = 126.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3) +
 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(1/1296*(6*sq
rt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^1
3 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018320*x^9 - 25445581
2*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 784
```

```
64672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 9
15953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 -
146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4
 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160
704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-672*sqrt(3) + 1164)
^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*x^12 - 289200*x^11
 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1849440*x^6 + 15584
64*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)*(142*x^15 + 4419*
x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 106
92*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108
288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 738528*x^13 + 186004
6*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*
x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 +
sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11
 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 55
19008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2
674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)
*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^1
4 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9
 - 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 186
1632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x
^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 4153
84*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78
784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x
^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x
^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^
4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 -
 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9
 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*
x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15
 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^
10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 11326953
6*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 5
5630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465
376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 2451
0368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571
584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 326169
6*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^
15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 10
51756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x
^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8
006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 +
 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^
3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) +
```

1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^
16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 -
55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6
+ 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x
^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 3
2149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 1
0910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 13834
24) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 -
30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7
- 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x
^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*
x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 1138002
32*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135
968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sqrt(-672*sqrt(3) +
1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 14
712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 324624*x^10 + 180816*x^
9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 - 1843584*x^4 + 1359
36*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7773*x^13 - 59447*x
^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 140472*x^7 + 127784
*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 + 62400*x - 24832)
- (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8423
784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405056
*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(2855*x^15 + 21635*
x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536*x^
9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 - 40
4416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(-67
2*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*
(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) + 6
*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 64440*x^12 + 1285200*
x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*x^7 + 3895488*x^6
- 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(25*x^1
6 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^11 - 1312*x^10 - 16
624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 15552*x^4 - 3712*x^
3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 - 246120*x^15 - 5181
72*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 6258120*x^10 - 21360
336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 15860544*x^5 + 4720704
*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 - 15*x^16 - 2960*x^1
5 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^10 - 256916*
x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x^4 - 72448*
x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108

944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3) + 1164) - 2035200*
x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 14286*x^15 + 39762*
x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 -
3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 -
108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 6
2584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 1
55648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 1158
4) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928*x - 239616)*sqrt(
-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 -
2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^
6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4
- 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 +
228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)
sqrt(2(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 11
64)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*
x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 +
sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) +
20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 +
16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 - 13*x^16 - 522*x^1
5 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^
9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 166
4*x^2 + 256*x)) + 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*a
rctan(1/1296*(6*sqrt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652
*x^13 - 997302*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 821253
6*x^8 - 84377808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 3643334
4*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*
x^13 - 575818*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*
x^8 - 48715600*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*
x^3 + 7280256*x^2 + 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*
x^14 - 1586477*x^13 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018
320*x^9 - 254455812*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 1
34645168*x^4 - 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15
- 310536*x^14 - 915953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10
- 41002448*x^9 - 146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*
x^5 + 77737424*x^4 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) -
21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-6
72*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*
x^12 - 289200*x^11 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1
849440*x^6 + 1558464*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)
*(142*x^15 + 4419*x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10
+ 128152*x^9 + 10692*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^
4 + 9632*x^3 - 108288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 73
8528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 69038

64*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^
3 + 3016704*x^2 + sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x
^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
+ 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 15439
36*x - 545536) + 2674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x +
235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqr
t(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 + 5976*x^16 + 5544
*x^15 - 299664*x^14 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x
^10 - 1046592*x^9 - 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 +
3699072*x^4 - 1861632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 +
238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10
- 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^
4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 260
52*x^15 - 332844*x^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696
*x^10 + 12621072*x^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256
*x^5 - 10936128*x^4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151
*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661
*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 26
3080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*
x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*
x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453
216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt
(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 11
21030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 6
1330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 1
5297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3
) + 1164) + 3261696*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 +
523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11
- 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680
688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x
^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x
^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 2630
80*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sq
rt(-672*sqrt(3) + 1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt
(x^3 - 1)*((459*x^16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12
- 12651948*x^11 - 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x
^7 + 99605088*x^6 + 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^
2 + sqrt(3)*(265*x^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12
- 7304604*x^11 - 32149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7
+ 57507040*x^6 + 10910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 -
1207168*x + 1383424) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13
+ 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x
^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 4554534
4*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 2600
15*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 8479

9980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26
295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sq
rt(-672*sqrt(3) + 1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4)
+ 3*(984*x^15 + 14712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 32462
4*x^10 + 180816*x^9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 -
1843584*x^4 + 135936*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7
773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 1
40472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 +
 62400*x - 24832) - (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 +
474132*x^11 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768
064*x^6 + 10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(
2855*x^15 + 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472
*x^10 + 3379536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5
+ 3826528*x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x
+ 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 11
64)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56
sqrt(3) + 97) - 6(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 644
40*x^12 + 1285200*x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*
x^7 + 3895488*x^6 - 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 1
08*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^1
1 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 1
5552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 -
246120*x^15 - 518172*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 62
58120*x^10 - 21360336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 1586
0544*x^5 + 4720704*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 -
15*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75
267*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 +
56784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x
^16 - 106410*x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*
x^11 - 59161524*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 452220
00*x^6 - 100598112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(
3)*(1351*x^17 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987
292*x^12 - 26727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46
883072*x^7 - 26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12
966272*x^2 + 4724480*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3)
+ 1164) - 2035200*x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 1
4286*x^15 + 39762*x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^
10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*
x^4 + 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^1
4 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^
8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^
2 - 37024*x + 11584) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928
*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720
*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^
2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2

016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278
*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x +
96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*
(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 -
5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*
x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8
+ 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 -
13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 239
44*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^
4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-67
2*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*(
-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 +
1440*x^4 - 2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36
sqrt(3)(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*s
qrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*
sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3
- 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 +
12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^
7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/5184*sqrt(
2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-
672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72
*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x
^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2
- 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*
x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt
(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*s
qrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x
^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6
+ 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 -
27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164)
+ 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 -
32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*log((x^8 + 16*x^7 + 112*x^6 + 16*
x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^6 + 54*x^5 + 96*x^4 + 56*x^3 - 36
*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 + 8*x^3 - 4*x^2 - 8*x) - 24*x - 16)
*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*
x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^
5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}}{3\sqrt{2}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x
^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4))
 + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 +
x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rubi [A] time = 0.0290123, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {488}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x
^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4))
 + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 +
x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
```

```
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\text{ta}}{}$$

Mathematica [C] time = 0.0531543, size = 68, normalized size = 0.32

$$\frac{x^2\sqrt{1-x^3}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(3\sqrt{3}-5\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [C] time = 0.169, size = 350, normalized size = 1.6

$$\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{-18+9\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\text{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
```

```
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x
)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alph
a-2)/(-3^(1/2)+2*_alpha+1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2
*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2)/(x^3-1)^(1/2)*(1+2*_alpha+3^(1/2)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*3^(1/2)*_alpha+_alph
a+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha
=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [B] time = 41.119, size = 28069, normalized size = 131.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*
(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/64
8*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 80158
4*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 -
2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x
^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 3214
12*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 3654
4) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17
+ 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 5
7963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7
```

```
 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^
2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x
^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280
*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360
*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt
(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584
*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2
*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^
12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 32141
2*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544
) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(
1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816
*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 53
9220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(1
81*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 9
4021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 13
9664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*(56
*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*x^15
 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17 - 104
*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x^11 -
 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 132171
2*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15*x^16
 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^1
0 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x
^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt(56*s
qrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)
*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x^14 -
 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 +
 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x
^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 549126
*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 544402
0*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360
864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x -
 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26415*x
^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 6283425
6*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 4249929
6*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15249*x
^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*
```

x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x
^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 239616
0)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqr
t(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8
423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405
056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15 + 216
35*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536
*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 -
404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(
x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 102933*
x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 + 2217
84*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(142*x^
15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x
^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*
x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*sqrt(
56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x^15 +
2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^9 - 4
9176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3 + 18
048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 +
6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 624
0*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6656)*
sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4
- 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x
^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2
+ 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(
5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4
- 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56
sqrt(3) + 97)(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(
3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/
(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 3
*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(
3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 771817
56*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7 + 12
0105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*x^2 -
sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*x^12
+ 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 113204536*
x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 45330624
*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(56*sqr
t(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
+ 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7

- 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2 + 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^14 + 738528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x + 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 88530*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 549408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2 - sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) - 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/648*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17 + 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -

321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
36544) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) -
sqrt(1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 2
16816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7
 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(
3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^1
1 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5
 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128
)*(56*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*
x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 591615
24*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 1005
98112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17
 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 26
727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 -
26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 +
4724480*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17
- 104*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x
^11 - 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 1
321712*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15
*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 7526
7*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56
784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt
(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(-4*sqrt(3)*sqrt(56*sqrt(3)
+ 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x
^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*
x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260
896*x^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 5
49126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5
444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 -
76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 1369644
8*x - 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26
415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62
834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42
499296*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15
249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 3627
7360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536
992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 2
396160)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6
(sqrt(3)(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^1
1 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 +
10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15
+ 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 33
79536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*
x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*
sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 10

2933*x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 +
 221784*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(1
42*x^15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26
308*x^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 1
9712*x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*
sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x
^15 + 2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^
9 - 49176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3
 + 18048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x
^12 + 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6
- 6240*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6
656)*sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720
*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24
*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 20
16*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*
x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97)
- 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 1
0*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*
sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1
152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)
) + 3*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt
(3)*(3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 7
7181756*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7
 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*
x^2 - sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*
x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 11320
4536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 453
30624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(5
6*sqrt(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 99730
2*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377
808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609
792*x^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818
*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 4871560
0*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256
*x^2 + 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sq
rt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^1
4 + 738528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 +
 6903864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155
968*x^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 107
3898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 891672
0*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 +
 1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97)
- 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^

10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728
*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13
+ 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^
7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x
+ 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt
(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 8853
0*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 5
49408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2
- sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^
12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 1256
00*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176)
 - 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 174
2*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 913
6*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 +
 256*x)) + 1/2592*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt
(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 116
4)^(1/4)*log(1/18*(18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3
 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^
2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4
) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x
^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3
+ 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27
*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3
+ 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) +
 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 -
4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/2592*(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt(-4*sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*log(1/18*(18*
x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3
)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 -
22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*
sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^
2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192
)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3
 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x
+ 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/36*sqrt(14*sqrt(3) + 24)*arctan
(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8) + 18*x - 12)*sqrt(14*sqrt(3) + 24)
/sqrt(x^3 - 1))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.128968, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2}\,dx, x, -\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 3.20489, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(\left(\frac{2\left(2i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{x+\sqrt{3}-1}+i\left(-1+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt
[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*
Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*(
(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[
3] + x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x)
)]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + S
qrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])
/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] +
 x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8
/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(
2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[
3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3]
```

```
)] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

Maple [C] time = 0.132, size = 327, normalized size = 5.

$$\frac{\mathrm{EllipticF}\left(x\left(\frac{i}{2}\sqrt{3}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\left(1+1/2\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3}-\frac{i}{2}}\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4+4\sqrt{3}x^2}}-2\sqrt{3}\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1
/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Fricas [B] time = 3.4669, size = 953, normalized size = 14.66

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^{9}+3708\,x^{8}-5472\,x^{7}+6432\,x^{6}+10944\,x^{5}+14832\,x^{4}+1926}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\,\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.12776, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2}\,dx, x, \frac{(}{\sqrt{-}}\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 7.54576, size = 876, normalized size = 13.9

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-x+\sqrt{3}+1\right)^2\left(\left(\left(\frac{2\left(2i\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}+\sqrt{6}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}+i\sqrt{3}-i+\frac{8i\left(-2+\sqrt{3}\right)}{-x+\sqrt{3}+1}}+\sqrt{-\frac{2i\left(\left(-1+\sqrt{3}\right)x-8\cdots\right.}{-x+\sqrt{3}+1}}\right.}{x-\sqrt{3}-1}\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I*Sqrt
[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 +
 Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(
1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3]
- x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
+ Sqrt[3] - x))] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]] + 2*S
qrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sq
rt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 +
 Sqrt[3])*x))/(1 + Sqrt[3] - x)]))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2
- Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 +
Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + S
qrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2
```

```
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt
[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2 -
 Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + S
qrt[3]))])])/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[
3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
))
```

Maple [C] time = 0.129, size = 311, normalized size = 4.9

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i}{2}\sqrt{3}\right), i\sqrt{1-4\sqrt{3}\left(1-1/2\sqrt{3}\right)}\right)}{\frac{i}{2}+\frac{i}{2}\sqrt{3}}\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4-4\sqrt{3}x^2}}+2\sqrt{3}\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/
2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/
2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Fricas [B] time = 2.83386, size = 301, normalized size = 4.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rubi [A] time = 0.0552897, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2151}

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) + \log(1+x) - \frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [F] time = 0.242957, size = 0, normalized size = 0.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{-1+x}{1+x}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=108

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rubi [A] time = 0.0925279, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2149, 239, 2151}

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right) - \frac{1}{4}\log\left(-\right.$$

Mathematica [F] time = 0.0572087, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] int(1/(1+x)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [B] time = 9.28147, size = 1422, normalized size = 13.17

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}\left(7114781247\,x^4+13663058416\,x^3-46178206896\,x^2-126842559344\,x-7\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
 + sqrt(3)*(93292570833559435663132301885*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 889426563183087468015580290
048*x^3 + 888876515195959220955879945824*x^2 + 351260598258508240019971964880*x - 476740009955972110578168843O4))/(78905434814564721745708464883*x^6 +
337746705836458222863347934450*x^5 + 15598952776058587894336070976*x^4 - 89
5430525315100108684787964824*x^3 + 361667862240477028869533375352*x^2 + 254
1802301011632510645972090336*x + 1554815286823334092314485968880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
```

```
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=98

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3)*x - (a + b*x^
3)^(1/3)]/(2*(a + b)^(1/3))
```

Rubi [A] time = 0.093337, antiderivative size = 135, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}}+\frac{\log\left(\frac{x^2(a+b)^{2/3}}{(a+bx^3)^{2/3}}+\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) - Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*(a + b)^(1/3)) +
 Log[1 + ((a + b)^(2/3)*x^2)/(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x
^3)^(1/3)]/(6*(a + b)^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \text{Subst}\left(\int \frac{1}{1-(a+b)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a+b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2+\sqrt[3]{a+b}x}{1+\sqrt[3]{a+b}x+(a+b)^{2/3}x^2}\,dx, x, \right.$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a+b}x+(a+b)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{a+b}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} - \frac{\log\left(1-\frac{\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}}$$

Mathematica [A] time = 0.155155, size = 120, normalized size = 1.22

$$\frac{-2\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)+\log\left(\frac{x^2(a+b)^{2/3}}{(a+bx^3)^{2/3}}+\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*
Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + ((a + b)^(2/3)*x^2)/
(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-x^3+1}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=154

$$\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3)
) - Log[(a + b)^(1/3)*x - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3))
```

Rubi [F] time = 0.299582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3)), x]
)/3 + ((3 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3
)), x])/3
```

Rubi steps

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx=\int\left(\frac{1-\frac{i}{\sqrt{3}}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}+\frac{1+\frac{i}{\sqrt{3}}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\right)dx$$
$$=\frac{1}{3}\left(3-i\sqrt{3}\right)\int\frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int\frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.28751, size = 0, normalized size = 0.

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1+x}{x^2+x+1}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=96

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rubi [A] time = 0.0781431, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 55, 617, 204, 31}

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(1-x^3\right)\sqrt[3]{a+bx^3}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)\\
&= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a+b}-x}\,dx,\right.}{2\sqrt[3]{a+b}}\\
&= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}\right)}{\sqrt[3]{a+b}}\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}
\end{aligned}
$$

Mathematica [A] time = 0.0808377, size = 80, normalized size = 0.83

$$\frac{-3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)+\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]] + Log
[1 - x^3] - 3*Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^2}{-x^3+1}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35095, size = 983, normalized size = 10.24

$$\left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(\left(bx^3+a\right)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3\,a-3\left(bx^3+a\right)^{\frac{1}{3}}(-a-b)^{\frac{2}{3}}+b}{x^3-1}\right)+(-a-b)^{\frac{2}{3}}\cdots}{6\,(a+b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0618137, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x\right.}{6\sqrt[3]{2}}\\
&= -\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}\\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 0.142763, size = 112, normalized size = 1.27

$$
\frac{-\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$
\int \frac{1}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] time = 26.9506, size = 660, normalized size = 7.5

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0108413, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx = \frac{1}{2} x^2 F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Mathematica [C] time = 0.0269249, size = 26, normalized size = 0.11

$$\frac{1}{2} x^2 F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; x^3, -x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x}{x^3+1} \frac{1}{\sqrt[3]{-x^3+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] time = 23.8683, size = 1049, normalized size = 4.5

$$-\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}(-1)^{\frac{1}{3}}\left(x^{16}-33x^{13}+11\right.}{6\left(x^{18}-102x^{15}+447x^{1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^{2}+x+1\right)}(x+1)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0578244, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0595163, size = 73, normalized size = 0.89

$$
\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.40032, size = 116, normalized size = 1.41

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 2.12013, size = 282, normalized size = 3.44

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.100 $\int \frac{1+x}{\left(1-x+x^2\right) \sqrt[3]{1-x^3}} \, dx$

Optimal. Leaf size=135

$$\frac{\log \left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2 \sqrt[3]{2}}-\frac{\log \left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{1-\frac{2 \sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)
 + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3
)^(1/3)]/(2*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/2^(1/3)
```

Rubi [C] time = 0.336726, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right) \log \left(2\, 2^{2/3} \sqrt[3]{1-x^3}+2 x-i \sqrt{3}+1\right)}{4 \sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right) \log \left(2\, 2^{2/3} \sqrt[3]{1-x^3}+2 x+i \sqrt{3}+1\right)}{4 \sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i \sqrt{3}\right) \tan ^{-1}\left(\cdots\right)}{2 \sqrt[3]{2}\left(\sqrt{3} \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx\\ &= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}+\frac{\left(i-\sqrt{3}\right)\log\left(-\left(1-\right.\right.}{4\sqrt[3]{}}\end{aligned}$$

Mathematica [F] time = 0.19016, size = 0, normalized size = 0.

$$\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]

[Out] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1+x}{x^2-x+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Fricas [B] time = 111.657, size = 882, normalized size = 6.53

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3+3x^2+x\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6x^2-9x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal. Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3)
 + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3
)^(1/3)]/(2*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/2^(1/3)
```

Rubi [C] time = 0.300503, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\right.}{2\sqrt[3]{2}\left(\sqrt{3}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

$$= \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)} + \frac{\left(i-\sqrt{3}\right)\log\left(-\left(1-i\sqrt{3}\right.\right.}{4\sqrt[3]{2}\left(i\right.}$$

Mathematica [C] time = 0.440747, size = 150, normalized size = 1.11

$$\frac{1}{3}x^3F_1\left(1;\frac{1}{3},1;2;x^3,-x^3\right) + x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right) + \frac{-\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}} + 1\right) + 2\sqrt{3}\tan^{-1}\left(\right.}{6\sqrt[3]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] + (x^3*AppellF1[1, 1/3, 1, 2, x^3
, -x^3])/3 + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3
]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)]
 + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(1+x)^2}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] time = 99.4621, size = 882, normalized size = 6.53

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3+3x^2+x\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6x^2-9x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^2}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.102 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{\left(x^3+1\right)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{2\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(1/
3)) - Log[1 + (2^(2/3)*(1 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)*(1 + x))/(1 +
x^3)^(1/3)]/(2*2^(1/3)) + Log[1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)]/2^(1/3
)
```

Rubi [C] time = 0.299049, antiderivative size = 399, normalized size of antiderivative = 3.35, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6728, 2148}

$$\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}+\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}+\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(2\ldots\right.}{2\sqrt[3]{2}\left(\sqrt{3}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3
))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) - ((3 + I*Sqrt[3])*ArcTan[(2 - (
2^(1/3)*(1 + I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I
 - Sqrt[3])) - ((I - Sqrt[3])*Log[(1 - I*Sqrt[3] - 2*x)*(1 - I*Sqrt[3] + 2*
x)^2])/(4*2^(1/3)*(I + Sqrt[3])) - ((I + Sqrt[3])*Log[(1 + I*Sqrt[3] - 2*x)
*(1 + I*Sqrt[3] + 2*x)^2])/(4*2^(1/3)*(I - Sqrt[3])) + (3*(I - Sqrt[3])*Log
[1 - I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/(4*2^(1/3)*(I + Sqrt[3])
) + (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/
(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx = \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\right)dx$$

$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx$$

$$= \frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}-\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}-\frac{\left(i-\sqrt{3}\right)\log\left(\left(1-i\sqrt{\right.\right.}{4\sqrt[3]{2}}$$

Mathematica [F] time = 0.162654, size = 0, normalized size = 0.

$$\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]

[Out] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)), x]
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1-x}{x^2+x+1}\frac{1}{\sqrt[3]{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x)
```

```
[Out] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [B] time = 93.4274, size = 737, normalized size = 6.19

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+7x^5+10x^4+7x^3+10x^2+7x+1\right)-4\sqrt{2}\left(x^5+x^4-3x^3-3x^2+x+1\right)\left(x^3+1\right)^{\frac{1}{3}}+\right.}{6\left(3x^6+9x^5+6x^4+x^3+6x^2+9x+3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
```

```
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx-\int -\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.103 $\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} \, dx$

Optimal. Leaf size=43

$$x^2 \left(-{}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3\right)\right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rubi [F] time = 0.425069, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)
*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[
Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int]
[(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} \, dx = \int \left(-\frac{4(1-x^3)^{2/3}}{3(-1+i\sqrt{3}-2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(-1+i\sqrt{3}-2x)} - \frac{4(1-x^3)^{2/3}}{3(1+i\sqrt{3}+2x)^2} + \frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(1+i\sqrt{3}+2x)}\right) d$$

$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}-2x)^2} \, dx\right) - \frac{4}{3}\int \frac{(1-x^3)^{2/3}}{(1+i\sqrt{3}+2x)^2} \, dx + \frac{(4i)\int \frac{(1-x^3)^{2/3}}{-1+i\sqrt{3}-2x} \, dx}{3\sqrt{3}} + \frac{(4i)\int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}+2x}}{3\sqrt{3}}$$

Mathematica [A] time = 0.150557, size = 43, normalized size = 1.

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Maple [A] time = 0.036, size = 34, normalized size = 0.8

$$-(-1+x)(1+2x)\frac{1}{\sqrt[3]{-x^3+1}}+x^2{}_2\mathrm{F}_1(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{4}+2\,x^{3}+3\,x^{2}+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{\left(x^{2}+x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{\left(x^{2}+x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=43

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rubi [F] time = 0.220072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((1 + I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x])
 - (1 - I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx\end{aligned}$$

Mathematica [A] time = 0.0745515, size = 43, normalized size = 1.

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Maple [A] time = 0.027, size = 34, normalized size = 0.8

$$-(-1+x)(1+2x)\frac{1}{\sqrt[3]{-x^3+1}}+x^2{}_2F_1(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^{2}\sqrt[3]{1-x^{3}}+x\sqrt[3]{1-x^{3}}+\sqrt[3]{1-x^{3}}}\,dx-\int -\frac{1}{x^{2}\sqrt[3]{1-x^{3}}+x\sqrt[3]{1-x^{3}}+\sqrt[3]{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.105 $\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}} \, dx$

Optimal. Leaf size=39

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rubi [A] time = 0.0289564, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1854, 12, 364}

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}}\,dx &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}-\int -\frac{2x}{\sqrt[3]{1-x^3}}\,dx\\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}+2\int \frac{x}{\sqrt[3]{1-x^3}}\,dx\\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)\end{aligned}$$

Mathematica [A] time = 0.0182633, size = 43, normalized size = 1.1

$$x^2\left(-\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Maple [A] time = 0.013, size = 34, normalized size = 0.9

$$-(-1+x)(1+2x)\frac{1}{\sqrt[3]{-x^3+1}}+x^2{}_2F_1(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^2/(-x^3+1)^(4/3),x)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}} - \int \frac{x^2-2\,x}{\left(x^3-1\right)\left(x^2+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
```

```
[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2\,x^3+3\,x^2+2\,x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)
```

```
[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)^2}{\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

3.106 $\int \left(1-x^3\right)^{2/3} dx$

Optimal. Leaf size=67

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rubi [A] time = 0.0098674, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 239}

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3),x]
```

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(1-x^3\right)^{2/3} dx = \frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\, dx$$
$$= \frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [C] time = 0.102595, size = 101, normalized size = 1.51

$$\frac{3(x-1)\left(1-x^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{x-1}{1-(-1)^{2/3}},-\frac{x-1}{1+\sqrt[3]{-1}}\right)}{5\left(\frac{x-1}{1+\sqrt[3]{-1}}+1\right)^{2/3}\left(\frac{x-1}{1-(-1)^{2/3}}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

Maple [C] time = 0.025, size = 12, normalized size = 0.2

$$x{}_2F_1(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

Maxima [B] time = 1.39687, size = 142, normalized size = 2.12

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Fricas [A] time = 1.59905, size = 258, normalized size = 3.85

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{2}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] time = 1.15963, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^3+1\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3), x)
```

3.107 $\int \frac{(1-x^3)^{2/3}}{x} \, dx$

Optimal. Leaf size=70

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rubi [A] time = 0.038365, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-x^3\right)^{2/3}}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log(x)}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0384837, size = 65, normalized size = 0.93

$$\frac{1}{2}\left(\left(1-x^3\right)^{2/3}+\log\left(1-\sqrt[3]{1-x^3}\right)-\log(x)\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/x,x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ((1 - x^3)^(2/3) - Log[x]
 + Log[1 - (1 - x^3)^(1/3)])/2
```

Maple [C] time = 0.035, size = 66, normalized size = 0.9

$$-\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{9\pi}\left(-\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)+\frac{2\pi\sqrt{3}x^3}{3\Gamma(2/3)}{}_3F_2(\frac{1}{3},1,1;\,2,2;\,x^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/x,x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi
*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,2]
,x^3))
```

Maxima [A] time = 1.39611, size = 99, normalized size = 1.41

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1
```

```
)^(1/3) - 1)
```

Fricas [A] time = 1.55922, size = 230, normalized size = 3.29

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

Sympy [C] time = 1.02588, size = 41, normalized size = 0.59

$$-\frac{x^2 e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

Giac [A] time = 1.09656, size = 100, normalized size = 1.43

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

3.108 $\int \frac{\left(1-x^3\right)^{2/3}}{a+bx}\,dx$

Optimal. Leaf size=384

$$-\frac{x^2\left(a^3+b^3\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}-\frac{a^2\log\left(\sqrt[3]{1-x^3}+x\right)}{2b^3}-\frac{\left(a^3+b^3\right)^{2/3}\log\left(a^3+b^3x^3\right)}{3b^3}+\frac{\left(a^3+b^3\right)^{2/3}\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}\right.}{2b^3}$$

```
[Out] (1 - x^3)^(2/3)/(2*b) - ((a^3 + b^3)*x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -(
(b^3*x^3)/a^3)])/(2*a^2*b^2) + (a^2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^3) - ((a^3 + b^3)^(2/3)*ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*x
)/(a*(1 - x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^3) + ((a^3 + b^3)^(2/3)*ArcTan[
(1 + (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^3) + (a*
x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b^2) - ((a^3 + b^3)^(2/3)*Log
[a^3 + b^3*x^3])/(3*b^3) + ((a^3 + b^3)^(2/3)*Log[-(((a^3 + b^3)^(1/3)*x)/a
) - (1 - x^3)^(1/3)])/(2*b^3) - (a^2*Log[x + (1 - x^3)^(1/3)])/(2*b^3) + ((
a^3 + b^3)^(2/3)*Log[(a^3 + b^3)^(1/3) - b*(1 - x^3)^(1/3)])/(2*b^3)
```

Rubi [F] time = 0.0682516, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+bx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(a + b*x), x]
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+bx}\,dx=\int \frac{\left(1-x^3\right)^{2/3}}{a+bx}\,dx$$

Mathematica [F] time = 0.34462, size = 0, normalized size = 0.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a}\left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
```

```
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

3.109 $\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$

Optimal. Leaf size=234

$$\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{2\left(1-x^3\right)^{2/3}x^2}{3\left(x^3+1\right)}+\frac{\left(1-x^3\right)^{2/3}x}{3\left(x^3+1\right)}-\frac{\left(1-x^3\right)^{2/3}}{3\left(x^3+1\right)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-$$

```
[Out] -(1 - x^3)^(2/3)/(3*(1 + x^3)) + (x*(1 - x^3)^(2/3))/(3*(1 + x^3)) + (2*x^2
*(1 - x^3)^(2/3))/(3*(1 + x^3)) - (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x
^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]])/(3*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3
 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3
)^(1/3)]/(3*2^(1/3))
```

Rubi [F] time = 0.414851, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)*
Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[In
t][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int][
(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx = \int \left(-\frac{4\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2} + \frac{4i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)} - \frac{4\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2} + \frac{4i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(-1+i\sqrt{3}+2x\right)} \right) dx$$

$$= -\left(\frac{4}{3} \int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2} dx \right) - \frac{4}{3} \int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2} dx + \frac{(4i) \int \frac{\left(1-x^3\right)^{2/3}}{1+i\sqrt{3}-2x} dx}{3\sqrt{3}} + \frac{(4i) \int \frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}+2x} d}{3\sqrt{3}}$$

Mathematica [F] time = 0.45237, size = 0, normalized size = 0.

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^2} \left(-x^3+1\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4-2\,x^3+3\,x^2-2\,x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

3.110 $\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$

Optimal. Leaf size=199

$$\frac{\left(1-x^3\right)^{2/3}}{x^2-x+1}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2}{3}\ldots}{\ldots}\right)}{\sqrt{3}}$$

```
[Out] (1 - x^3)^(2/3)/(1 - x + x^2) - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[
3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
)/Sqrt[3] + (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(1/3) - Log[-(2^(1/3)*x) - (1 - x^3)^(1
/3)]/2^(1/3) + Log[x + (1 - x^3)^(1/3)]
```

Rubi [F] time = 0.945257, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (4*(1 + I*S
qrt[3])*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 - (4*Defe
r[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (4*(1 - I*Sqrt[3])
*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3
```

Rubi steps

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx=\int \left(\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}-\frac{2x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\right)dx$$
$$=-\left(2\int \frac{x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx\right)+\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$
$$=-\left(2\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{2i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{2\left(1-i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}+\frac{2i\left(1\right.}{3\sqrt{3}\left(-1\right.}\right.\right.$$
$$=-\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx+\frac{1}{3}\left(4\left(1-i\sqrt{3}\right)\right)\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2\right.}$$

Mathematica [F] time = 0.34793, size = 0, normalized size = 0.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1-2\,x}{\left(x^2-x+1\right)^2}\left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}(2x-1)}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

Fricas [B] time = 14.5052, size = 5191, normalized size = 26.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^(2/3)*sqrt(3)*(5
0*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^(2/3) + 7644*4^(1/3)*sqrt
(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^(1/3) - 7*
sqrt(39)*(6*4^(1/3)*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898
)*(-x^3 + 1)^(2/3) - 4^(2/3)*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 1725
4*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*
x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^(1/3))*sqrt((6*4^(1/3)*(5*x^4 + 4
*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^
4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-
x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3
)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x
 - 29747))/(138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2
 - 70446*x - 256859)) + 8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^(
2/3)*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^(2/3) - 76
44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
 - 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
 8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
```

```
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3)) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
 - 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1
)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\,dx-\int \frac{2x\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2x-1)}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

3.111 $\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\tan^{-1}$$

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rubi [F] time = 0.0599141, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x),x]

[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx=\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Mathematica [F] time = 0.389881, size = 0, normalized size = 0.

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\left(-x^{3}+1\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(1+x),x)
```

```
[Out] int((-x^3+1)^(2/3)/(1+x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

3.112 $\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\ \text{tan}$$

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rubi [F] time = 0.0706373, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Mathematica [F] time = 0.150094, size = 0, normalized size = 0.

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{x^{2}-x+1}{x^{3}+1} \left(-x^{3}+1\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}} \left(x^{2}-x+1\right)}{x^{3}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{2}-x+1\right)}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx$

Optimal. Leaf size=132

$$-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)*ArcTan[(1 -
(2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - Log[1 + x^3]/(3*2^(1/3))
 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2
```

Rubi [C] time = 0.0082518, antiderivative size = 21, normalized size of antiderivative = 0.16, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx = xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0937368, size = 111, normalized size = 0.84

$$-\frac{4x\left(1-x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/
3, x^3, -x^3] + 2*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1}\left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

Fricas [A] time = 1.6769, size = 532, normalized size = 4.03

$$-\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\cdot 4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3} dx$

Optimal. Leaf size=250

$$-\frac{1}{2}x^2\, {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^{2/3}\tan}{}$$

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/
(6*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x
))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)])/3 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [C] time = 0.0119922, antiderivative size = 26, normalized size of antiderivative = 0.1, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0135238, size = 26, normalized size = 0.1

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x}{x^3+1}\left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} dx$

Optimal. Leaf size=383

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\right.}{\sqrt[3]{2}}$$

```
[Out] -((2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[
3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt
[3]) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)*ArcTa
n[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (x^2*Hypergeometr
ic2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 +
 x^3]/(3*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/
(1 - x^3)^(1/3)])/3 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x +
 (1 - x^3)^(1/3)]/2 + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [F] time = 0.398948, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (-2*Defer[Int][(1 - x^3)^(2/3)/(-1 - x), x])/3 - ((1 + (-1)^(2/3))*Defer[In
t][(1 - x^3)^(2/3)/(-1 + (-1)^(1/3)*x), x])/3 - ((1 - (-1)^(1/3))*Defer[Int
][(1 - x^3)^(2/3)/(-1 - (-1)^(2/3)*x), x])/3
```

Rubi steps

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = \int \left(-\frac{2\left(1-x^3\right)^{2/3}}{3(-1-x)} + \frac{\left(-1-(-1)^{2/3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+\sqrt[3]{-1}x\right)} + \frac{\left(-1+\sqrt[3]{-1}\right)\left(1-x^3\right)^{2/3}}{3\left(-1-(-1)^{2/3}x\right)}\right) dx$$

$$= -\left(\frac{2}{3}\int \frac{\left(1-x^3\right)^{2/3}}{-1-x}\,dx\right) + \frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{\left(1-x^3\right)^{2/3}}{-1-(-1)^{2/3}x}\,dx + \frac{1}{3}\left(-1-(-1)^{2/3}\right)\int \frac{\left(1-x^3\right)^{2/3}}{-1+\sqrt[3]{-1}x}$$

Mathematica [C] time = 0.15278, size = 138, normalized size = 0.36

$$-\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right) - \frac{4\left(1-x^3\right)^{2/3} xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2 - (4*x*(1 - x^3)^(2/3)*Appe
llF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1, 4
/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*AppellF1
[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1-x}{x^3+1}\left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\,dx - \int \frac{x\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

Optimal. Leaf size=272

$$\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{\sqrt[3]{2}\cdots}{\cdots}$$

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3)*(
1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))
+ (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3) +
(1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3))
```

Rubi [C] time = 0.0078488, antiderivative size = 21, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0882679, size = 109, normalized size = 0.4

$$-\frac{4x\sqrt[3]{1-x^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/
3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1}\sqrt[3]{-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

Fricas [A] time = 21.8302, size = 902, normalized size = 3.32

$$\frac{1}{18}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}2^{\frac{2}{3}}\left(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-24\sqrt{3}2^{\frac{1}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}\right.}{3\left(x^{18}-102x^{15}+447x^{12}-628x^{9}+447\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)\left(x^{2}+x+1\right)}}{(x+1)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```